Evanescent Wave Spectroscopy for Detection of Water and Water Treeing in Polymers

by

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Abstract

Water and water treeing affect the aging and electrical properties of polyethylene insulation in high voltage transmission cables that can eventually result in an electrical breakdown in the cable. Currently, only off-line tests are available to detect faults and degradation of the cable due to moisture and treeing. An evanescent wave methodology has been investigated to detect this moisture in polyethylene insulation. Dielectric waveguide and slow wave structures for use in the microwave and infrared frequency ranges were designed to produce evanescent fields which penetrate into the adjacent polymer.

The “penetration depth” of the evanescent fields in the infrared experimental apparatus changed with angular position of the dielectric crystal. In the microwave experiments, the “penetration depth” varied with frequency and thickness of the high dielectric constant crystal which generated evanescent waves. Polyethylene (PE) samples placed adjacent to the crystal have frequency (microwave) or angular (IR) dependent absorption of energy due to the lossy component of the dielectric constant for polyethylene with absorbed water, which depends on the moisture concentration and distribution.

In the IR, measurement results indicated the presence of water, but provided little spatial information about the moisture distribution due to the small penetration depth. Microwave measurements of attenuation were made in a sapphire-PE loaded rectangular waveguide with “water sheets” containing known amounts of moisture placed in various locations within the PE. Several models for the expected attenuation were developed and the microwave results were compared to theory.

The inverse problem was also investigated, where from attenuation measurements, it was desired to estimate the position and concentration of moisture. Three particular cases were studied: (1) a symmetric pair of water sheets, (2) multiple symmetric pairs of sheets, and (3) uniform moisture distribution. Measured and computer simulated theoretical attenuation data were used to test the estimation methods. In the first case, a non-linear least squares estimation was used to find the “best fit” of theory to both the measured and computer simulated attenuation data. The second case is an extension of the first, but only computer simulated attenuation data was available to test the algorithms. For the uniform moisture distribution, a linear least squares estimation was applicable and the algorithm was tested with computer simulated data.

Thesis Supervisor: Markus Zahn
Title: Professor of Electrical Engineering
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I’m grateful for the participation of my thesis committee, Prof. Frederic Morgenthaler and Prof. Peggy Cebe. Prof. Morgenthaler helped procure microwave equipment and his knowledge of microwave waveguides was invaluable. Prof. Cebe assisted in understanding the properties of polyethylene and other polymers.

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Contents

1 Introduction ....................................................... 15
  1.1 Background ................................................... 15
  1.2 Description of Problem .................................... 16
  1.3 Roadmap of Thesis ......................................... 16

2 Fourier Transform Infrared Spectroscopy (FTIR) ......................... 19
  2.1 Theory of FTIR ............................................... 19
    2.1.1 Water and Polyethylene Absorption Peaks in the IR .... 20
    2.1.2 Experimental Method .................................. 20
  2.2 Experiments and Results .................................... 23
    2.2.1 Water Cell Experiment ................................ 23
    2.2.2 PE Sheet Experiment .................................. 23
  2.3 Conclusions .................................................. 25

3 Dielectric Measurements, Moisture Models, and Tree Growth ............. 27
  3.1 Dielectric Measurements of Cable Samples at Microwave Frequencies . 27
    3.1.1 Basic Operation of Dielectric Measurement Apparatus ...... 27
    3.1.2 Measurement Results .................................. 30
  3.2 Models for “Moist” PE ....................................... 31
    3.2.1 Series Model .......................................... 31
    3.2.2 Parallel Model ....................................... 33
    3.2.3 Lorentz Sphere Model .................................. 34
    3.2.4 Independent Sphere Model .............................. 34
    3.2.5 Haus-Melcher Model .................................... 35
    3.2.6 Comparison of Models .................................. 36
3.3 Coax Cable Experiments ............................................. 37
  3.3.1 Loss in TEM mode ............................................. 37
  3.3.2 High Voltage Cable Losses in the TEM Mode ................. 40
  3.3.3 Microwave Waveguide Experimental Setup .................... 41
  3.3.4 Moisture Measurements ...................................... 42
  3.3.5 Unheated Cable ............................................. 43
  3.3.6 Heated Cable ............................................. 43
  3.3.7 Treed Cable ............................................. 44

3.4 Generation of Water Treed XLPE Sheets ......................... 46
  3.4.1 Methodology ............................................. 46
  3.4.2 Measurements of Complex Permittivities as Function of Moisture Concentration ............................................. 48

3.5 Conclusions ................................................... 48

4 Waveguide Modes In Dielectric Loaded Metal Waveguide 55
  4.1 Governing Equations ......................................... 56
  4.2 Power Conservation ......................................... 58
  4.3 Single Dielectric Region Case Studies ......................... 59
    4.3.1 Single Dielectric with Complex Permittivity ............ 59
    4.3.2 Effects of a Water Sheet ................................ 60
  4.4 Evanescent Mode Configurations ................................ 66
    4.4.1 Two Dielectrics ....................................... 66
    4.4.2 Effects of a Water Sheet ................................ 68
  4.5 Power Flow Calculations .................................... 72

5 Loss Theory ..................................................... 77
  5.1 Loss Determined from Dispersion Relations ..................... 77
  5.2 Exact Field Solutions ....................................... 79
    5.2.1 Calculation of Power Dissipation Integrals ............ 83
  5.3 Perturbation Approach ....................................... 85
  5.4 Comparison of Methods ...................................... 86
  5.5 Loss Due to Treed XLPE Sheets .............................. 87
# 6 Experimental Results

6.1 Description of Experimental Apparatus ........................................ 91
6.2 Quadratic Fit of Data ................................................................. 93
6.3 Measurements .............................................................................. 94
6.4 Theoretical Loss Calculations ....................................................... 97
6.5 Discussion of Differences Between Theory and Experiment ................. 98

# 7 Prediction of Distribution and Concentration of Moisture

7.1 Least Squares Estimation for Solving an Overdetermined System .......... 104
  7.1.1 Linear Systems ................................................................. 104
  7.1.2 Nonlinear Least Squares Estimation ..................................... 105
7.2 Application to the Water Sheet Problem ...................................... 107
  7.2.1 Rectangular Waveguide Filled with XLPE and Water Sheets, but no
       Sapphire .............................................................................. 107
  7.2.2 Rectangular Waveguide Loaded with Sapphire, XLPE, and Water
       Sheets .............................................................................. 109
7.3 Estimation of Location and Thickness of Symmetric Pairs of Sheets ...... 110
  7.3.1 From Measured Attenuation Data .......................................... 110
  7.3.2 From Simulated Attenuation Data .......................................... 113
  7.3.3 Sensitivity of the Algorithm ................................................. 117
7.4 Prediction of Position of Multiple Sheets from Simulated Data .......... 118
7.5 The Uniform Distribution "Moist" XLPE Problem ............................ 120
7.6 Conclusions .................................................................................. 122

# 8 Summary and Applications

8.1 Summary of Thesis ....................................................................... 125
8.2 Other Applications ........................................................................ 127

# A Three Region Solutions

# B Two and Four Region Solutions

# C Nonlinear Least Squares Solver

# D Uniform Moisture Solutions
List of Figures

2-1 Transmittance of water. Absorbance is $-\log_{10} \frac{T}{100}$, where $T$ is the percent transmittance. [15] ........................................... 21

2-2 Transmittance of dry polyethylene.[14] ........................................... 21

2-3 ATR sample holder. ................................................................. 22

2-4 Absorption spectra of water intrusion into polyethylene at 4, 26, and 48 hours, visible for wavenumbers $3100 \leq k \leq 3800$, $1500 \leq k \leq 1700$, and $k \leq 800 \text{ cm}^{-1}$. ................................................................. 24

2-5 The “wet” and “dry” sides of PE in an experiment with water in contact with one side for 30 days have similar IR absorption spectra. The IR spectra of an unused PE sample from the same sheet was subtracted out. .......... 26

3-1 Waveguide measurements for thin samples.[8] ........................................... 28

3-2 Packing arrangements for effective medium calculations. The models in order shown are the series, parallel, Lorentz sphere, independent sphere, and Haus-Melcher models. ................................................................. 33

3-3 Water data from [11] ................................................................. 37

3-4 Comparison of effective medium models as a function of moisture concentration at 8.5 GHz. ................................................................. 38

3-5 Comparison of the spherical effective medium models for low densities of water. 38

3-6 Model for TEM transmission ................................................................. 39

3-7 XLPE and semiconducting tapes taken from a high voltage transmission cable. 41

3-8 Microwave network analyzer system ................................................................. 42

3-9 Bow-tie trees grown in microwave cable insulation. The ruler markings at the bottom are separated by 1 mm. ................................................................. 45
3-10 Vented trees grown from needle punctures in microwave cable insulation. (a) after 21 days, (b) after 28 days, (c) after 35 days. .......................... 50

3-11 Treed samples from (a) VI-#5 and (b) VI-#2. The cell for VI-#5 contained 0.5M NaCl and was energized for 200 hours. Experiment VI-#2 was under identical conditions, but with soap added to both sides of the cell, and shows greater tree development. ................................. 51

3-12 Calculated real dielectric constant and loss factor ($\varepsilon''$) for various molarities of NaCl solutions using models from [29] and [30]. ............................ 52

3-13 Measured imaginary dielectric constant for dry XLPE and calculated (Haus-Melcher model) imaginary dielectric constant for 0.5M NaCl treed XLPE with 6000 ppm moisture content. .......................... 52

3-14 Loss factor ($\varepsilon''$) of treed XLPE at 8.5GHz as a function of water content.  
[Collaborative work with Tatsuyu Suzuki of TEPCO] .................. 53

4-1 Simple dielectric waveguide. .................................... 55

4-2 Reflections in waveguide experiment. .......................... 59

4-3 Rectangular waveguide with single dielectric region. ............ 60

4-4 Single dielectric region with water sheets assumed to have negligible thickness ($\Delta \ll a$). ............................................. 62

4-5 Water as a separate region within the dielectric. .................. 62

4-6 Comparison of electric field mode shapes when water sheets are added to the single region geometry for a waveguide with $a = 11.4$ mm at 8 GHz. The water sheets are 20$\mu$m thick and located at $d = \pm 5$ mm. ................. 65

4-7 Rectangular waveguide with two dielectric regions in evanescent mode configuration. .................................................. 66

4-8 Two dielectric regions with water sheets assumed to have negligible thickness ($\Delta \ll a$). ............................................. 68

4-9 Water as a separate region in the evanescent mode configuration. ....... 69

4-10 Comparison of electric field mode shapes when water sheets are added to the two region geometry. The sapphire is 2 mm thick ($b = 1$ mm), the waveguide has width $a = 11.4$ mm, and the frequency is 8 GHz. The water sheets are 50$\mu$m thick, located at $d = \pm 5$ mm. ................. 73
5-1 Change in attenuation when three different thicknesses of water sheets are placed inside the single region dielectric loaded waveguide. The dimensions are \( a = 11.4 \text{ mm}, \ d = 5 \text{ mm}, \) and (a) \( \Delta = 1\mu\text{m}, \) (b) \( \Delta = 5\mu\text{m}, \) (c) \( \Delta = 10\mu\text{m}. \)

5-2 Change in attenuation when three different thicknesses of water sheets are placed inside the two region dielectric loaded waveguide. The dimensions are \( a = 11.4 \text{ mm}, \ b = 1 \text{ mm}, \ d = 5 \text{ mm}, \) and (a) \( \Delta = 1\mu\text{m}, \) (b) \( \Delta = 5\mu\text{m}, \) (c) \( \Delta = 10\mu\text{m}. \)

5-3 Change in electric field mode shape with frequency for the evanescent mode configuration. The parameters are \( a = 11.4 \text{ mm}, \ b = 1 \text{ mm}, \) \( \epsilon_1 = 11\epsilon_0, \) and \( \epsilon_2 = 2.25\epsilon_0. \)

5-4 Change in attenuation when symmetric 0.5 mm and 1.0 mm thick treed XLPE sheets are placed inside the two region dielectric loaded waveguide. The dimensions are \( a = 11.4 \text{ mm}, \ b = 1 \text{ mm}, \ d = 2 \text{ mm}, \) and \( \Delta = 0.5 \text{ and } 1 \text{ mm}. \)

6-1 Dielectric constant for sapphire from data sheets provided by Saphikon.

6-2 Measured and fitted data for 1 slab dielectric waveguide with water sheets at \( d = \pm 9.4\text{mm} \) from center.

6-3 Summary of data fitted to attenuation measurements using a quadratic fit algorithm.

6-4 Theoretical loss corresponding to fitted data. The water sheet thicknesses used in the calculations are given by \( t. \)

7-1 Attenuation calculated from the position and thickness estimates given in Table 7.2 (dashed line) compared to measured data (solid line) for 2-slab \( (b = 0.4572 \text{ mm}) \) sapphire dielectric waveguide with XLPE and water sheets located in positions a, b, and c within the XLPE.
List of Tables

3.1 Description of waveguide measurements. .......................................................... 28
3.2 Notation for standing wave method equations. ................................................. 29
3.3 TEPCO splice material dielectric properties. .................................................... 32
3.4 Dielectric measurements of transmission cable material samples. ................. 32
3.5 Water content measurements for various water treeing experiments and condi-
tions. ....................................................................................................................... 47

6.1 Water sheet thickness calculations. ................................................................. 97
6.2 Water sheet thickness values used in the theoretical calculations. ............... 97
6.3 Errors between measured and theoretical loss due to water sheets using av-
erage water sheet thicknesses from Table 6.2 .................................................. 98
6.4 Errors between measured and theoretical loss due to water sheets corrected
for water sheet thickness variations in Table 6.2 ............................................... 100

7.1 Table of estimated and measured water sheet thicknesses, given the position,
the measured attenuation data, and $d = (a) \ 3 \text{mm} < d < 3.4 \text{mm}, (b) \ 6 \text{mm} <
d < 6.4 \text{mm} , \text{and} (c) \ 9 \text{mm} < d < 9.4 \text{mm}$. .................................................. 112
7.2 Table of estimated and measured water sheet thicknesses and locations in the
waveguide, given the fitted attenuation data. The estimate (*) was outside
the waveguide and corresponds to a measurement error. ............................... 112
7.3 Table of actual and estimated water sheet thicknesses and locations in the
waveguide, given simulated attenuation data calculated from the exact field
solutions. ................................................................................................................... 115
7.4 Table of actual and estimated water sheet thicknesses and locations for symmetric pairs of water sheets close to the edge of the waveguide, given simulated data calculated from the exact field solutions. The solver did not converge in the case of the $b = 0.6858$ mm waveguide. .......................... 116

7.5 Table of actual and estimated water sheet thicknesses and locations for water sheets close to the sapphire, given simulated data calculated from the exact field solutions. The estimates of $d$ marked with a (*) are not physical, as they are inside the sapphire region. .......................... 116

7.6 Results of parameter estimation when perturbation attenuation data is modified by random noise values for cases of ±1% and ±10% of the attenuation values. .................................................. 118

7.7 Results for the linear least squares estimate of uniform moisture, using the Haus-Melcher dielectric model (Equation 7.29) with 1000, 5000, and 10000 ppm of moisture. .................................................. 122
Chapter 1

Introduction

1.1 Background

Moisture intrusion into underground high voltage cross-linked polyethylene (XLPE) insulated power cables is a serious concern for utility companies. Water trees can initiate from voids or impurities in the XLPE and eventually create pathways for electrical breakdown [21, 26, 42, 44]. Therefore, it is desired to detect the moisture and water trees before catastrophic failure of the cable occurs. Currently, only off-line tests are available to detect faults and degradation of the cable due to moisture and treeing where it would be preferable to use an on-line test.

A logical location to monitor moisture and treeing is in or near cable splices which connect the segments of long distance cables. Each cable segment is several kilometers long [55]. In addition, information from Tokyo Electric Power Co. (TEPCO) indicated that much of the water enters the cable from the splices, even though the splices are carefully sealed [52, 53, 54]. The splices used by the sponsor consist of multiple layers of different tapes and sealants surrounded by a waterproof jacket. Because the splice geometry is very complicated, we chose to work with simple geometries in order to focus efforts on a water tree detection and location system that would be amenable to engineering analysis and experiments without geometric complexities.

Inherent to the discussion of moisture and treeing is a description of the source and types of trees. Water trees will form in the direction of electric field stress from a void or impurity in the polyethylene bulk, or defect in the polyethylene surface. Trees initiated from a void are called bow-tie trees. Those initiated from a surface defect are vented trees
and are usually "bushy" in shape. Enhancements of the electric field stress in water trees can initiate breakdown and create an electrical tree [21].

1.2 Description of Problem

A methodology was developed where spatially dependent microwave and infrared (IR) electric fields were used to probe the media such that the spatial distribution and concentration of moisture in polyethylene insulation could be determined. Evanescent fields generated by a dielectric or dielectrically loaded waveguide provided the sensing mechanism for the experiments. The penetration depth of the evanescent fields changes with frequency in the microwave experiments and with incident angle and wavelength in the infrared. Moisture in the polyethylene absorbs energy at specific frequencies from the microwave through the infrared range and the absorption vs. frequency characteristic were used to estimate the distribution and concentration of the moisture.

In the IR, measurement results indicated the presence of water, but provided little spatial information about the moisture distribution due to the small penetration depth. Microwave measurements of attenuation were made in a sapphire-PE loaded rectangular waveguide with "water sheets" containing known amounts of moisture placed in various locations within the PE. Several models for the expected attenuation were developed and the microwave results were compared to theory. The attenuation measurements were then used to predict the original moisture distribution by using a least squares algorithm to find the best fit of the experimental data to measurements.

1.3 Roadmap of Thesis

The theory of Fourier Transform Infrared (FTIR) spectroscopy and two experiments to measure moisture content are described in Chapter 2. The small penetration depth of IR into the polyethylene samples resulted in poor spatial resolution, therefore microwave measurement techniques were developed that have longer penetration depth evanescent fields. Chapter 3 details preliminary modelling and measurements needed to design the microwave experiments and computer simulations. The basic microwave waveguide configurations and the corresponding electromagnetic field equations are described in Chapter 4. Models for the attenuation due to moisture are developed in Chapter 5. In Chapter 6, the experimen-
tal setup and results are given for the "water sheet" experiments as well as the analysis of the results. Computer simulations of various experimental configurations were used in Chapter 7 to test the limits of our prediction methodology. An outline of how to approach the problem of other moisture distributions is also given in chapter 7.
Chapter 2

Fourier Transform Infrared Spectroscopy (FTIR)

2.1 Theory of FTIR

The design of FTIR spectroscopy equipment is based on the scanning two-beam interferometer first developed by Michelson in 1891 [16]. The interferometer is a device that divides the IR beam into two paths and then later recombines the beams after a disturbance has been added to one path. The one path length is changed by a mirror moving at constant velocity and the resulting combined beam intensity as a function of path length difference $\delta$ is the interference pattern, $I'(\delta)$.

For a monochromatic radiation beam, the interference pattern will repeat sinusoidally as $\delta$ changes by integer multiples of wavelengths $\lambda$, and is given by

$$I'(\delta) = 0.5S(\nu)(1 + \cos 2\pi\nu\delta)$$

(2.1)

where $\nu = \frac{1}{\lambda}$ is the wavenumber and $S(\nu)$ is the source intensity. If frequency dependent non-idealities in the optics and detector are included as a correction factor $H(\nu)$, the equation for the interferogram is

$$I(\delta) = 0.5S(\nu)H(\nu)\cos 2\pi\nu\delta = B(\nu)\cos 2\pi\nu\delta$$

(2.2)

and represents the contribution of Equation 2.1 dependent on the path length difference $\delta$. 

19
Therefore, the interferogram $I(\delta)$ is the cosine Fourier transform of the spectral intensity $B(\bar{\nu})$.

For a polychromatic source, the interferogram is given by

$$I(\delta) = \int_{-\infty}^{+\infty} C(\bar{\nu}) \cos 2\pi \bar{\nu} \delta d\bar{\nu}$$  \hfill (2.3)

where the spectral intensity $C(\bar{\nu})d\bar{\nu}$ is the contribution to the interferogram in the wavenumber interval $\bar{\nu}$ to $\bar{\nu} + d\bar{\nu}$. The spectral intensity wavenumber density $C(\bar{\nu})$ is then given by

$$C(\bar{\nu}) = \int_{-\infty}^{+\infty} I(\delta) \cos 2\pi \bar{\nu} \delta d\delta$$  \hfill (2.4)

Since the mirror moves at constant velocity $v$, $\delta = vt$ and $C(\bar{\nu})$ and $I(\delta)$ are a standard time and frequency Fourier transform pair. In reality, the interferometer can only accommodate a finite path length difference and a discrete Fourier transform is performed.

The major benefit of the FTIR method is that a large number of scans, typically 32, are taken very rapidly and added together. The peaks add while the noise averages to near zero, allowing a weak signal to increase in amplitude [18].

2.1.1 Water and Polyethylene Absorption Peaks in the IR

Every organic material has a characteristic energy absorption spectra corresponding to its molecular structure. The absorption bands correspond to loss due to different molecular vibrations (bending, stretching, or rotating) induced by the energy at IR frequencies.

Water has several broad absorption bands in the IR frequency range located around wavenumbers 3400, 1640, and 700 cm$^{-1}$. The height of these bands is a measure of the water concentration. Polyethylene has sharp peaks around wavenumber 2800-3200, 1500 and 700 cm$^{-1}$, and several smaller peaks. These absorption spectra are shown in Figures 2-1 and 2-2. Research on the effect of water treeing on the absorption of cross-linked polyethylene is described in [24, 25, 27, 28].

2.1.2 Experimental Method

The experimental apparatus used for measurements consisted of a Nicolet 20DX FTIR spectrometer system combined with a commercial ATR (attenuated total reflectance) accessory.
Figure 2-1: Transmittance of water. Absorbance is \(-\log_{10} \frac{T}{100}\), where T is the percent transmittance. [15]

Figure 2-2: Transmittance of dry polyethylene.[14]

The ATR device containing samples was placed in one beam of the interferometer and the IR absorption as a function of wavenumber \(\frac{\lambda}{A}\) was measured. A diagram of this ATR is shown in Figure 2-3.

Polyethylene samples were placed on either side of the IR crystal and held in place with pressure plates. The IR energy enters one end of the crystal, reflects off the inner surfaces of the crystal, and exits through the far end. The KRS-5 (thallous bromide-iodide) crystal has a higher index of refraction, \(n \approx 2.38\), than the sample (in our case, polyethylene with \(n \approx 1.50-1.54[10, \text{p C-729}]\)) such that incident IR energy undergoes total internal reflection of the electromagnetic waves at the crystal-sample interface. However, this total internal reflection produces evanescent fields outside the crystal. A sample placed in these fields absorbs energy at frequencies given by its characteristic absorption spectra, and reduces the intensity of the reflected IR at the absorbed frequencies.

The extent of the evanescent field penetration into the sample ("penetration depth") is dependent on both the angle of incidence of the IR beam and the frequency of the excitation
but generally is on the order of the infrared wavelengths. The governing equations are as follows:

\[
\theta = \theta_{IRE} + \sin^{-1}\left[\frac{\sin(\theta_{IRA} - \theta_{IRE})}{n_1}\right] \tag{2.5}
\]

\[
d_p = \frac{\lambda_1}{2\pi \left[\sin^2 \theta - \left(\frac{n_s}{n_1}\right)^2\right]^{\frac{1}{2}}} \tag{2.6}
\]

where \(\lambda_1\) is the wavelength inside the crystal, \(n_s = 2.38\) and \(n_1\) are the indices of refraction of the crystal and sample, and \(\theta\) is the angle of incidence inside the crystal and is calculated from the crystal face angle \((\theta_{IRE} = 45^\circ)\) and the settings on the ATR \((\theta_{IRA})\). By varying the penetration depth, a spatial profile of any contaminants (such as water) in the polyethylene can be calculated.

The absorption is measured on a logarithmic scale, which allows for the subtraction of spectra to calculate the change in absorption due to the addition or intrusion of moisture over time. In the first experiment, the spectral absorption of the PE sample was measured before water was added. This measurement was subtracted from the ensuing measurements, resulting in the change in absorption as a function of time. In the second experiment, measurements of control samples of dry PE were subtracted from those of wet PE taken from the water bath.
2.2 Experiments and Results

2.2.1 Water Cell Experiment

It was desired to observe the water intrusion into the polyethylene as a function of time. The ATR attachment was modified to replace one of the sample holder pressure plates with a water reservoir. A 2.6 mil (1 mil = 10^{-3} inches ≈ 25μm, 2.6 mil = 66μm) thick low density polyethylene sheet was placed between the crystal and the water reservoir. As a function of time, the absorption spectra is given in Figure 2-4. Water did not begin to appear in the IR spectra until after 3-4 hours as shown in Figure 2-4(a). After 26 hours, there was appreciable water present as shown in Figure 2-4(b). By 48 hours, it had reached a steady state as shown in Figure 2-4(c), and there was little change between 48 hours and 65 hours, when the final measurement was taken.

The calculated penetration depth for the 45° orientation of the ATR is only a few microns. With such a small penetration depth, it was unclear whether this water was building up at the crystal – polyethylene interface or whether it was water in the bulk of the polyethylene. The polyethylene sheet did show evidence of swelling, possibly caused by water absorption.

The nature of the crystal – polyethylene contact changed during the experiment. With increasing time, the polyethylene peaks became larger, indicating that the polyethylene was in better contact with the crystal. Water at the crystal - polyethylene interface possibly enhanced this contact effect. In fact, when the test setup was dismantled at the end of the experiment, the polyethylene was firmly stuck to the crystal.

2.2.2 PE Sheet Experiment

Since it was not known if the water was building up at the crystal - polyethylene interface or if the water was in the polyethylene bulk (as evidenced by the swelling of the polyethylene), a water-holding cell was designed which allowed water to come into contact with only one side of a large sheet of polyethylene. After a period of time, the polyethylene was removed from the cell, the surface dried with dry nitrogen, and was evaluated in the ATR setup (with normal sample holder plates).

The diffusion constant for low density polyethylene is roughly \( D \approx 10^{-12} \text{m}^2/\text{s} \) [32, 47]. For the \( d \approx 2.6 \) mil thick sheet of polyethylene, this results in a diffusion time constant of
Figure 2-4: Absorption spectra of water intrusion into polyethylene at 4, 26, and 48 hours, visible for wavenumbers $3100 \leq k \leq 3800$, $1500 \leq k \leq 1700$, and $k \leq 800\text{ cm}^{-1}$. 
\[
\tau = \frac{d^2}{D} \approx 72 \text{ minutes.}
\]

For this experiment, it was desired to saturate the polyethylene with moisture so that the heights of the water peaks could be compared with the previous experiment's results. Therefore, the experiment was run for one month while other measurements were being made with the FTIR.

After one month, a sample of polyethylene was removed and analyzed with the ATR. The height of the water peaks were similar for all samples, including samples examined from both the “wet” and “dry” sides as shown in Figure 2-5. (The “dry” side water peaks are slightly lower since this measurement was taken after the “wet” side and some moisture diffused out in the intervening time.) These measurements indicated that the polyethylene was saturated with water.

When comparing the plots in Figures 2-4 and 2-5, the experimental method must also be considered. In the first experiment, the initial measurement at time \( t = 0 \) was subtracted from the subsequent measurements. The sample holder plates were not moved during the time the experiment was run. The polyethylene peaks appeared with increased amplitude as the polyethylene sheet swelled, making better contact with the dielectric crystal. In the second experiment, the spectra of a dry control sample was subtracted from the wet samples. The contact between the polyethylene and the dielectric crystal is similar, but not the same, in these measurements, resulting in the change in the polyethylene peak amplitudes but not the wavelength locations. In addition, the plots in Figure 2-5 were auto-scaled to show more detail. Thus, the water peaks in Figure 2-5 are considerably smaller than those in Figure 2-4.

### 2.3 Conclusions

It is concluded that the water observed in the first experiment was primarily surface water which built up at the crystal – polyethylene interface rather than water diffused into the polyethylene. Using this method, the presence of water would be detectable, but not its spatial profile in the polyethylene. In addition, in the IR the penetration depth of the evanescent fields is small such that it is possible to only probe a few microns into the polyethylene even if the surface water were not an issue. The focus of this thesis therefore switched to the microwave frequency range where the penetration depth is longer and water has another absorption peak.
Figure 2-5: The "wet" and "dry" sides of PE in an experiment with water in contact with one side for 30 days have similar IR absorption spectra. The IR spectra of an unused PE sample from the same sheet was subtracted out.
Chapter 3

Dielectric Measurements, Moisture Models, and Tree Growth

This chapter details preliminary modeling and measurements needed to characterize material properties for design of the microwave experiments and computer simulations.

3.1 Dielectric Measurements of Cable Samples at Microwave Frequencies

Samples of cable and splice materials were obtained from TEPCO and the dielectric constant and loss were analyzed in a microwave waveguide system. The dielectric properties of these materials in the microwave frequency range were not known and were not available in the literature. The cross-linked polyethylene insulation (XLPE) cable samples show more loss than that of low density polyethylene in [3] due possibly to the migration of ions from the semiconducting shield into the XLPE [46] and from curing byproducts [44].

3.1.1 Basic Operation of Dielectric Measurement Apparatus

A microwave test system based on a shorted circular metallic waveguide cavity was used to calculate the dielectric properties of the test materials at discrete frequencies. Samples were machined or cut from cable sections and splicing tapes to fit snugly inside the waveguide.
The electric field nodal pattern was measured before and after the sample was inserted and change in the nodal pattern was used to calculate the complex dielectric constant. This technique is also known as the standing wave method.

Two types of experimental approaches are possible with this equipment. If the sample is fairly high loss, only a thin sample is used and is placed at one quarter wavelength ($\frac{\lambda}{4}$) from the shorted end where the electric field is maximum as shown in Figure 3-1. For a low loss material, a thicker sample is used for more accurate loss calculations. Typically, this sample is $\frac{3\lambda}{4}$ to $\frac{5\lambda}{4}$ long and is placed at the shorted end. The measurements are essentially the same as those for the thin sample and are defined in Table 3.1.

![Figure 3-1: Waveguide measurements for thin samples.][8]

| $x_o$ | Distance from face of sample to first minimum. |
| $\Delta x_a$ | Width of minimum measured at twice minimum power without sample, but with $\frac{\lambda}{4}$ spacer (if used). |
| $\Delta x_s$ | Width of minimum measured at twice minimum power with sample. |
| $N_q$ | A node position without sample, with $\frac{\lambda}{4}$ spacer. |
| $N_s$ | Corresponding node position with sample. |
| $n_q$ | Number of nodes from first minimum to $N_q$ node reading, without sample. |
| $n_s$ | Number of nodes from first minimum to $N_s$ node reading, with sample. |
| $\lambda$ | Wavelength in air-filled section of line. |
| $d$ | Sample thickness. |
| $d_q$ | Length of $\frac{\lambda}{4}$ spacer. |

Table 3.1: Description of waveguide measurements.
For measurements at 0.3 GHz, 1 GHz, and 3 GHz, the waveguide is configured with a center conductor and is operated in a TEM (transverse electric and magnetic) coaxial mode. At 8.5 GHz and 13.6 GHz, the hollow waveguide is excited in the $TE_{11}$ mode and the samples are solid disks. In this mode, material inhomogeneities in the polyethylene from diffused semiconductor particles were seen in some samples when they are rotated. (The cable samples had semiconducting tapes fused to the inner and outer surfaces.) These measurements were input to computer programs developed by Mr. William Westphal to calculate the complex dielectric constant of the samples, described in [8, 4] and briefly explained below. Additional notation used in the equations, in addition to the measurements from Table 3.1, are given in Table 3.2.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x$</td>
<td>Width of minimum measured at twice the minimum power corrected for line loss between minimum and sample face.</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Cutoff wavelength.</td>
</tr>
<tr>
<td>$u$</td>
<td>$(\lambda/\lambda_c)^2$ (=0 for TEM)</td>
</tr>
<tr>
<td>$w$</td>
<td>$1 + u$</td>
</tr>
<tr>
<td>$E_{min}/E_{max}$</td>
<td>Inverse standing wave ratio.</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>Intrinsic impedance of air-filled guide.</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>Intrinsic impedance of sample-filled guide.</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Propagation constant for air-filled guide.</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>Propagation constant for sample-filled guide.</td>
</tr>
</tbody>
</table>

Table 3.2: Notation for standing wave method equations.

For thick low-loss samples positioned at the shorted end, the calculations are as follows [4]. The distance from the sample face to the first minimum is

$$x_0 = N_s - N_q + n_s(\frac{\lambda}{2}) - n_s(\frac{\lambda}{2}) - d$$  \hspace{1cm} (3.1)

The width of the minimum at twice the minimum power corrected for the waveguide loss between the sample face and the minimum is given by

$$\Delta x = \Delta x_s - \left[ x_0 + n_s(\frac{\lambda}{2}) \right] \frac{\Delta x_a}{n_q(\frac{\lambda}{2})}$$ \hspace{1cm} (3.2)

The inverse standing wave ratio is

$$\frac{E_{min}}{E_{max}} = \frac{\sin \theta}{(2 - \cos^2 \theta)^{\frac{1}{2}}}$$ \hspace{1cm} (3.3)
where

\[ \theta = \frac{\pi \Delta x}{\lambda} \]  

(3.4)

The impedance ratio of the sample-filled guide to the empty guide is given by

\[ \frac{Z_2}{Z_1} = \frac{E_{\text{min}}}{E_{\text{max}}} - j \frac{\tan \frac{2\pi x_o}{\lambda}}{1 - j \frac{E_{\text{min}}}{E_{\text{max}}} \tan \frac{2\pi x_o}{\lambda}} \]  

(3.5)

The transcendental equation used to calculate the propagation constants \( \gamma_1 \) and \( \gamma_2 \) is

\[ \frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{1}{\gamma_1 d Z_1} \]  

(3.6)

This is solved numerically or by the tables provided in [4]. The resulting dielectric constant is given by

\[ \epsilon = \frac{\epsilon'}{\epsilon_o} - j \frac{\epsilon''}{\epsilon_o} = u - \left( \frac{\lambda}{2 \pi d} \gamma_2 d \right)^2 \frac{1}{1 + u} \]  

(3.7)

For thin high-loss samples placed at \( \frac{\lambda}{4} \) from the short, the equations change as follows [8, 4]. The distance from the sample face to the first minimum is now given by

\[ x_o = N_s - N_q + n_q \left( \frac{\lambda}{2} \right) - n_s \left( \frac{\lambda}{2} \right) + \frac{\lambda}{4} - d \]  

(3.8)

The transcendental equation used to solve for the propagation constants \( \gamma_1 \) and \( \gamma_2 \) becomes

\[ \frac{\coth \gamma_2 d}{\gamma_2 d} = \frac{1}{\gamma_1 d Z_1} \]  

(3.9)

The complex dielectric constant is calculated as in Equation 3.7.

### 3.1.2 Measurement Results

Using the above method, the dielectric constants for cable insulation samples, the semiconducting tapes, and ethylene-propylene rubber (EPR) tapes used in the manufacture of cable splices were measured and the results are given in Tables 3.3 and 3.4. The thin samples of the semiconducting tapes and semiconducting shield placed at the node peak (\( \frac{\lambda}{4} \) from the bottom) sagged in the center or did not sit well on the spacer and flipping the sample gave different results. In this case, the higher dielectric constant results were always used. If the
sample is not exactly at the node peak, the dielectric constant will be lower than expected. For the thick samples of XLPE and EPR tapes, the average of all measurements was used. If the complex dielectric constant is given by \( \varepsilon = \varepsilon' - j\varepsilon'' \), the values given in the tables are the relative dielectric constants \( \varepsilon'/\varepsilon_0 \) (real) and \( \varepsilon''/\varepsilon_0 \) (imaginary).

### 3.2 Models for “Moist” PE

Moisture diffuses into XLPE, changing the dielectric constant and affecting the attenuation of signals in the XLPE. A model for this new dielectric constant was needed to predict the attenuation due to the moisture in the XLPE cable insulation. Several effective medium models were considered. The first two, the series and parallel models, are shown in Figures 3-2(a) and (b) and are the lower and upper limits for the dielectric constant. Three additional models are shown in Figures 3-2(c), (d) and (e) and are the Lorentz sphere, Independent sphere, and the Haus-Melcher models. All models result in effective dielectric constants as a function of moisture concentration that lie between the series and parallel models.

#### 3.2.1 Series Model

The series model shown in Figure 3-2(a) is the lower limit for the effective dielectric constant. The water (\( \varepsilon_w \)) is packed in layers perpendicular to the applied electric field within the polyethylene (\( \varepsilon_{pe} \)). Each water layer is of height \( h_1, h_2, h_3 \ldots \), the total height of the water layers is \( h_w \), and the total height of the structure is \( h \). Let the volume fraction of water be \( x = \frac{h_w}{h} \) and the volume fraction of polyethylene be \( 1 - x \).

