DESIGN CRITERIA FOR AUTOMATIC CONTROL SYSTEMS
( Unclassified Title)
by
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The author is grateful for the association between the Department of Aeronautics and Astronautics and the Instrumentation Laboratory, which affords the opportunity to develop analytical concepts as well as an appreciation for an operating state-of-the-art system while performing this investigation.

The author wishes to acknowledge his appreciation to the members of the Skipper J. Group who developed the highly reliable system on which these data were recorded. In particular, the author wishes to thank the following persons.

Professor Winston R. Markey for his supervision, assistance, and criticisms of the initial draft while acting as technical faculty advisor on this thesis.

Mr. Richard E. Marshall for suggesting the topic, guiding its direction, and markedly commenting on the initial draft.

Mr. Thomas E. Reed and Mr. William T. McDonald for their many stimulating discussions of the subject matter and the preliminary draft.

Mrs. Elaine Walsh for typing this paper.
This foreword describes the goals and results of the thesis and the general situation that surrounded this work. It is not considered a regular part of the thesis.

On October 1, 1962, the 2-16 system had been gyrocompassing at the M.I.T. Skipper J Test Facility for 2000 hours. During this time, stable base gyrocompassing data had been recorded on the system operating in a benign environment. E-1231 (Ref. 3) contains the pertinent results recorded on the gyrocompassing system before the fourth quarter of 1962. The results which are fundamental to and which precede the investigations performed in this thesis are as follows.

1. The 2-16 gyrocompass demonstrated the existence of a memory-mode gyrocompassing capability accurate to better than five seconds of arc rms error in azimuth for periods of hundreds of hours.

2. Incremental angles below the order of 0.01 seconds of arc about the horizontal axes can be consistently detected in the 2-16 gyrocompass operating in a benign environment.

3. Gimbal static and sliding friction nonlinearities did not appear to be a major problem in the operation of a stable base gyrocompass.
During the first quarter of 1962, preliminary experiments on a wheel speed modulation technique of gyro error averaging demonstrated the feasibility of detecting very low level torque changes in a 2FBG-6F gyro with its input axis approximately east. The tests conducted on a single instrument mounted on a test table are the subject of T-315 (Ref. 1). The results indicated the possibility of utilizing wheel speed modulation to achieve a fixed base azimuth indicating system for long operational periods. First efforts on paper to devise an operational sample data procedure were carried on jointly by the author and other members of the Skipper J Group (Ref. 2).

The program for the 2-16 system during the fourth quarter of 1962 had as a first goal the demonstration of system feasibility of the wheel speed modulation technique. The author was in control of this effort as a research assistant.

The experimental part of this thesis was to record data on system performance versus bandwidth to reveal characteristics of instrument uncertainties. These data form one set of system inputs that limit frequency regions covered by practical solutions (Chapter IV).

Equal importance was attached to defining probable operating characteristics of such systems (Chapters II and III) and the development of adequate operational procedures (Chapter V).
Because the above aims were directly dependent upon demonstrating feasibility of the whole technique, it became reasonable to report these results. Thus, a third goal of the thesis is to present organized theories of operation and results achieved for the wheel speed modulated gyrocompass.

The value of having these related matters in a single document justifies the coverage which is often limited but manages at least one plausible solution in each area.

The results of these investigations are as follows.

1. The thesis analysis of system performance versus bandwidth reveals instrument uncertainties below the values previously anticipated. These data extend the frequency regions of realistic system performance. (Chapter IV).

2. Wheel speed modulation techniques are reported which appear to demonstrate the capability of detecting low level torque variations in the east gyro operating in a benign system environment (Chapter IV).

3. The thesis combines the data on the spectrum of system and instrument uncertainty with anticipated changes in system bandwidth and incremental angle measurements to define probable operating characteristics and gyro compensation procedures in a realistically perturbed environment (Chapter V).
4. The thesis reports stable base gyrocompassing characteristics and justifications for the simplifying assumptions which describe this system with its unconventional closely coupled loops and superior instrument performance (Chapter II).

5. The thesis reports accuracy requirements on instrument alignment and derives a possible alignment method (Chapter III).
DESIGN CRITERIA FOR A STABLE BASE GYROCOMPASS

by

John M. Vergoz

Submitted to the Department of Aeronautics and Astronautics on January 14, 1963 in partial fulfillment for the degree of Master of Science.

ABSTRACT

The current status of self-azimuthing capabilities of inertial guidance systems in a fixed base missile is reported. Empirical results and analyses are presented.

Data taken on an east gyro error averaging technique, wheel speed modulation, tentatively demonstrate the capability of detecting changes in the east gyro performance with sufficient accuracy to extend the operational period indefinitely.

The problems associated with absolute instrument alignment in a fixed base gyrocompass are discussed and one of the many possible solutions presented.

Data indicative of instrument uncertainty spectra in a benign environment are combined with estimates of the necessary changes in system bandwidth and recompensation techniques to operate the gyrocompassing system in an operational environment.

The data presented on gyro drift uncertainty spectra reveal that considerable latitude exists for optimizing system performance in the presence of horizontal acceleration inputs.

Thesis Supervisor: Winston R. Markey
Title: Associate Professor of Aeronautics and Astronautics
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Introduction</td>
<td>12</td>
</tr>
<tr>
<td>II</td>
<td>Gyrocompass Characteristics</td>
<td>21</td>
</tr>
<tr>
<td>III</td>
<td>Laboratory Alignment and Calibration</td>
<td>45</td>
</tr>
<tr>
<td>IV</td>
<td>System and Instrument Performance</td>
<td>70</td>
</tr>
<tr>
<td>V</td>
<td>Operational System Evaluation</td>
<td>105</td>
</tr>
<tr>
<td>VI</td>
<td>Conclusions</td>
<td>129</td>
</tr>
<tr>
<td>I</td>
<td>Nomenclature</td>
<td>133</td>
</tr>
<tr>
<td>II</td>
<td>Spectra of Gyrocompass Angles</td>
<td>139</td>
</tr>
<tr>
<td></td>
<td>BIBLIOGRAPHY</td>
<td>145</td>
</tr>
<tr>
<td>Figure 2-1</td>
<td>Stable Base Gyrocompass Coordinates</td>
<td>23</td>
</tr>
<tr>
<td>Figure 2-2</td>
<td>Gyrocompass Functional Block Diagram</td>
<td>26</td>
</tr>
<tr>
<td>Figure 2-3</td>
<td>East Azimuth Gyrocompass</td>
<td>29</td>
</tr>
<tr>
<td>Figure 2-4</td>
<td>East Azimuth Dynamic Errors</td>
<td>37</td>
</tr>
<tr>
<td>Figure 2-5</td>
<td>Azimuth Lateral Acceleration Errors</td>
<td>40</td>
</tr>
<tr>
<td>Figure 2-6</td>
<td>Gyrocompass Step Response</td>
<td>43</td>
</tr>
<tr>
<td>Figure 3-1</td>
<td>Deviation of East Gyro IA From Geographic East</td>
<td>47</td>
</tr>
<tr>
<td>Figure 4-1</td>
<td>East Azimuth Gyrocompass</td>
<td>75</td>
</tr>
<tr>
<td>Figure 4-2</td>
<td>Uncertainty Frequency</td>
<td>77</td>
</tr>
<tr>
<td>Figure 4-3</td>
<td>2-16 Gyrocompass Short-Term Stability</td>
<td>83</td>
</tr>
<tr>
<td>Figure 4-4</td>
<td>2-16 Gyrocompass Effect of Varying Bandwidth</td>
<td>86</td>
</tr>
<tr>
<td>Figure 4-5</td>
<td>2-16 Gyrocompass Effect of Wheel Speed Modulation</td>
<td>103</td>
</tr>
<tr>
<td>Figure 5-1</td>
<td>North Gyrocompass Loop</td>
<td>107</td>
</tr>
<tr>
<td>Figure 5-2</td>
<td>Open/Closed Loop - East Azimuth Gyrocompass</td>
<td>109</td>
</tr>
<tr>
<td>Figure 5-3</td>
<td>Open Loop East Vertical Sensor Output for Incorrect Azimuth Gyro Compensation</td>
<td>114</td>
</tr>
<tr>
<td>Figure 5-4</td>
<td>Open Loop East Vertical Sensor Output - East Gyro Drift (one-half normal wheel speed)</td>
<td>122</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>--------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1-1</td>
<td>Regions of Interest</td>
<td>17</td>
</tr>
<tr>
<td>2-1</td>
<td>Error Coefficients</td>
<td>35</td>
</tr>
<tr>
<td>4-1a</td>
<td>260 Hours - Low East/Azimuth Coupling</td>
<td>80</td>
</tr>
<tr>
<td>4-1b</td>
<td>168 Hours - High East/Azimuth Coupling</td>
<td>81</td>
</tr>
</tbody>
</table>
The effectiveness of an inertial guidance system is restricted by the accuracy to which it can be aligned in azimuth and to the vertical. Such guidance systems are presently aligned in azimuth with optical techniques utilizing complex ground installations and an accurately surveyed azimuth reference. The vertical is sensed and corrected with an accelerometer, pendulum, or other leveling device operating with the system gyros.

Since inertial guidance of missiles became a reality, the possibility of alignment by gyrocompassing has been realized and, at times, used. A deterrent to gyrocompassing has been stringent alignment requirements using available instruments. Recent advances in the development of single-degree-of-freedom integrating gyros, coupled with system optimization techniques, have developed the operational capability of self-aligning missile guidance systems.

This paper discusses gyrocompassing theory and characteristics, as well as laboratory alignment, calibration techniques, and operational concepts for a fixed base inertial guidance system. Data that are indicative of the uncertainty spectra of the gyrocompassing system form the basis of the calibration and compensation techniques developed.
1.1.0 **Gyrocompassing**

By definition, a gyrocompass includes any and all devices or systems that align themselves to a predetermined set of coordinates, using the earth's angular velocity and direction of gravity as basic references. Instruments which are sensitive to the earth's angular velocity, coupled with devices which determine the direction of gravity are, therefore, the core of a gyrocompassing system. The direction of gravity is defined by connecting the vertical sensing devices (pendulums, electrolytic levels, accelerometers, etc.) to the stabilized member integrating drives. In the three-gyro system, the gyro input axes are approximately vertical, north and east. Basically, the ideal east gyro torque summation is only satisfied when its input axis is parallel to geographic east. The azimuth orientation of the gyrocompassing system defines the angular velocity applied to the east gyro input axis. By coupling rotations about the east gyro input axis to the azimuth gyro torque generator via the east vertical sensor, a change in the angular velocity summation of the east gyro will produce a new azimuth angle. This new azimuth angle changes the horizontal component of earth rate applied to the east gyro input axis.

Accuracy capabilities of the gyrocompassing system are primarily determined by the stability of the east gyro. The closed loop gyrocompassing system cannot distinguish between
internal changes in the east gyro characteristics (uncertainty torques), making the angular velocity summation no longer applicable, and a change in azimuth angle coupling a new component of earth rate into the east gyro.

Gyrocompassing techniques differ mainly in the method of determining changes in performance of the east gyro with no external references. East-west averaging, gyro wheel reversal, and wheel speed modulation are three methods of east gyro evaluation which have been investigated in recent years. This paper limits attention to wheel speed modulation in a system environment because it has recently been used to determine low level torque variations on a single component basis.

1.2.0 Accuracy Requirements

Accurate long-term gyrocompassing imposes severe demands on system performance. The following discussion is included to develop a feeling for the magnitude of the problem. Justification for these regions of interest in the gyrocompassing system are included in the body of this report.

As an example, assume that the absolute azimuth stability of the gyrocompassing system shall be within ± 5 seconds of arc. This defines the maximum allowable uncompensated east gyro drift uncertainty at medium latitudes to be 0.018 meru (0.00027 degrees per hour). Allowable north and azimuth gyro uncertain-
ties are a function of system bandwidth which is a compromise between gyrocompassing uncertainties and lateral horizontal accelerations. In a benign environment, north and azimuth gyro drift rates in the 0.1 to 0.5 meru region can be tolerated in the closed loop gyrocompass. Changes in the performance of the north and azimuth gyro can be compensated for in the closed loop gyrocompass because they produce vertical as well as azimuth errors.

To detect changes in the performance of the east gyro, the wheel speed modulation technique requires the measurement of small angles by the east vertical sensor. A highly stabilization servo gain and low values of gimbal static friction will produce the most desirable (i.e., most nearly linear) operation. In the system operating in a benign environment, stabilization servo gains of 50 ft.lbs/mr, coupled with gimbal static friction in the 1 in. oz. region, have combined to develop a system in which 0.01 seconds of arc can be detected and used to recompensate the east gyro. Since gyro drift produces angle variations in the gyrocompassing system, the time required to measure small angles defines the bandwidth of gyro drift detection.

Latitude relocations change the vertical and horizontal components of earth rate. If the east gyro input axis is not parallel to geographic east, latitude relocations will change the value of the earth rate applied to the east gyro. This change in earth rate
applied to the east gyro would vary the azimuth angle of the gyrocompassing system. If the laboratory absolute azimuth reference and operating area are separated by five latitude degrees, seven seconds of arc error in the laboratory knowledge of the east gyro input axis will produce one second of arc azimuth error at the new latitude location.

These regions of interest are summarized in Table 1-1. The gyrocompassing system requirements are to maintain the angle stability over a maximum time interval which, hopefully, can be extended to several years. The basis of these numbers is the subject of this report.

1.3.0 Operational Integration

The unattended self-calibrating internal guidance system requiring no alignment or stability checks for several years after it leaves the laboratory is not a reality. However, the guidance system under test has the ability to memorize a set of coordinates over several days with sufficient accuracy for all presently known requirements for missile and satellite launchings. If the gyros are periodically recompensated, the system can hold the reference for an extended period of time with no external references. Integration of this system into a missile site containing optical references would immediately reduce the missile's vulnerability to ground disturbances by
*Regions of Interest

Angle Stability

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<th></th>
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</thead>
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<tr>
<td>Azimuth Variation</td>
<td>&lt; ± 5 seconds of arc</td>
</tr>
<tr>
<td>North Variation</td>
<td>&lt; ± 1 second of arc</td>
</tr>
<tr>
<td>East Variation</td>
<td>&lt; ± 1 second of arc</td>
</tr>
</tbody>
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Uncompensated Gyro Drift

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<th></th>
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</thead>
<tbody>
<tr>
<td>North</td>
<td>0.1 to 0.5 meru</td>
</tr>
<tr>
<td>Azimuth</td>
<td>0.1 to 0.5 meru</td>
</tr>
<tr>
<td>East</td>
<td>&lt; 0.018 meru</td>
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</tbody>
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Component Alignment

Knowledge of laboratory orientation of the east gyro input axis: better than 14 seconds of arc.

Miscellaneous

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<tbody>
<tr>
<td>STAB Servo Gain</td>
<td>50 lb. ft/mr</td>
</tr>
<tr>
<td>Gimbal Friction</td>
<td>1 in. oz.</td>
</tr>
<tr>
<td>Incremental Angles</td>
<td>0.01 second of arc (benign environment).</td>
</tr>
<tr>
<td></td>
<td>0.1 second of arc (operational environment).</td>
</tr>
<tr>
<td>Repeatability of Changing</td>
<td>better than 0.01 meru</td>
</tr>
<tr>
<td>East Gyro Wheel Speed:</td>
<td></td>
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* Table for 45° latitude.
memorizing the proper guidance system orientation, therefore, maintaining alignment after the optical reference is dislodged. The three main areas of interest for an operational system in the order of decreasing importance are (1) stability of the system as an absolute reference, which is primarily a function of gyro drift rates and the effectiveness of a recompensation procedure; (2) sensitivity of the system to latitude relocations, which depends on a knowledge of the absolute orientation of the east gyro input axis; (3) response time of the system, which is dependent upon the maximum allowable system bandwidth.

The system, data, and concepts are applicable in any area where accurate initial platform alignment by gyrocompassing techniques is required.

1.4.0 Design Criteria For A Stable Base Gyrocompass

This report is intended to carry the reader from gyrocompassing characteristics through operational concepts for a fixed base inertial guidance system. A slight detour in Chapter IV presents pertinent data on an operating system. The following section summarizes the organization of this report.

Chapter II reduces the complexity of the three-axis gyrocompass to a two-axis model suitable for analysis in the regions of interest. This model is analyzed to determine the linear relations between gyrocompass gains, error coefficients, response time,
Chapter III indicates the relation between knowledge of the east gyro input axis alignment and azimuth accuracy at various latitudes. A laboratory technique is developed to determine the orientation of the east gyro input axis. The wheel speed modulation concept of gyro error averaging is analyzed and a laboratory optimization technique presented. No change is anticipated in the laboratory determination of accelerometer bias scale factor and input axis location.

