22. Auction Theory

Most people are familiar with auctions involving, art, livestock or real estate, where an auctioneer (a seller) is looking for the *highest* price he can get. It is also generally known that most government business (military supplies, construction, and most other government-bought goods and services) are contracted for based a procurement auction, in which the auctioneer (buyer) is looking for the *lowest* price. Publicly-owned assets (such as airwave frequencies, timer rights, oil leases, or public companies on their way to privatization) are also sold off by the government in auctions. In addition, the US government sells treasury bills through an auction process every week.

> **Using the Lowest Bidder**
>
> On February 20th 1962, John Glen completed the first orbital flight by an American. He reportedly said that throughout the hours leading to the blast-off, he could only think that every component in his Friendship 7 Mercury capsule and the Atlas 6 rocket under it, were built by the lowest cost bidder.

Auctions, of course are not new. In fact, almost any buying and selling transaction can be viewed as the result of an auction process. A consumer looking at advertisements for automobiles, books or groceries can be viewed an auctioneer evaluating bids from competing suppliers. With the use of internet search engines such price comparisons are now commonplace. Many people are also familiar with Internet-based auction sites such as eBay, Amazon.com, UBid and others, where individuals and corporations sell a multitude of goods following specific auction rules.[1]

> **An Early Auction**
>
> The Praetorian Guard auctioned off the entire Roman Empire in193 AD. After mounting a coup against Emperor Pertinaz, they conducted a multiple-round auction between two rich Romans. The auction was won by Didius Julianus whose winning bid was 25,000 Sesterces/ guard. (Assuming 10,000 guardsmen, this was about a billion dollars in today's currency.) He was declared emperor but when he was unable to pay the price he bid, he was beheaded after only two month in the throne, in what could be the earliest documented case of the "winner's curse" (see Section 22.3.1).

Companies buy most of the goods and services they use through procurement auctions, where the auctioneer (buyer) is looking for the lowest price (among other things) from the bidders. In keeping with the auction theory literature, however, we will continue the exposition in this chapter assuming that the auctioneer is a seller who is looking for the highest possible price, but the

[1] Note that the example of the Praetorian Guard in the inset does not *necessarily* means that this is a case of the "winner's case" (discussed in section 22.3.1) since inability to pay does not by itself mean that the price is higher than the value. In addition, one can behave optimally (and take the winner's curse phenomenon into account when bidding) and still lose due to bad luck.

discussion is equivalent when the auctioneer is a buyer looking for the lowest price (the latter are sometimes called *reverse* auctions).

This chapter looks at various auction formats and lays the foundation for a discussion of procurement auctions (see Section 0_) and combinatorial auction in Chapter 23.

Why Auctions?

Both buyers and sellers use auction as a mechanism to accomplish the following:

- Price discovery – in many cases seller (or buyers) do not know what an item or service is “worth” and how much should they sell or buy it for. An auction serves as a “market test” (in fact, this very term is used by many companies to describe an auction process) to ascertain what are the prevailing prices.
- Winner determination – the auction process is used to determine who the object (contract, item, or whatever) should be allocated to, or who “wins” the auction.
- Payment mechanism – finally, the process can be used to determine how much the winner should pay. As shown later, the traditional process when participants pay what they bid is only one of many possible pricing mechanisms.

The popularity of government and corporate auctions is also rooted in their perceived fairness. Auction processes have rules which are explained before the auction and thus can avoid “special deals” between certain buyers and sellers, manipulation of the market, and wins by favorite bidders.

Television Broadcasting Rights for Sports

Television rights for sport games in the US used to be negotiated between television studios and the individual teams. In 1964 all the baseball team owners let the baseball league negotiate on their behalf and let the studios bid in an auction. The result was that the price went up to $28 million for a two year deal, and then to over $2 billion per year in 1998. All other sports followed the example set by baseball. Similarly the English soccer league sells the broadcasting rights at an auction. That auction fetched £537 million in 2000, up from £45 million in 1992. Another example is the Olympic Games which the International Olympic Committee is auctioning of the rights to televise the Olympic Games among television studios; NBC paid a record $705 million for U.S. broadcast rights to the 2000 Games in Sydney. They also paid $3.5 billion for the rights through the 2008 games.

In these cases, the moves from negotiated deals to competitive auctions have unlocked the value of sports broadcasting. It is difficult to imagine that savvy television executives will pay a price that is “too high” over and over again. Instead, it is a testament to the actual value of broadcasting popular sports events, which the teams, promoters, organizers, owners and athletes were able to obtain for themselves through competitive auctions.

Evaluating Auctions

When deciding between various auction mechanisms, the auctioneer has a very large number of auction designs to choose from. The most important criteria in the choice of auction format are the following:

- Revenue – auctioneers are looking for the auction that will yield the maximum revenue for the item sold. While this is an important tenet of auction theory and will be assumed to be the case in the bulk of this section, other considerations are also important not only to governments but to many corporations.

- Efficiency – an auction is successful if the bidder that values the item most *ex post* - actually gets it. In some contexts, such as government auctions this is important, especially when the government is selling public assets. When the sale involves future delivery of services, as in many procurement auctions, efficiency means that the contract is more likely to be carried out and the service provided at a high level.[2]

- Time and Effort – many B2B auctions involves the trading of many (sometimes tens of thousands) of items while soliciting bids from many (sometimes hundreds) of suppliers. Furthermore such auctions are conducted periodically, every year (for many MRO services) or every product model (for direct material). Auctioning organizations have to devote a great deal of time and effort to such auctions and thus mechanisms that minimize the time and effort involved are the ones that will be used.

- Simplicity – One of the objectives of auctioneers in most auctions is to get as many participants as possible. Keeping the rules simple, especially knowing that many suppliers have to respond to hundreds of auctions every month, helps participation in many situations.

While the literature on auction theory focuses on the first two criteria, the last two are very important in procurement auctions and are there for highlighted again in section 0

22.1 Standard Auctions

[2] Note that while efficiency can be enhanced through a secondary market. As argued by Krishna (2002), however, a secondary market cannot guarantee efficiency even in the absence of transaction costs, which are likely to be substantial in that context. Furthermore, in most procurement auctions reselling is simply not allowed.

Traditionally, auction theory deals with the sale of single and multiple items separately. While single item auctions are a special case of multiple items auctions, the theory regarding single items is more developed. We will explain multiple items auctions in section 2222.5 and continue in this section with the exposition of single unit auctions.

The basic paradigm is that of multiple bidders, each having a value to them for winning the item being auctioned off. Their goal is to buy the item below that value so the difference between the valuation and the price paid leaves the bidders with some profit (or *surplus* in economic terms). Naturally, that valuation is the maximum bid that any participant is willing to place.

In any auction process the seller does not know the value that bidders place on the item auctioned off and bidders do not know with certainty how other bidders value the item. The information available to bidders and the corresponding type of auctions can be classified as follows:

- Private value (PV) auctions – where each bidder knows only the value of the item to himself. Even if he would know what other bidders are willing to pay this would not affect his own valuation. Such a model is appropriate when the value of an item to a bidder is derived from its consumption alone and not from later resale.

- Interdependent value auctions – where the value of the items sold is not known to the bidders. Each bidder has only an estimate (signal) regarding the value (this may be an expert opinion or a test result). If a given bidder would have known the signals of other bidders, his own estimate of the true value may change.

- Common value (CV) auctions – a special case of interdependent values in which the value of the item *ex post* is the same for all bidders. For example, when oil leases are auctioned off, bidders have only their own test results regarding the actual amount of oil in the tract being leased. After the auction, however, the winner will find out exactly the amount of oil in the ground and this oil has a certain market value.

22.1.1 Auction Types

The exposition of auction theory typically starts with the introduction of four basic auction formats: two open auctions and two based on sealed bids:

Open bids

- <u>Ascending bid auction</u> (also called English Auction) – In this auction the price is successively raised until one bidder remains. This bidder wins the object at the final price. The auction can run by the auctioneer calling price, the bidder submitting prices, or electronic bids with the highest bid posted continuously.[3] Once somebody quits the process they are not allowed back in. This auction format is common in art, livestock, and some Internet-based procurement auctions.

- Descending bid auction (also called Dutch auction) – In this auction the price starts from a high level and called down. The first bidder who accepts the current price wins. This is how the Dutch flower auctions are managed, but there are not many other examples of Dutch auction.[4]

Sealed Bid Auctions

- First price sealed bid auction (1SB) – Each bidder submits a single bid (independently) and the item is sold to the highest bidder who pays the winning bid. This is the most common form in procurement auctions and in many government contract auctions.

- Second price sealed bid auction (2SB) – Each bidders submit a single bid (independently) and the item is sold to the highest bidder. Unlike the first price sealed bid auction, however, the price that the winner pays is the second highest price ("second price"). This type of auction was first suggested by Vickrey.[5] Although it is rarely practiced,[6] its value is the insight it offers to other auction mechanisms.[7]

Open auctions require the bidders to be all present (physically or digitally) at the same time and place, offering bidders the opportunity to observe the behavior of other participants. Sealed bids, on the other hand, are mailed in or submitted electronically with no opportunity for information feed-back. Interestingly, some of these differences do not matter to rational decision makers.

Note first that submitting the winning bid in a 1SB auction is equivalent to buying the item at that price through Dutch auction, provided the item is still available. This is true despite the fact that a (first price or otherwise) sealed bid auction offers no opportunity to observe other bidders behavior while the Dutch auction is "open." Naturally, the reason is that in a Dutch auction the first time that

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[3] When the price is changed continuously this auction is referred to as a "Japanese Auction." In a Japanese auction bidders keep indicating that they are still interested while the price goes continuously up (displayed on a central digital board). Bidders drop out and cannot enter the auction again. The winner is the last remaining bidder. By and large it is identical to English auction with the only difference being that the price increase increment are infinitesimal, while English auctions use a discrete increment which can be very small.

[4] Unfortunately, many of the Internet auction sites (such as eBay and Amazon.com) use the term "Dutch Auction" to describe a multiple item auction in which all the winners pay the lowest winning bid. Multiple items auctions are described in Section 2222.5 but even there we do not call these auctions "Dutch," a term that we reserve, in keeping with Auction Theory literature to the type of auction mentioned above.

[5] William Vickrey (1961) wrote the seminal paper on auctions and later got a Nobel Prize.

[6] It is used for auctions of stamps and sometimes for Internet-based auctions.

[7] Despite the introduction of the concept in 1961, second price sealed bid auction rarely used in practice – see comment in section __.

somebody enters, he wins and the auction terminates. Thus, in 1SB and Dutch auction bidders have to decide a-priori how much to bid and the auctioneer will get what the highest bidder submitted. In economic terms, 1SB auctions are strategically equivalent to Dutch auctions in the sense that for every realization of bidders' estimates the two auctions induce identical equilibrium outcomes (i.e., winners and prices).