The capacitance of a structure with cross sectional area \( A \), height \( h \), and dielectric constant \( \varepsilon_{eff} \) is given by \( C = \frac{\varepsilon_{eff}A}{h} \). Since capacitors in series add reciprocally, the total capacitance, neglecting fringing effects, can be written

\[
C(x) = \frac{\varepsilon_{pe}\varepsilon_w A}{h(\varepsilon_{pe}x + \varepsilon_w(1 - x))} \quad (3.10)
\]

Therefore, \( \varepsilon_{eff} \) is found to be

\[
\varepsilon_{eff} = \frac{\varepsilon_{pe}\varepsilon_w}{(\varepsilon_{pe}x + \varepsilon_w(1 - x))} \quad (3.11)
\]
### SPLICE MATERIALS

<table>
<thead>
<tr>
<th>Material</th>
<th>real dielectric constant</th>
<th>imaginary</th>
<th>loss tangent</th>
<th>imag/real</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEPCO EPR rubber tape .3 GHz</td>
<td>2.27078</td>
<td>0.00403</td>
<td>0.001774</td>
<td></td>
</tr>
<tr>
<td>1 GHz</td>
<td>2.29649</td>
<td>0.00202</td>
<td>0.000879</td>
<td></td>
</tr>
<tr>
<td>3 GHz</td>
<td>2.27415</td>
<td>0.00197</td>
<td>0.000868</td>
<td></td>
</tr>
<tr>
<td>8.5 GHz</td>
<td>2.27411</td>
<td>0.00192</td>
<td>0.000844</td>
<td></td>
</tr>
<tr>
<td>13.6 GHz</td>
<td>2.29718</td>
<td>0.00089</td>
<td>0.000386</td>
<td></td>
</tr>
<tr>
<td>TEPCO semi-cond. tape, narrow 8.5 GHz</td>
<td>2.87910</td>
<td>0.05263</td>
<td>0.018281</td>
<td></td>
</tr>
<tr>
<td>13.6 GHz</td>
<td>2.82648</td>
<td>0.04626</td>
<td>0.016368</td>
<td></td>
</tr>
<tr>
<td>TEPCO semi-cond. tape, wide .3 GHz</td>
<td>25.61965</td>
<td>53.82217</td>
<td>2.100816</td>
<td></td>
</tr>
<tr>
<td>1 GHz</td>
<td>26.00132</td>
<td>15.68650</td>
<td>0.603296</td>
<td></td>
</tr>
<tr>
<td>3 GHz</td>
<td>20.86396</td>
<td>8.60621</td>
<td>0.412492</td>
<td></td>
</tr>
<tr>
<td>8.5 GHz</td>
<td>22.55921</td>
<td>7.87116</td>
<td>0.348911</td>
<td></td>
</tr>
<tr>
<td>13.8 GHz</td>
<td>19.03898</td>
<td>4.56165</td>
<td>0.239596</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: TEPCO splice material dielectric properties.

### CABLE MATERIALS

<table>
<thead>
<tr>
<th>Material</th>
<th>real dielectric constant</th>
<th>imaginary</th>
<th>loss tangent</th>
<th>imag/real</th>
</tr>
</thead>
<tbody>
<tr>
<td>XLPE cable sample 300 MHz</td>
<td>2.23019</td>
<td>0.00905</td>
<td>0.004058</td>
<td></td>
</tr>
<tr>
<td>1 GHz</td>
<td>2.25567</td>
<td>0.01059</td>
<td>0.004696</td>
<td></td>
</tr>
<tr>
<td>3 GHz</td>
<td>2.25086</td>
<td>0.00979</td>
<td>0.004351</td>
<td></td>
</tr>
<tr>
<td>8.5 GHz</td>
<td>2.27060</td>
<td>0.00839</td>
<td>0.003676</td>
<td></td>
</tr>
<tr>
<td>13.6 GHz</td>
<td>2.27747</td>
<td>0.00685</td>
<td>0.003008</td>
<td></td>
</tr>
<tr>
<td>Outer semiconductor 300 MHz</td>
<td>53.94990</td>
<td>83.81287</td>
<td>1.553532</td>
<td></td>
</tr>
<tr>
<td>1 GHz</td>
<td>32.29750</td>
<td>33.87517</td>
<td>1.048848</td>
<td></td>
</tr>
<tr>
<td>3 GHz</td>
<td>24.22990</td>
<td>17.87808</td>
<td>0.737852</td>
<td></td>
</tr>
<tr>
<td>8.5 GHz</td>
<td>24.22528</td>
<td>14.92475</td>
<td>0.616081</td>
<td></td>
</tr>
<tr>
<td>13.6 GHz</td>
<td>13.23063</td>
<td>3.93115</td>
<td>0.297125</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Dielectric measurements of transmission cable material samples.
Figure 3-2: Packing arrangements for effective medium calculations. The models in order shown are the series, parallel, Lorentz sphere, independent sphere, and Haus-Melcher models.

3.2.2 Parallel Model

The parallel model shown in Figure 3-2(b) is the upper limit for the effective dielectric constant. The water ($\varepsilon_w$) is packed in layers parallel to the applied electric field within the polyethylene ($\varepsilon_{pe}$). Each water layer is of width $w_1, w_2, w_3 \ldots$, the total width of the water layers is $w_w$, and the total width of the structure is $w$. Let the volume fraction of water be $x = \frac{w_w}{w}$ and the volume fraction of polyethylene be $1 - x$.

The capacitance of a structure with height $h$, width $w$, depth $d$, and dielectric constant $\varepsilon_{eff}$ is given by $C = \frac{\varepsilon_{eff}wd}{h}$. Since capacitors in parallel add, the total capacitance,
neglecting fringing effects, can be written

\[ C(x) = \frac{wd}{h} (\epsilon_w x + \epsilon_{pe}(1 - x)) \quad (3.12) \]

Therefore, \( \epsilon_{eff} \) is found to be

\[ \epsilon_{eff} = \epsilon_w x + \epsilon_{pe}(1 - x) \quad (3.13) \]

### 3.2.3 Lorentz Sphere Model

The Lorentz sphere model, shown in Figure 3-2(c), allows for dielectric interactions between the molecules of water and polyethylene. Assume a spherical water particle with dielectric constant \( \epsilon_w \) has radius \( r_1 \) and is surrounded by a larger spherical shell of polyethylene with dielectric constant \( \epsilon_{pe} \) and radius \( r_2 \). The surrounding medium is assumed to be uniform with effective dielectric constant \( \epsilon_{eff} \). The volume fraction of water is \( x = (\frac{r_1}{r_2})^3 \).

If a uniform field \( E_0 \) is applied many radii away from the spheres, the resulting average fields in the polyethylene (\( E_{pe} \)) and water (\( E_w \)) regions are as follows:

\[ E_w = \frac{3\epsilon_{pe}E_{pe}}{2\epsilon_{pe} + \epsilon_w} \quad (3.14) \]

\[ E_{pe} = \frac{9\epsilon_{pe}\epsilon_{eff}E_0}{(2\epsilon_{eff} + \epsilon_{pe})(2\epsilon_{pe} + \epsilon_w) + 2x(\epsilon_{eff} - \epsilon_{pe})(\epsilon_{pe} - \epsilon_w)} \quad (3.15) \]

The dipole electric field terms will cancel out on average. In addition, the average field over the spherical region must equal the applied field \( E_0 \).

\[ E_0 =xE_w + (1 - x)E_{pe} \quad (3.16) \]

Using the above equations, \( \epsilon_{eff} \) can be solved for, giving

\[ \epsilon_{eff} = \frac{\epsilon_{pe}}{2}\frac{(2\epsilon_{pe} + \epsilon_w + 2x(\epsilon_w - \epsilon_{pe}))}{2\epsilon_{pe} + \epsilon_w - x(\epsilon_w - \epsilon_{pe})} \quad (3.17) \]

### 3.2.4 Independent Sphere Model

The independent sphere model, shown in Figure 3-2(d), assumes that both the water and polyethylene are spherical particles in the effective medium with dielectric constant \( \epsilon_{eff} \).
and that they are non-interacting. When a uniform field $E_o$ is applied many radii from the spheres, the average field inside each sphere is

$$E_w = \frac{3\varepsilon_{eff} E_o}{2\varepsilon_{eff} + \varepsilon_w}$$ (3.18)

and

$$E_{pe} = \frac{3\varepsilon_{eff} E_o}{2\varepsilon_{eff} + \varepsilon_{pe}}$$ (3.19)

The average field must be $E_o$ (Equation 3.16) and the volume ratio of the water is $x = \frac{r_1^3}{r_1^3 + r_2^3}$. These equations yield a quadratic equation for $\varepsilon_{eff}$

$$\varepsilon_{eff}^2 + \frac{1}{2}\varepsilon_{eff}(2\varepsilon_{pe} - \varepsilon_w + 3x(\varepsilon_w - \varepsilon_{pe})) - \frac{1}{2}\varepsilon_w\varepsilon_{pe} = 0$$ (3.20)

The solution of this equation is

$$\varepsilon_{eff} = \frac{1}{4} \left[\left(2\varepsilon_{pe} - \varepsilon_w + 3x(\varepsilon_w - \varepsilon_{pe})\right) + \left(2\varepsilon_{pe} - \varepsilon_w - 3x(\varepsilon_w - \varepsilon_{pe})\right)^2 + 8\varepsilon_w\varepsilon_{pe}\right]^{\frac{1}{2}}$$ (3.21)

### 3.2.5 Haus-Melcher Model

The Haus-Melcher model, shown in Figure 3-2(e) and described in [20], is the diffuse limit of the Lorentz sphere model where the concentration of water ($x$) is very small. The derivation is considerably different from those in previous sections. As before, assume there is an applied vertical uniform field $E_o$. The electric field for a single dielectric sphere with permittivity $\varepsilon_w$ in an infinite region of permittivity $\varepsilon_{pe}$ is given by

$$\bar{E} = E_o \hat{z} + E_o \left(\frac{R}{r}\right)^3 \left[\frac{\varepsilon_w - \varepsilon_{pe}}{\varepsilon_w + 2\varepsilon_{pe}}\right] \left(\hat{r} 2 \cos \theta + \hat{\theta} \sin \theta\right)$$ (3.22)

where $\hat{z} = \hat{r} \cos \theta - \hat{\theta} \sin \theta$ is the vertical axis, $\hat{r}$ is radially outward, and $\hat{\theta}$ is measured down from the $\hat{z}$ axis. This field expression is the original uniform field modified by a dipole field term. If this second term is compared to the field of an isolated permanent dipole in an $\varepsilon_{pe}$ medium,

$$\bar{E} = \frac{P}{4\pi\varepsilon_{pe} r^3} \left(\hat{r} 2 \cos \theta + \hat{\theta} \sin \theta\right)$$ (3.23)

35
the dipole moment can be solved for as
\[
p = 4\pi \epsilon_{pe} R^3 E_o \left[ \frac{\epsilon_w - \epsilon_{pe}}{\epsilon_w + 2\epsilon_{pe}} \right] \tag{3.24}
\]

If there are \( N \) of these dipoles with uniform center to center spacing \( S \), \( N = \frac{1}{S^3} \). The polarization density is given by \( P = Np \).

Therefore, since the polarization is linearly related to the applied electric field,
\[
\bar{P} = 4\pi \epsilon_{pe} \left( \frac{R}{S} \right)^3 \left[ \frac{\epsilon_w - \epsilon_{pe}}{\epsilon_w + 2\epsilon_{pe}} \right] \bar{E} \tag{3.25}
\]

The displacement flux density corrected for the dipole field from the water spheres is \( \bar{D} = \bar{P} + \epsilon_{pe} \bar{E} = \epsilon_{eff} \bar{E} \). Using Equation 3.25 gives
\[
\epsilon_{eff} = \epsilon_{pe} \left( 1 + 4\pi \left( \frac{R}{S} \right)^3 \left[ \frac{\epsilon_w - \epsilon_{pe}}{\epsilon_w + 2\epsilon_{pe}} \right] \right) = \epsilon_{pe} \left( 1 + 3x \left[ \frac{\epsilon_w - \epsilon_{pe}}{\epsilon_w + 2\epsilon_{pe}} \right] \right) \tag{3.26}
\]

where the volume density of the water in the polyethylene is \( x = \frac{4\pi}{3} \left( \frac{R}{S} \right)^3 \). If one were to take the limit of small \( x \) in Equation 3.17, this same result would be obtained.

### 3.2.6 Comparison of Models

Moisture is typically measured in parts per million (ppm) by weight, where ppm is the ratio of the mass of the water divided by the total mass. The density of polyethylene is roughly 925 kg/m³ and the density of water is 10³ kg/m³. Therefore, to convert ppm into the volume density \( x \), the following equation is used.
\[
x = \frac{\text{ppm} \times 10^{-6}}{\left( \frac{1-\text{ppm} \times 10^{-6}}{0.925} + \text{ppm} \times 10^{-6} \right)} \tag{3.27}
\]

A plot comparing the calculated real part of the dielectric constants vs. ppm for the various models using polyethylene data from Table 3.4 and water relative dielectric constant shown in Figure 3-3 evaluated at 8.5 GHz is shown in Figure 3-4. The dielectric constants are scaled by the permittivity of free space \( (\epsilon_o) \). The Haus-Melcher model is only valid for low densities of water \( (R \gg S) \) and the corresponding plot in Figure 3-4 is truncated at less than unity water concentration.

Undamaged polyethylene does not contain significant amounts of water, generally less
than $10^3$ ppm. A plot comparing the three spherical effective medium models for water densities up to $10^4$ ppm is shown in Figure 3-5. Note that all three models give comparable results for small moisture concentrations. The simpler Haus-Melcher model will be used when calculating the effective dielectric constants.

### 3.3 Coax Cable Experiments

As a preliminary test prior to developing the rectangular waveguide experiments, experience was gained investigating the attenuation due to moisture and treeing in microwave coaxial cables with polyethylene insulation operated in the TEM (transverse electric and magnetic) field mode. The possibility of using the high voltage power cable as the waveguide in a microwave TEM mode was also considered.

#### 3.3.1 Loss in TEM mode

The equations for the TEM transmission line model shown in Figure 3-6 are as follows:

$$\frac{dV}{dz} = -ZI$$  \hspace{1cm} (3.28)
Figure 3-4: Comparison of effective medium models as a function of moisture concentration at 8.5 GHz.

Figure 3-5: Comparison of the spherical effective medium models for low densities of water.
where \( Z = R + j \omega L \) and \( Y = G + j \omega C \). \( L \) and \( C \) are the characteristic inductance and capacitance per unit length of the lossless transmission line. \( V \) and \( I \) are the voltage between the conductors and the current in one conductor. Equations 3.28 and 3.29 can be combined to form a wave equation,

\[
\frac{d^2V}{dz^2} = ZYV = -k^2V \tag{3.30}
\]

The complex wavenumber \( k \) is given by

\[
k = \sqrt{-ZY} = \sqrt{-(R + j\omega L)(G + j\omega C)} \tag{3.31}
\]

Assuming any losses are small, that is \( R \ll \omega L \) and \( G \ll \omega C \) (likely at a frequency of 1 GHz), the wavenumber can be approximately written as

\[
k \approx \omega \sqrt{LC}(1 - \frac{1}{2}(\frac{R}{\omega L} + \frac{G}{\omega C})) = \omega \sqrt{LC} - \frac{1}{2} \sqrt{LC}(\frac{R}{L} + \frac{G}{C}) = k' - jk'' \tag{3.32}
\]

A solution to the wave equation for a voltage wave with amplitude \( V_0 \) at \( z = 0 \), propagating in the \(+z\) direction is

\[
V(z) = V_0 e^{-jkz} = V_0 e^{-jk'z - jk''z} \tag{3.33}
\]
Using a perturbation approach, where it is assumed that losses from \( R \) and \( G \) are small, such that the fields remain quasi-TEM, the impedances and conductances for a coaxial line are calculated from the following equations:

\[
L = \frac{\mu_0}{2\pi} \ln \left( \frac{r_o}{r_i} \right) \quad (3.34)
\]

\[
C = \frac{2\pi \epsilon'}{\ln \left( \frac{r_o}{r_i} \right)} \quad (3.35)
\]

\[
G = \frac{2\pi \omega \epsilon''}{\ln \left( \frac{r_o}{r_i} \right)} \quad (3.36)
\]

\[
R = \frac{1}{2\pi \sigma \delta} \left( \frac{1}{r_o} + \frac{1}{r_i} \right) \quad (3.37)
\]

\[
\delta = \sqrt{\frac{2}{\omega \mu_0 \sigma}} \quad (3.38)
\]

where \( r_i, r_o, \sigma, \) and \( \delta \) are the inner and outer conductor radii, conductivity, and skin depth of the conductors. Using these formulae, the attenuation coefficient \( k'' \) and the resulting attenuation can be calculated. Power attenuation is typically measured in dB/m (decibels per meter). The attenuation at a coordinate \( z \) with respect to the power at \( z = 0 \) is given by

\[
\text{Attenuation} = 20 \log_{10} \left| \frac{V(z = 0)}{V(z)} \right| = 20 \log_{10} \left| V(z) \right| = 20 \log_{10} V_o - 20 \log_{10} V_o e^{-k''z} = -20 \log_{10} e^{-k''z} = k''z 20 \log_{10} e \quad (3.39)
\]

The power attenuation per unit length is then

\[
\text{Attenuation/meter} = k'' 20 \log_{10} e \approx 8.69 k'' \quad (3.40)
\]

### 3.3.2 High Voltage Cable Losses in the TEM Mode

The dielectric properties of the components of a high voltage transmission cable were tabulated in Section 3.1. The cable, shown in Figure 3-7, is constructed of a inner copper conductor wrapped with a semiconducting tape, followed by extruded cross-linked polyethy-
lene, another layer of semiconducting tape, and the outer copper or aluminum conductor. Using the measured data, the TEM transmission losses of a typical cable were calculated. The admittances $Y$ of the two semiconducting layers and the polyethylene layer were added in series and the resistances of the conductors were added in parallel.

![XLPE and semiconducting tapes taken from a high voltage transmission cable.](image)

Figure 3-7: XLPE and semiconducting tapes taken from a high voltage transmission cable.

The inner radius of the inside semiconducting tape is 7.35 mm, the outer radius is 8.35 mm, the outer radius of the XLPE is 14.35 mm, and the outer radius of the outside semiconducting tape is 15.05 mm. At 1 GHz, the calculated attenuation is 2.5 dB/m and this attenuation grows with frequency. At 3 GHz, the attenuation is 8.6 dB/m, at 8.5 GHz, 22.5 dB/m and at 13.6 GHz, 36.5 dB/m.

Since the distance between splices is at least 100-200 meters [55], if a signal is applied at one splice in the cable, the signal at the next splice would be too small to measure. This approach could be used to analyze a splice, which is short (on the order of a meter in length). However, the splice geometry is somewhat more complicated and the splice is constructed with semiconducting tapes and sealants that are very lossy in the microwave region (see Table 3.3).

### 3.3.3 Microwave Waveguide Experimental Setup

This measurement technique uses a HP 8410A microwave network analyzer system, as shown in Figure 3-8. The sweep oscillator generates an RF signal which is provided to the transmission test unit. The test unit splits the signal into two paths and the waveguide (or TEM) experiment is placed in one path and attenuates this signal. The harmonic frequency converter takes the two signals from the test unit and generates a composite 278 kHz signal.
which is provided to the network analyzer. The network analyzer interprets the frequency, magnitude, and phase information encoded in this signal and displays the magnitude (in dB) and phase vs. frequency of the loss introduced by the waveguide experiment. This equipment works in the 2-12.4 GHz frequency range.

Measurements were made in a similar manner to the FTIR experiments. First, a reference measurement was taken using a dry cable or waveguide with dry polyethylene samples inserted. Water was then added to the polyethylene via various experimental techniques and another measurement was taken. The difference of these two measurements is the change in attenuation due to the addition of moisture. The absolute attenuation measurement is meaningless as it is very difficult to model the reflections at all connectors and material interfaces. It is assumed that these reflections remain the same throughout both experiments and their effect is canceled through subtraction of the reference measurement.

3.3.4 Moisture Measurements

The moisture in polyethylene samples was evaluated with a Mitsubishi Moisturemeter system using Karl Fischer coulometric titration. The samples were chopped into small pieces and placed in an oven at 120°C. Dry nitrogen was blown past the sample, collecting the
water vapor, and was then transported to a reaction vessel containing an iodine solution. The water reacted with the iodine, \( I_2 + SO_2 + H_2O = 2HI + SO_3 \). The water content is then determined from the charge required for electrolysis of the resulting iodine ions in the solution using \( 2I^- + 2(-e) = I_2 \). The moisture measurements were then used to calculate the dielectric constant and expected theoretical attenuation.

### 3.3.5 Unheated Cable

To examine the attenuation due to moisture in the TEM mode, two 10-foot microwave cables were constructed from RG214/U type coaxial cable. This cable has an inner dielectric radius of 1.09 mm and an outer radius of 3.62 mm and both conductors are silver-coated copper. The attenuation of each cable was measured as described in Section 3.3.3 for the frequency range of 8-12.4 GHz and the results were subtracted to give the difference in attenuation for the two cables. One cable was reserved as the reference cable and the other was immersed in a water bath, with its PVC coating removed, for a period of time. An additional section of cable was immersed in the same water bath and was used for the moisturemeter measurements described in Section 3.3.4.

Periodically, the cable was removed from the water bath, the water was dried from the outer braided conductor, and the attenuation of the cable was measured and compared to the reference cable. There was no noticeable frequency dependent change in the attenuation. At the same time, a small section of cable was removed from the second cable in the water bath and its moisture content measured. The moisture measurements were quite small, on the order of 50 ppm for wet cable and 25 ppm for dry cable after one month.

The theoretical dielectric constant was constructed using the Haus-Melcher model and the various moisture concentrations. At 12.4 GHz, the change in calculated attenuation using Equation 3.40 is 0.006 dB/m, well below the sensitivity of the microwave equipment. The room temperature saturation level of water in unoxidized low-density polyethylene is roughly 50 ppm [32]. The cable was saturated with moisture and it was apparent that no significant moisture levels would be obtained with this experiment.

### 3.3.6 Heated Cable

To attempt to get larger amounts of moisture into the cable, the above experiment was repeated with a cable in a heated 90°C water bath. After two weeks, the moisture content
of the polyethylene was measured at 240 ppm and remained near this level for subsequent measurements. Using extrapolated data from [32], the saturation level of moisture in unoxidized low density polyethylene at 90°C is 200 ppm. Again, it was unlikely that higher moisture levels would be obtained. The theoretical dielectric constant was constructed using the Haus-Melcher model with 240 ppm of moisture. At 12.4 GHz, the change in calculated attenuation using Equation 3.40 is 0.05 dB/m, still well below the sensitivity of the microwave equipment, as verified by measurements.

### 3.3.7 Treed Cable

Since undamaged polyethylene does not absorb enough water for its attenuation to be measurable with our microwave equipment, even when immersed in a hot water bath to raise the saturation level of water, water trees were grown in test samples of our microwave cable. Water trees have a higher water content and loss than water-saturated undamaged polyethylene. It was hoped that the attenuation from these trees would be large enough to be detected in our microwave measurement system.

The first experiment was to grow trees in undamaged new cable using both deionized water and 0.5 molar NaCl solutions. The addition of NaCl is known to accelerate tree initiation and growth [21]. A section of cable was immersed in a water bath and 2.5 kV RMS at 500 Hz was applied. New RG214 type cable has surface imperfections and grooves in the XLPE next to the cable braid. Since vented trees typically grow from surface imperfections [21], it was thought that this would be enough to initiate tree growth. However, because of electrical breakdown at the microwave connectors on the ends of the cable, a high enough electric field stress could not be applied in the cable to initiate tree growth.

The microwave connectors have a breakdown strength of about 2-3 kV and the cable insulation thickness (difference in outer and inner radii) was 2.5 mm. This meant that only about 1 kV/mm of electric field stress could be applied across the cable insulation, which was significantly less than typically needed to initiate tree growth. These microwave connectors are necessary to hook the cable into our microwave measurement system, and need to remain attached throughout the experiment. Changing the connectors can affect the loss attributed to the connectors, which could be difficult to distinguish from loss due to the growing water trees.

However, bow-tie trees were grown in the cable insulation with these applied voltages,
most likely from voids or impurities within the polyethylene. Figure 3-9 shows such a bow-tie tree made visible with methylene blue dye. The procedure used to dye the cable is described in [21].

Figure 3-9: Bow-tie trees grown in microwave cable insulation. The ruler markings at the bottom are separated by 1 mm.

In the next experiment, the outer surface of the XLPE insulation of a test cable was punctured with a needle to decrease the distance between the inner and outer electrodes. This resulted in a higher electric field stress for tree-initiation, due to both the shorter distance between the electrodes and the resulting pointed outer electrode. (When the needle is removed, it leaves a channel which is filled with water or the NaCl solution. This cavity creates the pointed electrode.) It was found that now the trees initially grow very fast from the needle puncture. As the tree grows, the electrical stress due to the pointed electrode decreases. Eventually, the tree stops growing or grows very slowly. These trees are shown in Figure 3-10.

Due to experimental difficulties in controlling corrosion of the cable conductors during tree growth and trouble with water wicking up the braid into the cable connectors, experiments to grow trees in a 10-foot section of the cable to test the attenuation were never carried out. These experiments were designed mainly to examine the attenuation of a cable
excited in a simple TEM mode and were auxiliary to the main thrust of the thesis, which is to use evanescent fields to probe the polyethylene insulation.

3.4 Generation of Water Treed XLPE Sheets

Trees were grown in cross-linked polyethylene (XLPE) sheets provided by TEPCO. These sheets were made using the same dry curing process as the cable samples tested in Section 3.1, however the dielectric constants of the two materials are not quite the same due to contamination from the semiconducting shield in the cable samples. These tree growing experiments were intended to produce samples that could be inserted into a waveguide and detected using evanescent fields. Extensive guidance in growing and characterizing treed XLPE was provided by Mr. Tatsuyu Suzuki, a visiting engineer from TEPCO.

3.4.1 Methodology

To saturate the test XLPE sheets with water trees, accelerated treeing conditions were employed. A treeing cell was designed which allowed water or salt solution to come in contact with both sides of the sheet, sealing against the sheet with gaskets. Electrodes were placed into each side of the water and an electric stress of 8 kV/mm at 500 Hz was applied. The sheets were 0.5 mm thick. Since higher molarity salt solutions encourages tree growth, this experiment was tried with 0.5, 1.0 and 2.0 molar solutions of NaCl. The polyethylene samples were prepared using the following method developed by Mr. Tatsuyu Suzuki:

1. Using #120 sandpaper, scratch both sides of the XLPE. Scratch each surface 40 times, rotating 90° after each set of ten scratches. Surface imperfections are tree-initiation sites.

2. Scrub the samples in water to remove particles.

3. Clean the samples in alcohol to remove any oils from handling. The cleaner surface has better contact with the water.

4. Dry the sample in a vacuum chamber for one day.

5. After inserting sample in the cell, pour 0.5 molar NaCl solution in slowly to discourage bubbles from forming on the surface. Remove bubbles with syringe.

In addition, since the wetting of the surface of the XLPE is apparently important for tree initiation, a few grains of laboratory soap were added to some of the samples in some
of the treeing cells. It was also found that letting the samples sit for several days in an unenergized treeing cell filled with solution also enhanced tree growth. Experimental results are summarized in the following table:

<table>
<thead>
<tr>
<th>Experiment and Cell Number</th>
<th>Treeing Cell Conditions</th>
<th>Water content (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>0.5M NaCl, 200 hr</td>
<td>2516</td>
</tr>
<tr>
<td>IV</td>
<td>1.0M NaCl, 200 hr</td>
<td>2353</td>
</tr>
<tr>
<td>V-#1</td>
<td>0.5M NaCl + side has soap 6254, 5030</td>
<td></td>
</tr>
<tr>
<td>V-#6</td>
<td>1.0M NaCl + side has soap 4201, 4968</td>
<td></td>
</tr>
<tr>
<td>V-#3</td>
<td>2.0M NaCl + side has soap 7016, 7489</td>
<td></td>
</tr>
<tr>
<td>V-#10</td>
<td>0.5M NaCl + side has soap 9090, 10325</td>
<td></td>
</tr>
<tr>
<td>VI-#2</td>
<td>0.5M NaCl + soap 200 hr</td>
<td>6081, 6680</td>
</tr>
<tr>
<td>VI-#5</td>
<td>0.5M NaCl 200 hr</td>
<td>4829, 5388</td>
</tr>
<tr>
<td>VI-#1</td>
<td>0.5M NaCl + soap 200+ hr 7129</td>
<td></td>
</tr>
<tr>
<td>VI-#6</td>
<td>0.5M NaCl 200+ hr</td>
<td>7767</td>
</tr>
</tbody>
</table>

Table 3.5: Water content measurements for various water treeing experiments and conditions.

Experiments III and IV took place before the sample cleaning procedure was modified to include the cleaning with alcohol. Note that the water content is quite low. For experiment V, the samples were cleaned with alcohol and placed in vacuum. In addition, a small amount of soap was added to the positive side of each cell. Most of the treeing cells experienced electrical breakdown before 200 hours had elapsed, so the water contents can't be compared directly. Thin samples were sliced from the treed regions and dyed with a methylene blue solution and examined under a microscope. In experiments V and VI, some samples were taken from two locations for moisture measurements, and both moisture values are given in Table 3.5. From comparing the trees on the hot and ground sides of the cell, it was discovered that adding soap created “bushier” trees, as shown in Figure 3-11. Also, the XLPE in cell V-#10 had been sitting in an unenergized cell filled with 0.5M NaCl for two weeks before the experiment. The moisture content is considerably higher than for the other cells. Possible reasons are that water saturated XLPE grows more dense trees, or that water molecules had additional time to enter tree initiation sites before energization.

For experiment VI, the cells were filled with solution 36 hours before the cells were energized. Two of the cells had soap added to both sides, the others had no soap. After 200 hours, one cell of each type were removed and the moisture in the XLPE was measured. Soaking the XLPE for only 36 hours before energization didn’t seem to enhance the tree
growth and moisture content as much as in experiment V-#10, but adding soap to the NaCl caused a slightly higher moisture content if experiments VI-#2 and VI-#5 are compared. The remaining two cells were allowed to run until breakdown (about another two days). These cells had an additional 2000 ppm of moisture in the treed XLPE. Photographs of VI-#2 and VI-#5 samples dyed with a methylene blue solution are given in Figure 3-11.

3.4.2 Measurements of Complex Permittivities as Function of Moisture Concentration

Since the best treed samples that are consistently produced with the above method have about 6000 ppm of moisture, this value was used for theoretical loss calculations of treed XLPE. Published values for the dielectric constant and loss factor for various molarities of NaCl solutions are given in Figure 3-12. Using the Haus-Melcher model, the theoretical dielectric constant for 0.5M NaCl treed XLPE with 6000 ppm of moisture is calculated and the original (dry cable sample) and calculated imaginary dielectric constants are plotted in Figure 3-13. Note that the cable XLPE dielectric constants are being used rather than the sheet XLPE values. The sheet XLPE dielectric constant was only measured at 8.5 GHz and 13.6 GHz by Mr. Suzuki. However, it is generally the change in the dielectric constant, rather than its actual value, that contributes to the change in attenuation.

Mr. Suzuki measured the loss factor ($\epsilon''$) of the treed XLPE samples at 8.5 GHz. He measured a water content of more than $10^4$ ppm for many of his treed samples, a level that our experiments only reached if they were allowed to run more than $\approx 200$ hours. The loss factor data that Mr. Suzuki measured is given in Figure 3-14. For dry sheet XLPE, containing roughly 50 ppm of moisture, Mr. Suzuki's imaginary dielectric constant is roughly a factor of 16 lower than that of dry cable XLPE measurement. The discrepancy is likely caused by semiconductor contaminants in the cable XLPE samples.

3.5 Conclusions

With the concentrations of moisture present in treed XLPE, one would expect the treed samples of XLPE to cause a significant change in attenuation when placed inside a metal waveguide as compared to untreed dry XLPE. The waveguide experiment using evanescent fields is described in the following chapters and the loss due to a 0.5 mm thick treed XLPE sheet with typical concentrations of water is calculated in Section 5.5. Calculations
show that a single sheet will be barely detectable with the available microwave system. In
addition, due to the difficulty of growing very large treed regions of XLPE needed for this
experiment, a "water sheet" experiment was designed, where measured amounts of water
were sealed between sheets of polyethylene to simulate a treed layer. These "water sheets"
were placed between layers of XLPE and then were inserted into the waveguide, producing
measurable attenuation.
Figure 3-10: Vented trees grown from needle punctures in microwave cable insulation. (a) after 21 days, (b) after 28 days, (c) after 35 days.
Figure 3-11: Treed samples from (a) VI-#5 and (b) VI-#2. The cell for VI-#5 contained 0.5M NaCl and was energized for 200 hours. Experiment VI-#2 was under identical conditions, but with soap added to both sides of the cell, and shows greater tree development.
Figure 3-12: Calculated real dielectric constant and loss factor ($\varepsilon''$) for various molarities of NaCl solutions using models from [29] and [30].

Figure 3-13: Measured imaginary dielectric constant for dry XLPE and calculated (Haus-Melcher model) imaginary dielectric constant for 0.5M NaCl treed XLPE with 6000 ppm moisture content.
Figure 3-14: Loss factor ($\varepsilon''$) of treed XLPE at 8.5GHz as a function of water content. [Collaborative work with Tatsuyu Suzuki of TEPCO]
Chapter 4

Waveguide Modes In Dielectric Loaded Metal Waveguide

First, a simple dielectric waveguide was considered, similar in concept to the dielectric crystal of the FTIR experiment and shown in cross section in Figure 4-1. If the higher dielectric constant material (dielectric waveguide) is surrounded by a region of lower dielectric constant, electromagnetic waves can be guided in the z direction by the waveguide material. The fields in the lower dielectric constant region will be evanescent, that is exponentially decaying, away from the waveguide.

There are several experimental difficulties with this approach in the microwave frequency region. One issue is the coupling from the microwave source and measurement system to the
dielectric waveguide. Dielectric waveguides are typically excited with a horn attached to the source. However, only some of the electromagnetic waves generated in this manner would be guided by the waveguide. Additional microwave energy propagating away from the horn outside the dielectric can be unsafe and also complicates measurement interpretation. In addition, there would be difficulty in supporting the dielectric as any dielectric or metallic support would affect the waveguide fields.

A rectangular metal waveguide addresses these issues. It is easily excited by the microwave system and there are minimal stray fields outside the waveguide. Dielectric materials can be inserted into the waveguide, creating evanescent fields. Methods developed to sense water with this geometry should be applicable to the simple dielectric waveguide geometry and a dielectric rod or fiber waveguide could be used to create evanescent fields to sense moisture in the cable or splice.

In this chapter the solutions for the various waveguide modes for the multiple dielectric loaded rectangular metal waveguide are given. Only symmetric $TE_{10}$ mode solutions are considered as this is the mode in the hollow waveguide which excites the dielectric loaded waveguide. The continuity conditions applied to the dielectric boundaries for each geometry result in transcendental expressions that are solved iteratively to find the propagation constants and field amplitudes. These are then used in Chapter 5 to calculate the attenuation of the electromagnetic fields as a function of moisture level and position.

While investigating techniques to grow water trees in XLPE with higher water content, a “water sheet” technique was developed to simulate treed layers by inserting spatial impulses of water in between pieces of XLPE placed inside the waveguide experiment. The XLPE was machined from 2 and 3 mm thick sheets provided by TEPCO. The “water sheets” consisted of two sheets of 1.6 mil polyethylene sealed together with a hot knife and filled with small amounts of water. These were then placed at known positions between the sheets of XLPE. Attenuation in the waveguide is primarily due to the loss in these water sheet layers.

4.1 Governing Equations

Maxwell’s equations in differential form are used to solve for the electromagnetic fields inside waveguides. Given that there are no electric charges or currents except in the waveguide walls and that there are no magnetic materials inside the waveguide, Maxwell’s equations
for the region inside the waveguide are

\[ \nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mu_0 \mathbf{H} \tag{4.1} \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t} \varepsilon \mathbf{E} = \frac{\partial}{\partial t} \varepsilon \mathbf{E} \tag{4.2} \]
\[ \nabla \cdot \varepsilon \mathbf{E} = \rho = 0 \tag{4.3} \]
\[ \nabla \cdot \mu_0 \mathbf{H} = \nabla \cdot \mathbf{B} = 0 \tag{4.4} \]

where \( \mathbf{E} \) [V/m] and \( \mathbf{H} \) [A/m] are the electric and magnetic field intensities, \( \mu_0 \) is the permeability of free space, \( \varepsilon \) is the permittivity of the dielectric inside the waveguide, \( \mathbf{J} \) [A/m\(^2\)] is the volume current density and \( \rho \) [coul/m\(^3\)] is the volume charge density.

The formulas for \( \mathbf{E} \) and \( \mathbf{H} \) in Equations 4.1-4.4 can be combined into the wave equations

\[ \nabla^2 \mathbf{E} = \mu_0 \varepsilon \frac{\partial^2}{\partial t^2} \mathbf{E} \tag{4.5} \]
\[ \nabla^2 \mathbf{H} = \mu_0 \varepsilon \frac{\partial^2}{\partial t^2} \mathbf{H} \tag{4.6} \]

In sinusoidal steady state, where \( \mathbf{E}(r, t) = \text{Real}\{\mathbf{E}(r) e^{\omega t}\} \) and \( \mathbf{H}(r, t) = \text{Real}\{\mathbf{H}(r) e^{\omega t}\} \), the wave equations can be rewritten in time-harmonic notation

\[ \nabla^2 \mathbf{E} = -\omega^2 \mu_0 \varepsilon \mathbf{E} = -k^2 \mathbf{E} \tag{4.7} \]
\[ \nabla^2 \mathbf{H} = -\omega^2 \mu_0 \varepsilon \mathbf{H} = -k^2 \mathbf{H} \tag{4.8} \]

where \( k = \frac{2\pi}{\lambda} = \omega \sqrt{\mu_0 \varepsilon} \) is the wavenumber or propagation constant and underbarred field quantities are complex amplitudes. General solutions to these equations for real \( \varepsilon \) are sines and cosines and complex exponentials. It will be assumed for the remainder of this thesis that all variables are in the sinusoidal steady state and the field quantities are the complex amplitudes. The underbar notation (\( \_ \)) and the \text{Real}\{\( e^{\omega t}\)\} will be omitted.

The corresponding boundary or continuity conditions at material interfaces are

\[ n \times \left[ \mathbf{E}^a - \mathbf{E}^b \right] = 0 \tag{4.9} \]
\[ n \times \left[ \mathbf{H}^a - \mathbf{H}^b \right] = \mathbf{K} \tag{4.10} \]
\[ n \cdot \left[ \varepsilon_a \mathbf{E}^a - \varepsilon_b \mathbf{E}^b \right] = \sigma_s \tag{4.11} \]
\[ n \cdot \mu_0 \left[ \mathbf{H}^a - \mathbf{H}^b \right] = 0 \tag{4.12} \]
where $K$ and $\sigma_s$ are the unknown induced surface current and surface charge densities in the waveguide walls and $n$ is a unit normal perpendicular to the boundary. There are no surface charges or currents at material interfaces inside the waveguide.