Chapter IV determines the spectrum of azimuth and east gyro drift on the basis of several hundred hours of stable base gyrocompassing data. Two system bandwidths were used in the recording of these data so that uncertainties in instrument performance can be properly identified. Results of wheel speed modulation over two-hundred and sixty-eight hours of system operation are also presented.

Chapter V develops a compensation technique for a fixed base gyrocompassing internal guidance system. This technique is based on instrument performance recorded in the previous chapter but anticipates the necessary changes in system bandwidth and incremental angle measurements when operating in a realistically perturbed environment.

Chapter VI summarizes the difficult problems associated with accurate stable base gyrocompassing and the results achieved in this paper.
Appendix I contains a glossary of symbols.

Appendix II assumes that wind gust spectrum is the primary interfering function between the benign laboratory and the operational environment. The spectrum analysis analytically confirms the orders of magnitude of the mean square azimuth as well as the vertical angles which must be detected in an operational gyro recompensation procedure.
II GYROCOMPASSING CHARACTERISTICS

Chapter II reduces the complexity of the three-axis gyrocompassing system to a model suitable for analysis of system characteristics. All simplifying assumptions are justified to produce a two-axis model which is used to develop the relationships between gyrocompass gains, dynamic and steady state errors, and response time of the system. In addition, the development considers the effect of a vertical sensor with and without a first order lag in the region of interest. Although friction and angle transducer nonlinearities exist, the linear approach is justifiable because it develops the necessary initial insight into stable base gyrocompassing characteristics. By referring to Appendix I for nomenclature, Chapter II may be bypassed by the reader who is familiar with gyrocompass dynamical relations with no loss in continuity, proceeding to "Laboratory Alignment and Calibration Techniques" of Chapter III.

2.1.0 Coordinates

Local geographic coordinates are designated $x_g$ - north, $y_g$ - east, and $z_g$ - vertical. In this chapter, platform coordinates are assumed to be perfectly aligned with instrument coordinates.
Since the instrumented coordinates and local geographic coordinates are at all times nearly coincident to a first approximation, a rotation about an instrumented coordinate is equivalent to a rotation about a geographic coordinate. Since these rotations are small, the angles developed by a rotation about two coordinates add cumulatively to define a new orientation of the third instrumented coordinate. These approximations greatly simplify the development to follow and do not seriously affect its validity.

Figure 2-1 is a line schematic of the gyrocompassing instrument geographic and instrumented coordinates. The gyro orientation is designated by the appropriate location of its input axis (i.e., east gyro IA is east). The vertical sensor's sensitive axis is parallel to the input axis of the gyro which accepts its output (i.e., east vertical sensor's sensitive axis is parallel to east gyro input axis). Using small angle approximations (cos $\theta = 1$ sin $\theta = 0$), Equation 2-1 indicates the total angular velocity input to each gyro.

\[
\omega_x = A_x - A_y \omega_{(ie)v} + \omega_{(ie)h} \\
\omega_y = A_y + A_x \omega_{(ie)v} - A_z \omega_{(ie)h} \\
\omega_z = A_z + A_y \omega_{(ie)h} + \omega_{(ie)v}
\]

* A glossary of symbols is given in Appendix I.
STABLE BASE GYROCOMPASS COORDINATES

NORTH VERTICAL SENSOR

EAST VERTICAL SENSOR

EAST Yg

NORTH Xg

\[ W_{(ie)h} = W_ie \cos \text{LAT} \]

VERTICAL Zg

\[ W_{(ie)v} = -W_ie \sin \text{LAT} \]
The steady state output of each vertical sensor is indicated in Equation 2-1. Where $S_{xv}$ and $S_{yv}$ are the sensitivities of the north and east vertical sensors respectively measured in volts per "g",

North Vertical Sensor Output: $g A_x S_{xv}$

East Vertical Sensor Output: $g A_y S_{yv}$

(2-2)

The north gyro and vertical sensor are connected together in such a manner that they ideally constrain the input axis of the east gyro in the horizontal plane. The east gyro torque summation is basically only satisfied when the input axis of the east gyro is east. If the east gyro input axis is not east, it will sense the horizontal component of earth rate, $-A_z \omega_{(ic)h}$, where $A_z$ is the angle between geographic east and the east gyro input axis. This angular velocity will cause the platform to tilt about the east axis, developing an angle, $A_y$. This tilt about the east axis will develop an east vertical sensor output equal to $g A_y S_{yv}$, which is coupled to the azimuth gyro torque generator and commands an azimuth angular rate in such a direction as to reduce the angle between the east gyro input axis and the east geographic coordinate. For our ideal system, the east gyro torque summation (satisfied only when
the input axis of the east gyro is east), coupled with the torque summation in the azimuth gyro (satisfied only when the output of the east vertical sensor is zero) combine to produce a gyro-compassing system which uses the direction of gravity and earth rate as the only external reference for initial platform alignment. A convergent solution is established by a coupling between the east vertical sensor and the east gyro.

2.2.0 Block Diagram

Figure 2-2 is a complete functional block diagram of the gyrocompassing system showing major mechanical, electrical, and earth rate couplings. In order to obtain a suitable working model for stability analysis, determination of error coefficients, and response time, the following six assumptions, together with their justification, are presented.

1. Typical stabilization servo time constants are 0.02 seconds. Gyro compass time constants are greater than one minute. The dynamics of the stabilization servos may be neglected in the study of gyrocompassing alignment dynamics and stability. (The platform is always aligned with the input axis of its gyros.)

2. The gyro time constants of approximately 0.001 seconds do not affect the stability or the solution time of the gyrocompassing system.
3. For this chapter static friction is assumed to be zero. Static friction levels are approximately 1 in. oz. Typical stabilization servo gains are 50 in. oz. per arc second. Therefore, 0.02 seconds of arc gyro signal generator error is sufficient to overcome static friction.

4. Alignment about north involves seeking the vertical only, and a sufficiently high gain loop can be closed so that north alignment errors have negligible effect on gyrocompassing accuracy and solution time. The upper boundary on the value of north gyrocompass loop gain is allowable system noise.

5. The horizontal component of earth rate coupled into the azimuth gyro, due to a rotation about east, is negligible compared to the azimuth gyro angular rate commanded via the east vertical sensor and $K_{AE}$ for all practical values of $K_{AE}$. ($K_{AE} > 1$ meru per second of arc, and gradient of earth rate at 45 degrees latitude equals 0.0034 meru per second of arc.)

* At 45 degrees latitude, steady state angle errors about the north gyro input axis, $A_x$, produce an equal azimuth angle, $A_z$, of the stabilized member. The steady state value of $A_x$ can be easily kept below one-half second of arc. The transient involved in north leveling can be completed faster than the gyrocompassing transients. Noise restrictions and vertical sensor dynamics are the only restrictions on the solution time of the north leveling loop.
6. Perfect earth rate compensation is assumed, and gyro drift or instabilities in the gyro torquing circuits or supply are included in total gyro drift rate, \((u)\omega_{x,y,z}\).

With the previous assumptions, the diagram of Figure 2-2 reduces to that of Figure 2-3. It is observed from Figure 2-3 that the azimuth gyro angular velocity summation is satisfied when \((u)\omega_{z} + A_y K_{AE}\) equal zero. The east gyro angular velocity summation is satisfied when \(-A_z \omega_{(ie)h} + (u)\omega_{y} - A_y K_{EE}\) equal zero. Therefore, the steady state equilibrium azimuth orientation of the gyrocompassing system is

\[
A_z = \frac{1}{\omega_{(ie)h}} \left[ (u)\omega_{y} + \frac{K_{EE}}{K_{AE}} (u)\omega_{z} \right]
\]  \hspace{1cm} (2-3)

The solution to Equation 2-3, in the absence of east gyro and azimuth gyro drift, is a gyrocompassing system with the instrumented east axis parallel to geographic east. The open chain angular velocity summation performed in the east gyro places a basic limitation on gyrocompassing accuracy. Steady state east gyro drift only produces a change in the azimuth angle of this stabilized member. Since the gyrocompassing system

*Gyro Drift:* Throughout this paper the term gyro drift is used to designate total gyro drift (i.e., any internal change in the torque summing member and/or torquing supply).
is the azimuth reference, east gyro drift cannot be detected in the closed loop system. Azimuth gyro drift, $(u)\omega_z$, produces an output from the east vertical sensor as well as an azimuth angle error. The effect of azimuth gyro drift on azimuth angle can be minimized by compensating the azimuth gyro and minimizing the output of the east vertical sensor.

2.3.0 Gains, Dynamic Errors, Response Time

Section 2.3.0 will consider system characteristics and how these characteristics are affected by the use of a vertical sensor with a time constant affecting the response time of the gyrocompassing system. Stability considerations affecting gyrocompass gains will be considered before dynamic and steady state errors because these errors depend on the gyrocompass gains. Response to lateral accelerations will be considered before transient response because the transient response depends on the amount of filtering to be done.

Equal roots to the gyrocompass characteristic equation are assumed for two reasons. First, the case of equal roots is easier to handle analytically, yet provides the necessary insight into stability, dynamic errors, and response time of the second as well as the third order gyrocompass.* Second, in an opera-

*Second Order Gyrocompass:- Vertical sensor time constant negligible compared to gyrocompass solution time.

Third Order Gyrocompass:- Vertical sensor time constant greater than one-third the gyrocompass solution time.
ting system, gains which are the product of sensitivities of electromechanical devices and amplifications often accumulate errors in excess of ten percent. Optimum gain can only be determined on the basis of system performance. The equal root solution gives the system designer an initial point from which gains can be optimized.

The development to follow determines system parameters for a critically damped system as a function of system bandwidth. These parameters are next used to analytically determine closed loop system response to gyro drift and horizontal accelerations. Armed with these analytical tools, the system designer must next determine a realistic system bandwidth based on design requirements, acceleration profile, gyro drift, and calibration techniques. Observing the operation of the critically damped system, the designer adjusts bandwidth so the system will have steady state errors within the initial design specifications. When the maximum allowable bandwidth for acceptable steady state errors is determined, the designer can optimize further between forced dynamic errors and minimum response time. This will, to a small degree, affect steady state errors so that system bandwidth may have to be decreased to attain acceptable steady state performance again. The development that follows determines gains, dynamic errors, and response times of the gyrocompassing system with and
without vertical sensor time constants in the region of interest.

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2.3.1 Gains

Figure 2-3 can be reduced to Matrix Equation 2-4.

\[
\begin{bmatrix}
1 + \frac{K_{EE}}{P(\tau_v P + 1)} & \frac{\omega_{(ie)h}}{P} & \frac{(u)\omega_y}{P} & -\frac{(u)A_y K_{EE}}{(\tau_v P + 1)P} \\
-\frac{K_{AE}}{P(\tau_v P + 1)} & 1 & \frac{(u)\omega_z}{P} & \frac{(u)A_y K_{AE}}{(\tau_v P + 1)P}
\end{bmatrix}
\]

(2-4)

The characteristic equation of Matrix 2-4 is

\[
P^2(\tau_v P + 1) + K_{EE} P + K_{AE} \omega_{(ie)h} = 0
\]

(2-5)

If \( \tau_v < \frac{1}{\sqrt{K_{AE} \omega_{(ie)h}}} \), a second order system very closely approximates the system characteristics. For a critically damped second
order system, the damping is proportional to the east vertical sensor to east gyro coupling ($K_{EE}$), and the natural frequency is equal to the square root of the east vertical sensor to azimuth gyro coupling times the horizontal component of earth rate,

$$\sqrt{K_{AE} \omega_{(ie)h}}.$$ 

$$\frac{1}{\tau_2} = \frac{1}{2} K_{EE}$$

(2-6)

If the time constant of the vertical sensor can be chosen, Roth's stability criterion shows that Equation 2-5 is free from right half-plane poles as long as $K_{EE} > K_{AE} \omega_{(ie)h} \tau_v$. The choice of a $\tau_v$ and $K_{AE}$ for this third order system depends on accuracy requirements, lateral acceleration profile, and required response time of the system. If all roots of Equation 2-5 are equal (critical damping), the following relations exist.

$$\frac{1}{\tau_3} = \frac{1}{3} \frac{1}{\tau_v} = \frac{1}{3} K_{EE} \omega_{(ie)h} = K_{EE}$$

(2-7)
2.3.2 Dynamic Errors

The primary error sources in the operation of a stable base gyrocompass are gyro drift and uncertainties in vertical sensor operation. The vertical sensor uncertainties are usually a result of horizontal accelerations. Minimization of the effects of horizontal acceleration on stable base gyrocompass performance is primarily a filter design problem. This section indicates the effect of gyro drift and uncertainties in vertical sensor operation on the closed loop performance of the gyrocompassing system.

By taking the partial derivatives of Matrix 2-4, with respect to gyro drift and uncertainties in vertical sensor operation, and substituting the values of $\tau_2$ from Equation 2-6, and $\tau_3^*$ from Equation 2-7, Table 2-1 is obtained. The error coefficients in Table 2-1 are expressed only as a function of the time constant of the gyrocompassing system and the horizontal component of earth rate so that they are easily convertible to any convenient units of measure. This paper will use the meru, which is equal to .001 earth rate, as the angular velocity unit of measure, and the second of arc as an angular measure. To become familiar with the meru and to convert from Table 2-1 into measurements other than reciprocal time, the following constants are noted.

* $\tau_2$ is the time constant of the second order gyrocompass.
* $\tau_3$ is the time constant of the third order gyrocompass.
TABLE 2-1

ERROR COEFFICIENTS

<table>
<thead>
<tr>
<th></th>
<th>Change in Azimuth Angle $\Delta A_z$</th>
<th>Change in East Angle $\Delta A_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncertainty in $(u)$</td>
<td>Uncertainty in $(u)$</td>
</tr>
<tr>
<td><strong>SECOND ORDER SYSTEM</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>East Gyro $(u)\omega_y$</td>
<td>$\frac{1}{\tau_2^2 \omega(ie)h} \frac{1}{(P + \frac{1}{\tau_2})^2}$</td>
<td>$P \frac{1}{(P + \frac{1}{\tau_2})^2}$</td>
</tr>
<tr>
<td>Azimuth Gyro $(u)\omega_z$</td>
<td>$(P + \frac{2}{\tau_2}) \frac{1}{(P + \frac{1}{\tau_2})^2}$</td>
<td>$- \frac{\omega(ie)h}{(P + \frac{1}{\tau_2})^2}$</td>
</tr>
<tr>
<td>East Angle $(u)A_y$</td>
<td>$\frac{P}{\tau_2^2 \omega(ie)h} \frac{1}{(P + \frac{1}{\tau_2})^2}$</td>
<td>$- \frac{2}{\tau_2} \frac{(P + \frac{1}{\tau_2})^2}{(P + \frac{1}{\tau_2})^2}$</td>
</tr>
<tr>
<td><strong>THIRD ORDER SYSTEM</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>East Gyro $(u)\omega_y$</td>
<td>$\frac{1}{\tau_3^3 \omega(ie)h} \frac{1}{(P + \frac{1}{\tau_3})^3}$</td>
<td>$P(P + \frac{3}{\tau_3}) \frac{1}{(P + \frac{1}{\tau_3})^3}$</td>
</tr>
<tr>
<td>Azimuth Gyro $(u)\omega_z$</td>
<td>$P^2 + \frac{3P}{\tau_3} + \frac{3}{\tau_2^2} \frac{1}{(P + \frac{1}{\tau_3})^3}$</td>
<td>$- \frac{\omega(ie)h(P + \frac{3}{\tau_3})}{(P + \frac{1}{\tau_3})^3}$</td>
</tr>
<tr>
<td>East Angle $(u)A_y$</td>
<td>$\frac{P}{\tau_3^3 \omega(ie)h} \frac{1}{(P + \frac{1}{\tau_3})^3}$</td>
<td>$- \frac{3}{\tau_3^2} \frac{(P + \frac{1}{\tau_3})^3}{(P + \frac{1}{\tau_3})^3}$</td>
</tr>
</tbody>
</table>
The error coefficients of Table 2-1 are plotted in Figure 2-4. Data recorded on system performance versus bandwidth (Chapter IV) indicates that gyro drift has a low frequency spectrum compared to the time constant of the gyrocompassing system. Therefore, \( (u)\omega_y \) and \( (u)\omega_z \) primarily affect the steady state performance of the gyrocompassing system. For both the second and third order systems, it is noted that azimuth gyro drift, \( (u)\omega_z \), shows up as an east vertical sensor output as well as an azimuth angle. Figure 2-4 also indicates that the steady state output of the east vertical sensor is only a function of azimuth gyro drift. Recompensation of the azimuth gyro on the basis of the output of the east vertical sensor is a problem in compensation techniques and will, therefore, be covered in Chapter V.
EAST AZIMUTH DYNAMIC ERRORS

FIGURE 2-4

--- 2nd Order

--- 3rd Order
It is noted (Figure 2-4) that steady state east gyro drift, \( (u)\omega_y \), only produces azimuth angle errors. Since the elimination of all external azimuth references is the primary objective in gyrocompassing, this open-loop characteristic of the east gyro places basic limitations on stable base gyrocompassing. With present state-of-the-art calibration techniques, the gyrocompass loops must be opened to detect east gyro drift rates. The detection and recompensation of the east gyro drift will also be covered in Chapter V.

