Lane Changing Models for Arterial Traffic

by

Varun Ramanujam

B.Tech in Civil Engineering (2005)
Indian Institute of Technology, Madras, India

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Signature of Author ..........................................................

Department of Civil and Environmental Engineering
May 30, 2007

Certified by ............................................................

Moshe E. Ben-Akiva
Edmund K. Turner Professor of Civil and Environmental Engineering
Thesis Supervisor

Accepted by ............................................................

Daniele Veneziano
Chairman, Departmental Committee for Graduate Students
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ABSTRACT

Driving behavior models for lane-changing and acceleration form an integral component of microscopic traffic simulators and determine its value in evaluation of different traffic management strategies. The state-of-art model for lane changing adopts a two-level framework: the first level involves a latent or unobserved choice of a target lane; the second level models the acceptance of adjacent gaps in the direction of the target lane. While this modeling approach has several advantages over past works, it assumes drivers to execute lane change within the same time step in which gap was found to be acceptable. In other words, under time steps typically adopted in model applications, the lane change duration is assumed to be negligibly small. However, past works report average lane change durations to the order of 5-6 seconds. Besides this practical maneuvering requirement, the assumption fails further in moderate or low density traffic conditions with ample gap sizes or low speed conditions, where lane changing maneuver can take longer than average. The work outlined in this thesis proposes an extension to the two-level framework for lane changing models through a third level that explicitly models the lane change duration. Traffic conditions in the driver’s neighborhood that are likely to influence lane change duration are accounted for in the third level.

The extended model is applied to data obtained from video observations on traffic on a stretch of an arterial corridor in California. Apart from possessing distinctive features including signalized intersections and multiple access locations that result in lower average speeds, the arterial dataset used in this study represents a relatively low density scenario in terms of gap availability, thereby presenting an ideal test-bed for the proposed model extension. Since arterial datasets have not received predominant attention in literature, this work uncovers some traffic aspects not encountered in past studies.

The model is estimated using a sample of the overall dataset available in the form of disaggregate vehicle trajectories. The estimated model is implemented in a microscopic traffic simulator MITSIMLab, and model validation is done using aggregated traffic data. Estimation and validation results showcase the improved modeling capabilities achieved through the proposed extension.

Thesis Supervisor: Moshe E. Ben-Akiva
Title: Edmund K. Turner Professor of Civil and Environmental Engineering
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CHAPTER 1

INTRODUCTION

1.1 Problem of traffic congestion
In the last two decades, traffic congestion has become the single most critical concern for road transportation planners and operators and researchers in general. Indeed, the rate of growth in urban and freeway congestion over different parts of the world has been very alarming. Given the influence road transportation exerts on national economy, especially in the developed nations, the rise in congestion has propagated several detrimental effects through the economic performance and turnover of several countries. Congestion also means greater time spent by automobiles on traffic, implying an increasing amount of fuel emissions, thereby raising serious environmental concern.

In United States alone, the total hours of congestion-induced delay in major urban areas amounted to 3.7 billion vehicle hours in the year 2003, an estimated increase of 50% from the corresponding figure 10 years before (Schrank et al., 2005). The total dollar value of this delay, including fuel costs and value of the time lost, was estimated to be about 0.6% of the US GDP at the time. Statistics also indicated that congestion was spreading over different urban cities, with an alarmingly increasing number of areas starting to suffer in severe proportions.

1.2 Solution strategies for congestion
It is now widely accepted in different transportation-related stakeholder circles that increasing roadway capacity is not a viable solution to the congestion problem, both from a feasibility as well as economic perspective. It was estimated that an additional 5000 lane miles of freeways and arterials would have been required to contain the contemporary congestion and meet the growing travel needs of people (Schrank et al., 2005). Given this projected rate of growth, such an endeavor would entail huge capital investments and therefore impose heavy financial burdens. Also, such an option forces the acquisition of land space for road construction, and the current land-use density numbers in many developed parts of the world make this further intractable.
Therefore, the focus of late has shifted to investing in operational infrastructure management that allows for a better and balanced utilization of current road space. Such strategies aim to enhance the realized roadway capacity and help mitigate current or forecasted congestion, which at most times is the result of a skewed demand for existing road resources.

Operation improvement strategies have grown in prominence over the last decade, and the results justify the interest and hope placed in them. The source for the above-mentioned statistics ranks operational treatments rank as the second-most effective strategy for containing congestion, behind public transportation. They had delivered a relief of almost 10% of the total congestion, in terms of vehicle delay hours, by 2003. There is significant promise in their future applications towards congestion mitigation.

1.3 Value of traffic simulation

It is in this domain of transportation operations planning that microscopic traffic simulation has been rapidly gaining prominence. The two broad and key services offered by microsimulation that adds tremendous value to the efforts along the direction of above described approach are:

- The study and simulation of local (traffic in immediate vicinity, etc) and global (travel times on different routes, etc) interactions governing the travel behavior of every individual on the road, including the effects of traffic control measures and road configurations, and its ultimate impact on traffic conditions;
- The consequent forecast of traffic conditions in reaction to proposed modifications to traffic control elements and strategies, and accurate estimation of key performance metrics used for the evaluation of these strategies.

As a consequence, several microscopic traffic simulators (AIMSUN, PARAMICS, VISSIM, MITSIM, etc.) have been developed in the recent past, each with their own distinct representation of the traffic world, and all for the purpose as described above. The ability of these simulators to accurately replicate the real traffic world is the single-most important determinant of its value to its intended functional purpose: the evaluation of traffic control and management strategies.
1.3.1 Driver behavior models for simulation
Driver behavior models are theoretical abstractions of actual driver behavior. They aim to explain in detail the motion of road users in traffic as influenced by various external stimulants and personal driving goals, through a decision/rule-based framework. Their nature and theoretical approach differ across simulators, but they invariably form the core amongst the various models that constitute each of these simulators. The quality of these models, assessed in terms of their ability to accurately replicate the various complex interactions dictating driver behavior in traffic, form the key to the overall ability of the simulator to replicate and forecast traffic conditions for their various applications. Development of advanced and behaviorally representative models has therefore been the focus of several dedicated research works in the past, and continues to be an active research area. It also forms the background for the work reported in this thesis.

Most microscopic simulators replicate driving as a discretized motion comprising of longitudinal and lateral components in each simulation time step. The longitudinal motion of a driver depends upon his/her instantaneous acceleration, while the lateral motion is captured through his/her lane-changing behavior. Literature is replete with models for both acceleration and lane-changing behavior of drivers. The scope of this thesis is the domain of models that characterize driver’s acceleration and lane-changing as the output of a conscious decision process. While it is understood and acknowledged that both these actions are inherently inter-related and need to be modeled jointly, the focus of this work has been devoted to the lane-changing aspect of driving behavior only.

1.4 Application scenario: Freeways vs Arterials
There are two major types of roadways that serve auto traffic: the freeways that serve inter-city traffic and the arterials that serve urban intra-city traffic. Freeway traffic has been the focus for driving behavior research in the past, for two major reasons:

- Freeway congestion has repeatedly resulted in severe breakdowns of transportation networks in several parts of the world, thereby motivating immediate research efforts towards mitigating them;
The availability of detailed observed data for freeway traffic that facilitated development and estimation of driving models.

While, perhaps spurred on by these two factors, there exists vast literature in the field of freeway driving models there is a scarcity among works specifically focusing on arterial traffic. Arterial traffic differs from that on freeways in more than one respect, and these differences in themselves entail the study and development of dedicated models. Also, there has been a significant growth in urban traffic in many large cities of late, further motivating the need to embark on a detailed study of their characteristics, and evaluate different traffic management and operational approaches suited to the arterial environment.

Traffic in arterials in general follow a start-stop mode, characterized by relatively low average speeds compared to a freeway owing to signalized intersections and multiple access points. The high proportion of turning vehicles, both entering and exiting off the road, result in significant merging and diverging movements. Therefore, the set of simple considerations that influence the longitudinal and lateral movements of drivers along a freeway stretch that serves only through traffic may well be extended in arterial situations. Most importantly, the average time spent by a vehicle in arterial traffic is significantly shorter than that in the freeway. Consequently, drivers’ focus tend to be on arriving at their respective exit points, rather than following a traffic stream desirable from the individual’s perspective on driving comfort. This tendency is also motivated by the absence of a significant speed advantage across lanes as typically observed on an arterial. Drivers therefore perceive little benefit in switching lanes for the sake of driving comfort.

1.5 Thesis work

This work deals with the study of arterial traffic and development of lane-changing driver behavior models for analyzing the same. It extends a model framework previously adopted for freeway traffic and applies it to arterial traffic. Apart from study of lane-changing behavior in arterials, the need to explore a previously ignored behavioral aspect motivates this proposed extension. Implicit assumptions on driver behavior exist within the model framework adopted in several past applications that have tended to focus on congested scenarios and/or freeway traffic. Specifically, it has been assumed that drivers execute lane change in a negligibly small time such
that the lane change maneuver coincides with the time instance during which they experience acceptable adjacent gaps. An argument in favor of this assumption might relate to the prevalent congestion in traffic, rendering opportunities for lane changing few and far between and forcing the driver to respond quickly. Additionally, in high speed traffic conditions like in freeways, the duration of a lane change maneuver may indeed be lower than average. However, this cannot be expected in small traffic pockets where gaps for lane change stay continually acceptable over a longer duration. Drivers in such conditions, pertaining to their individual characteristics, might execute their lane change in a more relaxed manner, since they perceive no compelling urgency to complete the lane change.

The proposed extension to the model framework is therefore an attempt to make it more general and overcome a significant assumption adopted during applications to congested and/or freeway environments. It is applied and tested under arterial driving conditions.

The availability of detailed and disaggregate vehicle trajectory data from video observations of traffic on an arterial road in California, U.S.A, facilitated the estimation of the developed model. The predominance of relatively large gaps in the dataset allows for testing the validity of the elements in the extended model framework that distinguish and enhance it in comparison to the two-level model framework applied to congested freeway situations.

Aggregate validation of the estimated model was carried out on MITSIMLab (Yang and Koutsopoulos, 1996), a microscopic traffic simulator developed at the Intelligent Transportation Systems Laboratory at MIT.

An overview of the overall model development process adopted in this study is presented in figure 1.1 below, enlisting the sequence in which different task modules including model estimation and validation were executed. This would help serve as a reference guide for the reader through the sequence of chapters that detail the application case study.
Data processed from disaggregate vehicle trajectories are used for estimating the model formulated adopting the above-discussed extension. Model estimation is an iterative process and involves the selection of best specification after the testing of different specifications. The estimated model is then implemented in a simulator and its applicability to a different scenario is tested. Data on the test scenario is prepared in aggregate form and segmented into two parts, typically based on time interval. The implemented model is calibrated using the first part of the aggregate data from the test scenario and validated on the second part. The final validation measures are used as performance metrics to assess the predictive quality of the developed model.

The outline of the thesis, following this introduction, is as follows: a set of relevant works on lane changing models and their various applications available in literature are discussed next. This is followed by a description of the model framework and structure as proposed in this work. The application of the proposed model to the aforementioned arterial traffic dataset is demonstrated, highlighting results of model estimation and validation and a comparative analysis.
with respect to the two-level model that serves as a reference base. The final chapter summarizes the contributions of this work and highlights some suggested directions for future research.
CHAPTER 2

LITERATURE REVIEW

The advent of microsimulation has stimulated intense research in the field of driver behavioral models. A great deal of emphasis in this line of research has been devoted towards the analysis of lane-changing and acceleration behavior. The application of these models has been predominantly limited to freeways, owing to issues concerning data availability and stakeholder focus. This chapter is presented in two parts: the first section presents a discussion of driver lane-changing models most relevant to the current work, including the recent developments. It identifies the limitation among current models and thereby develops the motivation for the proposed model extension. The second section outlines some past research works that have been applied in the domain of arterial driving analysis.

2.1 Lane-changing models: Trends and State-of-art

A comprehensive review on the state-of-art in lane changing models is available in Toledo (2007). A brief summary of the lane changing models discussed in that review and relevant to the current work is presented here.

Traditionally, lane changing models assume a two-step decision process, lane selection, and lane change execution. The desire to change lanes, in most models, is assumed to arise from either of two kinds of motivations: Mandatory (when the need for a lane change is necessitated by a lane drop, incident or approaching exit junction) or Discretionary (when a driver seeks better driving conditions compared to that experienced in his current lane). The lane selection process is influenced by the underlying nature of motivation for lane change. The lane change execution process is distinguished from the lane selection process, and modeled using gap acceptance models. The next two sections discuss the developments in each of these two model components leading to present state-of-art, as derived from literature.
2.1.1 Lane selection

The lane selection component of lane-changing models has undergone profound improvements since the first models were proposed.

Among the earliest lane changing models, the most popular one was that developed by Gipps (1986). Similar models were later developed by Hidas et al. (1999), Hidas et al. (2002) and Halati et al. (1997) and implemented in the microscopic simulators SITRAS and CORSIM respectively. The basic structure of these models treated lane selection as a rule-based process, where the set of lanes in the driver’s choice set were repetitively evaluated based on different considerations or rules prioritized through a deterministic sequence. This sequence was updated based on the driver location with respect to his trajectory and exit point. The rules/considerations governing lane changes could be broadly classified as belonging to either a Mandatory (MLC) or Discretionary (DLC) category. Due to the predetermined sequence of these considerations, the models failed to capture trade-offs between the different factors that could motivate or influence a lane change. They also did not consider variability in behavior among different drivers. These factors dented the realism of the models. Another critical shortcoming in these works was the absence of any proposed framework for model estimation.

Yang and Koutsopoulos (1996) were among the first to overcome the deterministic framework of the previous models. In their models, drivers would initiate an MLC action with certain probability that depends upon explanatory factors including distance from exit point, traffic density, etc. The DLC process, initiated when driver’s current speed is lower than his desired speed, involved the selection of the best among current and neighboring lanes, based on prevalent driving conditions. The key enhancement offered by these models was the introduction of a random utility framework for lane selection. It thereby helped capture trade-offs between various factors influencing lane choice at any given instant. These models were implemented in MITSIMLab, a microscopic traffic simulator developed at the Intelligent Transportation Systems Laboratory at MIT.
Ahmed (1999) provided an improved version of lane-changing models that captured both MLC and DLC situations. The model framework is presented in figure 2.1.

**Figure 2.1: Structure of the lane-changing model proposed by Ahmed (1999)**

Lane changing was modeled as a three-level decision process: decision to consider a lane change, selection of target lane if considering a lane change, and acceptance of gaps in the target lane to execute the lane change. The decisions in the first two levels were modeled using random utility models in a discrete choice framework. The first level involved a two-step decision process for determining driver’s motivation to change lanes. The driver first considers a mandatory lane change situation. Following the rejection of an MLC situation, the driver considers the DLC case. Following a decision to pursue either an MLC or DLC, the driver enters the lane selection level, wherein he evaluates his current and neighboring lanes on the basis of existing driving conditions. On choosing a lane other than his current lane, the driver evaluates the adjacent gaps available in the chosen lane. Gap acceptance decisions were modeled using critical gaps, which were random variables representing the minimum gap a driver would require to execute a safe lane change given certain driving conditions. Estimation for parameters at all levels was conducted separately.
for MLC and DLC situations, using detailed trajectory data from two different locations, one corresponding to a purely MLC situation, and other to purely DLC situations.

One drawback that persisted among these models was that MLC and DLC situations were still handled separately, with the former accorded precedence over the latter. Therefore, trade-offs between these two motivations for lane-changing could not be considered, and knowledge of the type of situation at hand (MLC or DLC) at every instant of a driver’s trajectory was assumed.

Toledo (2003) introduced an integrated driving behavior model that combined MLC and DLC considerations and incorporated them at the same level of driver’s lane selection process. The framework for the integrated model is presented in figure 2.2.

**Figure 2.2: Structure of the integrated driving behavior model proposed by Toledo (2003)**

![Diagram](image)

The utility function for lanes considered in the lane selection process accounts for both MLC and DLC related factors. Integration along another behavioral dimension was achieved through this model, wherein the interdependencies between the lane changing and acceleration decisions of a driver were captured. Lane-changing and acceleration were both considered to be actions governed by a latent planning process, executed by the driver with an objective of fulfilling latent goals (e.g., moving to a desired lane). Joint estimation was carried out for lane selection and gap acceptance model parameters, with results showcasing the significant improvement brought about by the explicit modeling of trade-offs between MLC and DLC considerations.
The integrated driving behavior model however continued to carry over one limitation from some of its predecessors, in that it assumed drivers to exhibit myopic behavior when it came to lane selection. The choice set only included the driver’s current lane along with his immediate left and right lanes. Choudhury (2005) extended the lane selection framework to include all lanes in the cross-section of the driver’s road segment in his choice set, and termed it the target lane selection model. The model framework is presented in figure 2.4, and analyzed in further detail in section 2.1.4. An immediate modeling advantage obtained by this enhancement was the ability to model lane changing in the presence of exclusive usage lanes like HOV or HOT. In such situations, vehicles in traffic could overlook the relative disutility of immediate neighboring lanes and target the exclusive lane, but this phenomenon cannot be replicated using the previous myopic lane selection models. Estimation and validation results confirmed the superiority of the target lane selection model, especially in scenarios involving exclusive usage lanes or other types of significant differentiation among lanes across the cross-section of a road segment.

Ahmed’s, Toledo’s and Choudhury’s models were each tested and validated in MITSIMLab. Choudhury’s target lane selection framework represents the state-of-art among all approaches to lane selection modeling, and was adopted for the model developed in this work.

2.1.2 Gap acceptance

Gap acceptance models first evolved in the context of vehicle behavior modeling at unsignalized intersections. They were used to explain the merging behavior of vehicles from minor streams that do not possess right-of-way into inter-vehicle gaps in the major stream. Subsequently, they were integrated with lane selection models resulting in the development of standard lane-changing models. Gap acceptance is typically modeled as a binary choice process. The driver compares a representative measure of the available adjacent gap with a critical threshold, also known as critical gaps. This threshold value is determined based on model parameters and/or given neighboring situation as per the adopted specification, and is assumed to follow a chosen random distribution, both of which vary across studies in literature.
Herman and Weiss (1961), Drew et al. (1967) and Miller (1972) were among the first to investigate the application of gap acceptance models in intersection behavior studies. They assumed the critical gaps to follow an exponential, a log-normal and a normal distribution respectively. They, similar to other contemporary researchers, used the time headway between the lead (or lag) and the subject vehicle as the representative measure for the lead (or lag) adjacent gap. Their specification of the critical gap only included the mean and the variance as model parameters requiring estimation.

Daganzo (1981) developed a model that allowed for variation of the mean of the critical gap across individuals and time instances, thereby facilitating complete usage of a panel dataset of merging observations that included consecutive instances of gap rejection before the merge execution. A normal distribution for critical gaps was assumed. Model estimation was carried out for a dataset comprising of roadside vehicle observations. Mahmassani and Sheffi (1981) extended the model specification to help differentiate between the effects of heterogeneity and temporal variance in the mean of the critical gap, which were aspects Daganzo had found difficult to estimate separately in his work. Driver impatience over the merge was found to have a significant impact resulting in the above-mentioned temporal variance.

The next phase of research development involved the application of gap acceptance models to non-mandatory lane changing situations (unlike those at intersection crossings or freeway merging). An explicit layer of decision regarding lane selection was introduced above that of the gap acceptance, resulting in the standard two-level lane changing model framework. Gipps (1986) was among the first to adopt this framework. In his model, gaps were represented by the deceleration required by the trailing vehicle to acquire its desired “car-following” speed, and were compared against a deterministic threshold that reflected vehicle capabilities and urgency for lane change. Both lead (comprising of the adjacent lead vehicle and subject vehicle) and lag gaps (comprising of the subject and the adjacent lag vehicle) needed to be acceptable for the gap to be acceptable.
The use of discrete choice framework for modeling gap acceptance provided tremendous potential for an enhancement in the specification for the mean of the critical gap, thereby capturing the effects of the immediate neighboring situation and individual-specific characteristics on gap acceptance behavior. The first instance of such an improvement was Kita's (1993) work, which adopted a logit model formulation for explaining gap acceptance behavior at freeway merging locations. Explanatory variables including length of adjacent gap, relative speed of subject w.r.t mainline vehicles, and remaining distance to end of merge lane were found to have significant impact. Ahmed et al. (1996) provided a two-level probabilistic lane changing model framework including lane selection and gap acceptance models. Critical gaps were assumed to follow log-normal distributions, in order to obviate the possibility of them attaining negative values. An adequate specification characterizing the impacts of immediate traffic situation as well as driver heterogeneity was adopted for the mean of critical gaps. A notable deviation from past works was that gaps were evaluated in terms of the space headway available between the leading and trailing vehicle, as opposed to time headway. Similar gap acceptance models were also adopted by Toledo (2003) and Choudhury (2005).