In English auctions it makes sense for a bidder to stay in the auction until his value has been reached (in fact, it is his dominant strategy, as discussed below). He should not drop beforehand and certainly should not stay longer. Thus the winning bidder will stay until the next-to-last bidder drops out, meaning that he will pay the price in which the last bidder dropped out, or the second highest bid. Thus English auctions are equivalent to 2SB auctions.[8] This equivalence is not as strong as the one between 1SB and Dutch auctions since with English auctions bidders do get information throughout the process when they see the prices in which other bidders drop out and can therefore adjust their estimate of the value of the item being sold and their strategy. This is not a consideration with private value auctions but important in common and interdependent value auctions. These equivalences are depicted in Figure 22.1.[9]

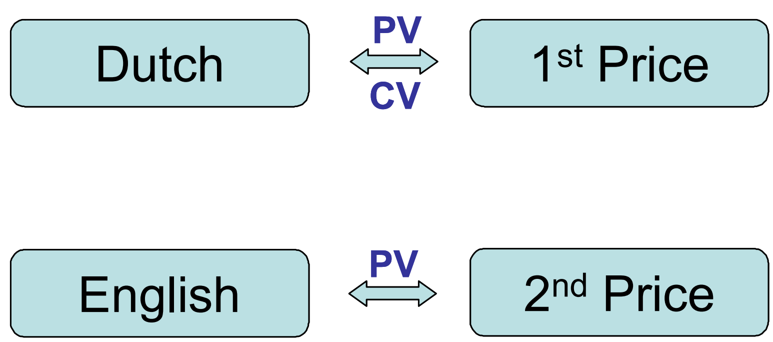


Figure 22.1 Equivalence of Open and Sealed Bids Auction

The most common types of *procurement* auctions (where the winner is the participant who offered the lowest bid) are the 1SB, where the winner gets to supply the item to the buyer at the lowest bid price and English auction, where the winner supplies the item or service at the second lowest price. English

[8] With Japanese auctions this will be the second highest bid exactly. With English auctions, in which either the auctioneer calls the price in discrete increments or the participants call in bids, the winner will pay the second highest price plus a bid increment. The final increment is typically relatively small as the auctioneer tries to elicit the last few bids.

[9] As an aside, note that one should distinguish between (statistically) *independent* private value auctions and (possibly affiliated) private values auctions With *independent* private value auctions the equivalence between second price and English auctions is in some sense stronger than that between first price and Dutch auctions since it is based on dominant strategy in second price – English while just on equilibrium in the other.

auctions became popular with procurement managers only during the 1990-s with the advent of Internet-based auctions (see Section 22.6).

22.2 Bidding Strategies and Auction Revenues

Auctions are typically analyzed by assuming that all bidders follow the same strategy, resulting in (Nash) equilibrium. At equilibrium no player can improve his expected profit by altering his strategy unilaterally. Note that the assumption is not that all bidders bid the same price – bidders have different information and different values. Just that they all use the same rationale to decide how to bid. Consequently, the analysis can proceed by looking at any bidder.

This section looks at bidding strategies for first and second price auctions under the assumption of private values.[10] In addition, the analysis assumes that there are n bidders, and each bidder i has a (private) value, v_i, which is a realization from of a random variable, V_i. All bidders' values are assumed to be independently[11] and identically (i.i.d.) distributed random variables drawn from a known probability density function, f(•) with cumulative distribution F(•).[12] The assumption that the bidders' values are drawn from identical distributions is referred as "symmetry" among bidders. In addition, we assume that bidders are risk-neutral do not exercise any collusion or predatory behavior, and are not subject to budget constraints.

22.2.1 Second Price Strategy

In a second price auction bidders should bid their true value. This strategy should be used regardless of whether other bidders are bidding truthfully or not. In fact, this strategy is the right one to use whether or not most of the assumptions above hold. To see this consider that your true value for the item being auctioned of is v.

The various possibilities are depicted in Figure 22.2.

[10] Note that second price auction are likely to be conducted in practice using the English format
[11] Independence of the random variables should not be confused with interdependence of the values for CV auctions. Values may be interdependent so one bidder's value depends on another bidder's signal yet the signals may be statistically independent and vice versa.
[12] The distribution function is assumed to be "well behaved" in several aspects.

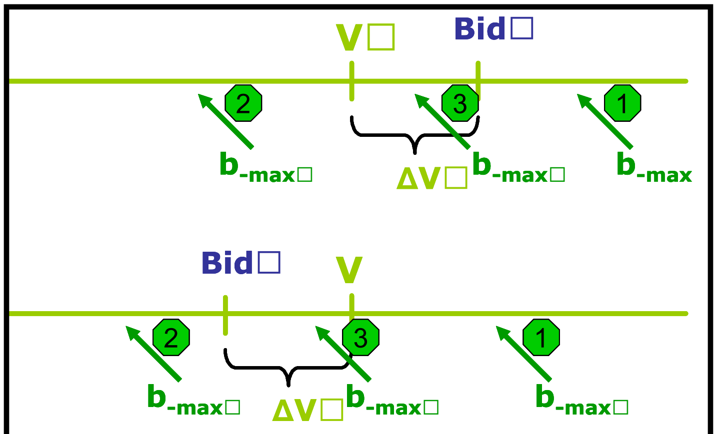


Fig. 22.2 Bidding Strategy in 2nd Price

- Consider bidding (v-Δ). If the highest bid other than yours is b_{-max}, then the following may occur:
  1. If $v < b_{-max}$, you lose (no different than bidding v)
  2. If (v-Δ) > b_{-max}, you win and pay b_{-max} (no different than bidding v)
  3. If $v > b_{-max} > (v-\Delta)$, bidding ($v$-$\Delta$) means you lose the auction, while bidding v would have meant winning and having "surplus value" of (v- b_{-max}).

- Consider bidding (v+Δ). If the highest value other than yours is b_{-max}, then the following may occur:
  1. If (v+Δ) < b_{-max}, you lose (no different than bidding v)
  2. If $v > b_{-max}$ you win and pay b_{-max} (no different than bidding v)
  3. If $v < b_{-max} < (v+\Delta)$, bidding ($v$+$\Delta$) means you "win" the auction, while bidding v would have meant losing, but you pay b_{-max}, which is higher than your value, v.

Clearly, bidding either (v-Δ) or (v+Δ) instead of v may only hurt you and will never help you.

Thus, in an ascending (English) auction, a bidder should raise the bid by "a little" as long as the current offer is lower than his valuation. The outcome is that the person with the highest valuation pays, but he pays the valuation of the second highest bidder (plus "a little").

22.2.2 Second Price Revenue

Let us calculate the expected revenue of a seller in a second price auction (which, as was mentioned above, is actually equivalent to an ascending, English auction). In this auction all participants will bid their true value (since this is the rational thing for them to do) and the winner will pay the bid of the second highest bidder.

Assume that there are n bidders and their values, $\{V_1, V_2, \ldots, V_n\}$ are drawn from independent and identical distributions, F(v) with density function f(v). Assume further, that these values will be ordered so that $V_{(1)}$ is the smallest, $V_{(2)}$ is the second smallest, and $V_{(n)}$ is the highest value. The variables $\{V_{(1)}, V_{(2)}, \ldots, V_{(n)}\}$ are *order statistics* and the density function of the k^{th} lowest value, f$(v_{(k)})$, is given by:[13]

$$f(v_{(k)}) = \frac{n!}{(k-1)! \bullet (n-k)!} \bullet f(v) \bullet [F(v)]^{k-1} \bullet [1-F(v)]^{n-k} \qquad [22.1]$$

Consider, for example, that the valuations are drawn for a uniform distribution between zero and one [i.e., U(0,1)]. In this case the distribution of the k^{th} order statistics is given by:

$$f(v_{(k)}) = \frac{n!}{(k-1)! \bullet (n-k)!} \bullet v^{k-1} \bullet (1-v)^{n-k} \qquad [22.2]$$

To see what the expected payment will be, note that Eq.[22.2] is a Beta distribution with parameters k and $(n-k+1)$. The mean of this distribution is:

$$E[V_{(k)}] = \frac{k}{n+1} \qquad [22.3]$$

Thus, in such an auction, the expected revenue for the auctioneer (before the auction has taken place) will be based on the second-highest order statistic, $V_{(n-1)}$, the expected value of which is:

$$E[V_{(n-1)}] = \frac{n-1}{n+1} \qquad [22.4]$$

22.2.3 First Price Strategy

In a first price sealed bid auction, participants' problem is somewhat more complicated. If a bidder will bid his valuation, there is no point in winning as there will be no surplus left. Thus bidders are likely to "shave" their bids by some amount.

[13]To see this result:

$$\Pr\{x < x_{(k)} \le x + dx\} =$$

$$= \Pr\{[x \le \text{one of the } X_i\text{'s} \le (x+dx)] \cap [X_i \le x \text{ for } (k-1)\ X_i\text{'s}] \cap [X_i > x \text{ for the remaining } (n-k)\ X_i\text{'s}]\} =$$

At the limit, when dx $\to$ 0:

$$\text{f}(\text{x}_{(\text{k})}) = \frac{n!}{(k-1)!(n-k)!} \bullet f(x) \bullet \left[F(x)\right]^{k-1} \bullet [(1-F(x)]^{n-k}$$

To develop the strategy, assume that each bidder wants to maximize his or her expected gain, which is the surplus associated with winning times the probability of winning:

$$E[Gain] = (v - b) \bullet P(b) \qquad [22.5]$$

Where:

v = the bidder's valuation of the object auctioned off
b = the bid
P(b) = the probability that a bid b will win the auction.

The optimal bid is the one that maximizes the expectation in Eq. [22.5].

Assume n bidders whose valuations are drawn from independent and identical (iid) distributions. Assume further that each bidder "shaves" his offer and bids a fraction of his value (so if he wins, he has a positive surplus). Thus the bid is $b = \alpha \cdot v$ where α is the same for all bidders and $db(v)/dv = \alpha$. Under these conditions (see Appendix 22.7), the optimal shaving and optimal bid can be given by:

$$\alpha = \frac{n-1}{n} \quad or \quad b^* = \frac{n-1}{n} \bullet v \qquad [22.6]$$

Thus, the best strategy for all bidders is "shave" their bid by $(1/n) \cdot v$.

22.2.4 First Price Revenue

To calculate the revenue that the auctioneer can expect in a first price auction, note that if all participants use the strategy mentioned above, the winner is the bidder with the highest value. In other words, the winner is the bidder with the highest order statistic, $V_{(n)}$. If the valuations are drawn from a U(0,1) probability density function, the expected value of the highest order statistics is (see Eq. [22.3]) is:

$$E[V_{(n)}] = \frac{n}{n+1} \qquad [22.7]$$

Furthermore, according to Eq.[22.6], the best bidding strategy is:

$$b^* = \frac{n-1}{n} \bullet E[V_{(n)}] \qquad [22.8]$$

where we substituted $E[V_{(n)}]$ for v in Eq. [22.6] since the auctioneer does not know the participants' valuations *before* the auction. Thus the expected winning bid is given by:

$$b^* = \frac{n-1}{n+1}, \qquad [22.9]$$

which is also the auctioneer's expected payoff.