The uncertainties in east vertical sensor operation which are induced by horizontal accelerations produce dynamic errors in the closed-loop gyrocompass. Each micro g of horizontal acceleration which has a frequency below the breakpoint of the vertical sensing device produces 0.206 seconds of arc error in the indicated vertical. The coupling of these uncertainties in the east vertical sensor into the gyrocompassing system is a function of the frequency of the disturbance, the time constant of the vertical sensing device, and the time constant of the gyrocompassing system. The effect of a vertical sensing device with a time constant close to that of the gyrocompassing system is to attenuate the higher frequencies of the horizontal accelerations. This is evident in Figure 2-4 which indicates that uncertainties in the east vertical sensor operation, \( (u)A_y \), are attenuated above the break frequency of the gyrocompassing system by a slope of
minus two in the third order system and a slope of minus one in the second order system.

In this section we are primarily interested in the effect of horizontal accelerations on the azimuth angle of the stable base gyrocompassing system. Figure 2-5 indicates the magnitude of dynamic azimuth errors in the gyrocompassing system as a function of the time constant of the gyrocompassing system and frequency of horizontal accelerations. This figure is obtained from $\frac{\partial A_y}{\partial (u)A_y}$ of Figure 2-4 and the 0.206 seconds of arc east vertical sensor uncertainty per micro g of horizontal acceleration.

The shortest allowable time constant** of the gyrocompassing system can be determined from a knowledge of the predominant frequencies*** of lateral horizontal accelerations and accuracy requirements on the azimuth angle of the gyrocompassing system.

For example, assume the stable base on which the gyrocompass is mounted is vibrating at ten micro g's with a frequency of 0.1 radians per second. The following azimuth errors would be induced.

---

* Superscript numerals denote references in the Bibliography

** Chapter V shows that the detection of gyro drift places a more severe restriction on the time constant of the east vertical sensing device. Consequently, the gyrocompass time constant is defined for a critically damped system.

*** This analysis assumes horizontal acceleration is sinusoidal. A more sophisticated approach would involve a statistical analysis of acceleration spectra. Appendix II presents pertinent results of such an analysis.
AZIMUTH LATERAL ACCELERATION ERRORS

![Graph showing lateral acceleration errors with varying acceleration frequencies and angles of arc.](image)

ACCELERATION FREQUENCY \( \frac{\text{RADIANS}}{\text{SECONDS}} \)

2nd Order

3rd Order

FIGURE: 2 - 5
2.3.3 Response Time

Although initial thermodynamic effects and final non-linearities in the small angle region make the gyrocompassing system response time deviate from that predicted by linear theory, certain insight into sequencing operations and solution times can be extracted from the linear model. Because the initial output of the east vertical sensor has a predominant effect on the azimuth transient, the most desirable response of the gyrocompassing system consists of two phases: (1) preliminary leveling; (2) gyrocompassing. Preliminary leveling can be accomplished with a high loop gain, the time constant of which is only a function of the time constant of the vertical sensing device. For the third order critically damped system, since $\tau$ equals $3 \tau_v$, one-fourth of the total alignment time will be used for leveling. If preliminary leveling is not used, the east vertical sensor output will often produce azimuth errors in excess of the...
initial azimuth angle. For example, in a system with an east azimuth coupling of 100 meru per second of arc, an initial east error of 10 seconds of arc in the wrong direction would produce over 300 seconds of arc azimuth error before the output of the east vertical sensor changed sense. Since initial azimuth angles are seldom above 60 seconds of arc, it is apparent that preliminary leveling will, in most cases, reduce the transient response time. The accuracy of preliminary leveling is a function of lateral horizontal acceleration. Since initial east error angles are around the 10 arc second region, three time constants of the leveling loops should reduce the output of the east vertical sensor to a small enough value so that the gyrocompass loops may be closed.

By substituting $A_z(0)$ for the uncertainties on the right hand side of Matrix Equation 2-4 and transforming the results, the linear azimuth time response of the gyrocompassing system is obtained. The response time for the second and third order critically damped systems with equal gyrocompass system time constants are indicated in Figure 2-6. It is noted that the responses are quite similar and, also, that if $\tau_3$ equals three-quarters of $\tau_2$, ninety-five percent of the azimuth response would be complete in both the second and third order systems in an equal amount of time. Referring to Figures 2-5 and 2-6, it is noted that for equal lateral horizontal acceleration errors the
GYROCOMPASS STEP RESPONSE

\[ \frac{Az(t)}{Az(0)} = \left( 1 + t + \frac{t^2}{2\tau_0} \right) e^{-\frac{t}{\tau_0}} \]

\[ \frac{Az(t)}{Az(0)} = (1 + t) e^{-\frac{t}{\tau_2}} \]

FIGURE: 2 - 6
third order system can have a wider bandwidth and will, therefore, have a much faster solution time than the second order system.
The knowledge required in an inertial guidance system to permit accurate guidance is the position of the input axis of the accelerometers (i.e., the measurement coordinates); relative to the local vertical and an azimuth reference in the defined horizontal plane (i.e., the geographic coordinates).

In sequence, one measures the position of the accelerometer axes with respect to the vertical sensing device and azimuth reference. In the laboratory an azimuth optical reference line can be surveyed in and used to erect and align the system. A mirror attached to the stabilized member is designated as the prime reference during the calibration sequence, and the measurement coordinates are located in relation to the mirror. If absolute optical azimuth and vertical references are available, at the system's operational sight, the measurement coordinates are defined in terms of local geographic coordinates by slaving the mirrored surface to the optical reference.

Stable base gyrocompassing substitutes a gyro input axis for the azimuth reference at the operational sight. Assuming that no absolute optical reference is available, the predetermined laboratory orientation of the stabilized member must coincide with that orientation acquired by gyrocompassing at
the operational sight. If geographic east is defined by the laboratory optical reference, one can write an equation describing the angular velocity summation performed by the east gyro of the gyrocompassing system in terms of gyro misalignments, residual interfering torques, and compensation.

\[
\dot{\omega}_{(ie)} [A_{(h-y)} \sin \text{Lat} - A_{(y-g-y)} \cos \text{Lat}] + \frac{R}{H} + i \frac{S_{TG}}{C} H = 0
\]

(3-1)

Because of latitude coupling into Equation 3-1, the gyro misalignment angles must be defined in the laboratory if the laboratory and operational sights are at separated latitudes. To determine the misalignment angle knowledge requirements as a function of latitude, Equation 3-1 is differentiated, holding the residual term and compensation current constant.

*\( A_{(h-y)} \) = angle between horizontal plane and east gyro input axis (Figure 3-1).

*\( A_{(y-g-y)} \) = angle between geographic east and east gyro input axis (Figure 3-1)
DEVIATION OF EAST GYRO IA FROM GEOGRAPHIC EAST

\[ A(-Yg-y) = A(-Yg-m) - A(m-y) \]

\[ A(Yg-y) = A(Yg-m) + A(m-y) \]

FIGURE: 3-1
The angle between the horizontal plane and east gyro input axis, $A_{(h-y)}$, is only defined by the null orientation of the north gyrocompass loop and is independent of latitude. The change in azimuth angle of the gyrocompassing system is equal to the change in angle between geographic east and the east gyro input axis. Rearranging Equation 3-2, the effects of latitude change on the azimuth angle of the gyrocompassing system are apparent.

$$
\omega_{(ie)} = (A_{(h-y)} \cos \text{Lat} + A_{(y_g-y)} \sin \text{Lat}) \, d \text{Lat} - \cos \text{Lat} \, d A_{(y_g-y)} = 0
$$

(3-2)

$$
\frac{d A_{(y_g-y)}}{d \text{Lat}} = A_{(h-y)} + A_{(y_g-y)} \tan \text{Lat}
$$

(3-3)

* The north gyrocompass loop is always operated close to a null. When the system changes latitude, the north gyro is recompenated. Section 2.2.0 and Section 5.1.0 discuss the north gyrocompass loop.
Equation 3-3 indicates that the change in azimuth angle of the system as a function of latitude only depends on the east gyro's misalignment angles. To develop a design requirement on the east gyro's misalignment angles as a function of latitude, assume that the calibration laboratory is located at 45 degrees latitude. For a ± 5 degree latitude zone around 45 degrees, the rms azimuth error is to be less than two seconds of arc. Under these conditions, $A_{h-y}$ and $A_{g-y}$ must be known to better than 14.1 seconds of arc.

The remainder of this chapter presents a method which can be used in the laboratory to determine the east gyro misalignment angles. Also presented is the initial laboratory optimization of wheel speed modulation. In order to determine these angles, the laboratory should be equipped with a rate table and two automatic autocollimators referred to geographic east and west. In addition, it is assumed that the system has one stationary mirror.*

* If the system permitted optical access to the east gyro case, advantages could be gained by locating a mirror with respect to the gyro input axis on a single component basis and then observing the position of the mirror when the system is gyrocompassing. However, the development that follows assumes the more general case of an arbitrary azimuth mirror on the stabilized member.
Several schemes have been proposed to reduce the angle between the horizontal plane and the east gyro input axis. One of these measures the azimuth orientation of the gyrocompassing system at several latitudes and determines $A_{(h-y)}$. This method uses the effect to calibrate the cause but is impractical because of the large latitude separation of surveyed sights required. Also, this method assumes all other system parameters remain constant during the time required for latitude change. A second method rotates the stable member exactly one-hundred and eighty degrees about a horizontal line, keeping the east gyro input and output axis in a plane perpendicular to the line. This method requires unrealistically accurate measurements of a large angle, and also assumes that changing the orientation of the east gyro by one-hundred and eighty degrees in the gravity field will not affect its torque summation. Method number three artificially applies a vertical rate to the stable member of the gyrocompassing system. When the east gyro is insensitive to the component of vertical table rate along its input axis, $A_{(h-y)}$ is equal to zero.

By rotating the stabilized member with a vertical table rate before the gyrocompassing system is assembled, $A_{(h-y)}$ can be measured to better than ten seconds of arc. Once $A_{(h-y)}$ is
known, it can be compensated for or reduced, depending on the alignment flexibility* of the gyrocompass instruments. The development that follows assumes a method is available for reducing $A_{(h-y)}$. This simplified the subsequent measurements of $A_{(y_{eq}-y)}$ and $R$, but does not reduce the validity of the technique if $A_{(h-y)}$ can only be compensated for (i.e., not reduced).

The vertical table rate is applied along an axis which is nearly parallel with the east gyro output axis. This table rate couples unwanted torques to the gyro float through other axes. The table angle changes the component of horizontal earth rate applied to the east gyro input axis. The accuracy of measuring $A_{(h-y)}$ relies firmly on the table rate and angle. The following detailed description of the calibration procedure defines the accuracy of determining $A_{(h-y)}$ and the optimum table rate and angle.

Mathematically, the moments about the output axis of a single-degree-of-freedom integrating gyro are as follows.

* If the north vertical sensor null orientation with respect to the stabilized member can be easily changed, $A_{(h-y)}$ can be reduced. In practice, machining tolerances will probably establish an $A_{(h-y)}$ of approximately 0.1 milliradians and, if measured, this angle can be compensated for.
Equation 3-4 assumes that the gyro angular momentum is orthogonal to its input axis, the case and the float input axis are perfectly aligned, and the spring force of the gyro float with respect to its case about the output axis is equal to zero. Solving Equation 3-4 for the angle of the gyro float with respect to the case about the output axis, \( \theta_{c-f(OA)} \),

\[
\theta_{c-f(OA)} = \frac{1}{PC_d(OA) \left( \frac{I_{g(OA)}}{P + 1} \right)} \left[ \frac{H}{C_d(OA)} \omega_{i-f(IA)} + \frac{S_{TG \ i_c}}{C_d(OA)} + \frac{(u)M(OA)}{C_d(OA)} \right]
\]  

(3-5)

* Appendix I contains a definition of terms.
The angular velocity of the gyro float with respect to inertial space about the gyro's input axis, $\omega_{i-f}(IA)$, is equal to the angular velocity of the gyro case with respect to inertial space plus the angular velocity of the gyro float with respect to the gyro case.

$$\omega_{i-f}(IA) = \omega_{i-c}(IA) + \omega_{c-f}(IA)$$  \hspace{1cm} (3-6)

The angular velocity of the gyro case with respect to inertial space, $\omega_{i-c}(IA)$, is due to the angular velocities external to the gyro case. The angular velocity of the gyro float with respect to the gyro case, $\omega_{c-f}(IA)$, is produced by inertial torques between the gyro float and case about the gyro input axis.

With the gyro output axis nearly vertical, a vertical table rate, $\omega_T$, is applied to the stable member. If the gyro input axis is not orthogonal to the vertical, the table rate applies $\omega_T A(h-y)$ to the gyro case. The problem is to minimize $A(h-y)$ by reducing the effect of the vertical table rate on the gyro input axis. This table motion couples two unwanted changing angular velocities into the gyro input axis. First, the horizontal component of earth rate depends on table angle. Second, the table
rate being nearly coincident with the gyro output axis, couples a torque equal to $\Omega_T \omega_T$ to the gyro float with respect to its case about the input axis. The problem is to separate the angular velocity applied to the gyro input axis due to $A_{(h-y)}$ from the other effects and reduce $A_{(h-y)}$.

To minimize the effects of varying earth rate applied to the gyro with $A_{(h-y)}$ constant, the gyro input axis is aligned approximately parallel to north. The gyro torquing signal, $i_{c \frac{S_{TG}}{H}}$, is adjusted to cancel the northerly component of earth rate plus any residual bias in the gyro. Under these conditions, when a vertical table rate, $\omega_T$, is applied to the stabilized member, the varying angular velocities applied to the gyro case about its input axis are

$$\omega_{i-c(IA)} = (\omega_{(ie)v} + \omega_T) A_{(h-y)} + \omega_{(ie)h} (1 - \cos A_T)$$

(3-7)

* The high ratio of output axis damping to inertia in a 2FBG-6F gyro forces the gyro float to very nearly follow the gyro case for output axis rates below five hundred radians per second.

** The horizontal gradient of earth rate around north is a minimum.
If the magnitude of the table angle, $A_T$, is less than 0.03 radians and the table rate, $\omega_T$, is greater than the vertical component of earth rate, Equation 3-7 reduces to

$$\omega_{i-c(I A)} = \omega_T A(h-y) + \omega^{(i e) h} \frac{(A_T)^2}{2!}$$

(3-8)

A sinusoidal table rate is chosen for reasons which will become clear as the alignment procedure proceeds.

$$A_T = B \sin \omega_o T$$

$$\omega_T = B \omega_o \cos \omega_o T$$

(3-9)

Substituting $A_T$ into Equation 3-8, and remembering that

$$\sin^2 x = 1/2[1 - \cos 2x]$$

Equation 3-9 changes to

$$\omega_{i-c(I A)} = A(h-y) B \omega_o \cos \omega_o T + \frac{\omega^{(i e) h} B^2}{4}[1 - \cos 2 \omega_o T]$$

(3-10)
Equation 3-10 indicates that the frequency of the horizontal component of earth rate applied to the gyro is twice that of the table rate. If the magnitude of these rates were equal, they could be separated by observing the output of the gyro ducosyn with the table rate as a reference. If the magnitude of the change in horizontal earth rate applied to the gyro input axis is equal the vertical rate coupled into the gyro, the frequency and amplitude of the table rate are related in the following manner.

$$B = 4 A_{(h-y)} \frac{w_0}{\omega_{(ie)h}}$$

(3-11)

Under the conditions of Equation 3-11, the effect of horizontal earth rate may be neglected because it can be separated out of the data.

