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The Facts of Light

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ABSTRACT

This is a random collection of facts about radiant and luminous energy. Some of this information may be useful in the design of photo-diode image sensors, in the set-up of lighting for television microscopes and the understanding of the characteristics of photographic image output devices. A definition of the units of measurement and the properties of lambertian surfaces is included.

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TABLE OF CONTENTS

SI UNIT DEFINITIONS (Systeme International):

Candela	luminous intensity of 1/600,000 m^2 of black-body at temperature of freezing platinum (about 2045°K).
Lumen	unit of luminous flux. A point-source of 1 candela (radiating uniformly in all directions) radiates 4π lumen.
Lux	unit of illumination. Equal to 1 lumen/m^2.

Candela is the basic unit, lumen and lux are derived units.

CALCULATING LUMINOUS FLUX

Luminous flux (F) in lumen = $685 \int V(\lambda)\, f(\lambda)\, d\lambda$

Radiant Flux (P) in watts = $\int f(\lambda)\, d\lambda$

Luminous efficiency = F/P lumen/watt (radiated)

"Overall" luminous efficiency = F/P_c lumen/watt (consumed)

$V(\lambda)$ - power per unit wave-length (in watts)

$f(\lambda)$ - C.I.E. Standard Observer (Photopic) Curve

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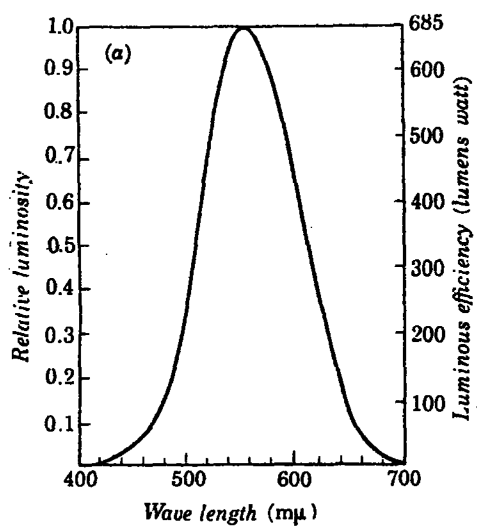


FIG. 13-2. Relative luminosity curve for the standard observer. Scale at left, relative luminosity; scale at right, luminous efficiency.

* From OPTICS, F. W. Sears, Addison-Wesley, 1958

UNITS FOR RADIANT ENERGY:

FLUX	Radiant flux	Watts	P
FLUX DENSITY (arriving)	Irradiance	Watts/m^2	H
FLUX DENSITY (departing)	Radiant emittance	Watts/m^2	W
INTENSITY	Radiant intensity	Watts/steradian	
"BRIGHTNESS"	Radiance	Watts/steradian/(projected)m^2	

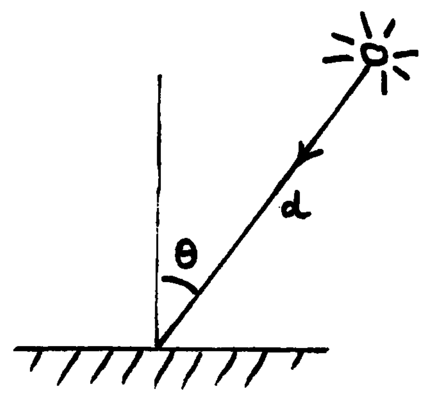
UNITS FOR LUMINOUS ENERGY:

FLUX	Luminous flux	Lumen	F
FLUX DENSITY (arriving)	Illuminance (Illumination)	Lumen/m^2 = Lux	E
FLUX DENSITY (departing)	Luminous emittance	Lumen/m^2	L
INTENSITY	Luminous intensity	Lumen/steradian = Candela	I
"BRIGHTNESS"	Luminance	Lumen/steradian/(projected)m^2 = Candela/(projected) m^2	B

POINT-SOURCE (Radiating Uniformly in all Directions):

$$F = 4\pi I$$

where F is the total flux emitted in lumen and I is the intensity in candelas.



$$E = \frac{I \cos(\theta)}{d^2}$$

where E is the illumance of the surface in lux, d is the distance to the source and θ the incident angle.

