Material and Product Design Integration: Establishing Relationships between Design Variables of Both Domains

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Abstract—Due to the increasing demand of application-specific and/or multi-functional materials, it is necessary to integrate material design and product design. To support such design integration, this paper proposes a methodology to establish the relationships of both material design variables and product design variables. These variables include the required system performances and/or other evaluation criteria, and the relevant system loadings and attributes, where the attributes include both the product structural attributes and the material properties. This is achieved by modeling the behaviors of the product and those of the used material, and identifying the dependencies between the relevant design variables from the behaviors. The variable relationships can then be used to solve various design problems, such as design evaluation, evaluation and optimization of critical design variables, and so on. A design case study is also conducted to illustrate the proposed methodology and its usefulness.

Index Terms—Design integration, material design, product design, variable relationship.

I. INTRODUCTION

MATERIAL selection is recognized as an important task in engineering design. Due to the increasing demand of application-specific and/or multi-functional materials used in various engineering fields, many materials have to be specifically designed to meet this kind of engineering requirement. Hence, both material selection and material design are necessary for product design and development. Because of the close correlation between the required material and the desired system performances, the selection or design of material should be integrated with the design of the product where the material is used. That is to say, material design (including material selection, which applies to the rest of this paper) and product design should be integrated.

Unfortunately, until present, the majorities of material design have not been directly linked with product design, especially at the early design stage. Judging from the literatures published in the journal “Materials & Design”, Edwards [1] has pointed out that “the development of materials in some areas is not always relevant to industrial need”, i.e. it is not oriented to specific applications. There are only a few researches that are relevant in tackling this problem. For example, Olson and his research group [2]-[4] proposed a framework in computational design of hierarchically structured materials, in which they model material design as a process from system analysis to design/synthesis, to implementation, and to operation. System analysis defines the material properties and performance objectives. The objectives are then prioritized and the necessary mechanistic models are determined, by which the material is designed in terms of its composition, processing and structure. To manage these design activities, the four elements of material design, i.e. processing, structure, properties and performance, are organized as a hierarchical system. Subsystem modeling can thus be carried out and interactions between different levels can be managed. By employing systems methodology, the proposed approach has effectively managed the interactions between properties, structure and processing. However, it does not incorporate the performances of the product where the material is used.

Raj and his research group [5]-[7] integrate materials science with systems engineering by proposing a systems methodology to manage the relationships between design variables. This is implemented by non-hierarchically partitioning the design variables. They employ a graph partition algorithm to derive different groups of variables (local variables), as well as the variables belonging to all groups (linking variables). Based on the segregation of local variables and linking variables, they state that the optimal values of the design variables, or their feasible bounds, can
be determined by analyzing each subsystem (corresponding to a group of local variables) on its own and then using the linking variables to overlay the results from each subsystem to reach overall results for the system. In a design example of tungsten filament, the authors used simple graphical (i.e. modeling) approach to evaluate the design variables. However, as the authors themselves have agreed, this approach is inappropriate for most of the design problems where the number of variables and functions are often too large to manage. Besides, there is no mention of how the design variables as well as their relationships may be identified.

Mistree et al. [8] address application- or product-oriented material design. Different from the work from Raj’s research group, their work focuses on the optimization of design variables involving both system performances and material behaviors, rather than on managing the interactions between them. They formulate material design as a robust design problem, for which a Robust Concept Exploration Method (RCEM) [9] is used. RCEM integrates a Response Surface Methodology (RSM) with compromise Decision Support Problem (DSP) [10].

In fact, to achieve material and product design integration, the two domains of design should be carried out in such a way that the interactions between them, including the relationships between design variables from both design domains, are properly formulated and managed, so that the design tasks involving the two domains can be conducted in an integrated manner. This not only increases design efficiency, but also ensures the consistencies among the design characteristics from the two domains.

The above literature overview shows that existing work applies systems approach in deriving subsystems, in order to manage different design aspects, such as design components, design variables to describe these components, and their interactions. However, there lacks a generic and unified method in formulating the relationships between design variables. These relationships may be characterized by the mechanistic models of the design variables, such as mathematical functions (explicit models) and CAE models or computer codes (implicit models); or they may be approximated by some experimentally established models (approximate models). By formulating these relationships, the designers can relate the required performances or other evaluation criteria to the design variables that are to be evaluated or optimized. Existing researches use only ad hoc methods to derive such relationships.