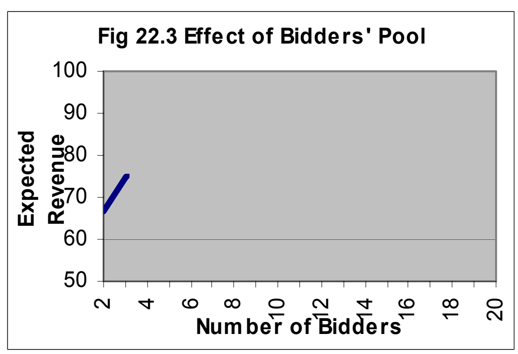
22.2.5 Revenue Equivalence

The payoff for the auctioneer is identical in both the second price auction (see Eq. [22.4]) and the first price auction (Eq. [22.9]). This is not a coincidence and is not unique to the use of a uniform distribution. Under the assumptions that bidders have private values[14], that their valuations are independently drawn from a continuous, "well behaved" distribution[15], that they are risk neutral and have no budget constraints, all auction mechanism where: (i) The object goes to the bidder with the highest value and (ii) Every bid above the reservation price is admissible, (iii) All bidders are treated equally (anonymity) and (iv) There exists a symmetric monotonic bidding function, yield the same expected revenue.[16]

In particular, all the auctions described above, as well as several others, will yield the same payoff. For example, "all pay" auctions, where each participant pays their bid regardless of who wins, will yield the results given by Eq. [22.9][17] when the valuations are drawn from a U(0,1) distribution. In addition, mechanisms such as "3rd price" auctions will have the same payoff.[18]

22.2.6 Number of Bidders

Figure 22.3 depicts the expected auctioneer's revenue for the case that the bidders' value are drawn from independent and identical distributions U(50,100).[19] Note that the revenue of the auctioneer will be higher the more participants are there in the auction, but this effect hits marginal returns. Also, by



[14] Revenue equivalence holds also for common value auctions, as long as bidders' signals are independent.
[15] The cumulative distribution is strictly increasing and atomless.
[16] Each bidder will also make the same expected payment as a function of his value or signal.
[17] Such auctions can be used to model lobbying activities where all participants have to bear the cost of the lobbying activities regardless of who wins.
[18] Third price auctions are not used in practice and are only a theoretical construct. Interestingly, in such auctions the optimal bid is higher that the bidder's valuation and the expected payoff will go *down* with the number of bidders.
[19] Recall that if x is a statistic drawn from a U[0,1] distribution, y=x·(b-a)+b is the corresponding statistics drawn from a U[a,b] distribution

the revenue equivalence theorem, this is true regardless of the type of auction used.

The conclusion is that in auctions with private values, auctioneers should invest in getting more bidders to participate in the auction, especially if the number of bidders is low. In a *procurement auction* the buyer's outlays are going to be lower with higher number of bidders.

22.2.7 Reservation Price

In order to increase revenue, auctioneers should set a reservation price below which they should not sell the item. If the item has a value, *w*, to the auctioneer, the reservation price clearly should not be set below *w*. The seller should, however, set a reservation price above his value in order to maximize the payoff. To see why this is the case, consider, again, an auction with a range of valuations drawn from U[0,1]. Assume that the auctioneers value is *w* = 0. Let the auctioneer sets a (low) positive reservation price, r, as shown in Figure 22.4.

For small *r*, the probability that any bidder's valuation is smaller than *r* is low, i.e., $F(r) << 1$.

Consider now a second price auction with the last two bidders still in the auction:

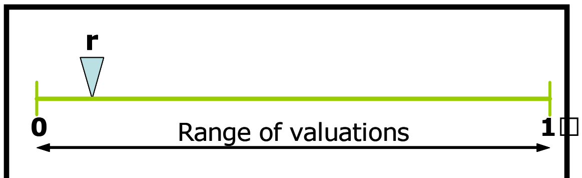


Fig. 22.4 Reservation price

- If both bids are above *r*, setting *r* did not hurt the auctioneer.

- If both bids are below *r*, the auctioneer loses due to setting *r* too high (there will be no sale). The probability of such an event is $[F(r)]^2$ and the maximum loss is *r*.

- If one bid is below *r* and one bid above it, the auctioneer come put ahead since the winner will have to pay *r* instead of the second highest price. The probability of this happening is $2 \bullet F(r) \bullet [1 - F(r)]$ and the expected win is ½*r*.

Thus, for small *r*, both the possible gain and the possible loss are on the order of r, while the probability of gain is proportional to *F(r)*, and the probability of a loss is proportional to $[F(r)]^2$.

As an example, assume now that the valuations are drawn from a U[0,1] distribution and *r*=0.1. This means that the probability of gaining something on the order of *r* is 0.1, while the probability of losing something on the order of *r* is 0.01. Thus it is clearly advantageous to set a reserve price.

In first price auctions, a reservation price would encourage bidders whose value is above the reservation price but whose "shaving" (see Eq.[22.6]) makes their bid just below the reservation price to raise their bid to the reservation price. Naturally, if bidders set a reservation price they should announce what it is, since otherwise it may not cause the participants to bid higher. The reason is that if the auctioneer announces that there is a reservation price, all participants can calculate what it will be (presumable set optimally by the auctioneer) and bid accordingly. If the auctioneer announces only that there is a reservation price but does not announce what it is participants may suspect that the rules will change after the bids are in and not bid aggressively.

Interestingly, these effects are the same for all the standard auction formats and thus the revenue equivalence theorem holds if the reserve price is set to be the same in all auctions.

For symmetric auctions in which the highest bidder wins, the optimal level of the reservation price can be developed using the following intuitive argument:

Assume that the auctioneer's value is zero, there are n bidders, the reserve price is set to r (where $r > 0$) and the auctioneer considers raising it by a small amount, δ.

- This is a good move if a single participant bids above ($r+\delta$). The probability of this happening is: $n \bullet F(r)^{n-1} \bullet [1 - F(r+\delta)]$. The gain in this case will be δ.

- This is a bad move if the highest bid turns out to be between r and ($r+\delta$). The probability of this event is: $n \bullet F(r)^{n-1} \bullet [F(r+\delta) - F(r)]$. The auctioneer will lose r in this case.

Thus the expected net gain per increment δ in r is:

$$G(\delta) := \frac{n \bullet F(r)^{n-1} \bullet [1 - F(r+\delta)]}{\delta} \bullet \delta - n \bullet F(r)^{n-1} \bullet \frac{F(r+\delta) - F(r)}{\delta} \bullet r$$

Simplifying and taking the limit as the increment of the reservation price goes to zero:

$$\lim_{\delta \to 0} G(\delta) = n \bullet F(r)^{n-1} \bullet \{[1 - F(r)] - f(r) \bullet r\}$$

Setting $G(\bullet) = 0$ (for the optimum) this simplifies to:

$$r^* = \frac{1-F(r^*)}{f(r^*)} \qquad [22.10]$$

The optimal reservation price, r^*, is the solution of Eq. [22.10].[20] Interestingly, the reservation price is independent of the number of bidders.

As a general rule, auctioneers should set a reservation price in most auctions not only because such reservation price may increase their revenue. The main reason for doing so is that many other assumptions imbedded in auction theory may not hold (for example, bidders may collude, intimidate others, or whatever). A proper reservation price can help avoid unpleasant surprises.

The Swiss 3GL Auction

In November 2000 the Swiss government auctioned off third generation cell phone licenses. For a variety of design failures only four bidders showed interest in the four licenses and since each bidder was restricted to a single license, the auction concluded at the government's reserve prices. Unfortunately, the reserve prices were only set at €20 per capita, resulting in revenue that was two orders of magnitude less than the Swiss government hoped for (The UK auction yielded €650 per capita and the German one €615 per capita).

In procurement auctions there may be cases in which not awarding the business to a vendor is not a realistic option – the buyer does not have the technology, cost structure, or capacity to consider "making" rather than "buying". In other cases, either the "make" decision is an option or the buyer can extend the existing contracts. In such cases considerations of reservation price should start with that cost (and be set *lower* for optimal auction results – as argued above).[21]

22.2.8 A Note on 2nd Price Auctions

Second price closed (Vickrey) auctions have been suggested in 1961 and analyzed extensively in the literature. They are, however, rarely used in practice. The reasons for this may be the following: (i) In the public arena, there may be a political fallout from a significant difference between the winning bid and the price paid. For example, if an oil company bids $100 million on the rights to drill in a new area and their closest competitor bids $10 Million, the first company will get

[20] The function f(v)/[1-F(v)] is known as the *hazard rate* and the right hand side of Eq. [22.10] is its inverse. The function *M(v) = v – [(1-F(v))/f(v)]* is called *virtual valuation* by Myerson (1981) and *marginal valuation* by Bulow and Roberts (1989). Bulow and Roberts show that *M(v)* plays the role of marginal revenue in their simple monopoly interpretation of the auctioneer problem. As in the monopoly problem, a seller ought not to sell a unit with marginal value below marginal cost (which is zero here since we assumed the seller's valuation to be zero.) Thus, the seller ought not sell to valuation below *v** such that *m(v*)* = 0 which implies *v* = [[(1-F(v*))/f(v*)]*, implying Eq. [22.10] above(!) A sufficient condition for standard auctions to be optimal is for the inverse of the hazard rate to be non-increasing. Optimal auction requires assigning the item in the independent private value case to the holder of the highest M(v). However under the above conditions there is a perfect order alignment between M(v) and v. Thus, assigning the item to the highest signal holder is the same to assigning it to the highest M(v) which is what most standard auctions do

[21] Recall that in procurement auction the buyer is the auctioneer and he is looking for a low price.

the rights for $10 Million. Since the bids will become public one can only imagine the outcry when the public may get (mis)informed that a $100 M license went for only $10 M… (ii) The auctioneer may be manipulating the auction by inserting bogus bids in. For example, he may ask a “friendly” company in the last example to submit a $95 M bid, just to make the winner pay more… (iii) Bidders may be reluctant to participate in an auction that requires them to bid their true value (thereby revealing it). The reasons for not wanting to reveal their true valuations are that it may put them at a disadvantage in the following *ex post* activities: (i) closing the deal with the auctioneer; (ii) negotiations with other bidders in a secondary market; and (iii) negotiations with suppliers.

22.3 Interdependent values and the Winner’s Curse

In auctions with interdependent values it is assumed that bidders have only partial information about the value of the item being auctioned off, in the form of a signal, which is a random variable. Many economists had analyzed the extreme case of common value auctions in which all bidders have the same *ex post* value for the item being auctioned off. Before the auction, however, each bidder has only a random signal regarding the value. Naturally, each bidder’s estimate of the true underlying value of the item will improve if he gets information about the other bidders’ signals – for example, when they drop from the process during an English auction.[22]

Interdependent value auctions are important to consider since it is difficult to imagine a situation in which there is not at least some portion of the value which is common. Even if bidders intend to purchase an item only for private consumption (such as a house or a painting), they may keep an eye towards selling it in the future, meaning that they would like also to consider the “future market value” of the item purchased, which is common, or at least depends on others’ valuations.

In procurement auctions bidders may have private values which stem from the way the business they are bidding on fits with their current business. The new business, however, is likely to have characteristics which are common regardless of the particular bidder. Such common values have to do with the business practices of the auctioneer, which may influence all bidders’ costs in the same way. Examples include the ease of doing business with the auctioneer and certain aspects of the service/project itself such as the final demand (in a bid on parts supply for an automobile model), or the soil characteristics and weather during the performance period for a construction contract. These are going to influence all bidders equally during the service period but are not known with certainty during the auction.

[22] The assumption that underlies this is that the bidding strategy is strictly monotone in the signal. In other words, a higher signal will mean a higher maximum bid.

Revenue Comparison

With interdependent values, it is no longer the case that English auctions are strategically equivalent to second price sealed bid auctions. The reason is that, as mentioned above, the English auction format allows bidders to know at what price other bidders have dropped out, inferring their signal and updating their own estimate of the true value of the item. Naturally, sealed bid auctions offer no opportunity to learn.

When analyzing interdependent and common value auctions, many economists also drop the assumption that the signals are independent and assume that bidders' signals are *affiliated*. Affiliation is a strong form of positive correlation.[23] Such treatment makes sense because in most cases the technology used to get the signals is similar. Since the revenue equivalence does not hold any more with affiliated signals, it is natural to question which auction format yields more revenue to the auctioneer.

