Essays in Behavioral Industrial Organization, Corruption, and Marketing

by

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Abstract

In Chapter 1, I propose a model in which consumers base their purchasing decisions upon their recollections of the product quality, and in which firms can use persuasive advertising in order to change these recollections. Although consumers are aware that such advertising has occurred and take this into account when updating their beliefs about the product, they cannot prevent their memories from being affected. I analyze which firms engage in persuasive advertising as well as the price level that these firms choose. I show that persuasive advertising may be used in equilibrium even though consumers are fully aware of it, and that persuasive advertising does not always signal high quality products. The model is then extended to incorporate both persuasive and informative advertising, where firms reveal some verifiable information about their products. In that case, persuasive advertising may block the full unraveling of information, and high quality products are not promoted with only one type of advertising – in some cases, persuasive advertising can signal a product of either higher or lower quality than a product promoted with informative advertising.

Chapter 2 is the product of joint work with Abhijit Banerjee and develops a model to study the effectiveness of complaints against corruption. A bureaucrat has to decide on a public infrastructure project in a village where a rich and a poor villagers live. A dishonest bureaucrat can be bribed not to choose the surplus maximizing project and instead to choose a project that favors the rich villager. Once the bureaucrat has chosen a project, the villagers can send a costly praising or complaining message to the bureaucrat’s supervisor who does not know whether the bureaucrat is honest or dishonest. From his point of view the messages are anonymous; the supervisor does not know who is rich or poor in the village. The only leverage of the supervisor is to transfer the bureaucrat and replace him with another one who will repeat the game in the following period. In any relevant equilibrium no complaints happen and more generally there are no complaints in equilibrium without bribery. We find that complaints will be observed only when they should not be and that the government cannot necessarily get people to complain by cutting the message cost. In addition, lowering that cost may hurt since, when the share of honest bureaucrat is low, the
poor are pessimistic about the benefit of complaints while the rich are optimistic and they respond more to a lower cost. Finally, the supervisor cannot fully decide to implement a particular equilibrium as multiple ones coexist.

Chapter 3 is the product of joint work with Elie Ofek. We model a duopoly in which ex-ante identical firms need to decide where to direct their innovation efforts. The firms face market uncertainty with respect to consumers’ preferences for innovation on two product attributes, and technology uncertainty with respect to the success of their R&D efforts. Firms can conduct costly research to resolve their market uncertainty before setting R&D strategy. We find that the value of market information to a firm depends on whether its rival is also expected to obtain this information in equilibrium. We show that, as a result, one firm may forgo market research even though its rival conducts such research and learns the true state of demand. We examine both vertical and horizontal demand structures. With vertical preferences, firms are a priori uncertain which attribute all consumers will value more. In this case, a firm that conducts market research will always innovate on the attribute it discovers that consumers prefer, and expend more on R&D than a rival that has not conducted market research. With horizontal preferences, distinct segments exist—each cares about innovation on only one attribute—and firms are a priori uncertain how many consumers are in each segment. In this case, a firm that conducts market research may follow a ‘niche’ strategy and innovate to serve the smaller segment to avoid intense price competition for the larger segment. Consequently, a firm that conducts market research may invest less in R&D and earn lower profits post-launch than a rival that has forgone such research.

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à Eric
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Chapter 1

Persuasive Advertising with Sophisticated but Impressionable Consumers

1.1 Introduction

Understanding how firms select their advertising strategy is an issue of both theoretical and practical interest. Firms decide not only how much to advertise but also how to advertise their products. The goals of advertising campaigns are diverse. They vary from creating awareness for a new product to affecting repeated purchases. Post-experience advertising, for example, affects what consumers remember about their past consumption. Braun (1999) documents that post-experience advertising has an effect on what consumers believe about a product and explains how it affects the formation of memories of consumers by creating an association between a positive feeling and a product. When consumers understand how advertising works, the choice of a firm of whether to advertise or not should reveal some information about the product of the firm to consumers. This approach is in contrast with the view that advertising is a pure money-burning device. \(^1\) Recent literature started to

\(^1\)Nelson (1974) describes the idea of advertising signaling quality by dissipating part of the profits and Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) formalize it.
incorporate the behavioral effects of advertising on consumers. In particular, Shapiro (2006) studies the level of advertising spending when advertising affects the memory of consumers while at the same time consumers are rational.

In this paper, I develop a framework to simultaneously analyze the advertising decisions and the pricing decisions of a firm. By taking into account the direct effect of advertising on consumers, a monopolist can signal the quality of its product through both its advertising and its pricing decisions. Persuasive advertising changes the prior of some consumers about the product quality. What these consumers believe about the product quality is more favorable with persuasive advertising-higher quality than without persuasive advertising-. Consumers are sophisticated, that is they understand the overall effect of persuasive advertising and correct for it as best as they can, but individually they do not know if they have been influenced or not. The firm, depending on its product quality, chooses its price as well as whether to engage in advertising or not. Consumers then form their belief about the product quality based on their prior, whether persuasive advertising was done, and the price. I first find that firms engage in persuasive advertising even if consumers are fully aware of their motivation for choosing this form of advertising. I then show that neither the relationship between advertising and quality nor the one between advertising and price are monotonic. Finally, when firms choose between persuasive advertising or informative advertising, the monopolist may prefer to withhold its information and instead engage in persuasive advertising.

I propose a model where consumers are uncertain about the quality of a product sold by a monopolist. At the time of purchase, consumers do not know the exact quality of a product but try to assess it using the information they have about it. This information includes the recollection they have of the product quality due to past personal experience with the product, word-of-mouth or official consumer reports, and also the price and the advertisements chosen by the seller. To model that consumers

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2Persuasive advertising in this paper does not refer to a shift in the demand of consumers as defined in Bagwell (2005).

3Informative advertising consists of truthful disclosure of a piece of information about the product quality.
have different priors about the product quality, I assume that they receive a signal about the product affecting their priors in a positive or in a negative way. The signal is linked to the quality of the product as the likelihood of having a good signal is an increasing function of quality.

Persuasive advertising increases the share of consumers with a good prior. An example of such a persuasive advertising is based on the limited memory of consumers. In the marketing and psychology literature, evidence exists that memory can be altered by post-experience advertising, i.e. exposure to advertising after the last time the product was consumed. Braun (1999) cites many studies documenting that post-experience advertising works. Her paper shows that in the case of orange juice, the memory of consumers is altered by advertisements viewed, even after an unpleasant experience. The recollection of the orange juice by consumers is closer to what the advertisement claims the orange juice is than to the true, potentially unpleasant, experience as remembered by the control group. If the signals are generated by consumers remembering whether their last experience with the product was good or not, their last experience does not have to be a personal one—that is from having actually used the product themselves—. The experience could simply be a suggested experience as consumers cannot distinguish between memories arising out of a suggested experience from memories generated by personal experiences. Marketers, aware of that effect, can generate more good memories than what the product would generate by itself.

Consumers are fully aware of the way persuasive advertising works. They know that persuasive advertising increases the likelihood of good signals. They know that they have seen the persuasive advertisements and that they might be affected by them. But they do not know if a good signal is the consequence of persuasive advertising or true experiences as, of course, they do not know the signal they would have had without persuasive advertising.

An important insight of this paper is that persuasive advertising is not naturally a signal of high quality nor it is naturally associated with high prices. Under some market conditions, the fact that consumers fully understand the effects of advertising allows firms to signal their high quality by choosing a high price and by not engag-
ing in advertising. Indeed if a firm chooses not to do any persuasive advertising, consumers with a positive signal about the product know that their signal comes only from the intrinsic quality of the product and was not artificially improved by a marketing campaign. Consumers with bad signals do not buy and therefore the quality of the firm has to be high enough to generate enough good signals and make choosing a high price profitable. As a result, choosing a high price and not engaging in persuasive advertising can help signal a high quality product. Finally, depending on the market conditions and on the degree of persuasion of the advertising, persuasive advertising can signal a high quality range, a low quality range or even an intermediate quality range. Similarly the result, that persuasive advertising is not always associated with high prices, is the consequence of two effects. The first one comes from the fact that firms use both price and advertising to signal their quality. Consequently, if advertising does not signal high quality it is not associated with high prices. Secondly, keeping everything else constant, exposure to persuasive advertising lowers the expectation of the product quality and therefore how much consumers are willing to pay for the product. The intuition is that a good signal with persuasive advertising does not carry as much good news as a good signal without persuasive advertising. The latter one was generated only through the quality of the product while the former one could be spurious and could have been bad without persuasive advertising. Similarly, after receiving a bad signal, it is more likely that the quality is low with persuasive advertising than with no persuasive advertising.

Allowing firms to choose both their advertising and their price is a crucial point, as advertising would have no signaling ability if the prices were fixed. The interplay between pricing and advertising decisions is especially interesting for firms choosing not to engage in persuasive advertising. Indeed, on the equilibrium path, consumers observing no advertising do not always know whether the quality of the firm is high or low, they need to check the price of the product to decide, while observing persuasive advertising pinpoints a quality range on the equilibrium path.

All the types of equilibrium that may arise can be characterized in the general case, without specifying any functional form. A set of necessary conditions is first
provided for each equilibrium. I then study when these equilibria can occur in the case of an uniform distribution of qualities with a particular persuasion technology. In particular, I find that when the degree of persuasion is high or when the range of qualities is small, advertising will never signal the high quality product. This result is intuitive as if advertising is really persuasive, it has an homogenizing effect. Firms do not look that different after engaging in persuasive advertising and therefore advertising cannot be used by high quality firms to signal their types. Similarly if the range of quality is small, the differential effect of advertising on firms is too small to allow firms to signal their quality.

Finally, I introduce a second type of advertising to study how firms can use the type of advertising strategically to reveal some information about them. This second type of advertising is referred to as informative advertising as it is based on a claim which is some hard/verifiable piece of information about the product. The monopolist chooses now between persuasive advertising, informative advertising and no advertising. The structure of the equilibria is then characterized. Interestingly, I find that the option of engaging in persuasive advertising might block the full unraveling of information in equilibrium; some firms could have engaged in informative advertising and released their information but instead they chose persuasive advertising.

**Literature Review**

This paper adds to the literature about the signaling ability of advertising by studying how the direct effect of advertising allows firms to signal their product quality when consumers are fully rational. The findings that advertising does not necessarily signal high quality and is not always associated with high prices is in sharp contrast with the series of papers where advertising is simply a burning money device. Nelson (1974) introduces the concept while Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) develop it formally. Milgrom and Roberts (1986) show how firms use both price and advertising to signal product quality when advertising has no direct effect on consumers. A high quality firm has a larger profit than a low quality firm.

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4For a definition of hard versus soft information see Tirole (1988a).
for a given price, as consumers repeat their purchase if the quality is high but not if it is low. Hence the signaling power of advertising stems from the fact that a firm with a large potential profit is willing to sacrifice some of it to signal its type while a firm with a smaller profit would not do so. There is an extensive literature on advertising as a signaling device, see Bagwell (2005) for a review.

This paper contributes also to the literature where advertising has a direct behavioral effect on consumers. These analyses vary in the degree of rationality that consumers are supposed to have. For example in Schmalensee (1978) consumers are irrational as they are not modeled as strategic players while in Shapiro (2006), consumers are rational. My work belongs to the second type of papers.

Schmalensee (1978) presents a model where advertising might signal the low quality products as consumers with bad experiences are more likely to switch to a brand the more it advertises. If consumers were sophisticated, they would understand that advertising is actually a signal of low quality and the equilibrium would break down. In my paper, advertising might signal the low quality products even if consumers fully understand how advertising works and take it into account when forming their beliefs.

Shapiro (2006) introduces a novel model of advertising in a limited memory set-up and shows that a number of interesting effects arise. Consumers are sophisticated but do not know the amount of advertising to which they were exposed, which introduces a signal-jamming story to his paper. One consequence of this assumption is that firms advertise in equilibrium due to a commitment problem. Firms would not advertise at all if they could commit ex-ante not to advertise as advertising is costly and does not change the expected level of demand nor the price. He finds that advertising levels might increase with consumer experience and that the relationship between advertising spending and quality is not necessarily monotonic.

This paper departs from the paper of Shapiro in several ways. First, consumers are aware of being exposed to persuasive advertising. One consequence is that advertising occurs in equilibrium without any commitment problem. Secondly, prices are chosen strategically by firms. Therefore firms use both their advertising choice and their
price level to signal quality. Interestingly, while the questions that we address are different, we both find a non-monotonic result. Indeed, I find that the relationships between the type of advertising and the quality and between the type of advertising and the price level are not necessarily monotonic. Finally I study how firms choose their advertising when two types of advertising are available.

In this paper, keeping everything else constant, exposure to persuasive advertising lowers the willingness to pay of consumers. This negative effect of advertising exposure is unusual in the literature, an exception is Anand and Shachar (2005) where the beliefs of a consumer are worsen by exposure to an advertisement revealing that the attributes of a promoted product are below his average match. The negative effect comes from what consumers learn from the content of an advertisement, whereas the negative effect of this paper does not come from the content but from the inferences of sophisticated consumers. Consumers adjust the value of the information contained in their prior, when exposed to advertising. They understand that a good signal with persuasive advertising does not carry as much good news as a good signal without persuasive advertising.

Finally this paper adds to the literature on advertising content. Anderson and Renault (2006) shows that a monopolist chooses to reveal only limited information about its product when consumers face a search cost. The last part of my paper finds a similar result that the monopolist might not reveal all its information but for a completely different reason, the firm prefers to engage in another type of advertising.

Empirical studies have tried to disentangle the different effects of advertising. Ackerberg (2001) and Ackerberg (2003) assume that advertising affects consumers differently depending on their level of experience with the product. These two papers find that, in the yogurt industry, the informative more than the persuasive effects of the advertisements affected consumers. Lastly, there is a general issue on how to classify the different types of advertising. Johnson and Myatt (2006) suggest to use a different taxonomy: hype versus real information. Their distinction comes from a vertical versus horizontal differentiation while the distinction that I used is based on the notion of soft versus hard information. The more appropriate taxonomy depends
of course on the context, but, the important claim is that the type of advertising matters and affects consumers differently and therefore it is a strategic tool that firms can use to communicate with their customers.

The rest of the paper is organized as follows. The next section presents the model and gives some support in favor of the different assumptions. Section 1.3 presents a benchmark model with no advertising shedding light on how firms choose their prices to signal their types. Section 1.4 presents the analysis of the full model and the full set of equilibria is characterized in the case of general functional forms. Then in Section 1.5, some specific functional forms are assumed and the existence of the equilibria are studied depending on the parameters of the model. In Section 1.6, firms can choose between two types of advertising, persuasive or informative. And finally I conclude.

1.2 Model

The model set-up is presented first and the notations are formally introduced. I then explain in more detail what quality represents in this context, how signals are generated and how an advertising campaign can be persuasive.

1.2.1 Model Set-up

A monopolist sells a product of quality $q \in [0,1]$, drawn from a distribution with probability distribution function $f$ and support $[q, \bar{q}] \subset (0,1]$. Marginal costs are assumed to be constant and, without loss of generality, equal to zero.

There is a unit-mass of consumers. Each consumer demands at most one unit of the product. Their outside option is normalized to zero. If they buy a product of quality $q$ and price $p$, their utility is $q - p$. Consumers do not know the quality of the product, but they know the distribution $f$. In addition, consumers receive a signal $s$ about the quality of the product. The signal is either good, $s = 1$, or bad, $s = 0$. The probability of a good signal, $P(s = 1| q) = \alpha(q) \in (0, 1)$, is an increasing function of the true quality of the product.
**Assumption 1** The distribution of qualities satisfies $\alpha(q)E[q|s = 1] < q$.

The worst quality is assumed to generate few good signals. More precisely the worst quality firm prefers selling to all consumers at the lowest price $p = q$ over selling only to consumers with good signals at $p = E[q|s = 1]$, the expected quality when consumers have a good signal, i.e. $\alpha(q)E[q|s = 1] < q$.

The firm cannot communicate the quality of its product directly to consumers. But it can run a persuasive advertising campaign to increase the probability that a consumer receives a positive signal about the product, $P(s = 1| q, a = P) = \varphi(q) \geq \alpha(q)$. If, for example, persuasive advertising converts a share $\theta$ of bad signals into good signals, then the probability of a good signal becomes $\varphi(q) = \alpha(q) + \theta(1 - \alpha(q))$. $\varphi$ is also assumed to be an increasing function of $q$. A persuasive advertising campaign covers the full market at no cost. 5

**Assumption 2** $\frac{\varphi(q)}{\alpha(q)}$ and $\frac{1 - \varphi(q)}{1 - \alpha(q)}$ are assumed to be non-increasing.

The first part of this assumption simply means that the share of created good signals relative to the good signals generated solely by the product does not become larger as the quality becomes higher. Similarly the second part means that the share of remaining bad signals after persuasive advertising relative to the bad signals generated solely by the product does not become larger as the quality becomes larger. This assumption is true in the previous example.

Finally, consumers are fully aware of the effects of persuasive advertising. They observe whether the firm engages in persuasive advertising or not and form Bayesian beliefs about the quality of the product.

The timing of the model is illustrated in Figure 1-1. The firm learns its type, it chooses its advertising campaign and its price. Consumers see the ad if any, then they receive their signal, observe the price and decide whether to buy or not. I analyze pure-strategy perfect Bayesian Nash equilibria, with a tie-breaking rule such that a firm indifferent between advertising and no advertising picks no advertising.

5The focus of this paper is on the demand side effects: how firms can use the direct effect of advertising on consumers to signal their quality. Introducing small fixed costs would not change the nature of the results. A more detailed discussion about costs can be found at the end of Section 1.4.
1.2.2 Motivation

In this paper, the quality of a product refers to the probability that the good delivers a satisfactory experience when it is consumed. The level of satisfaction may vary for several reasons. First, consumers do not always use the product in the same situations. In the case of cough medicine, for example, non-drowsiness is not crucial if the medicine is taken before going to bed, whereas it involves safety issues before driving or operating machinery. Secondly, consumers have heterogeneous needs. For example, some people take allergy drugs to treat a specific allergy while others need help for all possible sources of allergy. Finally, the quality of some goods, such as restaurant meals, varies from one production batch to the other.

I assume that the firm cannot communicate the quality of its product directly. This could be the case because the information is too difficult to process for a non-expert consumer and/or the information would not fit into the format of an advertisement or on the package of the good. On the other hand, consumers might have used the product in the past or might have talked to somebody who has some experience with it, they have a prior about the product at the time of purchasing. This is captured by the fact that consumers receive a good or a bad signal about the product. One example is that consumers remember whether their last experience with the product was satisfactory or not, and in that case $\alpha(q) = q$.

The firm might be able to interfere in the production of these signals with some persuasive advertising, which is modeled as a marketing technology increasing the

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The origin of the signal can be more complex, it could be the memory of one experience randomly drawn from all the past experiences, weighting them differently depending on whether the experience was good or bad, how far in the past it happened and on their ordering (first and last experiences might be more salient). The effects of confirmation bias can be incorporated, i.e. the fact that consumers put more weight on experiences confirming their first experience.
probability that a consumer receives a positive signal about the product. The characterization of how advertising affects consumers is consistent with behavioral research done in marketing and psychology about postexperience advertising. If the consumers are exposed to an ad after consuming a good but before asking them about what they thought of the product, it was proven that the ad might influence their memory of their experience. Braun (1999) and Braun, Ellis, and Loftus (2002) replicate this effect in several different settings and provide references of other papers describing and explaining how postexperience advertising works. Moreover consumers believe that their memory is their own, they can not distinguish what part comes from their experience from the part that was suggested by the advertisement.

Let me now briefly present a model providing some micro-foundations for postexperience advertising. First consumers use the product and their experience is either satisfactory or neutral, \( q \) is the probability of having a satisfactory experience. Then time elapses and consumers might forget about their satisfactory experience. At the time of the next purchase, \( \alpha(q) \) consumers remember a good experience while \( 1 - \alpha(q) \) consumers have a neutral memory, they either had a neutral experience or they forgot about their satisfactory experience. In that context, persuasive advertising serves as a reminder for consumers, it reminds them why their experience was satisfactory and decreases the number of consumers forgetting.\(^7\) Thus the share of consumers with a good memory is increased to \( \varphi(q) \).

Even if consumers are impressionable, they are fully aware of the effects of persuasive advertising. They observe whether the firm engages in persuasive advertising or not and form Bayesian beliefs about the quality of the product. In the literature, persuasive advertising is often modeled as a way to shift tastes. This is not exactly the case in my model. Here, persuasive advertising generates more positive signals about the product and shifts some consumers from having a bad signal to having a good memory.

\(^7\)In this example, advertising helps consumers remember their own experiential level of satisfaction. It could also, depending on one’s interpretation of the results on post-experience advertising, suggest a good memory to someone who had a neutral experience. It is easy to imagine that a consumer had a neutral experience because the last time he used the product he was under a lot of stress and nothing seemed pleasant, while an ad depicting a nice experience might remind him of what he could experience when consuming the product. In the end, that consumer might remember the good suggested memory over the neutral experienced one.
good signal, meaning that more consumers have a good prior about the product. But it does not make them more optimistic about the product as they understand that part of these positive signals are due to persuasive advertising, therefore a good signal conveys less good news about the product. The effects of persuasive advertising are described in more details in the next section.

1.2.3 Positive and Negative Effects of Persuasive Advertising

At this point, one might ask why a firm would forgo persuasive advertising as it is free and it increases the number of consumers with a good prior about the product. Before answering that point, let me first discuss the implications of Assumption 2, namely that \( \frac{\nu(q)}{\alpha(q)} \) and \( \frac{1-\nu(q)}{1-\alpha(q)} \) are non-increasing. It implies that holding the signal constant, persuasive advertising decreases the average willingness to pay of consumers, i.e. \( E[q|P, s, q_1 \leq q \leq q_2] \leq E[q|\emptyset, s, q_1 \leq q \leq q_2] \). I will refer to that as the negative direct effect of persuasive advertising. Sophisticated consumers understand that getting a good signal with persuasive advertising conveys less good information about the quality than getting a good signal without persuasive advertising as some good signals result from the marketing strategy and not the quality of the product.

Overall the negative direct effect of persuasive advertising has to be compared with the positive direct effect of persuasive advertising which is the generation of more consumers with good signals, translating into a demand expansion if the firm sells only to consumers with good signals. These two effects are illustrated in Figure 1-2. Consumers respond to persuasive advertising, hence the share of consumers with good signals increase with persuasive advertising. At the same time, they are sophisticated and take into account that their signals might be changed by persuasive advertising, thus their willingness to pay or expected quality decreases with persuasive advertising. Note that with or without persuasive advertising, consumers have the same ex-ante expected quality \( E[q] = E_s[E[q|\emptyset, s]] = E_s[E[q|P, s]] \). Persuasive advertising changes

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8 Indeed the distribution of \( q \) given that (1): \( q \in [q_1, q_2] \) (2): signal \( s \) was received and (3): the firm engaged in no persuasive advertising first-order stochastically dominates the distribution of \( q \) given that (1): \( q \in [q_1, q_2] \) (2): signal \( s \) was received and (3): the firm engaged in persuasive advertising: \( F(q|\emptyset, s, q_1 \leq q \leq q_2) \leq F(q|P, s, q_1 \leq q \leq q_2) \).
Figure 1-2: Distribution of the expected quality, i.e. willingness to pay, with and without persuasive advertising. The difference in the two distributions can be analyzed in two steps. First, the share of consumers with $s = 1$ increases as a direct effect of persuasive advertising. Secondly, sophisticated consumers understand that a good signal with persuasive advertising could have been generated by persuasion and not because of the true quality of the product. Hence a good signal with persuasive advertising carries less good news than without persuasive advertising, hence the decrease in the expected quality given a good signal. Similarly for a bad signal.

the dispersion around the mean, i.e. the variance. In the above example, where persuasive advertising converts a fixed share of bad memories into good memories, persuasive advertising reduces the variance.

### 1.3 No Advertising Case

I start by analyzing a simpler model, where firms cannot advertise. This provides some useful insight on the role played by prices in this model. Wolinsky (1983) and Bagwell and Riordan (1991) also study how prices signal product quality in settings different from the one presented in this paper.

In this section, a pure-strategy equilibrium is characterized by a triplet $(p(q), b(s,p), \mu(q|s,p))$, where $p(q)$ is the price chosen by the firm as a function of its quality, $b(s,p)$ is the decision of the consumers, buy or do not buy, and $\mu(q|s,p)$ are the beliefs of the consumers when they receive signal $s$ and observe price $p$. The
beliefs are a probability distribution over the set of qualities.

The first step of the analysis is to note that the support $A(p)$ of $\mu(q|s, p)$ when $p$ is on the equilibrium path in a pure-strategy equilibrium does not depend on $s$, as the choice of the firm $p(q)$ is independent of the signals received by consumers and each signal occurs with a positive probability for every quality. Therefore when $p$ is chosen in equilibrium, consumers with bad signals and consumers with good signals believe the same set of firms could have chosen that price, but the beliefs with a bad signal put more weight on the low quality firms than the beliefs with a good signal. This implies that, on the equilibrium path, if consumers with bad signals are willing to buy at $p$, consumers with good signals are also willing to buy, namely $b(s = 0, p) = \text{buy}$ $\Rightarrow b(s = 1, p) = \text{buy}$. In addition, as consumers always buy if the price is equal to the worst quality, the profits of the firms are larger than $q$ in equilibrium and, on the equilibrium path, firms either sell to everybody or to consumers with good signals only.

$$p \text{ is on the equilibrium path } \Rightarrow \begin{cases} b(s, p) = \text{buy if } s = 1 \\ \text{or} \\ b(s, p) = \text{buy for any } s \end{cases}$$

The next lemma shows formally that in equilibrium at most two prices are chosen. It proves that, when exactly two prices are on the equilibrium path, every consumer buys at the lower price while only consumers with good signals buy at the higher price. It also pins down the ordering in terms of quality; a low quality firm chooses the low price and a high quality firm chooses the high price.

**Lemma 1** *In the model with no advertising, in a pure-strategy equilibrium*

i. *At most two prices are chosen on the equilibrium path.*

ii. *When exactly two prices are on the equilibrium path,*

- *when the price is low, all the consumers buy and when the price is high, only the consumers with a good signal buy,*

- *a firm choosing the low price has a lower quality than a firm choosing the high price.*
Proof Consider two firms $q$ and $\bar{q}$ choosing $p$ and $\bar{p}$ in equilibrium respectively. First assume that consumers buy only when they receive a good signal for both prices or that all the consumers buy when they observe either price. It must be the case that the prices are equal. Otherwise the firm with the lower price deviates and chooses the higher price, its demand is unchanged but it now sells at a higher price. Assume now that consumers buy only when they receive a good signal when they see one price and they buy irrespectively of the value of their signal when they observe the other price, wlog let's assume that $b(s,p) =$ buy iff $s = 1$ and $b(s,\bar{p}) =$ buy for any $s$. The profits of firm $q$ after choosing $p$ must be at least as large as its profits if it had chosen $\bar{p}$, its no-deviation condition is $\alpha(q)p \geq \bar{p}$. Similarly for firm $\bar{q}$, the no-deviation condition is $\bar{p} \geq \alpha(\bar{q})p$. Recall that the signal function $\alpha$ is non-decreasing and smaller than 1, therefore these conditions imply that $q \geq \bar{q}$ and $p \geq \bar{p}$. Hence the results of the lemma.

Intuitively, firms choose between the highest price where all consumers buy and the highest price where only consumers with good signals buy, as firms always want to increase their prices when it does not lower the demand. The trade-off is then between selling at a low price to the full market or selling at a high price to one segment of the market. This effect is the usual monopoly trade-off between increasing the price and reducing the demand. Setting up a price high enough allows firms to signal their high quality by forgoing some demand. Indeed when consumers with good signals observe the high price, they know that consumers with bad signals never buy at that price. Thus it is profitable for a firm to choose that price only if the firm generates enough good signals. Therefore only high quality firms choose the high price over the low price and consumers buy at the high price.

Lemma 1 states that any equilibrium has exactly one or two prices on the equilibrium path. They are referred to as pooling equilibrium or as semi-separating equilibrium respectively. A pooling equilibrium is characterized by a price $p_L$ such that the strategies and beliefs along the equilibrium path are

\[ p(q) = p_L, \quad b(p_L, s) = \text{buy}, \quad \text{and} \quad \mu(q|p_L, s) = \frac{g_\theta(q|s)}{\int q g_\theta(y|s)dy}, \]

25
where \( g_0(q, s) = \alpha(q)^s (1 - \alpha(q))^{1-s} f(q) \) is up to a multiplicative constant the density function over the qualities given that signal \( s \) was received. And the previous lemma shows that a semi-separating equilibrium is characterized by two prices \((p_L, p_H)\) and one quality cutoff \( q_L \) such that the strategies and beliefs along the equilibrium path are

\[
p(q) = \begin{cases} 
  p_L & \text{if } q \leq q_L \\
  p_H & \text{if } q_L < q
\end{cases},
\]

\[
b(p_L, s) = \begin{cases} 
  \text{buy} & \text{if } s = 0 \\
  \text{don't buy} & \text{if } s = 1
\end{cases},
\]

\[
b(p_H, s) = \begin{cases} 
  \text{buy} & \text{if } s = 0 \\
  \text{don't buy} & \text{if } s = 1
\end{cases}.
\]

\( \mu(q|p_L, s) \) and \( \mu(q|p_H, s) \) are derived using Bayes' rule\(^9\). Figure 1-3 illustrates graphically these two types of equilibrium while proposition 1 states a set of necessary conditions that has to be satisfied by each equilibrium.

**Proposition 1** In the model with no advertising, any pure-strategy equilibrium satisfies one of the two following sets of conditions:

i. there exists a price \( p_L \) such that \( p_L \in \left[ q, E[q|s = 0] \right] \)

and the equilibrium is a pooling equilibrium characterized by \( p_L \);

ii. there exist two prices \((p_L, p_H)\) and one quality cutoff \( q_L \) such that:

\[
p_L = \alpha(q_L) p_H
\]

\[
p_L \in \left[ q, E[q|s = 0, q \leq q_L] \right]
\]

\[
p_H \in (E[q|s = 0, q_L < q], E[q|s = 1, q_L < q])
\]

and the equilibrium is a semi-separating equilibrium characterized by \((p_L, p_H)\) and \( q_L \);

These necessary conditions are also sufficient in the case of the worst out-of-equilibrium beliefs.

**Proof** The previous lemma states that an equilibrium has at most 2 prices on the equilibrium path. Firms can always choose to price at \( q \) and sell to everybody, hence

\[\mu(q|p_L, s) = \frac{g_0(q, s)}{\int_q g_0(y, s)dy} \text{ if } q \leq q_L, 0 \text{ otherwise}, \]

\[\mu(q|p_H, s) = \frac{g_0(q, s)}{\int_q g_0(y, s)dy} \text{ if } q < q_L, 0 \text{ otherwise.}\]
Figure 1-3: No advertising case: these graphs show the price as a function of the quality and whether consumers buy depending on their signals. The right graph illustrates a pooling equilibrium. The left graph illustrates a semi-separating equilibrium where high quality firms signal themselves by choosing a high price and forgoing selling to consumers with a bad signal.

it must be the case that any equilibrium price is larger than \( q \). (i) Consider first a pooling equilibrium around \( p_L \). Assume that consumers buy at \( p_L \) only if their signal is good, this is the case iff \( E[q|s = 0] < p_L \leq E[q|s = 1] \). Thus the profit of the worst quality firm is \( \pi(q) = \alpha(q)p_L \leq \alpha(q)E[q|s = 1] < q \), this firm can deviate to \( p = q \) and increase its profit. Therefore in a pooling equilibrium all the consumers buy irrespectively of their signals. Bayes' rule pins down their beliefs when they observe \( p_L \) and consumers with a bad signal are willing to buy iff \( p_L \leq E[q|s = 0] \). (ii) Consider now an equilibrium with two prices, the previous lemma shows that there exists two prices \( (p_L, p_H) \) and one quality cutoff \( q_L \) such that \( p(q) = \begin{cases} p_L & \text{if } q \leq q_L \\ p_H & \text{if } q < q \\ \end{cases} \), \( b(p_L, s) = \text{buy}, \quad b(p_H, s) = \begin{cases} \text{don't buy} & \text{if } s = 0 \\ \text{buy} & \text{if } s = 1 \\ \end{cases} \). The condition \( p_L = \alpha(q_L)p_H \) simply states that the firm with the cutoff quality is indifferent between the two prices. Bayes' rule delivers the beliefs of the consumers on the equilibrium path. Finally, the optimality of the purchasing decisions gives the range for the prices.  

Proposition 1 is general as it is stated with general functional forms \( f \) and \( \alpha \), the distribution of qualities and the signal function respectively. The next two corollaries present some additional results about which equilibrium has the highest profit or the

\(^{10}\text{The last inequality is one assumption of the model.}\)
highest welfare. The following corollary shows that the pooling equilibrium, where firms extract the full surplus of the consumers with bad signals, has the highest profits among all the pooling equilibria. Furthermore, this pooling equilibrium has the highest profits for all types of firms if the highest profit, achieved by the firm with the highest quality when selling only to consumers with good signals, is lower than the expected quality after receiving a bad signal. Finally the profits of any semi-separating equilibrium are smaller than the profits of a semi-separating equilibrium where the low quality firms extract the full surplus of the consumers with bad signals. 

These results give some support in favor of studying the equilibria with binding upper constraints on prices.

**Corollary 1** In the model with no advertising,

i. the pooling equilibrium with \( p_L = E[q | s = 0] \) has the highest profit among all the pooling equilibria,

ii. if \( \alpha(q)\bar{q} \leq E[q | s = 0] \), the pooling equilibrium with \( p_L = E[q | s = 0] \) has the highest profit for all types of firms,

iii. any semi-separating equilibrium with \( p_L < E[q | s = 0 , q \leq q_L] \) has smaller profit than a semi-separating equilibrium satisfying \( p_L = E[q | s = 0 , q \leq q_L] \).

**Proof** In a pooling equilibrium, the profits are \( \pi(q) = p_L \) implying the first result. In a semi-separating equilibrium, profits are non-decreasing with respect to quality and \( \pi(q) = \alpha(q)p_H \leq \alpha(q)\bar{q} \). Hence if \( \alpha(q)\bar{q} \leq E[q | s = 0] \), the equilibrium with the highest profits is the pooling equilibrium with \( p_L = E[q | s = 0] \). Consider a semi-separating equilibrium such that \( p_L < E[q | s = 0 , q \leq q_L] \) and \( p_H < E[q | s = 1 , q_L < q] \), then the semi-separating equilibrium with \( (p_L + \delta p_L, p_H + \delta p_H, q_L) \) such that \( \delta p_L > 0 \), \( \delta p_H > 0 \), \( \delta p_L = \alpha(q_L)\delta p_H \) and \( p_H + \delta p_H \leq E[q | s = 1 , q_L < q] \) leads to strictly higher profits for any quality. Consider a semi-separating equilibrium such that \( p_L < E[q | s = 0 , q \leq q_L] \) and \( p_H = E[q | s = 1 , q_L < q] \), then the semi-separating equilibrium with \( (p_L + \delta p_L, p_H, q_L + \delta q_L) \) such that \( \delta p_L > 0 \), \( \delta q_L > 0 \), \( p_L + \delta p_L = \alpha(q_L + \delta q_L)p_H \) and \( E[q | s = 0 , q_L + \delta q_L < q] < p_H \) leads to higher profits, strictly higher for the low
qualities. Therefore the semi-separating equilibrium with \( p_L = E[q | s = 0, q \leq q_L] \) has the highest profits among all the semi-separating equilibria.

