Synthesis of Electromagnetic Modes in Photonic Band Gap Fibers

by

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The author wrote this thesis with only his right eye open, he was recovering from a major eye surgery as a result of severe chronic retina detachment in his left eye. If you find any anomalies in this thesis, it is due to the left-right asymmetry of his vision, the theory itself is coherent.

Figure 1: The man himself

This paper is the culmination of nearly two years of research by the author at the Photonic Bandgap Fibers and Devices group led by Prof. Yoel Fink at MIT RLE.
Mode Synthesis
A theory of a lot of things

Qichao Hu

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Chapter 1

Abstract

In this paper, we report on the successful synthesis of three individual modes, $HE_{11}$, $TE_{01}$, and $TE_{02}$ for transmission in photonic band gap fibers at near infrared wavelengths. We measure the propagation losses of the $HE_{11}$ and $TE_{01}$ modes both inside and outside the band gap of the fiber, and show that the $TE_{01}$ is indeed the lowest loss mode, and is less lossy and has a much wider band gap than the $HE_{11}$. We study the superpositions of the $HE_{11}$ and $TE_{01}$ modes using the pure phase approach, and discuss the degeneracy problem that arises. We analyze these superpositions by decomposing the superposed images into low energy eigenmodes ($m < 3$), and compute each of the eigenmode's contribution in the superposition. We show that the contributions of the $HE_{11}$ and $TE_{01}$ behave sinusoidally in their superpositions. Finally we also explain the minor discrepancies between the superposition and decomposition results.
Chapter 2

Fundamentals of Beams

2.1 Gaussian and Bessel Beams

A plane wave and a spherical wave represent the two opposite extremes of angular and spatial confinement. The wavefront normals (rays) of a plane wave are parallel to the direction of the wave so that there is no angular spread, but the energy extends spatially over the entire space. The spherical wave, on the other hand, originates from a single point, but its wavefront normals (rays) diverge in all directions.

Waves with wavefront normals making small angles with the $z$ axis are called paraxial waves. One way of constructing a paraxial wave is to start with a plane wave $A \exp(-jkz)$, regard it as a carrier wave, and modify or modulate its complex envelope $A$, making it a slowly varying function of position $A(r)$ so that the complex amplitude of the modulated wave becomes

$$U(r) = A(r) \exp(-jkz)$$ (2.1)

The variation of $A(r)$ with position must be slow within the distance of a wavelength $\lambda = 2\pi/k$, so that the wave approximately maintains its underlying plane-wave nature.

In order for the paraxial wave equation Eqn. 2.1 to satisfy the Helmholtz equation, which is given by

$$(\nabla^2 + k^2) U(r) = 0$$ (2.2)

where $k = 2\pi\nu/c = \omega/c$ is the wave number, the complex envelope $A(r)$ must satisfy another partial differential equation obtained by substituting Eqn. 2.1 into Eqn. 2.2. The assumption that $A(r)$ varies slowly with respect to z signifies that within
a distance $\Delta z = \lambda$, the change $\Delta A$ is much smaller than $A$ itself, $\Delta A \ll A$. This inequality of complex variables applies to the magnitudes of the real and imaginary parts separately. Since

$$\Delta A = (\partial A/\partial z) \Delta A = (\partial A/\partial z) \lambda,$$

(2.3)

it follows that

$$\partial A/\partial z \ll A/\lambda = Ak/2\pi$$

(2.4)

and therefore,

$$\partial A/\partial z \ll kA$$

(2.5)

Similarly, the derivative $\partial A/\partial z$ varies slowly within the distance $\lambda$, so that

$$\partial^2 A/\partial z^2 \ll k\partial A/\partial z$$

(2.6)

and therefore,

$$\partial^2 A/\partial z^2 \ll k^2 A$$

(2.7)

Substituting Eqn. 2.1 into Eqn. 2.2 and neglecting $\partial^2 A/\partial z^2$ in comparison with $k\partial A/\partial z$ or $k^2 A$, we obtain

$$\nabla_T^2 A - 2jk\partial A/\partial z = 0$$

(2.8)

where $\nabla_T^2 = \partial_x^2 + \partial_y^2$ is the transverse Laplacian operator.

Eqn. 2.8 is the slowly varying envelope approximation of the Helmholtz equation. It is also referred to as the paraxial Helmholtz equation, it is a partial differential equation that resembles the Schrödinger equation. The simplest solution of the paraxial Helmholtz equation is the paraboloidal wave, which is the paraxial approximation of the spherical wave. However, the most interesting and useful solution, is the Gaussian beam [1].

The beam power of a Gaussian beam is principally concentrated within a small cylinder surrounding the beam axis. The intensity distribution in any transverse plane is a circularly symmetric Gaussian function centered about the beam axis. The width of the function is minimum at the beam waist and grows gradually in both directions. The wavefunctions are approximately planar near the beam waist, but they gradually curve and become approximately spherical far from the waist. The angular divergence of the wavefront normals is the minimum permitted by the wave equation for a given beam width. The wavefront normals are therefore much like a thin pencil of rays. Under ideal conditions, the light from a laser takes the form of a Gaussian beam.
2.2 The Gaussian Beam

Earlier we introduced the concept of paraxial wave, which is a plane wave \( e^{-j k z} \) modulated by a complex envelope \( A(r) \) that is a slowly varying function of position. The complex amplitude is \( U(r) = A(r) \exp(-j k z) \). The envelope is assumed to be approximately constant within a neighborhood of size \( \lambda \), so that the wave is locally like a plane wave with wavefront normals that are paraxial rays.

For the complex amplitude \( U(r) \) to satisfy the Helmholtz equation Eqn. 2.2, the complex envelope \( A(r) \) must satisfy the paraxial Helmholtz equation Eqn. 2.8. One simple solution to the paraxial Helmholtz equation provides the paraboloidal wave for which

\[
A(r) = \frac{A_1}{z} \exp \left( -j k \frac{\rho^2}{2z} \right) \tag{2.9}
\]

where \( A_1 \) is a constant, and \( \rho^2 = x^2 + y^2 \). The paraboloidal wave is the paraxial approximation of the spherical wave \( U(r) = (A_1/z) \exp(-j k r) \), where \( x \) and \( y \) are much smaller than \( z \).

Another solution of the paraxial Helmholtz equation provides the Gaussian beam. It is obtained from the paraboloidal wave by the use of a simple transformation. Since the complex envelope of the paraboloidal wave Eqn. 2.9 is a solution of the paraxial Helmholtz equation Eqn. 2.8, a shifted version of it, with \( z - \eta \) replacing \( z \) where \( \eta \) is a constant is also a solution. This provides a paraboloidal wave centered about the point \( z = \eta \) instead of \( z = 0 \). When \( \eta \) is complex, our solution to the paraxial Helmholtz equation acquires dramatically different properties. In particular, when \( \eta \) is purely imaginary, \( \eta = -j z_0 \), where \( z_0 \) is real, it gives rise to the complex envelope of the Gaussian beam

\[
A(r) = \frac{A_1}{z + j z_0} \exp \left( -j k \frac{\rho^2}{2(z + j z_0)} \right) \tag{2.10}
\]

the parameter \( z_0 \) is known as the Rayleigh range, it will be important to calculate later when we study the size and amplitude of the beam, and its effect on the modes that it can couple into \([1]\).

To separate the amplitude and phase of the complex envelope in Eqn. 2.10, we write the complex function in terms of its real and imaginary parts by defining two new real functions \( R(z) \) and \( W(z) \) such that

\[
\frac{1}{z + j z_0} = \frac{1}{R(z)} - j \frac{\lambda}{\pi W^2(z)} \tag{2.11}
\]
We will show later that \( W(z) \) and \( R(z) \) are measures of the beam width and wavefront radius of curvature respectively. We define these new functions as

\[
W(z) = W_0 \sqrt{1 + \left( \frac{z}{z_0} \right)^2}
\]

(2.12)

\[
R(z) = z \left[ 1 + \left( \frac{z_0}{z} \right) \right]
\]

(2.13)

\[
\eta(z) = \tan^{-1} \left( \frac{z}{z_0} \right)
\]

(2.14)

\[
W_0 = \sqrt{\frac{\lambda z_0}{\pi}}
\]

(2.15)

If we combine everything together, we obtain the new complex amplitude of the Gaussian beam

\[
U(r) = A_0 \frac{W_0}{W(z)} \exp \left[ -\frac{\rho^2}{W^2(z)} \right] \exp \left[ -jkz - jk \frac{\rho^2}{2R(z)} + j\eta(z) \right]
\]

(2.16)

where \( A_0 = A_1/jz_0 \) is the new constant. Eqn. 2.16 contains two parameters, \( A_0 \) and \( z_0 \), both of which are determined from boundary conditions.

### 2.2.1 Intensity of a Gaussian Beam

The optical intensity \( I(r) = |U(r)|^2 \) is a function of the axial and radial distances \( z \) and \( \rho \)

\[
I(\rho, z) = I_0 \left[ \frac{W_0}{W(z)} \right]^2 \exp \left[ -\frac{2\rho^2}{W^2(z)} \right]
\]

(2.17)

where \( I_0 = |A_0|^2 \). At each value of \( z \) the intensity is a Gaussian function of the radial distance \( \rho \). The Gaussian function has its peak at \( \rho = 0 \) and drops monotonically with increasing \( \rho \). The width \( W(z) \) of the Gaussian distribution increases with the axial distance \( z \). The normalized beam intensity \( I/I_0 \) and the corresponding Gaussian beam are shown in Fig. 2.1

On the beam axis (\( \rho = 0 \)) the intensity

\[
I(0, z) = I_0 \left[ \frac{W_0}{W(z)} \right]^2 = \frac{I_0}{1 + (z/z_0)^2}
\]

(2.18)

has its maximum value \( I_0 \) at \( z = 0 \) and drops gradually with increasing \( z \), reaching half its peak value at \( z = \pm z_0 \). When \( |z| \gg z_0 \), \( I(0, z) \approx I_0 z_0^2/z^2 \), so that the intensity decreases with the distance in accordance with an inverse-square law, as for spherical and paraboloidal waves. The overall peak intensity \( I(0,0) = I_0 \) occurs at the beam center (\( z = 0, \rho = 0 \)).
2.2.2 Power of a Gaussian Beam

The total optical power carried by the beam is the integral of the optical intensity over a transverse plane

\[ P = \int_0^\infty I(\rho, z) 2\pi \rho d\rho \]  

(2.19)

which gives

\[ P = \frac{1}{2} I_0 (\pi W_0^2) \]  

(2.20)

The result is independent of \( z \), as expected. Thus the beam power is one-half the peak intensity times the beam area. Since beams are often described by their power \( P \), it is useful to express \( I_0 \) in terms of \( P \)

\[ I(\rho, z) = \frac{2P}{\pi W^2(z)} \exp \left[ -\frac{2\rho^2}{W^2(z)} \right] \]  

(2.21)

The ratio of the power carried within circle of radius \( \rho_0 \) in the transverse plane at position \( z \) to the total power is

\[ \frac{1}{P} \int_0^{\rho_0} I(\rho, z) 2\pi \rho d\rho = 1 - \exp \left[ -\frac{2\rho_0^2}{W^2(z)} \right] \]  

(2.22)

For a Gaussian beam, the power contained within a circle of radius \( \rho_0 = W(z) \) is approximately 86% of the total power. About 99% of the power is contained within a circle of radius \( 1.5W(z) \).

2.2.3 Beam radius of a Gaussian Beam

Within any transverse plane, the beam intensity assumes its peak value on the beam axis, and drops by the factor \( 1/e^2 \) at the radial distance \( \rho = W(z) \). Since 86% of
the power is carried within a circle of radius \( W(z) \), we regard \( W(z) \) as the beam radius, or beam width. The rms width of the intensity distribution is \( \sigma = \frac{1}{2} W(z) \). As we showed earlier that \( W(z) \) depends on \( z \) and assumes its minimum value \( W_0 \) in the plane \( z = 0 \), and this is called the beam waist. Thus \( W_0 \) is the waist radius. The waist diameter \( 2W_0 \) is called the spot size. The beam radius increases gradually with \( z \), reaching \( \sqrt{2}W_0 \) at \( z = z_0 \), and continues increasing monotonically with \( z \). For \( z \gg z_0 \) \( W(z) \) can be approximated as a linear relation

\[
W(z) \approx \frac{W_0}{z_0} z = \theta_0 z \quad (2.23)
\]

where \( \theta_0 = W_0/z_0 = \lambda/\pi W_0 \).

Far from the beam center, when \( z \gg z_0 \), the beam radius increases approximately linearly with \( z \), defining a cone with half-angle \( \theta_0 \). About 86% of the beam power is confined within this cone. The angular divergence of the beam is therefore defined as the angle

\[
\theta_0 = \frac{2}{\pi} \frac{\lambda}{2W_0} \quad (2.24)
\]

The beam divergence is directly proportional to the ratio between the wavelength \( \lambda \) and the beam waist diameter \( 2W_0 \). If the waist is squeezed, the beam diverges. To obtain a highly directional beam, therefore, a short wavelength and a fat beam waist should be used.

