MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.033 October 3, 2003

Problem Set 5

Due: Thursday, October 16 (by 4:30 pm)

Reading: Chapter 3 & Chapter 5, sections 5.1–5.3 in Resnick & Halliday
Chapter 6 & start Chapter 7 in French.

Problem 1

"A Flying Brick"
Resnick & Halliday, problem #1, Chapter 3, page 117.

Problem 2

"A Small Change in Speed But a Large Change in Energy"
Resnick & Halliday, problem #5, Chapter 3, page 117.

Problem 3

"Limiting Rest Mass for a Neutrino"
In 1987 a star was observed to explode in our neighboring galaxy - The Large Magellanic Cloud (LMC). Neutrinos from the explosion arrived at the Earth and were detected within a few hours of the arrival of the optical light from the explosion. (Monitoring neutrinos was done on a continuous basis but, unfortunately, optical photographs of the LMC were taken only sporadically.) Take the average energy of the detected neutrinos to be 10 MeV, and the distance to the LMC to be 170,000 light years. Assume that the light and the neutrinos arrived at the Earth at the same time, but with an uncertainty of 3 hours. What limit can you place on the mass of the neutrinos? (See a related problem in Resnick & Halliday, #12, page 118.)

Problem 4

"The Fastest Particles?"
Resnick & Halliday, problem #13, chapter 3, page 118.
Note: the rest mass energy of a proton is ~ 940 MeV.

Problem 5

"Some Missing Algebra"
Resnick & Halliday, problem #18, Chapter 3, page 118.
Fill in any missing steps in the lecture derivation of $m = \gamma m_0$.

Problem 6

“Solving the Equation of Motion”
Resnick & Halliday, problem #33, Chapter 3, page 120.

Problem 7

“Not the Simplest Reference Frame”
Resnick & Halliday, problem #40, Chapter 3, page 120.
(Parts a, b, & c only.)

Problem 8

“The Great Neutron Shootout”
Resnick & Halliday, problem #43, Chapter 3, page 120.

Problem 9

“Cosmic-Ray Protons in the Galactic Magnetic Field”
Resnick & Halliday, problem #50, Chapter 3, page 121.
(Note: 1 light year = 9.5×10^{17} cm.)

Problem 10

“The Ultimate Terrestrial Accelerator”
Resnick & Halliday, problem #51, Chapter 3, page 121.

Problem 11

“The Decay of a Pion”
Resnick & Halliday, problem #56, Chapter 3, page 122.

Problem 12

“Water is Heavier Than Ice”
Resnick & Halliday, problem #66, Chapter 3, page 123.

Problem 13

“A Ton of Sunlight”
Resnick & Halliday, problem #69, Chapter 3, page 123.

Problem 14

“An Enormous Source of Energy”
Resnick & Halliday, problem #70, Chapter 3, page 123.

Problem 15

"Nuclear and TNT Explosions Compared"
Resnick & Halliday, problem #71, Chapter 3, page 123.

Problem 16

"Nuclear Reactions in the Sun"

Use the table of Atomic Mass Excesses (expressed in energy units of MeV) given on the next page to compute how much energy is liberated in each of the following reactions that can occur in the Sun:

$$\begin{aligned}
\mathrm{H}^1 + \mathrm{H}^2 &\rightarrow \mathrm{He}^3 + \gamma \\
\mathrm{He}^3 + \mathrm{He}^3 &\rightarrow \mathrm{He}^4 + 2\ \mathrm{H}^1 \\
\mathrm{Li}^7 + \mathrm{H}^1 &\rightarrow 2\ \mathrm{He}^4 \\
\mathrm{N}^{15} + \mathrm{H}^1 &\rightarrow \mathrm{C}^{12} + \mathrm{He}^4 \\
\mathrm{C}^{13} + \mathrm{H}^1 &\rightarrow \mathrm{N}^{14} + \gamma
\end{aligned} \tag{1}$$

[Hint: Add up the values of the mass excesses for the nuclei on the left-hand side of the reaction and subtract the sum of the mass excesses for the nuclei on the right-hand side. The rest mass of a γ-ray is zero.]

The following table is taken from the text “Principles of Stellar Evolution and Nucleosynthesis” by Donald C. Clayton (publisher: McGraw Hill):

The following optional problems are for discussion in recitation sections, and are not to be handed in:

Optional Problem A

"Human Power Output Per Unit Mass vs. That of the Sun"

Suppose that a "typical" person of mass ~ 60 kg, puts out 60 Watts of power while at rest. By contrast, the Sun has a luminosity (power output) of 4×10^{26} Watts, and mass of 2×10^{30} kg. Compare the power generated per unit mass of the Sun and a person. How do you make sense of this result?