The fact that the pooling equilibrium with the full consumer surplus being extracted leads to the highest profit among all the pooling equilibria is intuitive. Furthermore, if even the highest quality product generates so few good signals that, if consumers with good signals believed the product had the highest quality, the firm would still prefer the profits of this pooling equilibrium, then it must be the case that this pooling equilibrium has the highest profits for all firms. Finally if the monopolist does not extract the full surplus of the consumers with bad signals when its quality is low, then there is some scope for increasing that the profit by increasing that price. Indeed another equilibrium exists where that price has been increased and that leads to higher profits.

The highest welfare is achieved by any pooling equilibrium as all the consumers buy in these equilibria. The highest welfare among all the semi-separating equilibria is achieved by a semi-separating equilibrium where the low quality firms extract the full surplus of consumers with bad signals. These results are presented in the next corollary.

**Corollary 2** In the model with no advertising described above,

i. a pooling equilibrium leads to higher welfare than a semi-separating equilibrium,

ii. all the pooling equilibrium have the same welfare,

iii. the highest welfare among all the semi-separating equilibria is achieved by a semi-separating equilibrium satisfying \( p_L = E[q | s = 0, q \leq q_L] \).

**Proof** The total surplus in a pooling equilibrium is simply \( E[q] \) as all the consumers buy, while in a semi-separating equilibrium the total surplus is \( \int_{qL}^{qU} q f(q) dq + \int_{qL}^{\bar{q}} \alpha(q) q f(q) dq \leq \int_{qL}^{qU} q f(q) dq + \int_{qL}^{qU} q f(q) dq = E[q] \). Note that the inequality is an equality iff all the consumers of the high quality firms receive good signals, i.e. \( q > q_L \Rightarrow \alpha(q) = 1 \). The second part of the corollary is immediately derived from the last part of the proof of the previous corollary.
This section has illustrated how firms use price to signal their quality. A high quality firm can signal its type by choosing a price high enough to forgo the demand of consumers with bad signals while a low quality firm would never find it profitable. In the next section, I analyze how firms signal their quality when they can use not only their price but also their advertising decision to communicate with their customers.

1.4 Persuasive Advertising Case

In this section, I analyze the full model where firms choose both their advertising and their price. Two new insights about persuasive advertising are developed. First, persuasive advertising does not naturally signal high quality. Indeed when the price is high, a firm doing no persuasive advertising relies only on the intrinsic quality of its product to create good signals. And if the share of good signals has to be high to make that choice optimal, consumers understand that the product has a high quality. Secondly, persuasive advertising helps the monopolist extract rents. In the extreme case where the expected quality conditional on a signal does not depend on the value of the signal, the firm can extract the full surplus simply by charging a price equal to the expected quality. Nevertheless, when the expected quality conditional on a signal has some variance, the firm cannot extract the full surplus. The firm either sells to every consumer at a low price, leaving some surplus to the consumers with good signals, or it does not sell to consumers with bad signals, extracting only the surplus of consumers with good signals. When it is more profitable to sell only to consumers with good signals, persuasive advertising helps extract the surplus from more consumers.

An equilibrium is \(((p(q), a(q)), b(s, p, a), \mu(q|s, p, a))\), where \((p(q), a(q)) \in \mathbb{R}_+ \times \{\emptyset, P\}\) is the price and the advertising chosen by the firm as a function of its quality,\(^{11}\) \(b(s, p, a)\) is the decision of the consumers, buy or do not buy, and \(\mu(q|s, p, a)\) are the beliefs of the consumers when they receive signal \(s\) and observe price \(p\) and advertising \(a\). The beliefs are a probability distribution over the set of qualities. Note that the \(^{11}a = \emptyset\) represents no advertising while \(a = P\) represents persuasive advertising.
support \( A(p, a) \) of \( \mu(q|s, p, a) \) when \((p, a)\) is on the equilibrium path in a pure-strategy equilibrium does not depend on \( s \) as the choice of the firm \( p(q) \) is independent of the signals received by consumers and each signal occurs with a positive probability for every quality. Therefore, using a similar reasoning as the one in the no advertising case, firms either sell to everybody or only to consumers with good signals

\[
(p, a) \text{ is on the equilibrium path } \Rightarrow \begin{cases} 
    b(s, p, a) = \text{buy iff } s = 1 \\
    \text{or} \\
    b(s, p, a) = \text{buy for any } s 
\end{cases}
\]

Intuitively, firms face three kinds of demand: sell to all the consumers \( D = 1 \); sell to consumers with good signals with persuasive advertising \( D = \varphi(q) \); or sell to consumers with good signals without persuasive advertising \( D = \alpha(q) \). Obviously the highest price is always chosen when the demand is unchanged. Then the trade-off is between increasing the price and decreasing the demand. If these three kinds of demand occur in equilibrium, as they are ordered from the largest to the smallest, \( 1 \geq \varphi(q) \geq \alpha(q) \), they must be associated with prices with the reverse ordering.

The following lemma proves formally that a firm selling only to consumers with good signals without engaging in persuasive advertising chooses the highest price and that a firm selling to all the consumers chooses the lowest price. This lemma also shows that a firm selling only to consumers with good signals without engaging in persuasive advertising is part of the highest quality range and that a firm selling to all the consumers belongs to the lowest quality range.

**Lemma 2** In the model with persuasive advertising, in a pure-strategy equilibrium

i. at most 3 different prices are on the equilibrium path,

ii. if only the consumers with a good signal buy when they see \((p, 0)\) and if a firm with quality \( q \) chooses \((p, 0)\), then (1) all the firms with a higher quality than \( q \) choose \((p, 0)\) as well and (2) \( p \) is the highest price on the equilibrium path,

iii. if all the consumers buy when they see \((p, 0)\) and if a firm with quality \( q \) chooses
(p, θ), then (1) all the firms with a lower quality than q choose (p, θ) as well and
(2) p is the lowest price on the equilibrium path.

Proof Consider two firms q and q choosing (p, a) and (p, a) in equilibrium. If con-
sumers buy with any signal whenever they observe (p, a) or (p, a) then the prices must
be equal, otherwise the firm with the lower price could increase its profit by choosing
the same price and advertising as the other firm. Assume now that only consumers
with good signals buy when they observe (p, a) or (p, a). If the advertising choices
are equal, the prices must be equal, i.e. a = a ⇒ p = p. If the advertising choices are
different, wlog assume that a = θ and the profits of firm q after choosing (p, a)
must be at least as large as its profits if it had chosen (p, a), its no-deviation condition
is a(q)p ≥ a(q)p. Similarly for firm q, the no-deviation condition is a(q)p ≥ a(q)p.
Recall that the signal function p/a is non-increasing and larger than 1, therefore
these conditions imply that q > q and p > p. Finally, let’s consider the case where
only consumers with good signals buy when they observe (p, a) and all consumers
buy when they observe (p, a). The no-deviation conditions imply q > q and p > p.
Indeed firm q sells to a smaller share of consumers it must sell at a higher price. And
the share of consumers with good signals of firm q is too small to make the deviation
to a higher price profitable, in particular smaller than the share of consumers with
good signals of firm q. In the end, at most one price corresponds to firms selling to all
the consumers, and at most two prices correspond to firms selling only to consumers
with good signals with or without persuasive advertising. ■

Let me now describe how firms use persuasive advertising to signal their types.
Assume for the moment that a firm can choose to sell to consumers with good sig-
nals either at a high price without advertising or at a lower price with persuasive
advertising. The trade-off is once again between price and quantity. The firm chooses
no advertising and the high price if the proportional increase in quantity is smaller
than the proportional increase in price. Recall that \( \frac{\varphi(q)}{\alpha(q)} \), the share of good signals
with persuasive advertising relative to the share of good signals without persuasive
advertising, becomes smaller when the quality of the product increases. Therefore
by not engaging in persuasive advertising and choosing a high price, the high quality
firms can be separated from the firms engaging in persuasive advertising and selling at a lower price. In other words, forgoing the extra-demand created by persuasive advertising allow firms to signal their high quality. The exact nature of the equilibria are presented in the rest of the section.

According to the previous lemma, in an equilibrium with three different prices, the low quality firms choose the low price and no advertising, the intermediate quality firms choose the intermediate price and persuasive advertising while the high quality firms choose the high price and no advertising. In addition the low quality firms sell to all the consumers whereas the intermediate and high quality firms only sell to consumers with good signals. That equilibrium is characterized by three prices \((p_L, p_M, p_H)\) and two quality cutoffs \((q_L, q_M)\) such that the strategies and beliefs along the equilibrium path are

\[
\alpha(q) = \begin{cases} 
\emptyset & \text{if } q \leq q_L \\
 \delta & \text{if } q_L < q < q_M \\
\emptyset & \text{if } q_M \leq q 
\end{cases}
\]

\[
p(q) = \begin{cases} 
 p_L & \text{if } q \leq q_L \\
p_M & \text{if } q_L < q < q_M \\
p_H & \text{if } q_M \leq q 
\end{cases}
\]

\[
b(p_L, \emptyset, s) = \text{buy}, b(p_M, P, s) = b(p_H, \emptyset, s) = \begin{cases} 
 \text{don’t buy} & \text{if } s = 0 \\
 \text{buy} & \text{if } s = 1 
\end{cases}
\]

\[
\mu(q|p_L, \emptyset, s), \mu(q|p_M, P, s) \text{ and } \mu(q|p_H, \emptyset, s) \text{ are derived using Bayes’ rule}\textsuperscript{12}.
\]

Proposition 2 characterizes the set of necessary conditions of any equilibrium where persuasive advertising signals intermediate quality. The cutoff qualities are given by firms being indifferent between two strategies. And the prices are such that the purchasing decisions are rational. This equilibrium is shown graphically in Figure 1-4.

**Proposition 2** In the model with persuasive advertising, any equilibrium with 3 different prices satisfies the following set of conditions: the equilibrium is characterized

\textsuperscript{12}\(\mu(q|p_L, \emptyset, s) = \frac{q(q)(\varphi(q))^s}{\int_{q_L}^{q_M} g_M(q|s)dy} \text{ if } q \leq q_L, 0 \text{ otherwise, } \mu(q|p_M, P, s) = \frac{g_P(q|s)}{\int_{q_M}^{q_H} g_M(q|s)dy} \text{ if } q_L < q < q_M, 0 \text{ otherwise, } \mu(q|p_H, \emptyset, s) = \frac{g_H(q|s)}{\int_{q_M}^{q_H} g_M(q|s)dy} \text{ if } q_M \leq q, 0 \text{ otherwise, where } g_0(q|s) = \alpha(q)^{(1 - \alpha(q))^1-s} f(q) \text{ and } g_P(q|s) = \varphi(q)^{(1 - \varphi(q))^1-s} f(q).\)
by three prices \((p_L, p_M, p_H)\) and two quality cutoffs \((q_L, q_M)\) such that:

\[
p_L = \varphi(q_L)p_M \quad \text{and} \quad \varphi(q_M)p_M = \alpha(q_M)p_H
\]

\[
p_L \in \{q, E[q|\emptyset, 0, q < q_L]\}
\]

\[
p_M \in \{E[q|P, 0, q_L < q < q_M], E[q|P, 1, q_L < q < q_M]\}
\]

\[
p_H \in \{E[q|\emptyset, 0, q_M \leq q], E[q|\emptyset, 1, q_M \leq q]\},
\]

To be fully exhaustive, there also exists another set of conditions identical to the previous one except when \(q \leq q_L\), \(a(q) = P\) and \(p_L \in \{q, E[q|P, 0, q \leq q_L]\}\). When these conditions are satisfied, the previous ones are also satisfied.

These necessary conditions are also sufficient in the case of the worst out-of-equilibrium beliefs.

**Proof** The previous lemma implies that in an equilibrium with 3 prices, \(p_L < p_M < p_H\), the high quality firms, \(q_H \leq q\), choose the high price \(p_H\), no advertising and sell to consumers with good signals while the low quality firms, \(q \leq q_L\), choose the low price \(p_L\), no advertising and sell to every consumer. Hence the intermediate price \(p_M\) and persuasive advertising is chosen by the intermediate quality firms, \(q_L < q < q_M\), and only consumers with good signals buy. \(p_L = \varphi(q_L)p_M \quad \text{and} \quad \varphi(q_M)p_M = \alpha(q_M)p_H\) are the indifference conditions for cutoff firms \(q_L\) and \(q_M\) respectively. The beliefs of consumers are determined by Bayes’ rule on the equilibrium path. The inequalities on prices are derived from the rationality of the purchasing decision by consumers depending on their private signals and from the fact that prices have to be larger than \(q\) (otherwise firms could deviate, charge \(q\) and sell to every consumer).

In the equilibrium of Proposition 2, persuasive advertising signals the intermediate range of qualities. Indeed, firms have the choice between a high price with a regular demand or a medium price with an expanded demand. Moreover, the demand expansion, expressed as a share of the regular demand, becomes smaller as quality becomes larger.\(^{13}\) Hence the demand expansion created by persuasive advertising for

\(^{13}\)For example if the regular demand is \(\alpha(q) = .8\) the demand expansion cannot exceed 125% while if the regular demand is \(\alpha(q) = .5\) the demand expansion can go up to 200%.
Persuasive advertising case: Equilibrium with 3 Prices. Persuasive advertising with the intermediate price signals an intermediate range of qualities. No advertising with the low price signals low quality while no advertising with the high price signals high quality. By forgoing demand from consumers with bad signals, firms signal high quality. Top quality firms signal themselves from intermediate quality firms by not engaging in persuasive advertising. Consumers understand that the quality of a firm has to be high to be able to generate enough good signals without artificially creating more good signals with persuasive advertising.

The fact that persuasive advertising does not signal the high quality is in contrast with the previous literature about advertising as a means to burn money. If the content of the advertising is irrelevant and advertising matters only because it costs money, the firms with the higher profits can signal themselves by burning some of these profits as the firms with the lower profits cannot afford to burn the same amount of money. In that case, only high quality firms engage in advertising whereas the previous proposition shows that if the direct effect of advertising is taken into account, advertising signals an intermediate range of products and the high quality firms signal themselves by not engaging in persuasive advertising. The burning money story also implies that the relationship between price and advertising is monotonic, advertising
is associated with the highest price, while the previous proposition shows that a non-monotonic relationship between price and advertising can arise. No advertising is coupled with prices that are either higher or lower than the price associated with persuasive advertising.\textsuperscript{14}

I presented the equilibrium with three prices first as it includes the full set of regions. When some of these regions are empty, the equilibrium has one or two different prices on the equilibrium path. Persuasive advertising can signal high quality or signals low quality or signals nothing at all. An equilibrium, with persuasive advertising on the equilibrium path, is characterized by either 

i. two prices \((p_L, p_M)\) and one quality cutoff \(q_L\) such that:

\[
a(q) = \begin{cases} 0 & \text{if } q \leq q_L \\ P & \text{if } q_L < q \end{cases}, \quad p(q) = \begin{cases} p_L & \text{if } q \leq q_L \\ p_M & \text{if } q_L < q \end{cases}, \quad b(p_L, 0, s) = \text{buy}, b(p_M, P, s) = \begin{cases} \text{don't buy} & \text{if } s = 0 \\ \text{buy} & \text{if } s = 1 \end{cases}
\]

\(\mu(q|p_L, 0, s)\), and \(\mu(q|p_M, P, s)\) are derived using Bayes’ rule;

ii. two prices \((p_M, p_H)\) and one quality cutoff \(q_M\) such that:

\[
a(q) = \begin{cases} P & \text{if } q \leq q_M \\ 0 & \text{if } q_M < q \end{cases}, \quad p(q) = \begin{cases} p_M & \text{if } q \leq q_M \\ p_H & \text{if } q_M < q \end{cases}, \quad b(p_M, P, s) = b(p_H, 0, s) = \begin{cases} \text{don't buy} & \text{if } s = 0 \\ \text{buy} & \text{if } s = 1 \end{cases}
\]

\(\mu(q|p_M, P, s)\) and \(\mu(q|p_H, 0, s)\) are derived using Bayes’ rule;

iii. one price \(p_M\) such that:

\[
a(q) = P, \quad p(q) = p_M, \quad b(p_M, P, s) = \begin{cases} \text{don't buy} & \text{if } s = 0 \\ \text{buy} & \text{if } s = 1 \end{cases}
\]

\(\mu(q|p_M, P, s) = g_P(q|s)\).

\textsuperscript{14}It is difficult to come up with a clean example because of the monopoly assumption, nevertheless organic milk illustrate this result. Organic milk can be purchased in most of the supermarkets not just the ones specialized in organic food. It is not associated with any advertising and it is sold at a higher price than its closest substitute which is non-organic milk.
Proposition 3 gives some necessary conditions for any equilibrium with two different prices or a single price. In the first type of equilibrium, persuasive advertising signals high quality, while it signals low quality in the second type of equilibrium. The third type of equilibrium is a pooling equilibrium where all the firms engage in persuasive advertising. Finally the last types of equilibrium have no advertising occurring in equilibrium. In these last cases, advertising has no signaling value either because all the firms run a persuasive campaign or because none of them do.

**Proposition 3** In the model with persuasive advertising, any equilibrium with less than 3 different prices satisfies one of the following sets of conditions:

i. the equilibrium is characterized by two prices \((p_L, p_M)\) and one quality cutoff \(q_L\) such that:

\[
\begin{align*}
p_L &= \varphi(q_L)p_M \\
p_L &\in [q, E[q|\emptyset, 0, q \le q_L]] \\
p_M &\in (E[q|P, 0, q_L < q], E[q|P, 1, q_L < q])
\end{align*}
\]

ii. the equilibrium is characterized by two prices \((p_M, p_H)\) and one quality cutoff \(q_M\) such that:

\[
\begin{align*}
\varphi(q_M)p_M &= \alpha(q_M)p_H \text{ and } \varphi(q)p_M \ge q \\
p_M &\in (E[q|P, 0, q < q_M], E[q|P, 1, q < q_M]) \\
p_H &\in (E[q|\emptyset, 0, q_M \le q], E[q|\emptyset, 1, q_M \le q])
\end{align*}
\]

iii. the equilibrium is characterized by one price \(p_M\) such that:

\[
\begin{align*}
\varphi(q)p_M &\ge q \\
p_M &\in (E[q|P, 0], E[q|P, 1])
\end{align*}
\]
iv. conditions described in proposition 1.15

These necessary conditions are also sufficient in the case of the worst out-of-equilibrium beliefs.

This proposition is derived from lemma 2 in a similar way than proposition 2 is derived from lemma 2, therefore its proof is omitted.

In the equilibrium drawn on the left of Figure 1-5, persuasive advertising signals the high quality firms and is associated with the higher price while the low quality firms opt out of advertising and sell to the full market at the lower price. In the equilibrium drawn on the right of Figure 1-5, the reverse happens in terms of advertising signaling quality and pricing. Persuasive advertising signals the low quality firms and is coupled by the lower price while the high quality firms choose the higher price and no advertising.

Let me now come back on three important assumptions of the model, namely that in addition to the advertising decision, firms also choose their price, that advertising has no cost in this model, and that consumers are ex-ante homogeneous.

15To be fully rigorous, \( b(p, s) \) should be replaced by \( b(p, 0, s) \) and the expected qualities should also be conditional on no advertising.
Variable price

Assuming that firms choose both their advertising and their price is a key assumption in this model and reflects the fact that firms do use several instruments to communicate with their consumers. To illustrate this point, assume in this paragraph that prices are fixed and that the firm chooses only its advertising campaign. If the price is low, \( p \leq E[q|0,0] \), all the types of firms sell to all the consumers and do not engage in persuasive advertising. If the price is intermediate, \( E[q|0,0] < p < E[q|P,1] \), all the types of firms engage in persuasive advertising and sell to consumers with good signals. Persuasive advertising has no signaling power in that case, it is used only to take advantage of its demand expansion effect. Finally if the price is high, \( E[q|P,1] \leq p < E[q|0,1] \), all the types of firms choose not to do any persuasive advertising and are able to sell only to consumers with good signals. Indeed, the firm would like to expand its demand with persuasive advertising and have more consumers with good signals but it is not possible due to the negative direct effect of persuasive advertising. Recall that consumers are sophisticated and persuasive advertising lowers their willingness to pay. If the firm engages in persuasive advertising, the share of consumers will certainly be larger but none of them are willing to buy at that price. Therefore with a fixed price, advertising loses all its signaling power. It is important to study how firms use both instruments, advertising and price, when communicating with consumers.

Advertising costs

Advertising is assumed to come at no cost for the firms. If a small fixed cost for advertising is introduced in this model, the results are qualitatively unchanged. More specifically in a given type of equilibrium, the quality cutoffs would become larger and the prices would also be larger. By small cost, I mean that the cost does not completely overshadow the direct effect of advertising. Indeed if the cost becomes large, then the main effect is a burning money effect and the equilibria where persuasive advertising signals an intermediate quality or a low quality disappear. Firms with high quality have larger expected profits than firms with low quality and
they are willing to sacrifice some of their profits to signal their quality, whereas the cost of advertising is too large for low quality firms compared to their profits. In that case, advertising can only signal a high quality firm and is associated with a high price. I believe that advertising has to be more than a means to burn money as some advertising campaigns are famous for being a failure. They did not fail because consumers did not understand that they had cost a lot but they failed because of the message they were conveying or did not succeed to convey. Hence the message of the advertising is a strategic choice for firms.

**Consumer Homogeneity**

Consumers are supposed to be ex-ante identical. Once they receive their signal, the firm faces two types of consumers: the ones with a good prior about the product and the ones with a bad prior. If consumers are still assumed to all value quality but if their disutilities of price were drawn from a continuous distribution, the monopolist would be facing a continuous distribution of consumers instead of facing a distribution of consumers with two types. The drawback would be to introduce some equilibria where prices would be a continuous function of quality. The beliefs of consumers would be extremely sensitive to slight variations in prices, whereas the results presented above are still valid when the beliefs of consumers are the limit of beliefs when firms make some small errors when setting up their prices.

### 1.5 Uniform Distribution of Quality

In this section, I focus on the case of an uniform distribution of qualities coupled with a particular persuasive technology. More results are given about the sustainability of the different equilibria depending on the characteristics of the market. In particular, I analyze how the existence of an equilibrium depends on the degree of persuasion of the advertising technology.

Consider the model of the previous section with a uniform distribution of qualities between $q$ and $\bar{q}$, i.e. $f(q) = \frac{1}{\bar{q} - q}$. The signal without persuasive advertising is the
memory of the outcome of the last experience with the product, namely \( \alpha(q) = q \).
And persuasive advertising transforms a fixed share of bad experiential memories into good suggested memories, which simply leads to \( \varphi(q) = q + \theta(1-q) \). It is easily checked that the assumptions of the model are satisfied as \( \alpha(q) E[q|s = 1] = qE[q|s = 1] < q \), \( \frac{\varphi(q)}{\alpha(q)} = (1 - \theta) + \frac{\theta}{q} \) and \( \frac{1 - \varphi(q)}{1 - \alpha(q)} = 1 - \theta \) are non-increasing. The model has three parameters which are \( q \) and \( \bar{q} \) the range of qualities and \( \theta \) the degree of persuasion of the advertising technology.

In this section, I focus on the equilibria where the upper constraints on prices are binding, the prices are simply equal to the expected quality of the marginal consumers. And the worst out-of-equilibrium beliefs are used, that is to say, if an out-of-equilibrium action is observed, consumers believe they are facing the worst quality firm. If an equilibrium where persuasive advertising signals high quality firms does not exist with the worst out-of-equilibrium beliefs, it will not exist either with any other beliefs system. This is important for the following proposition which determines when persuasive advertising cannot signal the high quality firms. The next proposition shows that if the degree of persuasion is high, persuasive advertising cannot signal high quality. Indeed if a high share of bad signals are transformed into good signals, persuasive advertising has a homogenizing effect on firms, after engaging in advertising firms do not look that different as they mainly produce good signals. Therefore low quality firms and high quality firms have similar profits when engaging in persuasive advertising and as a consequence, persuasive advertising cannot signal high quality. The second reason why persuasive advertising cannot signal the high quality firms is if the range of quality is small and if the degree of persuasion is small. The intuition is similar as, if the range of quality is small, all firms generate a similar number of good signals. And if persuasive advertising has a small impact, even after engaging in persuasive advertising all the firms still have a similar share of good signals. Hence persuasive advertising does not help signal a high quality. The next proposition shows when the equilibrium where persuasive advertising signals the high quality does not exist. First let's define \( q^*(\bar{q}) = \frac{1}{4} \left( 3\bar{q}^2 - 2\bar{q} + 3 - \sqrt{9\bar{q}^4 + 12\bar{q}^3 - 42\bar{q}^2 + 12\bar{q} + 9} \right) \).

**Proposition 4** Consider the model with persuasive advertising with \( f(q) = \frac{1}{\bar{q}^2} \).
\( \alpha(q) = q \) and \( \varphi(q) = q + \theta(1-q) \).

There exist \( \theta^*(q, \bar{q}) \in (0, 1) \) and \( \theta^{**}(q, \bar{q}) \in [0, \theta^*(q, \bar{q})) \) such that persuasive advertising never signals the high quality if

\[
\theta < \theta^{**}(q, \bar{q}) \text{ or } \theta^*(q, \bar{q}) < \theta
\]

i. When \( \bar{q} < \frac{4-\sqrt{1}}{3} \) or \( \left( \frac{4-\sqrt{1}}{3} \leq \bar{q} \text{ and } q^*(\bar{q}) \leq \bar{q} \right) \), \( \theta^{**}(q, \bar{q}) > 0 \) and the equilibrium where persuasive advertising signals high-quality exists when

\[
\theta^{**}(q, \bar{q}) \leq \theta \leq \theta^*(q, \bar{q})
\]

ii. When \( \frac{4-\sqrt{1}}{3} \leq \bar{q} \) and \( q < q^*(\bar{q}) \), \( \theta^{**}(q, \bar{q}) = 0 \) and the equilibrium where persuasive advertising signals high-quality exists when

\[
\theta \leq \theta^*(q, \bar{q})
\]

**Proof** Equilibrium (i) in Proposition 1-5 is the only equilibrium where persuasive advertising signals the high quality firms. This equilibrium is sustainable iff there exists two prices \( (P_L, P_M) \) and one quality cutoff \( q_L \) such that \( P_L = \varphi(q_L)P_M \) and \( P_L = E[q|\emptyset, 0, q \leq q_L] \) and \( P_M = E[q|P, 1, q_L < q] \). Combining the three conditions leads to the condition: this equilibrium exists iff \( E[q|\emptyset, 0, q \leq q_L] = \varphi(q_L)E[q|P, 1, q_L < q] \) has a solution \( q_L \) in \( (q, \bar{q}) \). The LHS is an increasing function of \( q_L \) and it is concave as \( \frac{\partial^2 \varphi(q_L)E[q|P, 1, q_L < q]}{\partial q_L^2} = -\frac{4(1-q)^2}{3(1-\bar{q}+1-q_L)^3} \). The RHS is an increasing function of \( q_L \) and it is convex.\(^{16}\) Note that the LHS is actually independent of \( \theta \) and the RHS is increasing in \( \theta \).\(^{17}\) Besides when \( \theta = 1 \), no solution exists as the LHS is strictly smaller than the RHS. Indeed \( LHS = E[q|\emptyset, 0, q \leq q_L] \leq q_L < E[q|P, 1, q_L < q] = RHS|_{\theta=1} \). Let's

\[ \frac{\partial^2 \varphi(q_L)E[q|P, 1, q_L < q]}{\partial q_L^2} = \frac{\text{num}}{3((1-\theta)(\bar{q}+q_L)+2\theta)^3} \] where \( \text{num} \) is a polynomial function of degree 2 in \( \bar{q} \) with a positive coefficient in front of \( q^2 \). As \( \text{num}|_{q_L} \geq 0 \) and \( \frac{\partial \text{num}}{\partial q} \bigg|_{q_L} \geq 0 \), it implies \( \text{num} \geq 0 \) for any \( q_L \leq \bar{q} \) and any \( \theta \).

\[ \frac{\partial \text{RHS}}{\partial \theta} = \frac{\text{num}}{3(\bar{q})^2} \] where \( \text{num} \) is a polynomial function of degree 2 in \( \theta \) with a positive coefficient in front of \( \theta^2 \). As \( \text{num}|_{\theta=0} \geq 0 \) and \( \frac{\partial \text{num}}{\partial q} \bigg|_{\theta=0} \geq 0 \), it implies \( \text{num} \geq 0 \) for any \( q_L \) and any \( \theta \).
Figure 1-6: These figures illustrate when the equilibrium where persuasive advertising signals high quality exists. The figure on the left is drawn for $q = 0.25$ and the one on the right for $q = 0.75$. The region where persuasive advertising signals high quality is located in between the two curves. The upper range of quality $\bar{q} \in (q, 1)$ varies along the horizontal axis and the level of persuasion $\theta$ varies along the vertical axis.

look at what happens when $\theta = 0$. When $q_L$ tends to $q$, the LHS tends to $q$ and the RHS tends to $E[q|\theta, 1]$ which is smaller. When $q_L$ tends to $\bar{q}$, the LHS tends to $E[q|\theta, 0]$ and the RHS tends to $\bar{q}q$. Hence the LHS is larger than the RHS when $\theta = 0$ and when $q_L$ tends to $\bar{q}$ iff $E[q|\theta, 0] > \bar{q}^2 \iff \bar{q} > \frac{\bar{q}^2 + \bar{q}^2 + q^2}{3} - q^2(1 - \frac{\bar{q} + q}{2}) > 0$ which is a polynomial function of degree 2 in $q$. When $(\bar{q} < \frac{4 - \sqrt{7}}{3})$ or $(\frac{4 - \sqrt{7}}{3} \leq \bar{q} \text{ and } q^*(\bar{q}) \leq q)$, $E[q|\theta, 0] \geq \bar{q}^2$ which implies that no solution exists when $\theta = 0$. And when $\frac{4 - \sqrt{7}}{3} \leq \bar{q}$ and $q < q^*(\bar{q})$, $E[q|\theta, 0] < \bar{q}^2$ which implies that a unique solution exists when $\theta = 1$. Hence the results of the proposition. ■

The proposition states that for any distribution of quality, if the degree of persuasion is above a cutoff value, persuasive advertising cannot signal the high quality in equilibrium. When the range of quality is large, i.e. the highest quality is above $\frac{4 - \sqrt{7}}{3} \approx 0.45$ and the lowest quality is smaller than a threshold, persuasive advertising might signal high quality products for any degree of persuasion below the cutoff value. But, if the range of quality is small, persuasive advertising cannot signal the high quality in equilibrium when the degree of persuasion is below another cutoff value. In that case, persuasive advertising might signal high quality products only if the degree of persuasion is in between these two cutoff values. Figure 1-6 illustrates
Figure 1-7: These figures illustrate when the equilibrium where persuasive advertising signals low quality exists. The figure on the left is drawn for $q = 0.25$ and the one on the right for $q = 0.75$. The region where persuasive advertising signals low quality is located in between the two curves. The upper range of quality $q \in (q, 1)$ varies along the horizontal axis and the level of persuasion $\theta$ varies along the vertical axis.

when the equilibrium—where persuasive advertising signals high quality—exists or does not exist depending on the value of the highest quality $q$ (along the horizontal axis) and the degree of persuasion $\theta$ (along the vertical axis). The existence region is drawn for two values of the lowest quality $q = 0.25$ on the left and $q = 0.75$ on the right.

Similarly Figure 1-7 illustrates when the equilibrium—where persuasive advertising signals low quality—exists or does not exist depending on the value of the highest quality $q$ (along the horizontal axis) and the degree of persuasion $\theta$ (along the vertical axis). The existence region is drawn for two values of the lowest quality $q = 0.25$ on the left and $q = 0.75$ on the right. Recall that in this equilibrium all the firms sell only to consumers with good signals. Therefore when the degree of persuasion is low, by engaging in persuasive advertising the low quality firms do not get a large demand expansion and they prefer mimicking the high quality firms by selling at a higher price. This explains why persuasive advertising does not signal the low qualities when the degree of persuasion is low. On the other hand when the range of qualities is small, the share of good signals is similar across firms and high quality firms cannot signal themselves by not engaging in persuasive advertising.

The findings of this section are in sharp contrast with the results in the previous
literature. When advertising has no direct effect and is only a means to burn money, only the high quality firms signal their type with advertising and advertising is coupled with the highest price. But if the direct effect of advertising is taken into account and if consumers are sophisticated, high quality cannot be signaled with persuasive advertising when the degree of persuasion is high or when the range of quality is small. Furthermore, persuasive advertising does not necessarily signal low quality either, as when the degree of persuasion is low or when the range of quality is small, this advertising cannot signal the low quality products.

1.6 Two Types of Advertising

After studying how firms choose between persuasive advertising and no advertising, their decisions are analyzed when they have the choice between two types of advertising and no advertising. Informative advertising is introduced as a second type of advertising with a different direct effect on consumers than persuasive advertising.

I analyze the choice of a monopolist when it has a choice between two types of advertising that differ in how they affect consumers. Persuasive advertising interacts with the private signal received by consumers while informative advertising is a way to convey a piece of hard/verifiable information to consumers. Firms are assumed to choose only one type of advertising or not advertising at all.\footnote{This assumption is consistent with the fact that advertisements compete to grab the attention of consumers, therefore an ad conveys principally one message.} I start by presenting how informative advertising works in this model, then I analyze the equilibrium focusing on the interaction between persuasive advertising and informative advertising.

1.6.1 Informative Advertising

Suppose that prior to deciding on the advertising campaign, the monopolist does some marketing tests about its product, which might deliver a claim about the product. An example of a claim is “this toothpaste was clinically proven to fight germs for twelve hours”. The probability $P(\sigma = 1|q)$ that a firm gets a positive claim is an
increasing function of the product quality, where $\sigma = 1$ represents the fact that a claim was produced and $\sigma = 0$ means no claim was created. Note that for both types of advertising, the effect of advertising depends on the quality of the product. The share of extra good signals or the number of extra good signals created by persuasive advertising varies with the underlying quality of the product. Similarly, the probability to generate a positive claim about the product also depends on the quality of the product. The crucial point is that they affect consumers differently. Informative advertising adds an element to the information set of consumers while persuasive advertising changes the information set without allowing consumers to know what their information would have been without persuasive advertising. Hence informative advertising has only a positive direct effect—increase in willingness to pay—while persuasive advertising has both a positive and a negative direct effect—demand expansion and decrease in willingness to pay—.

The only difference between this model and the model analyzed in Section 1.4 is that when the firm has a positive claim it now chooses between informative advertising or persuasive advertising or no advertising while a firm with no claim chooses between persuasive advertising or no advertising. The timing of the model is unchanged and I analyze pure-strategy perfect Bayesian Nash equilibrium.

