The Inelastic Response of Multiple-Degree-of-Freedom Systems

by

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B.Sc., The University of the West Indies (1994)

Submitted to the Department of Civil and Environmental Engineering
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Abstract

In earthquake-resistant design it is usually assumed that the structure is linearly elastic and remains linearly elastic throughout the duration of the seismic activity. This is not always a realistic assumption. Indeed, even well designed structures will yield during very strong shaking and endure stresses well beyond the elastic limit of the design material. It is neither necessary nor economical to design structures to resist very strong shaking within the elastic limit of the construction materials used. Some damage can be tolerated, provided that there is no undue hazard to the occupants of the building. This idea is reflected in modern building codes. The accepted philosophy is that structures should be designed to resist moderate shaking without damage, while permitting some yielding and limited damage in strong shaking. In seismic design beyond the material elastic limit, elastoplastic behavior is usually assumed.

It is therefore of interest to the designer to study how a structure in which yielding is allowed to take place will behave under seismic loading. The aim of this research is to gain some insight into the inelastic behavior of a multistory building using finite element modeling.

The finite element analysis software ADINA (Automatic Dynamic Incremental Nonlinear Analysis) is used to obtain the 'true' structural response of a structural system with inelastic material properties. The results of any finite element analysis are only as accurate as the mathematical model used. Therefore, part of this work is devoted to assuring that the model used would indeed give as accurate a representation of the true response of the structure as possible.

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1. Introduction

Structural engineers are charged with the responsibility of providing structures and facilities which will safely withstand the worst forces they can be reasonably expected to undergo during their lifetime. For centuries, building codes have served to provide rules which govern and regulate design. Modern design codes provide guidelines on the use of different construction materials and techniques, together with recommendations on the loads that structures should be designed to resist.

The loads which it is likely to experience within its lifetime. These loads are usually classified as follows:

- Dead load
- Live load
- Floor load
- Roof load
- wind load
- Seismic load

There are provisions in all modern building codes for designing structures to handle these loads and any combination thereof. Some of the load types are better understood than others due to the fact that they occur frequently and information on how structures behave under these loads accumulates rapidly and is easily obtained. As we come to a better understanding of structural behavior, design codes evolve to reflect these developments.

Of the loads listed above, seismic loading is perhaps the least well understood. The understanding of the nature of earthquakes and how structures respond to them is all relatively recent, and provisions for earthquake resistant design were not included in American design codes until 1933. Modifications in seismic code provisions reflect the increasing experience gained from the study of actual earthquakes and structures, and from the results of research.
Modern earthquake-resistant design procedures are determined by zoning, site characteristics, occupancy, configuration, structural system, and building height. Although building codes contain recommendations for seismic design, the assessment of the site conditions, the soil type, and the material and structure type to be used are based on the professional judgment of the engineers. The main problems in seismic design are encountered at the planning stage in the determination of the design parameters.

It is reasonable to assume that a well designed building will resist earthquakes of moderate intensity without any structural damage. That is to say that the building can be expected to respond within the elastic range of the material. For the truly strong shaking of a severe earthquake or a nuclear blast, however, this is not a realistic assumption. Although it is possible to design structures to resist very strong earthquakes within the elastic range of the material, this is not considered to be economically feasible. The generally accepted philosophy is that structures can be permitted to experience some yielding and repairable structural damage under very strong shaking, provided that this damage does not pose undue hazard to the occupants of the building.

The inelastic one-degree-of-freedom system is reasonably well understood, and inelastic design spectra have been developed for use in the analysis of one-degree-of-freedom inelastic systems. The analysis and design of multiple-degree-of-freedom inelastic systems is not as straightforward. When the main response of a structure is from the fundamental mode, the building is modeled as a one-degree-of-freedom system and the inelastic response spectrum is used directly. In practice, the inelastic spectrum is also used in the analysis of multiple-degree-of-freedom systems by using the modal superposition method, even though this is not strictly valid for inelastic systems. The use of the method is considered justified since the contribution of the higher modes is relatively small and the error in calculating the response of the higher modes will not give rise to significant in the overall structural response of the structure.

