When a swift charged particle enters a materials medium it will interact with the electrons and nuclei in the medium and begins to lose energy as it penetrates into the medium. The interaction can be generally thought of as collisions between the charged particle and either the atomic electron or the nucleus (considered separately). The energy given off will result in ionization, production of ion-electron pairs, in the medium; also it can appear in the form of electromagnetic radiation, a process known as bremsstrahlung (braking radiation). We are interested in describing the energy loss per unit distance traveled by the charged particle, and the range of the particle in various materials, the latter being defined as the distance traveled from the point of entry to the point of being essentially rest.

A charged particle is called ‘heavy’ if its rest mass is large compared to the rest mass of the electron. Thus mesons, protons, α-particles, and of course fission fragments are all heavy charged particles. By the same token, electrons and positrons are ‘light’ particles.

If we ignore nuclear forces and consider only the interactions arising from Coulomb forces, then we can speak of four principal types of charged-particle interactions:

(i) Inelastic Collision with Atomic Electrons. This is the principal process of energy transfer, particularly if the particle velocity is below the level where bremsstrahlung is significant, it leads to excitation of the atomic electrons (still bound to the nucleus) and to ionization (electron stripped off the nucleus). Inelastic here refers to electronic levels.
(ii) Inelastic Collision with a Nucleus. This process can leave the nucleus in an excited state or the particle can radiate (bremsstrahlung).

(iii) Elastic Collision with a Nucleus. This process is known as Rutherford scattering. There is no excitation of the nucleus, nor radiation. The particle loses energy only through the recoil of the nucleus.

(iv) Elastic Collision with Atomic Electrons. The process is elastic deflection which results in a small amount of energy transfer. It is significant only for charged particles that are low-energy electrons.

In general interaction of type (i), which is sometimes simply called collision, is the dominant process of energy loss, unless the charged particle has a kinetic energy exceeding its rest mass energy in which case the radiation process, type (ii), becomes important. For heavy particles, radiation occurs only at such kinetic energies, \(~10^3\) Mev, that it is of no practical interest. The characteristic behavior of electron and proton energy loss in a high-Z medium like lead is shown in Fig. 13.1 [Meyerhof Fig. 3.7].

![Stopping Power: Energy Loss of Charged Particles in Matter](image)

**Stopping Power: Energy Loss of Charged Particles in Matter**

The kinetic energy loss per unit distance suffered by a charged particle, to be denoted as \(\text{d}T/\text{d}x\), is conventionally known as the *stopping power*. This is a positive quantity since \(\text{d}T/\text{d}x\) is \(<0\). There are quantum mechanical as well as classical theories for calculating this basic quantity. One wants to express \(-\text{d}T/\text{d}x\) in terms of the properties specifying the incident charged particle, such as its velocity \(v\) and charge \(ze\),
and the properties pertaining to the atomic medium, the charge of the atomic nucleus $Ze$, the density of atoms $n$, and the average ionization potential $\bar{I}$.

We consider only a crude, approximate derivation of the formula for $-dT/dx$. We begin with an estimate of the energy loss suffered by an incident charged particle when it interacts with a free and initially stationary electron. Referring to the collision cylinder whose radius is the impact parameter $b$ and whose length is the small distance traveled $dx$ shown in Fig. 13.2 [Meyerhof Fig. 3-1], we see that the net momentum transferred to the electron as the particle moves from one end of the cylinder to the other end is essentially entirely directed in the perpendicular direction (because $F_x$ changes sign so the net momentum along the horizontal direction vanishes) along the negative $y$-axis. So we write

\[
\int dtF_y(t) \approx 0 \quad (13.1)
\]

\[
p_e = \int dtF_y(t)
\]

\[
= \int \frac{ze^2}{x^2 + b^2} \frac{b}{\left(x^2 + b^2\right)^{1/2}} \frac{dx}{v}
\]
\[ \approx \frac{ze^2 b}{v} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + b^2)^{3/2}} = \frac{2ze^2}{vb} \quad (13.2) \]

The kinetic energy transferred to the electron is therefore

\[ \frac{p_e^2}{2m_e} = \frac{2(ze^2)^2}{m_e b^2 v^2} \quad (13.3) \]