### 1.6.2 Equilibrium Analysis

An equilibrium is $((p(q, \sigma), a(q, \sigma)), b(s, p, a), \mu(q|s, p, a))$, where $(p(q, \sigma), a(q, \sigma)) \in \mathbb{R}_+ \times \{\emptyset, P, I\}$ is the price and the advertising chosen by the firm as a function of its quality and its claim status, $b(s, p, a)$ is the decision of the consumers, it is either buy or do not buy, and $\mu(q|s, p, a)$ are the beliefs of the consumers when they receive message $s$ and observe price $p$ and advertising $a$. Using the same reasoning as in the previous models, on the equilibrium path in a pure-strategy equilibrium, firms either

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19 $a = I$ represents informative advertising.
sell to all the consumers or only to the consumers with good signals:

\[(p, a) \text{ is on the equilibrium path } \Rightarrow \begin{cases} b(s, p, a) = \text{buy iff } s = 1 \\ \text{or} \\ b(s, p, a) = \text{buy for any } s \end{cases}\]

In equilibrium, only five situations can arise. In two cases, firms choose a price high enough to sell only to consumers with good signals and engages in either informative advertising or no advertising. These prices are the highest and consumers believe the quality of the firm is high. Conversely in two other cases, firms choose a price low enough to sell to everybody and engage also in either informative advertising or no advertising. These prices are the lowest and consumers believe the quality is low. In the last case, persuasive advertising is chosen and firms sell only to consumers with good signals. These results are described formally in the next lemma.

**Lemma 3** In the model with persuasive and informative advertising, in a pure-strategy equilibrium,

i. at most 5 different prices are on the equilibrium path,

ii. if only the consumers with a good signal buy when they see \((p, I)\) and if a firm with quality \(q\) chooses \((p, I)\), then (i) all the firms with a higher quality than \(q\) and \(\sigma = 1\) choose \((p, I)\) as well and (ii) \(p\) is the highest price on the equilibrium path,

iii. if only the consumers with a good signal buy when they see \((p, 0)\) and if a firm with quality \(q\) chooses \((p, 0)\), then (i) all the firms with a higher quality than \(q\) and \(\sigma = 0\) choose \((p, 0)\) as well and (ii) the only price on the equilibrium path that can be higher than \(p\) is a price \(p'\) such that only the consumers with a good signal buy when they see \((p', I)\),

iv. if all the consumers buy when they see \((p, 0)\) and if a firm with quality \(q\) chooses \((p, 0)\), then (i) all the firms with a lower quality than \(q\) and \(\sigma = 0\) choose \((p, 0)\) as well and (ii) \(p\) is the lowest price on the equilibrium path,
v. if all the consumers buy when they see \((p, I)\) and if a firm with quality \(q\) chooses \((p, I)\), then (i) all the firms with a lower quality than \(q\) and \(\sigma = 1\) choose \((p, I)\) as well and (ii) the only price on the equilibrium path that can be lower than \(p\) is a price \(p'\) such that all the consumers buy when they see \((p', \emptyset)\).

The proofs of the previous lemma and the next proposition are omitted as they are derived by using a reasoning very similar to the ones of lemma 2 and proposition 2.

The previous lemma shows that an equilibrium with five different prices is characterized by five prices \((p_L^0, p_L^1, p_M, p_H^0, p_H^1)\) and four quality cutoffs \((q_L^0, q_L^1, q_M^0, q_M^1)\) such that along the equilibrium path

\[
a(q, 0) = \begin{cases} 
\emptyset & \text{if } q \leq q_L^0 \\
P & \text{if } q_L^0 < q < q_M^0, \ a(q, 1) = \begin{cases} 
I & \text{if } q \leq q_L^1 \\
P & \text{if } q_L^1 < q < q_M^1, \\
\emptyset & \text{if } q_M^1 \leq q 
\end{cases}
\end{cases}
\]

\[
p(q, \sigma) = \begin{cases} 
p_L^0 & \text{if } q \leq q_L^0 \\
p_M & \text{if } q_L^0 < q < q_M^0, \ \text{if s} = 1 \\
p_H^0 & \text{if } q_M^0 \leq q 
\end{cases}
\]

\[
b(p_M, P, s) = b(p_M^0, \emptyset, s) = b(p_M^1, I, s) = \begin{cases} 
don't \text{ buy} & \text{if } s = 0 \\
\text{buy} & \text{if } s = 1 
\end{cases}
\]

\(\mu\) on the equilibrium path is derived using Bayes' rule.

The low quality firms sell to all the consumers at a low price. If the firm has a claim, it reveals it through informative advertising and is able to charge a higher price than the firms with no claim, which chooses not to advertise. The intermediate

\[
\mu(q|p_L^0, \emptyset, s) = \frac{g_B(q|s, \sigma)}{\int_l^{q_L} g_B(y|s, \sigma)dy} \quad \text{if } q \leq q_L^0, \ 0 \ \text{otherwise}, \quad \mu(q|p_M, P, s) = \frac{g_B(q|s, \sigma=0)}{\int_l^{q_L} g_B(y|s, \sigma=0)dy + \int_q^{q_M} g_B(y|s, \sigma=0)dy} \quad \text{if } q_L^0 < q < q_L^1 \ \text{and } q_M^1 < q < q_M^0, = \frac{\varphi(q)^*(1-\varphi(q))}{\int_l^{q_L} g_B(y|s, \sigma=0)dy + \int_q^{q_M} g_B(y|s, \sigma=0)dy} \quad \text{if } q_L^1 < q < q_M^1, \ \text{otherwise}, \quad \mu(q|p_H^0, \emptyset, s) = \frac{g_B(q|s, \sigma=0)}{\int_l^{q_L} g_B(y|s, \sigma)dy} \quad \text{if } q_M^0 < q, \ 0 \ \text{otherwise}, \text{ where } g_B(q|s, \sigma) = \alpha(q)^*(1-\alpha(q))^{1-\sigma}(1-\eta(q))^\sigma f(q) \text{ and } g_B(q|s, \sigma) = \varphi(q)^*(1-\varphi(q))^{1-\sigma}(1-\eta(q))^\sigma f(q)}
quality firms choose an intermediate price, engage in persuasive advertising and sell only to consumers with good signals. Finally, the high quality firms with a claim choose an informative campaign and charge the highest price, while the high quality firms with no claim signal their type with a high price and no advertising. The following proposition gives the set of necessary conditions for an equilibrium with five different prices on the equilibrium path. The cutoff qualities are given by firms being indifferent between two strategies. And the prices are such that the purchasing decisions are rational. That equilibrium is also depicted graphically in Figure 1-8.

**Proposition 5** In the model with persuasive or informative advertising, any equilibrium with 5 different prices satisfies the following set of conditions: there exists five prices $(p_L^0, p_L^1, p_M^0, p_H^0, p_H^1)$ and four quality cutoffs $(q_L^0, q_L^1, q_M^0, q_M^1)$ such that\(^2\)

\[
\begin{align*}
p_L^0 &= \varphi(q_L^0)p_M^0 \text{ and } \varphi(q_M^0)p_M^0 = \alpha(q_M^0)p_H^0 \\
p_L^1 &\in [q, E[q|\emptyset, \sigma, 0, q \leq q_L^0]] \\
p_M^0 &\in (E[q|\{P, 0, 0, q_L^0 < q < q_M^0\} \cup \{P, 1, 0, q_L^1 < q < q_M^1\}], \\
&\quad E[q|\{P, 0, 1, q_L^0 < q < q_M^0\} \cup \{P, 1, 1, q_L^1 < q < q_M^1\}]], \\
p_H^0 &\in (E[q|\emptyset, \sigma, 0, q_M^0 \leq q], E[q|\emptyset, \sigma, 1, q_M^0 \leq q]) \\
p_H^1 &\in (E[q|\emptyset, \sigma, 0, q_M^0 \leq q], E[q|\emptyset, \sigma, 1, q_M^0 \leq q])
\end{align*}
\]

The low quality firms choose a low price, $p_L^0$ if they choose no advertising or or $p_L^1$ if they reveal a positive claim through informative advertising, these firms sell to all the consumers. The intermediate quality firms run a persuasive campaign and choose an intermediate price $p_M^0$, they sell only to consumers with good signals. The high quality firms also sell only to consumers with good signals, they choose a high price $p_H^0$ and no advertising if they have no claim to reveal and they choose the highest price $p_H^1$ and informative advertising if they can.

The set of firms with a claim choosing persuasive advertising over informative advertising is strictly included in the set of firms with no claim and engaging in persuasive advertising, in terms of quality cutoffs this translates into $q_L^0 < q_L^1 < q_M^0 < q_M^1 < q_H^0$.\(^2\)

\(^2\)The ordering of the elements in the conditional expectations is $a, \sigma, s, q$. For example $E[q|\emptyset, 0, 0, q \leq q_L^0]$ represents the expected quality when $a = \emptyset$, $\sigma = 0$, $s = 0$ and $q \leq q_L^0$. 

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\[ q_0^0. \] Indeed, the low price for a firm without a claim is lower than the low price for a firm with a claim as the firm with a claim can always forgo informative advertising and mimic the firm without the claim. On the other hand, with or without a claim there is a single price associated with persuasive advertising. Therefore the profit of the indifferent firm without a claim between no advertising with the lowest price and persuasive advertising with the intermediate price is equal to \( p_l^0 \) which is smaller than

![Diagram](image)

**Figure 1-8:** On the top graph, the pricing and advertising choices of the firm are drawn with respect to its quality when the firm does not have a positive claim about its product. On the bottom graph, the pricing and advertising choices of the firm are drawn with respect to its quality when the firm has a positive claim about its product.
$p_L$, hence the quality of the indifferent firm with a claim is higher than the quality of the indifferent firm without a claim. The intuition is similar in the case of the cutoff qualities between persuasive advertising with the intermediate price and no advertising with the high price.

It is striking that some firms with a claim choose persuasive advertising; these firms have a claim to reveal but choose not to reveal it to consumers. In this equilibrium, the information does not fully unravel. Consumers do not know whether a firm doing persuasive advertising has a claim or not. On the other hand, consumers rationally infer that a firm engaging in no advertising has no positive claim to reveal.

This equilibrium also illustrates the important role played by prices. When consumers observe an informative campaign, they cannot distinguish between the low and the high qualities. They need to check the price to be able to form their final beliefs. Depending on the level of the price, they think the product is from the bottom quality group or from the top quality group.

Finally, these results show that no clear ordering exists between the two types of advertising. Persuasive advertising can signal a higher or a lower quality than informative advertising.

1.7 Conclusions

Manufacturers of personal care products spent more than 5 billions dollars in 2003 on advertising in the United States,\textsuperscript{22} total advertising spending across all categories and media was estimated at 245 billions dollars. Firms have large advertising budgets and their choice of advertising is an important strategic decision. Advertising has not only a direct effect on consumers such as associating freshness to the product but consumers also use advertising to infer some information about the product, such as, for example, the high quality firms never engage in persuasive advertising.

I analyzed how firms can signal their product quality when consumers are sophisticated and understand that they can be affected by the persuasive side of advertising.

\textsuperscript{22}see Adage special report of June 28, 2004.
I found that no monotonic relationship between advertising and quality exists nor between advertising and price level. Depending on the market conditions, persuasive advertising can signal the low quality firms, the high quality firms and even an intermediate range of qualities. In particular, if the persuasion technology is very effective or if the range of qualities is small, persuasive advertising cannot signal high quality as firms engaging in persuasive advertising look very similar from the consumers point of view. When firms choose between several types of advertising, consumers draw inferences about what type of advertising was not chosen in addition to what type was chosen. A firm, with a positive claim about its product, might choose not to reveal the claim in order to signal a higher quality and achieve a higher profit.

These results were found by developing a model where persuasive advertising has a direct effect on consumers. Unlike in the previous literature where persuasive advertising was included as a direct element in the utility of consumers or where it shifted their preferences, persuasive advertising, in this paper, affects the memory of consumers in a way that is consistent with results found in the psychology and marketing literature on postexperience advertising.

The question of the effects of competition has yet to be studied in this context. It will raise some interesting issues about the effect on consumers of mentioning a competitive product in an advertisement. In particular, is this always a signal that the competitive product has a higher quality or will it depend on the advertising type chosen by the competitor?
Chapter 2

Corruption: Selection Effects in Popular Control

"A message [...] is that urban as well as rural resident have trouble being heard [...] and new institutional arrangements are evolving to overcome this problem"

2003 World Development Report

2.1 Introduction

Fighting corruption is a recurrent concern, specially in developing countries. Past empirical studies report mixed results about the effect of anti-corruption policies on corruption incidence. In this paper, we analyze how implementing a complaint system can help curb corruption in a bureaucracy by focusing on the role played by the incentives both at the villagers level and at the bureaucrats level. Furthermore, we investigate whether facilitating complaining always helps in fighting corruption and increasing social welfare.

In our model, we consider a central authority that supervises bureaucrats who are in charge of decision making in villages. The central authority wants the bureaucrats to maximize the total welfare of the villagers and instructs them to do so. However, bureaucrats make their decision in order to maximize their own utility. Some
bureaucrats are honest and will never accept a bribe that will affect their decisions, while other bureaucrats are dishonest and might allow bribes to alter their decisions. The central authority is concerned with the welfare loss due to corruption but does not know which bureaucrats are dishonest. Since all the villagers know whether the bureaucrat in their village is dishonest, the central authority wants to curb the corruption level by allowing the villagers to give her feedbacks about their bureaucrat. A negative feedback is called a complaint and a positive feedback a praise. The idea behind this system is that the poor villagers harmed by corruption could complain against a corrupt bureaucrat. The downside of it is that rich villagers willing to bribe dishonest bureaucrats could also threaten to complain against honest bureaucrats, as messages are soft information and villagers can lie. Additionally, if bureaucrats get some disutility from complaints, it might also lead honest bureaucrats to deviate from choosing the welfare maximizing decision because of the fear of a complaint. Therefore we study the overall effect of implementing a complaint system by focusing on the incentives of the villagers in deciding whether to send a message and on the incentives of the bureaucrats in making their decision. We focus on two types of complaint system. The first one is designed around self-selection where all the villagers can send messages about the bureaucrat, and will choose to do so if the benefit of a message is larger than the cost of sending it. The second system is designed around invitation, in which only selected villagers may send costly messages.

In the self-selection model, we find that the central authority can focus on the types of messages that are sent, complaint or praise. She can ignore whether she has received more than one complaint or more than one praise. If the central authority chooses a message cost sufficiently high, it is equivalent to shutting down the complaint system. We call the resulting equilibrium the no-voice equilibrium, and an equilibrium is considered better than the no-voice equilibrium if its expected surplus is higher. Interestingly, in such an equilibrium, no complaints are sent on the equilibrium path. The intuition behind that result comes from the changes in the incentives of bureaucrats induced by the complaint system. Now, on top of choosing the project, they also want to avoid complaints. If a villager sends a complaint on the equilibrium
path, he believes that he would be better off with the other type of bureaucrat, which means that the two types of bureaucrat choose two different projects, which is the case only if bribery has occurred.

One important insight in this paper is that it is not always optimal to make the complaint system completely costless. Indeed, the central authority would like the poor villagers to report back to her but cannot distinguish their messages from the messages sent by the rich villagers. Ideally, she would like to incite the poor villagers to report and discourage the rich villagers to do so. Because messages are soft information, both types of villager get the incentive to complain with a low message cost. Therefore, a low cost of complaint might not be optimal when the rich villagers get too much incentive to complain against an honest bureaucrat.

We also show that when the proportion of dishonest bureaucrats increases, it might be optimal to make complaining more costly. Without any complaint system, the rich villager has no leverage over the project chosen by an honest bureaucrat. Because an honest bureaucrat dislikes receiving a complaint about his work, the complaint system enables the rich villager to threaten to complain against the optimal project which changes the incentives of the honest bureaucrat. With a large proportion of dishonest bureaucrats, the likelihood of an honest bureaucrat being replaced by a dishonest bureaucrat is high and the likelihood of a dishonest bureaucrat being replaced by an honest bureaucrat is low. In this situation, the poor villagers benefit less from transferring a dishonest bureaucrat and the rich villagers benefit more from firing an honest bureaucrat, making the poor villagers less credible than the rich villagers in their respective threats to file complaints. Hence when the proportion of dishonest bureaucrat is high, the optimal system might be to shut down the complaint system, i.e. equivalent to make complaining extremely costly.

In the model with invited feedback, the central authority can exploit the fact that the poor villagers are more numerous than the rich villagers, by controlling how many messages the villagers can send. More specifically, each villager gets one invitation to send a single message. The idea behind this scheme is that the poor villagers can differentiate themselves from the rich villagers. Indeed if all the poor villagers send
the same message, the rich villagers cannot mimic them as they are not allowed to send as many messages. We investigate what happens when the central authority transfers the bureaucrat only when she believes that all the poor villagers agreed to complain with costless messages. As long as it is optimal not to distort the choice of honest bureaucrats, that complaint system is optimal if one ignores the coordination issues. Indeed, such a system creates a coordination problem, since if a poor villager believes that some poor villagers never complain, then the unique equilibrium is the no-voice equilibrium.

This paper relates to the large theoretical literature on corruption. Besley and McLaren (1993) studies whether efficiency wages is the best wage scheme to curb corruption when both moral hazard (observation of bribery requires costly monitoring) and adverse selection (honest or dishonest is private information) are present. Banerjee (1997) presents a theory of misgovernance explaining the occurrence of corruption and how the presence of poor people exacerbates it. The literature is discussed in detail by Laffont and Tirole (1993) and Banerjee, Benabou, and Mookherjee (2006). Our paper also relates to the literature on corruption and decentralization such as Bardhan and Mookherjee (2000). Banerjee (2007) reviews the anti-corruption policies. And finally, Olken (2007) investigates the accuracy of citizen perception of corruption.

The rest of the paper is organized as follows. The next section presents the model with self-selection. Section 2.3 presents the results about the optimal complaint system. Then Section 2.4 presents the model and the results in the case of invited complaints. Some concluding remarks are offered in Section 2.5. All the proofs can be found in the Appendix.

2.2 The Model

We consider a village where a bureaucrat has to decide between three alternative locations to build a public infrastructure. The locations are ranked according to the total surplus they create. The optimal location leads to the highest total surplus $2\xi$,
the neutral location to an intermediate total surplus $2\lambda \xi$, and the suboptimal location to the smallest total surplus $2\tau \xi$. Moving away from the optimal location reduces the total surplus, which is captured by the inequalities $1 > \lambda > \tau > 0$.

Two villagers live in the village, one is poor and the other is rich. Each villager gets half of the total surplus of the project and, if the project favors one villager over the other one, the favored villager gets an extra utility $\delta$ while the disadvantaged villager loses $\delta$. This extra utility or disutility can represent how convenient the public infrastructure is to use. For example, villagers benefit from having a water well close to their home. The optimal location favors the poor villager while the suboptimal location favors the rich villager. And the neutral location does not favor any villagers. The poor villager preferences are therefore aligned with the total surplus ranking. He prefers the optimal location (utility $\xi + \delta$) over the neutral one (utility $\lambda \xi$) over the suboptimal one (utility $\tau \xi - \delta$). The rich villager has reversed preferences as he prefers the suboptimal location (utility $\tau \xi + \delta$) over the neutral one (utility $\lambda \xi$) over the optimal one (utility $\xi - \delta$). The ranking of the three alternative locations is common knowledge in the village. Table 2.1 summarizes the utility levels of the villagers for the three different locations.

<table>
<thead>
<tr>
<th>Total villagers</th>
<th>Optimal</th>
<th>Neutral</th>
<th>Suboptimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor villager</td>
<td>$\xi + \delta$</td>
<td>$\lambda \xi$</td>
<td>$\tau \xi - \delta$</td>
</tr>
<tr>
<td>Rich villager</td>
<td>$\xi - \delta$</td>
<td>$\lambda \xi$</td>
<td>$\tau \xi + \delta$</td>
</tr>
</tbody>
</table>

Table 2.1: Utility levels for the three locations

The bureaucrat can be of two types, either honest or dishonest. An honest bureaucrat never accepts a bribe and cares about the social value of the project. A dishonest bureaucrat can be bribed to choose a specific location. If he is not corrupt, he cares about the social value of the project. When corrupt, he only cares about the project he has been bribed to implement. The bribe can be interpreted as how much he needs not to care about the social value of the project. The type of the bureaucrat is common knowledge in the village. Only the rich villager can pay a bribe $b$, the poor villager does not have enough resources to pay a bribe. Besides we assume that
the rich villager is always willing to bear the bribery cost to get a better location for him. In other words, the rich villager is ready to pay a bribe to go from the optimal project to the neutral project and to go from the neutral project to the suboptimal one. Therefore bribery would occur with a dishonest bureaucrat.

The bureaucrat is supervised by a central authority—CA thereafter—. The CA is in charge of thousands of villages and does not know much about them. In particular, she does not know the relative wealth of the villagers, the ranking of the locations and whether a specific project has been implemented or not. The bureaucrat is the link between the CA and the village. But the CA does not know the type of the bureaucrat. She only knows $p$ the proportion of honest bureaucrats in the administration. The objective of the CA is to maximize the total surplus of the villager. If the CA simply appoints a bureaucrat in the village and let him choose the location, an honest bureaucrat will choose the optimal location while a dishonest bureaucrat will be bribed by the rich villager to choose the suboptimal location. As the villagers know whether corruption occurs but the CA does not, the CA wants to design a complaint system to curb corruption in villages.

2.2.1 Self-selection complaint system

The villagers can choose to send some messages to the CA about the bureaucrat. Messages can either be a complaint such as "the bureaucrat has been bribed" or a praise such as "the bureaucrat has not been bribed". Messages are anonymous from the point of view of the CA as she does not know who sent it and who is rich or poor. Messages are soft information, the villagers can lie about what the bureaucrat did. There is plenty of evidence that people do lie in their complaints. The head of anti-corruption in Sri Lanka was quoted in the (Sri Lanka) Sunday Observer saying "... we also found that some complaints on bribery and corruption were based on personal animosity." In this paper, we take into account the fact that villagers can lie and we assume that it would be too costly for the CA to enforce a repressive policy.

\footnote{This implies that the rich villager is also willing to bribe to go from the optimal location to the suboptimal location.}
This model is called the self-selection complaint system as any villager can send a message about the bureaucrat to the CA, the only requirement is to pay the message cost $\Psi$. In principle, one villager could send several messages. Assume instead of one poor villager, several poor villagers live in the village but that the total of the surplus for the poor villagers is unchanged and divided equally among them. If the CA wanted to use the number of complaints versus praises to infer the messages sent by the poor villagers, then the rich villager could send several identical messages to take advantage of the beliefs of the CA. Consequently, the principal ideas of a self-selection system can be captured by considering a village where the poor villagers do not outnumber the rich villagers and each villager is allowed to send only one message per bureaucrat. In the last section of this paper, we study a complaint system based on invitation, where a villager cannot send a message without having received an invitation to do so and therefore the difference in the proportion of poor versus rich villagers is explicitly modeled.

The bureaucrat does not care about the messages per se, but he dislikes being transferred as this affects his career negatively. A bureaucrat wants first to minimize the probability of being replaced and then, conditional on that probability, he cares about the type of projects. In that sense, his preferences are lexicographic.

Given the limited knowledge of the CA about what is going on in the village, she has only one way to intervene which is to replace the bureaucrat by another bureaucrat drawn at random from the population of bureaucrats. Replacing a bureaucrat takes time, consequently the villagers have to wait at least to the next period to benefit from the project. We call $\theta$ the discount factor between two periods.

The CA chooses both the cost of complaining $\Psi$ and the decision rule that reacts to the number and the type of messages. So the CA picks the probabilities that the bureaucrat is transferred conditional on the reports on him. A priori she has six probabilities to choose: no message $\pi_0$, one complaint $\pi_C$, one praise $\pi_P$, two complaints $\pi_{CC}$, two praises $\pi_{PP}$ and one complaint coupled with one praise $\pi_{CP}$. We assume that the CA does not pay attention to the projects with no messages, i.e.

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2The subscript $P$ stands for praise while the subscript $C$ stands for complaint.
\( \pi_0 = 0. \) We will see that many of those probabilities are irrelevant in pure-strategy equilibrium.

### 2.2.2 Timing

The central authority chooses first the cost of sending a message \( \Psi \) and the probabilities of transferring the bureaucrat depending on the messages sent by the villagers. Then begins the stage game which is composed of five stages: it begins with the arrival of a new bureaucrat in the village, followed by the potential bribery, the project announcement, the potential messages and finally the potential transfer of the bureaucrat. The bribing stage occurs only if the bureaucrat is known to be dishonest. The rich villager can offer a bribe in return for choosing a particular project. The dishonest bureaucrat decides whether to accept the bribe or not. Then the bureaucrat announces his choice of project. If the bureaucrat is corrupt, he announces what the briber wants. During the message stage, both villagers can send a message simultaneously, i.e. they do not observe whether the other is sending a message when they send one, but in equilibrium they will infer the correct action. Finally if the central authority has received a message, she can decide to transfer the bureaucrat. The CA fires the bureaucrat according to the probabilities chosen at the beginning. The stage game is repeated until a bureaucrat is not transferred. If the bureaucrat is still in place, the announced project is implemented and the game ends.

### 2.2.3 Equilibria

We are solving for a subgame perfect Nash equilibrium, with stationary pure-strategy Nash equilibria in the stage game, i.e. the strategies do not depend on messages in the previous periods if the game is repeated.

**Strategies**

An honest bureaucrat simply chooses a project. The strategy of an honest bureaucrat is the project he announces \( a^h \in \{O, N, S\} \). A dishonest bureaucrat has to
choose a project if he is not corrupt and has to decide whether to accept a bribe to announce the neutral project and whether to accept a bribe to announce the suboptimal project.\(^3\) The strategy of a dishonest bureaucrat is \((a^d, b_N, b_S)\), where \(a^d \in \{O, N, S\}\) is the announced project with no bribe and \((b_N, b_S) \in \{yes, no\}\) are the answers to the bribe offers towards the neutral and suboptimal projects respectively. For example, \((O, yes, no)\) means that without a bribe the bureaucrat announces the optimal project, he accepts a bribe to implement the neutral but not to implement the suboptimal project.

At the message stage, the future does not depend on the type of the bureaucrat. Hence the message depends on the nature of the announced project but not on the type of the bureaucrat. Both villagers decide which message to send depending on the announced project. The strategy of the poor villager is \((m_p^O, m_p^N, m_p^S)\) where \(m_p \in \{C, P, \emptyset\}\) are the messages he sends depending on the announcement. The rich villager has to decide if he wants to offer a bribe for the suboptimal project, a bribe for the neutral project or no bribe at all. The strategy of the rich villager is \((bribe, m_r^O, m_r^N, m_r^S)\) where \(m_r \in \{C, P, \emptyset\}\) are the messages and \(bribe \in \{\emptyset, N, S\}\) is the bribe offer. For example, \((S, C, C, P)\) means the rich villager offers a bribe to implement the suboptimal project, he complains whenever the optimal or neutral project is announced and praises if the suboptimal project is announced. We assume without loss of generality that the rich villager does not offer a bribe if it will be rejected and that he never complains against a bureaucrat he has bribed.

### 2.3 Optimal Complaint System

The villagers' antagonist preferences allow the CA to reduce the number of probabilities to optimize over. Consider, for example, the case where one villager would like to get a new bureaucrat and the other would like him to stay. If a single message is sent in equilibrium, it is sent by the villager trying to replace the bureaucrat, as the bureaucrat will never want to pay a bribe to get the optimal project, which he dislikes the most.
The bureaucrat is not fired when no messages are sent. The anti-bureaucrat villager would select the message with the higher probability of transfer when sent alone. Hence, to respect the taxonomy of complaint versus praise, we can safely assume \( \pi_P \leq \pi_C \). If two messages are sent in equilibrium, the anti-bureaucrat villager and the pro-bureaucrat send the messages \( m^{\text{Anti}} \) and \( m^{\text{Pro}} \) respectively. The anti-bureaucrat villager is willing to bear the message cost if it increases the chance of transfer, while the pro-bureaucrat villager is willing to bear that cost if it decreases the chance of transfer. Therefore in order for the villagers to send two messages, the probability of transfer after both messages has to be larger than the probability of transfer with only \( m^{\text{Pro}} \) but smaller than the probability of transfer with only \( m^{\text{Anti}} \), i.e. \( \pi_{m^{\text{Pro}}} < \pi_{m^{\text{Anti}} m^{\text{Pro}}} < \pi_{m^{\text{Anti}}} \).

And given the ranking of the transfer probability with a single message, \( \pi_P \leq \pi_C \), it means their messages are different with the anti-bureaucrat villager complaining and the pro-bureaucrat villager praising. The following lemma shows that a praise tempers the effect of a complaint while a complaint stiffens the effect of a praise. It also shows that the CA can ignore how many times she receives the same message, in particular one complaint or two complaints have the same effect.

**Lemma 4** *Without loss of generality, the CA can optimize over three probabilities* \( \pi_P, \pi_C, \text{ and } \pi_{CP} \) *such that*

\[
\pi_P < \pi_{CP} < \pi_C
\]

*and choose* \( \pi_{PP} = \pi_P \) *and* \( \pi_{CC} = \pi_C \).

### 2.3.1 Equilibrium properties

A single praise is thus less effective than a single complaint in getting the bureaucrat fired. And when the other villager also sends a message, sending a praise never increases the transfer probability. Therefore if a villager wishes the bureaucrat were replaced, he will never want to praise for him. Only a villager who wants to keep his current bureaucrat would praise his behavior. As, firstly the bureaucrat is sure to stay if the CA receives no message, and secondly sending a message is costly, a single praise cannot be part of a subgame perfect Nash equilibrium. In addition, sending
a second praise is costly and has no effect, therefore two praises cannot be part of a subgame perfect Nash equilibrium either. The following lemma states that a villager praises only if the other villager complains.

**Lemma 5** After a project is announced, a villager praises only if the other villager complains.

The poor villager never wants to complain against the optimal project as that project leads to the best outcome for the poor villager. Therefore the previous lemma implies that the rich villager never wants to praise for the optimal project. Similarly the rich villager never wants to complain against the suboptimal project and therefore the poor villager never wants to praise for the suboptimal project. When the bureaucrat announces the optimal or the suboptimal project, the only possible messages part of a subgame perfect equilibrium are no messages, a single complaint, or a complaint with a praise, see next lemma.

**Lemma 6** If the optimal project or the suboptimal project is announced, the disadvantaged villager either complains or sends no message, and the favored villager either praises or sends no messages. In any subgame perfect equilibrium,

\[(m^p_0, m^r_0) \text{ and } (m^s_0, m^p_0) \in \{(\emptyset, \emptyset), (\emptyset, C), (P, C)\}\]

At the beginning of a stage game, when the new bureaucrat has not arrived to the village yet, the expected utility of each villager is independent of the past. Each time a bureaucrat is transferred, the villagers know the values of these expected utilities. If the poor villager is willing to wait to get his expected utility when the neutral project has been announced, he is willing to wait also when the suboptimal project is announced as his utility from the suboptimal project is lower than his utility from the neutral project. If the rich villager sends the same message in the neutral and suboptimal cases, the effect of complaining is the same in both cases and the cost is also identical. Hence if it is optimal for the poor villager to complain against the neutral project, it is also optimal to complain against the suboptimal case. This result
is formalized in the following lemma where three similar results are also given. If the poor villager sends the same message in the neutral and optimal case, and if it is optimal for the rich villager to complain against the neutral project then it is optimal to complain against the optimal one. Similarly, if the poor villager is not willing to wait to get his expected utility when the optimal project has been announced, he is not willing to wait also when the neutral project is announced as his utility from the neutral project is lower than his utility from the optimal project. If the rich villager sends the same message in the optimal and neutral cases, the effect of praising is the same in both cases and the cost is also identical. Hence if it is optimal for the poor villager not to praise for the optimal project, it is also optimal not to praise for the neutral case. And if the poor villager sends the same message in the neutral and suboptimal case, and if it is optimal for the rich villager not to praise for the suboptimal project then it is also optimal not to praise for the neutral one.

Lemma 7

i. If the rich villager sends the same message in the neutral and suboptimal case, and if the poor villager complains against the neutral project then he also complains against the suboptimal one:

if \( m_N^r = m_S^r \) then \( m_N^p = C \Rightarrow m_S^p = C \).

ii. If the poor villager sends the same message in the neutral and optimal case, and if the rich villager complains against the neutral project then he also complains against the optimal one:

if \( m_N^p = m_O^p \) then \( m_N^r = C \Rightarrow m_O^r = C \).

iii. If the rich villager sends the same message in the neutral and optimal case, and if the poor villager does not praise for the optimal project then he does not praise for the neutral one:

if \( m_N^r = m_O^p \) then \( m_O^p \neq P \Rightarrow m_N^p \neq P \).
iv. If the poor villager sends the same message in the neutral and suboptimal case, and if the rich villager does not praise for the suboptimal project then he does not praise for the neutral one:

\[ m^N_N = m^P_S \quad \text{then} \quad m^*_S \neq P \Rightarrow m^*_N \neq P. \]

As bureaucrats want to minimize their chance of being transferred before they care about the nature of the project, the nature of the messages on the equilibrium path sheds light on the nature of the messages out of the equilibrium path and also on whether a dishonest bureaucrat is corrupt. The first part of the next lemma states that if a complaint is on the equilibrium path, there is nothing a bureaucrat can do to avoid being complained against. Indeed if a bureaucrat can choose a project with no messages, he will do so as according to lemma 5 either no messages are sent or at least one complaint is sent in equilibrium. By consequence if a complaint is sent on the equilibrium path, all three projects must generate complaints. Besides if a villager is willing to bear the cost of complaining, this means that he prefers the delayed expected utility of starting over with a random bureaucrat over keeping the current bureaucrat. If both bureaucrat announce the same project, in equilibrium there is no gain for a villager from getting another bureaucrat who will do exactly the same but later. Hence if a villager complains on the equilibrium path, we can conclude that both bureaucrats announce two different projects. As an uncorrupt dishonest bureaucrat has the same strict preferences as an honest bureaucrat, they would choose the same project. Therefore the dishonest bureaucrat must be corrupt when a complaint is on the equilibrium path. In that case, the dishonest bureaucrat does not announce the optimal project and the honest bureaucrat does not choose the suboptimal as bribery would not occur otherwise.

Consider now the case where both a complaint and a praise are on the equilibrium path. We have just seen that this implies that all the projects would generate at least a complaint and that the bureaucrats choose different projects. As a praise tempers the effect of a complaint, see lemma 4, a bureaucrat never chooses a project generating a single complaint as he prefers a project generating both a complaint and a praise.
Thus the CA will receive a complaint and a praise for both bureaucrats. Suppose, now, that the honest bureaucrat chooses the neutral project in such an equilibrium. He did not choose the optimal project implies that the optimal project generates a single complaint, sent by the rich villager. It also means that the dishonest bureaucrat announces the suboptimal project, as they choose two different projects with both a complaint and a praise. Hence the rich villager complains against both the neutral and the optimal projects and the poor villager praises the neutral project but sends no message for the optimal project. This contradicts Lemma 7 showing that the poor villager would also praise the optimal project. The dishonest bureaucrat cannot choose the neutral project either for similar reasons. Consequently when both a complaint and a praise are on the equilibrium path, the honest bureaucrat chooses the optimal project and the dishonest bureaucrat the suboptimal one.

Finally, due to the tempering effect of praising, a bureaucrat never chooses a project creating a single complaint when there exists another project generating both a complaint and a praise. The next lemma states those results.

