

Measuring Impact of CONWIP Control On Production Rate and Inventory  
Distribution for a Flexible Production System Through Simulation

by

Erik Joseph Lampe

Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the  
Requirements for the Degree of

Bachelor of Science

at the

Massachusetts Institute of Technology

June 2007

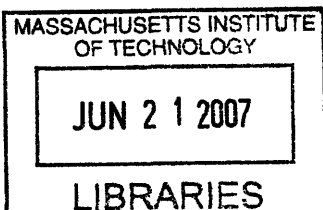
©2007 Erik Joseph Lampe  
All Rights Reserved

The author hereby grants to MIT permission to reproduce and to distribute publicly paper  
and electronic copies of this thesis document in whole or in part.

Signature of Author.....  
Department of Mechanical Engineering  
May 11, 2007

Certified by.....  
Stanley Gershwin  
Senior Research Scientist, Department of Mechanical Engineering  
Thesis Supervisor

Accepted by.....  
John H. Lienhard V  
Professor of Mechanical Engineering  
Chairman, Undergraduate Thesis Committee



ARCHIVES

# Measuring Impact of CONWIP Control On Production Rate and Inventory Distribution for a Flexible Production System Through Simulation

by

Erik Joseph Lampe

Submitted to the Department of mechanical Engineering  
on May 11, 2007, in Partial Fulfillment of the  
requirements for the degree of Bachelor of Science in  
Mechanical Engineering

## ABSTRACT

This research investigates the production output and inventory distribution of a manufacturing system that produces multiple part types on flexible machines while incorporating CONWIP inventory controls. The production machines, which are separated by individual storage areas for each part type, are modeled as unreliable and operate according to a strict priority sequence. The maximum total inventory of each part type in the entire system is limited a preset number of tokens in the system. While varying the token levels and the demand for finished parts, the average production output for each part type and the average inventory level for each buffer is monitored to understand their behaviors. The relationship between the inventory distributions and the production rates is also investigated. The goal of this research is to develop intuition behind the system behavior.

Thesis Supervisor: Stanley B. Gershwin

Title: Senior Research Scientist. Department of Mechanical Engineering

# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
1.1	Motivation.....	5
1.2	Flexible Production Systems.....	5
1.3	CONWIP Inventory Control Method.....	6
1.4	Research Approach .....	7
1.5	Literature Review.....	7
<b>2</b>	<b>System Model</b>	<b>7</b>
2.1	Model Descriptions and Assumptions.....	7
2.2	CONWIP in Different Demand Scenarios.....	10
<b>3</b>	<b>Discussion of Results</b>	<b>11</b>
3.1	Token Level Impact on Production Rates.....	11
3.1.1	Production Rates for High Demand.....	12
3.1.2	Production Rates for Medium Demand.....	13
3.1.3	Production Rates for Low Demand.....	15
3.2	Token Level Impact on Inventory Distribution.....	16
3.2.1	Inventory Distribution for High Demand.....	16
3.2.2	Inventory Distribution for Medium Demand.....	21
3.2.3	Inventory Distribution for Low Demand.....	24
<b>4</b>	<b>Conclusions</b>	<b>27</b>
4.1	Production Rates Intuition.....	27
4.2	Inventory Distribution Intuition.....	28
4.3	Future Research.....	28
<b>4</b>	<b>Appendix</b>	<b>29</b>
A.1	Number of Time Periods for Simulation.....	29

# List of Figures

2.1.1 Diagram of Production System Model.....	8
3.1.1 Production Rate vs Token Level for High Demand.....	12
3.1.2 Production Rate vs Token Level for Medium Demand.....	14
3.1.3 Production Rate vs Token Level for Low Demand.....	15
3.2.1 Type 1 Inventory Distribution for High Demand.....	17
3.2.2 Type 2 Inventory Distribution for High Demand.....	19
3.2.3 Type 3 Inventory Distribution for High Demand.....	20
3.2.4 Type 1 Inventory Distribution for Medium Demand.....	21
3.2.5 Type 2 Inventory Distribution for Medium Demand.....	22
3.2.6 Type 3 Inventory Distribution for Medium Demand.....	23
3.2.7 Type 1 Inventory Distribution for Low Demand.....	24
3.2.8 Type 2 Inventory Distribution for Low Demand.....	25
3.2.9 Type 3 Inventory Distribution for Low Demand.....	26

# **1. Introduction**

## **1.1 Motivation**

In manufacturing systems, companies must consider costs while maximizing their production output. One key source of cost that production systems face is inventory. When carrying inventory, companies must pay for the space to hold the inventory. Some inventory can also depreciate in value while sitting idly before it is used in the production line. A high inventory can also require a large investment of the company's capital which could be used for other profit-generating endeavors. In general, if machines are not perfectly reliable, a higher inventory level will increase the average production rate as downstream machines can pull parts from large inventory stores instead of always relying on a steady stream of parts coming from upstream machines. If machines are perfectly reliable, meaning that they never break down, inventory levels can be very low since stored parts are not required to keep downstream machines running when upstream machines are down. Machine reliability, however, can also require high investment costs. Managers must balance the various parameters of a system while trying to keep production levels high and total costs low. Understanding how to balance the different parameters can be difficult, especially for more complex production systems. Controlling a large number of parameters can also be a complex task for managers, so managerial intuition on system behavior is very important to making good decisions. This is especially true for flexible production systems, since they have the added complexity. Instead of only having to take into account the inventory levels and production output of a single type of part, managers must decide how to allocate resources to achieve high production output and low inventory levels for all of the part types. Currently, few analytical tools exist to help managers design flexible manufacturing systems that use a type of inventory control called CONWIP. The intuition developed here would serve to aid in designing these types of systems.

## **1.2 Flexible Production Systems**

Many of the most competitive production lines today make use of a flexible production system. In this type of production system, the same machines are used to

produce different types of products. With a single system that can produce different types of products, some companies can avoid having to invest in multiple production lines. System flexibility also provides the ability for a production line to rapidly respond to changing customer demands. The additional flexibility, however, provides greater system complexity, making system controls more difficult. Managers must meet desired production levels for each type of part, but they still must decide on an order of priority for the different types of parts. The higher priority types will tend to have higher production levels than the lower priority types. The priorities will also affect the inventory levels and distribution for each part type. In practice, the order of priority is not always obvious and often changes with varying demand.

### **1.3 CONWIP Inventory Control Method**

In order to decrease inventory levels and the associated costs, companies monitor and often set limits on inventory levels. Limits can be set at a micro level by controlling individual inventory holding areas, also called buffers. Inventory levels can also be controlled at a macro level by limiting the total inventory in the production line. CONWIP is a system that attempts to maintain a constant level of WIP, or work-in-progress in the production line. Using the CONWIP approach, managers can effectively control inventory levels through fairly simple controls of total WIP from a high level rather than controlling individual buffer levels. Using a less complex control system makes monitoring and controlling inventory levels easier.

While the total size of the inventory is important, it is not the only factor to consider for inventory. The location of the inventory is also important. Different inventory spaces can have different capacity costs. Controlling the location of the inventory can be important to ultimate cost of carrying the inventory. The location of the inventory can also have implications for the average production rate for a system with unreliable machines. Monitoring and controlling individual buffer levels, however, can take more time and attention than necessary. Developing an intuition of how the CONWIP approach affects the distribution of inventory over the buffer spaces for different relative demand and throughput rates can help managers determine how to best apply CONWIP controls in a manufacturing system design.