With affiliated signals and common values[24] the English auction may yield higher revenue for the auctioneer than a first price auction. The reason is that the transmission of information allows bidders to continuously upgrade the quality of their estimate of the true value of the item being sold, and thus they need to hedge less, bidding higher to the benefit of the auctioneer.[25]

Similar intuition means also that the auctioneer should release all the information he has about the item being sold. This may raise the bidding in all types of auctions, again to the benefit of the auctioneer.

Note that this discussion ignores bidders' preferences. In other words, this analysis assumes a fixed number of bidders. Auctions that are better for the seller may be worse for the bidders so less will participate. This creates a trade-off.

Efficiency of Auctions

[23] More or less it means that if some of the signals are high, it is more likely that the rest are high as well.
[24] This result also assumes that the bidders' values are symmetric. In other words, the signals are drawn from the same distribution.
[25] Milgrom and Webber (1982) and later Klemperer (1999) offers a more subtle argument: that with ascending auctions there is more opportunity for the bidders to learn about the valuations of the other bidders. In general, the winning bidder surplus (the difference between his bid and his valuation) is due to his private valuation. The iterations of the ascending auction reveal more of the winning bidder's information and thus are bound to reduce his surplus. The more the price paid depends on others' valuations the lower the surplus is. A low surplus will take place when the bids are high, meaning that the revenue of the auctioneer will be higher than with sealed bid (which does not give an opportunity for information exchange). (Note that this result applies to a single unit auction only.)

Under the assumptions of the revenue equivalence theorem and with private values, all the standard auction formats will allocate the auctioned item efficiently. In other words, if the item is sold (remember the possibility of a reservation price preventing a sale), it will be allocated to the bidder with the highest value. In pure common value auctions, there is no issue of efficiency since ex post all bidders value the item identically. In auction with interdependent values, however, the item sold will be awarded to the bidder with the highest signal, who may or may not be the bidder with the highest value.[26] Efficiency is assured only if the signals are such that the *ex post* values of all bidders can be ordered in the same way that their signals are ordered. Under this condition, all the standard auctions will be efficient, provided only that the signals are symmetric.

22.3.1 Winner's Curse

An interesting phenomenon which takes place when bidders fail to account for it and in the presence of interdependent values is the winner's curse. The winner's curse takes place when winners pay too much, due to their failure to anticipate and correct, in their bidding strategy, a bias in their estimate of the value of the item being auctioned off.

> **A Classroom Experiment**
>
> To demonstrate the winner's curse, Bazerman and Samuelson (1983) repeatedly auctioned off a jar filled with $8 worth of coins to different MBA classes at Boston University. The value of the jar was not revealed but the students could look at it and make their own inferences. The auction followed a 1SB format. In a large number of experiments, the average bid was $5.13, while the average *winning* bid was $10.01. Thus, on the average each winner lost $2.01. This is a pure common value auction (the value of the jar is the same for all bidders once the auction is over) which demonstrated clearly – the winner's curse.

The phenomenon arises since the winner of an auction with interdependent values is the bidder who submitted the highest bid, which is the bidder who had the highest signal. When notified of the win, the bidder knows one thing for sure – his signal was higher than the signal of all other (n -1) participants in the auction. This means that the winner is the bidder who most overestimated the value of the item sold. Economists sometimes refer to this phenomenon as the "adverse selection bias."[27]

Mathematically, this can be stated as follows:

$$E[V_0 \mid S_i = s_i] = s_i \qquad [22.11]$$

[26] In the interdependent auction case even when standard auctions *are* efficient they may not be optimal. (See, for example, Campbell and Levin, 2003.) In such cases and with multi-unit auctions, efficiency is a complicated issue.

[27] It can be argued that the Classroom example above does not conform exactly to the notion of the "winner's curse" since all the students observe the same jar and therefore get the same signal. The mistake in bidding here is rooted in different processing of the same information among the bidders, leading to different estimates of the content of the jar, rather than in different signals.

The best estimate of the true value of the item sold, v_0, before the auction is conducted, when the only information available to each bidder is s_i, the signal itself.[28] Consider now a sealed bid auction. When notified that he won, the winner knows that his signal was the highest signal, s_{max}. Based on this information, his estimate of the value of the item sold is that it is smaller than s_i since:

$$E[V_0 \mid S_i = s_{\max}] \leq s_i \qquad [22.12]$$

Thus, winning entails "bad news" about the value of the item just bought (in other words, "cold feet" after a win make perfect sense…) – unless the bidder realizes it and adjusts his bid a-priori.

22.3.2 A Case of the Winner's Curse

A probable case of the winner's curse took place in 1995 when Carolina Freight, a large LTL carrier won several large transportation services bids, notably from K-Mart and General Electric Company. At the time, these were some of the largest transportation auctions since many companies at the time (including GE and K-Mart) consolidated their LTL transportation needs and decided to award it to smaller number of carriers. (This was part of the general trend towards "core suppliers" in corporate procurement.) These auctions drew all the major LTL carriers in the US. Carolina won the bids even though it has not hauled for some of these customers before.

A few months later, Carolina Freight went bankrupt. Industry observers felt that winning these accounts was one of the more significant nails in Carolina Freight's coffin. It turned out that some of these accounts had elements that were difficult to service:[29] they required extra dock time; the freight was hard to mix with other companies' freight due to the way it was stacked; these customers required a lot of ancillary services which they expected the carriers to include in their transportation price; the freight tendering process involved was not accurate, creating extra work for the carrier; and many of these customers tended to pay late, increasing the financing burden of its carriers.

Some carriers participating in these auctions have served these customers or similar customers before and took these difficulties into account in their bid. Other carriers may have hedged their bets and bid high in order to account for the uncertainty of dealing with a new accounts. In any event, Carolina Freight

[28] Assuming that this is an unbiased estimator.
[29] Obviously not all the account had all the problems mentioned here.

was the lowest bidder and won several of these auctions in 1994 and 1995.[30] A few months later, Carolina Freight realized, for example, that hauling for K-Mart was unprofitable (by some industry accounts it cost them on the average $1.50 to haul $1.00 worth of revenue). Carolina Freight tried to raise prices, K-Mart resisted, and a minor public dispute ensued. Carolina Freight, wishing to appear fiscally responsible, told the trade press that they had dropped K-Mart. K-Mart claimed that they had dropped Carolina for service-related reasons. In any event, it was too-little-too-late and Carolina Freight filed for bankruptcy.

Fig. 22.5 ABF Stock Chart

Naturally, it is difficult to attribute all of Carolina Freight's difficulties to the winner's curse phenomenon. However, winning over other bidders who have more information (due to being incumbents) is even worse news than just winning when everybody has the same information…

A few months later, at the end of 1995, Carolina Freight was bought by another LTL carrier, ABF. Interestingly, that acquisition may have been another example of the winner's curse. Shortly after acquiring Carolina Freight, ABF has realized how many unprofitable accounts Carolina had, and the burden of the acquisition was one of the reasons the ABF stock price went as low as less than $5.00/ per share within five months of the acquisition, as shown in Figure 22.5...

The story does have a happy ending, though. In a decisive series of moves the ABF management team has walked away from a lot of the unprofitable business and raised prices on other. This, in combination with aggressive cost controls were responsible for ABF's recovery from the post-acquisition woes and for its continued profitability ever since. In fact, since that time, ABF has been the most profitable of the national US carriers.

22.3.3 Correcting for the winner's curse

The M&A example below[31] demonstrates, again, the winner's curse. The question is how should the bidder, in general, control for the winner's curse in his bid?

[30] It should be said that at the time Carolina's fortunes were already shaky and the bids could have represented a desperate move to gain business for the cash flow and to increase Wall-Street's confidence in the company.
[31] This example was suggested by Dan Levin.

An M&A Case of the Winner's Curse

Consider an acquiring company, BIG Inc., wishing to buy company SMALLCO. BIG Inc. knows that the value of SMALLCO is a U($0,$100M) random variable. (Naturally, the owner of SMALLCO knows its true value.) BIG Inc. also knows that due to its superior management and technology, it can increase that value by 50% after the purchase regardless of what that value turns out to be. The question is how much should BIG bid?

Without strategic analysis, most respondents would suggest a value between $50M and $75M. The logic for such biding, as argued by bidders in controlled experiments, is that the average value is $50M, thus $75M for the bidder. Bids between $50M and $75M are just a way to split the surplus. Such logic, however, is misleading.

Consider for example, a bid of $60M. If this bid is accepted, BIG knows only one thing – the true value is uniformly distributed somewhere between $0M and $60M since the owner of SMALLCO would have accepted only offers higher than the true value… In fact, the expected value at this point is only $30M and even a 50% performance improvement will mean that the buyer is expected to lose $15M – a case of the winner's curse.

Understanding this quandary, how much should BIG Inc. bid?

A bidder who wants to correct for the adverse selection bias should assume that he will be the winner (otherwise why bid?). Therefore he should assume that his signal is unusually high. In fact, he should assume that it is the highest signal and therefore it is a biased estimate of the true value of the item sold. Thus, rather than making the naive assumption in Eq. [22.11] bidders should "shave" their estimate of the value.

Note that this "shaving" of the bid should not be confused with the "shaving" discussed in connection with 1SB auction (see Eq.[22.6]). The shaving discussed here is due to the need to reduce the estimate of the value of the item and should be applied independent of the former.

Naturally, the more bidders there are the more each bidder should shave his bid since winning over many bidders is even worse news than winning over a few bidders.

Assume that a common value, v_0 is drawn from a U[L,H] distribution and that each bidder is given a signal, s, drawn from a U[v_0-ε, v_0+ ε], where L, U and ε are known to all the participants. The estimate of the common value is:[32]

$$E[V_0 \mid S = s_{\max}] = s - \frac{n-1}{n+1} \bullet \varepsilon \qquad [22.13]$$

[32] Under the information structure described, the solution is different in the three regions: L-$\varepsilon \leq L$+ε, L+$\varepsilon \leq H$-ε, and H-$\varepsilon \leq H$+ε. The solution depicted in Eq.[22.13] applies only to the middle region and assuming that the s is not "too close" to the lower boundary, L+ε. (See references in Section 22.8)

And each bidder should bid accordingly.

This means that by assuming that the value of the item auctioned off is equal to their signal, a winner in a first price auction will be expected to lose, on the average, $[(n-1)/(n+1)]\bullet\varepsilon$.

The more accurate the signal is (smaller ε), the smaller is the “shaving” of the estimate of the signal. In addition, the more bidders there are, the bigger should be this correction. The reason is that, without correction, winning over many bidders (having the highest signal among a many) is worse news than winning over a few bidders. Consequently, the correction for the value estimate has to be bigger with more bidders.

One might ask what the strategy should be if it is known that one of the bidders does not shave his estimate and bids as if his signal is an unbiased estimate of the common value. This may cause that candidate to win sometimes, to the detriment of the rational bidders. The answer is that if it is known that there is a bidder who acts in such irrational manner, bidders should shave their estimate down even more aggressively, since winning over such an irrational bidder is really bad news…

In English auctions with interdependent values, the bidders update their estimate of the value of the item sold every time a bidder drops out, since this provides another point of information. At any point in the auction, the remaining bidders’ current estimate of the value is a function of their own signal and their knowledge of the prices where other bidders dropped out, which is related to those bidders’ signal. Thus, for example, as the quality of their estimate improves, bidders may not need to shave their estimate of the value by as much as they did when they had no additional information.

