INSTABILITIES AND UNSTEADY FLOWS
IN CENTRIFUGAL PUMPS

by

JEFFREY PETER BONS

B.S., Aeronautical Engineering,
Massachusetts Institute of Technology

SUBMITTED TO THE DEPARTMENT OF AERONAUTICS AND ASTRONAUTICS
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE IN
AERONAUTICS AND ASTRONAUTICS

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May, 1990

© Massachusetts Institute of Technology, 1990. All rights reserved.

Signature of Author

Department of Aeronautics and Astronautics
May, 1990

Certified by

Dr. Begacem Jery

Assistant Professor of Aeronautics and Astronautics
Thesis Supervisor

Accepted by

Professor Harold Y. Wachman
Chairman, Department of Aeronautics and Astronautics
INSTABILITIES AND UNSTEADY FLOWS
IN CENTRIFUGAL PUMPS

by

JEFFREY PETER BONS

Submitted to the Department of Aeronautics and Astronautics on May 11, 1990
in partial fulfillment of the requirements for the Degree of
Master of Science in Aeronautics and Astronautics

ABSTRACT

The work discussed in this thesis represents the second phase in a multi-phase research program addressing pump instabilities. This report describes the construction and initial data from a test facility for investigating unsteady flow behavior in centrifugal pumping systems. The facility has been designed to give dynamic behavior similar to that found in practical environments for low specific speed pumps, and both steady and unsteady data have been obtained on a scaled up model of an existing pump of this type.

The central conclusion is that although the performance characteristic of the model pump was somewhat different than that of the original pump due to a geometric scaling error, the nature of instability observed in the model pump test loop is similar to that observed in the original centrifugal pump.

A matrix of system response data has been generated for the model pump. The performance characteristic shape and a stability parameter which depends on the volumes and lengths of the different component systems were shown to be the two dominant factors affecting pump stability. Changes in the pump stability boundary with operating speedline reflected the differences in characteristic shape and wheel speed variation between speedlines.

A simple linear model of the pumping system has been used to predict instability inception points and frequencies. The model accounts for variation in wheelspeed with pump mass flow, and employs a first order time lag in pressure rise to model the pump response. The model predictions matched the experimental results well, and captured the trends in the data. A nonlinear simulation using the same pumping system definition had similar results for predicting transient pump behavior during surge.

Preliminary flow visualization studies have revealed differences in the pump flowfield during steady and unsteady operation, pointing to the value of a detailed component by component investigation of the pump to improve the fundamental understanding of the unsteady behavior. This should be the focus of the next phase of research in this ongoing program.

Thesis Supervisor: Dr. Belgacem Jery

Title: Assistant Professor of Aeronautics and Astronautics
ACKNOWLEDGEMENTS

Naturally, this thesis represents the cumulative efforts of a great number of people. There are several of these individuals whom I wish to acknowledge as having been particularly influential in my work over these last two years.

I wish to thank my advisor, Dr. Belgacem Jery, for his support and guidance during my stay here. His good nature made it a pleasure to work with him. I would also like to thank Prof. Greitzer for his keen interest in my progress and his frequent words of encouragement. His ability to see quickly to the heart of any issue is complemented well by his patience in waiting for others to reach the same understanding.

This project was greatly benefitted by the professional advice and supervision of Tom Tyler, Paul Westhoff, and Paul Hermann of Sundstrand Corporation. Financial support for this work was provided by the National Science Foundation and Sundstrand Corporation.

I wish to thank my "partner in crime" and good friend, Nicolas Goulet, for his excellent work and lasting friendship. To Andrew Wo, I wish good luck with the continuation of an exciting project. I would also like to thank my former office and lab mate, Jon Simon, for many a stimulating conversation and shared insight. The experimental work presented herein would have been impossible without the expert workmanship and advice of GTL's technical staff, Viktor Dubrowski, Roy Andrew, and Jim Nash, who taught me the "ins and outs" of everything from electrical wiring to plumbing. Also, a special thanks to Holly, Diane, Nancy, and Karen for their constant assistance with paperwork and funding.

I would especially like to acknowledge the support and love manifest to me by my parents and family, particularly during the last two years. My father's academic and professional excellence has been a constant inspiration to me. Also, a note of appreciation in memory of my grandfather, Pieter Bons, who's love of higher learning influenced my original decision to attend MIT.

Finally, I cannot express in words the powerful influence that my wife Becky's love and encouragement have had on my life. Her patience and caring, along with the hugs and smiles from my darling daughter Cosette, have made all of this possible.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title Page</td>
<td>1</td>
</tr>
<tr>
<td>Abstract</td>
<td>2</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>3</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>4</td>
</tr>
<tr>
<td>List of Figures</td>
<td>7</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>12</td>
</tr>
<tr>
<td><strong>CHAPTER I: INTRODUCTION</strong></td>
<td>15</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>15</td>
</tr>
<tr>
<td>1.2 Background</td>
<td>16</td>
</tr>
<tr>
<td>1.3 Statement of Problem</td>
<td>17</td>
</tr>
<tr>
<td>1.4 Research Plan</td>
<td>18</td>
</tr>
<tr>
<td>1.5 Organization of this report</td>
<td>19</td>
</tr>
<tr>
<td><strong>CHAPTER II: THEORETICAL ANALYSES/MODELS</strong></td>
<td>21</td>
</tr>
<tr>
<td>2.1 Dynamic Systems Analysis Overview</td>
<td>21</td>
</tr>
<tr>
<td>2.2 The Lumped Parameter Model</td>
<td>23</td>
</tr>
<tr>
<td>2.2.1 Modification #1</td>
<td></td>
</tr>
<tr>
<td>Variable Wheel Speed</td>
<td>26</td>
</tr>
<tr>
<td>2.2.2 Modification #2</td>
<td></td>
</tr>
<tr>
<td>Time Lag in Pump Pressure Rise (A More Realistic Pump Response)</td>
<td>27</td>
</tr>
<tr>
<td>2.2.3 Summary of System Dynamic Model Features</td>
<td>30</td>
</tr>
<tr>
<td>2.3 Time-Resolved Dynamic Simulation</td>
<td>31</td>
</tr>
<tr>
<td><strong>CHAPTER III: DESCRIPTION OF EXPERIMENTAL FACILITY</strong></td>
<td>32</td>
</tr>
<tr>
<td>3.1 Design Considerations and Test Section Overview</td>
<td>32</td>
</tr>
<tr>
<td>3.1.1 Design Considerations</td>
<td>32</td>
</tr>
<tr>
<td>3.1.2 Facility Overview</td>
<td>33</td>
</tr>
<tr>
<td>3.2 Test Loop Components</td>
<td>34</td>
</tr>
<tr>
<td>3.2.1 Plenums and Airbags</td>
<td>34</td>
</tr>
<tr>
<td>3.2.2 Ducting</td>
<td>36</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.4.2 Predicted Ranges of Dynamic System Parameters</td>
<td>65</td>
</tr>
<tr>
<td>5.4.3 Predicted Instability Boundary</td>
<td>65</td>
</tr>
<tr>
<td>5.4.4 Summary of Linear Model Results</td>
<td>67</td>
</tr>
<tr>
<td>5.5 Comparison with Time-Resolved Simulation</td>
<td>67</td>
</tr>
<tr>
<td>5.5.1 Simulation Overview</td>
<td>67</td>
</tr>
<tr>
<td>5.5.2 Simulation Results</td>
<td>68</td>
</tr>
<tr>
<td>5.5.3 Summary of Simulation Results</td>
<td>70</td>
</tr>
<tr>
<td>5.6 Comparison with Development Pump Dynamic</td>
<td>70</td>
</tr>
<tr>
<td>5.7 Preliminary Flow Visualization Results During Unsteady Pump Operation</td>
<td>72</td>
</tr>
</tbody>
</table>

CHAPTER VI: CONCLUSIONS AND FUTURE WORK                                  | 74   |
| 6.1 Conclusions                                                        | 74   |
| 6.2 Recommendations for Future Work                                    | 75   |

REFERENCES                                                                | 76   |
FIGURES                                                                  | 79   |
APPENDIX A                                                               | 155  |
APPENDIX B                                                               | 173  |
APPENDIX C                                                               | 182  |
APPENDIX D                                                               | 187  |
LIST OF FIGURES

1.1 Performance characteristics and efficiency curves for various specific speed pumps [25].

2.1 Basic pumping system and analogies [9].
2.2 Instability modes and criterion [9].
2.3 Simple schematic of basic pump loop.
2.4 Experimental data: time trace of \( \Phi \) and \( \Omega \) (rpm) during surge for large B system.
2.5 Theoretical results: effect of variation in impeller wheel speedup term \( \left( \frac{dU_{tip}}{dm_p} \right) \) on predicted stability boundary.
2.6 Theoretical results: effect of variation in lag time constant \( (Z_{LAG}) \) on predicted stability boundary.

3.1 Schematic of MIT test loop.
3.2 Centrifugal pump used in MIT test facility.
3.3 Plenum layout with dimensions.
3.4 Plenum air bag filling procedure, part (a).
3.5 Plenum air bag filling procedure, part (b).
3.6 Loop transfer system.
3.7 Test section transmission.
3.8 Detail of transmission.
3.9 Side view: test section assembly.
3.10 Top view: test section assembly.
3.11 Schematic of MIT test loop with instrumentation indicated.
3.12 Pipe diffuser with pressure ports indicated.
3.13 Pipe diffuser with hydrogen bubble cross locations indicated.

4.1 Experimental data: MIT pump performance characteristics for 100%, 80%, 60%, 30%, and 20% speeds.
4.2 Experimental data: MIT pump efficiency curves for 100%, 80%, 60%, 30%, and 20% speeds.
4.3 Experimental data: comparison of performance characteristics for MIT pump and original.

4.4 Experimental data: comparison of efficiency curves for MIT pump and original.

4.5 Experimental data with analytical correlation results: Casey [26] surface roughness correction to bep performance of original pump superposed on Figure 4.3.

4.6 Experimental data: comparison of performance characteristics of original pump for two different Re (run numbers 458 and 448).

4.7 Experimental data: comparison of performance characteristics of MIT pump for two different Re (30% and 20% speedlines).

4.8 Experimental data with analytical correlation results: Osterwalder [18] surface roughness correction to bep performance of original pump superposed on Figure 4.3.

4.9 Experimental data: localized pressure data in pipe diffuser for 60% speedline.

4.10 Experimental observations: qualitative representation of flowfield in the diffuser and near the volute tongue for steady state operating point of $\Phi = 0.135$.

4.11 Experimental observations: qualitative representation of flowfield in the diffuser and near the volute tongue for steady state operating point of $\Phi = 0.065$.

4.12 Experimental observations: qualitative representation of flowfield in the diffuser and near the volute tongue for steady state operating point of $\Phi = 0.035$.

5.1 Unsteady test matrix.

5.2 Schematic of equivalent Helmholtz resonator for the simplified pumping system.

5.3 Experimental data: variation in reduced frequency of surge with air content in discharge plenum (100%, 80%, and 60% data).

5.4 Experimental data: variation in actual frequency (Hz) of surge with adjusted air content in discharge plenum (100%, 80%, and 60% data).

5.5 Experimental data: variation in experimental system B parameter with air content in discharge plenum (100%, 80%, and 60% data).

5.6 Experimental data: variation in inception point of surge with air content in discharge plenum (100%, 80%, and 60% data).

5.7 Experimental data: stability boundary (variation in inception point of surge with B parameter) (100%, 80%, and 60% data).

5.8 Experimental data: MIT pump performance characteristics for 100%, 80%, and 60% speeds.

5.9 Experimental data: detail of MIT pump performance characteristics for 100%, 80%, and 60% speeds.
5.10 Analytical fit to experimental data: performance characteristic slope vs. $\Phi$ for 100%, 80%, and 60% speeds.

5.11 Experimental data: variation in impeller wheel speed with pump mass flow for 100%, 80%, and 60% speeds.

5.12 Experimental data: time trace of $\Psi$ and $\Phi$ near bep, $\Phi = 0.102 \& 0.089$ (100% speed, $B = 0.35$).

5.13 Experimental data: time trace of $\Psi$ and $\Phi$ before surge inception, $\Phi = 0.046$ (100% speed, $B = 0.60$).

5.14 Experimental data: time trace of $\Psi$ and $\Phi$ after surge inception, $\Phi = 0.041$ (100% speed, $B = 0.60$).

5.15 Experimental data: time trace of $\Psi$ and $\Phi$ at maximum surge amplitude, $\Phi = 0.025$ (100% speed, $B = 0.60$).

5.16 Experimental data: time trace of $\Psi$ and $\Phi$ at shutoff, $\Phi = 0.00$ (100% speed, $B = 0.60$).

5.17 Experimental data: time trace of $\Psi$ and $\Phi$ at shutoff, $\Phi = 0.003$ (100% speed, $B = 0.35$).

5.18 Experimental data: time trace of $\Psi$ and $\Phi$ at $\Phi = 0.020 \& 0.014$ for $B < B_{\text{threshold}}$ (100% speed, $B = 0.26$).

5.19 Experimental data: time trace of $\Psi$ and $\Phi$ at $\Phi = 0.12 \& 0.18$ for $B < B_{\text{threshold}}$ (100% speed, $B = 0.08$).

5.20 Experimental data: variation in surge amplitude with $\Phi$ (100% speed and $B = 0.60$).

5.21 Experimental data: variation in surge amplitude and performance characteristic slope with $\Phi$ (100% speed and $B = 0.60$).

5.22 Experimental data: variation in maximum surge amplitude with system B parameter (100%, 80%, and 60% speed).

5.23 Experimental data: variation in reduced frequency of surge with $\Phi$ (100% speed and $B = 0.60$).

5.24 Theoretical results and experimental data: experimental stability boundary for 100% speed superposed on Figure 2.6.

5.25 Theoretical results and experimental data: comparison of theoretical and experimental variation in reduced frequency of surge with air content in discharge plenum (100%, 80%, and 60% speeds).

5.26 Theoretical results and experimental data: comparison of theoretical and experimental variation in system B parameter with air content in discharge plenum (100%, 80%, and 60% speeds).
5.27 Theoretical results and experimental data: comparison of theoretical and experimental variation in inception point of surge with air content in discharge plenum (100%, 80%, and 60% speeds).

5.28 Theoretical results and experimental data: comparison of theoretical and experimental stability boundary (variation in inception point of surge with B parameter) (100%, 80%, and 60% speeds).

5.29 Experimental data with analytical curve fit: detail of MIT pump performance characteristics for 100% and 60% speeds with third order curve fit.

5.30 Theoretical results: effect of variation in performance characteristic shape on predicted stability boundary (curve fits for 100% and 60% speeds).

5.31 Theoretical results: effect of variation in impeller wheel speedup term \( \frac{dU_{tip}}{d\rho} \) on predicted stability boundary (speedup values for 100%, 80%, and 60% speeds).

5.32 Experimental data and theoretical results: comparison of theoretical and experimental variation in reduced frequency of surge with \( \Phi \) (100% speed and \( B = 0.60 \)).

5.33 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at surge inception point (\( B = 0.60 \) and 100% speed).

5.34 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at \( \Phi = 0.32 \) (\( B = 0.60 \) and 100% speed).

5.35 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at \( \Phi = 0.25 \) (\( B = 0.60 \) and 100% speed).

5.36 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at \( \Phi = 0.17 \) (\( B = 0.60 \) and 100% speed).

5.37 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at \( \Phi = 0.12 \) (\( B = 0.60 \) and 100% speed).

5.38 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at shut off, \( \Phi = 0.00 \) (\( B = 0.60 \) and 100% speed).

5.39 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at surge inception point (\( B = 0.33 \) and 100% speed).

5.40 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at \( \Phi = 0.22 \) (\( B = 0.33 \) and 100% speed).

5.41 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at \( \Phi = 0.17 \) (\( B = 0.33 \) and 100% speed).

5.42 Experimental data and theoretical results: comparison of time-resolved simulation and experimental surge cycle at \( \Phi = 0.12 \) (\( B = 0.33 \) and 100% speed).
5.43 Experimental data and theoretical results: comparison of time-resolved simulation (with and without time lag) and experimental surge cycle at $\Phi = 0.32$ (B = 0.60 and 100% speed).

5.44 Experimental data and theoretical results: comparison of time-resolved simulation (with and without time lag) and experimental surge cycle at $\Phi = 0.22$ (B = 0.33 and 100% speed).
NOMENCLATURE

A: cross-sectional area (m²).
A(x): cross-sectional area at location x.
Amplp-p: peak to peak amplitude of oscillation.
b2: impeller blade discharge height.
B: system stability parameter.
Bthreshold: B parameter at which system instability first occurs.
Cm: impeller discharge meridional velocity.
D1: impeller inlet diameter.
D2: impeller discharge diameter.
H: head delivered by pump (feet of water).
K: head loss coefficient.
L: inertial length scale.

\[ L_{TH} = \frac{\int A(x) \mathrm{d}x}{A_{REF}} \]

m or m: mass flow (kg/s).
Ma: mach number.
N: number of impeller rotations (used in [8]).
Ns: specific speed.
Nss: suction specific speed.
NPSH: net positive suction head (feet of water).
P: pressure (Pa).
P1: pump inlet pressure.
Pv: liquid vapor pressure.
\( \Delta P \): pressure rise (Pa).
\( \Delta P_{throttle} \): pressure drop across throttle (Pa).
\( \Delta P_{throttle \ leg} \): pressure drop in throttle leg of piping (Pa).
\( \Delta P_{pump} \): pressure rise across pump (Pa).
\( \Delta P_{pump \ leg} \): pressure drop in pump leg of piping (Pa).
\( \Delta P_{pump \ STST} \): steady state pressure rise across pump (Pa).
\( \Delta P_{\text{STATIC}} \): static pressure rise (Pa).
\( \Delta P_{\text{TOTAL}} \): total pressure rise (Pa).
\( Q \): volume flow (gpm or m\(^3\)/s)
\( Ra \): mean surface roughness.
\( Re \): Reynolds number.
\( U \): impeller wheel speed (m/s).
\( U_{\text{TIP}} \): impeller tip wheel speed (m/s).
\( V \): air volume in plenums.
\( Z_{\text{LAG}} \): coefficient in time constant of time lag in pump pressure rise.

\( \beta_2 \): impeller discharge angle.
\( \Phi \): non-dimensional flow coefficient.
\[
\Phi = \frac{Q}{\pi D_2 b_2 U}
\]
\( \Phi_{\text{avg}} \): average \( \Phi \) value during surge cycle.
\( \Phi_{\text{ir}} \): surge inception point or transition point encountered by decreasing \( \Phi \) from \( \Phi_{\text{bep}} \).
\( \Phi_{\text{ir2}} \): surge inception point or transition point encountered by increasing \( \Phi \) from shut off.
\( \gamma \): specific heat ratio.
\( \eta \): overall pump efficiency.
\( \nu \): kinematic viscosity.
\( \rho \): density.
\( \sigma \): cavitation number.
\( \tau \): time constant in first ordertime lag in pump pressure rise.
\( \omega_{\text{HH}} \): Helmholtz frequency (Hz).
\( \omega_{\text{red}} \): reduced frequency of surge oscillation.
\[
\omega_{\text{red}} = \frac{\omega_{\text{unst}}}{\omega_{\text{shaft}}}
\]
\( \omega_{\text{shaft}} \): shaft frequency (Hz).
\( \omega_{\text{theory}} \): theoretical (predicted) frequency of surge oscillation (Hz).
\( \omega_{\text{unst}} \) or \( \omega_{\text{exp}} \): frequency of surge oscillation (Hz).
\( \Omega \): reduced frequency of pump operation during surge.
\[
\Omega = \frac{\text{pump throughflow time}}{\text{period of unsteadiness}} = \frac{L_{\text{TH}}/C_m}{1/\omega_{\text{unst}}}
\]
shaft rotational speed (rpm).
\( \Omega' \): effective reduced frequency of pump operation during surge imposed by the choice of \( Z_{\text{LAG}} \).
effective pump throughflow time = period of unsteadiness = \( Z_{LAG} \frac{L_{TH}/C_m}{1/\omega_{unst}} \)

\( \Psi \): non-dimensional pressure rise.
\[ \Psi = \frac{\Delta P}{\rho U^2} \]

\( \Psi_{STATIC} \): non-dimensional static pressure rise.

Subscripts:
- bep: best efficiency point.
- p-p: peak to peak amplitude
- P: pump leg or pump.
- REF: reference quantity.
- T: throttle leg or throttle.
- 1: large (inlet) plenum
- 2: small (discharge) plenum

Other:
- \( \Delta() \): difference or uncertainty of quantity.
- \( \delta() \): perturbation quantity.
- \( \text{Im}\{\} \): imaginary part of {}.
- \( \text{Re}\{\} \): real part of {}.
- \( \frac{dU_{Ti}}{dm_p} \): impeller wheel speed variation with pump mass flow.
- \( \frac{d\psi}{d\Phi} \): performance characteristic slope.
CHAPTER I

INTRODUCTION

1.1 Introduction

Industrial applications for high power density (low specific speed) centrifugal pumping systems are becoming increasingly more numerous and more demanding. The high pressure rise and small size of this type of pump are well suited for use in various components of high performance machines where power density is often the critical design parameter.

One major drawback, however, that is inherent to the use of low specific speed (Ns) pumps is their poor performance at low flow rates, i.e. flows well below the best efficiency point. It is well documented in the literature [1,2,3] that because of the positively sloped portion of their performance characteristics (cf. Figure 1.1), low Ns pumps and compressors have been observed to sustain large oscillations in flow and pressure rise at flows below the peak of their characteristic. These oscillations can in some cases grow in amplitude to exceed fifty percent of the time mean pressure rise [4] and over one hundred percent of the steady state flowrate. Sustained pump operation in such a volatile regime can be catastrophic for the pump and any parallel systems connected to it. To avoid this potentially damaging regime, therefore, pumping systems are generally operated at flowrates well above the instability inception point, resulting in a curtailed pump operating range.

Current and projected pump applications, however, tend to push one to access more of the overall pump performance envelope. Efficient operation over a wide throttling range, often without a bypass (to avoid overheating), and stability during startup and quick acceleration transients are typical requirements. Because of this, there is a need to expand the current understanding of off-design pump performance through more detailed research studies (both experimental and theoretical). Such a study forms the premise for the work presented in this thesis.
1.2 Background

Over half a century ago, water pump operators observed unstable oscillations similar to those mentioned above and witnessed their catastrophic effect on conventional pumping systems. Using a lumped parameter Helmholtz resonator analysis, models were constructed to describe the unstable behavior. At that time, the available data were too sparse and the modeling too primitive to afford significant advances. Since then, however, numerous studies of this global instability phenomenon in both axial and centrifugal compression systems have led to considerable insights into its causes and nature.

The precise nature of these unstable oscillations was first characterised by Emmons et al [1] in the early 1950's. In this landmark paper on unstable compressor operation, the global oscillations in compressor throughflow (surge) were first formally differentiated from local flow fluctuations (rotating stall). Subsequent efforts to better understand and model the global instability phenomenon or surge using lumped parameter representations by Taylor[3], Stenning[7], Greitzer[8,9], and others met with considerable success. The mechanism of instability was further determined, and a suitable non-dimensional parameter for judging the potential for instability in a given pumping system was developed [8]. This instability, or B, parameter weighs the relative magnitudes of compliant (or energy-storing) forces and inertial forces in the pumping environment. Although a positively sloped characteristic is a requisite for surge, the system B parameter determines where it occurs. Comparison of experimental results to lumped parameter based predictions of instability by Greitzer[10], Hansen et al [11], Fink [12], and others have validated the usefulness of this type of approach.

The simple 1-D models of compression systems were capable of predicting many of the observed trends in compressor instability onset. Due to the inherent non-linearity of the surge cycles themselves, however, these linear models lacked applicability beyond the instability inception point. Greitzer (for the axial compressor [8]) and subsequently Hansen et al (for the centrifugal compressor [11]) soon carried this 1-D analysis approach into the time domain. In so doing, they were able to obtain a remarkably accurate picture of compressor behavior during surge. The simple lumped parameter systems model was again successful in capturing much of the observed experimental behavior. Though considerably less research of this surge phenomenon has been applied directly to centrifugal pumps,
much of what has been gleaned through the study of gas compression systems is readily applicable here also.

While there have been many successful attempts at modeling the unsteady behavior of compression systems, there has been much less work on the flow character internal to the pump when it is operating at or near the instability inception point. Typically, the compressor, or pump, is treated as a black box, or function generator, that has a known steady state performance characteristic (a unique instantaneous $\Psi = f(\Phi)$). Only recently has a concerted attempt been made at explaining why the performance characteristic has the shape that it does. Among others, Dean [13], and Elder et al [14], undertook rigorous component by component investigations of compressor surge. By tracing the individual performance of each pump component, and determining which component is the most severely stalled at the inception point of surge, they were able to identify probable "triggers" of the global unsteadiness. The overall pump response is as or more complex, however, since surge is generally due to the interaction of the stalled and unstalled components. It does seem clear, though, that further advances in the understanding and control of compressor (pump) stability will come through this type of detailed investigation of local component behavior in the adverse flow conditions near surge. Evidently, there is yet much to be gained through researching and documenting this critical area of study.

1.3 Statement of Problem

In recent years, several low specific speed centrifugal pumps [20] have been known to encounter unstable oscillatory behavior at low flows (near 20%-30% of $\Phi_{bep}$). Preliminary analysis of the data from these pumps [21] led to the conclusion that the oscillations seen were due to a system instability. System modeling similar to that done in [1] and [3] revealed probable trends and behavior but could do little to decisively determine the underlying culprit. Typically, these pumps were extremely small (impeller diameters on the order of 3 cm) and local flow character was difficult if not impossible to determine accurately. Because of this lack of information, it was decided to construct an experimental test facility for studying this instability phenomenon in greater detail. The work presented in this thesis represents the second in a series of research reports dealing with the construction and operation of this facility.
1.4 Research Plan

The following section outlines the overall objective of the research effort being undertaken at the MIT Gas Turbine Laboratory on this problem as stated above. The scope of the project as well as an estimated methodology is also presented.

The experimental facility referred to in the preceding section was constructed with the intent of improving the knowledge of unstable performance in centrifugal pumps. The level of understanding achieved should enable the development of a reliable and efficient cure for the instability mentioned in Section 1.3. With this as the eventual goal, shorter range steps were fixed to direct the research in the near term. These are listed below in chronological order:

1) To facilitate a detailed investigation of this instability phenomenon, a larger model of the original developmental centrifugal pump should be constructed. This should be large enough to allow detailed flow and pressure measurements in each component of the pump (impeller, inlet, volute, etc...).

2) The scaled up experimental pump should exhibit a steady state performance characteristic similar to the original one. Considering the dominant influence of the pump characteristic shape in determining stability [9,14,19], this objective is necessary to insure transportability of resulting findings from the model to the actual hardware.

3) The experimental pump facility should be shown to exhibit the same unsteady flow phenomena as the original one. Any important trends should also be duplicated. To facilitate a parametric study of the unsteadiness, the test loop should be "tuneable", in the sense that by adjusting the morphology of the pumping system, its dynamic response can be modified. This has reference to the B parameter mentioned in Section 1.2.