Another line of research has focused on capturing state dependence, or in other words the impact of past decisions, in gap acceptance behavior. This is especially relevant in the context of MLC situations, where critical gap measures might change and reduce as the drivers spend more time awaiting lane change opportunities (first investigated in Mahmassani and Sheffi, 1981). Different merging mechanisms like forced and cooperative merging have been conceptualized and studied along this area of research, and have been shown to provide significant improvement in terms of explaining driver behavior at merge locations (Ahmed, 1999; Hidas, 2005; Choudhury et al., 2007). However, this topic is beyond the scope of the research outlined in this thesis.

2.1.3 Summary

The standard state-of-art lane changing model is an integration of two decision models: lane selection and gap acceptance, evaluated in succession by drivers while contemplating a lane-change action. Literature records significant enhancements that have been made in both these decision models individually and simultaneously since their inception. Results of their
application to different vehicle trajectory datasets in past studies have validated their superior performance in terms of modeling and predictive capabilities compared to more preliminary approaches.

The next section analyzes in detail the two-level model framework as proposed by Choudhury (2005), and explores the scope for further enhancing the same.

2.1.4 State-of-art two-level lane changing model (Choudhury, 2005)

The most recent lane-changing model framework, as adopted in Choudhury (2005), is shown in figure 2.4 below. It illustrates the decision process for a driver existing in a hypothetical situation involving a 4-lane roadway, as depicted in figure 2.3.

Figure 2.3: Hypothetical scenario for illustrating lane-changing decision process, involving subject vehicle in lane 2 of a 4-lane roadway
In the above framework, the first decision task for a driver involves the selection of a target lane. A random utility-based framework is used to model this decision, where the driver considers different aggregate lane-specific variables as well as his individual path plan in estimating utilities of all available lanes. The selection of the target lane among the available lanes follows a multinomial logit choice process.

Subject to the location of the selected target lane with respect to the driver’s current lane, the driver considers changing lane into the right or left adjacent lane (also termed his immediate target lane), moving laterally in the direction of the target lane. The next step in this process is the evaluation of the available adjacent gap in the lane that the driver considers moving into based on his target lane selection. The gap acceptance model is employed for this purpose, and the decision outcome results in the driver’s final observable action regarding lane change. If the adjacent gap is rejected, the driver does not enact a lane change, while a lane change is exhibited on acceptance of the adjacent gap.

### 2.1.5 Motivation for extension in model framework

While the above-described two-level model framework integrates several of the latest advancements in both lane selection and gap acceptance models, it suffers from a structural limitation at the second decision level. The gap acceptance level of this model structure contains...
an implicit assumption that every driver will complete a lane change execution at the same time step in which he finds the adjacent gap acceptable. The time step referred to here is the time resolution adopted in the application data that discretizes the continuous vehicle trajectories into a discrete set of lane changing observations. In previous applications of driving behavior models, this time step has been in the order of a second.

Now, an approach as described above overlooks the duration over which a lane change may occur. This duration might vary across different types of lane changes and would depend upon traffic conditions. It in reality will constitute a finite multiple of the time step adopted in the data on which the model is applied, especially when the adopted time step is very small.

Toledo and Zohar (2007) present a study on the duration of lane changing maneuver. They cite several past studies that report mean lane change duration in the range of 3-7 seconds which is significantly larger than typical unit time step values adopted in model applications. They estimate a regression model for the lane change duration using a disaggregate vehicle trajectory dataset wherefrom the instances of initiation and completion of lane changes for individual vehicles can be extracted to a reliable degree of accuracy. The lane-change duration was found to be dependent on factors like subject speed relative to neighboring vehicles, and traffic conditions on the adjacent lane being targeted. They claim two influences to be primarily responsible for prolonging lane change durations. The first influence is of the perceived risk on the physical safety of the lane change maneuver (captured through representative variables on neighboring conditions). A higher risk associated with a lane change ensures caution on the driver’s part and prolongs the maneuver. The other influence relates to the urgency of a lane change (also captured through representative variables). A more urgent lane change situation is expected to draw a quicker response from the driver and the lane change is expected to be of shorter duration.

The validity of ignoring the lane change duration therefore weakens greatly when applied to traffic conditions that don’t entail great urgency in lane changes. When traffic is heavily congested and there exists a scarcity of adequately sized adjacent gaps, it might be reasonable to expect that a driver would try to complete a lane change execution at the earliest. A similar case
of accentuated urgency might hold in case of MLC situations where the driver is less likely to exhibit slack when changing lanes.

However, this assumption is more severely challenged in the case of DLC situations and/or uncongested traffic conditions. Under such circumstances, a driver is less compelled to make an immediate lane change, and might prolong it, especially when the adjacent gap stays acceptable over a longer duration. Also, when driver speed is low, the lane change maneuver is expected to take longer.

Under such situations, depending upon the time step at which data is recorded or used, it is possible to experience a series of vehicle observations where the adjacent gap is in reality acceptable and yet a lane change is not recorded. These observations correspond to a lane change maneuver which has been initiated but not completed. In low to moderate density or low speed traffic, the number of such observations may be of significantly higher proportion in comparison to the number of lane changes, since durations of most lane changes are expected to be high.

The current two-level model framework would fail to explain this phenomenon in a behaviorally consistent way. A maximum likelihood approach to estimate the current model using a dataset having significant observations involving acceptable gaps not accepted prior to a lane change is likely to bias some estimates in the model specification. The critical gap parameters are most vulnerable to estimation errors under this phenomenon, since their maximum likelihood estimates would reflect the fact that gaps relatively larger/similar in comparison to the accepted gap were previously rejected. Also, variables in the target lane model that aim to capture the inertia of a driver against lane changing might also be erroneously affected, since the model can also interpret the rejection of acceptable gaps as a case of the driver not desiring to change lanes.

The application of the two-level lane changing model framework to low density or low speed traffic might therefore yield estimates that are not transferable to other situations. The work presented in this thesis aims at preserving the consistency of the parameter estimates in the critical gap specification under such applications. It extends the current two-level model framework and accounts for the delay in lane change completion through a distinct third level of driver decision-making following gap acceptance. This third level would explicitly model, through a binary logit formulation, the driver's decision to execute lane change in the current time step given adjacent gap acceptability. The time step when the driver decides to execute the
lane change would correspond to the completion of the lane change. The decision to execute lane change is hypothesized to depend upon factors influencing the urgency of lane change maneuver (the less urgent the lane change, greater the tendency of drivers to prolong it) and the speed at which the maneuver can be executed (greater the speed at which the lane change can be physically completed, shorter the expected duration of the lane change).

2.2 Lane-changing models for arterial traffic

Arterial traffic offers an interesting contrast to freeway traffic, through greater diversity in traffic mix, higher accessibility for entering vehicles and influence of periodic signalized intersections. Owing to the impacts of these elements and other characteristic features, drivers in arterial are expected to deviate in their behavior in comparison to that exhibited in freeways. However, literature is not replete with studies on arterial traffic and driver behavior models characterizing them. The following section provides a very brief overview of the broad classes of work in the topic of arterial driving behavior analysis.

2.2.1 Macroscopic models

A major proportion of research in the field of arterial traffic analysis has been based on macroscopic models for traffic flow. Traffic flow theory has been applied since 1950’s for the analysis of vehicle queuing and signal operations at urban intersections. Stochasticity of vehicle movements has also been introduced and studied within these frameworks. Geroliminis et al. (2005) develop an analytical methodology for modeling traffic flow between successive intersections. The methodology adopts the Markov assumption of memorylessness over the temporal domain. Combined with a platoon dispersion model for predicting the dispersion of traffic downstream of signals, the overall methodology is used to forecast arrival profiles and queue lengths at signalized intersections on arterials.

While simple to analyze and deploy owing to its amenability to analytical approaches, macroscopic models offer an overly simplistic alternative to replicating a complex process. Critical assumptions including the homogeneity among drivers’ motivations and other
characteristics that are inherent to a macroscopic approach severely weaken the accuracy and credibility of traffic predictions and subsequent evaluations.

2.2.2 Microscopic models
The author was unable to unearth a significant number of past works in literature that have focused on developing microscopic driver behavior models exclusively for arterial traffic situations. Among those that were found to be relevant to the topic of this work, a couple of studies are selectively discussed below. Wei et al. (2000) extracted detailed vehicle trajectory data from video observations of traffic in eight urban streets in Kansas City, Missouri, and used this data to characterize arterial lane changing behavior and propose rules that would form the basis for a more realistic and better-structured arterial lane changing model. They considered lane-changing to consist of three sequential components: a decision model, a condition model and an execution model. The decision model outlined three kinds of motivations for a lane change – mandatory, discretionary and preemptive. These motivations were however considered in a pre-determined order of priority (similar to some of the early freeway lane changing models discussed in the previous section), and no explicit trade-offs between them were captured. The condition model was a surrogate to the gap acceptance model discussed above in the context of freeway lane changing. The execution model was meant to account for the lane change duration given the driver decided to make a lane change and accepted the adjacent gap. Whilst no rigorous probabilistic model framework was developed to integrate the three components, and no estimation of model parameters was done using the available data, the work had two significant contributions. First, it acknowledged the dominance of preemptive lane changing behavior, an attempt by a driver to pre-position himself to avail his next exit, in arterial traffic conditions. Second, it introduced a third component following lane selection and gap acceptance that explicitly accounted for lane change duration. This is one of the aspects in the context of lane-changing maneuvers that the model extension proposed in this work aims to capture.

Other studies have focused exclusively on microscopic simulation of vehicles at signalized intersections, and used it as a tool to analyze intersection capacity. One example is that of Jin et
al. (1999), who use a cellular automata approach to simulate vehicles at intersections and develop car-following and lane-changing models that capture the effect of signal timings and conflicting turning movements of upstream vehicles in the approach to the intersection. However, no model estimation was done.

In this study, a lane-changing model that handles some distinctive characteristics of arterial traffic is developed. The model explicitly accounts for the duration of lane changing maneuver, a factor implicitly ignored in past works and expected to be prominent in arterial traffic conditions. A randomly selected sample of disaggregate vehicle trajectory data collected from a stretch of arterial corridor, with intermittent signalized intersections, located near US highway 101 in California, U.S.A, was used to estimate the model parameters. The model was implemented in a microscopic simulation tool (MITSIM) that has the capabilities to simulate signalized intersections and arterial corridors. Aggregate calibration was carried out using one portion of available trajectory data, while aggregate validation was carried out using the other portion. Results illustrate the improved performance of the extended model in terms of providing a better explanation of lane-changing mechanisms of the vehicles in the arterial.

The next chapter presents the formulation of the model, including details on framework and the structure of each decision-level within it.
CHAPTER 3

MODEL FORMULATION

Similar to current state-of-art driver behavior models, the model presented here conceptualizes lane changing action as an outcome of a sequence of choices made by the individual. These choices are modeled in a probabilistic framework and expressed as a function of certain explanatory variables governing driver behavior. The model is a direct extension to the two-level target lane model proposed by Choudhury (2005) which comprised of a target lane selection level followed by a gap acceptance decision level. The proposed extension introduces one additional level of choice-making concerning the execution of lane change following the gap acceptance decision. This extension endows the model framework with ability to explicitly handle lane change duration, a practical maneuverability-related aspect. Consideration of this aspect is critical for a comprehensive and structurally robust abstraction of driver behavior. The effect of unobserved driver/vehicle characteristics on the lane-changing process is captured by a latent driver-specific random term included at all three decision levels to capture the resulting correlations among successive observations of the same individual.

This chapter details the elements of the modeling framework and structure of this extended lane changing model. It discusses modeling assumptions and candidate explanatory variables for each decision level.

3.1 Modeling framework

The hierarchical structure of the 3-level extended lane changing model, with the two-level framework of Choudhury’s target lane model embedded within, is pictorially presented in the figure 3.1. The diagram illustrates the lane-changing decision tree that the model assumes a driver existing in the hypothetical scenario as described in figure 3.2 to consider.
The subject vehicle in the hypothetical scenario described above is driving currently in the 3rd lane of a 4-lane roadway, with 2 lanes available to the left and one available to the right of his direction of travel. The lane numbering follows an incremental order from the left to the right of the driver's travel direction.
The first two steps in the above decision framework of a driver are latent, with his/her final actions serving as mere indications to these decision outcomes. The first decision concerns the choice of a target lane for the driver. This decision is typically unobservable, and has been considered so in all models that have included this type of a decision in their framework.

The next decision concerns the acceptability of the adjacent gap available to a driver in the direction he/she targets heading into (right or left lane, depending upon the choice of the target lane). At any given instant, the fact that the available adjacent gap that is being targeted is acceptable to the driver is also not directly discernible from the observations. This represents the primary departure from the modeling assumptions in Choudhury (2005), which had gap acceptance as the final step in the decision sequence and therefore considered it to coincide with the observed lane-changing action of the driver. While it is still true that only the driver's lane-changing actions, constituting a lane-change execution to the left, right or neither, are observed, the current model acknowledges it as an outcome to a separate and explicit lane change execution decision that follows the gap acceptance decision.

In the above representation of the decision process, latent choices are shown as ovals, and observed ones are shown as rectangles.

*Description of each decision level:*

As mentioned above, the first two decision levels in the 3-level model are similar to the corresponding decision levels described in the target lane model (Choudhury 2005).

3.1.1 Target lane selection

The target lane is the lane the driver perceives as most desirable to drive in, depending upon the prevalent driving conditions and his immediate destination. A multinomial logit model is used to explain the choice of this target lane at each instant. The choice set for the target-lane selection includes all the lanes in the current road section along the direction of travel to which the vehicle can legitimately move. Each of the ovals shown at the first level represents a lane available for the driver to select as his target lane at the current-time step in the given hypothetical scenario of
figure 3.2. The choice of the target lane implicitly fixes the immediate adjacent lane the driver targets moving to (also referred to, for sake of brevity, as the immediate target lane), which is located on either left or the right of the current lane depending upon relative location of the target lane. Given that in the scenario considered the driver is in lane 3 in the current-time step, a choice of lane 3 as the target lane means that the driver decides not to pursue a lane-change and to continue in his current lane. This would directly take him to the end of his lane-changing decision process for the current time instant. If the driver perceives that there exists another available lane in the road section that would optimize his condition, he will choose that lane as his target lane. In the above example, the immediate target lane for the driver would be to his left if he chooses either lane 1 or 2 as his target lane, while it would be to his right if he chooses lane 4 as his target lane.

3.1.2. Gap acceptance
Subject to the selection of a target lane other than his current lane, the driver would evaluate the gaps in his neighborhood (the appropriate adjacent lane, i.e., left or right) adjacent to his current position. The gap acceptance framework involves an evaluation of the lead and lag gaps that define the overall adjacent gap considered by the driver. The figure 3.3 below, reproduced from Choudhury (2005), defines the adjacent lead and lag gap lengths considered by a driver at the gap acceptance decision level according to the model.

Figure 3.3: Description of Lead and Lag gaps

![Figure 3.3: Description of Lead and Lag gaps](image-url)
The available lead and lag gaps in the immediate target lane are compared with their respective latent critical gaps, based on which the driver decides to accept or reject the gap. The model assumptions entail that for the adjacent gap to be acceptable as a whole, both the lead and the lag gaps have to be individually acceptable, implying that they should be greater than the respective critical gap values. The critical gaps are random variables whose means are expressed as a function of appropriate neighborhood variables that characterize the risk of executing a lane change into the targeted adjacent gap.

3.1.3. Lane change execution

Given that the driver considers a lane other than his current lane to be most desirable to drive upon, and perceives that the adjacent gap in the direction towards the selected target lane is acceptable, he is assumed to begin the lane change. He then considers the final lane-change execution step during the lane change maneuver. This decision step indirectly accounts for the duration over which the driver would complete the lane change maneuver. It is modeled as a binary choice concerning his decision to execute the lane change in the given instant. He can either decide to execute the lane change (and thereby complete it) in the given instant (final observable decision captured as Change Right or Change Left respectively), or not to execute the lane-change (final observable decision captured as No Change) in the current instant. He may decide not to execute a lane change, or may appear not to be executing a lane change in the current time instant, if he is still happens to be in the process of completing the lane change. A decision on behalf of the driver to execute the lane change in the current time instant therefore represents the end of his lane change maneuver. In this manner, the third decision level helps capture the variations in lane change duration within the model framework.

In instances where the driver chooses his current lane as the target lane or does not find the adjacent gap in his immediate target lane acceptable, he does not reach this step in his decision tree, and the final observable decision reflects No Change.

The entire three-level decision framework is repeatedly applied over each time step of a driver's trajectory as recorded or processed in the application dataset. While this approach lacks an explicit handling of state dependency among successive driver actions (as proposed in Choudhury et al.,
2007), it is justified to an extent by the fact that the explanatory variables in the model specification capture elements of state dependency within their current values. As an example, the immediate neighboring conditions for a driver at the current instant, which govern his gap acceptance and lane change execution decisions, are in turn influenced by his past actions, and thereby help carry over the effect of past actions onto his current decisions.

### 3.2 Model structure

As explained earlier, the model adopts a probabilistic framework to explain the decisions made at each of the three levels of the driver’s decision tree. The probabilistic models chosen and the resultant mathematical expressions governing the choice at each of these decision levels are presented in this section. The first decision-level concerning the selection of a target lane is modeled as a multinomial choice model. The second level of decision concerning gap acceptance is modeled as a probabilistic binary choice comparing the deterministic available gap measure with a random threshold also known as the critical gap. The third level of decision concerning lane change execution is modeled as a binary choice. The dataset used for estimating this lane-changing model typically consists of multiple observations per individual describing his trajectory in discretized form, resulting in a panel dataset. As observed and studied in all datasets of such nature, the characteristics of an individual would have a consistent and time-invariant influence over his/her behavior. In this case, it would result in serial correlation among actions of a single driver over successive time steps. As mentioned earlier in this chapter, an attempt is made to capture this serial correlation through the introduction of latent driver-specific random terms within the probabilistic models for each decision level that together explain the driver actions.

#### 3.2.1 The Target-lane model

As described in the earlier section, the target lane selection process involves the selection of the most desirable lane (denoted as the target lane) among a set of candidate lanes available to a driver. This choice scenario is perfectly amenable to modeling using a utility maximization approach. Specifically, the multinomial logit choice model is chosen and adopted to explain the target lane selection process. For a rigorous description of the conceptual background and theoretical development of the multinomial logit and other random utility models, the reader is
referred to Ben-Akiva and Lerman (1985). The following description assumes the reader to possess a background on the fundamentals of discrete choice theory.

The target-lane (TL) choice set consists of all the lanes in the road cross-section that are available for the driver to move in given his current position on the roadway and direction of travel. The utilities of these lanes are given by:

\[ U^t_{in} = \beta X^t_i + \alpha' v^t_n + \epsilon^t_{in} \quad \text{lane } i \in I_{nt} \]  

(3.2.1)

where,

- \( I_{nt} \) describes the set of all lanes available to the driver \( n \) at time step \( t \);
- \( U^t_{in} \) is the utility of lane \( i \) to driver \( n \) at time \( t \);
- \( X^t_i \) is a vector of explanatory variables containing lane-specific attributes for lane \( i \) at time \( t \);
- \( \beta \) is the corresponding vector of parameters;
- \( \epsilon^t_{in} \) is the random term associated with the lane utility \( U^t_{in} \);
- \( v^t_n \) is a driver specific random term that represents unobservable characteristics of the driver \( n \) (in particular, characterizing driver’s aggressiveness); and
- \( \alpha' \) is the parameter of \( v^t_n \) specific to lane \( i \).

The logit model entails the assumption that the random terms \( \epsilon^t_{in} \) are independently and identically extreme value distributed over all the lanes in the target-lane choice set and over all observations. The driver specific random term is used to capture the effects of serial correlation among observations of the same individual over successive time instances that arise due to time-invariant influences of individual characteristics. As in past studies (Choudhury, 2005), it is assumed to be normally distributed over the driver population. Since it represents characteristics that are not observed nor can be extracted from the available dataset, it is considered a latent variable and treated accordingly in the model formulation.

