# SMA 6304
# Factory Planning and Scheduling
# Lecture 18: Single-stage, multiple part type, MTS

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# Make to Stock

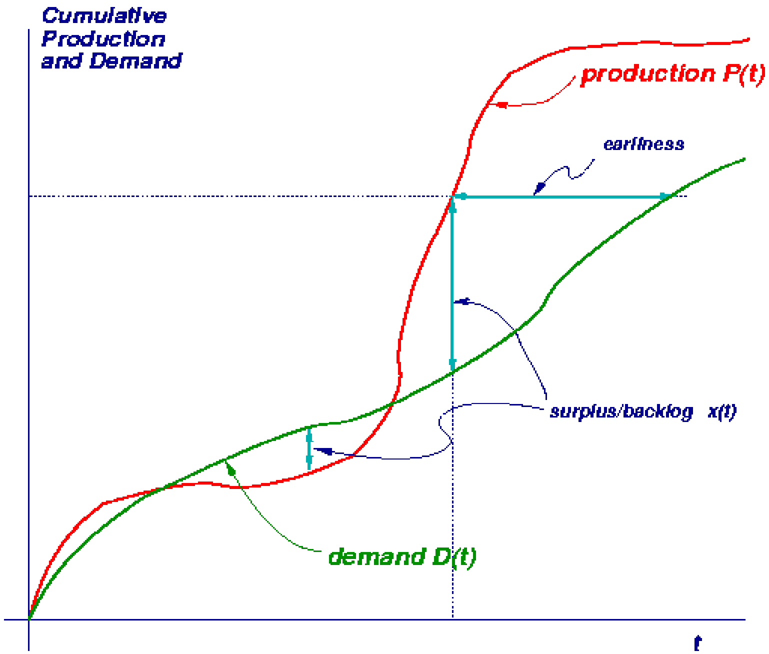
- *Ideal:* The product is available *(in stock)* when the order arrives.
- *Service rate:* the fraction of orders that are fulfilled with no waiting.
- *Costs:* Inventory, lost sales.

# Make to Stock

- *Trade-off:*
  - ⋆ The higher the service rate, the happier the customers *(so sales go up)* .
  - ⋆ The lower the service rate, the less inventory is required *(so costs go down)* .
- *The Quantification Dilemma:* Costs are easy to quantify. Benefits are not.

# Objective




Objective is to keep the cumulative production line close to the cumulative demand line.

# Setups

- *Setup:* It costs less to make a Type $i$ part after making a Type $i$ part than after making a Type $j$ part.
- Examples:
  - ⋆ Tool change
  - ⋆ Paint color change

# Setups

Setup costs can include

- Money costs, especially in labor. Also materials.
- Time, in loss of capacity and delay.
- Setup motivates *lots* or *batches:* a set of parts that are processed without interruption by setups.

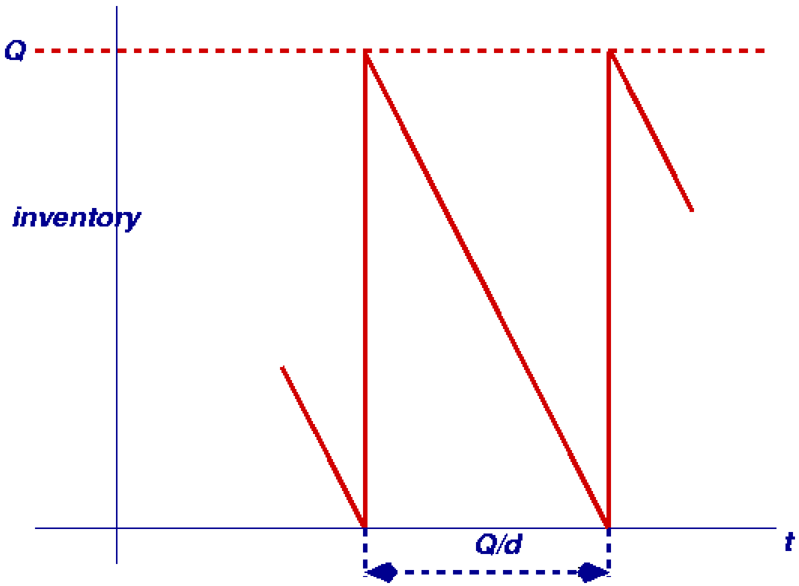
# Setups

- Money cost only.
- Each time the firm obtains a lot of $Q$ items, it must pay $A + cQ$ dollars for that lot.
  - ⋆ The unit cost is $c$, and the cost of ordering is $A$.
- The firm orders $Q$ items when its inventory level is 0, and it receives them instantly.

# Setups

## Economic Order Quantity

## (EOQ)




- Inventory is depleted at a constant rate $d$ until it is 0, at time $Q/d$.
- The average inventory level is $Q/2$.

*Holding cost*

- If $h$ is the dollar cost per time unit of holding an item in inventory, then $hQ/2$ is the average inventory holding cost per time unit.
- Over a long time interval of length $T$, the total holding cost is $hQT/2$.

*Acquisition cost*

- Over that interval, $Td$ units of material is acquired, in lot sizes of $Q$. The *number* of times it is ordered is $Td/Q$.
- The *total* cost of acquiring material is $(Td/Q)(A + cQ) = TdA/Q + Tdc$

- The total cost over the interval is

$$\frac{TdA}{Q} + Tdc + \frac{ThQ}{2} = T\left\{\frac{dA}{Q} + dc + \frac{hQ}{2}\right\}.$$

- The minimizing lot size is therefore

$$Q^{\star} = \left(\frac{2dA}{h}\right)^{\frac{1}{2}}.$$

In a random environment, EOQ can be converted into a real-time policy:

- When the inventory goes to 0, order $Q^\star$ units.

When delivery is not instantaneous, a variation is

- When the inventory goes below some level $Q_{min}$, order enough to bring it up to $Q^\star$ units.

# Setups

## Loss of Capacity

Assume

- there is one setup for every $Q$ parts ($Q$=lot size),
- the setup time is $S$,
- the time to process a part is $\tau$.

Then the time to process $Q$ parts is $S + Q\tau$. The average time to process one part is $\tau + S/Q$.

# Setups

## Loss of Capacity

If the demand rate is $d$ parts per time unit, then the demand is feasible only if

$$\tau + S/Q < 1/d \qquad \text{or} \qquad d < \frac{Q}{S + Q\tau} < \frac{1}{\tau}$$

This is not satisfied if $S$ is too large or $Q$ is too small.