The torque of the gyro float with respect to its case about the input axis, $H \omega_T$, is produced by the table rate about an axis nearly coincident with the gyro output axis. Summing moments of the gyro float with respect to its case about the input axis, Equation 3-12 is obtained.
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\[ \sum M_{(IA)} = \omega_T H + P^2 I_{g(IA)} A_{c-f(IA)} + P C_{d(IA)} A_{c-f(IA)} \]

\[ + K_{c-f(IA)} A_{c-f(IA)} = 0 \]

(3-12)

Equation 3-12 assumes that \( \omega_T \) is perpendicular to \( H \), and the total table rate is transmitted from the gyro case to float about the gyro's output axis. * Substituting \( \omega_{c-f(IA)} \) for \( P A_{c-f(IA)} \) and rearranging, Equation 3-12 describes the angular velocity of the gyro float with respect to its case about the input axis.

\[ \omega_{c-f(IA)} = \frac{-P \omega_T I_{g(IA)}}{P^2 + P C_{d(IA)} I_{g(IA)} + K_{(IA)} I_{g(IA)}} \]

(3-13)

The following numbers are applicable for a 2FBG-6F single-degree

* Gyro float tracks the case for output axis rates below five-hundred radians per second as previously noted.
of freedom integrating gyro.

\[ H = 2 \times 10^6 \text{ gram cm}^2 \text{ second} \]

\[ C_{d(IA)} = 5 \times 10^9 \text{ dyne cm radian}^{-1} \text{ second} \]

\[ C_{d(OA)} = 2 \times 10^6 \text{ dyne cm radian}^{-1} \text{ second} \]

\[ I_g(IA) = 5 \times 10^3 \text{ gram cm}^2 \]

\[ I_g(OA) = 2 \times 10^3 \text{ gram cm}^2 \]

\[ K_{(IA)} = 10^9 \text{ dyne cm radian} \]

\[ K_{(OA)} \approx 0 \]

Substituting numbers and combining Equations 3-5, 3-13, and 3-6,

\[ A_{c-f(OA)} = \frac{1}{P(10^{-3} P + 1)} [A(h-y) \omega_T] \]

\[ \frac{P \omega_T 4 \times 10^2}{P + 10^6} + \frac{(u) M_{(OA)}}{2 \times 10^6} \]
for $P < 10^3$.

\[
A_{c-f(OA)} \approx \frac{\omega_T}{P} [A_{(h-y)} - \frac{P \cdot 2 \times 10^{-3}}{5 \cdot P + 1} + \frac{(u) M(OA)}{2 \times 10^6}]
\]

(3-15)

Substituting $A_T$ for $\frac{\omega_T}{P}$ from Equation 3-9,

\[
A_{c-f(OA)} = -B \sin \omega T [A_{(h-y)} - \frac{2 \times 10^{-3} P}{5 \cdot P + 1} + \frac{(u) M(OA)}{2 \times 10^6}]
\]

(3-16)

Substituting the earth rate coupling constraint of Equation 3-11 into Equation 3-16,

\[
A_{c-f(OA)} = \frac{4 A_{(h-y)} \omega \sin \omega T}{\omega_{(ieh)}} [A_{(h-y)} - \frac{2 \times 10^{-3} P}{5 \cdot P + 1}]
\]

\[+ \frac{(u) M(OA)}{2 \times 10^6} \]

(3-17)
The angle between the gyro float and case about its output axis, $A_{c-f(OA)}$, is sensed with the gyro ducosyn. In a benign laboratory, incremental angles of 0.1 seconds of arc ($5 \times 10^{-7}$ radians) can be measured with the gyro ducosyn. Therefore, from Equation 3-17, in order to record the component of vertical table rate coupled to the gyro input axis at 45 degrees latitude,

$$\theta(3-18)$$

$$\frac{(A_{(h-y)})^2}{2} > \frac{5 \times 10^{-7} \frac{\omega_{(ie)h}}{\omega_o}}{4} = \frac{6.6 \times 10^{-12}}{\omega_o}$$

But, from Equation 3-17, in order to separate the component of vertical table rate applied to the gyro case from the component applied to the gyro float about its input axis,

$$A_{(h-y)} \geq 2 \times 10^{-3} \omega_o$$

$$\theta(3-19)$$
Substituting into Equation 3-18,

\[ \omega_o \leq (1.7 \times 10^{-6})^{1/3} = 1.2 \times 10^{-2} \text{ radians/second} \]  

(3-20)

Substituting into Equation 3-18, the minimum detectable \( A_{(h-y)} \) is

\[ A_{(h-y)\text{minimum}} = 2.4 \times 10^{-5} \text{ radians} = 4.9 \text{ arc seconds measurable} \]  

(3-21)

Substituting into Equations 3-9 and 3-11, the optimum table angle and angular rate are defined.

\[ A_T = 2.3 \times 10^{-2} \sin 1.2 \times 10^{-2} T \]  

(3-22)

Since the maximum table angle achieved is \( 2.3 \times 10^{-2} \) radians, the assumption of small table angles holds, and the expansion from Equation 3-7 to 3-8 is valid. The sinusoidal table rate allows the separation of the angular velocity components applied to the gyro.
input axis. The fact that the change in horizontal earth rate coupling is the second harmonic of table rate and the internal gyro coupling is in quadrature with table rate allows the amplitude of these effects to be made equal to the component of table rate proportional to $A_{(h-y)}$, and yet they can be separated.

The equations indicate that $A_{(h-y)}$ can be reduced to less than five seconds of arc by this method. When $A_{(h-y)}$ is minimized, the north vertical sensor should be brought to a null, or if this is not possible, the angle output of the north vertical sensor can be recorded. This is $A_{(h-y)}$ and should be used to recompensate the east gyro on the basis of Equation 3-1 for a known latitude relocation.

3.2.0 $A_{(y_g-y)}$

The angle between geographic east and the east gyro input axis can be determined by east-west averaging techniques with the gyrocompassing system. Single state-of-the-art gyros can east-west average with accuracies in the low arc second region. A complete gyrocompassing system should be able to gyrocompass east and west, yielding the same coefficients on the east seeking gyro as a single component. The development that follows applied east-west averaging techniques to an entire system to determine $A_{(y_g-y)}$ and $R$. Numerical calculations are
not presented because the process depends mainly on the stability of the east seeking gyro during the averaging interval. Mathematically, $A(y_g - y)$ is determined as follows.

Since $A(h - y)$ is small, the input axis of the east gyro is nearly in the horizontal plane. The angular velocity summation performed in the east gyro of the gyrocompassing system forces its input axis to deviate from geographic east by $A(y_g - y)$, due to residual "R" torques. Mathematically, the east gyro's angular velocity summation is

$$- A(y_g - y) \omega(ie)h + \frac{R_H}{H} = 0$$  \hspace{1cm} (3-23)

By observing the absolute orientation of a mirror surface when the gyrocompass is operating with the east gyro input axis approximately east, and then observing the same mirror surface when the gyro input axis is approximately west, $A(y_g - y)$ is determined.

With the system on the rate table, the gyrocompass is allowed to reach a steady state orientation. From Figure 3-1 and Equation 3-23, the east equilibrium orientation of the gyrocompassing system is

$$- \omega(ie)h A(m - y) - \omega(ie)h A(y_g - m) + \frac{R_H}{H} = 0$$  \hspace{1cm} (3-24)
The system is next changed from the gyrocompass mode to position mode of operation and rotated approximately one-hundred and eighty degrees. The system is then stabilized, \( K_{AE} \) reversed, and returned to the gyrocompass mode. From Figure 3-1 and Equation 3-23, it is seen that the east gyro equilibrium equation when gyrocompassing west is

\[
+ \omega_{(ie)} h A_{(m-y)} - \omega_{(ie)} h A_{(-y_g-m)} + \frac{R}{H} = 0
\]

(3-25)

Since \( A_{(m-y)} \) is a constant, Equations 3-24 and 3-25 may be added to yield \( \frac{R}{H} \).

\[
\frac{R}{H} = \frac{\omega_{(ie)} h}{2} \left[ A_{(y_g-m)} + A_{(-y_g-m)} \right]
\]

(3-26)

Substituting \( \frac{R}{H} \) into Equation 3-24 or Equation 3-25 yields \( A_{(m-y)} \). In order to align the east gyro input axis with geographic east, a compensation current is applied to the east gyro torque generator to reduce \( A_{(y_g-y)} \) to zero.

DECLASSIFIED

-64-
With the compensation current calculated by Equation 3-27 applied to the \( y \) gyro torque generator, the east-west averaging technique of Section 3.2.0 may be repeated to be sure that \( A(y_{g-y}) \) is within the design requirements.

3.3.0 Laboratory Optimization of Wheel Speed Modulation

In an operational system, wheel speed modulation is used with sample data techniques to determine low level torque changes in the east gyro and to recompensate the gyro. This technique depends on a low value of torque uncertainty being introduced when the east gyro wheel speed is cut in half. The wheel speed modulation technique depends on an initial laboratory calibration primarily optimizing gyro wheel power at both wheel speeds. This section discusses the initial laboratory optimization procedure.

Section 4.3.0 presents data recorded on a gyrocompassing system using wheel speed modulation.

Section 5.2.0 indicates a possible operational procedure which makes use of wheel speed modulation.

If the steady state gyrocompass east gyro wheel speed is
cut in half, Equation 3-27 changes to Equation 3-28.

\[
2 \frac{t}{c} \frac{S_{TG}}{H} + 2 \frac{R}{H} + \omega_m + A_m \omega_{(ie)h} = 0
\]  
(3-28)

\(\omega_m\) is the drift rate introduced in the east gyro due to the gyro wheel speed being cut in half. \(A_m \omega_{(ie)h}\) is the horizontal component of earth rate necessary to cancel \(\omega_m\). Since \(R\) and \(i_c\) have not changed, it is seen that

\[
A_m = -\frac{\omega_m}{\omega_{(ie)h}}
\]  
(3-29)

In the laboratory, the angle \(A_m\) can be measured with the autocollimator which senses the variation in azimuth mirror orientation for the gyrocompass at both gyro wheel speeds. Gyro wheel power can be properly equalized at both speeds so \(\omega_m\) will be almost zero. East gyro wheel conditions are optimized if the gyrocompass orientation does not vary when the east gyro wheel speed is cut in half.

To see how wheel speed modulation can be used to determine
Equation 3-30 describes the closed loop angular velocity summation performed by the east gyro in the gyrocompass mode at normal wheel speeds.

\[
\frac{R}{H} + i_c \frac{S_{TG}}{H} = 0 \tag{3-30}
\]

If \( R \) changes to \( R + \Delta R \), the following equation describes the closed-loop gyrocompass performance.

\[
\frac{R + \Delta R}{H} + i_c \frac{S_{TG}}{H} - A_{(y-g-y)} \omega_{(ie)h} = 0
\]

The \( \Delta R \) only produces an azimuth error in the gyrocompass system and, therefore, cannot be detected in a closed-loop basis. If the gyrocompass loops are opened and east gyro wheel speed cut in half, Equation 3-32 describes the angular velocity summation performed in the east gyro.
If the azimuth gyro is properly compensated, \( A(y_g - y) \) is constant. Combining Equations 3-30, 3-31, and 3-32 shows that the open-loop east gyro drift, \( \omega_{my}' \), at half wheel speed is equal to the change in the east gyro residual term

\[
\omega_{my} = \frac{\Delta R}{H} \tag{3-33}
\]

By recompensating the east gyro on the basis of open-loop drift rate and changing \( i_c \frac{S_{TG}}{H} \) to \( i_c \frac{S_{TG}}{H} + \frac{\Delta R}{H} \), the variation in the east gyro residual term is compensated, \( A(y_g - y) \) returns to zero, and Equation 3-30 again applies to the closed-loop east gyro performance. It is observed that changes in \( i_c \) and \( S_{TG} \) can also be detected and compensated for in this manner.

3.4.0 Accelerometer Location

The location of accelerometer input axis, scale factor, and bias determination can all be evaluated in the laboratory by conventional methods. With the autocollimator defining the azimuth
mirror orientation, a four-point calibration in the laboratory will determine the beta matrix which is stored in the guidance computer for future reference.
Representative data on the stable base performance of a gyrocompassing system consisting of three 2FBG-6F single-degree-of-freedom gyros and one Kearfott two-axis pendulum is analyzed in Chapter IV.

These data are presented to permit an evaluation of gyrocompassing uncertainties as a function of system bandwidth and to determine the effectiveness of the wheel speed modulation technique. Once gyrocompassing uncertainties as a function of system bandwidth are defined in a benign environment, an operational gyro drift detection and recompensation procedure can be derived for a more realistic environment. These data also permit an evaluation of the individual instruments in a system environment.

Two samples of data will be analyzed, (1) four-hundred and twenty-eight hours of steady state stable base data at two system bandwidths; (2) two-hundred and sixty-eight hours of steady state data utilizing wheel speed modulation to update the east gyro compensation.

4.0.1 Environment

Before proceeding to the analytical background and

DECLASSIFIED

-70-
analysis of data, the environment, which was chosen in order to determine component capabilities, must be defined. For the recording of these data, the system was operating on a seventeen-ton concrete slab experiencing lateral accelerations in the micro g region.* The system is designed to provide the proper environment for the operation of the 2FBG-6F gyros and Kearfott pendulums. The region of interest is torque uncertainties of 0.0012 dyne-centimeters, which correspond to angular velocity uncertainties of 0.01 meru, 0.00015 degrees per hour. In this area, gyro performance is a function of the electronics used to power and control the instrument as well as the accelerations and temperature present. Over-all system accuracy is a measure of the performance of the instruments in the controlled system environment. System data such as that of a stable base gyrocompass can often be separated to give an indication of the individual instrument's performance. These data, in turn, can be correlated with instrument history, age, and variations in environment to provide the basis for changes in instrument and system design.

The data that follow were recorded on the 2-16 gyrocompass where instrument temperature is controlled to better than 0.04 degree Fahrenheit. The east gyro is operating with its input

* Measurements with wide bandwidth accelerometers have defined the accelerations present.
axis approximately east and a total compensation of less than five meru. The fact that the east gyro's output axis is nearly vertical makes this instrument's torque summation less sensitive to gravity torques. It is a necessity in the operation of the three-gyro gyrocompass that the east gyro, whose performance requirements are two orders of magnitude better than the north or azimuth gyro, be positioned in the optimum orientation to act as the basic reference in the gyrocompassing system. The azimuth gyro is compensated for vertical earth rate which is approximately seven-hundred meru. One gravity acceleration is acting perpendicular to the azimuth gyro's spin and output axes, which are horizontal.

The north gyro's uncertainties are not analyzed because they do not seriously affect the accuracy of the gyrocompassing system.*

4.1.0 Bandwidth, Instruments, and Environment

The design of an effective gyrocompass is largely a compromise or optimization between the system's instrument uncertainties and the acceleration environment during operation. The interpretation of instrument errors on an uncertainty basis and estimation of specific spectral characteristics provide one

* Section 5.1.0 supports this statement.
set of bounds defining the lower limit on system bandwidth. Acceleration environment in a fixed base produces uncertainties in angular measurement* and induces dynamic errors** in system operation which define the upper frequency limits on system bandwidth. The data presented, together with an estimate of lateral acceleration inputs, are used to identify an area of general design interest.

In a system operating over a prolonged period of time, sample data techniques and an integrator*** are required to periodically recompensate the gyros. The frequency of the recompensation procedure depends on the accuracy of the process, the bandwidth of the gyrocompassing system, and the spectrum of instrument uncertainties. The spectrum of instrument uncertainties, and the effect of these uncertainties on system performance as a function of bandwidth, are defined in this chapter. Chapter V derives a gyro recompensation procedure which could be used in a more realistic environment to update the compensation to gyros with similar spectral characteristics.

---

* AppendixII defines the spectrum of angular measurement uncertainties as a function of lateral accelerations.

** Chapter V indicates that uncertainties in angular measurement, which limit the bandwidth of gyro drift evaluation, place more severe restrictions on system bandwidth than dynamic angle uncertainties. The uncertainties in angle measurement define the time constant of the vertical sensing device (i.e., the critically damped gyrocompass time constant).

*** The integrator's low sensitivity does not affect the stability of the gyrocompassing system.
4.1.1 Instrument Uncertainties

The effect of instrument uncertainties on east and azimuth angles can be determined from Figure 4-1.* The steady state azimuth angular velocity summation is satisfied when \((u)\omega_z + K_{AE} A_y = 0\). The steady state east gyro's angular velocity is satisfied only when \(-A_z \omega_{(ie)} + (u)\omega_y - K_{EE} A_y = 0\). Therefore, the equilibrium angles of the gyrocompassing system are

\[
A_y = -\frac{(u)\omega_z}{K_{AE}} \quad (4-1)
\]

\[
A_z = \frac{1}{\omega_{(ie)}h} \left[ (u)\omega_y + \frac{K_{EE}}{K_{AE}} (u)\omega_z \right] \quad (4-2)
\]

Equation 4-1 indicates that the output of the east vertical sensor is proportional to azimuth gyro drift. Therefore, changes in the output of the east vertical sensor which occur within the frequency range of the gyrocompassing system are proportional

* Chapter II contains the justification of Figure 4-1. In addition, Chapter II gives a detailed development of the dynamic as well as the steady state changes in east and azimuth angles due to instrument uncertainties. (Equation 4-2 is a repeat of Equation 2-3.)
EAST AZIMUTH: GYROCOMPASS

AUTOCOLLIMATOR

RECORDER

AMPLIFIER

KAE

-KEE

EAST VERTICAL SENSOR

\( \frac{1}{p} \)

\( 50 \frac{\text{lb-ft}}{\text{mr}} \)

-\( W(eh) \)

\( (u) W_z \)

\( (u) W_y \)

\( -75 \)

\( \theta \)
to uncertainties in azimuth gyro performance.