The existence of the winner’s curse in reality is still debated among auction theorists since it implies that bidders are not rational, which is one of the tenets of economics. While economists have offered many alternative explanations to apparent winner’s curse occurrences, we do not subscribe to the notion of rationality – our experience leads us to believe that not only are some bidders less than rational, many auctioneers know and expect apparently irrational behavior as part of their auction strategy. Even in cases of repeated auctions involving seasoned bidders, as is the case with procurement auctions, many factors combine to make each auction almost unique. Aside from the difficulties of the auctioneer to estimate the conditions during the period of performance such factors include the economic outlook at the time, the tenure of the bidder at the job, the inability of suppliers to manage and learn from their own accumulated knowledge, the sheer volume of bid responses required of many suppliers, the lack of accurate costing methods, and the ex post rationalization of using less than fully allocated cost to estimate the profit associated with a given business.

22.4 Managing Auctions

The revenue equivalence theorem means that many of the auction types discussed here have the same payoff. The assumption underlying this theorem, however, can be rarely expected to be met in practice. For example, we already mentioned that if the signals are affiliated, English auctions may outperform first price sealed bids auctions. It is thus interesting to see how relaxing some of the other assumptions of the revenue equivalence theorem will impact the choice of format.

The comparisons are focused on the two most common auction types: English auctions and first price sealed bid auctions. In general, it is instructive to note that since both formats are widely used in practice, one cannot expect either one to be totally dominant under all circumstances. As mentioned above, both auctions may provide similar payoff: in a first price sealed bid auction, participants are likely to bid a little less than their valuation (so they have "surplus"), but the payment is based on the first price. In an ascending auction, participants may bid their true valuation but the payment is based on the second price.

22.4.1 Relaxing the Assumptions

This section looks at relaxing some of the assumptions regarding the nature of the bidders imbedded in the analysis offered above.

Risk aversion

In first price auctions, an increase in any participant's bid will increase his chance of winning but reduce the value of a win. Thus, if participants are worried about not winning, they would bid higher in first-price auctions. This provides them with some "insurance" (at the cost of the incremental bid) against losing. Auctioneers, then, should prefer a first price sealed bid auction to an English auction.

With common values and random affiliated signals, participants may also be worried about bidding too high and winning with negative surplus. As mentioned in Sec. 22.3.3, however, such risk aversion may make bidders more comfortable with their own (continuously modified) estimate when participating in an English auction, due to the openness of the format.

Asymmetric Valuations

The revenue equivalence theorem assumes that bidders' valuations are drawn from the same distribution. If the valuations are drawn from different distributions, there will be "strong" bidders, whose values are drawn from higher distributions of valuations than "weak" bidders. This is very relevant to procurement auctions when some of the bidders may be incumbents and have information which is not available to new vendors. In general, strong bidders would prefer open ascending (such as English) auctions while weak bidders would prefer sealed bids auctions; strong bidders will always win in an open, ascending auction. The reason is that the sealed bid auctions are less certain and thus allow weak bidders some chance of winning. Furthermore, Weak bidders are likely to bid aggressively (closer to their valuation) and therefore should be encouraged to participate. Thus, sealed bid auctions may have a higher expected payoff and auctioneers may prefer them. On the other hand, the same argument means that such auction can be expected to be less efficient than English auctions (since by definition, weak bidders, who have lower valuations, may win).

If the bidders' valuations are also interdependent and their signals affiliated, the effect of asymmetries may be even more pronounced. The winning bidder in an English auction will have to pay a price that no other bidder was willing to pay. In a sealed bid auction, on the other hand, a bidder may win at a price that other bidders would have been willing to pay but they did not have an opportunity to express it. Consequently, there may be a lesser chance of a winner's curse. In particular, weaker firms will be cautious in English auctions since they will win only if they really overestimate the value. Knowing that their opponents have to hedge, strong firms may win in an open auction without bidding very high. Consequently, sealed bid auctions may have higher payoff, in contrast to our conclusion in Sec. 22.3. Even our assertion there that the release of public information benefits the auctioneer may not hold with asymmetric signals.

Reputation Advantage

A related but different phenomenon takes place when one of the bidders establishes a reputation for aggressiveness. In other words, he usually bids high. This is particularly relevant for procurement auctions since these auctions are repeated regularly with many of the same vendors bidding for the business. A reputation established in one auction can be carried to the bidder's advantage in the next auction.

To understand the advantage, which is relevant only in interdependent and common value auctions, consider two bidders, A and B, taking part in an English auction. If bidder A has a reputation for bidding aggressively, Bidder B may think that this is because Bidder A actually has better technology, lower cost structure, advantageous supply contracts, or whatever. Thus, Bidder B may lower his bid to account for the fact that winning over A may mean a particularly intense case of the winner's curse.

Bidder A, who may be just an ordinary bidder with no inherent advantage at all, realizes this and understands that now the danger of the winner's curse has lessened for him (winning over B who is bidding lower is not very scary). Thus, he can bid even higher and ensure three things: (i) he increases the probability that he wins, (ii) when he wins he pays less since the second highest bid is low (this being a second price auction), and (iii) he maintains his reputation for aggressive bidding. The reputation advantage is then self reinforcing and will result in lower revenues for the auctioneer.

In a first price auction such a posture is more expensive to maintain since bidders have to pay what they bid when they win. Thus, a first price auction may yield higher revenue when some bidders have even a small advantage (perceived or otherwise) over other bidders.

22.4.2. Practical Auction Design Considerations

The assumptions underlying auction theory do not hold in many real world cases. The theory does offer, however, invaluable insights into the basic behavior of auction participants. In particular it is difficult to imagine actual procurement auctions in which the information is symmetric and the values are anything but interdependent. While the existence if interdependent values may argue for English auctions, asymmetries and reputation advantages may call for sealed bid auctions. Thus, there is no clear-cut format preferred in all cases.

When designing actual auctions auctioneers should worry about several primary “common sense” issues, such as attracting enough bidders, preventing predatory behavior and avoiding collusion among bidders. The examples mentioned below are all taken from public sector auctions of the electromagnetic spectrum since in these cases the results were known (at least after the fact). These examples also deal, for the most part, with the auction of multiple items, a subject discussed only in Section 2222.5, but they illustrate the points made.

Number of bidders

As the example in Fig. 22.3 shows, the expected revenue of the auctioneer grows with the number of bidders. This is in line with intuition – the more bidders there are, the more competitive the process is. The question is what effect the auction format has on participation.

With interdependent values, English auctions may attract a smaller number of bidders. The reason is that weaker bidders may know that the strongest is likely to win and therefore not enter, allowing the strongest bidder to win at a lower price (lower than the price obtained

> **Holland – 3G Auction**
>
> Holland held its 3G auction in the summer of 2000. Even though there were only five incumbent operators and only five licenses were offered, creating five “natural” winners. The ascending price auction brought about the predictable disastrous result (for the government) with the auction raising less than a third than what was predicted.

if more bidders would have participated).

The argument is even stronger when the participants' valuations are affiliated. In this case the prospect of winner's curse may deter weak buyers from entering the auction process not only because they are afraid to lose but because they are afraid to *win*; winning over stronger bidders implies a severe over-estimation problem. The result is that the advantaged bidder always wins and that revenues are low. By contrast, sealed bids may encourage more participants to enter the auction, thereby increasing the expected revenue of the auctioneer.

As shown in the examples given in the insets, Holland botched its 3G auction by using an open ascending format, based on the belief that with affiliated signals English auctions tended to generate higher revenues, while neglecting the fundamental factor of attracting more bidders. Denmark, by contracts invested a lot in attracting more bidders, resulting in a very successful auction.

Denmark – 3G Auction

Denmark, which held its 3G auction in late 2001, faced a similar situation: four licenses and four incumbent bidders. To encourage entry it chose a sealed bid format that succeeded in attracting several foreign bidders. The result was that revenue far exceeded expectations. It was more than twice the payoff achieved by Switzerland, Belgium and Greece who conducted 3G auctions at around the same time.

Interestingly, some economists claim that when the number of bidders is very large, fear of the winner's curse may hold bidders back and that this effect may be even stronger than the competitive effect of having many bidders. The argument being: "There are a lot of smart people bidding, if the object is so good, why doesn't somebody else bid higher?[33]" This argument may have a secondary effect with common valuations.

Nevertheless, the important consideration is to have "enough" bidders. In fact, in many settings it is worthwhile for the auctioneer to pay bidders to actually enter the auction. This is done routinely in takeover battles in which "white nights" are encouraged to participate. On a more daily basis many leading firms pay bidders the cost of preparing a proposal in order to encourage more of them to bid, as do many research grants foundations, when trying to encourage researchers to participate in an RFP process for research.

[33] See, for example, Bulow and Klemperer (2002).

Predatory behavior

> **PacBell Telephone Bid**
>
> Before bidding on a California telephone license in 1994, Pacific Bell announced in the Wall Street Journal that "if somebody takes California from us they will never make any money." They also ran full page advertisements making the same point in newspapers where their main rivals were headquartered. Clearly this was meant to create an aggressive impression and deter other bidders.

Ascending auctions are more susceptible to predatory behavior than sealed bid auctions since buyers can, for example, act aggressively in the early rounds, causing other participants to drop too early, leaving the predator with a relatively low price. They can also employ high increment bidding ("jump bids") in order to cause other bidders to drop early, letting them win in a price that is lower than their limit.[34] Of course, as the PacBell, Glaxo, and the Dutch 3G auctions in the examples below demonstrate, the predatory behavior can be even more direct.

> **Glaxo Acquisition of Wellcome**
>
> When Glaxo made a bid to acquire Wellcome Inc. in early 1995, it made clear, through statements in the Financial Times, that it "would most certainly top a rival bid." Thus increasing rivals fear of the winner's curse and causing them not to enter into a bidding war.

> **Holland – 3G Auction**
>
> In the Dutch 3G auction mentioned before, one of the strong bidders, Telfort, sent a letter to a weaker bidder, Versatel, threatening it with legal action if it continued to bid. The government took no action so Versatel quit the auction and the sale raised a lot less revenue than predicted.

When there are many bidders predatory behavior is typically not a problem. It is a problem in cases where there are very few strong bidders. In many of these cases the auctioneer cannot really "punish" the offending bidder since they are either needed due to their technology or because dropping them would leave the field with too few bidders. In the electromagnetic spectrum auctions mentioned above, it is also difficult to imagine the practicality of the fine that would have to be levied on the offender – it will have to be in the billions of dollars to have a deterrent effect. In these cases the auction mechanism may not be the best vehicle – direct negotiations may produce better results, especially for private sector procurement situations. In the public sector, auction sometimes have to be used for the process to be seen as fair and open.

Collusion

[34] "Jump Bids" are common in takeover battles, which is basically an English auction with no set increments, and where bidders can come in and out of the auction

Many auctions (and in particular procurement auctions) involve the sale of multiple items simultaneously. Ascending auctions give bidders the opportunity to signal to each other in the early rounds and to “divide the pie” without driving the price very high. This is demonstrated in the Austrian example, where a natural “understanding” between the bidders was aided by statements from a strong participant. It is also demonstrated in the German example where the bidding process was used to divide the spoils between the two strong bidders.

