



16.423J/HST515J Space Biomedical Engineering and Life Support Extreme Environment & Bone Homework Assignment #1, Solutions

Question 1: Extreme Environments

(4 points) Describe two themes that most interested you in the reading/lectures on living in extreme environments. Why? Keep answers to 1/2 page for each theme.

(2 themes for 2 points each) All answers accepted.

Interestingly enough, the majority of folks mentioned social compatibility, crew selection, and group dynamics. The main question for thought revolves around homogeneous crew make up OR more diverse mixed cultural, and skill-set crews. This is always a major consideration for exploration in extreme environments. Numerous insights were reported on the following topics.

- Group dynamics, teams, compatibility
- The essence of extreme environments
- Quantification of human needs and emotions
- Physical limits
- Lack of human ability to dream and hope (psychosomatic issues)
- Unexpected problems
- Parallels and analogs to space exploration
- Living off the environment/land
- Stresses associated with isolation
- Mission leaders (behavior and psychology)

Question 2: Bone

(6 points total) The goal of this aspect of the assignment is to further investigate the changes that age and environmental loading (1G, microgravity, etc.) have on geometry and bone mineral density of tubular bone. You are to perform simulations using a computer spreadsheet or Matlab. Long bones can be simulated as hollow tubes, which are subjected in life mainly to bending stresses. Generally, the moment arms do not change with age although load magnitudes do change.

Starting conditions for young adult femur shafts.

Female outer radius, $r_o = 1.47$ cm Male outer radius, $r_o = 1.67$ cm

Female inner radius, $r_i = 0.97$ cm Male inner radius, $r_i = 1.16$ cm

Assume that remodeling normally increases inner radius at the rate of 0.004 cm/year (from measured data in males). The geometry of a hollow tube with effective mineral density of solid bone $\rho_m = 1.05$ gm/cm³.

$$I = \frac{\pi}{4} (r_o^4 - r_i^4)$$

$$Z = \frac{I}{r_o}$$

where I is cross-sectional Moment of Inertia and Z is Section Modulus

$$A = \pi (r_o^2 - r_i^2)$$

$$BMD = \frac{A \rho_m}{2 r_o}$$

Problems to Solve:

2.1. (3 points) Find the section modulus of young adult femoral shafts. Assume that this is maintained through life (assuming constant skeletal loading). A) Derive an expression for r_o as a function of changing r_i (this is messy). B) Show how the section modulus can be maintained through age 85. C) What happens to the BMD?

Expression for Z_0 :

$$Z_0 = \frac{\pi}{4} \cdot \frac{(r_{o_0}^4 - r_{i_0}^4)}{r_{o_0}}$$

Expression for $r_i(t)$:

$$r_i(t) = \dot{r}_i(t - 25) + r_{i_0}$$

Expression for $r_o(t)$:

$$r_o(t) = \sqrt[4]{r_i(t)^4 + \frac{4Z_0}{\pi} r_o(t)} \quad \text{(circular reference)}$$

where t = age (from 25 to 85 years old)

$r_i(t)$ = inner radius (cm)

r_{i_0} = initial inner radius (1.16 cm for male, 0.97 cm for female)

$r_o(t)$ = outer radius (cm)

r_{o_0} = initial outer radius (1.67 cm for male, 1.47 cm for female)

\dot{r}_i = inner radius increase rate (0.004 cm/year, male and female)

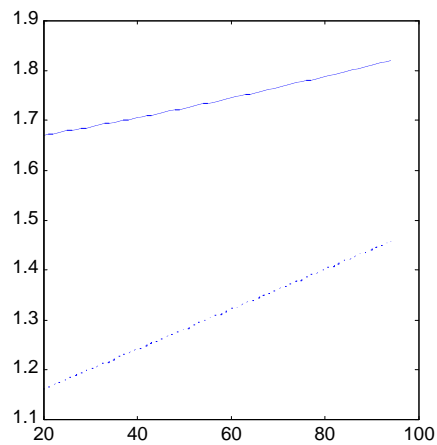
Z_0 = initial section modulus (cm³)