## **1.4 Research Approach**

Using simulation, this research builds intuition around the interaction between system inputs and the performance measures of production output and inventory levels. In particular, this research looks at the system effects of varying the maximum inventory level when using a CONWIP inventory control method. This control method will be applied to a flexible production system that manufactures three different types of parts on a single line of five machines. With the intuition developed here, managers can better understand how to design the production system that achieves high production levels at the lowest cost.

## **1.5 Literature Review**

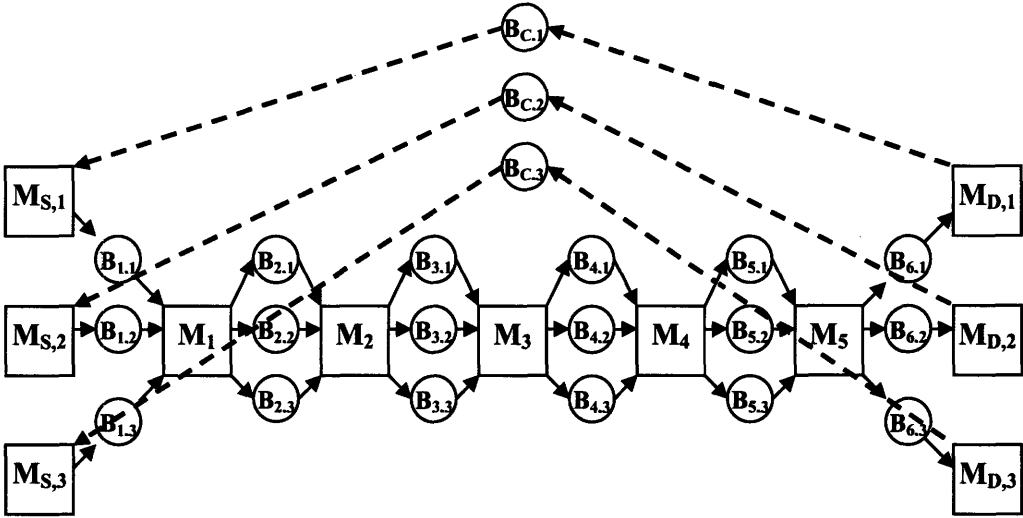
Stecke [1] describes problems in flexible manufacturing systems for researchers to work on, including determining appropriate buffer sizes and system inventory capacity. Tempelmeier and Kuhn [2] provide an overview of flexible manufacturing systems and design considerations including capacity optimization. Jang [3] provides a model and analysis of production rates and inventory levels for a flexible production line operating with a strict priority rule. His research, however, controls inventory through maximum buffer levels, rather than a CONWIP token control method.

# **2. System Model**

## **2.1 Model Description and Assumptions**

The system analyzed in this research as an example uses a flexible production system as described earlier. Figure 2.1.1 below shows the diagram of the system being analyzed. The production line manufactures three different types of parts on the same five-machine line. The part types are denoted as Type 1, Type 2, and Type 3. The parts enter the production line through supply machines labeled as  $M_{S,1}$ ,  $M_{S,2}$ , and  $M_{S,3}$ , modeling stochastic supply rates from suppliers. After coming through the supply machines, the parts are deposited in buffers, each of which holds a different part type. The production machines, labeled in the diagram as  $M_1$  through  $M_5$ , pull parts from the upstream buffers and deposit them in the downstream buffers labeled  $B_{1,1}$  through  $B_{1,3}$ .

After proceeding down the line through the production machines and buffers, the finished parts leave the production line through demand machines labeled as  $M_{D,1}$ ,  $M_{D,2}$ , and  $M_{D,3}$ , modeling stochastic demand from customers.



**Figure 2.1.1:** Diagram for production system analyzed in this research.

The system controls inventory levels using a CONWIP method, placing limits on the maximum inventory of each part type throughout the system. To model this control, three separate buffers are located in loops going from the demand machines to the supply machines, as shown in red in Figure 2.1.1. Each of these buffers begins with a preset number of tokens, which determines the total inventory allowed of each part type in the system. When a part enters the first buffer through a supply machine, a token is pulled from the corresponding control buffer for that part type and attached to the part. If there are no tokens left in the buffer, a part is not allowed to enter the system. When a part leaves the system through a demand machine, the attached token is removed from the part and placed back into the control buffer for that part type, making it possible for a new part of that type to enter the system.

The simulation used to model the system uses a time based approach. The system is given an initial state with all inventory buffers empty, all machines up and available to work on parts, and all control buffers full with the preset maximum number of tokens. The simulation runs for a preset number of periods. During each time period, each machine can work on up to 1 part. The machines in this model are not perfectly reliable,



which means that machine failures will occasionally occur. A machine failure is associated with a malfunction in the machine, preventing it from being able to work on parts. If a machine fails, it is said to be in a down state. If a machine is operating normally, it is in an up state. During the time series, each machine has an up or down state for each period, denoted by alpha. If a machine is up or down in a given period, the alpha for that machine is 1 or 0, respectively. An alpha of 0 for a given period means that the machine cannot work on a part. If the alpha is 1 and the inventory levels in the buffers before and after the machines allow, the machine can work on a part during that period.

The alpha state for each machine during each period is determined by geometric failure and repair processes. If a machine is up at the beginning of a period, it will fail with a probability  $p$  during that period. If the machine is down at the beginning of the period, it will be repaired with a probability  $r$  during that period. The future states of a machine are dependent only on the current state of the machine and have no dependence on any states before the current period. The theoretical throughput capacity of each machine is given by the ratio below.

$$P_{machine} = \frac{r}{r + p}$$

This ratio gives the average throughput of a machine, based on its repair rate and failure rate, if there is always inventory available in the upstream buffers and space available in the downstream buffers. The ratio indirectly comes from ratio of mean time to failure to the sum of mean time to repair and mean time to failure. If the probability of repairing a broken machine in a given period increases, the mean time to repair decreases. This increases the total up time of a machine, and therefore its potential throughput. Also, if the probability of failure for a working machine decreases, the mean time to failure decreases, causing the machine's total up time and potential throughput to increase. Similar to the production machines, the potential average supply rate for each supply machine is given by the same ratio as the production machines if space is always available in the downstream buffer for that part type. The same is true for the final production output of each demand machine if parts are always available in the last buffer in the line for that part type.

During each time period, each machine that has an alpha of 1 can pull one part from an upstream buffer and transfer it to the downstream buffer corresponding to its part type. Each of the buffers in the production line is given a maximum inventory capacity. A machine can only work on a part if one is available in a buffer immediately upstream and room is available in the downstream buffer for that part type. The machines operate on a strict priority rule in which a Type 1 part is always worked on first if possible, then Type 2, and finally Type 3. Anytime a Type 1 part is available in the upstream buffer and there is room in the downstream Type 1 buffer, the machine will work on a Type 1 part. If the machine cannot work on a Type 1 part, it will be available to work on a Type 2 part if upstream and downstream Type 2 buffer levels allow. Finally, a machine will only be available to work on a Type 3 part if the machine cannot work on either a Type 1 or a Type 2 part if the upstream and downstream Type 3 buffer levels allow. For the purposes of this research, we assume that there is no setup time required to switch a machine from one part type to another.

By varying the initial token levels in each of the control buffers, we can control the total inventory of each part type in the production line. The token levels of interest are those in which initial token level for a part type is set lower than the sum of the capacities of all the inventory buffers for that part type. Otherwise, the sizes of the control buffers are not actually placing any limit on the inventory levels, and the inventory levels are instead simply determined by the buffer capacities.