The expressions for the choice probabilities of lane \( i \in I_{nt} \) for driver \( n \) at time \( t \), conditional on the latent individual-specific error term \( (v^t_n) \), follows from the derivation for a multinomial logit choice model, and are given by:
\[ P_{nl}(\text{lane } i | v_n) = \frac{\exp(V_{nl}^i(v_n))}{\sum_{\mu \in I_n} \exp(V_{nl}^\mu(v_n))} \quad \text{lane } i \in I_{nl} \]  \hfill (3.2.2)

\( V_{nl}^i(v_n) \) are the systematic utilities of the alternatives (given individual specific term \( v_n \)), given by:

\[ V_{nl}^i = \beta X_i^i + \alpha v_n \quad \text{lane } i \in I_{nl} \]  \hfill (3.2.3)

Candidate explanatory variables that are hypothesized to be influential in the above model include:

1. Lane-specific attributes relevant to driving comfort: They include general aggregate lane attributes, such as queue length, average speed and traffic density. This category also subsumes other attributes that differentiate one lane from another in regard to driving comfort, including the number of lane changes required of the driver to reach the target lane from current lane, his/her general preference to continue in the current lane of travel at given instant, etc. Each of the above-described variables relate to the quality of driving experience for an individual driver. Lesser queues, higher average speeds and lower vehicular densities are expected to offer better driving conditions, while a lane closer to the current lane is more likely to be chosen as a target lane given all other conditions equal.

2. Path plan variables: These include variables linked to the path a driver has in mind as he/she travels through the roadway, and the mandatory turns/exits that get defined consequentially as the driver tries to follow this path. Some examples include the distance to the point in the current roadway where the driver must exit to continue along his path, the number of cross-sectional lanes separating the current lane of the driver from the lane which he/she needs to be in so as to make the targeted exit, etc. These variables capture the mandatory considerations of a driver during his travel through a roadway. Lanes closest to the exit would become more preferable as the driver nears his exit.
3. Driver-specific characteristics: These include individual characteristics like aggressiveness that influence the preference for any particular lane during travel through the main-stream traffic. These latent characteristics are captured by the individual-specific error term $v_i$. A typical implementation of this variable would intend to capture the incremental preference a driver possesses towards his current lane of travel as depending upon his relative aggressiveness compared to the average driver. This measure would be in addition to a general preference exhibited by all drivers irrespective of their aggressiveness (captured in the first class of explanatory variables as mentioned above).

Since the application case study discussed in this work involves arterial traffic, it is worth mentioning the following note at this point. In urban arterial conditions, owing to the relatively short duration a driver spends in the main traffic stream, the mandatory considerations related to a driver's path plan are expected to dominate over other comfort-related considerations in lane-changing decisions.

3.2.2. Gap acceptance model
In the target-lane model the driver chooses his/her target lane. The immediate target lane is determined as a consequence of this target-lane selection. If the target lane lies to the right/left of the current lane, the driver considers the right/left adjacent lane as his immediate target lane. Next, the driver decides whether or not a lane change into the immediate target lane can be undertaken by evaluating the corresponding adjacent gap. Conditional on the target-lane choice, the outcome of the gap acceptance model would indicate whether the driver perceives the existing adjacent gap in the current-time step safe enough to consider a lane change execution or not.

The adjacent gap in the immediate target lane is defined by the lead and lag vehicles in that lane, that are in turn identified based on their relative position with respect to the subject driver, as shown in figure 3.3 and reproduced in figure 3.4. The lead gap is the clear spacing between the rear of the lead vehicle in the immediate target lane and the front of the subject vehicle. Similarly, the lag gap is the clear spacing between the rear of the subject vehicle and the front of the lag
vehicle in the immediate target lane. Note that one or both of these gaps may be negative if the vehicles overlap.

**Figure 3.4: Definitions of the Front, Lead and Lag Vehicles and Their Relationship with the Subject Vehicle**

The driver compares each of the available lead and lag gaps to a corresponding threshold measure termed the critical gap, which is the minimum gap length needed to ensure gap acceptance. In other words, an available gap (lead or lag) is acceptable if it is greater than the corresponding critical gap measure (lead or lag). Critical gaps are modeled as random variables. Inheriting the latest advancements in gap acceptance models as outlined in the previous chapter, the means of the critical gaps are expressed as functions of explanatory variables related to the current driving situation. This approach helps capture and explain better the variation in critical gap measures for a driver over time. The individual-specific term is also included in the mean expression in order to capture correlations among the critical gap measures perceived by a driver over time. Critical gaps are assumed to follow lognormal distributions to ensure that they are always nonnegative. A typical form of the expression adopted for the critical gap measure is given below:

\[
\ln(G_{ngd}^{sd,cr}) = \beta^{sd} X_{ng}^{sd} + \alpha^{sd} \nu_n + \epsilon_{ng}^{sd} \quad g \in \{\text{lead, lag}\}, \quad d \in \{\text{right, left}\}
\]

where,

- \( G_{ngd}^{sd,cr} \) is the critical gap measure for gap \( g \) in the direction of change \( d \) perceived by driver \( n \) at time step \( t \), measured in a typical unit of distance;
\( X_{n|d}^{\text{ed}} \) is a vector of explanatory variables used to characterize the mean of the critical gap \( G_u^{\text{ed},cr} \); 
\( \beta^x \) is the corresponding vector of parameters; 
\( \varepsilon_{n|d}^{\text{ed}} \) is a random term: \( \varepsilon_{n|d}^{\text{ed}} \sim N(0, \sigma^2) \) following the log-normal assumption; and 
\( \alpha^x \) is the parameter of the driver-specific random term \( v_n \).

The gap acceptance model assumes that the driver must accept both the lead gap and the lag gap to find the total adjacent gap acceptable. The probability of accepting the adjacent gap, conditional on the individual specific term \( v_n \) and the choice of direction of change \( d_{nt} \), is therefore given by:

\[
P(\text{accepting adjacent gap in direction } d_{nt} \mid d_{nt}, v_n) = P(AG_{nt} = d_{nt} \mid d_{nt}, v_n) =
\]

\[
P(\text{accept lead gap} \mid d_{nt}, v_n) P(\text{accept lag gap} \mid d_{nt}, v_n) =
\]

\[
P(G_{nt}^{\text{lead},d_{nt}} > G_{nt}^{\text{lead},d_{nt},cr} \mid d_{nt}, v_n) P(G_{nt}^{\text{lag},d_{nt}} > G_{nt}^{\text{lag},d_{nt},cr} \mid d_{nt}, v_n)
\]

\( d_{nt} \in \{\text{Right, Current, Left}\} \) is the chosen direction of change for driver \( n \) at time \( t \), which is implied by the target-lane choice. The resulting gap acceptance decision variable \( AG_{nt} \) can be defined as the following:

\[
AG_{nt} = \begin{cases} 
1 & \text{if the available adjacent gaps to the left of the driver are acceptable, given the left lane is the immediate target lane} \\
-1 & \text{if the available adjacent gaps to the right of the driver are acceptable, given the right lane is the immediate target lane} \\
0 & \text{otherwise}
\end{cases}
\]

\( G_{nt}^{\text{lead},d_{nt}} \) and \( G_{nt}^{\text{lag},d_{nt}} \) are the available lead and lag gaps, and \( \gamma \), and \( G_{nt}^{\text{lead},d_{nt},cr} \) and \( G_{nt}^{\text{lag},d_{nt},cr} \) are the corresponding critical gaps in the chosen direction, respectively.

The assumption regarding the lognormal probability distribution of the critical gaps results in a well-defined functional form for the gap acceptance probability. The conditional probabilities that gap \( g \in \{\text{lead, lag}\} \) is acceptable is given by:

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The gap acceptance decision is hypothesized to be primarily affected by variables that characterize the nature of the corresponding gaps and the safety of a lane change maneuver into such a gap from a driver’s perspective. This set typically consists of neighborhood variables such as the subject speed relative to that of the lead and lag vehicles, current available gap lengths etc. The chances of accepting a bigger gap length are expected to be higher, but are expected to reduce if the gap is shrinking with time (a phenomenon of which the relative speed of the subject with respect to lead or lag vehicle would serve as a direct indicator). Also, driver-specific characteristics, such as aggressiveness, play a key role in deciding the lead or lag gap lengths that gets accepted. Aggressive drivers can be expected to accept smaller gaps by virtue of their risk-taking nature. The above variables are therefore considered in the specification of the mean of the critical gaps, in an attempt to capture their impacts on the gap acceptance decisions.

### 3.2.3. Lane change execution model

The lane change execution decision is the third decision considered by a driver in a given time instance, conditional on him selecting a target lane other than his current lane and accepting the corresponding adjacent gap in the direction towards the selected target lane. The first two decisions are latent, while the outcome of the third decision regarding lane change execution is translated to an observable driver action, namely a lane change or no lane change. The lane change execution decision involves a binary choice, with the alternatives being whether to execute a lane change or not to execute it in the current time instant. As in the case of the two earlier decision levels, this choice is modeled using a probabilistic framework. Binary logit is chosen as an ideal abstraction of this choice situation. It should be noted that the choice set for the given scenario includes a do-nothing alternative, whose utility is arbitrarily normalized to zero. The random utility (and corresponding systematic utility) for the execution alternative is as given below:

\[
P\left(G_{nt}^{ed} > G_{nt}^{ed,cr} \mid d_{nt}, v_{n}\right) = \Phi\left(\ln\left(G_{nt}^{ed}\right) - \ln\left(G_{nt}^{ed,cr}\right) \mid d_{nt}, v_{n}\right) = \\
\Phi\left[\frac{\ln\left(G_{nt}^{ed}\right) - \left(\beta^s X_{nt}^{ed} + \alpha^s v_{n}\right)}{\sigma_g}\right] 
\]

where \(\Phi[\cdot]\) denotes the cumulative standard normal distribution.

\[
3.2.6
\]
$$U_{nt}^{l_{nt}} = \begin{cases} V(\beta, X_t, \alpha^{l_{nt}}, \nu_n) + \varepsilon_{nt} & \text{if } l_{nt} = 1 \text{ or } -1 \\ 0 & \text{if } l_{nt} = 0 \end{cases}$$

and $$V_{nt}^{l_{nt}} = \begin{cases} V(\beta, X_t, \alpha^{l_{nt}}, \nu_n) & \text{if } l_{nt} = 1 \text{ or } -1 \\ 0 & \text{if } l_{nt} = 0 \end{cases}$$ (3.2.7)

Where $l_{nt}$ is an indicator variable for the choice at this decision level, and defined as

$$l_{nt} = \begin{cases} 1 \text{ or } -1 & \text{if executing lane change to left or right respectively} \\ 0 & \text{if not executing lane change} \end{cases}$$

$X_t$, a vector of explanatory variables characterizing the decision to execute a lane change at the current time instant $t$, 

$\beta$: the corresponding vector of parameters 

$\alpha^{l_{nt}}$: the parameter of the driver-specific random error term $\nu_n$ that is used across the three decision levels and represents his individual characteristics 

$\varepsilon_{nt}$: is the random term associated with this utility function, and following from the logit assumption, is independently and identically extreme value distributed over all observations.

The resulting probability of executing the lane change at current time instant, conditional on adjacent gap acceptance and driver-specific random error term, follows from the derivation for the binary logit model, and is given as:

$$P(l_{nt} \mid AG_{nt}, \nu_n) = \begin{cases} 1/(1+\exp(-V_{nt}^{l_{nt}})) & \text{if } AG_{nt} = \{1,-1\} \\ 0 & \text{if } AG_{nt} = 0 \end{cases}$$ (3.2.8)

_Nature of variables constituting lane change execution decision:_

In order to obtain a better understanding of the kind of variables that constitute the vector $X_t$, mentioned in (3.2.7), it is important to acknowledge the discrepancy between the theoretical conceptualization of the lane changing maneuver and its existence in reality. A lane change in field traffic is not an instantaneous process. It is executed over a finite time interval. However, most lane-changing model frameworks and application trajectory datasets treat them as
instantaneous, following the discretization of driver trajectory which is a continuous-time process. In the model framework, the final observable action can either belong to a no lane change category or a lane change category, implying that it can only conceive a given lane change at one, and only one, time instant. As mentioned earlier, the length of this time instant is decided by the time resolution in the application data. Detailed disaggregate datasets used in applications of the model frameworks discussed in this thesis describe vehicle trajectories as a sequence of discrete positions, and typically accommodate a lane change maneuver in only one such discrete observation. (It should be noted that Toledo and Zohar (2007) study lane change duration using a dataset that provides explicit observations of the beginning and completion of a lane change. However, such information could not be extracted from the study dataset.). However, practical maneuverability aspects including a finite duration for lane change imply that the lane change process in reality begins a few time instances prior to its recorded observation. In this respect, the explanatory variables typically incorporated in the first two decision levels may not be useful in differentiating between successive no lane change observations that precede a lane change observation and the lane change observation itself, thereby failing to explain adequately the occurrence of the lane change observation. Towards this objective, a model framework designed to capture operational level details including the above-described maneuverability aspects would possess better explanatory power in a context where lane change is perceived and modeled as instantaneous. Variables indicative of these operational aspects are considered as candidate options for the above-mentioned vector $X_t$.

**Candidate variables for lane change execution decision:**

In context of the overall model framework, these variables should help explain why a driver may not be observed to execute a lane change in the current time instant, although he targets another lane and finds the relevant adjacent gap acceptable. The reason for this phenomenon, as briefly discussed in the previous chapter, is the time duration of a lane change process, making it encompass several units of the discrete time step adopted in the application dataset. Therefore, the appearance of the lane changing record in the trajectory dataset might be preceded by a significant number of time instances leading back to the instance when the lane change maneuver was initiated. It should be understood that each of these time instances would contain a “no lane change” observation recorded against
them in the dataset, and would each correspond in reality to acceptable adjacent gaps. The number of such observations, depending upon the time resolution adopted, can be represented mathematically using the equation given below:

\[ N_{nlc_{AG}} = (Dur_{LC} / t_{res}) - 1 \]  

(3.2.9)

where \( N_{nlc_{AG}} \) refers to the number of observations involving acceptable adjacent gaps prior to the lane changing observation; \( Dur_{LC} \) refers to the duration of lane change in appropriate units of time, and \( t_{res} \) represents the unit time step in the dataset, expressed in same units as \( Dur_{LC} \). Other components influencing \( N_{nlc_{AG}} \) are not considered in this study.

The third decision level, concerning the execution of the lane change, is intended to capture and explain the variability in the term \( N_{nlc_{AG}} \), measured through \( Dur_{LC} \). For a driver who considers lane change execution, the probability of him executing the lane change in the given time instant and thereby completing it would be low if \( N_{nlc_{AG}} \) is large, i.e., the factors influencing \( Dur_{LC} \) render it long.

As outlined in the previous chapter, Toledo and Zohar (2007) analyze and estimate the influence of a list of driving factors on the duration of a lane change maneuver \( (Dur_{LC} \) in this context). The factors are believed to impact one of two major types of influences that govern lane change duration:

1) Risk associated with lane change, causing driver to exhibit greater caution and execute lane change slowly.

2) Urgency (or its lack) for lane change; a more urgent need for lane change would stimulate quicker driver response and therefore a faster lane change.

The factors considered in that study include subject speed relative to neighboring vehicles, adjacent gap length, traffic conditions in adjacent lane, etc. It is to be noted that some of these variables are already considered for the specification of lane utilities and critical gap means in the first two decision levels. In particular, the risk factor mentioned above is likely to be captured in the critical gap measures. Hence, variables representing the urgency of the lane change are sought.

Among the neighborhood driving conditions that can be expected to influence the urgency of a lane change are those that determine the extra leeway associated with the adjacent gap. When an adjacent gap length is significantly higher than the minimum threshold required by the driver
given current conditions, the gap is considered to offer significant leeway for him. This might in turn encourage some slack on part of the driver in executing the lane change, particularly in the case of a discretionary lane change, since he does not expect an imminent loss of the adjacent gap given such high leeway. A candidate variable for representing this gap leeway would be the difference between the available gap and the mean of the associated critical gap, expressed as $G_{nl}^{ed} - G_{nl,mean}^{ed,cr}$, with $g \in \{\text{lead,lag}\}$, with the terms as defined in eq (3.2.4). Other variables that might characterize the leeway for a lane change (and therefore its urgency) include, among others, the density and average speed of the lane the driver targets moving into.

Also, variables such as subject speed, that are reasonably correlated with some considered by Toledo and Zohar in their regression model and yet not completely similar to those already considered as candidate variables at higher decision-levels, are considered appropriate for inclusion at the lane change execution decision level as an indicator for the lane change duration. Given that all other factors remain equal, a driver is expected to complete a lane change faster if he is traveling at a higher current speed.

In summary, this chapter develops a lane changing model framework and structure as an extension to the previously applied two-level framework (Choudhury, 2005). It acknowledges the significance of the duration of a lane change and the structural limitation resulting from ignoring it. The model extension is developed with an objective of accounting for the lane change duration, and for capturing the various factors that are likely to influence it.

The model framework and structure as described in this chapter was applied to a dataset representing arterial traffic. The details of this application, and the different tasks executed as a part of it are presented in the following chapters. The next chapter describes in detail the dataset used in the application and the data processing tasks undertaken for model estimation and validation purposes. Also developed in the next chapter is the likelihood function as obtained from the proposed model framework and structure, specific to the driver lane changing observations recorded in the application dataset. The following chapter discusses the results of model estimation done using the processed data, while the subsequent chapter presents the model validation process and discusses the associated results.
CHAPTER 4

ARTERIAL DATASET DESCRIPTION

As mentioned before, a significant majority of the research in driver behavior models characterizing lane changing and acceleration actions have focused on applications to freeway traffic, and in particular, congested driving situations. As a consequence, most of the application datasets for these models have tended to represent either freeway mainline and/or merging traffic. The focus of this work has been on generalizing the state-of-art model and its application scope. Traffic in arterial corridors represents a scenario that possesses the characteristics to help test and develop the attempted model generalization. This study develops an extension to the state-of-art hierarchical two-level lane changing model framework previously applied to freeway traffic in Choudhury (2005) and applies the extended version to urban arterial traffic.

This chapter presents a description of the dataset used in this study. It also details the preparation of the data undertaken for subsequent tasks of estimation and validation, including the sampling procedure and subsequent processing operations adopted, and discusses some characteristics of the sample dataset that distinguish it from the dataset used in past studies.

Finally, it develops the likelihood function for the lane changing observations recorded in the dataset, as obtained from application of the extended 3-level model framework to explain the lane changing actions.

The following chapters describe respectively the estimation and validation of the proposed model using the study dataset, and discuss the associated results and their interpretations.

4.1 Dataset description

The study dataset represents traffic on an urban arterial corridor named Lankershim Boulevard, located in Los Angeles, California, U.S.A. It involves observations on individual vehicle trajectories, collected from traffic over a stretch of the arterial in close proximity to the
intersection with the US 101 highway, spanning a half-hour period in total. It was collected as a part of the Federal Highway Administration’s (FHWA) Next Generation Simulation (NGSIM) project on June 16, 2005, between 8:30 and 9:00 AM in the morning.

4.1.1 Data collection procedure
Five video cameras were used to collect the trajectory data. These cameras were mounted on the top of a 36-story building, 10 University Plaza, located adjacent to the U.S. 101 and Lankershim Boulevard interchange. These video cameras covered five distinct spatial regions or zones encompassing the entire arterial stretch. Videos of the traffic were recorded in conjunction by these five cameras, and each vehicle entering the study area was tagged using a unique ID that was maintained consistent across all 5 camera zones. These videos were later parsed and discretized at the resolution of an observation every 1/10th second and transcribed, using software developed exclusively for the NGSIM project, into detailed coordinate values that referred to the positions of different vehicles at different points in time. This temporal sequence of observations on vehicle positions in traffic constituted the individual trajectories of different vehicles. A detailed description of the data collection and processing tasks carried out to convert the captured videos to a dataset containing detailed and disaggregate vehicle trajectory information is available in the “NGSIM Data Analysis Reports”(2006), prepared by Cambridge Systematics, Inc. These reports serve as a basic reference for the description of the study site and dataset as presented in the following sections of this chapter.

4.1.2 Study area description
Figure 4.1 provides an aerial image of the arterial stretch over which data was obtained from the camera coverage. The study site is approximately 1,600 feet in length and is interspersed by four signalized intersections. In the dataset, each stretch of arterial roadway between two adjacent intersections has been referred to as sections. Each section consists of three to four through lanes along each direction. Almost every section is provided with exclusive turning bays, in one or both turning directions, along its approach to an intersection. The exclusive turning bays are meant to facilitate the diverging of turning vehicles from the mainline vehicles. As per the lane
use regulations, vehicles going through an intersection are not permitted to enter these exclusive bays.