# Setups

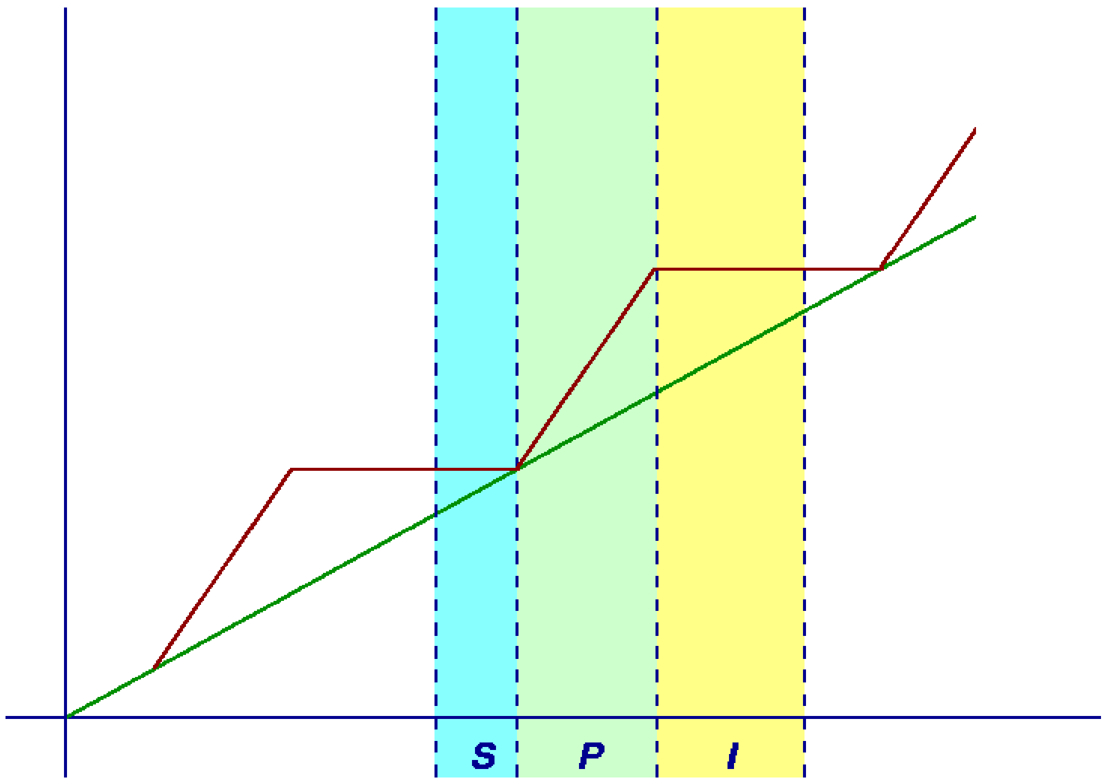
- Single part type *(simplification!)*
- Constant demand
- Deterministic setup and operation times.
- Setup/production/(idleness) cycles.
- *Policy:* Produce at maximum rate until the inventory is enough to last through the next setup time.

# Setups

Cycle:

# Setups

## Deterministic Example



# Setups

Cycle:

- *Setup period.* Duration: $S$. Production: 0. Demand: $dS$. Net change of *surplus* , ie of $P - D$ is $\Delta_S = -dS$.
- *Production period.* Duration: $t = Q\tau$. Production: $Q$. Demand: $dt$. Net change of $P - D$ is $\Delta_P = Q - dt = Q\tau(1/\tau - d) = Q(1 - d\tau)$.

# Setups

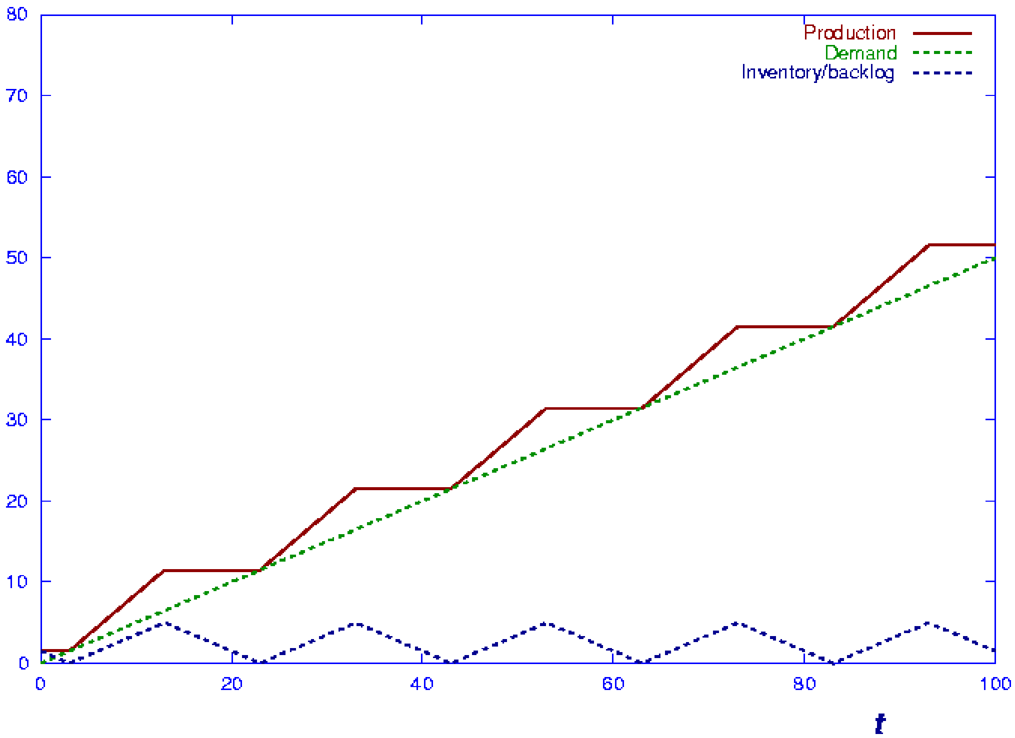
- *Idleness period.* Duration: $I$. Production: 0. Demand: $dI$. Net change of $P - D$ is $\Delta_I = -dI$.
- Total (desired) net change over a cycle: 0.
- Therefore, net change of $P - D$ over whole cycle is $\Delta_S + \Delta_P + \Delta_I = Q(1 - d\tau) - dS - dI = 0.$

# Setups

- Since $I \geq 0$, $Q(1 - d\tau) - dS \geq 0$.
- If $I = 0$, $Q(1 - d\tau) = dS$.
- If $d\tau > 1$, net change in $P - D$ will be negative.