The beam waist \( W(z) \) as a function of \( z \) is shown in Fig. 2.2. Since the beam has its minimum width at \( z = 0 \), it achieves its best focus at the plane \( z = 0 \). In either direction, the beam gradually grows out of focus. The axial distance within which the beam radius lies within a factor \( \sqrt{2} \) of the minimum value is known as the depth of focus or confocal parameter. It can be seen from Fig. 2.2 that the depth of focus is twice the Rayleigh range

\[
2z_0 = \frac{2\pi W_0^2}{\lambda} \quad (2.25)
\]

The depth of focus is directly proportional to the area of the beam at its waist, and inversely proportional to the wavelength. Thus when a beam is focused to a small spot size, the depth of focus is short and the plane of focus must be located with greater accuracy. A small spot size and a long depth of focus cannot be obtained simultaneously unless the wavelength of the light is short. Throughout our experiment, the wavelength and the spot size will be crucial in determining the mode that gets coupled into the photonic bandgap fiber. We will discuss the methods that we
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Figure 2.2: The beam waist $W(z)$ has its minimum $W_0$ at $z = 0$ plane.

used to measure the spot size experimentally and the range of wavelength that we investigated in later chapter.

2.3 Hermite-Gaussian Beams

The Gaussian beam is not the only beam-like solution of the paraxial Helmholtz equation. There are many other solutions including beams with non-Gaussian intensity distributions. Consider a Gaussian beam of complex envelope as in Eqn. 2.10 and the corresponding $W(z)$ and $R(z)$. Consider a second wave whose complex envelope is a modulated version of the Gaussian beam

$$A(x, y, z) = X \left[ \frac{x}{\sqrt{2} W(z)} \right] Y \left[ \frac{y}{\sqrt{2} W(z)} \right] \exp [jZ(z)] A_G(x, y, z)$$

(2.26)

where $X[\cdot]$, $Y[\cdot]$, and $Z[\cdot]$ are real functions, and $A_G(x, y, z)$ represents a Gaussian beam that we described earlier. The wave described by Eqn. 2.26 has the following properties.

1) The phase is the same as that of the underlying Gaussian wave, except for an excess phase $Z(z)$ that is independent of $x$ and $y$. If $Z(z)$ is a slowly varying function of $z$, the two waves have paraboloidal wavefronts with the same radius of curvature
There two waves are therefore focused by thin lenses and mirrors in precisely the same manner.

2) The magnitude

\[ A_0 x \left[ \sqrt{2} \frac{x}{W(z)} \right] Y \left[ \sqrt{2} \frac{y}{W(z)} \right] \left[ \frac{W_0}{W(z)} \right] \exp \left[ -\frac{x^2 + y^2}{W^2(z)} \right] \]  

(2.27)

is a function of both \( x/W(z) \) and \( y/W(z) \) whose widths in the \( x \) and \( y \) directions vary with \( z \) in accordance with the same scaling factor \( W(z) \). As \( z \) increases, the intensity distribution in the transverse plane remains fixed, except for a magnification factor \( W(z) \). This distribution is a Gaussian function modulated in the \( x \) and \( y \) directions by the functions \( X^2[.] \) and \( Y^2[.] \).

The modulated wave therefor represents a beam of non-Gaussian intensity distribution, but with the same wavefronts and angular divergence as the Gaussian beam. Let us examine the possibility of the existence of such wave by finding three real functions \( X[.] \), \( Y[.] \), and \( Z[.] \) such that the new modulated wave Eqn. 2.26 satisfies the paraxial Helmholtz equation. After substitution and defining two new variables \( u = \sqrt{2x/W(z)} \) and \( v = \sqrt{2y/W(z)} \), we obtain

\[
\frac{1}{X} \left( \frac{\partial^2 X}{\partial u^2} - 2u \frac{\partial X}{\partial u} \right) + \frac{1}{Y} \left( \frac{\partial^2 Y}{\partial v^2} - 2v \frac{\partial Y}{\partial v} \right) + kW^2(z) \frac{\partial Z}{\partial z} = 0
\]  

(2.28)

Since the left-hand side of this equation is the sum of three terms, each of which is a function of a single independent variable, \( u \), \( v \), or \( z \), respectively, each of these terms must be constant. Equating the first term to the constant \(-2\mu_1 \) and the second to \(-2\mu_2 \), the third must be equal to \( 2(\mu_1 + \mu_2) \). This separation of variables permits us to reduce the partial differential equation in Eqn. 2.28 into three ordinary differential equation for \( X(u) \), \( Y(v) \), and \( Z(z) \) respectively.

\[
-\frac{1}{2} \frac{d^2 X}{du^2} + u \frac{dX}{du} = \mu_1 X
\]  

(2.29)

\[
-\frac{1}{2} \frac{d^2 Y}{dv^2} + v \frac{dY}{dv} = \mu_2 Y
\]  

(2.30)

\[
z_0 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right] \frac{dZ}{dz} = \mu_1 + \mu_2
\]  

(2.31)

The first line in Eqn. 2.31 represents an eigenvalue problem whose eigenvalues are \( \mu_1 = l \), where \( l = 0, 1, 2, \ldots \) and whose eigenfunctions are the Hermite polynomials \( X(u) = H_l(u) \). There polynomials are defined by the recurrence relation

\[
H_{l+1}(u) = 2uH_l(u) - 2lH_{l-1}(u)
\]  

(2.32)
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thus

\[ H_0(u) = 1 \]  
\[ H_1(u) = 2u \]  
\[ H_2(u) = 4u^2 - 2 \]  
\[ H_3(u) = 8u^3 - 12u \]  

Similarly, the solution to the second line in Eqn. 2.31 are \( \mu_2 = m \) and \( Y(v) = H_m(v) \), where \( m = 0, 1, 2, \ldots \) There is therefore a family of solutions labeled by the indices \((l, m)\). In Eqn. 2.31, we can substitute \( \mu_1 = l \) and \( \mu_2 = m \) and integrate, we obtain

\[ Z(z) = (l + m) \eta(z) \]  

where \( \eta(z) = \tan^{-1}(z/z_0) \). The excess phase \( Z(z) \) varies slowly between \(- (l + m) \pi/2 \) and \((l + m) \pi/2 \), as \( z \) varies between \(- \infty \) and \( \infty \). Finally, if we piece everything together, and rearrange terms and multiply by \( \exp(-j k z) \), we obtain the new complex amplitude.

\[ U_{l,m}(x, y, z) = A_{l,m} \left[ \frac{W_0}{W(z)} \right] G_l \left[ \frac{\sqrt{2}x}{W(z)} \right] G_m \left[ \frac{\sqrt{2}y}{W(z)} \right] \times \exp \left[ -j k z - j k \frac{z^2 + x^2}{2R(z)} + j (l + m + 1) \eta(z) \right] \]  

where

\[ G_l(u) = H_l(u) \exp \left( \frac{-u^2}{2} \right) \]  

is the Hermite-Gaussian function of order \( l \), and \( A_{l,m} \) is a constant [2].

Since \( H_0(u) = 1 \), the Hermite-Gaussian function of order 0 is simply the Gaussian function. \( G_1(u) = 2u \exp(-u^2/2) \) is an odd function, \( G_2(u) = (4u^2 - 2) \exp(-u^2/2) \) is even, \( G_3(u) = (8u^3 - 12u) \exp(-u^2/2) \) is odd, and so on. These functions are shown in Fig. 2.3. An optical wave with complex amplitude given by Eqn. 2.38 is known as the Hermite-Gaussian beam of order \((l, m)\). The Hermite-Gaussian beam of order \((0, 0)\) is the Gaussian beam.

### 2.3.1 Intensity Distribution

The optical intensity of the \((l, m)\) Hermite-Gaussian beam is

\[ I_{l,m}(x, y, z) = |A_{l,m}|^2 \left[ \frac{W_0}{W(z)} \right]^2 G_l^2 \left[ \frac{\sqrt{2}x}{W(z)} \right] G_m^2 \left[ \frac{\sqrt{2}y}{W(z)} \right] \]
Fig. 2.4 illustrates the dependence of the intensity on the normalized transverse distances $u = \sqrt{2x}/W(z)$ and $v = \sqrt{2y}/W(z)$ for several values of $l$ and $m$. Beams of higher order have larger widths than those of lower order. Regardless of the order, however, the width of the beam is proportional to $W(z)$, so that as $z$ increases the intensity pattern is magnified by the factor $W(z)/W_0$ but otherwise maintains its profile. Among the family of Hermite-Gaussian beams, the only circularly symmetric member is the Gaussian beam.

### 2.4 Bessel Beams

The Hermite-Gaussian beams form a complete set of solutions to the paraxial Helmholtz equation. Any other solution can be written as a superposition of these beams. But this family is not the only one. Another complete set of solutions, known as Laguerre-Gaussian beams, may be obtained by writing the paraxial Helmholtz equation in cylindrical coordinates $(\rho, \phi, z)$ and using separation of variables in $\rho$ and $\phi$, instead of $x$ and $y$. The lowest-order Laguerre-Gaussian beam is the Gaussian beam.

In the search for beamlike waves, it is natural to examine the possibility of the existence of waves with planar wavefronts but with nonuniform intensity distributions
in the transverse plane. Consider a wave with the complex amplitude

\[ U (r) = A(x, y) e^{-j\beta z} \quad (2.41) \]

For this to satisfy the Helmholtz equation, \( A(x, y) \) must satisfy

\[ \nabla_T^2 A + k_T^2 A = 0 \quad (2.42) \]

where \( k_T^2 + \beta^2 = k^2 \) and \( \nabla_T^2 \) is the transverse Laplacian operator. The two-dimension Helmholtz equation, described in Eqn. 2.42, may be solved using the methods of separation of variables. Using polar coordinates \( (x = \rho \cos \phi, y = \rho \sin \phi) \), the result becomes

\[ A(x, y) = A_m J_m (k_T \rho) e^{im\phi} \quad (2.43) \]

where \( m = 0, \pm 1, \pm 2, \ldots \), and \( J_m (\cdot) \) is the Bessel function of the first kind and \( m \)th order, and \( A_m \) is a constant.

For \( m = 0 \), the wave has a complex amplitude

\[ U (r) = A_0 J_0 (k_T \rho) e^{-j\beta z} \quad (2.44) \]

and therefore has planar wavefronts. The wavefront normals (rays) are parallel to the \( z \) axis. The intensity distribution \( I(\rho, \phi, z) = |A_0|^2 J_0^2 (k_T \rho) \) is circularly symmetric, varies with \( \rho \). There is no spread of the optical power, this wave is called the Bessel beam.

It is interesting to compare the Bessel beam to the Gaussian beam. Whereas the complex amplitude of the Bessel beam is an exact solution of the Helmholtz equation,
the complex amplitude of the Gaussian beam is only an approximate solution (its complex envelope is an exact solution of the paraxial Helmholtz equation). The intensity distribution of these two beams are compared in Fig. 2.5. The asymptotic behavior of these distributions in the limit of large radial distances is significantly different. Whereas the intensity of the Gaussian beam decreases exponentially in proportionality to $\exp \left[-2p^2/W^2(z)\right]$, the intensity of the Bessel beam is proportional to

$$J_0^2 (k_T \rho) \approx (2/\pi k_T \rho) \cos^2 (k_T \rho - \pi/4)$$

which is an oscillatory function with slowly decaying magnitude. Whereas the rms width of the gaussian beam, $\sigma = W(z)/2$, is finite, the rms width of the Bessel beam is infinite at all $z$. There is a tradeoff between the minimum beam size and the divergence. Thus although the divergence of the Bessel beam is zero, its rms width is infinite. Since Gaussian beams are the modes of spherical resonators, they are created naturally by lasers, while the generation of Bessel beams requires special treatment, and this will be the focus of our research.
Chapter 3

Modes Superposition and Decomposition

In the previous chapter, we talked about the basics of beams and the functions that govern their profiles. Before we start studying electromagnetic modes and their interaction with each other, it is necessary to introduce the concept of polarization. Polarization plays an important role in studying the interaction of light with matter, absorption and scattering of light in waveguides, and the differential propagation velocities and phase shift. We will also discuss certain devices that we often use in optics such as polarizer later in this chapter.

3.1 Polarization

The polarization of light is determined by the time course of the direction of the electric field vector $E(r, t)$. In paraxial optics, light propagates along directions that lie within a narrow cone centered about the optical $z$ axis. Waves are approximately transverse electromagnetic (TEM) and the electric field vector therefore lies approximately in the transverse plane ($x - y$ plane). If the medium is isotropic, the polarization is in the form of ellipses. The polarization ellipses sometimes degenerate into a straight line or a circle, and the wave becomes either linearly polarized or circularly polarized respectively.