## **2.2 CONWIP in Different Demand Scenarios**

An ideal manufacturing system has a production capacity that exactly meets demand. If the supply rates and throughput rates of the machines in a production line exceed the demand rates, then the inventory levels can be reduced while still maintaining high output levels, effectively reducing costs. This research looks at the different scenarios for demand levels relative to the supply and production rates for each part type, in order to obtain an understanding on how inventory levels can be lowered without sacrificing output levels. The key scenarios to be examined are what we shall call the high demand, medium demand, and low demand scenarios. For the high demand scenario, the combined demand rates of the part types 1 and 2 are enough to exceed the

throughput rate of the production line. The medium demand scenario requires the combination of the demand rates of all three part types to exceed the production capacity. In the low demand scenario, the production capacity exceeds the combined demand rates of all three part types. By examining these different demand scenarios, we can gain insight into the relationship of the token buffer size with the production levels and inventory distribution, and how the relationship behaves differently based on the level of demand in comparison to the machine line throughput. This information will allow managers to better decide how to invest in machine and inventory capacity.

### **3. Discussion of Results**

#### **3.1 Token Level Impact on Production Rates**

When using the CONWIP control, the effect of the maximum inventory level on production level is very important. As mentioned earlier, the goal of the CONWIP system is to lower inventory levels, while still keeping production levels high. With this in mind, we wanted to measure the impact of the number of tokens made available in the control buffers on the production levels.

As mentioned earlier, the three important scenarios investigated here are high demand, medium demand, and low demand. Each of the scenarios use the same inputs of failure and repair probabilities for the five production machines, set at 0.01 and 0.10 respectively. Each of the three supply machines were also set with the failure and repair probabilities at 0.01 and 0.1 respectively. Based on the failure and repair probabilities, the throughput capacity of each production machine and each supply machine is 0.91. The capacities for each of the buffers in system were set at 10 parts. The three demand scenarios all used the above inputs. For each demand scenario, the sizes of the control buffers were all varied together from 1 to 100, by increments of 1. For each control buffer size, we measured the average steady state production output per period for each part type.

### 3.1.1 Production Rates for High Demand

In the high demand case, the failure and repair probabilities for each of the demand machines were both set at 0.10. The average demand rate resulting from those probabilities is 0.50 parts per period for each part type. This means that the combined demand rate for any two of the part types is enough to exceed the throughput capacity of the production machines. Figure 3.1.1 below shows the average steady state production rates for each token level.

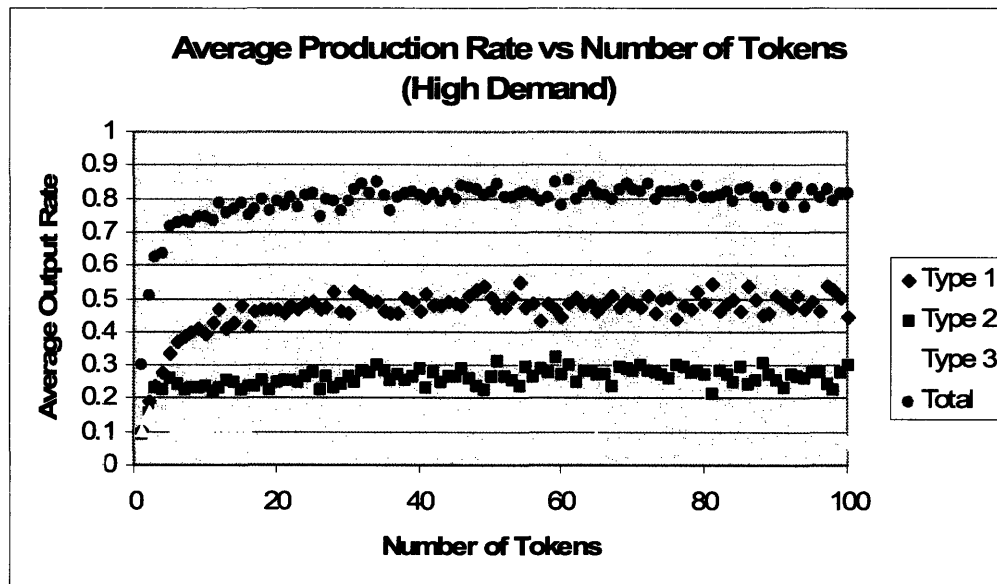


Figure 3.1.1: Manufacturing output levels for each part type with varying token levels for the high demand scenario, where average  $R_{dem}=0.50$  for each part type.

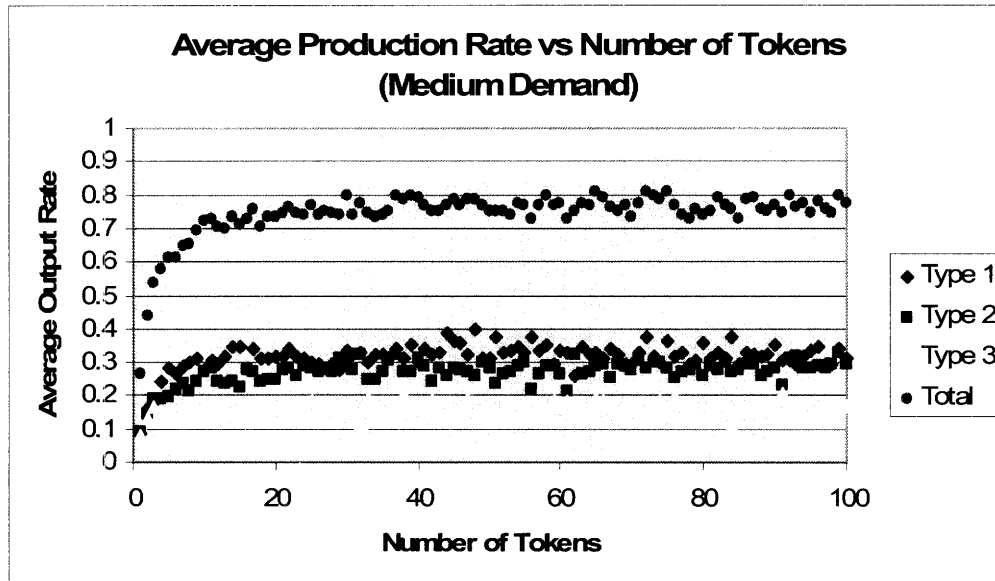
For the high demand case, we see that the production rates are dominated by the Type 1 parts. With the strict priority rules that we have set in place, the average production rate for Type 1 parts will always be at least as high as the other levels as long as the token levels, supply rates, and demand rates for Type 1 parts are the same as the other types. The graph in Figure 3.1.1 shows that as the number of tokens in the system increases, the output for Type 1 increases, but it plateaus around a level when the token level is about 30. The production output for Type 2 increases quickly at first, but then increases slowly until it plateaus when the token level is about 40. This is an interesting result because the minimum token level that would fill all the inventory capacity is 60, since there are 6 buffers for each part type and space for 10 parts in each buffer, but the production rates plateau at inventory levels that do not utilize full inventory capacity.

With this in mind, managers can lower inventory levels considerably from the maximum level and still achieve the same production rates. Using the CONWIP control rather than controlling individual buffer sizes allows the bulk of the inventory to be placed in the finished goods buffer before being pulled by the demand machine, which provides a higher production rate than if the inventory was kept in an upstream buffer before being pulled down to the end of the line.