II. METHODOLOGY

A. Research objective

The focus of the design integration described in this paper is targeted primarily at the stage between conceptual design and preliminary design. At this stage, designers may need to determine the critical design parameters so that the required design performances or other criteria can be achieved; or that they may need to evaluate conceptual variants so that the optimal one can be selected for further development. Our goal of design integration is to facilitate the determination of critical design parameters or design evaluation that involve both the material and the product, to ensure that the design can deliver the functions at the required or optimal level of performances, and meet other criteria, if any.

In order to achieve this goal, it is essential to model the various design characteristics, indicated as design variables, from both the material and the product, including their relationships. Our methodology follows three steps:

- Modeling the behaviors of the product and those of the material used in the product;
- Establishing the relationships between the design variables from these behaviors;
- Solving design problems by using the established variable relationships.

B. Modeling product and material behaviors

The required functions and desired performances of a product are achieved by its behaviors and those of the material used. In this paper, we regard the transformation of system inputs and system outputs as system behaviors (both product behaviors and material behaviors). Various design variables are used to characterize these behaviors, e.g. the required system performances are represented by performance variables, the attributes of structural elements and those of the material (i.e. material properties) are represented by attribute variables, and the loadings to the system are represented by loading variables. The loading variables can be regarded as the system inputs, while the performance variables as the system outputs.

The overall system behavior is implemented by various levels of individual behaviors. At the lowest level, each individual behavior is regulated by a certain physical principle, including material science principle. Hence, it is able to describe these individual behaviors by using their respective underlying physical/material principles. These bottom-level behaviors are referred to as elemental behaviors. Some of these elemental behaviors may be at the material’s microstructural (nanostructural) level, i.e. they are in fact material behaviors.

Each elemental behavior has its own input variables and output variables, where some of the input variables may be the loading variables of the system, and some of the output variables may be the performance variables of the system. It also has its own attribute variables, which describe the attributes of structural elements of this behavior, or those of the material. The relationships between the input, output and the attribute variables of an elemental behavior, which is characterized by the corresponding physical/material principle, may be expressed by one or more expressions. Such expressions are referred to as elemental expressions, where the word “elemental” signifies that the expressions...
only describe an elemental behavior of the whole system. The elemental behaviors are connected via the outputs from a preceding behavior and the inputs of a succeeding behavior, thus form a network of behaviors. Fig. 1 shows an example, where there are eight elemental behaviors connected via input-output pairs.

![Diagram of product and material behaviors](image)

**Fig. 1. Product and material behaviors**

As can be seen, apart from all the interior pairs of inputs and outputs, there are also some exterior inputs and exterior outputs. Obviously, the exterior inputs are the loadings of the system, while some of the exterior outputs are its performances. For example, in Fig. 1, there are three exterior inputs that are also the loadings to the system. There are also two exterior outputs that form part of the performances (another performance is an interior output, this is because it is possible that some performance requirements may be imposed on the interior outputs).

### C. Developing variable dependency graph

The developed network of behaviors can be used to establish the relationships between the performance variables and their dependent variables, where the dependent variables may be the attribute variables, the loading variables (i.e. the exterior inputs), or both of them.

Assume there are in total k counts of elemental behaviors. Each behavior B_i has m(i) counts of output variables, n(i) counts of input variables, p(i) counts of attribute variables, as well as q(i) counts of elemental expressions (i = 1 to k):

\[ B_i = \{O_{i1}, O_{i2}, \ldots, O_{i m(i)}; I_{i1}, I_{i2}, \ldots, I_{i n(i)}; A_{i1}, A_{i2}, \ldots, A_{i p(i)}; E_{i 1}, E_{i 2}, \ldots, E_{i q(i)}\} \]  

where O_{i1}, O_{i2}, \ldots, O_{im(i)} represent all the output variables of the behavior B_i; I_{i1}, I_{i2}, \ldots, I_{in(i)} represent all its input variables; A_{i1}, A_{i2}, \ldots, A_{ip(i)} represent all its attribute variables; and E_{i1}, E_{i2}, \ldots, E_{iq(i)} represent all its elemental expressions.