As previously noted, uncertainties in the east gyro performance only produce azimuth angle errors in the gyrocompassing system. By combining Equations 4-1 and 4-2, it is noted that steady state east gyro performance can be extracted from an appropriate summation of azimuth and east angle variations.

\[(u)\omega_{y} = \omega_{ie}h A_{z} + K_{EE} A_{y}\]  \hspace{1cm} (4-3)

4.1.2. Spectrum of Instrument Uncertainty

Section 4.1.1 has only discussed the steady state angular velocity summations performed in the azimuth and east gyros. The words "steady state" in the analysis of instrument uncertainties in the gyrocompassing system refer to uncertainties which occur at a frequency below the break frequency of the system. For data analysis, it is instructive to divide the total spectrum of possible instrument uncertainties into three regions (Figure 4-2).

Region 1: Instrument Uncertainty Frequency > \(\omega_{o}\)

When the frequency of instrument uncertainty is above the natural frequency of the gyrocompassing system,
UNCERTAINTY FREQUENCY

REGION 3
Low frequency region.
Instrument uncertainties produce essentially steady state angle errors in the gyrocompassing system. Instrument uncertainties can be separated in this region.

REGION 2
Medium frequency range.
Instrument uncertainties produce dynamic changes in east and azimuth angles. Individual instrument performances are difficult to separate in this region.

REGION 1
Frequencies above break-point of gyrocompass system. Individual instrument uncertainties attenuated by system and cannot be separated in these frequencies.

BREAK FREQUENCY OF GYROCOMPASS
0.001 \( \leq W_0 \leq 0.008 \\

FIGURE: 4-2
the effects will be attenuated in the closed-loop gyrocompass. These uncertainties do not seriously affect the performance of the system as long as the amplitude of the variations produced are acceptable. (The only exception to this is the east gyro drift rate which is detected in an open-loop basis in a short time interval.)

Region 2: Instrument Uncertainty Frequency \( \approx \omega_0 \)

When the frequency of instrument uncertainty is near the breakpoint of the gyrocompassing system, it is difficult to determine if any noted disturbance is due to a single instrument, environment, or a combination of these. In this region, the system is simultaneously responding to uncertainties in all instruments and environments.

If the variations in east and azimuth angles are within the design requirements of system performance, the uncertainties are acceptable. The total effect of all uncertainties in this region can be determined by observing repeatable data over several system time constants which is sampled at a frequency above the breakpoint of the gyrocompassing system. (The dynamic relations between instrument performance and gyro-compass performance are covered in Section 2.3.2 of this report.)
Region 3: Instrument Uncertainty Frequency < $\omega_0$

In the range of frequencies below the breakpoint of the gyrocompassing system, a steady state solution is obtained to all disturbances. In this region, instrument uncertainties produce steady state east and azimuth angle errors as indicated in Equations 4-1, 4-2, and 4-3. The analysis of data in this frequency range allows separation of the individual instrument's performance. The highest frequency of instrument uncertainty observable is equal to one-half the sample frequency. This region extends down to d-c values of instrument uncertainties.

4.2.0 Steady State Data

Table 4-1 summarizes the important system parameters for the recording of two bandwidths of steady state data which will be analyzed. The first two-hundred and sixty hours of data were recorded with an east-vertical-sensor-azimuth-gyro coupling, $K_{AE}$, of six meru per second of arc. This produced an over-damped, 0.0021 radian per second system, highly sensitive to azimuth gyro drift.* The

\[ \frac{\partial A_z}{\partial (u)\omega_z} = \frac{K_{EE}}{K_{AE}} \frac{1}{\omega_{(ie)}h}, \]  

the higher the ratio of $K_{EE}$ to $K_{AE}$, the more sensitive the azimuth angle of the gyrocompassing system is to azimuth gyro drift.

---

* \[ \frac{\partial A_z}{\partial (u)\omega_z} = \frac{K_{EE}}{K_{AE}} \frac{1}{\omega_{(ie)}h}, \]  

the higher the ratio of $K_{EE}$ to $K_{AE}$, the more sensitive the azimuth angle of the gyrocompassing system is to azimuth gyro drift.
260 HOURS - LOW EAST/AZIMUTH COUPLING

\[ K_{AE} = \text{6 meru per second of arc.} \]
\[ K_{EE} = \text{2 meru per second of arc.} \]

Roots: \[ p_1, p_2 = 1.5 \times 10^{-2} \left[ -1 \pm \sqrt{1 - 2.1 \times 10^{-2}} \right] \]

\[ \omega_o = 2.1 \times 10^{-3} \text{ radians per second} \]

\[ \frac{\Delta A_z}{(u)\omega_z} = \frac{K_{EE}}{K_{AE}} \frac{1}{\omega_{(ie)h}} \approx 100 \text{ seconds of arc per meru} \]

\[ \frac{\Delta A_z}{(u)\omega_y} = \frac{1}{\omega_{(ie)h}} = 286 \text{ seconds of arc per meru} \]

\[ \frac{\Delta A_E}{(u)\omega_z} = -\frac{1}{K_{AE}} = -0.165 \text{ seconds of arc per meru} \]
### TABLE 4-1b

**168 HOURS - HIGH EAST/AZIMUTH COUPLING**

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{AE}$</td>
<td>60 meru per second of arc.</td>
</tr>
<tr>
<td>$K_{EE}$</td>
<td>2 meru per second of arc.</td>
</tr>
<tr>
<td>Roots</td>
<td>$p_1, p_2 = 1.5 \times 10^{-2} [-1 \pm 1 - 2.1 \times 10^{-1}]$</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>$6.7 \times 10^{-3}$ radians per second</td>
</tr>
</tbody>
</table>

$$
\frac{\Delta A_z}{(u)\omega_z} = \frac{K_{EE}}{K_{AE}} \frac{1}{\omega_z^{(ie)h}} \approx 10 \text{ seconds of arc per meru}
$$

$$
\frac{\Delta A_z}{(u)\omega_y} = \frac{1}{\omega_z^{(ie)h}} = 286 \text{ seconds of arc per meru}
$$

$$
\frac{\Delta A_{AE}}{(u)\omega_z} = -\frac{1}{K_{AE}} = -0.0165 \text{ seconds of arc per meru}
$$
second one-hundred and sixty-eight hours of data were recorded with the east vertical sensor azimuth gyro coupling of sixty micro per second of arc. This produced a lighter damped system with an order of magnitude reduction in sensitivity to azimuth gyro drift.

The steady state data will be analyzed in two sections. First, one typical two-hour period of data at both system bandwidths will be studied to determine the short-term effect of all uncertainties on system performance. Second, the two-hundred and sixty hours of narrower bandwidth data will be compared with the one-hundred and sixty-eight hours of increased bandwidth data.

4.2.1 Short-Term Stability

Figure 4-3 is a typical two-hour recording of the azimuth orientation of the stabilized member and the output of the east vertical sensor. As one would expect, the short-term azimuth and east variations are a function of system bandwidth. Short-term oscillations are apparent in the lighter damped system of Figure 4-3a. These oscillations have a peak amplitude of five seconds of arc and occur at a frequency of $2 \times 10^{-3}$ radians per second, slightly below the breakpoint of the gyrocompassing system. Due to the large number of dynamic solutions taking place simultaneously in the frequency
Fig. 4-3a
2 HOURS

Fig. 4-3b

Fig. 4-3  2-16 Gyro Compass short-term stability.
range, it is difficult to attribute these oscillations to any particular instrument. However, from Table 4-1 and Figure 4-3 (unless uncertainties cancel during this period), by observing the total angle change over the two-hour interval an upper limit can be placed on instrument uncertainties.

Figure 4-3a

\[
(u)\omega_z < 0.3 \text{ meru} \\
(u)\omega_y < 0.01 \text{ meru}
\]

Figure 4-3b

\[
(u)\omega_z < 0.03 \text{ meru} \\
(u)\omega_y < 0.01 \text{ meru}
\]

The high value of azimuth gyro uncertainty, \((u)\omega_z\) (Figure 4-3a), is due to the uncertainty in east angular measurement. 0.3 meru corresponds to an output of 0.005 second of arc from the east vertical sensor in the wide bandwidth system with an east azimuth coupling of sixty meru per second of arc.

There is nothing unique about Figure 4-3. Thousands of hours of stable base gyrocompassing data similar to that of Figure 4-3 have been recorded. The two conclusions which we are primarily interested in from Figure 4-3 are, (1) the east and azimuth gyros short-term stability for both system
bandwidths are within the initial design criteria of Table 1-1; (2) the higher bandwidth system has larger short-term dynamic uncertainties.

4.2.2 Long-Term Stability

Data similar to that indicated in Figure 4-3 was averaged over a forty-two minute time interval for four-hundred and twenty-eight hours. These average values of the azimuth orientation of the gyrocompassing system and the output of the east vertical sensor are plotted in Figure 4-4. The first two-hundred and sixty hours were recorded under the conditions of Table 4-1a. The second one-hundred and sixty-eight hours were recorded under the conditions of Table 4-1b. It is noted (Figure 4-4) that system bandwidth has a marked effect on the long-term stability of the gyrocompass.

The first two-hundred and sixty hours of data indicate a strong correlation between the output of the east vertical sensor and azimuth angle of the gyrocompassing system. The period of these disturbances is approximately 10.6 hours, with an east vertical sensor amplitude variation of approximately 0.02 second of arc. With an east vertical sensor azimuth gyro coupling of six meru per second of arc, an azimuth gyro drift of 0.12 meru is large enough to produce the variations
Fig. 4-4  2-16 Gyro Compass effect of varying bandwidths.
indicated in the first two hundred and sixty hours of Figure 4-4. Some of the probable causes of an 0.12 meru azimuth gyro shift are indicated as follows:

Torquing supply variation: 0.01%
Torque generator sensitivity change: 0.02%
Output axis torque variation: 0.015 dyne-cm

Since the period of these variations do not correlate with any known disturbance, they are assumed to be caused by a change in the torque summation inside the azimuth gyro. The azimuth gyro output and spin axis are both in the horizontal plane. The torque summation is sensitive to mass shifts along the spin axis due to shifts in the spin axis bearing and/or the ball retainer.

When these data were recorded, the gyro wheel had over five-thousand operating hours. Equation 4-4 indicates the magnitude of the 257 gram gyro wheel shift along the spin reference axis, which would result in a 0.015 dyne-centimeters torque variation. (5)

\[
d = \frac{1.5 \times 10^{-2}}{(2.57)(9.80) \times 10^4} = 5.9 \times 10^{-8} \text{ centimeters}\]

[DECLASSIFIED] angstroms (4-4)

-87-
These periodic shifts which are apparent at the lower system bandwidth are quite small. They are reported here as a point of interest to show how powerful a tool the variation in system bandwidth is for determining the performance of a gyrocompassing system and its instruments in a benign environment. Whether or not the variations in azimuth angle noted at the reduced system bandwidth will affect the system's operation depends on the restrictions on system bandwidth as a result of lateral accelerations in a realistic environment. In any event, since these shifts show up as an output of the east vertical sensor as well as azimuth angle; their effect on the azimuth orientation of the gyrocompassing system can be reduced by sampling the output of the east vertical sensor periodically and updating the compensation to the azimuth gyro.

It is noted (Figure 4-4) that the peak variations in azimuth angle over the first two-hundred and sixty hours are less than $±11$ seconds of arc. The total azimuth gyro drift recorded during this period was less than 0.25 meru. It is difficult to determine an east gyro drift during this time because of the large variations in azimuth angle.

The second one-hundred and sixty-eight hours of data in Figure 4-4 were recorded under the conditions of Table 4-1b. It is obvious that increasing the coupling between the east

DECLASSIFIED

-88-

DECLASSIFIED
vertical sensor and azimuth gyro reduced the amplitude of the periodic variations previously noted. This is the expected result if these variations are a result of uncertainties in the performance of the azimuth gyro. The higher $K_{AE}$, the less tilt about east is necessary to compensate for a change in the torque summation inside the azimuth gyro. For equal east vertical sensor to east gyro gains, the lower the tilt about east, the smaller the change in azimuth angle.

The azimuth variations were smaller in amplitude for the wider bandwidth system, but an essentially constant azimuth angle variation of two and one-half seconds of arc per day is evident. This could not be a result of the azimuth gyro because a total azimuth gyro drift of 1.8 meru is required at this system bandwidth to produce eighteen seconds of arc azimuth error. With an east azimuth coupling of sixty meru per second of arc, a 1.8 meru azimuth gyro drift would result in an 0.030 second of arc output of the east vertical sensor. The d-c value of the output of the east vertical sensor did not vary by more than 0.005 second of arc, while the azimuth angle changed by eighteen seconds of arc.

Since the output of the east vertical sensor remained essentially constant while the azimuth angle had the constant variation noted, it is reasonable to assume that an east gyro
drift produced the azimuth angle variation during the second one-hundred and sixty-eight hours. From Table 4-1, it is noted that an east gyro drift of less than 0.01 meru per day is large enough to produce this constant azimuth angle change. The puzzling point from a comparison of the two samples of data in Figure 4-4 is that there is no reason why the east gyro drift, or the effects of this drift, should be a function of system bandwidth.

A computer analysis of the data indicated that the first two-hundred and sixty hours of data had a constant variation of 0.4 seconds of arc per day. In addition, a constant variation of 0.0028 seconds of arc per day was noted in the output of the east vertical sensor. The sense of the constant change in the output of the east vertical sensor was in such a direction that it canceled part of the constant azimuth variation above 0.4 seconds of arc per day. Using the azimuth east angle coupling of Table 4-1a and adding these effects, it is observed that 0.68 seconds of arc per day azimuth variation would have been realized if the output of the east vertical sensor had remained constant. This was in disagreement with the 2.5 seconds of arc per day data of the second one-hundred and sixty-eight hours and corresponds to a possible east gyro

* Equations 4-1 and 4-2 mathematically make this same statement.
constant variation of 0.0024 meru per day. In spite of this apparent discrepancy, conclusions on the amplitude and frequency of instrument uncertainties can be obtained from the data of Figure 4-4. From an instrument viewpoint, it is noted that the azimuth gyro should be recompen-sated hourly in the narrower bandwidth system. The low spectrum of what appears to be east gyro drift < 0.01 meru per day indicates that many hours can elapse between subsequent recompen-sations of this gyro.

4.3.0 Wheel Speed Modulation

Wheel speed modulation was used periodically over a two-hundred and sixty-eight hour interval to detect changes in the performance of the east gyro in the gyrocom-passing system. The east gyro compensation was updated alternately by two engineers following a procedure which could easily be programed into a computer. The procedure used to update the compensation to the east gyro and data recorded during this period is the subject of Sections 4.3.1 and 4.3.2.

Section 4.3.1 contains the analytical background of how wheel speed modulation was used in the benign laboratory to detect changes in the torque summation in the east gyro. The reader who is primarily interested in data and not background method may proceed directly to Section 4.3.2 with no loss in
the results obtained via wheel speed modulation technique.

This test appears to indicate that wheel speed modulation can detect changes in the performance of the east gyro in the gyrocompassing system. However, the test was conducted in an unrealistically benign environment and the transfer of this system and technique to an operational condition is a major step. The preliminary analytical background for the transfer is covered in Chapter V.

4.3.1 Wheel Speed Modulation Theory and Test

Before the test was started, east gyro compensation was adjusted so that the azimuth angle of the gyrocompassing system did not vary when the east gyro wheel speed was cut in half. Under these conditions, Equation 4-5 describes the angular velocity summation of the east gyro.

Full East Gyro Wheel Speed

\[-A_{zo} \omega (ie)h + \frac{R}{H} + i_c \frac{S_{TG}}{H} = 0\]  \hspace{1cm} (4-5a)

* Section 3.3.0 discusses the initial laboratory optimization of wheel speed modulation which this condition establishes.
East Gyro Half Wheel Speed

\[-A_{zo} \omega(ie)h + \frac{2R_o}{H} + 2i \frac{S_{TG}}{H} + \omega_m = 0\]

(4-5b)

Gyro wheel power was initially equalized at both wheel speeds so that \(\omega_m\), the modulation induced drift rate, is nearly zero.

In the closed loop gyrocompass, a change in the east gyro's residual term produces a change in azimuth angle.

\[-(A_{zo} + \Delta A_z) \omega(ie)h + \frac{R_o + \Delta R}{H} + i \frac{S_{TG}}{H} = 0\]

(4-6)

The following procedure was used over two-hundred and sixty-eight hours to determine if wheel speed modulation could detect changes in the east gyro residual term in the benign environment. During the test, an automatic auto-collimator monitored the azimuth orientation of the stabilized member; however, this independent data source was used as a check on the test, not as a source of information to recompensate the east gyro.
Step #1

Two times per day the output of the east vertical sensor was forced to zero by recompensating the azimuth gyro.