Another interesting result we see is that the production level for Type 3 parts actually increases at first for very low token levels but then decreases for a number of tokens. For very low token levels for each part type, the production machine capacity is more equally distributed among the three part types because production of the high priority part types are constrained by how little inventory is allowed in the system. When the lower priority parts are receiving a more equal portion of the machine capacity, an increase in token levels allows more of each part type into the system, making it more likely for a part to be in the finished goods buffer ready to be pulled by a demand machine. As the token levels increase, however, the higher priority parts, especially the Type 1 parts, receive a higher proportion of the production machine capacity because parts are more consistently available to be worked on. Type 2 parts receive the capacity unable to be utilized by Type 1 parts, and finally Type 3 parts receive whatever residual capacity is left after Type 2 parts utilize whatever capacity they can. Thus, for higher token levels, production rates for Type 3 parts decrease as production levels for Type 1 and Type 2 parts are increasing.

### **3.1.2 Production Rates Medium Demand**

In the medium demand case, the failure and repair probabilities for each of the demand machines were set at 0.10 and 0.05 respectively. The demand rate resulting from those probabilities is 0.33 parts per period for each part type. This means that the combined demand rate for all three of the part types is enough to exceed the throughput capacity of the production machines. Figure 3.1.2 below shows the average steady state production rates for each token level.



**Figure 3.1.2:** Manufacturing output levels for each part type with varying token levels for the medium demand scenario, where average  $R_{dem}=0.333$  for each part type.

Similar to the high demand scenario, the production rates for Type 1 parts increase with the number of tokens allowed in the system. The increase, however, is not as large as in the high demand scenario, which is expected because the steady state production rate is limited by the demand rate. The interesting result that follows is that the production rates for Type 2 and Type 3 parts are now much closer to the Type 1 rates. In fact, while Type 1 output rates decreased from the high demand scenario, Type 2 rates remained about the same, and Type 3 rates actually increased, despite a decrease in demand rates for each part type. With fewer Type 1 parts going through the production line, more machine capacity is left for Type 3 parts, providing a higher output.

In the medium demand scenario, the production rates plateau at a level for the number of tokens that is even less than the high demand scenario. This means that even fewer tokens are required to still achieve the maximum production level than the high demand scenario. With a lower average demand rate for finished parts, system inventory levels can be lower while still consistently having parts available in the finished goods buffer available for the demand machines. The token level for which the rate at which parts are pulled by a demand machine matches the rate at which the production line can restock the finished goods buffer does not need to be as high.

### 3.1.3 Production Rates for Low Demand

In the low demand case, the failure and repair probabilities for each of the demand machines were set at 0.10 and 0.02 respectively. The demand rate resulting from those probabilities is 0.167 parts per period for each part type. This means that the throughput capacity of the production machines is greater than the combined demand rate for all three of the part types. Figure 3.1.3 below shows the average steady state production rates for each token level.

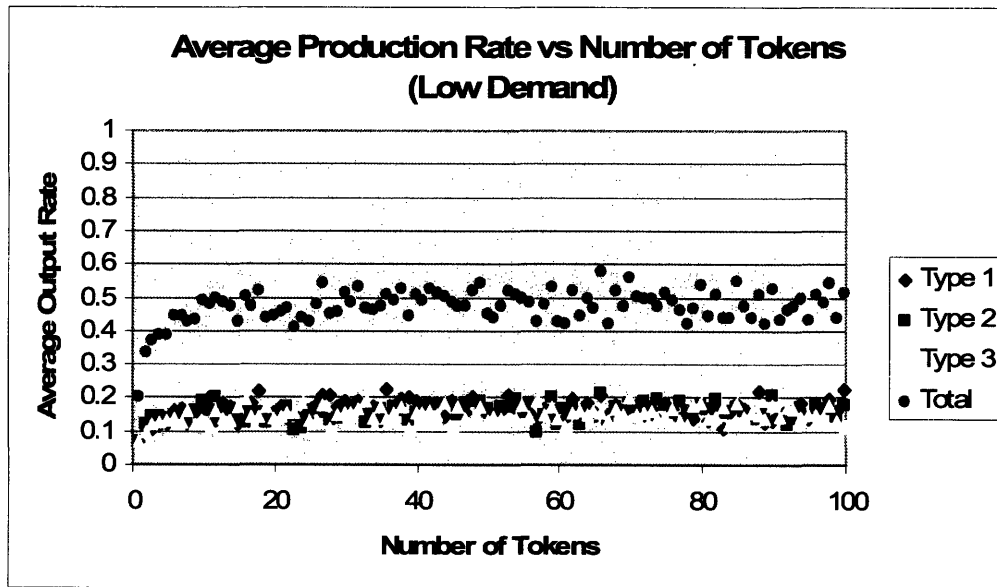


Figure 3.1.3: Average per period output levels for each part type with varying token levels for the low demand scenario, where average  $R_{dem}=0.167$  for each part type.

As the figure shows, the output rates are very close for all three part types at all token levels. For low token levels, the rates for all three types increase slightly with the number of tokens, but then the production rates plateau very quickly at around 10 tokens. It is interesting that at low demand levels, the production rates for the three part types are very similar, despite the strict priority that machine capacity should be used on Type 1 whenever possible, then Type 2, and finally Type 3. We are seeing that for such low demand rates, the residual capacity for Type 2 and even Type 3 parts is high enough to meet the demands for those types. The graph shows that the production rate at which all three part types plateau is only slightly less than the demand rate for each of those parts. The token level for which the output rate leveled off for all three types was noticeably lower than the two scenarios with higher demand. Therefore, in the case that production

machine throughput and supply rate are high enough to meet the combined demand of all three part types, the inventory levels in the system can be lower while still achieving maximum production levels.

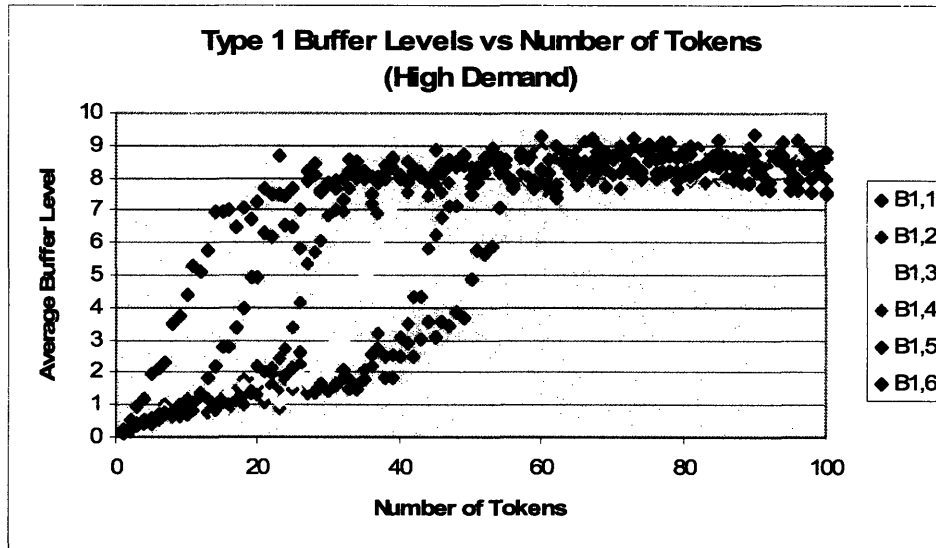
### **3.2 Token Level Impact on Inventory Distribution**

The way in which the inventory is distributed over the buffers in the system can be important to the cost considerations of inventory, as well as to the way inventory levels affect production. For some systems, different buffer spaces can have different carrying costs. With this in mind, it is important to understand how the number of tokens made available in a CONWIP system affects the way inventory is distributed. While investigating these effects, we used the same demand scenarios as in the previous section, so we can see how the effects change for different demand scenarios. Using the same demand scenarios also allows us to consider how the inventory distribution affects the production rates. To measure how the inventory was distributed, we measured the average inventory level for each buffer during steady state production.