**Figure 4.1: Lankershim Boulevard Arterial Section**

Figure 4.2 gives a schematic illustration of the entire arterial segment constituting the study area. It should be noted that this sketch is rotated to an angle of 90 degrees in the clockwise direction with respect to the image shown in figure 4.1. The sketch indicates various geographical details regarding the location and surroundings of the study area. It illustrates the lane configuration within each section, highlighting the presence of the exclusive turning bays in the latter part of most sections. It also provides details regarding the reference indices used for demarcating the origin and destination points (also termed as nodes) and the lanes within every section, as established during the data collection and preparation process. Lane numbering is assigned starting from the left-most lane. Reference indices are also adopted for sections and intersections, in simple numerical increments from 1 to 4 for intersections and 1 to 5 for sections, as indicated in the sketch. These indices are introduced for easing the preparation and interpretation of the dataset in the form of rows of time-specific observations of individual vehicles. Use of such reference indices makes the representation of successive vehicle positions in terms of different...
spatial elements more amenable for database storage in numeric form, and also facilitates subsequent analysis efforts.

Figure 4.2: Schematic Representation of the Arterial Stretch

As mentioned earlier, the dataset was prepared in order to provide detailed time-specific information on the trajectory of every vehicle observed during the specified time interval of data collection. It is in the form of rows and fields, with each row providing details on a vehicle’s status at a given point in time. This status description contains information on instantaneous details including vehicle position (stored and identified in cartesian (x,y) coordinates with reference to a local pre-specifed origin), IDs of section/intersection and lane where the vehicle is currently located, vehicle speed and acceleration at current time instant, etc. A detailed description of the trajectory dataset and the different fields provided within, following the data preparation tasks, is available in Cambridge Systematics’ data analysis reports (2006). The following section presents an overview of the dataset as drawn from these reports. A description is then provided of the sampling procedure and other processing steps carried out for the purpose of preparing data for model estimation. Characteristics of the estimation dataset obtained from the sampling task are then presented. Aspects that distinguish it from the freeway dataset considered in Choudhury (2005) and render it ideal for the current work are also discussed.

4.1.3 Dataset overview
The dataset contains detailed trajectory information of 2,442 vehicles in total, observed within the study region over a 32-minute period stretching from 8:28 am to 9:00 am. The processed dataset presents this information in two parts, the first part encompassing vehicles observed in the first 17-minutes from 8:28 a.m. to 8:45 a.m., and the second part for vehicles observed between 8:45 a.m. and 9:00 a.m. A significant proportion of these vehicles (97%) were automobiles, as can be seen from the vehicle distribution tables (Tables 4.1 and 4.2) presented below.
This preliminary statistic effectively implies that a heterogenous vehicle mix may not hold a significant influence on vehicle behavior within this dataset.

As shown in the schematic presented in the previous section (Figure 4.2), there are a total of 11 origins and 10 destinations demarcating the various entry and exit points along the perimeter of the study area. The configurations of the lanes within the sections restrict the number of origin-destination pairs that can be physically traversed by a vehicle within the study area to 64. Since each vehicle was individually tracked from the point of its entry into the study area to the point of its exit, the dataset also possesses information on the origin and destination of each vehicle. The distribution of vehicles across different origin-destination pairs over the observation time interval is presented in the appendix (Tables C.1 and C.2).
Based on the O-D pair defining their travel path and the corresponding turning movement that carries implications on their lane-changing behavior in traffic, vehicles can be classified into three distinct types:

1) Through vehicles that enter and exit the study area along the arterial,
2) Vehicles that enter the arterial from a side street within the study area and exit the study area along the arterial,
3) Vehicles that exit the arterial through one of the side streets within the study area.

Vehicles belonging to the first category don’t have any mandatory turning movements to consider. They are hence expected to be involved only in discretionary lane changing and lane pre-positioning activities that place them in a comfortable driving situation when faced with traffic merging from and diverging into the side streets. Vehicles in the second category are expected to undertake lane changing in an effort to attain a desirable position in the mainline traffic. Vehicles in the third category would be expected to make the greatest number of lane changes as they move towards the exclusive turning bays to make their desired exit. The number of vehicles belonging to each category in the overall dataset is presented in table 4.3 elaborated in section no. 4.1.5.

The original videos of the observed traffic were made available, and they were used to draw qualitative inferences on the traffic characteristics and its applicability for the current study. As described earlier, the key aspect sought in the application dataset for this study is low to moderate density in traffic, which would help test the validity of the extension proposed in the lane-changing model framework. The degree of congestion in the arterial traffic as observed from the videos was found to be moderate. The average speed for each vehicle, as calculated from the dataset, was found to range between 6 m/s and 20 m/s. Videos revealed queues that built up during red signal phases. However, all queues that evolved in such a manner were entirely dissipated during the subsequent green signal phases. The lane changes were through normal gap acceptance and no forced merge was apparent from the videos.

A significant number of lane-changes were observed to occur into open gaps involving no lead or lag vehicle. This very basic aspect of the study dataset differentiates it from the freeway
dataset used in the past studies, and suggests that this dataset might have very good potential to effectively test the model extension proposed.

4.1.4 Sampling and data processing
The original dataset consisting of 2,442 vehicles and 1,607,319 time instance observations proved computationally burdensome for the model estimation task. It especially hindered its iterative nature (since estimation had to be repeated till the best specification was obtained). Hence, it was important to sample out a representative part of the entire dataset to make the estimation process tractable. Vehicles were randomly selected at the rate of one per every five, with the objective of establishing a representative dataset for model estimation. This sampling step yielded a subset of the overall dataset consisting of 438 vehicles. It should be noted here that all variables representing aggregate traffic conditions (eg: average speed, density, queue length, etc.) and neighborhood conditions (eg: lead and lag vehicles statistics) as included in the model specifications that were estimated were extracted from the original unsampled dataset and used accordingly in the sampled version.

A multi-phase data processing task was then applied on the sampled dataset to prepare it for estimation purpose.

At this point, it must be noted that the model developed in this study is applicable only to vehicle behavior in mainline traffic. The study region encompassed by the observation cameras include the side streets connecting to the mainstream arterial (refer to figure 4.2). The dataset involves a continuous observation of vehicle trajectories from the point of entry into the study region up to its exit from the same. It therefore contains observations of vehicles at the side streets (prior to its entry into the mainline arterial or post its exit from it) and within the intersections. It is hypothesized that vehicle behavior in either of these two regions are characterized by complexities beyond the scope of the model presented in the current work. Vehicle behavior at the entry points of an arterial, including the selection of the lane on the mainstream arterial into which they move in directly from the side street, have been studied and modeled using another exclusive modeling framework. Details of this model framework, its application to the vehicle
behavior at intersections of the study dataset and the associated results can be obtained in Choudhury (2007). It needs to be mentioned here that the lane changing model for mainline traffic and the lane selection model for intersections are required in conjunction to explain and replicate the overall driver lane changing behavior in the arterial stretch. The focus of this study is however on the lane changing model for mainline traffic. Its use for driving simulation and forecasting, as elaborated in a future section on validation, involves joint implementation and operation with the intersection lane-selection model.

For the purpose of estimating the three-level lane-changing model framework developed in this study, it was required to eliminate the observations of vehicles at side-streets and intersections. The sampled dataset consisting of 438 vehicles, as inherited from the parent dataset, included observations of vehicles along the side streets and intersections, which, as per the model estimation requisites described above, had to be removed. This task constituted the first phase of data processing.

On further analysis and cross-referrals using the available traffic videos, the first-stage processed sample was revealed to possess some observations that were still not amenable for estimation purposes. Some of these observations were pure faulty ones that failed to conform with video observations for the corresponding time instant. Examples of such observations were those that wrongly indicated the lane or section IDs of a vehicle at a given time instant.

Other undesirable observations originated from non-conformant, uncommon behavior of drivers. Examples included observations of through vehicles (defined, by virtue of their paths, as those continuing along the arterial through an intersection) on exclusive turning bays, which contradicted the underlying lane use regulations. Since these regulations were also incorporated in defining the target lane choice set (eq 3.2.2) for the through vehicles, the model would find it difficult to explain the lane change of a through vehicle into a turning bay. A second stage of processing was therefore required to weed out these observations.
The final stage of processing was executed in the time dimension, at the rate of 1 per every 10 time instance observations of every sampled vehicle. As the original dataset was at one-tenth of a second time resolution, this processing step yielded data at one-second time resolution.

The final dataset obtained following the above-described sampling and processing operations consisted of 400 vehicles over a total of 16696 time instance observations. The characteristics of this sample dataset, which was used for estimation, are described in the following section.

### 4.1.5 Estimation dataset characteristics

Out of the 400 vehicles in the estimation sample, 160 vehicles were northbound, while 240 vehicles were southbound. The average vehicle observation duration was 51.3 seconds, with the maximum duration of observation being 170 seconds.

**Vehicle distribution:**

As described in the previous section, the dataset consists of vehicles across three categories, classified based on their mandatory turning movements as defined by the OD pair they travel across. The distribution of vehicles across these categories in the estimation sample and overall dataset is presented in the table 4.3 below.

| Table 4.3: Distribution of vehicles across categories based on their turning movements as defined by their O-D |
|---|---|---|---|
| Dataset | Through Vehicles | Vehicles turning into arterial | Vehicles turning off the arterial |
| Overall (2442 vehicles) | 864 (35.4%) | 578 (23.67%) | 1000 (40.95%) |
| Estimation Sample (400 vehicles) | 162 (40.5%) | 88 (22%) | 150 (37.5%) |
Aggregate lane-specific statistics:
Disaggregate details on each vehicle’s instantaneous status (e.g., lane ID, section ID, speed, position, time of observation, etc.) were aggregated and merged to obtain statistics on aggregate traffic characteristics. These statistics include lane-specific elements like average speed, density, queue length, etc. These statistics as observed by vehicles in the estimation sample are summarized in Table 4.4 below.

For data analysis and estimation purposes, lanes in the sections have been categorized into 6 types on the basis of permitted vehicular movements. Lane type 1 signifies a shared through flow, right- and left-turn lane; lane type 2 denotes a shared through and right-turn lane; lane type 3 denotes a shared through and left-turn lane; lane type 4 denotes an exclusive right turn bay; lane type 5 denotes an exclusive left-turn bay; and lane type 6 denotes an extra turn bay that is adjacent to another and to the interior of the section (as was observed within the study area).

The presence of turning vehicles, and the conflicts arising due to their movements in conjunction with through vehicles provide a reasonable explanation for the low average speeds observed in both the through and the turning lanes. The maximum queue length values are observed during red intervals at traffic signals. The presence of the exclusive turn bays, their lane use regulations and their relevance as an ultimate target lane for turning vehicles are key issues to be considered while developing the model specification.

Table 4.4: Aggregate Lane-Specific statistics by lane categories for sampled vehicles

<table>
<thead>
<tr>
<th>Lane Type</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Speed (m/s), among all vehicles</td>
<td>10.32</td>
<td>8.67</td>
<td>18.43</td>
<td>13.93</td>
<td>6.5</td>
</tr>
<tr>
<td>Average Queue Length (m), among all vehicles</td>
<td>1.071</td>
<td>1.93</td>
<td>0.182</td>
<td>1.44</td>
<td>2.085</td>
</tr>
<tr>
<td>Max Queue Length (m), among all vehicles</td>
<td>15</td>
<td>12</td>
<td>7</td>
<td>18</td>
<td>11</td>
</tr>
</tbody>
</table>

Note: No lane belonging to lane type 3 exists within the study area.

Lane changing statistics:
In the estimation sample, there were a total of 249 lane changes observed. Of these, 104 (41.8%) were made by turning vehicles, i.e., those belonging to the third category as defined in Table 4.3.
The table below (Table 4.5) shows the distribution of these lane changes by the distance from their exit point, in terms of number of sections, at which they occur.
Table 4.5: Distribution of locations of lane-change points for turning vehicles by the number of sections before the exit point

<table>
<thead>
<tr>
<th>Number of Sections from Exit Point</th>
<th>Number of Lane Changes (by Turning Vehicles)</th>
<th>Percentage of the Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Last Section</td>
<td>84</td>
<td>80.8</td>
</tr>
<tr>
<td>One Section from Exit</td>
<td>20</td>
<td>19.2</td>
</tr>
<tr>
<td>Two Sections from Exit</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>104</td>
<td>100</td>
</tr>
</tbody>
</table>

The statistics reveal a high proportion of lane changes that occur in the section immediately prior to the vehicle’s exit, implying the dominance of path plan considerations in the lane-changing decisions of turning vehicles.

Adjacent gaps: Observations with no lead or lag vehicles:
The most significant aspect of the arterial dataset was the prominence of observations in which adjacent gaps were characterized by the absence of either the lead or lag vehicle. In this study, it is assumed that a lead (or lag) vehicle in the adjacent lane influences the adjacent lead (or lag) gap only if it exists in the same section as the subject driver. Therefore, the observations with no lead (or lag) vehicle reflect instances where the subject vehicle faces no lead (or lag) vehicle in the adjacent lane within the section on which it currently moves.

The estimation sample inherited a significant proportion of such observations from the overall dataset. This aspect of the sample is summarized through the numbers presented in the table below.

Table 4.6: Summary statistics on number of vehicle observations without lead/lag vehicle

<table>
<thead>
<tr>
<th></th>
<th>All Observations</th>
<th>Lane-Changing Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>% (Total = 16696)</td>
</tr>
<tr>
<td>Lead Vehicle Absent</td>
<td>3749</td>
<td>22.45</td>
</tr>
<tr>
<td>Lag Vehicle Absent</td>
<td>3811</td>
<td>22.83</td>
</tr>
</tbody>
</table>
As can be inferred from these statistics, an inordinately large number of observations and lane changes don’t involve a lead or lag vehicle. To reiterate, lead and lag vehicles for a given subject vehicle contemplating a lane change are as defined in figure 3.3. The existence of the above observations (no lead or lag vehicle) is attributed to two factors:

a. The effect of signal operations at the intersections and the resulting stop-and-go nature of traffic,

b. The relatively lower density of traffic in the arterial stretch.

*Gap-acceptance for observations with no lead or lag vehicle – Pseudo Gap correction:*

To accommodate such observations as described above for the gap acceptance decision in the model framework, a pseudo gap length is assumed for lead or lag gaps whenever a lead or lag vehicle is respectively absent. It is hypothesized that the lead and lag gap lengths considered by the drivers in such instances are the distances from the nearest intersection boundaries lying within either gap. This approach represents a reasonable assumption given that traffic regulations do not allow vehicles to make a lane change within an intersection. As an illustrative example, consider vehicle A in a traffic situation depicted in figure 4.3 below (and magnified in figure 4.4).

**Figure 4.3: Traffic situation in an arterial section, with no lead or lag vehicle in left adjacent lane within the same section of the subject vehicle A**
Vehicle A’s nearest lead and lag vehicles in the left adjacent lane (B and C) don’t lie in the same section as A itself. Intersection 1 lies between the vehicle and its left lag (C), while intersection 2 lies between it and its left lead (B). Here, the pseudo lag gap is the clear spacing between the rear of vehicle A and the front edge of intersection 1, while the pseudo lead gap is the clear spacing between the front of vehicle A and rear edge of intersection 2. These pseudo gap measures serve as surrogates to the lead and lag gap lengths in the gap acceptance model. The use of the pseudo gap measure in driver’s evaluation of the lead gap makes behavioral sense, since the lane change has to be executed by vehicle A within its pseudo gap before it enters the intersection. From the perspective of gap acceptance model, such an approach is equivalent to assuming a pseudo lead vehicle for A that is stopped at the rear edge of intersection 2. The explanatory variables for the lead gaps (including relative speed of lead vehicle) under such instances are defined and calculated adopting this equivalent situation.

However, the definition of the pseudo gap does not hold any strong behavioral significance for lag gap evaluation. Therefore, a lag gap is by default assumed to be acceptable when there exists no lag vehicle within the subject’s section.
Comparison with I-395 freeway dataset (Choudhury, 2005):
The presence of the unique observations as described above motivates an interesting angle of comparison with the freeway dataset used in Choudhury (2005). As described earlier, the key aspect sought in the application dataset for this study is low to moderate density in traffic with large gap sizes, in contrast to the dataset on freeway traffic used in Toledo (2003) and Choudhury (2005). The freeway dataset contained vehicle trajectory observations collected by FHWA from southbound traffic on the I-395 in Airlington, VA. Details on traffic characteristics in this dataset are presented in Toledo (2003) and Choudhury (2005). The adjacent gap statistics of that dataset are referred here for comparison with the arterial dataset used for estimation in this study. The gap statistics for the freeway and arterial dataset are presented in tables 4.7(a) and 4.7(b) respectively. It should be noted that the statistics for the arterial dataset involve the use of pseudo gap lengths as described above for observations involving no lead vehicles. Observations with no lag vehicles are excluded from the lag gap statistics.

Table 4.7: Gap Statistics for Freeway and Arterial dataset (a – b)
(a): Gap Statistics for Freeway dataset

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std</th>
<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relations with Lead Vehicle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative speed (m/s)</td>
<td>0.2</td>
<td>2.6</td>
<td>0.5</td>
<td>-17.3</td>
<td>8.1</td>
</tr>
<tr>
<td>(0.0)</td>
<td>(2.9)</td>
<td>(0.1)</td>
<td>(-17.5)</td>
<td>(15.5)</td>
<td></td>
</tr>
<tr>
<td>Lead Spacing (m)</td>
<td>22.2</td>
<td>21.9</td>
<td>14.1</td>
<td>0.04</td>
<td>117.9</td>
</tr>
<tr>
<td>(19.6)</td>
<td>(39.9)</td>
<td>(13.0)</td>
<td>(-18.1)</td>
<td>(268.9)</td>
<td></td>
</tr>
<tr>
<td>Relations with Lag Vehicle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative speed (m/s)</td>
<td>-0.4</td>
<td>2.2</td>
<td>-0.3</td>
<td>-6.7</td>
<td>5.2</td>
</tr>
<tr>
<td>(0.0)</td>
<td>(2.7)</td>
<td>(0.0)</td>
<td>(-15.0)</td>
<td>(14.1)</td>
<td></td>
</tr>
<tr>
<td>Lag Spacing (m)</td>
<td>23.1</td>
<td>20.6</td>
<td>16.6</td>
<td>1.7</td>
<td>110.1</td>
</tr>
<tr>
<td>(18.6)</td>
<td>(23.0)</td>
<td>(12.0)</td>
<td>(-18.1)</td>
<td>(232.6)</td>
<td></td>
</tr>
</tbody>
</table>
(b): Gap Statistics for Arterial Estimation dataset

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std</th>
<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Relations with Lead Vehicle</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Speed (m/s)</td>
<td>-7.98</td>
<td>6.0</td>
<td>-8.98</td>
<td>-16.31</td>
<td>3.50</td>
</tr>
<tr>
<td></td>
<td>(-4.49)</td>
<td>(6.23)</td>
<td>(-2.07)</td>
<td>(-16.39)</td>
<td>(15.92)</td>
</tr>
<tr>
<td>Lead Spacing (m)</td>
<td>33.75</td>
<td>40.05</td>
<td>19.94</td>
<td>0.01</td>
<td>158.24</td>
</tr>
<tr>
<td></td>
<td>(34.11)</td>
<td>(41.73)</td>
<td>(13.73)</td>
<td>(-6.15)</td>
<td>(170.1)</td>
</tr>
<tr>
<td><strong>Relations with Lag Vehicle</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative speed (m/s)</td>
<td>-0.93</td>
<td>3.90</td>
<td>-0.9</td>
<td>-15.25</td>
<td>7.3</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(3.65)</td>
<td>(0.3)</td>
<td>(-15.73)</td>
<td>(15.62)</td>
</tr>
<tr>
<td>Lag Spacing (m)</td>
<td>9.18</td>
<td>23.47</td>
<td>10.1</td>
<td>0.75</td>
<td>128.5</td>
</tr>
<tr>
<td></td>
<td>(3.51)</td>
<td>(20.24)</td>
<td>(4.2)</td>
<td>(0.00001)</td>
<td>(152.28)</td>
</tr>
</tbody>
</table>

* Statistics in parantheses are for the entire dataset, others are for accepted gaps.