4) The unstable behavior of the pump should be thoroughly analyzed. This implies a global characterization to determine trends and ranges of dynamic performance parameters, and areas of further interest.
5) Building on the previous objective, it is desired to understand the performance characteristics of the individual components of the pump before and during instability inception (similar to [13,14]). This involves a more local study of the flow phenomenon and important trends of unsteadiness. It is hoped, at this point, that greater light will be shed on the reasons the pump performs as it does; or, more explicitly, why the pump performance characteristic looks the way it does for a low specific speed centrifugal pump.

6) In parallel with the above experimental effort, more advanced analytical models which better simulate the pump operation should be developed. This will necessitate breaking up the overall pump characteristic into the contributions from each individual component, as has been done in [12,13,14].

7) Finally, with respect to the original motivation for the research project, specific modifications should be determined to correct for the unwanted behavior without severely penalizing overall pump efficiency.

### 1.5 Organization of this report

The scope of this report does not cover the entire list of objectives just outlined. Inasmuch as the research effort is ongoing, the subject of this report is confined to that part of the research agenda which begins with step (1), the construction phase, and goes through step (4), the global characterization of unsteadiness. Parts of step (5), concerning primarily the pump diffuser performance, will be discussed as well. A previous work entitled "An Experimental Facility for the Study of Unsteady Flows in Turbopumps" by N. Goulet [22] dealt with the preliminary design and modeling of the facility components. It also discussed parts of the construction phase and developed the first-cut at a one-dimensional model of the pump performance at off design conditions (away from bep).

The following is a summary of the individual contents of each of the succeeding chapters.
• After this introductory section, Chapter II will briefly present the results of a literature search on the topic at hand. Refinements made to the preliminary lumped parameter models [22] will be advanced and discussed.

• Chapter III will deal with the construction of the experimental facility. Important aspects of the pump and test loop design will be mentioned, and a detailed description of the facility itself will be given.

• In Chapter IV the steady state performance results of the model pump will be presented. A comparison of the model pump performance to the original pump performance will also be shown. The issue of similarity will then be evaluated in an effort to determine how nearly the initial project objective [obj. (2)] was achieved.

• Chapter V will treat mainly the unsteady performance of the model pump. Trends and observations of interesting behavior will be made manifest and compared to findings from the original developmental unit. Results from the theoretical modeling of Chapter II will be presented and comparisons will be made between them and the experimental findings. The limitations and potential applications of the modeling techniques will then be discussed.

• Lastly, Chapter VI will summarize the findings of the previous chapters and present recommendations or provisions for subsequent work.
CHAPTER II

THEORETICAL ANALYSES/MODELS

2.1 Dynamic Systems Analysis Overview

The following chapter deals with the systems modeling of global instabilities (surge) in the facility introduced in Chapter I. A lumped parameter analysis commonly used for compression systems and more clearly defined in [9,22] is employed throughout. The chapter contains a description of this analysis and an outline of different solution procedures. A comparison of theoretical results to experimental results will occur in Chapter V.

Past studies have pointed repeatedly to the dominance of two principal factors in determining stability in pumping systems; the first being the pump performance characteristic, and the second being the pumping environment (represented by the B parameter).

The importance of the performance characteristic shape can be seen by evaluating the dynamic response of the equivalent mechanical (or electrical) analogue [9,22] (cf. Figure 2.1). The second order system for the dynamic response of the mass-spring-damper analogue reveals the existence of two types of potential instability. The solutions of this system take the form $e^{st}$. If $\text{Re}(s) > 0$ and $\text{Im}(s) = 0$, there exists a static instability (exponential growth). The corresponding situation in the pumping system occurs when the pump characteristic is locally steeper than the throttle characteristic (cf. Figure 2.2). A perturbation in mass flow, an increase say, at the statically unstable point B in Figure 2.2 causes a mismatch in pressures from pump to throttle. This further accelerates the fluid and causes the operating point to continue its shift from point B.

The characteristic slope portrayed under the heading "static instability" in Figure 2.2 might be typical of an axial compressor unit. A centrifugal pump (or compressor) can exhibit a more smoothly sloped (shallower) characteristic such as depicted under the
heading "dynamic instability" in Figure 2.2. Such a characteristic may have no potential for static instability due to its relatively flat slope, but, instabilities do arise and have been thoroughly reported [4,11,12,20]. In this case, the static stability criterion is not sufficient, and a second, more stringent, stability criterion must also be met. This is the case of \( \text{Re}\{s\} > 0 \) and \( \text{Im}\{s\} \neq 0 \), or dynamic instability, characterized by an oscillation of increasing magnitude.

Dynamic instability can occur in any region of positively sloped characteristic [1,7]. A supporting argument for this is presented in [9] and shown in Figure 2.2. On the negatively sloped portion of the performance curve, a perturbation in mass flow results in a net energy dissipation in the pump. For the positively sloped region, however, the mass flow and pressure perturbations are in phase, and so a net energy (greater than the steady state pump power input) is added to the system over the perturbation cycle. If this energy input matches the corresponding energy dissipation in the loop throttle, the system can maintain a periodic oscillation or limit cycle. This oscillation has been referred to as surge [1].

The above (dynamic instability) criterion appears straightforward until it is applied to an actual pumping system, at which point a question arises as to which performance characteristic measurement should be used in the analysis. In [23], Dean et al suggested that the pump characteristic measured in steady state may be quite different from the instantaneous performance characteristic near surge. Stability analyses conducted using solely the steady state slope may thus be inaccurate. Greitzer [8] also found this to be the case, and used a lag in pressure rise to represent an axial machine's transient response. This same approximation has been used with good results in centrifugal compressors [11,12]. Macdougal et al [24] underlined the importance of an exact knowledge of the characteristic in forward and reverse flows to properly predict transient behavior. Fink [12] has added further complexity (accuracy) by accounting for fluctuations in impeller wheel speed during transients. In summary, the actual pump operating characteristic is more complex than simple steady state measurements can provide, and is a dominant factor in determining a pump's propensity to go unstable.

As shown above, a pumping system has potential for dynamic instability provided the characteristic slope is positive. Once this condition is met, the determining factor for instability is the pumping environment (the amount of compliance available in the system vs. the accelerating fluid, or inertial forces). The B parameter is a convenient expression
representing this ratio of pressure forces to inertial forces in the pumping system. In [8], it is defined as,

\[ B = \frac{U_{\text{TIP}}}{2\omega_{\text{unstLP}}} = \frac{(\rho U_{\text{TIP}}^2/2)\lambda_{\text{P}}}{\rho U_{\text{TIP}}\omega_{\text{unstLP}}\lambda_{\text{P}}} \]

This instability parameter has been used for surge prediction in axial and centrifugal compressors with marked success. As \( B \) is increased, the pressure forces are greater relative to fluid inertia. This results in greater fluid accelerations in the facility ducting. Thus the tendency towards surge as \( B \) is increased for a given pumping system.

In conclusion, these two factors, characteristic slope and system \( B \) parameter, are the leading indicators of instability in any system which can be accurately represented by a lumped parameter, 1-D model. Their influence is evident in the literature and in all subsequent chapters of this document.

### 2.2 The Lumped Parameter Model

To construct a testing facility capable of providing the test pump with the desirable range of dynamic operation (i.e., reduced frequency of surge), a simple model was developed [22] to represent the important dynamics of a potential test loop. Using this model, a sensitivity analysis was conducted to determine the optimal loop dimensions required for maximum range of the reduced frequency (in the desired regime, between 5-20% of shaft speed). The actual design choices made were reported in [22] and will be discussed in more detail in Chapter III. For the present discussion, the important features of the test loop have been adequately represented on the loop schematic (cf. Figure 2.3).

Subsequent to the loop construction and preliminary unsteady testing, various refinements were added to this preliminary model. To facilitate the presentation of these modifications, the initial model will be briefly reviewed.

As Figure 2.3 suggests, the test loop consists primarily of four distinct components:
1) The piping sections.

These are modeled as inertial elements in which the fluid is incompressible. A lumped coefficient of resistance is also assigned to each pumping section, in which the pressure losses are assumed proportional to dynamic head. \( L_p \) represents the "inertial" length of piping from the exit of the large tank, or plenum, to the inlet of the small plenum. \( L_T \) represents the "inertial" length of piping from the exit of the small plenum to the inlet of the large plenum. \( K_p \) and \( K_T \) are the respective loss coefficients and \( A_p \) and \( A_T \) the corresponding reference (cross-sectional) areas.

2) The two plenums.

These two large tanks contain a specified volume of compressible gas (air), which can store energy through isentropic compression.

3) The throttle (in the actual facility there are two).

It is assumed that the throttle(s) exhibits a pressure drop characteristic which is proportional to dynamic head.

4) The pump.

The only active component which can add energy to the system is the pump. The steady state characteristic of the pump is considered known, and it is assumed that the pump operates in a quasi-steady manner along this characteristic. Note, however, that this assumption will be relaxed as one of the improvements to be discussed later in this chapter.

The relevant equations describing this system can now be constructed. The following convention for the lumped inertial length of \( L_p \) and \( L_T \) is employed as in [8] and [22]. Namely,

\[
L = A_{REF} \int_0^L \frac{dx}{A(x)}
\]

where \( A_{REF} \) is a convenient reference area for each duct.
Mass conservation in the plenums:

\[
\dot{m}_T - \dot{m}_P = \rho \frac{dV_2}{dt} \quad \text{and} \quad \dot{m}_P - \dot{m}_T = \rho \frac{dV_1}{dt} \quad (2.1, 2.2)
\]

Isentropic compression of the air in the plenums:

\[
dV_1 = \frac{V_1}{\gamma P_1} dP_1 \quad \text{and} \quad dV_2 = \frac{V_2}{\gamma P_2} dP_2 \quad (2.3, 2.4)
\]

Conservation of momentum in the piping ducts:

\[
\frac{d}{dt}(\dot{m}_P) = \frac{A_P}{L_P} (P_1 - P_2 - \Delta P_{\text{PUMP}} - \Delta P_{\text{PUMPLEG}}) \quad (2.5)
\]

and

\[
\frac{d}{dt}(\dot{m}_T) = \frac{A_T}{L_T} (P_2 - P_1 - \Delta P_{\text{THROTTLE}} - \Delta P_{\text{THROTTLELEG}}) \quad (2.6)
\]

Equations (2.1) and (2.2) can be combined with Equations (2.3) and (2.4) to give,

\[
\frac{d}{dt}(P_2) = \frac{\gamma P_2}{\rho V_2} (\dot{m}_P - \dot{m}_T) \quad \text{and} \quad \frac{d}{dt}(P_1) = \frac{\gamma P_1}{\rho V_1} (\dot{m}_T - \dot{m}_P) \quad (2.7,2.8)
\]

These four nonlinear equations [Eqns. (2.5),(2.6),(2.7), and (2.8)] can be linearized about an operating point of interest by introducing a perturbation quantity, \(X = \bar{X} + \delta X\). The dynamic response of the resulting set of linear differential equations to this small perturbation can be characterized by solving for the system eigenvalues. The real part of the eigenvalue determines stability, and the imaginary part determines the frequency of oscillatory behavior (cf. Section 2.1). This procedure was used in the design phase of the project [22] to predict instability inception points and frequencies of system oscillation.
2.2.1 Modification #1: Variable Wheel Speed

During pump operation, it was observed that, due to finite motor armature inertia, the rotational speed of the pump impeller varied during surge cycles (cf. Figure 2.4). This phenomenon has also been observed on at least one other experimental test facility during unsteady fluctuations in mass flow through a centrifugal compressor [12]. The variations in wheel speed actually serve to stabilize the system, as the pump operating point shifts to adapt to perturbations. The local characteristic slope is in essence flatter (more stable) due to this variation. From a systems perspective, the wheel inertia represents another system energy storage term which is coupled with the fluid inertia and can be lumped into the denominator of the B parameter. A given system will then have a smaller B (be more stable) with this extra inertia term due to wheel speedup. In either case, the stability of the pumping system is increased by this additional parameter and so it was incorporated into the preliminary model summarized in Section 2.2.

The wheel speed only affects the characteristic slope of the pump, which appears in Equation (2.5). The linearized version of this equation is as follows:

\[
\frac{d}{dt}(\delta \dot{m}_P) = \frac{A_p}{L_p} \left[ \delta P_1 - \delta P_2 - \delta \Delta P_{\text{PUMP}} - \left( \frac{K_p \delta \dot{m}_P}{2} \right) \delta \dot{m}_P \right] \tag{2.9}
\]

but, using a Taylor series expansion of \( \Delta P_{\text{PUMP}} \) about the mean operating point, \( \Delta P_{\text{PUMP}} \),

\[
\delta \Delta P_{\text{PUMP}} = \left( \frac{d\Delta P_{\text{PUMP}}}{d\dot{m}_P} \right) \delta \dot{m}_P \tag{2.10}
\]

The derivative, \( \frac{d\Delta P_{\text{PUMP}}}{d\dot{m}_P} \), must be defined in terms of the nondimensional pump characteristic slope (the known quantity obtained from experiment).

Nondimensionally,

\[
\psi_{\text{STATIC}} = \frac{\Delta P_{\text{PUMP}}}{\rho U_{\text{TIP}}^2} \quad \text{or} \quad \Delta P_{\text{PUMP}} = \rho U_{\text{TIP}}^2 \psi_{\text{STATIC}}
\]
Differentiating with respect to \( \dot{m}_\text{p} \) yields:

\[
\frac{d\Delta P_{\text{PUMP}}}{\dot{m}_\text{p}} = \rho U_{\text{TIP}}^2 \left( \frac{d\psi}{d\dot{m}_\text{p}} \right) + 2\psi_p U_{\text{TIP}} \left( \frac{dU_{\text{TIP}}}{d\dot{m}_\text{p}} \right) \tag{2.11}
\]

Combining the original Equation (2.9) with Equations (2.10) and (2.11),

\[
\frac{d}{dt}(\delta \dot{m}_\text{p}) = \frac{A_P}{L_P} \left\{ \delta P_1 \cdot \delta P_2 + \left( \frac{U_{\text{TIP}}}{\pi D_2 b_2} \left( \frac{d\psi}{d\Phi} \right) + 2\psi_p U_{\text{TIP}} \left( \frac{dU_{\text{TIP}}}{d\dot{m}_\text{p}} \right) \right) \delta \dot{m}_\text{p} \cdot \left( \frac{K_p \dot{m}_\text{p}}{r A_P} \right) \delta \dot{m}_\text{p} \right\}
\]

(2.12)

(where the pump pressure rise has been assumed to be positive).

In the linear analysis, \( U_{\text{TIP}} \), \( \frac{d\psi}{d\Phi} \), \( \dot{m}_\text{p} \), and \( \psi \) are all evaluated at the current operating point \( \Phi \). Experimentally, \( U_{\text{TIP}} \) was found to be approximately linear with \( \dot{m}_\text{p} \), thus the \( \frac{dU_{\text{TIP}}}{d\dot{m}_\text{p}} \) term is entered as an empirical constant, of negative sign. As claimed earlier, it serves to diminish the perceived characteristic slope at the operating point in question. Figure (2.5) shows the effect of this new term on an instability inception point \( \Phi_{\text{tr}} \) prediction.

### 2.2.2 Modification #2: Time Lag in Pump Pressure Rise
(A More Realistic Pump Response)

The second of the two refinements performed on the simple model outlined in Section 2.2 is the relaxing of one of the assumptions; namely, that the pump always operates on its steady state characteristic (in a quasi-steady manner during oscillations). This is far from realistic. If \( \Omega \) is the reduced frequency of pump performance during unsteadiness defined such that,

\[
\Omega = \frac{\text{pump throughflow time}}{\text{period of unsteadiness}} = \frac{L_{\text{TH}}/C_m}{1/\omega_{\text{unst}}}
\]
it is found that typical experimental values of this reduced frequency, $\Omega$, at the point of instability inception are on the order of 2. In contrast, the traditional quasi-steady limit is only approached as $\Omega << 1$, and the pump would not be expected to function quasi-steadily during surge. A simple attempt at correcting for this is made by adding a first-order lag in pump pressure rise.

$$\frac{d}{dt} (\Delta P_{\text{PUMP}}) = \frac{1}{\tau} (\Delta P_{\text{PUMP}}^{\text{stst}} - \Delta P_{\text{PUMP}})$$ (2.13)

This method was employed with satisfactory results by Fink (centrifugal compressor, [12]) and Greitzer (axial compressor, [8]).

In most axial compressors, the experimentally observed precursor to surge is rotating stall. Studies have shown that a stall cell forms and then makes several full revolutions before the entire global flow is affected and surge ensues. Greitzer, accordingly, used a time constant, $\tau$, based on rotor revolutions:

$$\tau = \frac{N \pi D_2}{U_{\text{TIP}}}$$

with an experimental value of $N = 2$ giving good agreement with experiment [10].

In work by Hansen et al [11] and Fink [12] on centrifugal compressors, rotating stall was not found to be a dominant factor in surge inception and a different definition of $\tau$ was employed. A time constant, $\tau$, based on compressor throughflow time yielded good agreement with experimental results in both cases. For a centrifugal pump, this is a more physically appealing definition. A finite amount of time is required for the pump to realistically adjust to a change in operating conditions. This amount of time is approximately equal to the time it takes for a fluid particle to transit the entire pump, and the following definition was adopted:

$$\tau = \frac{L_{\text{TH}}}{C_m} = \frac{L_{\text{TH}}}{\phi U_{\text{TIP}}} Z_{\text{LAG}}$$ (2.14)

$Z_{\text{LAG}}$ represents a factor that will be used to adjust $\tau$ in order to better mimic observed experimental trends (Chapter V). $L_{\text{TH}}$ is the average throughflow distance (taken to be approximately 2.3 m) for a fluid particle entering the model pump.
To incorporate this relationship into the system of equations from Section 2.2.1, the following steps are taken. The linearized version of Equation (2.13) is simply,

\[ \frac{d}{dt} (\delta \Delta P_{\text{PUMP}}) = \frac{1}{\tau} (\delta \Delta P_{\text{PUMP}} - \delta \Delta P_{\text{PUMP}}) \] (2.15)

\( \delta \Delta P_{\text{PUMP}} \) is a new variable to be solved for. \( \delta \Delta P_{\text{PUMP}}_{\text{sst}} \) is the same quantity that was previously referred to simply as \( \delta \Delta P_{\text{PUMP}} \) [cf. Equation (2.10)]. Explicitly,

\[ \delta \Delta P_{\text{PUMP}}_{\text{sst}} = \left( \frac{d\Delta P_{\text{PUMP}}}{d\dot{m}_p} \right) \delta \dot{m}_p = \left[ \frac{U_{\text{TIP}}}{\pi D_2 b_2} \left( \frac{d\Psi}{d\Phi} \right) + 2 \Psi \rho U_{\text{TIP}} \left( \frac{dU_{\text{TIP}}}{d\dot{m}_p} \right) \right] \delta \dot{m}_p \] (2.16)

Combining Equations (2.14), (2.15), and (2.16) yields,

\[ \frac{d}{dt} (\delta \Delta P_{\text{PUMP}}) = \frac{\Phi U_{\text{TIP}}}{L_{\text{THZLAG}}} \left\{ \left[ \frac{U_{\text{TIP}}}{\pi D_2 b_2} \left( \frac{d\Psi}{d\Phi} \right) + 2 \Psi \rho U_{\text{TIP}} \left( \frac{dU_{\text{TIP}}}{d\dot{m}_p} \right) \right] \delta \dot{m}_p - \delta \Delta P_{\text{PUMP}} \right\} \] (2.17)

Adding Equation (2.17) to the previous four equations [Equations (2.6), (2.7), (2.8), and (2.9)] produces a system of five coupled linear differential equations which can be analyzed in the same manner as done earlier.

A parameter that is not well known is the value of \( Z_{\text{LAG}} \) (the correction term to the lag time constant). Performing a sensitivity analysis on the dynamic response of the above system by varying this parameter shows the enormous effect that the lag can have on stability. Figure 2.6 shows the resulting stability boundaries for various values of \( Z_{\text{LAG}} \). Increasing \( Z_{\text{LAG}} \) (or rather, increasing \( \tau \)) makes the system more stable, as was reported in [12]. As \( Z_{\text{LAG}} \) is made to approach zero, the pump approaches the quasi-steady limit. It will be seen, in Chapter V, that the quasi-steady assumption appears to be unrealistic for the situation of interest here.
2.2.3 Summary of System Dynamic Model Features

The system dynamic behavior of the facility modelled in this chapter depends primarily on four important parameters:

1) \( \frac{d\psi}{d\phi} \), or the local slope of the nondimensional pump performance characteristic (in steady state).

2) \( B = \frac{U_{\text{TIP}}}{2\omega_{\text{unst}}L_{\text{P}}} \), (the B parameter), which is a measure of the ratio of pressure forces to inertial forces in the test loop. In the case at hand, B incorporates information about the ducting lengths and areas as well as the quantity of air in each plenum and the impeller wheel speed. It is therefore an environment dependent parameter (ie...given a pump operating speed, the B parameter is fixed for a set system).

3) \( \tau \), the time constant of the first order lag. This in effect alters the instantaneous shape of the steady state performance characteristic.

4) \( \frac{dU_{\text{TIP}}}{d\dot{m}_{\text{P}}} \), the wheel speed variation with mass flow, which is an empirical constant for a given pump and transmission. This could be driven to zero by adding a sufficiently large inertial mass to the pump drive shaft.

Given a set operating environment, the most critical determinant of stability is the transient pump characteristic (the steady state characteristic modified by a lag term to account for finite response time). The importance of this fact, and its bearing on research concerning the instability phenomenon of interest, will be further elucidated in Chapters IV and V. The Fortran code developed to calculate the eigenvalues (and hence the stability) of the above system can be found with a typical output in Appendix A. A comparison of results from this model and the findings from experiments will occur in Chapter V.
2.3 Time-Resolved Dynamic Simulation

During the preliminary modeling phase of this research project, an attempt was made to go beyond the linear model and compute the behavior of the dynamic system during surge [22]. To do this, a generic dynamic systems code was used to numerically integrate the non-linear equations outlined in Section 2.2 [Equations (2.5), (2.6), (2.7), and (2.8)] using a fourth order Runge-Kutta solver.

As in the case of the linear model, the impeller wheel speed variations and the time lag in pump pressure rise were added to the original simulation routine. The wheel speed variation was simply introduced by calculating the new wheel speed at each iteration. The same empirical correlation between \(U_{\text{TIP}}\) and \(m_p\) was employed. The lag just represents another first order equation which requires approximating by the solver. \(\Delta\text{PUMP}^{\text{stst}}\) is defined as before; it is the quasi-steady pressure rise of the pump which corresponds to the given operating point (mass flow). The same definition for \(\tau\) is employed, and \(Z_{\text{LAG}}\) is set to a value consistent with the linear model.

The only parameter that determines the steady state operating point of the pump (and system) is \(K\), the head loss coefficient of the throttle. In practice, \(K\) is first set to a value corresponding to pump operation in the stable regime. Thereupon, it is increased to simulate the closing of the throttle in the experimental test loop. This continues until a prearranged steady state operating point is reached (usually somewhere beyond the instability boundary). The system response (as predicted by the simulation) then typically develops into a finite amplitude limit cycle.

The code requires that initial conditions be specified for mass flows in the two ducts, pressures and air volumes in the two plenums, wheel speed of the pump impeller, and \(K\) (as mentioned above). In Chapter V, results of the dynamic simulation using initial values equivalent to those seen in the laboratory will be compared to experimental transients (surge or limit cycles). The equations used by the generic solver can be found in Appendix A.
CHAPTER III

DESCRIPTION OF EXPERIMENTAL FACILITY

3.1 Design Considerations and Test Section Overview

The test loop used for the experiments reported in this document (cf. Figure 3.1) was designed and built specifically for the experimental testing of the scaled up centrifugal pump shown in Figure 3.2. The present chapter will describe this facility in detail.

3.1.1 Design Considerations

The construction and design of this facility were guided by several key considerations [22]:

1) The primary reason for scaling up an existing developmental pump was to increase the accessibility of the internal components of the pump for detailed instrumentation. With this in mind, the pump impeller and casing were scaled approximately 20 times.

2) Given the required piping diameters and volume flows of the scaled-up pump, the decision was made to run the facility as a closed loop rather than an open loop, or blowdown rig.

3) Water was chosen as the pumping fluid because it allowed close Reynolds number matching with the original pump. In addition, the influence of cavitation could not be entirely ruled out as a potentially important factor in several of the original pumping units' performance tests. The use of water allows the investigation of cavitation on the model pump as well. Water is also convenient for flow visualization, and is safe and readily available.
4) To make the pump as accessible as possible to flow visualization and laser anemometry techniques, the pump impeller, volute, diffuser, and inlet were all fabricated using transparent plexiglass.

5) Piping lengths and tank volumes were chosen from dynamic considerations. Components of the test loop were sized to allow for the maximum range of dynamic oscillation frequencies. This was done to enhance the facility's ability to effectively reproduce the dynamic response of the original pumping system.

6) The test section was designed for ease of modification, in that all key components can be easily reconfigured without having to rebuild the entire unit. Potential instability cures can be incorporated and tested in a straightforward manner.

3.1.2 Facility Overview

As shown in Figure 3.1, the pump testing facility spans two floors of the Gas Turbine Laboratory at MIT. The pump is located upstairs with a vertical intake and horizontal discharge. The 8" PVC piping discharges into a small tank or plenum downstairs. This tank is fitted with inflatable tire innertubes (cf. Section 3.2.1) as part of the system dynamics. The 8" ducting leaving the small discharge tank is split into a main and a bypass channel, with a throttle located on each of these passages for controlling the flowrate of the pump. The two channels merge together again before entering the large inlet tank (plenum), which is also equipped with inflatable innertubes. The exit piping from the inlet plenum is 12" PVC, which leads into the vertical pump inlet ducting. All piping and both tanks are adequately supported to sustain the weight of the water and any dynamic loads.
3.2 Test Loop Components

3.2.1 Plenums and Airbags

The plenums were fabricated using 5/8" thick 5083 aluminum plate to the dimensions listed in Figure 3.3. The small plenum has a maximum compliant volume of approximately 415 liters, while the large plenum can support 1000 liters of air. The cover plate of each plenum is secured using an ASME standard bolt circle and a 1/4" thick, 40 durometer Neoprene gasket. The tanks were tested to 30 psig and 10 psig (small and large respectively) before operation with no water leakage. Each plenum was equipped with a center-core cylinder made of perforated 1/8" aluminum sheet (cf. Figure 3.3). The perforated cylinders support the deflated innertubes and prevent them from interfering with the flow path.

A possible drawback with using a closed loop facility is that perturbations or disturbances present in the pump discharge can pass through the throttle and reenter the pump. To avoid this, and thus to effectively decouple the discharge from the inlet, the inlet plenum was made as large as the laboratory space could accommodate.

The truck tire innertubes (four in the small plenum and five in the large plenum) are connected to the same pressure feed line in each tank. This line runs along the inside of each plenum and out the top through an o-ring fitting. In this way, all of the innertubes in each tank can be filled and emptied simultaneously.