➤ Z_0 for male: 2.81 cm³

➤ Z_0 for female: 2.02 cm³

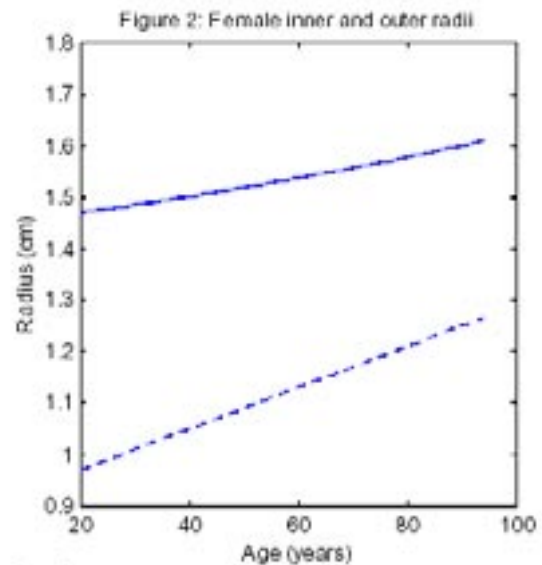
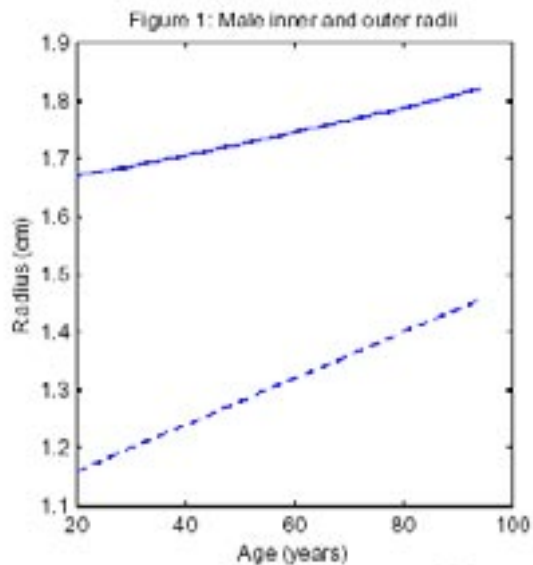
Note: I learned some math from you all. I won't go into more mathematical derivation here. This is good practice for you though, for qualifying exams! (-; Thanks for student contributions: "Quartic Formula" solver from the web (<http://www.sosmath.com/algebra/factor/fac12/fac12.html>), Mathematica and Maple solutions, and others just cranked through by hand and perseverance.

In these conditions, the **section modulus** is maintained constant if the outer radius increases with time:



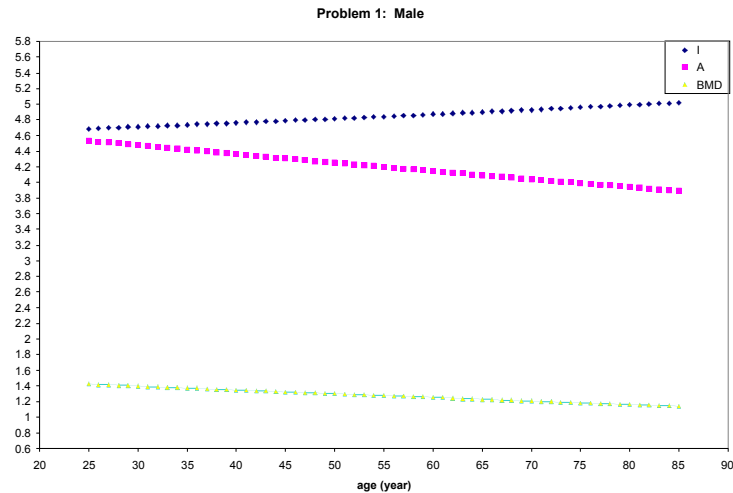
Source: Chris Sanders

Note: Correct answer, BUT could be improved by LABELING ALL AXES, PROVIDING UNITS AND LABELING ALL DATA AND SYMBOLS, less important, but more professional would be to also put a Figure caption on the plot (see below).