Lemma 8

i. If a complaint is on the equilibrium path, whatever the bureaucrat does a complaint will be sent, both types of bureaucrat announce a different project, and the dishonest bureaucrat is corrupt.

ii. If a complaint with a praise are on the equilibrium path, the honest bureaucrat announces the optimal project and the dishonest bureaucrat announces the suboptimal project.

iii. If a single complaint is sent on the equilibrium path, whatever the bureaucrat does a single complaint will be sent.

This lemma—part i.—shows that a message on the equilibrium path entails bribery on the equilibrium path. In other words, an equilibrium with no bribery on the equilibrium path means no messages on the equilibrium path. Nevertheless, the CA cannot conclude that receiving a message means that bribery has occurred. It
simply means that if the bureaucrat were dishonest then he would be corrupt. But
the bureaucrat could be honest and the message could have been sent by the rich
villager.

When both bureaucrats announce the same project, incurring the cost of sending
a message entails no gain as the same project would be chosen eventually. So on the
equilibrium path, the villagers do not send any messages. Both bureaucrats choose
the same project also imply that the other projects would generate a complaint.
Assume that this was not the case, and that two projects generate no messages. The
honest bureaucrat would choose the project with no message that create the larger
total surplus. And the rich villager could bribe a dishonest bureaucrat to choose
the project with no message that favors him the most, and the dishonest bureaucrat
would accept the bribe. Of course that project would be different from the one chosen
by an honest bureaucrat. The following lemma shows these results.

**Lemma 9**

i. If both bureaucrats announce the optimal project, no message is sent in equilib-
rium and the poor villager complains against both the neutral project and the
suboptimal projects,

\[ m^p_0 = m^o_0 = 0 \text{ and } m^p_0 = m^p_s = C. \]

ii. If both bureaucrats announce the suboptimal project, no message is sent in equi-
librium and the rich villager complains against both the neutral project and the
optimal projects,

\[ m^s_0 = m^s_s = 0 \text{ and } m^o_0 = m^r_s = C. \]

iii. If both announce the neutral project, no message is sent in equilibrium and
the rich villager complains against the optimal project while the poor villager
complains against the suboptimal project,

\[ m^r_0 = m^r_s = 0 \text{ and } m^p_0 = m^p_s = C. \]

The CA can always decide to ignore the messages she receives. Or she could set
up a message cost so high that no villager can afford to send a message. We call the
resulting equilibrium the no-voice equilibrium. In that case an honest bureaucrat picks up the optimal project and the dishonest one is corrupt to implement the suboptimal project. And of course no messages are sent either on or off the equilibrium path.\footnote{In the rest of the paper, we sometimes abuse notation and also call a no-voice equilibrium an equilibrium with the same equilibrium path as the no-voice equilibrium.}

The following proposition states that any equilibrium of interest, doing better than the no-voice equilibrium—with a larger total expected surplus—, has no complaint on the equilibrium path. The intuition is derived from lemma 8. Consider an equilibrium that has a complaint on the equilibrium path, two cases can occur. (i) This equilibrium may have a single complaint after all the projects. And in that case, an honest bureaucrat chooses the optimal project while a dishonest bureaucrat is bribed to choose the suboptimal project. That outcome is clearly worse than the no-voice equilibrium as it generates some costly complaints without improving anything. (ii) Or this equilibrium might have a complaint coupled with a praise on the equilibrium path. Once again it is worse than the no-voice equilibrium as the bureaucrats have the same strategies in both equilibria but now the villagers incur the cost of complaining and praising.

Proposition 6 Any equilibrium better than the no-voice equilibrium has no complaint on the equilibrium path.

2.3.2 Equilibria not dominated by the no-voice equilibrium

As the CA is always able to implement the no-voice equilibrium, we focus on the equilibria that are better than the no-voice equilibrium. Proposition 6 shows that these equilibria have no message sent on the equilibrium path. The no-voice equilibrium can be improved along two dimensions: corruption and project selection. In an equilibrium with no corruption, both bureaucrats choose the same project, as an uncorrupt dishonest bureaucrat and an honest bureaucrat have the same strict preferences. Three equilibria without corruption are possible, as the bureaucrats can announce one of the three projects. In the first-best equilibrium, both bureaucrats announce the optimal project and there is no corruption on the equilibrium path. Obviously,
the CA would like to implement that equilibrium as corruption has disappeared and both bureaucrats make the optimal decision. In the neutral-project equilibrium, both bureaucrats announce the neutral project and there is no corruption on the equilibrium path. Corruption has disappeared and dishonest bureaucrats announce the neutral project which is better than the suboptimal one. All this comes at a cost, which is a distortion in the incentives of honest bureaucrats. They now choose the neutral project in order to avoid complaints as rich villagers would complain after the optimal project. This equilibrium is better than the no-voice equilibrium if the distortion of the choice of honest bureaucrats is smaller than the gains of alleviating corruption and improving the choice of dishonest bureaucrats. This is the case only if the proportion of honest bureaucrats, $p$, is not too large. In the suboptimal-project equilibrium, both bureaucrats announce the suboptimal project and there is no corruption on the equilibrium path. In this equilibrium, no corruption occurs as well. But in terms of project selection, there is no improvement for dishonest bureaucrats and the choice of honest bureaucrats is completely distorted. By fear of complaints, they choose the worst project. Therefore the rich villager does not even need to pay a bribe to get their favorite project. This equilibrium constitute an improvement also if the proportion of honest bureaucrat is low. And finally, an equilibrium with corruption can be better than the no-voice equilibrium if dishonest bureaucrats choose the neutral project instead of the suboptimal one. In the corrupt-to-neutral equilibrium, the honest bureaucrat announces the optimal project while the dishonest bureaucrat is corrupt to announce the neutral project. This equilibrium is obviously better than the no-voice equilibrium.

The first-best equilibrium cannot exist if the poor villager is too impatient. Indeed, the first-best is supported by the threat of the poor villager to complain against the non optimal projects. But if the poor villager prefers the neutral project in the current period over the optimal project in the following period, he cannot credibly threaten to complain against the neutral project. The exact necessary condition for the existence of the first best equilibrium is given in the next lemma.
Lemma 10 A necessary condition for the existence of the first-best equilibrium is that the poor villager prefers the optimal project later to the neutral project right away:

\[ \theta \geq \frac{\lambda \xi}{\xi + \delta} \]

In the next proposition, we characterize the second-best equilibrium when the first-best equilibrium does not exist, i.e. when the poor villager prefers the neutral project to waiting for the optimal project. As both villagers derive the same utility from the neutral project and the utility of the poor villager for the optimal project is the largest, the rich villager prefers the neutral project over waiting for the suboptimal project. Thus when the first-best equilibrium does not exist, the suboptimal-project equilibrium does not exist either. The second-best equilibrium is the no-voice equilibrium, the neutral-project equilibrium or the corrupt-to-neutral equilibrium. In the neutral equilibrium, the rich villager threatens to complain against the optimal project which is true only if he prefers the neutral project later to the optimal project right away. Similarly, the corrupt-to-neutral equilibrium exists if the poor villager prefers the neutral project later to the suboptimal project right away. Indeed, even if the new bureaucrat is dishonest, the poor villager is better off than keeping the suboptimal project. Besides, if that is not the case but if the poor villager prefers the optimal project later to the suboptimal project right away, then the corrupt-to-neutral equilibrium exists only if the proportion of honest bureaucrat is large enough. Indeed, if the next bureaucrat is dishonest and chooses the neutral project, the poor villager would rather get the suboptimal project now, but if the next bureaucrat is honest and announces the optimal project, the poor villager prefers waiting. When both the neutral and the corrupt-to-neutral equilibria exist, the neutral equilibrium is better only if the proportion of honest bureaucrat is low. Intuitively, in the neutral equilibrium, the loss comes from the honest bureaucrats not choosing the optimal project which is small when there are few honest bureaucrats, while in the corrupt-to-neutral equilibrium, the loss comes from the bribery which is small when there are few dishonest bureaucrats. The second best equilibrium in terms of the discount
factor—or how patient the villagers are—is depicted in Figure 2-1.

Proposition 7 When the first-best equilibrium never exists, $\lambda \xi > \theta (\xi + \delta)$,

i. if the poor villager prefers the suboptimal project right away to the optimal project later, $\tau \xi - \delta \geq \theta (\xi + \delta)$, the second best is the no-voice equilibrium

ii. if the poor villager prefers the optimal project later to the suboptimal project right away to the neutral project later, $\theta (\xi + \delta) > \tau \xi - \delta \geq \theta \lambda \xi$, the second best is

- the no-voice equilibrium if the proportion of honest bureaucrat is low
- the corrupt-to-neutral equilibrium if the proportion of honest bureaucrat is high

iii. if the poor villager prefers the neutral project later to the suboptimal project right away and the rich villager prefers the optimal project right away to the neutral project later, $\xi - \delta \geq \theta \lambda \xi > \tau \xi - \delta$, the second best equilibrium is the corrupt-to-neutral equilibrium

iv. if the rich villager prefers the neutral project later to the optimal project right away, $\theta \lambda \xi > \xi - \delta$, the second best equilibrium is

- the neutral-project equilibrium if the proportion of honest bureaucrat is low
- the corrupt-to-neutral equilibrium if the proportion of honest bureaucrat is high

In the corrupt-to-neutral equilibrium, the dishonest bureaucrat does not accept a bribe to implement the suboptimal project. The complaint system is fulfilling its role in the sense that it limits the impact of corruption. It does not erase it completely though. In the neutral-project equilibrium, no bribery occurs but both bureaucrats choose the neutral project. By fear of facing a complaint, the dishonest bureaucrat does not accept a bribe. But this comes at the cost of distorting the incentives of the honest bureaucrat. By introducing the feedback system, the honest bureaucrat also cares about the messages sent about him, which gives some leverage to the rich bureaucrat. Indeed, without any feedback system, the rich villager could not influence
Figure 2-1: Second-best equilibrium as a function of the discount factor \( \theta \)–reflecting how patient the villagers are–when \( \lambda \) is large enough. When \( \lambda \) becomes smaller, the top region ceases to exist.

the decision of the honest bureaucrat at all, whereas he can threat to complain against him once the complaint system is set up.

Ideally, the CA would like to encourage the poor villager to send messages and would like to discourage the rich villager to send anything. This would allow the CA to find out whether corruption happened. But, due to the preferences of the villagers, the complain system gives the same kind of incentives to both villagers. For example, if sending a message is cheap, both villagers tend to send complaints and praises. Consequently, encouraging the poor villager to report bribery also encourages the rich villager to lie and claim the optimal project is the result of bribery. If the CA had more tools, and not just the option of firing the bureaucrat, she would be able to achieve a higher surplus. Indeed, she could try to infer not only the type of the bureaucrat but also the type of the villager because the messages could contain the level of interest in the project in addition to the level of corruption. And based on the implementation literature, see Maskin (1999), the truth would be revealed. Nevertheless the CA needs to be able to reward or punish the villagers to achieve that goal. Fighting strategic
false reporting happens in reality but is difficult. Hong Kong has something called ICAC Complaints Committee, which investigates complaints against those who accuse government officials of corruption. Singapore has a law: "Any person who makes a complaint against anyone or gives any information in writing knowing this to be false, commits an offence under Section 28 of the Prevention of Corruption Act and will not go unpunished. Such a person when convicted, can be fined up to $10,000 or jailed up to one year, or both. The CPIB (Corrupt Practices Investigation Bureau) takes a very serious view of malicious complaints and will spare no effort to trace the "poison-pen writer" and prosecute him." Our paper studies the case when such investigations are too costly or not reliable and the CA cannot reward or punish the villagers directly, so her only leverage is to be able transfer bureaucrats.

Simply setting up a complaint system and claiming that it helps curbing corruption might not be true. As was previously noted, if complaining is cheap and easy, the rich villager will have more leverage against a honest bureaucrat by credibly threatening to complain. And the resulting equilibrium could be worse than the equilibrium without any complaint system. This is the case when the second-best is the corrupt-to-neutral equilibrium and when the rich villager is willing to gamble on getting a dishonest bureaucrat in the next period. The non-optimality of always choosing a costless complaint system is stated in the next proposition.

**Proposition 8** Setting the cost of complaint to zero may not be optimal.

And finally, when the level of dishonesty in the bureaucrat population increases, it is not necessarily optimal to make complaining easier. Indeed the incentives of the villager depends on the degree of dishonesty of the bureaucrat population. A poor villager has more incentives to complain against a dishonest bureaucrat if the probability of getting an honest bureaucrat in the following period increases. And the incentives of the rich villager work the other way, the rich villager has fewer incentives to complain against an honest bureaucrat if the chance of getting a dishonest bureaucrat decreases. As the CA would like to encourage the poor villager to report and would like to discourage the rich villager to send a message, the complaint system
works better when the proportion of honest bureaucrat increases. The next proposition shows that, when the proportion of dishonest bureaucrat increases, it might be optimal to shut down the complaint system.

**Proposition 9** It might be optimal to make complaining more costly when the proportion of dishonest bureaucrat increases: the optimal \( \frac{w}{ \pi_c} \), ratio of cost of complaint over probability of firing the bureaucrat after a single complaint, can be decreasing with respect to \( p \), the proportion of honest bureaucrat.

### 2.4 Selection Effects

In this section, we investigate the case of villages with more poor inhabitants than rich ones. If anybody can send a message to the CA, the rich villagers could send several messages, and the CA would not be able to rely on the number of complaints to infer the wealth level of the villagers who sent them. Nevertheless, if a villager needs an invitation in order to send a message, the CA could bar the rich villagers from mimicking the poor population by controlling the number of messages that they can send.

#### 2.4.1 The Model

The model is very similar to the one in the previous section. The main difference is that more poor villagers live in a village than rich villagers; three villagers live in the village, two are poor and one is rich. The total surplus of each project remains the same: \( 2\xi \) for the optimal project, \( 2\lambda \xi \) for the neutral one and \( 2\tau \xi \) for the suboptimal one. The preferences of the rich villager are also unchanged, he prefers the suboptimal location (utility \( \tau \xi + \delta \)) over the neutral one (utility \( \lambda \xi \)) over the optimal one (utility \( \xi - \delta \)). The utility of a poor villager is now half what it was with a single poor villager. Each poor villager gets \( \frac{\xi + \delta}{2} \) for the optimal project, \( \frac{\lambda \xi}{2} \) for the neutral project and \( \frac{\tau \xi - \delta}{2} \) for the suboptimal one. Thus the preferences of poor villagers are still aligned with the social preferences, while the rich villager has reversed preferences. Table 2.2
summarizes the utility levels of the villagers for the three different locations.

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|}
\hline
 & Optimal & Neutral & Suboptimal \\
\hline
Total villagers & $2\xi$ & $2\lambda \xi$ & $2\tau \xi$ \\
Poor villager & $\frac{\xi + \delta}{2}$ & $\lambda \xi$ & $\frac{\xi - \delta}{2}$ \\
Rich villager & $\xi - \delta$ & $\lambda \xi$ & $\tau \xi + \delta$ \\
\hline
\end{tabular}
\caption{Utility levels for the three locations}
\end{table}

The rest of the model and the timeline remains the same.

2.4.2 Equilibrium with Invitations

The CA can exploit the fact that the poor villagers outnumber the rich villagers, thanks to the limited number of messages, by ignoring any messages that are not sent by all the poor villagers. For example, if the CA receives two complaints, she believes that the poor villagers sent them, and that belief will be correct in equilibrium. If that is the case, the CA can ignore any single message, fire a bureaucrat after at least 2 complaints and not fire him after at least 2 praises. The following proposition lists all the subgame perfect Nash equilibria, with stationary pure-strategy Nash equilibria in the stage game, when the CA chooses these probabilities and a message cost close to zero.

**Proposition 10** If the CA ignores any single message, fires a bureaucrat after at least 2 complaints and does not fire him after at least 2 praises and chooses a message that is almost free, the only equilibria are the first best equilibrium, the corrupt-to-neutral equilibrium, and the no-voice equilibrium.

\[
\begin{cases}
\pi_C = \pi_P = \pi_{2P} = \pi_{3P} = \pi_{2P,C} = 0 \\
\pi_{2C} = \pi_{3C} = \pi_{2C,P} = 1 \\
\psi = 0
\end{cases}
\]

i. the first best equilibrium exists when the poor villagers prefer the optimal project later than the neutral project right away, $\theta(\xi + \delta) > \lambda \xi$;
ii. *the corrupt-to-neutral equilibrium exists when the poor villager prefers the neutral project right away to expectation of the optimal or the neutral project later to the suboptimal project right away, \( \lambda \xi > \theta(p\xi \delta) + (1 - p)\lambda \xi > \tau \xi - \delta; \)

iii. *the no-voice equilibrium exists when the poor villager prefers the suboptimal project right away to the optimal project later, \( \tau \xi - \delta > \theta(\xi + \delta)\).

Firstly, when the corrupt-to-neutral equilibrium is the second best, it will exist when the complaint cost is very small. This is to be contrasted with the self-selection model where the corrupt-to-neutral equilibrium is not always sustainable for a small complaint cost. Nevertheless, a coordination issue arises as both villagers have to complain in that equilibrium. Both poor villager have to believe that the other one will complain, otherwise they might play the no-voice equilibrium. Secondly, the neutral-project equilibrium does not exist with the complaint system if Proposition 10. Hence when the second-best equilibrium is the neutral-project equilibrium, giving full power to the poor villagers is not optimal. And designing the optimal complaint system with invitation has the similar problem as designing the optimal complaint system with self-selection. The CA want to encourage the villagers to complain in some circumstances but discourage them in others, which is difficult as the same incentives are given to the villagers in both cases.

Interestingly, varying the degree of selection—who is allowed to send a message—changes the impact of the complaint system. With costless messages in both cases, the equilibrium with invitation can only be an improvement over the no-voice equilibrium whereas the equilibrium with self-selection can be worse than the no-voice equilibrium.

### 2.5 Concluding Remarks

Complaint systems have been implemented in different countries in the world. Many states in India are adopting web-based complaint systems, e.g. www.keralavigilance.org. The World Bank financed randomized evaluation in Indonesia of complaint boxes. This paper investigates the effectiveness of complaints against corruption. In partic-
ular, we emphasizes the role played by incentives as introducing a scheme to fight corruption can potentially change them. The CA would like to get a feedback from the poor villagers, since they are the ones suffering from the corruption. The difficulty is to encourage them to report back to the CA without also allowing the rich villagers to do the same. Consequently, lowering the cost of sending a message may not be optimal. That is true in particular when the proportion of dishonest bureaucrats is high, because the rich villagers are more eager to transfer an honest bureaucrat than the poor villager are to transfer a dishonest bureaucrat. Indeed the probability of getting a dishonest bureaucrat in the next period is high, which is good for the rich villagers but unfortunate for the poor villagers. In addition to the incentives of the villagers, the incentives of the bureaucrats play also a key role. Restricting the incentive of dishonest bureaucrats to accept bribes by fear of being transferred also affects the incentives of an honest bureaucrat. Without the threat of transfer, a rich villager had no leverage over the choice of an honest bureaucrat. Once the complaint system is introduced, the actions of honest bureaucrats can be distorted if rich villagers can credibly threaten to complain against them.

Finally, we contrast the effectiveness of different complaint systems in fighting corruption depending on their level of selection. The first model studies a system with self-selection. As the villagers can send several messages, the CA cannot infer who complains depending on how many complaints have been sent. In the second model, we investigate the effectiveness of a complaint system based on invitations. A villager needs an invitation in order to be able to send his feedback. We explicitly take into account that the poor villagers outnumber the rich villagers and that the CA can use that information. Costless messages can only improve the situation in the invited complaint system, even though the second-best equilibrium might not be sustainable, whereas costless messages may hurt in the self-selected complaint system. Therefore, our model sheds some light on the fact that empirical studies on anti-corruption policies have mixed result depending on their degree of selection, for a discussion on this topic see Banerjee (2007).
2.6 Appendix

Proof of Lemma 4 Let’s look at the three possible cases at the message stage, i.e. after a project has been announced. (i) One villager would like to fire the bureaucrat and the other one would like to keep him. Let’s call for the purpose of this proof the pro-bureaucrat villager Mr. Pro and the anti-bureaucrat villager Mr. Anti. If Mr. Pro does not send a message, the best response of Mr. Anti might be to send a message to try to replace the bureaucrat. In that case, he would send the message with the highest firing probability as all messages have the same cost. In order to respect the taxonomy, we call the higher one \( \pi_C \) so that \( 0 \leq \pi_P \leq \pi_C \). At most one complaint and no praise is sent in equilibrium except when \( \pi_P < \pi_{CP} < \pi_C \) and \( \pi_{PP} \leq \pi_{CP} \leq \pi_{CC} \). (If \( \pi_P = \pi_C \), Mr. Anti praising and Mr. Pro sending nothing could also be an equilibrium). Therefore the only interesting case is when \( \pi_P < \pi_{CP} < \pi_C \) and \( \pi_{PP} \leq \pi_{CP} \leq \pi_{CC} \), and the three possible candidates are Mr. Pro sending nothing and Mr. Anti sending nothing or complaining, or Mr. Pro praising while Mr. Anti complains. (ii) When both villagers want the bureaucrat to stay, \( \pi_{CP} < \pi_{CC} \) implies that the only candidates are no message or two praises. (iii) When both villagers want to get a new bureaucrat,\(^5\) \( \pi_{PP} < \pi_{CP} \) implies that the only candidates are no message, one complaint or two complaints. As \( \pi_{PP} \) and \( \pi_{CC} \) are irrelevant both in and out of equilibrium, wlog we choose them equal to \( \pi_P \) and \( \pi_C \) respectively. ■

Proof of Lemma 5 Derived from the fact that all the messages have the same cost and that \( \pi_P < \pi_C \) and \( \pi_{PP} = \pi_P \). ■

Proof of Lemma 6 The optimal project is the best outcome the poor villager can get. He does not want the bureaucrat to be transferred. Sending a complaint has either no effect on the probability of firing the bureaucrat or increases this probability. And sending a complaint is costly. Therefore the poor villager never complains if the project is optimal. As a praise can only be a best response to a complaint (see lemma 5), the rich villager never praises the optimal project. The three equilibrium

\(^5\)This can be the case only out-of-the-equilibrium path.
candidates are derived from the best responses. A similar reasoning leads to the result about the suboptimal project.

**Proof of Lemma 7** In a stationary equilibrium, each time a bureaucrat is transferred, the villagers know the values of their expected utilities. If the poor villager is willing to wait to get this expected utility when the neutral project has been announced by the current bureaucrat, he is willing to wait also when the suboptimal project is announced as his utility from the suboptimal project is lower than his utility from the neutral project. The poor villager can increase the probability of firing the bureaucrat by complaining. If the rich villager sends the same message in the neutral and suboptimal cases ($0$ or $P$ see lemma 6), the effect of complaining is the same in both cases and the cost is also identical. Hence if it is optimal for the poor villager to complain against the neutral project, it is also optimal to complain against the suboptimal case. The proof of the other three results is similar.

**Proof of Lemma 8** (i) Given the preferences of the bureaucrats, if some projects will induce a complaint while some projects will not, the bureaucrats pick a project with no complaint. Hence if on the equilibrium path a complaint is sent, it means that the bureaucrat was not able to avoid that complaint by announcing another project: whatever project the bureaucrat chooses, one villager will complain. On top of that, in a stationary equilibrium, if both bureaucrats announce the same project, sending a message is costly and does not change anything for the villagers. Hence if a villager complains on the equilibrium path, we can conclude that both bureaucrats announce two different projects. An uncorrupt dishonest bureaucrat has exactly the same preferences than an honest bureaucrat. And those preferences are strict, there is no indifference. Therefore they cannot choose two different projects. If a dishonest bureaucrat is willing to choose another project, this means he has been corrupt to do so.

(ii) If a complaint associated with a praise is on the equilibrium path, the dishonest bureaucrat is corrupt and picks a project for which the poor villager complains and the rich villager praises, while the honest bureaucrat chooses another project for which the poor villager praises and the rich villager complains. For each project, one villager
sends a complaint. If the poor villager is the one sending the complaint against the neutral project, as he is already sending a complaint against the suboptimal project, the rich villager will necessarily praise for the suboptimal project see lemma 7. This implies that the dishonest bureaucrat accepts a bribe to implement the suboptimal project. Finally the poor villager praises the optimal project while the rich villager complains against it, and the honest bureaucrat chooses the optimal project. Similarly if the rich villager is the one sending the complaint against the neutral project, as he is already sending a complaint against the optimal project, the poor villager praises for the optimal project see lemma 7. This implies that the honest bureaucrat implements the optimal project. And the dishonest bureaucrat is bribed to choose the suboptimal project and the rich villager praises for it.

(iii) If a single complaint is on the equilibrium path, that project is associated with the highest transfer probability and the bureaucrat could not choose a project with a lower transfer probability. All the projects generate a single complaint. ■

Proof of Lemma 9 If both types of bureaucrats announce the same project, there is no point in sending a costly message. An uncorrupt dishonest bureaucrat has the same preferences than an honest bureaucrat. Both bureaucrats choosing the same project implies that the dishonest bureaucrat is not corrupt, as the rich villager would be bribing him to do what he would have done in any case. If several projects generate a zero transfer probability, the rich bureaucrat could bribe a dishonest bureaucrat to choose the project among these projects which is best for him and that would be different from the project chosen by an honest bureaucrat. ■

Proof of Proposition 6 If a single complaint is on the equilibrium path, lemma 8 shows that a single complaint happens whatever the bureaucrat does. In that case, the honest bureaucrat chooses the optimal project and the dishonest bureaucrat is corrupt to implement the suboptimal project. This is clearly inferior to the no-voice equilibrium. Lemma 8 shows also that an equilibrium with a praise and a complaint on the equilibrium path is clearly inferior to the no-voice equilibrium. ■

Proof of Lemma 10 From lemma 9, the poor villager complains against the neutral project in the first-best equilibrium. This is optimal only if the poor villager prefers
Proof of Proposition 7 We assume that the poor villager prefers the neutral project right away to the optimal project later. Therefore the rich villager prefers the neutral project right away to the suboptimal project later and the suboptimal-project equilibrium does not exist. The rich villager cannot credibly threaten to complain against the neutral project. (i.) The poor villager is the villager who benefits the most from getting from his least favorite project to his favorite project. If he is too impatient to wait to get the optimal project, no villager will ever complain. The only equilibrium is the no-voice equilibrium. Let’s assume for the rest of the proof that the poor villager prefers the optimal project later to the suboptimal project right away.

(ii.) If the poor villager prefers the suboptimal project right away to the neutral project later, then the rich villager prefers the optimal project right away to the neutral project later. And the neutral-project equilibrium does not exist as the rich villager cannot threaten to complain against the optimal project. Then if the proportion of honest bureaucrat is low, a new bureaucrat is likely to be dishonest and announce a neutral project in the corrupt-to-neutral equilibrium. And the poor villager would not complain against the suboptimal project. Once again the only equilibrium is the no-voice equilibrium. On the other hand if the proportion of honest bureaucrat is high, the corrupt-to-neutral equilibrium exists (for a range of $\frac{\psi}{\pi_C}$ and $\frac{\pi_C \rho}{\pi_C}$) and is better than the no-voice equilibrium.

(iii.) If the poor villager prefers the neutral project later to the suboptimal project right away, the corrupt-to-neutral equilibrium exists (for a range of $\frac{\psi}{\pi_C}$ and $\frac{\pi_C \rho}{\pi_C}$). If the rich villager prefers the optimal project right away to the neutral project later, the neutral-project equilibrium still does not exist.

(iv.) If the rich villager prefers the neutral project later to the optimal project right away, both the corrupt-to-neutral and the neutral-project equilibria exist. If the proportion of honest bureaucrat is low, the cost of corruption is prevalent and the neutral-project equilibrium is better than the corrupt-to-neutral equilibrium. And if the proportion of honest bureaucrat is high, the corrupt-to-neutral equilibrium is better than the neutral-project equilibrium as the choice of the honest bureaucrat is
Proof of Proposition 8 When the first-best equilibrium never exists and the second best is the corrupt-to-neutral equilibrium, this equilibrium exists only if the rich villager does not complain against the optimal project. If complaining is costless and the rich villager prefers waiting for the next bureaucrat over getting the optimal project right away, i.e. \( \theta \geq \frac{\xi - \delta}{p(\xi - \delta) + (1 - p)(\lambda \xi - b)} \), the corrupt-to-neutral equilibrium does not exist.

Proof of Proposition 9 This is derived from the results of proposition 7 part ii.. When the proportion of dishonest bureaucrat is low (high \( p \)), in the second best, the rich villager should not be able to bribe a bureaucrat to announce the suboptimal project. The poor villager must be able to complain against the suboptimal project, hence the cost of complaint cannot be too high. On the contrary, when the proportion of dishonest bureaucrat becomes large, the second-best is the no-voice equilibrium that can be implemented uniquely by choosing a costly complaint.

Proof of Proposition 10 If the other players send no message, the best response is not to send any message as it would have no effect. If the other players send the same message, the best response is also not to send any message. Thus, due to the opposite preferences, the rich villager never sends a message and both poor villagers either send no message or send one complaint. Hence the no-voice equilibrium always exists. The corrupt-to-neutral equilibrium exists iff the poor villagers prefers the neutral project right away to their expected utility before a bureaucrat arrives to the suboptimal project right away. Finally the first-best equilibrium exists iff the poor villagers prefer the optimal project later to the neutral project right away.
Chapter 3

Market Research and Innovation Strategy

3.1 Introduction

New product strategy often boils down to determining what novel feature to develop or which attribute to significantly improve upon. In deciding where to direct R&D effort, firms face uncertainty not only with respect to the technical feasibility of each innovation being considered, but also with respect to market demand for it. Market uncertainty arises because firms may not know a priori how much or how many consumers would value a planned new feature or a proposed attribute improvement.

Consider the following examples. In the mid 90s, Compaq and Toshiba faced a dilemma in the development of laptops for the consumer market. Because this market was still emerging and consumer preferences for various concepts was unclear, the dilemma was whether to focus on reducing weight by allowing only light and interchangeable peripherals (the CD-ROM and floppy drive would be modular and could not be used simultaneously) or, alternatively, to focus on offering greater convenience by attempting to integrate all the peripherals internally and including high performance components that further added weight Bell and Leamon (1999). In the late 80s, firms in the disk drive industry, such as Seagate and Conner, were not sure whether the market wanted to shift to smaller drives and were contemplating whether
to dedicate R&D effort to reducing drive size or to increasing memory capacity Christensen (1997). In the early 2000s, firms in the cell phone handset market (Nokia and Motorola) were not sure about consumers’ tastes for different designs and faced a decision whether to invest in the “candy-bar” or “clam-shell” styles Economist (2004); firms in the anti-depressant category (Eli Lilly, Pfizer, GlaxoSmithKline) were not sure about demand for new treatments and had to decide whether to develop a drug with much better efficacy in treating acute depressive symptoms, or a drug that would treat physical symptoms of pain associated with mild depression Hirschfeld (2001).

Interestingly, in several of these examples firms in the same industry pursued different product development paths (that is, they chose to improve different attributes, incorporate different new features, or focus on distinct designs), while in other cases all firms chose the same development path. Moreover, documented records indicate that substantial research to resolve the market uncertainty and guide development was sometimes, but not always, conducted. In the laptop example given above, it is documented that only Compaq conducted market research prior to development, in the form of numerous focus group testing. In the cell phone example, Nokia continued to develop new candy-bar handsets without having conducted extensive market research upfront.1 Hence, there seems to be variance in firms’ strategies with respect to conducting market research, the choice of which innovation path to pursue, or both.

In this paper, we seek to shed light on how firms make such innovation related decisions and to explore the strategic implications of market research using a game-theoretic model. In a duopoly context, identical firms need to decide which of two product attributes (or features) to innovate upon. Firms face market uncertainty and only have a prior probability distribution regarding consumer preferences. Our analysis addresses the following research questions.

1Compaq’s market research revealed that consumers did not respond favorably to the concept of interchangeability in order to reduce laptop weight by a few pounds but wanted the convenience of all add-ons integrated internally and better performance. This information led to the development of the Presario notebook line, which garnered a dominant share shortly after being introduced in 1996. In Nokia’s case, it bet on developing the wrong style of hand-sets and lost significant share in 2004 to Motorola that introduced clam-shell phones. In the antidepressant case, market research by Lilly revealed that about 25% of patients had mild depressive symptoms and suffered from related pain Ofek (2006); Lilly indeed pursued an FDA indication for pain for its drug.
• Given ex-ante identical firms, should we always expect symmetric equilibria in the decision to conduct market research? Can it be optimal for one firm to forgo market research while its rival conducts such research?

• Under what conditions do firms innovate on the same vs. different attributes? Will a firm that conducts market research always choose the attribute that yields greater demand?

• What is the relationship between conducting market research and the R&D level a firm selects? Does market research induce a greater or lower R&D level?

In addressing these questions, we distinguish between two types of consumer preference uncertainty for new products. In the first, which we call ‘vertical’, all consumers value innovation on both attributes but prefer one over the other. Firms are initially uncertain which innovation customers will value more, though they may have a prior belief that one attribute is more likely to be preferred. In the disk drive example, for all customers improvement along drive capacity would be relevant Christensen (1997), but firms were unsure whether smaller drives, that were more stable and could be used in smaller machines, were actually more critical for end users and would become the standard; the prevailing prior was that capacity was more important. In the laptop example, Compaq had originally conjectured that the concept of modularity in order to reduce weight would be highly valued by consumers Bell and Leamon (1999).

To model such vertical market uncertainty, we assume that firms assign a probability that each attribute is the one more valued by consumers. The attribute with higher ex-ante likelihood of being preferred is called the “safer” attribute. Our analysis reveals two countervailing incentives for firms to undertake upfront market research. On the one hand, a market study has a direct benefit of allowing a firm to foresee the attribute most highly valued by consumers. On the other hand, if both firms are informed about the demand there is a competitive “correlating” drawback. Specifically, when both firms discover which attribute consumers prefer this will result in the firms innovating head to head on the same attribute. Therefore, the value of information
from market research to a firm depends on whether its rival possesses this information as well. These considerations give rise to three primary equilibria depending on the cost (or difficulty) of conducting market research: (i) both firms conduct market research and innovate on the attribute discovered to be preferred by consumers, (ii) only one firm conducts market research and its rival that forgoes market research pursues the safer attribute, and (iii) neither firm conducts market research, and both innovate on the safe attribute or each firm selects a different attribute to innovate upon. Hence, asymmetric equilibria in firm strategies can endogenously arise even though the two firms are initially identical in every respect. The analysis further shows that in case (iii), when both firms have foregone market research, the decision of which attribute to innovate upon is driven by two factors. The first reflects a desire to avoid market uncertainty as much as possible and drives both firms to select the attribute perceived to be safer ex-ante. The second reflects a desire to soften future competition and drives firms to differentiate by selecting separate attributes to innovate upon, with each firm betting that it has chosen the preferred attribute. This second force is linked to the degree of technical uncertainty, and dominates when the cost of product development is relatively small. Hence, when neither firm conducts market research we can expect more divergence in their innovation path selection.