It is assumed that the waveguide walls are perfectly conducting with conductivity $\sigma = \infty$. Metals are ohmic conductors with $J = \sigma E$, and perfect conductors can have no electric field since the current density $J$ must remain finite. If there is no electric field, from Equation 4.1 there can be no time varying magnetic field. Surface currents and charges arise on the surfaces of the conductors to eliminate the fields inside the conductors. The boundary conditions at the walls reduce to

$$n \times E = 0 \quad (4.13)$$
$$n \cdot H = 0 \quad (4.14)$$

Equations 4.13 and 4.14 state that there can be no tangential $E$ or normal $H$ at the surface of a perfect conductor.

### 4.2 Power Conservation

Before solutions are developed for the different dielectric configurations, assumptions about the power flowing in the waveguide need to be addressed. Using complex notation, the time averaged power density flowing in the waveguide is

$$\langle S \rangle = \frac{1}{2} \text{Real} \{E \times H^*\} \left[ \frac{\text{watts}}{m^2} \right] \quad (4.15)$$

The total time averaged power flowing in the z direction is therefore

$$P = \int \int_{A} \langle S \rangle \cdot \hat{z} \, da \quad (4.16)$$

where $A$ is the waveguide cross sectional area.

At material interfaces, such as between the empty rectangular waveguide and the dielectric loaded waveguide, there will be reflections as shown in Figure 4-2. Some of the transmitted power will be reflected and some transmitted into the new region at each material interface. It is assumed that the addition of the water sheets into the dielectric region
will not significantly change the mode shape of the electromagnetic fields or the reflection and transmission coefficients at each interface. Therefore the power flowing in each region will remain the same as in the case with no water sheets, to first order. Power flow remaining constant will be used to calculate the field amplitudes when the water sheets are added.

![Diagram](image)

Figure 4-2: Reflections in waveguide experiment.

### 4.3 Single Dielectric Region Case Studies

#### 4.3.1 Single Dielectric with Complex Permittivity

A hollow rectangular metal waveguide is completely filled with a dielectric material and is excited in the $TE_{10}$ mode as shown in Figure 4-3. The designation $TE_{mn}$ indicates that the electric field pattern has $m - 1$ nulls, excluding those at the walls, in the $x$ (wider) direction and $n - 1$ nulls in the $y$ (narrow) direction. In the $TE_{10}$ case, the electric field has no nulls except at the walls in the $x$ direction and does not vary in the $y$ direction. Using this and the boundary condition from Equation 4.13, the electric field must be zero at $x = \pm a$ and can be only in the $y$ direction.

The solutions to Equations 4.1-4.4 and 4.7-4.8 which match these boundary conditions
Figure 4-3: Rectangular waveguide with single dielectric region.

\[ E = \hat{y} E_0 \cos k_x x e^{-j k_z z} \]  
\[ H = -\frac{E_0}{\omega \mu_0} [\hat{x} j k_x \sin k_x x + \hat{y} k_z \cos k_x x] e^{-j k_z z} \]

where \( k^2 = k_x^2 + k_z^2 = \omega^2 \mu_0 \varepsilon_{pe} \) and \( k_x = \frac{\pi}{2a} \) is fixed by the boundary conditions. If \( \varepsilon_{pe} \) is complex, \( k_z \) would likewise be complex with \( k_z = k'_z - j k''_z \) and the electromagnetic waves would attenuate with position. The various methods of calculating the attenuation are discussed in more detail in Chapter 5. The time averaged power flowing in this mode is given by

\[ P_1 = \frac{1}{2} \int_{-a}^{a} \int_{0}^{h} \frac{1}{2} \text{Real} \{ \mathbf{E} \times \mathbf{H}^* \} \cdot \hat{z} \, dx \, dy \]
\[ = \frac{|E_0|^2}{2\omega \mu_0} \text{Real} \{ k_z \} ah e^{-2k''_z z} \]

### 4.3.2 Effects of a Water Sheet

Two approaches can be used to solve for the mode shape in the case where a symmetric pair of water sheets is placed inside the waveguide, as shown in Figure 4-4. The first is a
perturbation approach which assumes that the electromagnetic field mode and amplitude are not affected by the addition of the water sheets. The attenuation due to the addition of the water sheets is calculated in Section 5.3 and the field solutions are those given in Equations 4.17-4.18.

If the water sheets are thick enough to affect the mode shape or attenuation significantly, they must be considered as separate regions as shown in Figure 4-5. In this case, the electromagnetic fields in the three regions are given by

\[
\begin{align*}
E_I &= \hat{y} E_1 \cos k_{x_1} x e^{-j k_{z_1} z} & 0 \leq x \leq d & (4.20) \\
E_{II} &= \hat{y} (E_{21} \cos k_{x_1} x + E_{22} \sin k_{x_2} x) e^{-j k_{z_1} z} & d \leq x \leq d + \Delta & (4.21) \\
E_{III} &= \hat{y} E_3 \sin k_{x_1} (a - x) e^{-j k_{z_1} z} & d + \Delta \leq x \leq a & (4.22) \\
H_I &= -\frac{E_1}{\omega \mu_o} \left[ \hat{x} k_{x_1} \sin k_{x_1} x + \hat{z} k_{z_1} \cos k_{x_1} x \right] e^{-j k_{z_1} z} & 0 \leq x \leq d & (4.23) \\
H_{II} &= \left[ \hat{x} k_{x_2} \left( E_{21} \cos k_{x_2} x + E_{22} \sin k_{x_2} x \right) - \hat{z} k_{x_1} \left( E_{21} \cos k_{x_2} x + E_{22} \sin k_{x_2} x \right) \right] e^{-j k_{z_2} z} & d \leq x \leq d + \Delta & (4.24) \\
H_{III} &= -\frac{E_3}{\omega \mu_o} \left[ \hat{x} k_{x_1} \cos k_{x_1} (a - x) + \hat{z} k_{z_1} \sin k_{x_1} (a - x) \right] e^{-j k_{z_1} z} & d + \Delta \leq x \leq a & (4.25)
\end{align*}
\]

The solutions for \( x < 0 \) are similar but are not needed due to the even symmetry. The \( k_z \)'s in the three regions must all be the same due to phase matching at the material interfaces at \( x = \pm d \) and \( x = \pm (d + \Delta) \). The tangential \( E \) and \( H \) must be continuous (Equation 4.9) and therefore the phase \( (e^{-j k_{z_1} z}) \) of the electric field must be the same in all regions. The dispersion relation for regions I and III is

\[
k_{x_1}^2 + k_z^2 = \omega^2 \mu_o \epsilon_1
\]

and in region II is

\[
k_{x_2}^2 + k_z^2 = \omega^2 \mu_o \epsilon_2
\]

where \( \epsilon_1 = \epsilon_{pe} \) and \( \epsilon_2 = \epsilon_w \). These can be combined and solved for \( k_{x_2} \)

\[
k_{x_2} = \sqrt{\omega^2 \mu_o (\epsilon_2 - \epsilon_1) + k_{x_1}^2}
\]

Another equation relating \( k_{x_1} \) to \( k_{x_2} \) is needed and this comes from applying the boundary conditions (Equations 4.9 and 4.10) at \( x = d \) and \( x = d + \Delta \), where \( \Delta \) is the thickness
Figure 4-4: Single dielectric region with water sheets assumed to have negligible thickness ($\Delta \ll a$).

Figure 4-5: Water as a separate region within the dielectric.
of the water sheet. These boundary conditions applied at $x = d$ result in

\[
E_1 \cos kx_1 d = E_{21} \cos kx_2 d + E_{22} \sin kx_2 d \quad (4.29)
\]

\[
E_1 kx_1 \sin kx_1 d = E_{21} kx_2 \sin kx_2 d - E_{22} kx_2 \cos kx_2 d \quad (4.30)
\]

and at $x = d + \Delta$ give

\[
E_3 \sin kx_1 (a - d - \Delta) = E_{21} \cos kx_2 (d + \Delta) + E_{22} \sin kx_2 (d + \Delta) \quad (4.31)
\]

\[
E_3 kx_1 \cos kx_1 (a - d - \Delta) = E_{21} kx_2 \sin kx_2 (d + \Delta) - E_{22} kx_2 \cos kx_2 (d + \Delta) \quad (4.32)
\]

Equations 4.29 and 4.30 can be used to solve for $E_{21}$ and $E_{22}$, resulting in

\[
\begin{bmatrix}
E_{21} \\
E_{22}
\end{bmatrix}
= E_1
\begin{bmatrix}
\cos kx_2 d & \frac{kx_1}{kx_2} \sin kx_2 d \\
\sin kx_2 d & -\frac{kx_1}{kx_2} \cos kx_2 d
\end{bmatrix}
\begin{bmatrix}
\cos kx_1 d \\
\sin kx_1 d
\end{bmatrix}
\quad (4.33)
\]

Likewise, Equations 4.31 and 4.32 give us another set of equations

\[
\begin{bmatrix}
E_{21} \\
E_{22}
\end{bmatrix}
= E_3
\begin{bmatrix}
\cos kx_2 (d + \Delta) & \frac{kx_1}{kx_2} \sin kx_2 (d + \Delta) \\
\sin kx_2 (d + \Delta) & -\frac{kx_1}{kx_2} \cos kx_2 (d + \Delta)
\end{bmatrix}
\begin{bmatrix}
\sin kx_1 (a - d - \Delta) \\
\cos kx_1 (a - d - \Delta)
\end{bmatrix}
\quad (4.34)
\]

Equations 4.33 and 4.34 are set equal to each other, yielding two equations for $E_{21}$ and $E_3$. These are divided, providing the second equation for $kx_1$ in terms of $kx_2$.

\[
f(kx_1, kx_2) = f_1 f_4 - f_2 f_3 = 0 \quad (4.35)
\]

where

\[
f_1 = \cos kx_2 d \cos kx_1 d + \frac{kx_1}{kx_2} \sin kx_2 d \sin kx_1 d \quad (4.36)
\]

\[
f_2 = \sin kx_2 d \cos kx_1 d - \frac{kx_1}{kx_2} \cos kx_2 d \sin kx_1 d \quad (4.37)
\]

\[
f_3 = \cos kx_2 (d + \Delta) \sin kx_1 (a - d - \Delta) + \frac{kx_1}{kx_2} \sin kx_2 (d + \Delta) \cos kx_1 (a - d - \Delta) \quad (4.38)
\]

\[
f_4 = \sin kx_2 (d + \Delta) \sin kx_1 (a - d - \Delta) - \frac{kx_1}{kx_2} \cos kx_2 (d + \Delta) \cos kx_1 (a - d - \Delta) \quad (4.39)
\]
Equations 4.28 and 4.35 are solved for \( k_{x1} \) and \( k_{x2} \) using an iterative Newton's method to solve for \( f(k_{x1}, k_{x2}) = 0 \) and implemented in Matlab programs given in Appendix A.

In addition, once \( k_{x1} \) and \( k_{x2} \) are found, Equation 4.33 can be used to solve for \( E_{21} \) and \( E_{22} \) in terms of \( E_1 \). Either Equation 4.31 or 4.32 can then be used to solve for \( E_3 \) in terms of \( E_1 \). \( E_1 \) is related to the amplitude \( E_0 \) of the original mode by equating the power flowing in the two cases as described in Section 4.2.

The time averaged power intensity flowing in each region is given by

\[
\langle S_x \rangle = \frac{1}{2} \text{Real}\{E_y H_x^*\}
\]  

(4.40)

where \( H_x = \frac{-k_x}{\omega \mu_0} E_y \). In terms of the parameters for each region,

\[
\langle S_{zI} \rangle = \left| E_1 \right|^2 \frac{\text{Real}\{k_x\}}{2\omega \mu_0} e^{-2k_x^2z} \cos k_{x1} x \cos k_{x1}^* x \quad 0 \leq x \leq d \quad (4.41)
\]

\[
\langle S_{zII} \rangle = \left| E_{21} \right|^2 \frac{\text{Real}\{k_x\}}{2\omega \mu_0} e^{-2k_{x2}^2z} \left( \cos k_{x2} x \cos k_{x2}^* x + |E_{22}|^2 \sin k_{x2} x \sin k_{x2}^* x + 2 \text{Real}\{E_{21} E_{22}^* \cos k_{x2} x \sin k_{x2}^* x\} \right) \quad d \leq x \leq d + \Delta \quad (4.42)
\]

\[
\langle S_{zIII} \rangle = \left| E_3 \right|^2 \frac{\text{Real}\{k_x\}}{2\omega \mu_0} e^{-2k_x^2z} \sin k_{x1} (a - x) \sin k_{x1}^* (a - x) \quad d + \Delta \leq x \leq a \quad (4.43)
\]

The total time averaged power flowing is the integral of \( \langle S_x \rangle \) integrated over the cross section. Due to symmetry, the power can be integrated for only \( x > 0 \) and then doubled. In general, the power flow can be written

\[
P_3 = P_{31} + P_{32} + P_{33} \quad (4.44)
\]

\[
P_3 = P_A(E_1, k_{x1}, d) + P_B(E_{21}, E_{22}, k_{x2}, d, d + \Delta) + P_C(E_3, k_{x1}, d + \Delta) \quad (4.45)
\]

where \( P_{31}, P_{32}, \) and \( P_{33} \) are the time averaged powers flowing respectively in regions I, II, and III and "3" designates the three region geometry. The calculation of \( P_A, P_B, \) and \( P_C \) is described in Section 4.5. The form of the integral used to solve for \( P_A, P_B, \) and \( P_C \) depends on whether the \( k_x \) is real, imaginary, or complex.

To find the amplitudes of \( E_1, E_{21}, E_{22}, \) and \( E_3 \) in terms of \( E_0 \), Equations 4.19, 4.31, and 4.33 are used in conjunction with \( P_1 = P_3 = P_{31} + P_{32} + P_{33} \). Note that the \( k_x \)'s and \( k_z \)'s are different between the two modes, even though similar notation is used. An example plot showing the effects of the addition of the water sheets is given in Figure 4-6. Note that
since the water has a higher dielectric constant than the XLPE, the $k_{x2}$ in the water sheet is larger than $k_{x1}$ and the field amplitude rises slightly. Given the hypothetical case where $\varepsilon_w < \varepsilon_{pe}$, the field amplitude would be lower at the location of the water sheet.

![Figure 4-6: Comparison of electric field mode shapes when water sheets are added to the single region geometry for a waveguide with $a = 11.4$ mm at 8 GHz. The water sheets are 20$\mu$m thick and located at $d = \pm 5$ mm.](image)

One might expect that the area under the two curves in Figure 4-6 would be the same since the power flowing in the two cases is the same. Indeed, the power does depend on the square of the magnitude of the electric field, but it also depends on the longitudinal wavenumber $\text{Real}\{k_z\}$. The fields in the very thin water sheet do not contribute significantly to the power integral. However, the addition of the water sheets does cause the $k_z$ to change. Notice how the electric field curve flattens out near $x = 0$ in the case with the water sheets resulting in a lower transverse wavenumber ($k_z$) in the XLPE regions. From the dispersion relation $k_z^2 + \kappa_z^2 = \omega\mu_0\varepsilon$, the $k_z$ must therefore be larger in the case with the water sheets. For the plot shown in Figure 4-6, the real part of the $k_z$ changes from $211(\frac{1}{m})$ to $221(\frac{1}{m})$, an increase of 4.7%. Since the power flowing remains the same, the electric field magnitude decreases when $k_z$ increases.
4.4 Evanescent Mode Configurations

4.4.1 Two Dielectrics

With the sapphire dielectric positioned inside the waveguide as shown in Figure 4-7, the electric field in the surrounding XLPE can be sinusoidal or evanescent (exponentially decaying) depending on the frequency and the thickness of the sapphire dielectric. For the frequencies and sapphire thicknesses used here, the mode will be primarily evanescent and not sinusoidal. In addition, the shape of the evanescent curve in the XLPE changes with frequency, decaying faster away from the sapphire at higher frequencies.

\[ E_I = \hat{y} E_{10} \cos k_{x1}x \, e^{-j k_z z} \quad 0 \leq x \leq b \]  

\[ E_{II} = \hat{y} E_{20} \sin k_{x2}(a - x) \, e^{-j k_z z} \quad b \leq x \leq a \]  

\[ H_I = -\frac{E_{10}}{\omega \mu_0} [k_jk_{x1} \sin k_{x1}x + \hat{k}k_z \cos k_{x1}x] \, e^{-j k_z z} \quad 0 \leq x \leq b \]  

Figure 4-7: Rectangular waveguide with two dielectric regions in evanescent mode configuration.

Since the source excites the waveguide with a symmetric vertical electric field \((E_y)\), the fields in the dielectric regions will also be symmetric and in the \(\hat{y}\) direction. This electric field must be zero at \(x = \pm a\), since the walls are metallic and there can be no tangential electric fields adjacent to perfect conductors. Solutions to Equations 4.1-4.4 and 4.7-4.8 for the two region geometry which match these boundary conditions are
\[ \mathbf{H}_{II} = -\frac{E_{20}}{\omega \mu_0} [\hat{x}_0 k_{x2} \cos k_{x2}(a-x) + \hat{y}_0 k_x \sin k_{x2}(a-x)] e^{-jk_{x2}z} \quad b \leq x \leq a \quad (4.49) \]

The solutions for \( x < 0 \) are similar but are not needed due to symmetry. Note that these are the most general solutions that incorporate both sinusoidal and evanescent modes. The sinusoidal and hyperbolic (exponential) functions are related by \( \sin j\alpha = j \sinh \alpha \) and \( \cos j\alpha = \cosh \alpha \). A mode that is evanescent in region II would have a \( k_{x2} \) which is imaginary.

To solve for the unknown \( k_{x1}, k_{x2} \), and the field amplitudes, the boundary conditions are applied at \( x = \pm b \). Tangential \( \mathbf{E} (E_y) \) and \( \mathbf{H} (H_z) \) continuous at \( x = \pm b \) give

\[ E_{10} \cos k_{x1} b = E_{20} \sin k_{x2}(a-b) \quad (4.50) \]
\[ k_{x1} E_{10} \sin k_{x1} b = k_{x2} E_{20} \cos k_{x2}(a-b) \quad (4.51) \]

Dividing these two equations results in

\[ f(k_{x1}, k_{x2}) = 0 = k_{x1} \tan k_{x1} b - k_{x2} \cot k_{x2}(a-b) \quad (4.52) \]

Equation 4.52 is one equation relating \( k_{x1} \) to \( k_{x2} \). The other comes from the dispersion relations which are the same as Equations 4.26-4.28 with \( \epsilon_1 = \epsilon_s \) of the sapphire and \( \epsilon_2 = \epsilon_{pe} \) of the polyethylene. These equations are then solved for \( f(k_{x1}, k_{x2}) = 0 \) using an iterative Newton's method implemented in Matlab programs given in Appendix B. Once the \( k_x \)'s are found, \( E_{20} \) can be solved for in terms of \( E_{10} \).

The power flowing in the waveguide must also be found so that it can be compared with the power flowing once water sheets are added. The time averaged power intensities are

\[ \langle S_{zz} \rangle = |E_{10}|^2 \frac{\text{Re} \{k_z\}}{2\omega \mu_0} e^{-2k_z^2z} \cos k_{x1} x \cos k_{x1} x \quad 0 \leq x \leq b \quad (4.53) \]
\[ \langle S_{zz} \rangle = |E_{20}|^2 \frac{\text{Re} \{k_z\}}{2\omega \mu_0} e^{-2k_z^2z} \sin k_{x2}(a-x) \sin k_{x2}^*(a-x) \quad b \leq x \leq a \quad (4.54) \]

The total time averaged power flowing is the integral of \( \langle S_z \rangle \) integrated over the cross section. Due to symmetry, the power can be integrated for only \( x > 0 \) and then doubled. The total power is given as

\[ P_2 = P_{21} + P_{22} \quad (4.55) \]
\[ = P_A(E_1, k_{x1}, b) + P_C(E_2, k_{x2}, b) \quad (4.56) \]
where $P_{21}$ and $P_{22}$ are the time averaged powers flowing in regions I and II. The calculation of $P_A$ and $P_C$ is described in Section 4.5.

### 4.4.2 Effects of a Water Sheet

Two approaches can be used to solve for the mode shape in the case where a symmetric pair of water sheets is placed inside the dielectric loaded waveguide, as shown in Figure 4-8. The first is a perturbation approach which assumes that the electromagnetic field mode and amplitude are not affected by the addition of the water sheets. The attenuation due to the addition of the water sheets in this case is calculated in Section 5.3 and the field solutions are those given in Equations 4.46-4.49.

If the water sheets are thick enough to affect the mode shape or attenuation significantly, they must be considered as separate regions as shown in Figure 4-9. In this case, the electromagnetic fields in the four regions are given by

\begin{align*}
E_I &= \hat{y} E_1 \cos k_{21} x \ e^{-jk_z z} & 0 \leq x \leq b \\
E_{II} &= \hat{y} (E_{21} \cos k_{22} x + E_{22} \sin k_{22} x) \ e^{-jk_z z} & b \leq x \leq d \\
E_{III} &= \hat{y} (E_{31} \cos k_{33} x + E_{32} \sin k_{33} x) \ e^{-jk_z z} & d \leq x \leq d + \Delta \\
E_{IV} &= \hat{y} E_3 \sin k_{32} (a - x) \ e^{-jk_z z} & d + \Delta \leq x \leq a
\end{align*}

Figure 4-8: Two dielectric regions with water sheets assumed to have negligible thickness ($\Delta \ll a$).
Figure 4-9: Water as a separate region in the evanescent mode configuration.

\[
\begin{align*}
H_I &= -\frac{E_1}{\omega \mu_o} \left[ \hat{\alpha} j k_{x1} \sin k_{x1} x + \hat{\xi} k_z \cos k_{x1} x \right] e^{-j k_z z} \quad 0 \leq x \leq b \quad (4.61) \\
H_{II} &= \left[ \hat{\alpha} j k_{x2} \frac{k_z}{\omega \mu_o} \left( -E_{21} \sin k_{x2} x + E_{22} \cos k_{x2} x \right) - \hat{\xi} k_z \left( E_{21} \cos k_{x2} x + E_{22} \sin k_{x2} x \right) \right] e^{-j k_z z} \quad b \leq x \leq d \quad (4.62) \\
H_{III} &= \left[ \hat{\alpha} j k_{x3} \frac{k_z}{\omega \mu_o} \left( -E_{31} \sin k_{x3} x + E_{32} \cos k_{x3} x \right) - \hat{\xi} k_z \left( E_{31} \cos k_{x3} x + E_{32} \sin k_{x3} x \right) \right] e^{-j k_z z} \quad d \leq x \leq d + \Delta \quad (4.63) \\
H_{IV} &= -\frac{E_4}{\omega \mu_o} \left[ \hat{\alpha} j k_{x2} \cos k_{x2} (a - x) + \hat{\xi} k_z \sin k_{x2} (a - x) \right] e^{-j k_z z} \quad d + \Delta \leq x \leq a \quad (4.64)
\end{align*}
\]

As before, the solutions for \( x < 0 \) are similar but are not needed due to symmetry. Also, these are the most general solutions that incorporate both sinusoidal and evanescent modes.

The sinusoidal and hyperbolic (exponential) functions are related by \( \sin j \alpha = j \sinh \alpha \) and \( \cos j \alpha = \cosh \alpha \). A mode that is evanescent in regions II, III, and IV would have a \( k_{x2} \) and \( k_{x3} \) which are imaginary.

The dispersion relations in the three different materials are

\[
\begin{align*}
k_{x1}^2 + k_z^2 &= \omega^2 \mu_o \epsilon_1 \quad (4.65) \\
k_{x2}^2 + k_z^2 &= \omega^2 \mu_o \epsilon_2 \quad (4.66) \\
k_{x3}^2 + k_z^2 &= \omega^2 \mu_o \epsilon_3 \quad (4.67)
\end{align*}
\]
where \( \epsilon_1 = \epsilon_s \) and \( \epsilon_2 = \epsilon_{pe} \), and \( \epsilon_3 = \epsilon_w \). Subtracting, these result in two equations for \( k_{x2} \) and \( k_{x3} \) in terms of \( k_{x1} \)

\[
\begin{align*}
k_{x2} &= \sqrt{\omega^2 \mu_0 (\epsilon_2 - \epsilon_1) + k_{x1}^2} \\
k_{x3} &= \sqrt{\omega^2 \mu_0 (\epsilon_3 - \epsilon_1) + k_{x1}^2}
\end{align*}
\] (4.68)  (4.69)

Another equation relating \( k_{x2} \) and \( k_{x3} \) to \( k_{x1} \) is needed and this comes from applying the boundary conditions (Equations 4.9 and 4.10) at \( x = b, x = d \) and \( x = d + \Delta \), where \( \Delta \) is the thickness of the water sheet. These boundary conditions applied at \( x = b \) result in

\[
\begin{align*}
E_1 \cos k_{x1} b &= E_{21} \cos k_{x2} b + E_{22} \sin k_{x2} b \\
E_1 k_{x1} \sin k_{x1} b &= E_{21} k_{x2} \sin k_{x2} b - E_{22} k_{x2} \cos k_{x2} b
\end{align*}
\] (4.70)  (4.71)

and at \( x = d \)

\[
\begin{align*}
E_{21} \cos k_{x2} d + E_{22} \sin k_{x2} d &= E_{31} \cos k_{x3} d + E_{32} \sin k_{x3} d \\
E_{21} k_{x2} \sin k_{x2} d - E_{22} k_{x2} \cos k_{x2} d &= E_{31} k_{x3} \sin k_{x3} d - E_{32} k_{x3} \cos k_{x3} d
\end{align*}
\] (4.72)  (4.73)

and at \( x = d + \Delta \) give

\[
\begin{align*}
E_4 \sin k_{x2} (a - d - \Delta) &= E_{31} \cos k_{x3} (d + \Delta) + E_{32} \sin k_{x3} (d + \Delta) \\
E_4 k_{x2} \cos k_{x2} (a - d - \Delta) &= E_{31} k_{x3} \sin k_{x3} (d + \Delta) - E_{32} k_{x3} \cos k_{x3} (d + \Delta)
\end{align*}
\] (4.74)  (4.75)

Equations 4.70 and 4.71 can be used to solve for \( E_{21} \) and \( E_{22} \) in terms of \( E_1 \), resulting in

\[
\begin{bmatrix}
E_{21} \\
E_{22}
\end{bmatrix} = E_1 \begin{bmatrix}
\cos k_{x2} b & \frac{k_{x1}}{k_{x2}} \sin k_{x2} b \\
\sin k_{x2} b & -\frac{k_{x1}}{k_{x2}} \cos k_{x2} b
\end{bmatrix} \begin{bmatrix}
\cos k_{x1} b \\
\sin k_{x1} b
\end{bmatrix}
\] (4.76)

Likewise, Equations 4.74 and 4.75 give us another set of equations

\[
\begin{bmatrix}
E_{31} \\
E_{32}
\end{bmatrix} = E_4 \begin{bmatrix}
\cos k_{x3} (d + \Delta) & \frac{k_{x2}}{k_{x3}} \sin k_{x3} (d + \Delta) \\
\sin k_{x3} (d + \Delta) & -\frac{k_{x2}}{k_{x3}} \cos k_{x3} (d + \Delta)
\end{bmatrix} \begin{bmatrix}
\sin k_{x2} (a - d - \Delta) \\
\cos k_{x2} (a - d - \Delta)
\end{bmatrix}
\] (4.77)
Equations 4.72 and 4.73 give us a third set of equations

\[
\begin{bmatrix}
E_{31} \\
E_{32}
\end{bmatrix} =
\begin{bmatrix}
\cos k_{x3}d & \frac{k_{x2}}{k_{x3}} \sin k_{x3}d \\
\sin k_{x3}d & -\frac{k_{x2}}{k_{x3}} \cos k_{x3}d
\end{bmatrix}
\begin{bmatrix}
\cos k_{x2}d & \sin k_{x2}d \\
\sin k_{x2}d & -\cos k_{x2}d
\end{bmatrix}
\begin{bmatrix}
E_{21} \\
E_{22}
\end{bmatrix}
\]  
(4.78)

Equations 4.76-4.78 are set equal to each other, yielding two equations for \(E_1\) and \(E_4\). These are divided, providing the third equation relating \(k_{x1}\) to \(k_{x2}\) and \(k_{x3}\).

\[
f(k_{x1}, k_{x2}, k_{x3}) = f_1 f_4 - f_2 f_3 = 0 \tag{4.79}
\]

where

\[
f_1 = \cos k_{x3}d \left( \cos k_{x2}d \left( \cos k_{x2}b \cos k_{x1}b + \frac{k_{x1}}{k_{x2}} \sin k_{x2}b \sin k_{x1}b \right) + \sin k_{x2}d \left( \sin k_{x2}b \cos k_{x1}b - \frac{k_{x1}}{k_{x2}} \cos k_{x2}b \sin k_{x1}b \right) \right) + \frac{k_{x2}}{k_{x3}} \sin k_{x3}d \left( \sin k_{x2}d \left( \cos k_{x2}b \cos k_{x1}b + \frac{k_{x1}}{k_{x2}} \sin k_{x2}b \sin k_{x1}b \right) + \cos k_{x2}d \left( \sin k_{x2}b \cos k_{x1}b - \frac{k_{x1}}{k_{x2}} \cos k_{x2}b \sin k_{x1}b \right) \right) \tag{4.80}
\]

\[
f_2 = \sin k_{x3}d \left( \cos k_{x2}d \left( \cos k_{x2}b \cos k_{x1}b + \frac{k_{x1}}{k_{x2}} \sin k_{x2}b \sin k_{x1}b \right) + \sin k_{x2}d \left( \sin k_{x2}b \cos k_{x1}b - \frac{k_{x1}}{k_{x2}} \cos k_{x2}b \sin k_{x1}b \right) \right) - \frac{k_{x2}}{k_{x3}} \cos k_{x3}d \left( \sin k_{x2}d \left( \cos k_{x2}b \cos k_{x1}b + \frac{k_{x1}}{k_{x2}} \sin k_{x2}b \sin k_{x1}b \right) + \cos k_{x2}d \left( \sin k_{x2}b \cos k_{x1}b - \frac{k_{x1}}{k_{x2}} \cos k_{x2}b \sin k_{x1}b \right) \right) \tag{4.81}
\]

\[
f_3 = \cos k_{x3}(d + \Delta) \sin k_{x2}(a - d - \Delta) + \frac{k_{x2}}{k_{x3}} \sin k_{x3}(d + \Delta) \cos k_{x2}(a - d - \Delta) \tag{4.82}
\]

\[
f_4 = \sin k_{x3}(d + \Delta) \sin k_{x2}(a - d - \Delta) - \frac{k_{x2}}{k_{x3}} \cos k_{x3}(d + \Delta) \cos k_{x2}(a - d - \Delta) \tag{4.83}
\]

Equations 4.68-4.69 and 4.79 are solved for \(k_{x1}\), \(k_{x2}\), and \(k_{x3}\) using an iterative Newton’s method to solve for \(f(k_{x1}, k_{x2}, k_{x3}) = 0\) and implemented in Matlab programs given in Appendix B. Once the \(k_{x}\)'s are found, the field amplitudes can be related to \(E_1\) using the boundary conditions. \(E_1\) is related to the amplitudes \(E_{10}\) and \(E_{20}\) of the original mode by equating the power flowing in the two cases.
The time averaged power intensity flowing in each region is given by

\[
\langle S_{zi} \rangle = |E_z|^2 \frac{\text{Real}\{k_z\}}{2\omega \mu_o} e^{-2k_z^2 z} \cos k_{x1} x \cos k_{x1} x \cos k_{x2} x \cos k_{x2} x \quad 0 \leq x \leq b \quad (4.84)
\]

\[
\langle S_{zII} \rangle = \frac{\text{Real}\{k_z\}}{2\omega \mu_o} e^{-2k_z^2 z} \left( |E_{21}|^2 \cos k_{x2} x \cos k_{x2} x + |E_{22}|^2 \sin k_{x2} x \sin k_{x2} x + 2 \text{Real}\{E_{21} E_{22}^{*} \cos k_{x2} x \sin k_{x2} x \} \right) \quad b \leq x \leq d \quad (4.85)
\]

\[
\langle S_{zIII} \rangle = \frac{\text{Real}\{k_z\}}{2\omega \mu_o} e^{-2k_z^2 z} \left( |E_{31}|^2 \cos k_{x3} x \cos k_{x3} x + |E_{32}|^2 \sin k_{x3} x \sin k_{x3} x + 2 \text{Real}\{E_{31} E_{32}^{*} \cos k_{x3} x \sin k_{x3} x \} \right) \quad d \leq x \leq d + \Delta \quad (4.86)
\]

\[
\langle S_{zIV} \rangle = |E_4|^2 \frac{\text{Real}\{k_z\}}{2\omega \mu_o} e^{-2k_z^2 z} \sin k_{x2} (a - x) \sin k_{x2} (a - x) \quad d + \Delta \leq x \leq a \quad (4.87)
\]

The total time averaged power flowing is the integral of \( \langle S_z \rangle \) integrated over the cross section. Due to symmetry, the power can be integrated for only \( x > 0 \) and then doubled. In general, the power flow can be written

\[
P_4 = P_{41} + P_{42} + P_{43} + P_{44} \quad (4.88)
\]

\[
P_4 = P_A(E_1, k_{x1}, b) + P_B(E_{21}, E_{22}, k_{x2}, b, d) + P_B(E_{31}, E_{32}, k_{x3}, d, d + \Delta) + P_C(E_4, k_{x2}, d + \Delta) \quad (4.89)
\]

where \( P_{41}, P_{42}, P_{43}, \) and \( P_{44} \) are the time averaged powers flowing respectively in regions I, II, III, and IV. The calculation of \( P_A, P_B, \) and \( P_C \) is described in Section 4.5.

To find the amplitudes of \( E_1, E_{21}, E_{22}, E_{31}, E_{32}, \) and \( E_4 \) in terms of \( E_{10} \) and \( E_{20} \), Equations 4.55, and 4.76-4.78 are used in conjunction with \( P_2 = P_4 = P_{41} + P_{42} + P_{43} + P_{44} \). Note that the \( k_x \)'s and \( k_z \)'s are different between the two modes, even though similar notation is used. An example plot showing the effects of the addition of the water sheets is given in Figure 4-10. Note that since the water has a higher dielectric constant than the XLPE, the \( k_{x3} \) in the water sheet is larger than \( k_{x2} \) and the field amplitude rises slightly just as it did in the three region case (Figure 4-6). Given the hypothetical case where \( \epsilon_w < \epsilon_{pe} \), the field amplitude would be lower at the location of the water sheet.

### 4.5 Power Flow Calculations

The time averaged power intensity is integrated over the cross section to find the total power flowing in the waveguide. The three types of expressions for time average power intensity
Figure 4-10: Comparison of electric field mode shapes when water sheets are added to the two region geometry. The sapphire is 2 mm thick \( (b = 1 \text{ mm}) \), the waveguide has width \( a = 11.4 \text{ mm} \), and the frequency is 8 GHz. The water sheets are 50\(\mu\text{m} \) thick, located at \( d = \pm 5 \text{ mm} \).

are

\[
\langle S_{zA} \rangle = |A|^2 \frac{\text{Real}\{k_z\}}{2\omega \mu_0} e^{-2k_z'^2} \cos k_z x \cos k_z^* x
\]

\[
\langle S_{zB} \rangle = \frac{\text{Real}\{k_z\}}{2\omega \mu_0} e^{-2k_z'^2} \left( |B_1|^2 \cos k_z x \cos k_z^* x + |B_2|^2 \sin k_z x \sin k_z^* x + 2 \text{Real}\{B_1 B_2^* \cos k_z x \sin k_z^* x\} \right)
\]

\[
\langle S_{zC} \rangle = |C|^2 \frac{\text{Real}\{k_z\}}{2\omega \mu_0} e^{-2k_z'^2} \sin k_z (a - x) \sin k_z^* (a - x)
\]

The time averaged power flowing in the three types of regions characterized by Equations 4.90-4.92 can be found by integrating over the cross section of each region. Due to symmetry, the power can be integrated for only \( x > 0 \) and then doubled. The power for each region can be characterized by the various amplitudes, \( k_z \)'s and limits of integration. Particularly,

\[
P_A = P_A(A, k_z, x_1)
\]

\[
P_B = P_B(B_1, B_2, k_z, x_1, x_2)
\]

\[
P_C = P_C(C, k_z, x_2)
\]

73
The form of the integrals used to solve for $P_A$, $P_B$, and $P_C$ depends on whether the $k_x$ is real, imaginary, or complex.