Some of the above statistics are along expected lines, adhering to our hypothesis on gap acceptance behavior. In case of the freeway dataset, average accepted lead and lag gaps both exceed the average over the entire dataset, implying a larger gap length is more likely to be accepted for a lane change. The lead relative speed for accepted gaps exceed the overall average while the lag relative speed for accepted gaps are lower than the overall average. These are also along expected lines, as higher lead relative speeds and lower lag relative speeds imply increasing gap lengths for the corresponding cases and therefore safer lane change execution.

For the arterial dataset, the lag gap statistics are along similar lines (especially since they don’t contain the instances with no lag vehicles). However, the pseudo gap correction for the lead gaps result in some marked changes to the lead gap statistics, which when compared with the corresponding freeway statistics provide two significant inferences:

1) The available lead gaps are of typically higher value in the arterial dataset, indicating that density in this dataset is significantly lower, as required of the study intended to be done in this work, since this statistic implies higher lane change duration.
2) The sample distribution of the accepted lead gaps has similar means and standard deviations to that of the available gaps in the arterial dataset, indicating a strong possibility for existence of higher lengths of available lead gaps prior to the instance when lane change was completed. This is especially likely to be the case when a vehicle changes lane into a gap that has no lead vehicle.

As discussed in an earlier section, the second implication is one phenomenon that cannot be explained by the 2-level model structure used in Choudhury (2005). Its presence in the arterial dataset hence supplies a strong case for use of the model extension.

Another notable anomaly for the arterial dataset is that the average lead relative speed for accepted lead gaps are lower than the average lead relative speed for all available gaps, which is contrary to expectation. This contradiction is partially caused by the pseudo-gap correction, where lane changes made with no lead vehicle actually record a negative lead relative speed due to the abstraction of a pseudo stationery lead vehicle. On recalculation without adopting the pseudo gap correction, the average lead relative speed for accepted lead gaps is found to be -2.05 m/s, which is still lower than the average lead relative speed for all gaps which is calculated as 0.30 m/s. This peculiar phenomenon is attributed to the distinctive features of the dataset. It is reasoned that when faced with large lead gaps, drivers in this dataset tend to take much longer to execute a lane change, and complete it only when they come close to a lead vehicle. The instantaneous lead relative speed recorded at the instance of completion of such a prolonged lane change may thus prove misleading, as revealed by the measured statistic. This phenomenon offers another case for the proposed extension.

Another aspect distinguishing the arterial from the freeway dataset is that of average vehicle speeds. The lane-specific average speeds as presented in table 4.4 are reproduced here in table 4.8 in comparison to the equivalent figures reported in Choudhury (2005) for the dataset on traffic over I-395, which comprised of 4-lanes.
Table 4.8: Lane-specific average speeds for Arterial (current study) and Freeway (Choudhury, 2005) dataset

<table>
<thead>
<tr>
<th>Arterial dataset</th>
<th>Lane type 1</th>
<th>Lane type 2</th>
<th>Lane type 3</th>
<th>Lane type 4</th>
<th>Lane type 5</th>
<th>Lane type 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freeway dataset</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lane 1</td>
<td>10.32</td>
<td>14.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lane 2</td>
<td>8.67</td>
<td>15.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lane 3</td>
<td>18.43</td>
<td>16.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lane 4</td>
<td>13.93</td>
<td>17.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lane 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.5</td>
</tr>
<tr>
<td>Lane 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As indicated from these statistics, the average speed in the arterial dataset is in general lower than that in the freeway dataset. As per the hypothesis, this might also be responsible for longer lane change durations in the arterial, causing greater inconsistencies in instantaneous gap statistics, and providing further motivation for the current study.

In summary, modeling instantaneous gap acceptance as the final decision step leading to a lane change action, as done in the 2-level model, is likely to give erroneous parameter estimates under the presence of the above-described situations that result in somewhat misleading gap statistics.

### 4.2 Likelihood function

This section develops the likelihood function associated with observed lane-changing actions in the dataset, based on the framework and structure of the extended 3-level model presented in the previous chapter.

If \( l_{nt} \) denotes the lane-changing action observed of driver \( n \) at time \( t \) \((l_{nt} \in \{1, -1, 0\})\), as defined in eq (3.2.7)), its probability, conditioned on the latent individual-specific error term \( v_n \), integrates that of the choices made at each of the three levels in the driver’s decision tree, and is given by the joint probability expression below:
\[ P(l_n | v_n) = \sum_{j \in TL} P(TL^{j}_{nt}, AG_{nt}, l_n | v_n) \]  

(4.2.1)

where \( TL^{j}_{nt} \) denotes the event of driver \( n \) selecting lane \( j \) as the target lane during time instant \( t \), with \( TL \) denoting the choice set for target lanes, and the term \( AG_{nt} \) is as defined in eq (3.2.5).

Multiplicative rule is then employed to break down the joint probability term in (4.2.1) into individual components representing the conditional probabilities for choices made at each of the decision levels of the model framework.

\[ P(TL^{j}_{nt}, l_n, AG_{nt} | v_n) = P(l_n | AG_{nt}, v_n) P(AG_{nt} | d_{nt}, v_n) P(TL^{j}_{nt} | v_n) \]  

(4.2.2)

where,

- \( d_{nt} \) is the direction of lane change for driver \( n \) at time \( t \) as chosen based on the choice of target lane \( j \);
- \( P(TL^{j}_{nt} | v_n) \) is given by equation (3.2.2), and represents the probability for a driver \( n \) selecting lane \( j \) as the target lane at time \( t \);
- \( P(AG_{nt} | d_{nt}, v_n) \) is given by equation (3.2.5), and represents the probability of the driver \( n \) accepting the relevant adjacent gap in direction \( d_{nt} \) conditioned on his choice of target lane; and
- \( P(l_n | AG_{nt}, v_n) \) is given by equation (3.2.8), and represents the probability of driver \( n \) executing a lane change in the current time instant, conditioned on the fact that he is looking for a lane change and finds the adjacent gap acceptable.

Given a driver \( n \) and his actions are observed over a succession of \( T_n \) time instances, and going by the assumption that these actions, conditioned on the his latent individual characteristics and chosen explanatory variables, are independent of each other, the joint conditional probability for his entire sequence of observations, termed \( l_n \), is given by:

\[ P(l_n | v_n) = \prod_{t=1}^{T_n} \sum_{j \in TL} P(TL^{j}_{nt}, l_n, AG_{nt} | v_n) \]  

(4.2.3)

The assumption of conditional independence of successive observations of an individual’s actions, given his/her characteristics, extends from the original motivation behind introducing
the latent driver-specific error term \( v_n \), which was to capture the serial correlations among the observations of the same individual. This assumption also implies that the current approach ignores the effect of any other sources of correlation that may exist between successive driver observations.

As stated in an earlier section, the individual-specific characteristic \( v_n \) has typically been assumed to follow a normal distribution over the entire population of drivers in past works, and the current approach adheres to that assumption. The unconditional likelihood \( L_n \) for the sequence of observations \( l_n \) that characterize the driving trajectory of individual \( n \) can therefore be obtained by integrating the conditional probability term in eq (4.2.3) over the distributional space of \( v_n \).

\[
L_n = P(l_n) = \int P(l_n \mid v) f(v) dv
\]  

Another assumption commonly made in past approaches, and also adopted in this work, is the independence among actions of different drivers, given the chosen explanatory variables. Under such an assumption, the likelihood of observing the complete set of observations (\( L \)) in a detailed disaggregate vehicle trajectory dataset involving multiple drivers (say \( N \) in number), can be given as a product of the unconditional likelihoods of every individual driver expressed in equation (4.2.4).

\[
L = \prod_{n=1}^{N} L_n
\]  

The log-likelihood function for an entire dataset of observations involving \( N \) individuals can hence be given as:

\[
LL = \sum_{n=1}^{N} \ln(L_n)
\]  

The next chapter details the implementation of this likelihood function for model estimation using the sample dataset prepared for the same. Estimation results are presented and their interpretations are discussed.
CHAPTER 5

MODEL ESTIMATION

The proposed 3-level lane changing model as presented in the model formulation chapter was estimated using the sampled dataset described in the previous chapter. As the 3-level model is an extension to the state-of-art 2-level lane changing model, its estimation results are compared with those of the original model as obtained on estimation with the same sample. This comparative analysis is expected to justify the superior ability of the extended model in explaining lane changing behavior in low density traffic, and in particular, under the presence of observations without lead or lag vehicles.

Model estimation is an iterative process, with the specifications being appropriately modified based on intermediate estimation results till the best model specification is arrived at. The estimation results discussed below for the 3-level and 2-level models are from their best specifications. All instantaneous explanatory variables for these specifications were extracted from the sample dataset through scripts written in MATLAB. Estimation for both models was done using GAUSS software. Details of the estimation procedure adopted are presented in the following section.

5.1 Estimation approach

The log-likelihood function, as expressed in eq (4.2.6), is maximized to jointly estimate all the parameters specified over the three levels of the proposed model framework. In this case-study, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimization algorithm implemented in the statistical estimation software GAUSS, Version 3.6 (Aptech Systems 2003) has been used for estimating the entire model. It is noted here that BFGS is a quasi-Newton method, which maintains and updates an approximation of the Hessian matrix based on first-order derivative information (see, for example, Bertsekas 1999). GAUSS implements
a variant of BFGS attributed to Gill and Murray (1972), which updates the Cholesky decomposition of the Hessian (Aptech Systems 2005). For further information on the optimization algorithms provided by GAUSS for likelihood maximization purposes, the reader is referred to the GAUSS user manual (Version 5.0) and related documents.

The integration of each individual likelihood function \( l_n \) over the individual-specific characteristic \( z_i \) is performed using the numerical integration technique “Legendre quadrature method” in GAUSS.

Another important implementation-related note mentioned here is that the likelihood function of eq(4.2.6) is not globally concave. It owes this property specifically to the use of the random driver-specific error term \( v_n \), which during implementation is assumed to follow a normal distribution. As an illustrative example, if the signs of all of the coefficients of the individual-specific error term were reversed, the log-likelihood value would remain unchanged due to the symmetric nature of the normal distribution about its mean. This unique property directly implies that there exist multiple solutions of parameter estimates that result in the same optimal likelihood function value, confirming the concavity of the likelihood function. To avoid obtaining a local solution under this situation, different starting points need to be used in the optimization procedure.

The remainder of this chapter is presented in three sections. The first section presents and analyzes the estimation results for the 3-level model. The second section presents the estimation results for the 2-level model, obtained on the same estimation dataset. The final section compares the estimation results of the above two models in order to better understand the improvement in modeling capabilities that is achieved by the proposed model extension.
5.2 Extended 3-level lane-changing model

Table 5.1 presents the estimates and t-statistics obtained for the parameters used in the best specification of the 3-level model. It also provides the final log-likelihood value for these set of estimates. The final log-likelihood is the maximum value of the log-likelihood as obtained at the convergence of the maximum likelihood algorithm used for estimation. It represents a quantitative measure of the fit of the model to the empirical data used for estimation. A higher log-likelihood signifies an improved model fit. This measure can therefore be used for comparing the ability of different models to explain the sequence of actions observed in the estimation dataset.

A description of the explanatory variables associated with the specified parameters is presented next in table 5.2. It is followed by the expressions for the specifications and analysis of the estimation results for each decision level of the model framework.

5.2.1 Estimation results
Table 5.1: 3-level lane-changing model estimation results

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Parameter</th>
<th>t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level 1: Target-Lane Selection Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current-Lane Dummy</td>
<td>1.43</td>
<td>1.16</td>
</tr>
<tr>
<td>Path Plan impact: No. of lane changes to exit lane</td>
<td>-0.729</td>
<td>-1.015</td>
</tr>
<tr>
<td>Path plan impact: No. of lane changes to exit interacted with distance from exit (m&lt;sup&gt;-1.33&lt;/sup&gt;)</td>
<td>-2.20</td>
<td>-2.33</td>
</tr>
<tr>
<td>Exponent of dist. to exit in no. of lanes to exit- dist. to exit interaction</td>
<td>0.289</td>
<td>0.685</td>
</tr>
<tr>
<td>No. of lane changes from current lane &lt;= 3</td>
<td>-2.42</td>
<td>-2.047</td>
</tr>
<tr>
<td>No. of lane changes from current lane &gt;= 4</td>
<td>-5.68</td>
<td>-0.006</td>
</tr>
<tr>
<td>Queue length ahead in lane (number of vehicles)</td>
<td>-0.352</td>
<td>-4.11</td>
</tr>
<tr>
<td>Front vehicle rel. speed negative, interacted with front veh. Gap (m/s per m)</td>
<td>0.0378</td>
<td>0.742</td>
</tr>
<tr>
<td>σ&lt;sup&gt;CL&lt;/sup&gt;</td>
<td>-0.449</td>
<td>-2.91</td>
</tr>
<tr>
<td><strong>Level 2: Gap Acceptance Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead Critical Gap</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead gap constant</td>
<td>2.31</td>
<td>71.29</td>
</tr>
<tr>
<td>ΔV&lt;sub&gt;lead,TL&lt;/sub&gt; (m/s)</td>
<td>-0.0482</td>
<td>-7.81</td>
</tr>
<tr>
<td>σ&lt;sub&gt; lead&lt;/sub&gt;</td>
<td>0.00745</td>
<td>0.225</td>
</tr>
<tr>
<td>σ&lt;sup&gt; lead&lt;/sup&gt;</td>
<td>-1.78</td>
<td>-104.69</td>
</tr>
<tr>
<td>Lag Critical Gap</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lag gap constant</td>
<td>1.51</td>
<td>82.72</td>
</tr>
<tr>
<td>ΔV&lt;sub&gt;lag,TL&lt;/sub&gt; (m/s)</td>
<td>0.0314</td>
<td>8.0</td>
</tr>
<tr>
<td>σ&lt;sub&gt; lag&lt;/sub&gt;</td>
<td>0.0085</td>
<td>0.242</td>
</tr>
<tr>
<td>σ&lt;sup&gt; lag&lt;/sup&gt;</td>
<td>-1.74</td>
<td>-47.61</td>
</tr>
<tr>
<td><strong>Level 3: Execution Decision Level</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.52</td>
<td>-2.97</td>
</tr>
<tr>
<td>Gap Leeway (m)</td>
<td>-0.308</td>
<td>-1.63</td>
</tr>
<tr>
<td>Lead Vehicle Absent</td>
<td>-0.88</td>
<td>-1.1</td>
</tr>
<tr>
<td>V&lt;sub&gt;ext&lt;/sub&gt; (m/s)</td>
<td>0.60</td>
<td>5.36</td>
</tr>
</tbody>
</table>
### 5.2.2 Variable Definitions

#### Table 5.2: 3-level lane-changing model variable definitions

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current-Lane Dummy</td>
<td>1 if the lane is the current lane of the driver, 0 otherwise</td>
</tr>
<tr>
<td>Path Plan impact: No. of lane changes to exit lane</td>
<td>Number of lane changes the driver has to make from the target lane in order to follow his path (to take the turn/exit)</td>
</tr>
<tr>
<td>Path plan impact: No. of lane changes to exit interacted with distance from exit</td>
<td>Number of lane changes the driver has to make from the target lane in order to follow his path (to take the turn/exit)(^*) (remaining longitudinal distance to the turn/exit) raised to an estimated exponent</td>
</tr>
<tr>
<td>Exponent of dist. to exit in no. of lanes to exit- dist. to exit interaction</td>
<td>The exponent of the remaining longitudinal distance in the preceding variable</td>
</tr>
<tr>
<td>Number of lane changes from current lane &lt;= 3</td>
<td>Dummy variable. One if Number of lane changes required to reach the target lane from the current lane of the driver &lt;=3, zero otherwise</td>
</tr>
<tr>
<td>Number of lane changes from current lane &gt;= 4</td>
<td>Number of lane changes required to reach the target lane from the current lane of the driver if the number of lane changes required to reach the target lane&gt;=4, 0 otherwise</td>
</tr>
<tr>
<td>Queue length ahead in lane</td>
<td>Number of vehicles ahead in target lane</td>
</tr>
<tr>
<td>Front vehicle rel. speed negative, interacted with front veh. Gap (m/s per m)</td>
<td>Interaction of relative speed of front vehicle in the current lane with the available front spacing (Reference Equation 5.2.1)</td>
</tr>
<tr>
<td>(\sigma_{CL}^2)</td>
<td>Heterogeneity term for inertia</td>
</tr>
<tr>
<td>Lead gap constant</td>
<td>Constant in mean of lead critical gap function</td>
</tr>
<tr>
<td>(\Delta V_{ni}^{lead,TL} ) (m/s)</td>
<td>Relative speed difference with lead vehicle (Lead vehicle speed - subject vehicle speed)</td>
</tr>
<tr>
<td>(\sigma_{lead})</td>
<td>Standard deviation of lead critical gap</td>
</tr>
<tr>
<td>(\phi_{lead})</td>
<td>Heterogeneity term for mean of lead critical gap</td>
</tr>
<tr>
<td>Lag gap constant</td>
<td>Constant in mean of lag critical gap function</td>
</tr>
<tr>
<td>(\Delta V_{ni}^{lag,TL} ) (m/s)</td>
<td>Relative speed difference with lag vehicle (Lag vehicle speed - subject vehicle speed)</td>
</tr>
<tr>
<td>(\sigma_{lag})</td>
<td>Standard deviation of lag critical gap</td>
</tr>
<tr>
<td>(\phi_{lag})</td>
<td>Heterogeneity term for mean of lag critical gap</td>
</tr>
<tr>
<td>Intercept</td>
<td>Intercept of execution level</td>
</tr>
</tbody>
</table>
| Gap Leeway (m) | (Difference between available and mean critical lead gap, if lead vehicle present, 0 otherwise) +
{ Difference between available and mean critical lag gap, if lag vehicle present, 0 otherwise} |
| Lead Vehicle Absent | 1 if no lead vehicle in current section; 0 otherwise |
| \(V_{nt} \) (m/s) | Current Speed of subject vehicle |
5.2.3 Model specification and results analysis

Target-Lane Model

The expression for the systematic utility of lane i at time t as considered by driver n in the target lane model can be expressed as follows:

\[ V_{nt} = 1.43 \delta_{nt}^i - 0.729 e_{nt}^i - 2.20 e_{nt}^i (d_{nt}^{exit})^{0.289} - 2.42 (k_{nt}^i \leq 3)k_{nt}^i \]
\[ - 5.68 (k_{nt}^i > 4)k_{nt}^i - 0.449 d_{nt}^i \delta_{nt}^i - 0.352 (q_{nt}^i) (q_{nt}^i \leq 3) \]
\[ - 1.056 (q_{nt}^i > 4) + (\delta_{nt}^i)^* \left[ 0.0378 \min(0, f_{nt}^c) / (1 + \exp (f_{nt}^c)) \right] \]

Where, \( \delta_{nt}^i \) = current lane dummy, 1 if lane i is current lane for vehicle n at time t, 0 o/w
\( q_{nt}^i \) = queue ahead in lane i
\( k_{nt}^i \) = number of lane changes required from current lane to lane i (\( i-j_{nt}^i \))
where \( j_{nt} \) = current lane of individual n at time t

Where, \( e_{nt}^i \) = number of lane changes reqd. to take desired exit/turn from lane i
\( d_{nt}^{exit} \) = remaining dist. to exit/turn (m)
\( f_{nt}^c \) = front vehicle relative speed (m/s)
\( f_{nt}^e \) = front vehicle speed (m/s)
\( \nu_n \) = latent error term specific to individual n

The target lane utility specification contains variables of two main categories:

a. Path-plan variables, including ‘number of lane changes to exit lane’, and ‘number of lane changes to exit lane interacted with the remaining distance to mandatory lane-changing point’; and

b. Current –lane inertia variables, including ‘current lane dummy’ and ‘lane changes required from current-lane to the target lane’.

The interaction variable between number of lanes to exit lane and the remaining distance to exit takes a special functional form as shown in the utility expression (5.2.1). This is done to ensure that remaining distance to exit has a decisively diminishing effect on utilities of lanes away from the exit lane as the driver gets closer to his exit.

The signs of the coefficients for all path plan variables are along expected lines, implying that a driver always feels a greater affinity for the exit lane he/she needs to reach ultimately, and that this affinity increases as the distance to the exit reduces. The coefficients of the current lane inertia variables indicate, as expected, that a driver always prefers a lane closer to the current lane as his/her target lane.
Other variables include queue length as an aggregate lane-specific characteristic, whose coefficient sign conveys the disutility a driver feels for a lane that has vehicles held up in queue. Its functional form ensures that the driver perceives no incremental disutility for lanes with queue lengths greater than 3, which was found to be a threshold value for this dataset. The relative speed of the front vehicle with respect to the subject vehicle, interacted with the spacing between the two, captures the sensitivity of the driver to his immediate surrounding conditions. Its positive coefficient indicates that a driver prefers to change lanes if his current front vehicle is moving slower than he/she, and his sensitivity to this speed difference increases with reduction in front spacing. This variable effectively accounts for the freedom for accelerating that a driver possesses in his current lane.

