Philosophical issues

1. Mathematically, continuous and discrete random variables are very different.
2. *Quantitatively* , however, some continuous models are very close to some discrete models.
3. Therefore, which kind of model to use for a given system is a matter of *convenience* .

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Continuous random variables

Philosophical issues

Example: The production process for small metal parts (nuts, bolts, washers, etc.) might better be modeled as a continuous flow than a large number of discrete parts.

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- Continuous random variables can be defined
  - ⋆ in one, two, three, ..., infinite dimensional spaces;
  - ⋆ in finite or infinite regions of the spaces.
- Continuous random variables can have
  - ⋆ probability measures with the same dimensionality as the space;
  - ⋆ lower dimensionality than the space;
  - ⋆ a mix of dimensions.

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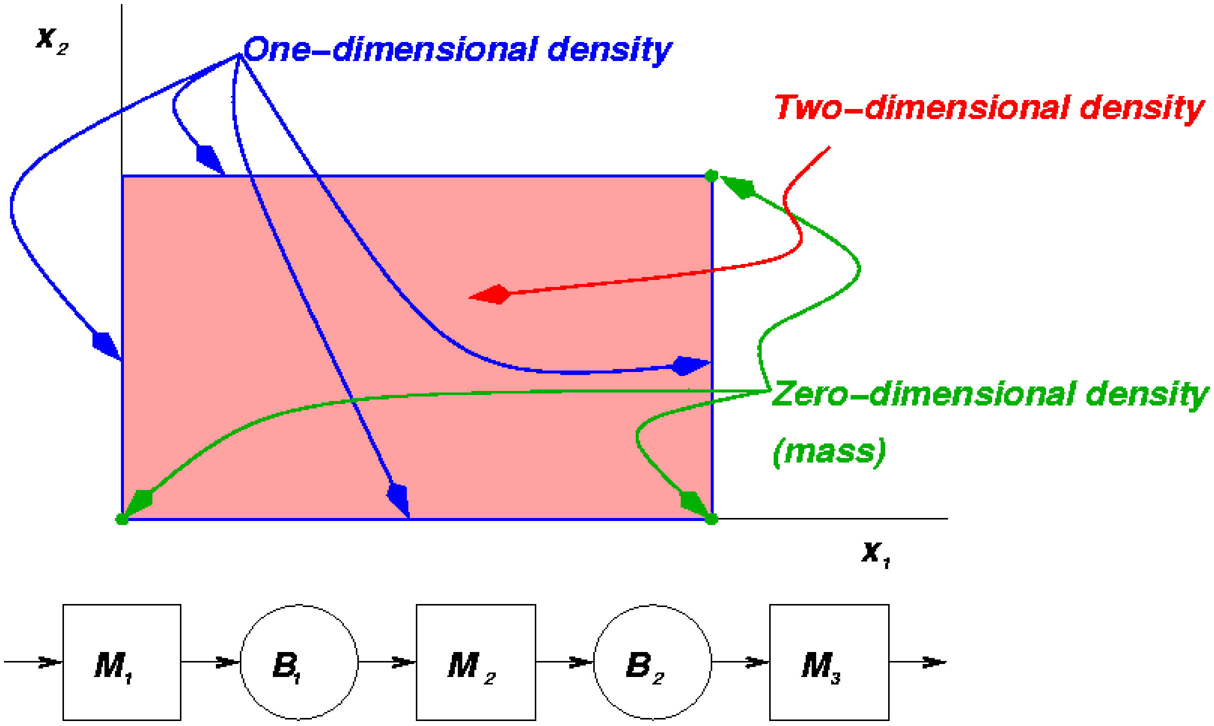

Probability distribution of the amount of material in each of the two buffers.