LAMBERTIAN SURFACE (Diffusing Perfectly):

$$B = E/\pi \quad \text{(independent of direction)}$$

where B is the luminance in candelas/m^2, while E is the illuminance in lux.

$$L = \pi B = E$$

where L is the luminous emittance in lux.

IMAGING SYSTEM (Ignoring Light-Losses in Lens):

$$E_i = \frac{\pi}{4} \frac{1}{N_e^2} B_o$$

where E_i is the image illuminance in lux and B_o is the object luminance in candelas/m^2. N_e is the effective f-number and is defined by:

$$N_e = N_o(1 + M) = (f/d)(1 + M)$$

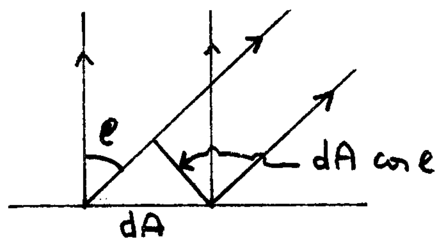
Here N_o is the nominal f-number, f the focal length of the lens and d the diameter (of the entrance pupil). The magnificiation M is the ratio of the linear size of an image to the linear size of the corresponding object oriented at right angles to the optical axis. For normal photographic practice, M is small and $N_e \approx N_o = (f/d)$.

If the object surface is lambertian we have:

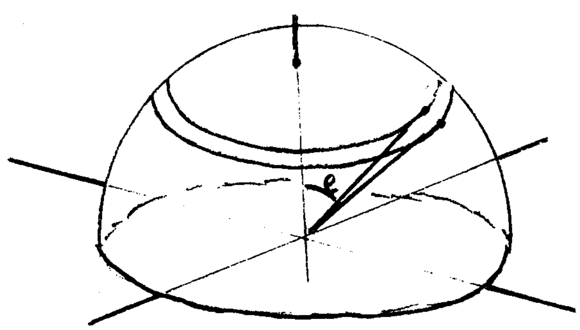
$$E_i = \frac{1}{4} \frac{1}{N_e^2} L_o = \frac{1}{4} \frac{1}{N_e^2} E_o$$

LIGHT EMITTED FROM A LAMBERTIAN SURFACE:

A lambertian surface looks equally bright from all viewpoints. It follows that its luminance B (candelas/m^2) is independent of the direction. What about the flux emitted per unit solid angle per unit surface area? Since luminance is flux emitted per unit solid angle per unit projected area, we need to compensate for foreshortening in calculating this quantity.



Evidently the flux per unit solid angle per unit surface area must be B cos(e). We are now ready to calculate how much is emitted into the cone $0 \leq e \leq e_o$. This is a useful quantity to know since it will allow us to calculate the flux entering a camera's lens, for example.



The strip shown on the unit hemi-sphere has radius sin(e) and width de. The integral we are looking for then becomes:

$$\int_o^{e_o} B\ (2\pi)\ \sin(e)\ \cos(e)\ de = \pi B\ \sin^2 e_o$$

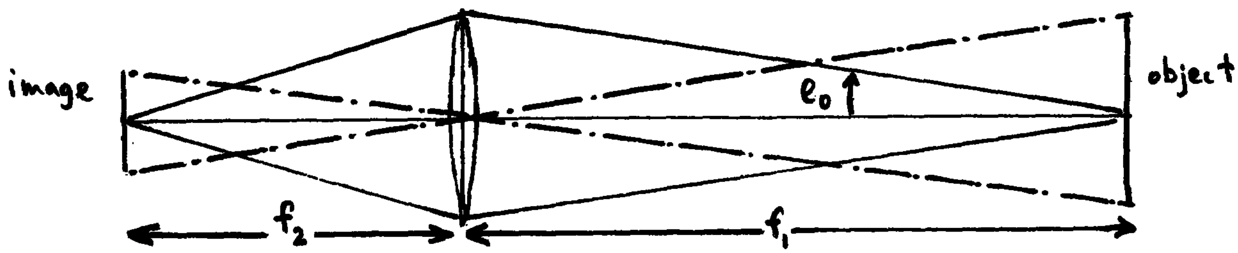
The total amount of light emitted per unit surface area then, the luminous emittance L (lux) is πB (setting $e_o = \pi/2$). If no light is absorbed by the surface, this must also equal the illuminance E (lux). So we then have $B = E/\pi$, for a perfect lambertian surface.