As such, the design as a whole has:

- All output variables: \(OV = \{O_{11}, O_{12}, \ldots, O_{1 m(1)}; O_{21}, O_{22}, \ldots, O_{2 m(2)}; \ldots; O_{k1}, O_{k2}, \ldots, O_{k m(k)}\}\)
- All input variables: \(IV = \{I_{11}, I_{12}, \ldots, I_{1 n(1)}; I_{21}, I_{22}, \ldots, I_{2 n(2)}; \ldots; I_{k1}, I_{k2}, \ldots, I_{k n(k)}\}\)
- All attribute variables: \(AV = \{A_{11}, A_{12}, \ldots, A_{1 p(1)}; A_{21}, A_{22}, \ldots, A_{2 p(2)}; \ldots; A_{k1}, A_{k2}, \ldots, A_{k p(k)}\}\)
- All elemental expressions: \(FE = \{E_{11}, E_{12}, \ldots, E_{1 q(1)}; E_{21}, E_{22}, \ldots, E_{2 q(2)}; \ldots; E_{k1}, E_{k2}, \ldots, E_{k q(k)}\}\)

Each pair of the interiorly connected input-output may be simply represented by one variable. For example, in Fig. 1, the interior output of behavior B_2 is also the interior input of behavior B_5. Hence, we opt not to use all the interior input variables. The remaining input variables will be exclusively the exterior input variables, i.e. the loading variables of the system. We use LV to represent the set of all loading variables, where \(LV \subseteq IV\).

As a result, the actual design variables include all the loading variables LV, all the attribute variables AV and all the output variables OV (including both interior and exterior). The performance variables are among the output variables, and are in general the exterior output variables. Hence, we only need to establish the relationships of OV with LV and AV by making use of all the elemental expressions FE.

We adopt the method of variable dependency graph (VDG) proposed by Deng et al. [11] to derive the relationships between the design variables. This is achieved by examining all the elemental expressions. The basic idea is to separate all design variables into a number of levels (assuming the levels are organized from left to right). Any variable in such a VDG is dependent only on the variables at its immediate left level. All the variables at the leftmost level are not dependent on any other variable, which obviously include all the loading variables and all the attribute variables. All variables at the other levels are output variables, among which are the performance variables. The performance variables are in general at the rightmost level.

The VDG development process starts from the identification of all loading variables and attribute variables, that is, the variables at the first level of the VDG. After that, all the elemental expressions are examined to derive other variables (output variables), level by level, which are dependent on the variables at their left-side levels. Assume that \(V_{\text{known}}\) represents the union of the set of loading variables and attribute variables, that is, \(V_{\text{known}} = LV \cup AV\); \(F_{\text{remain}}\) represents the set of remaining elemental expressions that are yet to be processed. The detailed procedure for VDG development is as follows:

**Step 1:** Initialize \(V_{\text{known}}\) with the union of loading variables and attribute variables, and \(F_{\text{remain}}\) with the set of all elemental expressions.

\(V_{\text{known}} = LV \cup AV\)

\(F_{\text{remain}} = FE\)

Initialize the VDG graph with one level of nodes (the leftmost level), where each node corresponds to a variable in the set \(V_{\text{known}}\).

**Step 2:** Examine all elemental expressions in the set \(F_{\text{remain}}\). Assume that an elemental expression \(FE_j\) is found that relates to only one output variable \(O_j\) not included in the set \(V_{\text{known}}\), that is, \(O_j \notin V_{\text{known}}\), and it also relates to a set of variables \(I_k\) that are all included in the set \(V_{\text{known}}\), that is, \(I_k \subseteq V_{\text{known}}\). Hence the dependency between the output variable...
Oj and the set of variables Ik is established.

Step 3: In the VDG graph, create a node for Oj at the level that is immediate right to the nodes of variables Ik and connect this new node with the nodes of variables Ik. Add Oj to Vknown and subtract Fei from Fremain.