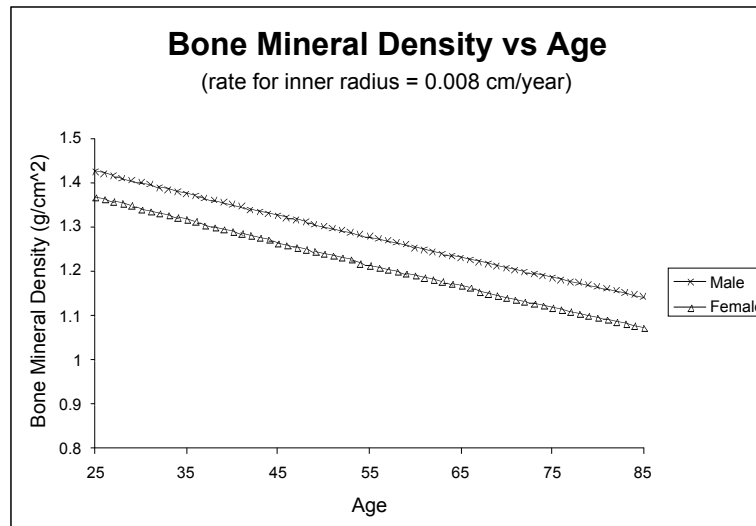
Source: Chris Sanders

In the graph, the outer radius increases for both male and female at a rate of about 0.002 cm/year. The following graph illustrates the variations of **A, I and BMD (Area, Inertia, and Bone Mineral Density)** for male:



Source: Viengvilay Oudonesom

The Bone Mineral Density is decreasing with time as shown in the figure below:



Source: Simon Nolet

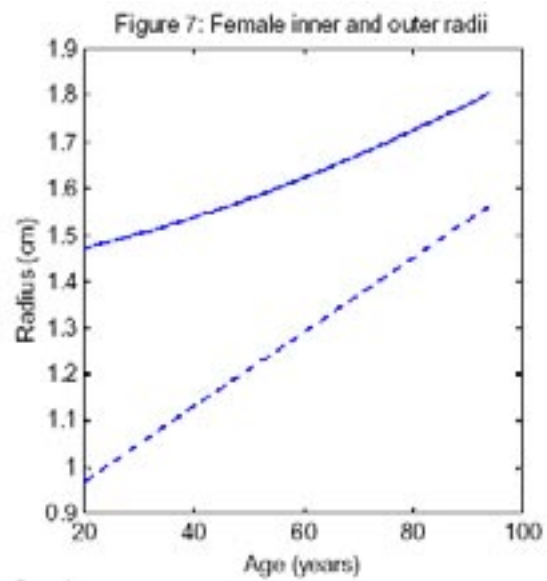
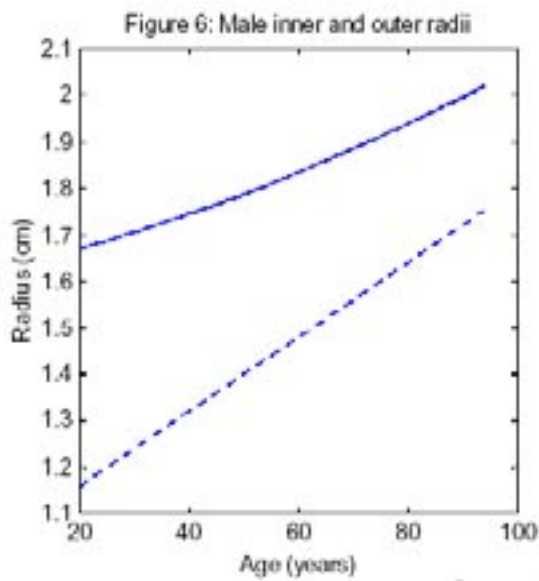
Note: This graph shows the correct BMD solution and is presented professionally.

Conclusions/Discussion: With remodeling (increasing inner radius), in order to keep the section modulus constant through the age of 85, the outer radius must increase also. The relationship between outer and inner radius is not linear from the equations (though from the graphs, it resembles an almost linear relationship). Also, from the graphs, the BMD decreases with increasing inner radii by 20% for males and 21% for females with time, even though the outer radius also increases¹.