Optional Problem B

"Powering the Sun by Gravitational Potential Energy"

Suppose that the luminosity of the Sun were powered by the release of gravitational potential energy only. Its power output is 4×10^{26} Watts. If the Sun contracted from its current radius of $R_\odot = 7 \times 10^8$ m, to half that value, how long would the contraction have to last in order to supply the observed power output? Assume that the gravitational potential energy of the Sun is $\sim GM_\odot^2/R$, where $M_\odot = 2 \times 10^{30}$ kg, and R is the radius at any given time.

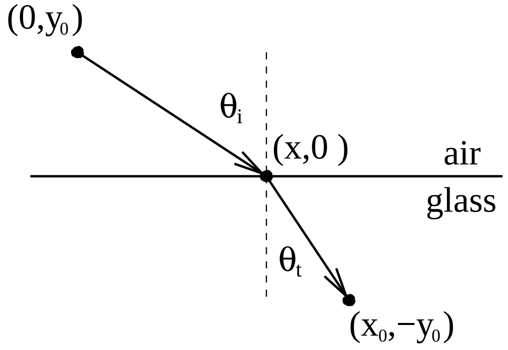
Optional Problem C

"Snell's Law Derived From Fermat's Principle"

Fermat's Principle states that the path of a light ray from one point to another is that which requires the least time. Suppose that light is emitted by a source at $(x = 0, y = y_0)$ in air, heads in the direction of a semi-infinite slab of glass located at $y = 0$, and enters the glass such as to reach the point $(x = x_0, y = -y_0)$. The geometry is shown in the diagram. Take the speed of light to be c in air, and c/n in glass, where n is called the "index of refraction", and is typically a number in the range of 1.4–1.6.

Assume, from Fermat's Principle, the obvious result that *within* any given medium (i.e., of fixed n) light travels in a straight line. Then use Fermat's Principle to prove Snell's law: $\sin\theta_i = n \sin\theta_t$. You need not bother with the calculus of variations for this problem.

Hint: take the distances traveled to be $y_0/\cos\theta_i$ and $y_0/\cos\theta_t$ in the air and glass, respectively. Then note that $x_0 = y_0(\tan\theta_i + \tan\theta_t)$, which defines the constraint between θ_i and θ_t.



Optional Problem D

“Lagrange’s Equations of Motion”

Marion & Thornton’s book on “Classical Dynamics” introduces Lagrangian mechanics with the following two instructive statements:
1. “Minimal principles in physics have a long and interesting history. The search for such principles is predicated on the notion that nature always minimizes certain important quantities when a physical process takes place.”
2. In two papers in 1834 and 1835, Hamilton announced the dynamical principle on which it is possible to base all of mechanics and, indeed, most of classical physics. Hamilton’s Principle may be stated as follows: “Of all the possible paths along which a dynamical system may move from one point to another within a specified time interval (consistent with any constraints), the actual path followed is that which minimizes the time integral of the difference between the kinetic and potential energies.”

Let’s use this principle and the Euler equations for extremizing path integrals to derive Lagrange’s equations of motion and solve a couple of simple mechanics problems. First, identify the “Lagrangian” as the difference between the kinetic and potential energies:

$$\mathcal{L} \equiv K - V \quad ,$$

where K and V are the kinetic and potential energies, respectively. Show that if K and V depend on coordinates q_k, then the equations that satisfy Hamilton’s principle are:

$$\frac{\partial \mathcal{L}}{\partial q_k} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} = 0 \quad .$$

a. As a first illustrative mechanics problem, consider a mass M suspended (in gravity) on a spring with a spring constant k. Call the vertical direction z. Take the position of the mass to be $z = 0$ when the spring is neither stretched nor compressed. What is the potential energy as a function of z. Write down the kinetic energy in terms of $\dot{z}$. Use Lagrange’s equation to find the equation of motion of the mass. How does the equation compare with that found by starting with $F = ma$?

b. As a second illustrative problem, consider a pendulum consisting of a mass, M, suspended on a string of length l. Dispace the pendulum by an angle ϕ with respect to the vertical and release the mass. Use Lagrange’s equation to describe the subsequent motion of the mass. The problem is effectively one-dimensional in that a complete description of the motion is given in terms of a single dependent variable (ϕ) and its derivative ($\dot{\phi}$). Simple analytic solutions to this equation are found by making a small-angle approximation, i.e., $\sin\phi \simeq \phi$. You also made the same approximation in the 8.01 version of this problem.