#### **3.2.1 Inventory Distribution for High Demand**

As in the high demand scenario in Section 4.1.1, the average rate for each of the demand machines was 0.50 parts per period, while the each of the supply and production machines had a theoretical throughput capacity of about 0.91 parts per period. Figures 3.2.1, 3.2.2, and 3.2.3 show the average inventory levels for each individual buffer for Type 1, Type 2, and Type 3 respectively in the high demand scenario.





**Figure 3.2.1:** Average inventory levels for each buffer for Type 1 parts with varying token levels for the high demand scenario, where average  $R_{dem}=0.50$  for each part type.

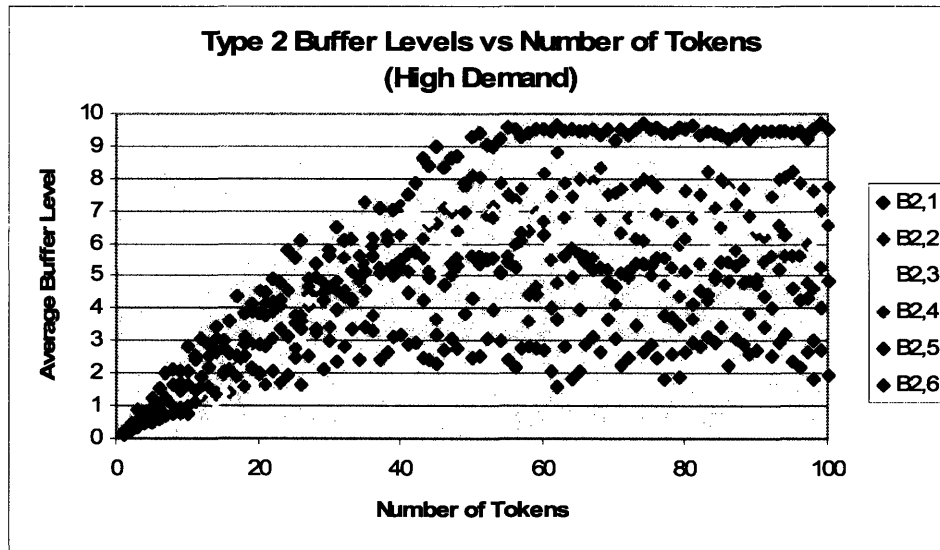
The graph in Figure 3.2.1 shows that the inventory distribution favors the downstream buffers, especially for lower token levels. This means that the last buffer in the line will generally have higher average inventory than buffers 1-5, buffer 5 will have higher average levels than buffers 1-4, and so on. Since Type 1 parts receive the highest priority, the machine capacities are focused on Type 1 parts first. Thus, when a Type 1 part enters the system through a supply machine, the production machines push it down to the last buffer, where it reaches a bottleneck and sits until it is pulled by the Type 1 demand machine.

Since the number of tokens provides the upper limit on the inventory in the system and the distribution favors downstream buffers, we see that as we increase the number of tokens, the buffers furthest downstream fill up close to their capacities first. As we continue to increase token levels, the inventory distribution works its way backward, filling upstream buffers one by one until all the buffers are nearly full. The graph shows that for every increment of 10 tokens, there is a sharp increase in the average inventory level for a new buffer. After increasing the token level by 10, the average inventory for that buffer then reaches a plateau at a level similar to the downstream buffers. Thus, we can expect that for parts that are achieving machine throughput rates that are higher than the demand rate for those parts, the inventory distribution will favor

downstream buffers, especially when applying a more stringent control on the inventory level by only allowing the system a low level of tokens.

When relating the inventory distribution and token levels back to the production rates of type 1 parts, we can understand why the production rate leveled off when the token level was only 30 parts. In the graph for average inventory, we see that when the token level was about 30 parts, the average inventories in the last and next to last buffers were each approximately 8 parts. With inventory levels that high in the last buffers, there is almost always a part available to be pulled by a demand machine in the up state. This means that only the demand rate would be limiting production rate for a high priority part, since the inventory level allowed is high enough.

The inventory distributions for the Type 2 and Type 3 parts show some clear distinctions from the Type 1 distribution, largely due to the priority rules set in place for the production machines. While the Type 1 inventory distribution favors downstream buffers, the Type 2 and 3 distributions in Figures 3.2.2 and 3.2.3 favor upstream buffers. This means that buffer 1 generally has more inventory than buffers 2-6, buffer 2 has more inventory than buffers 3-6, and so on. With most of the production machine capacity going to the Type 1 parts, there is not enough capacity left to keep pushing Type 2 and Type 3 parts downstream. While the Type 1 inventory buffers appeared to fill up one at a time for an increasing number of tokens, the inventory levels in each of the Type 2 and 3 buffers appeared to increase linearly with the number of tokens until they each stabilized at their respective maximum inventory level.



**Figure 3.2.2:** Average inventory levels for each buffer for Type 2 parts with varying token levels for the high demand scenario, where average  $R_{dem}=0.50$  for each part type.

As with the Type 1 inventory distribution and production rates, we see some relationships between the distribution and output for Type 2. In the high demand scenario, the Type 2 production rate reached a plateau when the number of tokens was about 40. In Figure 3.2.2 above, we see that the average inventory levels for last two buffers in the line stabilized at that same number of tokens. As for Type 1, when the downstream buffers reached their peak average inventory level, there is a higher probability of Type 2 inventory being available to be pulled when the demand machine was up, resulting in higher Type 2 output. Increasing the token level beyond 40, however, does not provide gains in production because the average inventory levels in the last two buffers do not increase any more.

Overall, in the high demand scenario, the behaviors of the inventory distributions for Type 2 and Type 3 were fairly similar, but the polarization of the inventory was noticeably higher for Type 3 as seen below in Figure 3.2.3. For Type 2, the average inventories of each of the buffers were closer to each other than for Type 3.