- \( V_{\text{known}} = V_{\text{known}} \cup O_j \)
- \( F_{\text{remain}} = F_{\text{remain}} - F_e \)

Step 4: If \( F_{\text{remain}} \neq \text{NULL} \), then go back to Step 2. If \( F_{\text{remain}} = \text{NULL} \), then stop.

The resultant graph is just the desired VDG, where each node represents a design variable, and each link establishes a dependency between two variables. All the links to the left side of a node correspond to an elemental expression formed by the variables of the linked nodes. Fig. 2 shows an example of a VDG, where there are two loading variables (L1 and L2), three attribute variables (A1, A2, and A3), and six output variables (O1-O6, among which O6 is the performance variable). Four levels of dependencies are developed for these variables.

![Variable Dependency Graph](image)

**Fig. 2.** A variable dependency graph characterizing variable relationships

Note that the established variable relationships may include design evaluation criteria other than the required performances, such as the cost of manufacturing, the lifetime of the engineering system, and so on. These criteria also depend on the loading variables and attribute variables, hence may be handled in the same way as performance variables. As such, the performances, the performances requirements or the performance variables mentioned in the rest of the paper cover all design evaluation criteria.

### III. SOLVE DESIGN PROBLEMS

The developed variable relationships and the elemental expressions associated with them can be used to solve various design problems, such as

- design evaluation,
- evaluation of critical design variables,
- optimization of critical design variables.

Critical design variables refer to those design variables that affect the performances of a design at the overall (i.e., system) level and their values need to be determined at the early design stage. These include loading variables and attribute variables, where evaluation/optimization of loading variables means that the product may only be used to undertake or be optimized to undertake certain among of loadings; and evaluation/optimization of attribute variables means that these structural parameters or material properties should be determined so that the system can achieve the required performances at their targeted or optimized values.

To solve these design problems, the developed VDG for each performance variable will have to be a tree, that is, each performance variable will be dependent on various levels of other output variables and eventually on the relevant loading and attribute variables. Besides, there can only exist one hierarchy of such dependencies, i.e., only one tree, for each performance variable. We call such a VDG a complete VDG. If there is more than one tree for a performance variable, which is highly unlikely to be possible, we will have to leave out additional ones and retain only one tree for this performance variable in the VDG. If any of the variable dependencies is not available, which makes it impossible for the dependencies of a performance variable to form a tree, we will have to find ways to make the VDG a complete one before solving the corresponding design problems. Similarly, there should exist mechanistic models for all the elemental expressions, either explicitly such as mathematical functions, e.g., algebraic expressions or ordinary differential expressions (ODEs), or implicitly such as CAE models or computer codes. When any of the variable dependencies or any of the elemental expressions is not available to the designers, meta-modeling techniques [12] can be applied to generate approximate model first, such as using DOE (design of experiment) and RSM (response surface methodology).

For design evaluation, the known values of the loading variables and attribute variables can be propagated via the VDG to the performance variables so that their values can be derived. The different designs consisting of different sets of loading parameters and attribute parameters (for material, this would mean different material properties thus possibly different materials) can then be evaluated to determine which one gives best performances. If there are multiple performances to be compared, then weighting factors may be specified on them. The best one can then be selected for further development.

For evaluating/optimizing critical design variables, the requirements on the performance variables, or their targeted values, can be propagated via the VDG back to the critical design variables, by which these critical design variables
can be evaluated, either as a single value, or a set of values, or a range of values. Optimization of critical design variables may be carried out if the performance requirements can be formulated as an optimization objective function.

For all of the problems, if the mechanistic models are in the form of simple mathematical functions, such as algebraic expressions, then a direct relationship between the performance variables and the loading and attribute variables, including critical design variables, can be established by following the sequence as set forth by the VDG. In this situation, either design evaluation or evaluation/optimization of critical design variables can be directly accomplished without propagation of variable values over the VDG.

IV. CASE STUDY

A. Problem description

In this section, we study a design case to illustrate the proposed methodology and also to demonstrate its usefulness. The design case is borrowed from [5]-[7]. It is a design of an incandescent lamp system, which consists of structural components such as filament, bulb, base, inert air inside the bulb (argon, halogen or nitrogen), etc.