$S = 3,\ Q = 10,\ \tau = 1,\ d = .5$

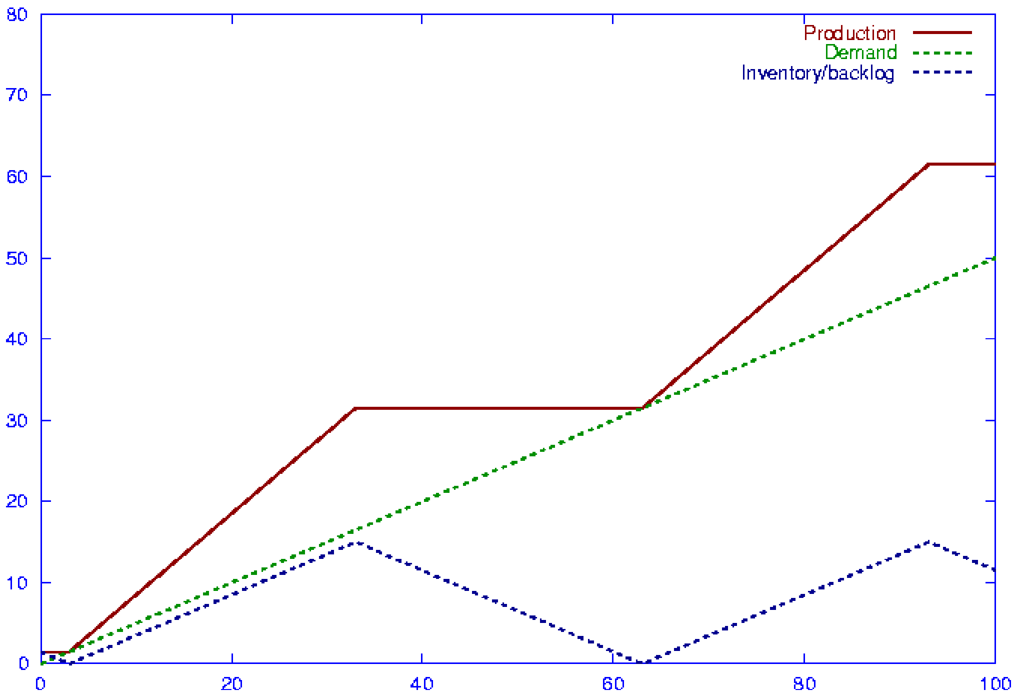



- Production period duration = $Q\tau = 10$.
- Idle period duration = 7.
- Total cycle duration = 20.
- Maximum inventory is $Q(1 - \tau d) = 5$.

# Setups

## Not frequent enough

$$S = 3, Q = 30, \tau = 1, d = .5$$



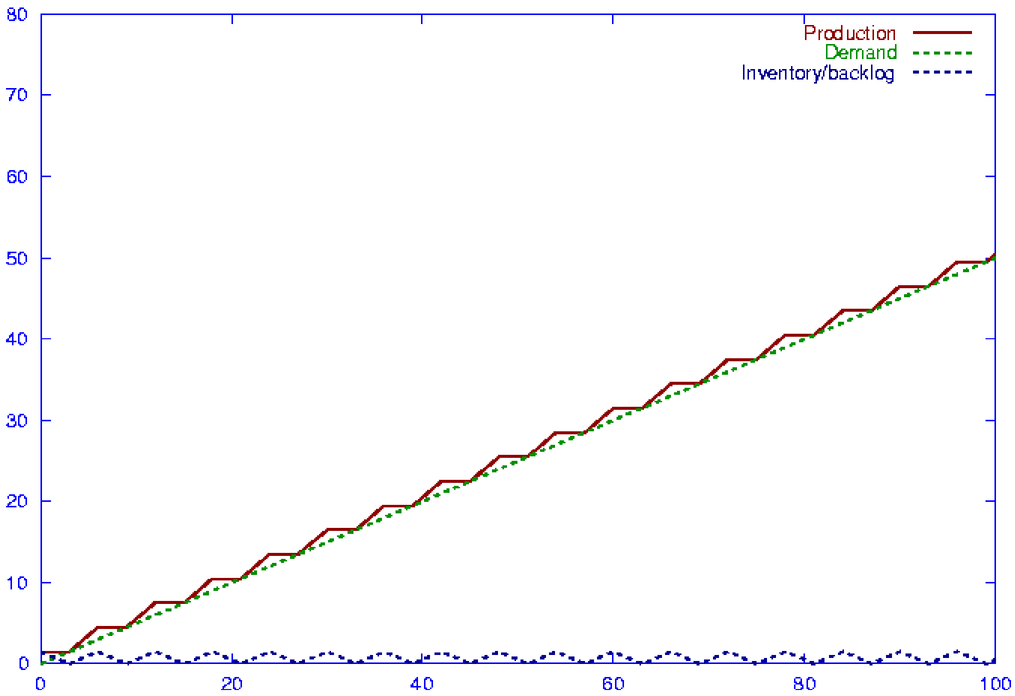

- *Large batches – big inventories.*
- Production period duration = $Q\tau = 30$.
- Idle period duration = 27.
- Total cycle duration = 60.
- Maximum inventory is $Q(1 - \tau d) = 15$.

# Setups

## Just right!

$$S = 3,\ Q = 3,\ \tau = 1,\ d = .5$$

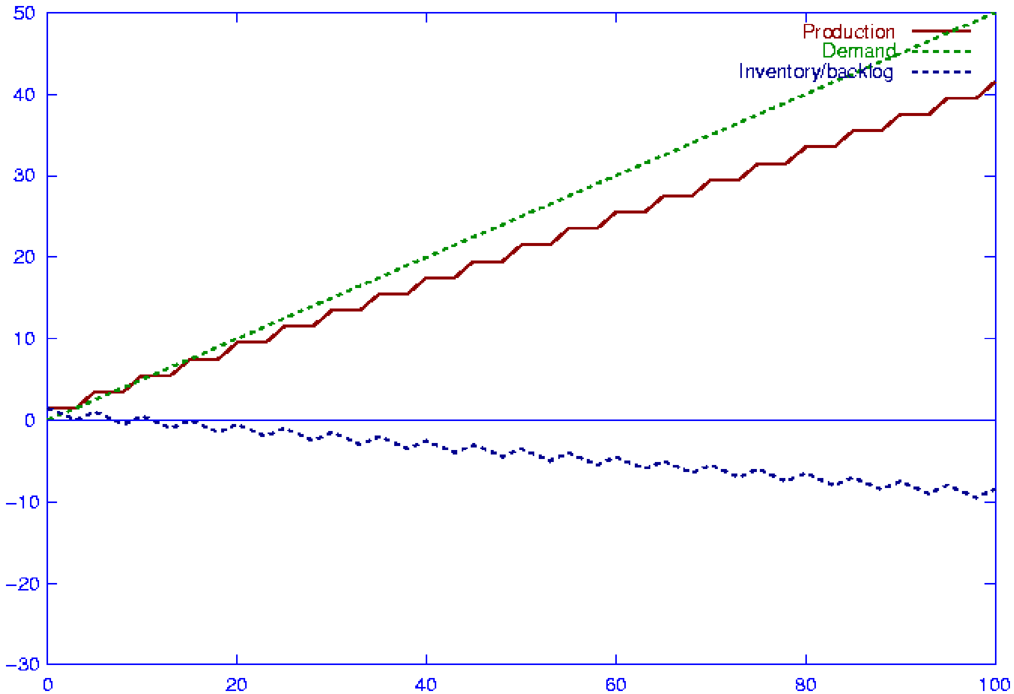



- *Small batches – small inventories.*
- Maximum inventory is $Q(1 - \tau d) = 1.5$.