> **Austria’s 3G Auction**
>
> In 1999 Austria put out to bid twelve spectrum blocks. Six bidders participated in the auction which concluded very quickly with each bidder winning two blocks. Of course, the bidders could have arrived to this “understanding” with no help but just to ensure the outcome, Telekom Austria (one of the strongest bidders) announced just before the auction that “they would be satisfied with just two blocks” and “if all bidders behave sensibly it would be possible to get the frequencies on sensible terms” but “it would bid on a 3rd block of any rival would…”

> **Germany’s 3G Auction**
>
> In 1999 Germany auctioned off ten spectrum blocks in its DCS-1800 auction. It instituted a rule that each bid had to be 10% higher than the last one. There were only two credible bidders: T-mobile and Mannesman. The first Mannesman bid was DM 20 million for block 1-5 and DM 18.18 million for block 6-10. Since 18.18 plus 10% exactly equals 20, T-Mobile understood this as a signal. They bid DM 20 million for block 6 – 10 and lower for blocks 1 – 5. The auction was over at that price.

English auctions also give the participants opportunity to “punish” aggressive behavior by bidding high on something that the aggressive firm really cares about – as shown by the US West and Mcleod example below. All this can be done in the early rounds without the participants having to actually pay their bids.

> **US West and Mcleod**
>
> In 1996 US West was competing against Mcleod for several spectrum licenses. It really wanted the Rochester Minnesota license and when Mcleod bid on it, US West bid $313,378 and $62,378 for two Iowa licenses which Mcleod was bidding aggressively on but US West did not show any previous interest in. Since all other bids were in exact thousands of dollars and “378” was the area code of the Iowa license area, Mcleod understood the signal and the implicit threat and dropped out of the Rochester competition. When it then raised its bid for the Iowa licenses, US West did not respond, as expected.

In many cases the opportunity for collusion and predatory behavior embedded in the English auction is quite subtle. It allows bidders to enforce a tacit understanding by knowing who is bidding what and these winners knowing that they know…

Collusion is much harder to organize covertly in sealed bid auctions since they provide no opportunity for signaling and punitive bids. This was one of the reasons that the Danish government chose a sealed bid format in its highly successful 3G auction.

Also, in a sealed bid format new entrants may be attracted when "bidding rings" are formed since these rings will try to keep the price down, allowing for an outsider to win at an advantageous price.

Summary of Practical Considerations

As argued above, sealed bids have many advantages in that they can deter predatory behavior and collusion, and they can attract more bidders leading to higher revenue for the auctioneer. Yet because they attract weaker bidders they may lead to less efficient results. They also require more information gathering from the bidders (since the process itself reveals less) and thus may deter some bidders.

Denmark, 2001

As mentioned before, Denmark in its summer 2001 3G auction used a sealed bid format in order to attract entries. In addition, however, it allowed bidders to submit multiple bids in multiple locations. The idea was to help increase the fear of cheating among colluding partners by making it more difficult for "bidding rings" to enforce their internal agreement or understanding. The auction was a success.

In addition, the competitive process imbedded in the English auction may lead to (sometimes irrational) higher bids and the process may seem more fair and less open to manipulation by the auctioneer, who may change the rules after the bidding, invite more bids, negotiate with bidders individually offering them private deals, etc. While the integrity of the auctioneer may be less of an issue in government auctions (where the results are know *ex post*), it is an important issue with business-to-business procurement auctions.

To avoid some of the pitfalls of ascending auctions, the bidders can be made to bid in pre-specified increments. This makes collusion and signaling more costly. Also, bids can be made anonymous, with only the winning bid showing at each round without identifying the owner of that bid. Again, this increases the hurdle to signaling and collusion.

Many auction variants were suggested in order to create more efficient revenue-maximizing auctions; most of them involve multiple auction phases. For example, working for the UK government, Klemperer (1998) designed an "Ascending-Sealed Bid" hybrid auction. This two-stage auction starts with an English (ascending) auction until two bidders remain. These two bidders are then required to submit final sealed bid offers which are no lower than the current asking price. This type of auction is somewhat similar to eBay auctions which terminate at a fix time. On eBay, when the auction is about to terminate, many bidders will submit a final bid knowing that there is no time to counter and thus the last few seconds somewhat resemble a sealed bid process.

22.5 Multiple unit Auctions

Most auctions involve the sale of several items rather than a single one. The auctioneer can, of course sell them in multiple separate auctions of a single item each, and sell them at separate time frame or in separate places. Setting up an auction, however, involves significant fixed costs and thus invariably most auctions involve the sale of multiple items.

In this section we will look at the sale of *m* identical items and we assume that the marginal values of the items are declining for all bidders. Thus, the value of having two items is less than twice the value of having one; the value of having three is less than 3/2 of the value of having two; etc.

22.5.1 Sealed Bid Auctions

We analyze three auction formats:

- Discriminatory auction
- Uniform auction
- Vickerey auction

Each participant submits a *bid set*: $\mathbf{b^i} = b_1^i \geq b_2^i \geq ... \geq b_m^i$ where b_j^i is the amount that bidder *i* is willing to pay for the j^{th} item. Once all the bids are received, the auctioneer ranks them all and awards the units to the highest m bids from among all $\{b_j^i\}$ where $i = 1,\ldots,n$ and $j = 1,\ldots,m$.[35]

As an example, consider the bid set submitted by three bidders for five auctioned items shown in table 22.1. These three bids (which are, in effect, three demand functions) are depicted in Figure 22.6. Note that the bids in the table are used only to illustrate the different auction rules. In other words, we use the same set of bids for all auctions. Obviously, with different auctions the bids will be different as bidders adjust their strategy to the different auction rules.

Table 22.1 Multiple Items Bids

	Bidder 1	Bidder 2	Bidder 2
Item	b^1	b^2	b^3
1	$120	$100	$106
2	$115	$70	$85
3	$82	$55	$80
4	$26	$37	$48
5	$20	$24	$33

The aggregate demand faced by the auctioneer can be obtained from combining all the bids, as shown in Figure 22.7. This figure also depicts the winning five bids.

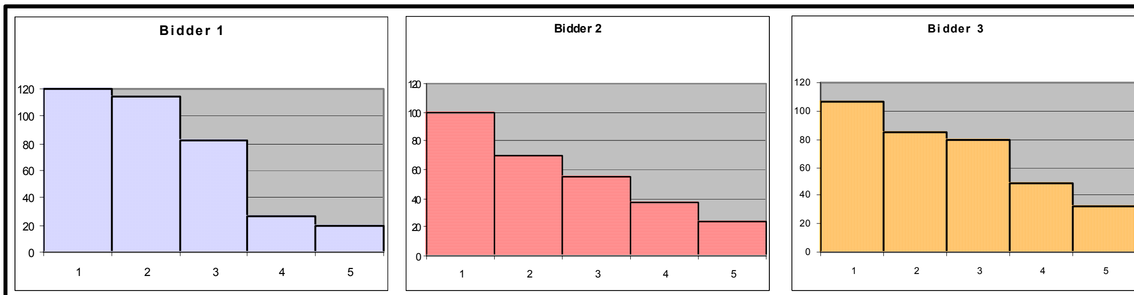
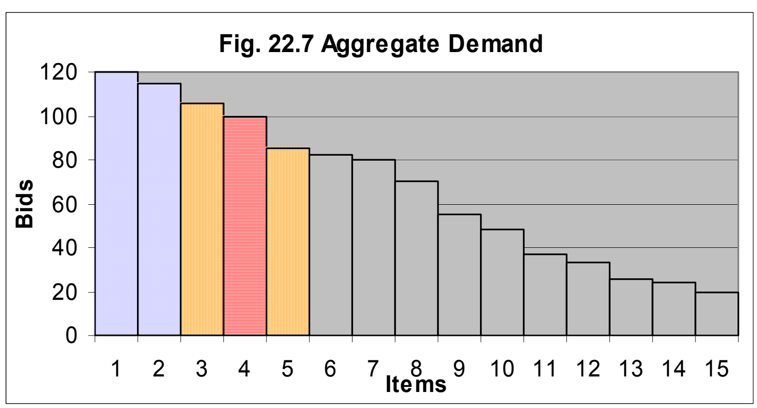


Fig. 22.6 Bid Sets

As evident from Fig. 22.7, Bidder 1 wins two items, bidder 3 wins two items and Bidder 2 wins a single item. Then three auctions mentioned above differ in how much the winners pay.



[35] Auctions such as the ones discussed here where the m highest bids win the m items are referred to in the literature as standard auctions.

Discriminatory auction

In a discriminatory auction each bidder pays what he bid – what he declared he is willing to pay for that number of units. Thus, if bidder *i* wins *k* items out of the m offered, he pays $\sum_{j=1}^{k} b_j^i$. In the example discussed here Bidder 1 will pay $b_1^1 + b_2^1 = \$120 + \$115 = \$235$, Bidder 2 will pay \$100 and Bidder 3 will pay\$191.

Uniform Price Auction

In a uniform price auction all units are sold at the same price. Different schemes are used to determine what this price is. The most common uniform price is the "market clearing" price. In this scheme all bidders pay either the lowest winning bid (as done, for example, by Amazon.com, eBay and UBid), or the highest losing bid.[36] In our example, paying the highest losing bid would mean that all winners pay \$82 per item while paying the lowest winning bid would mean that they all pay \$85 per item won. In keeping with the economics literature, we will use the highest losing bid as the market clearing price.

For a single item the uniform price auction reduces to a second price auction. As shown below, however, it does not share most of the important characteristics of second price auctions.

One way to see how much each bidder should pay is to construct a composite "market bid set" for each bidder. This bid consist of the highest bids of all the other players in the market – these are the bids each bidder has to defeat in order to win. For Bidder 1, for example, this composite market bid set is given by:

$$\mathbf{c^1} = \{\$106, \$100, \$85, \$80, \$70\}.$$

Recall that his bid set is:

$$\mathbf{b^1} = \{\$120, \$115, \$82, \$26, \$20\}.$$

As evident from the two bid set above, Bidder 1 wins two items, his competition wins three and he pays \$82·2=\$164.

Vickrey Auctions

A bidder who wins *k* items in a Vickrey auction pays the *k* highest losing bids of the other bidders, that is: the *k* highest losing bids excluding his own. In other words, bidder *i* pays the auctioneer what the auctioneer would have been getting for the items that bidder *i* won, had bidder *i* not participated in the auction.

[36] In treasury bills auction, small players all pay the average price instead of participating in the auction.

Going back to our example, consider the bid set of Bidder 1 and his composite market bid set:

$$\mathbf{b^1} = \{\$120, \$115, \$82, \$26, \$20\}.$$

And

$$\mathbf{c^1} = \{\$106, \$100, \$85, \$80, \$70\}.$$

If Bidder 1 would have not participated, the two items he won would have been won by others at prices of $80 and $70, respectively. (Bidders 2 and 3 would have won one item each – see the bids in Table 22.1.) Thus Bidder 1 pays $80+$70=$150.

Bidder 2 is facing a composite market bid set of $\mathbf{c}^2 = \{\$120, \$115, \$106, \$85, \$82\}$. He won a single item and thus if he would not have participated, somebody else (in this case Bidder 1) would have one that item. Thus he pays $82 for his item. Similar considerations mean that Bidder 3 will pay $70+$82=$152 for his two items.

Note that the examples depicted here do not mean that a discriminatory auction will yield higher revenue than a uniform or Vickrey auction. What we have not yet discussed is the bidding strategies that are likely to be deployed by the bidders in these cases. For example, sometimes participants in uniform auctions would bid higher than in discriminatory auctions since they do not have to pay their bid but a lower price.[37]

22.5.2 Open Auctions

This section looks at the following three open auction formats, which parallel, in some sense and under certain conditions, the sealed bid formats discussed in the previous section. The three formats are:

- Dutch auction
- English auctions
- Ausubel auctions

Dutch auctions

In a multiunit dutch auction, the auctioneer starts with a very high price and lowers it continuously. When a bidder decides to buy a unit at a certain price this unit is sold to him at that price. The auction ends when all units are gone.