Consider a plane wave of frequency $\nu$ traveling in the $z$ direction with velocity $c$. The electric field lies in the $x - y$ plane and is described as

$$E(z, t) = Re \left\{ A \exp \left[ j2\pi \nu \left( t - \frac{z}{c} \right) \right] \right\}$$

(3.1)
where the complex envelope \( A = A_x \hat{x} + A_y \hat{y} \), is a vector with complex components \( A_x \) and \( A_y \). If we trace each endpoint of \( E(z,t) \) at each position \( z \) as a function of time, we obtain a description of the polarization of the wave.

Similarly, we can also separate the electric field vector \( E(z,t) \) into its \( x \) and \( y \) components

\[
E_x = a_x \cos \left[ 2\pi \nu \left( t - z \over c \right) + \phi_x \right] \tag{3.2}
\]

\[
E_y = a_y \cos \left[ 2\pi \nu \left( t - z \over c \right) + \phi_y \right] \tag{3.3}
\]

where \( A_x = a_x \exp(j \phi_x) \), \( A_y = a_y \exp(j \phi_y) \), and \( E(z,t) = E_x \hat{x} + E_y \hat{y} \). The components \( E_x \) and \( E_y \) are periodic functions of \( t - z/c \) oscillating at frequency \( \nu \). Eqn. 3.3 can be rewritten in the following form

\[
\frac{E_x^2}{a_x^2} + \frac{E_y^2}{a_y^2} - 2 \cos \phi \frac{E_x E_y}{a_x a_y} = \sin^2 \phi \tag{3.4}
\]

where \( \phi = \phi_1 - \phi_2 \) is the phase difference.

When one of the components in Eqn. 3.3, for instance \( a_x \), the light is linearly polarized in the direction of the other component \( a_y \) in this case. The wave is also linearly polarized if the phase difference \( \phi = 0 \) or \( \pi \).

If \( \phi = \pm \pi/2 \) and \( a_x = a_y = a_0 \), Eqn. 3.3 becomes

\[
E_x = a_0 \cos \left[ 2\pi \nu \left( t - z \over c \right) + \phi_x \right] \tag{3.5}
\]

\[
E_y = \mp a_0 \sin \left[ 2\pi \nu \left( t - z \over c \right) + \phi_x \right] \tag{3.6}
\]

this leads to

\[
E_x^2 + E_y^2 = a_0^2 \tag{3.7}
\]

which is the equation of a circle. In this case, the wave is circularly polarized. When \( \phi = +\pi/2 \), the electric field at a fixed position \( z \) rotates in a clockwise direction when viewed from the direction toward which the wave is approaching, thus this light is right circularly polarized. When \( \phi = -\pi/2 \), the light is left circularly polarized.

### 3.1.1 Representations

A more convenient way of describing the \( x \) and \( y \) component of a field is to use the Jones matrix. For a wave that is linearly polarized with the \( x \) axis, its Jones vector is simply

\[
\begin{bmatrix}
1 \\
0
\end{bmatrix} \tag{3.8}
\]
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For a linearly polarized wave, whose plane of polarization is making an angle $\theta$ with the $x$ axis, its Jones vector is given by

$$
\begin{bmatrix}
\cos \theta \\
\sin \theta
\end{bmatrix}
$$

(3.9)

A right circularly polarized wave has

$$
\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}
$$

(3.10)

and a left circularly polarized wave has

$$
\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}
$$

(3.11)

where the intensities are normalized such that $|A_x|^2 + |A_y|^2 = 1$.

Several devices that help us to manipulate with the polarization of beams have the following Jones matrices. A linear polarizer has

$$
T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}
$$

(3.12)

it transforms a wave of components $(A_{1x}, A_{1y})$ into a wave of components $(A_{1x}, 0)$. A wave retarder has

$$
T = \begin{bmatrix} 1 & 0 \\ 0 & \exp(-j\Gamma) \end{bmatrix}
$$

(3.13)

it transforms a wave with field components $(A_{1x}, A_{1y})$ into another with components $(A_{1x}, e^{j\Gamma}A_{1y})$. When $\Gamma = \pi/2$, we have a quarter-wave retarder. When $\Gamma = \pi$, we have a half-wave retarder. Lastly, a polarization rotator has

$$
T = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}
$$

(3.14)

it converts a linearly polarized wave $\begin{bmatrix} \cos \theta_1 \\ \sin \theta_1 \end{bmatrix}$ into a linearly polarized wave $\begin{bmatrix} \cos \theta_2 \\ \sin \theta_2 \end{bmatrix}$ where $\theta = \theta_2 - \theta_1$. 
3.2 Modes Supported by an OmniGuide Fiber

The demands for high bit rates, dense wavelength-division multiplexing, and long distances, have pushed optical fibers towards ever-more demanding levels. A particularly exciting departure from traditional fibers are fibers based on photonic band gaps, forbidden frequency ranges in periodic dielectric structures that can confine light even in low-index or hollow regions. Two main classes of fibers have emerged using photonic band gaps: photonic crystal fibers that use a two-dimensional transverse periodicity, and Bragg fibers that use a one-dimensional periodicity of concentric rings. Here we focus on the propagation of light in a novel class of Bragg fibers "OmniGuide" fibers with a hollow core, which use a multilayer cladding that exhibits omnidirectional reflection in the planar limit [4].

In this section we will briefly introduce the concept of omnidirectional reflection and the design and fabrication of multilayered macroscopic fiber preforms of omnidirectional dielectric mirror fibers. The Bragg mirror is sometimes called the perfect mirror because it reflects light at any angle with virtually no loss of energy. As a result it makes possible a number of applications in optical technology, the most significant to date being flexible optical fiber that can transmit the high-powered lasers.

3.2.1 Fiber Fabrication Technique

Polymer fibers are ubiquitous in applications such as textile fabrics because of their excellent mechanical properties and the availability of low-cost, high-volume processing techniques; however, control over their optical properties has so far remained relatively limited. Conversely, dielectric mirrors are used to precisely control and manipulate light in high-performance optical applications, but the fabrication of these typically fragile mirrors has been mostly restricted to planar geometries and remains costly. We combined some of the advantages of each of these seemingly dissimilar products in the fabrication of polymeric fibers with an exterior multilayer dielectric mirror. Thermal processing techniques were used to reduce a macroscopic layered dielectric structure to submicrometer length scales, creating a fiber having a photonic band gap in the mid-infrared (mid-IR).

We employed a three-pronged approach in omnidirectional dielectric mirror fiber production, consisting of materials identification, fiber preform construction, and fiber draw. Materials selection involved the empirical identification of a pair of amorphous materials, polyether sulfone (PES) and arsenic triselenide (As$_2$Se$_3$), which have sub-
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Substantially different refractive indices, yet similar thermomechanical properties within a certain thermal processing window. In order to use similar processes in the fabrication of omnidirectional mirror structures, the selected materials should exhibit low optical absorption over a common wavelength band, very similar viscosities at the processing temperatures of interest, and good adhesion/wetting without cracking, even when subjected to thermal quenching [6]. The choice of a high-temperature polymer, PES, and a simple chalcogenide glass, $\text{As}_2\text{Se}_3$, resulted in excellent thermal co-deformation without film cracking or delamination. One advantage in choosing $\text{As}_2\text{Se}_3$ for this application is that not only is it a stable glass, but it is a stoichiometric compound that can be readily deposited in thin films through thermal evaporation or sputtering without dissociation. Additionally, $\text{As}_2\text{Se}_3$ is transparent to IR radiation from approximately 0.8 to 17 $\mu$m and has a refractive index of 2.8 in the mid-IR. PES is a high-performance, dimensionally stable thermoplastic with a refractive index of 1.55 and good transparency to EM waves in a range extending from the visible regime into the mid-IR.

The selected materials were used to construct a multilayer preform rod, which essentially is a macroscale version of the final fiber. In Fig. 3.1, we measure the transmission spectrum of the PES material using Fourier Transform Infra Red (FTIR) spectroscopy. The transmission spectrum of PES shown in Fig. 3.1 will be important later when we measure the transmission loss of modes in fibers, because it gives us a picture of transmission through the cladding multilayer, and we can use this as a reference when we want to separate the transmission loss due to the cladding from the core. We can see from Fig. ??, that there are 3 local minima at 1132 nm, 1665 nm, and 1905 nm. This implies that at all other wavelengths, the transmission loss in the fiber is dominated by the loss due to the core, but at these 3 particular wavelengths, we need to consider both the loss due to the core and the cladding [7].

To fabricate the dielectric mirror fiber preform, we deposited an $\text{As}_2\text{Se}_3$ film through thermal evaporation on either side of a free-standing PES film, which was then rolled on top of a PES tube substrate, forming a structure with 21 alternating layers of PES and $\text{As}_2\text{Se}_3$. The resulting multilayer fiber preform was subsequently thermomechanically drawn down with an optical fiber draw tower Fig. 3.2 into hundreds of meters of multilayer fiber with precisely controlled submicrometer layer thickness, creating a photonic band gap in the mid-IR [13]. The spectral position of the photonic band gap was controlled by the optical monitoring of the OD of the fiber during draw, which was later verified by reflectivity measurements on single
and multiple fibers of different diameters.

Mirror fiber reflectivity was measured from both single fibers and parallel fiber arrays with a Nicolet/SpectraTech NicPlan infrared microscope and Fourier transform infrared spectrometer (Magna 860). These reflectivity results are strongly indicative of uniform layer thickness control, good interlayer adhesion, and low interdiffusion through multiple thermal treatments. This was confirmed by scanning electron microscope (SEM) inspection of fiber cross sections, shown in Fig. 3.3. Fig. 3.3 shows three cross-sectional views (at increasing magnification) of a hollow cylindrical fiber, that can transmit laser beams with minimal energy loss. Layers of a chalcogenide glass (white), arsenic triselenide (As$_2$Se$_3$), alternating with a thermoplastic polymer (gray) surround the hollow core (black). The same polymer is used to coat the fiber [8].

Bragg mirrors have different band-gaps for TE ($E_z = 0$) and TM ($H_z = 0$) polarizations, referring to the fields purely parallel to the interface and fields with a normal component, respectively. Here we use the convention that modes are labeled by their angular momentum integer $m$, and the $l$-th mode of a given $m$ is labeled $TX_{ml}$.The modes supported by any cylindrical waveguide, can be computed by the
transfer-matrix method. Here the longitudinal fields ($E_z$ and $H_z$) of a given $(m, \omega, \beta)$ in an annular region of index $n_j$ are expanded in Bessel functions $J_m(k_j r)$ and $Y_m(k_j r)$, with 

$$k_j \equiv \sqrt{n_j^2 \omega^2 / c^2 - \beta^2}$$ 

(3.15)

Here we are primarily interested in the modes that lie within the band gap of the one-dimensional Bragg mirrors. Such modes must decay exponentially with $r$ in the cladding, and therefore are truly guided modes in the limit of infinitely many cladding layers (21 layers in our preforms). In the dielectric wavguide, the modes are only purely TE and TM for $m = 0$, but for $m \neq 0$ they are strongly TE-like or TM-like, and are called HE and EH, respectively. In Omni-Guide fibers, the nonzero $m$ modes ($HE_{11}$) are doubly degenerate, while the $m = 0$ (TE$_{01}$) are non-degenerate. In this report, we will consider the 4 most common and fundamental modes individually, and also in superpositions. These modes include, $HE_{11}$, TE$_{01}$, TE$_{02}$, and TM$_{01}$ [11].
Figure 3.3: SEM images of hollow-core fiber

3.3 Mode Pictures

In this section we will study the 4 modes mentioned earlier, $HE_{11}$, $TE_{01}$, $TE_{02}$, and $TM_{01}$, but with special emphasis on the first three, because the polarization of $TM_{01}$ is orthogonal to the polarization of $TE_{01}$, thus the fiber can only support one of these two modes. The profiles and polarizations of these 4 modes are illustrated in Fig. 3.4.

In the first column, there shows the complete intensity profile of each mode, the red color corresponds to the high positive intensity regions, the yellow color corresponds to the zero intensity regions, and the blue color corresponds to the negative intensity regions. We can see from the first column that the $HE_{11}$ mode is governed by a Gaussian function with maximum intensity at the center and decays gradually as $\exp(-ar^2)$, where $a$ contains information about the width of the beam. While $TE_{01}$,
Figure 3.4: Polarizations and phase distributions of 4 fundamental modes.