The latent driver-specific error term is also included to capture correlations among successive decisions of same driver, and can be interpreted as representing his/her aggressiveness in making lane changing decisions. The negative sign of the associated coefficient implies that in the given dataset, aggressive drivers tend to be more active in seeking lane changing opportunities compared to the average driver.

The path plan and current-lane inertia variables clearly have a counteracting influence on the target lane selection of drivers, with the current-lane variables emphasizing affinity to the current lane, while the path plan variables drawing the vehicle to his/her exit lane. The tradeoff between these effects is illustrated in the following figures (figures 5.1 and 5.2), for an example situation of four lanes (which represents the typical case in the current dataset), with lane 4 representing the exit lane. The lane-specific probabilities perceived by a given driver during target lane selection depending upon his current lane, is plotted, and the changes to these probabilities as he moves closer to his exit are studied.
Figure 5.1: Tradeoff between Current-Lane Inertia and Path-Plan Effect (Lane specific probabilities for target lane selection)

Note: Distance to exit = 410m, Turning/exit lane = Lane 4.

Figure 5.2: Tradeoff between Current-Lane Inertia and Path-Plan Effect (Lane specific probabilities for target lane selection)

Note: Distance to exit = 75m, Turning/exit lane = Lane 4.

When the driver is far from his exit, current-lane inertia dominates his target-lane selection, while as the driver approaches his turn, the path-plan effects take over. Figure 5.1 illustrates
the higher affinity of the driver for lanes in the immediate vicinity of his current lane, all other things being equal. This is expected since he is still reasonably far from his exit point (a distance of 410 m implies over 2 sections’ distance in the study dataset). The order of preference shifts to lanes in the vicinity of the exit lane (Lane 4) as the driver comes closer to his exit (his remaining distance to exit is 75 m) and path plan considerations take over, as exhibited in Figure 5.2. In this situation, one can observe that there is a high probability for choosing Lane 4 irrespective of the driver’s current lane.

*Gap Acceptance Model*

The specifications for the mean of lead and lag critical gaps for driver n at given time t is presented in the following set of equations:

\[
G_m^{\text{lead TLcr}} = \exp(2.31 - 0.0482AV_m^{\text{lead,TL}} - 1.78 v_n + \varepsilon_m^{\text{lead}}),
\]

\[
\varepsilon_m^{\text{lead}} \sim N(0,0.00745^2)
\]

\[
G_m^{\text{lag TLcr}} = \exp(1.51 + 0.0314 AV_m^{\text{lag,TL}} - 1.74 v_n + \varepsilon_m^{\text{lag}})
\]

\[
\varepsilon_m^{\text{lag}} \sim N(0,0.0085^2)
\]

\[
P(\text{accepting gap}) = P(\text{accepting lead gap}) \times P(\text{accepting lag gap})
\]

Where,

\[
\Delta V_{m}^{g_i} = V_{m}^{g_i} - V_m
\]

\[
V_m = \text{speed of subject vehicle n at time t}
\]

\[
V_{m}^{g_i} = \text{speed of vehicle associated with gap g of subject n at time t in direction of target lane i}
\]

Critical gap lengths can be conceived as the minimum threshold gap length that a driver considers safe enough to accept given current conditions. The adopted specification is a simple functional form that hypothesizes the critical gap mean to vary only with respect to the relative speed of the vehicle defining the gap (lead or lag) and the individual specific error term.

As the relative speed of the lead vehicle with respect to the subject vehicle increases, the lead gap is expected to expand with time and is considered safer to move into at the present moment, implying a lower threshold or lead critical gap. The negative coefficient for the lead vehicle’s relative speed corroborates this understanding. A similar argument justifies the
positive coefficient of the lag relative speed in the expression for the mean of the lag critical gap.

An aggressive driver is expected to find a given adjacent gap length more acceptable compared to an average driver. This hypothesis is supported by the negative coefficient for the latent driver-specific error term in lead and lag critical gap mean expressions, implying lower critical gap lengths for aggressive drivers.

The influence of the above-discussed explanatory variables on the critical gap lengths are summarized through the figures 5.3 and 5.4 below.

**Figure 5.3: Variation of Lead Critical Gap with Relative Lead Speed and Alpha (Driver Aggressiveness)**

![Critical Lead Gap Variation](image-url)
Figure 5.4: Variation of Lag Critical Gap with Relative Lag Speed and Alpha (Driver Aggressiveness)

Note: the term $niu$ represents driver aggressiveness ($niu = 0$ represents an average driver, whereas $niu = 1$ represents an aggressive driver)

The term $relv$ represents relative speed of the associated adjacent vehicle.

**Execution Model:**

The third decision level on lane change execution is a binary choice model, with the do-nothing alternative’s utility normalized to zero. The expression for the systematic utility of the lane change execution alternative ($V_{nt}^{lw}$) for driver $n$ at time $t$ is as follows:

$$
V_{nt}^{lw} = -2.52 + 0.60*V_{nt} - 0.308*(G_{nt}^{lead TL} - G_{nt,mean}^{lead TL,cr})*K_{nt}^{lead TL} + (G_{nt}^{lag TL} - G_{nt,mean}^{lag TL,cr})*K_{nt}^{lag TL}) - 0.88*(1-K_{nt}^{lead TL})
$$ (5.2.3)
\( V_{nt} \) refers to speed of driver \( n \) at time \( t \)

\[ K_{nt}^{lead\ TL} = \begin{cases} 
1 & \text{if lead vehicle for driver } n \text{ present in target lane } TL \text{ at time } t \\
0 & \text{otherwise} 
\end{cases} \]

Where

\[ K_{nt}^{lag\ TL} = \begin{cases} 
1 & \text{if lag vehicle for driver } n \text{ present in target lane } TL \text{ at time } t \\
0 & \text{otherwise} 
\end{cases} \]

\[ l_{nt} = \begin{cases} 
1 & \text{if } TL \text{ is left lane} \\
-1 & \text{if } TL \text{ is right lane} \\
0 & \text{otherwise} 
\end{cases} \]

As mentioned in the model formulation chapter (chapter no. 3), the execution level aims to capture the duration of the lane change post its initiation following gap acceptance. The lane change duration was hypothesized to be influenced by two aspects:

a) The urgency of the lane change.

b) The speed at which it could be completed.

Variables included in the utility specification at the third level aim to represent these two aspects. The gap leeway variable (difference between available and mean critical gap) accounts for the first aspect, under the hypothesis that a high gap leeway might reduce the urgency of a lane change, and encourage a driver to prolong the completion of an initiated lane change. It should be noted that the mean critical gap is a latent variable that is not physically observed. It is also hypothesized that drivers might treat situation with no lead vehicle different from that with a lead vehicle when evaluating the lead gap leeway. Hence a dummy variable representing the presence of a lead vehicle in the immediate target lane is adopted in the utility specification. It should be noted that the current approach assumes a lag adjacent gap to be acceptable by default under absence of a lag vehicle. A corresponding dummy variable denoting the absence of a lag vehicle hence does not enter the utility specification since the lag’s absence is assumed not to influence the gap leeway a driver considers in such situations.

The current speed of a driver is used to represent the rate at which a driver can complete an initiated lane change. Drivers moving at high speeds are expected to complete a lane change faster than slow-moving vehicles. The parameter estimate fully corroborates this hypothesis. The positive coefficient for the subject speed produces a higher probability of lane change execution, and therefore completion, in the current instant for faster drivers. When conceived
over a continuous time domain, a higher instantaneous probability for lane change execution implies a lower time duration over which the lane change maneuver is completed.

Following the same line of understanding, the negative coefficient for the gap leeway variable implies a lower probability of instantaneous lane change execution for a higher gap leeway and therefore greater lane change duration. The inclination to prolong lane change execution is increased under the absence of a lead vehicle in the targeted lane, as indicated by the estimate of the parameter for the corresponding dummy variable.

The signs and statistical significance of the parameter estimates at this third decision level help justify the introduction of this additional decision level. Further tests to confirm the improved modeling capabilities rendered by this extension would involve a comparison with the estimates obtained for the 2-level model on the same estimation sample.

5.3 Estimation results for 2-level lane changing model

The 2-level lane changing model was estimated using the same sample as that used for the estimation of the 3-level model. The specification adopted for the first two levels of the 3-level model are replicated for the 2-level model in the estimation process so as to facilitate comparison with the 3-level model through a simple likelihood ratio test. The estimation results are presented in table 5.3 below.
Table 5.3: 2-level lane-changing model estimation results

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Parameter</th>
<th>t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level 1: Target-Lane Selection Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current-Lane Dummy</td>
<td>2.09</td>
<td>1.36</td>
</tr>
<tr>
<td>Path Plan impact: No. of lane changes to exit</td>
<td>-0.468</td>
<td>-0.15</td>
</tr>
<tr>
<td>Path plan impact: No. of lane changes to exit interacted with distance from exit (m^-1.186)</td>
<td>-2.59</td>
<td>-2.33</td>
</tr>
<tr>
<td>Exponent of dist. to exit in no. of lanes to exit - dist. to exit interaction</td>
<td>0.171</td>
<td>0.15</td>
</tr>
<tr>
<td>No. of lane changes from current lane &lt;= 3</td>
<td>-2.18</td>
<td>-1.47</td>
</tr>
<tr>
<td>No. of lane changes from current lane &gt;= 4</td>
<td>-5.71</td>
<td>-0.003</td>
</tr>
<tr>
<td>Queue length ahead in lane (number of vehicles)</td>
<td>-0.182</td>
<td>-2.76</td>
</tr>
<tr>
<td>Front vehicle rel. speed negative, interacted with front veh. Gap (m/s per m)</td>
<td>0.0576</td>
<td>1.19</td>
</tr>
<tr>
<td>σ^CL</td>
<td>-0.662</td>
<td>-5.4</td>
</tr>
<tr>
<td><strong>Level 2: Gap Acceptance Model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead Critical Gap</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead gap constant</td>
<td>2.23</td>
<td>167.8</td>
</tr>
<tr>
<td>ΔV^lead,TL (m/s)</td>
<td>0.348</td>
<td>140.38</td>
</tr>
<tr>
<td>σ^lead</td>
<td>0.00745</td>
<td>0.942</td>
</tr>
<tr>
<td>σ^dead</td>
<td>-1.68</td>
<td>-97.44</td>
</tr>
<tr>
<td>Lag Critical Gap</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lag gap constant</td>
<td>1.44</td>
<td>165.99</td>
</tr>
<tr>
<td>ΔV^lag,TL (m/s)</td>
<td>0.265</td>
<td>193.47</td>
</tr>
<tr>
<td>σ^lag</td>
<td>0.0085</td>
<td>1.59</td>
</tr>
<tr>
<td>σ^lag</td>
<td>-1.86</td>
<td>-137.99</td>
</tr>
</tbody>
</table>

The variable definitions are the same as that presented in table 5.2.
5.4 Model comparison

The signs of the estimates of all but one parameter in the 2-level model match the corresponding estimates in the 3-level model. The one parameter whose signs do not match over the two models is the coefficient for lead vehicle relative speed in lead critical gap. It is estimated to be -0.0482 for the 3-level model and +0.348 for the 2-level model. The positive sign for this coefficient as obtained for the 2-level model refutes the hypothesis presented earlier regarding the expected sign for this coefficient and seems behaviorally inconsistent. This result is a direct consequence of the anomaly in gap statistics for the sample dataset (Table 4.7(b)) discussed in the last chapter. It is observed that in the sample dataset, the average lead relative speed for accepted gaps is lower than that for all available gaps, which is contrary to expectation. The inability of the 2-level model to account for the mechanisms that are believed to cause this anomaly is responsible for the erroneous sign of the above-discussed coefficient. This result offers another example of the improvement in driver lane change modeling brought in through the introduction of the third decision level capturing lane-change duration.

5.4.1 Likelihood test

Since the 2-level model is a fully nested version of the 3-level model, a likelihood ratio test can be done to compare the goodness of fit achieved by the models on the estimation dataset.

\[ \mathcal{L}_U = -1000.3 \] (the 3-level model is the unrestricted model, with 21 parameters estimated)

\[ \mathcal{L}_R = -1044.61 \] (the 2-level model is the restricted model, with 17 parameters estimated)

Degrees of freedom = 21 - 17 = 4

\[ \chi^2_{0.05, 4} \text{ (Chi-square test statistic at } 4 \text{ degrees of freedom and } 95\% \text{ level of confidence)} = 9.488 \]

Test statistic = \(-2*(\mathcal{L}_R - \mathcal{L}_U) = 88.62 > 9.488\), implying that the 3-level model does indeed provide a significant improvement in fit over the 2-level model on the sample dataset.
The above-discussed estimation results and model comparison strongly support the enhanced modeling ability offered by the extended 3-level lane changing model. This improvement is emphasized further by its application on an arterial dataset possessing distinctive features that resulted in misleading instantaneous statistics (due possibly to prolonged lane change durations) which could not be explained by the original 2-level model.

In summary, the 3-level model provided a significant improvement in fit compared to the 2-level model. It was also seen that the 3-level model was able to correct the signs of the critical gap parameter estimates that were obtained erroneously for the 2-level model. The estimates of parameters for the third decision level on lane change execution were found to corroborate the original hypothesis on lane change execution behavior: large gap leeways cause drivers to prolong their lane change completion, while faster drivers complete lane change in shorter time.

The next chapter presents the validation process undertaken to study the ability of the extended model to forecast driver behavior, and assess its superiority in the same in comparison to the 2-level model.
CHAPTER 6

MODEL VALIDATION

As explained in the Introduction chapter, the primary purpose of traffic simulators in transportation planning and traffic management studies is to help provide an accurate and detailed forecast of the impacts of different operational and planning changes being studied. The task of model validation aims to test the extensibility of an estimated model to scenarios beyond that used for estimation, and assess its value as a reliable tool for forecasting driving behavior.

The evaluation criterion is the accuracy with which the forecasted behavior rendered by the model for a chosen validation dataset reflects that observed in the field for that dataset. Validation is typically a comparative process, and relies on measures that facilitate comparisons with the predictive performance of another reference model. A consistent improvement in prediction quality is sought from the new model to validate its superiority. The new model being validated here is the 3-level model, with parameters estimates as presented in table 5.1. The reference model used in this study is the 2-level lane changing model presented in Choudhury (2005), with model estimates as presented in table 5.3. Owing to the unavailability of a good candidate for validation dataset, the unsampled version of the original dataset had to be used for validation purposes.

Model Validation is typically preceded by a sub-task of Implementation, while it in itself comprises of two sub-tasks: Aggregate calibration and validation. The execution of each of these sub-tasks for the current study is outlined through the following sections.

6.1 Implementation

The task of validation requires the simulation of driver choices as predicted by the estimated models. For this purpose, the developed lane changing model needs to be implemented in a traffic simulator that replicates driver behavior at a microscopic level of detail. This task is
called Implementation, and the traffic simulator used for this purpose in the current study was MITSIMLab. This sub-section presents an overview of MITSIMLab, its features and components.

MITSIMLab is a simulation-based laboratory developed for evaluating the impacts of alternative traffic management system designs at the operational level and assisting in subsequent refinement. Examples of systems that can be evaluated with MITSIMLab include advanced traffic management systems (ATMS) and route guidance systems. MITSIMLab is a synthesis of a number of different models and represents a wide range of traffic management system designs. It has the ability to model the response of drivers to real-time traffic information and controls and can incorporate the dynamic interaction between the traffic management system and the drivers on the network.

The various components of MITSIMLab are organized in three modules:

1. Microscopic Traffic Simulator (MITSIM)
2. Traffic Management Simulator (TMS)
3. Graphical User Interface (GUI)

A microscopic simulation approach, in which movements of individual vehicles are represented, is adopted for modeling traffic flow in the traffic flow simulator MITSIM. The traffic and network elements are represented in detail in order to capture the sensitivity of traffic flows to the control and routing strategies. The road network is represented by nodes, links, segments (links are divided into segments with uniform geometric characteristics) and lanes. Traffic controls and surveillance devices are represented at the microscopic level.

The traffic simulator accepts time-dependent origin to destination trip tables as inputs. The OD tables represent either expected conditions or are defined as part of a scenario for evaluation. A probabilistic route choice model is used to capture drivers' route choice decisions. The origin/destination flows are translated into individual vehicles wishing to enter the network at a specific time. Behavior parameters (e.g., desired speed, aggressiveness) and vehicle characteristics are assigned to each vehicle/driver combination. MITSIM moves
vehicles according to car-following and lane-changing models. The car-following model captures the response of a driver to conditions ahead as a function of relative speed, headway and other traffic measures. The lane changing model distinguishes between mandatory and discretionary lane changes and simulates driver action as an output of a complex decision-making framework. Merging, drivers' responses to traffic signals, speed limits, incidents, and toll booths are also captured.

The traffic management simulator (TMS) mimics the traffic control system under evaluation. A wide range of traffic control and route guidance systems can be evaluated. These include regular traffic signals, ramp control, freeway mainline control, lane control signs, variable speed limit signs, portal signals at tunnel entrances, intersection control, variable Message Signs and in-vehicle route guidance. TMS has a generic structure that can represent different designs of such systems with logic at varying levels of sophistication (pre-timed, actuated or adaptive). An extensive graphical user interface is used for both debugging purposes and demonstration of traffic impacts through vehicle animation. A detailed description of MITSIMLab appears in Yang and Koutsopoulos (1996) and Yang et al (2000).

In this study, the 3-level lane changing model framework was implemented as the active lane changing model adopted by MITSIM in simulating driver decisions regarding lane change action at every simulator time step. Tests were conducted following the model implementation to verify whether it was working and simulating lane change action as desired. Details of this verification tests are available in the final report on Arterial Lane Selection model submitted to NGSIM as a part of this study (Choudhury et al. (2007)). The arterial also contained intersections that allowed for vehicles to enter mainstream traffic from side streets, which were simulated using the intersection lane-selection model developed and presented in Choudhury (2007). Appendix A presents the framework adopted for the intersection lane-selection model and explains how it works in conjunction with the mainline lane changing model developed in this study. For a comprehensive treatment of this model’s development and application, the reader is referred to Choudhury (2007).
MITSIMLab’s driving behavioral models also include acceleration models that simulate the longitudinal motion of vehicles. The different kinds of scenario-specific acceleration models in MITSIMLab that work in conjunction with lane changing models to simulate driver motion are presented in Appendix B.

Other notable implementation details on the supply side of the simulator include:

- The development of a network structure representative of the arterial stretch within the study area, with appropriate incorporations of lane and intersection configurations and specifications of lane use regulations to be used within the simulator.
- The development of a signal file containing information on signal phase timings and control logic behind their operations at each intersection, as best deciphered from the data provided on signal interval lengths over the 32 minutes of data collection.

### 6.2 Model Validation

The model validation module typically includes within it the task of aggregate calibration and validation.

There usually exist model parameters whose estimates reflect characteristics specific to the estimation dataset and are hence not extensible to another scenario. Hence, prior to applying the estimated model for forecasting purposes to another scenario, these situation-specific parameters need to be re-estimated. Also, other simulator parameters of demand and supply, including O-D flows, parameters of other behavioral models including route choice and acceleration, etc., need to be estimated so as to reflect the conditions in the new scenario.

This task of jointly re-estimating the above parameters in order to tune the simulator to reflect the prevailing traffic conditions in the new application scenario is called calibration. If the observed traffic conditions on the new scenario are used in aggregate form for this re-estimation process, the process is called aggregate calibration. For this purpose, a part of the validation dataset is used and the above-mentioned parameters are adjusted in order to obtain a good fit between the simulated and observed traffic. Traffic characteristics like speeds, counts, density etc describing traffic at an aggregate level are chosen as basis for comparison.
and for evaluating fit. Different goodness-of-fit measures suitable for use as a measure of calibration are available in statistical literature.

It must be mentioned here that statistics from traffic in the north bound direction alone were considered for generating calibration and validation measures. This step was taken in view of the poor quality of operational data available for signals controlling the south-bound traffic. This deficiency was expected to introduce significant errors in measurements for the traffic simulated in the south-bound direction, and render it unrepresentative of the model performance.