Market research also has implications for the R&D levels firms select. In particular, an informed firm facing an uninformed rival is induced to select the highest R&D level. This is because the informed firm knows through market research that it is pursuing the most valued attribute, and hence its expected rewards are higher. The uninformed rival responds by selecting a relatively low R&D level due to strategic substitutability. As such, we find that a predisposition to be market oriented with respect to innovation causes the firm to expend more on R&D relative to a rival that is not market oriented. Moreover, with the vertical preference structure, a firm that conducts market research and successfully innovates can never be worse off in terms of post-launch profits than a firm that forgoes market research.

In the second type of consumer preference structure, which we call ‘horizontal’, there exist two distinct segments and each values innovation on only one of the
attributes. Firms are initially uncertain as to the size of these segments. In the pharmaceutical example, there was anecdotal evidence that some patients had mild depression symptoms yet suffered considerably from related physical pain, vis-a-vis other patients (classified as “hard-to-treat”) for whom existing drugs were relatively ineffective in alleviating the depression symptoms but had no associated pain. An innovative drug that offered comparable efficacy on depression to existing drugs and treated the co-morbid pain would appeal only to the former segment and a new drug that offered greater efficacy in treating acute depression would appeal only to the latter segment. But it was not clear ex-ante how big these segments were.

To model such horizontal demand uncertainty, the size of the segments is assumed to be a random variable that can take on any value between zero and one. Segments are of ex-ante equal expected size, and a firm that conducts market research learns the true size of the segments. In this case, we find that market research can have the property of “negatively correlating” firms’ innovation paths. Specifically, a firm that conducts market research may actually target the ‘niche’ segment with its innovative efforts as a way to avoid harsh price competition for the larger segment. This result holds in an asymmetric equilibrium where a single firm conducts market research, as well as for one of the firms in an equilibrium where both conduct market research. Such a niche innovation strategy entails a relatively low R&D level. Consequently, a firm that conducts market research may earn lower post-launch profits than its rival. This type of ‘segmented’ equilibrium is more prevalent the lower the technical uncertainty, because when development effort is very likely to succeed the need to differentiate is more acute. We note that both in the vertical and horizontal cases we establish the existence of equilibria where ex-ante identical firms behave asymmetrically in the decision to conduct market research. But although in the vertical case the informed firm would always pursue the attribute discovered to be preferred by consumers, in the horizontal case the informed firm needs to take into account not only which attribute is valued by more consumers but also which attribute its rival is pursuing.

In terms of prior related work, limited analytic research has examined firms’ incen-
tives to choose among different innovation paths. Jovanovic and Rob (1987) assume uncertain consumer preferences and let firms gather market information before selecting a new product location. However, in their model firms face no technology development hurdle and do not compete with their new products (they act as price takers). Therefore, strategic considerations in selecting innovation direction are not explicitly modeled. The focus of Cabral (2002) and Cabral (2003) is to show that an initial asymmetry in industry position (technology gap between a leader and follower) results in an asymmetry in the technology path each firm pursues. In both these papers, firms have fixed R&D budgets and the rewards to each path are known ex-ante. By contrast, firms in our model are in the same industry position at the outset, R&D budgets are endogenously determined, and, importantly, firms are uncertain about the payoffs that will result from each path. In marketing, Iyer and Soberman (2000) study the incentives of competing firms to purchase information relevant for product modifications from a strategic vendor. In their model, firms do not make decisions under demand uncertainty and acquiring the information automatically enables offering the modified product (i.e., no product development needs to be undertaken). Ofek and Turut (2007) examine the incentives to conduct market research when an entrant needs to decide between innovative vs. imitative market entry. Their focus is on the signaling properties of entry strategy, i.e., characterizing when an incumbent may be able to forgo market research by drawing an inference from the entrant’s actions. By contrast, we model ex-ante symmetric firms that face market uncertainty regarding consumer preferences for innovation on two separate attributes. We do not incorporate signaling aspects of innovation strategy, and the decision of a firm to forego market research in our model is linked to the decrease in the value of information when a rival conducts such research; this leads to markedly different patterns of behavior across firms. Moreover, we micromodel two types of demand structures—vertical and horizontal—and hence our findings produce richer implications in terms of the incentives to conduct market research for purposes of new product differentiation. Furthermore, Ofek and Turut (2007) use only reduced form profit levels and firms are not able to differentiate in the type of innovation they
pursue based on the outcome of market research; the implications of our findings in this regard constitute a central contribution of this paper.

The extant literature thus largely ignores the confluence of demand uncertainty and technology uncertainty as it pertains to setting innovation strategy, and does not jointly examine the incentives to reduce them in a competitive context. This is despite evidence that points to the dual source of uncertainty competing firms face in new product development Cooper and Kleinschmidt (1987). Moreover, marketers have stressed the need to incorporate input on consumer desires prior to expending R&D effort Griffin and Hauser (1993)–in the form of conjoint analysis, concept testing, etc.–and have recently offered new techniques to achieve this Toubia, Simester, Hauser, and Dahan (2003). However, the strategic consequences of having a customer-centric orientation to innovation when your rival is (or is not) embracing such an orientation, and the implications for R&D investment, are not well understood.

Finally, it is relevant to mention prior work that has examined strategic information acquisition that aids in subsequent pricing or quantity decisions. For example, the incentives to resolve demand uncertainty have been studied by Li, McKelvey, and Page (1987) with quantity competition and by Raju and Roy (2000) with price competition. At a substantive level, market research in our model sheds light on the potential rewards to various innovation paths and impacts the return on risky R&D investment; thereby introducing different considerations relative to this literature.

The rest of the paper is organized as follows: Section 3.2 describes the model setup. We then investigates firms’ innovation strategies when confronted with vertical and horizontal market uncertainty in Sections 3.3 and 3.4, respectively. We discuss extensions and limitations of the analysis presented in the paper in Section 3.5. The paper concludes by summarizing the key results and offering managerial implications in Section 3.6. All proofs are given in the Appendix.

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2Reviewing a number of studies on the matter, they report that on average 46% of R&D projects fail to result in a working product and 35% of new products successfully developed fail to gain consumer acceptance.
3.2 Model Setup

Two identical firms, indexed \( i \in (1, 2) \), are planning to introduce a new product into a given market in the next period. They face a decision regarding the direction of their innovative effort. In particular, there are two attributes or features, indexed \( j \in (a, b) \), and each firm has to decide to which of these attributes it will devote R&D effort. This characteristic of our model captures the undesirability (or inability) of pursuing an unlimited number of innovation paths in a given time frame, and therefore the need to select among them.\(^3\)

For a given innovation path chosen, the R&D intensity a firm selects will affect the probability of development effort success. The greater a firm wishes to ensure product development success, the more costly it is at an increasing marginal rate. We use a quadratic cost function to capture this notion. When a firm attempts innovation on attribute \( j \), the R&D success probability it selects will be denoted by \( \varphi_j \) and it incurs a cost of \( \frac{1}{2} K \varphi_j^2 \), where \( \varphi_j \in [0, 1) \) and \( K \) is the development cost factor. Note that from a technology development standpoint the two attributes are equivalent— in the sense that the likelihood of R&D success is the same for a given expenditure. If firm \( i \) is successful in R&D effort, it introduces a new product and prices it at \( p(i) > 0 \). If both firms introduce new products, they compete à la Bertrand. Consumers buy at most one good and maximize their utility given the new products and prices offered. If a firm offers a product with no innovation on either attribute, we assume that such a product is identical to existing offerings and yields zero profits.\(^4\)

\(^3\)The assumption we make is realistic as in many cases trying to pursue multiple paths results in the diversion of management attention or in the need to incur significant set-up costs for each path so that a firm would not find it optimal do so (see Thomke and Krishnan 1999, for an example). Furthermore, there are typically diseconomies of scope in improving multiple attributes Economist (2002). The examples given in the Introduction are consistent with this characterization and the IO literature cited earlier Jovanovic and Rob (1987), Cabral (2002 2003) has assumed firms choose only one development path.

\(^4\)Specifically, we assume both firms can offer such products (or there are other suppliers of these commoditized products) such that Bertrand competition drives their price to zero. Mathematically, let existing products yield consumers utility of \( u_0 \) and be priced at close to zero and let an innovation increase this base utility by \( u_{inn} \) and be priced at \( p \). Then in comparing the utility from the existing product to that from the innovation \( u_0 \) cancels out. Thus our normalization of \( u_0 \equiv 0 \) is without loss of generality.
improvement on each attribute or to pursue multiple innovations impacts our findings.

Beyond the technology development uncertainty, there is also ex-ante uncertainty about consumer preferences. We examine two forms of market uncertainty. In the first, all consumers prefer innovation on one attribute over the other, and firms are ex-ante uncertain as to which attribute consumers value more. We call this demand structure "vertical". In the second, a portion of consumers only desire innovation on attribute \(a\) and the other portion only on attribute \(b\), and firms are ex-ante uncertain as to the size of these segments. We call this demand structure "horizontal". Our designation of these demand structures as vertical and horizontal is consistent with standard definitions in the IO literature on product differentiation.\(^5\)

Given the uncertainty about consumer preferences, firms may decide to conduct upfront market research to help guide their innovation decisions. Market research comes at a cost of \(C > 0\), and for simplicity we assume that it perfectly resolves the demand uncertainty. If a firm decides to conduct market research, we will say the firm is 'informed' and denote this by \(mr = 1\), and if a firm forgoes such research we will say the firm is 'uninformed' and denote this by \(mr = 0\). When a firm conducts market research it learns which attribute is 'preferred', i.e., more valued in the vertical case or valued by the larger segment in the horizontal case. When it chooses to attempt innovation on the preferred attribute, we denote this choice by \(c\), and by \(\bar{c}\) otherwise \((c, \bar{c} \in \{a, b\})\). This notation allows referring to the attribute strategy of an informed firm without knowing the results of the market research.

The timing of the game is as follows: in the first stage firms simultaneously decide whether to commission market research or not \((mr)\). A firm does not observe whether its rival has conducted market research.\(^6\) In the second stage, firms simultaneously

\(^5\)In particular, the defining characteristic of vertical models is that consumers agree on the rank ordering of product attribute importance such that "were any two goods in question offered at the same price, then all consumers would agree in choosing the same one". The defining characteristic of horizontal models is that "consumers would differ as to their preferred choice if all the goods in question were offered at the same price" (for these definitions see Shaked and Sutton 1983, Schmalensee and Thisse 1988, p. 1469 and p. 228, respectively). Our characterizations are specific versions of the vertical and horizontal demand structures. In reality, of course, there may be contexts where both types of differentiation are present at the same time. Our analysis here examines each separately in order to better understand the forces that arise in each case.

\(^6\)In reality, it may be quite difficult for firms to observe whether a rival has conducted market
choose which attribute to innovate on \((j)\) and their R&D intensity \((\varphi_j)\). In the third stage, firms’ development efforts conclude and they set prices \((p)\) for their new products. Note that if a firm conducts market research it resolves demand uncertainty in stage 2, but if it forgoes market research it learns the realization of demand only in stage 3. An example of a possible strategy vector for firm \(i\) is:
1. Conduct market research \((mr = 1)\).
2. Attempt innovation on the attribute discovered to be preferred and select R&D level \(\varphi_c\).
3. When R&D effort is successful
   - if the competitor has failed, charge price \(p^u_c\) (price a monopolist would charge having innovated on the preferred attribute).
   - if the competitor has also succeeded, charge price \(p^d_c\) (duopoly price depending on the new product of the rival).

Firms are risk neutral and maximize their expected payoffs. We denote the profit level of firm \(i\) when it chooses to innovate on attribute \(j\) as \(\pi_j(i)\). The game is solved through backward induction starting with the pricing subgame at \(t=3\).

### 3.3 Vertical Preferences Over the Attribute Space

In this section, we study the case where all consumers are assumed to have similar preferences over the product attribute space. Although consumers place value on both attributes, ex-ante firms do not know if they would prefer to adopt a new product that offers innovation on attribute \(a\) or \(b\). To capture this type of market uncertainty, we assume that with probability \(0 < \alpha < \frac{1}{2}\) consumers prefer innovation on attribute \(a\), and with probability \(1 - \alpha > \frac{1}{2}\) they prefer innovation on attribute \(b\).\(^7\) Thus, without research, particularly if such research is done by the firm’s own employees. Many market research firms also guarantee they will not divulge having provided services to rivals (Orme 2007). (On the unobservability of market research, see also Hauk and Hurkens 2001). Relaxing this assumption to allow the observability of conducting market research (but not the results of the effort) would not qualitatively impact our results. This is because in our set-up, in equilibrium a firm will correctly conjecture whether its rival is conducting market research or not at \(t=2\). Quantitatively, of course, regions where the equilibria exist will shift.

\(^7\) Allowing \(\frac{1}{2} < \alpha < 1\) would merely reverse the roles of attributes \(a\) and \(b\).
loss of generality, attribute \( b \) is the “safer” attribute (the limiting case of \( \alpha = \frac{1}{2} \) is studied in Section 3.3.1). This captures the fact that in real life firms may have an a priori belief that certain attributes are more likely to matter for consumers than others. When a firm successfully innovates on the attribute or feature that is more valued, consumers derive utility \( V \) from the new product. When a firm successfully innovates on the attribute that is less valued, consumers derive utility \( V - D \) from the new product, where \( D \in (0, V) \) captures the degree of consumer disutility. Hence, a new product with innovation on attribute \( a \) will be vertically differentiated from a new product with innovation on attribute \( b \). To summarize, the utility that a consumer derives from purchasing a new product from firm \( i \), conditional on the success of its R&D efforts, is given by:

\[
u(i) = V - I(i) \cdot D - p(i),\]

where

\[
I(i) = \begin{cases} 
0 & \text{if firm } i \text{ innovates on the attribute more valued by consumers,} \\
1 & \text{if firm } i \text{ innovates on the attribute less valued by consumers.}
\end{cases}
\]

The following Lemma characterizes the outcome of the final subgame depending on which new products are introduced and on consumer preferences.

**Lemma 11** In the pricing subgame, having chosen to invest in attribute \( j \), firm \( i \)'s price \( p_j(i) \) and profits \( \pi_j(i) \) will be

- \( V \), if it is the sole innovator and consumers prefer attribute \( j \),
- \( V - D \), if it is the sole innovator and consumers prefer attribute \( j' \) (\( j' \neq j \)),
- \( D \), if firm \( i \) innovates on attribute \( j \), its rival innovates on attribute \( j' \), and consumers prefer attribute \( j \), and
- 0, otherwise.

A firm can thus attain one of four profit levels depending on the outcome at \( t = 3 \).

Note that if \( D > V/2 \), profits from innovating on the preferred attribute and facing competition from a firm that innovated on the less preferred attribute are higher than

\[8\] In Section 3.5 we discuss an extension that allows for heterogeneity in consumer willingness to pay (or price sensitivity). But the uncertainty would still be regarding which attribute all consumers prefer.
having innovated on the less favored attribute and facing no competition. In the limit
\( D \to V \), innovation on the less preferred attribute is worthless.

By conducting market research a firm discovers which attribute consumers value
more. Based on Lemma 11, we establish the following result regarding an informed
firm's innovation strategy.

**Lemma 12** A dominant strategy for an informed firm is to attempt innovation on
the attribute it discovers to be more valued by consumers.

A firm that conducts market research will therefore always pursue innovation on
the attribute discovered through market research to yield maximal value to consumers.
This is because regardless of the attribute pursued by the rival, a firm's profits will
always be greater with a new product that embodies innovation on the most valued
attribute.

Before characterizing the full equilibria in the case of duopolistic competition, we
seek to gain insights by solving for the optimal actions of a monopolist. This allows
us to isolate a firm's incentives when competitive concerns are absent.

**Proposition 11** There exists a cutoff value
\( C^m = \frac{aD(2V-aD)}{2K} \) such that for \( C \leq C^m \) a
monopolist would conduct market research and select R&D level \( \varphi_c = \frac{V}{K} \). Its expected
payoffs in this case are \( \pi_c^m = \frac{V^2}{2K} - C \). For \( C^m \leq C \) a monopolist forgoes market
research and attempts innovation on the safer attribute \( b \) with \( \varphi_b = \frac{V-aD}{K} \). Its expected
payoffs are \( \pi_b^m = \frac{(V-aD)^2}{2K} \).

For a monopolist, the trade-off in conducting market research is clear, a mar-
ket study is costly but guarantees innovating on the attribute consumers prefer and
reaping the highest possible rents. Hence, if the benefit to being informed is greater
than the cost, the monopolist commissions market research. Because the informed
monopolist always attempts innovation on the preferred attribute (Lemma 12), its
R&D level will be the same \( (\frac{V}{K}) \) regardless of which attribute that is discovered to
be. Proposition 11 also reveals that an uninformed monopolist \((mr = 0)\) always finds
it beneficial to innovate on the safer attribute \( (b) \). The solution thus demonstrates
an interplay between market research and the level of R&D chosen. Market research ensures that R&D effort is directed towards a reward of $V$, rather than a smaller expected reward of $V - \alpha D$ when the firm is uninformed, and hence leads to a higher R&D level ($\varphi_c > \varphi_b$). This productivity enhancing feature of market research is what distinguishes its undertaking from merely pouring more money into reducing technical uncertainty through a higher R&D level.

We make the following additional observations regarding the equilibrium R&D levels $\varphi_c$ and $\varphi_b$ given in Proposition 11. Both R&D levels increase in $V$, the maximal value of innovation to consumers, and decrease in $K$, the difficulty of achieving R&D success. Intuitively, $V$ increases the expected payoffs from R&D success and $K$ decreases them regardless of whether the monopolist is informed or not. The parameters $\alpha$ and $D$, however, are only relevant for the uninformed monopolist. The uninformed monopolist chooses to innovate on the safer attribute $b$, yet with probability $\alpha$ it attempts development of the less valued new product and will only be able to price at $V - D$. Hence, the uninformed monopolist lowers its R&D level as $\alpha$ and $D$ increase. By contrast, the informed monopolist knows the preferred attribute and, conditional on R&D success, will extract the full surplus $V$ from consumers.

### 3.3.1 Market Research and Competition

We now turn to the case of two competing firms planning to develop new products. We expand our notation such that for a given variable the first subscript denotes the attribute the firm has chosen to innovate upon while the second subscript denotes the attribute the rival has chosen. Following a similar convention, we use two superscripts to denote whether firms have chosen to conduct market research or not. For example, we write $\varphi_{bc}^{01}$ for the R&D level of a firm that did not conduct market research ($mr = 0$) and chose to innovate on attribute $j = b$, facing a rival that has conducted market research ($mr' = 1$) and that innovates on the attribute it discovers to be preferred by consumers ($j' = c$). In this case, the R&D level of the rival firm that conducted market research will be denoted $\varphi_{bc}^{10}$. We solve for pure-strategy subgame-perfect Bayesian equilibria of the game Fudenberg and Tirole (1991).
Market Research and the Direction of R&D Effort

In Lemma 11, we established the pricing equilibrium in the final subgame depending on which new products were introduced and on consumer preferences. We now characterize the equilibria of the entire game in terms of the decision to conduct market research and the choice of which attribute to innovate upon. In the next subsection we provide more details regarding the R&D levels selected. A strategy profile $\sigma = \{\sigma(1), \sigma(2)\}$, $\sigma(i) = \{mr(i), j(i), \varphi_j(i), p(i)\}$ $i \in \{1, 2\}$, will form an equilibrium if, conditional on its rival's strategy, each firm's expected profits can not be increased by unilaterally deviating to an alternative strategy.

Proposition 12 For any values of the parameters $K$, $V$, $D$ and $\alpha$, there exist cutoff values $0 < C^{01} < C^{11} < C^{00} < C^{10}$ such that the following equilibria of the game exist

(i) Dual-market-research equilibrium: If $C < C^{11}$, both firms conduct market research and innovate on the most valuable attribute.

(ii) Single-market-research equilibrium: If $C^{01} \leq C \leq C^{10}$, only one firm conducts market research and innovates on the most valuable attribute. The uninformed firm innovating on the safer attribute $b$ is always an equilibrium; under certain conditions, innovating on attribute $a$ may also be feasible.

(iii) No-market-research equilibrium: If $C^{00} \leq C$, neither firm conducts market research. Firms either both innovate on the safer attribute or innovate on separate attributes.

The various equilibria in terms of conducting market research are depicted in Figure 3-1. Note that multiple equilibria may exist as the single-market-research region overlaps the other two.

In understanding the intuition behind Proposition 12 we point out that, as in the monopoly case, the trade-off between the benefit of learning which attribute is preferred by consumers (related to $\alpha D$) and the cost of acquiring the information ($C$) remains. However, with competition there is an added consideration— the benefit from conducting market research goes down if the rival firm conducts market research as well. This is because of the perfectly "correlating" feature of market research with
vertical preferences: both firms will unavoidably attempt innovation on the same attribute (consistent with Lemma 12), and if the R&D efforts of both are successful the new products introduced will not be differentiated (leading to 0 profits per Lemma 11). We can now explain in greater detail the intuition behind each equilibrium region and formalize firms’ choice of which attribute to attempt innovation on. We begin with regions (i) and (iii) of Proposition 12, and then discuss region (ii).

(i) **Dual-market-research equilibrium** ($C \leq C^{11}$): the cost of market research is low and an equilibrium whereby each firm conducts market research can be sustained. The informational benefit of innovating on the preferred attribute outweighs the costs. However, because market research will correlate the two firms’ innovation paths, which lowers the benefit of being informed relative to a monopolist, we will have $C^{11} < C^m$.

(iii) **No-market-research equilibrium** ($C^{00} \leq C$): When market research is prohibitively costly, an equilibrium in which neither firm conducts market research can be sustained. In this case, two types of equilibria may arise in the choice of which attribute to innovate on. The following result establishes when each will occur.

**Result 1** If neither firm conducts market research, there exist values $K$ and $\bar{K}$ ($K < \bar{K}$) such that

- for $K < K$, the unique equilibrium is asymmetric with one firm innovating on attribute $a$ and the other on $b$,
- for $K \leq K \leq \bar{K}$, two equilibria coexist: a symmetric one with both firms innovating on attribute $b$, and an asymmetric one with each firm innovating on a separate attribute.
tribute,
for $\bar{K} < K$, the unique equilibrium is symmetric with both firms innovating on attribute $b$.

$K$ and $\bar{K}$ are non-decreasing functions of $\alpha$, with
$$\lim_{\alpha \to \frac{1}{2}} \bar{K} = \lim_{\alpha \to \frac{1}{2}} K = +\infty.$$  

When neither firm is willing to incur the high cost of market research, Result 1 reveals an interesting pattern for the attribute each firm attempts to innovate upon. Like the monopolist (Proposition 11), an uninformed firm has an incentive to innovate on the safer attribute ($b$) given that it is a priori more likely to be the preferred attribute— we call this the uncertainty-avoidance effect. But duopolists also have to consider the implications of attribute choice on downstream product market competition. In particular, there is a force driving firms to innovate on separate attributes so that they don’t end up competing profits away in the event that both of their R&D efforts are successful— we call this the differentiation effect. If $K$ is small, all else equal, firms tend to select high R&D levels and hence their innovation efforts will likely succeed. Choosing to innovate on the same attribute will result in both firms introducing identical new products with high probability, leading to 0 profits (Lemma 11). Thus, when technical uncertainty is low ($K < K$), the differentiation effect dominates and the uninformed firms prefer to ‘gamble’ on a distinct attribute, each hoping it has made the correct choice. However, when $K$ is large firms tend to select relatively low R&D levels and technical uncertainty is substantial. Each firm is then less concerned with the success of its rival’s efforts and would rather innovate on the safer attribute, i.e., uncertainty avoidance dominates.

We would like to highlight the interplay between the degree of market uncertainty and the degree of technology development difficulty. As $\alpha$ increases, uncertainty-avoidance becomes less pronounced and we can sustain differentiated innovation paths for higher values of the development cost factor $K$. Figure 3-2 depicts Result 1 graphically.

(ii) **Single-market-research equilibrium** ($C^{01} \leq C \leq C^{10}$): in this range we get the intriguing result whereby the two, otherwise identical, firms pursue asymmetric strategies at the outset. This equilibrium occurs due to a combination of
Both firms choose the safer attribute (b)

<table>
<thead>
<tr>
<th>K</th>
<th>0</th>
<th>0.5</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>(unique)</td>
<td></td>
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<td></td>
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</table>

Each firm chooses a different attribute (one a the other b)

The factors outlined above. When one firm conducts market research, the benefit to that firm of being informed is relatively high and outweighs the costs (C) that are in a mid-range. However, the uninformed firm does not want to deviate and conduct market research because the value of the information will be relatively low since, as explained earlier, the two firms will then surely innovate on the same attribute. Thus, while the benefit of conducting market research is worth it for one firm, the benefit for two firms is lower and is outweighed by the cost.

The firm that conducts market research does not need to gamble on which innovation path to pursue as it will always direct its R&D efforts to the attribute that is more valued by consumers (Lemma 12). However, the uninformed firm needs to choose which innovation path to pursue using only the ex-ante prior. Although uncertainty avoidance will always make the choice of attribute b an equilibrium strategy, under certain conditions the uninformed firm may take a chance on attribute a.

**Result 2** There exists a single-market-research equilibrium where the uninformed firm:

(i) chooses to innovate on the safer attribute \( (j = b) \) \( \forall \alpha \); this equilibrium is unique for \( \alpha < \bar{\alpha} \), (ii) chooses to innovate on the riskier attribute \( (j = a) \) if \( \bar{\alpha} \leq \alpha < 1/2 \).

To understand the intuition, note that the uninformed firm makes positive profits only when the informed firm’s R&D efforts fail (Lemma 11). Also note that the
informed firm invests less when it conjectures, according to the equilibrium being played, that its uninformed rival has chosen to innovate on the same attribute (because in the event that both succeed in R&D they make no profits). Now, assume first that the two attributes are ex-ante almost equally likely to be more valued by consumers (α close to 1/2) and that the uninformed firm is expected in equilibrium to innovate on attribute a. In considering whether to deviate to attribute b, the uninformed firm takes into account two possibilities: (1) The informed firm discovers that attribute a is the preferred attribute. The uninformed firm would thus be deviating to the less preferred attribute, reducing potential profits by D, (2) The informed firm discovers that attribute b is the preferred attribute. Now, the uninformed firm would be deviating to the preferred attribute, increasing its potential profits by D. But in this case, the informed firm will invest heavily because it believes (according to the equilibrium) that the attribute it has chosen is different than that of the uninformed rival. This, in turn, reduces the chances of the uninformed firm being the sole innovator and making positive profits. These considerations create an overall disincentive to deviate. Another way to think about why this equilibrium can hold is that the uninformed firm believes (given the prior) that it is more likely that its informed rival will attempt innovation on attribute b, and hence it might be better off betting on a different attribute. This is similar to the dilemma facing a participant in a lottery jackpot when there is a pre-determined lucky number that many are expected to select. If the odds aren’t too unfavorable (in our case α not too close to 0), it may be more beneficial to gamble on a different number and avoid having to share the prize with other participants.

If, according to the equilibrium being played, the uninformed firm is expected to innovate on attribute b and is considering deviation to attribute a, then uncertainty avoidance creates an added desire to not deviate. As α moves away from 1/2, uncertainty avoidance reinforces these effects when attribute b is chosen in the proposed equilibrium, but works against them when attribute a is chosen. Therefore, we can always sustain the single-market-research equilibrium with the uninformed firm.

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9We thank an anonymous reviewer for providing us with this intuitive analogy.
choosing the safer attribute \( b \), but we require \( \alpha \) close to \( 1/2 \) to sustain the choice of attribute \( a \).

**Market Research and the Level of R&D Effort**

We now turn to examining the R&D levels firms select in the different equilibria of Proposition 12. In particular, we are interested in understanding how facing an informed rival affects the incentives to undertake R&D, and whether two informed firms invest more or less than two uninformed firms. Lastly, we want to understand how the different R&D levels depend on the model’s parameters. Recall that \( \varphi_{ji}^{mr, mr'} \) denotes the R&D level of a firm given its market research decision \( (mr) \) and attribute choice \( (j) \) as well as those of its rival \( (mr' \text{ and } j') \).

To gain insight into the strategic considerations of R&D level choice, we first establish the following Lemma.

**Lemma 13** R&D levels always form strategic substitutes.

Strategic substitutability Bulow, Geanakopolos,, and Klemperer (1985) means that the marginal return to a firm from its own R&D level is decreasing in the rival’s R&D level (mathematically, \( \frac{\partial^2 E(i)}{\partial \varphi(i) \partial \varphi(j)} < 0 \)). This is intuitive because when the rival’s R&D level increases, the firm is more likely to earn duopoly rather than monopoly rewards. As this reduces expected profits, the firm’s marginal return on R&D goes down. The implication of Lemma 13 is that when a competitor is more aggressive in R&D, the firm will have an incentive to react by being less aggressive in R&D.

The following Proposition provides the ranking of R&D intensities.

**Proposition 13** The R&D levels in the different equilibria are ranked as follows:

(i) in the uninformed firm cases: \( \varphi_{ac}^{11} < \varphi_{bc}^{01} < \varphi_{ab}^{00} < \varphi_{bb}^{00} < \varphi_{ba}^{00} \),

(ii) in the informed firm cases: \( \varphi_{cc}^{11} < \varphi_{bb}^{10} < \varphi_{aa}^{10} < \varphi_{ab}^{10} < \varphi_{ba}^{10} \),

(iii) All the R&D levels of the uninformed firm cases are smaller than the R&D levels of the informed firm cases, except that when \( \alpha < \alpha^* \), \( \alpha^* \in (0, \frac{1}{2}) \), we have \( \varphi_{ba}^{00} > \varphi_{cc}^{11} \).
First, it is apparent from Proposition 13 that an informed firm always selects a higher R&D level than an uninformed rival. This is true even when the informed firm ends up choosing the same attribute it would have selected in a no-market-research equilibrium (for example, \( \varphi_{ba}^{00} < \varphi_{ba}^{10} \)), and it happens because the market research indicates with certainty that the return on this attribute is higher and hence the firm allocates more effort. In addition, an informed firm knows from its market research which attribute is most valued and selects a higher R&D level when, according to the equilibrium being played, the uninformed rival’s strategy is to attempt innovation on the less preferred attribute (see levels \( \varphi_{ab}^{10} \) and \( \varphi_{ba}^{10} \) in part (ii)). In this case, the informed firm is guaranteed profits of \( V \) or \( D \) if its R&D efforts succeed, compared to \( V \) or 0 if it had turned out that the uninformed rival was also attempting innovation on the preferred attribute. The highest R&D level is \( \varphi_{ba}^{10} \), because when the uninformed firm chooses attribute \( a \) rather than \( b \), the smaller expected returns (since \( \alpha < \frac{1}{2} \)) induce it to select a lower R&D level. Due to strategic substitutability, the informed firm reacts by investing more.

Second, and more surprising, when both firms are uninformed and choose separate attributes (part (iii)), the R&D level \( \varphi_{ba}^{00} \) can be higher than the R&D level when both are informed (\( \varphi_{cc}^{11} \)). This happens because the firm innovating on the safer attribute \( b \) has a higher expected payoff than the firm innovating on attribute \( a \), hence in equilibrium \( \varphi_{ab}^{00} < \varphi_{ba}^{00} \). As \( \alpha \) moves away from 1/2, the firm innovating on the safer attribute faces an even softer competitor, and as R&D intensities are strategic substitutes, it selects an even more aggressive R&D level, i.e., \( \varphi_{ba}^{00} \) increases. If \( \alpha \) is below some threshold, this level will be higher than that of the firms in the dual-market-research equilibrium. In the latter case, though both firms know for sure which attribute is preferred, the prospects of head-to-head competition have a dampening effect on the incentives to undertake R&D.

We wish to reflect on how the R&D levels are affected by changes in the parameters of the model. The equilibrium adjustment of an R&D level can be decomposed into a direct effect and a strategic effect Tirole (1988b). The direct effect is related to the sensitivity of a firm’s expected profits to a change in a parameter, while keeping the
R&D intensity of the rival fixed. The strategic effect is related to the sensitivity of a firm’s expected profits to the rival’s response to the parameter change. We highlight the key comparative statics (a complete list appears in Table 3.1 in the Appendix).

The direct effect of increasing the maximal consumer value for innovation \((V)\) is always positive, while that of increasing the development cost factor \((K)\) is always negative. But in the single-market-research equilibrium, these direct effects are more pronounced for the informed firm that knows the consumer preferred attribute at the stage of selecting its R&D level and does not take an expectation over possible market rewards. This disparity in the magnitude of direct effects across the firms will result in a negative strategic effect for the uninformed firm, which can be large enough to overshadow its positive direct effect, so that \(\varphi_{1k}^{01}\) decreases in \(V\) and increases in \(K\). Similarly, in the no-market-research equilibrium when firms differentiate, the same logic explains why the firm that chooses the more uncertain attribute \(a\) may decrease (increase) its R&D level as \(V\) (\(K\)) increases.

With respect to the disutility parameter, in the single-market-research equilibrium the two firms react differently to an increase in \(D\). This is because the direct effect of an increase in \(D\) is negative for the uninformed firm (a payoff of \(V - D\) if the informed rival fails and it has innovated successfully on the less preferred attribute) and this produces a positive strategic effect for the informed firm (for the levels \(\varphi_{ab}^{00}\) and \(\varphi_{ba}^{00}\) there is also a positive direct effect; a payoff of \(D\) when the uninformed rival innovates successfully). With respect to the degree of market uncertainty, an increase in \(\alpha\) makes attribute \(a\) more likely to be preferred. For an uninformed firm, this produces a positive direct effect when innovating on attribute \(a\) and a negative direct effect when innovating on attribute \(b\). In the no-market-research equilibria, the direct effect dominates, which explains why \(\varphi_{ab}^{00}\) increases while \(\varphi_{ba}^{00}\) and \(\varphi_{bc}^{00}\) decrease in \(\alpha\). In the single-market-research equilibria, the direct effect is not relevant for the informed firm, which has resolved the market uncertainty. As the informed firm only has a strategic effect, its R&D sensitivity to \(\alpha\) will in all cases be opposite to that of the uninformed firm. For example, in an equilibrium in which the uninformed firm attempts innovation on attribute \(b\), the informed firm’s R&D level \((\varphi_{cb}^{10})\) will actually
increase in $\alpha$.

It is instructive to compare these last findings with the monopoly solution. An informed monopolist's R&D level does not at all depend on the degree of market uncertainty $\alpha$ nor on the disutility parameter $D$. However, as explained above, with competition this is no longer true; when a firm is informed but its rival is not, strategic effects arise and create a dependence on these parameters.

The Case of $\alpha = \frac{1}{2}$

We conclude this section by examining the special case of $\alpha = \frac{1}{2}$. Because neither attribute has ex-ante greater likelihood of being more valued by consumers, uncertainty avoidance does not play a role. The only change in the monopoly case is that when uninformed, the firm would be indifferent between attributes $a$ and $b$ and would attain equal expected payoffs from attempting innovation on either. As for the competitive context, all three types of equilibria in market research will exist, as laid out in Proposition 12. They will have the following characteristics:

*Dual-market-research equilibrium.* Both firms will choose the attribute discovered through market research to be most valued by consumers. (no change).

*Single-market-research equilibrium.* The informed firm chooses to innovate on the attribute discovered through market research to be most valued by consumers. Two equilibria coexist, the uninformed firm attempts innovation on attribute $a$ or alternatively on attribute $b$.

*No-market-research equilibrium.* Each firm attempts innovation on a different attribute.