TE_{02} and TM_{01} are all governed by the 1st-order Bessel function \( J_1(r) \), thus have a zero at the center, and oscillate between peaks and valleys, with each peak lower than its preceding one, and the oscillation is modulated by a Gaussian envelope. It is interesting to note that if we normalize the maximum intensities and \( 1/e^2 \) widths of the HE_{11} and TE_{01} modes, the TE_{01} is wider than the HE_{11}, and the TE_{01} has very little overlap with the HE_{11} if we multiply the two modes together, since the HE_{11} has its maximum where the TE_{01} has its 0, and when the TE_{01} reaches its maximum the HE_{11} is approaching asymptotically to 0. On the other hand, the TE_{02} is also a 1st-order Bessel function like the TE_{01}, but its width is smaller than the TE_{01}, thus we see a secondary ring in TE_{02} but only the primary ring in TE_{01}. As a result, the TE_{02} has more overlap with the HE_{11} than TE_{01} does, because its primary ring is smaller and forms a better approximation with the HE_{11}.

Column 1 of Fig. 3.4 also shows that the HE_{11} is linearly polarized, the TE_{01} and
The $\text{TE}_{02}$ are azimuthally circularly polarized (the second ring of $\text{TE}_{02}$ has the opposite orientation than the primary ring), and the $\text{TM}_{01}$, although has the exact same intensity profile as the $\text{TE}_{01}$, but is radially circularly polarized (can be either radially pointed inward or outward). Despite the exactness in intensity, the $\text{TM}_{01}$ actually has 0 overlap with the $\text{TE}_{01}$, due to the fact that at every point in the profile, the polarization of the $\text{TM}_{01}$ is exactly $\pi/2$ out of phase with the polarization of the $\text{TE}_{01}$. Columns 2 and 3 of Fig. 3.4 show two polarizations of each mode, with the difference in the two polarization angles being $\pi/2$. In this case, since the Gaussian is already linearly polarized, a polarization angle of 0 would simply recovered the original profile, while a polarization angle of $\pi/2$ would completely kill the beam, since it has no component in this polarization. For the TE and TM, both of the two polarizations are antiparallel, with a phase difference of $\pi$, but the orientation of the polarization compared to the two semicircles of the intensity profiles differ between them by a phase of $\pi/2$. In the last column of Fig. 3.4, we introduced the phase diagrams of the modes, these are used to characterize one particular polarization of the modes. The two gray scales correspond to a phase of either 0 or $\pi$. Since the $\text{HE}_{11}$ is linearly polarized, thus it has a uniform phase diagram, with no internal phase difference. Whereas, the phase diagrams of the TE and TM are split into two halves corresponding to the internal phase difference of $\pi$. The orientation of the TM phase diagram is $\pi/2$ out of phase with the TE to signify their 0 overlapping. In the phase diagram of the $\text{TE}_{02}$, we used a similar diagram from $\text{TE}_{01}$, but it has alternating phase shift of $\pi$.

These arguments seem hand-waving and qualitative at the moment, but we will quantify them later in the experiment section, where we will show the relation between the size of the modes in relation to both the size of the beam and the fiber core-diameter, and their coupling. These phase diagrams, especially the gray scale that represents each phase will be fundamental in synthesizing different modes. The instrument that we use to implement these computer generated phase diagrams is called a Spatial Light Modulator (SLM), which is an object that imposes a spatially-varying modulation on the phase of the beam. Since the natural laser beam has a Gaussian profile, we can use the SLM to modulate the beam to the desire mode, such as the $\text{TE}_{01}$. The SLM has a pixellated structure and a resolution of $640 \times 480$, and accepts phase diagram colors with gray-scale ranging from 0 to 256. This requires that our phase diagrams have dimensions that match the resolution of the SLM, and the correct colors or brightness value for the phases. Each brightness value on the
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phase diagram (any value between 0 and 256), corresponds to a phase shift (from 0 to $2\pi$), this correspondence, however, also depends on the wavelength of the incoming beam. To find the 2 brightness values that give us a phase shift of exactly $\pi$, we calibrate the SLM by selecting the wavelength of the input beam, and measuring the output intensity while implementing a series of the HE$_{11}$ phase diagrams on the computer with brightness value changing from 0 to 256. An example of this is shown in Fig. 3.5. In this example, we picked the wavelength to be 1550 nm. The plotted

![Figure 3.5: SLM Calibration](image)

function is a sinusoidal wave, thus we can easily locate the two points on the curve that give us a difference of $\pi$, and the $x$ component or brightness value of these two points will be our gray-scale for the 0 and $\pi$ phase on all phase diagrams.

### 3.4 Mode Superposition

In this section, we will discuss two different methods of achieving the superposition of two different modes using an SLM. One approach is to modulate the amplitude, and the other is to modulate the phase. We first show that the complete superposition of two modes requires modulating both the phase and the amplitude of the two modes. But in this paper, we will only use the phase modulation approach, even though this leads to degeneracies in the superposition, this gives a close approximation of the real superposition with only one SLM.

In general the superposition of two states $|\psi_1\rangle$ and $\psi_2\rangle$ is given by,

$$|\psi\rangle = \alpha|\psi_1\rangle + \beta|\psi_2\rangle$$  \hspace{1cm} (3.16)
Q. Hu

where $\alpha$ and $\beta$ are the amplitude coefficients of the individual phases. In position space, Eqn 3.16 can be written as

$$\vec{\psi}(r) = \alpha \vec{f}_1(r)e^{i\psi_1} + \beta \vec{f}_2(r)e^{i\psi_2} \tag{3.17}$$

where $\vec{f}(r)$ encodes information about the amplitude of the state. For instance, in $HE_{11}$ mode, $\vec{f}(r)$ is a Gaussian function, thus

$$\vec{f}_{HE_{11}}(r) = e^{-r^2/(2\sigma^2)/(\sigma\sqrt{2\pi})} \tag{3.18}$$

where $\sigma$ is the standard deviation of the distribution. In both $TE_{01}$ and $TE_{02}$, $\vec{f}(r)$ is a Bessel function, thus

$$\vec{f}_{TE_{01}}(r) = A_i J_m(k,r) \hat{\theta} \tag{3.19}$$

where $k$ determines the position of the first minimum of the Bessel function.

If we are given two states $|\psi_1\rangle$ and $|\psi_2\rangle$ with phase distributions $\phi_1$ and $\phi_2$ respectively, and we are interested in studying the phase superposition of these two states, we can calculate the resultant phase $\Phi$ by using Eqn 3.20,

$$\alpha e^{i\phi_1} + \beta e^{i\phi_2} = e^{i\Phi} \tag{3.20}$$

Since in Eqn 3.20 we fix the amplitude of the output to be 1, choices of $\alpha$ and $\beta$ must then satisfy the following normalization condition,

$$|\alpha e^{i\phi_1} + \beta e^{i\phi_2}|^2 = 1 \tag{3.21a}$$

$$|\alpha|^2 + |\beta|^2 + 2\alpha\beta Re\left\{\iint dx dy e^{i(\phi_1 - \phi_2)}\right\} = 1 \tag{3.21b}$$

$$\beta = (\sqrt{(4\alpha^2\Psi^2 - 4(\alpha^2 - 1)) - 2\alpha\Psi})/2 \tag{3.22}$$

where

$$\Psi = \Re\left\{\iint dx dy e^{i(\phi_1 - \phi_2)}\iint dx dy\right\} \tag{3.23}$$

and $\iint dx dy$ is the resolution of the SLM screen, which is a 640 x 480 matrix. From Eqn 3.21, $\beta$ is automatically determined once $\alpha$ is specified, and both $\alpha$ and $\beta$ are real numbers ranging from 0 to 1. It is interesting to notice that when the modes in superposition are orthogonal to each other i.e. they are eigenmodes, then the cross term in Eqn 3.21 vanishes, $\Psi = 0$. Thus we should expect $\beta = \sqrt{1 - |\alpha|^2}$.

Ideally, we would like Eqn 3.20 to output only a phase $e^{i\Phi} \leq 1$, because this allows us to model the phase and amplitude involved in a superposition separately, and this
would require $\Phi$ to be real. However, because we only assign real values to $\alpha$ and $\beta$, $\Phi$ actually takes on a complex value,

$$\Phi = \Phi_r + i\Phi_i$$

(3.24)

therefore $e^{i\Phi}$ has an amplitude associated with it as well as a phase.

$$e^{i\Phi} = e^{i(\Phi_r + i\Phi_i)} = e^{-\Phi_i}e^{i\Phi_r}$$

(3.25)

As we see in Eqn 3.25, a non-zero $\Phi_i$ deviates the amplitude away from 1. In order to eliminate $\Phi_i$, we need to set either $\alpha = 1$ or $\alpha = 0$. But if we restrict $\alpha$ such that $\Phi_i$ has a non-vanishing value, we can still achieve the specific output of $e^{-\Phi_i}e^{i\Phi_r}$ by using the apparatus shown in Fig 3.6.

As shown in Fig. 3.6, an input beam, which is linearly polarized and Gaussian by nature, is first treated with a polarizer, thus it picks up a polarization of $\psi$. It then passes through the SLM, where we implement the phase pattern. Here $\phi$ denotes the phase value on the phase pattern, thus it is actually a 2-dimensional matrix, whose value depends on the coordinate, $\phi(x, y)$.

The output beam is again past through another polarizer with polarization angle of $\delta$. Here each of the three angles is a degree of freedom, and our objective is to construct a beam at the output of the second polarizer such that it has a phase distribution of $\Phi$, which may also depend on the coordinate $\Phi(x, y)$. Each of the two polarization angles and the SLM can be treated as an operator acting on the beam, and they are related by the following

$$
\begin{pmatrix}
\cos \psi \\
\sin \psi
\end{pmatrix}
\begin{pmatrix}
e^{i\phi} & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos^2 \delta & \sin \delta \cos \delta \\
\sin \delta \cos \delta & \sin^2 \delta
\end{pmatrix}
\begin{pmatrix}
\cos \delta \\
\sin \delta
\end{pmatrix}
= e^{i\Phi}
\begin{pmatrix}
\cos \delta \\
\sin \delta
\end{pmatrix}.
$$

(3.26)
if we rearrange terms, Eqn 3.26 can be simplified into its real and imaginary parts separately.

\[
\cos \phi \cos \psi \cos \delta + \sin \psi \sin \delta = e^{-\Phi_r} \cos \Phi_r \tag{3.27a}
\]

\[
\sin \phi \cos \psi \cos \delta = e^{-\Phi_r} \sin \Phi_r \tag{3.27b}
\]

Eqn. 3.27 gives us a guideline when we choose the angles of the two polarizers and design our phase patterns.

The drawback of the above approach is that whenever we need to compute the superposition of two modes, we need to reconfigure all three parameters, the two polarization angles and the phase distribution on the SLM. Here we introduce an alternative approach, where only the SLM is implemented in our setup without the use of polarizers, and we will also see that this introduces degeneracy into our approach. In this approach we only look at the phase of the modes, and primarily focus on the HE_{11} and TE_{01} modes. Since the HE_{11} mode has a uniform phase distribution, whether it has a uniform 0 or \( \pi \), the SLM does not recognize any internal phase shift. Whereas the TE_{01} has an internal phase shift of \( \pi \). As we discussed earlier about the basics of an SLM, we can first find the two gray scales that correspond to a phase difference of \( \pi \). Then the superposition of two phase distributions, will be linear superpositions of these two gray scales.

\[
\begin{array}{cccccccc}
-1 & -0.8 & -0.6 & -0.4 & -0.2 & 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1
\end{array}
\]

Figure 3.7: Phase superposition SLM patterns

In Fig. 3.7, we show how the phase distributions vary in a purely-phase superposition. At the two extrema of Fig. 3.7 we have two uniform phase diagrams, with different gray scale, but both give the HE_{11} mode. At the center, we have the phase diagram for the TE_{01}, where the upper and bottom rectangles correspond to the two extrema in the gray scale, thus creating a phase shift of \( \pi \). From the left-most phase diagram to the center one, we gradually reduce the gray scale of the upper rectangle, thus slowly increase the phase shift from 0 to \( \pi \). From the center phase diagram to the right-most one, we gradually reduce the gray scale of the bottom rectangle, and
similarly reduce the phase shift from $\pi$ back to 0. As a result the entire spectrum in Fig. 3.7 represents a transition from the $HE_{11}$ mode to the $TE_{01}$ and back to the $HE_{11}$ mode. We label each phase diagram at a stage with $\alpha$. However, the degeneracy problem occurs here. If we look at the values of $\alpha$, the phase diagram at $\alpha = -1$ and $\alpha = 1$ both give the same $HE_{11}$, because the SLM only recognizes the phase difference, and not the absolute amplitude of each. Therefore, if we run the superposition from $\alpha = -1$ to $\alpha = 0$, this is the exact same superposition if we run from $\alpha = 1$ to $\alpha = 0$. In this method we lose the information about the amplitude. When we change the gray scale of the upper rectangle, it is the same as changing the gray scale of the lower rectangle. Later in the experimental results chapter, we will show the consequence of this degeneracy in pictures. Despite this degeneracy problem, we will use this approach throughout our measurements, because we are interested in studying the losses of the modes, and mode decompositions, this degeneracy does not affect either of those.