CALCULATING IMAGE ILLUMINANCE:

If the (linear) image magnification is M, then the ratio between image area and the corresponding (projected) object area is M^2. The image illumination E_i can be calculated from the object luminance B_o:

$$E_i = (\pi B_o) \sin^2 e_o / M^2$$

The M^2 reflects the fact that the flux is concentrated into an area M^2 as large and the $\sin^2 e_o$ measures the amount of the flux intercepted by the lens.



Now the magnification $M = f_2/f_1$. Then the effective f-number $N_e = f_2/d$. If the diameter of the lens is small compared to the working distance f_1, $\sin(e_o) \approx \tan(e_o) = d/(2f_1)$. So,

$$E_i \approx (\pi B_o)\left(\frac{d}{2f_1}\right)^2 \left(\frac{f_1}{f_2}\right)^2 = \frac{\pi}{4}\frac{1}{N_e^2} B_o$$

If in addition the surface is lambertian we have $L_o = \pi B_o = E_o$ and further, the image illumination will not vary if the surface is tilted away from being perpendicular with respect to the optical axis as shown in the sketch. E_o is the object illuminance and L_o is the object luminous emittance.

$$\frac{E_i}{E_o} \approx \frac{1}{4}\frac{1}{N_e^2} \qquad \text{for lambertian surfaces}$$

For microscopy, the above approximation for $\sin(e_o)$ is not reasonable, since M is large and f_1 is now near f instead of f_2. For lenses used in this fashion the numerical aperture is usually specified, this is simply $\sin(e_o)$.

$$E_i = (\pi B_o)(NA/M)^2$$

For lambertian surfaces we have simply:

$$\frac{E_i}{E_o} = \frac{NA^2}{M^2}$$

CALCULATION OF PHOTO-DIODE CURRENT IN IMAGE SENSING SYSTEM:

$$i = \frac{1}{\pi} * PR * SA * t\phi = \frac{1}{4} * PR * \frac{a}{N^2_e} * t\phi$$

Under the assumption of a lambertian object surface one gets:

i	photo-diode current	amp
PR	$= \int p(\lambda)\ r(\lambda)\ d\lambda$	
$p(\lambda)$	spectral irradiance of scene	watt/m^2-unit wave-length interval
$r(\lambda)$	spectral responsivity of diode	amps/watt
S	solid angle per picture cell	steradian
A	area of lens (entrance pupil)	m^2
a	area of aperture in image	m^2
N_e	effective focal length $(f/d)*(1 + M)$	
	$\frac{\pi}{4} \frac{a}{N^2_e} = SA$	
t	lens transmission (< 1)	
ϕ	reflectance of object surface (< 1)	
$r(\lambda) =$	$(e/hc)\lambda q(\lambda) = 806{,}560\lambda q(\lambda)$	amps/watt
$q(\lambda)$	quantum efficiency (< 1)	
f	focal length of lens	m
d	diameter of lens	m
M	(linear) image magnification	

$$T = (e/i)(S/N)^2$$

T	time to integrate current for desired Signal to Noise ratio
e	charge of electron (1.6021×10^{-19} coulomb)
i	photo-diode current
S/N	Signal to Noise ratio

This is ignoring any noise contribution of the diode or op-amp!

EXAMPLE FOR PHOTO-DIODE CURRENT CALCULATION:

Sun-light at noon is about 570 w/m^2 in the range that the diode is sensitive to. Good indoor lighting is about a hundreths of that, lets say 5.7 w/m^2. The average responsivity of the diode in the visible frequency range is around .35 amp/watt. So PR $\approx$ 2 amp/m^2.

For mirrors deflect a total of .88 radian (50°) and we would like a thousand by thousand pixel image. Then a pixel will be 1.2 milliradians wide. If it is circular, which makes it easier to set up the aperture, its area as seen from the lens will be 1.1×10^{-6} steradians. The mirrors are about an inch wide, so the entrance pupil of the lens better be no more than 14 mm, so its area will be $1.5 \times 10^{-4} m^2$. Evidently SA = 1.6×10^{-10} steradian-m^2.