Thus, when at least one firm chooses not to conduct market research, because only the differentiation effect is present, we should expect more divergence in terms of the new products firms attempt to develop.\(^\text{10}\)

\(^{10}\)In terms of R&D levels, we will have a similar ranking to that in Proposition 13: $\varphi_{cc}^{11} < \varphi_{jj}^{10} < \varphi_{cc}^{01} < \varphi_{jj}^{00}$. Note that in the dual-market-research case, both firms unequivocally invest more in R&D than in the no-market-research case; consistent with the condition on $\alpha$ in the Proposition.
3.4 Horizontal Preferences Over the Attribute Space

In this section, we study the case where a portion $s_b$ of consumers only value innovation on attribute $b$ and the remaining $s_a = 1 - s_b$ of consumers only value innovation on attribute $a$. For consumers in the former segment, innovation on attribute $b$ yields utility (net of price) of $V$ and innovation on attribute $a$ yields 0 utility. The reverse is true for consumers in the latter segment, for which innovation on attribute $a$ yields utility of $V$ and innovation on attribute $b$ yields 0 utility. Hence, a new product with innovation on attribute $a$ is horizontally differentiated from a new product with innovation on attribute $b$.

Ex-ante firms do not know the size of the segments, i.e., they face market uncertainty regarding how many consumers value each type of innovation. At the outset of the game firms treat $s_b$ as a random variable. Let $\tilde{s}_b$ be distributed over $[0, 1]$ with cdf $F$ and pdf $f$. We assume that $F$ is differentiable and that ex-ante firms’ prior is that the segments are of equal size, $E[s_b] = \frac{1}{2}$.\(^{11}\) If a firm conducts market research, at cost $C_s$, it learns the value of $s_b$.

The structure of the game remains the same as described in the model setup (Section 3.2), and we solve for the pure strategy equilibria as before (starting from the pricing subgame backward). The following Lemma gives the prices and profits that arise in equilibrium in the final subgame.

**Lemma 14** In the pricing subgame, having chosen to invest in attribute $j \in \{a, b\}$, firm $i$’s price $p_j(i)$ and profits $\pi_j(i)$ will be

- $p_b(i) = V$, $\pi_b(i) = s_bV$, if firm $i$ is the sole innovator on attribute $b$,
- $p_a(i) = V$, $\pi_a(i) = (1 - s_b)V$, if firm $i$ is the sole innovator on attribute $a$,
- $p_j(i) = 0$, $\pi_j(i) = 0$, otherwise.

From Lemma 16, we see that with segments that differ in their horizontal tastes both firms will make positive profits when they successfully innovate on separate

\(^{11}\)Assuming $E[\tilde{s}_b] \neq \frac{1}{2}$ does not affect our main results.
attributes. In particular, the firm that innovates on attribute \( b \) caters to a segment of size \( s_b \) and earn profits of \( s_bV \), while its rival caters to a segment of size \( s_a \) and earn profits of \( (1-s_b)V \). Recall that both firms earning positive profits in equilibrium was not feasible in the previous type of preference structure. It is also evident that only three profit levels are possible in this case. The reason is that when firms choose to innovate on different attributes their profits are independent from each other.

We proceed in a similar fashion to the previous section, by first examining the monopoly benchmark case and then examining the competitive context.

**Proposition 14** There exists a cutoff value \( C_s^m \) such that for \( C_s \leq C_s^m \) a monopolist conducts market research, pursues innovation on the attribute that targets the larger segment and selects an R&D level \( \varphi_c = \frac{\max(s_b, 1-s_b)V}{K} \). Its expected payoffs are \( \pi_c^m = E\left[\max(s_b, 1-s_b)^2\right]V^2 - C_s \). For \( C_s^m < C_s \) a monopolist forgoes market research, attempts innovation on either attribute, and sets \( \varphi_j = \frac{V}{2K} \). Its expected payoffs are \( \pi^m = \frac{(\varphi_j)^2}{2K} \).

Thus, the monopolist's decision rule regarding market research has a very similar flavor to that in Proposition 11. The cutoff \( C_s^m \) is determined by the expected benefit of learning the segment sizes (proportional to \( E\left[\max(s_b, 1-s_b)^2\right] - \frac{1}{4} \)). It is clear that an informed monopolist will unequivocally choose to innovate on the attribute valued by more consumers (in analogy to the previous outcome of the monopolist having a dominant strategy to innovate on the preferred attribute). We also see that the informed monopolist's R&D level will depend on the exact value of \( s_b \). Therefore, the value of market research for the monopolist is twofold, first it allows choosing the attribute that appeals to the larger segment, and second it allows gauging the exact return from an innovation along that attribute so as to set the optimal R&D level.

### 3.4.1 Market Research and Competition

We now turn our attention to the case of competition. As the next proposition will show, with horizontal tastes we can still sustain the three types of equilibria in market research as in the previous analysis. However, importantly, a firm conducting market
research may in equilibrium choose to develop an innovation that targets the smaller of the two segments, yielding lower market share and profits than its rival post launch.

**Proposition 15** For any values of the parameters $K$ and $V$, there exist cutoff values $0 < C_{s1}^0 < C_{s1}^{11} < C_{s0}^0 < C_{s0}^{10}$ such that the equilibria of the game are

(i) Dual-market-research equilibrium. If $C_s \leq C_{s1}^{11}$, both firms conduct market research. Firms either both pursue the attribute that appeals to the larger segment or pursue separate attributes.

(ii) Single-market-research equilibrium. If $C_{s1}^0 \leq C_s \leq C_{s0}^{10}$, only one firm conducts market research. At least one firm pursues the attribute that appeals to the larger segment; it is not necessarily the informed firm.

(iii) No-market-research equilibrium. If $C_{s0}^{00} \leq C_s$, neither firm conducts market research. Firms pursue separate attributes: one firm attempts to innovate on attribute $a$ and the other on $b$.

To understand the intuition for the different equilibria, in particular the attributes firms choose to innovate on, we need to recognize that with the horizontal demand structure two forces exist: A *segment-size* effect and a *differentiation* effect. The former is relevant only for an informed firm, and drives it to target the larger segment because of the higher rewards if it is the sole innovator on that attribute. The latter effect drives a firm to select a different attribute to innovate upon than its rival in order to avoid harsh price competition in case both firms successfully develop a new product; by serving a distinct segment a firm is guaranteed positive profits if its R&D efforts succeed (Lemma 16). An important point to note about the differentiation effect is that it is also relevant for an informed firm. By contrast, recall that with the vertical preference structure, once a firm learned the preferred attribute it had a dominant strategy to pursue that attribute regardless of its rival’s actions (Lemma 11). Using these forces, we now explain the intuition for each of the equilibria in greater detail.

(i) **Dual-market-research equilibrium**: When the cost of market research is small, both firms will conduct market research as there is always a benefit in learning
the value of $s_b$. Due to the segment-size effect, at least one of the firms will target the larger of the two segments. But the best response of the other firm depends on the interplay between the segment-size and differentiation effects as characterized by the following result.

**Result 3** If both firms conduct market research, there exist $s_b, \bar{s}_b (\frac{1}{2} < s_b < \bar{s}_b < 1)$ such that
- Both firms attempting innovation on attribute $b$ is an equilibrium iff $s_b \leq s_b$;
- Each firm attempting innovation on a separate attribute is an equilibrium iff $1 - \bar{s}_b \leq s_b \leq \bar{s}_b$;
- Both firms attempting innovation on attribute $a$ is an equilibrium iff $s_b \leq 1 - s_b$.

$s_b$ and $\bar{s}_b$ increase as $K$ decreases.

Figure 3-3 depicts the regions for the various equilibria to hold. Intuitively, when the firms learn that one segment is substantially greater, the segment-size effect dominates and both firms will target their innovative efforts to serve this segment. If $s_b$ is close to 1 they will attempt innovation on attribute $b$, and if $s_b$ is close to 0 on attribute $a$. Market research thus has the effect of correlating the firms’ innovation paths when it reveals that the vast majority of consumers fall in one of the segments.

The interesting equilibrium occurs when the segments are not too dissimilar in size. In this case, market research actually negatively correlates the firms’ innovation paths. In particular, if the firms learn that $s_b \in (\frac{1}{2}, \bar{s}_b)$, one firm will definitely choose to attempt innovation on attribute $b$; the attribute valued by the larger segment. Given that the rewards are higher from innovation that serves this segment, this firm will select an aggressive R&D level and likely succeed in introducing a new product. Consequently, an equilibrium will exist where the rival, who is also informed, prefers as a best response to avoid competing for the same set of customers (with profits driven to 0) and hence pursues innovation that targets the other segment, even though it is smaller and generates less profits than can be earned by exclusively serving the large segment. The mirror situation will occur, of course, if the firms learn that $s_b \in (1 - \bar{s}_b, \frac{1}{2})$, in which case one firm will target the larger segment that values
Firms target different segments

\[ 0 \quad \begin{cases} 1 - \bar{s}_b & \text{both attempt innovation on attribute } a \\ \bar{s}_b & \text{both attempt innovation on attribute } b \end{cases} \]
\[ 1 - s_b \quad s_b \quad 1 \]

Firms target the same segment

Figure 3.3: The Possible Dual-Market-Research Equilibria Depending on Segment Sizes

innovation on attribute \( a \) and the rival’s best response is to target the smaller segment that values attribute \( b \).

To summarize, if firms discover that \( s_b \) in an intermediate range, the segment-size effect is not very strong and the differentiation effect prevails. Hence, it is possible for identical firms facing ex-ante equally profitable innovative paths to both conduct market research, but end up differentiating in their choice of which innovation path to pursue. As is clear from Figure 3.3, this type of equilibrium is unique when the segment sizes are discovered to be in the range \( 1 - \bar{s}_b < s_b < \bar{s}_b \). The cutoffs \( \bar{s}_b \) and \( \bar{s}_b \) increase as development becomes less costly. As \( K \) decreases, the firms are induced to select higher R&D levels and are more likely to succeed in their development efforts; hence the chances of making positive profits by targeting the same segment, even if large, diminish and the differentiation effect becomes even stronger. Notably, for low values of \( K \) we have \( \bar{s}_b \to 1 \) and we can sustain the asymmetric equilibrium in attribute choice even when one of the segments is quite small compared to the other.

(ii) Single-market-research equilibrium. Market research may reveal that one segment is much larger, i.e., the majority of consumers only value innovation on a particular attribute (\( s_b \) close to 1 or zero). In that case, as we know from Result 3, if both firms are informed they will target the large segment with their innovation efforts. If their R&D efforts succeed, we know from Lemma 16 that their profits will be zero. Hence, even with horizontal preferences, when one firm is informed this reduces the expected value of information for the rival. Consequently, as \( C_s \) increases, at some point one firm will forgo market research. In this case, we can
sustain two pure strategy equilibria, in one the uninformed firm attempts innovation on attribute \( a \) and in the other it attempts innovation on attribute \( b \). The informed firm learns the value of \( s_b \), and has to decide whether to pursue the same attribute as the uninformed firm is expected to choose in equilibrium.\(^{12}\) The following result characterizes the equilibrium in terms of the attribute each firm chooses to attempt innovation on.

**Result 4** In a single market-research equilibrium, there exists a value \( \hat{s}_b \left( \frac{1}{2} < \hat{s}_b < 1 \right) \) such that the informed firm will attempt to innovate on

- attribute \( b \): if \( \hat{s}_b < s_b \),
- the attribute not chosen by the uninformed firm: if \( 1 - \hat{s}_b < s_b < \hat{s}_b \),
- attribute \( a \): if \( s_b < 1 - \hat{s}_b \).

The informed firm's decision is governed by the trade-off between the segment-size effect and the differentiation effect. If it discovers that one segment is very large compared to the other, then the segment-size effect dominates and it will target that segment irrespective of the uninformed firm's strategy. If the segment that values innovation on attribute \( b \) is neither too large nor too small relative to the segment that values attribute \( a \) (\( 1 - \hat{s}_b < s_b < \hat{s}_b \)) then the informed firm's best response is to differentiate. The informed firm is better off ensuring positive profits—though smaller than what it could earn as a monopolist serving the larger segment—to avoid a situation where competition drives profits to zero if its rival also succeeds in development.

We make two observations. First, recall that an informed monopolist (Proposition 14) would always target the larger of the two segments. However, with competition we see that a single informed firm, that has incurred the cost of market research and learns the structure of demand, may knowingly follow a 'niche' innovation strategy.

\(^{12}\)Recall from Section 3.2 that in any equilibrium, each firm takes the strategy of the rival as given and must prefer to follow its proposed equilibrium strategy. In this case, the uninformed firm's strategy is to innovate on one of the attributes and select an R&D level: \( \sigma^*(\text{uninformed}) = \{ mr = 0, j', \varphi' \} \). The informed firm's strategy must be a best response to this strategy. As we now explain, and formally show in the Appendix, the informed firm's best response will be contingent on what it learns from market research: \( \sigma^*(\text{informed}) = \{ mr = 1, j(s_b), \varphi_j(s_b) \} \).
and target the smaller segment with its innovative efforts. This happens when it turns out that the uninformed firm has bet on the attribute valued by the majority of consumers (and the niche segment is not too small). Second, this result is in contrast with the vertical model where an informed firm always chooses the attribute discovered to be valued by consumers irrespective of the uninformed rival's strategy.

(iii) No-market-research equilibrium. The value of being informed is bounded from above, so when the cost is high neither firm conducts market research in equilibrium. Given that the segments are of a priori equal size, the only force relevant is the differentiation effect and each firm attempts innovation on a separate attribute. Thus, when neither firm conducts market research and R&D efforts succeed, we should expect distinct new products to be launched – each catering to a different segment. Ex-post, except for knife-edge cases, one firm will do better than its rival given the realization of consumer preferences; but both firms will earn positive profits in equilibrium.

Lastly, we note that the situation here is similar in some respects to that analyzed in Section 3.3.1 with each attribute equally likely to be more valued by all consumers ($\alpha = \frac{1}{2}$). There, when neither firm conducted market research firms also chose differentiated innovation paths. However, unlike with horizontal preferences, ex-post only one firm would have bet correctly on the most valued attribute and earned positive profits.

Market Research and the Level of R&D Effort

Up to now we have focused on the value of being informed in terms of the innovation path selected. Two benefits emerged: Market research may reveal that one segment is so large that it only makes sense to target that segment, even if the rival is innovating to the same segment, or it may reveal that segments are not too dissimilar and hence it makes sense to differentiate. A third benefit of market research is the ability to adjust the R&D level itself to the exact size of the segments ($s_j$), which results in an investment that optimally matches the return. We now turn to understanding how R&D levels are affected by the results of market research.
Result 5 In a single-market-research equilibrium, the informed firm will see its R&D level initially decreasing and then increasing in $s_b$. In the dual-market-research equilibrium, there exists a region where the R&D levels of the firms exhibit diverging patterns as $s_b$ increases.

The intuition for this result is as follows. When an informed firm discovers that $s_b = 0$, it knows that all consumers only value innovation on attribute $a$, and hence innovative effort will always be directed to that attribute. As $s_b$ initially increases, this implies that the segment being targeted is decreasing in size and this lowers the R&D level. At some point, as $s_b$ increases further the informed firm will switch to attempting innovation on attribute $b$. From there on, the greater $s_b$ is discovered to be the more the firm will invest in R&D. When both firms conduct market research, recall from Result 3 that there exists a range of $s_b$ ($s_b \in [1 - \bar{s}_b, \bar{s}_b]$) in which each firm targets a different segment with its innovative efforts. In this region, the firm attempting innovation on attribute $a$ will select an R&D level $\varphi_{ab}^{11} = (1-s_b) \frac{v}{K}$ while its rival selects an R&D level of $\varphi_{ba}^{11} = s_b \frac{v}{K}$, which will clearly lead to diverging patterns, i.e., decreasing for the former level while increasing for the latter as $s_b$ increases.

To examine how the R&D levels for the various equilibria of Proposition 15 compare to one another, it is useful to understand the strategic interaction between firms’ R&D incentives.

Lemma 15 In any equilibrium in which the firms choose the same attribute to innovate on, R&D levels will form strategic substitutes. In any equilibrium in which the firms choose different attributes to innovate on, R&D levels will be strategically independent.

The first part of this lemma reflects the fact that when both firms attempt innovation on the same attribute, each firm is strictly worse off when the rival’s R&D efforts succeed (because profits will be zero). Hence, the higher the R&D level of the rival the less incentive a firm has to invest in R&D. The second part reflects the fact that when each firm attempts to innovate for a different segment their payoffs are
independent of each other. This latter outcome was not possible with vertical preferences because there, even when firms chose different attributes, each firm’s payoffs would still depend on the success or failure of its rival’s R&D.

In presenting how the various R&D levels compare to one another we use the following notation. When a firm conducts market research, we denote with subscript $c$ the attribute it learns is valued by the larger segment and by $\bar{c}$ the attribute valued by the smaller segment. For example, if the firm learns through market research that $\frac{1}{2} < s_b$ then $c = b$ and $\bar{c} = a$.\footnote{Note that in forming a best response the informed firm takes the strategy of the uninformed firm as given, and in equilibrium correctly conjectures whether the uninformed firm’s attribute choice corresponds to the segment that is larger ($c$) or smaller ($\bar{c}$).} In addition, let $s_c$ be the size of the larger segment. When a firm does not conduct market research, its R&D level does not depend on the attribute it chooses (given that $E[\tilde{s}_b] = \frac{1}{2}$) and for simplicity no subscripts are used.

**Proposition 16** For any given values of the parameters $K$ and $V$, there exists a $s^*_c$ ($\frac{1}{2} < s^*_c < 1$) such that the following relationships hold for the R&D levels in the different equilibria:

(i) in the uninformed firm cases: $\varphi^{01} < \varphi^{00}$,

(ii) in the informed firm cases: $\varphi^{11}_{cc} = \varphi^{00}_{cc} < \varphi^{11}_{cc} < \varphi^{11}_{cc} = \varphi^{10}_{cc}$,

(iii) for $s_c < s^*_c$ we have $\varphi^{10}_{cc} < \varphi^{00}$.

We highlight two R&D orderings that sharply contrast with the vertical preferences case. First, an informed firm facing an uninformed rival may allocate less to R&D than two informed firms. This occurs when the former ends up selecting the niche strategy of innovating to serve the small segment (see first inequality in part (ii)). The intuition is clear, exactly because the firm is informed, and knowingly decides to serve the small segment, it is prompted to select a relatively low R&D level. Second, and more surprising, if market research reveals that the large segment is not too big, then the informed firm in the single-market research equilibrium will invest less than two uninformed firms; even though the informed firm is attempting innovation on the larger segment (part (iii)). The intuition is that the informed firm
discovers that $s_c$ is large enough so that it does not want to pursue a niche strategy due to the segment-size effect. But at the same time it realizes, according to the equilibrium being played, that its rival has chosen to innovate for the same segment and the prospects of downstream competition dampen its R&D incentives. By contrast, two uninformed firms will always choose distinct segments and earn positive profits if their development efforts succeed, and hence select a greater R&D level. In sum, while with vertical preferences market research increased the return to R&D because the firm’s efforts were always directed to the more profitable innovation, with horizontal preferences this effect is qualified; market research may induce the firm to direct effort in a way that decreases the return on R&D relative to what an uninformed firm expects.

3.5 Model Extensions and Limitations

The setup of our model entails a number of assumptions on the demand structure, the nature of R&D, and firm behavior. We have made these assumptions in an attempt to best capture the phenomena of interest while at the same time maintaining simplicity and tractability. When relevant, we have justified the assumptions based on evidence from practice. That said, we have examined how modifying a number of key assumptions affects our results. The formal analysis of these extensions appears in the separate Technical Appendix. We also discuss several remaining limitations at the end of this section.

**Vertical Preferences with Heterogeneity in Willingness to Pay**—In the case of vertical preferences, all consumers agree on the ordering of which products they derive more utility from (net of price). In our setup, we have followed this convention but also assumed for simplicity that consumers are homogeneous with respect to willingness to pay for innovation on each type of attribute. One might wonder how allowing heterogeneity in consumers’ willingness to pay (as in Shaked and Sutton
1982) affects our results. In this case, a firm that innovates on the less valued attribute can make positive profits by charging a low enough price and serving the low willingness to pay consumers. Our analysis shows that all reported equilibria in Proposition 12 can hold under some conditions. Notwithstanding, because the desire to differentiate can be stronger with heterogeneity, we can also sustain equilibria in which a firm undertakes market research and chooses the attribute that is less preferred. For such equilibria to exist, the development cost factor ($K$) has to be low (so that each firm worries that its rival will succeed in R&D) and the disutility parameter ($D$) can neither be too high (otherwise the profits from innovation on the less preferred attribute are very small) nor too low (otherwise the two attributes are of similar value and duopoly competition is fierce). It is important to note that in these equilibria a firm’s attribute strategy is determined irrespective of what the market research reveals (in the sense that on the equilibrium path a firm is expected to choose attribute $j$, the preferred attribute $c$, or the non-preferred attribute $\bar{c}$), and in many cases the main value of market research is in allowing the firm to adjust its R&D level. However, with horizontal preferences, the attribute strategy will itself heavily depend on the specific results of the market research. In particular, only after the firm learns the true segment sizes does it determine if its innovation path will be positively or negatively correlated with that of its rival. We also note that R&D levels with vertical preferences form strategic substitutes even when there is heterogeneity in willingness to pay. By contrast, with horizontal preferences there are cases where R&D levels are strategically independent (see Lemma 15). Hence, the R&D comparative statics differ across the two settings.

**Sequential Asymmetric Market Research Equilibrium**— In our set-up, we assumed that firms simultaneously decide at $t=1$ whether to conduct market research. We found that asymmetric equilibria can arise in the market research decision even though the two firms are identical at the outset. Although such an analysis does not designate which specific firm will be the one to conduct market research, from a game-
theoretic standpoint our results imply that when firms play out the game they will
correctly anticipate each other’s actions and act accordingly (one firm will conduct
market research and the other firm’s best response is not to conduct). Prescriptively
then, our results tell a firm what it should do given what it expects its rival will
do. This interpretation is consistent with the standard Nash Equilibrium concept.
We further note that this feature is common in many product positioning models.
For example, in Shaked and Sutton (1982) and Moorthy (1988) one firm (arbitrarily
denoted firm 1) selects the high quality level and the other (arbitrarily denoted firm
2) selects the low quality in equilibrium. Such asymmetric equilibria with ex-ante
symmetric firms is characteristic of multi-stage games in other domains as well (e.g.,
Desai and Purohit 2004, in the context of selecting one of two pricing policies). From
an empirical standpoint, the way to interpret our asymmetric equilibria is that, under
the appropriate conditions, we should expect to observe heterogeneity in firm behavior
with respect to market research. For example, in the hiring of a market research firm
or in the allocation of funds to this activity.

One way in which the “coordination” issue of the asymmetric equilibrium can
be more explicitly modeled is by letting firms make their decisions sequentially. In
an extension, we have assumed that at time t=1’ the first-mover decides whether to
conduct market research. After observing this decision, the second mover decides
whether to conduct market research. We show the existence of equilibria where only
the first-mover conducts market research (and all other decisions are qualitatively
similar to those in our basic set-up). Of course, this kind of extension could still raise
the issue of which firm goes first. But assuming any random process that prompts
one firm to go first, our analysis would prescribe what the actions of the first and
second mover would be. This also captures the notion that in practice it may be rare
for both firms to make this kind of decision at exactly the same time.\footnote{The issue of which asymmetric equilibrium will be selected given symmetric parties is common in the literature on preemption and war of attrition games (see Fudenberg and Tirole 1991, on these types of games). We thank an anonymous reviewer for asking us to clarify these points.}
Pursuing Multiple Innovations— Firms in our model were restricted to choosing one attribute to innovate upon. In the model set-up section we have justified scenarios where such an assumption is realistic (see footnote 3). We have relaxed this assumption in the case of vertical preferences. Qualitatively, our key results continue to hold if we allow firms to simultaneously pursue multiple innovations. Firms would now select a positive R&D level to innovate on each attribute, but our basic findings would hold with respect to which attribute receives a greater R&D allocation. To see why, note that an informed firm will always select a much higher R&D level on the attribute it discovers to be preferred by consumers (since it knows for sure the return on it is higher). Uncertainty avoidance is still relevant because an uninformed firm would always select a higher R&D level for the safer attribute $b$, and differentiation is still a driving force under competition. Therefore, given that the same forces remain relevant for the R&D allocation decision, allowing firms to pursue multiple innovative routes complicates the exposition and analysis without offering much added insight.

Innovation on a Given Attribute Can be Radical or Incremental— To capture technical uncertainty, we have assumed that a firm’s R&D level impacts the success/failure of development. But the degree of improvement along the chosen attribute was fixed. We have examined the implications of accommodating this possibility by modifying the R&D structure in the vertical preferences case. In particular, given the decision to innovate on attribute $j$, a firm could attempt either ‘radical’ or ‘incremental’ innovation on that attribute. The benefit to consumers would be $V$ in the radical case and $v$ in the incremental case ($V > v$) and the R&D cost factors would be $K$ and $\gamma K$ ($\gamma < 1$), respectively. Hence, radical innovation yields greater consumer benefits but is more difficult to develop with a given R&D budget. Our results show that the nature of the equilibria in terms whether to conduct market research and the choice of which attribute to attempt innovation on are identical to the ones from the model presented in the paper. Furthermore, if the incremental cost factor is relatively high ($\gamma \to 1$), firms will always choose radical improvement. If, however, the incremental R&D cost factor is low enough, in all types of market
research equilibria there exist conditions for firms to attempt different degrees of attribute improvement. For example, there exists a dual-market-research equilibrium where both firms choose to innovate on the attribute preferred by consumers, but one firm will go for a radical improvement and the other for an incremental improvement.

Limitations— Although we believe our model captures many of the important issues firms face in setting innovation strategy under market uncertainty, and a number of extensions have been examined, we acknowledge several limitations of our study. We discuss these in turn.

In order to focus on the incentives to conduct market research and to invest in R&D effort, we have assumed no differences across firms in the ability to conduct either of these activities. Our approach has been to show that asymmetric equilibria can arise even if we restrict attention to identical firms. In reality, asymmetries in firms’ abilities may exist, and may be linked to differences in their industry position. These could affect the findings presented by creating greater or lower incentives for one of the firms to undertake market research or R&D.

The nature of consumer demand for innovation in our model along with Bertrand competition implied that a firm could never be better off when its rival’s R&D efforts succeeded (see profit levels in Lemmas 12 and 16). Hence, R&D investments in our model were never strategic complements. Indeed, our demand and price markup functions are consistent with the broad conditions in Athey and Schmutzler (2001), such that investments are strategic substitutes in the vertical preferences case and in the horizontal preferences case when firms target the same segment. However, one could envisage situations where a rival’s introduction of a similar innovation serves to boost the firm’s own profits. This can happen if multiple new products increase demand or speed up adoption (for example, by reducing consumer uncertainty or by

\textsuperscript{16}Specifically, see Lemma 1 in Athey and Schmutzler (2001). We note that if there is an initial asymmetry among firms, and the leader has no stand-alone incentive to innovate (Katz and Shapiro 1987), i.e., it would only be cannibalizing its own current sales by innovating, then R&D investment would be strategic complements for the leader. We also note that if there are R&D capability differences among firms, for example if the leader has more efficient development skills, then R&D levels can be strategic complements (see Ofek and Sarvary 2003).
increasing the amount of advertising that creates awareness for the category), or if network externalities are relevant and multiple competing products result in a greater base and higher consumer willingness to pay (Conner 1995, Sun, Xie, and Cao 2004). If such factors are present, they could lead to strategic complementarity of R&D investments. One would need a different demand and product market competition structure than in our model to accommodate these factors. How these issues relate to market uncertainty and the ability to resolve it, which are the focus of our research, are not entirely obvious and would require further research that is beyond the scope of this paper.

Lastly, we have made several assumptions regarding the market uncertainty firms face and how that is resolved through market research. First, we assumed that firms know the potentially relevant attributes or features but are uncertain either about their relative importance to consumers or how many consumers care about each one. In practice, discovering the relevant attributes may itself require some form of early market research (such as talking to retailers or observing consumers, see Srinivasan, Lovejoy, and Beach 1997). Second, for simplicity, market research was modeled as a binary decision that provides perfect information regarding consumer preferences. This captures many real-life situations where the nature of the study or consulting project needed to get a reliable market forecast is known and can be commissioned at the outset. It can also be interpreted as a decision to set-up a substantial marketing arm in the new product planning phase. Although market research techniques keep improving, and increasingly provide firms with accurate information about consumer preferences, in some instances such accuracy may be limited. Analysis we conducted reveals that if one holds the cost of market research constant but lets the precision of the results vary, we can sustain the following equilibria. Market research is conducted by: a) both firms if it is very precise, b) neither firm if it is very imprecise, and c) only

\[17\] This characterization is also consistent with our examples. In the case of laptop computers in the mid 90s, Compaq had been given the proposal for the number and type of focus groups to run and their cost. Management had to decide whether to conduct them or just go with the safer development path Bell and Leamon (1999). In the case of anti-depressants, Eli Lilly set up a dedicated team to conduct market research for evaluating development opportunities of a new drug Ofek (2006).
one firm for mid-levels of precision. We note that in the case of mid-level precision, the informed firm does not entirely trust the findings and may prefer to differentiate rather than attempt innovation on the attribute the (noisy) market research indicates is more valued.

3.6 Conclusion and Implications

Setting the course for new product development is a critical managerial decision in a range of industries. In making this decision, firms need to take into account the implications of demand and technology uncertainties along with competitive pressures. Indeed, a recent business practice is to set-up a separate entity within the organization—often called the New Product Planning group—that tries to integrate assessments of market potential, estimates of R&D feasibility, and the anticipation of rival actions Morin (2005). In this paper, we have attempted to shed light on how firms behave in such an environment by examining the front-end incentives to undertake market research (which resolves demand uncertainty) and the factors that influence the selection of R&D level (which reduces technical uncertainty)—all of which feed into the choice of which innovation strategy to pursue in the face of competition.

In Figure 3-4 we provide a summary of our main findings by indicating what actions a firm should take given the market and competitive conditions it finds itself in. We highlight the key managerial implications. First, when the cost (or difficulty) of conducting market research is non-negligible, a firm should not rush to conduct such research. Basically, if rivals are expected to undertake market research, this has the effect of lowering the value of information about consumer preferences and the best course of action could be to forgo market research. Second, regarding the decision of which attribute to innovate on and the R&D level to select, we find that the results depend on the nature of market uncertainty. If firms are uncertain as to which attribute is more important for all consumers (vertical case)—e.g., does the market want to move to smaller disk drives or is more capacity still the main purchase criterion—then a firm that conducts market research should always allocate its R&D to
the attribute it discovers is more valued. Because in this case market information has the effect of directing a firm’s investment to the more profitable attribute, conducting market research should be accompanied by a high R&D level. When it is prohibitively costly to conduct market research, firms should be very mindful of the development challenges. If technical uncertainty is fairly low and R&D likely to succeed, the firm should select a different innovation path than its rival.

If the type of uncertainty firms face is with respect to the size of segments that each predominantly prefer one attribute over the other (horizontal case)—e.g., what fraction of patients need greater efficacy in treating depression versus others for which co-treating physical pain is lacking with existing drugs—then a firm that conducts market research may want to act “counter” to the findings regarding which innovation the majority of consumers prefer. In particular, if the rival is expected to choose the attribute that appeals to the larger segment, a firm may consciously opt for a niche strategy to avoid head-to-head competition and, as a result, it should invest relatively lightly in R&D. Of course, the market research should reveal that the niche segment is not too small, otherwise the firm should attempt innovation on the larger segment. Taken together, we believe our findings provide important guidelines for how firms should approach new product planning that involves a confluence of customer, technology and competitor considerations Hauser, Tellis, and Griffin (2006).

At an industry level, our work is relevant for understanding certain patterns of evolution. For example, when firms forgo costly market research we can expect distinct innovation paths to be selected more often. This can occur even though one path may be initially perceived as more likely to achieve market acceptance than the other. Although we have purposefully focused on ex-ante symmetric firms, one could imagine situations where incumbents are better positioned to preempt the safer innovation route, particularly if it is of a sustaining nature Christensen (1997).\[^{18}\] This might help explain why when industry leadership changes hands, it is achieved through entrant innovation on dimensions that were initially dismissed by incumbents and industry.

\[^{18}\text{For example, incumbents may be better positioned from the R&D capability standpoint to pursue a sustaining path. Or the rewards to a sustaining path can be higher for incumbents due to reputation in that domain.}\]
experts as being less likely.
What Type of Market Uncertainty do I Face?

Vertical
(Which attribute do consumers value more?)

Horizontal
(How big is the segment that values each attribute?)

Should I Conduct Market Research?

If market research cost:
- Low: Always conduct mr
- Intermediate: Conduct mr if rival doesn't
- High: Never conduct mr

Which Attribute Should I Choose to Innovate on?
- If forgo mr, go for safe attribute (or other attribute if no strong prior)
- If conduct mr, go for most valued attribute

If R&D cost:
- Low:
  - if one segment: Much larger
    - choose to serve it,
  - Otherwise
    - differentiate from rival
- High:
  - choose safe attribute

How Much Should I Invest in R&D?
- If safe attribute
  - if segment chosen: Large
    - invest moderately
    - if segment chosen: Small
      - invest lightly
- If risky attribute
  - if segment chosen: Large
    - invest heavily
    - if segment chosen: Small
      - invest lightly
- If forgo mr
  - invest lightly

If conduct mr and segment chosen:
- Large
  - invest heavily
- Small
  - invest lightly

Always attempt innovation on most valued attribute

Always conduct mr if rival doesn't

If one segment: Much larger
- choose to serve it,
- Otherwise
  - differentiate from rival

Always conduct mr if rival doesn't

Always differentiate from rival

Figure 3-4: Summary of Main Findings
3.7 Appendix

Proof of Lemma 11 Let firm $i$ innovate successfully on attribute $j$. If firm $i$ is a monopolist, i.e., the only firm introducing a new product, its optimal price is: $p^m(i) = V$ if $j$ is the preferred attribute and $p^m(i) = V - D$ otherwise. When both firms introduce a new product, Bertrand price competition implies that the only case in which firm $i$ prices above zero is when $j$ is the preferred attribute and the rival firm innovated on attribute $j' \neq j$ – in which case $p^d(i) = D$. Otherwise, when both firms sell non-differentiated products, the equilibrium price is $p^d(i) = 0$. Lastly, when firm $i$ offers a new product that is inferior in the eyes of customers, its rival with a superior new product prices at $D$, which forces it to price at $p^d(i) = 0$. Profits follow these prices given that demand is normalized to 1. ■

Proof of Lemma 12 Fix the R&D intensities of the firms in the interval $[0, 1)$. Recall that we denote by $j = c$ innovation on the preferred attribute and let $j = \bar{c}$ denote innovation on the less preferred attribute. If a firm conducts market research then, conditional on its R&D effort being successful at $t=3$, it can be: (i) A monopolist, in which case choosing $j = c$ leads to profits of $V > V - D$, which are the profits it would get by choosing $j = \bar{c}$. (ii) A duopolist with a rival that innovates on the preferred attribute. Regardless of whether $c = a$ or $b$, the informed firm will get zero profits. (iii) A duopolist with a rival that innovates on the less favored attribute: in that case choosing $j = c$ leads to a profit equal to $D > 0$, which are the profits it would get by choosing $j = \bar{c}$. Hence,

$$E\pi(i)[j=c \mid \varphi(i), \varphi(i')] > E\pi(i)[j = \bar{c} \mid \varphi(i), \varphi(i')]
\Rightarrow \max_{\varphi(i)} E\pi(i)[j = c \mid \varphi(i')] \geq \max_{\varphi(i)} E\pi(i)[j = \bar{c} \mid \varphi(i')] \Rightarrow j = c \text{ is a dominant strategy.}$$

■

Proof of Proposition 11 From Lemma 12, an informed monopolist innovates on the preferred attribute and maximizes $\varphi V - K_2 \varphi^2$, which leads to $\varphi_c = \frac{V}{K}$ and $\pi^m_c = \frac{V^2}{2K} - C$. An uninformed monopolist that chooses to innovate on the safer attribute maximizes $\varphi_b(V - \alpha D) - K_2 \varphi_b^2$ which leads to $\varphi_b = \frac{V - \alpha D}{K}$ and $\pi^m_b =$
An uninformed monopolist never innovates on attribute $a$ as it leads to 
$\pi^m_a = \frac{(V-(1-a)D)^2}{2K} < \pi^m_b$. Hence, a monopolist chooses $mr = 1$ iff $\pi^m_c \geq \pi^m_b$, or 
$C \leq C^m = \frac{V^2}{2K} - \frac{(V-aD)^2}{2K}.