3.5 Mode Decomposition

In waveguides that support a multiple of modes, eigenmode decomposition of the output fields provide fundamental insights into the nature of electromagnetic wave propagation. The comparison of the modes present at the input to those exiting the waveguide can not only shed light on the loss mechanism in the structure and also allows quantitative analysis of mode coupling. The approach that we use is to measure the output intensity and thus indirectly obtaining the phase of the optical wave front. It involves the use of the Gerchberg-Saxton (GS) phase retrieval algorithm that results in a unique and also noise tolerant solution by iterating back and forth between the two-dimensional (2D) field distributions of the object and the Fourier planes.

Before we get into the details of mode decomposition, it is important to review some of the basics of Fourier optics. The Fourier transform $F(\nu_x, \nu_y)$ is related to the object $f(x, y)$ through the following transform

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\nu_x, \nu_y) \exp[-j2\pi(\nu_xx + \nu_yy)] d\nu_x d\nu_y$$  \hspace{1cm} (3.28)

In this section, we will show two ways of coming the Fourier transform in optics. One is that at sufficiently long distance, only a single plane wave contributes to the total amplitude at each point in the output plane, and the Fourier components are
eventually separated naturally. The other approach is to use a lens to focus each of
the plane waves into a single point.

When the propagation distance $d$ is sufficiently long, the only plane wave that
contributes to the complex amplitude at a point $(x, y)$ in the output plane is the
wave with direction making angles $\theta_x \approx x/d$ and $\theta_y \approx y/d$ with the optical axis. This
corresponds to the wave vector $k_x \approx (x/d)k$ and $k_y \approx (y/d)k$ and amplitude $F(\nu_x, \nu_y)$
with $\nu_x = x/\lambda d$, and $\nu_y = y/\lambda d$. The complex amplitude $g(x, y)$ and $f(x, y)$ of the
wave at the $z = d$ and $z = 0$ planes are related by

$$g(x, y) \approx h_0 F\left(\frac{x}{\lambda d}, \frac{y}{\lambda d}\right)$$

(3.29)

where $h_0 = (j/\lambda d) \exp(-jkd)$. Contributions of all other waves cancel out as a
result of destructive interference. This is known as the Fraunhofer approximation,
which simply states that the complex amplitude $g(x, y)$ of a wave of wavelength $\lambda$
in the $z = d$ plane is proportional to the Fourier transform $F(\nu_x, \nu_y)$ of the complex
amplitude $f(x, y)$ in the $z = 0$ plane, evaluated at the spatial frequencies $\nu_x$ and $\nu_y$.
We call the $z = 0$ plane "near field" and the $z = d$ plane "far field".

The alternative is to use a lens. Here the same plane wave has a complex amplitude

$$U(x, y, 0) = F(\nu_x, \nu_y) \exp[-j2\pi(\nu_x x + \nu_y y)]$$

(3.30)
in the $z = 0$ plane, and

$$U(x, y, d) = H(\nu_x, \nu_y)F(\nu_x, \nu_y) \exp[-j2\pi(\nu_x x + \nu_y y)]$$

(3.31)
in the $z = d$ plane, where

$$H(\nu_x, \nu_y) = H_0 \exp\left[j\pi\lambda d (\nu_x^2 + \nu_y^2)\right]$$

(3.32)
is the transfer function of a distance $d$ of free space and $H_0 = \exp(-jkd)$. When
crossing the lens, the complex amplitude is multiplied by the lens phase factor

$$U(x, y, d + \Delta) = H_0 \exp\left(j\pi\frac{x^2 + y^2}{\lambda f}\right)$$

$$\times \exp\left[j\pi\lambda d (\nu_x^2 + \nu_y^2)\right] F(\nu_x, \nu_y) \exp[-j2\pi(\nu_x x + \nu_y y)]$$

(3.33)

At the $z = d + \Delta + f$ plane, if we integrate over all the plane waves, we obtain the relation

$$g(x, y) = h_f \exp\left[j\pi\frac{(x^2 + y^2)(d - f)}{\lambda f^2}\right] F\left(\frac{x}{\lambda d'}, \frac{y}{\lambda d}\right)$$

(3.34)
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where \( h = (j/\lambda f) \exp[-jk(d + f)] \), and when \( d = f \), we obtain

\[
g(x, y) = h F\left(\frac{x}{\lambda d}, \frac{y}{\lambda d}\right)
\]

Here we obtain the Fourier transform using a lens by taking advantage of the fact the the complex amplitude of light at a point \((x, y)\) in the back focal plane of a lens of focal length \( f \) is proportional to the Fourier transform of the complex amplitude in the front focal plane evaluated at the frequencies \( \nu_x \) and \( \nu_y \).

The decomposition of a field in a waveguide is a one-dimensional problem since the field is constrained to be a linear superposition of the waveguide eigenmodes that can be determined from the waveguide structure. The only unknowns are the expansion coefficients in the superposition, which form a 1D complex space [10].

\[
I = |\sum c_n |n\rangle|^2
\]

where \( I \) is the measured intensity of the image, and \( c_n \) is the coefficient of each eigenmode \( |n\rangle \). An example of such superposition of eigenmodes is shown in Fig. 3.8. By

![Figure 3.8: An example of mode decomposition](image)

mapping the problem from a 2D image space into an abstract 1D space of waveguide eigenmodes, we significantly reduce the number of independent variables, and completely remove the dependency on the number of pixels in the iterative process when we carry out the GS algorithm, which can be computationally intensive. Furthermore, since the higher-order modes in multimode waveguides tend to have higher losses, we can also set a modal cutoff, and deal with only a finite number of modes.

We construct our algorithm to minimize an error function with respect to a set of independent variables representing the expansion coefficients of a basis set constructed of the waveguide eigenmodes. Since the vectorial aspect of the waveguide
modes is essential we define four squared-error functions for two orthogonally polarized components in both the near and far fields [5]

\[ \Delta_{a,b} = \int_{\text{core}} \left( I_{r}^{a,b}(r) - I_{me}^{a,b}(r) \right)^2 dA \] (3.37)

where \( a = 1, 2 \) defines the plane of measurement (near or far field), \( b = 1, 2 \) defines one of two perpendicular polarizations, \( I_{me} \) is the measured intensity, while \( I_{r} \) is the intensity of a reconstructed estimate of this field.

If the electric and magnetic field vectors of the nth waveguide mode are \( \phi_{n}^{(E)} \) and \( \phi_{n}^{(H)} \), respectively, and their scalar projections in a fixed direction are \( e_{n} \) and \( h_{n} \), then the total field vectors are

\[ E(r) = \sum_{n} c_{n} \phi_{n}^{(E)}(r) \] (3.38)

\[ H(r) = \sum_{n} c_{n} \phi_{n}^{(H)}(r) \] (3.39)

where \( c_{n} = |c_{n}|e^{i\phi_{n}} \) are the expansion coefficients. Using this notation, the reconstructed intensity is

\[ I_{r}(r) = \frac{1}{N} \Re \sum_{i,j} c_{i}^{*} c_{j}^{*} e_{i}(r) h_{i}(r) \] (3.41)

where \( N \) is a normalization factor, and we note that \( e_{i} \) and \( h_{j} \) may be chosen to be real functions in two-dimensional structures. By rearranging, we may write any of the four error functions \( \Delta_{a,b} \) as follows

\[ \frac{1}{N^2} \sum_{ijpq} c_{i} c_{j} c_{p} c_{q}^{*} \Lambda_{ijpq} - \frac{2}{N} \sum_{ij} c_{i} c_{j}^{*} \Gamma_{ij} + P \] (3.42)

where \( P \) is the integral over space of \( I_{me} \) squared, and

\[ \Lambda_{ijpq} = \int e_{i}(r) e_{j}(r) h_{p}(r) h_{q}(r) dA \] (3.43)

\[ \Gamma_{ij} = \int I_{me}(r) e_{i}(r) h_{j}(r) dA \] (3.44)

The tensors \( \Lambda \) and \( \Gamma \) may be contracted significantly by exploiting the symmetry of the waveguide modes. In our case, we use a circularly symmetric cylindrical waveguide throughout our experiment. Here we only perform decompositions using a basis consisting of the 16 lowest-energy modes with angular momentum \( m < 4 \); these 16 modes are listed in Table. 3.1
Table 3.1: 16 lowest-energy modes

| \( m = 0 \) | \( \text{TE}_01 \) | \( \text{TE}_02 \) | \( \text{TM}_01 \) | \( \text{TM}_02 \) |
| \( m = 1 \) | \( \text{EH}_{11} \) | \( \text{EH}_{12} \) | \( \text{HE}_{11} \) | \( \text{HE}_{12} \) |
| \( m = 2 \) | \( \text{EH}_{21} \) | \( \text{EH}_{22} \) | \( \text{HE}_{21} \) | \( \text{HE}_{22} \) |
| \( m = 3 \) | \( \text{EH}_{31} \) | \( \text{EH}_{32} \) | \( \text{HE}_{31} \) | \( \text{HE}_{32} \) |
Chapter 4

Analysis of Low-Loss Modes

In this chapter we present the light-propagation characteristics of Omni-Guide fibers, which guide light by concentric multi-layer dielectric mirrors having the property of omnidirectional reflection. We also show that the lowest-loss TE$_{01}$ mode can propagate in a single-mode fashion through hollow-core fibers, with other modes eliminated asymptotically by their higher losses and poor coupling. Like hollow metallic waveguides, there is substantial loss discrimination between a single lowest-loss mode, the TE$_{01}$, and other guided modes, this produces modal filtering that allows even a highly multimode Omni-Guide fiber to operate in an effectively single mode fashion. Because the TE$_{01}$ is cylindrically symmetrical and non-degenerate, it has the additional benefit of immunity to polarization mode dispersion from fiber birefringence.

As mentioned earlier, we can compute the supported modes in an Omni-Guide fiber by finding solution to the following equation

\[
\begin{pmatrix}
E_z(r, z) \\
H_z(r, z)
\end{pmatrix} =
\begin{pmatrix}
A_i & B_i \\
C_i & D_i
\end{pmatrix}
\begin{pmatrix}
H^{(1)}_m(k_{T,i} r) \\
H^{(2)}_m(k_{T,i} r)
\end{pmatrix}
\exp[i(k_z z - \omega t + m\theta)]
\]  

(4.1)

In this chapter, we are primarily interested in modes that lie within the band gap of the one-dimensional Bragg mirrors. Such modes must decay exponentially with $r$ in the cladding, and therefore are truly guided modes in the limit of infinitely many cladding layers (in our experiment we only used 20 layers). In Fig. 4.1, we show the computed guided modes of the Omni-Guide fiber. The colored lines correspond to the 3 different modes in the band gap, and the black line is the light line $\omega = ck_z$. The white region represents the continuum of modes that propagate within the multilayer cladding. In the upper left corner of Fig. 4.1, we also showed the simulated beam profile of the 3 modes being strongly confined within the core, with minor leakage.
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FIGURE 4.1: Guided modes supported by a hollowing Omni-Guide fiber

into the cladding [9].

Although in the differential losses between the $\text{TE}_{01}$ and the other modes create a modal-filtering effect that allows these waveguides to operate in an effectively single-mode fashion, in large-core multi-mode Omni-Guide fibers, the losses become dominated by scattering into other closely-spaced modes, for instance the $\text{TE}_{01}$ might scatter into the degenerate $\text{TM}_{11}$ mode. Throughout our research, our objective is to synthesize a multiple number of modes, such as the $\text{TE}_{01}$ and $\text{HE}_{11}$, using a multi-mode Omni-Guide fiber, but also to limit the number of modes supported by the fiber to those two. As a consequence of this, we need to choose a fiber with just the right core size, such that it is large enough to support those two modes, but also small enough that it eliminates the high-order or degenerate modes.

Previously we only considered the situation where there is an infinite number of cladding layers surrounding the inner core of the fiber, but in reality, there is only a finite number of cladding layers in the omnidirectional mirrors. Because of this the field power will slowly leak out in a process akin to quantum mechanical tunneling. However, this radiation loss decreases exponentially with the number of cladding layers. It has been reported earlier that only a small number of layers is
required to achieve leakage rates well below 0.1 dB/km. Also the good news is that the radiation leakage strongly differs between modes, thus inducing a modal-filtering effect that allows a large-core and multi-mode supporting Omni-Guide fiber to operate in an effectively single-mode fashion. When we had infinitely many cladding layers, the modes in the Omni-Guide fibers are truly confined due to the band gap and have discrete real eigenvalues $k_n$, as shown in Fig. 4.1. But in the case of finitely many layers, the modes are no longer truly confined, this leads to a continuum of $k$ values with infinitely extended eigenstates. The confined modes become leaky resonances, which are superpositions of real-$k$ eigenmodes centered at $k_n$ but with a width $\Delta k$ proportional to the radiative decay rate $\alpha_n$.