## Too frequent

$S = 3$, $Q = 2$, $\tau = 1$, $d = .5$

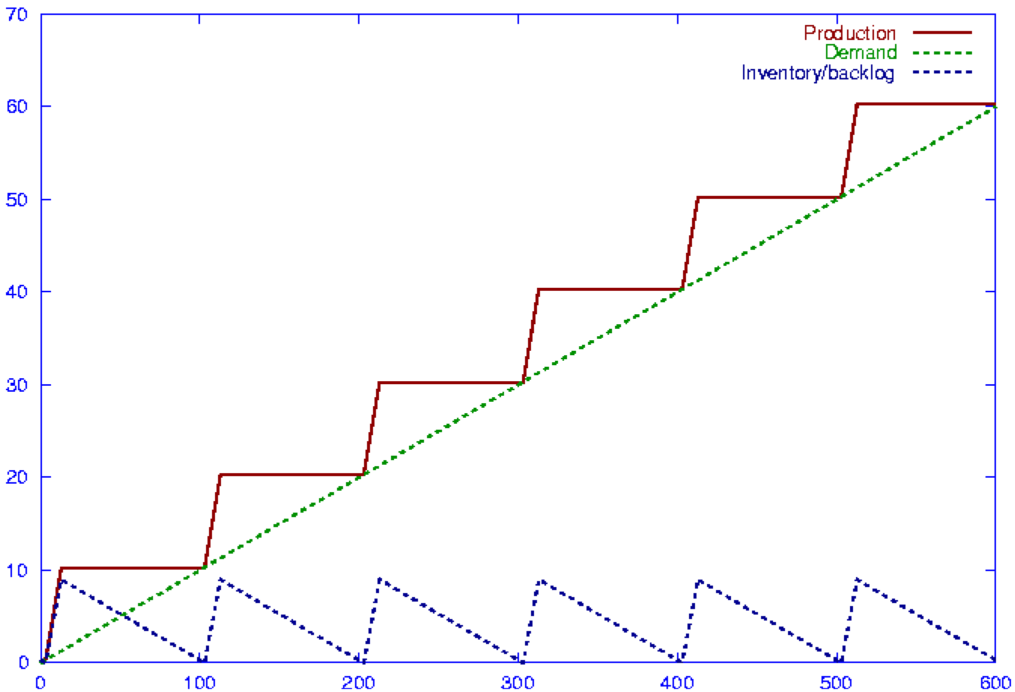



- *Batches too small – demand not met.*
- $Q(1 - d\tau) - dS = -0.5$
- Backlog grows.
- Too much capacity spent on setups.

# Setups

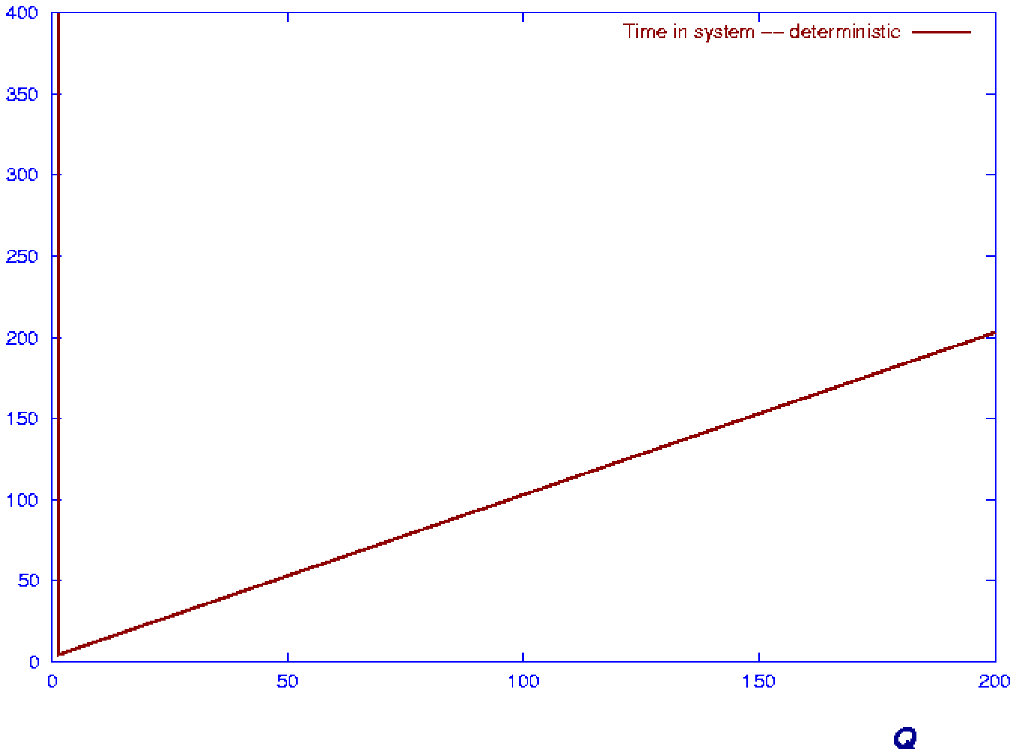
## Other parameters

$$S = 3, Q = 10, \tau = 1, d = .1$$




- Not always symmetrical.



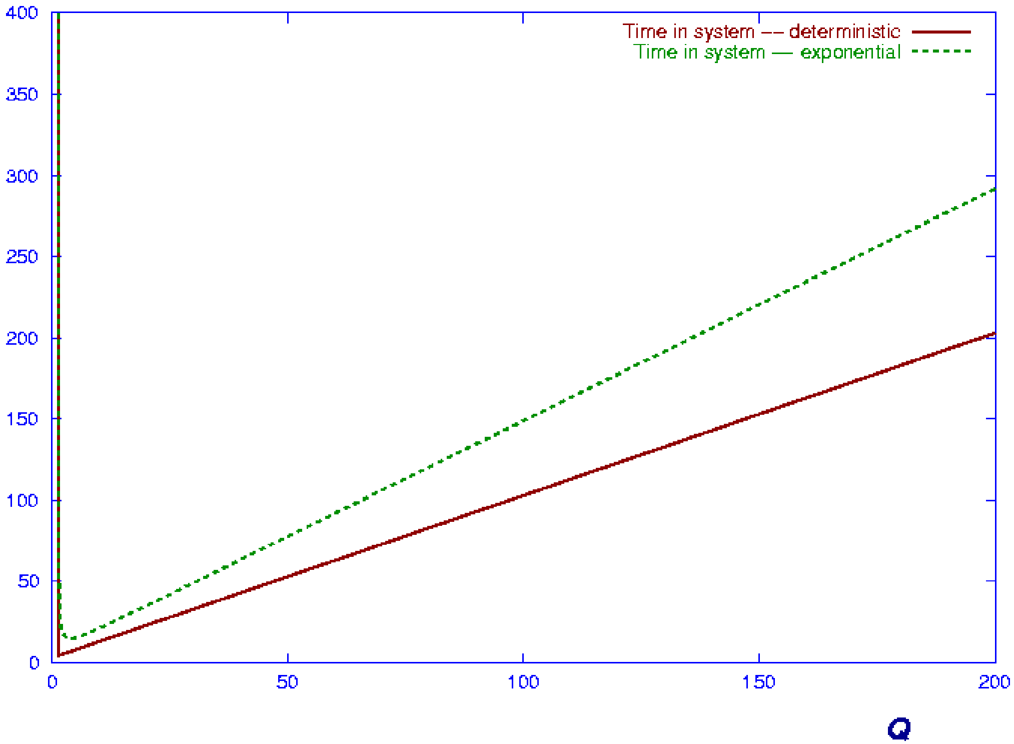

- Each batch spends $Q\tau + S$ time units in the system *if* $Q(1 - d\tau) - dS \geq 0$.
- Optimal batch size: $Q = dS/(1 - d\tau)$