Ignoring lens-losses and taking the white lambertian surface as a standard we get t$\emptyset$ $\approx$ 1.

This then makes the full scale current expected out of the diode 100 pA. That is 100×10^{-12} amp.

USEFUL PHYSICAL CONSTANTS:

e	charge of electron	1.6021×10^{-19}	coulomb
h	Planck's constant	6.6257×10^{-34}	joule-seconds
c	speed of light	2.9979×10^{8}	meter/second
k	Boltzmann constant	1.3806×10^{-23}	joule/°K

BLACK-BODY RADIATORS (Where does that factor of 685 come from?):

According to Planck's law the amount of energy emitted at wavelength λ by a black-body at temperature T (Kelvin), per unit wavelength interval per unit area is:

$$\frac{c_1\lambda^{-5}}{e^{c_2/\lambda T} - 1} = f(\lambda)$$

Where $c_1 = 2\pi c^2 h = 3.740 \times 10^{-16}$ and $c_2 = hc/k = 1.4385 \times 10^{-2}$.

This function is a maximum at $c_2/\lambda mT = 4.965114$.
The total amount of energy emitted per unit area is σT^4, where $\sigma = (\pi^4 c_1)/(15 c_2{}^4) = (2\pi^5 k^4)/(15c^2h^3) = 5.672 \times 10^{-8}$.

So we can calculate the radiant energy produced by a black-body. In order to calculate the luminous energy we have to weigh the wave-length distribution of energy with the C.I.E. Standard Observer Curve $V(\lambda)$. For T equal to the freezing temperature of platinum (about 2045°K) one gets:

$$\int V(\lambda)\ f(\lambda)\ d\lambda = 2750$$

Since the candela is defined to be the luminous intensity of a 600,000th of a m^2 of a black-body at this temperature and using the fact that the luminous emittance of such a body is π times the luminance (since it is like a lambertian surface) we find the factor

$$600{,}000\ \pi/2750 = 685$$

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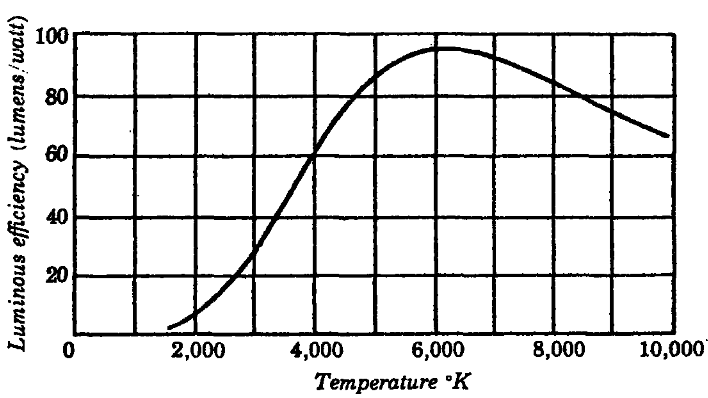

Luminous efficiency of the radiant flux from a blackbody as a function of temperature.

* From OPTICS , F. W. Sears, Addison-Wesley, 1958.

ALL THOSE OTHER (ABSURD) UNITS:	(SI UNITS UNDERLINED)
Luminous Flux	Lumen
Illuminance	Lumen/m^2 = Lux = Meter-Candle Lumen/cm^2 = Phot (old cgs unit) Lumen/ft^2 = foot-candle (imperial unit)
Luminous emittance	ditto
Luminous intensity	Lumen/steradian = candela
Luminance	Lumen/steradian/(projected)m^2 = Candela/m^2 = Nit (!) Candela/cm^2 = Stilb (old cgs unit) Candela/ft^2 (imperial unit) ($1/\pi$) Candelas/m^2 = Meter-Lambert = Apostilb (!) (mks?) ($1/\pi$) Candelas/cm^2 = Lambert (Lambertian cgs) ($1/\pi$) Candelas/ft^2 = Foot-Lambert (lambertian imperial)

IT IS STRONGLY RECOMMENDED THAT ONLY SI UNITS BE USED!