During pump testing, it was found that substantial volumes of dissolved gases came out of solution (across the throttles, for example) and collected in the upper portion of the tanks. This is undesirable as it prevents the precise measurement of the air volumes. To avoid this situation, air bleeds were installed at the top of each tank. Before and after each run, the bleed valves were opened to allow trapped air to escape. It was found that after prolonged operation (one to two hours), the water would lose most or all of its dissolved air. The pump could then be run for a considerable length of time without significant air accumulation in the large tank (less than 0.5 liters, or 0.3% of a typical compliant air volume, per one hour run).
To adequately model the dynamics of the test facility, a knowledge of the exact volume of air in each plenum is essential. Accordingly, a procedure (cf. Figure 3.4) was devised for filling the innertubes. First, the entire facility is filled with water and run for a prolonged period of time (one to two hours). Any accumulated air is then bled from the plenums. The innertubes are also emptied of all air. Then the loop water level is marked at some point on the discharge stack. Water is then released from the bottom of the discharge tank, fed through a water meter, and emptied into the supply tank (cf. Figure 3.5). The water meter accuracy was measured to be ±0.25 liters. This is done until a predetermined volume of water has left the test loop.

Next, a compressed air line (at 50 psig) is attached to the air intake line for the tubes in the small tank. The tubes are filled until the loop water level reaches the height recorded originally. In this way, the volume of air is measured by volume displacement. After the tubes in the small tank are filled to the appropriate level, the same procedure is repeated for filling the tubes in the large tank.

During testing, the large plenum air content was always kept at a prescribed value with the small plenum air content as the variable. 300 liters of air in the large and 0 to 300 liters of air in the small plenum were found to provide the desired range of response frequency. These are the values used in the testing discussed in Chapter V. After several tests, it was found that the top of the large plenum could be filled with 300 liters of air without any air leaking or escaping up the inlet during pump operation. Since this was much simpler, in practice, than filling the innertubes, it was adopted as the standard procedure during the majority of the tests reported herein. The discharge plenum innertubes were still filled as described earlier.

The air volumes mentioned above are all measured with the pump off. With the pump on, however, these volumes vary with the corresponding plenum pressures in a manner outlined in detail in Appendix D. To simplify the comparison of data from different speedlines (and thus different plenum static pressure levels), all of the plots in Chapter V which show air volumes refer to volumes as measured with the pump off (unless otherwise designated). The linear model, though, uses the corrected volumes in its calculation of \( \Phi_{tr} \), B, and frequency \( (\omega_{unst}) \).
3.2.2 Ducting

As depicted in the loop schematic (Figure 3.1), there is approximately 7.7 m of 8" PVC pipe between the exit of the pump diffuser and the inlet to the discharge plenum. This was one of the design parameters specifically chosen to set the range of oscillation frequencies. Approximately 3 m of 12" PVC piping is between the exit of the inlet plenum and the pump inlet and 3.3 m of 8" PVC piping is located between the plenums (the throttle leg). The protruding stub of piping at the 'tee' in the 8" discharge piping (referred to as the discharge stack, cf. Figure 3.1) was used to mark the loop water level during air filling. This was done with 1/4" transparent tubing attached in parallel to the discharge stack piping.

The linear model presented in Chapter II uses three inputs to fully characterize each of the two piping sections (L_p and L_T, cf. Section 2.2). These three inputs are: L, A_REF and K. L is the area weighted inertial length defined earlier as:

\[ L = \frac{\int_{0}^{L} \frac{dx}{A(x)}}{ \int_{0}^{L} A(x) dx} \]

Values of L were calculated for both the pump and the throttle legs. They were found to be 41.92 m and 3.28 m respectively. Appendix B contains the detailed calculation of these two lengths. A_REF was chosen to be the pump inlet area for the pump leg (0.0572 m²) and the 8" pipe cross-sectional area for the throttle leg (0.029 m²).

Preliminary estimates of the head loss coefficients, K, for each leg showed them to be negligibly small compared to the resistance of the throttles near \( \Phi_{tr} \). For this reason, no effort was made to measure them experimentally. Instead, estimates using ASME standard head loss correlations for turbulent pipe flow were used to obtain values of 4 and 1.5 respectively (pump leg and throttle leg). Appendix B contains the details of this calculation. The compliance of the PVC piping was also calculated [22] and found to be negligible in comparison to the air bag compliance.
3.2.3 Throttles

Two throttles are used in the pump testing facility to allow for coarse and fine control of the pump flowrate. Both are controlled by electric actuators which employ a feedback control loop to determine valve position. The 8" main throttle is a butterfly valve and the 1" bypass throttle is a ball valve. The large throttle has a closing time of approximately 10 seconds and the bypass valve has a closing time of 5 seconds.

3.2.4 Accessory Systems

Figure 3.6 shows the loop water transfer and storage system. The water used in the system is filtered tap water (filtered to 5 μm and then to 0.4 μm). 1500 ppm sodium nitrite in solution is also added to the water to act as a corrosion inhibitor; this additive worked quite well. The water is changed periodically (every 3 to 6 months) as the need arises.

The test facility was designed to operate at various levels of overall pressure. This is accomplished by closing off the loop from atmosphere, and either filling or emptying air from the innertubes until the desired loop pressure level is reached. Runs at elevated pressures of 1, 2, and 3 psig were quite successful. Attempts at lowering the loop pressure below atmospheric, however, resulted in several leaks. This was subsequently avoided.

3.3 The Test Section

3.3.1 Pump Transmission

The motor used to power the pump is a 15 hp, 4 pole (900 rpm) A/C drive. An adjustable frequency A/C drive controller is used to vary the wheel speed over the desired range (0 to 450 rpm).
To avoid vibrations and noise from a belt transmission, the motor was mounted to the ceiling above the pump shaft and run in a direct drive configuration as shown in Figure 3.7. Two couplings, a fixed sleeve coupling and a flexible coupling, are used to transmit torque, first to the torque sensor, and then to the impeller shaft itself. The stainless steel impeller shaft is hollow to allow for rotating frame pressure measurements via a slip ring assembly mounted to the shaft. The shaft is set in a bearing housing with one Double Row Conrad bearing to absorb radial loads and two tandem 15 degree bearings to absorb thrust loads. Thrust loads and shaft critical speeds were calculated in the predesign phase [22]. A circumferential lip seal at the shaft lower end prevents water from reaching the bearings. The exact configuration is described in detail in [22] and is shown in Figure 3.8.

A double eccentric mounting wheel at the top of the bearing housing will allow parallel offsets (up to 1/2" radially) of the pump impeller away from the natural centerline of the volute. Axial offsets are possible as well [22].

### 3.3.2 The Pump

The dimensions of the model pump impeller are as follows:

\[
\begin{align*}
D_2 &= 0.610 \text{ m (24.00 in)} \\
b_2 &= 0.0119 \text{ m (0.468 in)} \\
D_1 &= 0.201 \text{ m (7.93 in)} \\
\beta_2 &= 34 \text{ degrees}
\end{align*}
\]

The impeller is of the single suction, shrouded variety and is overhung. It is composed of eight blades, four splitters and four full. The diffuser has an included angle of 8 degrees and begins with a transition from the rectangular volute cross section to the circular pipe cross section. The volute consists of a single vaneless scroll of constant tangential velocity design wrapped about the impeller. Figures 3.9 and 3.10 show the entire test section in greater detail. As mentioned earlier the impeller, diffuser, inlet, and volute were all fabricated from plexiglass to provide optical openness.

Three 3/8" ports on the bearing housing below the lip seal serve as back leakage ports. 1/2" plastic tubing from each of these three ports is collected in a manifold, and the liquid is reinjected into the flow at the 90 degree turn below the pump inlet. This flow is
referred to as the back plate leakage flow and is done to simulate the actual operating conditions of the original developmental pump. Near shutoff, this leakage flow was a substantial percentage of the total flow through the original pump.

A circumferential "labyrinth" seal is situated between the shrouded impeller (the rotating frame) and the pipe wall (the stationary frame) near the pump inlet. During pump operation this "labyrinth" seal moves only slightly (one full rotation in the direction of impeller rotation in 5 to 10 minutes). This implies a properly functioning labyrinth seal.

3.4 Instrumentation

3.4.1 Global Performance Measurements

The majority of the test results presented in Chapters IV and V were obtained using only six measurement devices. These are listed below and shown in Figure 3.11:

1) A Yokogawa ADMAG (Series AM220) 8" magnetic flowmeter was placed in the discharge piping about 2 m from the diffuser exit. The ADMAG internal electrodes are sampled at 75 Hz and the measured electric field has a magnitude which is proportional to the water volume flowrate. The response time is adequate for the low frequency flow oscillations which are characteristic of the test loop (approx. 0.4 Hz). Accuracy as stated by the manufacturer is 0.5% of flowrate between 150 and 1500 gpm and ±0.75 gpm below 150 gpm.

A typical histogram of \( \Phi = \frac{Q}{\pi D_2 b_2 U_{tip}} \) measurements taken over 7 seconds at the same throttle setting can be found in Appendix C. It shows the meter reading to be repeatable to within ±1%. The magnetic flowmeter is well suited for flow measurement in unsteady or oscillatory flows as it is only weakly profile dependent (it assumes some flow symmetry). It is also capable of measuring reverse flows. The 4 to 20 mA output is easily accessible by the facility's data acquisition system.
2) The back leakage flow is measured using a 1¼" FP5300 Omega turbine flowmeter. The meter is located after the collecting manifold, before the leakage flow reenters the loop downstairs. The signal from the turbine flowmeter is processed using the FPM704A Omega Digital Indicator. During steady state testing (time averaged measurements) the meter output is manually keyed into the data acquisition system. It was found that during flow oscillations, the leakage flow varied only by ±10% (ΔΦ = ±0.0005). Given the inherent difficulty associated with sampling the turbine flowmeter output at high frequencies during surge, and also given the relatively small variation in leakage flow mentioned above, an average leakage flow value was input manually for each string of time-resolved data points.

To obtain the overall flow through the pump at any moment, the leakage flow and the ADMAG flow measurements were simply added. As the dissolved air content of the water between the pump and the meters is negligibly small, this method of calculating pump throughflow gives sufficient accuracy.

3) and 4) Static pressure taps with wall mounted transducers are located at equal distances from the pump at about 1.5 m (one in the inlet pipe and one in the discharge, cf. Figure 3.11). Both transducers were calibrated using pressurized air and a mercury manometer. Each transducer output is amplified and sampled using the facility's data acquisition system. The signal voltages are subtracted internally, so that the differential pressure rise of the pump is the quantity measured. The repeatability is shown in the histogram of \( \psi = \frac{\Delta P}{\rho U^2} \) in Appendix C.

5) The impeller shaft rotational frequency is measured with the Lebow 1604 Rotary Transformer. The signal is processed by the Lebow 7540 Strain Gage Indicator and fed into the data acquisition system. The output was calibrated using a standard stroboscope. The unamplified rpm signal is quite small, and a lack of extra amplification resulted in a resolution of ± 4 rpm. This was found to be adequate for the current phase of experimental testing, but will be upgraded in subsequent tests.
6) Finally, the shaft torque is measured using the Lebow 1604 Torque Sensor and the same strain gage indicator. The torque sensor was calibrated through approximately one half of its full scale (around 600 in lb) using a weight and pulley system. Torque tares were determined by running the pump without water and were found to be small (maximum of 10 in lb). The calibration curve and actual torque tares measured are found in Appendix C.

These six measurements allow the calculation of all relevant performance parameters: $\eta$, $\Psi$, and $\Phi$. The total pressure rise is calculated using the $\Delta P_{\text{static}}$ from the transducers and applying continuity from pump inlet to discharge.

### 3.4.2 Local Pressure Measurements

Six ports situated along the underside of the pump diffuser are available for static pressure measurements (cf. Figure 3.12). Flush mounted Kulite XTM-190 ruggedized transducers were fitted into four of the six ports to give local pressure data. All four were calibrated in place using the discharge static pressure tap cited earlier as reference. This was done by running the pump at shutoff over a range of wheel speeds. With both throttles fully closed, and no air in the plenums, flow visualization studies showed little fluid motion in the diffuser. Thus, the static pressure seen by all five transducers (the four in the diffuser plus the one downstream) can be considered approximately equal. The results were quite linear and repeatable (again pointing to the validity of the calibration method). The four signals were amplified and then sampled by the data acquisition system. The inlet transducer signal is subtracted from each of these four transducer signals, and the static pressures are represented by $\Psi_{\text{static}} = \frac{\Delta P_{\text{static}}}{\rho U^2}$ at each port.

### 3.4.3 Flow Visualization Techniques

Two methods of preliminary flow visualization have been applied to the test facility to date: dye injection and hydrogen bubble. The dye injection was done through the pressure
port in the volute nearest the tongue (or cutwater). A syringe filled with standard red ink was sufficient to give a satisfactory representation of flow character in this region. Due to the high flowrates and prevailing turbulence at the impeller exit, the dye diffused rather quickly (in approximately 10 cm). Observations made from using this method will be briefly discussed in Chapter V.

Thin (4 mil) platinum wires were installed as crosses at four locations along the diffuser and at one location in the pump inlet. The transformer supplying the necessary voltage to the wires could be operated in either pulse or continuous mode. Fig 3.13 shows the wire locations down the diffuser. Preliminary findings from this method will be discussed in Chapters IV and V.

3.4.4 Data Acquisition

The voltage signals from the above devices were all sampled by the data acquisition software. The A/D signal processing was done using the Data Acquisition DT2801 board and a software package known as PCLAB. A data collection and processing program entitled SPDLN2 was written using the building block acquisition subroutines available through PCLAB. The software (SPDLN2) offers the flexibility of taking both time-averaged and time-resolved data. A Fast Fourier Transform of the time-resolved data is automatically taken to determine the two dominant frequencies of the system. Time-averaged measurements were used during steady state testing. Typically, the program was preset to sample and average 2 seconds of 50 Hz data for each steady state data point. The transient measurement results shown in Chapter V were obtained using the time-resolved option. Typical sampling frequencies were between 50 and 10 Hz (depending on the anticipated system frequency).
CHAPTER IV

EXPERIMENTAL RESULTS: STEADY OPERATION

4.1 Chapter Preview

As discussed in Section 1.4, it is important that the steady state performance characteristics for the original and scaled up pumps be identical. Chapters II and V emphasize the dominant role that the performance characteristic shape has in determining pump stability. If the shape of the characteristic is not similar, the dynamic behavior will not be either. This chapter will discuss to what degree the pump similarity objective was realized. The implications of the discrepancies between the two units for the overall research project will also be explored.

4.2 Model Pump Performance

4.2.1 Performance Characteristic

Figure 4.1 shows the performance characteristics of the MIT centrifugal pump in a non-dimensional $\Psi$-$\Phi$ format. Curves are presented for several different speedlines. These are referenced by their corresponding percentage of pump design speed (design speed is approximately 420 rpm at bep). The curves are relatively flat over the majority of the operating range shown. The noticeable decrease in pressure rise as $\Phi$ tends towards zero is characteristic of low specific speed pumps. This is the positively sloped region referred to earlier which renders the pump susceptible to dynamic instability. Note that the shallowness of this positively sloped region eliminates the possibility of static instability. Despite the relatively small positive slope, however, the pump can easily be made to sustain dynamic oscillations, as will be shown in Chapter V. A significant degradation in pump performance with decreasing Reynolds number is also discernible. This will be touched on again in section 4.2.3 of this chapter. The pump motor controller is not of the constant
speed variety, and the wheel speed varies over the operating characteristic. This is accounted for in the non-dimensional phase plots.

Figure 4.2 shows corresponding overall efficiency plots for a variety of speedlines. The efficiency curves are flat near their peak, and there is a certain amount of subjectivity in choosing a best efficiency point (bep) for pump operation at a given speed. For the data presented in this report, the best efficiency point was chosen by estimating the center point of the nearly flat, maximum efficiency portion of the efficiency curve.

The following table lists typical values of the major pump parameters at bep for two different speeds.

<table>
<thead>
<tr>
<th></th>
<th>100% SPEED</th>
<th>30% SPEED</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_s$</td>
<td>836</td>
<td>804</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>0.1469</td>
<td>0.1346</td>
</tr>
<tr>
<td>Q (gpm)</td>
<td>687.5</td>
<td>193</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>0.5265</td>
<td>0.5153</td>
</tr>
<tr>
<td>$\Delta P_{\text{TOTAL}}$ (psi)</td>
<td>13.35</td>
<td>1.249</td>
</tr>
<tr>
<td>H (ft)</td>
<td>30.79</td>
<td>2.88</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.7286</td>
<td>0.6848</td>
</tr>
<tr>
<td>Re</td>
<td>1.583 E5</td>
<td>4.86 E4</td>
</tr>
<tr>
<td>$\Omega$ (rpm)</td>
<td>417</td>
<td>128</td>
</tr>
<tr>
<td>$U_{\text{TIP}}$ (m/s)</td>
<td>13.31</td>
<td>4.09</td>
</tr>
<tr>
<td>$\sigma$ (cavitation number)</td>
<td>1.12</td>
<td>12.94</td>
</tr>
<tr>
<td>$N_{ss}$</td>
<td>785</td>
<td>118.8</td>
</tr>
<tr>
<td>$\tau$ (torque, in lb)</td>
<td>1 110</td>
<td>101.1</td>
</tr>
</tbody>
</table>

Table 4.1

The definitions used are as follows [25]:

$$\Phi = \frac{Q}{\pi D^2 b^2 U_{\text{TIP}}}$$
$$\Psi = \frac{\Delta P}{\rho U^2_{\text{TIP}}}$$
$$\eta = \frac{Q \Delta P}{\Omega \tau} \text{ const}$$
$$\text{Re} = \frac{U_{\text{TIP}} b^2}{\nu}$$
$$\sigma = \frac{P_i - P_v}{\frac{1}{2} \rho U^2_{\text{TIP}}}$$

$$N_s = \frac{\Omega \sqrt{Q}}{H^{3/4}}$$
and
$$N_{ss} = \frac{\Omega \sqrt{Q}}{NPSH^{3/4}}$$
4.2.2 Comparison with Developmental Pump Performance

From dimensional analysis, two geometrically similar pumps will have identical performance (as measured in $\Psi$-$\Phi$ space), if the following parameters are maintained at comparable levels: Mach number (Ma), cavitation number ($\sigma$) or suction specific speed ($N_{SS}$), specific speed ($N_s$), and Reynolds number (Re). Figure 4.3 is a comparison of performance characteristics for the original developmental pump [4,20] and the scaled up pump at MIT. The 30% speedline from the model pump was used in the comparison because it matched the Re of the actual unit.

The performance of the two pumps is significantly different. At the bep point, for example, the model pump has 5% greater $\Psi$ and 25% greater $\Phi$ than the original. As $\Phi$ is brought to shutoff, the gap in performance decreases, until near $\Phi = 0.075$ the curves cross.

The original pump data end before reaching shutoff because the unit was stopped at the inception of unstable oscillations. The model pump steady state characteristic was measured without air in the plenums; with no compliance in the pumping loop, the pump can be run stably down to shutoff. Even with the lack of data for a direct comparison at low flows, it is apparent that the peak pressure rise for the original unit is 6% greater than the peak pressure rise for the model. The original pump characteristic also peaks at lower $\Phi$. The region of positive slope is thus smaller and occurs at lower $\Phi$. The magnitude of the slope may also be different (the data are lacking). All of these features have a direct effect on the nature of the unstable behavior in the corresponding pump. These effects will be discussed further in Chapter V but are mentioned here in order to emphasize the premise stated in section 4.1; namely, that the dissimilarity in pump characteristics implies a corresponding disparity in dynamic response.

A comparison of the overall pump efficiency curves for the two pumps reveals similar trends (cf. Figure 4.4). The MIT pump efficiency is greater at high flows (nearly 12 efficiency points higher at bep) and the two become comparable at lower flows.

The following table shows a comparison of pump performance at bep [4].
Given the differences presented above, the obvious question is, "why?". To answer this, a step by step evaluation of parameter matching between the pumps was conducted. The results of this investigation are presented in the succeeding section.

### 4.2.3 Discussion of Discrepancies in Scaling

The two performance curves in Figure 4.3 are at matching Reynolds numbers. The Mach number in both cases is sufficiently low to neglect compressibility effects. Table 4.2 above shows that the model specific speed is slightly lower than that of the original, but this is a result of the dissimilar performance rather than a cause. The two remaining parameters are cavitation number and geometry. Though there is a factor of nearly 200 between the values of $\sigma$, cavitation was not evident in either of the pump runs shown. The influence of this parameter can also be ruled out.

Upon closer inspection of the model impeller, an error was discovered in the scaling of the blade height. This dimension is approximately 20% shorter in the model impeller. In other words, the $\frac{b_2}{D_2}$ ratio for the model pump is about 20% less than for the original pump. The impeller passages are narrower and, though $Re$ is matched, the flow sees a substantially altered pump.

One implication of the narrower passage width is a smaller $Ns$. Specific speed is commonly used to classify different types of pumps (cf. Figure 1.1, [25]). A smaller $Ns$
pump typically has smaller blade passages (smaller $\frac{b_2}{D_2}$). It performs better at high flows, and less efficiently at low flows (cf. Figure 1.1). These same trends are apparent for the comparison in question. On these grounds alone, the discrepancy in performance is understandable. Further investigation, however, produced a second discrepancy.

One feature which enters into any estimate of frictional losses in a pump or compressor is the roughness of the wetted surface. This is an integral part of the pump geometry and must be accounted for in any scaling attempt. A recent paper on Re scaling in centrifugal compressors by Casey [26] addresses the issue of surface roughness at considerable length. The paper emphasizes several conclusions regarding flows in low specific speed compressors.

1) The relevant Reynolds number for a centrifugal compressor is defined by

$$\text{Re} = \frac{U T I P b_2}{v}.$$ 

2) The flow in narrow impeller passages is similar to a turbulent pipe flow.

3) All frictional losses in a compressor can be lumped together when considering Re scaling. This is possible without significant loss of accuracy in the prediction of the model's performance.

With these as a base, the paper develops a correlation for Reynolds number which employs the familiar Colebrook-White formula [27] for the friction factor of a turbulent pipe flow. This friction factor is a function of Re and the relative roughness of the passage, $\frac{Ra}{b_2}$, where Ra is the mean surface roughness of the impeller, the arithmetic centerline average of aberrations in wall surface. Estimates of the difference in surface roughness between the two impellers in question are $15 \mu m < Ra < 25 \mu m$ for the actual pump and $5 \mu m < Ra < 15 \mu m$ for the model pump. The corresponding ranges of relative roughness are $2.27 \times 10^{-2} < \frac{Ra}{b_2} < 3.79 \times 10^{-2}$ and $4.21 \times 10^{-4} < \frac{Ra}{b_2} < 1.26 \times 10^{-3}$.

The correlation used in the reference [26] is not touted as a roughness correction. It is suggested rather for use as a Reynolds number scaling tool for constant $\frac{Ra}{b_2}$. 

47
Nevertheless, a rough estimate can be made using the ranges of $\frac{Ra}{b_2}$ above. Figure 4.5 shows the calculated range of adjustment in performance from the original pump’s bep to the model’s bep. The trend matches that shown experimentally.

Another trend which can be extracted from the correlation in [26] is the following: for two comparable low specific speed compressors, such that $Re_1 = Re_2$ and $\left(\frac{Ra}{b_2}\right)_1 > \left(\frac{Ra}{b_2}\right)_2$, an equivalent change in $Re$ will result in a greater change in performance for the smooth pump than for the rough one.

Figures 4.6 and 4.7 show runs at two Reynolds numbers for the actual and the model pump respectively. The order of magnitude decrement in $Re$ appears to have almost no noticeable effect on the actual pump performance. In contrast, a smaller decrement in $Re$ for the MIT pump has a marked effect on performance. Using the findings from reference [26], this seems to be attributable to a hydraulically smoother MIT pump (lower relative roughness). The surface finish, then, appears to be one candidate for explaining the discrepancy in the two pump characteristics.

Several other reports [6,17,18] corroborate the Casey findings (though through alternative correlation equations). In one such paper by Osterwalder et al [18], the notion of lumping all Reynolds number dependent losses together into one correction term is dismissed in favor of a more rigorous approach. Each mechanism for friction loss is considered independently (component by component) and their sum becomes the sought after pump $Re$ correlation. Figure 4.8 shows the adjustment in performance predicted by this alternate method. Even with this more detailed analysis, the same trends are borne out. Surface roughness is an important factor which must be considered in geometric scalings.

So, in summary, the dissimilarity between the reference pump and the scaled up version can be explained by accounting for the differences in impeller outlet height and relative surface roughness.
4.2.4 Implications of Discrepancies on Research Effort

The pump used in this study falls short of proper geometrical scaling. The resulting performance characteristic is, therefore, not identical. As Chapter V will show, however, the basic phenomenon of instability seen in this model pump is identical to that seen in the original unit. Generic findings and modeling from the use of this "imperfect" impeller are thus expected to contribute to the fundamental understanding of the phenomenon of interest, and thus to the overall goal of the research project.

At present, a second impeller has been modified to match the original pump geometry. Subsequent testing will be performed on this more similar configuration. The surface roughness issue, however, will still be present (the new impeller will have the same finish as the one used in this report). So the performance is still not expected to match exactly (the larger model exhibiting the greater pressure rise for a given flow rate). The performance discrepancy, though, should be less with the new impeller.

4.3 Diffuser Steady State Performance

Figure 4.9 shows the diffuser performance profile for a 60% speedline. The static pressure tap positions are shown in Figure 3.12. The trends at 60% speed are similar to those obtained at higher speeds, only more pronounced. From the data, it is apparent that the static pressure does not rise continuously through the pipe diffuser, indicating separation and possibly other unfavorable flow phenomena over parts of the pump operating range. In an effort to pinpoint more precisely the diffuser behavior at various operating points, the hydrogen bubble technique was employed to obtain qualitative information about separation and velocity profiles. By observing the streaklines marked by the hydrogen bubbles for several minutes, regions of full forward flow could be distinguished from separating or reverse flow. Circles representing the cross sectional area at each wire location are shown with a qualitative representation of the flow character at several operating points (Figures 4.10, 4.11, 4.12). The locations of wire crosses used for the hydrogen bubble work are noted on Figure 3.13. Data for three operating points are discussed in this section. The positions of these operating points on the 60% speed steady state pump characteristic are noted on Figure 4.9.
Above bep, the flow is full and generally attached up to the last two wire crosses (cf. Figure 4.10) where intermittent separation occurs primarily on the tongue-side wall. The asymmetry of the flow seems to point to a separation streamline at the tongue like that pictured. The high velocity fluid ejected from the impeller appears to hug the outer wall down the diffuser.

Near the peak of the characteristic, the flow has shifted (cf. Figure 4.11). The static pressure falls off sooner, implying a largely separated diffuser. The hydrogen bubbles also indicate separation over at least the last half of the diffuser. They also show that the flow is now fuller near the tongue-side wall. This seems to indicate a distorted separation streamline near the tongue as shown. The diffusion begins much earlier now, and the higher velocity fluid is drawn to the tongue side of the diffuser.

At low flow, separation and reverse flow dominate over the full diffuser (cf. Figure 4.12). The pressure is nearly constant throughout, with only a slight mixing-out benefit at the far downstream static tap. These are, more or less, the prevalent conditions over the entire positively sloped portion of the characteristic.

Dominant frequencies extracted from time-resolved data in the diffuser show multiples of shaft frequency over much of the characteristic, with frequencies around 25-30% of shaft speed appearing only at low flowrates. This could be a result of a rotating stall cell in the impeller at off design flows. Future measurements in the volute will be able to explain this more convincingly.