# Setups

- Batch sizes equal ($Q$); processing times random.
  - ⋆ Average time to process a batch is $Q\tau + S = 1/\mu$.
- Random arrival times (exponential inter-arrival times)
  - ⋆ Average time between arrivals of batches is $Q/d = 1/\lambda$.
- Infinite buffer for waiting batches

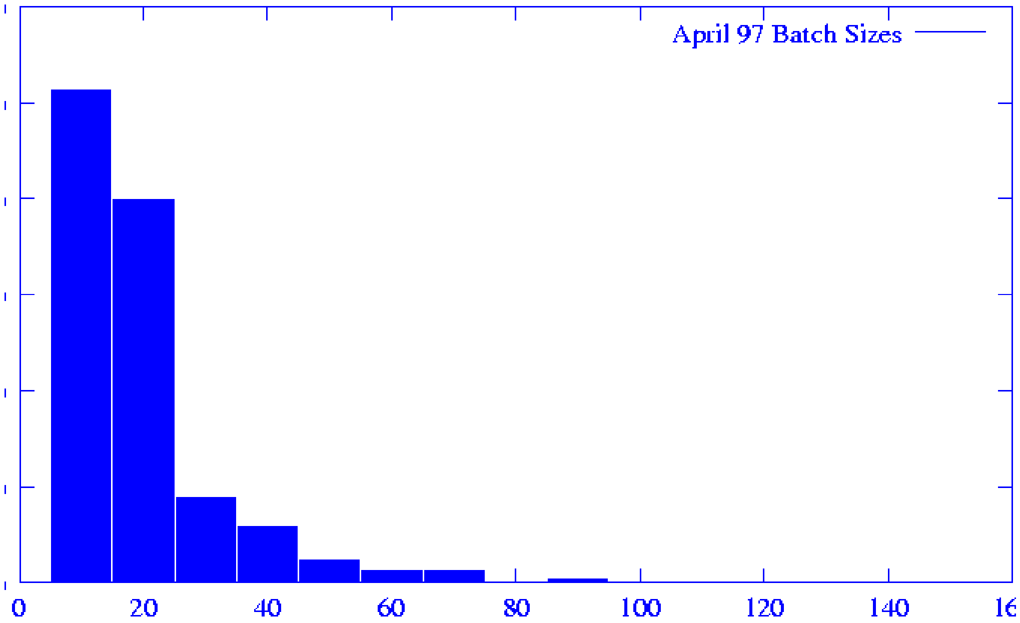
# Setups




- Treat system as an $M/M/1$ queue in batches.
- Average delay for a batch is $1/(\mu - \lambda)$.
- *Variability increases delay* .