For all modes, the radiative decay $\alpha$ decreases exponentially with increasing numbers of cladding layers, due to the exponential decay of fields in the Bragg band gap, eventually to the point where other losses, such as absorption, dominate. At the telecommunication wavelength, $\lambda=1.55\mu m$, the TE loss decreases by a factor of about 10 per cladding bilayer. In Fig. 4.2 panel (a), we show the computed radiation leakage rate $\alpha$ for the lowest-loss TE$_{01}$, and the more lossier modes HE$_{11}$ and TM$_{01}$. In the right 2 panels of Fig. 4.2, we also show the loss as functions of the number of layers in the cladding, and also the core diameter of the fiber. It is clear from these two figures that the losses of the three modes decrease substantially when the core radius is increased. Moreover, in our analysis, if we assume that the number of layers in the dielectric mirror is large enough, we can neglect the radiation losses.

From Fig. 4.2, we clearly see that these differential losses create a mode-filtering effect that allows the TE$_{01}$ mode to operate as effectively a single mode even in large-core and multi-mode supporting Omni-Guide fibers. The radiation losses are proportional to the field amplitude or intensity in the cladding, which goes like $1/R^4$ for the TE$_{0l}$ modes, and multiplied by the surface area, which scales as $R$, thus the radiation leakage rate $\alpha$ is proportional to $1/R^3$. This $1/R^3$ dependence of the TE$_{01}$ dissipation loss is due to the presence of a node in the electric field of this mode near the interface between the hollow core and the first layer of the dielectric mirror. The only nonzero component of the electric field for the TE$_{01}$ mode is $E_\phi$. TE$_{01}$ is the lowest-loss mode within the TE$_{0l}$ family of modes. There is a monotonic increase of the loss with the order $l$. In particular, the loss of the TE$_{02}$ is about 3.3 times larger than that of the TE$_{01}$ for almost all frequencies and core radii. For mode other than the TE$_{0l}$ modes, the absence of the node in the electric field near the core-layer interface results in more penetration of the field into the cladding, and also different scaling
laws for the \( R \) dependence. We can see from Fig. 4.2 that the TM_{01} dissipation loss follows a \( 1/R \) dependence, while the HE_{11} has some intermediate dependence with a slope that approaches \( 1/R \) for large \( R \) [12].

Another reason for the low losses of the TE_{01} mode is the fact that this mode is confined in the core only by the TE gap. TM modes and mixed modes have a component that is confined by the weaker TM gap, and thus they penetrate deeper into the multilayer cladding. The fact that the TE_{01} loss is smaller than that of the other modes is important for the transmission properties of Omni-Guide fibers. It implies that even if several modes are excited at the input of a waveguide, after a certain distance only the TE_{01} mode will remain in the waveguide. This loss-induced modal discrimination results in an effectively single-mode operation in a multi-mode
supporting waveguide.

Now we consider the effect of fiber bend on the propagation of modes, this is interesting to take into account, because when we study a piece of fiber, the fiber may not be held perfectly straight and may have some curvature to it. In conventional optical fibers, since there is typically only a single mode, all of the coupling in a bend is to the radiation continuum. On the other hand, the Omni-Guide fibers are highly multimode, with many other guided modes at $k$ values nearby to TE$_{01}$, also the cladding modes lie at distant $k$ due to the large band gap. Since mode coupling varies inversely with $\Delta k$, Omni-Guide fiber bend effects will be dominated by coupling/scattering into other guided modes in the core. The scattering power is proportional to $1/(R_b \Delta k)^2$, where $R_b$ is the radius of curvature [9]. This scattered power will be dominated by the closest $m = 1$ mode, and is significant when $R_b$ approaches $1/\Delta k$. In ordinary hollow metallic waveguide, the TE$_{01}$ mode is degenerate with the TM$_{11}$ mode, since the selection rule, which states that for a waveguide with a cylindrical symmetry, a bend can only directly couple modes with $|\Delta m| = 1$, allows these two modes to couple, and can produce significant scattering for any bend radius. Fortunately, the Omni-Guide fibers break such degeneracy, however, there is commonly a scattering of the TE$_{01}$ into the EH$_{11}$. As a result, throughout our experiment we need to also worry about the bend of the fiber, especially when the bend continues for a long distance, it would significantly increase the lossy rate of the modes.
Chapter 5

Experimental Results

5.1 Overview of Procedures and Techniques

The objective of the experiments is to control the excitation and propagation of either a single mode or a superposition of modes. The process is in five stages. We here give a general picture first, a more detailed setup will follow.

1. First we generate a laser beam, which naturally has a Gaussian distribution. The source through which we generate the beam also allows us to control the wavelength and power of the beam.

2. This Gaussian beam is then sent into a mode synthesis system, which modulates either the amplitude or the phase of the incoming beam (in our experiment, we only had success with phase modulation). This system transforms the Gaussian beam into the mode that we are interested.

3. The new beam generated by the mode synthesis system, with its new phase or amplitude, is sent into a photonic band gap fiber.

4. The beam propagates down the fiber. We only know that at the input of the fiber the beam has the desired profile. Ideally, the beam will only excite the modes that the mode synthesis system gave to it. But as the beam travels inside the fiber, different eigenmode picks up a different phase shift, and the beam at the output of the fiber will be different from the beam at the input.

5. The output beam then enters a mode analysis system, where we calculate the different eigenmodes that contribute to the beam that we see, and compare
them with the eigenmodes that we assigned to the beam in the mode synthesis system.

We show this process pictorially in Fig. 5.1. The detailed optical setup that implements this process is shown in Fig. 5.2.

![Figure 5.1: Generation and Transmission of Modes](image)

As shown in Fig. 5.2, the source that we use includes a Mira\textsuperscript{TM} 900 Mode-locked Titanium:Sapphire Ultrafast Oscillating Laser System (Ti:Sa) and an Optical Parametric Oscillator (OPO), both are manufactured by Coherent Inc. The OPO is an effective nonlinear technique to significantly extend the wavelength coverage of Ti:Sa-based ultrafast oscillators. Coherent’s Mira\textsuperscript{TM} OPO is a synchronously pumped system, seeded by a Mira Ti:Sa laser that provides high-repetition rate, near-transform-limited femtosecond and picosecond pulses in the visible and infrared. In our experiment, we will be primarily concerned with near-infrared wavelength (1000 nm to 1600 nm).

We monitor the wavelength of the beam coming out of the OPO using an Optical Spectra Analysis (OSA). The beam coming out of the OPO is collimated using a lens, and its size is reduced using a 300 μm pinhole. Afterwards a nicely collimated beam, which has a size or around 1 inch is split into two paths using a beam splitter. One path continues along the original optical path, the other path enters the SLM. The SLM is connected to a nearby workstation whose monitor has a resolution of 640×480 pixels. The SLM treats the entire screen on the monitor as the phase input pattern (here we only use the phase modulation method). We construct phase diagrams of those modes, such as the ones shown in Fig. 3.4, and set those phase diagrams in full-screen mode. The modulated beam is then reflected back along its original path.
and passes the beam splitter and recombines with the other path. At this point, the beam is no longer Gaussian, it already has its new phase pattern encoded on its beam profile. The beam is then shrunk to a size of approximately 50 μm using a 40 cm lens (we will explain the technique that was used to measure the size of the beam in the later section). We also have the option of rerouting the input beam to a VidiCon camera or power detector to study the profile and power intensity of the input beam. The beam now enters the photonic band gap fiber. In our measurement, we used a piece that is 30 cm in length and 68 μm in core diameter. At this core diameter, a 50 μm input beam achieves optimal coupling to the TE\(_{01}\). It has been measured that the optimal TE\(_{01}\) coupling happens when the input beam is roughly 60% of the core diameter, the optical HE\(_{11}\) coupling happens when the ratio is around 50%, and the ratio is even lower, around 40%, for the optimal TE\(_{02}\) coupling. The fiber itself is held as straight and loose as possible to prevent any unevenly distributed pressure from causing changes in the absorption or propagation loss of the modes. The output beam, which of course has a beam size the same as the fiber core diameter, is magnified 60× using a microscope objective lens. The beam after the 60× lens is then allowed to travel in free space a distance of a few inches, thus further magnifying itself, before...
it enters an InGaAs CCD camera, which has a detector chip size of 1.6 by 1.2 cm. Typically, by the time the beam reaches the chip of the camera, it has a size of around 3 mm.

5.1.1 Beam Size Measurement

The size of the beam changes constantly throughout this optical setup. However, the plane where the size of the beam plays a crucial role is at the fiber input. As we discussed in previous chapter, the relative size of the beam compared to the fiber core diameter and cladding layers determines the rate of loss. When measuring the beam size, we turn off all phase modulation, and only look at the size of the natural Gaussian beam at that plane. Here we characterize the size of a beam by the Full-Width-at-Half-Maximum (FWHM) of the Gaussian distribution. Here we use a beam stopper with a sharp edge, and mount it on an x-y-z stage, which allows us to move the stopper with μm precision. We start out with the beam completely unblocked, and measure the power intensity, which is the maximum. We then slowly move the stopper and gradually cover the entire beam, the power intensity detected therefore is an erf function as shown in Fig. 5.3. We can see that the power detected changes from maximum (when the beam is completely unblocked), to almost 0 (when the

![Figure 5.3: FWHM of input beam](image)

Fitting Eqn: 
\[ f(x) = a \cdot \text{erf}((x-b)/c) + d \]

\[ c = 29.26 \]

FWHM = 2\(c\sqrt{\ln 2}\)

FWHM = 48.72 μm
beam is completely blocked). The transition has a steep slope. When we fit the
data, we obtain an erf function, whose width can be used to calculate the FWHM. Alternatively, if we differentiate the erf function, we recover the Gaussian distribution as shown in the little panel. The Gaussian function has the same width or standard deviation as the erf function, and the FWHM can be computed using the following

\[
\frac{1}{2} = \exp \left( -\frac{x^2}{c^2} \right) \quad (5.1)
\]

\[
\text{FWHM} = 2c\sqrt{\ln 2} \quad (5.2)
\]

where \( c \) is the width of the Gaussian or the erf function. As a result, we calculated the size of the beam at the fiber input to be approximately 49 \( \mu \text{m} \).

5.2 Transmission Spectrum

As discussed in earlier chapter that we expect optimal transmission of modes when the incoming beam has an energy that is within the band gap of the fiber. We have calculated that when the incoming beam is outside of the band gap the loss rate becomes so high that very few modes can successfully propagate inside the fiber, maybe with the exception of the TE\(_{01}\) mode, which is the lowest loss. Whereas if the beam wavelength is within the band gap, the loss rates are much lower for all the modes, thus we expect that the fiber will be able to support a multiple number of modes, we will mainly look at the HE\(_{11}\), TE\(_{01}\) and the TE\(_{02}\). The challenge to synthesize modes within the band gap is the effect of scattering and coupling into other modes. In the later section, we will explain the steps that we took to generate both a pure single-mode, and a controllable superposition of two modes.

However, the question remains, how do we measure the location of the band gap. This is done by measuring the transmission spectrum of the fiber as a function of wavelength. Here we used Fourier Transform Infra Red spectroscopy (FTIR). FTIR is a measurement technique whereby spectra are collected based on measurements of the temporal coherence of a radiative source, using time-domain measurements of the electromagnetic radiation or other type of radiation. The spectrum that we measured for our fiber is shown in Fig. 5.4. Fig. 5.4 shows the transmission through the fiber core. Transmission through the cladding layers was also measured, but it was subtracted later, since the polymer that we used for all fiber cladding has the same transmission spectrum and therefore can be treated as a reference. Alternatively,
we can also sputter the input cross section of the fiber with gold, thus the IR beam can only propagate through the fiber core and not the cladding. From Fig. 5.4, we can see that the band gap is centered around 1.55 \( \mu \text{m} \), which is standard for telecommunication wavelength. The band gap has a width of approximately 2 \( \mu \text{m} \), ranging from 1.45 \( \mu \text{m} \) to 1.65 \( \mu \text{m} \). Due to the limitation of our Ti:Sa laser source, we can only reach wavelength up to 1.63 \( \mu \text{m} \). We will use Fig. 5.4 as a guide throughout our measurement.

5.3 Individual Mode Synthesis

Once we calibrate the optical setup shown in Fig. 5.2 and have the transmission spectrum shown in Fig. 5.4 as a guide, we are ready to synthesize modes. Without any mode synthesis system, or in our case, the SLM, the natural beam at the fiber output is TE\(_{01}\) if we are outside of the band gap (from below the band gap to the lower end of the band gap, 1.4 \( \mu \text{m} \) to 1.52 \( \mu \text{m} \)), however, we can also couple into the TE\(_{02}\) at the lower end of the band gap (1.48 \( \mu \text{m} \) to 1.52 \( \mu \text{m} \)). When we are inside the band gap (1.5 \( \mu \text{m} \) to 1.62 \( \mu \text{m} \)), we observe predominately the HE\(_{11}\), and can also couple into the TE\(_{01}\).