As explained above, the arterial dataset whose 20% sample was used for estimation was chosen as the validation dataset in the absence of any other suitable dataset. An overview of this dataset and the extraction of aggregate traffic characteristics chosen to serve as benchmarks for aggregate calibration are described next. Common goodness-of-fit measures typically used in the calibration efforts are then enlisted. The calibration problem is then outlined, followed by the list of parameters chosen for calibration in this study and their calibrated values on implementation of each of the two lane changing models (2- and 3-level models). The traffic measures chosen for aggregate validation, and their results and associated discussions are presented next.

### 6.2.1 Dataset description

The trajectory data from Lankershim Boulevard, Los Angeles, California collected by the NGSIM Team was used for aggregate calibration and validation of the model. Lane-specific vehicle counts aggregated over 5-minute time intervals were chosen as the traffic characteristics to serve as a benchmark for calibrating both estimated models (2- and 3-level models). Since the dataset was available in disaggregate trajectory format, the individual observations had to be aggregated to calculate the above traffic characteristics at the level of aggregation required of the calibration task. Synthetic sensors at different locations of the arterial stretch were introduced for this purpose, and counts observed at these locations were extracted from the trajectory data at the desired level of aggregation. Three sets of sensors
were placed for each arterial section at the beginning, end, and midpoints, respectively. The locations of the synthetic sensors are shown schematically in figure 6.1 below.

**Figure 6.1: Locations of synthetic sensors in study area stretch**

The trajectory dataset provided information on the path taken by each vehicle through the study area. Hence, there was no route choice decision that was required of the simulator for each driver. Exact vehicle O-D flows were also available from the trajectory dataset.

Details of the O-D data are provided in Appendix C. The calibration process therefore only involved adjustment of the driving behavior parameters to better fit the observed traffic characteristics.

The total dataset was available for a 32-minute period (8:28 a.m. to 9:00 a.m.). The first 22 minutes of data was used for calibration and the remaining 10 minutes was used for validation.

### 6.2.2 Goodness-of-fit measures

A number of goodness-of-fit measures can be used to evaluate the overall performance of a simulation model. Popular among them are the root mean square error \((RMSE)\) and the root mean square percent error \((RMSPE)\). These statistics quantify the overall error of the simulator. Percent error measures directly provide information on the magnitude of the errors relative to the average measurement.

The two measures are given by:
\[
RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (Y_{n}^{\text{sim}} - Y_{n}^{\text{obs}})^2} \tag{6.2.1}
\]

\[
RMSPE = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left( \frac{Y_{n}^{\text{sim}} - Y_{n}^{\text{obs}}}{Y_{n}^{\text{obs}}} \right)^2} \tag{6.2.2}
\]

\(Y_{n}^{\text{obs}}\) and \(Y_{n}^{\text{sim}}\) are the averages of observed and simulated measurements at space-time point \(n\), respectively calculated from all available data (observations and/or multiple simulation replications).

\(RMSE\) and \(RMSPE\), however, penalize large errors at a higher rate relative to small errors.

Other measures include: Mean error (\(ME\)); and Mean percent error (\(MPE\)).

\(ME\) and \(MPE\) indicate the existence of systematic under- or over-prediction in the simulated measurements. These measures are given by:

\[
ME = \frac{1}{N} \sum_{n=1}^{N} (Y_{n}^{\text{sim}} - Y_{n}^{\text{obs}}) \tag{6.2.3}
\]

\[
MPE = \frac{1}{N} \sum_{n=1}^{N} \frac{Y_{n}^{\text{sim}} - Y_{n}^{\text{obs}}}{Y_{n}^{\text{obs}}} \tag{6.2.4}
\]

where \(Y_{n}^{\text{obs}}\) and \(Y_{n}^{\text{sim}}\) are the averages of observed and simulated measurements at space-time point \(n\), respectively calculated from all available data.

### 6.2.3 Aggregate Calibration process

Aggregate calibration can be formulated as an optimization problem that seeks to minimize a function of the deviation of the simulated measurements of chosen aggregate traffic characteristics from the observed measurements of the same. The formulation presented here assumes that the driving behavior parameters are stable over the period of observation.

The formulation is shown below. The objective function is in the form of sum of error squares, with the error being the deviation of the each simulated measurement from the observed measurement. The first constraint shows the dependence of simulated
measurements on the driving behavior parameters and the network conditions which govern the simulation.

\[
\min_{\beta, \Omega} \sum_{i=1}^{N} (M_{i}^{\text{sim}} - M_{i}^{\text{obs}})^T W^{-1} (M_{i}^{\text{sim}} - M_{i}^{\text{obs}}) \\
\text{s.t. } M_{i}^{\text{sim}} = S(\beta)
\]  

(6.2.5)

where,

\[\beta = \text{Driving behavior parameters;}\]
\[N = \text{Number of days for which sensor data is available;}\]
\[M_{i}^{\text{sim}} = \text{Simulated measurements;}\]
\[M_{i}^{\text{obs}} = \text{Observed measurements for day } i;\]
\[S = \text{The simulation model function, which generates simulated traffic measurements; and}\]
\[W = \text{Variance-covariance matrix of the sensor measurements.}\]

The simulated measurements that were used as benchmark for calibration in this study were the "synthetic" sensor counts generated using the first 22 minutes of available trajectory data (8:28 a.m. to 8:50 a.m.).

The number of behavioral parameters in the simulation model is very large. It is never feasible to calibrate all of them. Also, some of them have typically been proven to be transferable across different driving conditions. Therefore, a few parameters that typically have the most significant effect on the simulation results, and are also expected to be sensitive to driving conditions, are usually selected. These parameters are selected from a pool of parameters belonging to all the driving behavior models (lane changing, acceleration, etc) that work in conjunction to simulate the traffic. The focus rests on calibrating these parameters, while fixing the other parameters to their estimated (default) values. Previous experience has shown that the simulation results are most sensitive to the following parameters:

- Sensitivity parameters of the acceleration and deceleration function;
- Parameters of the desired speed distribution;
- Intercepts and variances (constants and sigmas) in the critical gap functions;
- Path plan variables.
The optimization in this study was done in MATLAB using Box’s Complex algorithm (Box, 1965).

6.2.3.1 Calibration parameters
Based on previous experience and sensitivity test results, the following parameters of the behavioral models in MITSIMLab were selected for calibration in this study:

- Acceleration and deceleration constants;
- Desired speed mean and standard deviation;
- Current lane dummy,
- Lanes away from exit lane
- Intercepts (constants) and variance (sigmas) of critical gap,
- Constant in the execution level; and
- Intercept (constant) of outer lane in intersection lane choice model

The acceleration and deceleration constants, along with mean and standard deviation of desired speed are parameters of the car-following model implemented in MITSIMLab. Further details of this car-following model and associated parameters are available in Ahmed (1999) and Toledo(2003). They are summarized in Appendix B.

The current lane dummy and the lanes away from the exit lane are two parameters used in the target lane selection model of the 2- and 3-level lane changing model structure. The intercepts and standard deviations of the lead and lag critical gaps are parameters in the second decision level (gap acceptance) of the 2- and 3-level lane changing model structure, while the execution level constant is a parameter present only in the 3-level lane changing model, at the third level (lane change execution).

The intercept of outer lane is a parameter used in the Intersection lane-selection model presented in Choudhury (2007).

The current lane dummy and the constant in the execution level were expected to be sensitive to the new dataset, because of the discrepancy between the estimation sample and the overall dataset in terms of the number of lane changes per north-bound vehicle. Out of the 400 vehicles in the estimation sample, 160 were north-bound. The total number of lane changes
aggregated over all north-bound vehicles in the sample was 132, implying a figure of 0.825 lane change per vehicle. In the overall dataset, 945 vehicles were observed in the north-bound direction, and a total of 1169 lane changes were observed to be executed by these vehicles within the study area. This amounted to 1.237 lane change per vehicle, which was significantly higher than that observed in the estimation sample. This fact implied that drivers in the overall dataset were less averse to lane change on average. Therefore, the current lane dummy, which captures the affinity of a driver for his current lane, and execution level constant, which represents his proclivity for a lane change, were both expected to significantly vary when extending the estimated model to the overall dataset.

6.2.3.2 Calibration results

A. 3-Level lane changing model:
The calibrated values of the parameters, when using the 3-level model for lane changing in MITSIMLab, are listed in table 6.1.
Table 6.1: Calibrated parameter values for the selected parameters (3-Level lane changing model implementation)

<table>
<thead>
<tr>
<th>Behavior Model/Variable</th>
<th>Calibrated Parameter</th>
<th>Explanation of Calibrated Parameter</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car-Following*</td>
<td>Acceleration constant</td>
<td>Constant term for acceleration in MITSIMLab car-following model (Ahmed 1999)</td>
<td>0.040 0.06</td>
</tr>
<tr>
<td></td>
<td>Deceleration constant</td>
<td>Constant term for deceleration in MITSIMLab car-following model (Ahmed 1999)</td>
<td>-0.042 -0.04</td>
</tr>
<tr>
<td>Desired Speed*</td>
<td>Mean</td>
<td>Mean of the normally distributed desired speed of the driver</td>
<td>0.100 0.056</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>Variance of the normally distributed desired speed of the driver</td>
<td>0.150 0.540</td>
</tr>
<tr>
<td>Mainline Lane Change:</td>
<td>Lead gap constant</td>
<td>Constant term for critical normal lead gap</td>
<td>2.31 2.16</td>
</tr>
<tr>
<td>Gap Acceptance</td>
<td>Lead gap sigma</td>
<td>Variance for critical normal lead gap (log normally distributed)</td>
<td>0.00751 0.0406</td>
</tr>
<tr>
<td></td>
<td>Lag gap constant</td>
<td>Constant term for critical normal lag gap</td>
<td>1.51 1.06</td>
</tr>
<tr>
<td></td>
<td>Lag gap sigma</td>
<td>Variance for critical normal lag gap (log normally distributed)</td>
<td>0.0085 0.0517</td>
</tr>
<tr>
<td>Intersection-Lane Choice</td>
<td>Lane 3 constant</td>
<td>Constant term for lane 3</td>
<td>1.31 1.10</td>
</tr>
<tr>
<td></td>
<td>Target-lane dummy</td>
<td>One, if immediate lane and target lane are the same</td>
<td>3.16 2.13</td>
</tr>
<tr>
<td>Mainline Lane Change – Target Lane Selection</td>
<td>Away from exit lane</td>
<td>Number of lane changes needed for getting to the exit lane of the driver</td>
<td>-0.73 -0.43</td>
</tr>
<tr>
<td></td>
<td>Current-lane dummy</td>
<td>One, if the current lane is the target lane</td>
<td>1.43 0.5</td>
</tr>
<tr>
<td>Mainline Lane Change – Execution Level</td>
<td>Execution constant</td>
<td>Constant for execution decision</td>
<td>-2.52 -1.37</td>
</tr>
</tbody>
</table>

* General parameters of MITSIMLab. These variables are described in Ahmed (1999).

Among these parameters, the desired speed parameter, the current-lane dummy, and the execution constant made the most significant contribution in improving the performance of
the model. When these parameters were unconstrained, the model performed better (objective function for calibration improved significantly) compared to the case when these parameters were fixed to the originally estimated values. As had been expected, the current lane dummy was reduced, and the execution level constant increased by the calibration process, to reflect the greater propensity for drivers to make a lane change in the overall dataset as compared to the estimation sample.

The improvements after the calibration are presented in table 6.2 below.

Table 6.2: Improvement resulting from calibration of 3-level model

<table>
<thead>
<tr>
<th></th>
<th>Calibration</th>
<th>Percent Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>After</td>
<td></td>
</tr>
<tr>
<td>Lane-Specific Counts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE (over 20 minutes)</td>
<td>18.80</td>
<td>15.10</td>
</tr>
<tr>
<td>RMSPE (over 20 minutes)</td>
<td>0.83</td>
<td>0.81</td>
</tr>
</tbody>
</table>

B. 2-Level lane changing model

Since the 2-level lane changing model was used as a reference model for assessing the predictive value of the 3-level lane changing model, it was also calibrated using the same procedure as described above. The calibrated parameters of the 2-level model included the following:

- Acceleration and deceleration constants;
- Desired speed mean and sigma;
- Current-lane dummy;
- Lanes away from exit lane; and
- Intercepts (constants) and variance (sigmas) of lead/lag critical gaps

The calibration results when using the 2-level model for lane changing in MITSIMLab is presented in table 6.3 below.
Table 6.3: Calibrated parameter values for the selected parameters (2-level lane changing model implementation)

<table>
<thead>
<tr>
<th>Behavior Model/Variable</th>
<th>Calibrated Parameter</th>
<th>Explanation of Calibrated Parameter</th>
<th>Parameter Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car-Following(^a)</td>
<td>Acceleration constant</td>
<td>Constant term for acceleration in MITSIMLab car-following model (Ahmed 1999)</td>
<td>0.040 0.042</td>
</tr>
<tr>
<td></td>
<td>Deceleration constant</td>
<td>Constant term for deceleration in MITSIMLab car-following model (Ahmed 1999)</td>
<td>-0.042 -0.029</td>
</tr>
<tr>
<td>Desired Speed(^a)</td>
<td>Mean</td>
<td>Mean of the normally distributed desired speed of the driver</td>
<td>0.100 0.056</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>Variance of the normally distributed desired speed of the driver</td>
<td>0.150 0.540</td>
</tr>
<tr>
<td>Mainline Lane Change: Gap Acceptance</td>
<td>Lead gap constant</td>
<td>Constant term for critical normal lead gap</td>
<td>2.23 2.2</td>
</tr>
<tr>
<td></td>
<td>Lead gap sigma</td>
<td>Variance for critical normal lead gap (log normally distributed)</td>
<td>.00751 0.009</td>
</tr>
<tr>
<td></td>
<td>Lag gap constant</td>
<td>Constant term for critical normal lag gap</td>
<td>1.44 1.40</td>
</tr>
<tr>
<td></td>
<td>Lag gap sigma</td>
<td>Variance for critical normal lag gap (log normally distributed)</td>
<td>.0085 0.009</td>
</tr>
<tr>
<td>Intersection-Lane Choice</td>
<td>Lane 3 constant</td>
<td>Constant term for lane 3</td>
<td>1.31 1.10</td>
</tr>
<tr>
<td></td>
<td>Target-lane dummy</td>
<td>One, if immediate lane and target lane are the same</td>
<td>3.16 2.13</td>
</tr>
<tr>
<td>Mainline Lane Change – Target Lane Selection</td>
<td>Away from exit lane</td>
<td>Number of lane changes needed for getting to the exit lane of the driver</td>
<td>-0.468 -0.104</td>
</tr>
<tr>
<td></td>
<td>Current-lane dummy</td>
<td>One, if the current lane is the target lane</td>
<td>2.09 0.99</td>
</tr>
</tbody>
</table>

\(^a\) General parameters of MITSIMLab. These variables are described in Ahmed (1999).

The improvements after the calibration are presented in table 6.4.
### Table 6.4: Improvement resulting from calibration of 2-level model

<table>
<thead>
<tr>
<th>Calibration Percent</th>
<th>Before</th>
<th>After</th>
<th>Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lane-Specific Counts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE (vehicles per 20 minutes)</td>
<td>18.80</td>
<td>14.80</td>
<td>21.3%</td>
</tr>
<tr>
<td>RMSPE</td>
<td>0.83</td>
<td>0.77</td>
<td>7.2%</td>
</tr>
</tbody>
</table>

It can be observed that the final fit as obtained from the calibration process for the first 22 minute time-interval is better for the 2-level model in comparison to the 3-level model. This could be attributed to possible over-fit. The measures of deviation over the validation interval should also be considered before making a conclusive assessment of model performance.

6.2.4 Aggregate Validation

This final sub-task involves the application of the calibrated model to the dataset specifically earmarked for the purpose of assessing the quality of forecasts rendered by the developed model. In context of this study, aggregate validation involved a comparison of the traffic simulated by use of the new model for the chosen time period with that observed for the same. The basis for this comparison comprised of different traffic elements/features most relevant to the current study, adopted in some aggregated form.

As mentioned in the calibration section, the time period between 8:50 a.m to 9:00 a.m of the arterial traffic had been sidelined for validation purposes. The model as estimated and then re-calibrated over the previously described efforts was applied to simulate traffic over this 10 minute interval. The vehicle origin-destination matrix and details of signal operations served as the only external information provided to the simulator.

The following features/elements of traffic are selected to serve as validation measures, i.e, measures of model performance:

1. Aggregate lane-specific counts
2. Lane distribution of vehicles in mainline arterial
3. Number of lane changes per vehicle across different turning movement categories

As the validation process is comparative, the degree of fit with the observed traffic measured for the traffic simulated by the 3-level model is analyzed relative to that measured for the 2-level model. An analysis of the relative performance of these two models in replicating the observed traffic behavior, evaluated in terms of the above selected traffic characteristics, is presented next.

1. Aggregate lane-specific counts:
The fit between the simulated and observed traffic in terms of the aggregate lane-specific counts, as obtained for the 3- and 2-level models over the validation time interval, are summarized in Table 6.5 below.

<table>
<thead>
<tr>
<th>Validation time interval (8:50 – 9:00 am)</th>
<th>Percent Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Level</td>
<td>3-Level</td>
</tr>
<tr>
<td>Lane-Specific Counts</td>
<td></td>
</tr>
<tr>
<td>RMSE (vehicles per 20 minutes)</td>
<td>19.40</td>
</tr>
<tr>
<td>RMSPE</td>
<td>0.783</td>
</tr>
</tbody>
</table>

As can be seen, the 3-level model provides a consistently better fit with regards to lane-specific counts over the validation time interval in comparison to the 2-level model. The observed improvement in degree of fit in counts over the validation interval for the 3-level model occurs in spite of the lower fit relative to the 2-level model over the calibration interval. This result provides support for the extensibility of the 3-level model.
2. Lane distribution of vehicles in mainline arterial:

The distribution of vehicles over different lanes in the cross-section of a roadway is an ideal measure to verify the quality of performance of a lane changing model. Since the model extension aims to capture the physical duration of the lane change, the chief contribution is expected to occur with regard to the actual location of the lane changes. While the 2-level model treats lane changes as an instantaneous action following gap acceptance, the 3-level model is expected to simulate lane changing actions closer to practical reality and replicate the lane change locations better. By considering lane-specific synthetic sensors at different reaches along a single arterial section (refer figure 6.1), it is possible to investigate the actual locations of simulated lane changes within the section by comparing the lane distributions of vehicles across the different reaches of the section. The lane distributions across the left extreme lane, middle/through lanes and the right extreme lane for the section between intersection nos. 2 and 3 (refer demarcation in figure 4.2), as simulated by both models, are presented in the following figures. The equivalent measures observed from the dataset are also presented in the figures to serve as a benchmark.

Lane-specific vehicle counts used to deliver these statistics are aggregated over 5 minute intervals. The percentage distribution of vehicles across the lanes over different cross-sectional reaches of the selected section have been presented in the following figures (6.2 (a)-(f)). Two 5-minute intervals have been selected for presentation, one chosen from the calibration period (8:45 – 8:50 am) and another chosen from the validation period (8:55 – 9:00 am).

These figures offer a sample of comparison between the performances of the 2- and 3-level models with regards to lane distributions of vehicles, and thereby lane change locations.
Figure 6.2: Lane distribution of vehicles over different reaches of section 3 (between Intersections 2 and 3) (a) – (f)

(a)  

Lane Distribution - Section beginning (8:45 - 8:50 am)

(b)  

Lane Distribution - Section middle (8:45 - 8:50 am)
As can be observed from the list of the figures above, the 3-level model provides a consistent albeit small improvement in quality of predicted lane distributions as compared to the 2-level model. This in turn can be inferred as an indication of an improvement in the predicted location of lane changes.