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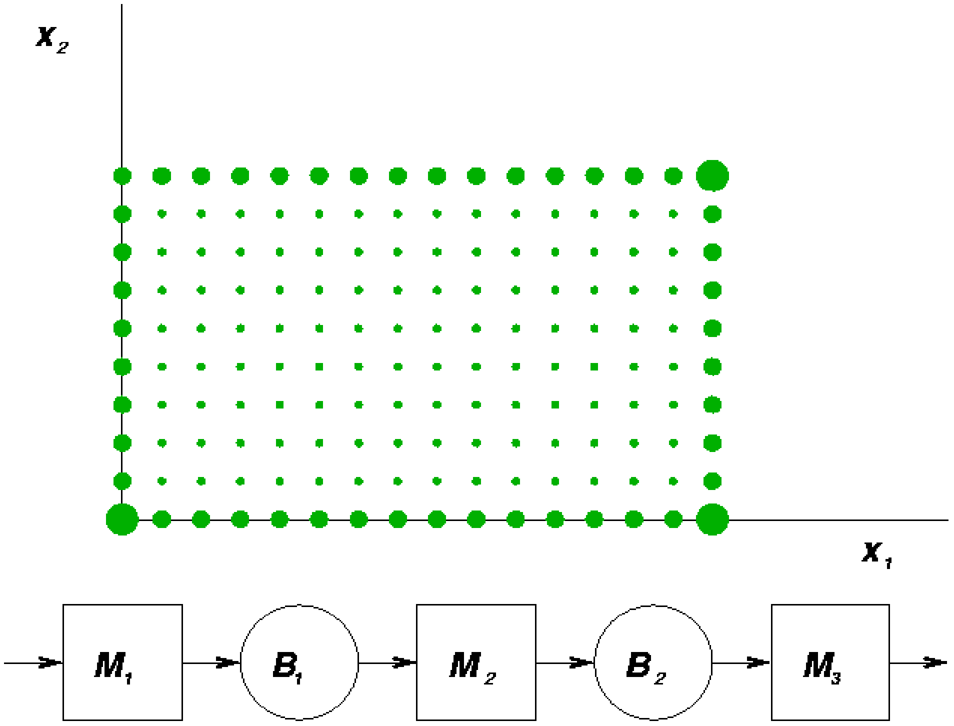
Discrete approximation



Probability distribution of the amount of material in each of the two buffers.

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Continuous Random Variables

cumulative distribution function (cdf) is

$$F(t) = P(X \le t) \text{ for all } t$$

probability density function (pdf) is

$$f(t) = \frac{dF(t)}{dt}$$

$$\Pr(a \le X \le b) = \int_a^b f(t)dt = F(b) - f(a)$$

E[X] **and** ***VAR(X)*** **are similar to discrete case, except you replace sums by integrals**

$$E(X) = \int xf(x)dx$$

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Example:

IQ test scores are normally distributed with $\mu = 100$ and $\sigma = 10$

What is $P(X > 125)$?

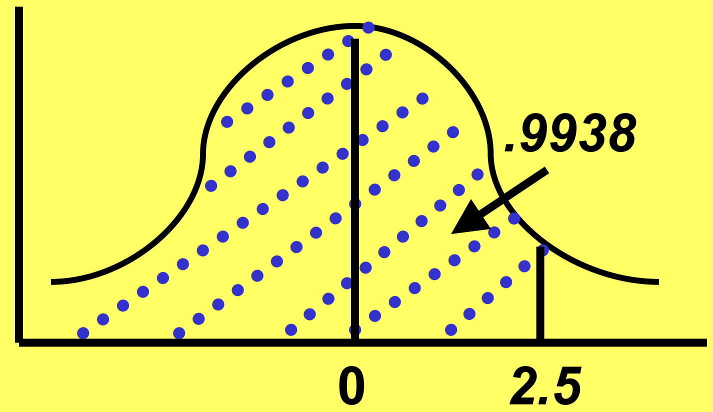
$$= P\left(\frac{X - 100}{10} > \frac{125 - 100}{10}\right)$$

$$= P(Z > 2.5)$$

$$= 1 - P(Z \leq 2.5)$$

$$= 1 - .9938$$

$$= .0062$$



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Example:

Manufacturing cycle times are normal with $\mu = 100$ days and $\sigma = 10$ days

You want to quote delivery lead times (= delivery date - current date) so that you achieve 90% on-time delivery

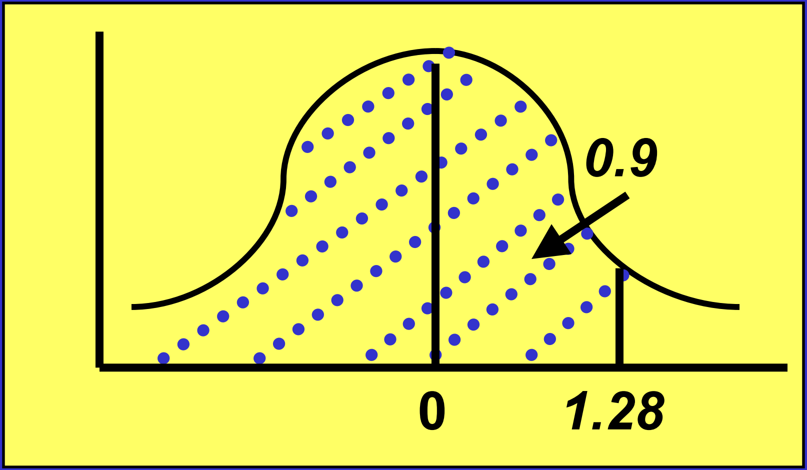
Q: What delivery lead time should you quote?

Choose x so that $P(X > x) = .1$

or
$$P\left(\frac{X - 100}{10} > \frac{x - 100}{10}\right) = .1$$

$$P\left(Z > \frac{x - 100}{10}\right) = .1$$

or
$$P\left(Z \le \frac{x - 100}{10}\right) = .9$$



$P(Z \le 1.28) = .9$ so $\frac{x - 100}{10} = 1.28$ $X = 112.8$

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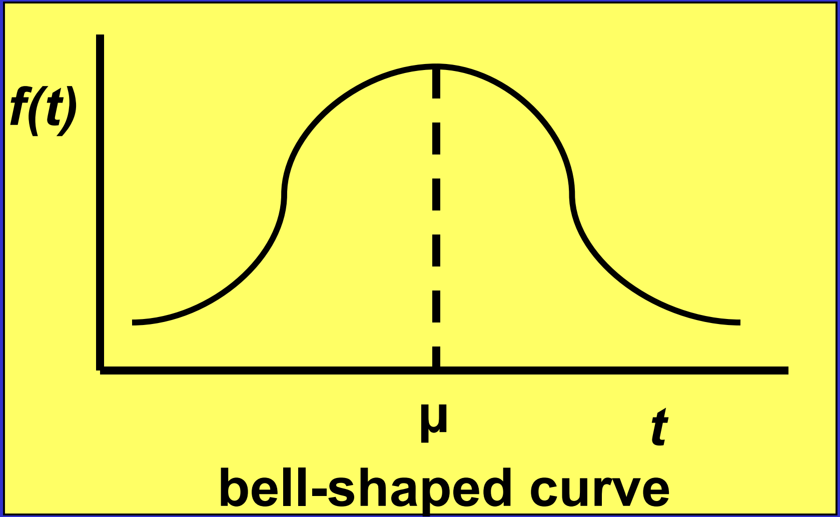
Normal (or Gaussian) Distribution