4.4 Inlet Steady State Performance

Using the same hydrogen bubble method, qualitative flowfield information was generated for the pump inlet. The single inlet wire cross is approximately 10 cm upstream of the pump inlet bell.

Above bep, the flow is predominantly axial, with very little swirl (tangential velocity) near the wall. Near the peak of the characteristic, the axial core is still present but greatly diminished. Swirl, caused by the nearby rotating shroud, is now the predominant flow feature. Its effect is more noticeable nearer to the wall. At lower flow
coefficients, only a small portion of flow near the center of the cross section is still axial. Flow elsewhere is completely tangential and even reversed in some locations nearest the stationary wall. Though strictly quantitative measurements are not yet available, the three dimensional nature of the inlet flowfield (especially at low flow coefficients near $\phi_{17}$) is clearly evident from these preliminary observations.
CHAPTER V

EXPERIMENTAL RESULTS: UNSTEADY OPERATION

5.1 Chapter Preview

The primary motivation for the construction of the experimental facility introduced in the preceding four chapters is the comprehensive study of unsteady pump operation. Before initiating an intensive component by component investigation of the unsteady phenomena, however, a series of tests were conducted with the object of mapping out the global pump behavior during surge. The envelope of attainable values of reduced frequency and B parameter was also established. The results of these tests are presented in this chapter. Theoretical results from the linear model (Section 2.2) and the dynamic simulation (Section 2.3) are also presented, and a comparison is made between the experimental and the analytical results. Finally, the dynamic response of the MIT pump facility is compared with available dynamic data from the original pump facility. The degree to which the dynamics are similar is assessed and implications for future work are discussed.

5.2 The Instability Boundary

5.2.1 Unsteady Test Matrix

Figure 5.1 shows the test matrix used for the experimental study of global pump behavior during unsteadiness. As discussed earlier, it was anticipated that the B parameter would have a dominant role in determining the nature of loop unsteadiness. For this reason, the test matrix shows variations in two loop variables: the rotational speed of the pump ($U_{T1P}$) and the volume of air in the small (discharge) plenum ($V_2$). Results from the linear model show that the frequency of system oscillation, $\omega_{\text{unst}}$, varies as the inverse square root
of the volume $V_2$ (this will be discussed in detail in the Section 5.2.2). Thus, from the definition of $B$ employed in Chapter II,

$$B = \frac{U_{TIP}}{2\omega_{unst} L_P} \alpha \frac{U_{TIP} \sqrt{V_2}}{2L_P}$$

it is clear that by varying $U_{TIP}$ and $V_2$, it is possible to obtain the same value of $B$ for two different values of speed. In this way, the dependence of the instability in question (surge) on the system $B$ parameter can be accurately evaluated (independently of speedline).

During the testing reported in this chapter, the air volume in the inlet plenum was held constant at 300 liters. The inertial lengths were not varied and cavitation was never allowed to occur. Though the facility has the ability to operate at elevated pressures (Section 3.2.4), all tests were accomplished without varying the overall loop pressure. The water level was always brought to the same height before testing to insure a consistent hydrostatic pressure between runs.

The testing procedure was as follows. The innertubes in the discharge plenum were first filled with the desired volume of air (cf. Figure 5.1) following the steps outlined in Section 3.2.1. With the discharge stack valve closed, the pump wheel speed was brought up to the speedline of interest (60%, 80%, or 100%). Then, beginning above the bep point on the pump characteristic, steady state data were collected at progressively smaller throttle settings. Measurements were made of the following quantities: pump discharge pressure, pump inlet pressure, discharge volume flow, shaft torque, shaft rpm, and backplate leakage flow (cf. Section 3.4.1).

The instability inception point ($\Phi_{tr}$) was indicated by the formation of a constant amplitude oscillation of significant magnitude ($\text{Ampl}_{p-p} \geq 3\%$ of $\Psi_{bep}$) while closing the large throttle in small increments along the positively sloped portion of the performance characteristic. This "minimum amplitude for surge" designation corresponds to an oscillation amplitude greater than the steady state noise level of the pump at low flows. The error in $\Phi_{tr}$ is estimated at $\Delta \Phi = \pm 0.002$ (roughly 3% of $\Phi_{bep}$).

Time-resolved data at 10-20 Hz (i.e. about 30 times greater than the typical surge frequency of 0.4 Hz) were collected at successively smaller throttle settings, after the surge onset, until shutoff was reached. The surge cycles are referenced by their equivalent
steady state operating point, determined by averaging the value of $\Phi$ over one surge cycle. This quantity is referred to as $\Phi_{avg}$ for the surge cycle in question. The frequency of oscillation was determined by an FFT of the transient data. Because of the desire to compare the dynamic performance of this pump to the smaller developmental unit (which turns at speeds nearly 200 times greater), the frequency is usually presented as the reduced frequency, or $\omega_{red}$, where

$$\omega_{red} = \frac{\omega_{unst}}{\omega_{shaft}}$$

This reduced frequency, $\omega_{red}$ non-dimensionalized by shaft frequency, is to be distinguished from $\Omega$, the reduced frequency based on pump throughflow time, which was introduced in Section 2.2.2 and appears again in Section 5.4.1. $\Omega$ is used to evaluate the pump response to unsteadiness. It is expected that the component which influences this response the most is the pump impeller, and the relevant velocity in the impeller passages is $C_m = \Phi U_{tip}$. When referencing the surge frequency, on the other hand, the entire flow between the two plenums is concerned, and the relevant velocity is associated with the pump wheel speed, or $\omega_{shaft}$. Unless otherwise indicated, discussions of reduced frequency in this report will refer to $\omega_{red}$.

As used in this document, the B parameter is a function of the experimentally derived frequency, $\omega_{unst}$, discussed above. In other studies, B has been based on a theoretically derived frequency ($\omega_{theory}$) [9] or on the system natural frequency [12] as determined by exciting the inactive facility with random noise and evaluating the frequency of resonance. It is difficult to say which quantity (B based on $\omega_{unst}$ or B based on $\omega_{theory}$) has more relevance for an engineer. For a new pump design, $\omega_{theory}$ would be more useful, whereas for an operational unit, $\omega_{exp}$ might be more readily available. Furthermore, any experimentally observed effects not accounted for by a theoretical model might cause a significant discrepancy between predicted and actual frequencies. In such a case, the B parameter based on experimental frequency is a more reliable indicator of the actual pumping system's instability potential. Since, in this study, the linear model was used to "predict" stability after the experimental results had already been obtained (and not as a design tool), the B parameter based on experimental frequency has been used.
5.2.2 Experimentally Determined Ranges of Dynamic System Parameters

Before presenting the parametric study results, it is instructive to consider a greatly simplified model of the MIT test facility. The linear model outlined in Chapter II uses the linearized equations which fully define the pumping system of interest to predict the frequency of oscillation. Results from this analysis will be shown in Section 5.4 to be quite accurate in predicting both the magnitude and trends of the experimental oscillation frequency. A much simpler model, however, can be constructed which provides a basic feel for the expected trends in system frequency without predicting the exact magnitude. This is done by modeling the test loop as a Helmholtz resonator [9]. Figure 5.2 shows the equivalent resonator in schematic form. The throttle leg is neglected in the analysis, and the water fills only the \( L_p \) length of piping in its equilibrium position. The inlet plenum air content \( (V_1) \) is typically much greater than the discharge plenum air content \( (V_2) \) and varies very little (±5%) during surge. Thus, the pump inlet is assumed open to atmosphere. Also, the pump is assumed to be operating at a point of zero characteristic slope, \( \frac{d\Phi}{d\Phi} = 0 \). The frequency of small amplitude oscillations (of the water column) in this system is called the system Helmholtz frequency, \( \omega_{hh} \), defined by:

\[
\omega_{hh} = a \sqrt{\frac{\rho_a Ap}{\rho_w L_p}} \left( \frac{1}{V_2} \right)
\]

Here "a" is the speed of sound in air, \( \rho_a \) and \( \rho_w \) are the densities of air and water respectively, \( Ap \) and \( L_p \) are the reference area and inertial length of the pump leg (Section 3.2.2), and \( V_2 \) is the discharge plenum air volume at \( \Phi_{tr} \).

In the test matrix being discussed, \( V_2 \) and \( UTIP \) are the only variables. From the above equation, it is evident that increasing \( V_2 \) will cause the frequency of oscillation to decrease by the square root law as shown. The actual frequency values predicted by this simple resonator model, are about twice as large as the experimentally measured frequencies. The trends, however, are instructive when trying to understand the experimental results.
Figure 5.3 shows the experimental reduced frequency of surge plotted vs. the air content in the discharge plenum (as measured with the pump off). The frequency used in the \( \omega_{\text{red}} \) determination here is from the inception surge cycle (at \( \Phi_{\text{tr}} \)). Results for 100%, 80%, and 60% speedlines are presented. As expected, the frequency of oscillation decreases with increasing air volume by what appears to be a square root relation. The use of the reduced frequency here lumps the effects of differences in wheel speed and oscillation frequency into one parameter. To see the frequency trend more clearly, the dimensional frequency, \( \omega_{\text{unst}} \) (Hz), is plotted in Figure 5.4. In addition, this plot uses the adjusted volume of air in the discharge plenum (instead of the volume as measured with the pump off, cf. Appendix D) to more clearly demonstrate the dependence of the oscillation frequency, \( \omega_{\text{unst}} \), on \( V_2 \). The square root dependency is apparent, as predicted by the Helmholtz analysis, independently of speed.

The range of reduced frequencies shown on Figure 5.3 is \( 8.3\% > \omega_{\text{red}} > 3.7\% \). During other tests with larger volumes, reduced frequencies as low as 2.8\% have been obtained. Oscillations at frequencies as large as 21.8\%, using smaller volumes than those shown, have also been produced. These frequencies, however, correspond to oscillations that were intermittent in nature, and thus were not classified as "surge" frequencies and are not shown.

Once again, in this report the B parameter is based on \( \omega_{\text{unst}} \), and the resulting range of experimental B parameter (cf. Figure 5.5) can be readily calculated from a knowledge of wheel speed and experimental frequency of oscillation. The same trends extracted from Figure 5.3 are evident here. The experimental range of B parameter is \( 0.28 < B < 0.60 \).

5.2.3 The Stability Boundary (\( \Phi_{\text{tr}} \))

The operating point at which the pump operation first became unstable (while throttling from bep towards shutoff) is plotted in Figure 5.6 vs. the air content in the discharge plenum (there is a certain amount of subjectivity in choosing \( \Phi_{\text{tr}} \) and the data are somewhat more irregular than the reduced frequency and B parameter data). On a given speedline, increasing the small plenum air volume results in an increase in B (cf. Figure 5.5) and it is thus anticipated that the system should become unstable earlier. This trend is apparent from the figure. Also, since the higher speedlines have a higher B parameter (cf.
Figure 5.5), it is expected that the degree of instability should increase with speedline; this is also shown in the data.

According to theoretical modeling (Chapter II and [9]), the three curves in Figure 5.6 should collapse to one when $\Phi_{IR}$ is plotted vs. the $B$ parameter. This is done in Figure 5.7. The three do not collapse exactly and it appears that the 80% data are more unstable than the 100% data, and that the 60% data are more unstable than both.

There are at least two possible sources of this deviation from theory. First, the local characteristic slope is a key determinant of stability. The phase portraits of the 100%, 80%, and 60% steady state speedlines (cf. Figure 5.8) appear roughly similar, but the curves do vary slightly (cf. Figure 5.9). The 100% speedline is noticeably less smooth, with several "kinks", while the 60% and 80% speedlines both exhibit a gradual change in slope down to shutoff. By using a third order fit to the positively sloped portion of the experimental curves (only the positively sloped portion has potential for instability), it is possible to plot the characteristic slope vs. operating point (cf. Figure 5.10). The slope for the 60% speed is slightly more positive (unstable) than for 80% and is similar in shape. The slope for the 100% speed, however, has a markedly different shape and is much less positive (less unstable). In Section 5.4, when comparisons are made with the linear model predictions, a sensitivity analysis will show quantitatively the effect of a moderate change in characteristic slope (such as those shown in Figure 5.10) on stability predictions.

Second, as was shown in Section 2.2.1 (cf. Figure 2.4), there is significant wheel speedup during surge. This is a stabilizing factor (cf. Figure 2.5) since it effectively alters the local value of the characteristic slope. Figure 5.11 shows the variation in $U_{TIP}$ with pump leg mass flow, $m_p$, as determined experimentally for the three speedlines. The slope, $\frac{dU_{TIP}}{dm_p}$, is noticeably steeper for the higher speedlines. This suggests that the 100% data should be more stable than the 60% data (the trend shown on Figure 5.8) for comparable values of $B$. A more quantitative sensitivity analysis showing the effect of the different experimental values of $\frac{dU_{TIP}}{dm_p}$ will be done using the linear predictor model in Section 5.4, and it will be seen that the experimental trend of Figure 5.8 is confirmed by the model.
5.2.4 Features of the Instability Boundary

Figure 5.7 has several other interesting features which warrant attention. It was found from experiments on an axial compressor [9,10] that a minimum $B$ was necessary (independent of speedline) for instability. Below this threshold, a limit cycle behavior (surge) never occurred. This same conclusion has also been derived from work on a centrifugal compressor [12]. Though there is clearly some deviation from a $B_{\text{threshold}}$ which is completely independent of speedline (for the reasons outlined in Section 5.2.3), the results in this report do support the same conclusion. There is clearly a cutoff value of $B$ somewhere in the range $0.28 < B_{\text{threshold}} < 0.33$. Below this value of $B$, there is occasional unsteadiness which becomes less and less predominant until at $B = 0$ (with no air) a steady state speedline is attained. This establishes the applicability of the simple linear model and thus the $B$ parameter as a reliable predictor of instability.

In addition, it was observed that, under certain conditions, the pump operation would become unstable at $\Phi_{tr}$ and then become stable again with continued throttling toward shut off. If the throttle were slowly opened from the region of stable operation near shutoff the unsteady operation would resume. This "lower" stability limit, or $\Phi_{tr2}$, is apparent from the shape of the stability curves in Figure 5.7. The same behavior has been seen elsewhere [10] and can be explained with reference to the steady state characteristic. From Figure 5.8, it is clear that the slope of the pump performance characteristic reaches a maximum at some positive value of $\Phi$, and then typically goes to zero near shutoff, with the reverse flow slope strongly negative [10,11,24]. Assuming this to be the case for the pump in question, the slope near shutoff becomes small enough that for the lower values of $B$ ($B_{\text{threshold}} < B < 0.40$) the system can no longer sustain the limit cycle behavior. For $B > 0.40$, the loop compliance is large enough to support surge oscillations even at shutoff. This feature will be discussed in more detail in Section 5.3.2.

It is apparent, then, that the $B$ parameter is a stability indicator of significant merit. The value of $B_{\text{threshold}}$ is machine specific, so that once it is determined (through a variable $B$ laboratory test, for example) for a specific pump configuration, it is independent of a new pumping environment or wheel speed. It thus has considerable potential as a design tool.
5.3 The Experimental Surge Cycle

Up to this point, little has been said about the form of the transient pump behavior during surge. The following sections give a documentation of this phenomenon.

5.3.1 Evolution and Shape

Figure 5.12 is a transient time trace of $\Psi$ and $\Phi$ after a throttle adjustment on the negatively sloped portion of the characteristic near bep. Some low level noise is apparent, however, no sustained oscillation is observed. The behavior of the pump becomes increasingly unsteady as the transition point is approached. Typically, when the system has been throttled to within 10 to 20% of $\Phi_{tr}$, the pump operating point begins to oscillate intermittently (cf. Figure 5.13). The amplitude of this motion can become as large as 5% of $\Psi$ ($\Delta \Psi_{p-p}$) and then subsequently die away. This behavior becomes more prevalent as the system approaches $\Phi_{tr}$. This is the so-called "presurge" instability referred to in [5,12,23,24]. It is commonly observed near the centrifugal pump (compressor) surge line, and is due to the nearly balanced unstable and resistive forces in the pumping loop. If the local characteristic slope is locally unstable, a small perturbation begins to grow, but is damped out by the loop resistive elements (throttles).

When the system is throttled to an operating point with a sufficiently large local slope (for $B > B_{threshold}$), there is no longer enough system damping to extinguish the presurge perturbations and they escalate quickly (within 4-7 cycles) into a constant amplitude limit cycle. The frequency of this initial limit cycle was used in the determination of $\omega_{red}$ and $B$ (as mentioned earlier) because it is typically of small amplitude, and thus comes closest to the small perturbation assumption of the linear model. In addition, the frequency and amplitude of the transition point surge oscillation constitute the dynamic data available from the developmental pump, so the behavior at this point provides a fair comparison of dynamic response between the model and the original. Figure 5.14 shows a final amplitude limit cycle at $\Phi_{tr}$ after the transients have died away.

With no further throttle adjustment (from the transition point), the system will continue surging at constant frequency and amplitude. Throttling beyond the transition point
causes the oscillation amplitude to grow, until a maximum amplitude surge cycle is attained (cf. Figure 5.15). Subsequent lowering of the operating point results in a decreased amplitude, until (depending on the magnitude of B) either a shutoff limit cycle is attained or the oscillation ceases. These two phenomena are shown in Figure 5.16 (B = 0.64) and Figure 5.17 (B = 0.36) respectively. Opening the throttle from shutoff, reproduces the variation in amplitude trend, but in reverse order.

As was mentioned earlier, if B is less than the $B_{\text{threshold}}$, the pumping system cannot support a constant amplitude limit cycle (surge). Figure 5.18 shows a time trace of typical pump behavior for $B = 0.28$, at two different operating points. On the exploded scale, the $\Phi$ trace is noticeably erratic. Though the amplitudes are not constant and never exceed 4-5% of $\Psi$, operation in this region could not be considered truly stable. Only as the air content of the discharge plenum is brought close to zero ($B < 0.1$, cf. Figure 5.19) does the pump behavior approximate the steady state operation.

5.3.2 Variations in Surge Amplitude

The variation in surge cycle amplitude with operating point (at constant B), mentioned in Section 5.3.1, has been documented by others [10,12,15]. In the literature, adjectives such as "mild", "deep", and "classic" have been used to differentiate between the stages of surge cycle development. The changes in the limit cycle with operating point can all be understood by analysis of the pump characteristic. Delimiters which differentiate between types of surge cycles will not be used here.

The performance characteristic slope is nearly flat at shutoff and again just above the unstable transition point. At the $\Phi_{\text{fr}}$ point, then, the surge cycle amplitude is limited by the close proximity of a stable portion of the characteristic (which has a strong dampening influence). When the oscillation amplitude starts to grow, the pump operating point begins to swing further and further into this stable region until the cycle is constrained or held to a finite excursion. If the average operating point ($\Phi_{\text{avg}}$ over a cycle) is moved to a lower value of $\Phi$ than $\Phi_{\text{fr}}$, the instability can then increase in amplitude. Further throttling continues this trend, until a large amplitude oscillation brings the transient pump operating point close to $\Phi_{\text{shutoff}}$. The steady state characteristic flattens out again near $\Phi_{\text{shutoff}}$, so
this serves to suppress the large amplitude surge cycle, with the effect increasing as $\Phi_{\text{avg}}$ approaches shutoff.

Figure 5.20 shows a typical trend in oscillation amplitude with operating point. The ordinate shown is the amplitude of oscillation in $\Phi$ rather than $\Psi$, though the trends are similar for both. Using a fifth order curve fit to the entire steady state pump characteristic, it is possible to plot surge amplitude vs. local characteristic slope (cf. Figure 5.21). Interestingly, the point of maximum amplitude does not occur at the point of maximum positive slope, as might be expected. Instead, it occurs at a location just above the midpoint between the two regions of zero slope (near shutoff and at the characteristic peak). This implies that the unsteady pump behavior is not determined by the local steady state slope, but rather the averaged, or "smoothed-out" slope, over a significant region around the $\Phi_{\text{avg}}$ operating point (cf. [15,16]).

This same "smoothed-out" slope effect has been noted before in reference to the instability transition point. Two different studies [15,16] concluded that because there are significant random 'presurge' oscillations just before the transition point, local features of the steady state characteristic are effectively 'smoothed out' by the wandering pump operating point. Thus, the instantaneous slope, which actually figures into the determination of unsteadiness (cf. Section 2.1), is really quite smooth (representing an average of the steady state slopes over a sizeable region). Since these same 'presurge' oscillations were also seen in the facility in question, the influence of "kinks" in the steady state speedline data on stability is uncertain. This bolsters the case for the use of third and fifth order curve fits (Sections 5.2.3 and 5.3.2), which smooth out local characteristic features, in stability prediction models (Section 5.4).

Another conclusion which can be drawn from the trend in amplitude variation shown on Figure 5.21 is the importance of the backflow characteristic in determining transient surge behavior at flows near shut off. A steep reverse flow characteristic can stabilize surge at low flow coefficients by damping the transient amplitude of oscillation. Both of these factors (the backflow characteristic and the "smoothed-out" slope effect) underline the dominance of the instantaneous pump performance characteristic in determining the nature of the transient pump behavior before and during surge.

The surge cycle amplitude is found, experimentally, to be a strong function of both operating point and system B parameter. The more compliance available in a pumping
system, the greater the attainable oscillation amplitude. This trend has been documented elsewhere [10,12,15] and is evident from a plot of maximum surge cycle amplitude vs. $B$ in Figure 5.22. This figure brings up the same features which were discussed at length in Section 5.2.3, concerning an unexpected speedline dependence observed on the $\Phi_{tr}$ vs. $B$ plot (cf. Figure 5.7). The arguments employed there, about differences in wheel speed variation and different slopes between speedlines, both point to increasing stability with speed (the trend seen in Figure 5.22).

5.3.3 Variations in Surge Frequency

A second phenomenon which occurs by throttling past $\Phi_{tr}$ is a decrease in the frequency of oscillation. This trend is shown in Figure 5.23. The pump discharge pressure decreases as the system is throttled from $\Phi_{tr}$ to shutoff. This decrease in pressure, causes the air volume in the small plenum to expand (and the air volume in the inlet plenum to contract, cf. Appendix D). Thus, from the Helmholtz resonator analysis, it is expected that the frequency of oscillation will also decrease. This trend is convincingly reproduced by the linear model in Section 5.4.

5.3.4 Hysteresis at the Stability Boundary

A characteristic feature of non-linear systems such as the facility of interest is the hysteresis encountered at the instability transition points ($\Phi_{tr}$ and $\Phi_{tr2}$). There is some difference in the experimentally observed value of $\Phi_{tr}$ depending on the direction of approach. If approached from a stable operating regime, $\Phi_{tr}$ is lower than if approached from an unstable regime. Typically the hysteresis band width was about $0.002 < \Delta \Phi < 0.004$. Similar hysteresis has been observed elsewhere [10] and is not unexpected. The surge cycle at the transition point is a finite amplitude limit cycle with transient excursions well into the previously stable portion of the performance curve. If the system, which is surging at $\Phi_{tr}$, is throttled away from the unstable regime, it encounters operating points which were previously stable enough to resist small perturbations. Because of the large amplitude perturbations, however, an operating point further from $\Phi_{tr}$ is required to resume stable operation. Experimental transition points referred to in this document were

62
always determined by approaching $\Phi_I$ from the stable regime, as this is the typical procedure in operational pumps.

5.4 Comparison with the Linear Model

5.4.1 Model Overview

The linear model outlined in Chapter II was not only used in the preliminary phase [22], but was also employed as an instability predictor after experimental data were available. This was done to determine the model's utility and limitations by comparing its predictions with experimental results. Experimentally derived inputs were used in order to more closely model the actual test facility. These are discussed below in detail.

A third order curve fit to the positively sloped portion of the steady state pump performance data was used to determine the operating point, $\Psi = f(\Phi)$, and the local characteristic slope, $\frac{d\Psi}{d\Phi}$. This third order fit gave a much closer approximation to the unstable portion of the characteristic than did the corresponding fifth order fit to the entire characteristic. Due to differences in the nondimensional pump performance curves at different speedlines (cf. Section 5.2.3), three different curve fits were employed, one for each of the 100%, 80%, and 60% speedlines. The independent variable in the linear program (cf. Appendix A) is $\Phi$. The characteristic is scanned from $\Phi = 0.001$ to 0.070 (incrementing by 0.001) and the eigenvalues are determined at each operating point. Empirical relations were used to determine $UTIP$, $P_1$, and $P_2$ for the $\Phi$ in question. Using the relation outlined in Appendix D, the compliant volumes ($V_1$ and $V_2$) were also calculated at each operating point. The values used for the lengths, areas, and head loss factors of each piping leg are mentioned in Chapter III. The experimental value of $\frac{dUTIP}{d\Phi}$ was found to vary between speedlines, and values corresponding to each speedline were used as measured from Figure 5.11.

The only variable in the linear model is $Z_{LAG}$, the correction term to the lag time constant, $\tau$ (cf. Section 2.2.2). A sensitivity analysis was conducted in an attempt to see the effect of $Z_{LAG}$ on the model stability predictions. Figure 5.24 shows the predicted
instability boundary for several values of $Z_{LAG}$ vs. the 100% speedline experimental findings. The quasi-steady results correspond to $Z_{LAG} = 0.0$. Best agreement was found for $Z_{LAG} = 0.03$.

This result seems plausible. A typical value for $\Phi_{tr}$ is 0.035. The 100% speedline has an average wheel tip speed of 13.7 m/s, and for a moderate $B$, the frequency of oscillation is roughly 0.4 Hz ($\omega_{\text{red}} = 5.5\%$). Using the relationship outlined in Section 2.2.2 for $\Omega$, the reduced frequency,

$$\Omega = \frac{\text{pump throughflow time}}{\text{period of unsteadiness}} = \frac{L_{TH}/C_m}{1/\omega_{\text{unst}}}$$

$\Omega$ (for the case outlined above) is found to be around 1.8 (using $L_{TH} = 2.3$ m and $C_m = \Phi U_{\text{tip}}$). Physically, this means that in the time it takes for an average fluid particle to traverse the entire pump (inducer + impeller + volute + diffuser), the surge oscillation has gone through nearly two complete cycles. Setting the lag time constant equal to the throughflow time, then, is equivalent to ignoring the oscillation altogether. The perturbation in mass flow gets buried in the lag, and, as a result, the pump will never go unstable because its response time is much slower than the period of changing operating conditions. This trend is visible in Figure 5.24. Even $Z_{LAG} = 0.15$ shows almost no susceptibility to unsteadiness.

The $Z_{LAG}$ value of 0.03 represents a pumping system which reacts in approximately one twentieth of the period of unsteadiness:

$$\Omega' = \Omega (0.03) = \frac{\left(\frac{L_{TH}}{C_x}\right) 0.03}{1/\omega_{\text{unst}}} = 0.053$$

$\Omega'$ is the effective reduced frequency of the pump imposed by the empirical choice of $Z_{LAG}$. From another perspective, this choice of $Z_{LAG}$ implies that the relevant length scale, $L_{TH}$, is really 7 cm rather than 2.3 m (as has been used). This new length scale is approximately equal to $\frac{D_1}{2}$, the impeller inlet eye radius. The distance $\frac{D_1}{2}$ may point to the inducer and impeller blade leading edges as the critical region of performance deterioration near $\Phi_{tr}$.