Assume that after the early stage of conceptual design, the design concepts have been derived. Subject to the specified performance requirements, such as luminosity and lifetime of the lamp, the further design tasks may include solving the following design problems:

• Evaluate the design concepts consisting of different structural attributes and material properties, e.g. different filament materials (characterized by different material properties), different inert gases, different filament wire diameters and coil diameters, different amount of inert gas, different bulb sizes, etc.

• Evaluate or optimize some of the above parameters, such as determining appropriate filament material and inert gas, optimizing filament wire diameter and coil diameter, and so on.

Solving these design problems requires the designers to deal with both the product design (e.g. determining filament wire diameter, coil diameter and bulb size) and material design (e.g. determining filament material and inert gas material). This is where there are close interactions and correlation between the two domains of design characteristics. Hence, we can apply the proposed methodology of developing variable relationships to support design integration. In the following, we will take filament design as an example because this is the most critical part of an incandescent lamp.

B. Modeling behaviors for filament design

Following the proposed methodology, we should establish the detailed relationships between the required performances and their dependent design variables. Assume a performance has been specified: the filament lifetime, which indicates that it should not be less than a certain number of hours. The structural element is the “filament”, and the material to be designed is the “filament material”.

The network of individual behaviors is developed by modeling the behaviors of the filament and those of its material in achieving the required performance, i.e. the filament lifetime; in other word, in analyzing how the failure of the filament is caused and how long it will take for this failure to occur.

Using a tungsten material as an example, we adopt the explanation given by [5]-[7] regarding the failure mechanism of tungsten filament. Before we elaborate on this mechanism, it is necessary to illustrate the filament structure. Briefly, this is a coiled or doubly coiled structure. Such a structure is to reduce convective heat loss as well as to house long wire in a confined space (i.e. the bulb). Fig. 3 shows a section of filament between two adjacent supports, where two dashed curves indicate the sagging state of the filament. Its attribute variables include wire radius \( r_0 \), coil radius \( R \), spacing between adjacent turns of the wire \( S \), and length of coil between the two supports \( l \).

![Fig. 3. Illustration of a filament coil](image)

The failure is postulated to be related to grain boundary sliding. This is a material phenomenon, which is a shear process occurring in the direction of grain boundary. The torsional deformation caused by the weight of the filament coil and the grain boundary sliding will lead to the adjacent turns of the coil to touch, forming a hot spot. The filament failure will then occur.

A more detailed exploration of this process will reveal a number of physical and material principles involved, thus the network of behaviors can be developed. To facilitate elaboration, we present the developed network of behaviors first, see Fig. 4.

![Fig. 4. The network of behaviors for the filament design](image)
The network consists of six behaviors B1-B6. Each behavior is characterized by its input variables (IV), output variables (OV) and attribute variables (AV). All the variables of these behaviors are shown in Fig. 4 and explained in Table 1.

<table>
<thead>
<tr>
<th>Input variables (IV)</th>
<th>Attribute variables (AV)</th>
<th>Output variables (OV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1 ( I_{11} = V ) (applied voltage)</td>
<td>( A_{11} = r_0 ) (wire radius)</td>
<td>( O_{11} = T ) (filament working temperature in Kelvin)</td>
</tr>
<tr>
<td>B2 ( I_{21} = T )</td>
<td>( A_{21} = D_{O,V} ) (pre-exponential for lattice diffusion coefficient) ( A_{22} = Q_v ) (activation energy for lattice diffusion coefficient) ( A_{23} = R ) (coil radius)</td>
<td>( O_{21} = D_v ) (lattice diffusion coefficient)</td>
</tr>
<tr>
<td>B3 ( I_{31} = T )</td>
<td>( A_{31} = \delta D_{O,B} ) (pre-exponential for boundary diffusion) ( A_{32} = Q_0 ) (activation energy for boundary diffusion coefficient) ( A_{33} = \Omega ) (atom volume)</td>
<td>( O_{31} = \delta D_B ) (boundary diffusion)</td>
</tr>
<tr>
<td>B4 ( I_{41} = r_0 ) (wire radius)</td>
<td>( A_{41} = r_0 ) (wire radius)</td>
<td>( O_{41} = \lambda ) (wavelength of grain boundary shape) ( O_{42} = h ) (amplitude of grain boundary shape)</td>
</tr>
<tr>
<td>B5 ( I_{51} = D_v ) ( I_{52} = T ) ( I_{53} = \delta D_B ) ( I_{54} = \lambda ) ( I_{55} = h )</td>
<td>( A_{51} = \Omega ) (atom volume)</td>
<td>( O_{51} = f_s ) (sliding friction under a uniform applied shear stress)</td>
</tr>
<tr>
<td>B6 ( I_{61} = f_s )</td>
<td>( A_{61} = 1 ) (spacing between adjacent supports from which the coil is suspended) ( A_{62} = M ) (mass of the coil between adjacent supports) ( A_{63} = r_0 ) (wire radius) ( A_{64} = R ) (coil radius) ( A_{65} = S ) (spacing between adjacent turns)</td>
<td>( O_{61} = \tau ) (frictional viscosity for torsional sliding rate) ( O_{62} = \tau ) (torque exerted at the grain boundary) ( O_{63} = \hat{\theta} ) (rotation rate at the grain boundary) ( O_{64} = \delta \theta ) (filament failure criterion) ( O_{65} = t_f ) (filament lifetime)</td>
</tr>
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</table>

