A random variable is a function that assigns a numerical value to each possible outcome of a probability experiment

Example 1: X = **the number that comes up on the roll of a dice**

Example 2: X = **the number of heads in 3 flips of a coin**

Outcome	Probability	Value of X
(h,h,h)	.125	X = 3
(h,h,t)	.125	X = 2
(h,t,h)	.125	X = 2
(h,t,t)	.125	X = 1
(t,h,h)	.125	X = 2
(t,h,t)	.125	X = 1
(t,t,h)	.125	X = 1
(t,t,t)	.125	X = 0

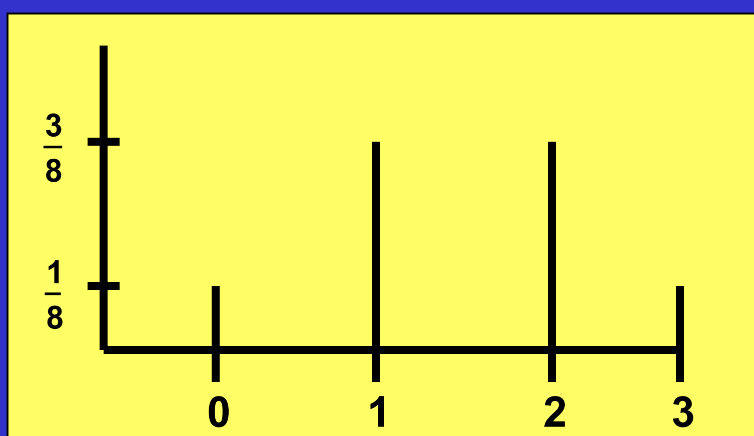
Random variables are either

- **discrete:** can only assume a finite set of values
- **continuous:** can take on any value within some interval of real numbers

Today: discrete random variables
Next Class: continuous random variables

The probability distribution of a discrete random variable X is the possible values $x_1, \ldots, x_n$ **and corresponding probabilities** $p_i, \ldots, p_n$ **where** $\sum_{i=1}^{n} p_i = 1$

Example 3:



Number of heads in 3 flips

Summary Measures

Mean =

E [X] = expected value of X

= mean of X

= the average outcome

= μ_X

E [X] = $\sum_{i=1}^{n} x_i P(X = x_i)$

Var (x) = variance of x

= σ_x^2

= $E\left[(x - \mu_x)^2\right]$

= average squared deviation from the mean

= $\sum_{i=1}^{n} P(X = x_i)(x_i - \mu_x)^2$

Example 3: X = number of heads in 3 flips of a coin

$$E(X) = \sum_i x_i P(X = x_i) = 1(\tfrac{3}{8}) + 2(\tfrac{3}{8}) + 3(\tfrac{1}{8}) = 1.5$$

$$\sigma^2(x) = \sum_i P(X = x_i)(x_i - \mu_x)^2 =$$

$$= \frac{1}{8}(0 - 1.5)^2 + \frac{3}{8}(1 - 1.5)^2 + \frac{3}{8}(2 - 1.5)^2 + \frac{1}{8}(3 - 1.5)^2 = \frac{3}{4}$$

Variance is in "squared" units, so we take square root

$\sigma(x) = \sqrt{\sigma^2(x)} =$ **standard deviation of x**

The bigger $\sigma \Rightarrow$ the more spread out the distribution

$\Rightarrow$ the more uncertainty in the random variable

Linear Functions of a Random Variable

Y=aX+b (a and b are known numbers)

$$E[f(X)] = \sum_i p_i f(x_i)$$

$$E(Y) = \sum_i p_i (a x_i + b) = a\sum_i p_i x_i + b\sum_i p_i$$

$$= aE(X) + b$$

$$Var(Y) = E\left(aX + b - [a\mu_x + b]\right)^2$$

$$= E\left(a[X - \mu_x]\right)^2$$

$$= \sum_i p_i \left(a[x_i - \mu_x]\right)^2$$

$$= a^2 \sum_i p_i (x_i - \mu_x)^2$$

$$= a^2 Var(x)$$

Binomial Distribution

Each trial is

- a success with probability P
- a failure with probability 1- P

X= number of successes in **n** independent trials

$$P(X = x) = \frac{n!}{x!(n-x)!} p^{x} (1-p)^{n-x} \qquad x = 0, \ldots, n$$

$$E(x) = np \qquad Var(x) = np(1-p)$$

Example: **X** = number of heads in 3 tosses of a coin

$$n = 3 \quad P = \frac{1}{2}$$

$$P(X = x) = \frac{3!}{x!(3-x)!}\left(\frac{1}{2}\right)^{3}$$

$$P(X = 0) = P(X = 3) = \frac{1}{8}$$

$$P(X = 1) = P(X = 2) = \frac{3}{8}$$

Manufacturing application: X=number of defective parts in a lot

Two Random Variables

$X = x_i$ and $Y = y_i$ with probability P_i

COV(x,y)= covariance of X and Y

$$= E\left[\left(X - \mu_x\right)\left(Y - \mu_y\right)\right]$$

$$= \sum_{i=1}^{n} p_i \left(x_i - \mu_x\right)\left(y_i - \mu_y\right)$$

if X and Y are independent then COV (X,Y) = 0

if X and Y tend to vary in the same direction, then COV (X,Y) > 0

if X and Y tend to vary in the opposite direction, then COV (X,Y) < 0

$$E\left(aX + bY\right) = aE\left(X\right) + bE\left(Y\right)$$

VAR(X+Y)= VAR(X) + VAR(Y) + 2COV(X,Y)

if X and Y are independent, then *VAR(X+Y)= VAR(X)+VAR(Y)*

CORR(X,Y) = correlation of X and Y

$$= \frac{COV(X,Y)}{\sigma_x \sigma_y} \in [-1, 1]$$

is "NORMALIZED COVARIANCE"