In this context, it is interesting to compare the value of $Z_{LAG}$ used here to that used to match the unsteady performance in a centrifugal compressor [12]. Fink [12] found that
a value of $Z_{LAG} = 2.3$ best correlated the modeling predictions of unsteady behavior with the experimental results. His definition of the first order lag time constant, $\tau$, was identical to that used in this report (based on the throughflow time). The difference in $Z_{LAG}$ appears to be large, but the typical reduced frequency of pump performance during unsteadiness, $\Omega$, for Fink's compressor was approximately 0.044 (except during "deep" surge). His compressor operation is, to a good approximation, quasi-steady during surge. The effective pump reduced frequency, $\Omega'$, represented by the use of $Z_{LAG} = 2.3$ is calculated:

$$\Omega' = \Omega (2.3) = 0.101 \left( = \frac{\text{effective pump throughflow time}}{\text{period of unsteadiness}} \right)$$

The value of $\Omega'$, which is the effective pump response time imposed by the empirical choice of $Z_{LAG}$ divided by the period of unsteadiness, has similar magnitude in the two cases shown. This is reassuring, but clearly the actual pump response is more complicated than has been assumed.

5.4.2 Predicted Ranges of Dynamic System Parameters

In general, the linear model predicted the dynamic behavior well. Figure 5.25 shows the predicted values of reduced frequency (for $Z_{LAG} = 0.03$) alongside the experimental data. Results are shown for the three speedlines of interest: 100%, 80%, and 60%. Agreement is quite good for the 100% and 80% speedlines, although the 60% predictions are as much as 15% lower than the experimental results. The linear model shows the same trends as the experimental data: decreasing frequency with increasing $V_2$ and increasing reduced frequency with decreasing wheel speed. The trends in $B$ are likewise matched (cf. Figure 5.26). The results, then, are encouraging, and the approximations outlined in Chapter II appear to have validity.

5.4.3 Predicted Instability Boundary

Figure 5.27 shows the predicted values of $\Phi_{lr}$ alongside the experimentally obtained results. The degree of agreement is somewhat less, but the trends are clearly reproduced.
Stability decreases with increasing compliance and increases with decreasing speed. Figure 5.28 shows the predicted stability boundary vs. the experimental result. The experimentally observed differences between speedlines are well captured by the linear model. This is made possible by accounting for the two factors which were introduced in Section 5.2.3. These are outlined below as they pertain to the present discussion.

1) The first factor centered on the inherent differences in the steady state pump performance characteristics. Inasmuch as a different third order curve fit was input for each speedline, the linear model predictions have accounted for this factor. From Figure 5.28 alone, it is difficult to separate the effect of these characteristic differences from other effects which are present. For this reason, the linear model was run with two different performance curves, as shown in Figure 5.29, everything else being the same. These performance curves are representative of the 100% and 60% curve fits. The corresponding stability boundaries are shown in Figure 5.30. The implication is that differences in local characteristic slope, typical of those seen in the experimental data, can have an effect on stability.

2) The second factor from Section 5.2.3 exposed the different degrees of wheel speedup observed in the three speedlines. This feature is also incorporated into the linear model predictions shown in Figure 5.28. Once again, its direct effect is difficult to discern because the figure shows the effect of other speedline differences as well. To demonstrate more clearly the influence of the different values of \( \frac{d\text{UTIP}}{d\text{mp}} \), several cases were run using three different inertial influence slopes (corresponding to 100%, 80%, and 60% speedline operation) with the same pump characteristic (100% speedline). The resulting stability boundaries are shown in Figure 5.31. The differences in wheel speedup do have an effect, and the stability decreases with speed.

By accounting for these two differences between speedlines, the model is able to predict the system dynamics satisfactorily.
As was pointed out in Section 5.2.4, for certain values of B, there exist two transition points (\( \Phi_{tr} \) and \( \Phi_{tr2} \)). The predicted stability boundaries also exhibit this trend (cf. Figure 5.28).

In addition, the predicted frequency of oscillation does vary with operating point (as seen experimentally, cf. Section 5.3.3). Figure 5.32 shows a comparison between the predicted and experimental variations in frequency with operating point. The good agreement is made possible by recalculating the pressures and volumes at each new operating point.

5.4.4 Summary of Linear Model Results

The simple linear model is able to capture most of the experimentally observed unstable behavior. The lumped parameter and pump performance approximations result in a simplified system which is remarkably accurate. The influence of the B parameter as an instability indicator is also verified.

The uses of this linear model as a potential predictive design tool are tempered by the fact that the results are sensitive to the magnitude of the time constant in the lag expression. It would be difficult to accurately predict this value for a new generation of pumps. The model could be used rather effectively to evaluate the effect of a new pumping environment on the stability of a pump somewhat similar to one which had been used and tested previously.

5.5 Comparison with Time-Resolved Simulation

5.5.1 Simulation Overview

The motivation behind using the dynamic simulation outlined in Chapter II was to obtain time-resolved pump performance predictions to be compared with experimental data. This was done for several different initial conditions, corresponding to typical experimental operating conditions.
The fifth order curve fit to the steady state pump performance characteristic was employed here instead of the third order fit, since the former approximated a typical reverse flow characteristic better. All other parameters used in the linear model code were used as well ($Z_{LAG}$, $L_P$, $\frac{dU_{tip}}{d\dot{m}_P}$, etc...). Each simulation was given initial conditions corresponding to an operating point in the stable regime of the pump curve (near $\Phi = 0.055$). By timing the closing time of the electronically actuated throttles used in the actual test facility, an estimate of throttling speed was available. Accordingly, the throttle head loss coefficient, $K$, (cf. Section 2.3), was transiently increased to move the operating point ($\Phi_{avg}$) into the unstable regime. $\Phi_{avg}$ values similar to those for which time-resolved data were available were employed. The system was then left at this operating point until a final amplitude was reached (typically in 5 to 15 cycles) and the final "limit cycle" was then compared to the corresponding experimental data.

5.5.2 Simulation Results

Figures 5.33 through 5.38 show simulation results vs. data for a "high B" system ($B = 0.60$) at several operating points. The 100% steady state characteristic was used for these plots and is shown as the solid line. At the transition point, the simulation matches the experimental results quite well. The actual values of transition from the model and the experiment differ slightly, as was shown in Figure 5.28, but the surge cycle shape is well captured. As the operating point ($\Phi_{avg}$) decreases, the surge cycle amplitude increases in both the simulation and experiment. Further decreases in operating point result in a decrease in the oscillation amplitude, until at shutoff both the simulation and experiment have been reduced to approximately one fourth their maximum amplitudes.

Surge cycle comparisons were also made at other B parameters. Figures 5.39 through 5.42 show the results for $B = 0.33$ at four different operating points (again the 100% characteristic curve fit was used). The trends in amplitude are reasonably well portrayed, although the simulation overestimates the limit cycle magnitude by roughly fifty percent (in $\Psi$ and $\Phi$) in all four cases.

The simulation thus appears to reproduce much of the observed unsteady behavior. Trends in amplitude near $\Phi_{tr}$ and shutoff are well captured. The simulation generally
overestimates the amplitude of oscillation, and this discrepancy is more pronounced for low B and $\Phi_{avg}$ near shutoff. Two simplifications employed by the model appear to be directly responsible for this:

1) In the literature [11,24], time-resolved simulations of transient pump (compressor) behavior rely on estimates of the backflow characteristic. The reverse flow (steady state) performance of the model pump is currently unknown, and has only been guessed in the simulation. For the $B = 0.60$ comparisons, this is the region in which the simulation begins to severely underpredict the experimental results.

2) The pump performance has been modeled as a black box function generator, $\Psi = f(\Phi)$, with an added first order lag. This assumption resulted in good agreement between the experimental findings and the linear model near the instability transition point ($0.025 < \Phi < 0.045$) (cf. Section 5.4.3). Once the surge cycle amplitude outgrows the small perturbation approximation, however, it is unclear that the pump performance model retains its applicability. In the axial compressor time-resolved simulation developed by Greitzer [8], two different values of the lag time constant, $\tau$, were necessary for accurate predictions. A smaller value was employed when the compressor was operating on the unstalled portion of its characteristic, and a much larger value was employed to model the transition to rotating stall. Perhaps a new $Z_{LAG}$ should be employed when the pump behavior becomes non-linear. A more sophisticated model, then, would be necessary when the pump operation begins to depart noticeably from the linear model assumptions. These issues will not be addressed in any further detail in this report, but are recommended as topics for future research.

The first order lag does improve predictions over the quasi-steady ($Z_{LAG} = 0.0$) pump performance model. This was shown to be the case in the linear model results (cf. Section 5.4). It is also valid for the simulation routine as shown in Figures 5.43 and 5.44, which compare the simulation surge cycle predictions at two different B's, with and without lag ($Z_{LAG} = 0.03$ and $Z_{LAG} = 0.00$, respectively).
5.5.3 Summary of Simulation Results

The time-resolved simulation gives fair prediction of transient pump operation. Even employing some rudimentary approximations to the actual component behavior, the model is able to capture much of the system response. The importance of the pump performance characteristic in determining unstable behavior has been established. Discrepancies between the simulation results and the corresponding experimental findings have been examined by exploring the limitations in the model. The simulation results show that a greater understanding of the pump response during unsteadiness is necessary in order to improve the predictive power of the models used. This, in turn, points to the value of a detailed, component by component analysis of the pump performance at and near the transition point. This will be discussed again in Chapter VI.

As a potential tool for the pump designer, this simulation could provide realistic information about the transient pump performance during various throttling operations. An estimate of the maximum operating pressures and flow swings could also be made available, although the same limitations which apply to the linear code (cf. Section 5.4.4) are also applicable for the time-resolved simulation.

5.6 Comparison with Development Pump Dynamic Performance

In light of the discrepancy in steady state pump characteristics between the MIT and the development pump, it is not useful to perform a quantitative comparison between their respective dynamics. A different performance characteristic shape implies a different B_{threshold}, different amplitudes of oscillation, and different transition points, and precise matching between the two pumps' dynamic response is not expected. There are, however, at least two important reasons for a qualitative comparison of available dynamic data from the two pumps. These are outlined below.

1) The two pump performance curves are not extremely different, and it is expected that the trends and the approximate range of parameters should be comparable.
2) The fundamental premise for this entire project is the ability to reproduce the dynamic response of the original pump in the larger model. The instability which is studied and "cured" in the model pump must be the same instability exhibited by the original pump. Accordingly, it must be convincingly shown that the basic instability phenomenon encountered in the two pumps is the same.

The following table lists the range of dynamic parameters observed in the MIT facility and in the family of low specific speed development pumps under consideration:

<table>
<thead>
<tr>
<th></th>
<th>MIT</th>
<th>DEVELOPMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instability Transition Point ($\Phi_{tr}$)</td>
<td>0.025-0.045</td>
<td>0.005-0.045</td>
</tr>
<tr>
<td>Reduced Frequency of Surge ($\omega_{red}$)</td>
<td>2.8%-8.3%</td>
<td>2.0%-20.0%</td>
</tr>
<tr>
<td>B parameter</td>
<td>0.28-0.82</td>
<td>0.10-1.15</td>
</tr>
<tr>
<td>Amplitude of Surge Oscillation (%$\Psi_{bep}$)</td>
<td>3.0%-28.0% (peak to peak)</td>
<td>10%-40%</td>
</tr>
</tbody>
</table>

Table 5.1

The development pump data reveal a surge type limit cycle occurring just to the left of the peak of the performance characteristic (cf. Figure 4.3). The associated frequencies of oscillation and amplitudes have been observed as shown. Wheel speed variations were sensed in the developmental units, but calculations show that they should be negligible. Thus, it is expected that $\Phi_{tr}$ and oscillation amplitudes would be higher (more unstable for same B) than in the MIT model. As the table shows, the data compare quite well. From all appearances, the two pumps do exhibit the same system instability phenomenon. This point is further bolstered by previous analyses [21] of the development unit dynamic data. In this study [21], all observed trends of the unsteadiness could be explained by using a system analysis similar to that shown in Chapter II.

In summary, all indications are that the unstable behavior seen in the pumps is of the same basic nature. Thus, the system dynamics methodology which was employed in an effort to better understand the instability appears to have been the correct approach on both counts (Section 5.4 and [21]).
5.7 Preliminary Flow Visualization Results During Unsteady Pump Operation

In Sections 4.3 and 4.4, the use of the hydrogen bubble technique during steady pump operation was discussed. This technique was also implemented, although in a more cursory fashion, to highlight the flow structure during surge.

Near the surge inception point, the flowfield in the diffuser showed signs of low amplitude pulsations. As the surge amplitude increased, the whole flow began to oscillate at the surge frequency. The observed behavior during surge was similar to that seen during steady state operation for the equivalent instantaneous flow coefficient \( \Phi_{\text{inst}} \). For large values of \( \Phi_{\text{inst}} \), the flow in the diffuser was full and generally well attached to the diffuser walls, and for low values of \( \Phi_{\text{inst}} \), intermittent separation and reverse flow were predominant. However, the flow field behavior exhibited a slightly phase lagged response to the surge oscillation (by roughly 5% of the surge cycle period). It appears that the flow accelerations, for \( \frac{d\Phi}{dt} > 0 \), had the effect of making the flow more uniform and attached than the steady state flowfield at the same \( \Phi \). Also, when \( \frac{d\Phi}{dt} < 0 \), the flow would stall at higher \( \Phi \) than was observed in the steady state. This deviation from a true quasi-steady behavior was more evident during large amplitude surge cycles.

The inlet flowfield during surge was also investigated. During surge, a central core of forward flowing fluid similar to that seen under steady state operation was always visible (even when the overall pump throughflow was instantaneously reversed). When \( \frac{d\Phi}{dt} < 0 \), the azimuthal component of velocity (swirl) would slowly decrease and the flow away from the center of the inlet would become strongly reversed. Then as the flow began to swing forward (when \( \frac{d\Phi}{dt} > 0 \)) the fluid would "spin up" again as it prepared to enter the pump. This "spinning up" created a larger swirl effect than was observed during steady state operation for the same flow coefficient.

In addition to the hydrogen bubble work performed on the pump inlet and diffuser, dye injection was used to visualize the flow near the volute tongue. The injection point was situated such that during steady forward flow the dye would flow directly down the center of the diffuser inlet. This flowpath was also observed during surge, for large values of \( \Phi_{\text{inst}} \).
As $\Phi_{\text{inst}}$ decreased, however, the dye streak would slowly shift from the center of the diffuser to the tongue side wall. If the overall pump throughflow became reversed, the streak would shift further until it remained entirely in the volute (not entering the diffuser at all). This same shifting of streamlines near the tongue is shown in the steady state figures from Section 4.3. The flow near the tongue appears to adjust quasi-steadily during surge.

From the above observations, it appears that the flowfield in the pump (and its components) during surge is different from the steady state flowfield. This suggests that using the pump steady state characteristic to model pump unsteady behavior is inadequate, and that the one-dimensional model presented in Chapter II is limited in its ability to capture the dynamic behavior of a pumping system. The first order lag in pump response introduced in Section 2.2.2 was a rudimentary method of correcting for this. More in depth studies of component performance will improve the understanding of unsteady pump behavior and hopefully bridge the gap between the model and the actual pump.
CHAPTER VI

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

The construction of a large scale, dynamically tuneable pumping facility has been satisfactorily completed and the subsequent unsteady behavior exhibited by the model pump is similar to that of the original unit. The ranges of dynamic indicators ($\omega_{\text{red}}$, $B$, $\Phi_{\text{tr}}$, and oscillation amplitude) achieved in the experimental facility are closely similar to those seen in the original developmental unit. Due to an error in the scaling of the pump impeller, the steady state performance curve of the model is different from that of the original; so, differences in corresponding dynamic behavior were expected.

A full set of data on unsteady pump operation has been generated and the instantaneous pump performance characteristic and system B parameter were shown to be the two dominant parameters determining instability.

The lumped parameter systems model can accurately describe the unsteady behavior of the pumping system under investigation. Results from both the linear model and the time-resolved simulation match the experimental findings convincingly, giving an accurate reproduction of parametric trends seen in the data.

From preliminary component investigations (diffuser and inlet), pump performance during surge appears to be different from quasi-steady. To improve the accuracy of future dynamic system models, this effect should be accounted for by increasing the sophistication of the pump response model beyond that of a first order phase lag. This necessitates a detailed analysis of each pump component before and during surge, in order to understand its behavior and evaluate its contribution to the overall behavior of the pump.
6.2 Recommendations for Future Work

The next step in further understanding the instability mechanism(s) is a detailed, component by component investigation of the centrifugal pump described in Chapter III. Efforts in this direction have already been initiated with the qualitative diffuser and inlet data presented in this report. The large pump dimensions and its inherent openness are well suited for this type of detailed flow characterization. These features should be further exploited with continued flow visualization studies (hydrogen bubble) and localized pressure and velocity measurements.

As mentioned in Chapter IV, a modified impeller which matches more closely the original impeller geometry has been installed in the test section, and will be tested to determine its steady and unsteady behavior. Continued investigations with this more similar pump are expected to improve comparisons of unsteady behavior between the two pumps.

Lastly, it is recommended that a more concerted effort be placed on improving the analytical models presented in this report, including more sophisticated pump component performance models.
REFERENCES


Figure 5.1: Pump efficiency versus specific speed and pump size (Worthington).

Figure 9.1: Head-capacity curves for several specific speeds.

Figure 1.1: Performance characteristics and efficiency curves for various specific speed pumps.
General Pumping System Configuration

Simplified Pumping System Configuration

Mass/Spring/Damper Model

Figure 2.1: Basic pumping system and analogies (from [9]).
DYNAMIC INSTABILITY

Throttle Lines

Unstable if slope of pump characteristic is greater than slope of throttle line (Point B)

NET ENERGY INPUT

Pumping Characteristic

NET ENERGY DISSIPATION

Even if statically stable, system can be dynamically unstable (Point D)

Figure 2.2: Instability modes and criterion (from [9]).
Figure 2.3: Simple Schematic of Closed Loop Facility
Figure 2.4: Experimental data: time trace of impeller wheel speed (rpm) and non-dimensional flow coefficient during typical large B (B = 0.60) surge cycle.
Figure 2.5: Results from theory: effect of variation in wheel speedup on the system stability boundary (inception point of surge, $\Phi_{tr}$, vs. $B$ parameter).
Figure 2.6: Results from theory: effect of variations of the coefficient in the time lag time constant ($Z_{LAG}$) on the system stability boundary (inception point of surge, $\Phi_{tr}$, vs. $B$ parameter).
Figure 3.1: Sketch of loop layout.
Figure 3.2: The MIT centrifugal pump
Removable flange

Flow in Flow out

DIMENSION LARGE PLENUM SMALL PLENUM
Length (L) 50" 40"
Plenum radius (R_o) 28" 20"
Perforated aluminum radius (R_i) 9.5" 7.5"

(all dimensions in inches)

Figure 3.3: Plenum layout.
(1) Close water meter valve.

(2) Deflate air bags.

(3) Fill loop to desired level using transfer pump.

Figure 3.4: Part A: Filling procedure for plenum air bags.
Release desired
volume of water through
water meter.

Inflate discharge air bags
to resume desired water level.

Repeat process with inlet
plenum air bags.

Figure 3.5: Part B: Filling procedure for plenum air bags.
Figure 3.6: Transfer system.
Figure 3.7: Pump test section with motor.
Figure 3.8: Shaft and bearing assembly detail
Figure 3.9: Test section assembly: side view
Figure 3.10: Test section assembly: top view.
Figure 3.11: Loop schematic with instrumentation noted.
Figure 3.12: Pipe diffuser with static tap locations. Only the four furthest to the right were used for the results discussed in Chapter IV. These are numbered as referred to in the Figure 4.9.
Figure 3.13: Pipe diffuser with hydrogen bubble wire cross locations.
Figure 4.1: Experimental data: non-dimensional phase portrait of MIT pump performance characteristics. Five different speedlines are referenced by their corresponding percentage of design speed (design speed = 420 rpm). Reynolds numbers indicated for each speedline.
Figure 4.2: Experimental data: efficiency plots for MIT pump. Five different speedlines are referenced by their corresponding percentage of design speed (design speed = 420 rpm). Reynolds numbers indicated for each speedline.
Figure 4.3: Experimental data: phase portrait comparison of MIT and original pump's performance characteristics at matching Reynolds number (@4.8 E4 based on blade discharge height). Best efficiency points and Reynolds numbers noted for each plot.
Figure 4.4: Experimental data: comparison of MIT and original pump’s efficiency plots at matching Reynolds number (@4.8 E4 based on blade discharge height). Best efficiency points and Reynolds numbers noted for each plot.
Figure 4.5: Experimental data with analytical adjustments added: phase portrait comparison of MIT and original pump's performance characteristics at matching Reynolds number (@4.8 E4 based on blade discharge height). Best efficiency points and Reynolds numbers noted for each plot. Analytical adjustments to original pump's bep shown using Casey correlation [26] for differences in relative surface roughness. Two arrows show range of predicted bep adjustment given the range of surface roughness estimates used (cf. Section 4.2.3).
Figure 4.6: Experimental data: phase portrait comparison of original pump's performance characteristic at two different Reynolds numbers. Reynolds numbers noted for each plot.
Figure 4.7: Experimental data: phase portrait comparison of MIT pump's performance characteristic at two different Reynolds numbers. Reynolds numbers noted for each plot.
Figure 4.8: Experimental data with analytical adjustments added: phase portrait comparison of MIT and original pump's performance characteristics at matching Reynolds number (@4.8 E4 based on blade discharge height). Best efficiency points and Reynolds numbers noted for each plot. Analytical adjustment to original pump's bep shown using Osterwalder correlation [18] for differences in relative surface roughness. Arrow shows predicted bep adjustment given the range of surface roughness estimates used (cf. Section 4.2.3).
Figure 4.9: Experimental data: non-dimensional static pressure rise data for MIT pump at 60% speed \((\text{Re} = 9.50 \text{ E}4)\). Four diffuser ports and one port far downstream used for measuring pressure rise across pump (inlet static pressure transducer is reference). Diffuser port locations shown on Figure 3.12. Operating points for which hydrogen bubble flow visualization results are available are indicated on the characteristic by the corresponding figure numbers (Figures 4.10-4.12).
Figure 4.10: Experimental observations at $\Phi=0.135$: qualitative flow representation in the pipe diffuser at four axial locations using hydrogen bubble crosses. Regions of intermittent separation or backflow indicated. Pressure profile in pipe diffuser with static pressure from flush mounted transducers non-dimensionalized by the static pressure far downstream. Qualitative representation of expected flowfield near volute tongue.
Figure 4.11: Experimental observations at $\phi=0.065$: qualitative flow representation in the pipe diffuser at four axial locations using hydrogen bubble crosses. Regions of intermittent separation or backflow indicated. Pressure profile in pipe diffuser with static pressure from flush mounted transducers non-dimensionalized by the static pressure far downstream. Qualitative representation of expected flowfield near volute tongue.
HYDROGEN BUBBLE RESULTS

DIFFUSER INLET

DIFFUSER EXIT

Region of intermittent separation or backflow

Tongue-side wall

DIFFUSER PRESSURE FIELD

○ 60% SPEED DATA

\[ \frac{P_{\text{local}}}{P_{\text{downstream}}} \]

0.8

0.9

1.0

downstream

Port #1
Port #2
Port #3
Port #4

AXIAL LOCATION IN PIPE DIFFUSER

FLOWFIELD NEAR VOLUTE TONGUE

Figure 4.12: Experimental observations at Φ=0.035: qualitative flow representation in the pipe diffuser at four axial locations using hydrogen bubble crosses. Regions of intermittent separation or backflow indicated. Pressure profile in pipe diffuser with static pressure from flush mounted transducers non-dimensionalized by the static pressure far downstream. Qualitative representation of expected flowfield near volute tongue.
<table>
<thead>
<tr>
<th>Air content in discharge plenum (liters)</th>
<th>Speedline in percent of design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>0</td>
<td>X</td>
</tr>
<tr>
<td>30</td>
<td>X</td>
</tr>
<tr>
<td>76</td>
<td>X</td>
</tr>
<tr>
<td>114</td>
<td>X</td>
</tr>
<tr>
<td>151</td>
<td>X</td>
</tr>
<tr>
<td>189</td>
<td>X</td>
</tr>
<tr>
<td>227</td>
<td>X</td>
</tr>
<tr>
<td>265</td>
<td>X</td>
</tr>
<tr>
<td>303</td>
<td>X</td>
</tr>
</tbody>
</table>

- Air content in inlet plenum held constant at 300 liters.
- Loop pressure held constant at 0 psig.

Figure 5.1: Unsteady test matrix.
Figure 5.2: Equivalent Helmholtz resonator for simplified MIT test loop.
Figure 5.3: Experimental data: variation in reduced frequency of surge oscillation with air content of discharge plenum. Reduced frequency, defined as oscillation frequency normalized by shaft frequency, is shown here as a percentage. Experimental data for three different speedlines is shown, each referenced by its corresponding percent of design speed. Air content in discharge plenum air bags is measured with pump off. Air content of inlet plenum is constant at 300 liters (measured with pump off).
Figure 5.4: Experimental data: variation in actual frequency of surge oscillation (Hz) with air content of discharge plenum. Experimental data for three different speedlines is shown, each referenced by its corresponding percent of design speed. Air content in discharge plenum air bags has been adjusted to account for variations in volume due to operating pressures. Air content of inlet plenum is constant at 300 liters (measured with pump off).
Figure 5.5: Experimental data: variation in experimental B parameter with air content of discharge plenum. Experimental data for three different speedlines is shown, each referenced by its corresponding percent of design speed. Air content in discharge plenum air bags is measured with pump off. Air content of inlet plenum is constant at 300 liters (measured with pump off).
Figure 5.6: Experimental data: variation in surge cycle inception point ($\Phi_{tr}$) with air content of discharge plenum. Experimental data for three different speedlines is shown, each referenced by its corresponding percent of design speed. Air content in discharge plenum air bags is measured with pump off. Air content of inlet plenum is constant at 300 liters (measured with pump off).
Figure 5.7: Experimental data: stability boundary (surge cycle inception point, $\Phi_{tr}$ vs. $B$ parameter) for three different speedlines. Regions of stable and unstable pump operation noted. Each speedline is referenced by its percent of design speed.
Figure 5.8: Experimental data: phase portrait of MIT pump performance characteristic at three different speeds. Each speedline is referenced by its percent of design speed.
Figure 5.9: Experimental data: phase portrait detail of MIT pump performance characteristic at three different speeds (i.e. detail of Figure 5.8). Each speedline is referenced by its percent of design speed. Third order curve fits to each speedline also shown.
Figure 5.10: Slope of analytical curve fit to experimental data: MIT pump characteristic slope for three different speedlines. Slope calculated using $\Phi$ derivative of third order curve fit to experimental speedline detail shown in Figure 5.9. Each speedline is referenced by its percent of design speed.
Figure 5.11: Linear fits to experimental data: variation in impeller tip speed (m/s) with pump mass flow (kg/s) for three different speedlines. 80% speedline data shifted by 2.91 m/s and 60% speedline data shifted by 5.66 m/s to coincide with 100% data at \( m_p = 0 \). Each speedline referenced by its percent of design speed. Data taken during steady pump operation, with constant motor frequency setting. The range of \( m_p \) shown corresponds to traversing the entire steady state characteristic for each speed. Linear fits to each set of data also shown.
Figure 5.12: Experimental data: transient time trace of non-dimensional pressure rise and flow coefficients during throttle closing from $\Phi = 0.102$ to $\Phi = 0.089$. Data taken on 100% speedline, for a system B parameter of 0.35.
Figure 5.13: Experimental data: time trace of non-dimensional pressure rise and flow coefficients at constant throttle setting of $\Phi = 0.046$ (presurge). Data taken on 100% speedline, for a system B parameter of 0.60.
Figure 5.14: Experimental data: time trace of non-dimensional pressure rise and flow coefficients at constant throttle setting of $\Phi_{\text{avg}} = 0.044$ (surge cycle inception point, $\Phi_{\text{tr}}$). Data taken on 100% speedline, for a system B parameter of 0.60, and after transients have disappeared.
Figure 5.15: Experimental data: time trace of non-dimensional pressure rise and flow coefficients at constant throttle setting of $\Phi_{avg} = 0.025$ (maximum surge cycle amplitude). Data taken on 100% speedline, for a system B parameter of 0.60, and after throttling transients have disappeared. Note flow reversal in $\Phi$ trace.
Figure 5.16: Experimental data: time trace of non-dimensional pressure rise and flow coefficients at constant throttle setting of $\Phi_{\text{avg}} = 0.000$ (shut off surge cycle). Data taken on 100% speedline, for a system B parameter of 0.60, and after throttling transients have disappeared. Note flow reversal in $\Phi$ trace.
Figure 5.17: Experimental data: transient time trace of non-dimensional pressure rise and flow coefficients at two consecutive throttle settings of $\Phi_{avg} = 0.013$ and $\Phi_{avg} = 0.003$.