---

The structural analysis software now available make it possible to create and analyze reasonably accurate models of actual structures. In this way we can come to a better understanding of their actual behavior. The aim of this research is to use one such program, ADINA (Automatic Dynamic Incremental Nonlinear Analysis) to find the ‘true’ response of a multistory building with some inelastic properties to seismic loading.

The elastic one-degree-of-freedom system is first discussed in Chapter 2. The equilibrium equations are presented and the concept of response spectrum is discussed.

Chapter 3 introduces the elastoplastic beam element in ADINA. It describes the element and outlines some simple static tests performed using ADINA and compares the results obtained with the theoretical results.

The inelastic one degree of freedom system under seismic loading is discussed in Chapter 4. Newmark’s inelastic spectrum is discussed. Analyses of inelastic single degree of freedom systems were performed using ADINA. These results are also presented.

The most common methods used in the seismic design of multistory buildings are discussed in Chapter 5. A simple multistory structure is used to illustrate the response spectrum method.

The same building considered in Chapter 5 is analyzed inelastically using ADINA in Chapter 6. The beams of the structure were modeled as elastoplastic and a dynamic analysis is performed. The accelerogram of the 1940 El Centro earthquake (SE component) is used with increasing values of peak ground acceleration. The analysis is repeated with the first two floors alone modeled as elastoplastic. The entire structure is then modeled as elastic and a large displacement dynamic analysis performed using the same ground acceleration but including the effects of gravity. Finally, both material and geometric
nonlinearities are included in a single analysis. The results of the above analyses are presented and discussed in Chapter 6.
2. The Elastic One-Degree-of-Freedom System under Seismic Loading

2.1 The elastic one-degree-of-freedom-system

Figure 2.1 is a schematic diagram of a typical one-degree-of-freedom system under seismic loading.

The relative motion, $y$, is defined as

$$y = u - u_g$$

where $u$ is the absolute displacement and $u_g$ is the ground acceleration.

The dynamic equilibrium equation of the system is

$$m\ddot{u} + c\dot{y} + ky = 0$$

Substituting $\ddot{u} = \dot{y} - \ddot{u}_g$ in equation 2.2 gives
\[ m\ddot{y} + c\dot{y} + ky = -m\ddot{y}_g \] 2.3

We define

\[ \frac{c}{m} = 2\xi \omega \] 2.4

where \( \xi \) is the fraction of critical damping, and

\[ \frac{k}{m} = \omega^2 \] 2.5

Dividing equation 2.2 by \( m \), and substituting equations 2.4 and 2.5 we get

\[ \ddot{y} + 2\xi \omega \dot{y} + \omega^2 y = 0 \] 2.6

For undamped systems (i.e. \( \xi = 0 \))

\[ \ddot{y} = -\omega^2 y \] 2.7

and

\[ \ddot{y}_{\text{max}} = -y_{\text{max}} \] 2.8

For damped systems, when \( y = \ddot{y} \) is a maximum, the relative velocity, \( \dot{y} = 0 \)
The acceleration associated with the maximum relative acceleration is given by

\[ \ddot{u} = -\omega^2 \ddot{y} \]  

While this value is not the actual maximum absolute acceleration, it is a very good approximation.