MOST COMMON CONVERSION FACTORS:

Illuminance:	1 Foot-Candle	is 10.76 Lux (Lumens/m^2)
Luminance	1 Foot-Lambert	is 3.426 Candelas/m^2

SOME TYPICAL LUMINOUS EFFICIENCIES:

	lumen/watt
All power at 556nm (max of C.I.E. curve)	685
Uniform distribution ("White light") 400nm - 800nm	220
Sun-like distribution for 400nm - 800nm only	200
Sodium Discharge Tube - Low Pressure (Monochromatic)	175
Sodium Discharge Tube - High Pressure	110
Blackbody at 6500°K (Optimal Temperature)	93
Multi-Vapour (GE) and High-efficiency Fluorescents	90
Fluorescent, 40W 48" long 1-1/2" diameter	80
Sun-light at surface of earth	77
High-Pressure Mercury Discharge Lamp (1000 Watt)	63
High-Pressure Mercury Discharge Lamp (175 Watt)	48
Tungsten 3475° K (Short-Life)	35
Xenon Arc	30
Tungsten 3400° K (500 Watt Sun-Gun)	28
Tungsten 3125° K (250 Watt Household)	22
Tungsten 3000° K (200 Watt)	20
Tungsten 2850° (150 Watt)	18.6
Tungsten (100 Watt)	17.1
Tungsten (75 Watt)	15.6
Tungsten 2700° K (60 Watt)	14.3
Tungsten (40 Watt)	11
Tungsten (25 Watt)	9
Tungsten (15 Watt)	8
Candle	5

For a high intensity bulb (20 lumen/watt), 14% of the power is radiated in the 350nm - 800nm range, 77% in the infra-red and 9% dissipated. The luminous efficiency of the radiated energy is about 142 lumen/watt.

<u>LUMINOUS EFFICIENCY OF FLUORESCENCE LAMPS</u> (40 Watt, 48" long, 1-1/2 diameter)

	Apparent Color Temperature	Radiated Luminous Efficiency	Overall Luminous Efficiency
High Efficiency (Yellow-Green)	-	-	92 lumen/Watt
Cool White	4200°K	328 lumen/Watt	78
White	3500°K	351	75
Warm White	3000°K	371	73
Daylight	7000°K	293	64
Cool White Deluxe	4200°K	295	56
Warm White Deluxe	2900°K	316	54
Vita-Light, Chroma 50	5000°K	228	50
Gro-Lux Wide Spectrum	-	220	45
Plant-Lights, Gro-Lux	-	112	22

For a 40 Watt lamp, about 22% of the power is radiated in the 350nm - 800nm range, 27% in the infra-red and 50% dissipated.

The excitation of the phospor-coatings is due to the line spectrum of the mercury discharge (<u>253.7</u>, '300' '312' '335' 365.0, 404.7 '415' <u>435.8</u>, <u>546.1</u> and 578.0 nm). Most of the power goes into the line at 253.7 nm. The discharge also produces a bit of continuous power in the red.

High-Output lamps consume 60 Watt and Very-High-Output lamps 110 Watt, (in the same 48" size) and produce correspondingly more light).

SOME (APPROXIMATE) VALUES OF ILLUMINANCE AT SURFACE OF EARTH:

	Lux (Lumen/m^2)
Full Sun plus Sky	100,000
Dull day - heavy clouds	3,000
Recommended lighting of work surfaces	1,000
Interior (Day)	300
Interior (Artificial light at night)	100
Full Moon	.16
Moon, First or Third Quarter	.02
Clear Night (Star-light)	.000,3
Very Cloudy Moon-less Night	.000,03
Contribution of single 0th magnitude star	.000,002
Contribution of single 6th magnitude star	.000,000,008

SOME APPROXIMATE VALUES OF LUMINANCE:

	Candelas/m^2 (Lumen/steradian/(projected)m
Surface of Sun	2,000,000,000
Tungsten 2700°K	10,000,000
Black-body 2045°K (freezing platinum)	600,000 (exactly)
White Paper in Sun-Light (80% reflectance)	25,000
Fluorescent Tube Surface	6,500
Candle Flame	5,000
Clear Sky	3,200
Surface of Moon (9% reflectance)	2,900
Surface of totally eclipsed moon	.3
White Paper in Moon-Light	.036
Space Background	.000,01

ILLUMINANCE DUE TO STAR OF VISUAL MAGNITUDE m_v:

$$\text{Illuminance (lux)} = 2.09 \times 10^{-(6 + .4m_v)}$$

So a five magnitude difference corresponds to a factor of one hundred.