[37] in the forward to the US Treasury report, **Uniform Price Auctions, Update of the Treasury Experience** (Malvey and Archibald, 1998), Dr. Lawrence Summers, then Deputy Secretary of the Treasury made the following arguments: “Auction participants will bid more aggressively in uniform-price auction since successful bidders in uniform price auctions pay only the price of the lowest accepted bid, rather than the actual price they bid, as in the multiple-price approach.”

This auction is *outcome equivalent* to the discriminatory price auction in the sense that each bidder pays his bid price. It is not, however, *strategically equivalent* since the Dutch auction is open. Thus when valuations are interdependent a Dutch auction allows bidders to update their own valuations as they watch the prices in which other bidders decide to buy. With either private value or with a single item auction first price sealed bid and Dutch auctions are equivalent since in the first case the information gathered during the open auction is meaningless and in the second the first bit of information available also means that the auction is over.

English auctions[38]

In a multiunit English auction the auctioneer starts with a low price and raises it. At every price point, bidders indicate how many items they are willing to buy at that price. Naturally, when the price is low the total demand is high and it drops as the price increases. When the total demand equals the number of units auctioned off the auction terminates (this happens when the demand changes from m+1 to m units). This is the price that all the bidders who are still active pay.

The English auction is outcome equivalent to the uniform price auction. Just as the equivalence between the English auction and the second price sealed bid auction of a single item does not extend to the interdependent values case, the English auction is not strategically equivalent to the uniform price auction. The reason is that the English auction is open and allows bidders to update their valuation as they watch the behavior of others.

Ausubel auctions

The Ausubel auction is an open, ascending auction that is outcome-equivalent to the (sealed bid) Vickrey auction for multiunit auctions. It starts with the auctioneer calling a low price and raising it while bidders indicate how many items they are willing to buy at each given price. At certain price points bidders reduce the number they are willing to buy at that price.

	Bidder 1	Bidder 2	Bidder 2
Item	b^1	b^2	b^3
1	\$120	\$100	\$106
2	\$115	\$70	\$85
3	\$82	\$55	\$80
4	\$26	\$37	\$48
5	\$20	\$24	\$33

This auction format means that at a certain price point in the auction, bidder *i* will "clinch" an item. This will happen when somebody (other than *i*) drops their demand and the auctioneer realizes that even if all the remaining items will go to all the other bidders, bidder *i* would still win one. At that point one item is sold to Bidder *i* at the current price. The process then repeats and it terminates when all items are sold.

[38] As mentioned before, it is unfortunate but the popular auction sites (Amazon, eBay, UBid) all call this type of auction,), "Dutch Auction" (albeit with the lowest winning bid as the market clearing price).

To see how this process works, consider the example presented at the beginning of this section.

- At a price of zero each participant is willing to buy all the items.

- At a price of $20 Bidder 1 drops his demand from five to four but this does not affect anything as the total supply is still high.

- When the price crosses $55, Bidder 1 is willing to buy three items, Bidder 2 two items and Bidder 3 three items.[39] The available supply is still five items. The situation is then as shown in the table:

Bidder	1	2	3
# Demanded @ $55	3	2	3
Total Supply:	5		

- At a price of $70 Bidder 2 drops his demand from two to one. At this point the total demand by Bidders 2 and 3 together is four items. Thus, Bidder 1 will win at least one item. This item is then sold to him at $70. This, however, is the same situation that Bidder 3 is in. The total demand by Bidders 2 and 3 is four items so he also clinches and gets to buy an item at $70. The process continues with three items left to sell as shown in the table.

Bidder	1	2	3
# Demanded @ $70+	2	1	2
Total Supply	3		

- At a price of $80, Bidder 3 drops his demand from two to one. At this point Bidder 1 clinches another item – the total supply is three and the total demanded by Bidder 2 and 3 together is only two. (Note that Bidder 2 does not clinch at this point since Bidders 1 and 3 together still demand the whole three remaining supply.) Bidder 1 now pays $80 for his second item and the process continues with two items remaining.

Bidder	1	2	3
# Demanded @ $80+	1	1	1
Total Supply	2		

- At $82 Bidder 1 drops from the picture completely (his demand drops from one to zero). At this point both Bidder 2 and Bidder 3 are facing a situation in which the total supply is two and the total demand by the remaining bidder is one. Thus each clinches one and the auction is over. They each pay $82 for the item they get at this point. The auction is over.

Bidder	1	2	3
# Demanded @ $82+	0	0	0
Total Supply	0		

The results of the Ausubel auction are

- Bidder 1 wins two items and pays $70+$80=$150
- Bidder 2 wins one item   and pays $82
- Bidder 3 wins two items and pays $70+$82=$152

[39] To use the baseball terminology used by Ausubel, at this point the "magic number" of both bidders 1 and 3 is one. This is the case since both of them are in a situation where if one of the other bidders reduces their demand, they would clinch an item.

Just like in the Vickrey auction.

In another version of the Ausubel auctions, which is mentioned here just for completeness, participants get feedback on the prices at which others have dropped. This creates a truly dynamic environment with bidders possibly using a rich set of strategies, depending on the information provided. In this case bidding one's value may no longer be the best response to every situation (se Sec.

The six auction formats discussed in this section are outcome equivalent to each other as follows:

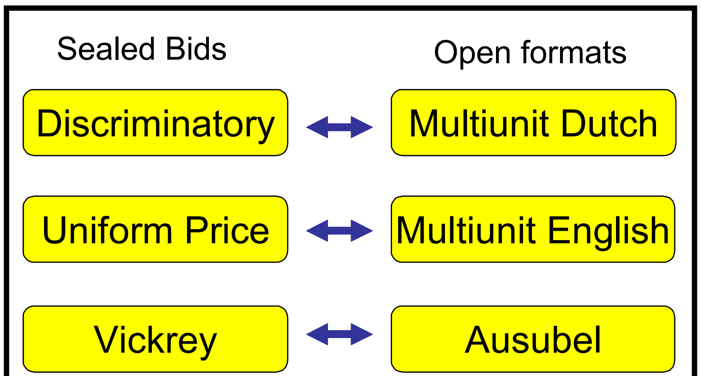


Fig. 22.8 Equivalence of Multi-unit Auctions with Private Values

22.5.3 Bidding Strategies

It is easy to show[40] that with private values, in both the Vickrey and its open equivalent – Ausubel auction -- bidders should bid their values truthfully. They cannot win and can only lose by deviating from such strategy. This, however, is not the case with uniform price and discriminatory auctions.

In uniform price auctions, bidders have incentive to shave all their bids beyond the first one. The reason is that for a given bidder, there is always a chance that one of his own bids, beyond the first will be the "market clearing" price and will therefore set his payments for all the items he will be buying. This reasoning does not apply to the first item. On the first item each bidder should bid his value, just like in a second price auction on a single item. If he does not, he can only lose. A high bid may cause him to win at a price higher than his value and a low

[40] To see this, use the example discussed earlier in this section. Look at a bidder and his composite bid set. Assume that he deviates from bidding his true value while all others do bid their values. See what happens if he wins more than before and when he wins less than before – the bidder under consideration will either lose or have the same surplus as before, but he can never improve his situation by deviating from bidding his value.

bid may cause him to lose an item for which he could have won with positive surplus.[41]

In a discriminatory auction, bidders shave all their bids for the same reasons that bidders shave their bids in a first price auction for a single item. Since they are paying the bidding amount for each of the items, the only way to guarantee positive surplus is to bid below their value.

The shading of the various bids in the case of uniform and discriminatory auctions means that they do not necessarily lead to an efficient allocation of the awards to the bidders. In other words, there is no guarantee that the bidders with the highest values will get the awards. The Vickrey auction, however, will allocate the items efficiently (but it does not assure the highest revenue for the auctioneer).

Interdependent Values

When the bidders' values are not private but interdependent, open auctions defer from closed formats. Open auctions allow bidders the opportunity to observe the prices at which other bidders change the quantity they want (in an English and Ausubel auctions – see related comment in the previous section), or the price at which they buy (in a Dutch auction). It then makes sense for bidders to continuously adjust their valuations throughout the auction process.

Another adjustment that bidders should be thinking about is an extension of the winner's curse discussed in Section 22.3.1. Recall that when bidding for a single item when values are interdependent (or common), the news that a person won meant that his signal was higher than all other signals. Without correcting for this *adverse selection bias*, winning mean that he probably over-estimated the value of the item.

With multiple items, interdependent values mean that each bidder has also an *internal* adverse selection bias. Consider, for example, a case of multiple items all with the same common value and bidders having random signals. Winning more items is, in that sense, worse news than winning fewer items.[42]

22.6 Procurement Auctions

Many firms, mainly in the US, have for many years relied on competitive mechanisms to choose among suppliers, especially in MRO and services procurement. Even when buying direct materials, many firms divide the

[41] Note that if each bidder is looking to buy only a single item, the uniform auction will have the same "bid your value" strategy that the Vickrey auction exhibits.
[42] Ausubel refers to this phenomenon as the "Champion's Plague," to parallel the winner's curse…

purchasing between (i) long term suppliers with whom they may work on joint product design and from whom they may buy highly engineered parts and subassemblies, and (ii) suppliers of commodities such as chemicals or standard parts. In many cases suppliers in the first category would not be subject to a competitive bid. Instead, buyers may rely on benchmarking studies and even "open book" agreement with suppliers to determine a fair price. Suppliers in the latter category, however, would be subject to bids frequently.

22.6.1 Characteristics of Procurement Auctions

Procurement auctions have certain characteristics which distinguishes them somewhat from auctions for electromagnetic spectrums or durable goods.

Future transaction

Many procurement auctions do not end up in a transaction. What the winner gets is the potential for selling its services in the future. Consider, for example procurement auctions for parts for a given model of an automobile or a certain appliance. The auctioneer will only buy as many parts as needed to go into the product made. In other words, if I Goodyear wins the right to supply Ford with tires for an upcoming SUV model, it will sell Ford only five tires per SUV manufactured and no more; there is no absolute tire volume commitment on Ford's part. It all depends on how many SUV-s will Ford sell.

Absolute volume commitments are sometimes made, however, when the supplier has to make a large specific investment in order to serve the buyer. Typically those are in the form of a minimum volume or a minimum dollar amount spent over a specified period. In addition to minimizing the future costs of the material bought, three other criteria are important in procurement auctions:

- Efficiency – While efficiency is important in any auction (in the public sector for economic welfare reasons), in procurement auctions another argument is at work. In an auction for future services, the quality of that future service may be a function of how well the business won in any auction fits with the winner's other business. If the new business does not really fit the winner's capability, the service will likely be poor.

- Robustness – a criterion that is not typically used in standard auction theory is that of robustness. The auction results are robust if a change to the set of suppliers – such as a supplier going out of business or consistently failing to meet commitments – does not result in a large cost increase. Such a situation may arise if the winner is a weak supplier while the others who are bidding on the same piece of business are expensive, and also if there is no alternative short term spot market for the item or part under consideration.

- Simplicity and Speed - Many procurement departments have to conduct large and complicated auctions very frequently. In addition, many suppliers have to respond to many auction - sometimes hundred every week. Consequently simple format that can be executed quickly are preferable.