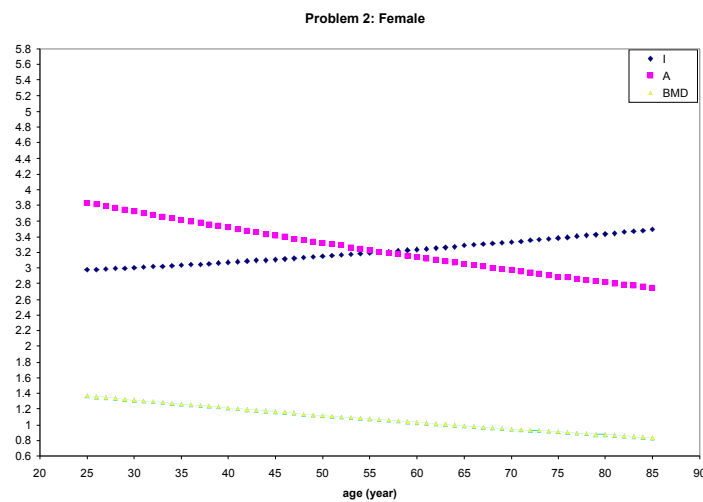
2.2. (1 point) What happens to BMD (and other parameters) if remodeling rates (increase in r_i) double? (assume constant Z).

If the remodeling rate is doubled, the outer radius will increase faster to keep the section modulus constant:

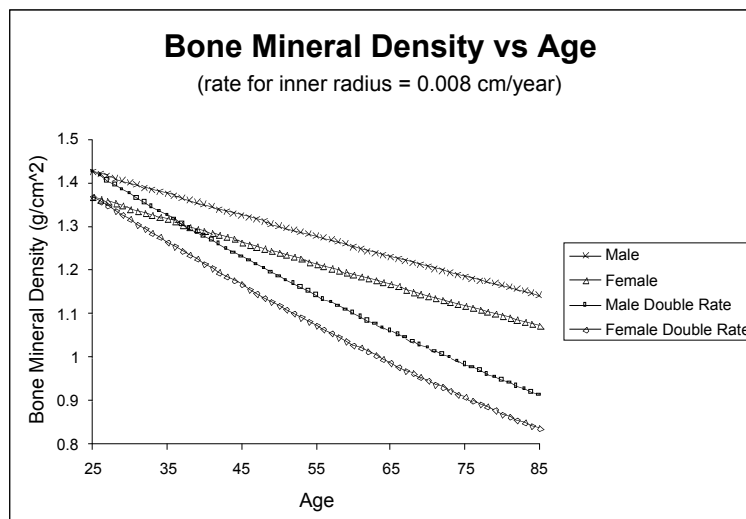
¹ Thanks to Jessica Marquez.



In the graph, the outer radius increases for both male and female at a rate of about 0.0044 cm/year. The following graph illustrates the variations of A, I and BMD for female:



The Bone Mineral Density is decreasing with time at a faster rate:

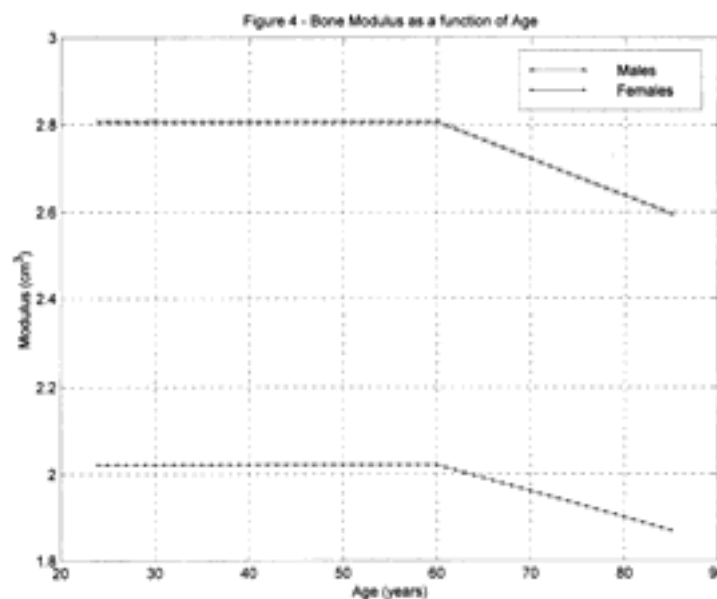


Conclusions: With remodeling rates doubled, in order to maintain constant sectional modulus, the rate at which the outer radius grew increased, as expected. The BMD also decreased as in the previous exercise. At the end of the lifetime, this BMD is 20% lower for males and 22% lower for females than if the remodeling rate is 0.004 cm/yr. Therefore, even though section modulus is kept constant with increased remodeling rates, the bone mineral density decreases even more².