Therefore, for both undamped and moderately damped systems the maximum absolute acceleration can be considered to be proportional to the maximum relative displacement.

\[ |\ddot{u}_{\text{max}}| = \alpha^2 |\gamma_{\text{max}}| \]  

The idea of spectral or “pseudo” quantities is based on this relationship. The relative displacement, \( S_d \) and pseudo-acceleration, \( S_a \) are defined as

\[ S_d = |\gamma_{\text{max}}| \]  

\[ S_a = \alpha^2 S_d \]  

A third quantity, the pseudo-velocity, \( S_v \), which is not the actual maximum relative velocity, but is related to it, is defined as

\[ S_v = \alpha S_d \]
2.2 Response Spectra

A response spectrum is a plot of the maximum response (typically displacement, velocity, or acceleration) of single degree of freedom systems with a specific level of damping to a specified load versus the natural frequency of the system. To construct a response spectrum, each frequency of interest is considered individually. The response of a single degree of freedom system to a known earthquake acceleration time history is evaluated and the maximum value of the response being considered is recorded. The results of several such analyses are presented in one chart called the response spectrum. Response spectra can be plotted for any level of damping, provided the acceleration time history of the earthquake is known. Figure 2.2 below is the response spectrum for true relative velocity for the south east component of the 1940 El Centro earthquake.

Figure 2.2 Relative velocity response spectrum for El Centro earthquake (SE component)
The maximum response of a one-degree-of-freedom system to a particular load function can thus be obtained simply by reading the response corresponding to the natural frequency from the response spectrum. Response spectra are also used in the analysis of elastic multiple-degree-of-freedom systems in the response spectrum method. This method is discussed further in chapter 5.

2.2.1 Tripartite Spectra

It is possible to represent the values of spectral displacement, velocity and acceleration on a single chart using logarithmic scales.

Substituting $\omega = \sqrt{k/m} = 2\pi f$ in equation 2.13 and taking the base ten logarithm of this expression, we get

$$\log S_v = \log f + \log(2\pi S_D)$$  \hspace{1cm} 2.14

For constant values of $S_D$, the graph of $\log S_v$ versus frequency, $f$ is therefore a straight line with a gradient of one.

Similarly

$$S_v = \frac{S_S}{\omega} = \frac{S_s}{2\pi f}$$  \hspace{1cm} 2.15

Taking the base ten logarithm of this expression gives

$$\log S_v = -\log f + \log \frac{S_S}{2\pi}$$  \hspace{1cm} 2.16
Therefore, for a constant value of $S_a$ the graph of pseudo-velocity versus frequency is a straight line with a gradient of negative one.

It is thus possible to draw diagonal logarithmic scales sloping at angles equivalent to a gradient of plus and minus one to the abscissa for spectral acceleration and spectral displacement respectively, with frequency on the abscissa and spectral velocity on the vertical axis also with logarithmic scales. A response spectrum plotted on such a grid is called a tripartite spectrum. Figure 2.3 is an example of tripartite spectrum.

**Undamped Response Spectrum for El Centro Earthquake (SE component)**

![Undamped tripartite spectrum for El Centro earthquake](image)

*Figure 2.3 Undamped tripartite spectrum for El Centro earthquake (SE component)*
2.2.2 Factors affecting Response Spectra
Several factors have been found to influence the shape and magnitude of response spectra. Clough and Penzien\(^3\) list these factors as

1. Source mechanism
2. Epicentral distance
3. Focal depth
4. Geological conditions
5. Richter magnitude
6. Soil conditions
7. Damping ratio
8. Period

Of these, only the effects of soil condition, damping ratio, and of course period are considered in the construction of design spectra. This is due to the fact that the effect of the other parameters are not well understood, and cannot be quantified in a manner that the soil conditions and damping ratio can.

Design spectra, which are described fully in the following section are thus specified in terms of these factors only, although it is understood that the other factors have some influence on the shape of the spectra.

2.3 Elastic Design Spectra

The response spectrum is very useful in the seismic analysis of both single and multiple degree of freedom systems, provided that the acceleration record of the earthquake is available. In structural design, however, a measure of the maximum response to an earthquake that a structure is likely to experience in
its lifetime is required. Since there is no way at present to predict the exact acceleration time history of future earthquakes, a response spectrum cannot be plotted for the expected earthquake. Basic spectra have been developed which can be used in design in much the same way as response spectra. These plots are known as design spectra.