For the case where $k_x$ is real, these powers are calculated to be

$$P_A = |A|^2 \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \int_0^{x_1} \cos k_x x \cos k_x^* x \, dx$$

$$P_A = |A|^2 \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \left[ \frac{x_1}{2} + \frac{1}{4 k_x^2} \sin 2 k_x x \right] \tag{4.96}$$

$$P_B = \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \int_{x_1}^{x_2} |B_1|^2 \cos k_x x \cos k_x^* x + |B_2|^2 \sin k_x x \sin k_x^* x +$$

$$2 \text{Re} \left\{ B_1 B_2^* \cos k_x x \sin k_x^* x \right\} \, dx$$

$$P_B = \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \left[ |B_1|^2 \left( \frac{x_2 - x_1}{2} + \frac{1}{4 k_x^2} \left( \sin 2 k_x x_2 - \sin 2 k_x x_1 \right) \right) +
\frac{|B_2|^2}{4 k_x^2} \left( \sin 2 k_x x_2 - \sin 2 k_x x_1 \right) \right] +$$

$$2 \text{Re} \left\{ B_1 B_2^* \frac{1}{2 k_x} \left( \sin^2 k_x x_2 - \sin^2 k_x x_1 \right) \right\} \tag{4.97}$$

$$P_C = |C|^2 \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \int_{x_2}^a \sin k_x (a - x) \sin k_x^* (a - x) \, dx$$

$$P_C = |C|^2 \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \left[ \frac{a - x_2}{2} - \frac{1}{4 k_x^2} \sin 2 k_x (a - x_2) \right] \tag{4.98}$$

If $k_x$ is entirely imaginary, the integrals are calculated similarly to Equations 4.96-4.98 and the results differ by only sign changes.

$$P_A = |A|^2 \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \left[ \frac{x_1}{2} + \frac{1}{4 k_x^2} \sin 2 k_x x \right] \tag{4.99}$$

$$P_B = \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \left[ |B_1|^2 \left( \frac{x_2 - x_1}{2} + \frac{1}{4 k_x^2} \left( \sin 2 k_x x_2 - \sin 2 k_x x_1 \right) \right) +
\frac{|B_2|^2}{4 k_x^2} \left( \sin 2 k_x x_2 - \sin 2 k_x x_1 \right) \right] -$$

$$2 \text{Re} \left\{ B_1 B_2^* \frac{1}{2 k_x} \left( \sin^2 k_x x_2 - \sin^2 k_x x_1 \right) \right\} \tag{4.100}$$

$$P_C = |C|^2 \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \left[ \frac{1}{4 k_x^2} \sin 2 k_x (a - x_2) - \frac{a - x_2}{2} \right] \tag{4.101}$$

Note that $\sin \alpha = j \sinh \alpha$, $\cos \alpha = \cosh \alpha$, and that $\frac{\sin \alpha}{\alpha}$ is real.

When $k_x$ is complex with both real and imaginary terms, the integrals become

$$P_A = |A|^2 \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \left[ \frac{\sin \left( k_x - k_x^* \right) x_1}{2 \left( k_x - k_x^* \right)} + \frac{\sin \left( k_x + k_x^* \right) x_1}{2 \left( k_x + k_x^* \right)} \right] \tag{4.102}$$

$$P_B = \frac{\text{Re}(k_x)}{\omega \mu_0} e^{-2 k_y^2} \left[ |B_1|^2 \left( \frac{\sin \left( k_x - k_x^* \right) x_2}{2 \left( k_x - k_x^* \right)} + \frac{\sin \left( k_x + k_x^* \right) x_2}{2 \left( k_x + k_x^* \right)} \right) - \right.$$

$$\left[ \frac{\sin \left( k_x - k_x^* \right) x_1}{2 \left( k_x - k_x^* \right)} + \frac{\sin \left( k_x + k_x^* \right) x_1}{2 \left( k_x + k_x^* \right)} \right]$$
\[
\frac{\sin(k_x - k_x^*) x_1}{2(k_x - k_x^*)} - \frac{\sin(k_x + k_x^*) x_1}{2(k_x + k_x^*)} + \\
2(k_x - k_x^*) \left( \frac{\sin(k_x - k_x^*) x_2}{2(k_x - k_x^*)} - \frac{\sin(k_x + k_x^*) x_2}{2(k_x + k_x^*)} - \\
\frac{\sin(k_x - k_x^*) x_1}{2(k_x - k_x^*)} + \frac{\sin(k_x + k_x^*) x_1}{2(k_x + k_x^*)} \right) + \\
2 \text{Re} \left\{ B_1 B_2^* \left( \frac{\cos(k_x - k_x^*) x_2}{2(k_x - k_x^*)} - \frac{\cos(k_x + k_x^*) x_2}{2(k_x + k_x^*)} - \\
\frac{\cos(k_x - k_x^*) x_1}{2(k_x - k_x^*)} + \frac{\cos(k_x + k_x^*) x_1}{2(k_x + k_x^*)} \right) \right\} \right) \] (4.103)

\[
P_C = |C|^2 \frac{\text{Re}\{k_z\}}{\omega \mu_0} e^{-2 k_y' z} \left[ \frac{\sin(k_x - k_x^*)(a - x_2)}{2(k_x - k_x^*)} - \frac{\sin(k_x + k_x^*)(a - x_2)}{2(k_x + k_x^*)} \right] \] (4.104)
Chapter 5

Loss Theory

The addition of symmetric pairs of water sheets into the waveguide to simulate treed layers within the XLPE results in attenuation of the power flowing in the waveguide. These water sheets consist of two sheets of 1.6 mil polyethylene sealed together with a hot knife and filled with small amounts of water which are then placed at known positions in between sections of XLPE. The power dissipated in these water sheets can be calculated and used to find the change in attenuation due to the water sheets.

As described in Section 3.3.3, the microwave measurement system can only measure the difference in attenuation between measurements such as the “wet” and “dry” configurations. The absolute attenuation of a single measurement is meaningless as it is very difficult to model the reflections at all connectors and material interfaces within the waveguide experiment. It is assumed that these reflections remain the same for the measurements with and without the water sheets, and the effect of the reflections is canceled through subtraction of the two measurements. This chapter describes the various methods of calculating the attenuation due to lossy media inside the waveguide and the change in attenuation when the water sheets are added.

5.1 Loss Determined from Dispersion Relations

One method to calculate the attenuation inside the waveguide uses the calculated dispersion relations (the $k$'s) for the waveguide modes. Attenuation of the electromagnetic fields inside the waveguide can be caused by either operation below “cut-off” or by lossy materials placed inside the waveguide. As a simple case study, consider the single dielectric region with
dielectric constant $\varepsilon$ and electric field mode shape shown in Figure 4-3. The field solutions and time averaged power flowing are given in Equations 4.17-4.19 and are repeated here

\begin{align}
\mathbf{E} &= \hat{y} E_0 \cos k_z x \ e^{-jk_z'z} \ e^{-jk_z''z} \\
\mathbf{H} &= -\frac{E_0}{\omega \mu_0} \left[ \hat{x} j k_z \sin k_z x + \hat{z} k_z \cos k_z x \right] \ e^{-jk_z'z} \ e^{-jk_z''z} \\
P_1 &= \int_0^h \int_{-a}^a \frac{1}{2} \text{Real} \{ \mathbf{E} \times \mathbf{H}^* \} \cdot \hat{z} \ dz \ dy = \frac{E_0^2}{2\omega \mu_0} \text{Real} \{ k_z \} ah \ e^{-2k_z''z}
\end{align}

The dispersion relation for the single region dielectric is $k^2 = k_z^2 + k_z''^2 = \omega^2 \mu_0 \varepsilon$, where $k_z = \frac{\pi}{2a}$ is fixed by the boundary conditions.

Assuming the dielectric constant $\varepsilon$ is real, the cut-off frequency for this mode is given by $\omega_c = \frac{\pi}{2a} \sqrt{\frac{1}{\mu_0 \varepsilon}}$. If a signal is applied at a frequency lower than $\omega_c$, the longitudinal wavenumber $k_z$ will be imaginary. The field quantities will decay and there will be no real power flow, as $k_z$ will have no real part. Geometries and frequencies below cut-off where $k_z$ is entirely imaginary are not considered here, as this imaginary $k_z$ is not due to lossy media and the waveguide mode is not propagating.

When the waveguide is operated above cut-off and the dielectric constant $\varepsilon$ is complex, $k_z$ is also complex, with $k_z = k_z' - jk_z''$. The fields now propagate with wavenumber $k_z'$ and decay as $e^{-k_z''z}$. The attenuation of the time averaged power at a coordinate $z$ with respect to the power at $z = 0$ is given by

\begin{align}
\text{Attenuation} &= 10 \log_{10} P(z = 0) - 10 \log_{10} P(z) \\
&= -10 \log_{10} e^{-2k_z''z} \\
&= k_z'' z 20 \log_{10} e
\end{align}

The power attenuation per unit length is then

\begin{align}
\text{Attenuation/meter} &= k_z'' 20 \log_{10} e \\
&\approx 8.69 k_z''
\end{align}

Now consider any of the pairs of configurations of dielectrics with and without symmetric water sheets discussed in Chapter 4. The wavenumbers ($k_z$'s and $k_z'$'s) can be calculated for each pair of configurations. Assume that the $k_z$ for the geometry with the water sheets
is $k_z^w$ and the $k_z$ for the "dry" or geometry without the water sheets is $k_z^d$. The change in attenuation per unit length due to the addition of the water sheets is then given by

$$\Delta \text{Attenuation/meter} = (k_z^{\text{w''}} - k_z^{\text{d''}}) \cdot 20 \log_{10} e \quad (5.6)$$

where $k_z'' = -\text{Im}\{k_z\}$.

### 5.2 Exact Field Solutions

Another method to compute the change in attenuation uses the ratio of power dissipated per unit length to the power flowing in the waveguide. This method is more complicated than the method of Section 5.1 because of the integrations required, but should yield the same results as Equation 5.6 and is used to compute the change in attenuation due to moisture intruding into the XLPE region in Section 7.5.

When lossy materials are added to the single region waveguide, the expression for the time averaged power flowing is

$$P(z) = \frac{|E_o|^2}{2\omega \mu_0} k_z' ah e^{-2k_z''z} = P_0 e^{-\alpha z} \quad (5.7)$$

where $\alpha = 2k_z''$. The power dissipated per unit length can be written

$$P_d = -\frac{dP}{dz} = \alpha P(z) \quad (5.8)$$

The negative derivative is used because as the power is decreasing, positive power loss occurs. The power attenuation coefficient and the attenuation per unit length are then

$$\alpha = \frac{P_d}{P(z)} \quad (5.9)$$

$$\text{Attenuation/meter} = \frac{P_d}{P(z)} \cdot 10 \log_{10} e \quad (5.10)$$

As before, consider pairs of configurations with and without symmetric water sheets. The time averaged power flowing is assumed the same for both cases (Section 4.2). The $w$ and
$d$ superscripts refer to the “wet” and “dry” cases. The change in attenuation is then

$$\Delta \text{Attenuation/meter} = \left( \frac{P_d^w}{P(z)} - \frac{P_d^d}{P(z)} \right) 10 \log_{10} e \quad (5.11)$$

Once the power dissipation per unit length (calculated below) and the total power flowing in the waveguide are found for each pair of geometries, they can be used with Equation 5.11 to find the change in attenuation.

The total time averaged power flowing for the four geometries considered in Chapter 4 are given in Equations 4.19, 4.44, 4.55, and 4.88, with the integrals calculated in Section 4.5. The total time averaged power dissipated per unit length is the time averaged power dissipation density integrated over the waveguide cross section. The time averaged power dissipation density associated with lossy dielectrics and dielectric heating is given by ([20, p 483])

$$\langle p_d \rangle = \langle E \cdot \frac{\partial D}{\partial t} \rangle \frac{[\text{Watts}]}{m^3} \quad (5.12)$$

$$= \frac{1}{2} \text{Real} \{ j\omega D \cdot E^* \} \quad (5.13)$$

For linear dielectrics the electric flux density $D$ is given by

$$D = \epsilon E = (\epsilon' - j\epsilon'')E \quad (5.14)$$

Therefore,

$$\langle p_d \rangle = \frac{1}{2} \omega \epsilon'' E \cdot E^* \quad (5.15)$$

For the single region dielectric, using Equation 5.1, the power dissipation density is given by

$$\langle p_{d1} \rangle = \frac{1}{2} \omega \epsilon'' e^{-2k_x z} |E_o|^2 \cos^2 k_x x \quad (5.16)$$

where $k_x = \frac{\pi}{2a}$ and the total time averaged power dissipation per meter is

$$P_{d1} = \int_0^h \int_{-a}^a \langle p_{d1} \rangle \, dx \, dy$$

$$= \frac{\omega \epsilon''}{2} |E_o|^2 ah e^{-2k_x z} \quad (5.17)$$

For the three region configuration (single region with water sheets), using Equations 4.20-
4.22 the power dissipation density in each region is given by

\[
\langle p_{d1} \rangle = |E_{10}|^2 \frac{1}{2} \omega \varepsilon'' \epsilon_s e^{-2k''_{1z}x} \cos k_{x1}x \cos k_{z1}^*x \quad 0 \leq x \leq b
\]  
\[
\langle p_{dII} \rangle = \frac{1}{2} \omega \varepsilon'' \epsilon_s e^{-2k''_{1z}x} \left( |E_{12}|^2 \cos k_{x2}x \cos k_{z2}^*x + |E_{22}|^2 \sin k_{x2}x \sin k_{z2}^*x + 2 \text{Re} \{E_{12}E_{22}^* \cos k_{x2}x \sin k_{z2}^*x \} \right) \quad d \leq x \leq d + \Delta
\]  
\[
\langle p_{dIII} \rangle = |E_{30}|^2 \frac{1}{2} \omega \varepsilon'' \epsilon_s e^{-2k''_{1z}x} \sin k_{x1}(a-x) \sin k_{z1}^*(a-x) \quad d + \Delta \leq x \leq a
\]

The total time averaged power dissipation per unit length is the integral of \( \langle p_d \rangle \) over the waveguide cross section. In general, the power dissipation per unit length can be written

\[
P_{d3} = P_{d31} + P_{d32} + P_{d33}
\]

\[
P_{d3} = P_{dA}(E_1, k_{x1}, d, \varepsilon''_{pe}) + P_{dB}(E_{12}, k_{x2}, d, d + \Delta, \varepsilon''_{pe}) + P_{dC}(E_3, k_{x1}, d + \Delta, \varepsilon''_{pe})
\]

where \( P_{d31}, P_{d32}, \) and \( P_{d33} \) are the time averaged power dissipation per unit length respectively in regions I, II, and III and "3" designates the three region geometry. The calculation of \( P_{dA}, P_{dB}, \) and \( P_{dC} \) is described in Section 5.2.1.

For the two region evanescent configuration, using Equations 4.46 and 4.47 the power dissipation density in each region is given by

\[
\langle p_{d1} \rangle = |E_{10}|^2 \frac{1}{2} \omega \varepsilon'' \epsilon_s e^{-2k''_{1z}x} \cos k_{x1}x \cos k_{z1}^*x \quad 0 \leq x \leq b
\]  
\[
\langle p_{dII} \rangle = |E_{20}|^2 \frac{1}{2} \omega \varepsilon'' \epsilon_s e^{-2k''_{1z}x} \sin k_{x2}(a-x) \sin k_{z2}^*(a-x) \quad b \leq x \leq a
\]

The total time averaged power dissipation per unit length is then given by

\[
P_{d2} = P_{d21} + P_{d22}
\]

\[
P_{d2} = P_{dA}(E_1, k_{x1}, b, \varepsilon''_{s}) + P_{dC}(E_2, k_{x2}, b, \varepsilon''_{pe})
\]

where \( P_{d21} \) and \( P_{d22} \) are the time averaged power dissipation per unit length in regions I and II. The calculation of \( P_{dA} \) and \( P_{dC} \) is described in Section 5.2.1.

For the four region configuration (two region dielectric with water sheets), using Equa-
tions 4.57-4.60, the power dissipation density in each region is given by

\[ \langle S_{zI} \rangle = |E_1|^2 \frac{1}{2} \omega \epsilon_s'' e^{-2k_s'z} \cos k_{x1}x \cos k_{x2}^*x, \quad 0 \leq x \leq b \]  
(5.27)

\[ \langle S_{zII} \rangle = \frac{1}{2} \omega \epsilon_{pe}'' e^{-2k_{x2}'z} \left( |E_{21}|^2 \cos k_{x2}x \cos k_{x2}^*x + |E_{22}|^2 \sin k_{x2}x \sin k_{x2}^*x + 2 \text{Real}\{E_{21}E_{22}^* \cos k_{x2}x \sin k_{x2}^*x}\right) \quad b \leq x \leq d \]  
(5.28)

\[ \langle S_{zIII} \rangle = \frac{1}{2} \omega \epsilon_{w}'' e^{-2k_{x3}'z} \left( |E_{31}|^2 \cos k_{x3}x \cos k_{x3}^*x + |E_{32}|^2 \sin k_{x3}x \sin k_{x3}^*x + 2 \text{Real}\{E_{31}E_{32}^* \cos k_{x3}x \sin k_{x3}^*x}\right) \quad d \leq x \leq d + \Delta \]  
(5.29)

\[ \langle S_{zIV} \rangle = |E_4|^2 \frac{1}{2} \omega \epsilon_{pe}'' e^{-2k_{x4}'z} \sin k_{x2}(a-x) \sin k_{x2}^*(a-x) \quad d + \Delta \leq x \leq a \]  
(5.30)

The time averaged power dissipation per unit length is then

\[ P_{d4} = P_{d41} + P_{d42} + P_{d43} + P_{d44} \]  
(5.31)

\[ = P_{dA}(E_1, k_{x1}, b, \epsilon_s'') + P_{dB}(E_{21}, E_{22}, k_{x2}, b, \epsilon_{pe}'') + P_{dB}(E_{31}, E_{32}, k_{x3}, d, d + \Delta, \epsilon_{w}'') + P_{dC}(E_4, k_{x4}, d + \Delta, \epsilon_{pe}'') \]  
(5.32)

where \( P_{d41}, P_{d42}, P_{d43}, \) and \( P_{d44} \) are the time averaged power dissipation per meter respectively in regions I, II, III, and IV. The calculation of \( P_{dA}, P_{dB}, \) and \( P_{dC} \) is described in Section 5.2.1.

The change in attenuation when a symmetric pair of water sheets is inserted into the single region geometry is then given by

\[ \Delta \text{Attenuation/meter} = \left( \frac{P_{d3}^w}{P_3(z)} - \frac{P_{d4}^d}{P_4(z)} \right) 10 \log_{10} e \]  
(5.33)

where \( P_3(z) = P_1(z) \). When a symmetric pair of water sheets is inserted into the two region evanescent configuration, the change in attenuation is

\[ \Delta \text{Attenuation/meter} = \left( \frac{P_{d3}^w}{P_3(z)} - \frac{P_{d2}^d}{P_2(z)} \right) 10 \log_{10} e \]  
(5.34)

where \( P_4(z) = P_2(z) \).
5.2.1 Calculation of Power Dissipation Integrals

The time averaged power dissipation density is integrated over the cross section to find the total power lost per unit length in the waveguide. The three types of expressions for time average power dissipation density are

\[ \langle P_{dA} \rangle = |A|^2 \frac{1}{2} \omega \epsilon'' e^{-2 k''_z} \cos k_x x \cos k^*_x x \]  \hspace{1cm} (5.35)

\[ \langle P_{dB} \rangle = \frac{1}{2} \omega \epsilon'' e^{-2 k''_z} \left( |B_1|^2 \cos k_x x \cos k^*_x x + |B_2|^2 \sin k_x x \sin k^*_x x + 2 \text{ Real} \{B_1 B^*_2 \cos k_x x \sin k^*_x x \} \right) \]  \hspace{1cm} (5.36)

\[ \langle P_{dC} \rangle = |C|^2 \frac{1}{2} \omega \epsilon'' e^{-2 k''_z} \sin k_x (a - x) \sin k^*_x (a - x) \]  \hspace{1cm} (5.37)

The time averaged power dissipation per unit length in the three types of regions characterized by Equations 5.35-5.37 can be found by integrating over the cross section of each region. Due to symmetry, the power dissipation can be integrated for only \( x > 0 \) and then doubled. The power dissipation for each region can be characterized by the various amplitudes, \( k_x \)'s, \( \epsilon'' \)'s and limits of integration. Particularly,

\[ P_{dA} = P_{dA}(A, k_x, x_1, \epsilon'') \]  \hspace{1cm} (5.38)

\[ P_{dB} = P_{dB}(B_1, B_2, k_x, x_1, x_2, \epsilon'') \]  \hspace{1cm} (5.39)

\[ P_{dC} = P_{dC}(C, k_x, x_2, \epsilon'') \]  \hspace{1cm} (5.40)

The form of the integrals used to solve for \( P_{dA} \), \( P_{dB} \), and \( P_{dC} \) depends on whether the \( k_x \) is real, imaginary, or complex.

For the case where \( k_x \) is real, the power dissipation for each region is calculated to be

\[ P_{dA} = |A|^2 \omega \epsilon'' h \int_0^{x_1} \cos k_x x \cos k^*_x x \, dx \]
\[ = |A|^2 \omega \epsilon'' h \left[ \frac{x_1}{2} + \frac{1}{4k_x} \sin 2k_x x_1 \right] \]  \hspace{1cm} (5.41)

\[ P_{dB} = \omega \epsilon'' h e^{-2 k''_z} \int_{x_1}^{x_2} \left| B_1 \right|^2 \cos k_x x \cos k^*_x x + \left| B_2 \right|^2 \sin k_x x \sin k^*_x x + \right. \]
\[ \left. 2 \text{ Real} \{B_1 B^*_2 \cos k_x x \sin k^*_x x \} \right) \, dx \]
\[ = \omega \epsilon'' h e^{-2 k''_z} \left[ \frac{x_2 - x_1}{2} + \frac{1}{4k_x} \left( \sin 2k_x x_2 - \sin 2k_x x_1 \right) \right] + \]
\[ |B_1|^2 \left( \frac{x_2 - x_1}{2} - \frac{1}{4k_x} \left( \sin 2k_x x_2 - \sin 2k_x x_1 \right) \right) + \]
\[ |B_2|^2 \left( \frac{x_2 - x_1}{2} + \frac{1}{4k_x} \left( \sin 2k_x x_2 - \sin 2k_x x_1 \right) \right) + \]
If $k_x$ is entirely imaginary, the integrals are calculated similarly to Equations 5.41-5.43 and the results differ by only sign changes.

\[ P_{dA} = |A|^2 \omega'' h e^{-2k_x^2} \left[ \frac{x_1}{2} + \frac{1}{4k_x} \sin 2x_1 k_x \right] \]  
(5.44)

\[ P_{dB} = \omega'' h e^{-2k_x^2} \left[ |B_1|^2 \left( \frac{x_2 - x_1}{2} + \frac{1}{4k_x} (\sin 2k_x x_2 - \sin 2k_x x_1) \right) + \right. \]
\[ \left. |B_2|^2 \left( \frac{1}{4k_x} (\sin 2k_x x_2 - \sin 2k_x x_1) - \frac{x_2 - x_1}{2} \right) - 2 \text{ Real} \left\{ B_1 B_2^* \left( \sin^2 k_x x_2 - \sin^2 k_x x_1 \right) \right\} \right] \]  
(5.45)

\[ P_{dC} = |C|^2 \omega'' h e^{-2k_x^2} \left[ \frac{1}{4k_x} \sin 2k_x (a - x_2) - \frac{a - x_2}{2} \right] \]  
(5.46)

If the mode were evanescent with an imaginary $k_x$, these power dissipations are still real since $\sin j \alpha = \sinh \alpha$, $\cos j \alpha = \cosh \alpha$, and $\sin^2 j \alpha$ is real.

When $k_x$ is complex with both real and imaginary terms, the integrals become

\[ P_{dA} = |A|^2 \omega'' h e^{-2k_x^2} \left[ \frac{\sin(k_x - k_x^*) x_1}{2(k_x - k_x^*)} + \frac{\sin(k_x + k_x^*) x_1}{2(k_x + k_x^*)} \right] \]  
(5.47)

\[ P_{dB} = \omega'' h e^{-2k_x^2} \left[ |B_1|^2 \left( \frac{\sin(k_x - k_x^*) x_2}{2(k_x - k_x^*)} + \frac{\sin(k_x + k_x^*) x_2}{2(k_x + k_x^*)} \right) - \right. \]
\[ \left. \frac{\sin(k_x - k_x^*) x_1}{2(k_x - k_x^*)} + \frac{\sin(k_x + k_x^*) x_1}{2(k_x + k_x^*)} \right] + \]
\[ |B_2|^2 \left( \frac{\sin(k_x - k_x^*) x_2}{2(k_x - k_x^*)} - \frac{\sin(k_x + k_x^*) x_2}{2(k_x + k_x^*)} \right) - \]
\[ \frac{\sin(k_x - k_x^*) x_1}{2(k_x - k_x^*)} + \frac{\sin(k_x + k_x^*) x_1}{2(k_x + k_x^*)} \right] + \]
\[ 2 \text{ Real} \left\{ B_1 B_2^* \left( \frac{\cos(k_x - k_x^*) x_2}{2(k_x - k_x^*)} - \frac{\cos(k_x + k_x^*) x_2}{2(k_x + k_x^*)} \right) - \right. \]
\[ \left. \frac{\cos(k_x - k_x^*) x_1}{2(k_x - k_x^*)} + \frac{\cos(k_x + k_x^*) x_1}{2(k_x + k_x^*)} \right\} \right] \]  
(5.48)

\[ P_{dC} = |C|^2 \omega'' h e^{-2k_x^2} \left[ \frac{\sin(k_x - k_x^*) (a - x_2)}{2(k_x - k_x^*)} - \frac{\sin(k_x + k_x^*) (a - x_2)}{2(k_x + k_x^*)} \right] \]  
(5.49)

Note that these integrals are very similar to those calculated in Section 4.5 with $\frac{\text{Real}(k_x)}{\omega \mu_0}$.
replaced by $\omega \varepsilon''$, where $\varepsilon''$ is different for each region.

5.3 Perturbation Approach

If it can be assumed that the water sheets are very thin ($\Delta \ll a$) such that the electromagnetic field mode shapes and amplitudes for the cases with and without the water sheets are roughly the same, a perturbation approach can be used to find the change in attenuation. If the mode shape and amplitudes remain the same, the loss in the dielectrics (sapphire and XLPE) will also remain the same. Since it is the change in power dissipation per unit length that is important (Equation 5.11), only the power dissipation per unit length in the symmetric pair of water sheets need be considered. That is

$$\Delta \text{Attenuation/meter} = \frac{\Delta P_d}{P(z)} 10 \log_{10} e \quad (5.50)$$

For the single region dielectric, the electric field at the location of the water sheet at $x = d$ is given by

$$E_y(x = d) = E_o \cos k_x d e^{-jk_z z} \quad (5.51)$$

The sheet has thickness $\Delta$ and there is a symmetric sheet located at $x = -d$. Since the sheets are very thin, the electric field is assumed to be uniform over the sheet thickness. The power dissipated per unit length in these sheets is then

$$\langle P_d \rangle = \frac{1}{2} \omega \varepsilon'' \Delta h \cos^2 k_x d e^{-2k'' z} \quad (5.52)$$

and the power flowing in the waveguide is given by

$$P_1 = \frac{|E_o|^2}{2 \omega \mu_o} \Re\{k_z\} ah e^{-2k'' z} \quad (5.53)$$

The resulting change in attenuation is

$$\Delta \text{Attenuation/meter} = \frac{2 \omega^2 \mu_o \varepsilon'' \Delta \cos^2 k_x d}{\Re\{k_z\} a} 10 \log_{10} e \quad (5.54)$$

In the two region evanescent configuration, the electric field at the location of the water
Again, the sheet has thickness Δ and there is a symmetric sheet located at \( x = -d \). The power dissipated per unit length in these sheets is

\[
\langle P_d \rangle = 2 \cdot \frac{1}{2} \omega \epsilon''_w \Delta h |E_{20}|^2 \sin k_{x2}(a - d) \sin k_{x2}^*(a - d) e^{-2k''_z z} \tag{5.56}
\]

The resulting change in attenuation is

\[
\Delta \text{Attenuation/meter} = \frac{\omega \epsilon''_w \Delta h |E_{20}|^2 \sin k_{x2}(a - d) \sin k_{x2}^*(a - d) e^{-2k''_z z}}{P_2(z)} \tag{5.57}
\]

where \( P_2(z) = P_{21} + P_{22} = P_A(E_1, k_{x1}, b) + P_C(E_2, k_{x2}, b) \). \( P_{21} \) and \( P_{22} \) are the time averaged powers flowing in the sapphire and XLPE regions and \( P_A \) and \( P_C \) are calculated in Section 4.5.

### 5.4 Comparison of Methods

Using typical dimensions for the waveguide and dielectrics, this section compares the calculated attenuation using the dispersion relation, power ratio, and perturbation methods. The dispersion relation and power ratio methods, both exact calculations, yield the same results, as expected. All curves labeled "exact field expressions" are the results for these two methods superimposed and no difference can be detected.

The plots for the case where symmetric pairs of water sheets are added to the single region dielectric are shown in Figure 5-1. Three sheet thicknesses are considered, \( \Delta = 1 \mu m \), \( \Delta = 5 \mu m \), and \( \Delta = 10 \mu m \). The discrepancy between the exact and perturbation calculations grows with both the thickness and excitation frequency. At 12.5 GHz, the error between the perturbation and exact methods is 0.23%, 2.4%, and 5.0% for the three cases. For thicker water sheets, the mode shape is no longer purely sinusoidal, as shown in Figure 4-6. The water sheets cause the field amplitude to rise slightly at the location of the sheets and the calculated loss is slightly higher than calculated with the perturbation method. The loss increases with frequency because the imaginary component of the water dielectric constant also increases with frequency, as shown in Figure 3-3.

The case where symmetric pairs of water sheets are added to the two region evanescent...
configuration is somewhat more complicated and is shown in Figure 5-2. The discrepancy between the exact and perturbation calculations increases with thickness as before, but is less dependent on the frequency. At 8 GHz, the error between the perturbation and exact methods is 0.7%, 1.8%, and 3.6% for the three cases. As before, the perturbation method predicts lower attenuation than the exact methods because the water sheets raise the field amplitudes slightly (Figure 4-10), increasing the loss. However, here the loss decreases with frequency. The reason for this is shown in Figure 5-3. As the frequency is raised, the field amplitudes drop off more sharply in the evanescent region. With a lower field amplitude, the attenuation is lowered, even though the water has more loss at the higher frequencies. This change in mode shape with frequency is what makes the evanescent approach unique.

Since the perturbation approach provides results that are very similar to those using the exact expressions at a great reduction in complexity, these will be used in all further computations involving symmetric pairs of thin water sheets.

5.5 Loss Due to Treed XLPE Sheets

In Section 3.4, the procedure to grow water trees in sheet XLPE samples was described. Generally, the maximum concentration of moisture in these treed sheets was about 6000 ppm. Using the Haus-Melcher Model (Section 3.2.5), the dielectric constant of the XLPE sheet containing 6000 ppm 0.5M NaCl solution was calculated and plotted in Figure 3-13.

The loss due to these treed XLPE sheets can be calculated in the same manner as the loss due to the water sheets. The only differences are the thickness and the dielectric constant, both of which can be accommodated with the model. The change in attenuation is calculated for the case when symmetric 0.5 mm and 1.0 mm thick treed XLPE sheets are inserted into the two region dielectric loaded waveguide at $d = 2\text{ mm}$, a position fairly close to the center where the loss would be largest. The exact expressions are used, rather than a perturbation approach, and this attenuation is plotted in Figure 5-4. (The perturbation approach assumes that the sheets are very thin and does not compensate for the material removed to make room for thicker sheets.) The loss due to one or two thicknesses of treed XLPE sheets is estimated to be quite small, and probably is not detectable with the microwave measurement system. The attenuation of even very thin water sheets is much greater (see Figures 5-1 and 5-2) than that of the treed XLPE sheets.
Figure 5-1: Change in attenuation when three different thicknesses of water sheets are placed inside the single region dielectric loaded waveguide. The dimensions are $a = 11.4$ mm, $d = 5$ mm, and (a) $\Delta = 1\mu$m, (b) $\Delta = 5\mu$m, (c) $\Delta = 10\mu$m.
Figure 5-2: Change in attenuation when three different thicknesses of water sheets are placed inside the two region dielectric loaded waveguide. The dimensions are $a = 11.4$ mm, $b = 1$ mm, $d = 5$ mm, and (a) $\Delta = 1\mu$m, (b) $\Delta = 5\mu$m, (c) $\Delta = 10\mu$m.
Figure 5-3: Change in electric field mode shape with frequency for the evanescent mode configuration. The parameters are $a = 11.4$ mm, $b = 1$ mm, $\epsilon_1 = 11\epsilon_0$, and $\epsilon_2 = 2.25\epsilon_0$.

Figure 5-4: Change in attenuation when symmetric 0.5 mm and 1.0 mm thick treed XLPE sheets are placed inside the two region dielectric loaded waveguide. The dimensions are $a = 11.4$ mm, $b = 1$ mm, $d = 2$ mm, and $\Delta = 0.5$ and 1 mm.
Chapter 6

Experimental Results

Several experiments were carried out with a hollow rectangular waveguide loaded with XLPE slabs, sapphire, and a symmetric pair of water sheets. The attenuation with and without the water sheets was measured for several configurations and the difference is compared to theoretical attenuation calculations.

6.1 Description of Experimental Apparatus

Since microwave equipment was available in the 2-12.4 GHz range, experiments were designed around the X-band (8-12.4 GHz) microwave waveguide. Lower frequencies were not considered as the water losses are higher at the higher frequencies. This X-band rectangular waveguide has inside dimensions of 0.9 x 0.4 inches (22.8 x 10.2 mm) and is designed for single mode ($TE_{10}$) operation in the 8-12.4 GHz range without dielectric loading. The cut-off frequency for the $TE_{10}$ mode is 6.6 GHz, for the $TE_{20}$ mode is 26.3 GHz, and for the $TE_{01}$ mode is 14.7 GHz. See sections 4.3.1 and 5.1 for discussions of mode designations and cut-off frequencies.

When a dielectric such as polyethylene is inserted into the waveguide, the cut-off frequency is lowered by the square root of the dielectric constant. In the case of XLPE, with a nominal dielectric constant of 2.25, these cut-off frequencies would be lowered by a factor of 1.5. Note that now the $TE_{01}$ mode has a cutoff frequency of 9.8 GHz, within the measurement frequency range. However, the waveguide is excited with a $TE_{10}$ mode vertically directed electric field (Figure 4-3). The $TE_{01}$ mode has a horizontally directed field which will not be excited in this configuration which only excites vertically directed electric field
modes. Thus, in this experiment, the waveguide still operates in the single mode regime.

To generate evanescent fields, a material with dielectric constant larger than the nominal 2.25 of XLPE is needed. Evanescent fields can only be generated when the dielectric constant in the center is considerably larger than the material on the outside. Sapphire (aluminum oxide, Al$_2$O$_3$) was chosen due to its low loss and dielectric constant of 11.5, which is constant over a wide frequency range.

Sapphire is a birefringent crystalline material, where the dielectric constant varies depending on the orientation of the electric field. Along one axis of the material (parallel to the C-Axis), the dielectric constant is 11.5 and along the other two axes (perpendicular to the C-Axis), the dielectric constant is 9.3. Sapphire is commercially available in sheets from Saphikon with the C-Axis in the plane of the sheet. Specifications for this sapphire are given in Figure 6-1.

![Figure 6-1: Dielectric constant for sapphire from data sheets provided by Saphikon.](image)

These sapphire sheets are 0.018 inches (0.4572 mm) thick and were cut and polished to 0.4 inches high and 2.0 inches long to fit inside the waveguide. Several sapphire sheets were placed end-on-end to match the length of the XLPE samples. The thicker sapphire dielectric slabs were created by putting two or more sapphire sheets side-by-side.

The XLPE was provided by TEPCO in 2 and 3 mm thick sheets. These sheets were cut into thin strips and machined to 0.4 inches high and 10 inches long. Some were sanded to various thicknesses to accommodate differing sapphire widths inside the waveguide. This XLPE was processed with the same cross-linking chemicals as the transmission cables. The dielectric properties of the cable samples were analyzed and summarized in Table 3.4.
The water sheets simulate treed layers by inserting spatial impulses of water in between pieces of XLPE placed inside the waveguide experiment. These water sheets consist of two sheets of 1.6 mil polyethylene sealed together with a hot knife and filled with small amounts of water. The dimensions of the water sheets matched those of the XLPE. Empty water sheets were constructed for the dry measurements. These water sheets were then placed symmetrically at known positions between the sheets of XLPE.

To measure the thickness of the water in the water sheet, the sheet was weighed before each set of experiments. The scale used was a Mettler AE163, accurate to 0.1 mg. Since water diffuses through the thin polyethylene fairly quickly (the diffusion time for 1.6 mil (40μm) polyethylene is about 27 minutes, see Section 2.2.2), the experiments were performed as quickly as possible. At the end, the water sheet was broken open, dried, and the polyethylene was again weighed. The difference in the weight measurements is the weight of the water. The density of water is 1.0 \( \text{g/cm}^3 \), so the thickness (in cm) is the weight divided by the height times length (in cm\(^2\)). In most cases, the water sheets were between 5 and 15 μm thick.

The polyethylene slabs, sapphire, and water sheets were inserted into the X-band waveguide in exactly the same location for each experiment. The waveguide was then placed into the microwave measurement system shown in Figure 3-8. Between each experiment, only the waveguide was disturbed. No microwave connectors were removed and reattached, in order to minimize complicating reflection effects. The storage oscilloscope sampled 4000 data points of attenuation measurements generated by the network analyzer between 8 and 12.4 GHz. Attenuation measurements for each geometry and water sheet position were recorded and downloaded to a computer. These results are presented in the following sections.

### 6.2 Quadratic Fit of Data

Before the data is presented, a discussion of curve fitting is necessary. The 4000 data points contain considerable noise and oscillations due to reflections inside the waveguide. Fitting a smooth curve to these data points is needed in order to compare the measurements to theoretical values and to present the measured data in a compact format. The Matlab function ‘polyfit’ was used to fit a polynomial of specified order to the data points [23]. A quadratic polynomial was chosen because it matched both the theoretical and measured
curves best (see Figures 5-1 and 5-2). There was no indication that a cubic polynomial was necessary.

The linearly increasing frequency can be considered the x-axis and the measured data the y-axis. Given an x vector, 'polyfit' finds a polynomial p, such that p(x) fits the data vector y in a least squares sense. That is

\[ p(x) = c_n x^n + c_{n-1} x^{n-1} + \cdots + c_0 \]  

(6.1)

where n is the order of the polynomial. The least squares method takes the difference between each value of the estimate p(x) and the corresponding measured data point y. The difference is then squared and the result for all points is summed. The value of the summation is then minimized. The implementation of the least squares method is discussed extensively in Section 7.1.