The behavior B1 maintains the working temperature \( T \) for the filament by balancing between the heat generated and the heat loss. It associates with the physical principles relevant to the filament. According to the explanation given by [6], the generated heat is related to the electrical resistance of the filament (thus wire radius \( r_0 \) and total wire length) and the applied voltage \( V \) (an exterior input, \( i.e. \) a loading variable). The heat loss is proportional to \( T^4 \) (due to radiation) and wire length. Hence, the input to B1 is the voltage \( V \), its attribute is the wire radius \( r_0 \) (the wire length is not considered as an attribute because it is related to both the generated heat and the heat loss in the same magnitude), and the output from B1 is the temperature \( T \). The elemental expression characterizing the relationship between these variables [6] is:

\[
E_{11}: T = c \sqrt[4]{V r_0} \quad (c \text{ is a constant})
\]

Behaviors B2, B3, B4 and B5 are all associated with material science principles, such as the diffusion and the grain boundary sliding. The temperature \( T \) from B1 is required by behaviors B2, B3 and B5, hence B1 is connected to these three behaviors, as can be seen in Fig. 4 and Table 1:

- \( I_{21} = I_{31} = I_{51} = O_{11} = T \) (the first input to B2, the first input to B3 and the second input to B5 are all provided by the first output of B1).

Behaviors B2, B3 and B4 are connected with B5 by the following input-output pairs:

- \( I_{51} = O_{21} = D_v \) (the first input to B5 is provided by the first output of B2);
- \( I_{52} = O_{31} = \delta D_B \) (the third input to B5 is provided by the first output of B3);
- \( I_{54} = O_{41} = \lambda \) (the fourth input to B5 is provided by the first output of B4);
- \( I_{55} = O_{42} = h \) (the fifth input to B5 is provided by the second output of B4).

Some of the elemental expressions from the associated material principles are identified as (see [5], [6] for a detailed explanation of these principles):

\[
E_{21}: D_v = D_{O,V} e^{\frac{Q_v}{RT}} \quad \text{(from behaviour B2)}
\]

\[
E_{31}: \delta D_B = \delta D_{O,B} e^{\frac{Q_B}{RT}} \quad \text{(from behaviour B3)}
\]

\[
E_{41}: \lambda = r_0 \quad \text{(from behaviour B4)}
\]

\[
E_{42}: h = 2r_0 \quad \text{(from behaviour B4)}
\]

\[
E_{51}: \frac{f_s}{8\Omega} = \frac{\pi k T}{\lambda D_v + \pi \delta D_B} \quad \text{(from behaviour B5, } k \text{ is Boltzmann constant)}
\]

The output from B5 is used as an input to behavior B6, hence they are connected:

\( I_{61} = O_{51} = f_s \) (the first input to B6 is provided by the first output of B5).