Step #2

After the azimuth gyro was recompensated, the system was transferred from the gyrocompass mode to the gyro stabilized mode of operation. With the east gyro operating at normal wheel speed, the loops were left open for $T_1$ seconds. At the end of $T_1$ seconds, the output of the east vertical sensor is equal to

$$
**A_m(T_1) = 1.5 \times 10^{-2} \int_0^{T_1} \left( -A_{z1} \omega_{(ie)}h \ + \ \frac{R_1}{H} + i_c \frac{S_{TG}}{H} \right) \ dt
\ + \ i_{sw} \frac{S_{TG}}{H} \ dt - \left( 1.5 \times 10^{-2} \right)^2 \int_0^{T_1} \int_0^{T_1} \omega_z \omega_{(ie)}h \ dt^2
\]

\[(4-7)\]

* Gyro stabilized. There is no connection between the vertical sensors and the gyro torque generators in the gyro stabilized mode of operation, but all gyro drift and compensation remain as inputs to the gyros.

** $\omega_{(ie)}h = \frac{1}{286}$ meru per second of arc at 45 degrees latitude.
Meru $\times 1.5 \times 10^{-2} = \text{seconds of arc per second of time.}$
The first integral is a result of angular velocity in the east gyro at time zero. The second double integral is derived from the open-loop azimuth gyro drift coupling a horizontal component of earth rate into the east gyro. $i_{sw}$ is, principally, the uncertainty in the signal applied to the east gyro torque generator between the closed and open-loop gyrocompass. Prior to opening the gyrocompass loops, Equation 4-6 described the angular velocity summation performed in the east gyro, where $A_{z_1} = A_{z_0} + \Delta A_z$, and $R_1 = R_0 + \Delta R$.

Substituting Equation 4-6 into Equation 4-7,

\[
A_m(T_1) = 1.5 \times 10^{-2} \int_{0}^{T_1} i_{sw} \frac{S_{TG}}{H} \, dt
\]

\[
- (1.5 \times 10^{-2})^2 \int_{0}^{T_1} \int_{0}^{T_1} \omega_z \omega_{(ie)} \, dt
\]

(4-8)

For a constant azimuth gyro drift rate during $T_1$ and a constant change in the east gyro due to opening the gyrocom-
pass loops,

\[ A_m(T_1) = 1.5 \times 10^{-2} \left[ i_{sw} \frac{S_{TG}}{H} T_1 - 1.5 \times 10^{-2} \omega_{(ie)h} \frac{\omega z T_1^2}{z} \right] \]

(4-9)

Equation 4-9 describes the output of the east vertical sensor after \( T_1 \) seconds. At normal wheel speed, this angle is the result of an angular velocity component switched in when the system is transferred from a closed-loop to an open-loop system and an angular velocity component proportional to the azimuth gyro drift.

**Step #3**

After \( T_1 \) seconds were used to evaluate the open-loop output of the east vertical sensor at normal wheel speed, a tight leveling loop* was closed between the east vertical sensor and the east gyro. The gyro wheel speed was cut in half and \( T_2 \) seconds were required until the east gyro reached an equilibrium condition. Since the azimuth gyro has been the azimuth reference during Steps #2 and #3, a change in azimuth angle has coupled a change in the horizontal earth rate applied to the

* Leveling Loop: east gyro torque generator signal derived from the east vertical sensor. Azimuth gyro open-loop.
east gyro. At the end of $T_2'$, the total change in azimuth angle is

$$A_z(T_2) = 1.5 \times 10^{-2} \int_{0}^{T_1+T_2} \omega_z \, dt$$

(4-10)

Evaluation of Equation 4-10 for constant azimuth gyro drift is

$$A_z(T_2) = 1.5 \times 10^{-2} \omega_z(T_1 + T_2)$$

(4-11)

Step #4

At time $T_2$, the east leveling loop is opened. With the east gyro operating at half wheel speed, Equation 4-12 describes the output of the east vertical sensor.

$$A_m(T_3) = 1.5 \times 10^{-2} \int_{T_1+T_2}^{T_3} \left[ (-A_z \omega_z \omega_{(ie)}h + \frac{2R_1}{H} + 2i_c \frac{S_{TG}}{H} ight.$$ 

$$+ 2i_{sw} \frac{S_{TG}}{H} + \omega_m) \, dt - 1.5 \times 10^{-2} \int_{T_1+T_2}^{T_3} \omega_z \omega_{(ie)}h \, dt^2 \right]$$

(4-12)
Now, $A_{z_3}$ is equal to the azimuth angle at the beginning of the wheel speed modulation test plus any angle accumulated since the test started.

$$A_{z_3} = A_{z_1} + A_z(T_2) \quad (4-13)$$

Also, if any change in the east gyro's residual term occurred since Equations 4-5b and 4-6 applied,

$$- (A_{z_0}) \omega_{(ie)h} + \frac{2 R_o}{H} + 2 i \frac{S_{TG}}{cH} + \omega_m = 0$$

$$\quad (4-14)$$

Remembering that $A_{z_1} = A_{z_0} + \Delta A_z$, and that $\Delta A_z \omega_{(ie)h} = \frac{\Delta R}{H}$, Equations 4-11 and 4-14 combine to

$$A_m(T_3) = 1.5 \times 10^{-2} \int \omega_{(ie)h} \omega_z(T_1 + T_2) \, dT \frac{S_{TG}}{H} \right) dt - (1.5 \times 10^{-2})^2 \int \omega_z \omega_{(ie)h} \, dT \quad (4-15)$$
Integrating Equation 4-15, and making \((T_3 - (T_1 + T_2))\) equal to \(T_1\) (i.e., times to evaluate east angle at both wheel speeds are equal),

\[
A_m(T_3) = 1.5 \times 10^{-2} \left[ (-1.5 \times 10^{-2} \omega_{(ie)h} \omega_z (T_1 + T_2) T_1
\right.
\]

\[
+ \frac{\Delta R}{H} T_1 + 2 \frac{S_{TG}}{H} \frac{T_2}{T_1} \left] - (1.5 \times 10^{-2})^2 \omega_z \omega_{(ie)h} \frac{T_1^2}{2}\right.
\]

(4-16)

\(A_E(T_3)\) is the output of the east vertical sensor when the east gyro is operating at half wheel speed and the leveling loop is open for \(T_3\) seconds. \(A_E(T_1)\) is the output of the east vertical sensor when the east gyro is operating at full wheel speed and the leveling loop is open for \(T_1\) seconds. Adding \(-2 \ A_E(T_1)\) to \(A_E(T_3)\), Equation 4-17 is obtained.

\[
A_m(T_3) - 2 \ A_E(T_1) = 1.5 \times 10^{-2} \left[ (-1.5 \times 10^{-2} \omega_{(ie)h} \omega_z (T_1 + T_2) T_1
\right.
\]

\[
+ \frac{\Delta R}{H} T_1 + 1.5 \times 10^{-2} \omega_z \omega_{(ie)h} \frac{T_1^2}{2}\right]\]

(4-17)
Rearranging Equation 4-17,

\[ A_m(T_3) - 2 A_E(T_1) = (1.5 \times 10^{-2})^2 \omega_{(ie)h} \omega_z \left[ \frac{T_1}{2} + T_1 T_2 \right] + 1.5 \times 10^{-2} \frac{\Delta R}{H} T_1 \] (4-18)

Since we are trying to detect a change in the gyro residual term, \( \Delta R \), it is seen from Equation 4-18 that by adding two times the angle measured at the full wheel speed to the angle measured at half east gyro wheel speed, the switching current has canceled. Also, the coupling of horizontal earth rate into the east gyro, due to azimuth gyro drift, is partially canceled. The sum of the angles indicated in Equation 4-18 is proportional to \( \frac{\Delta R}{H} \).

Step #5

On the basis of the angles measured in Equation 4-18, the east gyro compensation was changed by \( \frac{\Delta R}{H} \). East gyro wheel speed changed to normal and the gyrocompass loops closed.

This technique cannot distinguish between changes in the east gyro residual term, \( \Delta R \), and the earth rate coupled into the east gyro due to the azimuth gyro drift. From Equation 4-18, the minimum detectable \( \frac{\Delta R}{H} \), as a function of the open
loop azimuth gyro drift rate \( \omega_z \) can be calculated.

\[
\frac{\Delta R}{H \omega_z} = \frac{1.5 \times 10^{-2}}{2} \omega(\text{e})h[T_1 + 2 T_2]
\]  

(4-19)

Typical values of \( T_1 \), \( T_2 \), and \( T_3 \) in a benign environment are three minutes each. Substituting into Equation 4-19,

\[
\frac{\Delta R}{H \omega_z} \approx 1.42 \times 10^{-2}
\]  

(4-20)

The possibility of utilizing this method in an environment other than the benign laboratory depends upon the open-loop azimuth gyro drift as well as uncertainty in angular measurements. These are the subjects of Chapter V.

4.3.2 Wheel Speed Modulation Data

Section 4.3.1 developed the wheel speed modulation technique used for the recording of these data. During the wheel speed modulation test, the system was operating under the wide bandwidth conditions and the parameters indicated in Table 4-1b.
Figure 4-5a is a graph of the azimuth variation of the gyrocompassing system during the eleven day wheel speed modulation test. It is noted that the system had a peak azimuth variation of ± six seconds of arc due to recompensation uncertainties. However, the wheel speed modulation east gyro recompensation procedure did remove the steady state azimuth angle variation previously noted. Figure 4-5b is a plot of the total east gyro compensation change applied as a result of the wheel speed modulation test. It is noted that this compensation has a slope of 0.007 meru per day, which is similar to the steady state east gyro performance previously predicted from the wide bandwidth system data of Figure 4-4. These data are redrawn in Figure 4-5c as a source of data comparison of the azimuth angle variation of the gyrocompassing system with and without wheel speed modulation.

It is noted that wheel speed modulation removed the steady state component of azimuth angle variation. However, the recompensation technique introduced more short-term variations in azimuth angle.

This was the first wheel speed modulation test attempted on a system basis, and it is believed that future tests can reduce the azimuth variations. These future tests will utilize a vertical sensor with a longer time constant, reducing the
2-16 GYROCOMPASS - EFFECT OF WHEEL SPEED MODULATION

AZIMUTH VARIATION USING WHEEL SPEED MODULATION

EAST GYRO COMPENSATION APPLIED USING WHEEL SPEED MODULATION

AZIMUTH VARIATION - NO WHEEL SPEED MODULATION

FIGURE: 4-5a

FIGURE: 4-5b

FIGURE: 4-5c
uncertainty in angular measurement. In addition, a computer will replace the engineers used for this initial test, with a resultant reduction in many uncertainties.

The initial wheel speed modulation test appears to demonstrate the capability of detecting low level torque variations in the east gyro of a gyrocompassing system operating in a benign laboratory.
V OPERATIONAL SYSTEM EVALUATION

In order to use the gyrocompassing system as a long-term reference, gyro drift must be detected and compensated for in an operational system. Chapter IV has presented data which indicate that the short-term stability of state-of-the-art gyros does not produce appreciable errors in the gyrocompassing system.* Therefore, the system can be used as an absolute reference over a time interval of a few days with no gyro compensation. However, since the proposed system's operational requirements could be extended over a period of several years, long-term gyro drift must be compensated for in the gyrocompassing system.

Compensation for gyro drift is primarily a problem of detecting the gyro drift by incremental measurements of small angles. In the gyrocompassing system operating in a benign environment, vertical sensing devices such as pendulums and electrolytic levels are capable of incremental angle measurements below 0.01 seconds of arc. The sensitivity of these vertical sensing devices to horizontal accelerations will increase the noise in the gyrocompassing system.

* If the system bandwidth is such that the steady state azimuth gyro drift - azimuth angle coupling is less than one-hundred seconds of arc per meru.
system when operating in a perturbed environment. This will increase the size of an angle necessary to produce an equivalent signal to noise ratio. A vertical sensing device which has a first order lag will be used to filter out the higher frequencies of lateral acceleration. If the time constant of the vertical sensing device is one-third that of the gyrocompassing system, a third order critically damped system closely approximates the system characteristics.** Typical gyrocompass time constants between 100 and 500 seconds will be investigated.***

Chapter V discusses the parameters to be considered in the development of a recompensation procedure in a realistic environment. The chapter provides insight into the problem, but a closed form solution is not obtained for two reasons, (1) sufficient data on the low frequency spectrum of horizontal accelerations is not available; (2) the system is the best environment analyzer available; optimization of the gyrocompassing system's performance in the perturbed environment provides the firmest solution to this problem.

5.1.0 North Gyro Compensation

Figure 5-1 is a simplified drawing of the north loop

---

* Chapter II develops this theory.
** Vertical sensors with time constants between 0.1 and 500 seconds are commercially available.
*** Throughout this entire paper we have assumed that the north gyro is used primarily for leveling purposes and, therefore, is of negligible importance in the analysis of the gyrocompassing properties of the system.
NORTH GYROCOMPASS LOOP

\[
\begin{align*}
\frac{1}{TvP + 1} & \quad \Rightarrow & \quad -K_{NN} \\
(u)W_x & \quad \Rightarrow & \quad \text{North Stabilization Servo} \\
\frac{1}{P} & \quad \Rightarrow & \quad A_x
\end{align*}
\]

FIGURE: 5-1
of the stable base gyrocompassing system. The figure assumes that the vertical component of earth rate coupled into the north gyro due to rotations about east is negligible. Since rotations about east will always be less than ten seconds of arc, \( K_{NN} \) can be adjusted so that the 0.03 meru of earth rate coupled into the north gyro at medium latitudes will not produce appreciable rotation about the north gyro input axis, \( A_x \).

Stability requirements on the north loop indicate that \( K_{NN} \) must be less than one-quarter \( \tau_v \) for the poles to be in the left half plane. North gyro recompenensation involves detecting a change in the output of the north vertical sensor and forcing this signal to zero by recompenensation the north gyro. With available instruments and instrumentation techniques, \( A_x \) can easily be kept below one-half second of arc. At middle latitudes, rotations about the north gyro input axis, \( A_x \), produce a nearly equal azimuth angle change of the stabilized member. Since \( A_x \) can be kept below one-half second of arc, the azimuth variations which are a result of north gyro drift are negligible.

5.2.0 East Azimuth Loop

Figure 5-2 is a simplified diagram of the east azimuth

* A sample data technique and a low sensitivity integrator could be used. The low sensitivity of the integrator does not affect the stability of the system.
OPEN/CLOSED LOOP
EAST AZIMUTH GYROCOMPASS

\[ a_i(P) \]

\[ \frac{I}{TVP+1} \]

\[ A_m \]

\[ K_{AE} \]

\[ -K_{EE} \]

\[ (u)W_z \]

\[ -V_z \]

\[ +V_y \]

\[ (u)W_y \]

\[ A_z \]

\[ A_y \]

*3rd Order Critically Damped Gyrocompass

\[ \tau = \text{Time Constant of Gyrocompass} \]

\[ \tau = 3\tau_V \]

\[ \tau = \frac{1}{K_{EE}} \]

\[ \tau = \frac{1}{\sqrt{3K_{AE}\omega(ie)h}} \]

\[ \frac{\text{meru}}{\text{sec. of arc}} \times 1.5 \times 10^{-2} = \text{sec. of arc} \]

\[ \frac{\text{sec. of time}}{\text{meru}} \]

\[ \omega(ie)h = 0.0034 \frac{\text{meru}}{\text{sec. of arc}} \]

Closed Loop Gyrocompass

\[ -V_z = \text{Signal applied to azimuth gyro torque generator in closed loop gyrocompass.} \]

\[ V_z = +(u)\omega_z \text{ meru} \]

\[ V_y = \text{Signal applied to east gyro torque generator in closed loop gyrocompass.} \]

\[ V_y = (u)\omega_z \frac{K_{EE}}{K_{AE}} = +3(1.5 \times 10^{-2})\tau\omega(ie)h(u)\omega_z \text{ meru} \]

* Relations developed in Section 2.3.1

FIGURE: 5-2
loops of the gyrocompassing system. Gyro drift can be detected by observing the output of the east vertical sensor, $A_m$, with switches $S_1$ and $S_2$ open. The accuracy of the gyro drift evaluation depends upon getting a clearly defined angle indication in the presence of acceleration induced angle errors. The sample technique permitting the system drift while a relatively clear indication is obtained, depends directly upon the other drifts and error effects occurring during the sample period. The data of Chapter IV indicate that the frequency of gyro drift is low compared to what will turn out to be a sample period.