We can see that we can couple into the TE\(_{01}\) at almost all wavelengths by simply changing the alignment of the fiber input with respect to the incoming Gaussian beam, in other words, by misaligning the Gaussian beam at the input such that only
a portion of the beam is coupled into the fiber, this effectively creates a phase shift in the beam, the condition for coupling into the TE_{01}. Moreover, due to its low loss, the TE_{01} is not as limited by the band gap as the other modes. However, this misalignment trick does not allow us to couple into the TE_{02}. The optimal beam size for TE_{02} coupling is much smaller than those of the TE_{01} or the HE_{11}. Since throughout our measurement, the size of the input beam is fixed, we can only achieve TE_{02} coupling at low wavelength. In addition, due to the higher loss of the TE_{02} compared to the TE_{01} outside of the band gap, we only observe the TE_{02} at the lower end of the band gap. When the input beam is perfectly aligned with the fiber, and has a wavelength that is inside the band gap, we observe the HE_{11}.

If we decrease the size of the input beam to around 30 μm (less than 50% of the fiber core diameter), then we observe the TE_{02} throughout the entire band gap, and can not couple into the HE_{11}. Similarly if we slightly increase the size of the input beam to around 50 μm, we only observe the HE_{11} throughout the band gap and not the TE_{02}. At all beam sizes and wavelengths, we observe the TE_{01}, while the TE02 and HE_{11} can only be observed inside the band gap and are limited by the beam size.

An alternative and more formal approach than this qualitative method as described above is to use a mode synthesis system, or an SLM. Here we use the phase approach that we mentioned in the mode superposition chapter, but in this section we will only look at pure individual modes. The phase diagrams that we encode onto the beam using the SLM are shown in Fig. 3.4. The images at the fiber output, after further magnification, are shown in Fig. 5.5. Here we chose the size of the beam such that we observe both the TE_{02} at the lower end of the band gap and the HE_{11} at the higher end of the band gap, and of course the TE_{01} throughout. These images were captured when the wavelength of the input beam was at 1520 nm, the demarcation line between the lower and higher end of the band gap.

In Fig. 5.5, we only showed the images of the HE_{11}, TE_{01}, and TE_{02}, and not the TM_{01}. This is due to the orthogonality between the TE and the TM, since we wanted to achieve optimal coupling to the TE, the TM components are thus suppressed. In the first column of Fig. 5.5, we show the entire beam image of the 3 modes. The colors correspond to the intensity, as expected, we see a Gaussian distribution for the HE_{11}, and Bessel distributions for the TE's, where the TE_{02} has a secondary ring, and both the TE's have a zero at the center. However, these images only tell us the intensity of the modes, not enough to completely characterize them, we also need to look at their polarizations. To verify that those modes are indeed HE_{11}, TE_{01}, and TE_{02}, and
not some degenerate ones, we can perform a simple test using a polarizer. We look at 5 different polarization angles of the beam images (0°, 45°, 90°, 135°, and 180°), and the images at these polarizations are shown in the other columns. These polarized images confirm our expectations that the HE_{11} is a linearly polarized Gaussian beam, thus we see the intensity is at minimum at 0° and maximum at 90° and back to minimum at 180°. It is interesting that when the HE_{11} is at its minimum intensity at 0° and 180°, the beam does not completely vanish, in fact we see some remnants of the TE_{01}. This implies that the HE_{11} that we observed is not 100% pure, and it has some contributions from the TE_{01} due to its strong survivability in the fiber. But since the intensity difference is large, we can ignore the TE_{01} contribution here, but later when we decompose and analyze modes more rigorously, we will discuss this more quantitatively. In addition, since the TE’s are both circularly polarized in the azimuthal direction, in other words, it has equal components in polarizations, we expect the intensity of the polarized beam to be the same for all polarization angles. Indeed we see that in Fig. 5.5 the polarized beams have the same intensities and the orientation of the polarizations rotates as the polarization angle changes.

In Fig. 5.6, we measure the power intensity of the polarized beam for the HE_{11} and
TE$_{01}$ modes as a function of polarization angles. Since the HE$_{11}$ is linearly polarized, we see that its intensity varies in a sinusoidal function as we change the polarization angle. Whereas the TE$_{01}$ is circularly polarized, which implies equal intensity in all angles, we should expect the intensity to stay constant as we change the polarization. In Fig. 5.6, we do see some sinusoidal vibration in the TE$_{01}$ data, instead of a perfect horizontal line, that is due to the imperfection in the TE$_{01}$ coupling. It is interesting to note that the extrema of the HE$_{11}$ and TE$_{01}$ occur at the same polarization angle. It indicates that the TE$_{01}$ that we observed is not 100% pure and has some contribution from the HE$_{11}$, thus the TE−01 is slightly stronger in the polarization parallel to the HE$_{11}$ and weaker in the orthogonal polarization. However, the amplitudes of the two waves are vastly different, and we can treat them as effectively single modes.

5.4 Propagation Loss Measurement

In this section we measure the transmission losses of the HE$_{11}$ and TE$_{01}$ modes, and show that the TE$_{01}$ is indeed lower loss than the HE$_{11}$. The ideal way to measure the transmission loss of a mode at a particular wavelength in a fiber is to use the cutback technique. This technique involves the following steps
1. measuring the power at the output of the complete piece of fiber \( L \).

2. cutting the fiber at a point a small incremental distance \( \Delta l \) away from the fiber output

3. repeating the power measurement at the output of the new shorter piece of the same fiber \( L - \Delta l \).

We repeat the above process iteratively until the power at the output reaches a steady state level. The slope of the power vs. distance data plot gives information about the transmission loss. The power is an exponential function of the distance, it decays to its minimum value at the very end of the longest fiber.

However, the drawback of this method is that it involves doing cutbacks and is destructive, not very practical or efficient if we want to measure the losses of more than one modes and at a long range of wavelengths. Therefore, we adopted an alternative technique, and that is to measure the radial emission loss of the modes in the fibers. The concept of radial emission in fibers is shown in the left side of Fig. 5.7. Measuring the radial emission does not allow us to directly compute the transmission loss of a mode, because when the radiation leaks radially, we need to take into account the effect of cladding, and the radial loss is proportional to the longitudinal transmission loss. However, since the effect of cladding on radial emission is the same for all modes, we can treat it as a constant offset, and here we are only interested to show the relative losses between the \( \text{HE}_{11} \) and the \( \text{TE}_{01} \), it suffices to just measure the radial losses.

The radial emission power is measured by scanning the fiber using a fiber optics integrating sphere, like the one shown in the right side of Fig. 5.7. The sphere has
2 small openings that allow the fiber to go through, and is connected to a power detector. As a result, all the power emitted radially by the segment of fiber that is inside the sphere is collected by the detector. At each wavelength, the integrating sphere measures the power of the two modes. In Fig. 5.8, we show an example of the scanning result. Here the power at each point of the scanning distance is plotted in log scale in the vertical axis. We see that the power of both the HE\textsubscript{11} and TE\textsubscript{01} decays exponentially as we approach the end of the fiber. To reduce the errors in our measurement, we fit each data set with two exponential functions, and the parameters of the fitting functions contain information about the loss. We see that the slope of the HE\textsubscript{11} is steeper than that of the TE\textsubscript{01}, this implies that over the same length of propagation distance, the power of the HE\textsubscript{11} decays more than the power of the TE\textsubscript{01} does. This is exactly what we would expect, since the HE\textsubscript{11} is more lossy, thus its power decays faster. From Fig. 5.8, we can measure the slopes $s$ of the two lines, they give us the radial loss. Similarly, the inverse of the slope $1/s$, or the inverse of the loss, can be interpreted as the transmission easiness. Therefore, using this technique, we obtain two values at each wavelength, each corresponds to the transmission easiness of one of the two modes.

Figure 5.8: Power decay (log scale) at 1520 nm
We repeated the above measurement at all tunable wavelengths allowed by the Ti:Sa, and Fig. 5.9 shows the easiness values of the HE_{11} and TE_{01} modes as functions of wavelength. By purging the Ti:Sa with liquid nitrogen, we are able to tune the Ti:Sa from 1100 nm to 1630 nm. The easiness of the TE_{01} is plotted in blue, and that of the HE_{11} is in red. The easiness is in the unit of cm, and immediately we see that the TE_{01} lies above the HE_{11}. This is in agreement with Fig. 5.8, it simply indicates that the TE_{01} requires a longer propagation distance than HE_{11} does in order for the power to decay by the same amount in both. We see in the TE_{01} spectrum, several interesting phenomena, since it covers such a large range. One is the effect of cladding at low wavelengths (1100 nm to 1250 nm), here we mostly see the beam transmitting through the cladding multilayer instead of the core, the wavelengths at which the cladding takes place is also reasonable, since the polymer that we use (PES) to make the cladding has a transmission peak around 1130 nm, as shown in Fig. 3.1. Fortunately, we do not need to worry about the other two transmission peaks indicated by Fig. 3.1, since our wavelength tunable range stops at 1630 nm. Another interesting feature is the oscillation in the TE_{01} data set, this is evident in the two ”mountains” in the TE_{01} data, one around 1350 nm, and another around 1550 nm. This has also been predicted in simulation that the secondary ”mountain” in the

Figure 5.9: Transmission easiness of the HE_{11} and TE_{01} modes [3]
transmission loss could potentially reach as high as the primary one. The essence of Fig. 5.9 is obviously in the wavelength range from 1450 nm to 1630 nm, which is also the band gap of the fiber, as we showed previously in Fig. 5.4. This is the only region where the HE_{11} can be observed, and even though near the center of the band gap (1550 nm), the transmission easiness of the HE_{11} becomes comparable to that of the TE_{01}, the TE_{01} spectrum remains dominant, and has a band gap width about twice that of the HE_{11}. Although according to simulation, the TE_{01} spectrum should be much higher relative to the HE_{11} than what is shown in Fig. 5.9 (other factors may need to be considered, such as the imperfections in the coupling, and the fact that we did not directly measure the output power), Fig. 5.9 clearly shows that it is easier to transmit the TE_{01} in the fiber, or conversely, the TE_{01} is lower loss compared to the HE_{11}.

5.5 Mode Superposition and Decomposition Measurement

In the previous chapter where we discussed both the amplitude and the phase approach in mode superposition, but here we only show experimental implementation of the phase approach. In this section we will only discuss the mode superposition experiment involving the HE_{11} and TE_{01} modes. Although we have experimented with superpositions involving modes such as the TE_{02} and the EH21, it was difficult to couple into these two modes without changing the size of the input beam. Here we modulate the phase of the incoming Gaussian laser beam using the phase patterns shown in Fig. 3.7. The two phase diagrams located at both ends of Fig. 3.7 represent the HE_{11}, and the one in the center represents the TE_{01} mode. The two colors of the TE_{01} phase correspond to the 0 and π phase value at the particular wavelength under investigation, in this case, the wavelength is around 1500 nm, and the two grayscale values are 80/255 and 180/255. The optical setup that we used for this measurement similar to the one shown in Fig. 5.2, but with minor modifications at the output of the fiber, as shown in Fig. 5.10.

As we discussed earlier in Eqn. 3.37, that mode decomposition of an image requires knowledge of 4 parameters, two orthogonal polarizations in the near field, and the same two orthogonal polarizations in the far field. At the near field, we simply obtain the image itself, and at the far field, we obtain the Fourier transform of the image. In
Fig. 5.10, we showed in the upper panel that the image of the beam can be measured by magnifying the output beam at the fiber and captured using a CCD camera. To measure the image at the far field, we initially tried to focus the beam after the magnifying lens using a combination of lenses (using the lens focusing technique that we described in the Fourier optics section of the mode decomposition chapter), however, because the magnifying lens that we used was a microscope objective lens, the output beam can no longer be described as a pure plane wave, and by simply focusing the beam with another lens, it was difficult to find the correct plane where we can measure the Fourier transform of the initial image coming out of the fiber. Therefore we adopted the alternative technique that we also introduced earlier, and that is to let the beam propagate for a sufficiently long distance. This is shown in the bottom panel of Fig. 5.10, where the magnifying microscope objective lens is removed, and we measured the output beam directly with a camera, while allowing a sufficiently long propagation distance between the fiber output and the camera, in this case, it was approximately 4 inches. A polarizer is used in both the near and the far field measurements, it is placed between the camera and the beam to measure the two orthogonal polarizations.