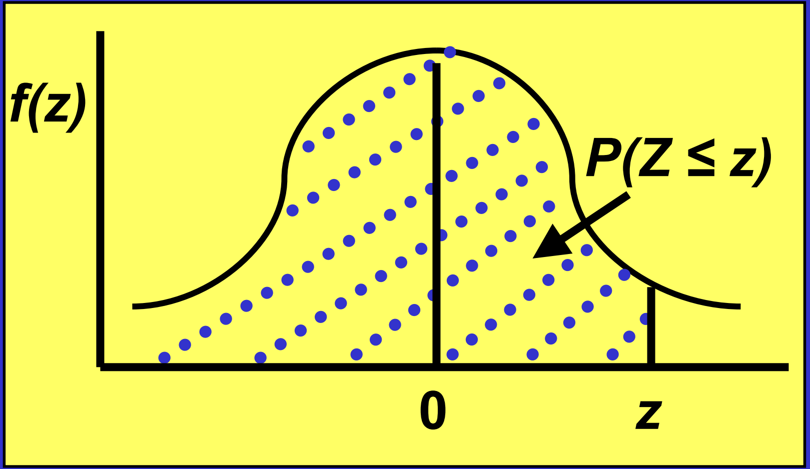
$$f(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(t-\mu)^2 / 2\sigma^2}$$

mean = μ variance = σ^2

X is a $N(\mu,\sigma)$ random variable

Fact: if X and Y are normal, so is $aX+bY+c$

Statistics books have tables for $Z = N(0,1)$



Fact: if X is $N(\mu,\sigma)$, then $\frac{X-\mu}{\sigma}$ is $N(0,1)$

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Central Limit Theorem

$X_1, \ldots, X_n$ are independent and identically distributed with

$E[X_i] = \mu \quad VAR(X_i) = \sigma^2$

X_i's are not normal!

Let $S_n = X_1 + \ldots + X_n$

Central Limit Theorem for sum: if n is large, then S_n is approximately normal with mean $n\mu$ and standard deviation $\sigma\sqrt{n}$

Let $m_n = \dfrac{X_1 + \ldots + X_n}{n}$

Central Limit Theorem for mean: if n is large, then m_n is approximately normal with mean μ and standard deviation $\dfrac{\sigma}{\sqrt{n}}$