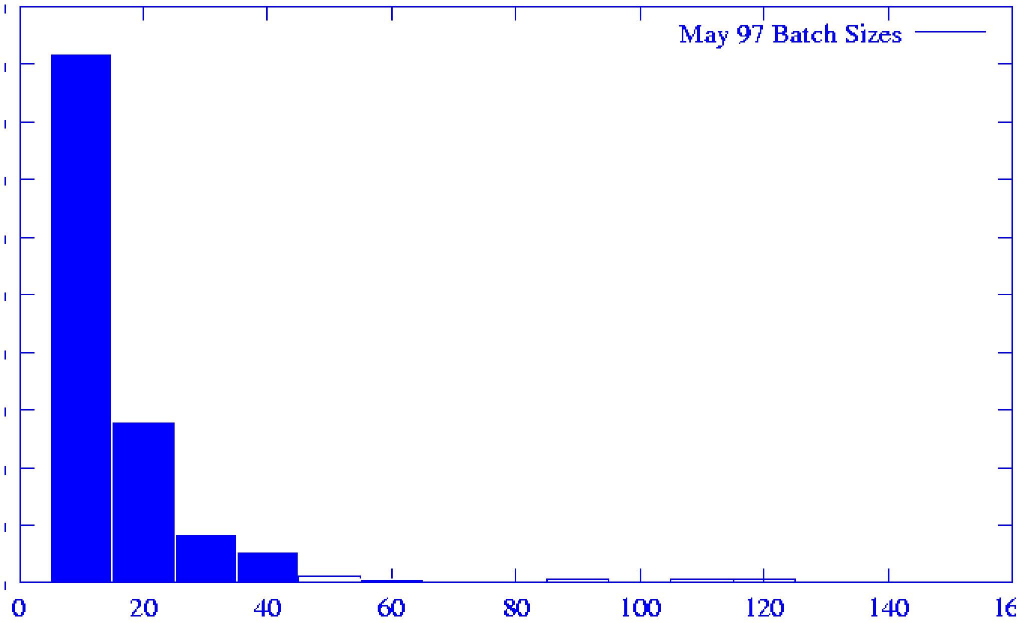
# Setups

## Batch size data from a factory



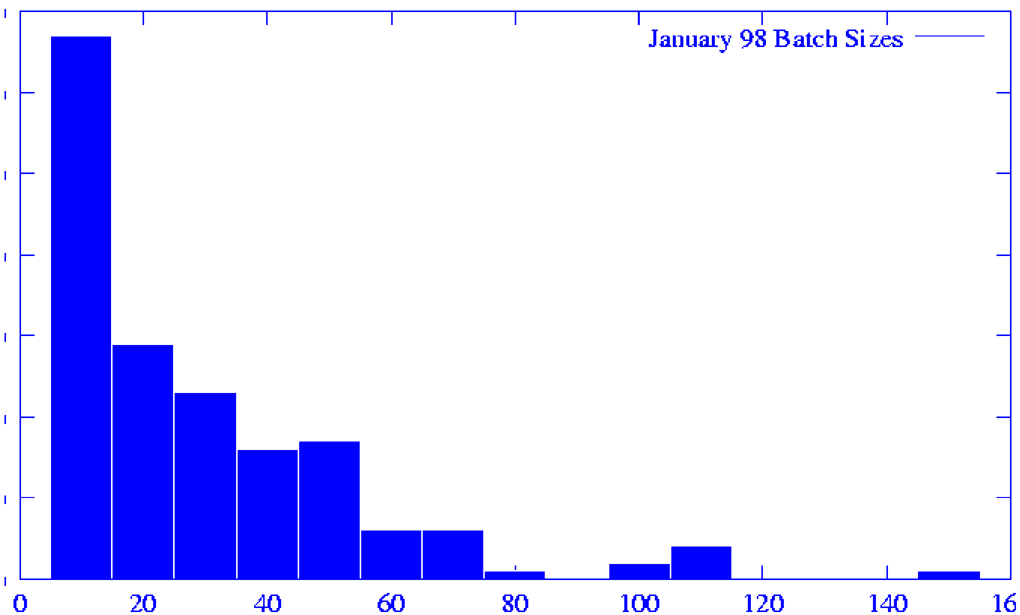
# Setups

## Batch size data from a factory



# Setups

Batch size data from a factory




Avg Lot Size=25

Std Dev=27

# Setups

- *Wagner-Whitin (1958)* problem
- Assumptions:
  - ⋆ Discrete time periods (weeks, months, etc.); $t = 1, 2, ..., T$.
  - ⋆ Known, but non-constant demand $D_1$, $D_2$, ..., $D_T$.
  - ⋆ Production, setup, and holding cost.
  - ⋆ Infinite capacity.

## Other notation

- $c_t$ = production cost (dollars per unit) in period $t$
- $A_t$ = setup or order cost (dollars) in period $t$
- $h_t$ = holding cost; cost to hold one item in inventory from period $t$ to period $t+1$
- $I_t$ = inventory at the end of period $t$ — the state variable
- $Q_t$ = lot size in period $t$ — the decision variable

minimize $\sum_{t=1}^{T}(A_t\delta(Q_t) + c_tQ_t + h_tI_t)$

(where $\delta(Q) = 1$ if $Q \geq 0$; $\delta(Q) = 0$ if $Q = 0$)

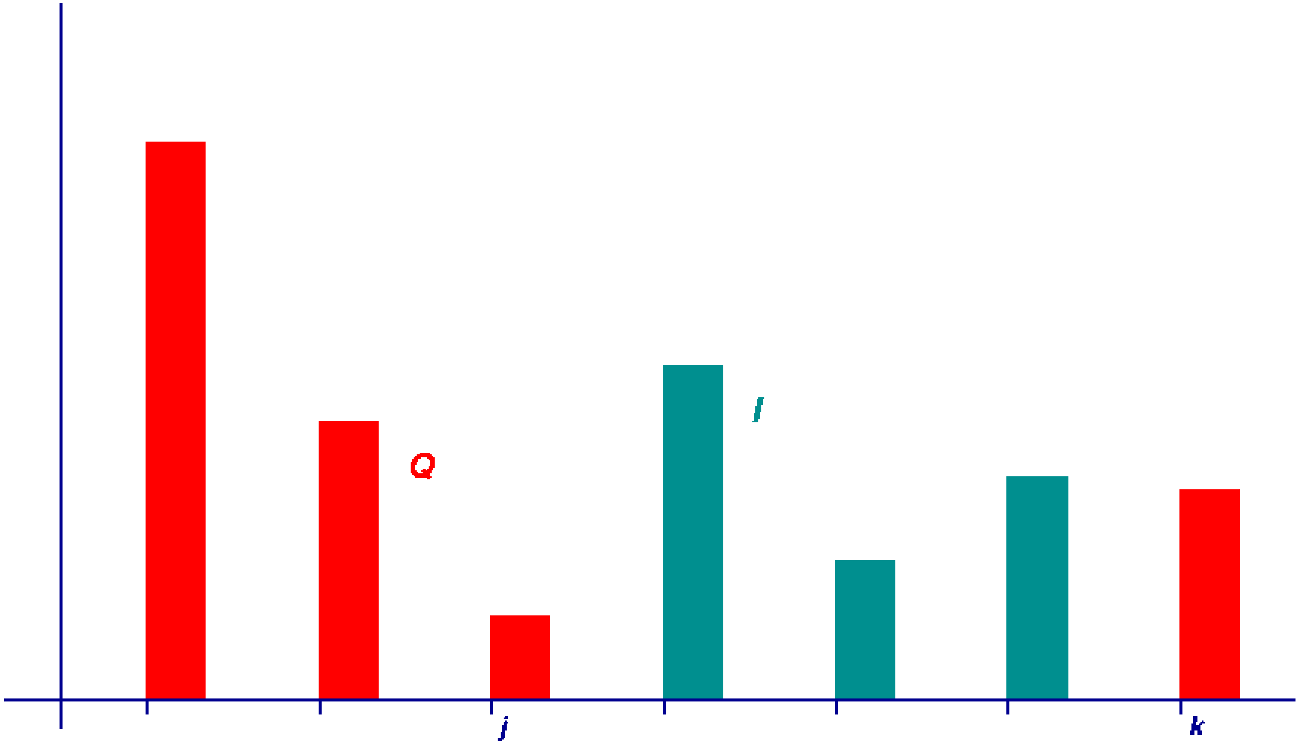
subject to

- $I_{t+1} = I_t + Q_t - D_t$
- $I_t \geq 0$

# Setups

## Wagner-Whitin Property

Characteristic of Solution:



Characteristic of Solution:

- *Either* $I_t = 0$ *or* $Q_{t+1} = 0$. *That is, produce only when inventory is zero.* Or,
  - ⋆ If we assume $I_j = 0$ and $I_k = 0$ $(k > j)$ and $I_t > 0, t = j + 1, ..., k,$
  - ⋆ then $Q_j > 0$, $Q_k > 0$, and $Q_t = 0, t = j + 1, ..., k.$

# Setups

## Wagner-Whitin Property

Then

- $I_{j+1} = Q_j - D_j$,
- $I_{j+2} = Q_j - D_j - D_{j+1}$, ...
- $I_k = 0 = Q_j - D_j - D_{j+1} - \ldots - D_k$

Or, $Q_j = D_j + D_{j+1} + \ldots + D_k$

which means *produce enough to exactly satisfy demands for some number of periods, starting now.*

- This is not enough to determine the solution, but it means that the search for the optimal is limited.
- It also gives a qualitative insight.

# Setups

- *Problem:* How to decide on batch sizes (ie, setup change times) in response to events.
- *Issue:* Same as before.
  - ⋆ Changing too often causes capacity loss; changing too infrequently leads to excess inventory and lead time.