Both east and azimuth gyro drift are evaluated by observing the output of the east vertical sensor, $A_m$, with switches $S_1$ and $S_2$ open. The angular velocity summation in the east gyro is always satisfied by inputs other than those derived from the east vertical sensor; therefore, when the switches are open, the ideal output of the east vertical sensor is only due to azimuth gyro drift. However, when the east gyro wheel speed is cut in half and the switches $S_1$ and $S_2$ are open, the output of the east vertical sensor is proportional to east as well as azimuth gyro drift. The optimization of gyro drift compensation involves choosing a system time constant and sampling interval so that in spite of limitations in angular measurements the uncertainty in gyro compensation has a minimum effect on the azimuth angle of the gyrocompassing system.
By writing a general expression for the output of the east vertical sensor, \( A_m \), after switches \( S_1 \) and \( S_2 \) are opened (Figure 5-2), the relative importance of the parameters will be developed. For this analysis, it is assumed that the gyro-compassing system has reached the steady state when the switches are opened. In addition, it is assumed that gyro drift does not change while the switches are open.* When switches \( S_1 \) and \( S_2 \) are opened, the east and azimuth gyro torque generators are shorted out so a step change in angular velocity equal to \( V_z \) is applied to the azimuth gyro and \(-V_y\) to the east gyro. Equation 5-2 describes the output of the east vertical sensor after switches \( S_1 \) and \( S_2 \) are opened and the signals applied to the east and azimuth gyro torque generators respectively are \(-\frac{V_y}{P}\) and \(\frac{V_z}{P}\).

\[
A_m(P) = \frac{a_1(P)}{(\tau_v P + 1)} + \frac{1.5 \times 10^{-2} V_y}{P^2(\tau_v P + 1)} + (1.5 \times 10^{-2})^2 \frac{V_z \omega(ie)h}{P^3(\tau_v P + 1)}
\]  

(5-2)

* Chapter IV indicates that the highest frequency of important gyro drift is in azimuth which has a period of approximately ten hours. Since the loops will be open for less than one-half hour, this is a good assumption.
Substituting $V_z = (u)\omega_z$, $V_y = (u)\omega_z \frac{K_{EE}}{K_{AE}} = 3(1.5 \times 10^{-2})\tau \omega_{(ie)h}(u)\omega_z$,

$$A_m(P) = \frac{a_1(P)}{(\tau_v P + 1)} + (1.5 \times 10^{-2})^2 \omega_{(ie)h}(u)\omega_z \left[ \frac{3\tau}{P^2(\tau_v P + 1)} \right]$$

$$+ \frac{1}{P^3(\tau_v P + 1)}$$

(5-3)

Transforming Equation 5-3 to the time domain and expressing the time constant in terms of the gyrocompass time constant, $\tau = 3\tau_v$,

$$A_m(t) = f(a_1, t, \tau_v) + (1.5 \times 10^{-2})^2 \omega_{(ie)h}(u)\omega_z \{ 3\tau[t + \frac{\tau}{3}(e^{\frac{3t}{\tau}} - 1)]$$

$$+ \frac{t^2}{2} - \frac{\tau t}{3} + \frac{\tau^2}{9}(1 - e^{\frac{3t}{\tau}}) \}$$

(5-4)

Substituting $K(t)\tau$ for $t$, Equation 5-5 is obtained.
Equation 5-5 indicates that the signal appearing at the output of the east vertical sensor is composed of the component at the east gyro torque generator at \( t = 0 \) and a component at the azimuth gyro torque generator at \( t = 0 \). Assuming the average \( a_i(t) = 0 \), Equation 5-5 is plotted in Figure 5-3.

Three points should be noted from Equation 5-5 and Figure 5-3:

1. after the gyrocompass loops are opened, the output of the east vertical sensor is ideally proportional to uncompensated azimuth gyro drift;
2. the amplitude of the output of the east vertical sensor varies as the square of the time constant of the gyrocompassing system for the time ordinate which varies linearly (for a given uncompensated azimuth gyro drift, narrower bandwidth system produces a larger output from
OPEN LOOP EAST VERTICAL SENSOR OUTPUT FOR INCORRECT AZIMUTH GYRO COMPENSATION
the east vertical sensor in an equal time interval); (3) the relative shape of the contribution of the east and azimuth loops to the output of the east vertical sensor is independent of the time constant of the gyrocompassing system (for reasons which will become clear later, $\frac{t}{\tau} = 3.3$ will be chosen to evaluate the output of the east vertical sensor).

5.2.1 Azimuth Gyro Drift Compensation

From Section 5.2.0, it is noted that after the gyrocompass loops are opened, the output of the east vertical sensor is proportional to the uncompensated azimuth gyro drift, $(u)\omega_z$. From Figure 5-3, the output of the east vertical sensor at $t = 3.3\tau$ is equal to

$$A_m = (u)\omega_z \tau^2 (3.5 \times 10^{-6} + 7.0 \times 10^{-6})$$

(5-6)

The uncertainty in azimuth gyro drift detection is found from Equation 5-6 to be

$$(u)\omega_z \text{ (measurement uncertainty)} \approx \frac{A_m}{\tau^2} \text{ (minimum detectable)} \times 10^5$$

(5-7)
For $\frac{t}{T} = 3.3$, Equation 5-7 defines the relation between the uncertainty in azimuth gyro drift in meru, minimum detectable angle in seconds of arc, and the time constant of the gyrocompass.

As a realistic example, a vertical sensor with a 110 second lag is assumed. For a critically damped system, this requires a gyrocompassing system time constant of 330 seconds. Substituting the value into Equation 5-7 and remembering that the gyrocompass loops are open for $3.3 \frac{t}{T}$, the following relations are apparent.

- Time to evaluate azimuth gyro drift: 16.6 minutes
- East Vertical Sensor Output
  - Azimuth Gyro Drift: 1 second of arc
  - Change in Azimuth Angle: 15 seconds of arc

With the system operating in a benign environment, 0.01 seconds of arc is easily detected at the output of the east vertical sensor. This defines the vertical sensor's capability. When the system is transferred to a perturbed environment, it is assumed that noise introduced by lateral accelerations will change this angle to close to 0.1 seconds of arc, averaged over the total time the gyrocompass loops were opened. *

* Appendix II presents the results of a spectrum analysis of gyrocompass error angles which indicate that detecting 0.1 seconds of arc output from the vertical sensor is in agreement with less than a five second of arc mean square azimuth angle error.
This would produce an uncertainty of 0.1 meru in the azimuth gyro compensation and a 1.5 seconds of arc azimuth error while the gyrocompass loops were opened.

Sample data techniques and a low gain integrator can be used to bypass $K_{AE}$ and recompensate the azimuth gyro on the basis of its open loop drift rate. There are many variations on the method used to detect drift in the azimuth gyro and update its compensation. The three most important things to consider when determining a recompensation procedure are simplicity of implementation, accuracy, and speed of convergence. Although these subjects are extremely interesting and tempting, they are beyond the scope of this paper. This chapter illustrates a method which could be used to detect gyro drift and leaves the resultant recompensation of the gyros as a subject for future discussions.

Before we proceed to the detection of east gyro drift, the effect of 0.1 meru azimuth gyro uncertainty on steady state azimuth angle in the closed loop gyrocompass must be considered. The coupling between azimuth gyro drift and azimuth angle is $\frac{K_{EE}}{K_{AE}} \frac{(u)\omega_z}{\omega_{(ie)h}}$. From Figure 5-2, this is equal to $4.5 \times 10^{-2} \tau (u)\omega_z$. For a system time constant of 330 seconds, 0.1 meru uncompensated azimuth gyro drift would produce a steady state 1.5 seconds of arc azimuth angle error.
5.2.2 **East Gyro Drift Compensation**

In the steady state closed loop gyrocompass, the angular velocity summation performed by the east gyro is always satisfied. East gyro drift is counteracted via a change in the azimuth orientation of the stabilized member coupling a new horizontal component of earth rate to the east gyro input axis. Wheel speed modulation reduces the sensitivity of the east gyro to this new component of earth rate. If the gyrocompass loops are opened and the east gyro wheel speed cut in half, the east gyro drift rate is equal to the change in earth rate applied to the east gyro input axis. Under the aforementioned conditions, observing the output of the east vertical sensor yields data on the change in performance of the east gyro. Updating the east gyro compensation on the basis of these data and closing the gyrocompass loops should return the stabilized member to its original azimuth orientation.

Sections 3.3.0, 4.3.1, and 4.3.2 have developed a laboratory optimization of the wheel speed modulation technique as well as a method which was used to determine the effectiveness of wheel speed modulation on a system operating in a benign environment. These sections show that when the azimuth angle does not change and the east gyro wheel speed is cut in half, the east gyro drift rate is equal to the change in the residual term inside the east gyro.

\[
(u)\omega_E^{(h\frac{\Delta R}{H})} = \frac{\Delta R}{H} = \Delta A_z^{(ie)h} \omega^{(ie)h}
\]
In this section we shall see how well $\frac{\Delta R}{H}$ can be determined in an environment where 0.1 seconds of arc is the minimum detectable angle. The following sequence illustrates the procedure for evaluating the east gyro drift rate by opening switches $S_1$ and $S_2$ (Figure 5-2) and cutting the east gyro wheel speed in half.

Step #1
 Disconnect the east vertical sensor azimuth gyro coupling (open switch $S_1$, Figure 5-2).

Step #2
 Increase the east vertical sensor east gyro coupling, $K_{EE}$, and reduce the east gyro wheel speed to half its normal value. (Less than three minutes are necessary for this transient to settle.)

Step #3
 Open the east vertical sensor east gyro coupling, $S_2$ (Figure 5-2), and record the output of the east vertical sensor. During the maximum three minutes required to reduce the east gyro wheel speed, the azimuth gyro has been the azimuth reference. Drift of the azimuth gyro has changed the horizontal component of earth rate applied to the east gyro. The loops will be opened for over fifteen minutes

* Reducing the east gyro wheel speed couples an inertia reaction torque to the gyro output axis. To minimize the effects of this transient, the east vertical sensor east gyro coupling is increased.
during which time the azimuth drift produces an output of the east vertical sensor as indicated in Section 5.2.1. Because the gyrocompass loops are opened for over fifteen minutes time, the three minutes necessary to change east gyro wheel speed can be neglected, simplifying the calculations considerably and introducing less than a ten percent error.

Under the aforementioned conditions, Equation 5-9 describes the output of the east vertical sensor after $S_1$ and $S_2$ are opened and the east gyro is operating at half wheel speed.

$$A_m(P) = \frac{a_1(P)}{(\tau_P P + 1)} + (1.5 \times 10^{-2})^2 \omega_{(ie)h} (u)\omega_z$$

$$[\frac{3 \tau}{P^2(\tau_P P + 1)} + \frac{1}{P^3(\tau_P P + 1)}] + \frac{1.5 \times 10^{-2} (u)\omega_y}{P^2(\tau_P P + 1)}$$

(5-9)

This equation is exactly the same as Equation 5-3 which was used to evaluate the azimuth gyro drift except for a term proportional to the east gyro drift, $(u)\omega_y$. The term proportional to $(u)\omega_y$ is included here because the east gyro is operating at half its normal wheel speed. We want to determine $(u)\omega_y$ from the output of the east verti-
cal sensor described by Equation 5-9.

Assuming that the average interfering acceleration term is equal to zero, the part of Equation 5-9 proportional to the azimuth gyro drift has been previously plotted in Figure 5-3. The last term of Equation 5-9 which is proportional to east gyro drift at the reduced wheel speed is plotted in Figure 5-4.

It should be noted (Figures 5-3 and 5-4) that the time constant of the gyrocompassing system is the only system parameter used to evaluate the east and azimuth gyro open loop drifts.

In order to show how these figures should be used, the example of Section 5.2.1 is continued. In this example, azimuth gyro drift was detected and recompensated for by opening the 330 second gyrocompass loops and observing the output of the east vertical sensor. It was noted that if 0.1 seconds of arc is the minimum detectable angle, there would be an uncertainty of less than ± 0.1 meru of azimuth gyro compensation. Under these conditions, the azimuth gyro drift would produce a maximum of 0.1 seconds of arc output of the east vertical sensor in 16.5 minutes of time. If the recompensation of the azimuth gyro is immediately followed by a test to determine the necessity of recompensating the east gyro, it is assumed that azimuth gyro drift does not vary appreciably during this time interval.* Therefore,

* This is a good assumption because the highest period of measurable azimuth gyro drift was over ten hours (Chapter IV).
OPEN LOOP EAST VERTICAL SENSOR OUTPUT
EAST GYRO DRIFT (\(\frac{1}{2}\) NORMAL WHEEL SPEED)

FIGURE: 5-4
0.1 seconds of arc is the maximum output of the east vertical sensor due to azimuth gyro drift during the time interval required to evaluate the east gyro.

From Figure 5-4, it is noted that the output of the east vertical sensor produced by the east gyro drift during $3.3\frac{t}{\tau}$ is equal to

$$A_m \text{ (east gyro drift)} = 15 \text{ seconds of arc}$$

During this test the total output of the east vertical sensor is assumed to be produced by the east gyro drift. There is an uncertainty of 0.1 seconds of arc angle measurement and an uncertainty of 0.1 seconds of arc output from the east vertical sensor due to azimuth gyro drift. Therefore, the rms uncertainty in the output of the east vertical sensor, assuming it is all a result of the east gyro drift, is equal to

$$A_m \text{ (uncertainty when evaluating the east gyro drift)} = 0.14 \text{ seconds of arc}$$
Combining Equations 5-8 and 5-9, the uncertainty in the east gyro drift measurement is obtained.

\[(u)\omega_y \text{ (uncertainty in measurement)} = 0.01 \text{ meru}\]

(5-10)

At 45 degrees latitude, this corresponds to a steady state azimuth error of 2.9 seconds of arc.

Step #4
After the gyrocompass loops have been opened for 16.5 minutes of time, the east gyro is compensated on the basis of the angle output of the east vertical sensor (Equation 5-8). (The uncertainty in this compensation is indicated by Equation 5-10.)

Step #5
A tight east leveling loop is closed while the east gyro wheel speed is returned to its normal value.

Step #6
After waiting approximately four minutes for all transients to settle, the gyrocompass loops are closed.

During this entire procedure the azimuth gyro has been the azimuth reference. The compensation procedure relies firmly on an open loop
azimuth gyro drift rate of less than 0.1 meru. Using this number, it is noted that a total azimuth angle of less than 2.2 seconds of arc developed during the 23.5 minutes the gyro-compass loops were opened for east gyro recompensation.

5.3.0 Compensation Sequence

Before a compensation sequence can be developed, data must be available on the spectrum of instrument uncertainty and the accuracy of the recompensation process. Representative data on spectrum of azimuth and east gyro uncertainties has been presented in Chapter IV. These data indicate that the east gyro has, essentially, a maximum steady state uncertainty of 0.01 meru per day. Since 0.01 meru corresponds to three seconds of arc azimuth error at the medium latitudes, the east gyro could be recompensated every twelve hours with very little effect on the accuracy of the gyrocompassing system. The azimuth gyro has an uncertainty of 0.2 meru periodic with a period of approximately ten hours. For the 330 second system, 0.2 meru azimuth gyro uncertainty corresponds to three seconds of arc azimuth error. But, the east gyro recompensation procedure requires an azimuth gyro uncertainty of less than 0.1 meru.

The accuracy of the recompensation process is difficult to define because of the large number of variables involved. From the expected recompensation accuracies of Section 5.2.1 and Section 5.2.2, the
rms azimuth error due to east and azimuth gyro uncertainty is

\[
(u)A_z = \sqrt{\frac{K_{EE}}{K_{AE}} \left( \frac{(u)\omega_y}{\omega_{(ie)}h} \right)^2 + \left( \frac{(u)\omega_y}{\omega_{(ie)}h} \right)^2} = \sqrt{(1.5)^2 + (2.8)^2} = 3.2 \text{ seconds of arc}
\]

(5-11)

Gyro drift is the major source of azimuth error in the stable base gyrocompassing system. Although many simplifying assumptions have been employed in the derivation of the expected rms azimuth uncertainty, it is representative of the performance which would result with azimuth gyro uncertainties of 0.1 meru and east gyro uncertainties of 0.01 meru. If properly executed, the compensation procedure should keep the uncertainty in gyro drift below the aforementioned value over the maximum period of time. It is important to have the system operating with sufficient accuracy that it may be transferred from the gyrocompass to the gyro stabilized mode of operation over the maximum time interval. It was noted in the previous sections that large azimuth angle errors did not result when the gyrocompass loops were opened and the output of the east vertical sensor monitored to detect changes in the performance of the azimuth and east gyros. Therefore, gyro recompensation can be a continual process because
it does not affect the operational readiness of the system.