**Proof of Proposition 12** Using the equilibrium of the $t=3$ subgame (Lemma 11), 
we now characterize the equilibrium in remaining strategies $\sigma(i) = \{mr = \{0,1\}, j = \{a,b,c\}, \varphi_j(i)\}$. A firm forms a best response to its rival’s actions, denoted 
$\sigma(i') = \{mr' = \{0,1\}, j' = \{a,b,c\}, \varphi_j(i')\}$.

**Case 1**- Rival strategy is $\sigma(i') = \{mr' = 0, j' = b, \varphi_j(i') = \varphi_b\}$. The best response 
for Firm $i$ is determined by comparing the expected profits from each of the following strategies

i. $\sigma(i) = \{mr = 0, j = b, \varphi_b(i) = \varphi\}$.

$$\pi(i) = \max_{\varphi} \varphi(1 - \varphi_b)(V - \alpha D) - \frac{K}{2} \varphi^2 = \frac{(1-\varphi_b)(V-\alpha D)^2}{2K}. $$

ii. $\sigma(i) = \{mr = 0, j = a, \varphi_a(i) = \varphi\}$.

$$\pi(i) = \max_{\varphi} \varphi [(1 - \varphi_b)(V - (1 - \alpha)D) + \varphi_b \alpha D] - \frac{K}{2} \varphi^2 = \frac{(1-\varphi_b)(V-(1-a)D+\varphi_aD)^2}{2K}. $$

iii. $\sigma(i) = \{mr = 1, j = c, \varphi_c(i) = \varphi_a \text{ if } c = a, \varphi_c(i) = \varphi_b \text{ if } c = b\}$.

$$\pi(i) = -C + (1 - \alpha) \max_{\varphi_a} \left\{\varphi_b (1 - \varphi_b) V - \frac{K}{2} \varphi_b^2 \right\}$$

$$+ \alpha \max_{\varphi_a} \left\{\varphi_a [(1 - \varphi_b)V + \varphi_b D] - \frac{K}{2} \varphi_a^2 \right\} = -C + \frac{(1-\varphi_b)(1-\varphi_a)V+\varphi_aD+\alpha(1-\varphi_b)V+\varphi_bD)^2}{2K}. $$

**Case 2**- Rival strategy is $\sigma(i') = \{mr' = 0, j' = a, \varphi_j(i') = \varphi_a\}$. The best 
response for firm $i$ is determined by comparing the expected profits from each of the following strategies

i. $\sigma(i) = \{mr = 0, j = b, \varphi_b(i) = \varphi\}$.

$$\pi(i) = \max_{\varphi} \varphi [(1 - \varphi_a)(V - \alpha D)+\varphi_a(1 - \alpha)D] - \frac{K}{2} \varphi^2 = \frac{(1-\varphi_a)(V-\alpha D)+\varphi_a(1-a)D)^2}{2K}. $$

ii. $\sigma(i) = \{mr = 0, j = a, \varphi_a(i) = \varphi\}$.

$$\pi(i) = \max_{\varphi} \varphi(1 - \varphi_a)(V - (1 - \alpha)D) - \frac{K}{2} \varphi^2 = \frac{(1-\varphi_a)(V-(1-a)D)^2}{2K}. $$

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iii. $\sigma(i) = \{mr = 1, j = c, \varphi_c(i) = \hat{\varphi}_a \text{ if } c = a, \varphi_c(i) = \hat{\varphi}_b \text{ if } c = b\}.$

$$\pi(i) = -C + (1 - \alpha) \max_{\hat{\varphi}_b} \left\{ \hat{\varphi}_b \left[ (1 - \varphi_a) V + \varphi_a D - \frac{K}{2} \hat{\varphi}_b^2 \right] \right\} + \alpha \max_{\hat{\varphi}_a} \left\{ \hat{\varphi}_a \left[ (1 - \varphi_a) V - \frac{K}{2} \hat{\varphi}_a^2 \right] \right\} = -C + \frac{(1 - \alpha)(\varphi_a V + \varphi_a D)^2 + \alpha(1 - \varphi_a V)^2}{2K}.$$  

**Case 3-** Rival strategy is $\sigma(i') = \{mr' = 1, j' = c, \varphi_c(i') = \varphi_a \text{ if } c = a, \varphi_c(i') = \varphi_b \text{ if } c = b\}$. The best response for firm $i$ is determined by comparing expected profits from each of the following strategies

i. $\sigma(i) = \{mr = 0, j = b, \varphi_b(i) = \varphi\}.$

$$\pi(i) = \max_{\varphi} \varphi \left[ (1 - \alpha)((1 - \varphi_a) V + \varphi_a D) \right] - \frac{K}{2} \varphi^2$$

$$= \frac{(1 - \alpha)(1 - \varphi_a V^2 + \alpha(1 - \varphi_a)(V - D))^2}{2K}.$$  

ii. $\sigma(i) = \{mr = 0, j = a, \varphi_a(i) = \varphi\}.$

$$\pi(i) = \max_{\varphi} \varphi \left[ \alpha(1 - \varphi_a V) + (1 - \alpha)(1 - \varphi_b)(V - D) \right] - \frac{K}{2} \varphi^2$$

$$= \frac{(\alpha(1 - \varphi_a V^2 + (1 - \alpha)(1 - \varphi_b)(V - D))^2}{2K}.$$  

iii. $\sigma(i) = \{mr = 1, j = c, \varphi_c(i) = \hat{\varphi}_a \text{ if } c = a, \varphi_c(i) = \hat{\varphi}_b \text{ if } c = b\}.$

$$\pi(i) = -C + (1 - \alpha) \max_{\hat{\varphi}_b} \left\{ \hat{\varphi}_b ((1 - \varphi_a) V - \frac{K}{2} \hat{\varphi}_b^2 \right\} + \alpha \max_{\hat{\varphi}_a} \left\{ \hat{\varphi}_a (1 - \varphi_a) V - \frac{K}{2} \hat{\varphi}_a^2 \right\} = -C + \frac{(1 - \alpha)(1 - \varphi_b V)^2 + \alpha(1 - \varphi_a V)^2}{2K}.$$ 

A strategy profile $\sigma^*$ will constitute a subgame perfect Bayesian Nash equilibrium of the game if for $i = \{1, 2\}$ we have $\pi(i)[\sigma^*(i), \sigma^*(i')] \geq \pi(i)[\hat{\sigma}(i), \sigma^*(i')]$, $\forall \hat{\sigma}(i)$, $i \neq i'$ (where $\hat{\sigma}(i)$ is any alternative strategy firm $i$ can select). Using the profit levels established in the above three cases, we can specify the conditions required for each equilibrium to hold.

**No-market-research equilibrium with both firms innovating on attribute $b$:** The R&D intensity is $\varphi_{bb}^{00} = \frac{(1 - \varphi_{bb}^{00})(V - aD)}{K} \iff \varphi_{bb}^{00} = \frac{V - aD}{K + V - aD}$. The conditions for this equilibrium are given below and are derived by establishing when the profits in Case 1(i) are greater than those in Case 1(ii) and Case 1(iii), respectively.

1. $\varphi_{bb}^{00} \leq \frac{1 - 2\alpha}{1 - \alpha} \iff \alpha(V - aD) \leq K.$

2. $C \geq C_{bb}^{00} = \frac{(1 - \alpha)((1 - \varphi_{bb}^{00})(V - aD))^2 + \alpha(1 - \varphi_{bb}^{00})(V + \varphi_{bb}^{00}D)^2 - (1 - \varphi_{bb}^{00})(V - aD))^2}{2K}$. 

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No-market-research equilibrium, one firm chooses to innovate on attribute a and the other on b: The R&D intensities are
\[
\varphi_{ab}^{00} = \frac{K(V-(1-a)D)-(V-D)(V-aD)}{K^2-(V-D)^2}, \quad \varphi_{ba}^{00} = \frac{K(V-aD)-(V-D)(V-(1-a)D)}{K^2-(V-D)^2}.
\]

Note that \( \varphi_{ab}^{00} < \varphi_{ba}^{00} \). The conditions for this equilibrium are given below and are derived by establishing when the profits in Case 1(ii) are greater than those in Case 1(i) and Case 1(iii), and when the profits in Case 2(i) are greater than those in Case 2(iii). The profits in Case 2(ii) are always smaller than those in Case 2(i).

1. \( \varphi_{ba}^{00} \geq \frac{1-2a}{1-a} \).

2. \( C \geq C_{asym}^{00} = \max\{C_{ab}^{00}, C_{ba}^{00}\} \), where
\[
C_{ab}^{00} = \frac{(1-\alpha)(1-\varphi_{ba}^{00}V)^2+\alpha((1-\varphi_{ba}^{00})V+\varphi_{ba}^{00}D)^2-(1-\varphi_{ba}^{00})(V-(1-a)D)+\varphi_{ba}^{00}aD)^2}{2K},
\]
\[
C_{ba}^{00} = \frac{(1-\alpha)((1-\varphi_{ab}^{00})V+\varphi_{ab}^{00}D)^2+\alpha((1-\varphi_{ab}^{00})V)^2-(1-\varphi_{ab}^{00})(V-aD)+\varphi_{ab}^{00}(1-a)D)^2}{2K}.
\]

Dual-market-research equilibrium: The R&D intensities satisfy \( \varphi_{aa}^{11} = \varphi_{bb}^{11} = \varphi_{cc}^{11} \), and solve \( \varphi_{cc}^{11} = \frac{(1-\varphi_{cc}^{11})V}{K} \Leftrightarrow \varphi_{cc}^{11} = \frac{V}{V+K} \). The condition for this equilibrium is given below and is derived by establishing when the profits in Case 3(iii) are greater than those in Case 3(i). The profits in Case 3(i) are always greater than those in Case 3(ii).

1. \( C \leq C_{11}^{11} = \frac{((1-\varphi_{cc}^{11}V)^2-(1-\varphi_{cc}^{11})(V-aD))^2}{2K} \).

Single-market-research equilibrium, uninformed firm chooses attribute b: The R&D intensities are
\[
\varphi_{bc}^{10} = \frac{(K-V)(V-aD)}{K^2-(1-a)V^2-a(V-D)^2}, \quad \varphi_{bb}^{10} = \frac{(1-\varphi_{bb}^{10})V}{K}, \quad \varphi_{ab}^{10} = \frac{(1-\varphi_{ab}^{10})V+\varphi_{ab}^{10}D}{K}.
\]

Note that \( \varphi_{ab}^{10} > \varphi_{bb}^{10} \). The conditions for this equilibrium are given below and are derived by establishing when the profits in Case 1(iii) are greater than those in Case 1(iii).
1(i) and Case 1(ii), and when the profits in Case 3(i) are greater than those in Case 3(iii).

1. \( C \geq C_{bc}^{01} = \frac{(1-\alpha)((1-\varphi_{bb}^{10})V)^2 + \alpha((1-\varphi_{bb}^{10})V)^2 - ((1-\alpha)(1-\varphi_{bb}^{10})V + \alpha(1-\varphi_{bb}^{10})(V-D))^2}{2K} \).

2. \( C \leq C_{cb}^{10} \), where \( C_{cb}^{10} = \min\{C_{cb}^{10}, C_{ab}^{10}\} \) and
   \[
   C_{bb}^{10} = \frac{(1-\alpha)((1-\varphi_{bb}^{10})V)^2 + \alpha((1-\varphi_{bb}^{10})V + \varphi_{bb}^{01}D)^2 - ((1-\varphi_{bb}^{10})(V-\alpha D))^2}{2K},
   \]
   \[
   C_{ab}^{10} = \frac{(1-\alpha)((1-\varphi_{bb}^{10})V)^2 + \alpha((1-\varphi_{bb}^{10})V + \varphi_{bb}^{01}D)^2 - ((1-\varphi_{bb}^{10})(V-(1-\alpha)D) + \varphi_{bc}^{01}D)^2}{2K}.
   \]
   Note that \( C_{cb}^{10} = C_{bb}^{10} \iff \varphi_{bc}^{01} \leq \frac{1-2\alpha}{1-\alpha} \).

**Single-market-research equilibrium with uninformed firm innovating on attribute a:** The R&D intensities are

\[
\varphi_{ac}^{01} = \frac{(K-V)(V-(1-\alpha)D)}{K^2 - \alpha V^2 - (1-\alpha)(V-D)^2},
\varphi_{ba}^{10} = \frac{(1-\varphi_{ac}^{01})V + \varphi_{ac}^{01}D}{K},
\varphi_{aa}^{10} = \frac{(1-\varphi_{ac}^{01})V}{K}.
\]

Note that \( \varphi_{aa}^{10} \leq \varphi_{ba}^{10} \). The conditions for this equilibrium are given below and are derived by establishing when the profits in Case 2(iii) are greater than those in Case 2(i), and when the profits in Case 3(ii) are greater than those in Case 3(i) and Case 3(iii).

1. \( \alpha \geq \frac{1-\varphi_{bb}^{10}}{2-\varphi_{aa}^{10} - \varphi_{bc}^{10}} \).
2. \( C \geq C_{bc}^{01} = \frac{(1-\alpha)((1-\varphi_{bb}^{10})V)^2 + \alpha((1-\varphi_{bb}^{10})V)^2 - ((1-\alpha)(1-\varphi_{bb}^{10})V + \alpha(1-\varphi_{bb}^{10})(V-D))^2}{2K} \).
3. \( C \leq C_{ba}^{10} = \frac{(1-\alpha)((1-\varphi_{bb}^{10})V + \varphi_{bb}^{01}D)^2 + \alpha((1-\varphi_{ac}^{01})V)^2 - ((1-\varphi_{ac}^{01})(V-(1-\alpha)D) + \varphi_{bc}^{01}(1-\alpha)D)^2}{2K} \).

Let \( (C_{bc}^{01}, C_{ba}^{10}) = (C_{bc}^{01}, C_{ab}^{10}) \) if the equilibrium \( (C_{ac}^{01}) \) does not exist, and \( \{\min\{C_{bc}^{01}, C_{ac}^{01}\}, \min\{C_{ab}^{10}, C_{bc}^{10}\}\} \) otherwise. And let \( C_{ab}^{00} = C_{bb}^{00} \) if the equilibrium \( (\varphi_{ab}^{00}) \) does not exist, \( = C_{ab}^{00} \) if equilibrium \( (\varphi_{bb}^{00}) \) does not exist and \( = \min\{C_{bb}^{00}, C_{ab}^{00}\} \) otherwise. Then from the expressions in the various steps of the proof: \( 0 < C_{ab}^{01} < C_{ab}^{11} < C_{ab}^{10} < C_{ab}^{00} \).

**Proof of Result 1** From the characterization of the no-market-research equilibrium (proof of Proposition 12) with \( j = j' = b \), this equilibrium requires \( K \geq K \) where \( K = \)
\[ \min(V, a(V - aD)). \]

From the characterization of the no-market-research equilibrium with \( j \neq j' \), this equilibrium requires

\[
\frac{K(V - aD) - (V - D)(V - (1 - a)D)}{K^2 - (V - D)^2} > \frac{1 - 2a}{1 - a} \quad \iff \quad 0 \geq K^2 - (V - D)^2 - \frac{1 - a}{1 - 2a} (K(V - aD) - (V - D)(V - (1 - a)D)).
\]

Note that this parabola in \( K \) has a root greater than \( V - D \) and a root smaller than \( V - D \). Call \( K_1 \) the greater root. \( K_1 \) can be larger or smaller than \( V \) (when \( \alpha = 0 \), it is smaller, and when \( \alpha = \frac{1}{2} \) it is larger). Hence the existence of this equilibrium requires \( K \leq K_1 \) where \( K_1 = \max(V, K_1) \). It follows from the definitions that \( K < K \) and that, as a function of \( \alpha \), both first equal \( V \) and then increase with an infinite limit at \( \alpha = \frac{1}{2} \).

**Proof of Result** 2 The first part follows directly from \( C_{bc}^{01} < C_{cb}^{10} \). For the second part of the result, the sustainability of this equilibrium requires \( \alpha V^2 + (1 - \alpha)(V - D)^2 + \frac{V - (1 - a)D}{1 - 2a}(\alpha V - (1 - \alpha)(V - D)) \geq K^2 \) which is not satisfied for \( \alpha = 0 \) but is for \( \alpha = \frac{1}{2} \). As the LHS is an increasing function of \( \alpha \), there must exist an \( \alpha \) as stated in the result.

**Proof of Lemma 13** Using the expected profit expressions from the proof of Proposition 12, we have \( \frac{\partial^2 \pi(t)}{\partial \phi(i) \partial \phi(i)} < 0 \) for all R&D intensities.

**Proof of Proposition 13** Ranking the R&D levels follows from the equilibrium expressions used in the proof of Proposition 12. This involves tedious algebra and the details are available from the authors. The value of \( \alpha^* \) is found by solving the condition for \( \varphi_{ba}^{00} > \varphi_{bc}^{11} \iff \frac{K}{D} = \frac{V^2 - (V - D)^2}{(V + K)(K + (V - D))} = \alpha^* > \alpha \). Table 3.1 below gives the comparative statics that arise from differentiating each of the R&D levels by the appropriate parameter.

<table>
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<tr>
<th>( \alpha )</th>
<th>( \varphi_{ba}^{10} )</th>
<th>( \varphi_{ab}^{00} )</th>
<th>( \varphi_{ba}^{10} )</th>
<th>( \varphi_{bc}^{10} )</th>
<th>( \varphi_{bc}^{11} )</th>
<th>( \varphi_{cc}^{11} )</th>
<th>( \varphi_{ba}^{00} )</th>
<th>( \varphi_{bb}^{00} )</th>
<th>( \varphi_{ab}^{00} )</th>
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<td>+</td>
<td>+ ( \vdots ) then ( (b) )</td>
<td>+ then ( (c) )</td>
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</table>

\( (a) \) if \( \alpha \) small, \( - \) otherwise \( (b) \) if \( \alpha \) small, + otherwise \( (c) \) if \( \alpha \) small or \( \frac{K}{D} \) large, \(-\) otherwise
Proof of Lemma 16 If firm \( i \) innovates on attribute \( j \) and its rival does not, firm \( i \) sells to all the consumers of its segment at price \( V \). Its profits are then \( s_b V \) if \( j = b \) and \((1 - s_b)V \) if \( j = a \). If both introduce identical new products, the profits are zero due to Bertrand competition. If the firm does not innovate (its R&D fails), its profits are zero as well.

Proof of Proposition 14 An informed monopolist always attempts innovation on the large-segment attribute and then maximizes \( \varphi \max\{s_b, 1 - s_b\} V - \frac{K}{2} \varphi^2 \) which leads to \( \varphi_c = \frac{\max\{s_b, 1 - s_b\} V}{K} \) and \( \pi^m_c = \frac{(\max\{s_b, 1 - s_b\} V)^2}{K} - C \). An uninformed monopolist is indifferent between the two attributes and maximizes \( \varphi \frac{V}{2} - \frac{K}{2} \varphi^2 \) which leads to \( \varphi_j = \frac{1}{2} \frac{V}{K} \) and \( \pi^m_j = \frac{(\frac{V}{2})^2}{K} - C \).

Proof of Proposition 15

No-market-research equilibrium with one firm innovating on attribute \( a \) and the other on \( b \): The R&D intensities are \( \varphi_{ab}^{00} = \varphi_{ba}^{00} = \frac{V/2}{K} \equiv \varphi^{00} \). The condition for this equilibrium is

1. \( C \geq C_s^{00} = \int_0^{1-\varphi^{00}} \frac{1}{2K} \frac{((1 - s_b) V)^2}{K} f(s_b) ds_b + \int_{1-\varphi^{00}}^1 \frac{1}{2K} \frac{((1 - \varphi^{00}) s_b V)^2}{K} f(s_b) ds_b - \frac{V^2}{8K} \).

Dual-market-research equilibrium: The attribute chosen and the R&D level depend on the segment size revealed by market research. We use the notations introduced prior to Result 3. When \( s_b < 1 - \bar{s}_b \) or \( \bar{s}_b < s_b \), both firms target the large segment \( c \) and set \( \varphi_{cc}^{11} = \frac{s_b V}{K + s_c V} \). When \( 1 - s_b < s_b < s_b \), the firms target different segments, one targets the large segment \( c \) and sets \( \varphi_{cc}^{11} = \frac{s_b V}{K} \), and one the small segment \( \bar{c} \) and sets \( \varphi_{cc}^{11} = \frac{(1 - s_b) V}{K} \). When \( 1 - \bar{s}_b \leq s_b \leq 1 - s_b \) or \( s_b \leq s_b \leq \bar{s}_b \), both equilibria are sustainable. Let \( \{(j'(s_b), \varphi'(s_b))\} \) be the strategy of a firm in a dual-market-equilibrium. We have: \( \{(j'(s_b), \varphi'(s_b))\} = (c, \varphi_{cc}^{11}) \) if \( s_b < s_c \), \( (c, \varphi_{cc}^{11}) \) or \( (\bar{c}, \varphi_{cc}^{11}) \) if \( \frac{1}{2} \leq s_c < s_b \), and \( (c, \varphi_{cc}^{11}) \) or \( (\bar{c}, \varphi_{cc}^{11}) \) if \( s_b \leq s_c \leq \bar{s}_b \). The condition for this equilibrium is

1. \( C \leq C_s^{11} = \max_{j'(s_b), \varphi'(s_b)} \left\{ \int_0^1 \max_{j(s_b), \varphi(s_b)} \pi(j(s_b), \varphi(s_b)) j'(s_b), \varphi'(s_b)) f(s_b) ds_b \right. \\
- \left. \max_{j, \varphi} \int_0^1 \pi(j, \varphi j'(s_b), \varphi'(s_b)) f(s_b) ds_b \right\}. \)
Single-market-research equilibrium, uninformed firm chooses attribute $b$: The single-market-research equilibrium where the uninformed firm chooses attribute $a$ is derived by symmetry. The R&D intensity of the uninformed firm is $\varphi^{01} = \frac{K^2V}{K^2 - \int_{\frac{1}{2}}^{1} s_b^2V^2 f(s_b) ds_b} \cdot 20$ Define $\hat{s}_b = \frac{1}{2 - \varphi^{01}}$. The attribute choice and R&D level of the informed firm depend on the segment size revealed by market research. When $s_b \leq \frac{1}{2}$, the informed firm invests in attribute $a$ and $\varphi^{01} = \frac{(1-s_b)V}{K}$. When $\frac{1}{2} \leq s_b \leq \hat{s}_b$, the informed firm still invests in attribute $a$ and $\varphi^{01} = \frac{(1-s_b)V}{K}$. When $s_b \leq \hat{s}_b$, the informed firm invests in attribute $b$ and $\varphi^{01} = \frac{(1-s_b)V}{K}$. Let $(j^{10}(s_b), \varphi^{10}(s_b))$ be the R&D strategy of the informed firm in a single-market-equilibrium. The condition for this equilibrium is

1. $C \geq C^{01}_s = \int_0^1 \max_{j(s_b), \varphi(s_b)} \pi(j(s_b), \varphi(s_b)) [\varphi^{10}(s_b), \varphi^{10}(s_b)] f(s_b) ds_b - \frac{K}{2} (\varphi^{01})^2$.

2. $C \leq C^{10}_s = \int_0^1 \hat{s}_b \frac{(1-s_b)^2V^2}{2K} f(s_b) ds_b + \int_{\hat{s}_b}^{1} \frac{(1-s_b)^2V^2}{2K} f(s_b) ds_b - \frac{(V/2)^2}{2K}$.

Proof of Result 3 If both firms choose to innovate on attribute $b$ in equilibrium, they invest $\frac{s_bV}{K+s_bV}$. One firm could deviate and invest $\frac{(1-s_b)V}{K}$ on attribute $a$. This deviation is not profitable iff $\frac{(1-s_b)V}{K} \leq \frac{s_bV}{K+s_bV} \iff 0 \leq s_b^2V + s_b(2K-V) - K$. Hence the condition $s_b \geq s_b$ where $s_b = \frac{-K+\frac{1}{2}V+\frac{1}{2}\sqrt{4K^2+4V^2}}{V}$ is decreasing from $\frac{\sqrt{5}-1}{2}$ to $\frac{1}{2}$ wrt $K$.

The equilibrium with both firms attempting innovation on attribute $a$ is derived by symmetry. In an equilibrium where the firms choose different attributes, the optimal investment on attribute $b$ is $\frac{s_bV}{K}$. The optimal investment on attribute $a$ is $\frac{(1-s_b)V}{K}$.

The firm investing on $a$ does not want to deviate to $b$ iff $\frac{(1-s_b)V}{K} \geq (1 - \frac{s_bV}{K}) \frac{s_bV}{K} \iff s_b^2V - 2Ks_b + K \geq 0$. Hence the condition $s_b \leq \overline{s}_b$ where $\overline{s}_b = \frac{K-\sqrt{K^2-KV}}{V}$ is decreasing from 1 to $\frac{1}{2}$ wrt $K$. The non-deviation condition for the firm on attribute $b$ is derived by symmetry. This yields the conditions in the result. ■

Proof of Result 4 If the uninformed firm invests $\varphi_j'$ on attribute $j'$, the informed firm chooses between attribute $j$, with optimal R&D level $\frac{(1-s_j)V}{K}$, and attribute $j'$,
with optimal R&D level \((1 - \varphi_{j'}) \frac{s_{j'}V}{K}\). The informed firm chooses \(j'\) iff \(s_{j'} \geq \frac{1}{2 - \varphi_{j'}}\).

Hence if \(j' = b\), \(\hat{s}_b = \frac{1}{2 - \varphi_b}\), and if \(j' = a\), \(\hat{s}_b = 1 - \frac{1}{2 - \varphi_a}\). 

**Proof of Result 5** From Result 4, when the informed firm attempts innovation on \(a\), its R&D level is decreasing wrt to \(s_b\), and when it attempts innovation on \(b\), it is increasing. Result 3 shows that when \(1 - s_b < s_b < \hat{s}_b\), one firm attempts innovation on \(a\) and the other on \(b\). So when \(s_b\) increases, they have diverging R&D patterns.

**Proof of Lemma 15** From the R&D levels established in the proof of Proposition 15, when firms choose the same attribute we have \(\frac{\partial^2 \pi(i)}{\partial \varphi(i) \partial \varphi(i')} < 0\), but when they choose different attribute \(\frac{\partial^2 \pi(i)}{\partial \varphi(i) \partial \varphi(i')} = 0\).

**Proof of Proposition 16**

(i) We have \(\varphi^{00} = \frac{V/2}{K} = \int_0^{s_b} sVf(s)ds\)

\[\int_0^{s_b} sVf(s)ds + \int_{s_b}^{1} (1 - \varphi_{cc}^{10}(s))sVf(s)ds > \frac{1}{2 - \varphi^{01}}\]. 

Effectively, each uninformed firm in the no-market-research equilibrium serves a different segment and is like a monopolist, while the uninformed firm in a single-market-research equilibrium will face a competitor when the size of the segment it has chosen to innovate for is large. (ii) We have \(\varphi^{11} = \varphi^{10} = \frac{s_cV}{K} > \frac{(1 - \varphi^{01})s_cV}{K} = \varphi^{10}_{cc}\). Effectively, an informed firm that exclusively targets the large segment (regardless of the rival’s information level) invests more than an informed firm competing with an uninformed firm for the large segment. We also have \(\varphi^{11}_{cc} < \varphi^{11} \iff \frac{(1 - s_c)V}{K} < \frac{s_cV}{K + s_cV} \iff s_c > s_b\). Effectively, in the dual-market-research equilibrium, when the two equilibria in the R&D stage exist, each firm invests more when they both innovate for the large segment than the firm that innovates to server the small segment. (iii) As \(\varphi^{00} > \varphi^{10}\), we can define the cutoff value \(s^{*}_c\) by \(s^{*}_c = \frac{1}{2(1 - \varphi^{01})} > \frac{1}{2}\).
Appendix A

Technical Appendix for:
Market Research and Innovation Strategy

In this Technical Appendix we provide details of the model extensions discussed in the Limitations section of Chapter 3. Throughout this appendix, we will refer to the model from Section 3.2 as the basic model.

A.1 Heterogeneity in Consumer Demand and Segmentation through Innovation

We extend the basic model to reflect consumer heterogeneity in willingness to pay for innovation quality. As will become clear, such heterogeneity allows for the case in which both types of innovations (one on attribute $a$ and the other on attribute $b$) achieve positive profits in equilibrium if successfully developed.

As in the basic model, all consumers have the same taste in terms of which attribute they would value more, but are heterogeneous in their willingness to pay for innovation quality. Specifically, each consumer is characterized by a marginal valuation of quality $\theta$, which is uniformly distributed between 0 and 1. The utility derived
from consuming a new product of quality $q$ and price $p$ is $q\theta - p$. We will call a new product embodying innovation on the preferred attribute a 'high-value' innovation and a new product embodying innovation on the less preferred attribute a 'low-value' innovation. Note that for the high-value innovation $q = V$, and for the low-value innovation $q = V - D$. As in the basic model, firms face market uncertainty as to which attribute is preferred (in this case whether innovation on attribute $j$ will be high or low-value), but know the distribution of willingness to pay for quality conditional on the realized preference. The utility of not buying is still normalized to 0 and consumers buy at most one product. The timeline of the game is identical to that in basic model and firms choose actions from the same strategy space.

The following Lemma gives the profits that arise in equilibrium in the final subgame.\footnote{In the vertical set-up we analyze here, a monopolist with an innovation will choose to serve only half of the market and extract the full surplus of the marginal consumer at $\theta = 1/2$. This introduces a factor of $1/4$ everywhere compared to the basic model where all consumers were homogeneous. In the basic model the monopolist serves the entire market and extracts the surplus of every consumer.}

**Lemma 16** In the pricing subgame firm $i$'s profits $\pi_j(i)$ will be

- $\frac{V}{4}$, if it is the sole innovator and attribute $j$ is high-value,
- $\frac{V-D}{4}$, if it is the sole innovator and attribute $j$ is low-value,
- $\frac{4V^2D}{(3V+D)^2}$, if firm $i$ innovates on the high-value attribute and its rival innovates on low-value attribute,
- $\frac{VD(V-D)}{(3V+D)^2}$, if firm $i$ innovates on the low-value attribute and its rival innovates on the high-value attribute, and
- 0, otherwise.

**Proof** A consumer buys a product of quality $q$ and price $p$ from a monopolist iff $\theta \geq \frac{p}{q}$. The monopolist maximizes its profit $\pi = (1 - \frac{p}{q})p$ leading to $p^* = \frac{q}{2}$ and $\pi^* = \frac{q^2}{4}$.

In a duopoly, if the new products are identical the profits are zero (as in standard vertical differentiation models). When each firm introduces a different innovation, without loss of generalitly let firm 1 sell a new product of quality $V$ at price $p_1$ and
firm 2 a new product of quality $V - D$ at price $p_2$. The marginal consumers are
\[ \theta_1 = \frac{p_1 - p_2}{V - D} \quad \text{and} \quad \theta_2 = \frac{p_2}{V - D}. \]
The best responses are $p_1^* = \frac{D + p_2}{2}$ and $p_2^* = \frac{p_1(V - D)}{2}$. From which one can solve for the profits in equilibrium that are
\[ \pi_1^* = \frac{4V^2D}{(3V + D)^2} \quad \text{and} \quad \pi_2^* = \frac{VD(V - D)}{(3V + D)^2}. \]

**Monopoly**

It is quite evident from Lemma 16 that the monopoly case is unchanged in spirit from the basic model (since only the first two profit levels are relevant and those are identical up to a scaling constant). Therefore we can state

**Proposition 17** There exists a cutoff value $C_m = \frac{aD(2V - aD)}{32K}$ such that for $C \leq C_m$ a monopolist would conduct market research and select R&D level $\varphi_c = \frac{V}{4K}$. Its expected payoffs in this case are $\pi_c^m = \frac{V^2}{32K} - C$. For $C_m \leq C$ the monopolist forgoes market research and chooses to innovate on the safer attribute $b$ with $\varphi_b = \frac{V - aD}{4K}$. Its expected payoffs are $\pi_b^m = \frac{(V - aD)^2}{32K}$.

**Proof** The proof is identical to the proof of Proposition 1.

**Duopoly**

It should be obvious that we can qualitatively sustain all the equilibria from the basic model for certain regions of the parameter space. For firms to be able to segment the market with two distinct innovations (one on attribute $a$ and the other on attribute $b$) and achieve positive profits, $D$ cannot be too close to $V$. When $D$ is close to $V$, consumers' utility from the less preferred attribute approaches zero. We therefore state

**Lemma 17** There exists a $\bar{D}$ such that for $\bar{D} < D$, the 5 equilibria $(\frac{1}{1c}, \frac{1}{1b}, \frac{10}{ca}, \frac{10}{bb}, \frac{00}{ab})$ of the basic model exist and the R&D levels are unchanged. The cutoff values for $C$ have the same rank ordering, but take on different values from the basic model.

**Proof** From Lemma 16, at the limit when $D \rightarrow V$ profits from introducing an innovation on the less preferred attribute approach zero. In this case the payoff...
matrix approaches the one described in Lemma 1 from the main paper. Hence by continuity all the equilibria of the basic model hold.

New equilibria with Segmentation

In the basic model, an informed firm (i.e., that has conducted market research) has a dominant strategy to innovate on the preferred attribute, regardless of the attribute its rival is innovating upon (Lemma 2). This result is driven by the fact that in the basic model a firm selling the less preferred innovation makes no profit when both firms introduce new products. In the vertical differentiation setting we analyze here, when $D$ is not too close to $V$, this has to be re-examined because when both types of innovation are introduced, the low-value innovation can yield positive profits. It is still the case that an informed firm always chooses to invest on the high-value attribute if its rival is expected to innovate on the low-value attribute. However, to avoid head to head competition (leading to zero profits per Lemma 16) an informed firm might settle for the low-value attribute if its rival is expected to innovate on the high-value attribute. The desire to differentiate is driven by duopolistic competition in the product market, therefore, new equilibria will emerge when the development cost factor $K$ is relatively small (because then R&D levels tend to be high and hence successful; and a firm might be better off pursuing the low-value innovation that allows segmenting the market than risking fierce price competition with identical high-value innovations). Before presenting the new equilibria that arise when segmentation is possible, we state the following Lemma

**Lemma 18** In an equilibrium where at least one firm undertakes market research, at least one firm will pursue the high-value attribute. It does not have to be an informed firm.