### 6.3 Measurements

Four sets of measurements were taken: (1) with no sapphire slab (a simple one-region dielectric problem), (2) one sapphire slab 0.4572 mm thick, (3) two sapphire slabs for a total of 0.9144 mm thick, and (4) three sapphire slabs 1.3716 mm thick. The water sheets were placed symmetrically at three locations in the waveguide between sections of XLPE at \( d = 3.4 \) mm, \( d = 6.4 \) mm, and \( d = 9.4 \) mm. The attenuations with and without the water sheets were measured and subtracted to find the net additional attenuation due to only the water sheets.

The measured and fitted data for the one sapphire slab geometry with one pair of symmetric water sheets is shown in Figure 6-2. The first plot is the dry measurement with empty polyethylene water bags at symmetric positions inside the waveguide, the second has water sheets inserted at those positions, and the third is the difference in loss with and without the water. The fitted (smooth) curve is a quadratic least squares fit to the data. Figure 6-3 summarizes all of the measured data. Only the attenuation due to the addition of the water sheets is shown. Note that the network analyzer displays the output signal strength referenced to the input signal strength, and therefore displays the negative attenuation. The difference plots are therefore inverted to display positive attenuation so that they can be easily compared to theory.
Figure 6-2: Measured and fitted data for 1 slab dielectric waveguide with water sheets at \( d = \pm 9.4 \text{mm} \) from center.
Figure 6-3: Summary of data fitted to attenuation measurements using a quadratic fit algorithm.
6.4 Theoretical Loss Calculations

First, the water sheet thicknesses must be calculated before the theoretical loss can be estimated. The two water sheets (left and right) in Table 6.1 are denoted I and II. Water sheet weight measurements 0-3 were taken before each waveguide (0-3 sapphire slabs) experiment, and measurement 4 was taken after the last experiment. The dry measurement was taken after the water sheet was opened and dried. The net weight of the water in each water sheet is given in the “Net” column. Roughly a half hour elapsed between each weight measurement. Note that the water weight decreases with time, possibly due to moisture diffusion or handling of the water sheet during the experiments.

The water sheet thickness used in the theoretical calculations is the average of the thicknesses of the two sheets and the average of the measurements before and after the experiment. This is reasonable given that in the perturbation model for loss, the loss is a linear function of the water sheet thickness. Therefore, for the four experiments, the water sheet thicknesses used with the theoretical model are

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Sheet</th>
<th>Weight (g)</th>
<th>Net (g)</th>
<th>Thickness (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>I</td>
<td>0.2874</td>
<td>0.0381</td>
<td>14.8</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.2725</td>
<td>0.0258</td>
<td>10.0</td>
</tr>
<tr>
<td>1</td>
<td>I</td>
<td>0.2858</td>
<td>0.0365</td>
<td>14.1</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.2708</td>
<td>0.0240</td>
<td>9.3</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>0.2811</td>
<td>0.0318</td>
<td>12.3</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.2678</td>
<td>0.0210</td>
<td>8.1</td>
</tr>
<tr>
<td>3</td>
<td>I</td>
<td>0.2743</td>
<td>0.0250</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.2633</td>
<td>0.0165</td>
<td>6.4</td>
</tr>
<tr>
<td>4</td>
<td>I</td>
<td>0.2699</td>
<td>0.0206</td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.2604</td>
<td>0.0136</td>
<td>5.2</td>
</tr>
<tr>
<td>Dry</td>
<td>I</td>
<td>0.2493</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.2468</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Water sheet thickness calculations.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>maximum thickness (µm)</th>
<th>minimum thickness (µm)</th>
<th>average thickness (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (0 slab)</td>
<td>12.4</td>
<td>11.7</td>
<td>12.1</td>
</tr>
<tr>
<td>2 (1 slab)</td>
<td>11.7</td>
<td>10.2</td>
<td>11.0</td>
</tr>
<tr>
<td>3 (2 slabs)</td>
<td>10.2</td>
<td>8.0</td>
<td>9.1</td>
</tr>
<tr>
<td>4 (3 slabs)</td>
<td>8.0</td>
<td>6.5</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Table 6.2: Water sheet thickness values used in the theoretical calculations.
The attenuation is then calculated using the perturbation method described in Section 5.3. Figure 6-4 shows the theoretical attenuation due to the symmetric water sheets. In most cases, the theoretical calculations and the measurements are similar in shape and within a factor of two. In general, the theoretical attenuation was larger than the measured attenuation.

6.5 Discussion of Differences Between Theory and Experiment

The theoretical data (Figure 6-4) corresponds fairly well in curvature and slope to the curves fitted to the measured data (Figure 6-3). The data values are very similar in some cases, and differ by as much as a factor of two (in dB) in others. To get bounds on the error, the minimum and maximum error for each set of curves is calculated and tabulated in the following table. The average value of the water sheet thickness from Table 6.2 is used to calculate the theoretical attenuation. Errors are expressed in percentage error referenced to the measurements. A positive percentage error is an overestimate of the theoretical model. The X indicates a measurement problem, as the measured curve has negative attenuation (growth rather than decay), which is not physically realizable.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>9.4 mm</th>
<th>6.4 mm</th>
<th>3.4 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>no slab</td>
<td>max</td>
<td>8.5%</td>
<td>53%</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>-25%</td>
<td>29%</td>
</tr>
<tr>
<td>1 slab</td>
<td>max</td>
<td>7.8%</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>-45.8%</td>
<td>-6.8%</td>
</tr>
<tr>
<td>2 slabs</td>
<td>max</td>
<td>-19%</td>
<td>2.9%</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>-68%</td>
<td>-49%</td>
</tr>
<tr>
<td>3 slabs</td>
<td>max</td>
<td>X</td>
<td>-16%</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>X</td>
<td>-51%</td>
</tr>
</tbody>
</table>

Table 6.3: Errors between measured and theoretical loss due to water sheets using average water sheet thicknesses from Table 6.2.

In Section 6.4, the calculation of the water sheet thickness for each set of experiments was discussed. It is the average of the two symmetric sheet thicknesses, as well as the average of the thicknesses measured before and after each set of experiments. (A set of experiments all have the same sapphire width.) The d = 9.4 mm (first column) measurements were made first and the d = 3.4 mm (third column) measurements were made several minutes
Figure 6-4: Theoretical loss corresponding to fitted data. The water sheet thicknesses used in the calculations are given by t.
later. It would be reasonable to assume that the actual water sheet was a little thicker for
the earlier measurements and a little thinner for the later measurements, given the trend
in the net water content of the sheets listed in Table 6.1.

This also corresponds to the trends in the errors calculated in Table 6.3. In the first
column (9.4 mm, earlier measurements), the theoretical predictions were too low, indicating
that the actual water sheet was thicker than used in the theoretical models. In the last
column (3.4 mm, later measurements), the theoretical predictions are too high. More water
has diffused or leaked out, and the actual water sheet was a little thinner than used in
the models. The middle column errors were in between these two extremes. If, rather
than using the average water sheet thickness, the maximum and minimum values of water
sheet thickness of Table 6.2 were used to recalculate the theoretical loss and errors of the
earlier and later measurements (first and third columns of Table 6.3), the following table
is obtained. Adjusting the water sheet thickness has reduced the extremes in the errors
compared to Table 6.3.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>9.4 mm</th>
<th>6.4 mm</th>
<th>3.4 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>no slab</td>
<td>max</td>
<td>11%</td>
<td>53%</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>-23%</td>
<td>29%</td>
</tr>
<tr>
<td>1 slab</td>
<td>max</td>
<td>15%</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>-42%</td>
<td>-6.8%</td>
</tr>
<tr>
<td>2 slabs</td>
<td>max</td>
<td>-9%</td>
<td>2.9%</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>-64%</td>
<td>-49%</td>
</tr>
<tr>
<td>3 slabs</td>
<td>max</td>
<td>x</td>
<td>-16%</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>x</td>
<td>-51%</td>
</tr>
</tbody>
</table>

Table 6.4: Errors between measured and theoretical loss due to water sheets corrected for
water sheet thickness variations in Table 6.2.

There are several sources of error in the measurements and theory. There are small
air gaps between the XLPE sheets, as much as 0.8 mm cumulative for the width of the
waveguide. Since the dielectric constant of air (1) is less than that of XLPE (2.25), the
electric fields will be lowered slightly in the area of the air gap, resulting in less power loss
if the water sheet were also at that location. However, the dielectric constants of air and
XLPE are fairly similar so the effect of the air gaps is small, only a few percent at most.

There is also uncertainty in the location of the water sheets. The three positions used
in the theoretical modelling were 9.4, 6.4, and 3.4 mm. These water sheets were actually
somewhere within the ranges of $9.0 < x < 9.4\text{ mm}$, $6.0 < x < 6.4\text{ mm}$, $3.0 < x < 3.4\text{ mm}$. 

100
If the sheet were located closer to the center where the fields are greater, the model would predict a larger loss. The amount of change in attenuation depends on the electric field mode shape of the particular waveguide experiment and is explored in Section 7.3.1. Both the slight difference in the water sheet thicknesses (left and right, Table 6.1) and the uncertainty in the position could affect the symmetry of the problem, which would in turn affect the attenuation.

In the next chapter, the reverse problem is solved. Given the attenuation as a function of frequency, solve for the water sheet thickness and position. Both the measured data and generated theoretical data are used to test these inverse algorithms.
Chapter 7

Prediction of Distribution and Concentration of Moisture

The experimental measurements presented in the previous chapter consisted of 4000 data points of attenuation due to the introduction of symmetric water sheets for frequencies between 8 and 12.4 GHz. In those experiments, the position and thickness of the water sheets were fairly well known. Given those parameters, the attenuation could be predicted with a theoretical model and compared to the experimental results.

Here, the inverse problem is investigated. Given only the attenuation measurements and knowledge of the geometry, estimate the positions and thicknesses of the symmetric pairs of water sheets or continuous distribution of moisture that would have generated that attenuation. In the case of symmetric water sheets, there are only a few unknown positions and thicknesses of the water sheets. For a continuous distribution of moisture, the moisture concentration is unknown. Both cases are overdetermined systems, with many more measurements (and therefore equations) than variables. The method of least squares estimation is therefore used to find the location and thickness parameters or moisture concentration that gives the “best fit” to the measurements.
7.1 Least Squares Estimation for Solving an Overdetermined System

The attenuation due to symmetric pairs of water sheets is a linear function of the sheet thickness, but a nonlinear function of the position. Therefore, a nonlinear least squares method must be applied to the water sheet problem. The attenuation due to low concentrations of moisture in XLPE is a linear function of the concentration and in this case linear least squares will be used. This section outlines how an overdetermined system is solved with both the methods of linear and nonlinear least squares using simple examples.

7.1.1 Linear Systems

Least squares estimation is used to solve for the unknown parameters of a mathematical model using measured physical data. There is often a degree of uncertainty in measured data, often due to measurement or modelling errors, and least squares estimation will provide the “best fit” of the parameters to the measured data.

A simple example of a problem where least squares estimation can be used is that of a mass moving with constant acceleration. The position of the mass is given by $y(t) = x_1 + x_2 t + x_3 t^2/2$. If the position is measured at $m$ different times, we can organize our results in matrix form:

$$
\begin{bmatrix}
  y(t_1) \\
  \vdots \\
  y(t_m)
\end{bmatrix}
= 
\begin{bmatrix}
  1 & t_1 & t_1^2/2 \\
  \vdots & \vdots & \vdots \\
  1 & t_m & t_m^2/2
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3
\end{bmatrix}
$$

In traditional matrix notation, $M = Cx$, where $C$ is a known $m \times n$ matrix, $M$ is a known $m$-element vector of measurements, and $x$ is an $n$-element vector of model parameters that are to be determined. The concern here is with the case $m \geq n$ where there is at least as many measurements as unknown parameters. Typically, since there is always noise and uncertainty in measurements, there will be no exact solution $x$ that solves $M = Cx$. Therefore, an error vector is defined $e = M - Cx$. The least squares estimate is the solution $x$ which minimizes the quantity $e^T e$, or the squared length of the error vector.

In linear algebra terminology, the columns of the matrix $C$ span an $n$-dimensional subspace $\text{Ra}(C)$. The vector $M$ is not in this subspace due to measurement or modelling errors.
The minimum length of the error vector is found when the vector \( e \) is orthogonal to the subspace \( Ra(C) \), or equivalently, \( M \) is projected onto the subspace \( Ra(C) \). In two-dimensions you can envision this as a right triangle. Let "M" be the hypotenuse and "Cx" be one side. The third side "e" will have minimum length when it is orthogonal to "Cx".

Since \( e \) is orthogonal to \( Ra(C) \), \( C^Te = 0 \). Substituting in for \( e \) results in \( (CTC)x = C^TM \), or

\[ x = (CTC)^{-1}C^TM \quad (7.2) \]

### 7.1.2 Nonlinear Least Squares Estimation

The water sheet attenuation problem cannot be described so simply. The measured absorption is a linear function of the thickness of the water sheet, but it is a nonlinear function of water sheet position. As in the previous section, a nonlinear function can be defined to use as an example. Let \( y(t) = f_1(t, x_1) + f_2(t, x_2) + f_3(t, x_3) \), where \( f_1 \), \( f_2 \), and \( f_3 \) are nonlinear functions of \( x_1 \), \( x_2 \), and \( x_3 \), respectively. In matrix notation,

\[
\begin{bmatrix}
y(t_1) \\
\vdots \\
y(t_m)
\end{bmatrix} =
\begin{bmatrix}
f_1(t_1, x_1) + f_2(t_1, x_2) + f_3(t_1, x_3) \\
\vdots \\
f_1(t_m, x_1) + f_2(t_m, x_2) + f_3(t_m, x_3)
\end{bmatrix} \quad (7.3)
\]

For this example, one could write \( M = C(x) \), where \( M \) is the m-element vector of measurements and \( C(x) \) is the m-element vector of nonlinear functions of the parameters \( x_1 \), \( x_2 \), and \( x_3 \) (\( n=3 \)).

To solve this, the equation is linearized around an initial estimate \( x = \hat{x}_o \) using a Taylor series expansion. The (\( \cdot \)) is used to denote an estimate of a parameter.

\[
y(t, x) \approx f_1(t, \hat{x}_{1,0}) + f_2(t, \hat{x}_{2,0}) + f_3(t, \hat{x}_{3,0}) + \\
\frac{\partial f_1}{\partial x_1} \bigg|_{\hat{x}_o} (x_1 - \hat{x}_{1,0}) + \frac{\partial f_2}{\partial x_2} \bigg|_{\hat{x}_o} (x_2 - \hat{x}_{2,0}) + \frac{\partial f_3}{\partial x_3} \bigg|_{\hat{x}_o} (x_3 - \hat{x}_{3,0}) \quad (7.4)
\]

Since there are m measurements of \( y(t) \), this expansion can be written in matrix form,

\[
M \approx C(\hat{x}_o) + \left[ \frac{dC(x)}{dx} \right]_{x=\hat{x}_o} (x - \hat{x}_o) \quad (7.5)
\]

105
or

\[ M - C(\hat{x}_0) \approx \left[ \frac{dC(x)}{dx} \right]_{x=\hat{x}_0} (x - \hat{x}_0) \quad (7.6) \]

where, if \( i \) is the row and \( j \) is the column,

\[ \left[ \frac{dC(x)}{dx} \right]_{ij} = \frac{dC_i}{dx_j} \quad (7.7) \]

\( M \) is the \( m \)-element vector of measurements, \( C(\hat{x}_0) \) is the \( m \)-element vector calculated from our initial estimate for \( x = \hat{x}_0 \), and \( x \) is the \( n \)-element vector of parameters to estimate (here, \( n = 3 \)). A new matrix can be defined

\[ A(\hat{x}_0) = A_0 = \left[ \frac{dC(x)}{dx} \right]_{x=\hat{x}_0} \quad (7.8) \]

which is the \( m \times n \) matrix of derivatives of the nonlinear functions evaluated at \( x = \hat{x}_0 \). The columns correspond to the derivatives of different nonlinear functions, and the rows to different data points (in this case, different values of \( t \)).

As before, it is unlikely that an exact solution can be found for \( x \) which satisfies these equations, and an error vector is defined \( e = M - C(\hat{x}_0) - A_0[x - \hat{x}_0] \). This is the same form of equation as found in the linear least squares estimation, and using linear least squares, a new estimate for the parameter vector \( \hat{x} \) can be found.

\[ \hat{x}_1 = \hat{x}_0 + (A_0^T A_0)^{-1} A_0^T [M - C(\hat{x}_0)] \quad (7.9) \]

Given this new estimate for \( x \), the nonlinear least squares algorithm is applied again,

\[ \hat{x}_{i+1} = \hat{x}_i + (A_i^T A_i)^{-1} A_i^T [M - C(\hat{x}_i)] \quad (7.10) \]

The solution is found in an iterative process, terminating when the change in the parameters \( \hat{x}_{i+1} - \hat{x}_i \) is acceptably small.
7.2 Application to the Water Sheet Problem

The method of nonlinear least squares estimation described in the previous section is now applied to the symmetric water sheet problem. The attenuation of the water sheets predicted by the perturbation method (Section 5.3) is linear in thickness and nonlinear in position for both the single region dielectric and the evanescent two-region dielectric problems. A nonlinear least squares solver is developed which incorporates these perturbation models.

The attenuation predicted by exact field solutions is not used here except to test the least squares solver in later sections. With the exact solutions, where the water sheet is considered an independent region, all of the field parameters (amplitudes, wavenumbers, etc.) depend nonlinearly on the thickness and position of the water sheet. The derivatives used in the nonlinear least squares method would be very complicated and computationally intensive.

7.2.1 Rectangular Waveguide Filled with XLPE and Water Sheets, but no Sapphire

In this case, the waveguide is a simple rectangular metal waveguide containing XLPE and a symmetric pair of water sheets. The mode shape for the electric field in the waveguide is sinusoidal as shown in Figure 4-3 rather than the evanescent (decaying) fields of the two region case. Using a perturbation approach, the water sheet is assumed to be very thin and the loss small enough that the electromagnetic fields are unchanged by the presence of the water sheets. The solutions for the electric field mode shape and attenuation due to a symmetric pair of water sheets are given in Sections 4.3.1 and 5.3.

For the case of one pair of water sheets embedded in the XLPE, the perturbation model for the attenuation of the applied microwave signal is

\[
C(\omega, \delta, d) = \frac{2\omega^2 \mu_0 \epsilon''_w(\omega) \delta}{a \text{Real}\{k_x(\omega)\}} \cos^2(k_x d) 10 \log_{10} e \\
= F(\omega) \delta \cos^2(k_x d)
\]  \hspace{1cm} (7.11)

(7.12)

where \( \omega = 2\pi f \) is the signal radian frequency and \( f \) varies from 8 to 12.4 GHz, \( \epsilon''_w \) is the imaginary component of the permittivity of the water which causes the signal attenuation, \( \delta \) and \( d \) are the thickness and position of the water sheet, as shown in Figure 4-3. Since
the problem is symmetric, the attenuation of the water sheet at \( x = -d \) is incorporated as a factor of two in \( C(\omega, \delta, d) \). All of the constants and frequency dependent variables not related to the thickness or position of the water sheets are combined into \( F(\omega) \). Note that \( k_x = \frac{\pi}{2a} \) is not dependent on frequency and is fixed by the geometry. If there are multiple pairs of water sheets, Equation 7.12 would become

\[
C(\omega, \delta_1, \ldots \delta_n, d_1, \ldots d_n) = \sum_{i=1}^{n} F(\omega)\delta_i \cos^2(k_x d_i) \tag{7.13}
\]

For the case with a single symmetric pair of water sheets, let the vector of estimates of the parameters be

\[
\hat{x} = \begin{bmatrix} \hat{\delta} \\ \hat{d} \end{bmatrix} \tag{7.14}
\]

The matrix of the nonlinear derivatives \( \frac{\partial C}{\partial \delta} \) and \( \frac{\partial C}{\partial d} \) evaluated at the initial estimate of the parameters \( x = \hat{x}_o \) is given by

\[
A_o = \left[ \frac{dC(x)}{dx} \right]_{x=\hat{x}_o}
\]

\[
= \begin{bmatrix}
F(\omega_1) \cos^2(k_x \hat{d}_o) & -F(\omega_1)k_x \hat{\delta}_o \cos(k_x \hat{d}_o) \sin(k_x \hat{d}_o) \\
\vdots & \vdots \\
F(\omega_m) \cos^2(k_x \hat{d}_o) & -F(\omega_m)k_x \hat{\delta}_o \cos(k_x \hat{d}_o) \sin(k_x \hat{d}_o)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
F(\omega_1) \\
\vdots \\
F(\omega_m)
\end{bmatrix}
\begin{bmatrix}
\cos^2(k_x \hat{d}_o) & -k_x \hat{\delta}_o \sin(2k_x \hat{d}_o)
\end{bmatrix} \tag{7.16}
\]

Unfortunately, since \( k_x \) is the same for all frequencies, the columns of \( A_o \) are multiples of each other and \( A_o \) does not have full column rank. \( A_o^T A_o \) is singular and cannot be inverted. The parameters \( d \) and \( \delta \) cannot be solved for independently of each other, although if one of the two variables is known fairly well, an estimate can be obtained for the other. Further, if multiple water sheets are placed at different positions, ALL of the columns of the resulting \( A_o \) matrix will be linearly dependent. At most, only the estimate of only one sheet’s position or thickness can be found.
### 7.2.2 Rectangular Waveguide Loaded with Sapphire, XLPE, and Water Sheets

With the sapphire dielectric positioned inside the waveguide as shown in Figure 4-7, the electric field in the surrounding XLPE can be sinusoidal or evanescent (decaying) depending on the frequency and the thickness of the sapphire dielectric. The transverse wavenumbers \(k_x\)'s) in both of these cases varies with frequency, unlike the previous case, and the columns of the matrices will not be linearly dependent. When there is one pair of symmetric water sheets embedded in the XLPE, the perturbation model for the attenuation is

\[
C(\omega, \delta, d) = \frac{\omega \epsilon''_w \delta h E_{20}^2 |\sin k_{x2}(a - d)|^2 e^{-2k''_z z}}{P_2(z)} 10 \log_{10} e \tag{7.17}
\]

\[
= F(\omega)\delta |\sin k_{x2}(a - d)|^2 \tag{7.18}
\]

where

\[
F(\omega) = \frac{\omega \epsilon''_w \delta h E_{20}^2 e^{-2k''_z z}}{P_2(z)} 10 \log_{10} e \tag{7.19}
\]

and \(P_2(z) = P_{21} + P_{22} = P_A(E_1, k_{x1}, b) + P_C(E_2, k_{x2}, b)\). \(P_{21}\) and \(P_{22}\) are the time averaged powers flowing in the sapphire and XLPE regions and \(P_A\) and \(P_C\) are calculated in Section 4.5. Since the problem is symmetric, the attenuation of the water sheet at \(x = -d\) is incorporated as a factor of two in \(C(\omega, \delta, d)\). If are were multiple pairs of water sheets, Equation 7.18 would become

\[
C(\omega, \delta, d_1, d_2, \ldots, d_n) = \sum_{i=1}^{n} F(\omega)\delta_i |\sin k_{x2}(a - d_i)|^2 \tag{7.20}
\]

The solutions for the electric field mode shape and attenuation due to one symmetric pair of water sheets are given in Sections 4.4.1 and 5.3. In this case, the transverse wavenumbers \(k_{x1}\) (sapphire) and \(k_{x2}\) (XLPE) are not fixed and vary with frequency. For the evanescent modes, \(k_{x2}\) is primarily imaginary, indicating a decaying rather than a sinusoidal mode shape in the XLPE region.

Analogous to Equations 7.14 and 7.15, the vector of parameter estimates is defined as

\[
\hat{x} = \begin{bmatrix}
\hat{\delta} \\
\hat{d}
\end{bmatrix} \tag{7.21}
\]
and a matrix of the nonlinear derivatives evaluated at the initial estimate \( x = \mathbf{x}_0 \) is,

\[
A_o = \left[ \frac{dC(x)}{dx} \right]_{x=\mathbf{x}_0} = \begin{bmatrix}
F(\omega_1)\sin k_{x2}(a-d_o)\cos k_{x2}(a-d_o) & -F(\omega_1)d_o2\text{Real}\{k_{x2}k_{x2}^*(a-d_o)\cos k_{x2}(a-d_o)\}
\vdots & \vdots 
F(\omega_m)\sin k_{x2}(a-d_o)\cos k_{x2}(a-d_o) & -F(\omega_m)d_o2\text{Real}\{k_{x2}k_{x2}^*(a-d_o)\cos k_{x2}(a-d_o)\}
\end{bmatrix}
\]

(7.23)

where \( k_{x2} \) is a function of frequency \( \omega \). Using Equation 7.10 and an initial estimate for \( \mathbf{x}_0 \), the nonlinear least squares algorithm can be used to successively approximate our parameter vector \( x \)

\[
\mathbf{x}_{i+1} = \mathbf{x}_i + (A_i^T A_i)^{-1}A_i^T[M - C(\mathbf{x}_i)]
\]

(7.24)

The solution is found in an iterative process, terminating when the change in the parameters \( \mathbf{x}_{i+1} - \mathbf{x}_i \) is acceptably small. This nonlinear least squares algorithm is implemented in Matlab and provided in Appendix C.

### 7.3 Estimation of Location and Thickness of Symmetric Pairs of Sheets

#### 7.3.1 From Measured Attenuation Data

Since position and thickness of the symmetric water sheets in the single region dielectric geometry cannot be estimated with the nonlinear least squares method, a linear algorithm is used to predict the thickness given the location. In the experiments detailed in Chapter 6, the position of the symmetric pair of water sheets is known with reasonably accuracy, and the attenuation results for 4000 frequencies with water sheets at three positions inside the waveguide are given in Figure 6-3. In reality, the water sheets were located in the regions (a) \( 3 \text{ mm} < d < 3.4 \text{ mm} \), (b) \( 6 \text{ mm} < d < 6.4 \text{ mm} \), and (c) \( 9 \text{ mm} < d < 9.4 \text{ mm} \). The position isn’t known with any greater accuracy due to the air gaps needed for the XLPE to slide easily into the waveguide.

Using a linear least squares algorithm on the measured attenuation data with these ranges of positions, the estimated water sheet thicknesses were (a) \( 7.70 \mu \text{m} < \delta < 8.11 \mu \text{m} \), (b) \( 8.14 \mu \text{m} < \delta < 9.24 \mu \text{m} \), and (c) \( 10.5 \mu \text{m} < \delta < 14.9 \mu \text{m} \). These results essentially are the
same regardless of whether the measured attenuation data or the quadratic curves fit to the measured data were used as the input to the linear least squares algorithm. The weight of the water in the water sheet results in a calculated thickness of $11.7\mu m < \delta < 12.4\mu m$ (Table 6.2). The measured attenuation is slightly lower in some cases than predicted by the theoretical models given the amount of water present in the sheets, and it is therefore not unexpected that the estimation of the sheet thickness is low as well.

For the two region dielectric geometry, where the electromagnetic field mode shape changes with frequency, the nonlinear least squares method can be applied to estimate the thickness and position of symmetric water sheets. Attenuation results for 4000 frequencies are given in Figure 6-3 for three different thicknesses of sapphire placed in the center of the waveguide filled with XLPE. The water sheets were placed within the XLPE in the regions (a) $3\, \text{mm} < d < 3.4\, \text{mm}$, (b) $6\, \text{mm} < d < 6.4\, \text{mm}$, and (c) $9\, \text{mm} < d < 9.4\, \text{mm}$. Again, the uncertainty in the position is due to the air gaps necessary for the XLPE to slide easily into the waveguide.

In the first test on the data, the position $d$ was provided to the least squares algorithm and was not allowed to change. The water sheet thickness was then calculated from the measured attenuation data using linear least squares, since the loss is linear in water sheet thickness. The estimated water sheet thicknesses, as well as the thickness measured from the moisture content in the water sheet are given in Table 7.1. If the fitted (rather than measured) data were used, the results are essentially identical. As in the case with the simple dielectric loaded waveguide, the estimated sheet thicknesses are generally in the right range or slightly lower than the measured thicknesses. The errors between the measured thicknesses and the closest extreme of the estimated thickness range between a 21% overestimate to a 33% underestimate. As discussed in Section 6.5, the order of the attenuation measurements within the same width sapphire experiments was c-b-a. The measured water sheet thicknesses can be corrected for the timing of the attenuation measurements, rather than using just the average thickness. In this case, the errors are reduced to a 12% overestimate and a 28% underestimate. The (*) denotes a measurement problem where the measured curve has negative attenuation, which is not physically realizable. The field strength at the location of the water sheet in this case is very small and the measured attenuation was close to zero.

Next, both $\delta$ and $d$ were allowed to vary in the nonlinear least squares algorithm. The
Table 7.1: Table of estimated and measured water sheet thicknesses, given the position, the measured attenuation data, and \( d = (a) \, 3 \text{ mm} < d < 3.4 \text{ mm}, \) \( (b) \, 6 \text{ mm} < d < 6.4 \text{ mm}, \) and \( (c) \, 9 \text{ mm} < d < 9.4 \text{ mm}. \)

<table>
<thead>
<tr>
<th>( \frac{1}{2} ) Sapphire thickness (b)</th>
<th>position in waveguide</th>
<th>range of estimated ( \delta ) (( \mu \text{m} ))</th>
<th>measured ( \delta ) (( \mu \text{m} )) from moisture content</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2286 mm ( a )</td>
<td>6.66 ( &lt; \delta &lt; 7.38 )</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>0.2286 mm ( b )</td>
<td>8.75 ( &lt; \delta &lt; 10.2 )</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>0.2286 mm ( c )</td>
<td>9.91 ( &lt; \delta &lt; 14.3 )</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>0.4572 mm ( a )</td>
<td>5.98 ( &lt; \delta &lt; 6.99 )</td>
<td>9.1</td>
<td></td>
</tr>
<tr>
<td>0.4572 mm ( b )</td>
<td>8.98 ( &lt; \delta &lt; 10.8 )</td>
<td>9.1</td>
<td></td>
</tr>
<tr>
<td>0.4572 mm ( c )</td>
<td>11.4 ( &lt; \delta &lt; 16.5 )</td>
<td>9.1</td>
<td></td>
</tr>
<tr>
<td>0.6858 mm ( a )</td>
<td>4.61 ( &lt; \delta &lt; 5.66 )</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>0.6858 mm ( b )</td>
<td>6.85 ( &lt; \delta &lt; 8.49 )</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>0.6858 mm ( c )</td>
<td>0.314 ( &lt; \delta &lt; 0.618^* )</td>
<td>7.3</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2: Table of estimated and measured water sheet thicknesses and locations in the waveguide, given the fitted attenuation data. The estimate (*) was outside the waveguide and corresponds to a measurement error.

<table>
<thead>
<tr>
<th>( \frac{1}{2} ) Sapphire thickness (b)</th>
<th>position in waveguide</th>
<th>measured ( \delta ) (( \mu \text{m} ))</th>
<th>nominal ( d ) (mm)</th>
<th>estimated ( \delta ) (( \mu \text{m} ))</th>
<th>estimated ( d ) (mm)</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2286 mm ( a )</td>
<td>11.0</td>
<td>3.2</td>
<td>5.36</td>
<td>2.10</td>
<td>220</td>
<td></td>
</tr>
<tr>
<td>0.2286 mm ( b )</td>
<td>11.0</td>
<td>6.2</td>
<td>4.04</td>
<td>3.54</td>
<td>192</td>
<td></td>
</tr>
<tr>
<td>0.2286 mm ( c )</td>
<td>11.0</td>
<td>9.2</td>
<td>0.597</td>
<td>1.87</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>0.4572 mm ( a )</td>
<td>9.1</td>
<td>3.2</td>
<td>4.95</td>
<td>2.52</td>
<td>433</td>
<td></td>
</tr>
<tr>
<td>0.4572 mm ( b )</td>
<td>9.1</td>
<td>6.2</td>
<td>3.14</td>
<td>3.47</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>0.4572 mm ( c )</td>
<td>9.1</td>
<td>9.2</td>
<td>0.967</td>
<td>4.44</td>
<td>19.2</td>
<td></td>
</tr>
<tr>
<td>0.6858 mm ( a )</td>
<td>7.3</td>
<td>3.2</td>
<td>3.94</td>
<td>2.69</td>
<td>78.8</td>
<td></td>
</tr>
<tr>
<td>0.6858 mm ( b )</td>
<td>7.3</td>
<td>6.2</td>
<td>4.42</td>
<td>5.16</td>
<td>28.8</td>
<td></td>
</tr>
<tr>
<td>0.6858 mm ( c )</td>
<td>7.3</td>
<td>9.2</td>
<td>4.36</td>
<td>11.4*</td>
<td>806</td>
<td></td>
</tr>
</tbody>
</table>

resulting estimates for thickness and position are given in Table 7.2. In general, all of the estimates for the position \( d \) are too close to the center of the waveguide, and the estimates for the thickness \( \delta \) are too small. (The estimates for \( \delta \) are smaller than before since, as the water sheet moves closer to the center, the effect on the attenuation is more intense.) However, with the exception of the (*)'d estimate, where the estimate was outside the waveguide, the general trend for the estimate of the location \( d \) is to get further from the center in the same manner as the actual position. The difference in the estimated parameters when the measured attenuation data is used with the algorithm rather than the fitted data is at most 3%, indicating that the quadratic curve fit to the measured data was a good match.

A slight error in the measured curve shape can strongly affect the nonlinear estimate of...
the position, and this is investigated in Section 7.3.2. The nonlinear least squares algorithm had difficulty converging unless the initial estimate for the position \( d \) was within 1-2 mm of the final value. A version of the nonlinear solver that tries several initial estimates within the allowable range is given in Appendix C. The solver is not as sensitive to the initial estimate for the thickness \( \delta \). Any guess within an order of magnitude will work.

The least squares error in the last column of Table 7.2 is calculated as follows. The error vector, given the parameter estimate \( x_i \) is \( e = M - C(\bar{x}_i) - A_i[x - \bar{x}_i] \), where \( M \) is the measured (or simulated) data, \( C(\bar{x}_i) \) is calculated by the nonlinear solver, and \( A_i \) is the matrix of derivatives of \( C(x_i) \). The nonlinear least squares solver is run until the estimates for \( x_i \) no longer change, and it is then assumed that \( x - \bar{x}_i = 0 \). Therefore the error vector is \( e = M - C(\bar{x}_i) \), or the difference between the measured and theoretical attenuation vectors. The calculated numerical error is \( e^T e \), or the sum of the square of these differences, and represents how well the attenuation model used by the least squares algorithm matches the measured attenuation data. It is this numerical error that the least squares algorithm is minimizing. Note that the error is summed over 4000 data points in the case of Table 7.2. When the measured data is used as the input to the nonlinear least squares solver rather than the smooth curve of the fitted data, the error is much larger (roughly a factor or 100) due to the oscillations in the data.

Once \( \delta \) and \( d \) are estimated from the measured attenuation data, these values can then be used to recalculate the theoretical attenuation and this can be compared to the original measured attenuation. This comparison is given in Figure 7-1 for the \( b = 0.4572 \) mm sapphire width experiments. The curves match quite well, indicating that the algorithm is estimating \( \delta \) and \( d \) correctly from the measured data, even though the values are not what one would expect from the known locations and measurements of the water content in the water sheets. For Figure 7-1, the maximum error between the measured and estimated curves is 16%.

7.3.2 From Simulated Attenuation Data

Since the measured attenuation data has noise and measurement errors, the nonlinear least squares solver can be further tested with simulated attenuation data to examine its behavior. Given the geometry, materials, water sheet thickness and position, two types of simulated data can be generated. The first is attenuation data calculated by the perturbation method
Figure 7-1: Attenuation calculated from the position and thickness estimates given in Table 7.2 (dashed line) compared to measured data (solid line) for 2-slab \( (b = 0.4572 \text{ mm}) \) sapphire dielectric waveguide with XLPE and water sheets located in positions a, b, and c within the XLPE.

of Section 5.3. The nonlinear least squares solver also uses this perturbation model to calculate the attenuation in Equation 7.18. A set of perturbation attenuation data was generated for the three sapphire thicknesses, with 5\( \mu \text{m} \) thick symmetric pairs of water sheets located at \( d = 2, 4, 6, 8, \) and 10 mm within XLPE region in the waveguide. To decrease computation time, only 20 frequencies uniformly distributed between 8 and 12.4 GHz were used. For all cases, the solver was given initial estimates of 10\( \mu \text{m} \) for the thickness and 5 mm for the position. The solver converged to the correct value in all cases within 12 iterations and with a least squares error of less than \( 2 \times 10^{-18} \), where the error is the sum of the squared differences between the simulated data and the least squares calculation of the attenuation. The values of \( d \) and \( \delta \) found by the solver were the same as the original values to 14 decimal places.

The exact field solutions and corresponding attenuation data were also calculated (Section 5.1) and used with the nonlinear least squares solver. The exact attenuation is slightly higher than calculated by the perturbation method due to the slight increase in field ampli-
tude at the location of the water sheet. Since the least squares solver uses the perturbation method to calculate the attenuation, it will no longer converge to the correct solution. A new set of data using the exact attenuation was generated for the three sapphire thicknesses, again using 20 frequencies, with 5μm thick symmetric water sheets located at \( d = 2, 4, 6, 8, \) and 10 mm within XLPE region in the waveguide. The solver was then given initial estimates of 10μm for the thickness and 5 mm for the position. The parameters estimated by the solver are given in Table 7.3.