2.3. (2 points) Assume that loading is reduced by 30% at age 60, remaining constant thereafter. Assume a linear effect on Z. What happens to BMD and other parameters?

There are 2 ways to solve this problem. The first way assume a step reduction in loading pattern. If you assumed a step, necessary rational is required.

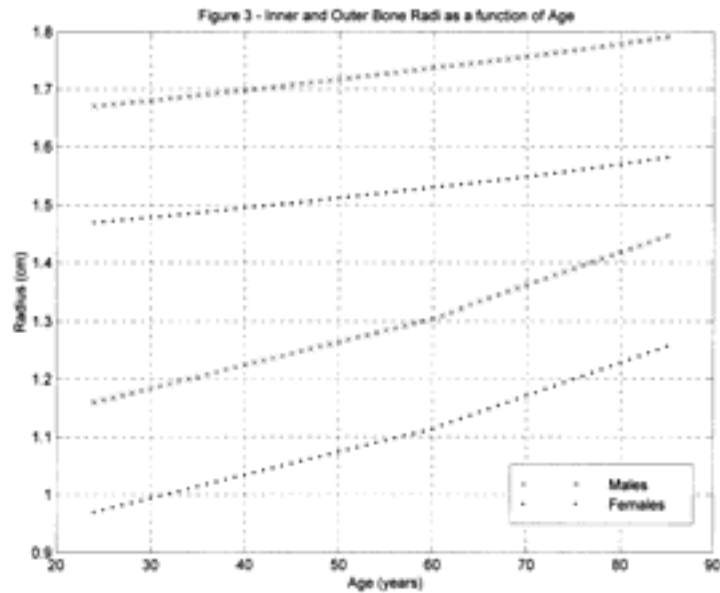
First, it is assumed that the section modulus would be reduced over 25 years:



Source: Chris Carr

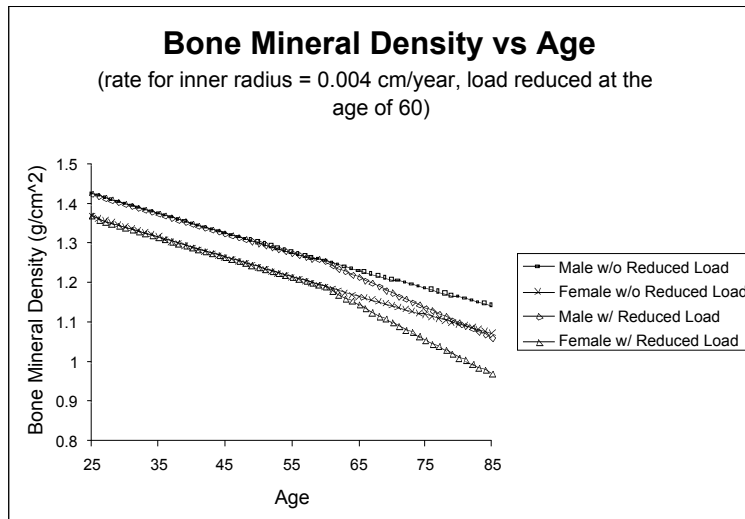
The radii would increase following this pattern (discontinuity at 60 years old):

² Thanks to Jessica Marquez



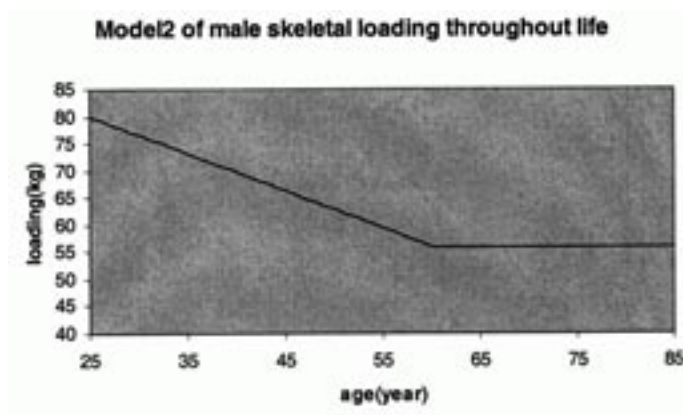
Source: Chris Carr

The trend would be a faster decrease of the Bone Mineral Density from 60 years old:

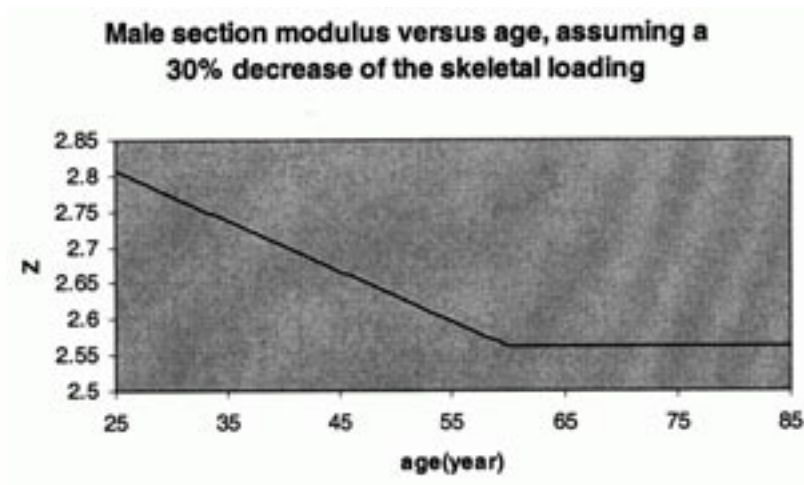


Source: Simon Nolet

The second way to solve the problem assumes the following loading and section modulus pattern:

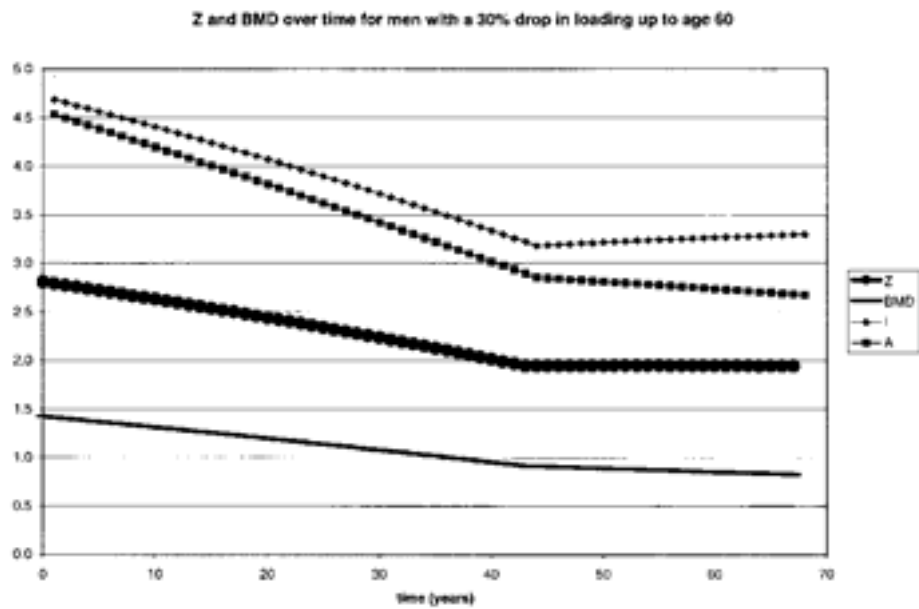


Source: Berengere Houdou



Source: Berengere Houdou

This will give the following pattern on the other parameters:



Source: Erika Brown

This unloading profile produces ~40% loss in BMD for male and female subjects where most of the loss is accounted for during the years leading up to 60. The outer radii exhibit slight decreases during the 25-60 year timeframe, but you must remember that remodeling has been held constant at 0.004 cm/yr. The reductions in area are very substantial. In sum, the normal loading condition leads to a gradual thinning of the femoral cortical shell with constant section modulus whereas the unloading condition results in significant physiological deconditioning and strength loss. The significant effect of section modulus (i.e., derivative) is highlighted in these simulations in contrast to the conventional emphasis on bone mineral density.

Note that it is unrealistic to assume a step function on the section modulus, since the bones cannot react instantaneously to a load change.