The complete output images (without the polarizer) at the near field are shown in Fig. 5.11, where they are also referenced with the corresponding SLM phase patterns. The images shown in Fig. 5.11 were captured at a wavelength of 1500 nm, near the center of the band gap. In Fig. 5.11, the far left column represents a pure HE_{11},
the far right column represents a pure TE\textsubscript{01}, and the ones in the center represent a gradual transition from the HE\textsubscript{11} to the TE\textsubscript{01}, where each phase pattern in upper row is a superposition of the two modes. We notice that when the SLM phase pattern is the HE\textsubscript{11}, i.e. no phase modulation on the incoming beam, we indeed observe a Gaussian beam at the fiber output (maximum intensity at the center and decays exponentially in radial direction).

At this moment, we will only qualitatively argue that since the output beam resembles that of a circularly symmetric HE\textsubscript{11}, therefore the input HE\textsubscript{11} beam only coupled into the HE\textsubscript{11} mode in the fiber, and very little scattering into the other modes (more intensive mode decomposition will follow later). On the other hand, when the SLM phase pattern is the TE\textsubscript{01}, i.e. a phase shift of \( \pi \) is encoded onto the input beam, we do not observe a perfect TE\textsubscript{01} mode, or a donut mode, at the output. Instead we observe something that resembles the TE\textsubscript{01}. Like the TE\textsubscript{01}, it has zero intensity at the center, but unlike the TE\textsubscript{01}, the ring, where it has maximum intensity, is not circularly symmetric, and has a peculiar preference in one polarization. This implies that although we sent int a pure TE\textsubscript{01} at the input, the TE\textsubscript{01} did not couple only to the TE\textsubscript{01} in the fiber, but also some other "mysterious modes". Later we will use the mode decomposition algorithm to identify the other modes that the TE\textsubscript{01} coupled into, but at this point, we can also qualitatively state, that these "mysterious modes" have one polarization that is parallel to the TE\textsubscript{01}, and another polarization orthogonal to the TE\textsubscript{01}. In regions where the two polarizations are parallel, we see an increase in the intensity and where the two polarizations are orthogonal, we see a decrease in the intensity. From this, we can also deduce that the "mysterious modes"
Figure 5.12: Sample mode decomposition analysis results [10]
are both TE- and TM-like, thus must be one of those \( m \neq 0 \) degenerate modes.

From Fig. 5.11, we can also pictorially characterize the evolution of the transition as the following. The HE\(_{11}\) Gaussian mode first moves to the bottom half of the beam, and shrinks in width in one direction while elongates in the other direction, and eventually forms a semicircular ring. At the same time another semicircular ring emerges at the upper half of the beam. These two semicircular rings form the entire ring of the TE\(_{01}\). Here is where we find evidence of the degeneracy effect that we mentioned earlier. When we run superposition in the other direction (with \( \alpha \) going from 0 to 1), we see the exact same evolution here except in time-reversed mode, namely the bottom semicircular ring grows and becomes the HE\(_{11}\), while the upper semicircular ring decays and vanishes. The ideal superposition with no degeneracy would be that when \( \alpha \) grows in the positive direction, the upper semicircular ring would grow and become the HE\(_{11}\) while the bottom one vanishes. In other words, in non-degenerate superposition, the transition from \( \alpha = -1 \) to 0 should be the time-reversed and upside down version of the transition from \( \alpha = 0 \) to 1. We also notice that the superposed images do not vary linearly with \( \alpha \), we see that the image stays approximately constant for low values of \( \alpha \), but experiences a drastic change when \( \alpha \) is near \(-0.2\), we will show later that indeed the transition actually is a sinusoidal function of \( \alpha \).

To analyze this more quantitatively, we process the measured images using the mode decomposition algorithm. The setup shown in Fig. 5.10 is only the hardware portion of the entire decomposition system. Some sample results, extreme cases, are illustrated in Fig. 5.12. Here we showed the cases for the pure HE\(_{11}\) (\( \alpha = -1 \)), pure TE\(_{01}\) (\( \alpha = 0 \)), and one in between (\( \alpha = -0.5 \)). In each case, we show the near field polarized images at the left panel, the far field polarized images at the right panel, and the unpolarized images at the center panel, where the unpolarized near field image is at the top and the unpolarized far field image is the bottom. All the near field images are boxed in red, and the far field images are boxed in blue. In each field, we show four smaller panels. The top two panels correspond to the measured intensities at the two orthogonal polarizations, and the bottom two panels are for the reconstructed intensities.

As we showed earlier, that both the measured and the reconstructed intensities are important in computing the error function. As we would expect, the measured intensities at the far field appear smoother than those at the near field, this is due to the fact that at each point on the wave in the far field, we have contributions from
every point on the wave in the near field, this is equivalent to perform a 2D averaging of the near field image. We also observe that in the HE_{11} case (α = -1), there is a significant different in the intensities of the two polarizations, this is expected due to the linear polarization of the HE_{11}. Similarly, in the TE_{01}, we see that the intensities in the two polarizations are comparable, this is due to the circular polarization of the TE_{01}. Another interesting feature in Fig. 5.12 is that we see very faint rings in the measured intensities of the near field, especially in the TE_{01} case, this is an indication of the presence of higher frequency or higher energy modes. This further confirms our initial argument that when we try to couple into the TE_{01} at the input, there is significant scattering into other higher energy modes.

Once we have both the measured and reconstructed intensities at the two polarizations, we project these onto the 16 lowest order eigenmodes, as listed in Table 3.1. Actually here we only looked at the first 12 modes, since contributions from later 4 were insignificant and can be ignored. Since the beam that we measured is a superposition of the eigenmodes, with each eigenmode has its own coefficient, by projecting the measured beam onto each eigenmode, we can compute the coefficient.
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The superposition is a 1D problem with the eigenmodes form its basis. And we define the contribution from each eigenmode as the square of its coefficient. In Fig. 5.13, we list some examples of the decomposition results, the three extreme cases discussed earlier.

In Fig. 5.13, we showed the contributions from the 12 lowest energy modes. In the first column, we have modes with \( m = 0 \), in the second column, we have modes with \( m = 1 \), and the modes in column 3 have \( m = 2 \). The two key modes that we focus our attention on are the TE\(_{01}\) (second bar of the first panel), and the HE\(_{11}\) (first bar of the second panel). We also normalize the contribution of all modes to 1. We notice that in the case of the pure HE\(_{11}\) (\( \alpha = -1 \)), the result beautifully agrees with our prediction, that the HE\(_{11}\) bar simply towers above the rest, there is some contribution due to the HE\(_{12}\), but we indeed coupled very efficiently to the HE\(_{11}\) mode without exciting the other modes too much. We also see that the contribution from the TE\(_{01}\) is nearly zero, this confirms our prediction that the HE\(_{11}\) and the TE\(_{01}\) are indeed orthogonal.

As we increase \( \alpha \) or the contribution of the TE\(_{01}\), we see that the HE\(_{11}\) contribution decreases, and the contributions from the TE\(_{01}\), TE\(_{02}\), and the EH\(_{21}\) rise. However when we reach the pure TE\(_{01}\) case (\( \alpha = 0 \)), the result deviates from our theory a little bit. Although we see a strong contribution from the TE\(_{01}\), we also see an equal contribution from the EH\(_{21}\). This indicates that the peculiar asymmetry that we observed in the TE\(_{01}\) earlier in Fig. 5.11 is due to the fact that it has contribution only partially from the TE\(_{01}\) and another significant portion from the EH\(_{21}\). Furthermore, we see that the contribution from the HE\(_{11}\) does not go to zero when the TE\(_{01}\) reaches its maximum, this contradicts with what we said earlier about the orthogonality between the TE\(_{01}\) and the HE\(_{01}\). However, overall as we switch from the HE\(_{11}\) to the TE\(_{01}\) on the SLM, we do see that the contribution from the HE\(_{11}\) decreases and the contribution from the TE\(_{01}\) increases.

Here we use the SLM to implement a complete set of phase patterns, the entire collection as shown in Fig. 3.7. At both ends, the uniform phase diagrams give uniform Gaussian distributions, thus the HE\(_{11}\) mode (\( \alpha = \pm 1 \)). The two colors only create relative phase shift. Therefore we expect to see a transition from the HE\(_{11}\) to the TE\(_{01}\) when \( \alpha \) goes from \(-1\) to 0, and a transition from the TE\(_{01}\) back to the HE\(_{11}\) when \( \alpha \) goes from 0 to 1. We computed the coefficients of all the eigenmodes of interests at different superpositions of HE\(_{11}\) and TE\(_{01}\), and plotted the contributions of the HE\(_{11}\) and TE\(_{01}\) in Fig. 5.14. Here we see that near the two ends of the spectrum,
where we both have the pure \( HE_{11} \), the two curves behavior like two nice sinusoidal functions. But near \( \alpha = 0 \), where we have the pure \( TE_{01} \), the sinusoidal behavior is not as clear. This is due to the reason that we discussed earlier, the emergence of the \( EH_{21} \) mode. Ideally, if we couple only to \( HE_{11} \) and \( TE_{01} \) modes, we should only excite these two modes and no other ones, and should expect to see two sinusoidal waves that are \( \pi/2 \) out of phase, or one reaches its minimum when the other reaches its maximum. Here we see that the maximum of the \( TE_{01} \) is lower than that of the \( HE_{11} \), and also the minimum of the \( HE_{11} \) is higher than that of the \( TE_{01} \). Thus we can deduce that the presence of the \( EH_{21} \) adds to the contribution of the \( HE_{11} \) and reduces that of the \( TE_{01} \). Overall, the rise and fall of the contributions from the \( HE_{11} \) and \( TE_{01} \) modes agree with the transition of the SLM phase patterns.

With the help of the decomposition analysis, we figured out that the "mysterious mode" is in fact the \( EH_{21} \) mode (along with a few other modes). It is interesting to look at the profile of such mode, and we showed the intensity of an \( EH_{21} \) in Fig. 5.15. We see 4 lobes in the \( EH_{21} \) beam, and they are arranged in an orientation similar to the \( TE_{01} \) that we observed, the intensity is higher in the upper right and lower left quadrants, and lower in the upper left and lower right quadrants. When we study the \( EH_{21} \) mode with a polarizer, we see that it has the same polarization as the \( TE_{01} \) at
0 and $\pi/2$ (in the azimuthal direction), but when we rotate it by $\pi/4$, it has the same polarization as the TM$_{01}$ (in the radial direction). Since the TM$_{01}$ is orthogonal to the TE$_{01}$, when EH$_{01}$ superposes with the TE$_{01}$, they combine constructively at the angle where they have the same polarizations, and destructively at the angle where they have the opposite polarizations. Since the difference in these two angles is $\pi/4$, we see in the TE$_{01}$ images, the two lobes with high intensity are rotated by $\pi/4$ with respect to the horizontal axis.
Chapter 6

Conclusion

In this paper, we demonstrated that we successfully produced both individual pure modes and controllable superpositions of modes at near infrared wavelengths using photonic band gap fibers. This was achieved using an SLM and by modulating the phase of the beams only. We also looked into the problem with degeneracy in this pure phase approach, and concluded that this does not affect the loss of the mode or the decomposition results. We reported on the synthesis of three pure modes, HE_{11}, TE_{01}, and TE_{02}, and studied their polarizations, and they agreed with our prediction, such as the HE_{11} is linearly polarized, and the TEs are circularly polarized, this was done by measuring the intensity of the modes at different polarization angles.

After successfully synthesizing the individual modes, we proceeded to study the propagation losses of the modes in the photonic band gap fibers. Here we mainly focused on the HE_{11} and TE_{01} modes. We measured the radial power output of the modes in the fiber at wavelengths both inside and outside of the band gap, and calculated that indeed the TE_{01} is the lowest loss mode, and has a transmission band gap about twice that of the HE_{11}.

We saved the most fascinating part of our research in the last section of our report, and that is on the superposition and decomposition of modes. In this experiment, we studied the superpositions of the HE_{11} and TE_{01} modes. Here we measured the intensity at different polarizations of both the images and their Fourier transforms, and decomposed the output images into 12 low energy $m < 3$ eigenmodes, and computed their contributions in the superpositions. We focused on the contributions due to the HE_{11} and TE_{01} modes, and showed that the contribution from the HE_{11} decreases and the contribution from the TE_{01} increases as we transform from the HE_{11} to the TE_{01}, and similarly for the other direction, just as we expected. We showed
that indeed the contribution from the two modes behave sinusoidally as a function of the superposition. We also used the decomposition algorithm to understand the discrepancies in the superposition between the $HE_{11}$ and the $TE_{01}$, and discovered that it was due to the excitation of other higher energy modes, most notably the $EH_{21}$ mode.

The ability to controllably synthesize these modes has a tremendous amount of potential applications, both in telecommunication as information carriers, and in atomics physics, where it can be used to guide or trap atoms in hollow core fibers.
Bibliography