<table>
<thead>
<tr>
<th>( \frac{1}{2} ) Sapphire thickness (b)</th>
<th>( \delta ) (μm)</th>
<th>( d ) (mm)</th>
<th>estimated ( \delta ) (μm)</th>
<th>estimated ( d ) (mm)</th>
<th>iterations</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2286 mm</td>
<td>5</td>
<td>2</td>
<td>4.91</td>
<td>1.90</td>
<td>9</td>
<td>3.2e-3</td>
</tr>
<tr>
<td>0.2286 mm</td>
<td>5</td>
<td>4</td>
<td>4.77</td>
<td>3.79</td>
<td>6</td>
<td>8.7e-4</td>
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<tr>
<td>0.2286 mm</td>
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<td>4.44</td>
<td>5.59</td>
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<tr>
<td>0.2286 mm</td>
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<td>3.70</td>
<td>7.38</td>
<td>8</td>
<td>2.0e-4</td>
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<tr>
<td>0.2286 mm</td>
<td>5</td>
<td>10</td>
<td>1.34</td>
<td>8.67</td>
<td>10</td>
<td>1.9e-4</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>2</td>
<td>4.94</td>
<td>1.96</td>
<td>9</td>
<td>7.4e-4</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>4</td>
<td>4.87</td>
<td>3.90</td>
<td>6</td>
<td>3.4e-4</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>6</td>
<td>4.80</td>
<td>5.84</td>
<td>7</td>
<td>3.3e-4</td>
</tr>
<tr>
<td>0.4572 mm</td>
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<td>8</td>
<td>4.40</td>
<td>7.73</td>
<td>9</td>
<td>1.0e-4</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>10</td>
<td>3.55</td>
<td>9.72</td>
<td>9</td>
<td>1.3e-7</td>
</tr>
<tr>
<td>0.6858 mm</td>
<td>5</td>
<td>2</td>
<td>4.94</td>
<td>1.97</td>
<td>7</td>
<td>2.3e-5</td>
</tr>
<tr>
<td>0.6858 mm</td>
<td>5</td>
<td>4</td>
<td>4.92</td>
<td>3.93</td>
<td>7</td>
<td>2.9e-4</td>
</tr>
<tr>
<td>0.6858 mm</td>
<td>5</td>
<td>6</td>
<td>4.91</td>
<td>5.91</td>
<td>7</td>
<td>6.7e-5</td>
</tr>
<tr>
<td>0.6858 mm</td>
<td>5</td>
<td>8</td>
<td>4.84</td>
<td>7.90</td>
<td>9</td>
<td>5.4e-5</td>
</tr>
<tr>
<td>0.6858 mm</td>
<td>5</td>
<td>10</td>
<td>3.99</td>
<td>9.82</td>
<td>11</td>
<td>6.0e-8</td>
</tr>
</tbody>
</table>

Table 7.3: Table of actual and estimated water sheet thicknesses and locations in the waveguide, given simulated attenuation data calculated from the exact field solutions.

The least squares error in Table 7.3 is now quite a bit larger than the \( 2 \times 10^{-18} \) obtained when the simulated perturbation attenuation data was used, showing that these estimated parameters no longer predict an attenuation that exactly matches the simulated data. It is interesting to note that in all cases, the estimated position is closer to the center and the estimated thickness is smaller. This same pattern was found when the solver was used with the measured data (Table 7.2). In this case, the maximum error of the position \( d \) is 13.3% and the thickness \( \delta \) is 73%, and this occurs for the single sapphire slab, with the water sheet at \( d = 10 \) mm. The exact shape of the attenuation curve is apparently quite important for the correct estimation of the position and thickness of the water sheets. Also, results are best for the thickest sapphire dielectric which has the largest change in electric field mode shape with frequency. Note that when comparing the least squares error between Tables 7.2
and 7.3, 4000 data points were used in the first case, and only 20 in the second.

Consider next the extreme cases with symmetric water sheets placed either very close to the sapphire dielectric or the outside edge of the waveguide. In both cases, the solver is much more sensitive to the initial estimate for $d$, and in general it requires that this estimate be within 1-2 mm of the final value. If a 5 $\mu$m thick sheet is placed at $d = 11$ mm (the waveguide width is 11.4 mm), the solver finds the correct position and thickness given the perturbation data. The results when using the exact attenuation data are given in Table 7.4. The electric fields are close to zero near the outside of the waveguide, and would be smallest in the $b = 0.6858$ mm case, for which the solver did not converge on a solution.

<table>
<thead>
<tr>
<th>$\frac{1}{2}$ Sapphire thickness (b)</th>
<th>$\delta$ (um)</th>
<th>$d$ (mm)</th>
<th>estimated $\delta$ (um)</th>
<th>estimated $d$ (mm)</th>
<th>iterations</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2286 mm</td>
<td>5</td>
<td>11</td>
<td>0.42</td>
<td>10.0</td>
<td>5</td>
<td>2.4e-7</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>11</td>
<td>1.17</td>
<td>10.6</td>
<td>8</td>
<td>5.3e-10</td>
</tr>
<tr>
<td>0.6858 mm</td>
<td>5</td>
<td>11</td>
<td>X</td>
<td>X</td>
<td>2</td>
<td>X</td>
</tr>
</tbody>
</table>

Table 7.4: Table of actual and estimated water sheet thicknesses and locations for symmetric pairs of water sheets close to the edge of the waveguide, given simulated data calculated from the exact field solutions. The solver did not converge in the case of the $b = 0.6858$ mm waveguide.

The tendency for the solver to find solutions with a smaller $d$ and $\delta$ than the actual positions when given simulated exact attenuation data causes difficulty at the other extreme, where the symmetric water sheets are placed very close to the sapphire. Table 7.5 shows the results for the solver when the water sheets are placed directly adjacent to and a small distance away from the sapphire. The least squares errors are somewhat larger than in

<table>
<thead>
<tr>
<th>$\frac{1}{2}$ Sapphire thickness (b)</th>
<th>$\delta$ (um)</th>
<th>$d$ (mm)</th>
<th>estimated $\delta$ (um)</th>
<th>estimated $d$ (mm)</th>
<th>iterations</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2286 mm</td>
<td>5</td>
<td>0.2286</td>
<td>4.97</td>
<td>0.0433*</td>
<td>6</td>
<td>2.30e-2</td>
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<tr>
<td>0.2286 mm</td>
<td>5</td>
<td>0.4</td>
<td>4.97</td>
<td>0.237</td>
<td>9</td>
<td>2.03e-2</td>
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<td>0.4371*</td>
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<td>2.85e-2</td>
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<td>0.4816</td>
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<td>2.62e-2</td>
</tr>
<tr>
<td>0.6858 mm</td>
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<td>0.6858</td>
<td>5.03</td>
<td>0.6914</td>
<td>6</td>
<td>6.24e-3</td>
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<tr>
<td>0.6858 mm</td>
<td>5</td>
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<td>5.03</td>
<td>0.7056</td>
<td>6</td>
<td>5.94e-3</td>
</tr>
</tbody>
</table>

Table 7.5: Table of actual and estimated water sheet thicknesses and locations for water sheets close to the sapphire, given simulated data calculated from the exact field solutions. The estimates of $d$ marked with a (*) are not physical, as they are inside the sapphire region.
the previous tests, indicating that there is a larger difference between the simulated exact attenuation data and that calculated by the least squares solver using the perturbation method. In the case of the thicker \( b = 0.6858 \) mm sapphire dielectric, the estimated position is slightly larger than the actual position for water sheets placed adjacent to the sapphire dielectric and therefore is a better choice in detecting surface water near the interface between the sapphire and XLPE. There is clearly a trade-off between using thin sapphire dielectric having a larger penetration depth and the thick sapphire dielectric giving better position estimates for surface moisture.

7.3.3 Sensitivity of the Algorithm

To test the algorithm in the presence of noisy data, the simulated perturbation attenuation data of the previous section is modified with noise. The perturbation data is used since this data (without noise) yields exact parameter values when used with the nonlinear least squares algorithm. First, all attenuation values were modified by a fixed percentage of 10%. The algorithm estimates the position correctly, however, there is a 5% error in the thickness of the sheets since the algorithm is linear in sheet thickness and there are two symmetric sheets.

Next, the simulated perturbation data was modified by random noise values between ±1% or ±10% of the attenuation values. The same random noise is used to calculate the noisy attenuation in all cases so that they may be compared. The results of parameter estimation with this noisy data are summarized in Table 7.6. It is interesting to note that the errors in the parameter estimates for the \( b = 0.2286 \) mm and \( b = 0.4572 \) mm sapphire dielectrics increase as the water sheets move further away from the center of the waveguide, but in the case of the \( b = 0.6858 \) mm, the difference between the actual and estimated positions actually decreases further from the center. For this last case, the maximum errors for both \( d \) and \( \delta \) with the 1% noise data is \(~1\%\) and with the 10% noise data, the maximum error in \( d \) is \(~8\%\) and in \( \delta \) is \(~7\%\). The thicker sapphire dielectric geometry is clearly advantageous in the presence of noise.
<table>
<thead>
<tr>
<th>( \frac{1}{2} ) Sapphire thickness (b) (mm)</th>
<th>( \delta ) (( \mu )m)</th>
<th>( d ) (mm)</th>
<th>( \delta ) (( \mu )m) ( \pm 1% ) noise</th>
<th>( d ) (mm) ( \pm 1% ) noise</th>
<th>( \delta ) (( \mu )m) ( \pm 10% ) noise</th>
<th>( d ) (mm) ( \pm 10% ) noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2286 mm</td>
<td>5</td>
<td>2</td>
<td>4.95</td>
<td>1.95</td>
<td>4.53</td>
<td>1.54</td>
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<td>3.95</td>
<td>4.36</td>
<td>3.48</td>
</tr>
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<td>4.88</td>
<td>5.93</td>
<td>3.99</td>
<td>5.36</td>
</tr>
<tr>
<td>0.2286 mm</td>
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<td>8</td>
<td>4.72</td>
<td>7.90</td>
<td>3.13</td>
<td>7.11</td>
</tr>
<tr>
<td>0.2286 mm</td>
<td>5</td>
<td>10</td>
<td>3.74</td>
<td>6.78</td>
<td>1.14</td>
<td>4.87</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>2</td>
<td>4.96</td>
<td>1.98</td>
<td>4.95</td>
<td>1.78</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>4</td>
<td>4.96</td>
<td>3.98</td>
<td>4.62</td>
<td>3.79</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>6</td>
<td>4.95</td>
<td>5.98</td>
<td>4.55</td>
<td>5.97</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>8</td>
<td>4.92</td>
<td>7.98</td>
<td>4.29</td>
<td>7.76</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5</td>
<td>10</td>
<td>4.66</td>
<td>9.95</td>
<td>2.86</td>
<td>9.56</td>
</tr>
<tr>
<td>0.6858 mm</td>
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<td>1.98</td>
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<td>1.85</td>
</tr>
<tr>
<td>0.6858 mm</td>
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<td>3.99</td>
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<td>3.89</td>
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<tr>
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<td>6</td>
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<td>5.99</td>
<td>4.79</td>
<td>5.92</td>
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<tr>
<td>0.6858 mm</td>
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<td>8.00</td>
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<tr>
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<td>4.98</td>
<td>10.00</td>
<td>4.87</td>
<td>9.98</td>
</tr>
</tbody>
</table>

Table 7.6: Results of parameter estimation when perturbation attenuation data is modified by random noise values for cases of \( \pm 1\% \) and \( \pm 10\% \) of the attenuation values.

### 7.4 Prediction of Position of Multiple Sheets from Simulated Data

When the waveguide contains multiple symmetric pairs of water sheets, the nonlinear solver computes the attenuation with the perturbation method for each pair of sheets and the total attenuation is the sum over all pairs, as given in Equation 7.20. The matrix of derivatives for the solver (Equation 7.23) has more columns, two for each pair of water sheets. To find the values of \( \delta_i \) and \( d_i \) which provide the best fit to the provided attenuation data, the following method is used:

1. For multiple sheets, the initial estimates for position become more important. The XLPE region (where the water sheets are located) is broken into \( n \) initial values, which increases with the number of pairs of sheets. Generally, \( n \) is some constant factor multiplied by the number of pairs of sheets.

2. Each sheet’s position \( d_i \) can have any of the \( n \) initial values, creating a set of initial values to try.

3. For each set of initial positions, try three different initial thicknesses, 1, 10, and 100 \( \mu \)m.

4. Given a maximum number of sheets \( N \), call the solver with all the initial estimates for each of the \( N \) sheets.
5. For every solution found, compare with previous solutions. If the calculated least squares error $e^Te$ is less than that of the previous solution, replace the previous solution with the new one.

This search method is implemented in Appendix C. The algorithm was tested only with simulated perturbation data since the evanescent configuration was only solved exactly for the case of a single pair of sheets. Solving exactly for the multiple symmetric pairs of sheets is quite complicated since each water sheet pair adds two regions to the original two region problem.

The first test of the algorithm was with waveguides loaded with 5 different sapphire widths, with $b = 0.2286, 0.4572, 0.6858, 0.9144, \text{ and } 1.0 \text{ mm, where } b$ is 1/2 the width of the sapphire. In each case, simulated attenuation data was generated for pairs of 5$\mu$m thick water sheets placed at $b + 0.1, b + 1.1, b + 2.1, \ldots, b + 10.1 \text{ mm symmetrically in the waveguide. The attenuations were added for various combinations of sheets and provided to the least squares estimator. In general, with } n = 3 \text{ (3 initial estimates for } d \text{ for one pair, 6 initial estimates for each } d_i \text{ with two pairs of sheets, etc.), up to four pairs of sheets can be estimated without difficulty. Occasionally, the solution with the correct number of sheets was not found and the best parameter fit was found with one fewer pair of sheets than were used to generate the simulated data. In no case was any solution found with more water sheets than were used to generate the data.}

Given combinations of two pairs of sheets, the only difficulty in estimating the position and thickness occurred when one pair was placed at $b + 0.1 \text{ mm and the other at } b + 10.1 \text{ mm, particularly in the cases with the thicker sapphire dielectrics. This was likely due to the small field amplitudes beyond } x = 11 \text{ mm inside the waveguide. With combinations of three pairs, a similar pattern occurred. Given one pair at } b + 0.1 \text{ mm, the remaining sheets were placed in various combinations within the waveguide. If these sheets were placed at } b + 9.1 \text{ mm and } b + 10.1 \text{ mm, the solver had difficulty finding both in the cases with thicker sapphire dielectrics. Occasional difficulty in finding solutions, particularly if two of the pairs of sheets were placed at } b + 0.1 \text{ mm and } b + 1.1 \text{ mm were solved if more initial guesses were allowed for the initial estimates of } d_i.\]

The next test of the algorithm was with waveguides loaded with 2 different sapphire widths, with $b = 0.2286, \text{ and } 1.0 \text{ mm. In these cases, simulated attenuation data was generated for 5$\mu$m thick water sheets placed at } b + 0.1, \ldots, b + 0.9, \text{ and } 10.3, \ldots, 11.3 \text{ mm in}
0.1 mm increments. The attenuations of adjacent pairs of sheets were added and provided to the nonlinear least squares solver. In the $b = 0.2286$ mm case, the adjacent pairs of sheets between 10.3 and 11.3 mm were indistinguishable and the solver only found one of the two pair of water sheets. This estimated position was closer to the center and thicker than the original two positions. With $b = 1.0$ mm, no solution was found in the XLPE region for pairs of sheets less than $b + 0.7$ mm (the solution was in the sapphire region, which the algorithm does not count as a solution). For all other adjacent pairs, only one solution was found, and again, this solution was closer to the center and thicker than the individual pairs of sheets.

The results of these tests show that estimating the position of single symmetric pairs of water sheets is most reliable. When two or more pairs of sheets are present, difficulty in predicting their positions and thicknesses arises when the sheets are closely spaced or placed in regions where the electric field strength is low.

### 7.5 The Uniform Distribution “Moist” XLPE Problem

Consider the problem where the XLPE region has a uniform distribution of moisture, where it is desired to quantify the amount of moisture in parts per million (ppm). Since small amounts of moisture change the attenuation by only fractions of a dB, sensitive equipment would be needed to measure the attenuation. The attenuation in dB for a single region dielectric is given by

$$\text{Attenuation/meter} = \frac{\omega^2 \mu_e \varepsilon''_{\text{PE}}(\omega)}{\text{Real}(k_z(\omega))} 10 \log_{10} e$$  \hfill (7.25)

where Equations 4.19, 5.10, and 5.17 were used. For the two region (sapphire and XLPE) problem, the attenuation is given by

$$\text{Attenuation/meter} = \frac{P_{d2}(z)}{P_2(z)} 10 \log_{10} e$$  \hfill (7.26)

where $P_2$ is the power flowing in the waveguide, calculated in Equation 4.55, and $P_{d2}$ is the power dissipated in both the sapphire and XLPE regions, calculated in Equation 5.25. In the discussion that follows, the two region configuration is used as the example.

Since sapphire will not absorb moisture, the attenuation due to the sapphire dielectric
loss given in Figure 6-1 can only change due to the change in mode shape resulting from
the moisture in the XLPE. The change in attenuation in the waveguide when moisture is
added to the XLPE can be described as

\[
\text{Change} = \text{atten}(\text{sapphire, wet}) + \text{atten}(\text{XLPE, wet}) - \text{atten}(\text{sapphire, dry}) - \text{atten}(\text{XLPE, dry})
\] (7.27)

Assuming the change in the dielectric constant of the XLPE does not affect the mode
shape significantly, the perturbation method can be applied here. The attenuation in the
sapphire with and without the moisture in the XLPE will be the same and these terms will
cancel. The attenuation in the XLPE is proportional to the imaginary part of the dielectric
constant. Therefore, the change in attenuation can be written

\[
\frac{\text{Change}}{\text{meter}} = (\varepsilon_{pe, wet}'' - \varepsilon_{pe}'') \frac{P_{d22}(z)}{\varepsilon_{pe}''} P_2(z) 10 \log_{10} e
\] (7.28)

where \( P_{d22} \) is the power dissipated only in the XLPE region and both \( P_{d22} \) and \( P_2 \) are
calculated using the known dry dielectric constant for the XLPE (\( \varepsilon_{pe} \)).

Since polyethylene does not absorb large amounts of moisture, the Haus-Melcher model
(Section 3.2.5) can be used to model the dielectric constant of the moist XLPE (\( \varepsilon_{pe, wet} \)).
The dielectric constant is given by

\[
\varepsilon_{pe, wet} = \varepsilon_{pe} \left( 1 + 3x \left[ \frac{\varepsilon_w - \varepsilon_{pe}}{\varepsilon_w + 2\varepsilon_{pe}} \right] \right)
\] (7.29)

where \( x \) the volume density of water in the XLPE. The equation that relates the volume
density to parts per million (ppm) is Equation 3.27, which can be simplified to \( x \approx \text{ppm} \times 10^{-6} \times 0.925 \) and results in less than 1% error with concentrations up to \( 10^5 \) ppm.

Given the dielectric constant \( \varepsilon = \varepsilon' - j\varepsilon'' \), the change in imaginary dielectric constant is

\[
\varepsilon_{pe, wet}'' - \varepsilon_{pe}'' = -\text{Imag} \{ \varepsilon_{pe, wet} - \varepsilon_{pe} \} \quad (7.30)
\]
\[
= -\text{ppm} \times 10^{-6} \times 0.925 \times 3 \text{Imag} \left\{ \varepsilon_{pe} \left[ \frac{\varepsilon_w - \varepsilon_{pe}}{\varepsilon_w + 2\varepsilon_{pe}} \right] \right\} \quad (7.31)
\]

Therefore, Equation 7.28 is linear in moisture concentration (ppm) and the linear least
squares algorithm can be used to find the best value of concentration, given an attenuation
measurement. Programs that implement this algorithm can be found in Appendix D. The
same methodology can be used to find the concentration of moisture in the single region dielectric configuration.

For the two region geometry, XLPE dielectric data was generated using the Haus-Melcher model for moisture concentrations of 1000, 5000, and 10000 ppm. This data was then used to calculate the loss using the perturbation method (Equation 7.28) and exact field calculations. These simulated measurements were then used by the linear least squares solver to estimate the original moisture concentration and the results are tabulated in Table 7.7. The maximum error using the perturbation model in the least squares estimator rather than the exact model is 2.8%.

<table>
<thead>
<tr>
<th>( \frac{1}{2} ) Sapphire thickness (b)</th>
<th>actual ppm</th>
<th>perturbation model</th>
<th>exact fields</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4572 mm</td>
<td>1000</td>
<td>1000</td>
<td>978</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>5000</td>
<td>5002</td>
<td>488</td>
</tr>
<tr>
<td>0.4572 mm</td>
<td>10000</td>
<td>10008</td>
<td>9722</td>
</tr>
</tbody>
</table>

Table 7.7: Results for the linear least squares estimate of uniform moisture, using the Haus-Melcher dielectric model (Equation 7.29) with 1000, 5000, and 10000 ppm of moisture.

### 7.6 Conclusions

An algorithm was developed to estimate the thickness and position of symmetric pairs of water sheets and uniform moisture using the least squares method to find the parameters that result in the best fit of the experimental results to theoretical values. The nonlinear least squares solver correctly estimated the locations and thicknesses of symmetric pairs of water sheets given simulated attenuation data calculated using the perturbation method. When multiple sheets were present, initial estimates of the positions must be given which were close to the actual values in order for the algorithm to converge. This is typical of nonlinear systems.

When the measured data or simulated data from exact calculations were used, the algorithm estimated that the water sheet positions were closer than the actual and the thicknesses are smaller. The best results using simulated data were for the waveguides with the thicker sapphire dielectrics, indicating that the more the electric field mode shape changes with frequency, the easier it is to estimate the correct position. However, the dielectric cannot be too thick, as the dielectric tends to concentrate the fields, resulting in
a much smaller penetration depth into the XLPE samples.

To refine the nonlinear least squares algorithm, it should be possible to characterize the discrepancy between the attenuation calculated by the perturbation method and exact field expressions. For example, in Figure 5-2, the difference in the attenuations calculated with these two methods appears to be a simple scaling factor. A correction factor dependent on the dielectric thickness and estimated water sheet thickness could be added to the algorithm. Alternatively, a weighted least squares algorithm could be investigated, where the estimate of one of the parameters is weighted more heavily than the others in the algorithm. A more difficult approach would be to build a nonlinear least squares estimator based on the exact field solutions and attenuation calculations.

The uniform moisture distribution problem can be solved with a linear least squares algorithm, assuming that the moisture concentrations are below $5 \times 10^4$ ppm where the Haus-Melcher model is valid for calculating the effective dielectric constant. Using simulated attenuation data calculated by the perturbation method and using exact field expressions, this method worked quite well at estimating the moisture concentration. To estimate non-uniform moisture measurements, a nonlinear least squares algorithm would be needed, along with refined effective medium models and modified attenuation calculations using the perturbation method.
Chapter 8

Summary and Applications

8.1 Summary of Thesis

A methodology was developed where spatially dependent microwave electric fields were used to probe cross-linked polyethylene (XLPE) insulation such that the spatial distribution and concentration of moisture in polyethylene insulation could be determined. Evanescent fields generated by a dielectrically loaded rectangular waveguide provided the sensing mechanism for the experiments, where the penetration depth of the evanescent fields changes with frequency in the microwave experiments. Moisture in the polyethylene absorbs energy at specific frequencies from the microwave through the infrared range and the absorption vs. frequency characteristic were used to estimate the distribution and concentration of the moisture.

A similar method (FTIR spectroscopy) was used to probe for moisture in the infrared. However, although measurement results indicated the presence of water, little spatial information about the moisture distribution resulted due to the small penetration depth of the fields in the infrared frequency range. The microwave experiments were designed around the dielectric waveguide with longer wavelength evanescent fields analogous to the FTIR experiments.

Significant background work was necessary to characterize the materials used in the microwave experiments. The dielectric constants of the cable and splice XLPE and semiconducting tapes were measured at discrete frequencies using a hollow waveguide and the standing wave method described in Section 3.1. Effective medium models were explored to model the dielectric constant of the XLPE into which moisture has diffused. It was found
that the simple Haus-Melcher model (Section 3.2.5) gives good results when the moisture is below $5 \times 10^4$ parts per million (ppm). Water trees were grown sheet samples of the XLPE and the Haus-Melcher model was used to calculate the effective complex dielectric constant for treed XLPE insulation. The attenuation due to these treed sheets was estimated to be below the sensitivity of the microwave equipment used in the experiments. Therefore, a "water sheet" method was developed to insert spatial impulses of water between XLPE sheets in the waveguide.

Microwave measurements of attenuation were made in a sapphire-PE loaded rectangular waveguide with symmetric pairs of "water sheets" containing known amounts of moisture placed in various locations within the XLPE. The sapphire provided the high dielectric constant needed to generate evanescent fields in the XLPE region. Perturbation and exact models were developed to model the expected attenuation of symmetric pairs of water sheets and these theoretical results were compared to the measured attenuation data. It was found that the measured attenuations were similar to those predicted by theory, generally within a factor of two in dB and having similar curve shapes and slopes.

An "inverse" algorithm was developed to estimate the thickness and position of symmetric pairs of water sheets and uniform moisture using the least squares method to find the parameters that result in the best fit of the experimental results to theoretical values. The method of nonlinear least squares correctly estimated the locations and thicknesses of symmetric pairs of water sheets given simulated attenuation data calculated using the perturbation method. However, when the measured data or simulated data from exact calculations was used, the algorithm estimated that the water sheet positions were closer than the actual and the thicknesses were smaller. Also, the initial estimates of the position provided to the nonlinear least squares solver must be within 1-2 mm of the final values in order for the algorithm to converge. This is typical of nonlinear systems.

A very important result was found when the single region dielectric and two region dielectric results are compared. The mode shape does not change with frequency in the case of the single dielectric region, and the nonlinear least squares algorithm cannot be used. The nonlinear least squares algorithm can only be used in the case of evanescent fields, such as those generated in the XLPE regions of the sapphire loaded rectangular waveguide. These evanescent fields change with frequency, resulting in a smaller penetration depth at higher frequencies. Effects of this changing penetration depth can be seen in Figures 6-3
and 6-4, where the slope of the attenuation decreases with frequency in the case of the thicker dielectrics. As the frequency is raised, the field amplitudes drop off more sharply in the evanescent region. With a lower field amplitude, the attenuation is lowered, even though the water has more loss at the higher frequencies. This change in mode shape with frequency is what makes the evanescent approach unique.

The best results using simulated data are for the waveguides with the thicker sapphire dielectrics, indicating that the more the electric field mode shape changes with frequency, the easier it is to estimate the correct position. However, the sapphire dielectric cannot be too thick, as the high dielectric constant tends to concentrate the fields, resulting in a much smaller penetration depth into the XLPE samples.

Given the discrepancy between the attenuation calculated with the perturbation method and exact field expressions, it will be necessary to refine the nonlinear least squares algorithm. This modification could take the form of a correction factor added into the perturbation method that is a function of frequency, dielectric thickness, and/or estimated water sheet thickness or position. A more difficult approach would be to build a nonlinear least squares estimator based on the exact field solutions and attenuation calculations. The thickness and position of the water sheets is used to calculate the wavenumbers \(k_x\)'s and \(k_z\)'s), and the derivatives used for the nonlinear least squares method would therefore be very complicated.

In the case of a uniform moisture distribution in the XLPE region, the moisture content can be estimated with good results using a linear least squares algorithm, assuming that the moisture concentrations are below \(5 \times 10^4\) ppm where the Haus-Melcher model is valid for calculating the effective dielectric constant. To extend this to the estimation of non-uniform moisture measurements, a nonlinear least squares algorithm would be needed, along with refined effective medium models and modified attenuation calculations using the perturbation method.

### 8.2 Other Applications

The evanescent probing technique developed here is not limited to locating water or moisture in polyethylene. It could also be used to detect the distribution and concentration of any dielectric in a two dielectric mixture, such as that of liquid crystal particles embedded in
polymers, which resembles the structure of the Haus-Melcher dielectric model of Figure 3-2. Other applications include monitoring for surface and volume contaminants, such as moisture or pollution. The only requirement for this method to work is that the imaginary part of the dielectric constant (associated with the attenuation) of the material one wishes to sense be larger than that of the substrate material.

The evanescent fields used here are not only found adjacent to dielectric waveguides, but are also found in low frequency geometries where the quasistatic electric or magnetic fields can be described by a scalar Laplacian potential function. One such geometry is that of interdigital electrodes described in [50]. The planar electrodes have a spatial periodicity with \( \lambda = 2\pi/k \), which forces the electric potential (and therefore the electric field) to decay exponentially away from the surface of the electrodes. Since these evanescent fields do not change with frequency, a set of electrodes each with different periodicity is needed in order to provide the several measurements required for spatial resolution. Such multiple wavelength dielectrometry sensors are already used to measure the conductivity and permittivity profiles of moisture in oil impregnated pressboard. The estimation methods developed in the thesis could be also implemented with such sensors.

Similar to the interdigital electrodes are the meandering winding magnetometers (MWM) described in [22], which generate quasistatic magnetic evanescent fields using coils rather than electrodes to examine the properties of conducting and magnetic media. In this case, the penetration depth depends on both the spatial wavelength of the coils and the frequency dependent skin depth of the material in the evanescent field. Therefore, only a single wavelength sensor would provide the necessary data for estimating spatial profiles using methods developed in this thesis.

Mechanical ultrasonic waves also exhibit evanescence when propagating near the surface of a slightly compressible solid. These surface acoustic waves can propagate across a piezoelectric substrate, and materials placed adjacent to the substrate (such as a thin film) change the velocity and phase of the surface wave. As a gas or chemical sensor, this film is made of a material which reacts with the gas, and in turn changes the properties of the surface wave. Acoustic surface wave characteristics change with frequency in a similar manner to the fields adjacent to the dielectric waveguide and the data generated by such a sensor would be amenable to parameter estimation.
Appendix A

Three Region Solutions

This appendix provides the Matlab routines that solve for the attenuation and mode shape for the single region dielectric with symmetric water sheets (resulting in a three region symmetric problem). The main programs are atten3.m and graphe3.m, which compute the attenuation and mode shape, respectively. The function findkx3.m finds \( k_{x1} \) iteratively using \( f(k_{x1}) = f1 \cdot f4 - f3 \cdot f2 = 0 \) (Equation 4.35) and a Newton’s method given in newton3.m, ffunc3.m, and ffunc3d.m. Efactor3.m computes the scaling factor for the electric field amplitudes so that the power flowing in the three and one region geometries (with and without the water sheets) is the same. Efunc3.m computes the amplitudes of the electric fields for the three region geometry in terms of the center amplitude \( E_0 \). The attenuation is calculated by loss3.m. The integrals without their scaling factors are computed in inte3.m which are used in computing both the power flowing and power dissipated in the waveguide.

atten3.m

% This function computes the theoretical attenuation of a pair of
% water sheets for the three region geometry. The results and the
% data needed for the least squares estimation algorithms are stored
% in a file called 'atten3.mat'. attendb3 is calculated via the
% perturbation method. attendb1 and origattendb1 are calculated
% using the dispersion relations and their difference (diff1) is
% the change in attenuation due to the water sheet. attendb2 and
% origattendb2 are similar, except that the field expressions are used
% to calculate the attenuation.

% atten3.m calls findkx3.m, efunc3.m, and loss3.m
function atten3(a,d,delta,filename)

if (nargin ~= 4)
    disp ('usage: atten3(a,d,delta,filename) ');
    return;
end

a = a*1e-3;
d = d*1e-3;
delta = delta*1e-6;

if (d > a)
    disp ('Error in size of a and d.');
    return
elseif (d+delta > a)
    disp ('Error in size of a and d+delta.');
    return
end

% data file should have the following for each
% frequency (not necessarily all used in this version)
% frequency, e.sap', tandelta, epe', tandelta, e_w', tandelta

fullfilename = [filename '.dat'];
eval(['load ' fullfilename]);
data = eval(filename);

if (size(data,2) ~= 7)
    disp('data file wrong size');
    return;
end

mu_o = 4 * pi * 1e-7;
ep_o = 8.85e-12;
nfreqs = size(data,1);
freq = data(:,1);
ep1 = ep_o*(data(:,4) - j*data(:,5) .* data(:,4) * 1e-4);
ep2 = ep_o*(data(:,6) - j*data(:,7) .* data(:,6) * 1e-4);

origkx = pi/(2*a);
kx1_init = origkx;  % should be able to use this as the initial guess
                    % since the mode shape hasn't changed by much
kx1 = zeros(size(freq));
kx2 = zeros(size(freq));
kz = zeros(size(freq));
origkz = zeros(size(freq));
attendb1 = zeros(size(freq));
attendb2 = zeros(size(freq));
attendb3 = zeros(size(freq));
origattendb1 = zeros(size(freq));
origattendb2 = zeros(size(freq));

% Assume we will get roughly the same mode shape as the original fields
% so that we aren't moving into some funky higher order mode.

% Compute the attenuation 3 ways.
% First, use the imaginary part of kz. "alpha" = imag(kz).
% Second, use the field expressions to calculate the attenuation.
% Third, use the perturbation approach to find the attenuation (alpha).

for (index = 1:nfreqs)
    omega = 2 * pi * freq(index) * 1e9;

    [kx1(index), kx2(index), kz(index), error] = findkx3(omega, a, d, delta, ...
        ep1(index), ep2(index), kxl_init);

origkz(index) = sqrt(omega^2 * mu_o * ep1(index) - origkx * origkx);
origattendb1(index) = 20.0 * 0.434 * (-imag(origkz(index)));
origattendb2(index) = 20.0 * 0.434 * (0.5*(-imag(ep1(index))) * ...
    sqr(omega) * mu_o / real(origkz(index)));

delta_kz = sqr(omega) * mu_o * (-imag(ep2(index))) * delta * ...
    sqr(cos(origkxz)) / (a * real(origkz(index)));
attendb3(index) = 20.0 * 0.434 * delta_kz;

if ((real(kz(index)) ~ = 0) & ~error)
    attendb1(index) = 20.0 * 0.434 * (-imag(kz(index))); % per meter

    [e21, e22, e31, e32] = efunc3(kx1(index), kx2(index), a, d, delta);
    delta_kz = loss3(a, d, delta, e21, e22, e31, kx1(index), kx2(index), ...
        kx(index), omega, mu_o, ep1(index), ep2(index));
    attendb2(index) = 20.0 * 0.434 * delta_kz;

    kxl_init = kx1(index); % make first guess closer next time around
else
    attendb1(index) = 0;
    attendb2(index) = 0;
diffl = attendbi-origattendbi;
diff2 = attendb2-origattendb2;
save 'atten3' freq origattendbi origattendb2 attendb1 attendb2 attendb3...
    diff1 diff2 kx1 kx2 kz origkx origkz ep1 ep2

graphe3.m

% graphe3 plots the electric field for x > 0 when a water sheet
% is inserted into the waveguide in the single region dielectric case.
% The fields are calculated for the given frequency and dielectric
% constants that are provided by the user. The program is a loop,
% allowing the user to try different initial guesses to examine their
% effect on the solution and resulting mode shape.

% graphe3.m calls findkx3.m, efunc3.m, and efactor3.m

function graphe3(freq,ep1,ep2,a,d,delta)

if (nargin ~= 6)
    disp ('usage: graphe3(freq,ep1,ep2,a,d,delta) ');
    return;
end

a = a*1e-3;
d = d*1e-3;
delta = delta*1e-6;

if (d > a)
    disp ('Error in size of a and d.');
    return
elseif (d+delta > a)
    disp ('Error in size of a and d+delta.');
    return
end

omega = 2 * pi * freq * 1e9;
mu_o = 4 * pi * 1e-7;
ep_o = 8.85e-12;
ep1 = ep1*ep_o;
ep2 = ep2*ep_o;

done = 0;
while (~done)

kx1_max = pi/(2*d)
origkx = pi/(2*a)
origkz = sqrt(sqr(omega)*mu_o*ep1 - sqr(origkx))

clear kx1_init;
disp('Chose the starting value for kx1:');
disp(' 1 origkx = pi/2a');
disp(' 2 j * origkx');
disp(' 3 kx1_max = pi/2d');
disp(' 4 input value');
disp(' 5 enter program to check and set values');

pickstart = input('Choice: ');
if (pickstart == 1)
    kx1_init = origkx;
elseif (pickstart == 2)
    kx1_init = origkx * j;
elseif (pickstart == 3)
    kx1_init = kx1_max;
elseif (pickstart == 4)
    kx1_init = input('Enter starting guess: ');
elseif (pickstart == 5)
    disp('Set the value of kx1_init, then type return.');
    keyboard;
else
    disp('Not a valid choice.');
    return;
end

if (exist('kx1_init')==0)
    disp('kx1_init not set.');
    return;
end

[kx1,kx2,kz,error] = findkx3(omega,a,d,delta,ep1,ep2,kx1_init)
if (~error & (imag(kz) > 0)) % this sometimes happens on our probs
    disp ('conjugate search');
    kx1 = conj(kx1);
    [kx1,kx2,kz,error] = findkx3(omega,a,b,ep1,ep2,kx1)
end
doscale = 1;
if ((real(kz) == 0) & (error ~= 1))
doiscale = input('Non propagating mode, plot anyway? (y/n) ','s');
if (isempty(doiscale) | (doiscale ~= 'y'))
    error = 1;
else
doscale = 0;
disp('Note: plots will not be scaled.');
end
end
if (~error)

    atten = exp(2.0*imag(kz))
    attendb = 20.0 * 0.434 * (- imag(kz))

    [e21,e22,e31,e32] = efunc3(kxl, kx2, a,d,delta)
    if (doscale == 1)
        escale = efactor3(a,d,delta,e21,e22,e31,kx1,kx2,kz,origkx,...
                       origkz);
    else
        escale = 1;
    end

    orige = cos(origkx * d)
    olde = cos(kx1*d)
    newe = escale * cos(kx1 * d)
    for n=1:500
        x(n) = n * a / 500;
        origfield(n) = cos(origkx * x(n));
        if (x(n) <= d)
            field(n) = escale * cos(kx1 * x(n));
        elseif (x(n) <= d + delta)
            field(n) = escale*(e21*cos(kx2*x(n)) + e22*sin(kx2*x(n)));
        else
            field(n) = escale*e31 * sin(kxl*(a - x(n))); 
        end
    end
    subplot(311)
    plot(x, origfield)
    title('Plot of original E vs x')
    if (escale > 1)
        axis([0 x(500) 0 2]);
    end
end
end;
subplot(312)
plot(x,field)
title('Plot of E vs x with dielectric')
if (escale > 1)
    axis([0 x(500) 0 2]);
end;
subplot(313)
plot(x,field-origfield)
title('Difference, with dielectric - orig')

% this lets the user enter the program and save the plot
% and other data

saveplot = input('Save plot? (y/n) ','s');
if (~isempty(saveplot) & (saveplot == 'y'))
    disp('Enter print commands, then type return.');
    keyboard;
end
end

doicontinue = input('Try another guess? (y/n) ','s');
if (isempty(doicontinue) | (doicontinue ~= 'y'))
    done = 1;
end
end

findkx3.m

% Given an initial guess for kx1, this function finds the zero crossing
% of the function f(kx1) = f1 * f4 - f3 * f2 = 0 via newton's method for
% the three region geometry.