Data taken on 100% speedline, for a system B parameter of 0.35. Note surge cycle amplitude decay when system is throttled below the low flow transition point ($\Phi_{tr2}$). $\Phi_{tr2}$ is roughly 0.007 for this B.
Figure 5.18: Experimental data: transient time trace of non-dimensional pressure rise and flow coefficients at two consecutive throttle settings of $\Phi_{avg} = 0.020$ and $\Phi_{avg} = 0.014$. Data taken on 100% speedline, for a system B parameter of 0.26 (less than $B_{threshold}$). Note absence of constant amplitude oscillation (limit cycle behavior), even on expanded $\Phi$ scale.
Figure 5.19: Experimental data: transient time trace of non-dimensional pressure rise and flow coefficients at two consecutive throttle settings of $\Phi = 0.012$ and $\Phi = 0.018$. Data taken on 100% speedline, for a system B parameter of 0.08 (much less than $B_{\text{threshold}}$). Note absence of constant amplitude oscillation (limit cycle behavior), even on expanded $\Phi$ scale.
Figure 5.20: Experimental data: surge cycle amplitude variation with pump operating point (Φ). Peak to peak amplitude shown as percentage of Φbep. Data for 100% speedline, with system B parameter of 0.60.
Figure 5.21: Experimental data and analytical fits to experimental data: three plots showing the relative positions of the points of maximum surge cycle amplitude, and maximum characteristic slope vs. the pump performance characteristic. The topmost plot is from a fifth order curve fit to the 100% experimental speedline shown in Figure 5.8. The middle plot is the $\Phi$ derivative of the topmost plot. The lowest plot is simply Figure 5.20, showing surge cycle amplitude variation with $\Phi$ for system B parameter of 0.60.
Figure 5.22: Experimental data: variations in maximum surge cycle amplitude with $B$ parameter for three different speedlines. Each speedline referenced by its percent of design speed. Maximum amplitude determined by throttling over the full range of unstable operating points for each speedline, and for each $B$ parameter setting. Peak to peak amplitude shown as percent of $\Phi_{bep}$. 
Figure 5.23: Experimental data: variation in reduced frequency of surge oscillation with operating point ($\phi$). Data shown for 100% speedline and $B = 0.60$. Reduced frequency, defined as surge frequency normalized by shaft frequency, is shown as a percentage.
Figure 5.24: Experimental data and theoretical results: comparison of predicted (theoretical) stability boundaries for various values of $Z_{\text{LAG}}$ and the experimental stability boundary for the 100% speedline data. $Z_{\text{LAG}}$ is the coefficient to the time constant in the pump pressure rise lag. Regions of stable and unstable pump operation are indicated. The stability boundary is the surge inception point, $\Phi_{\text{tr}}$, plotted vs. the associated system B parameter.
Figure 5.25: Experimental data and theoretical results: predicted (theoretical) and experimental reduced frequencies of surge oscillation plotted vs. the air content of the discharge plenum. Theoretical and experimental frequencies shown for three different speedlines, referenced by percent of design speed. The reduced frequency (surge frequency normalized by shaft frequency) is shown as a percentage. Air content in discharge plenum shown as measured with the pump off. Air content in inlet plenum held constant at 300 liters (measured with pump off).
Figure 5.26: Experimental data and theoretical results: predicted (theoretical) and experimental B parameters plotted vs. the air content of the discharge plenum. Theoretical and experimental B shown for three different speedlines, referenced by percent of design speed. Air content in discharge plenum shown as measured with the pump off. Air content in inlet plenum held constant at 300 liters (measured with pump off).
Figure 5.27: Experimental data and theoretical results: predicted (theoretical) and experimental surge inception points ($\Phi_{tr}$) plotted vs. the air content of the discharge plenum. Theoretical and experimental $\Phi_{tr}$ shown for three different speedlines, referenced by percent of design speed. Air content in discharge plenum shown as measured with the pump off. Air content in inlet plenum held constant at 300 liters (measured with pump off).
Figure 5.28: Experimental data and theoretical results: comparison of predicted (theoretical) stability boundaries and experimental stability boundaries for three different speedlines. Each speedline is referenced by its percent of design speed. Regions of stable and unstable pump operation are indicated. The stability boundary is the surge inception point, $\Phi_{tr}$, plotted vs. the associated system B parameter.
Figure 5.29: Analytical curve fits to experimental data: third order curve fits to a detail of the MIT pump performance characteristic. Two speedlines shown, referenced by their percent of design speed.
Figure 5.30: Theoretical results: comparison of predicted (theoretical) stability boundaries using two different performance characteristic curve fits (see Figure 5.29), everything else being the same. Each stability boundary is referenced by the curve fit used, shown as a percent of design speed. Regions of stable and unstable pump operation are indicated. The stability boundary is the surge inception point, $\Phi_{tr}$, plotted vs. the associated system B parameter.
Figure 5.31: Theoretical results: comparison of predicted (theoretical) stability boundaries using three different wheel speedup slopes, $\frac{dU_{TIP}}{dm_p}$ (see Figure 5.11), everything else being the same. Each stability boundary is referenced by the experimental speedline corresponding to the wheel speedup slope used, shown as a percent of design speed. Regions of stable and unstable pump operation are indicated. The stability boundary is the surge inception point, $\Phi_{tr}$, plotted vs. the associated system B parameter.
Figure 5.32: Experimental data and theoretical results: variation of predicted (theoretical) and experimental reduced frequencies with average operating point during surge, $\Phi_{avg}$. Data shown for 100% speedline and system B parameter of 0.60. Reduced frequency (surge frequency normalized by shaft frequency) shown as a percentage.
Figure 5.33: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{\text{avg}} = \Phi_{\text{tr}}$, the surge cycle inception point ($\Phi_{\text{tr}} = 0.041$ and 0.046 for experiment and theory respectively). Data is for 100% speedline and experimental system B parameter of 0.60. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.34: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{\text{avg}} = 0.032$. Data is for 100% speedline and experimental system B parameter of 0.60. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.35: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{\text{avg}} = 0.025$. Data is for 100% speedline and experimental system B parameter of 0.60. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.36: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{\text{avg}} = 0.017$. Data is for 100% speedline and experimental system B parameter of 0.60. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.37: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{\text{avg}} = 0.012$. Data is for 100% speedline and experimental system B parameter of 0.60. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.38: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at shut off, $\Phi_{\text{avg}} = 0.000$. Data is for 100% speedline and experimental system B parameter of 0.60. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.39: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{avg} = \Phi_{ir}$, the surge cycle inception point ($\Phi_{ir} = 0.025$ and 0.032 for experiment and theory respectively). Data is for 100% speedline and experimental system B parameter of 0.33. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.40: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{\text{avg}} = 0.022$. Data is for 100% speedline and experimental system B parameter of 0.33. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.41: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{avg} = 0.017$. Data is for 100% speedline and experimental system B parameter of 0.33. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.42: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{avg} = 0.012$. Data is for 100% speedline and experimental system B parameter of 0.33. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.43: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{avg} = 0.032$. Data is for 100% speedline and experimental system B parameter of 0.60. Theoretical surge cycle predictions obtained with and without the first order lag in pump pressure rise ($Z_{LAG} = 0.03$ and 0.00 respectively) are represented. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
Figure 5.44: Experimental data and theoretical results: phase portraits of experimental and theoretical surge cycles at $\Phi_{\text{avg}} = 0.022$. Data is for 100% speedline and experimental system B parameter of 0.33. Theoretical surge cycle predictions obtained with and without the first order lag in pump pressure rise ($Z_{\text{LAG}} = 0.03$ and 0.00 respectively) are represented. The fifth order curve fit to the 100% experimental steady state speedline is shown in the background.
APPENDIX A

LINEAR MODEL CODE

WITH SAMPLE OUTPUT

AND

TIME-RESOLVED SIMULATION EQUATIONS
Dynamic simulation of the MIT pump loop (see J. Bons' or N. Goulet's thesis for schematic).

The MIT test facility is a closed loop consisting of 2 legs, one inlet plenum (#2), and one discharge plenum (#1). The four equations which define this system are: conservation of momentum in the pump and throttle legs, and conservation of mass in the two tanks (plenums). A fifth equation representing a time lag in pump pressure rise has been used as well. All five equations have been linearized by assuming only small perturbations about a known operating point.

This program determines the instability point and frequency of unstable oscillations by calculating the system eigenvalues. It uses inputs corresponding to predetermined experimental conditions. The operating range from $\Phi = 0.001$ to $0.07$ is scanned.

The original code was developed by N. Goulet and is presented in his Master's thesis. His program allowed for the use of multiple plenums. In its present state, the code assumes only two plenums. Other modifications have been implemented as follows:

Version 10: This version of the linear analysis code accounts for variations in impeller wheelspeed with mass flow. Also, the actual b2 of 0.01189 m is used instead of the design value (0.01438 m). Finally, the pressure and volume adjustments of the air bags which occur when the pump is turned on are accounted for in the proper manner (see thesis, J. Bons).

--- J. Bons, 23 April 1990

PROGRAM DATA ACQUISITION

The number of legs is set to $N = 2$, and the following parameters are defined:

- $\text{DYN}(i)$, this is the dynamic head loss coefficient for each leg.
- $\text{ZLAG}$, this is the correction term to the pressure lag time constant.
- $\text{VCOM}(i)$, this is the air volume in each plenum, measured in cubic meters.
- $\text{PCOM}(i)$, this is the pressure of the air volumes in each plenum, measured in Pascals.
- $\text{AREA}(i)$, this is the reference area used for each leg (m).
- $\text{LEN}(i)$, this is the lumped inertial length of each leg (m).

```
PARAMETER (NMAX=2,NMAX2=5,GAMMA=1.4)
PARAMETER (R=1000,PI=3.14159,D2=.6096)
INTEGER N,Z,NSPDLN,NFILE
REAL PSLOPE,TSLOPE,SPEED,PHI,PSI,INC,CSPD,ZLAG,DLAG
REAL AREA(NMAX),VCOM(NMAX),PCOM(NMAX),LEN(NMAX),DYN(NMAX)
REAL MATRIX(NMAX2,NMAX2),WR(NMAX2),WI(NMAX2),PDAMP(NMAX)
REAL B(NMAX2),FREQ(NMAX2),UPAR(NMAX2),ZAPR(NMAX2),ZAPI(NMAX2)
REAL REDFREQ(NMAX2),SHFREQ,PSMI,PLGI,VSMI,VLGI
CHARACTER*1 CONF
CHARACTER*8 DATFIL
COMMON LEN(NMAX),AREA(NMAX),VCOM(NMAX),PCOM(NMAX),MATRIX(NMAX2,NMAX2),
// DYN(NMAX),PDAMP(NMAX),PSMI,PLGI,VSMI,VLGI

2 N = 2
DYN(1) = 6
```
DYN(2) = 1.5
ZLAG = 0.03
AREA(2) = .0290
LEN(2) = 3.277
AREA(1) = .0572
LEN(1) = 41.92

* These air volumes and pressures are as measured with the pump off. They will be adjusted later for the operating point of interest. #1 is the small plenum and #2 is the large.

3 VCOM(2) = .300
PCOM(1) = 1.427E5
PCOM(2) = 1.389E5
VCOM(2) = VCOM(2)*1000

* The program can be run without accounting for variations in impeller wheelspeed. If the constant speed value (CSPD) is set to zero, then the program assumes the speed does vary with massflow (for a given speedline). This is prompted for.

CSPD = 0.0

* If a data file is desired, set NFILE = 1. Important data is spooled to the screen so this is not always necessary. NFILE = 0 will bypass the datafile creation.

NFILE = 1

Experimental curve fits for three different speedlines are available in the present version.

WRITE(9,*)'ENTER THE TYPE OF SPEEDLINE BEING TESTED'
WRITE(9,*)'100% = 1, 80% = 2, 60% = 3'
READ(9,*)NSPDLN
WRITE(9,*)'VOLUMES AS MEASURED WITH PUMP OFF'
WRITE(9,*)'AIR VOLUME OF LARGE PLENUM (liters): ',VCOM(2)
WRITE(9,*)'WHEEL SPEED IS A VARIABLE'
WRITE(9,*)'ZLAG = ',ZLAG
WRITE(9,*)'DO YOU WISH TO CHANGE ANY OF THESE (Y/N),'
READ (9,1011) CONF
IF (CONF='N') GO TO 7
WRITE (9,*)'ENTER PHASE LAG FACTOR, ZLAG'
READ (9,*)ZLAG
WRITE(9,*)'ANY FURTHER CHANGES?'
READ (9,1011) CONF
IF (CONF='N') GO TO 7
WRITE (9,*)'ENTER LG PLENUM VOLUME (liters):'
READ (9,*)VCOM(2)
WRITE (9,*)'ENTER SMALL PLENUM PRESSURE W PUMP OFF (Pa)'
READ(9,*) PCOM(1)
WRITE (9,*) 'ENTER LARGE PLENUM PRESSURE W PUMP OFF (Pa)'
READ(9,*) PCOM(2)
WRITE (9,*) 'ENTER CONSTANT WHEEL SPEED VALUE (m/s), 0=VARIABLE'
READ(9,*) CSPD

* Enter volume for pump leg (measured with pump off). Experimentally
* this is one of the primary loop variables (wheelspeed being
* the other).

WRITE (9,*) 'ENTER SM PLENUM VOLUME (liters):'
READ (9,*) VCOM(1)
VCOM(1) = VCOM(1)/1000.0
VCOM(2) = VCOM(2)/1000.0

* The pump leg (NP) is #1 and the throttle leg is #2.

NP = 1
NT = 2

* A datafile can be generated containing the important program
* output for each run.

WRITE(9,*) 'DO YOU WANT A DATA FILE CREATED? (Y/N)'
READ (9,1011) CONF
IF (CONF='N') THEN
  NFILE = 0
  GOTO 10
ENDIF
WRITE(9,*)'ENTER FILE NAME FOR RESULTS'
READ(9,'(A8)') DATFIL
OPEN(UNIT=6,FILE=DATFIL,STATUS='NEW')
CALL HEADER (N,NSPDLN,NP,NT,ZLAG)
WRITE (6,*)
WRITE (6,1028)

* TREATMENT
* The loop, #600, determines the system eigenvalues over the interval
* 0.001 to 0.07, stepping by 0.001. At each new operating point,
* the subroutine SLOPE is called to calculate the necessary phi-
* dependent parameters: Utip, psi, and dpsi/dphi. Then the
* subroutine STIFMATRIX assembles the 5x5 matrix elements.

WRITE(9,*) 'COMPUTING EIGENVALUES'
DO 600 PHI=0.001,0.07,0.001
CALL SLOPE(PHI,PSI,PSLOPE,TSLOPE,SPEED,NSPDLN,CSPD,DLAG)
CALL STIFMATRIX(N,NSPDLN,PSLOPE,TSLOPE,SPEED,PHI,DLAG,ZLAG)

*
The matrix is reduced to upper Hessenberg form using the subroutine ELMHES. Then the system eigenvalues are extracted by employing a QR algorithm (subroutine HQR).

N=NMAX2
CALL ELMHES(MATRIX,N,NMAX2)
CALL HQR(MATRIX,N,NMAX2,WR,WI)
N=2

Now the program calculates the frequency of oscillation and the associated B parameter and spools the results to the screen (and to a datafile, if so requested). REDFREQ is the reduced frequency of the oscillation (normalized by the shaft frequency).

DO 12 I=1,NMAX2
   IF (WI(I).NE.0) THEN
      SHFREQ = SPEED/(PI*D2)
      FREQ(I)=ABS(WI(I)/(2*PI))
      REDFREQ(I) = (FREQ(I)/SHFREQ)*100.0
      B(I)=SPEED/(2*FREQ(I)**2*PSLOPE*TSLOPE)
      ELSE
         B(I)=0
         REDFREQ(I) =0
         FREQ(I)=0
         UPAR(I)=0
   ENDIF
12 CONTINUE

DO 15 Z=I1,NMAX2
   ZAPI(Z)=0
   ZAPR(Z)=0
15 CONTINUE

The program ignores the following eigenvalues:
1) Those which have very small (<1E-8) real parts.
2) Those which are equal to other eigenvalues.
3) Those which have imaginary parts = 0.
4) Those with negative real parts.

The others are spooled to the screen, along with the air volumes, pressures, and wheelspeed at each operating point.

DO 13 I=1,NMAX2
   IF (((WI(I).EQ.0).AND.(WR(I).EQ.0)).OR.
      (ABS(WR(I)).LE.1E-8)) GOTO 13
   DO 16 J=1,Z
      IF (((ABS(WI(I)).EQ.ABS(ZAPI(J)))).
         .AND.(WR(I).EQ.ZAPR(J))) GOTO 13
16 CONTINUE
   IF (WI(I).EQ.0) GOTO13
   IF (WR(I).GT.0) THEN
WRITE (9,1026) PHI, PSI, WI(I), WR(I)
/, REDFREQ(I), B(I), UPAR(I)
WRITE (9,1040) VCOM(1), PCOM(1), VCOM(2)
/, PCOM(2), SPEED

ENDIF
IF (NFILE.EQ.1) THEN
WRITE (6,1026) PHI, PSI, WI(I), WR(I), REDFREQ(I)
/, B(I), UPAR(I)
ENDIF
Z = Z + 1
ZAPI(Z) = WI(I)
ZAPR(Z) = WR(I)

CONTINUE

ZAPI(Z) = WI(I)
ZAPR(Z) = WR(I)

CONTINUE

******************************************************************************
*                                                                        *
* The program can now be terminated or continued with new initial          *
* values.                                                                *
*                                                                        *
******************************************************************************

WRITE (9,1023)
READ (9,1011) CONF
IF (CONF = 'N') GO TO 3

*****************************************************************************
*
* FORMAT statements and END
*
*****************************************************************************

1011 FORMAT (A1)
1022 FORMAT (I2,5X,1PE10.3,4X,1PE10.3,5X,1PE10.3,5X
/, 1PE10.3,5X,1PE10.3)
1023 FORMAT ('DO YOU WANT TO END (Y/N)?: ')
1026 FORMAT (F4.3,X,F4.3,X,1PE10.3,X,1PE10.3,X,1PE10.3
/, X,1PE10.3,X,1PE10.3)
1028 FORMAT (' PHI PSI WI WR
/, REDFREQ B PAR UPAR ')
1040 FORMAT (2X,F5.3,1X,F8.1,1X,F5.3,1X,F8.1,1X,F5.2)

200 END

*****************************************************************************
*
* SUBROUTINES
*
*****************************************************************************
*
* The first subroutine, HEADER, writes the header for the present          *
* configuration in a datafile.                                            *
*
*****************************************************************************

SUBROUTINE HEADER (NNSPDNL, NP, NT, ZLAG)
PARAMETER (NMAX=2, NMAX2=5)
REAL AREA(NMAX), VCOM(NMAX), PCOM(NMAX), ZLAG
REAL MATRIX(NMAX, NMAX), DYN(NMAX)
COMMON LEN(NMAX), AREA(NMAX), VCOM(NMAX),
/, PCOM(NMAX), MATRIX(NMAX2, NMAX2),
/, DYN(NMAX), PDAMP(NMAX)
WRITE (6,2003)
WRITE (6,3002) VCOM(2),PCOM(2),ZLAG
WRITE (6,2005)

WRITE (6,3004) 1,LEN(1),AREA(1),DYN(1),
VCOM(1),PCOM(1)
WRITE (6,3005) 2,LEN(2),AREA(2),DYN(2)
WRITE (6,2006)
IF (NSPDLN.EQ.1) THEN
   NSPD = 100
ELSEIF (NSPDLN.EQ.2) THEN
   NSPD = 80
ELSE
   NSPD = 60
ENDIF
WRITE (6,3006) NSPD,NP,NT
RETURN
END

* The subroutine SLOPE calculates the steady state operating point
* and the various damping coefficients. The pump and throttle slopes
* are also computed.
*
*****************************************************************************

SUBROUTINE SLOPE(PHI,PSI,PSLOPE,TSLOPE,SPEED,NSPDLN,CSPD,
DLAG)
INTEGER NSPDLN
REAL PHI,PSI,PSLOPE,TSLOPE,SPEED,DUDM,CSPD
PARAMETER (NMAX=2,NMAX2=5,RO= 1000,PI=3.1415926)
PARAMETER (D2=0.6096,B2=1.189E-2)
REAL LEN(NMAX),AREA(NMAX),DYN(NMAX),PDAMP(NMAX),DLAG
COMMON LEN(NMAX),AREA(NMAX),VCOM(NMAX),
PCOM(NMAX),MATRIX(NMAX2,NMAX2),
DYN(NMAX),PDAMP(NMAX)
*****************************************************************************

* Each speedline has a corresponding equation for the pump
* performance characteristic. The pump wheelspeed and speed
* variation slope with mass flow (dU/dm), as well as the local
* pump characteristic slope (dpsi/dphi) are also determined.
*
IF (NSPDLN.EQ.1) THEN
  PSI = 0.53972+1.5835*PHI-19.715*PHI**2+115.53*PHI**3
  C PSI=0.54289+0.93162*PHI+11.034*PHI**2-349.85*PHI**3
  C +2332.7*PHI**4-5412.9*PHI**5
  IF (CSPD.EQ.0) THEN
    SPEED = 13.812-3.4823*PHI+1.2017*PHI**2
    DUDM = -1.116E-2
    ELSE
    SPEED = CSPD
    DUDM = 0.0
    ENDIF
  PSLOPE = 1.5835-2*19.715*PHI+3*115.53*PHI**2
  C PSLOPE = 0.93162+2*1.034*PHI-3*349.85*PHI**2
  C +4*2332.7*PHI**3-5*5412.9*PHI**4
ELSEIF (NSPDLN.EQ.2) THEN
  PSI = 0.54045+1.4774*PHI-15.218*PHI**2+42.775*PHI**3
  C PSI = 0.54291+1.0539*PHI+1.2051*PHI**2-169.4*PHI**3
  C +1021.2*PHI**4-2056.4*PHI**5
  IF (CSPD.EQ.0) THEN
    SPEED = 10.876-0.80951*PHI-6.6898*PHI**2
    DUDM = -8.037E-3
    ELSE
    SPEED = CSPD
    DUDM = 0.0
    ENDIF
  PSLOPE = 1.4774-2*15.218*PHI+3*42.775*PHI**2
  C PSLOPE = 1.0539+2*1.2051*PHI-3*169.4*PHI**2
  C +4*1021.2*PHI**3-5*2056.4*PHI**4
ELSE
  PSI = 0.53906+1.2090*PHI-5.7289*PHI**2-33.151*PHI**3
  C PSI = 0.5389+1.1659*PHI+1.3398*PHI**2-237.67*PHI**3
  C +1848*PHI**4-4760.9*PHI**5
  IF (CSPD.EQ.0) THEN
    SPEED = 8.1296-0.45335*PHI-4.3236*PHI**2
    DUDM = -6.38E-3
    ELSE
    SPEED = CSPD
    DUDM = 0.0
    ENDIF
  PSLOPE = 1.2090-2*5.7289*PHI+3*33.151*PHI**2
  C PSLOPE = 1.1659+2*1.3398*PHI-3*237.67*PHI**2
  C +4*1848*PHI**3-5*4760.9*PHI**4
ENDIF

The pump exit and inlet areas are used to convert from the total
pressure rise of the pump characteristic to the static pressure
rise required by the linearized equations. Also the local throttle
characteristic slope is calculated by equating the pressure loss in
the throttle and the piping legs to the pressure rise in the pump.
Differentiating this with respect to mass flow, gives the
local throttle curve slope, PDAMP(2), which is derived in
two steps: first TSLOPE then PDAMP(1).

AEX = AREA(2)
AIN = AREA(1)
TSLOPE = 2*SPEED*PSI/(PI*D2*B2*PHI)-PHI*PI*D2*B2*SPEED*
PDAMP(1) = DYN(1)*(PHI/AREA(1)**2)*SPEED*PI*D2*B2
PDAMP(2) = PDAMP(1)-TSLOPE

* The nondimensional steady state pump slope is first dimensionalyzed
and then corrected for wheelspeed variations (DUDM). The dynamic
pressure rise is also subtracted, the result being DLAG.

DLAG = (PSLOPE*SPEED/(PI*D2*B2)+PSI*2*RO
*SPEED*DUDM-PI*D2*B2*SPEED*PHI*(1/AEX**2-
1/AIN**2))

RETURN
END

* The subroutine, STIFMATRIX, calculates the system matrix for
* the subsequent eigenvalue extraction.

SUBROUTINE STIFMATRIX(N,NSPD,NSPDLN,PSLOPE,TSLOPE,SPEED,PHI,DLAG,ZLAG)
PARAMETER (NMAX=2,NMAX2=5,GAMMA=1.4,RO=1000,PI=3.14159,
EPS=0.001)
INTEGER NSPD
REAL PSMIPLGI,VLGI,G M,LENLG,RLG,HIN
REAL NUMB,VGSS,DVOL,VSF,VSFM,VLGF,PLGF
REAL NUMER,DEM1,DEM2,DENTOT,VRES,DVRES,DPIN
REAL VSMI,ALGF,LSMT,RISM,ASMF
REAL AREA(NMAX),VCOM(NMAX),PCOM(NMAX),LEN(NMAX)
REAL MATRIX(NMAX2,NMAX2),PDAMP(NMAX),PHI,DLAG,ZLAG
COMMON LEN(NMAX),AREA(NMAX),VCOM(NMAX),
PCOM(NMAX),MATRIX(NMAX2,NMAX2),DYN(NMAX),PDAMP(NMAX),PSMI,PLGI,VSMI,VLGI

DO 6 I=1,NMAX2
   DO 7 J=1,NMAX2
      MATRIX(I,J)=0
   6 CONTINUE
   7 CONTINUE

* The plenum air volumes and pressures (as input earlier) are
* measured with the pump off. At the operating point of interest,
* the air volumes adjust to equalize the pressure drops across each
* air-water interface. So the equilibrium values of air pressure and
* volume are calculated at each new operating point in the manner
* outlined below (see J. Bons thesis).