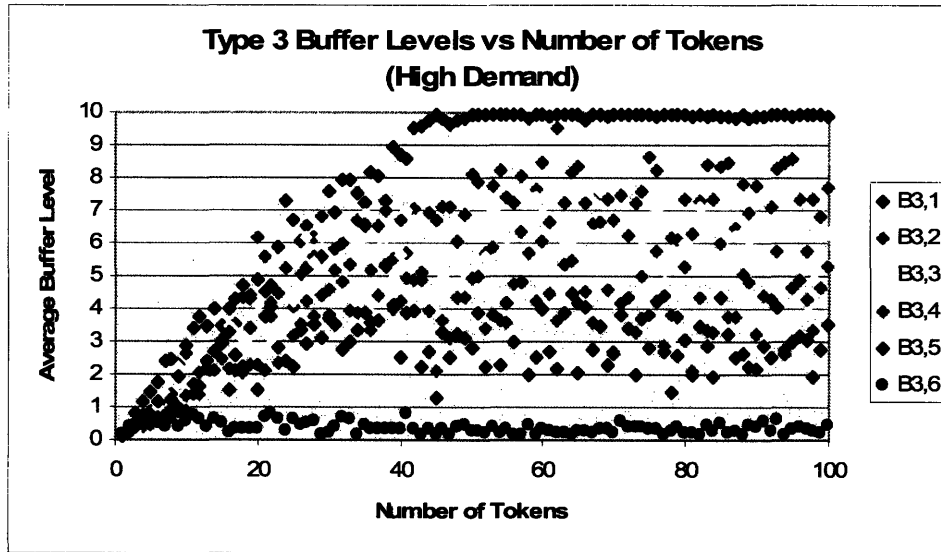


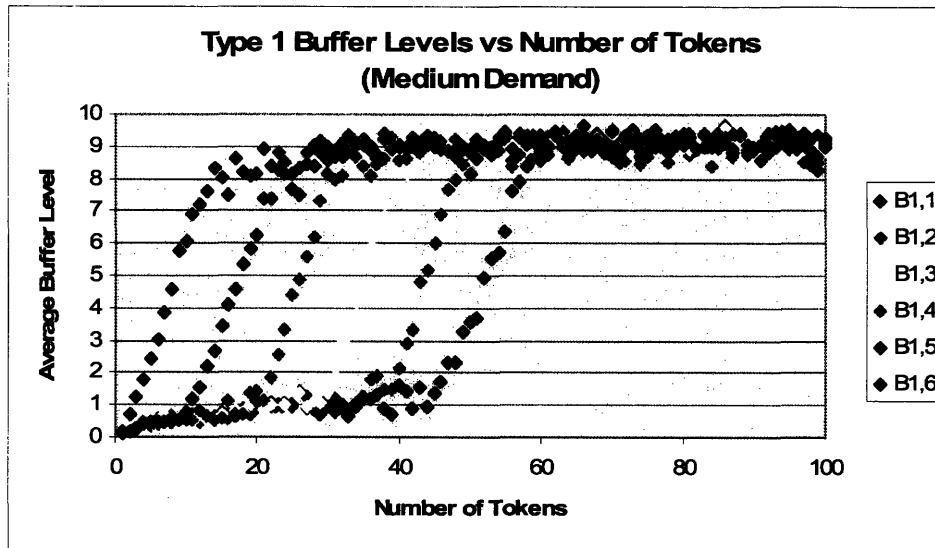
Figure 3.2.3: Average inventory levels for each buffer for Type 3 parts with varying token levels for the high demand scenario, where average  $R_{dem}=0.50$  for each part type.

For a high enough token level, the average inventory in buffer 1 for Type 3 was almost 10, which means that any time a Type 3 part was pulled from buffer 1, the supply machine almost generally replaced a new one very quickly. Buffer 6 on the other hand, has a very low average inventory, which means that when a part reached the final buffer, it was usually pulled very quickly by the demand machine. The greater polarization for Type 3 is another result of the priority rules. Since the Type 3 parts are receiving the residual capacity of the production machines, there is even less pressure from the production machines pushing the Type 3 parts downstream. The demand rate for Type 3 is greater than residual capacity of the production machines left for Type 3, causing any inventory in buffers near the end of the line to be pulled out by the demand machine.

When looking at the inventory distribution for this token level, we see that the average inventory sizes in the last buffer are actually slightly higher for very low token levels, around 10 or so, which would appear to be counterintuitive at first. However, in Section 4.1.1, the production rate for Type 3 parts peaked for a low token level because the inventory restrictions on Type 1 and 2 parts freed up machine capacity for Type 3. Because the residual machine capacity for Type 3 was higher, more parts could be sent downstream to the final buffer before being pulled by the demand machine, thus slightly increasing the average inventory level in that final buffer.

### 3.2.2 Inventory Distribution for Medium Demand

As in the medium demand scenario in Section 4.1.2, the average rate for each of the demand machines was 0.33 parts per period, while each of the supply and production machines had a theoretical throughput capacity of about 0.91 parts per period. Figures 3.2.4, 3.2.5, and 3.2.6 show the average inventory levels for each individual buffer for Type 1, Type 2, and Type 3 respectively.



**Figure 3.2.4:** Average inventory levels for each buffer for Type 1 parts with varying token levels for the high demand scenario, where average  $R_{dem}=0.33$  for each part type.

In the medium demand scenario, the Type 1 buffer distributions for different token levels turns out to be quite similar to the Type 1 distributions in the high demand scenario. Like in the high demand scenario, as the number of tokens allowed in the system increased, the buffers appeared to fill up one by one, starting at the last buffer and moving upstream until finally reaching the first buffer. Since the Type 1 parts receive the highest priority, they were receiving the necessary production capacities to meet demand, resulting in this behavior. However, this behavior was even clearer for the medium demand case because the production rate was even higher relative to the demand rate. In the high demand scenario, there was some tendency for the inventory of the upstream buffers to start increasing before each of the downstream buffers had reached their peaks. The medium demand case showed less of this tendency, so inventory would not start to build up as much in upstream buffers until the downstream buffers had leveled off. The more orderly distribution of inventory over the buffers is a result of the lower demand in

this scenario. Since the demand rate is lower, the pressure from the production machines pushing the parts downstream is higher relative to the pulling pressure from the demand machines. Thus, for a given token level, parts are pushed into the downstream buffers and spend more time sitting there before being pulled, rather than spending as much relative time in the upstream buffers. This results in less creeping of the average inventory levels upstream until all of the downstream buffers have been filled to their peak levels.

The Type 2 inventory distributions for the medium demand scenario show some important distinctions from the high demand scenario. In medium demand scenario, we see what appears to be a transition in behavior from what we saw in the Type 1 and the Type 2 distribution behaviors for high demand. Figure 3.2.5 below shows how the inventory distribution varied with token level.

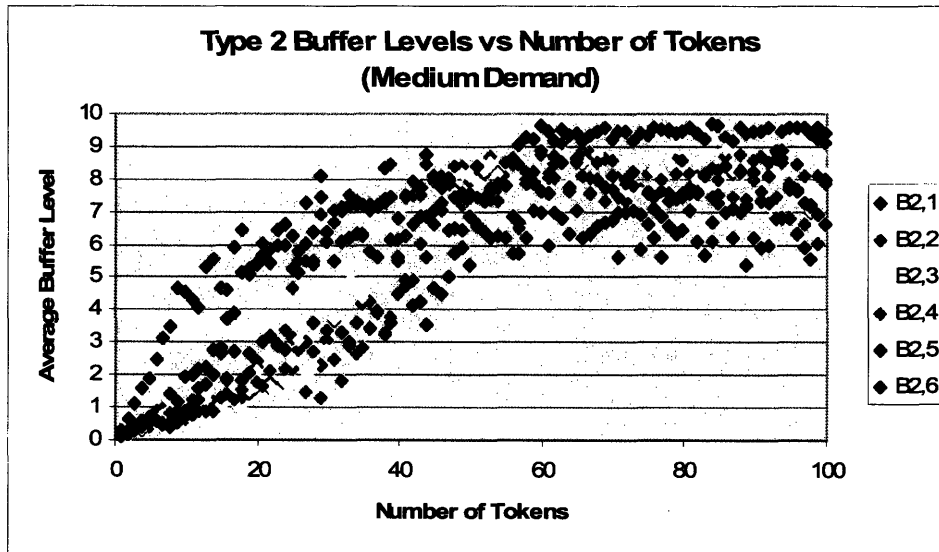
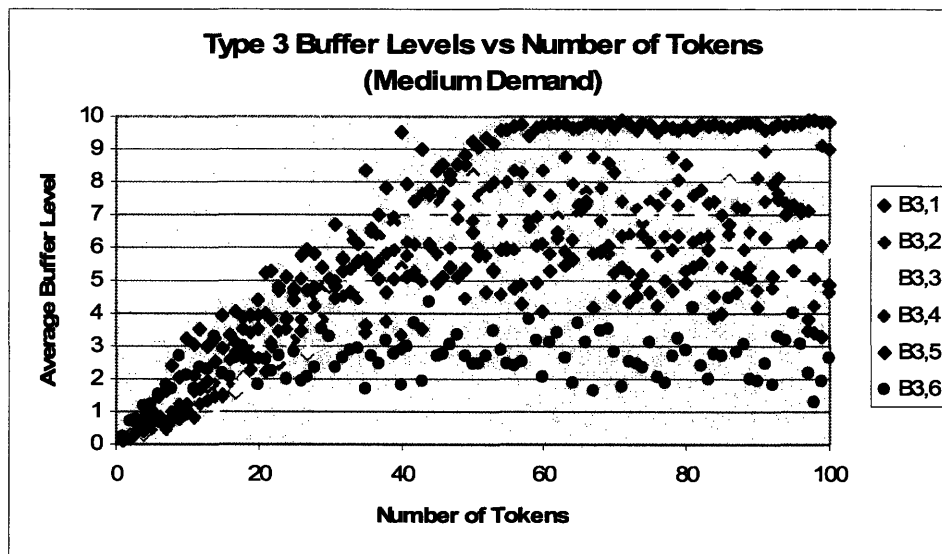