If we assume this is equal to the energy loss of the charged particle, then multiplying by \( nZ(2\pi \rho db dx) \), the number of electrons in the collision cylinder, we obtain

\[ -\frac{dT}{dx} = \frac{h\nu}{b_{\text{min}}} \int_{b_{\text{min}}}^{b_{\text{max}}} \frac{nZ 2\pi \rho db}{2 m_e} \left( \frac{ze^2}{vb} \right) ^2 \]

\[ = \frac{4\pi(ze^2)^2 nZ}{m_e v^2} \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right) \quad (13.4) \]

where \( b_{\text{max}} \) and \( b_{\text{min}} \) are the maximum and minimum impact parameters which one should specify according to the physical description he wishes to treat.

In reality the atomic electrons are of course not free electrons, so the charged particle must transfer at least an amount of energy equal to the first excited state of the atom. If we take the time interval of energy transfer to be \( \Delta t \approx b/v \), then \( (\Delta t)_{\text{max}} \approx 1/v \), where \( h\nu \approx \bar{I} \) is the mean ionization potential. Then

\[ b_{\text{max}} \approx h\nu / \bar{I} \quad (13.5) \]

An empirical expression for \( \bar{I} \) is \( \bar{I} \approx kZ \), with \( k \approx 19 \text{ ev} \) for H and \( \approx 10 \text{ ev} \) for Pb. We estimate \( b_{\text{min}} \) by using the uncertainty principle to say that the electron position cannot be specified more precisely than its de Broglie wavelength in the relative coordinate
system of the electron and the charged particle. Since electron momentum in the relative coordinate system is $m_e v$, we find

$$b_{\text{min}} \approx \frac{h}{m_e v}$$

Combining these two estimates we obtain

$$-\frac{dT}{dx} = \frac{4\pi^2 e^4 nZ}{m_e v^2} \ln\left(\frac{2m_e v^2}{I}\right)$$

In (13.7) we have inserted a factor of 2 in the argument of the logarithm, this is to make our formula agree with the result of quantum mechanical calculation which was first carried out by H. Bethe using the Born approximation.

Eq.(13.7) describes the energy loss due to particle collisions in the nonrelativistic regime. One can include relativistic effects by replacing the logarithm by

$$\ln\left(\frac{2m_e v^2}{I}\right) - \ln\left(\frac{v^2}{c^2}\right) - \frac{v^2}{c^2}$$

This correction can be important in the case of electrons and positrons.

Eq.(13.7) is a relatively simple expression, yet one can gain much insight into the factors that govern the energy loss of a charged particle by collisions with the atomic electrons. We can see why one can usually neglect the contributions due to collisions with nuclei. In a collision with a nucleus the stopping power would increase by a factor $Z$, because of the charge of the target with which the incident charged particle is colliding, and decrease by a factor of $m_e/M(Z)$, where $M(Z)$ is the mass of the atomic nucleus. The decrease is a result of the larger mass of the recoiling target. Since $Z$ is always less than $10^2$ whereas $M(Z)$ is at least a factor $2 \times 10^3$ greater than $m_e$, the mass factor always dominates over the charge factor. Another useful observation is that (13.7) is independent of the mass of the incident charged particle. This means that
nonrelativistic electrons and protons of the same velocity would lose energy at the same rate, or equivalently the stopper power of a proton at energy $T$ is about the same as that of an electron at energy $\sim T/2000$. This scaling is more or less correct as seen in Fig. 13.1.

Fig. 13.2 shows an experimentally determined energy loss curve (stopping power) for a heavy charged particle (proton), on two energy scales, an expanded low-energy region where the stopping power decreases smoothly with increasing kinetic energy of the charged particle $T$ below a certain peak centered about 0.1 Mev, and a more compressed high-energy region where the stopping power reaches a broad minimum around $10^3$ Mev. Notice also a slight upturn as one goes to higher energies past the broad minimum which we expect is associated with relativistic corrections. One should regard Fig. 13.2 as the extension at both ends of the energy of the curve for proton in Fig. 13.1.