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Dice Graphs

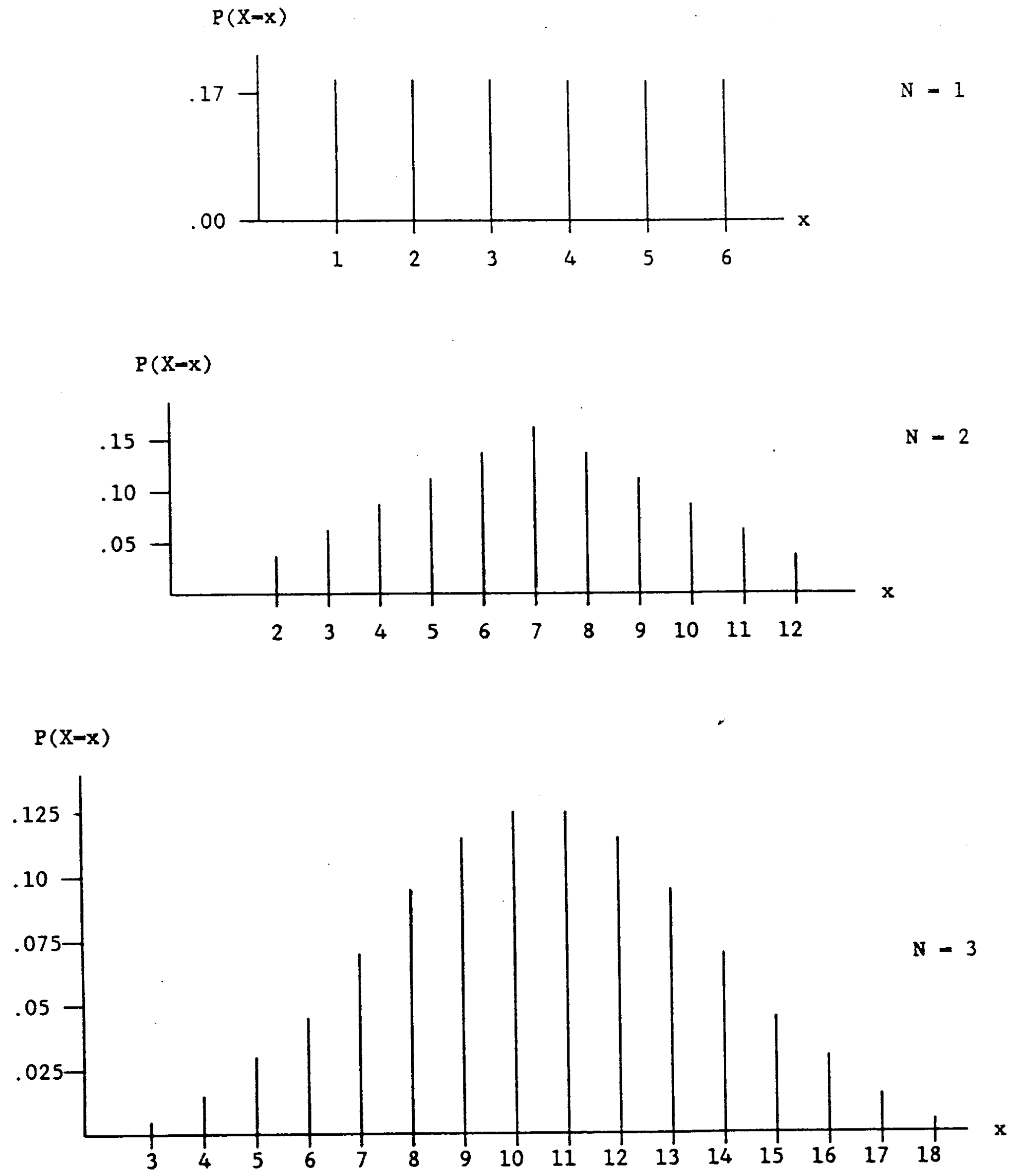

Figure 1. Probability distribution function for the sum of numbers, X , obtained when rolling N dice [N = 1, 2, 3] .

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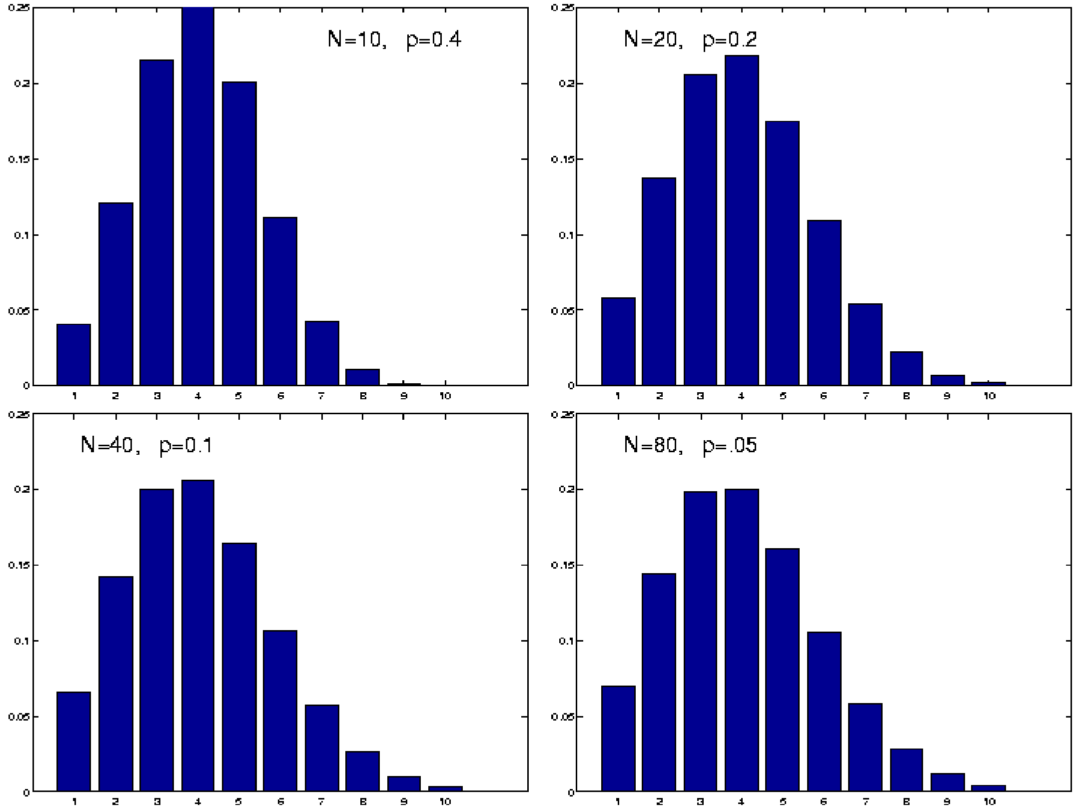
Binomial distributions