The above results contribute to validating the improvement in behavioral realism of lane change modeling brought about by the model extension and explicit handling of lane change duration.
3. Number of lane changes per vehicle across different turning movement categories

Vehicle categorization based on their turning movements, and its relevance to lane-changing behavior, was discussed in the data description section (4.1.3). Statistics on number of vehicles across the three defined turning categories (through, turning into arterial and turning off arterial) observed in estimation sample and overall dataset were presented in table 4.3. The number of simulated lane changes by each vehicle turning category offers another pertinent validation measure for assessing the performance of the lane changing models being studied. The following figures (6.3 – 6.5) present the distribution of vehicles based on the number of lane changes per vehicle as simulated by either models for each vehicle turning category. The presented statistics are aggregated over the entire validation period. The equivalent statistics as observed from the validation dataset for the same time interval are used as benchmark.
Figure 6.3: Distribution of vehicles by lane change per vehicle – Through Vehicles

Validation Period (8:50 - 9:00 am)

Figure 6.4: Distribution of vehicles by lane change per vehicle – Vehicles turning into Arterial

Validation Period (8:50 - 9:00 am)
Figure 6.5: Distribution of vehicles by lane change per vehicle – Vehicles Turning off Arterial

Validation Period (8:50 - 9:00 am)

The figures reveal a consistently better fit achieved by the 3-level model with regards to the number of lane changes per vehicle for each turning movement category. In particular, it is observed that the 2-level model generally tends to under-predict the total number of lane changes for vehicles across all turning categories (since the number of vehicles with more than 2 lane changes simulated by the 2-level model is lower than the corresponding number simulated by the 3-level model for every vehicle turning category). The source of this discrepancy is tracked back to the model estimation stage. The series of vehicle observations in the study dataset that recorded huge adjacent gaps (some with no lead/lag vehicle) but no lane changes were expected to have caused error in critical gap estimates for the 2-level model owing to its modeling deficiency. These errors in critical gap measures are likely to have been responsible for the observed under-prediction of lane changes by the 2-level model.

In summary, the presented validation measures indicate a modest but consistent improvement in traffic prediction exhibited by the 3-level model in comparison to the 2-level model. Validation measures were chosen so as to reflect aspects of simulated driving behavior that were expected to show improvement based on the motivation surrounding the model.
extension. The corresponding results substantiate this hypothesis, and indeed justify the introduction of the model extension.
CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 Thesis summary

This thesis work dealt with a study of driver behavior modeling, and focused on introducing an enhancement to the state-of-art lane changing model. The current lane changing model treated driver’s lane changing action to be the output of a two-level decision framework: target lane selection and gap acceptance decision. The conceptualization of lane change as an instantaneous driver action following gap acceptance was considered to be inadequate from a practical perspective since it ignored away the magnitude of lane change duration. This aspect governing a practical lane change maneuver would result in a time lag between the actual completion of a lane change and its initiation following the discovery of an acceptable adjacent gap.

An extension in model framework was proposed for overcoming this deficiency. The current study introduced a third decision level regarding the execution of a lane change that drivers are hypothesized to consider following acceptance of the adjacent gap. A decision to execute a lane change at a given instant would correspond to the completion of an initiated lane change at that instant in reality. This decision level on lane change execution into an acceptable gap would therefore help account for the duration of the lane change maneuver.

The lane change duration is expected to be influenced by two predominant factors, one characterizing the urgency of the lane change, and other characterizing the speed at which it can be completed. List of traffic and individual-specific variables that would best capture these two factors were discussed.

The extended lane changing model framework was then applied to study lane changing behavior in arterial corridors. Detailed vehicle trajectory collected over 32 minutes of observation of traffic at Lankershim Boulevard, California, U.S, was used as the application
dataset. The study area was an arterial corridor having signalized intersections that allowed controlled mainline access to side streets.

The application scenario was in itself unique, since driving behavior in arterials have not received significant attention in literature. In addition, the arterial dataset was found to possess some distinctive characteristics that strongly distinguished it from previous study datasets. Apart from representing traffic of lower density and speeds compared to those encountered in previous datasets (mostly freeways), it contained a significant proportion of observations that recorded no adjacent lead or lag vehicles for the subject vehicle, and therefore no measurement of adjacent gap length. A pseudo gap correction was applied to generate adjacent gap lengths for such observations. Instantaneous gap statistics indicated that vehicles were not moving into such gaps immediately, implying a prolonged lane change process. This phenomenon could not be explained in the original 2-level model framework, and hence offered an ideal case to showcase the applicability of the model extension.

A sample of the overall study dataset was used for model estimation purposes. The parameters for the original and extended models were jointly estimated in GAUSS. The best model specification for the extended 3-level model was arrived upon following an iterative process, and was also adopted for the 2-level model baring the execution level. A comparison of the parameter estimates and the likelihood function indicated that the extended 3-level model explained the lane changing behavior in the sample dataset significantly better. It was discovered in particular that the extended model was able to correct the errors in signs of some of the critical gap parameter estimates that infested the original 2-level model. Also, the estimates of parameters in the third level of the extended model suggested that:

a) Drivers tend to prolong lane change completion when they experience adjacent gaps significantly greater than the minimum threshold they deem as required for a safe lane change.

b) Drivers moving at a faster speed are likely to complete their lane changes quicker than others.

Owing to the unavailability of another suitable dataset, the unsampled version of the study dataset was chosen for model validation purposes. The newly developed lane changing
model was implemented in a microscopic traffic simulator MITSIMLab, which replicates individual driver behavior through a sequence of discrete decisions on lane changing and acceleration, and has the capability of handling signalized corridors. The model was calibrated using the first 22 minutes of the observation interval, and validated using the remaining 10 minutes.

Both the original 2-level and the extended 3-level models were calibrated and validated using MITSIMLab to help assess the value offered by the proposed model extension with regards to driver behavior prediction. Aggregate measures that were expected to reflect the specific improvements in traffic prediction attributable to the model extension were chosen as validation measures. It was seen that the 3-level model delivered a consistent improvement over the 2-level model in each of these measures. Although not stark, these improvements lead to a reliable inference regarding the superiority in performance of the extended model.

### 7.2 Research contributions

The research work outlined in this thesis is believed to make the following two contributions in the field of driver behavior modeling.

#### 7.2.1 Enhanced behavioral realism in lane changing models

The work identifies a limitation in the state-of-art two-level lane changing model framework that would in particular weaken its applicability to low density, low speed traffic situations. The treatment of lane change as an instantaneous action following gap acceptance ignores a critical maneuverability-related aspect: lane change duration. The actual time duration of a lane change execution that extends over some finite number of time instances (as per the time resolution adopted in the dataset), and the tendency of drivers to prolong it under situations where large adjacent gaps are available, are both overlooked in the current state-of-art model. These elements are captured and incorporated within the extended model framework through the additional level of decision following gap acceptance that models driver’s decision to execute lane change at the current instant. While the model still treats lane change as an
instantaneous process, the additional level gives it flexibility in capturing the time delay in lane change completion post its initiation. This extension helps bridge the gap between model conceptualization and behavioral execution of lane changing actions.

7.2.2 Application to lane changing behavior in arterial corridors
As mentioned throughout this thesis, past studies on driver lane changing models have tended to exclusively focus on traffic in congested situations. Owing to data availability and interest, most of the models that were developed in the past were applied to freeway scenarios. Studies on arterial lane changing behavior are indeed very limited. This study dealt with traffic from an arterial corridor that also included within it signalized intersections. It was discovered that the arterial dataset indeed differed from freeways in certain crucial respects, further weakening the applicability of models previously tested on freeway datasets. The arterial dataset was therefore found to offer an interesting and strong case for implementation of the proposed model extension. This study represents one of the few works involving the development and successful application of lane changing models for arterial traffic.

7.3 Directions for future research
This section outlines some of the unexplored research topics that are believed to be of worth pursuing in future in order to propel modeling improvements for driver lane changing behavior.

7.3.1 State dependency
Lane changing behavior is modeled as the output of a sequence of decisions taken by drivers over successive time intervals. The state-of-art lane changing models, along with the extended model developed in this study, assume independence among latent driver decisions across different time instances. This ignorance of state dependency in driver lane changing actions is a major flaw among the current established models. It constitutes a most interesting line of research that is being strongly pursued by current researchers. A robust yet
computationally tractable implementation of features that help incorporate state dependency in lane changing model framework has not yet been accomplished, and would represent a fundamental improvement in lane change modeling when achieved.

7.3.2 Integration with acceleration model
This study incorporated an extended model framework that focused primarily on the lane changing decisions of the driver. However, driver behavior comprises of, in the simplest scenario, two components: acceleration and lane-changing. As already discussed in the literature review chapter and established in driving behavior literature, these actions tend to be the joint output of the same decision-making framework. Toledo (2003) had developed an integrated driver behavior model that jointly modeled lane-changing and acceleration decisions. However, applications of subsequent enhancements to the lane changing component of that integrated model, as done by Choudhury (2005) and further attempted in this work, do not integrate lane changing with acceleration decisions. Appending appropriate decision levels to the discussed lane changing model framework that help capture the influence of lane changing decisions on acceleration decisions, and applying them on the study dataset would represent a significant improvement over the current work.

7.3.3 Effect of time resolution
Every one of the models discussed in this thesis, and most of those developed in literature, are applied to explain the driver actions occurring across discrete time instances as adopted in the application trajectory dataset. The conceptualization of the process they describe involves a discretization of a continuous-time process. In such cases, the time resolution adopted in the discretization step would have a significant impact on the model parameters. In general, the alternative specific intercepts tend to absorb the effect of time resolution of the estimation dataset. For example, the original dataset as provided through video coverage was at a time resolution of 1/10th of a second, and was later aggregated to one-second resolution. The parameter estimates, especially of the intercepts at the three decision levels, would differ when using either of the two time resolutions for estimation purposes.
The unit time step at which driver decisions are simulated is expected to vary across microscopic traffic simulators. It would be hence be inappropriate to apply a model estimated using a trajectory dataset adopting a given time resolution onto a simulator that works at a different unit time step. In order for this estimated model to work within the simulator’s time step, one would need to recalibrate the model parameters, in particular the alternative specific intercepts. This requirement for re-calibration before use within every new simulator can be obviated if one could explicitly estimate the effect of the time resolution within the lane changing model framework. The presence of this parameter estimate would allow an estimated model to be transferable across different simulators irrespective of the simulation time step adopted in the simulator.

7.3.4 Transferability
As mentioned in the study, another suitable dataset could not be found to serve for the validation task. Hence, an unsampled version of the arterial dataset had to be used instead. One of the key concerns about microscopic behavior models is the applicability of their parameters to different traffic scenarios. It would be interesting to conduct systematic and rigorous transferability tests that would help identify the parameters whose estimates within the model framework stay consistent across different traffic situations. Availability of different datasets representing arterial traffic would permit the execution of this task. This would provide an empirical basis for identifying model parameters that require calibration as the application scenario changes.

Another interesting aspect that was not tested during the course of this study was the extensibility of the model to freeway dataset. The extended 3-level model is expected to be a generalized version of the original model. This claim can be tested by assessing its performance on freeway datasets on whom the original 2-level model was previously applied and tested.
APPENDIX A

INTERSECTION LANE-SELECTION MODEL

The intersection lane-choice model involves the lane selection of drivers entering the arterial from a side street (Figure A.1). The model also is applicable for lane selection of drivers entering the side street from arterials. It should be noted that since vehicles traveling from an arterial link to another arterial link are not allowed to make lane changes within the intersection, the intersection model is not applicable there.

Figure A.1: Intersection Lane Selection

A.1 Modeling Framework

The intersection lane-selection model consists of two steps: choice of target lane and choice of immediate lane. The structure of the model is shown in Figure A.2.

The first step in the decision process is latent since the target-lane choice is unobservable and only the driver's actual chosen lanes are observed. Latent choices are shown as ovals; observed ones are shown as rectangles.
The target lane is the lane the driver perceives as best to be in considering the attributes of the lane and the path plan considerations. However, the driver may not be able to maneuver to his target lane immediately after crossing the intersection and the observed lane of the driver may be different from his target lane.

The choice of target lane is a tactical decision of the driver whereas the choice of immediate lane is governed by maneuverability considerations.

It should be noted that once the driver enters the arterial, the mainline lane-changing model will take over and be used for predicting his decisions.

A.2 Model Structure

Target-Lane Choice

The target-lane choice of the driver can be modeled as a multinomial logit (MNL) model. The target-lane choice is significantly affected by the planning capability of the driver and his familiarity with the network. The choice set of the driver can thus depend on driver characteristics. The drivers can belong to either of the two classes:
• **Class 1 - Myopic Drivers** – Those who consider the immediate section only (Figure A.3); and

• **Class 2 - Drivers Who Plan Ahead** – Those who consider more than one section ahead (Figure A.4).

Variables associated with the target lane of the driver also may vary depending on the driver class.

**Figure A.3: Perspective of Myopic Drivers**

**Figure A.4: Perspective of Drivers who Plan Ahead**

The probability that driver \( n \) selects lane \( l \) as the target lane, conditional on individual-specific characteristics, can be expressed as follows:

\[
P_n(l | v, \tau) = \frac{\exp(\beta X_{nl}^l(\tau_n) + \alpha' v_n)}{\sum_{j \in C_n} \exp(\beta X_{nj}^l(\tau_n) + \alpha' v_n)}
\]  
(A.1)
where,

\[ X'_{nt} = \text{attributes of lane } l \text{ for vehicle } n \text{ at time } t, \text{ can be function of } \tau_n \]

\[ \tau_n = \text{individual specific lookahead distance} \]

\[ \beta = \text{coefficients} \]

\[ \nu_n = \text{individual specific random effect, } \nu_n \sim N(0,1) \]

\[ \alpha' = \text{coefficient of individual specific random effect for lane } l \]

Variables likely to influence the target-lane choice of the driver include:

- **Path Plan Variables** – Distance to the point when the driver needs to be in a specific lane to follow his path, and the number of lane changes required to be in the correct lane;

- **Lane Attributes** – Queue lengths, average speeds, and queue discharge rates; and

- **Driving Style and Capabilities** – Individual driver/vehicle characteristics, such as the look-ahead distance of the driver and aggressiveness of the driver.

These influencing variables can differ among drivers in the same intersection with the same path-plan, depending on their network knowledge and experience.

**Immediate-Lane Choice**

The immediate-lane choice of the driver also can be modeled as an MNL model. The immediate-lane choice is affected by the driving effort needed to reach a particular lane and maneuverability considerations, and is conditional on the choice of target lane.

The probability that driver \( n \) selects lane \( i \) as the immediate lane, conditional on target lane \( l \) and individual-specific characteristics, can be described as follows:

\[
P_n(i \mid l, \nu) = \frac{\exp(\beta^T X'_{ni} + \alpha' \nu_n)}{\sum_{k \in C_l} \exp(\beta^T X'_{nk} + \alpha' \nu_n)}
\]  

(A.2)

where,

\[ X'_{ni} = \text{attributes of lane } i \text{ for vehicle } n \text{ at time } t \]

\[ \alpha' = \text{coefficient of individual specific random effect for lane } i \]

Variables likely to influence the immediate-lane choice of the driver include:

- Current position of the driver: Proximity of a given lane to the receiving lane closest to the driver;
• Neighborhood variables: Presence of other vehicles and their actions, relative position and speed of the subject vehicle with respect to vehicles surrounding it, geometric elements of the roadway, signals and signs, and available capacity of the lane; and

• Driving style and capabilities: Individual driver/vehicle characteristics, such as the aggressiveness of the driver and performance capabilities of the vehicle (e.g., required turning radius).
APPENDIX B

Acceleration Models in MITSIM

The acceleration models implemented in MITSIM (Ahmed, 1999) are described in this section.

MITSIM considers two acceleration regimes: free-flow and car following. The free flow acceleration regime, in which the vehicle travels at his/her desired maximum speed, prevails when there is no lead vehicle or the lead vehicle in front is far enough ahead that it has no impact on the subject vehicle. The free-flow acceleration model has the following form:

\[ a_n^{\text{ff}}(t) = \hat{\lambda}^{\text{ff}} \times \left[ X_n^{\text{DS}}(t - \tau_n) \beta^{\text{DS}} - V_n(t - \tau_n) \right] + \varepsilon_n^{\text{ff}}(t) \]  

(B.1)

Where, \( a_n^{\text{ff}} \) is the free flow acceleration of driver \( n \) at time \( t \), \( \hat{\lambda}^{\text{ff}} \) is the constant sensitivity, \( X_n^{\text{DS}}(t - \tau_n) \) is the vector of explanatory variables affecting the desired speed, \( \tau_n \) is the reaction time of the driver, \( \beta^{\text{DS}} \) is the corresponding coefficient, \( [V_n(t - \tau_n)] \) is the current speed of the driver at time \( t - \tau_n \), \( \varepsilon_n^{\text{ff}}(t) \) is the random term associated with free flow acceleration.

The model was estimated to contain the following behavioral parameters:

\[ a_n^{\text{ff}}(t) = \beta_{\text{sens}} \cdot \left[ \alpha + \beta_{\text{frs}} \cdot V_n^{\text{front}}(t - \tau_n) - \beta_{\text{h}} \cdot \delta_{n}^{\text{heavy}} + \beta_{d} \cdot \delta[k_n(t - \tau_n)] - V_n(t - \tau_n) \right] + \varepsilon_n^{\text{ff}}(t) \]  

(B.2)

Where,

- \( \beta_{\text{sens}} \) is the sensitivity constant, \( \alpha \) is a constant, \( V_n^{\text{front}}(t - \tau_n) \) is the front vehicle speed at time \( t - \tau_n \),
- \( \delta_{n}^{\text{heavy}} \) is the heavy vehicle dummy, \( \delta[k_n(t - \tau_n)] \) is the indicator for density.
If the headway is less than the threshold, the car-following model dictates acceleration decisions when a lead vehicle is near enough to the subject vehicle that the subject must accelerate or decelerate to maintain a safe following distance. The car-following acceleration (when the relative speed is positive, deceleration when the relative speed is negative) is shown in Equation B.3.

\[ a_n^{cf} (t) = s \left[ X_n^{cf} \left( t - \tau_n \right) \right] f \left[ \Delta V_n (t - \tau_n) \right] + \varepsilon_n^{cf} (t) \]  

(B.3)

Where,

\[ s \left[ X_n^{cf} \left( t - \tau_n \right) \right] \]  

is sensitivity, a function of a vector of explanatory variables affecting the car following acceleration sensitivity at time \( t - \tau_n \), \( f \left[ \Delta V_n (t - \tau_n) \right] \) is the stimulus, a function of relative speed between front vehicle and subject vehicle at time \( t - \tau_n \) and \( \varepsilon_n^{cf} (t) \) is the random term associated with car following acceleration of driver \( n \) at time \( t \).

The model was estimated to contain the following behavioral parameters:

\[ a_n^{cf, acc} (t) = \alpha \cdot \frac{V_n(t)^{\beta_1}}{\Delta X_n(t)^{\beta_2}} \cdot k_n(t)^{\beta_3} \left| \Delta V_n (t - \tau_n) \right|^{\beta_4} + \varepsilon_n^{cf, acc} (t) \]  

(B.4)

\[ a_n^{cf, dec} (t) = \alpha \cdot \frac{1}{\Delta X_n(t)^{\beta_2}} \cdot k_n(t)^{\beta_3} \left| \Delta V_n (t - \tau_n) \right|^{\beta_4} + \varepsilon_n^{cf, dec} (t) \]

where,

\[ \alpha \]  

is a constant, \( V_n(t) \) is the subject speed at time \( t \), \( \Delta X_n(t) \) is the space headway at time \( t \), \( k_n(t) \) is an indicator for density ahead of subject and \( \Delta V_n(t - \tau_n) \) is the difference between the front vehicle speed and the subject speed.
The estimation results of the model structure using the trajectory data from Arlington, VA (Toledo 2003) are presented in Table B.1.
Table B.1: Estimation results for the acceleration model

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<tr>
<th>Variable</th>
<th>Parameter value</th>
<th>t-statistic</th>
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<td>Speed, m/sec.</td>
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<td>Space headway, m.</td>
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<tr>
<td>Density, veh/km/lane</td>
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<tr>
<td>Relative speed, m/sec.</td>
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<tr>
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<td>Relative speed, m/sec.</td>
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<tr>
<td>( \ln(\sigma_{h}) )</td>
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These parameters were used in the current study. Some of them (acceleration and deceleration constants) were recalibrated for the new dataset.
APPENDIX C

LANKERSHIM ORIGIN-DESTINATION DATA

For validation, exact vehicle O-D flows were calculated using the Lankershim trajectory data. The O-D files can be downloaded from: http://mit.edu/its/papers/OD_Lankershim.zip

The node locations and numbering associated with the O-D files are presented in Figure C.1, which is a reproduction of Figure 4.2. Summary of OD is given in Tables C.1 and C.2.
Figure C.1: Node Locations and Numbering for Lankershim Boulevard
### Table C.1: Vehicle Distribution by O-D Pair

**Period 8:28 A.M. to 8:45 A.M.**

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Source: NGSIM Data Analysis Report – Cambridge Systematics Inc.

### Table C.2: Vehicle Distribution by O-D Pair

**Period 8:45 A.M. to 9:00 A.M.**

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Source: NGSIM Data Analysis Report – Cambridge Systematics Inc.
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