Sun at earth appears as a	-26.7 magnitude star	100,000 lumen/m
One Candela at one meter	-14.2 magnitude star	1 (exactly)
Moon at earth appears as	-12.2 magnitude star	.16
Venus (at its brightest)	- 4.28	.000,1
Jupiter (mean opposition)	- 2.25	.000,016
Sirius (brightest star)	- 1.58	.000,009
0 magnitude star	0.00	.000,002
Limit of human vision	+ 6.00 magnitude star	.000,000,008

SOME FACTS ABOUT THE SUN: (On a clear day...)

Mean total radiated flux = 3.92×10^{26} watts. Distance to earth 1.49×10^{11} m. Flux density at earth 1405 watts/m^2. Of this about 1340 watts/m^2 reaches the surface - the solar constant is 1.92 calories/cm^2-minute. (One calorie is 4.19 joule). About 495 watts/m^2 of this energy is in the band 300 nm - 800 nm. This results in an illuminance of about 100,000 lux (lumen/m^2). So the luminous efficiency at the surface is about 77 lumen/watt. (Counting only the energy in the 300 nm - 800 nm one gets about 200 lumen/watt).

EXPOSURE INDEX (ASA):

Exposure-Index = 16/exposure in lux-seconds
Daylight or Tungsten Light is specified. The exposure is that required to produce a density of .90 above minimum density. The development has to be specified as well. This is for subjects of normal contrast.

RECOMMENDED EXPOSURE: 4/(ASA rating) lux-seconds

PHOTO-RECORDING SENSITIVITY:

Photo-Recording Sensitivity = 1/exposure in lux-seconds.
Tungsten light is used. The exposure is that required to produce a density of 0.1 above the gross fog level. The exposure time is short and the development has to be specified as well. (T = 2870°K).
Under most conditions (such as gamma near one), the photo-recording sensitivity is about equal to the exposure index.

DIN-rating = $3 \times \log_2$[ASA-rating] + 1
(Doubling ASA-rating, increases DIN-rating by 3)

EXAMPLE: AGFACHROME or KODACHROME ASA 64 (DIN 19) (REVERSAL FILMS)

Takes .25 lux-seconds	to expose for ~ .1 density	16/ASA
Takes .062 lux-seconds	for recommended exposure	4/ASA
Takes .016 lux-seconds	to expose for .9 density	1/ASA

FLASH EXPOSURE:

Let E be the flash output in candela-seconds (i.e., lumen-seconds/steradian) in the direction of the object. The object (illuminance x time) is then E/r^2 (lumen-seconds/m^2). Here r is the distance between the flash and the object. If the object is a perfect lambertian reflector oriented at right angles to the line from it to the flash, its (luminance x time) will be $E/(\pi r^2)$ candela-seconds/m^2. The image exposure finally comes to

$$\frac{E}{4\, r^2 N_e^{\,2}} \qquad \text{(lux-seconds)}$$

Where N_e is the effective f-number. The recommended exposure is 4/(ASA-rating) lux-seconds. Equating these two quantities, we get:

$$\sqrt{\frac{E \times ASA}{16}} = r\, N_e$$

This is the metric guide-number. If r is expressed in feet one obtains the "English" guide-number (divide the above by .3).

Note: The average object reflects less than the lambertian surface assumed so the guide number is usually reduced somewhat ($1/\sqrt{2}$ about).

EXAMPLE: BRAUN F270 has an output of 270 candela-seconds in the forward direction. AGFACHROME or KODACHROME film have an ASA-rating of 64. This gives a metric Guide-Number of 33 x .7 = 23. This comes to an "English" Guide-Number of about 80.

That is, the f-number one should use is 80/(distance-in-feet).