The design spectrum is widely used in earthquake resistant design, and is a central item of most design codes. The design spectrum is a smooth curve, or a series of straight line segments, and gives an estimate of the maximum acceleration, displacement, and velocity as a function of natural period or frequency, and damping. Each curve corresponds to a specific damping level. Design spectra were developed by smoothing the response spectra of several actual recorded earthquakes, thereby arriving at an 'average' response.

Design spectral shapes have been developed for general applications. These general spectra can typically be used for the design of different types of structures, at different sites, to resist earthquakes of various sizes. The design spectra need only be modified for each site and expected level of damping and maximum ground acceleration. This usually involves modification of the overall amplitude, and not the shape of the spectrum.

Several independent studies were conducted in the development of design spectra with similar results. The Newmark design spectrum is considered here.

---

4 G. W. Housner and P.C. Jennings, "Earthquake Design Criteria," Earthquake Engineering Research Institute, 1982
2.3.1 Construction of Design Spectra

Newmark Design Spectrum

Newmark and his colleagues found that the general shape of the tripartite spectra was similar for the variety of recorded ground motions in their study. They found that basic design spectrum can be obtained by amplifying the ground motion spectrum associated with the soil type under consideration. For high frequencies the spectral acceleration is equal to the maximum ground acceleration and for low frequencies the spectral displacement is equal to the maximum ground displacement. The intermediate frequency range can be divided into the five regions outlined by Gupta\(^5\): (1) an amplified velocity region in the middle flanked by (2) amplified displacement and (3) acceleration regions and two regions of transition, (4) from the maximum ground displacement to the maximum ground displacement to the amplified spectral displacement and (5) from the amplified spectral acceleration to the maximum ground acceleration. Figure 2.4 shows the general shape of the elastic design spectrum.

Design spectra can thus be developed by amplifying ground motion spectra. In their analyses, Newmark et al considered two types of soil conditions on which they based the ground motion spectra. For a maximum ground acceleration of $1g$, it was found that the maximum ground displacement and velocity were 0.9144 m (36 in) and 1.2192 m/s (48 in/sec) for alluvial soil, and 0.3048 m (12 in) and 0.7112 m/s (28 in/sec) for rock. These values are in general correct for practical applications.

The recommended amplification factors for displacement, velocity, and acceleration for alluvial soil are based on the response spectra for past earthquakes available. These factors are thus periodically updated as more data becomes available over time. The amplification factors suggested in reference texts for construction of the Newmark design spectrum reflects this evolving nature. Table 2.1 is presented by Paz. Table 2.2 is presented by Gupta.

---

When the amplification factors in Table 2.2 the values for the 84.1% probability level are usually used.
The recommendation for the transition from the amplified acceleration to the ground acceleration is that this region begin at a frequency of 6 Hz for all values of damping and slope downwards until it intersects the line of maximum ground acceleration. For a system with 2% damping these lines intersect at a frequency of 20 Hz. The lines corresponding to other damping values are drawn parallel to this line. This corresponds to the transition zone ending at 40, 20, and 20 Hz for damping ratios of 0.5%, 5% and 10% respectively.
3. The Inelastic Beam Element in ADINA

3.1 Introduction

Before a finite element analysis of a problem can be attempted, it is necessary to idealize the physical problem with an appropriate mathematical model. This requires making certain assumptions which will together lead to the formulation of the governing differential equations. Assumptions are made on the following:

- Geometry
- Kinematics
- Material Law
- Loading
- Boundary conditions

The finite element software, ADINA (Automatic Dynamic Incremental Nonlinear Analysis) offers a wide range of element, material, and analysis types. While this research does not involve a detailed evaluation or study of the finite element procedures employed by ADINA, it is necessary to ensure that the problem is modeled in such a way as to yield the best possible results.