Figure 3.2.5: Average inventory levels for each buffer for Type 2 parts with varying token levels for the high demand scenario, where average  $R_{dem}=0.33$  for each part type.

In the high and medium demand scenarios, we saw that the Type 1 inventory levels were more concentrated in downstream buffers. However, the high demand scenario had Type 2 inventory levels favoring upstream buffers for all token levels. The medium demand scenario shown in Figure 3.2.5 above, shows a transition between these two different behaviors. For lower token levels, the distribution of Type 2 inventory favors downstream buffers, yet it favors upstream buffers for higher token levels. Since

demand is lower in this scenario, when the token level is high, the pulling pressure from the demand machines is lower relative to the pushing pressure on Type 2 parts from the production machines, because the Type 2 parts are receiving less residual capacity from the production machines than they do for lower token levels. For lower token levels, however, the Type 2 parts receive more of the production capacity, causing them to behave more like Type 1 parts in the way that inventory is pushed downstream.

The effect of the number of tokens on inventory distribution for Type 3 in the medium demand case shows a similar behavior to that of Types 2 and 3 in the high demand case, as shown in Figure 3.2.6 below.



**Figure 3.2.6:** Average inventory levels for each buffer for Type 3 parts with varying token levels for the high demand scenario, where average  $R_{dem}=0.33$  for each part type.

With the lower demand rate, the residual capacity of the production machines for Type 3 is higher, since production rates on Type 1 and 2 are limited by their demand rates. The greater residual capacity available for Type 3 causes the polarity of the inventory distribution to be less than it was in the high demand scenario. Figure 3.2.6 shows that the average inventory for buffer 6 is noticeably higher than in the high demand scenario that we saw in Figure 3.2.3. This is a result of the increased downstream pressure from the production machines as well as a lower pulling pressure from the demand machine, since the average demand rate is lower.

### 3.2.3 Inventory Distribution for Low Demand

As in the low demand scenario in Section 4.1.3, the average rate for each of the demand machines was 0.167 parts per period, while each of the supply and production machines again had a theoretical throughput capacity of about 0.91 parts per period. Figures 3.2.7, 3.2.8, and 3.2.9 show the average inventory levels for each individual buffer for Type 1, Type 2, and Type 3 respectively.

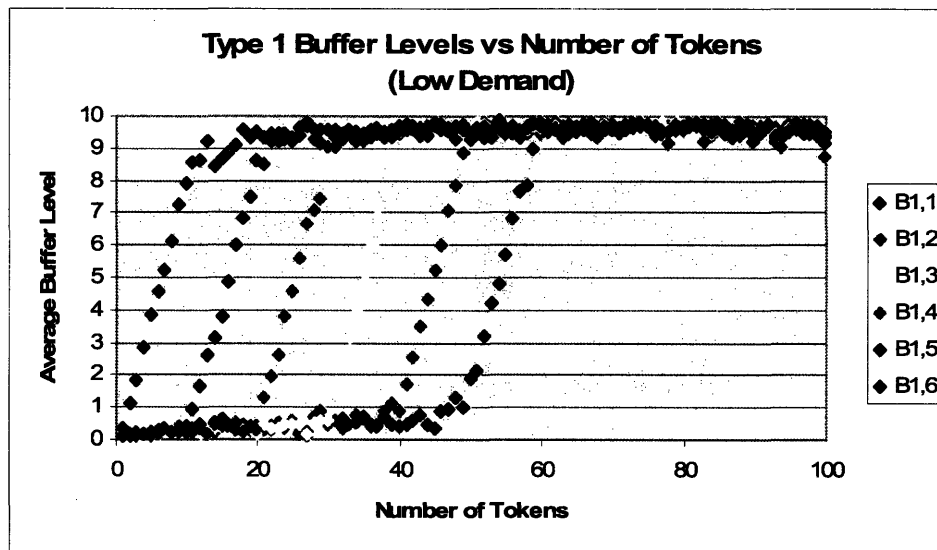
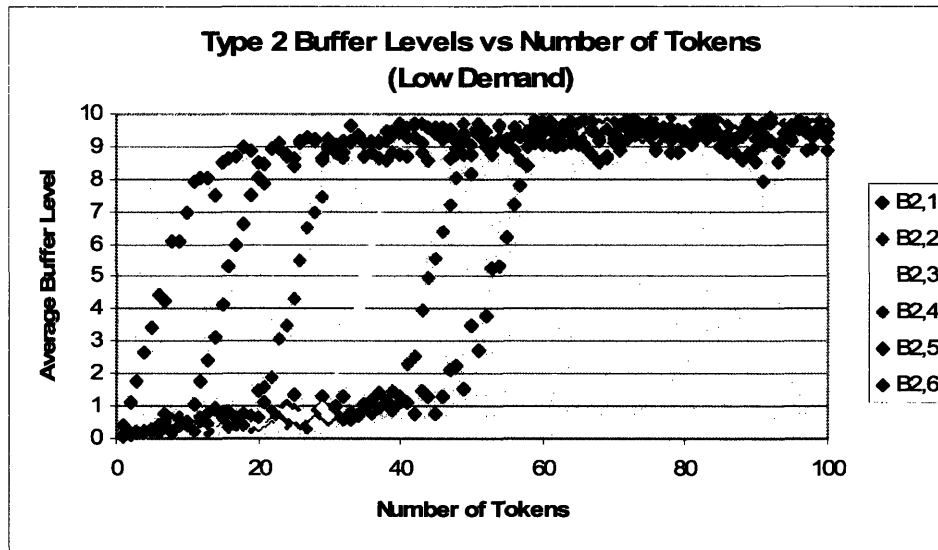


Figure 3.2.7: Average inventory levels for each buffer for Type 1 parts with varying token levels for the high demand scenario, where average  $R_{dem}=0.167$  for each part type.