**Proof** If an informed firm learns after conducting market research that its competitor is pursuing the low-value attribute, it will target its R&D towards the high-value attribute. This is because from Lemma 16 selling the high-value innovation leads to a higher profit whether the firm ends up being a monopolist or facing a competitor.
selling the low-value innovation. Hence, it cannot be the case that both firms pursue the low-value innovation when at least one is informed and knows which attribute is more valued.

We now describe the new equilibria that emerge in this extended set-up, and start with the single-market research case. We characterize an equilibrium outcome where the informed firm innovates on attribute $a$, regardless of what it learns from market research, while the uninformed firm innovates on attribute $b$. Such an equilibrium could never exist in the basic set-up because there the informed firm would always direct innovative effort contingent on the outcome of its market research.

**Proposition 18** A new single-market-research equilibrium exists where the uninformed firm innovates on attribute $b$ while the informed firm innovates on attribute $a$ regardless of the market research results. This equilibrium exists when $\alpha \leq \hat{\alpha}$, $K \leq \bar{K}(\alpha)$, and $D(K, \alpha) \leq D \leq \bar{D}(K, \alpha)$.

**Proof** The R&D intensities of this equilibrium are characterized by

$$
\varphi_{a|a} = \frac{1}{K} \left[ (1 - \varphi_b) \frac{V}{4} + \varphi_b \frac{4V^2D}{(3V + D)^2} \right],
\varphi_{a|b} = \frac{1}{K} \left[ (1 - \varphi_b) \frac{V - D}{4} + \varphi_b \frac{VD(V-D)}{(3V + D)^2} \right],
$$

where $\varphi_{a|a}$ denotes an informed firm’s R&D level when innovating on attribute $a$ contingent on discovering that attribute $a$ is the high-value attribute, while $\varphi_{a|b}$ denotes the informed firm’s R&D level when innovating on attribute $a$ contingent on discovering that attribute $b$ is the high-value attribute. The R&D level of the uninformed firm is

$$
\varphi_b = \frac{1}{K} \left[ \alpha \left( (1 - \varphi_{a|a}) \frac{V - D}{4} + \varphi_{a|a} \frac{VD(V-D)}{(3V + D)^2} \right) + (1 - \alpha) \left( (1 - \varphi_{a|b}) \frac{V}{4} + \varphi_{a|b} \frac{4V^2D}{(3V + D)^2} \right) \right].
$$

When the informed firm learns that attribute $b$ is the high-value attribute, it chooses to innovate on $a$ iff its competitor’s R&D level is high $\varphi_b \geq \frac{1}{1 + \frac{4V^2(V-D)}{(3V + D)^2}}$. The uninformed firm pursues attribute $b$ iff $\alpha(1 - \varphi_{a|a}) \leq (1 - \alpha)(1 - \varphi_{a|b}) + \alpha \varphi_{a|a} \frac{4V(V-D)}{(3V + D)^2} + (1 - \alpha) \varphi_{a|b} \frac{16V^2}{(3V + D)^2}$. The second inequality is satisfied when $\alpha$ is lower than a threshold.
(satisfied for $\alpha = 0$ but not satisfied for $\alpha = .5$). $\varphi_b$ is decreasing with respect to $\alpha$ as both the direct effect of $\alpha$ is negative and $\varphi_b$ is strategic substitute with $\varphi_{a/b}$ and $\varphi_{a/a}$ which are not directly affected by $\alpha$. When $\alpha = 0$, $\varphi_b = \frac{\sqrt{\gamma} - (\frac{\gamma - 4\gamma^2D}{(3\gamma + D)^2})^\frac{\gamma - D}{4}}{K^2 - (\frac{\gamma - 4\gamma^2D}{(3\gamma + D)^2})} \frac{\gamma - D}{4}$ which is decreasing with respect to $K$. And when $K = \frac{\gamma}{4}$, $\varphi_b$ is increasing with respect to $D$ and $\varphi_b$ is smaller than $\frac{1}{1 + \frac{4}{D}(3\gamma + D)^2}$ when $D = 0$ and equal to it when $D = V$. When $K = V$, $\varphi_b$ is always smaller than $\frac{1}{1 + \frac{4}{D}(3\gamma + D)^2}$. This implies the existence of $\dot{K}(\alpha = 0)$, $\dot{D}(K, \alpha = 0)$, and $\dot{D}(K, \alpha = 0)$ such that, when $\alpha = 0$, this equilibrium exists only if $K \leq \dot{K}(\alpha = 0)$ and $\dot{D}(K, \alpha = 0) \leq D \leq \dot{D}(K, \alpha = 0)$. By contuinity, we can define $\dot{K}(\alpha)$, $\dot{D}(K, \alpha)$, and $\dot{D}(K, \alpha)$.

The intuition for this equilibrium is that when the cost of R&D is low, both firms will select relatively high R&D levels and will be concerned about head to head competition that drives profits to zero. Hence, a differentiation effect exists. Furthermore, $D$ is in a mid-range so that innovating on attribute $a$, even if discovered to be the low-value attribute, still yields decent returns. At the same time, by always being anticipated to innovate on attribute $a$, the informed firm lowers the R&D level of the uninformed firm, which knows it is exclusively innovating on attribute $b$. The informed firm benefits from conducting market research because it can adjust its R&D level based on the information (which is valuable here because firms tend to select a high level).\(^2\) In the notation of the main paper, this equilibrium would be denoted $(01)_{ba}$. Of course, this equilibrium would require that the cost of market research not be too low nor too high.\(^3\)

When the cost of market-research is low, both firms engage in market-research. If a duopoly setting is very likely, i.e., both firms succeed in their R&D efforts, the firms would rather sell differentiated products to soften product market competition. Hence, even with perfect knowledge of consumer tastes, one firm may wish to pursue the high-value innovation while the other firm focuses on the low-value innovation. In

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\(^2\)For example this equilibrium exists when $K = .26V$, $\alpha = 0.02$ and $D = 0.2V$.

\(^3\)We have checked that the reverse equilibrium, in which the uninformed firm innovates on the ex-ante more uncertain attribute $a$ while the informed firm is always anticipated to innovate on the safer attribute $b$, does not exist.
particular, when the development cost factor is small, a firm can invest aggressively on the high-value attribute forcing the other firm to opt for innovation on the low-value attribute. This leads to a new type of dual-market-research equilibrium with differentiated products. The next proposition details those equilibria.

**Proposition 19** A new type of dual-market-research equilibrium exists where each of the informed firms innovates on a different attribute when \( K \leq \hat{K}(0) \), and \( \hat{D}(K, 0) \leq D \leq \hat{D}(K, 0) \)

**Proof** Let \( \varphi_c \) and \( \varphi_{c'} \) be the R&D intensities of the firm pursuing the high-value attribute and the firm on the low-value attribute respectively. These intensities are characterized by \( \varphi_c = \frac{1}{K} \left[ (1 - \varphi_{c'}) \frac{V}{4} + \varphi_{c'} \frac{4V^2D}{(3V+D)^2} \right] \) and \( \varphi_{c'} = \frac{1}{K} \left[ (1 - \varphi_c) \frac{V-D}{4} + \varphi_c \frac{VD(V-D)}{(3V+D)^2} \right] \). Using Lemma 18, the firm facing a competitor innovating on the low-value attribute innovates on the high-value attribute. But the firm facing a competitor innovating on the high-value attribute, responds by innovating on the low-value attribute iff
\[
\varphi_c \geq \frac{1}{1+\frac{4VD(V-D)}{(3V+D)^2}}.
\]
Note that this is identical to the case \( \alpha = 0 \) in the proof of the previous proposition. ■

**A.2 Sequential Asymmetric Market Research Equilibrium**

In our basic model, we analyze a fully symmetric situation, and we find the existence of an asymmetric equilibrium, which is the single-market-research equilibrium. In this section, we analyze the case where the firms choose their market research decision sequentially in order to break the initial symmetry. We assume that at time \( t = 1' \) the first-mover decides whether to conduct market research. After observing this decision, the second mover decides whether to conduct market research. Once the market research phase is done, both firms observe whether its competitor is informed, then they simultaneously choose an attribute and how much to invest in R&D. Finally, both the technical and the demand uncertainties are resolved and the firms compete in price à la Bertrand. By consequence, only the market research phase differs from
the basic model. We are interested in understanding how being first mover or second mover affects the market research decisions. Therefore we focus our attention on $\alpha$ not too close to $\frac{1}{2}$, which corresponds to the case in the basic model when only the R&D phase can be asymmetric.\footnote{The only equilibria in the basic model are the dual-market-research equilibrium $\left(\frac{1}{2}\right)$, the single-market-research equilibrium with the uninformed firm on the safe attribute $\left(\frac{1}{2}\right)$, and the no-market-research equilibrium with both firms on the safe attribute $\left(\frac{3}{2}\right)$.}

The analysis of this model differs from the analysis of the basic model. Nevertheless, we exploit their similarities to prove that the first-mover always chooses the market research in the single-market-research equilibrium. In particular, the profits–on the equilibrium path– have already been computed in the basic model. If both firms engage in market-research, their profits are $\pi_{cc}^{11}$ for both firms. If one firm is informed while the other is not, the profits of the informed firm and uninformed firm are $\pi_{cb}^{10}$ and $\pi_{bc}^{01}$ respectively. And if no firm chooses the market research, the profits are $\pi_{bb}^{00}$. The game tree is given in Figure A-1.

In the basic model, in the region of existence of the single-market-research equilibrium, an informed firm facing an uninformed competitor has a higher profit than its uninformed competitor. This result is stated in the following lemma and helps
Lemma 19 In our basic model, in the region where the single-market-research equilibrium exists, i.e. $C \in [C^{01}, C^{10}]$, the profit of the informed firm in the single-market-research equilibrium is larger than the profit of its competitor

$$\pi^{10}_{cb} > \pi^{01}_{bc}$$

Proof When $C = C^{10}$, the informed firm is exactly in different between being informed or uninformed, hence $\pi^{10}_{cb} = \frac{(1-\varphi^{01}_{bc})(V-aD)^2}{2K}$.

Recall that $\pi^{01}_{bc} = \frac{(1-\alpha)(1-\varphi^{01}_{cb})V+\alpha(1-\varphi^{10}_{ab})(V-D))^2}{2K}$ and that $\varphi^{01}_{bc} < \varphi^{10}_{bb} < \varphi^{10}_{ab}$. Therefore when $C = C^{10}$, $\pi^{01}_{bc} < \pi^{10}_{cb}$. As the profit $\pi^{01}_{bc}$ is independent of the market research cost $C$, while the profit $\pi^{10}_{cb}$ is a decreasing function of $C$, we conclude that $\pi^{01}_{bc} < \pi^{10}_{cb}$ when $C < C^{10}$. ■

We can now state the main result of this section. If the firms do not make their market research decision simultaneously, the first mover is always the informed firm in the single-market-research equilibrium.

Proposition 20 In the sequential model, when $\alpha$ is not too close to $1/2$, the first-mover is always the informed firm in the single-market-research equilibrium.

Proof Let's assume first that $\pi^{10}_{cb} > \pi^{00}_{bb}$. This implies that if the first mover forgoes market research, the second mover engages in market research. We then need to analyze two cases. (i) If $\pi^{11}_{cc} > \pi^{01}_{bc}$, the second mover engages in market research. In that case, the first mover chooses between engaging in market research to get $\pi^{11}_{cc}$ and forgoing market research to get $\pi^{01}_{bc}$. Therefore the first mover chooses to engage in market research. (ii) If $\pi^{11}_{cc} < \pi^{01}_{bc}$, the second mover forgoes market research. In that case, the first mover chooses between engaging in market research to get $\pi^{10}_{cb}$ and forgoing market research to get $\pi^{01}_{bc}$. Therefore, using the previous lemma, the first mover chooses to engage in market research as well.

Let's assume now that $\pi^{10}_{cb} < \pi^{00}_{bb}$. This implies that if the first mover forgoes market research, the second mover forgoes market research as well.
Therefore, in a single-market-research equilibrium, the first-mover is always the informed firm.

A.3 Pursuing Multiple Innovations

The main paper assumes that each firm is restricted to attempting a single innovation, with all R&D effort dedicated to pursuing either attribute $a$ or $b$. In effect, we implicitly assumed that the R&D cost function had a fixed component that deterred firms from pursuing multiple new product projects and simultaneously trying to innovate on each of the attributes. We now lift this restriction and study the opposite case in which firms can select a positive R&D level to pursue multiple innovations (and there are no diseconomies of scope or fixed costs associated with pursuing multiple innovation paths). The two new product projects, one attempting innovation on attribute $a$ and the other on attribute $b$, will be referred to as the $a$-innovation and $b$-innovation respectively. Because $\frac{\partial \tilde{R}_j}{\partial \varphi_j} \big|_{\varphi_j=0} > 0$, $j = \{a, b\}$, a firm will always choose a positive R&D level for developing each of these innovations.

The timing of the game and the relevant parameters of the model remain unchanged, the only difference is that at $t=2$ the strategy space for firms is enlarged to allow allocating R&D effort to each of the possible innovations. We will show that the nature of our findings from the main paper, now expressed in terms of the relative R&D allocation across the two innovations, will be similar in this extended set-up. Specifically, we determine on which innovation, the $a$-innovation or the $b$-innovation, each firm devotes more R&D effort to in the extended model, and compare that to the attribute upon which the firms would exclusively innovate in the basic model. The reason we will get a correspondence between the two models is that firms’ incentives

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\[ 5 \text{Obviously one might contemplate yet another scenario where a firm tries to develop a single new product that embodies innovation on both attributes. One would then have to specify consumers’ utility from a product that has successfully innovated on both attributes. Though we believe our qualitative results would hold, since again a firm needs to decide how much R&D to allocate to each attribute with the same forces at play, we leave this issue for future research. In addition, such a scenario is more likely to involve significant diseconomies of scope on the development side as it is typically difficult to offer improvement/new features on two dimensions within one product (for example, a faster plane that also holds more passengers, a faster car that is also more fuel efficient, etc.)} \]
regarding how much to allocate to each attribute are still driven by a combination
of the uncertainty avoidance effect and the differentiation effect. In particular, when
a firm does not know which attribute is preferred, the uncertainty avoidance effect
drives it to select a higher R&D level on the b-innovation. The differentiation effect
helps soften competition—so that when a firm selects a very high R&D level on one
of the innovations, the rival may be better off concentrating its R&D effort on the
other attribute, to reduce the possibility that they end up with the same innovations
in the marketplace and compete head to head.

Monopoly

In the final stage pricing game, even if the monopolist has the potential to market
both innovations (the a—innovation and the b—innovation), it optimally chooses to
sell only the most valued innovation to achieve a profit equal to \( V \).\(^6\) Therefore, the
monopolist markets the less valued of the two innovations only when its R&D efforts
on the preferred attribute have failed.\(^7\)

In the basic model, we showed that an informed monopolist always chose to inno-
vate on the preferred attribute. Similarly we can now show that it will select a higher
R&D level for the more valued of the two innovations. It will be optimal to devote
positive R&D to innovate on the less preferred attribute, in the event that the R&D
efforts to innovate on the preferred attribute fail.

**Proposition 21** An informed monopolist selects a higher R&D level on the preferred
attribute than on the less preferred attribute.

**Proof** Let \( c \) be the preferred attribute and \( \bar{c} \) the less preferred attribute. The R&D
levels are \( \varphi_c = \frac{KV - (V-D)^2}{K^2 - (V-D)^2} > \varphi_{\bar{c}} = \frac{(K-V)(V-D)}{K^2 - (V-D)^2} \). \( \blacksquare \)

\(^6\) We assume that the monopolist would not market a product that no one buys. Otherwise, it
could always market both innovations, with the less preferred innovation priced above its valuation
of \( V - D \) and the preferred innovation priced exactly at \( V \), which would yield the same profits as
only marketing the preferred innovation. Given that there are usually set-up and marketing costs
involved— it is unlikely that this would ever occur.

\(^7\) This result remains true in the vertical differentiation model analyzed elsewhere in this Technical
Appendix. If the monopolist sells only the superior product, its profit is equal to \( V \). If the monopolist
wants to have a positive demand for both products, its maximal profit is \( \frac{VD(V-D)}{4(V+D)^2} < \frac{V}{4} \).
In the basic model, an uninformed monopolist always innovates on the safe attribute \((b)\). Similarly it now selects a higher R&D level in pursuing the \(b\)-innovation. The monopolist here will also pursue innovation on the more uncertain attribute \(a\) for two reasons: i) it may turn out to be the preferred attribute, ii) R&D effort on the safe attribute could still fail.

**Proposition 22** An uninformed monopolist selects a higher R&D level on the \(b\)-innovation than on the \(a\)-innovation.

**Proof** The investment levels are 
\[
\varphi_b = \frac{K(V-(1-a)D)-(V-aD)(V-D)}{K^2-(V-D)^2}, \\
\varphi_a = \frac{K(V-aD)-(V-(1-a)D)(V-D)}{K^2-(V-D)^2}.
\]

**Duopoly**

Given the innovations introduced in the final stage, it is straightforward to calculate equilibrium price strategies. Based on the equilibrium price strategies, the next table gives the equilibrium profit levels that emerge for the case that consumers prefer attribute \(a\) (a similar table can be constructed for the case that they prefer attribute \(b\)). Note that in this case the success of R&D effort on the \(b\)–innovation is only relevant if the R&D effort on the \(a\)–innovation has failed.

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>(a)</th>
<th>(\overline{a}b)</th>
<th>(\overline{a}\overline{b})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(0, 0)</td>
<td>(D, 0)</td>
<td>((V, 0))</td>
</tr>
<tr>
<td>(\overline{a}b)</td>
<td>(0, 0)</td>
<td>0</td>
<td>((V-D, 0))</td>
</tr>
<tr>
<td>(\overline{a}\overline{b})</td>
<td>(0, (V))</td>
<td>((0, V-D))</td>
<td>((0, 0))</td>
</tr>
</tbody>
</table>

\(a = \) R&D effort on \(a\) was successful
\(\overline{a}b = \) R&D effort on \(a\) was not successful and R&D effort on \(b\) was successful
\(\overline{a}\overline{b} = \) R&D effort on \(a\) was not successful and R&D effort on \(b\) was not successful
Order of profits are (firm1, firm2).

In the basic model, it is a dominant strategy for an informed firm to invest on the preferred attribute (Lemma 2). Similarly, it is now a dominant strategy to select a higher R&D level on the innovation embodying the preferred attribute.
Lemma 20 A dominant strategy for an informed firm is to allocate a higher R&D level to innovation on the preferred attribute than to innovation on the less preferred attribute.

Proof Let \( c \) be the preferred attribute and \( \bar{c} \) the less preferred attribute. Let \( \varphi'_c \) and \( \varphi'_\bar{c} \) be the R&D levels of the competitor on attributes \( c \) and \( \bar{c} \) respectively. We have

\[
\varphi_c = \frac{K(1-\varphi'_c)^2((1-\varphi'_c)V+\varphi'_c D)-(1-\varphi'_c)^2(1-\varphi'_c)^2(V-D)^2}{K^2-(1-\varphi'_c)^2(1-\varphi'_c)^2(V-D)^2} \\
> \varphi_\bar{c} = \frac{K(1-\varphi'_\bar{c})((1-\varphi'_\bar{c})V+\varphi'_\bar{c} D)-(1-\varphi'_\bar{c})^2(1-\varphi'_\bar{c})^2(V-D)}{K^2-(1-\varphi'_\bar{c})^2(1-\varphi'_\bar{c})^2(V-D)^2}.
\]

The single-market-research equilibria in this extension are similar to the single-market-research equilibria of the basic model. The next Lemma states that the uninformed firm uses the same condition in both models to decide how much R&D to direct to each innovation. In particular, this condition depends only on the informed firm's R&D level that is directed towards the preferred attribute. We call \( \varphi_{a}^{01} \) and \( \varphi_{b}^{01} \) the R&D levels of the uninformed firm. We denote \( \varphi_{k,j}^{10} \) the investment level of the informed firm on attribute \( k \) when attribute \( j \) is preferred.\(^8\)

Lemma 21 In a single-market-research equilibrium, the uninformed firm selects a higher R&D level for the b-innovation iff \((1 - \alpha)(1 - \varphi_{b/b}^{10}) \geq \alpha(1 - \varphi_{b/a}^{10})\). In the basic model, we had a similar result: In a single-market-research equilibrium, the uninformed firm innovates on attribute \( b \) iff \((1 - \alpha)(1 - \varphi_{b}^{10}) \geq \alpha(1 - \varphi_{a}^{10})\).

Proof The best response of the uninformed firm leads to

\[
\varphi_{a}^{01} - \varphi_{b}^{01} = D\frac{\alpha(1 - \varphi_{a/a}^{10}) - (1 - \alpha)(1 - \varphi_{b/b}^{10})}{K - [\alpha(1 - \varphi_{a/a}^{10})(1 - \varphi_{b/a}^{10}) + (1 - \alpha)(1 - \varphi_{b/b}^{10})(1 - \varphi_{a/b}^{10})](V - D)}.
\]

which is positive iff \( \alpha (1 - \varphi_{a/a}^{10}) - (1 - \alpha)(1 - \varphi_{b/b}^{10}) > 0 \). In the basic model, the condition follows from the proof of Proposition 2 case 3. In particular, if an informed firm selects \( \varphi_{c}^{10} \), the uninformed firm innovates on attribute \( b \) iff \((1 - \alpha)(1 - \varphi_{b}^{10})V +

\(^8\)There are four possible levels to consider \( \varphi_{a/a}^{10}, \varphi_{b/a}^{10}, \varphi_{a/b}^{10} \) and \( \varphi_{b/b}^{10} \).
\[ \alpha(1 - \varphi_a^{10})(V - D) \geq \alpha(1 - \varphi_a^{10})V + (1 - \alpha)(1 - \varphi_b^{10})(V - D) \iff (1 - \alpha)(1 - \varphi_b^{10}) \geq \alpha(1 - \varphi_a^{10}). \]

The no-market-research equilibria in this extension are similar to the no-market-research equilibria of the basic model. The next Lemma states that the firms use the same condition to decide how much R&D effort allocate. When a competitor allocates more to the \( a \)-innovation \((\varphi'_a > \varphi'_b)\), the best response for the firm is to always allocate more R&D to the \( b \)-innovation. We had a very similar result in the basic model where the best response of an uninformed firm facing an uninformed competitor innovating on attribute \( a \) was to innovate on attribute \( b \). When the competitor allocates more to the \( b \)-innovation \((\varphi'_b > \varphi'_a)\), the best response for the firm is to allocate more R&D to: the \( a \)-innovation if demand uncertainty is high \((\alpha \text{ close to } \frac{1}{2})\), and to the \( b \)-innovation if demand uncertainty is small \((\alpha \text{ close to } 0)\). These results are in line with the basic model (per Result 2) and are driven precisely by the same interplay between the uncertainty avoidance effect (which is more pronounced when \( \alpha \) close to 0) and the differentiation effect (which can kick in when \( K \) is small enough and \( \alpha \) close to \( 1/2 \)).

\textbf{Lemma 22} When neither firm conducts market research, let \( \varphi'_a \) and \( \varphi'_b \) be the R&D levels of an uninformed competitor, and \( \varphi_a \) and \( \varphi_b \) the best responses of the uninformed firm.

\[ \varphi_a > \varphi_b \iff \alpha(1 - \varphi'_a) > (1 - \alpha)(1 - \varphi'_b). \]

In the basic model, the only difference is \( \varphi'_a = 0 \) if the competitor innovates on \( b \) and \( \varphi'_b = 0 \) if the competitor innovates on \( a \).

\textbf{Proof} \[ \varphi_a - \varphi_b = D \frac{\alpha(1 - \varphi'_a) - (1 - \alpha)(1 - \varphi'_b)}{K - (1 - \varphi'_a)(1 - \varphi'_b)(V - D)} \]

The development cost factor \( K \) does not appear explicitly in this Lemma that is expressed in terms of best responses. In equilibrium, of course, the R&D intensities are functions of the parameters, and the explicit condition is expressed in terms of \( \alpha, K, V \) and \( D \). For the basic model, Figure 2 in the main paper illustrates the condition in the \((\alpha, K)\) plane.
A.4 Radical versus Incremental Innovations

To capture technical uncertainty, we have assumed that a firm’s R&D level impacts the success/failure of development. But the degree of improvement along the chosen attribute was fixed. We now examine the implications of accommodating this possibility by modifying the R&D structure in the vertical preferences case. In particular, given the decision to innovate on attribute $j$, a firm could attempt either ‘radical’ or ‘incremental’ innovation on that attribute. The benefit of the preferred attribute to consumers would be $V$ in the radical case and $v$ in the incremental case ($V > v$) and the R&D cost factors would be $K$ and $\gamma K$ ($\gamma < 1$), respectively. The value of an innovation on the less preferred attribute is 0 regardless of the type of the innovation. Hence, radical innovation yields greater consumer benefits but is more difficult to develop with a given R&D budget.

The following Lemma gives the profits that arise in equilibrium in the final pricing subgame. The main results of this section are driven by the fact that, when both firms innovate on the preferred attribute, a firm with an incremental innovation has always zero profit while a firm with a radical innovation might have a positive profit if the competitive product is an incremental innovation.

**Lemma 23** In the pricing subgame firm $i$’s profits $\pi_j(i)$ will be

- $V$, if it is the sole innovator on $j$, its innovation is radical and consumers prefer attribute $j$,
- $v$, if it is the sole innovator on $j$, its innovation is incremental and consumers prefer attribute $j$,
- $V - v$, if both firms innovate on $j$, firm $i$’s innovation is radical while the other innovation is incremental and consumers prefer attribute $j$,
- 0, otherwise.

**Proof** If consumers do not prefer attribute $j$, firm $i$ makes no profit. Assume now that consumers prefer attribute $j$. If firm $i$ innovates on attribute $j$ and its rival does not, firm $i$ sells at price $V$ a radical innovation and at price $v$ an incremental innovation. If both firms innovates on attribute $j$, firm $i$ makes a positive profit iff its
innovation is radical while its competitor’s is radical. In that case, its profit is \( V - \nu \).

\[ \text{Monopoly} \]

Unlike in the duopoly case, the monopolist does not care about the competition effects, it always choose the type of innovation with the higher benefit to cost ratio. Hence, when the R&D cost factors for the incremental and radical innovations are close, the monopolist always chooses the radical innovation. And when the incremental innovation is much cheaper than the radical innovation, the monopolist always chooses the incremental innovation. In all other aspects, the monopolist behaves like in the basic model (it always chooses the preferred attribute when informed and the safer attribute when uninformed, and it engages in market research when the cost is low enough).

**Proposition 23 .**

(i) If \( \gamma \geq \left( \frac{\nu}{V} \right)^2 \): A monopolist always selects the radical innovation. And there exists a cutoff value \( C_m^\tau \left( C_m^\tau = \frac{\alpha(2-\alpha)V^2}{2K} \right) \) such that for \( C \leq C_m^\tau \) a monopolist would conduct market research and select R&D level \( \varphi_c = \frac{V}{K} \). Its expected payoffs in this case are \( \pi_c^m = \frac{V^2}{2K} - C \). For \( C_m^\tau \leq C \) the monopolist forgoes market research and attempts innovation on the safer attribute \( b \) with \( \varphi_b = \frac{(1-\alpha)V}{K} \). Its expected payoffs are \( \pi_b^m = \frac{(1-\alpha)V^2}{2K} \).

(ii) If \( \gamma \leq \left( \frac{\nu}{V} \right)^2 \): A monopolist always selects the incremental innovation. And there exists a cutoff value \( C_m^i \left( C_m^i = \frac{\alpha(2-\alpha)V^2}{2\gamma K} \right) \) such that for \( C \leq C_m^i \) a monopolist would conduct market research and select R&D level \( \varphi_c = \frac{V}{2\gamma K} \). Its expected payoffs in this case are \( \pi_c^m = \frac{\nu^2}{2\gamma K} - C \). For \( C_m^i \leq C \) the monopolist forgoes market research and attempts innovation on the safer attribute \( b \) with \( \varphi_b = \frac{(1-\alpha)V}{\gamma K} \). Its expected payoffs are \( \pi_b^m = \frac{(1-\alpha)V^2}{2\gamma K} \).

**Proof** An informed monopolist attempting a radical innovation on the preferred attribute chooses \( \varphi_c = \frac{V}{K} \) and gets \( \pi_c^m = \frac{V^2}{2K} - C \). An informed monopolist attempting an incremental innovation on the preferred attribute chooses \( \varphi_c = \frac{V}{\gamma K} \) and gets \( \pi_c^m = \)
An uninformed monopolist attempting a radical innovation on the safe attribute chooses \( \varphi_c = \frac{(1-\alpha)\gamma}{K} \) and gets \( \pi^m_c = \frac{(1-\alpha)^2v^2}{2K} \). An uninformed monopolist attempting an incremental innovation on the safe attribute chooses \( \varphi_c = \frac{(1-\alpha)\gamma}{\gamma K} \) and gets \( \pi^m_c = \frac{(1-\alpha)^2v^2}{2\gamma K} \). Hence, when \( \gamma > \frac{v^2}{\gamma} \), the monopolist always prefers the radical innovation to the incremental innovation. And the cutoff values are derived from the expressions of the profits. ■

**Duopoly**

When the radical innovation is better than the incremental innovation (higher benefit to cost ratio), we have just seen that a monopolist always invests towards the radical innovation. Firms in a duopoly behaves similarly as there is no benefit in investing in an incremental innovation as it is less efficient and competition drives the profits to zero.

**Lemma 24** When \( \gamma \geq \left(\frac{v}{\gamma}\right)^2 \), the incremental innovation is never chosen in equilibrium.

**Proof** A firm investing on an incremental innovation makes a positive profit iff it turns out to be a monopolist on the preferred attribute. In that case, the firm could have had a larger profit by investing on a radical innovation. Indeed, suppose that the firm invests \( \varphi \) on an incremental innovation on attribute \( j \). Call \( q \) the probability that \( j \) is the preferred attribute and that its competitor does not have a product with attribute \( j \). Its profit is \( \varphi qv - \gamma \frac{v^2K}{2} \). By investing \( \sqrt{\gamma} \varphi \) on attribute \( j \) but on the radical innovation, the profit would have been at least \( \sqrt{\gamma} \varphi qv - \gamma \frac{v^2K}{2} \) which is a larger profit. ■

Therefore when \( \gamma \geq \left(\frac{v}{\gamma}\right)^2 \), the incremental innovation can be ignored and we are back to the basic model.

*Assume now that* \( \gamma < \left(\frac{v}{\gamma}\right)^2 \). The radical innovation could be chosen in a duopoly, as a radical innovation allows the firm to have a positive profit even if the other firm has an incremental innovation on the preferred attribute. The following proposition
shows that the market research decision and the attribute choice are identical to the ones in the basic model. When the market research cost is low, a dual market research equilibrium exists where both firms engage in market research. For higher market research cost only one firm chooses to be informed in a single market research equilibrium. And finally, when the market research cost is high, the no-market-research equilibrium becomes sustainable. In terms of attribute selection, an informed firm always chooses the preferred attribute while an uninformed firm can invest either on the safe or risky attribute. The main difference with the main model is that, now, firms can choose different level of investment after selecting an attribute. Both types of firms, informed or uninformed, can either pursue an incremental or a radical innovation. A firm chooses a radical innovation to avoid the competition on the incremental innovation.

**Proposition 24** For any values of the parameters $K$, $V$, $v$, $\gamma$ and $\alpha$, such that $\frac{v}{K} \leq \gamma < \left(\frac{v}{V}\right)^2$, there exist cutoff values $0 < C^{11} < C^{10} < C^{00} < C^{01}$ such that the equilibria of the game are

(i) Dual-market-research equilibrium. If $C < C^{11}$, both firms conduct market research and innovate on the most valued attribute. Firms will either

- both pursue the incremental innovation or
- one firm invests on the incremental innovation and the other invests on the radical innovation.

(ii) Single-market-research equilibrium. If $C^{01} < C < C^{10}$, only one firm conducts market research and innovates on the most valued attribute. The uninformed firm innovates on the safer attribute $b$; under certain conditions innovating on attribute $a$ may also be feasible. At least one firm invests on the incremental innovation, it does not have to be the informed firm.

(iii) No-market-research equilibrium. If $C^{00} \leq C$, neither firm conducts market research. Firms will either

- both pursue the incremental innovation on the safer attribute or
- invest incrementally on separate attributes (one incremental innovation on $a$ and one incremental innovation on $b$) or
both choose the safer attribute, one firm investing on the incremental innovation and the other on the radical innovation.

**Proof** The proof is similar to the proof of Proposition 2, hence we focus on the differences between the two proofs. In the no-market-research equilibrium (when $C$ is high), at least one firm attempts an incremental innovation on the safe attribute. Indeed, if the competitor gambles on the risky attribute, the safe attribute is always the best investment and, on top of that, we are in the case when the incremental innovation leads to a higher profit than the radical innovation. Hence the firms are either both attempting the incremental innovation on two different attributes or are both investing on the safe attribute. Let’s focus now on the case when both firms invest on the safe attribute, one attempting a radical innovation and the other an incremental innovation. Their R&D investments are respectively $\varphi_{rb}^{00}$ and $\varphi_{ib}^{00}$, where the subscript $rb$ stands for radical on $b$ and the subscript $ib$ for incremental on $b$.

The conditions for this equilibrium are (i) $\varphi_{rb}^{00} \leq \frac{1-2a}{1-\alpha}$, (ii) $\varphi_{ib}^{00} \leq \frac{(1-\alpha)V-\alpha}{(1-\alpha)V}$, and (iii) $\varphi_{ib}^{00} \geq \frac{v-\sqrt{V}}{(1-\sqrt{V})V}$. Conditions (i) and (ii) means that neither firms invest so much as to push its competitor to gamble on the risky attribute. And condition (iii) simply means that the firm attempting the incremental innovation invests sufficiently so that its competitor is better off attempting the radical innovation to take advantage of its superiority in the duopoly competition. These conditions can be satisfied only when $\alpha$ is not too close to .5, otherwise the firms would differentiate in terms of attributes, and when $K$ is not too large, otherwise the competition effect is not important.

The rest of the proposition is derived from the following result. An informed firm chooses to invest on the radical innovation on the preferred attribute if and only if its competitor invests on the incremental innovation on the preferred attribute and its R&D level $\varphi'$ is larger than $\frac{v-\sqrt{V}}{(1-\sqrt{V})V}$. Therefore, in the dual-market-research equilibrium with one firm attempting the incremental innovation and the other firm attempting the radical innovation, the R&D level on the incremental innovation is $\frac{(K-V)v}{\gamma K^{2}-v^{2}}$. This investment has to be larger than $\frac{v-\sqrt{V}}{(1-\sqrt{V})V}$, which is the case when $K = \frac{v}{\gamma}$. Hence by contuinity, this equilibrium exists when the R&D cost is not too large and the competition effect is prevalent. ♦
Bibliography


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