IF (PHIEQ.0.001) THEN
   PSMI = PCOM(1)
   PLGI = PCOM(2)
   VSMI = VCOM(1)
   VLGI = VCOM(2)
ENDIF
The added pressure of the water in each plenum (from pump off to current operating point) is determined from empirical fits to experimental data (as a function of speedline, of course).

```
IF (NSPDLN.EQ.1) THEN
    DPEX = 90631+2.3286E5*PHI-2.4107E6*PHI**2
    / +2.8722E6*PHI**3
    DPIN = -11408-18329*PHI+2.1705E5*PHI**2
    / -1.5318E5*PHI**3
ELSEIF (NSPDLN.EQ.2) THEN
    DPEX = 62190+1.6366E5*PHI-1.6497E6*PHI**2
    / +1.961E6*PHI**3
    DPIN = -1072.9-3309.8*PHI+24825*PHI**2
    / -4568.8*PHI**3
ELSE
    DPEX = 34494+89893*PHI-8.5243E5*PHI**2
    / +7.7986E5*PHI**3
    DPIN = -588.01-2099.9*PHI+22133*PHI**2
    / -36778*PHI**3
ENDIF
```

When the large plenum air volume is 300 liters, the surface area of the air water interface has been computed to be 1.276 square meters. The small plenum air volume interface area is computed for the initial volume specified (see J. Bons' thesis for complete derivation). RISM and LSMT are relevant length scales for the small plenum air volume.

```
ALGF = 1.276
RISM = 0.19
LSMT = 1.016
ASMF = 4*PI*RISM*(VSMI/(LSMT*2*PI*RISM) + LSMT)
```

If VLGI is not 300 liters, we need to calculate the new corresponding surface area of the air water interface in the inlet plenum. This is done with the following iteration.

```
IF (VLGI.EQ.0.300) GOTO 35
LENLG = 1.27
RLG = 0.55
HIN = 0.2239
30 NUMB = SQRT(RLG**2-HIN**2)
    VGSS = LENLG*(RLG**2*ASIN(NUMB/RLG)-HIN*NUMB)
    DVOL = ABS(VGSS-VLGI)
    IF (DVOL.LT.EPS) GOTO 32
    HIN = HIN*(VGSS/VLGI)**2
    GOTO 30
32 ALGF = 2*LENLG*NUMB
```

*
The program now iterates to find the final volumes in both
the small and large plenums at the new operating point
(first the small then the large). To simplify the iteration, the
surface areas are assumed constant (at their value with the pump
off). The initial guess for the large plenum air volume at the
operating point in question is the initial volume.

***************************************************************************

35 VGSS = VLGI
40 NUMB = VSMI/(((DPEX+PSMI)/PSMI)-ALGF*(DPIN+PLGI-
PLGI*(VLGI/VGSS)**GAMMA)/(ASMF*PSMI))**(1/GAMMA)
VRES = VLGI + VSMI - NUMB
DVRES = ABS(VRES-VGSS)
IF (DVRES.LT.EPS) GOTO 50
VGSS = VGSS - 0.5*(VGSS-VRES)
GOTO 40
50 VCOM(2) = VRES
VCOM(1) = VLGI + VSMI - VCOM(2)
PCOM(1) = PSMI*(VSMI/VCOM(1))**GAMMA
PCOM(2) = PLGI*(VLGI/VCOM(2))**GAMMA
***************************************************************************

These volumes and pressures, along with information from the
other parts of the program, are now used to construct the system
matrix. The non-zero entries are filled below.

***************************************************************************

MATRIX(1,1) = -PDAMP(1)*AREA(1)/LEN(1)
MATRIX(1,3) = AREA(1)/LEN(1)
MATRIX(1,4) = -AREA(1)/LEN(1)
MATRIX(1,5) = AREA(1)/LEN(1)
MATRIX(2,2) = PDAMP(2)*AREA(2)/LEN(2)
MATRIX(2,3) = -AREA(2)/LEN(2)
MATRIX(2,4) = AREA(2)/LEN(2)
MATRIX(3,1) = -(GAMMA*PCOM(1))/(RO*VCOM(1))
MATRIX(3,2) = -MATRIX(3,1)
MATRIX(4,1) = (GAMMA*PCOM(2))/(RO*VCOM(2))
MATRIX(4,2) = -MATRIX(4,1)
MATRIX(5,1) = DLAG*PHI*SPEED/(ZLAG*2.3)
MATRIX(5,5) = -PHI*SPEED/(ZLAG*2.3)
RETURN
END
***************************************************************************

This subroutine, ELMHES, reduces the system matrix to Upper
Hessenberg form. The algorithm shown was taken from a standard
numerical recipes text.

***************************************************************************

SUBROUTINE ELMHES(A,N,NP)
DIMENSION A(NP,NP)
IF (N.GT.2) THEN
DO 17 M=2,N-1
   X=0
   I=M
   DO 11 J=M,N
      IF (ABS(A(J,M-1)).GT.ABS(X)) THEN
         X=A(J,M-1)
         I=J
      ENDIF
      CONTINUE
   IF (I.NE.M) THEN
      DO 12 J=M-1,N
         Y=A(IJ)
         A(IJ)=A(MJ)
         A(MJ)=Y
      CONTINUE
      DO 13 J=1,N
         Y=A(IJ)
         A(IJ)=A(JM)
         A(JM)=Y
      CONTINUE
      ENDIF
      IF (X.NE.0) THEN
         DO 16 I=M+1,N
            Y=A(I,M-1)
            IF (Y.NE.0) THEN
               Y=Y/X
               A(I,M-1)=Y
               DO 14 J=M,N
                  A(IJ)=A(IJ)-Y*A(MJ)
               CONTINUE
               DO 15 J=1,N
                  A(J,M)=A(J,M)+Y*A(J,I)
               CONTINUE
            ENDIF
         CONTINUE
      ENDIF
   CONTINUE
17 CONTINUE
END

*********************************************************
****************************************************************
*  This subroutine, HQR, extracts the system eigenvalues from the  *
*  upper hessenberg matrix created above. This algorithm is also   *
*  from a standard numerical recipes text.                        *
****************************************************************
*********************************************************
SUBROUTINE HQR(A,NP,NP,WR,NI)
DIMENSION A(NP,NP),WR(NP),NI(NP)
ANORM=ABS(A(1,1))
DO 12 I=2,N
   DO 11 J=I-1,N
      ANORM=ANORM+ABS(A(IJ))
   CONTINUE
11 CONTINUE
12 CONTINUE
NN=N
T=0
166
1 IF (NN.GE.1) THEN
   ITS=0
2   DO 13 L=NN,2,-1
      S=ABS(A(L-1,L-1))+ABS(A(L,L))
      IF (S.EQ.0.) S=ANORM
      IF (ABS(A(L,L-1))+S.EQ.S) GO TO 3
13   CONTINUE
   L=1
3   X=A(NN,NN)
   IF (L.EQ.NN) THEN
      WR(NN)=X+T
      WI(NN)=0
      NN=NN-1
   ELSE
      Y=A(NN-1,NN-1)
      W=A(NN,NN-1)*A(NN-1,NN)
      IF (L.EQ.NN-1) THEN
         P=0.5*(Y-X)
         Q=P**2+W
         Z=SQRT(ABS(Q))
         X=X+T
         IF (Q.GE.0.) THEN
            Z=P+SIGN(Z,P)
            WR(NN)=X+Z
            WR(NN-1)=WR(NN)
            IF (Z.NE.0.) WR(NN)=X-W/Z
            WI(NN)=Z
            WI(NN-1)=-Z
            ELSE
               WR(NN)=X+P
               WR(NN-1)=WR(NN)
               WI(NN)=Z
               WI(NN-1)=-Z
            ENDIF
            NN=NN-2
      ELSE
         IF(ITS.EQ.30)PAUSE 'Too many its.'
         IF(ITS.EQ.10.OR.ITS.EQ.20)THEN
            T=T+X
            DO 14 I=1,NN
               A(I,I)=A(I,I)-X
14         CONTINUE
            S=ABS(A(NN,NN-1))+ABS(A(NN-1,NN-2))
            X=0.75*S
            Y=X
            W=-0.4375*S**2
            ELSE
               P=(R*S-W)/A(M+1,M)+A(M,M+1)
               Q=A(M+1,M+1)-Z-R-S
               R=A(M+2,M+1)
               S=ABS(P)+ABS(Q)+ABS(R)
               P=P/S
               Q=Q/S
            ENDIF
            ITS=ITS+1
            DO 15 M=NN-2,L,-1
               Z=A(M,M)
               R=X-Z
               S=Y-Z
               P=(R*S-W)/A(M+1,M)+A(M,M+1)
               Q=A(M+1,M+1)-Z-R-S
               R=A(M+2,M+1)
               S=ABS(P)+ABS(Q)+ABS(R)
               P=P/S
               Q=Q/S
15          CONTINUE
   ENDIF
   NN=NN-2
ELSE
   IF(ITS.EQ.30)PAUSE 'Too many its.'
   IF(ITS.EQ.10.OR.ITS.EQ.20)THEN
      T=T+X
      DO 14 I=1,NN
         A(I,I)=A(I,I)-X
14   CONTINUE
      S=ABS(A(NN,NN-1))+ABS(A(NN-1,NN-2))
      X=0.75*S
      Y=X
      W=-0.4375*S**2
   ELSE
      P=(R*S-W)/A(M+1,M)+A(M,M+1)
      Q=A(M+1,M+1)-Z-R-S
      R=A(M+2,M+1)
      S=ABS(P)+ABS(Q)+ABS(R)
      P=P/S
      Q=Q/S
   ENDIF
   ITS=ITS+1
   DO 15 M=NN-2,L,-1
      Z=A(M,M)
      R=X-Z
      S=Y-Z
      P=(R*S-W)/A(M+1,M)+A(M,M+1)
      Q=A(M+1,M+1)-Z-R-S
      R=A(M+2,M+1)
      S=ABS(P)+ABS(Q)+ABS(R)
      P=P/S
      Q=Q/S
15 CONTINUE
R=R/S

IF (M.EQ.L) GO TO 4

U=ABS(A(M,M-1))*ABS(Q)+ABS(R)
V=ABS(P)*ABS(A(M-1,M-1))+ABS(Z)

IF (U+V.EQ.V) GO TO 4

CONTINUE

DO 16 I=M+2,NN
A(I,I-2)=0
IF (I.NE.M+2) A(I,I-3)=0

16 CONTINUE

DO 19 K=M,NN-1
IF (K.NE.M) THEN
P=A(K,K-1)
Q=A(K+1,K-1)
R=0
IF (K.NE.NN-1) R=A(K+2,K-1)
X=ABS(P)+ABS(Q)+ABS(R)
IF (X.NE.0) THEN
P=P/X
Q=Q/X
R=R/X
ENDIF
S=SIGN(SQRT(P**2+Q**2+R**2),P)
IF (S.NE.0) THEN
BALA(K,K-1)=
ELSE
A(K,K-1)=S*X
ENDIF
P=P+S
X=P/S
Y=Q/S
Z=R/S
Q=Q/P
R=R/P
DO 17 J=K,NN
P=A(K,J)+Q*A(K+1,J)
IF (K.NE.NN-1) THEN
P=P+R*A(K+2,J)
A(K+2,J)=A(K+2,J)-P*Z
ENDIF
A(K+1,J)=A(K+1,J)-P*Y
A(K,J)=A(K,J)-P*X

17 CONTINUE

DO 18 I=L,MIN(NN,K+3)
P=X*A(I,K)+Y*A(I,K+1)
IF (K.NE.NN-1) THEN
P=P+Z*A(I,K+2)
A(I,K+2)=A(I,K+2)-P*R
ENDIF
A(I,K+1)=A(I,K+1)-P*Q
A(I,K)=A(I,K)-P

18 CONTINUE

19 CONTINUE
ENDIF

ENDIF

GO TO 1

ENDIF

RETURN

END
### SAMPLE OUTPUT OF LINEAR MODEL CODE

<table>
<thead>
<tr>
<th>PHI</th>
<th>PSI</th>
<th>VOI</th>
<th>WR</th>
<th>REDFREQ</th>
<th>B</th>
<th>PAR</th>
<th>UPAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.541 -1.486E+00</td>
<td>5.022E-03</td>
<td>3.281E+00</td>
<td>6.963E-01</td>
<td>4.915E+05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.002</td>
<td>0.543 -1.430E+00</td>
<td>3.233E-02</td>
<td>3.157E+00</td>
<td>7.236E-01</td>
<td>2.595E+05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.003</td>
<td>0.544 -1.391E+00</td>
<td>6.863E-02</td>
<td>3.072E+00</td>
<td>7.435E-01</td>
<td>1.785E+05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.004</td>
<td>0.546 -1.370E+00</td>
<td>1.050E-01</td>
<td>3.027E+00</td>
<td>7.546E-01</td>
<td>1.348E+05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>0.547 -1.362E+00</td>
<td>1.360E-01</td>
<td>3.010E+00</td>
<td>7.590E-01</td>
<td>1.065E+05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.006</td>
<td>0.549 -1.362E+00</td>
<td>1.601E-01</td>
<td>3.010E+00</td>
<td>7.590E-01</td>
<td>8.671E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.007</td>
<td>0.550 -1.366E+00</td>
<td>1.787E-01</td>
<td>3.020E+00</td>
<td>7.565E-01</td>
<td>7.210E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>0.551 -1.373E+00</td>
<td>1.919E-01</td>
<td>3.035E+00</td>
<td>7.525E-01</td>
<td>6.095E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.009</td>
<td>0.552 -1.381E+00</td>
<td>2.013E-01</td>
<td>3.055E+00</td>
<td>7.477E-01</td>
<td>5.220E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.010</td>
<td>0.554 -1.390E+00</td>
<td>2.074E-01</td>
<td>3.076E+00</td>
<td>7.426E-01</td>
<td>4.522E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.011</td>
<td>0.555 -1.399E+00</td>
<td>2.102E-01</td>
<td>3.096E+00</td>
<td>7.378E-01</td>
<td>3.959E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.012</td>
<td>0.556 -1.408E+00</td>
<td>2.114E-01</td>
<td>3.117E+00</td>
<td>7.328E-01</td>
<td>3.492E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.013</td>
<td>0.557 -1.418E+00</td>
<td>2.106E-01</td>
<td>3.138E+00</td>
<td>7.278E-01</td>
<td>3.101E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.014</td>
<td>0.558 -1.426E+00</td>
<td>2.084E-01</td>
<td>3.158E+00</td>
<td>7.234E-01</td>
<td>2.774E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.015</td>
<td>0.559 -1.435E+00</td>
<td>2.052E-01</td>
<td>3.178E+00</td>
<td>7.188E-01</td>
<td>2.491E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.016</td>
<td>0.560 -1.443E+00</td>
<td>2.007E-01</td>
<td>3.196E+00</td>
<td>7.147E-01</td>
<td>2.251E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.017</td>
<td>0.562 -1.451E+00</td>
<td>1.957E-01</td>
<td>3.215E+00</td>
<td>7.105E-01</td>
<td>2.041E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.018</td>
<td>0.563 -1.458E+00</td>
<td>1.898E-01</td>
<td>3.232E+00</td>
<td>7.067E-01</td>
<td>1.858E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.019</td>
<td>0.563 -1.465E+00</td>
<td>1.834E-01</td>
<td>3.248E+00</td>
<td>7.032E-01</td>
<td>1.698E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.020</td>
<td>0.564 -1.472E+00</td>
<td>1.768E-01</td>
<td>3.265E+00</td>
<td>6.995E-01</td>
<td>1.556E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.021</td>
<td>0.565 -1.479E+00</td>
<td>1.697E-01</td>
<td>3.280E+00</td>
<td>6.963E-01</td>
<td>1.431E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.022</td>
<td>0.566 -1.485E+00</td>
<td>1.621E-01</td>
<td>3.295E+00</td>
<td>6.933E-01</td>
<td>1.319E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.023</td>
<td>0.567 -1.491E+00</td>
<td>1.545E-01</td>
<td>3.309E+00</td>
<td>6.904E-01</td>
<td>1.219E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.024</td>
<td>0.568 -1.497E+00</td>
<td>1.468E-01</td>
<td>3.322E+00</td>
<td>6.875E-01</td>
<td>1.129E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.025</td>
<td>0.569 -1.502E+00</td>
<td>1.390E-01</td>
<td>3.334E+00</td>
<td>6.851E-01</td>
<td>1.048E+04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.026</td>
<td>0.570 -1.506E+00</td>
<td>1.309E-01</td>
<td>3.346E+00</td>
<td>6.827E-01</td>
<td>9.751E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.027</td>
<td>0.570 -1.511E+00</td>
<td>1.231E-01</td>
<td>3.357E+00</td>
<td>6.804E-01</td>
<td>9.090E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.028</td>
<td>0.571 -1.515E+00</td>
<td>1.149E-01</td>
<td>3.367E+00</td>
<td>6.784E-01</td>
<td>8.492E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.029</td>
<td>0.572 -1.520E+00</td>
<td>1.069E-01</td>
<td>3.378E+00</td>
<td>6.763E-01</td>
<td>7.943E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.030</td>
<td>0.573 -1.524E+00</td>
<td>9.921E-02</td>
<td>3.387E+00</td>
<td>6.743E-01</td>
<td>7.443E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.031</td>
<td>0.573 -1.527E+00</td>
<td>9.136E-02</td>
<td>3.396E+00</td>
<td>6.726E-01</td>
<td>6.988E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.032</td>
<td>0.574 -1.531E+00</td>
<td>8.350E-02</td>
<td>3.406E+00</td>
<td>6.707E-01</td>
<td>6.568E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.033</td>
<td>0.575 -1.534E+00</td>
<td>7.620E-02</td>
<td>3.412E+00</td>
<td>6.694E-01</td>
<td>6.192E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.034</td>
<td>0.575 -1.537E+00</td>
<td>6.855E-02</td>
<td>3.421E+00</td>
<td>6.677E-01</td>
<td>5.836E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.035</td>
<td>0.576 -1.539E+00</td>
<td>6.148E-02</td>
<td>3.427E+00</td>
<td>6.666E-01</td>
<td>5.520E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.036</td>
<td>0.577 -1.542E+00</td>
<td>5.435E-02</td>
<td>3.434E+00</td>
<td>6.651E-01</td>
<td>5.221E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.037</td>
<td>0.577 -1.544E+00</td>
<td>4.733E-02</td>
<td>3.440E+00</td>
<td>6.640E-01</td>
<td>4.950E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.038</td>
<td>0.578 -1.547E+00</td>
<td>4.067E-02</td>
<td>3.446E+00</td>
<td>6.628E-01</td>
<td>4.698E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.039</td>
<td>0.578 -1.549E+00</td>
<td>3.422E-02</td>
<td>3.452E+00</td>
<td>6.617E-01</td>
<td>4.467E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.040</td>
<td>0.579 -1.551E+00</td>
<td>2.769E-02</td>
<td>3.457E+00</td>
<td>6.608E-01</td>
<td>4.254E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.041</td>
<td>0.579 -1.553E+00</td>
<td>2.187E-02</td>
<td>3.462E+00</td>
<td>6.598E-01</td>
<td>4.057E+03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.042</td>
<td>.580</td>
<td>-1.555E+00</td>
<td>1.612E-02</td>
<td>3.467E+00</td>
<td>6.589E-01</td>
<td>3.875E+03</td>
<td></td>
</tr>
<tr>
<td>.043</td>
<td>.581</td>
<td>-1.556E+00</td>
<td>1.035E-02</td>
<td>3.472E+00</td>
<td>6.579E-01</td>
<td>3.707E+03</td>
<td></td>
</tr>
<tr>
<td>.044</td>
<td>.581</td>
<td>-1.558E+00</td>
<td>4.996E-03</td>
<td>3.476E+00</td>
<td>6.572E-01</td>
<td>3.553E+03</td>
<td></td>
</tr>
<tr>
<td>.045</td>
<td>.582</td>
<td>-1.559E+00</td>
<td>3.195E-03</td>
<td>3.479E+00</td>
<td>6.565E-01</td>
<td>3.412E+03</td>
<td></td>
</tr>
<tr>
<td>.046</td>
<td>.582</td>
<td>-1.560E+00</td>
<td>-5.126E-03</td>
<td>3.482E+00</td>
<td>6.560E-01</td>
<td>3.283E+03</td>
<td></td>
</tr>
<tr>
<td>.047</td>
<td>.583</td>
<td>-1.561E+00</td>
<td>-9.675E-03</td>
<td>3.485E+00</td>
<td>6.555E-01</td>
<td>3.164E+03</td>
<td></td>
</tr>
<tr>
<td>.048</td>
<td>.583</td>
<td>-1.561E+00</td>
<td>-1.412E-02</td>
<td>3.487E+00</td>
<td>6.550E-01</td>
<td>3.055E+03</td>
<td></td>
</tr>
<tr>
<td>.049</td>
<td>.584</td>
<td>-1.562E+00</td>
<td>-1.835E-02</td>
<td>3.490E+00</td>
<td>6.546E-01</td>
<td>2.955E+03</td>
<td></td>
</tr>
<tr>
<td>.050</td>
<td>.584</td>
<td>-1.563E+00</td>
<td>-2.233E-02</td>
<td>3.493E+00</td>
<td>6.539E-01</td>
<td>2.860E+03</td>
<td></td>
</tr>
<tr>
<td>.051</td>
<td>.585</td>
<td>-1.564E+00</td>
<td>-2.574E-02</td>
<td>3.496E+00</td>
<td>6.534E-01</td>
<td>2.775E+03</td>
<td></td>
</tr>
<tr>
<td>.052</td>
<td>.585</td>
<td>-1.565E+00</td>
<td>-2.891E-02</td>
<td>3.499E+00</td>
<td>6.528E-01</td>
<td>2.698E+03</td>
<td></td>
</tr>
<tr>
<td>.053</td>
<td>.585</td>
<td>-1.565E+00</td>
<td>-3.221E-02</td>
<td>3.500E+00</td>
<td>6.527E-01</td>
<td>2.630E+03</td>
<td></td>
</tr>
<tr>
<td>.054</td>
<td>.586</td>
<td>-1.566E+00</td>
<td>-3.496E-02</td>
<td>3.503E+00</td>
<td>6.521E-01</td>
<td>2.564E+03</td>
<td></td>
</tr>
<tr>
<td>.055</td>
<td>.586</td>
<td>-1.566E+00</td>
<td>-3.746E-02</td>
<td>3.504E+00</td>
<td>6.518E-01</td>
<td>2.508E+03</td>
<td></td>
</tr>
<tr>
<td>.056</td>
<td>.587</td>
<td>-1.566E+00</td>
<td>-3.977E-02</td>
<td>3.504E+00</td>
<td>6.520E-01</td>
<td>2.461E+03</td>
<td></td>
</tr>
<tr>
<td>.057</td>
<td>.587</td>
<td>-1.567E+00</td>
<td>-4.172E-02</td>
<td>3.507E+00</td>
<td>6.514E-01</td>
<td>2.413E+03</td>
<td></td>
</tr>
<tr>
<td>.058</td>
<td>.588</td>
<td>-1.566E+00</td>
<td>-4.344E-02</td>
<td>3.506E+00</td>
<td>6.515E-01</td>
<td>2.376E+03</td>
<td></td>
</tr>
<tr>
<td>.059</td>
<td>.588</td>
<td>-1.565E+00</td>
<td>-4.488E-02</td>
<td>3.505E+00</td>
<td>6.517E-01</td>
<td>2.344E+03</td>
<td></td>
</tr>
<tr>
<td>.060</td>
<td>.589</td>
<td>-1.566E+00</td>
<td>-4.605E-02</td>
<td>3.507E+00</td>
<td>6.513E-01</td>
<td>2.312E+03</td>
<td></td>
</tr>
<tr>
<td>.061</td>
<td>.589</td>
<td>-1.568E+00</td>
<td>-4.691E-02</td>
<td>3.512E+00</td>
<td>6.503E-01</td>
<td>2.281E+03</td>
<td></td>
</tr>
<tr>
<td>.062</td>
<td>.590</td>
<td>-1.565E+00</td>
<td>-4.763E-02</td>
<td>3.508E+00</td>
<td>6.512E-01</td>
<td>2.266E+03</td>
<td></td>
</tr>
<tr>
<td>.063</td>
<td>.590</td>
<td>-1.565E+00</td>
<td>-4.810E-02</td>
<td>3.508E+00</td>
<td>6.512E-01</td>
<td>2.250E+03</td>
<td></td>
</tr>
<tr>
<td>.064</td>
<td>.591</td>
<td>-1.565E+00</td>
<td>-4.821E-02</td>
<td>3.508E+00</td>
<td>6.511E-01</td>
<td>2.236E+03</td>
<td></td>
</tr>
<tr>
<td>.065</td>
<td>.591</td>
<td>-1.564E+00</td>
<td>-4.803E-02</td>
<td>3.508E+00</td>
<td>6.512E-01</td>
<td>2.228E+03</td>
<td></td>
</tr>
<tr>
<td>.066</td>
<td>.592</td>
<td>-1.564E+00</td>
<td>-4.731E-02</td>
<td>3.508E+00</td>
<td>6.511E-01</td>
<td>2.222E+03</td>
<td></td>
</tr>
<tr>
<td>.067</td>
<td>.592</td>
<td>-1.563E+00</td>
<td>-4.666E-02</td>
<td>3.508E+00</td>
<td>6.512E-01</td>
<td>2.220E+03</td>
<td></td>
</tr>
<tr>
<td>.068</td>
<td>.593</td>
<td>-1.561E+00</td>
<td>-4.556E-02</td>
<td>3.504E+00</td>
<td>6.518E-01</td>
<td>2.226E+03</td>
<td></td>
</tr>
<tr>
<td>.069</td>
<td>.593</td>
<td>-1.562E+00</td>
<td>-4.449E-02</td>
<td>3.506E+00</td>
<td>6.516E-01</td>
<td>2.228E+03</td>
<td></td>
</tr>
<tr>
<td>.070</td>
<td>.594</td>
<td>-1.561E+00</td>
<td>-4.288E-02</td>
<td>3.506E+00</td>
<td>6.516E-01</td>
<td>2.235E+03</td>
<td></td>
</tr>
</tbody>
</table>
TIME-RESOLVED SIMULATION EQUATIONS

\[ D_{p\_pump} = D_{p\_pump} + dt \times (D_{p\_pump dt}) \]
\[ \text{INIT}(D_{p\_pump}) = 1.089e5 \]
\[ m1 = m1 + dt \times (dm1dt) \]
\[ \text{INIT}(m1) = 17.06 \]
\[ m2 = m2 + dt \times (dm2dt) \]
\[ \text{INIT}(m2) = 17.06 \]
\[ P1 = P1 + dt \times (dP1dt) \]
\[ \text{INIT}(P1) = 2.374E5 \]
\[ P2 = P2 + dt \times (dP2dt) \]
\[ \text{INIT}(P2) = 1.2517E5 \]
\[ V1 = V1 + dt \times (dV1dt) \]
\[ \text{INIT}(V1) = .053 \]
\[ V2 = V2 + dt \times (dV2dt) \]
\[ \text{INIT}(V2) = .323 \]
\[ A1 = .0572 \]
\[ A2 = .029 \]
\[ b2 = .01189 \]
\[ D2 = 0.6096 \]
\[ dm1dt = (A1/I1)*(P2-P1+D_{p\_pump-D_{p\_leg1}}) \]
\[ dm2dt = (A2/I2)*(P1-P2-D_{\_throttle}) \]
\[ dP1dt = (1.4*P1/(V1*ro2))*(m1-m2) \]
\[ dP2dt = (1.4*P2/(V2*ro2))*(m2-m1) \]
\[ D_{p\_leg1} = (m1/SQRT(m1^2))*(K1*m1^2)/(2*ro*A1^2) \]
\[ D_{p\_pumpst} = (D_{p\_pumpst-D_{p\_pump}})*U*Phi/(2.3*.03) \]
\[ D_{\_throttle} = (m2/SQRT(m2^2))*(Kt*m2^2)/(2*ro*(A2^2)) \]
\[ dV1dt = -(V1/(1.4*P1))*dP1dt \]
\[ dV2dt = -(V2/(1.4*P2))*dP2dt \]
\[ K1 = 6 \]
\[ I1 = 38.53 \]
\[ I2 = 3.277 \]
\[ Phi = m1/(ro*PI*D2*b2*U) \]
\[ Psi = 0.54298+0.93162*Phi+11.034*(Phi^2)-349.85*(Phi^3)+2332.7*(Phi^4)-5412.9*(Phi^5) \]
\[ Psi2 = (P1-P2+m1/SQRT(m1^2)*m1^2*883.4/2.0/ro2)/(ro2*U^2) \]
\[ ro = 1000 \]
\[ ro2 = ro \]
\[ t = \text{TIME} \]
\[ U = 13.812-1.116E-2*m1 \]
APPENDIX B

INERTIAL LENGTH CALCULATION
AND
PIPING HEAD LOSS ESTIMATES

B.1 Calculation of the Lumped Inertial Lengths

The system of equations outlined in Chapter II uses an area weighted inertial length scale for each leg of piping. The length is defined in [9] as:

\[ L = A_{\text{REF}} \int_0^L \frac{dx}{A(x)} \]

As in [9,22], the reference area for the pump leg was chosen to be the pump inlet area, 0.0572 m². The reference area for the throttle leg of piping was chosen to be the cross-sectional area of the 8 inch PVC pipe (0.029 m²). The only term left to evaluate is the integral. For a straight piping section, this calculation is quite simple. For the passages in and around the pump, however, the integral is less straightforward. This appendix will step through each section of the MIT experimental test facility, determining the contribution to the overall integral for each step. This is done first for the pump leg and then for the throttle leg.