**Fig. 13.2.** The experimentally determined stopping power, $(-dT/dx)$, for protons in air, (a) low-energy region where the Bethe formula applies down to $T \sim 0.3$ Mev with $T \sim 80$ ev. Below this range charge loss due to electron capture causes the stopper power to
reach a peak and start to decrease (see Fig. 13.3), (b) high-energy region where a broad minimum occurs at $T \sim 1500$ Mev. [from Meyerhof]

Experimentally, collisional energy loss is measured through the number of ion pairs formed along the trajectory path of the charged particle. Suppose a heavy charged particle loses on the average an amount of energy $w$ in producing an ion pair, an electron and an ion, the residual nucleus which is now charged. Then the number of ion pairs produced per unit path is

$$i = \frac{1}{w} \left( -\frac{dT}{dx} \right)$$

Eq.(13.7) is valid only in a certain energy range because of the assumptions we have made in its derivation. We have seen from Figs. 13.1 and 13.2 that the atomic stopping power varies with energy in the manner sketched below. In the intermediate energy region, $500 \tilde{T} < T \leq Mc^2$, where $M$ is the mass of the charged particle, the stopping power behaves like $1/T$, which is roughly what is predicted by (13.7). In this region the relativistic correction is small and the logarithm factor varies slowly. At higher energies the logarithm factor along with the relativistic correction terms give rise to a gradual increase so that a broad minimum is set up in the neighborhood of $\sim 3$ Mc2. At energies below the maximum in the stopping power, $T < 500 \tilde{T}$, Eq.(13.7) is not valid
because the charged particle is moving slow enough to capture electrons and begin to lose its charge. Fig. 13.3 [Meyerhof Fig. 3-3] shows the correlation between the mean charge

![Figure 3-3](image)

of a charged particle and its velocity. This is a difficult region to analyze theoretically. For $\alpha$-particles and protons the range begins at $\sim 1$ Mev and 0.1 Mev respectively [H. A. Bethe and J. Ashkin, “Passage of Radiation Through Matter”, in *Experimental Nuclear Physics*, E. Segrè, ed (Wiley, New York, 1953), Vol. I, p.166].

Eq.(13.7) is generally known as the Bethe formula. It is a quantum mechanical result derived on the basis of the Born approximation which is essentially an assumption of weak scattering [E. J. Williams, Rev. Mod. Phys. 17, 217 (1945)]. The result is valid provided

$$\frac{ze^2}{\hbar v} = \left(\frac{e^2}{\hbar c}\right) \frac{z}{v/c} = \frac{z}{137(v/c)} \ll 1 \quad (13.9)$$

On the other hand, Bohr has used classical theory to derive an expression for the stopping power,

$$-\left(\frac{dT}{dx}\right)_{\text{class}} = \frac{4\pi^2 e^4 nZ}{m_e v^2} \left[n\left(\frac{M\hbar v}{2ze^2(m_e + M)}\frac{2m_e v^2}{T}\right) \right] \quad (13.10)$$

which holds if
Thus (13.7) and the classical formula apply to opposite conditions. Notice that the two expressions agree when the arguments of the logarithms are equal, that is, $\frac{2ze^2}{hv} = 1$, which is another way of saying that their regions of validity do not overlap. According to Evans (p. 584), the error tends to be an overestimate, so the expression that gives the smaller energy loss is likely to be the more correct. This turns out to be the classical expression when $z > 137(v/c)$, and (13.7) when $2z < 137(v/c)$. Knowing the charge of the incident particle and its velocity, one can use this criterion to choose the appropriate stopper power formula. In the case of fission fragments (high Z nuclides) the classical result should be used. Also, it should be noted that a quantum mechanical theory has been developed by Bloch that gives the Bohr and Bethe results as appropriate limiting cases.

The Bethe formula, (13.7), is appropriate for heavy charged particles. For fast electrons (relativistic) one should use

$$-\frac{dT}{dx} = \frac{2\pi e^4 nZ}{m_e v^2} \left[ \ln \left( \frac{m_e v^2 T}{I^2 (1 - \beta^2)} \right) - \beta^2 \right]$$

(13.12)

where $\beta = v/c$. For further discussions see Evans.