% This routine is called by atten3.m and graphe3.m and calls
% newton3.m

function [kx1,kx2,kz,error] = findkx3(omega,a,d,delta,ep1,ep2,kx1_init);

format long e;

done = 0;
tol = 1e-6;
mu_o = 4 * pi * 1e-7;
ep_o = 8.85e-12;
kx1_max = pi/(2*d);
kx1_guess = kx1_init;
kx2_guess = sqrt(sqr(omega) * mu_o * (ep2-ep1) + sqr(kx1_guess));

iters = 0;
error = 0;
while (done == 0)
    iters = iters+1;
    newkx1_guess = newton3(kx1_guess, kx2_guess, a, d, delta);
    newkx2_guess = sqrt(sqr(omega) * mu_o * (ep2-ep1) + sqr(newkx1_guess));
    if (abs((newkx1_guess - kx1_guess)/kx1_guess) < tol)
        done = 1;
    else
        kx1_guess = newkx1_guess;
        kx2_guess = newkx2_guess;
    end

if (iters > 500)
    disp('Findkx3 not converging."
    disp(omega/(2.0*pi*1e9));
    done = 1;
    error = 1;
end

if (abs(real(kx1_guess)) > 5*kx1_max)
    done = 1;
    error = 1;
    disp('findkx3: kx1 out of range, high');
    disp(omega/(2.0*pi*1e9));
    disp(kx1_guess);
    kx1_guess = kx1_max;
    kx2_guess = sqrt(sqr(omega) * mu_o * (ep2-ep1) + sqr(kx1_guess));
elseif (kx1_guess == 0)
    done = 1;
    error = 1;
    disp('findkx3: kx1 out of range, zero');
    disp(omega/(2.0*pi*1e9));
    disp(kx1_guess);
    kx1_guess = 0;
    kx2_guess = sqrt(sqr(omega) * mu_o * (ep2-ep1) + sqr(kx1_guess));
end
end

kx1 = kx1_guess;
kx2 = kx2_guess;
\[ kz = \sqrt{\text{sqr}(\omega) \cdot \mu_o \cdot \epsilon_p - \text{sqr}(kx)}; \]

\textbf{newton3.m}

\% This solves for one step in a newton’s method for finding
\% the zero crossing of a function.

\% newton3.m calls ffunc3.m and ffuncd3.m and is called by findkx3.m

function [new_kx1] = newton3(kx1, kx2, a, d, delta);

\[ [f1, f2, f3, f4] = \text{ffunc3}(kx1, kx2, a, d, delta); \]
\[ [df1, df2, df3, df4] = \text{ffuncd3}(kx1, kx2, a, d, delta); \]

\[ f = f1 \cdot f4 - f3 \cdot f2; \]

\[ \text{slope} = f1 \cdot df4 + f4 \cdot df1 - f3 \cdot df2 - f2 \cdot df3; \]

if (slope == 0)
    disp('newton3: slope 0');
    disp(f1); disp(f2); disp(f3); disp(f4);
    disp(df1); disp(df2); disp(df3); disp(df4);
    disp(f);
end

\% old slope = (fdelta - f)/(kx1_delta - kx1);

new_kx1 = kx1 - (f/slope);

\textbf{ffunc3.m}

\% This calculates the functions f1, f2, f3, and f4 for the
\% three region geometry, and is similar to efunc3.m

\% ffunc3.m is called by newton3.m

function [f1, f2, f3, f4] = ffunc3(kx1, kx2, a, d, delta);

\[ f1 = \cos(kx1 \cdot d) \cdot \cos(kx2 \cdot d) + \sin(kx1 \cdot d) \cdot kx1/kx2 \cdot \sin(kx2 \cdot d); \]
\[ f2 = \cos(kx1 \cdot d) \cdot \sin(kx2 \cdot d) - \sin(kx1 \cdot d) \cdot kx1/kx2 \cdot \cos(kx2 \cdot d); \]
\[ f3 = \sin(kx1 \cdot (a-d-delta)) \cdot \cos(kx2 \cdot (d+delta)) + ... \]
\[ \cos(kx1 \cdot (a-d-delta)) \cdot kx1/kx2 \cdot \sin(kx2 \cdot (d+delta)) ; \]
\[ f4 = \sin(kx1 \cdot (a-d-delta)) \cdot \sin(kx2 \cdot (d+delta)) - ... \]
\[ \cos(kx1 \cdot (a-d-delta)) \cdot kx1/kx2 \cdot \cos(kx2 \cdot (d+delta)); \]
ffuncd3.m

% This calculates the derivatives of the functions f1, f2, f3, and f4
% for the three region geometry.

% ffuncd3.m is called by newton3.m

function [df1, df2, df3, df4] = ffuncd3(kx1, kx2, a,d,delta);

% derivatives of f1, f2, f3, f4 with respect to kx1

fact1 = (sqr(kx1/kx2) - 1);
fact2 = - (1/kx2) * fact1;
x1 = a - d - delta;
x2 = d + delta;

df1 = d * sin(kx1*d)*cos(kx2*d)*fact1 + sin(kx1*d)*sin(kx2*d)*fact2;
df2 = d * sin(kx1*d)*sin(kx2*d)*fact1 - sin(kx1*d)*cos(kx2*d)*fact2;

df3 = (a + x2 * fact1) * cos(kx1*x1)*cos(kx2*x2) - ...  
    a * sin(kx1*x1)*sin(kx2*x2) * (kx1/kx2) + ...  
    cos(kx1*x1)*sin(kx2*x2) * fact2;

df4 = (a + x2 * fact1) * cos(kx1*x1)*sin(kx2*x2) + ...  
    a * sin(kx1*x1)*cos(kx2*x2) * (kx1/kx2) - ...  
    cos(kx1*x1)*cos(kx2*x2) * fact2;

efactor3.m

% Using the fact that we are assuming the power flowing in
% the unperturbed and perturbed cases is the same, use this to
% find the scaling factor for the electric fields in the three
% region geometry.

% efactor3.m is called by graphe3.m and calls inte3.m

function escale = efactor3(a,d,delta,e21,e22,e31,kx1,kx2,kz,origkx,origkz);

if (real(kz) == 0)
    escale = 0;
    return
end
% assume the original mode was sinusoidal with possibly a complex part
% but not pure evanescent

if (origkx == real(origkx))
    int0 = a/2;
else
    temp1 = origkx + conj(origkx);
    temp2 = origkx - conj(origkx);
    int0 = sin(temp2 * a)/(2*temp2) + sin(temp1*a)/(2*temp1);
end

% the integrations for the three region geometry

[int1,int2,int3,int4,int5] = inte3(a,d,delta,kxl,kx2);

escale = sqrt(real(origkz) * int0/...
    (real(kz)*(int1 + sqr(abs(e21))*int2 + ...
    sqr(abs(e22))*int3 + 2*real(e21*conj(e22)*int4) +...
    sqr(abs(e31))*int5)));

efunc3.m

% This calculates the scaling factors for the electric field
% components in terms of E_o for the three region geometry.

% efunc3.m is called by atten3.m and graphe3.m

function [e21, e22, e31, e32] = efunc3(kxl, kx2, a,d,delta);

f1 = cos (kxl * d) * cos(kx2 * d) + sin(kxl*d) * kxl/kx2 * sin(kx2 * d);

f2 = cos (kxl * d) * sin(kx2 * d) - sin(kxl*d) * kxl/kx2 * cos(kx2 * d);

f3 = sin (kxl * (a-d-delta)) * cos(kx2 * (d+delta)) + ...
    cos(kxl * (a-d-delta)) * kxl/kx2 * sin( kx2 * (d+delta));

f4 = sin (kxl * (a-d-delta)) * sin(kx2 * (d+delta)) - ...
    cos(kxl * (a-d-delta)) * kxl/kx2 * cos(kx2 * (d+delta));

e21 = f1;
e22 = f2;
e31 = f1/f3;
if (f4 == 0)
    e32 = e31;
else
    e32 = f2/f4;  % 0/0 if kx1=kx2
end
inte3.m

% This calculates the integrals used for the power flowing and
% power dissipation integrals for the three region geometry.
% The constants are not included here, only the actual integrals,
% which allows it to be used in both cases.

% inte3.m is called by efactor3.m and loss3.m

function [int1,int2,int3,int4,int5] = inte3(a,d,delta,kx1,kx2)

% kx1 and kx2 can be anything, real, imaginary, complex

if (kx1 == real(kx1))
    int1 = d/2 + sin(2*d*kx1)/(4*kx1);
    int5 = (a-d-delta)/2 - sin(2*(a-d-delta)*kx1)/(4*kx1);
elseif (real(kx1) == 0)
    int1 = d/2 + sin(2*d*kx1)/(4*kx1);
    int5 = -(a-d-delta)/2 + sin(2*(a-d-delta)*kx1)/(4*kx1);
else
    temp1 = kx1 + conj(kx1);
    temp2 = kx1 - conj(kx1);
    int1 = sin(temp2 * d)/(2*temp2) + sin(temp1*d)/(2*temp1);
    int5 = sin(temp2 * (a-d-delta))/(2*temp2) - ...
           sin(temp1*(a-d-delta))/(2*temp1);
end

if (kx2 == real(kx2))
    int2 = delta/2+(sin(2*(d+delta)*kx2)-sin(2*d*kx2))/(4*kx2);
    int3 = delta/2+(-sin(2*(d+delta)*kx2)+sin(2*d*kx2))/(4*kx2);
    int4 = 1/(2*kx2)*sqr(sin(kx2*(d+delta))) - sqr(sin(kx2*d));
elseif (real(kx2) == 0)
    int2 = delta/2+(sin(2*(d+delta)*kx2)-sin(2*d*kx2))/(4*kx2);
    int3 = -delta/2+(sin(2*(d+delta)*kx2)-sin(2*d*kx2))/(4*kx2);
    int4 = -1/(2*kx2)*(sqr(sin(kx2*(d+delta))) - sqr(sin(kx2*d)));
else
    temp1 = kx2 + conj(kx2);
    temp2 = kx2 - conj(kx2);
    int2 = sin(temp2*(d+delta))/(2*temp2)+sin(temp1*(d+delta))/(2*temp1)...
           - sin(temp2*d)/(2*temp2)-sin(temp1*d)/(2*temp1);
    int3 = sin(temp2*(d+delta))/(2*temp2)-sin(temp1*(d+delta))/(2*temp1)...
           - sin(temp2*d)/(2*temp2)+sin(temp1*d)/(2*temp1);
    int4 = cos(temp2*(d+delta))/(2*temp2)-cos(temp1*(d+delta))/(2*temp1)...
           - cos(temp2*d)/(2*temp2)+cos(temp1*d)/(2*temp1);
end
loss3.m

% Find the loss in the three region geometry by calculating the power
% dissipated and dividing by the total power flowing. This is similar
% to the function efactor3.m. There is no need to scale to the original
% field, since the scale factor cancels.

% loss3.m is called byatten3.m and calls inte3.m

function deltakz = loss3(a,d,delta,e21,e22,e31,kx1,kx2,kz,omega,...
    mu_o,ep1,ep2);

[int1,int2,int3,int4,int5] = inte3(a,d,delta,kx1,kx2);

  deltakz = 0.5 * omega * omega * mu_o / real(kz) * ...
  (-imag(ep1)*(int1 + abs(e31)*abs(e31)*int5) ...  
   -imag(ep2)*(abs(e21)*abs(e21)*int2 + ... 
         abs(e22)*abs(e22)*int3 + 2*real(e21*conj(e22)*int4))/... 
 (int1 + abs(e21)*abs(e21)*int2 + ... 
    abs(e22)*abs(e22)*int3 + 2*real(e21*conj(e22)*int4) +... 
    abs(e31)*abs(e31)*int5);
Appendix B

Two and Four Region Solutions

This appendix provides the Matlab routines that solve for the attenuation and mode shape for the two region dielectric with symmetric water sheets (resulting in a four region symmetric problem). The main programs are atten2.m, which calculates only the perturbation attenuation of the water sheets using the two region solution, graphe2.m, which solves for the mode shape in the two region case, and atten4.m and graphe4.m, which calculate the attenuation and mode shape for the four region geometry, respectively. The two and four region solutions reduce to the one and three region solutions when the first and second regions are specified with the same dielectric constants.

Findkx2.m and findkx4.m solve iteratively for $k_{z1}$ in the two and four region geometries using $f(k_{z1}) = f1 \cdot f4 - f3 \cdot f2 = 0$ (Equations 4.52 and 4.79) and a Newton’s method given in newton2.m, newton4.m, ffunc4.m, and ffuncd4.m. In the four region geometry, efactor4.m sets the scaling factors for the electric field amplitudes so that the power flowing in the two and four region geometries is the same. Efunc4.m calculates the field amplitudes in terms of the center amplitude $E_o$.

The routines inte2.m and inte4.m calculate the integrals used by both the power flowing and power loss calculations. Loss2.m and loss4.m solve for the attenuation in the two and four region geometry. Pertloss.m calculates the perturbation attenuation of the water sheets in the two region geometry.

atten2.m

% This function computes the theoretical attenuation via the
% perturbation method of a pair of water sheets in the two region
% geometry. (The water sheets are not considered as a separate region.)
% The results and the data needed for the least squares
% estimation algorithms are stored in a file called 'atten2.mat'.

% atten2.m calls findkx2.m and pertloss.m

function atten2(a,b,d,delta,filename)

if (nargin ~= 5)
    disp ('usage: atten2(a,b,d,delta,filename) ');
    return;
end

a = a*1e-3;
b = b*1e-3;
d = d*1e-3;
delta = delta*1e-6;

if (d > a)
    disp ('Error in size of a and d.');
    return
elseif (d+delta > a)
    disp ('Error in size of a and d+delta.');
    return
end
if (b > d)
    disp ('Error in size of b and d.');
    return
end

% data file should have the following for each
% frequency (not necessarily all used in this version)
% frequency, esap', tandelta, e_pe', tandelta, e_w', tandelta

fullfilename = [filename ' .dat'];
eval(['load ' fullfilename]);
data = eval(filename);

if (size(data,2) ~= 7)
    disp('data file wrong size');
    return;
end

mu_o = 4 * pi * 1e-7;
ep_o = 8.85e-12;

nfreqs = size(data,1);
freq = data(:,1);
ep1 = ep_o*(data(:,2) - j*data(:,3).*data(:,2) * 1e-4);
ep2 = ep_o*(data(:,4) - j*data(:,5).*data(:,4) * 1e-4);
ep3 = ep_o*(data(:,6) - j*data(:,7).*data(:,6) * 1e-4);

kx1 = zeros(size(freq));
kx2 = zeros(size(freq));
kz = zeros(size(freq));
attendb = zeros(size(freq));

% Assume we will get roughly the same mode shape as the original fields
% so that we aren't moving into some funky higher order mode.
% These should have a complex region 2, so start with a guess that would
% send it that way.

kx2_max = pi/(2*(a-b)) * j;

omega = 2 * pi * freq(1) * 1e9;
kx1_init = sqrt(sqr(kx2_max)-sqr(omega)*mu_o*(ep2(1)-epl(1)));

for (index = 1:nfreqs)
    omega = 2 * pi * freq(index) * 1e9;
    [kx1(index), kx2(index),kz(index),error] = ...
        findkx2(omega,a,b,epl(index),ep2(index),kx1_init);
    if (~error & (imag(kz(index)) > 0))
        disp ('Conjugate search for kx1');
        [kx1(index), kx2(index),kz(index),error] = ...
            findkx2(omega,a,b,epl(index),ep2(index),conj(kx1(index)));
    end

    kx1_init = kx1(index); % next time

% Use the perturbation approach to find the attenuation coefficient.

delta_kz = pertloss(a,b,d,delta,kx1(index),kx2(index),...
    kz(index),omega,mu_o,ep1(index),ep2(index),ep3(index));

    attendb(index) = 20.0 * 0.434 * delta_kz;
end

save 'atten2' attendb kx1 kx2 kz ep1 ep2 ep3 freq
graphe2.m

% graphe2 plots the electric field for x > 0 for the two region
% dielectric geometry. The fields are calculated for the given
% frequency and dielectric constants that are provided by the
% user. The program is a loop, allowing the user to try
% different initial guesses to examine their effect on the
% solution and resulting mode shape. The electric field is not
% scaled in reference to any other solutions.

% graphe2.m calls findkx2.m

function graphe2(freq,ep1,ep2,a,b)

if (nargin ~= 5)
    disp ('usage: graphe2a(freq,ep1,ep2,a,b) ');
    return;
end

if (b > a)
    disp ('Error in size of a and b.');
    return
end

a = a*1e-3;
b = b*1e-3;

omega = 2 * pi * freq * 1e9;
mu_o = 4 * pi * 1e-7;
ep_o = 8.85e-12;
ep2 = ep2 * ep_o;
ep1 = ep1 * ep_o;
done = 0;

while (~done)
    kx1_max = pi/(2*b)
kx2_max = pi/(2*(a-b))
origkx = pi/(2*a)
origkz = sqrt(sqr(omega)*mu_o*ep1 - sqr(origkx));

    clear kx1_init;
    disp('Chose the starting value for kx1:');
    disp('1 origkx = pi/2a');
disp(' 2  j * origkx');
disp(' 3  kx1_max = pi/2b');
disp(' 4  kx2 = kx2_max = pi/2(a-b)');
disp(' 5  j * kx2_max');
disp(' 6  input value for kx1');
disp(' 7  input value for kx2');
disp(' 8  enter program to check and set values');

pickstart = input('Choice: ');
if (pickstart == 1)
    kx1_init = origkx;
elseif (pickstart == 2)
    kx1_init = origkx * j;
elseif (pickstart == 3)
    kx1_init = kx1_max;
elseif (pickstart == 4)
    kx1_init = sqrt(sqr(kx2_max) - sqr(omega) * mu_o*(ep2-ep1));
elseif (pickstart == 5)
    kx1_init = sqrt(sqr(j*kx2_max) - sqr(omega) * mu_o*(ep2-ep1));
elseif (pickstart == 6)
    kx1_init = input('Enter starting guess for kx1: ');
elseif (pickstart == 7)
    kx2 = input('Enter starting guess for kx2: ');
    kx1_init = sqrt(sqr(kx2) - sqr(omega) * mu_o*(ep2-ep1));
elseif (pickstart == 8)
    disp('Set the value of kx1_init, then type return.');
    keyboard;
else
    disp('Not a valid choice. ');
    return;
end

if (exist('kx1_init')==0)
    disp('kx1_init not set. ');
    return;
end

[kx1,kx2,kz,error] = findkx2(omega,a,b,ep1,ep2,kx1_init);

if (~error & (imag(kz) > 0))  % this sometimes happens on our probs
    disp('conjugate search');
    kx1 = conj(kx1);
    [kx1,kx2,kz,error] = findkx2(omega,a,b,ep1,ep2,kx1);
end
if (~error)

    atten = exp(2.0*imag(kz))
    attendb = 20.0 * 0.434 * (- imag(kz))

    escale = 1;
    c = cos(kx1*b)/sin(kx2*(a-b))

    for n=1:500
        x(n) = n * a / 500;
        if (x(n) <= b)
            field(n) = escale * cos(kx1 * x(n));
        else
            field(n) = escale * c * sin(kx2*(a - x(n)));
        end
    end

    subplot(212)
    plot(x,field)
    title('Plot of E vs x with dielectric')

    saveplot = input('Save plot? (y/n) ','s');
    if ~(isempty(saveplot) & (saveplot == 'y'))
        disp('Enter print commands, then type return.');
        keyboard;
    end
end
end
end

doicontinue = input('Try another guess? (y/n) ','s' );
if ~(isempty(doicontinue) | (doicontinue ~= 'y'))
    done = 1;
end
end

atten4.m

% This function computes the theoretical attenuation of a pair of
% water sheets for the four region geometry. The results and the
% data needed for the least squares estimation algorithms are stored
% in a file called 'atten4.mat'. attendb3 is calculated via the
perturbation method. attendb1 and origattendb1 are calculated using the dispersion relations and their difference (diff1) is the change in attenuation due to the water sheet. attendb2 and origattendb2 are similar, except that the field expressions are used to calculate the attenuation.

atten4.m calls findkx4.m, efunc4.m, pertloss.m, loss2.m, and loss4.m

```matlab
function atten4(a,b,d,delta,filename)
    if (nargin ~= 5)
        disp('usage: atten4(a,b,d,delta,filename)');
        return;
    end

    a = a*1e-3;
    b = b*1e-3;
    d = d*1e-3;
    delta = delta*1e-6;

    if (d > a)
        disp('Error in size of a and d.');
        return
    elseif (d+delta > a)
        disp('Error in size of a and d+delta.');
        return
    end

    if (b > d)
        disp('Error in size of b and d.');
        return
    end

    fullfilename = [filename '.dat'];
    eval(['load ' fullfilename]);
    data = eval(filename);

    if (size(data,2) ~= 7)
        disp('data file wrong size');
        return;
    end
```

% data file should have the following for each
% frequency (not necessarily all used in this version)
% frequency, e_sap', tandelta, e_pe', tandelta, e_w', tandelta

149
mu_o = 4 * pi * 1e-7;
ep_o = 8.85e-12;

nfreqs = size(data,1);
freq = data(:,1);
ep1 = ep_o*(data(:,2) - j*data(:,3) .* data(:,2) * 1e-4);
ep2 = ep_o*(data(:,4) - j*data(:,5) .* data(:,4) * 1e-4);
ep3 = ep_o*(data(:,6) - j*data(:,7) .* data(:,6) * 1e-4);

kx1 = zeros(size(freq));
kx2 = zeros(size(freq));
kx3 = zeros(size(freq));
kh = zeros(size(freq));
origkx1 = zeros(size(freq));
origkx2 = zeros(size(freq));
origkh = zeros(size(freq));
origattendb1 = zeros(size(freq));
origattendb2 = zeros(size(freq));
attendb1 = zeros(size(freq));
attendb2 = zeros(size(freq));
attendb3 = zeros(size(freq));

% Assume we will get roughly the same mode shape as the original fields
% so that we aren't moving into some funky higher order mode.

% Compute the attenuation 3 ways.
% First, use the imaginary part of kh.  "alpha" = imag(kh).
% Second, use a method similar to above, but corrected for field strengths.
% Third, use the perturbation approach to find the attenuation (alpha).

% these should have a complex region 2, so start with a guess that would
% send it that way.

origkx2_max = pi/(2*(a-b)) * j;

omega = 2 * pi * freq(1) * 1e9;
origkx1_init = sqrt(sqr(origkx2_max)-sqr(omega)*mu_o*(ep2(1)-ep1(1)));

for (index = 1:nfreqs)
    omega = 2 * pi * freq(index) * 1e9;
    [origkx1(index), origkx2(index),origkh(index),error] = ...  
        findkx2(omega,a,b,ep1(index),ep2(index),origkx1_init);
if (~error & (imag(origkz(index)) > 0))
    % this sometimes happens on our probs
    disp ('Conjugate search for origkx1');
    [origkx1(index), origkx2(index), origkz(index), error] = ... 
        findkx2(omega, a, b, ep1(index), ep2(index), conj(origkx1(index)));
end

origkx1_init = origkx1(index); % next time
origattendbi(index) = 20.0 * 0.434 * (-imag(origkz(index)));

delta_kz = loss2(a, b, origkx1(index), origkx2(index), ... 
    origkz(index), omega, mu_o, ep1(index), ep2(index)); 
origattendb2(index) = 20.0 * 0.434 * delta_kz;

delta_kz = pertloss(a, b, d, delta, origkx1(index), origkx2(index), ... 
    origkz(index), omega, mu_o, ep1(index), ep2(index), ep3(index));
attendb3(index) = 20.0 * 0.434 * delta_kz;

kxl_init = origkx1(index);

[kx1(index), kx2(index), kx3(index), kz(index), error] = findkx4(omega, ... 
    a, b, d, delta, ep1(index), ep2(index), ep3(index), kxl_init);

% problems converging, try another initial guess
if (error & (index > 2))
    kxl_init = kx1(index-1) + (kx1(index-1) - kx1(index-2))
    [kx1(index), kx2(index), kx3(index), kz(index), error] = ... 
        findkx4(omega, a, b, d, delta, ep1(index), ep2(index), ep3(index), ... 
            kx1_init);
end

if (~error & (imag(kz(index)) > 0))% this sometimes happens on our probs
    disp ('Conjugate search for kxl');
    kxl_init = conj(kx1(index));
    [kx1(index), kx2(index), kx3(index), kz(index), error] = findkx4(omega, ... 
        a, b, d, delta, ep1(index), ep2(index), ep3(index), kx1_init);
end

if ((real(kz(index)) ~= 0) & ~error)
    attendbl(index) = 20.0 * 0.434 * (-imag(kz(index)));
    % per meter
    [e21, e22, e31, e32, e4] = efunc4(kx1(index), kx2(index), kx3(index), ...
a,b,d,delta);
delta_kz = loss4(a,b,d,delta,e21,e22,e31,e32,e4,kx1(index),...
kx2(index),kx3(index),kz(index),omega,mu_o,ep1(index), ...
ep2(index),ep3(index));
attendb2(index) = 20.0 * 0.434 * delta_kz;
else
    attendb1(index) = 0;
    attendb2(index) = 0;
end
end
diff1 = attendb1-origattendb1;
diff2 = attendb2-origattendb2;
save 'atten4' freq origattendb1 origattendb2 attendb1 attendb2 attendb3 ...
diff1 diff2 kx1 kx2 kx3 kz origkx1 origkx2 origkz epl ep2 ep3

graphe4.m

% graphe4 plots the electric field for x > 0 when a water sheet
% is inserted into the waveguide in the two region dielectric case.
% The fields are calculated for the given frequency and dielectric
% constants that are provided by the user. The program is a loop,
% allowing the user to try different initial guesses to examine their
% effect on the solution and resulting mode shape. It assumes that
% the original two region solution is close and uses this as one of
% the initial guesses.

% graphe4.m calls findkx2.m, findkx4.m, efunc4.m, and efactor4.m

function graphe4(freq,ep1,ep2,ep3,a,b,d,delta)

if (nargin ~= 8)
    disp ('usage: graphe4(freq,ep1,ep2,ep3,a,b,d,delta) ');
    return;
end

a = a*le-3;
d = d*le-3;
b = b*le-3;
delta = delta*le-6;

if (d > a)
    disp ('Error in size of a and d.');
    return
elseif (d+delta > a)
disp ('Error in size of a and d+delta.'); return
if (b > d)
    disp ('Error in size of b and d.'); return
end

omega = 2 * pi * freq * 1e9;
mu_o = 4 * pi * 1e-7;
ep_o = 8.85e-12;
ep1 = ep1*ep_o;
ep2 = ep2*ep_o;
ep3 = ep3*ep_o;

% these should have a complex region 2, so start with a guess that would
% send it that way.

quest = input('Assume evanescent in original region 2? (y/n) [y]', 's');

if (isempty(quest) | (quest == 'y'))
    origkx2_max = pi/(2*(a-b)) * j;
else
    origkx2_max = pi/(2*(a-b));
end

origkxi_init = sqrt(sqr(origkx2_max)-sqr(omega)*mu_o*(ep2-ep1));

[origkx1, origkx2, origkz, error] = findkx2(omega, a, b, ep1, ep2, origkxi_init);

if (~error & (imag(origkz) > 0)) % this sometimes happens on our probs
    disp ('Conjugate search for origkx1');
    origkx1 = conj(origkx1);
    [origkx1, origkx2, origkz, error] = findkx2(omega, a, b, ep1, ep2, origkx1);
end

if (error)
    disp('Error in finding original kx1.')
end

done = 0;

while (~done)
    origkx1
origkx2
origkz
kx1_max = pi/(2*b)

clear kx1_init;
disp('Chose the starting value for kx1:');
disp('1 kx1 from 2 region problem');
disp('3 kx1_max = pi/2b');
disp('5 input value');
disp('6 enter program to check and set values');

pickstart = input('Choice: ');
if (pickstart == 1)
    kx1_init = origkx1;
elseif (pickstart == 3)
    kx1_init = kx1_max;
elseif (pickstart == 5)
    kx1_init = input('Enter starting guess: ');
elseif (pickstart == 6)
    disp('Set the value of kx1_init, then type return.');
    keyboard;
else
    disp('Not a valid choice.'):
    return;
end

if (exist('kx1_init')==0)
    disp('kx1_init not set.');
    return;
end

[kx1,kx2,kx3,kz,error]=findkx4(omega,a,b,d,delta,ep1,ep2,ep3,kx1_init)
if (~error & (imag(kz) > 0)) % this sometimes happens on our probs
    disp ('Conjugate search for kx1');
    kx1 = conj(kx1);
    [kx1,kx2,kx3,kz,error]=findkx4(omega,a,b,d,delta,ep1,ep2,ep3,kx1)
end

doscale = 1;
if ((real(kz) == 0) & (error ~= 1))
    doiscale = input('Non propagating mode, plot anyway? (y/n) ','s');
    if (isempty(doiscale) | (doiscale ~= 'y'))
        error = 1;
    else

doscale = 0;
disp('Note: plots will not be scaled.');
end

if (~error)

atten = exp(2.0*imag(kz))
attendb = 20.0 * 0.434 * (- imag(kz))

[e21,e22,e31,e32,e4] = efunc4(kx1, kx2, kx3, a,b,d,delta)
if (doscale == 1)
escale = efactor4(a,b,d,delta,e21,e22,e31,e32,e4,kx1,kx2,...
               kx3,kz,origkx1,origkx2,origkz)
else
escale = 1;
end

c = cos(origkx1*b)/sin(origkx2*(a-b));
orige = c * sin(origkx2*(a - d))
olde = e21*cos(kx2 * d)+e22*sin(kx2*d)
newe = escale * (e21*cos(kx2 * d)+e22*sin(kx2*d))
for n=1:500
    x(n) = n * a / 500;
    if (x(n) <= b)
        origfield(n) = cos(origkx1 * x(n));
    else
        origfield(n) = c * sin(origkx2*(a - x(n)));
    end
    if (x(n) <= b)
        field(n) = escale * cos(kx1 * x(n));
    elseif (x(n) <= d)
        field(n) = escale*(e21*cos(kx2*x(n)) + e22*sin(kx2*x(n)));
    elseif (x(n) <= d + delta)
        field(n) = escale*(e31*cos(kx3*x(n)) + e32*sin(kx3*x(n)));
    else
        field(n) = escale*e4* sin(kx2*(a - x(n)));
    end
end

subplot(311)
plot(x, origfield)
title('Plot of original E vs x')
if (escale > 1)
axis([0 x(500) 0 2]);
end;
subplot(312)
plot(x,field)
title('Plot of E vs x with dielectric')
if (escale > 1)
    axis([0 x(500) 0 2]);
end;
subplot(313)
plot(x,field-origfield)
title('Difference, with dielectric - orig')

% this lets the user enter the program and save the plot
% and other data

saveplot = input('Save plot? (y/n) ', 's');
if (~isempty(saveplot) & (saveplot == 'y'))
    disp('Enter print commands, then type return. ');
    keyboard;
end
end

doicontinue = input('Try another guess? (y/n) ', 's');
if (isempty(doicontinue) | (doicontinue ~= 'y'))
    done = 1;
end
end

findkx2.m

% Given an initial guess for kx1, this function finds the zero crossing
% of the function f(kx1) = f1 * f4 - f3 * f2 = 0 via newton's method for
% the two region geometry.

% This routine is called by atten2.m, attenmoist.m, atten4.m, graphe2.m,
% and graphe4.m and calls newton2.m

function [kx1,kx2,kz,error] = findkx2(omega,a,b,ep1,ep2,kx1_init);

format long e;

done = 0;
tol = 1e-8;
mu_o = 4 * pi * 1e-7;
ep_o = 8.85e-12;
\[ k_{x1_{\text{max}}} = \frac{\pi}{2b}; \]
\[ k_{x1_{\text{guess}}} = k_{x1_{\text{init}}}; \]
\[ k_{x2_{\text{guess}}} = \sqrt{\text{sqr}(\omega) \times \mu_o \times (\epsilon_2 - \epsilon_1) + \text{sqr}(k_{x1_{\text{guess}}})}; \]

\text{iters} = 0;
\text{error} = 0;
\text{while (done} \neq 0) \text{)
\text{iters} = \text{iters} + 1;
\text{newkx1_{guess} = newton2(kx1_{guess},kx2_{guess},a,b); }
\text{newkx2_{guess} = sqrt(sqr(\omega) \times \mu_o \times (\epsilon_2 - \epsilon_1) + \text{sqr}(newkx1_{guess})});
\text{if (abs((newkx1_{guess} - kx1_{guess})/kx1_{guess}) < tol) }
\text{done} = 1;
\text{else }
\text{kx1_{guess} = newkx1_{guess};}
\text{kx2_{guess} = newkx2_{guess};}
\text{end}
\text{if (iters} > 500) \text{)
\text{disp('Findkx2 not converging.'))}
\text{disp(\omega/(2.0*pi*le9)); }
\text{done} = 1;
\text{error} = 1;
\text{end}
\text{if (abs(real(kx1_{guess})) > 5kx1_{max}) }
\text{done} = 1;
\text{error} = 1;
\text{disp('findkx2: kx1 out of range, high');}
\text{disp(\omega/(2.0*pi*le9)); }
\text{disp(kx1_{guess}); }
\text{kx1_{guess} = kx1_{max};}
\text{kx2_{guess} = sqrt(sqr(\omega) \times \mu_o \times (\epsilon_2 - \epsilon_1)+sqr(kx1_{guess})});
\text{elseif kx1_{guess} == 0 }
\text{done} = 1;
\text{error} = 1;
\text{disp('findkx2: kx1 out of range, zero');}
\text{disp(\omega/(2.0*pi*le9)); }
\text{disp(kx1_{guess}); }
\text{kx1_{guess} = 0;}
\text{kx2_{guess} = sqrt(sqr(\omega) \times \mu_o \times (\epsilon_2 - \epsilon_1)+sqr(kx1_{guess})});
\text{end}
\text{end}
\text{kx1 = kx1_{guess};}
\text{kx2 = kx2_{guess};}
kz = sqrt(sqr(omega)*mu_o*ep1 - sqr(kx1));
return;

**newton2.m**

% This solves for one step in a newton's method for finding
% the zero crossing of a function.

% newton2.m is called by findkx2.m

function [new_kxl] = newton2(kxl, kx2, a,b);

    temp1 = tan(kxl*b);
    temp2 = tan(kx2*(a-b));
    temp3 = 1.0/cos(kxl*b);
    temp4 = 1.0/sin(kx2*(a-b));

    f = kxl * temp1 - kx2 / temp2;
    df = temp1 + kxl*b*sqr(temp3) - (kxl/kx2) / temp2 + kxl * (a-b)*sqr(temp4);

    new_kxl = kxl - (f/df);

**inte2.m**

% This calculates the integrals used for the power flowing and
% power dissipation integrals for the two region geometry.
% The constants are not included here, only the actual integrals,
% which allows it to be used in both cases.

% inte2.m is called by loss2.m and pertloss.m

function [int1,int2] = inte2(a,b,kx1,kx2);

% mode, can be anything, real, imag, complex

if ((kx1 == real(kx1)) | (real(kx1) == 0))
    int1 = b/2 + sin(2*b*kx1)/(4*kx1);
else
    temp1 = kx1 + conj(kx1);
    temp2 = kx1 - conj(kx1);
    int1 = sin(temp2 * b)/(2*temp2) + sin(temp1*b)/(2*temp1);
end
if (kx2 == real(kx2))
    int2 = (a-b)/2 - sin(2*(a-b)*kx2)/(4*kx2);
elseif (real(kx2) == 0)
    int2 = -(a-b)/2 + sin(2*(a-b)*kx2)/(4*kx2);
else
    temp1 = kx2 + conj(kx2);
    temp2 = kx2 - conj(kx2);
    int2 = sin(temp2 * (a-b))/(2*temp2) - sin(temp1*(a-b))/(2*temp1);
end

loss2.m

% Find the loss in the two region geometry by calculating the power
% dissipated and dividing by the total power flowing.

% loss2.m is called by attemoist.m, atten4.m, and calls inte2.m
% it is also called by fomega.m and fomega1.m, which are part
% of the least squares solver routines.

function deltakz = loss2(a,b,kx1,kx2,kz,omega,mu_o,epl,ep2);
%
% crunch out the integrals

[int01,int02] = inte2(a,b,kx1,kx2);

c = cos(kx1*b)/sin(kx2*(a-b)); % for original region 2

deltakz = 0.5*sqr(omega)*mu_o/real(kz)*...
    (-imag(ep1)*int01 -imag(ep2)*sqr(abs(c))*int02)/...
    (int01 + sqr(abs(c))*int02);

pertloss.m

% This finds the loss by using a perturbation approach on the
% original two region fields, calculating the power dissipated
% and dividing by the total power flowing.