B.1.1 The Pump Leg Inertial Length

The pump leg, defined in Chapter II, extends from the exit of the inlet (or large) plenum to the entrance of the discharge (or small) plenum, cf. Figure 3.1. In the analysis below, it has been divided into manageable sections. The contributions from each section to the overall inertial influence integral are summed to determine the final value of \( L_P \).
1) The pump leg contains 7.70 m of 8 inch (nominal diameter) PVC piping between the end of the pipe diffuser cone and the entrance to the discharge plenum.

\[
\int_0^L \frac{dx}{A(x)} = \frac{L}{A} = 267.5 \text{ m}^{-1}
\]

Length \((L) = 7.76\) m \quad Area \((A) = 0.0290\) m\(^2\)

2) The pump leg contains 2.99 m of 12 inch (nominal diameter) PVC piping between the exit of the inlet plenum and the beginning of the pump inlet aluminum fairing piece.

\[
\int_0^L \frac{dx}{A(x)} = \frac{L}{A} = 45.52 \text{ m}^{-1}
\]

Length \((L) = 2.99\) m \quad Area \((A) = 0.0656\) m\(^2\)

3) The inlet fairing and aluminum inlet pipe have the same cross-sectional area, and contribute as follows:

\[
\int_0^L \frac{dx}{A(x)} = \frac{L}{A} = 17.11 \text{ m}^{-1}
\]

Length \((L) = 0.98\) m \quad Area \((A) = 0.0572\) m\(^2\)

For conical sections, the following formulation is employed:
For a standard cone, as pictured above, where \( \alpha = \tan^{-1}\frac{R_2-R_1}{L} \):

\[
R(x) = R_1 + x \tan(\alpha) \\
A(x) = \pi R(x)^2 = \pi (R_1 + x \tan(\alpha))^2
\]

and the integral becomes:

\[
\int_0^L \frac{dx}{A(x)} = \int_0^L \frac{dx}{\pi (R_1 + x \tan(\alpha))^2} = \frac{1}{\pi \tan(\alpha)} \left( \frac{1}{R_1} - \frac{1}{R_1 + L \tan(\alpha)} \right)
\]

4) The pipe diffuser has a conical section with the following dimensions:

\[
\alpha = 4 \text{ degrees} \quad R_1 = 0.0547 \text{ m} \quad L = 0.591 \text{ m}
\]

\[
\int_0^L \frac{dx}{A(x)} = 35.82 \text{ m}^{-1}
\]

5) The inlet contraction has three sections of different dimensions:
   a) conical section,

\[
\alpha = 16 \text{ degrees} \quad R_1 = 0.101 \text{ m} \quad L = 0.119 \text{ m}
\]

\[
\int_0^L \frac{dx}{A(x)} = 2.78 \text{ m}^{-1}
\]

b) constant area section,

Length (L) = 0.025 m \quad Area (A) = 0.0572 m^2

\[
\int_0^L \frac{dx}{A(x)} = \frac{L}{A} = 0.44 \text{ m}^{-1}
\]
c) and another constant area section,

\[
\text{Length (L) = 0.025 m} \quad \text{Area (A) = 0.032 m}^2
\]

\[
\frac{L}{\int_{0}^{A(x)} \frac{dx}{A(x)}} = \frac{L}{A} = 0.79 \text{ m}^{-1}
\]

6) The transition insert can be approximated as a cone with the same \( \alpha \) as the pipe diffuser:

\[
\alpha = 4 \text{ degrees} \quad R_1 = 0.0313 \text{ m} \quad L = 0.335 \text{ m}
\]

\[
\frac{L}{\int_{0}^{A(x)} \frac{dx}{A(x)}} = 62.25 \text{ m}^{-1}
\]

7) The impeller can be divided into three sections:

a) The contraction at the tip of the inlet pipe shroud,

\[
\alpha = 30 \text{ degrees} \quad R_1 = 0.101 \text{ m} \quad L = 0.044 \text{ m}
\]

\[
\frac{L}{\int_{0}^{A(x)} \frac{dx}{A(x)}} = 1.09 \text{ m}^{-1}
\]

b) The inlet pipe shroud and impeller eye can be approximated as one constant area section with the following dimensions,

\[
\text{Length (L) = 0.175 m} \quad \text{Area (A) = 0.032 m}^2
\]

\[
\frac{L}{\int_{0}^{A(x)} \frac{dx}{A(x)}} = \frac{L}{A} = 5.47 \text{ m}^{-1}
\]
c) In the impeller passages, the total area is approximately constant with radius. This area is equal to \( \pi D_2 b_2 \) and the radial length is \( 0.5(D_2 - D_1) \).

Length (\( L \)) = 0.204 m \hspace{1em} \text{Area (\( A \)) = 0.0228 m}^2

\[
\frac{\int_{0}^{L} \frac{dx}{A(x)}}{L} = \frac{L}{A} = 8.96 \text{ m}^{-1}
\]

8) To determine the inertial length of the volute scroll, the following approximations are made:

- neglect backplate leakage flows.
- neglect recirculating flows past the tongue.
- neglect frontplate leakage flows (past the labyrinth seal).
- assume that the flowrate leaving the impeller is constant about its circumference.

It is convenient, for this analysis, to unwrap the volute and divide it into four sections (corresponding to the four arcs which define the outer wall). Each of these four sections can then be approximated as a constant height, variable width, rectangular section of piping (see figure below),

\[
\alpha = \tan^{-1} \frac{w_2 - w_1}{2L}, \quad w(x) = 2 \left( \frac{w_1}{2} + x \tan \alpha \right)
\]

and, \( h \) = constant = 0.0627 m.
So, the area as a function of $x$ is simply,

$$A(x) = h w(x) = 2h \left( \frac{w_1}{2} + x \tan \alpha \right)$$

If the recirculatory flows are neglected, then the flow entering the first section (at its $x=0$ face) is zero. And, following the assumption that the flow leaving the impeller is uniform about its circumference, the flow exiting the first section (at its $x=L$ face) is only 25% of the total flow leaving the impeller at any moment. This same analysis can be applied to each of the four sections. Defining $F_0$ as the fraction of the total volume flow (from the pump) which enters any section, and $F_1$ as the fraction leaving the section, the inertia integral must be weighted by the following relation:

$$\left( \frac{X}{L} \Delta F + F_0 \right)$$

where the increase in volume flow in a given section is taken to be linear with $x$ and $\Delta F = F_1 - F_0$. The integral for the inertial contribution of each section is then:

$$\int_0^L \frac{dx}{A(x)} = \frac{1}{2h} \int_0^L \frac{dx}{\left( \frac{w_1}{2} + x \tan \alpha \right)} \left( \frac{X}{L} \Delta F + F_0 \right)$$

$$= \frac{F_0}{2htan \alpha} \left\{ \ln \left( \frac{w_1}{2} + Ltan \alpha \right) - \ln \left( \frac{w_1}{2} \right) \right\} +$$

$$\frac{\Delta F}{2hLtan \alpha} \left\{ L + \frac{w_1}{2tan \alpha} \left[ \ln \left( \frac{w_1}{2} \right) - \ln \left( \frac{w_1}{2} + Ltan \alpha \right) \right] \right\}$$

The four sections will now be treated sequentially:

a) The first section has,
$$\alpha = 1.226 \text{ degrees}, \ L = 0.510 \text{ m}, \ F_0 = 0.00, \ F_1 = 0.25,$$
$$\text{and} \ \frac{w_1}{2} = 9.21 \ E^{-3} \text{ m}.$$
\[ \int_{0}^{L} \frac{dx}{A(x)} = 31.71 \, \text{m}^{-1} \]

b) The second section has,
\[
\alpha = 1.186 \, \text{degrees}, \quad L = 0.527 \, \text{m}, \quad F_0 = 0.25, \quad F_1 = 0.50,
\]
and \[ \frac{w_1}{2} = 2.01 \times 10^{-2} \, \text{m}. \]

\[ \int_{0}^{L} \frac{dx}{A(x)} = 61.12 \, \text{m}^{-1} \]

c) The third section has,
\[
\alpha = 1.150 \, \text{degrees}, \quad L = 0.545 \, \text{m}, \quad F_0 = 0.50, \quad F_1 = 0.75,
\]
and \[ \frac{w_1}{2} = 3.11 \times 10^{-2} \, \text{m}. \]

\[ \int_{0}^{L} \frac{dx}{A(x)} = 74.09 \, \text{m}^{-1} \]

d) The fourth section has,
\[
\alpha = 1.114 \, \text{degrees}, \quad L = 0.562 \, \text{m}, \quad F_0 = 0.75, \quad F_1 = 1.00,
\]
and \[ \frac{w_1}{2} = 4.20 \times 10^{-2} \, \text{m}. \]

\[ \int_{0}^{L} \frac{dx}{A(x)} = 82.52 \, \text{m}^{-1} \]

e) Finally, there is a constant area section between the last volute arc and the beginning of the transition insert,

Length (L) = 0.196 m \hspace{2cm} Area (A) = 0.00548 \, \text{m}^2

\[ \int_{0}^{L} \frac{dx}{A(x)} = \frac{L}{A} = 35.77 \, \text{m}^{-1} \]
Taking all of these into account, the summation of the inertial integral terms above yields:

\[
\int_0^L \frac{dx}{A(x)} = 732.94 \text{ m}^{-1}
\]

And finally,

\[
L_p = \frac{A_{\text{REF}}}{A(x)} = 41.92 \text{ m}
\]

**B.1.1 The Throttle Leg Inertial Length**

The throttle leg, defined in Chapter II, extends from the exit of the discharge plenum to the entrance of the inlet plenum, cf. Figure 3.1. The piping is of constant area (equal to the \(A_{\text{REF}}\) for this leg) and the calculation of \(L_T\) is relatively simple:

1) The piping has an 8 inch (nominal) diameter and is 3.277 m long.

Length \((L) = 3.277 \text{ m}\)  
Area \((A) = 0.0290 \text{ m}^2\)

\[
L_T = \frac{A_{\text{REF}}}{A(x)} = \frac{L}{A_{\text{REF}}} = 3.277 \text{ m}
\]

**B.2 Lumped Head Loss Factor Determination**

The head loss in the piping was estimated using loss correlations available in [33]. The losses in both piping legs were assumed to be proportional to the dynamic head of the flowing water:

\[
\Delta P = K \frac{1}{2} \rho V^2
\]
Conventional practise uses loss correlations based on the friction factor, $f$, commonly defined as:

$$f = K \frac{L}{D}$$

The friction factor is a function of Reynolds' number and the relative surface roughness, $\frac{\varepsilon}{D}$. The flow in the PVC piping sections can be assumed turbulent at flows near $\Phi_{tr}$ since the typical Reynolds number level is around 5E4 and there are very few straight sections of piping. The roughness factor, $\varepsilon$, for PVC is estimated around 3E-6 m. Using the Colebrook equation for turbulent pipe flow, the friction factor is bracketed between 0.02 and 0.03. This gives $K = 0.3$ for the 12" inlet piping, $K = 1.2$ for the 8" discharge piping, and $K = 0.5$ for the 8" throttle leg piping.

The above estimates are accurate only for straight sections of piping. Bends and tanks in the piping line add to the losses. Using typical corrections, the $K$ for the 12" inlet can be adjusted to 2.0, the discharge piping $K$ can be adjusted to 4.0, and the throttle leg $K$ to 1.5. The lumped loss coefficient factors employed in the linear model are then: $K_p = 6.0$ and $K_T = 1.5$. 

181
APPENDIX C

ψ AND φ HISTOGRAMS

AND

TORQUE CALIBRATION

WITH

TORQUE TARES
Figure C.1: Experimental data: histogram of 140 $\Psi$ data points collected at 20 Hz for constant throttle setting during steady pump operation (no air in plenums). $\Phi_{\text{avg}} = 0.16$
Figure C.2: Experimental data: histogram of 140 $\phi$ data points collected at 20 Hz for constant throttle setting during steady pump operation (no air in plenums). $\Phi_{\text{avg}} = 0.16$
Figure C.3: Experimental data: torque calibration of Lebow torquemeter with linear fit to data. Procedure for calibration outlined in Section 3.4.1. Maximum operational torque on 100% speedline is 1200 in lbs.
Figure C.4: Experimental data: torque tares measured for MIT test pump without water. Range of impeller rotational speeds represents operational range of test facility. Maximum operational torque on 100% speedline is 1200 in lbs.
APPENDIX D

AIR VOLUME AND PRESSURE ADJUSTMENT

DURING PUMP OPERATION

D.1 Introduction

The linear model predictions of oscillation inception point and frequency (cf. Chapter II) depend upon an accurate estimate of the loop compliance. Since the pumping fluid, water, is incompressible, the compliance is only a function of the pressure and displaced volume of the two air bags (one in the large plenum and one in the small). As part of the standard testing procedure, the air bags are always filled with the pumping loop open to atmosphere (cf. Section 3.2.1). Thus, with the pump off, the pressure in the air bags is approximately equal to that of the surrounding fluid. The air volumes are also measured at this pressure. Then, before starting the pump, the test loop is shut off from atmosphere by closing the valve at the top of the discharge stack piping. From continuity, the total volume (of water and air) in the loop remains unchanged during pump operation.

As soon as the pump is set in motion, however, the pressure field in the loop is significantly altered. The air trapped in the two plenums sees a different environment and adjusts to reach an equilibrium level of volume and pressure which corresponds to the new operating point of the pump. The adjustment process, though intuitively clear, deserves some attention, for it greatly influences the inputs to the linear code. This appendix will explain the air adjustment in some detail, beginning with a simple thought experiment for proof of concept and ending with the derivation of the equations employed by the linear model code (cf. Appendix A).
D.2 A Thought Experiment

A tank containing both air and water is configured as shown in the figure below:

Initially, the massless plates which hold the water in place are pinned in the positions shown. The valve linking the two volumes of air is open and the air is allowed to reach an equilibrium state with the surrounding atmosphere. The water column has a height, $h$, and a constant cross-sectional area, $A$. In this initial position, the pressure on the underside of the top plate is simply the atmospheric pressure or $P_a$. The pressure on the topside of the bottom plate is $P_a + \rho gh$.

At some time well after the air has reached equilibrium (such that the pressure of the air in the top and bottom of the tank is approximately equal to $P_a$), the valve connecting the two air volumes is shut and the pins are simultaneously removed. The plates are constrained to translate only vertically and do so in a frictionless manner (with an adequate seal so that the water is prevented from escaping). The final equilibrium position of the water column is represented in the following figure.
Since the water is incompressible (and h cannot vary) the equation governing the final volumes of the two air pockets is:

\[ V_{T0} + V_{Bo} = V_{Tf} + V_{Bf} \]

Using a vertical force balance on the water column, the final equilibrium position is defined by:

\[ \sum F_v = [P_{Bf} - (P_a + \rho gh)] A + [P_a - P_{Tf}] A = 0 \]

And, the isothermal relation governs the expansion and compression of the air volumes:

\[ P_{T0} V_{T0}^\gamma = P_{Tf} V_{Tf}^\gamma \quad \text{and} \quad P_{Bo} V_{Bo}^\gamma = P_{Bf} V_{Bf}^\gamma \]

Substituting these two into the force balance equation yields (using \( P_{T0} = P_{Bo} = P_a \)):

\[ V_{Bf} = \frac{V_{Bo}}{\left[ \frac{\rho gh}{P_a} + \left( \frac{V_{T0}}{V_{Tf}} \right) \right]^{1/\gamma}} \]

Finally, employing the volume conservation equation, the final air volumes (and then pressures) can be calculated.

\[ V_{Tf} = V_{T0} + V_{Bo} - \frac{V_{Bo}}{\left[ \frac{\rho gh}{P_a} + \left( \frac{V_{T0}}{V_{Tf}} \right) \right]^{1/\gamma}} \]
D.3 Volume Adjustments in the Actual Test Facility

The thought experiment outlined above shows that the adjustments in volume and pressure merit closer investigation. To do this for the case of the actual pump test loop, several of the simplifications employed above must be relaxed.

The first important assumption is that of constant area. The surface area of the area-water interface in the two plenums is not the same. In addition, the surface area changes with included volume (unlike the example above). The air volume in each plenum is considered below.

As mentioned in Section 3.2.1, the air in the large plenum was stored in the top of the tank instead of using the air bag system. The surface area of the water to air interface is solely a function of the air volume. Looking at a cross-section of the large plenum,

\[ x^2 + (y + h)^2 = R^2 \]
The area of the air-filled region in this 2-D cross-section is determined by the integral:

\[
A_{\text{air}} = 2 \int_{0}^{\sqrt{R^2-h^2}} \left( \sqrt{R^2-x^2} - h \right) dx
\]

\[
= R^2 \sin^{-1}\left(\frac{\sqrt{R^2-h^2}}{R}\right) - h \sqrt{R^2-h^2}
\]

Then, the total volume of air in the tank is defined by \( V_{\text{air}} = A_{\text{air}} L \), where \( L \) is the length of the large tank (\( L = 1.27 \) m). The surface area of the air-water interface is simply,

\[
A_{s} = 2L \sqrt{R^2-h^2}
\]

For a given air volume, \( V_{\text{air}} \), the surface area, \( A_{s} \), can then be determined by iteratively solving for the corresponding value of \( h \). At \( V_{\text{air}} = 300 \) liters (the standard "pump-off" air volume of the large plenum used experimentally), this surface area is 1.460 \( m^2 \) (\( R = 0.71 \) m).

A sensitivity analysis of the above relationship shows that a typical (experimental) air volume change of 45 liters (from 300 liters with pump off to 345 liters with pump on) produces only a 3% increase in surface area. This being the case, the change in surface area with volume in the large plenum is neglected in the linear code.

In the small plenum, the air is stored in four innertubes which have been placed around a perforated aluminum core cylinder in the center of the plenum (cf. Section 3.2.1). For ease of analysis, the surface area of the four inflated tubes is approximated by that of a cylinder which contains the total air volume of the four innertubes (see figure below).
It is assumed that this "equivalent-volume" flexible wall cylinder has a constant inner diameter during fluctuations in volume. This inner diameter is taken to be that of the plenum's (perforated aluminum) core cylinder. During volume adjustments, then, the only dimension of the cylinder which changes is D, the thickness. It is further assumed that the cylinder ends are at all times touching the end faces of the plenum. With the above simplifications, it is possible to write the equations for the included volume and the surface area of this cylinder.

\[ V_{\text{air}} = DL (2\pi R_m) \text{ (approx.)} \quad \text{and} \quad A_s = 4\pi LR_m \]

From geometry, \( D = 2 (R_m - R_i) \), and the result is an expression for \( A_s \) in terms of \( V_{\text{air}} \):

\[ A_s = 4\pi L \left( R_i + \sqrt{R_i^2 + \frac{V_{\text{air}}}{\pi L}} \right) \]

For a typical value of \( V_{\text{air}} \)(150 liters), the surface area is 3.16 m² \((L = 1.016 \text{ m and } R_i = 0.20 \text{ m})\). As with the surface area in the large plenum, a 45 liter adjustment in volume here (from 300 liters at pump off to 255 liters with pump on) causes only a 3% adjustment in surface area. It is again assumed, then, that the surface area of the air-water interface (the exposed innertube walls) in the small plenum is approximately constant.
The second assumption from the thought experiment which needs to be relaxed in the present analysis is that the initial pressures of the two air volumes are equal. The air in the large plenum collects in the top of the tank, whereas the innertubes in the small plenum are constrained to the center of the tank. The hydrostatic head in the two locations is significantly different. Accordingly, the initial pressures used by the linear model code for the two air volumes are: \( P_{sm} = 1.427 \times 10^5 \text{ Pa} \) and \( P_{lg} = 1.389 \times 10^5 \text{ Pa} \).

Third, the water pressure difference between the two plenums doesn't vary as simply as the \( \rho gh \) relation for the water column in Section D.2. Rather, this pressure difference is equal to the pressure rise of the pump added to the initial differences in pressure. Or, more simply put, the operating pressure (with pump on) of the water at the air water interface in both plenums is:

\[
P_{SM} = 1.427 \times 10^5 + \Delta P_{exit}
\quad \text{and} \quad
P_{LG} = 1.389 \times 10^5 + \Delta P_{inlet}
\]

Using these assumptions, and also neglecting innertube stiffness, compression losses, and surface tension effects, the equivalent relations for volume and pressure adjustments in the experimental test loop can be formulated.

Once again, volume conservation for the two air volumes requires that ("i" signifies initial conditions with pump off and "f" signifies final equilibrium conditions at the operating point),

\[
V_{SMi} + V_{LGi} = V_{SMf} + V_{LGf}
\]

The force balance equation is,

\[
[ P_{SMf} - (\Delta P_{ex} + P_{SMi}) ] A_{SM} + [ (\Delta P_{in} + P_{LGi}) - P_{LGf} ] A_{LG} = 0
\]

The isothermal compression and expansion relations for the air volumes are,

\[
P_{SMi} V_{SMi}^\gamma = P_{SMf} V_{SMf}^\gamma \quad \text{and} \quad
P_{LGi} V_{LGi}^\gamma = P_{LGf} V_{LGf}^\gamma
\]
Combining these, as in Section D.2, yields the following expression for $V_{LGf}$:

$$V_{LGf} = V_{LGi} + V_{SMi}$$

$$- \left\{ \frac{\Delta P_{ex} + P_{SMi}}{P_{SMi}} - \frac{A_{LG}}{P_{SMi} A_{SM}} \left[ \Delta P_{in} + P_{LGi} - P_{LGi} \left( \frac{V_{LGf}}{V_{LGf}} \right) \right]^{1/\gamma} \right\}$$

This is the relation used in the linear model code to determine the air volumes and pressures at each successive operating point (as the pump characteristic is scanned from $\Phi = 0.001$ to 0.07). The additional accuracy gained by this procedure is quite substantial. The following table shows the pressure and volume adjustments calculated (using the above procedure) for a change from pump off to pump operation at $\Phi = 0.055$ on the 100% speedline.

<table>
<thead>
<tr>
<th></th>
<th>SMALL PLENUM</th>
<th>LARGE PLENUM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volume</td>
<td>Pressure</td>
</tr>
<tr>
<td>Pump off</td>
<td>300 l</td>
<td>1.427E5 Pa</td>
</tr>
<tr>
<td>$\Phi = 0.055, 100%$</td>
<td>220 l</td>
<td>2.208E5 Pa</td>
</tr>
</tbody>
</table>

And more importantly, the linear model oscillation frequency prediction for the above case (using adjustments) is much closer to the experimentally derived frequency than the model predictions where no volume/pressure adjustment is considered. More precisely, the reduced frequency predictions are 3.63% and 3.09% for the two different methods, respectively, versus the experimental reduced frequency of 3.87%.

One final item worth mentioning is the effect of the above discussion on the system equations presented in Chapter II. It is obvious from the equations that the pressures referred to in the conservation of momentum equations and those used in the continuity expressions are not the same. The conservation of momentum refers to the static pressure of the water in the plenums, whereas continuity uses the plenum air pressures (via the isothermal compression and expansion relation). Thus, when solving for the perturbation quantities, the equations are actually decoupled, and other relations are required to express
the air pressures in terms of the water pressures. To avoid this complexity, the following assumption is made.

The force balance expression used in the preceding section (cf. Section D.2) was the following:

\[ \sum F_v = \left[ P_{Bf} - (P_a + pgh) \right] A + \left[ P_a - P_{Tf} \right] A = 0 \]

Substituting the actual test facility variables in the above expression gives,

\[ [ P_{\text{air}2} - P_{\text{water}2} ] A_2 + [ P_{\text{water}1} - P_{\text{air}1} ] A_1 = 0 \]

where 2 refers to the small plenum and 1 refers to the large plenum. If the areas are equal, \( A_1 = A_2 \), then the above expression becomes simply,

\[ P_{\text{water}1} - P_{\text{water}2} = P_{\text{air}1} - P_{\text{air}2} \]

The momentum equation only uses the water pressures in the above form (as a difference). Thus, if it is possible to assume that the areas, \( A_2 \) and \( A_1 \), are nearly equal (to a good approximation), then the air pressure difference can be substituted for the water pressure difference in the momentum equation. The perturbation pressures in the final linearized system would then be the air pressures instead of the water pressures and the analysis would be virtually unchanged.

Accordingly, the effect of small pressure perturbations (on the order of typical experimental presurge oscillations) on the air volumes and pressures was determined using the equal area assumption. This was compared to the same effect using the actual areas (as determined earlier). For a small perturbation \( \Delta P_{\text{ex}} = 1400 \text{ Pa} \) and \( \Delta P_{\text{in}} = -160 \text{ Pa} \), the difference in the effects between the two methods was around 3%. Thus, for the small perturbation analysis, the assumption of equal surface areas for the two plenum air volumes is quite good.