Why are these distributions so similar?



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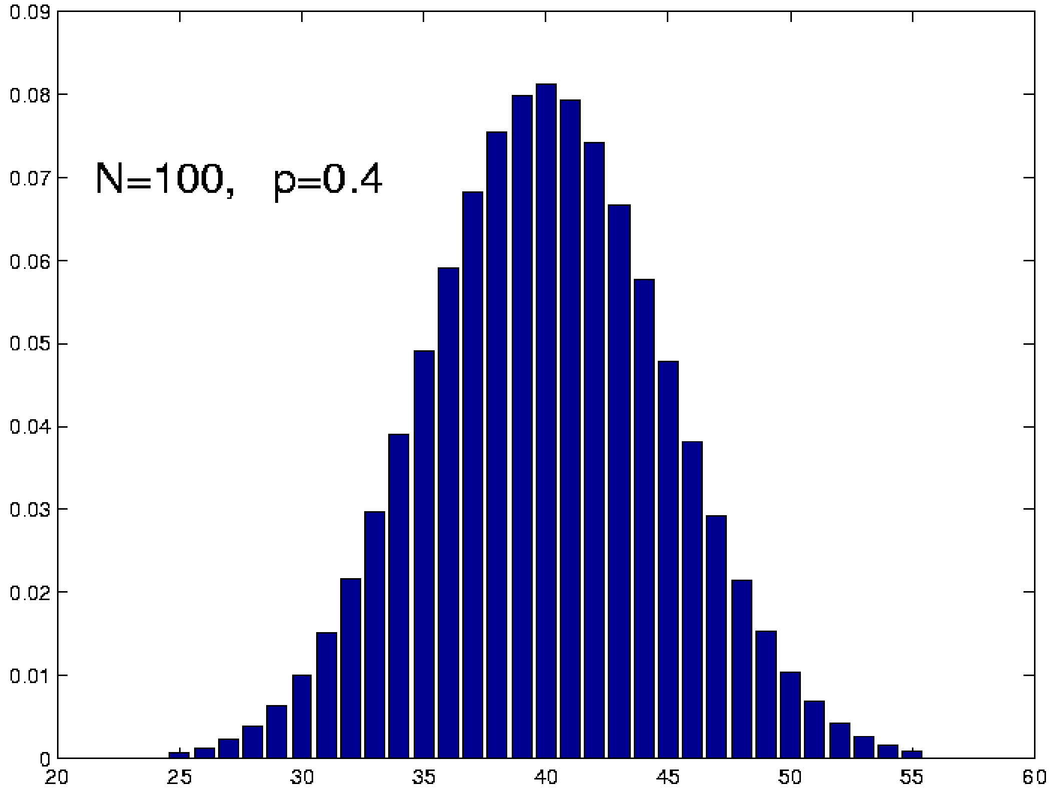
Binomial distributions

Binomial for large N approaches normal.



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Normal distribution has 3 uses

1) Models many physical processes

2) Sum of normal random variables

3) Sum or mean of many iid random variables

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