Long term relationships

Buyers and sellers (auctioneers and bidders) develop long term relationships in the sense that the large suppliers and the large buyers depend on each other for business and capacity, respectively. The implications of this are the following:

- Sequential auctions - The auctions themselves are typically repeated every one to two years and suppliers do get to know the nature of their customers' business and the strategies of their main competitors.

- Asymmetric information – In every auction some suppliers are incumbents on a significant portion of the business. This means that they understand many of the processes of the customer's operation, may have electronic data interchange links already established, or may be located nearby. In addition they may be aware of contract details that may not be mentioned in the request for proposal such as extra requirements, the actual payment lead time, etc.

- Encouraging weak bidders – Another, related consequence of the long term aspect of the relationships between buyers and seller is that auctioneers have an interest in encouraging weak bidders. Over the long term procurement departments need more qualified suppliers in order to keep long term prices low and capabilities available. Thus, encouraging weak bidders to participate in auctions and awarding business to weak bidders are important considerations.

- Auctioneer's knowledge – Since the auctions are repeated periodically, procurement managers are generally aware of the range of prices and capabilities prevailing in the market place.

- Auctioneer's reputation – Auctioneers (customers) do enjoy or suffer from reputation developed *ex post*. Auctioneers who do not stand by their commitments or who are difficult to do business with find that suppliers not only bid higher (remember – in a procurement auction the auctioneer looks for the *lowest* price) the next time around but also are not reluctant to share their experience. On the other hand, customers who pay on time and are fair in dispute resolution may see aggressive bids.

- Collusion is not an important issue - By and large business-to-business suppliers do not seem to collude, at least in the US and Western Europe. The reasons are that when there is a small number of bidders, in many cases procurement department do not rely on auction mechanisms but rather on benchmarking and direct negotiations. When auctions are used, it is usually the case that the number of bidders is relatively large and there is less opportunity to collude. Other reason for the lack of widespread collusion may be the following: (i) the familiarity associated with the repeated nature of the auctions, (ii) the expertise of the auctioneers who know more or less what to expect, (iii) the ease with which human resources can move between companies and the existence whistle blowers, both of which make detection easy, and (iv) the generally high ethical standards of business to business dealings in the US and Western Europe.

  Consequently the issue of bidding rings and rigged bids which occupies some of the auction theory literature is not evident in many business-to-business procurement auctions. A very important exception to this statement is bidding on government contracts, and in particular local and city government contracts. Criminal bid rigging charges have been filed in New York and Chicago for contracts involving school construction, bridge repair, road pavement and many other construction and service contracts (see, for example Bajari, 2001)

Complexity of Auctions

Procurement departments handle a very large number of auctions, some of which are very large, involving thousands of items. Two of the many important characteristics of such auctions are:

- Interdependent of values - Procurement auctions should be thought of as interdependent value auctions, in which the bidders have both private and common values. The private component of the value is rooted in each supplier's own cost structure and the way the business auctioned off fits with the other commitments that the supplier may have. The common portion of the value results from characteristics of the customer and the business which are unknown at the time of the bid.

- Multiple Criteria – Procurement auctions are rarely determined based on price alone. In many procurement auctions, service quality and product quality are important determinants of the auction winner. In addition, incumbent bidders have an advantage since they are also likely to know the business and be ready to perform immediately rather than follow a learning curve. Different bidders score differently among dimensions of their offerings other than the price.

22.6.2 The Choice of Procurement Auction Format

As a result of the drive for simplifying the auction process itself, most procurement department are content with sealed bid first price auctions – mainly because of their inherent simplicity for the auctioneer: (i) they involve only one round of bidding, (ii) they are simpler to administer, and (iii) they take less time. Most vendors also prefer a single round bid since they have to contend with many (sometimes hundreds) of RFP-s per week. Keeping on top of multi-round bids is too time-consuming for these vendors. An auctioneer who wants to get many of these vendor to participate, has to simplify the process for them by sticking to a one-round format. Thus, interestingly, even though the auction theory literature suggests that sealed bid auctions require more preparation from the bidders in terms of preparation and market research, in procurement bids when bidders know the market, the burden of multiple rounds more than offsets this consideration.

As mentioned above, collusion and entry-deterrence are not significant issues in business-to-business procurement auctions. In addition, as we have argued, efficiency is an important consideration. In such cases one might think, based on our discussion in section22.4.2 that English auction would be used most of the time. Yet, it is the very fact that a large number of bidders have to be qualified and managed that pushes most procurement managers to a first price sealed bid auction. Again, simplicity and speed of the process seem to be driving these managers.

In addition, the need to evaluate multiple dimensions of service means that auctions are difficult to automate and the simplicity of the sealed bid one-round auction is attractive to procurement managers.

Another argument for the use of sealed bid auction is that they encourage weaker player to enter the bid since there is a realistic chance for them to win. As mentioned above, procurement managers have a long term interest in developing weaker bidders. This argument, however, is not completely one-sided.

Since the late 1990, several companies have used the services of Internet-based procurement and bidding service providers such as Free Markets Inc. and ICG Commerce Inc.. These companies manage the auction process claiming billions of dollars in savings. For the most part they use a descending (these being procurement auctions) English-type auctions to get these results. Other companies use management consultants or internal teams powered with software from providers such as Ariba Inc. and Commerce One Inc. to accomplish much of the same.

The use of multiple round auctions in this context just proves the point that until such technology, including both software and processes was developed, the burden of multi-round auction was the determinant factor in procurement auction design.

Even with this technology widely available, many procurement departments are sticking with sealed bid auctions, even as they use these consultants. The probable reason is that the main balue of the software and service providers was not so much in the use of descending rather than sealed bid auction mechanism. Instead, it is the fact that they are bringing into the process many new suppliers with whom they themselves have developed relationships through their repeated auctions. This is the reason that the market for the provision of procurement auction services is being stratified along industry lines with some companies stronger in one industry and others in other industries. It is also the reason that many companies use the services of these providers once and then move to use them on a different product.

Many procurement auctions combine several involve multiple phases of bidding. For example, some Free Markets and ICG Commerce auctions are conducted as descending (English) auctions with a final round of sealed bids, or face-to-face negotiations to which a few of the highest bidders are invited. In other cases a sealed bid auction is followed by face-to-face negotiations between the buyer and the bidding vendors, looking to pit them against other vendors in an effort to secure further concessions. This effectively amount to conducting a partial English auction on the heals of a sealed bid process.

22.7 Technical Appendix

22.7.1 First Price bidding

To develop the strategy for bidding in first price auction, assume that each bidder wants to maximize his or her expected gain, which is the surplus associated with winning times the probability of winning (see Eq.[22.5]):

$$E[Gain] = (v - b) \bullet P(b) \qquad [22.14]$$

The optimal bid is the one that maximizes the expectation in Eq.[22.14].

Consider the case with n bidders whose valuations are drawn from independent and identical (iid) distributions $F(\bullet)$. Assume further that the equilibrium of such an auction has bidders using a symmetric and monotonic bidding function. This is:

$$b_i(v_i) = b(v) \quad \forall i, \quad \text{with } db(v)/dv > 0.$$

Let the inverse bidding function be $\upsilon(\bullet)$; in other words, $\upsilon(b_j) \equiv v_j$. A bidder j who submits a bid b_j , wins against a rival bidder k when $b_j > b_k(v_k)$, or when $v_k < \upsilon(b_j)$ with probability given by $F(\upsilon(b_j))$. Thus, the probability of winning against $n-1$ rivals with a bid of b is (due to the i.i.d assumption) $[F(\upsilon(b))]^{n-1}$. Using this notation, the maximization problem in Eq. [22.14] can be written as:

$$\text{Max}_{wrt\ b>0} E[Gains] = [v-b]\bullet\left[F\left(\nu(b)\right)\right]^{n-1}\bullet \qquad [22.15]$$

The first order conditions yield:

$$[v-b]\bullet(n-1)\bullet\left[F\left(\nu(b)\right)\right]^{n-2}\bullet f\left(\nu(b)\right)\frac{d\nu(b)}{db}-\left[F\left(\nu(b)\right)\right]^{n-1}=0$$

or:

$$[v-b]\bullet(n-1)\bullet f\left(\nu(b)\right)\frac{d\nu(b)}{db}-F\left(\nu(b)\right)=0$$

Since in symmetric equilibrium $\upsilon(b) = v$, this differential equation can be written as:

$$\frac{db(v)}{dv}=[v-b]\bullet(n-1)\bullet\frac{f(v)}{F(v)} \qquad [22.16]$$

Initial condition can be established independently by assuming that the lowest possible valuation gets 0 payoffs at equilibrium. If we assume further that the valuations are distributes U[0,1], we get:

$$\frac{db(v)}{dv}=[v-b]\bullet(n-1)\bullet\frac{1}{v} \qquad [22.17]$$

Assume now that the equilibrium solution is such that each bidder will "shave" his offer and bid a fraction of his value (so if he wins, he has a positive surplus). Thus the bid is $b = \alpha\cdot v$ where α is the same for all bidders and $db(v)/dv = \alpha$. Eq. [22.17] now becomes:

$$\alpha=[v-\alpha\bullet v]\bullet(n-1)\bullet\frac{1}{v} \qquad [22.18]$$

Leading to:

$$\alpha=\frac{n-1}{n} \quad or \quad b^*=\frac{n-1}{n}\bullet v \qquad [22.19]$$

The result conforms to our assumption that the bid is a constant times the valuation and thus the solution holds.

22.8 Literature Review

The discussion in this chapter was influenced strongly from the excellent exposition by Krishna (2002a) which is recommended for readers who are interested in auction theory. The book surveys some of the most important contributions in the field. Klemperer (2000) provides a very illuminating and detailed literature review of auction theory, including the seminal papers of Vickrey (1961), Milgrom and Webber (1982) and many others, as well as a relatively straight-forward proof of the revenue equivalence theorem. The conditions for revenue equivalence outlined in section 22.2.5 follow the classic paper by Riley and Samuelson (1981). A general treatment developed independently which makes that paper a simple special case was developed by is by Myerson (1981) Myerson's paper also shows that all the auctions referred to as "standard" in this chapter (and many more simple auctions) are optimal not just among auctions but among all possible mechanisms of exchange (revenue equivalence is a consequence of this result.) Extending Myerson approach Campbell and Levine (2003) developed it for the IID case but with interdependent valuations. Some of the auction examples mentioned in the chapter were taken from Cassidy (1967) and McMillan (2002), while all the spectrum auction examples were taken from a series of publications by Klemperer (2002a) and (2002b) where Klemperer discusses the nine Western European auctions in detail, and from references he gives there. The discussion of practical consideration in auction design given in section22.4.2 is influenced to a large extent by these publications of Klemperer. Some of the explanations of first and second price bidding strategies follow chapters 8 and 9 of Wolfstetter (1999). The discussion of the winner curse seem to have started with the arguments of Capen, Clapp and Campbell (1977), three petroleum engineers, who used the argument to explain why oil companies suffered unexpected low returns from low returns on OCS lease rates, year in and year out, during 1960 and 1970s. The phenomenon is described in detail in the book by Kagel and Levine (2002), which includes a series of papers by the authors, many of which describe laboratory auction experiments which help calibrate much of the theory in the field. In particular, Levin and Kagel (1996) give the proofs of the correction required in first and second price auctions to account for the winner's curse (following Wilson, 1977 and Milgrom and Webber, 1982). Kagel (1995) gives an earlier survey of experimental research in auction theory. A survey of applied work with auction data is given by Laffont (1997).

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