Armed with the data of Chapter IV and the compensation procedures previously developed, the following compensation sequence is suggested. Each time the gyrocompass loops are opened, the sample data system averages the output of the east vertical sensor and, at the end of the indicated time interval, applies a compensation to the azimuth or east gyro. The sample data technique applies azimuth gyro compensation in 0.1 meru steps, east gyro compensation in 0.0066 meru steps.
# Compensation Sequence

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00:00:00</td>
<td>Open east-azimuth gyrocompass loops evaluate ( (u)\omega_z ).</td>
</tr>
<tr>
<td></td>
<td>00:16:30</td>
<td>Record ( A_m ) and compensate azimuth gyro (0.1 seconds of arc = 0.1 meru). Close gyrocompass loops.</td>
</tr>
<tr>
<td>2 to 11</td>
<td>01:00:00</td>
<td>Repeat of Test #1 at one hour intervals. If the azimuth gyro drift in Test #11 is ( \leq 0.1 ) meru, go on to Test #12. If the azimuth gyro drift is ( &gt; 0.1 ) meru, repeat Test #11 until azimuth gyro drift is ( \leq 0.1 ) meru.</td>
</tr>
<tr>
<td>12</td>
<td>11:00:00</td>
<td>Open east azimuth loop. Tighten east vertical sensor east gyro loop. Reduce east gyro wheel speed to one-half normal value.</td>
</tr>
<tr>
<td></td>
<td>11:03:00</td>
<td>Open east vertical sensor east gyro loop.</td>
</tr>
<tr>
<td></td>
<td>11:19:30</td>
<td>Record ( A_m ) and compensate east gyro (0.1 seconds of arc = 0.0066 meru).</td>
</tr>
<tr>
<td></td>
<td>11:20:00</td>
<td>Close tight east vertical sensor east gyro loop. Change east gyro wheel speed to normal.</td>
</tr>
<tr>
<td></td>
<td>11:24:00</td>
<td>Close normal gyrocompass loops</td>
</tr>
<tr>
<td>13</td>
<td>12:00:00</td>
<td>Repeat Test #1, etc.</td>
</tr>
</tbody>
</table>
One year ago a preliminary analysis of the arduous problems involved in stable base gyrocompassing with a three-gyro system indicated that present instruments, systems, and instrumentation could not perform the job. First, the combined short-term uncertainty of all instruments operating in the system environment must be small enough to produce acceptable dynamic errors in the gyrocompassing system. The only method available to reduce the effects of short-term instrument uncertainties is to change the coupling between the various instruments and gyrocompassing angles by decreasing the bandwidth of the system. But, the narrower the bandwidth of the system, the more susceptible it is to steady state changes in instrument performance. The second deterrent to attempting high accuracy stable base gyrocompassing was the minute incremental angles which must be consistently detected in the gyrocompassing system to recompensate the primary long-term references, gyros. Because of friction and angle transducer nonlinearities, considerable doubt existed regarding angular measurements below 0.1 seconds of arc. The third formidable problem was recompensation

* Short-term: all time up to approximately five time constants of the gyrocompassing system.
of the east gyro. Regardless of the gyrocompassing method employed (east-west averaging, gyro wheel reversal, wheel speed modulation, etc.), 0.01 muer, approximately 0.0015 dyne-centimeters, is of paramount importance in the evaluation of the east gyro. Because of recent preceding tests performed on a single component, this paper was interested in the repeatability of the wheel speed modulation technique in a system environment. Although gyros are temperature sensitive and gyro wheel hysteresis nonlinearities exist, the gyro wheel speed must be consistently changed from 12,000 to 6,000 rpm, introducing less than 0.0015 dyne-centimeters of uncertainty in the torque summation.

Laboratory testing of a prototype gyrocompassing system has produced data on system performance in a benign environment which removes many of the previous seemingly insurmountable problems. These tests produced data on instrument uncertainties, system parameters, and over-all system performance. A flexible test program, coupled with realistic evaluations of proposed initial system characteristics, can remove the previously assumed artificial limits on the performance of state-of-the-art instruments and systems. Many of the answers to future system designs can be obtained only from a combined analytical design and laboratory analysis of prototype models.

Four conclusions can be derived from the experimental data and
1. Superior gyro performance (Chapter IV) in the 2-16 gyrocompass is the primary reason that stable base gyrocompassing is currently realizable. Two reasons for the improved performance are, (1) the 2FBG-6F single-degree-of-freedom integrating gyro design is superior to any previous gyro design in its class; (2) the 2-16 system provides the proper temperature control, power supplies, and supporting circuitry to derive the optimum performance from its instruments.

What appeared to be an east gyro torque uncertainty in the 2-16 gyrocompass was a ramp with a slope of less than 0.0013 dyne-centimeters per day. The azimuth gyro torque uncertainty was approximately periodic with a period of ten hours and an amplitude of 0.026 dyne-centimeters.

2. Two-hundred and sixty-eight hours of data indicate wheel speed modulation to be a usable technique in the 2-16 gyrocompass. This initial test on the feasibility of wheel speed modulation (Chapter IV) produced encouraging results; future tests over a longer time duration employing refined procedures are necessary before any firm commitments can be made on the operational capability of the technique.

3. The transferring of the stable base gyrocompassing system
from the benign environment to a more realistic surrounding is a major change. However, a preliminary analysis (Chapter V) indicates that the system has considerable capability to permit its being adapted to a more realistic environment.

4. Absolute instrument alignment and calibration can be accomplished with available instrumentation techniques so that reasonable latitude relocations do not affect the absolute accuracy of the gyrocompassing system.

Data recorded in the benign environment indicate that the gyrocompassing system is capable of being adapted to an operational environment with acceptable degradation in performance. The preliminary concepts tested in the laboratory also indicate the possibility of extending the accuracy of the self-azimuthing guidance system over an indefinite period of time. Until the system is actually tested in a realistic environment, it is dangerous to extrapolate these data, yet, unrealistic to ignore the data and possibilities it implies.
NOMENCLATURE

\(x_g\) ................. Geographic North

\(y_g\) .................. Geographic East

\(z_g\) .................. Direction of Gravity

\(\omega_x\) ................ Angular Velocity into North Gyro

\((u)\omega_x\) ............. North Gyro Drift

\(\omega_y\) ................ Angular Velocity into East Gyro

\((u)\omega_y\) ............. East Gyro Drift

\(\omega_z\) ................ Angular Velocity into Azimuth Gyro

\((u)\omega_x\) ............. Azimuth Gyro Drift

\(A_x\) .................. Rotation About x Gyro Input Axis

\(A_y\) .................. Rotation About y Gyro Input Axis

\(A_z\) .................. Rotation About z Gyro Input Axis

\((u)A_y\) ................. Uncertainty in East Angle
**NOMENCLATURE**

\( \omega_{(ie)h} \) ............... Horizontal Component of Earth Rate

\( \omega_{(ie)v} \) ............... Vertical Component of Earth Rate

\( g \) ............... Magnitude of Gravity,

\( S_{xv} \) ............... Sensitivity of North Vertical Sensor volts/g

\( S_{yv} \) ............... Sensitivity of East Vertical Sensor volts/g

\( K_{EE} \) ............... East Vertical Sensor - East Gyro Coupling, meru/arc second

\( K_{AE} \) ............... East Vertical Sensor - Azimuth Gyro Coupling, meru/arc second

\( \tau_v \) ............... Vertical Sensor Time Constant

\( \tau_2 \) ............... Characteristic Time of Second Order Gyrocompass

\( \tau_3 \) ............... Characteristic Time of Third Order Gyrocompass

\( A_{(h-y)} \) ............... Angle Between the Horizontal Plane and East Gyro Input Axis
NOMENCLATURE

\( A_{(y,-y)} \) .......... Angle Between the Horizontal Plane and East Gyro Input Axis

\( A_{(-y,-y)} \) .......... Angle Between Geographic West and East Gyro Input Axis

\( i_c \) .................. East Gyro Torquing (Compensation) Command

\( S_{TG} \) ................. Torque Generator Sensitivity meru/ampere'

\( H \) ...................... Full Speed Angular Momenta of Gyro Wheel

\( R \) ...................... All non-acceleration Dependent Torques in the East Gyro

\( A_{(y,-m)} \) ............... Angle Between Geographic East and Mirror Normal

\( A_{(-y,-m)} \) ............... Angle Between Geographic West and Mirror Normal

\( A_{(m,y)} \) ................. Angle Between Mirror Normal and East Gyro Input Axis

\( (u)A_{(h,-y)} \) ............... Uncertainty in Measurement of \( A_{(h,y)} \)

\( (u)R \) ...................... Uncertainty in Measurement of \( R \)
NOMENCLATURE

\( (u)A(y - y) \) Uncertainty in Measurement of \( A(y - y) \)

\( \Delta R \) Change in \( R \)

\( \Delta i_c \) Change in \( i_c \)

\( \Delta S_{TG} \) Change in Torque Generator Sensitivity

\( \omega_m \) Modulation Induced Drift Rate

\( \omega_{my} \) East Gyro Drift Rate of Normal Wheel Speed.

\( \omega_{(ie) \cos \text{Lat}} \) Horizontal Component of Earth Rate

\( \omega_{(ie) \sin \text{Lat}} \) Vertical Component of Earth Rate

\( \omega_{i-f(IA)} \) Input Axis Angular Velocity of Gyro Float With Respect to Inertial Space

\( I_{g(OA)} \) Output Axis Inertia of Gyro Float With Respect to Case

\( A_{c-f(OA)} \) Output Axis Angle of Gyro Float With Respect to Case

\( C_{d(OA)} \) Output Axis Damping of Gyro Float With Respect to Case
NOMENCLATURE

\( (u)M(OA) \) ............ Output Axis Torque Uncertainty

\( \omega_{i-c}(IA) \) ........ Input Axis Angular Velocity of Gyro Case With Respect to Inertial Space

\( \omega_{c-f}(IA) \) ........ Input Axis Angular Velocity of Gyro Float With Respect to Case

\( \omega_{T} \) .............. Table Rate

\( A_{T} \) ................. Table Angle

\( \omega_{o} \) ............... Sinusoidal Frequency of Table Angle

\( B \) ..................... Amplitude of Table Angle

\( \Sigma M(IA) \) ........... Input Axis Total Moments of Gyro Float With Respect to Case

\( I_{g}(IA) \) ............. Input Axis Inertia of Gyro Float With Respect to Case

\( A_{c-f}(IA) \) ............ Input Axis Angle of Gyro Float With Respect to Case

\( C_{d}(IA) \) .............. Input Axis Damping of Gyro Float With Respect to Case

\( K_{c-f}(IA) \) ............ Input Axis Spring Constant of Gyro Float With Respect to Case
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{zo}$</td>
<td>Initial Azimuth Angle of Gyrocompassing System</td>
</tr>
<tr>
<td>$R_o$</td>
<td>Initial East Gyro Residual Term</td>
</tr>
<tr>
<td>$\Delta A_{z}$</td>
<td>Change in Azimuth Angle of Gyrocompassing System</td>
</tr>
<tr>
<td>$A_m$</td>
<td>Output of East Vertical Sensor</td>
</tr>
<tr>
<td>$V_z$</td>
<td>Z Gyro Angular Velocity Command Proportional to Uncompensated Azimuth Gyro Drift</td>
</tr>
<tr>
<td>$V_y$</td>
<td>Y Gyro Angular Velocity Command Proportional to Uncompensated Azimuth Gyro Drift</td>
</tr>
<tr>
<td>$a_i(P)$</td>
<td>Interferring Acceleration</td>
</tr>
</tbody>
</table>
APPENDIX II
DECLASSIFIED

SPECTRA OF GYROCOMPASS ANGLES

When the stable base gyrocompass is transferred from the benign laboratory to an operational environment, questions of angle uncertainties produced primarily by lateral accelerations arise. In the course of this paper, wind gust acceleration spectra were assumed to be acting upon the two-axis stable base gyrocompass of Figure 5-2. The results of this analysis indicate that the relative orders of magnitude of mean square value gyrocompass error angles assumed in this paper are realizable.

The spectra of stable base acceleration frequencies are poorly defined because of the wide variations in driving functions and base dynamics. In order to derive the relative orders of gyrocompass error angles, wind power density acceleration spectra was assumed to be impressed on the stable base gyrocompassing system, which is mounted in a structure of uniform unit transmissibility over all frequencies. Reference (6) provides a source of experimental data on the power density spectra of wind velocity which may be written in terms of acceleration spectra for our case.

\[ \Omega(S) = \frac{\gamma \beta^2 S^2}{\pi (-S^2 + \gamma^2)} \] (II-1)
\( \beta^2 \) is the amplitude of the wind autocorrelation function, and \( \gamma \) is the average frequency of wind occurrence.

This function was used in conjunction with the system transfer function (Figure 5-2) and Parceval's Theorem to derive the mean square values of the following gyrocompass error angles: (1) \( A_m \), output of east vertical sensor, (2) \( A_y \), east angle, (3) \( A_z \), azimuth angle. From this analysis the mean square value of these gyrocompass error angles were found to be

\[
\lambda_{AA}(0) = \frac{\beta^2 (K_{AE})^2}{\tau_v} - \frac{K_{AE} \omega (ie) h \tau_v \gamma}{K_{EE}} - K_{EE} + \gamma
\]  \hspace{1cm} (II-2)

\[
\lambda_{YY}(0) = \frac{\beta^2 K_{EE}}{\tau_v} \left[ \frac{K_{EE} + \left( \frac{K_{AE} \omega (ie) h}{K_{EE}} \right)^2 \tau_v}{K_{EE} \omega (ie) h \tau_v \gamma} - K_{EE} + \gamma \right]
\]  \hspace{1cm} (II-3)
These equations all have the same denominator function due to feedback in the gyrocompassing system. The mean square value of azimuth error angle, $\lambda_{AA}(0)$, is only proportional to the value of the east azimuth coupling squared, $(K_{AE})^2$. The mean square value of the east error angle, $\lambda_{YY}(0)$, is composed of two terms. One is proportional to east vertical sensor east gyro coupling squared, $(K_{EE})^2$, and the other is proportional to the coupling via the azimuth gyro and horizontal earth rate squared. This second term can be neglected for most ratios of $K_{AE}$ to $K_{EE}$.

The mean square output of the east vertical sensor, $\lambda_{MM}(0)$, is produced by the transfer function of the first order lag vertical sensor plus a feedback via the east gyro loop. The term proportional to a feedback via the azimuth gyrocompass loop is negligible in magnitude and has been disregarded.

For typical wind gusts, $0.4 \text{ radians/second} < \gamma < 3 \text{ radians/second}$.
Therefore, $\gamma >> K_{EE}$. Also, submitting the critically damped third order gyrocompass relations from Equation 2-7 and assuming that the system is operating at a medium latitude, the equations reduce to

\[
\lambda_{AA}(0) = \frac{3}{2} \times 10^8 \frac{\beta^2}{\gamma} \left(\frac{1}{\tau}\right)^5 \tag{II-5}
\]

\[
\lambda_{YY}(0) = \frac{10}{3} \frac{\beta^2}{\gamma} \left(\frac{1}{\tau}\right)^3 \tag{II-6}
\]

\[
\lambda_{MM}(0) = 9 \beta^2 \left(\frac{1}{\tau}\right)^2 \tag{II-7}
\]

Equations II-5, II-6, and II-7 express the mean square value of the gyrocompass error angles as a function of the time constant of the third order critically damped gyrocompass. It should be noted that the assumption that $\gamma >> K_{EE}$ has reduced Equation II-4 to Equation II-7 by neglecting the feedback to the east vertical sensor via the east loop and is, therefore, applicable when the gyrocompass loops are opened in Figure 5-2.

* For $100 < \tau < 500$, $K_{EE} < < 1 \text{ second}^{-1}$
Equations II-5, II-6, and II-7 are generally may be used for any third order critically damped gyrocompass. The mean square value of these error angles can be evaluated from a knowledge of the amplitude of the wind autocorrelation function, \( \frac{\beta^2}{2} \), and average frequency of wind occurrence, \( \gamma \). We do not know \( \frac{\beta^2}{2} \) or \( \gamma \), and they can only be determined experimentally. However, for the system time constants discussed in this paper, 100 seconds < \( \tau \) < 500 seconds. The relative orders of magnitude of these error angles turn out to be

\[
\begin{align*}
\tau = 100 & \\
\frac{3}{2} \times 10^{-2} \frac{\beta^2}{\gamma} & < \lambda_{AA}(0) < \frac{1}{3} \times 10^{-5} \frac{\beta^2}{\gamma} \\
\frac{1}{3} \times 10^{-6} \frac{\beta^2}{\gamma} & < \lambda_{YY}(0) < \frac{5}{2} \times 10^{-8} \frac{\beta^2}{\gamma} \\
9 \times 10^{-4} \beta^2 & < \lambda_{MM}(0) < \frac{9}{25} \times 10^{-4} \beta^2
\end{align*}
\]

The important point to be noted from this analysis is that the mean square value of azimuth angle and the output of the east
vertical sensor appear to be consistent with those values assumed throughout this paper. $\lambda_{YY}(0)$ is so small compared to the other angles that it is below our region of interest. In the course of this paper a design criteria has been rms azimuth variations of five seconds of arc. A compensation procedure was developed by assuming that we could detect an average output of the east vertical sensor of 0.1 seconds of arc. Since $\lambda_{MM}(0)$ is between one and two orders of magnitude below $\lambda_{AA}(0)$, this analysis indicates that reasonable relative values have been assumed for these angles.

2. Reed, T. E., *Wheel Speed Modulation*, (Unclassified Title), (Massachusetts Institute of Technology, Instrumentation Laboratory, Cambridge, Massachusetts, August, 1962). CONFIDENTIAL