Like behavior B1, behavior B6 associates with the physical principles of the filament rather than those of the material. It has a number of outputs, and the filament itself also has a number of attributes. According to the relevant physical principles, some of the outputs are only dependent on some attributes; some depend on both the input and some attributes; some even depend on the other outputs (\( i.e. \) there
are interactions between the outputs). Below is a list of the relevant elemental expressions to characterize their relationships:

\[ E_{61}: f = \pi r_0^4 / 2 \]
\[ E_{62}: \tau = gMl/2 \]
\[ E_{63}: \theta = \tau / f \]
\[ E_{64}: \Delta \theta_f = \left( \frac{s}{2r_0} - 1 \right) \frac{r_0}{R} \]
\[ E_{65}: t_f = \Delta \theta_f / \theta \]

The actual lifetime given by the elemental expression \( E_{65} \) will be used to evaluate designs as well as to examine whether the designed filament satisfies the required lifetime constraint.

**C. Developing VDG and solving filament design problems**

Following the proposed VDG development algorithm, the VDG for the filament design is derived and is shown in Fig. 5, where there are seven levels of nodes for all the design variables, with the first level consisting of a loading variable (V) and all attribute variables (\( D_{OV}, Q_V, \delta D_{OB}, Q_B, \Omega, r_0, R, S, l, M \)), and all other levels consisting of all output variables (T, \( \lambda, h, \Delta \theta_f, \tau, D_V, \delta D_B, f, \theta, t_f \)). The performance variable (\( t_f \)) is at the rightmost level.

This is a complete VDG, hence the aforementioned design problems can be solved without using meta-modeling techniques. For example, if we have a filament design with specified values for all attribute variables, and we have a number of options in determining the voltage to be supplied to the filament, then we can propagate the voltage \( V \) on the VDG to derive the value of \( t_f \) (lifetime of the filament), so as to determine the best working voltage in terms of the longest filament lifetime.

We may also evaluate any attribute variable or loading variable, with all other variables being pre-specified, by propagating the targeted value of \( t_f \) over the VDG back to this variable at the first level of the VDG.

We may also optimize a number of attribute variables and loading variable by formulating the design as an optimization problem. To this end, the design space (i.e. the optimization constraints) can be determined by the bounds of each of these variables, and the optimization objective function can be implicitly determined by the variable relationships and their associated elemental expressions incorporated in the VDG. To be more detailed, for a candidate design solution within the design space (i.e. satisfying the optimization constraints), the objective function value can be determined by propagating the values of the first-level design variables (determinable from this design solution) to the performance variable, i.e. \( t_f \), which has been used to specify the objective function. Appropriate optimization algorithm, e.g. gradient method, Genetic Algorithm, etc., can then be applied to derive the optimal design solution, namely, the optimal values for this group of design variables.

In fact, for the tungsten filament design, since all the elemental expressions are in the form of simple mathematical functions (algebraic expressions), we can derive the direct relationships between the performance variable and the loading and attribute variables. This can be achieved by following the sequence as set forth in the developed VDG. We do not give out this direct relationship because our intention of this case study is to illustrate the more general use of the proposed methodology.

**V. CONCLUSIONS**

In the previous sections, we have proposed a methodology to support the integration of material design and product design. A number of methods have been presented, including (1) modeling the behaviors of the product and material, where the relevant physical/material principles associated with each behavior are characterized by a number of design variables and their elemental expressions; (2) establishing the variable relationships from the developed network of elemental behaviors by a variable dependency graph; (3) solving various design problems by propagating the values of design variables both from the loading and attribute variables to the performance variables
(e.g. for design evaluation and for calculating objective function value in design optimization), and vice versa (e.g. for evaluation of some of the loading or attribute variables).

The proposed methodology provides a generic framework to develop the relationships between the required performance variables and their related loading and attribute variables, where the attribute variables characterize both the attributes of the structure of a product (thus for product design) and the material properties (thus for material design). The framework can also guide designers in identifying physical and material principles, identifying design variables and representing these principles by the relevant elemental expressions.

Our continual work will be enhancing the methodology to support more design situations and problems, such as when a VDG contains non-tree structure for a performance variable (e.g. more than one variable is dependent on the other variables). We will also investigate the appropriate optimization algorithms for design optimization involving both material design and product design, based on the proposed methodology.

REFERENCES