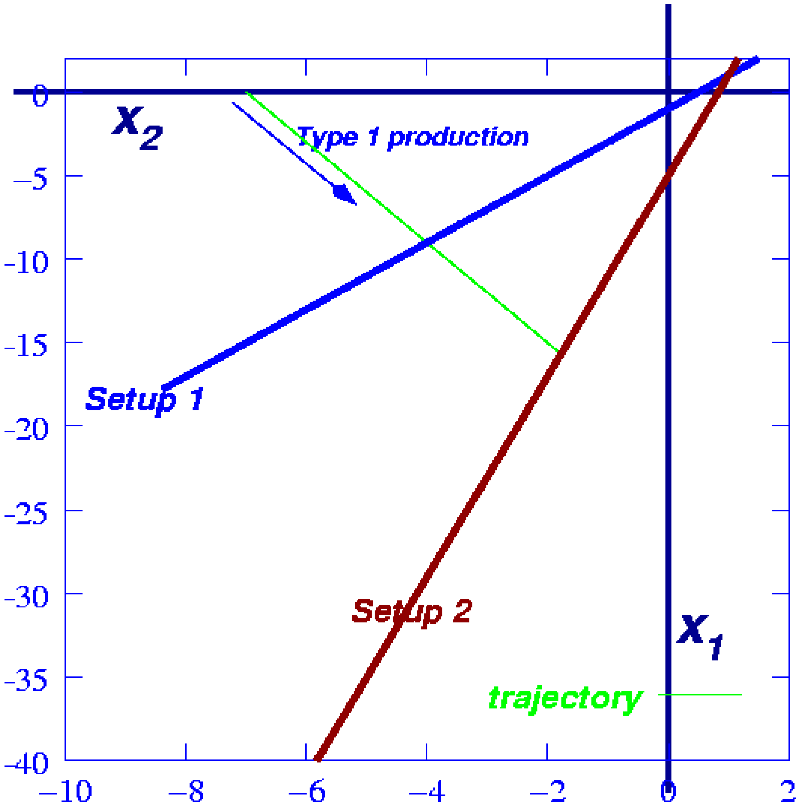
# Setups

Model:

- $d_i$ = demand rate of type $i$
- $\mu_i = 1/\tau_i$ = maximum production rate of type $i$
- $S$ = setup time
- $u_i(t)$ = production rate of type $i$ at time $t$
- $x_i(t)$ = surplus (inventory or backlog) of type $i$
- $\dfrac{dx_i}{dt} = u_i(t) - d_i, i = 1, 2$

# Setups

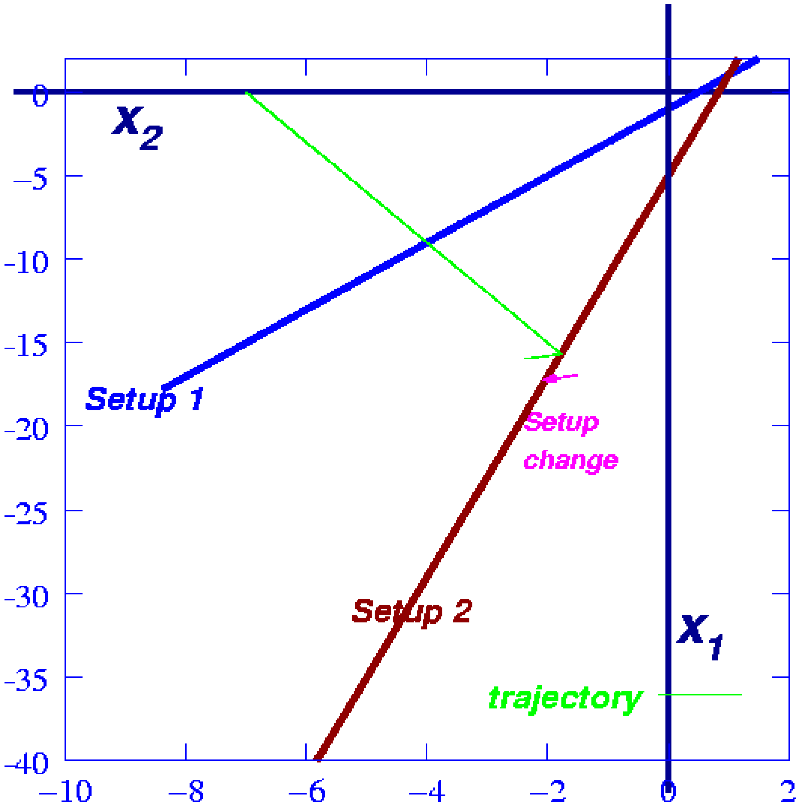
## Corridor Policy



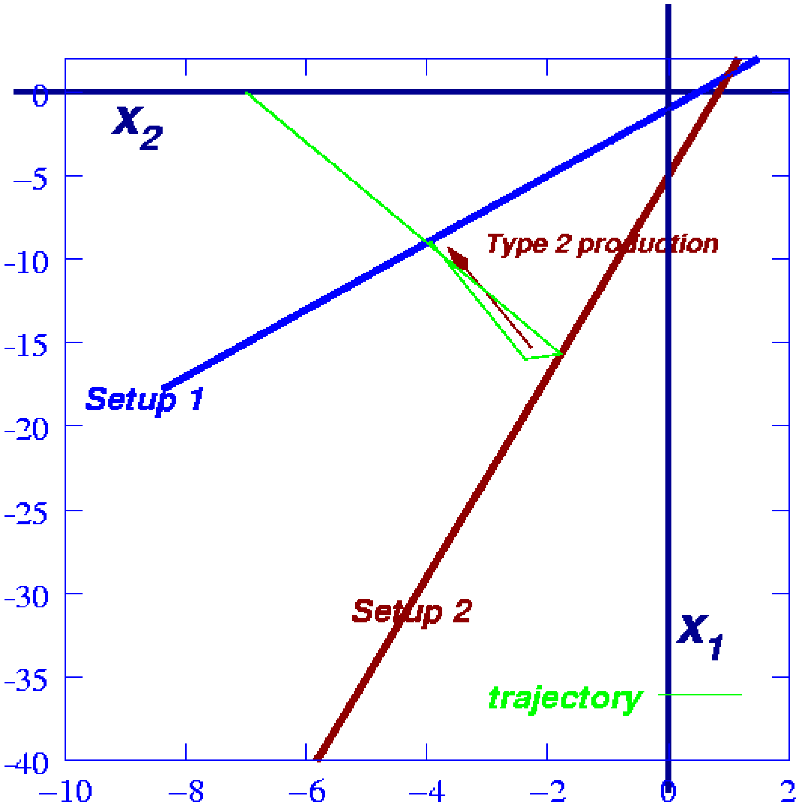

- Draw two lines, labeled *Setup 1* and *Setup 2.*
- Keep the system in setup $i$ until $x(t)$ hits the *Setup* $j$ line.
- Change to setup $j$.
- Etc.