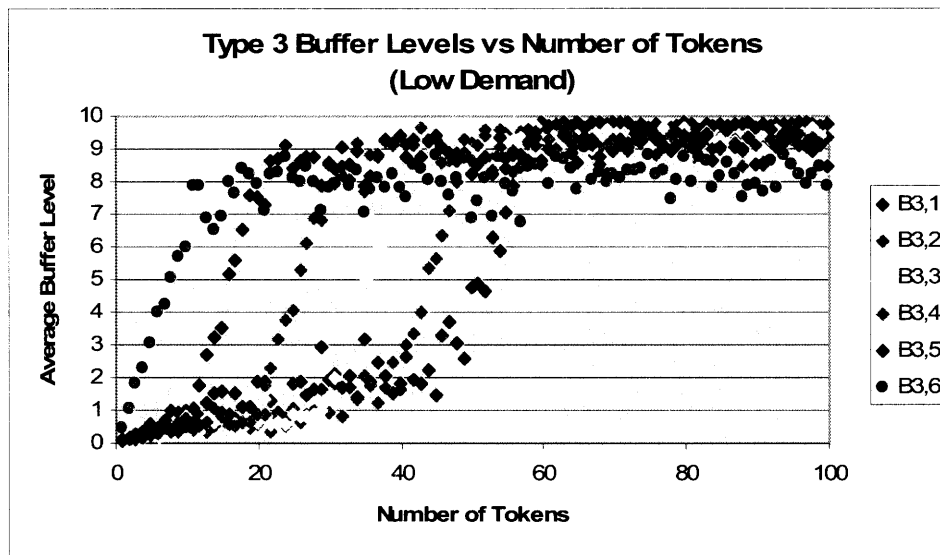
The inventory distribution for the Type 1 parts in the low demand scenario differs only slightly from the medium and high demand scenario, in the same way that the medium demand scenario differed from the high demand scenario. As for the other scenarios, we see here that the distribution of inventory favors downstream buffers. However, this tendency is even clearer for the low demand scenario. We see even less creeping of inventory into the upstream buffers. On average, there is very little inventory in a buffer until the token level is high enough to fill all of the buffers that are downstream from that point. The Type 2 buffer shows a very similar behavior in Figure 3.2.8 below.





**Figure 3.2.8:** Average inventory levels for each buffer for Type 2 parts with varying token levels for the high demand scenario, where average  $R_{dem}=0.167$  for each part type.

With demand rates so low in this scenario, the Type 1 parts only consume a small portion of the capacity of the five production machines. This leaves enough capacity for the Type 2 parts to meet the demand rates, causing an inventory distribution behavior similar to each of the Type 1 scenarios. The average inventory in a given buffer does not increase by much until the token level is high enough to fill all of the downstream buffers. The Type 2, and also the Type 3, distributions do differ slightly from Type 1 behavior in that the inventory levels of upstream buffers do increase slightly before all of the downstream buffers have been filled. Also, for high token levels, the inventories in the upstream buffers are slightly higher than the downstream buffers for the lower priority types. The Type 1 parts are still receiving slightly more of the production machine capacity than the Type 2 and 3 parts, causing more pressure pushing parts downstream for the higher priority parts. Since the downstream pressure is less for the lower priority part types, the upstream levels remain slightly higher for high token levels.



**Figure 3.2.9:** Average inventory levels for each buffer for Type 3 parts with varying token levels for the high demand scenario, where average  $R_{dem}=0.167$  for each part type.

When relating the inventory distribution to the output for the different types, we find that the peak output rates correspond to the peak average inventory levels in the last buffers in the same way that they did for the high and medium demand scenarios. The peak production rates stabilized at token levels of about 15 for all three part types, which corresponds to the token level where the average inventory of the last buffer for each of the types leveled off at its maximum level.

In the low demand scenario, we see an interesting behavior of in the random variation of the average inventory levels for the buffers after they have reached their stable levels. When comparing the variation in the levels for each of the buffers over the different part types, we find that the lower priority types show larger random variation than the higher priority types. Even for the highest priority Type 1 parts, the system has some built in random variation in inventory levels due to the randomness in the up and down states of all of the machines. The Type 2 parts see increased random variation from the system because they receive the residual capacity from the Type 1 parts, so the randomness of the system is compounded with the randomness from the Type 1 parts. The Type 3 parts face even greater random variation because they face the randomness from the system plus the compounded randomness of the Type 1 and Type 2 parts. This behavior is important to recognize because the randomness affects how accurately and

confidently we can predict inventory distribution for each part type. With higher random variation for the lower priority parts, accurately predicting the inventory distribution becomes more difficult. Thus designing the production system and inventory controls for lower priority parts also becomes more difficult.

## **4. Conclusion**

This research studies the effect of maximum allowed inventory level through CONWIP control on the performance of a flexible manufacturing system producing different part types according to a strict priority rule. The CONWIP control method applied individual limits on the system inventory level for each of the part types. The individual token limits were set at the same level as they were varied simultaneously while running simulations to determine the average per period output and average inventory level of each buffer for each part type.

### **4.1 Production Rates Intuition**

In a flexible production system, the spread between the production rates of the different part types is larger for higher levels of demand. Given the same demand level, the spread between production levels also increases with token level, as the throughput rate of the higher priority parts are less constrained by limits on inventory. Interestingly, for high demand rates, the highest production rate of the lowest priority type actually peaks at a very low token level because it receives the largest residual capacity when throughput of higher priority parts are constrained by inventory limits. When the demand rates decrease from high levels, the production rates of the higher priority parts decrease while the lower priority part types actually tend to increase due to larger residual production machine capacities for those types. As demand rate continues to decrease, the production rates of all the part types eventually meet at the same level and then decrease together with further drops in demand rates. For low levels of demand, the average output of each of the part types move together with changing token levels. The output of each part type increases with token level until reaching a plateau at a fairly low token level. Also, the token level necessary to reach the maximum production rate was lower

for lower demand rates. Intuitively, the inventory in the system does not need to be as high if demand rates are lower in relation to the throughput capacity of the system.

## **4.2 Inventory Distribution Intuition**

The application of the CONWIP controls showed some interesting implications for the inventory distributions in different demand scenarios. The inventory distributions also appeared to have implications on the output rates. For each part type in each scenario, the production levels consistently reached their maximum level when the furthest downstream buffer also reached its maximum level. For each demand level, the part types that received enough machine capacity to meet each of their demand rates showed a similar behavior in their inventory distributions. These types tend to fill their buffers one at a time, starting with the buffers furthest downstream and working backwards up the line from there. On the other hand, the types not receiving enough capacity from the production machines to meet their demand rates would tend to have higher inventory in the upstream buffers.

## **4.3 Future Research**

With the intuitions developed in this research, production managers can more intelligently design flexible production lines with inventory controls to achieve the desired results. The intuition also serves as a good starting point for further research in design optimization for systems using similar control methods. Eventually, for a given production system, a manager should be able to balance inventory costs, machine capacities, supply rates, and demand rates to achieve an optimum production system design.

## **Appendix**

### **A.1 Number of Time Periods in Simulation**

When running the simulation, it is important to run it for a long enough time that it reaches a steady state production rate and inventory distribution. This way we are measuring the steady state behaviors instead of any behavior that is affected by ramping up time in the transient periods. For this research, we ran the simulation for a length of 10800 time periods, and recorded the inventory levels and production output for the last 3600 periods. After running some initial trials, we compared the results of simulations as long as 21600 periods to ones as short as 3600 periods and found that over a measurement window of 3600 periods for each simulation length, we obtained the same behavior. The simulations, however, showed different results for trials up to 3600 periods than for longer ones. When looking at behavior beyond the initial 3600 periods, the results became consistent, so the ramp up time is approximately 3600.

## **Bibliography**

- [1] K.E. Stecke. Design, planning, scheduling, and control problems of flexible manufacturing systems. *Annals of Operations Research*, 3(1):1-12, January 1985.
  
- [2] H. Tempelmeier and H. Kuhn. *Flexible Manufacturing Systems: Decision Support for Design and Operation*. Wiley-Interscience, New York, NY 1993.
  
- [3] Y.J. Jang. Mathematical Modeling and Analysis of Flexible Production Lines. PhD thesis, Massachusetts Institute of Technology, Laboratory for Manufacturing and Productivity, June 2007.