% pertloss.m is called by atten2.m, atten4.m, and calls inte2.m

function deltakz = pertloss(a,b,d,delta,kx1,kx2,kz,omega,mu_o,epl,...
                  ep2,ep3);
%
% original mode, for power flowing

[int01,int02] = inte2(a,b,kx1,kx2);

c = cos(kx1*b)/sin(kx2*(a-b)); % for original region 2
\[
\text{delta}_kz = 0.5 * \text{sqr}(\omega) * \text{mu}_o * (-\text{imag}(\text{ep3})) * \text{delta} / \text{real}(kz) * ... \\
\text{(sqr}(\text{abs}(c*\sin(kx2*(a-d))))/((\text{int}01 + \text{sqr}(\text{abs}(c))*\text{int}02));
\]

\textbf{findkx4.m}

% Given an initial guess for kx1, this function finds the zero crossing % of the function \(f(kx1) = f_1 * f_4 - f_3 * f_2 = 0\) via newton’s method for % the four region geometry.

% This routine is called by atten4.m and graphe4.m and calls % newton4.m

function [kx1,kx2,kx3,kz,\text{error}] = findkx4(\omega,a,b,d,\text{delta},...
\text{ep1},\text{ep2},\text{ep3},kx1_{\text{init}});

\text{format long e};

done = 0;
tol = 1e-6;
\text{mu}_0 = 4 * \text{pi} * 1e-7;
\text{ep}_0 = 8.85e-12;
kx1_{\text{max}} = \pi/(2*b);
kx1_{\text{guess}} = kx1_{\text{init}};
kx2_{\text{guess}} = \text{sqr}(\text{sqr}(\omega) * \text{mu}_0 * (\text{ep}_2-\text{ep}_1) + kx1_{\text{guess}}*kx1_{\text{guess}});
kx3_{\text{guess}} = \text{sqr}(\text{sqr}(\omega) * \text{mu}_0 * (\text{ep}_3-\text{ep}_1) + kx1_{\text{guess}}*kx1_{\text{guess}});

\text{iters} = 0;
\text{error} = 0;
\text{while } (\text{done} == 0)
\text{iters} = \text{iters}+1;
\text{newkx1}_{\text{guess}} = \text{newton}4(kx1_{\text{guess}},kx2_{\text{guess}},kx3_{\text{guess}},a,b,d,\text{delta});
\text{newkx2}_{\text{guess}} = \text{sqr}(\text{sqr}(\omega) * \text{mu}_0 * (\text{ep}_2-\text{ep}_1) + \text{sqr}(\text{newkx1}_{\text{guess}}));
\text{newkx3}_{\text{guess}} = \text{sqr}(\text{sqr}(\omega) * \text{mu}_0 * (\text{ep}_3-\text{ep}_1) + \text{sqr}(\text{newkx1}_{\text{guess}}));
\text{if } (\text{abs}((\text{newkx1}_{\text{guess}} - kx1_{\text{guess}})/kx1_{\text{guess}}) < \text{tol})
\text{done} = 1;
\text{else}
\text{kx1}_{\text{guess}} = \text{newkx1}_{\text{guess}};
\text{kx2}_{\text{guess}} = \text{newkx2}_{\text{guess}};
\text{kx3}_{\text{guess}} = \text{newkx3}_{\text{guess}};
\text{end}

\text{if } (\text{iters} > 500)
\text{disp('Findkx4 not converging.'})
\text{disp(\omega/(2.0*\text{pi}*1e9))};
\text{done} = 1;
error = 1;
end

if (abs(real(kxi_guess)) > 5*kxl_max)
done = 1;
error = 1;
disp('findkx4: kxi out of range, high');
disp(omega/(2.0*pi*e9));
disp(kxi_guess);
kxi_guess = kxl_max;
kx2_guess = sqrt(sqr(omega) * mu_o * (ep2-ep1)+sqr(kxi_guess));
kx3_guess = sqrt(sqr(omega) * mu_o * (ep3-ep1)+sqr(kxi_guess));
elseif (kxi_guess == 0)
done = 1;
error = 1;
disp('findkx4: kxl out of range, zero');
disp(omega/(2.0*pi*e9));
disp(kxi_guess);
kxi_guess = 0;
kx2_guess = sqrt(sqr(omega) * mu_o * (ep2-ep1)+sqr(kxi_guess));
kx3_guess = sqrt(sqr(omega) * mu_o * (ep3-ep1)+sqr(kxi_guess));
end
end

kx1 = kxi_guess;
kx2 = kx2_guess;
kx3 = kx3_guess;
kz = sqrt(sqr(omega)*mu_o*ep1 - sqr(kx1));
return;

newton4.m

% This solves for one step in a newton’s method for finding
% the zero crossing of a function.

% newton4.m calls ffunc4.m and ffuncd4.m and is called by findkx4.m

function [new_kx1] = newton4(kx1, kx2, kx3,a,b,d,delta);

[f1, f2, f3, f4] = ffunc4(kx1, kx2, kx3, a, b, d, delta);
[df1, df2, df3, df4] = ffuncd4(kx1, kx2, kx3, a, b, d, delta);

f = f1 * f4 - f3 * f2;
slope = f1 * df4 + f4 * df1 - f3 * df2 - f2 * df3;
if (slope == 0)
    disp('newton4: slope 0');
    disp(f1); disp(f2); disp(f3); disp(f4);
    disp(df1); disp(df2); disp(df3); disp(df4);
    disp(f);
end

new_kx1 = kx1 - (f/slope);

**ffunc4.m**

% This calculates the functions f1, f2, f3, and f4 for the
% four region geometry, and is similar to efunc4.m

% ffunc4.m is called by newton4.m

function [f1, f2, f3, f4] = ffunc4(kx1, kx2, kx3, a,b,d,delta);

    f3 = sin (kx2 * (a-d-delta)) * cos(kx3 * (d+delta)) + ...    
        cos(kx2 * (a-d-delta)) * kx2/kx3 * sin( kx3 * (d+delta));
    f4 = sin (kx2 * (a-d-delta)) * sin(kx3 * (d+delta)) - ...     
        cos(kx2 * (a-d-delta)) * kx2/kx3 * cos(kx3 * (d+delta));

    e21 = cos (kx1 * b) * cos(kx2 * b) + sin(kx1*b) * kx1/kx2 * sin(kx2 * b);
    e22 = cos (kx1 * b) * sin(kx2 * b) - sin(kx1*b) * kx1/kx2 * cos(kx2 * b);

    temp1 = cos(kx2*d) * e21 + sin(kx2*d) * e22;
    temp2 = sin(kx2*d) * e21 - cos(kx2*d) * e22;

    e31 = cos(kx3*d) * temp1 + (kx2/kx3) * sin(kx3*d) * temp2;
    e32 = sin(kx3*d) * temp1 - (kx2/kx3) * cos(kx3*d) * temp2;

    f1 = e31;
    f2 = e32;

**ffuncd4.m**

% This calculates the derivatives of the functions f1, f2, f3, and f4
% for the four region geometry.

% ffuncd4.m is called by newton4.m

function [df1, df2, df3, df4] = ffuncd4(kx1, kx2, kx3, a,b,d,delta);
derivatives of $f_1, f_2, f_3, f_4$ with respect to $k_{x_1}$

```matlab
% fact1 = (1-sqr(kx1/kx2));
fact2 = (1-sqr(kx2/kx3));
fact3 = (kx1/(kx2*kx3)) * fact2;

x1 = a - d - delta;
x2 = d + delta;

df3 = (kx1/kx2) * (a - x2 * fact2) * cos(kx2*x1)*cos(kx3*x2) - ...
    a * (kx1/kx3) * sin(kx2*x1)*sin(kx3*x2) + ...
    cos(kx2*x1)*sin(kx3*x2) * fact3;

df4 = (kx1/kx2) * (a - x2 * fact2) * cos(kx2*x1)*sin(kx3*x2) + ...
    a * (kx1/kx3) * sin(kx2*x1)*cos(kx3*x2) - ...
    cos(kx2*x1)*cos(kx3*x2) * fact3;

e21 = cos (kx1 * b) * cos(kx2 * b) + sin(kx1*b) * kx1/kx2 * sin(kx2 * b);
e22 = cos (kx1 * b) * sin(kx2 * b) - sin(kx1*b) * kx1/kx2 * cos(kx2 * b);

de21 = (1/kx2)*fact1*sin(kx1*b)*sin(kx2*b) - fact1*b*sin(kx1*b)*cos(kx2*b);
de22 =-(1/kx2)*fact1*sin(kx1*b)*cos(kx2*b) - fact1*b*sin(kx1*b)*sin(kx2*b);

temp1 = cos(kx2*d) * e21 + sin(kx2*d) * e22;
temp2 = sin(kx2*d) * e21 - cos(kx2*d) * e22;

dtemp1 = sin(kx2*d) * (de22 - (kx1/kx2)*d * e21) + ...
    cos(kx2*d) * (de21 + (kx1/kx2)*d * e22);
dtemp2 = sin(kx2*d) * (de21 + (kx1/kx2)*d * e22) +...
    cos(kx2*d) * (-de22 + (kx1/kx2)*d * e21);

f1 = cos(kx3*d) * temp1 + (kx2/kx3) * sin(kx3*d) * temp2; %e31
f2 = sin(kx3*d) * temp1 - (kx2/kx3) * cos(kx3*d) * temp2; %e32

df1 =sin(kx3*d) * ((kx2/kx3)*dtemp2 - (kx1/kx3)*d* temp1 + temp2*fact3)+...
    cos(kx3*d) * (dtemp1 + (kx2/kx3)*(kx1/kx3)*d*temp2);
df2 = sin(kx3*d) * (dtemp1 + (kx2/kx3)*(kx1/kx3)*d * temp2) +...
    cos(kx3*d) * (-((kx2/kx3)*dtemp2 + (kx1/kx3)*d*temp1-temp2*fact3));
```

% Using the fact that we are assuming the power flowing in
% the unperturbed and perturbed cases is the same, use this to
% find the scaling factor for the electric fields in the four
% region geometry.
% efactor4.m is called by graphe4.m and calls inte2.m and inte4.m

function escale = efactor4(a,b,d,delta,e21,e22,e31,e32,e4,kx1,kx2,kx3,...
    kz,origkx1,origkx2,origkz);

    if (real(kz) == 0)
        escale = 0;
        return
    end

    [intO1,intO2] = inte2(a,b,origkx1,origkx2);

    c = cos(origkx1*b)/sin(origkx2*(a-b));  % for original region 2

    [int1,int2,int3,int4,int5,int6,int7,int8] = inte4(a,b,d,delta,kx1,kx2,kx3);

    escale = sqrt(real(origkz) * (intO1 + sqr(abs(c))*intO2)/...
        (real(kz)*(int1 + sqr(abs(e21))*int2 + ...
            sqr(abs(e22))*int3 + 2*real(e21*conj(e22)*int4) +...
            sqr(abs(e31))*int5 + ...
            sqr(abs(e32))*int6 + 2*real(e31*conj(e32)*int7) +...
            sqr(abs(e4))*int8)));

efunc4.m

% This calculates the scaling factors for the electric field components in terms of E_o for the four region geometry.

% efunc4.m is called by atten4.m and graphe4.m

function [e21,e22,e31,e32,e4] = efunc4(kx1, kx2, kx3, a,b,d,delta);

    e21 = cos (kx1 * b) * cos(kx2 * b) + sin(kx1*b) * kx1/kx2 * sin(kx2 * b);
    e22 = cos (kx1 * b) * sin(kx2 * b) - sin(kx1*b) * kx1/kx2 * cos(kx2 * b);

    temp1 = cos(kx2*d) * e21 + sin(kx2*d) * e22;
    temp2 = sin(kx2*d) * e21 - cos(kx2*d) * e22;

    e31 = cos(kx3*d) * temp1 + (kx2/kx3) * sin(kx3*d) * temp2;
    e32 = sin(kx3*d) * temp1 - (kx2/kx3) * cos(kx3*d) * temp2;

    f3 = sin (kx2 * (a-d-delta)) * cos(kx3 * (d+delta)) + ...  
         cos(kx2 * (a-d-delta)) * kx2/kx3 * sin( kx3 * (d+delta));
    f4 = sin (kx2 * (a-d-delta)) * sin(kx3 * (d+delta)) - ...
\[
\cos(kx_2 * (a-d-delta)) * kx_2/kx_3 * \cos(kx_3 * (d+delta));
\]

e4 = e31 / f3;
if (f4 == 0)
e42 = e4;
else
  e42 = e32/f4;  \% 0/0 if kx2=kx3
end

**inte4.m**

\% This calculates the integrals used for the power flowing and
\% power dissipation integrals for the four region geometry.
\% The constants are not included here, only the actual integrals,
\% which allows it to be used in both cases.

\% inte4.m is called by efactor4.m and loss4.m

function [int1,int2,int3,int4,int5,int6,int7,int8] = ...
  inte4(a,b,d,delta,kx1,kx2,kx3);

\% crunch out the integrals
\% kx1, kx2, and kx3 can be anything, real, imag, complex

if (kx1 == real(kx1))
  int1 = b/2 + sin(2*b*kx1)/(4*kx1);
elseif (real(kx1) == 0)
  int1 = b/2 + sin(2*b*kx1)/(4*kx1);
else
  temp1 = kx1 + conj(kx1);
  temp2 = kx1 - conj(kx1);
  int1 = sin(temp2 * b)/(2*temp2) + sin(temp1*b)/(2*temp1);
end

if (kx2 == real(kx2))
  int2 = (d-b)/2+(sin(2*d*kx2)-sin(2*b*kx2))/(4*kx2);
  int3 = -d-b/2+(sin(2*d*kx2)-sin(2*b*kx2))/(4*kx2);
  int4 = 1/(2*kx2)*((sqr(sin(kx2*d)) - sqr(sin(kx2*b))));
  int8 = -(a-d-delta)/2 - sin(2*(a-d-delta)*kx2)/(4*kx2);
elseif (real(kx2) == 0)
  int2 = (d-b)/2+(sin(2*d*kx2)-sin(2*b*kx2))/(4*kx2);
  int3 = -(d-b)/2+(sin(2*d*kx2)-sin(2*b*kx2))/(4*kx2);
  int4 = -1/(2*kx2)*((sqr(sin(kx2*d)) - sqr(sin(kx2*b))));
  int8 = -(a-d-delta)/2 + sin(2*(a-d-delta)*kx2)/(4*kx2);
else

temp1 = kx2 + conj(kx2);
temp2 = kx2 - conj(kx2);
int2 = sin(temp2*d)/(2*temp2) + sin(temp1*d)/(2*temp1)...
   - sin(temp2*b)/(2*temp2) - sin(temp1*b)/(2*temp1);
int3 = sin(temp2*d)/(2*temp2) - sin(temp1*d)/(2*temp1)...
   - sin(temp2*b)/(2*temp2) + sin(temp1*b)/(2*temp1);
int4 = cos(temp2*d)/(2*temp2) - cos(temp1*d)/(2*temp1)...
   - cos(temp2*b)/(2*temp2) + cos(temp1*b)/(2*temp1);
int8 = sin(temp2 * (a-d-delta))/(2*temp2) - ...
   sin(temp1*(a-d-delta))/(2*temp1);
end

if (kx3 == real(kx3))
int5 = delta/2 + (sin(2*(d+delta)*kx3) - sin(2*d*kx3))/(4*kx3);
int6 = delta/2 + (-sin(2*(d+delta)*kx3) + sin(2*d*kx3))/(4*kx3);
int7 = 1/(2*kx3)*(sqr(sin(kx3*(d+delta))) - sqr(sin(kx3*d)));
elseif (real(kx3) == 0)
int5 = delta/2 + (sin(2*(d+delta)*kx3) - sin(2*d*kx3))/(4*kx3);
int6 = -delta/2 + (sin(2*(d+delta)*kx3) - sin(2*d*kx3))/(4*kx3);
int7 = - 1/(2*kx3)*(sqr(sin(kx3*(d+delta))) - sqr(sin(kx3*d)));
else
  temp1 = kx3 + conj(kx3);
  temp2 = kx3 - conj(kx3);
  int5 = sin(temp2*(d+delta))/(2*temp2) + sin(temp1*(d+delta))/(2*temp1)...
     - sin(temp2*d)/(2*temp2) - sin(temp1*d)/(2*temp1);
  int6 = sin(temp2*(d+delta))/(2*temp2) - sin(temp1*(d+delta))/(2*temp1)...
     - sin(temp2*d)/(2*temp2) + sin(temp1*d)/(2*temp1);
  int7 = cos(temp2*(d+delta))/(2*temp2) - cos(temp1*(d+delta))/(2*temp1)...
     - cos(temp2*d)/(2*temp2) + cos(temp1*d)/(2*temp1);
end

loss4.m

% Find the loss in the four region geometry by calculating the power
% dissipated and dividing by the total power flowing. This is similar
% to the function efactor4.m. There is no need to scale to the original
% field, since the scale factor cancels.

% loss4.m is called by atten4.m and calls inte4.m

function delta_kz = loss4(a,b,d,delta,e21,e22,e31,e32,e4,kx1,kx2,kx3,kz,...
     omega,mu_o,ep1,ep2,ep3);

[int1,int2,int3,int4,int5,int6,int7,int8] = inte4(a,b,d,delta,kx1,kx2,kx3);

delta_kz = 0.5 * sqr(omega) * mu_o / real(kz) * ...
(-\text{imag}(\text{ep1})*\text{int1} + ... \\
-\text{imag}(\text{ep2})*(\text{sqr}(\text{abs}(\text{e21}))*\text{int2} + ... \\
\text{sqr}(\text{abs}(\text{e22}))*\text{int3} + 2*\text{real}(\text{e21}*\text{conj}(\text{e22}))*\text{int4}) + ... \\
\text{sqr}(\text{abs}(\text{e4}))*\text{int8} ... \\
-\text{imag}(\text{ep3})*(\text{sqr}(\text{abs}(\text{e31}))*\text{int5} + ... \\
\text{sqr}(\text{abs}(\text{e32}))*\text{int6} + 2*\text{real}(\text{e31}*\text{conj}(\text{e32}))*\text{int7})))/... \\
(\text{int1} + \text{sqr}(\text{abs}(\text{e21}))*\text{int2} + ... \\
\text{sqr}(\text{abs}(\text{e22}))*\text{int3} + 2*\text{real}(\text{e21}*\text{conj}(\text{e22}))*\text{int4}) + ... \\
\text{sqr}(\text{abs}(\text{e4}))*\text{int8} + \text{sqr}(\text{abs}(\text{e31}))*\text{int5} + ... \\
\text{sqr}(\text{abs}(\text{e32}))*\text{int6} + 2*\text{real}(\text{e31}*\text{conj}(\text{e32}))*\text{int7});
Appendix C

Nonlinear Least Squares Solver

This appendix contains the Matlab code for the nonlinear least squares solver that estimates
the position and thickness of one or more pairs of water sheets given either measured or
simulated data (such as that generated by atten4.m). In the case of a single pair of sym-
metric water sheets placed in the two region geometry, nllse2.m will estimate the position
and thickness of the water sheets by comparing the provided attenuation to that calcu-
lated by nllse2.m via the perturbation method (Equation 7.18). For multiple water sheet
pairs, nllse2main.m serves as the engine calling nllse2N.m (which is essentially the same as
nllse2.m, but handles multiple pairs of sheets). Nllse2main.m breaks the XLPE region into
initial estimates for d and calculates all the unique permutations of initial estimates using
nestfunc.m given N pairs of water sheets. It compares each solution found by nllse2N.m
with previous solutions, picking the one with the smallest least squares error. Fomega.m
calculates the part of estimated attenuation that does not depend on either d or delta, and
the result is provided to nllse2.m and nllse2main.m in order to cut down on computation
time.

nllse2.m

% nllse2.m estimates the position and thickness of one pair of
% water sheets. f_omega is the frequency dependent parts of the
% perturbation attenuation model which don’t change with position
% or thickness, and is provided by the user to speed computations.
% M is the measured data.

% This gives the solution to the following problem:
% $x_{i+1} = x_i + (A_i^T A_i)^{-1}A_i^T [M - C(x_i)]$
% where the $A_i = A(x_i)$ is the matrix of derivatives of $C(x_i)$
% and the algorithm terminates when $x_i$ no longer changes. ($A_i$
% is called D_mat here.) Part of the equation is solved with
% the matlab linear least squares function ($\backslash$).

function [delta,d,error] = nllse2(a,b,d,delta,M,f_omega,kx2);

if (nargin ~= 7)
    disp ('usage: nllse2(a,b,d,delta,M,f_omega,kx2) ');
    return;
end

if (size(f_omega) ~= size(M))
    disp ('Error in size of M and f_omega.');
    return
end

a = a*1e-3;
b = b*1e-3;
d = d*1e-3;
delta = delta*1e-6;

small = 1e-12;

npts = size(M,1);
g_omega = zeros(size(M));
dg_omega = zeros(size(M));

bump = 0; % the d estimates hit a boundary, terminate if it does it twice
done = 0;

while done == 0
    g_omega = sin(kx2*(a-d)).*sin(conj(kx2)*(a-d));
    dg_omega = -2 * real(kx2 .* cos(kx2*(a-d)) .* sin(conj(kx2)*(a-d))) ;

    C_mat = delta .* f_omega .* g_omega;
    D_mat = [f_omega .* g_omega f_omega * delta .* dg_omega];

    error = M - C_mat;

    change = D_mat \ error;

    z = [delta d]';
\[
z = z + \text{change};
\]
\[
delta = z(1); \ d = z(2);
\]
\[
\text{if} \ (\text{abs}(d) > a) \ % \text{only check outside}
\]
\[
\text{if} \ (\text{bump} == 1)
\begin{align*}
\text{done} &= 1; \\
\text{else}
\begin{align*}
\text{d} &= a * 0.99; \\
\text{bump} &= 1;
\end{align*}
\end{align*}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{tolerance} = \text{change} ./ \ z; \ % \text{scale by the values}
\]
\[
\text{ntol} = 0;
\]
\[
\% \text{if value heads to zero, so does change, don’t count it}
\text{for} \ i=1:2
\begin{align*}
\text{if} \ (\text{abs}(z(i)) > \text{small})
\begin{align*}
\text{ntol} &= \text{ntol} + \text{tolerance}(i) * \text{tolerance}(i);
\end{align*}
\end{align*}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{if} \ \text{ntol} < \text{small}
\begin{align*}
\text{done} &= 1;
\end{align*}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\% \text{least squares error value}
\]
\[
\text{rerror} = \text{error'} * \text{error};
\]

\text{nllse2main.m}

\% This routine sets up the initial values of d and delta for the
\% case of multiple pairs of water sheets. It then calls nllse2N.m
\% with each of the initial values. It searches for up to maxsheets
\% pairs of water sheets, picking the solution with the smallest
\% least squares error.

\% nestfunc.m is used to generate the set of initial values of d,
\% and is basically used to find all possible combinations given the
\% number of water sheets, where order doesn’t matter.
function nllse2main(sheets,maxiters,divi,a,b,M,f_omega,kx2);

if (nargin ~= 8)
    disp ('usage: nllse2main(maxsheets,maxiters,divi,a,b,M,f_omega,kx2) ');
    return;
end

if (size(f_omega) ~= size(M))
    disp ('Error in size of M and f_omega.');
    return
end

ERROR = 1e6;
d = zeros(sheets,1);
error = zeros(sheets,1);
newd = zeros(sheets,1);
newdelta = zeros(sheets,sheets);
good = zeros(sheets,1);

for i=1:sheets
    error(i) = ERROR;
end

% Search over range of possible initial values. d is important, delta is % not. Iterate up to sheets times.

for k=1:3 % Guesses for thickness
    for i=1:sheets
        delta = zeros(sheets,1);
        for j = 1:i
            delta(j) = a/1e4 * 10^(k-1);
        end

        % number of possible guesses
        num = divi*i;
        inc = (a-b)/(divi*i+1);

        % find number of guesses for this loop. Num taken i at a time, order % doesn't matter.
        count = 1;

        % code continues here
for j = 1:i
    count = count * (num+1-j)/j;
end

d1 = zeros(1,sheets);
dlist = zeros(count,sheets);

[dlist,count] = nestfunc(i,1,1,num,b,inc,0,d1,dlist);

for j=1:count
    d = dlist(j,:);
    [testdelta,testd,testerror,iter] = nllse2N(i,a,b,d,
           delta,maxiters,M,f_omega,kx2);
    if ((iter < maxiters) & (testerror < ERROR))
        good(i) = good(i) + 1;  % number of solutions found
    end
    if testerror < error(i)
        newdelta(:,i) = testdelta;
        newd(:,i) = testd;
        error(i) = testerror;
    end
end
end
end

% run another set of passes on the "best solutions" to fine tune them

for i=1:sheets
    if error(i) < ERROR
        [testdelta,testd,testerror,iter] = nllse2N(i,a,b,newd(:,i),
           newdelta(:,i),maxiters*2,M,f_omega,kx2);
        if testerror < error(i)
            newdelta(:,i) = testdelta;
            newd(:,i) = testd;
            error(i) = testerror;
        end
    end
end

testerror = ERROR;

for i=1:sheets
    if error(i) < testerror

best = i;
testerror = error(i);
end

if testerror == ERROR
disp('no solution');
else
disp('number of sheets: ');
best
disp('d: ');
newd(1:best,best)
disp('delta: ');
newdelta(1:best,best)
end

disp('other solutions:');
newd
newdelta
error
good

nllse2N.m

% nllse2N.m estimates the position and thickness of N pairs of
% water sheets. f_omega is the frequency dependent parts of the
% perturbation attenuation model which don't change with position
% or thickness, and is provided by the user to speed computations.
% M is the measured data. This is called by nllse2main, which sets up
% the initial guesses for delta and d, but could be called in
% a similar manner to nllse2.m.

% This gives the solution to the following problem:
% x_{i+1} = x_i + (A_i^{-T} A_i)^{-1}A_i^{-T} [M -C(x_i)]
% where the A_i = A(x_i) is the matrix of derivatives of C(x_i)
% and the algorithm terminates when x_i no longer changes. (A_i
% is called D_mat here.) Part of the equation is solved with
% the matlab linear least squares function (\).

function [delta,d,rror,count]=nllse2N(N,a,b,d,delta,maxiters,M,...
   f_omega,kx2);

if (nargin ~= 9)
disp ('usage: nllse2N(N,a,b,d,delta,maxiters,M,f_omega,kx2) ');
return;
end
if (size(f_omega) ~= size(M))
    disp ('Error in size of M and f_omega.');
    return
end

small = 1e-20;
ERROR = 1e6;

maxN = size(d,1);
npts = size(M,1);
g_omega = zeros(size(M));
dg_omega = zeros(size(M));
D_mat = zeros(npts,2*N);
z(1:N,1) = delta(1:N,1);
z(N+1:2*N,1) = d(1:N,1);

count = 0;
done = 0;
bump = 0; % the d estimates hit a boundary, stop when happens twice.

while done == 0
    count = count + 1;
    if (count > maxiters)
        done = 1;
    end
    C_mat = zeros(size(M));
    for i=1:N
        g_omega = sin(kx2*(a-d(i))).*sin(conj(kx2)*(a-d(i)));
        dg_omega = - 2 * real(kx2 .* cos(kx2*(a-d(i))).*...
            sin(conj(kx2)*(a-d(i))));
        C_mat = C_mat + delta(i) * f_omega .* g_omega;
        D_mat(:,i) = f_omega .* g_omega;
        D_mat(:,i+N) = delta(i) * f_omega .* dg_omega;
    end

    error = M - C_mat;
    badmat = 0;

    % check for singular matrices.
    if (any(any(isnan(D_mat))) | ~all(all(finite(D_mat))) | ...)
        any(any(D_mat > 1e20)))
badmat = 1;
done = 1;
else
    ranksize = rank(D_mat);
    if ranksize ~= 2*N
        badmat = 1;
        done = 1;
    end
end
if (badmat == 0)
    change = D_mat \ error;

    z = z + change;

delta(1:N,1) = z(1:N,1);
d(1:N,1) = z(N+1:2*N,1);

    % check for out of range values for d, correct once

    for j=1:N
        if (abs(d(j)) > a) % only check outside
            if (bump == 1)
                done = 1;
            else
                d(j) = a*0.99;
                bump = 1;
            end
        end
    end

tolerance = change ./ z; % scale by the values
ntol = 0;

    % if value heads to zero, so does change, don’t count it

    for i=1:N
        if (abs(z(i)) > small)
            ntol = ntol + tolerance(i)*tolerance(i);
        end
    end
    if ntol < small
        done = 1;
    end
% single out condition of too many sheets predicted or problems
% finding solution with initial guess. In these cases, the D_mat will
% likely be singular or one of the sheets will be of very small thickness.

if badmat
    rerror = ERROR;
else
    for j=1:N
        if (delta(j) < small)
            rerror = ERROR;
        end
    end

% check for out of range values for d

for j=1:N
    if ((d(j) < b) | (d(j) > a))
        rerror = ERROR;
    end
end
end

% If no other errors, set rerror to the squared measurement error.
% rerror tells us how good this solution is

if rerror ~= ERROR
    rerror = error' * error;
end

fomega.m

% This computes the frequency dependent part of the attenuation
% used in the nllse2 programs.
% f_omega is the frequency dependent part of the attenuation calculated
% via the perturbation method, without the terms dependent on the
% delta and d.

% fomega.m is not called by any program. The data it generates
% (f_omega) is used by nllse2.m, nllse2N.m and nllse2main.m. It calls
% inte2.m to calculate the integrals for the two region case.

function [f_omega] = fomega(a,b,kx1,kx2,kz,ep3,freq);
if (nargin != 7)
    disp ('usage: fomega(a,b,kx1,kx2,kz,ep3,freq) ');
    return;
end

mu_o = 4 * pi * 1e-7;
npts = size(freq,1);
int = zeros(size(freq));

omega = freq * pi * 2.0 * 1e9;
c = cos(kx1*b)./sin(kx2*(a-b));

for i=1:npts,
    [int01,int02] = inte2(a,b,kx1(i),kx2(i));
    int(i) = int01 + sqr(abs(c(i)))*int02;
end

% attenuation loss without the delta and the sin terms

f_omega = 10.0 * 0.434 * omega .* omega .* muo .* (-imag(ep3)) .* ...
        abs(c) .* abs(c) ./ (real(kz) .* int);

nestfunc.m

% This is called by nllse2main.m to calculate recursively the
% initial values of the positions of multiple water sheets.
% This is a combinatorial problem, where order does not matter.

function [testd,count] = nestfunc(i,index,snum,fnum,b,inc,count,d,testd)
if (index == i)
    for j=snum:fnum
        d(index) = b+j*inc;
        count = count + 1;
        testd(count,:) = d;
    end
else
    for j=snum:fnum
        d(index) = b+j*inc;
        [testd,count] = nestfunc(i,index+1,1+j,fnum,b,inc,count,d,testd);
    end
end
Appendix D

Uniform Moisture Solutions

This appendix contains the Matlab code used to estimate the moisture concentration of the XLPE containing a uniform moisture distribution, as described in Section 7.5. Fomega1.m calculates the part of the estimated attenuation used by findppm.m that does not depend on the moisture concentration. Attenmoist.m generates the simulated exact and perturbation attenuation data that tests the findppm.m routine.

findppm.m

% This uses linear least squares to estimate the moisture concentration % in the XLPE region of the two dielectric geometry waveguide. % f_omega is the frequency dependent part of the attenuation calculated % via the perturbation method, without the dielectric constant e''.
%
% M is the measured data, provided by the user. ep2 and epw are the % dielectric constants of the XLPE and water, and f_omega is described % above.

function [bestppm] = findppm(M,ep2,epw,f_omega);

if (nargin ~= 4)
    disp ("usage: findppm(M,ep2,epw,f_omega) ');
    return;
end

if (size(f_omega) ~= size(M))
    disp ('Error in size of M and f_omega.');
    return
end
stuffs = -1e-6 * .925 * 3 * imag(ep2.*(epw-ep2)./(2*ep2+epw)).* f_omega;

bestppm = stuff \ M;

fomegal.m

% This computes the frequency dependent part of the attenuation
% used in the uniform moisture estimator.
% f_omegal is the frequency dependent part of the attenuation calculated
% via the perturbation method, without the dielectric constant e''.

% fomegal.m is not called by any program. The data it generates
% (f_omegal) is used by findppm.m. It calls inte2.m to calculate
% the integrals for the two region case.

function [f_omega] = fomegal(a,b,kx1,kx2,kz,freq);

if (nargin ~= 6)
    disp ('usage: fomegal(a,b,kx1,kx2,kz,freq) ');
    return;
end

a = a*le-3;
b = b*le-3;

mu_o = 4 * pi * 1e-7;
ep_o = 8.85 * 1e-12;

npts = size(freq,1);
int = zeros(size(freq));
int01 = zeros(size(freq));
int02 = zeros(size(freq));

omega = freq * pi * 2.0 * 1e9;
c = cos(kx1*b)./sin(kx2*(a-b));

for i=1:npts,
    [int01(i),int02(i)] = inte2(a,b,kx1(i),kx2(i));
    int(i) = int01(i) + sqr(abs(c(i)))*int02(i);
end

% attenuation loss without the epsilon or the first region terms (which
% will cancel. This is really attendb = f_omegal * (ep3''-ep2'')
attenmoist.m

% This computes the attenuations attendb1 and attendb2, given two
% different values for the dielectric constant of the XLPE.
% attendb1 is calculated with a perturbation approach and
% attendb2 is calculated by using exact field expressions.
% The results, which could be used with the findppm.m program
% are stored in the matrix 'attenmoist.mat'.

% attenmoist.m calls findkx2.m and loss2.m

function attenmoist(a,b,fdry,fwet)

if (nargin ~= 4)
    disp ('usage: attenmoist(a,b,fdry,fwet) ');
    return;
end

a = a*1e-3;
b = b*1e-3;

% dry data file should have the following for each
% frequency (not necessarily all used in this version)
% frequency, esap', tandelta, e_pe', tandelta, e_w', tandelta
% wet data file should have frequency, e_pe', tandelta

fullfilename = [fdry '.dat'];
eval(['load ' fullfilename]);
drydata = eval(fdry);

if (size(drydata,2) ~= 7)
    disp('dry data file wrong size');
    return;
end
fullfilename = [fwet '.dat'];
eval(['load ' fullfilename]);
wetdata = eval(fwet);

if (size(wetdata,2) ~= 3)
    disp('wet data file wrong size');
    return;
end
if (size(wetdata,1) == size(drydata,1))
    disp('data files must be same size');
    return;
end

mu_o = 4 * pi * 1e-7;
ep_o = 8.85e-12;

nfreqs = size(drydata,1);
freq = drydata(:,1);
ep1 = ep_o*(drydata(:,2) - j*drydata(:,3) .* drydata(:,2) * 1e-4);
ep2 = ep_o*(drydata(:,4) - j*drydata(:,5) .* drydata(:,4) * 1e-4);
ep3 = ep_o*(wetdata(:,2) - j*wetdata(:,3) .* wetdata(:,2) * 1e-4);

kx1 = zeros(size(freq));
kx2 = zeros(size(freq));
kz = zeros(size(freq));
kxw = zeros(size(freq));
kx2w = zeros(size(freq));
kzw = zeros(size(freq));
attendb1 = zeros(size(freq));
attendb2 = zeros(size(freq));

% Assume we will get roughly the same mode shape as the original fields
% so that we aren't moving into some funky higher order mode.

% compute the attenuation difference between wet and dry.
% these should have a complex region 2, so start with a guess that would
% send it that way.

kx2_max = pi/(2*(a-b)) * j;

omega = 2 * pi * freq(1) * 1e9;
kx1_init = sqrt(sqr(kx2_max)-sqr(omega)*mu_o*(ep2(1)-ep1(1)));

for (index = 1:nfreqs)
    omega = 2 * pi * freq(index) * 1e9;

    % dry

    [kx1(index), kx2(index),kz(index),error] = ...
        findkx2(omega,a,b,ep1(index),ep2(index),kx1_init);
if (~error & (imag(kz(index)) > 0)) % this sometimes happens on our probs
   disp ('Conjugate search for kx1');
    [kx1(index), kx2(index), kz(index), error] = ...
        findkx2(omega, a, b, ep1(index), ep2(index), conj(kx1(index)));
end

kx1_init = kx1(index);

% wet

[kx1w(index), kx2w(index), kzw(index), error] = ...
    findkx2(omega, a, b, ep1(index), ep3(index), kx1_init);

if (~error & (imag(kzw(index)) > 0)) % this sometimes happens on our probs
    disp ('Conjugate search for kx1w');
    [kx1w(index), kx2w(index), kzw(index), error] = ...
        findkx2(omega, a, b, ep1(index), ep3(index), conj(kx1w(index)));
end

% perturbation approach, assume mode shape doesn't change

delta_kz1 = loss2(a, b, kx1(index), kx2(index),...
            kz(index), omega, mu_o, ep1(index), ep2(index));
delta_kz2 = loss2(a, b, kx1(index), kx2(index),...
            kz(index), omega, mu_o, ep1(index), ep3(index));

attendb1(index) = 20.0 * 0.434 * (delta_kz2 - delta_kz1);

% calculate exact attenuation

delta_kz2 = loss2(a, b, kx1w(index), kx2w(index),...
            kzw(index), omega, mu_o, ep1(index), ep3(index));

attendb2(index) = 20.0 * 0.434 * (delta_kz2 - delta_kz1);

end

save 'attenmoist' kx1 kx2 kx1w kx2w kz kzw freq ep1 ep2 ep3 ...
    attendb1 attendb2
References

[1.] **Books and Technical Reports**


[3.] **Journal and Conference Papers: Polyethylene, Moisture, and Water Treeing**


[5.] **Journal and Conference Papers: Cable and Splice Manufacture**


[6.] **Journal and Conference Papers: Microwave Measurements and Techniques**