# Setups

## Corridor Policy

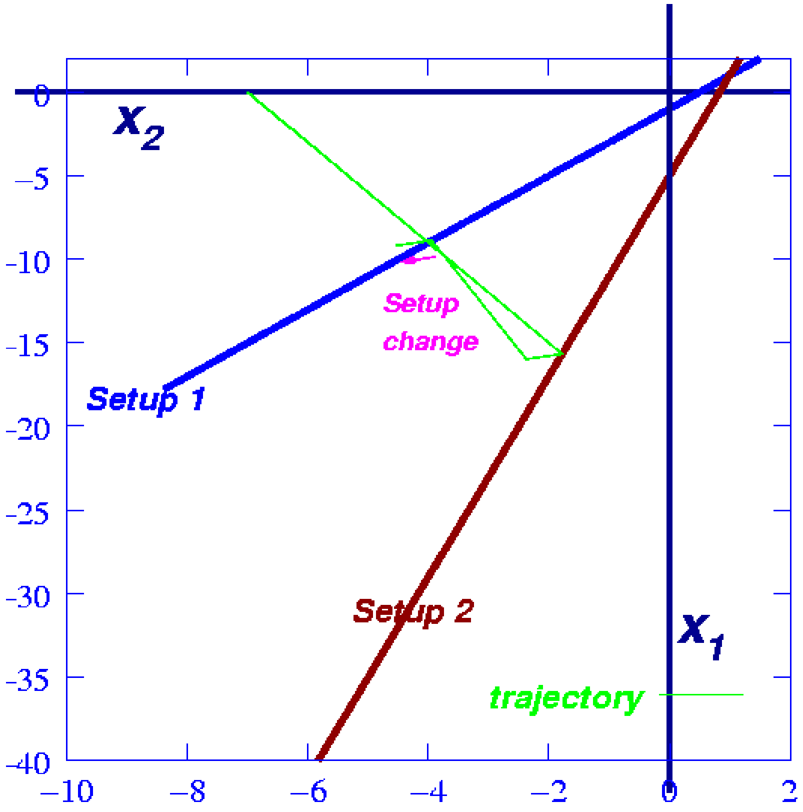


# Setups


0
-5
-10
-15
-20
-25
-30
-35
-40
−10
−8
−6
−4
−2
0
2
x2
x1
Type 2 production
Setup 1
Setup 2
trajectory

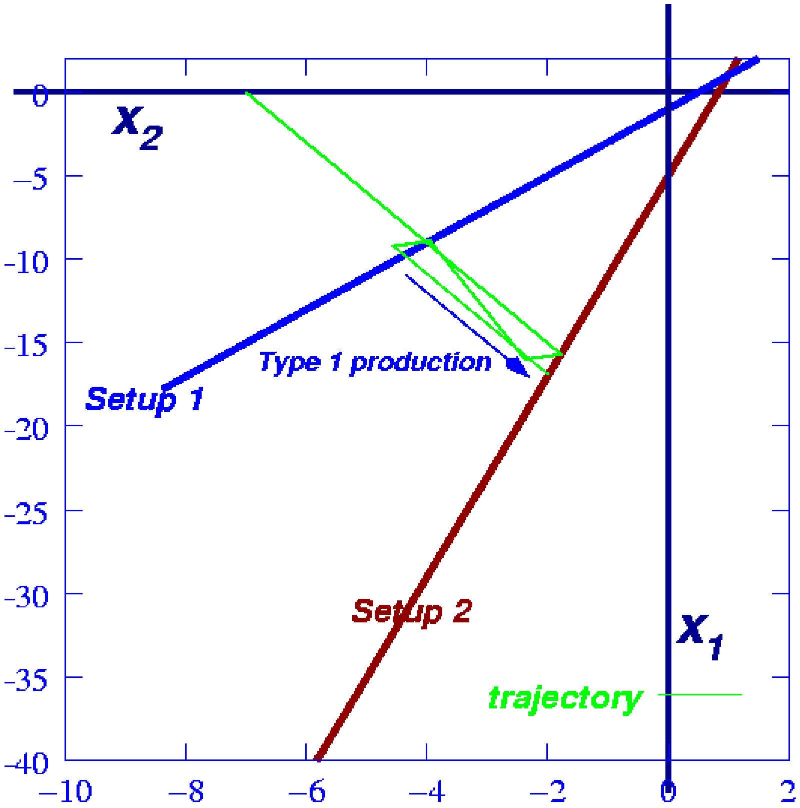
# Setups

## Corridor Policy



# Setups

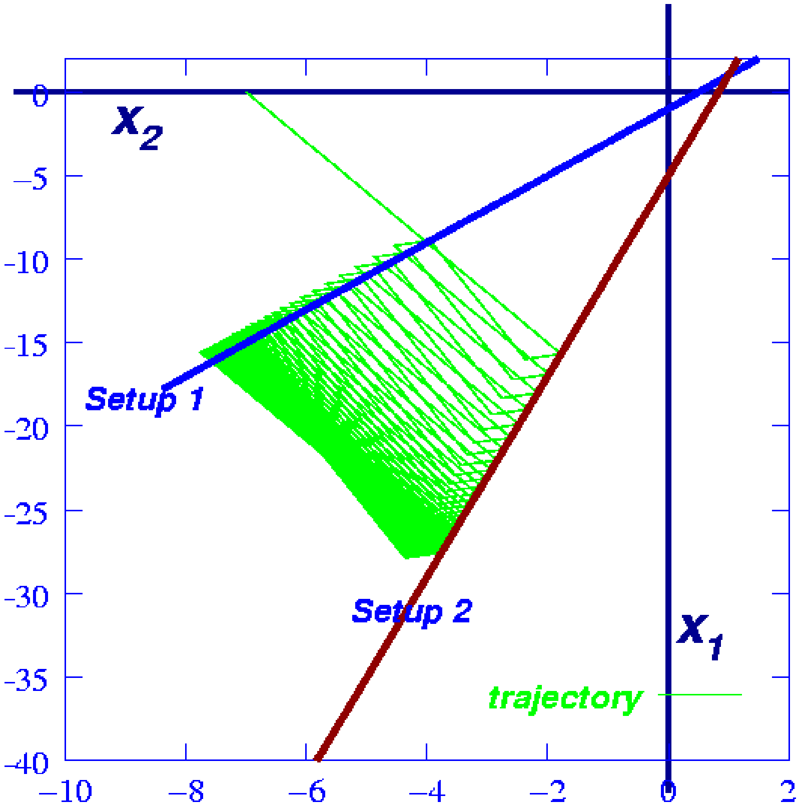
## Corridor Policy



# Setups

## Corridor Policy



# Setups

- In this version, batch size is a function of time.
- Also possible to pick parallel boundaries, with an upper limit. Then batch size is constant until upper limit reached.

# Setups

Two possibilities:

- Converges to limit cycle — only if demand is within capacity, ie if $\sum_i \tau_i d_i < 1$.
- Diverges — if
  - ⋆ demand is not within capacity, or
  - ⋆ corridor boundaries are poorly chosen.

# Setups

Three possibilities for the corridor policy:

- Limit cycle — only if demand is within capacity,
- Divergence — if
  - ⋆ demand is not within capacity, or
  - ⋆ corridor boundaries are poorly chosen.
- *Chaos* if demand is within capacity, and corridor boundaries chosen ... not well?