To this end, some simple primary analyses were performed using ADINA, and the results obtained compared with the known theoretical results. These results, together with material presented in the ADINA Theory and Modeling Guide, were used to arrive at the mathematical model to be used in this research.

The ultimate aim of this research is to examine how a multistory building with inelastic material properties behaves under seismic loading.
In an elastoplastic analysis of a multistory building, it is expected that plastic hinges will form at the points of greatest moment. It is desirable that these hinges form in the beams rather than in the columns. This philosophy will be reflected in the finite element analysis by modeling the columns as elastic and the beams elastoplastic.

### 3.2 Performance of the elastoplastic beam element in ADINA

Some simple tests were performed on the elastoplastic beam element shown in Figure 3.1 using ADINA. The material was considered to be elastic-perfectly plastic and the stress-strain diagram is shown in Figure 3.2. In each case the theoretical results were compared to the results obtained using this program.

![Figure 3.1 Cantilever beam element tested in ADINA](image1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>200,000 MPa</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>250 MPa</td>
</tr>
<tr>
<td>L</td>
<td>10 m</td>
</tr>
<tr>
<td>b</td>
<td>1 m</td>
</tr>
<tr>
<td>d</td>
<td>2 m</td>
</tr>
</tbody>
</table>

![Figure 3.2 stress-strain diagram for element](image2)
3.2.1 Theoretical Results

3.2.1.1 Elastoplastic beam in pure bending \( (V = 0) \)

For a rectangular cross section

\[
I_{xx} = \frac{bd^3}{12}
\]  

3.1

where \( I_{xx} \) is the second moment of area about the \( x \) axis.

Within the elastic range, the bending moment, \( M \) is given by

\[
M = S\sigma
\]  

3.2

where \( \sigma \) is the bending stress and \( S \) is the elastic modulus and is given by

\[
S = \frac{I_{xx}}{d/2}
\]  

3.3

Substituting \( \sigma_y \) in equation 3.2 gives the expression for yield moment, \( M_y \)

\[
M_y = S\sigma_y
\]  

3.4

In the above model

\[
I_{xx} = \frac{2^3}{12} = 0.667m^4
\]  

3.5
\[ M_y = 0.667 \times 250 = 166.75 \text{MNm} \]

The force \( H_y \) at which initial yielding occurs is therefore given as

\[ H_y = \frac{M_y}{L} = 16.67 \text{GN} \]

A graph of applied load, \( H \) versus displacement can thus be expected to become nonlinear at this point.

The ultimate moment \( M_u \) that will cause the section to become fully plastic is given by

\[ M_u = \sigma_y Z \]

where \( Z \) is the plastic section modulus and for a rectangular section

\[ Z = \frac{bd^2}{4} \]

For our model

\[ M_u = 250 \times 1 = 250 \text{GNm} = F_u L \]

where \( F_u \) is the ultimate load. A graph of applied load, \( H \) versus displacement, \( u \) will have an asymptote of 25GN.
3.2.1.2 Elastoplastic beam under lateral load plus constant axial load

The above diagram shows the stress distribution in the section due to both axial and lateral load. The total stress diagram is obtained by adding the stresses obtained if the loads are applied independently.

Let stress in member due to axial load, \( V \) be \( \sigma_a \) where

\[
\sigma_a = \frac{V}{A}
\]

and \( A \) is the cross sectional area. Let \( \sigma_b \) be the bending stress; the yield stress is reached when

\[
\sigma_1 = \sigma_a + \sigma_b = \sigma_y
\]

The yield moment is given by

\[
M_y = (\sigma_y - \sigma_a)S
\]
Substituting different values for \( V \) in equation 3.11, and solving equations 3.13 and 3.14 yields the values listed in table 1.1 below.

### Table 3.1 Theoretical Results

<table>
<thead>
<tr>
<th>Applied constant vertical load, ( V ) (GN)</th>
<th>Horizontal load, ( H_y ) (GN) at which yielding occurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>13.33</td>
</tr>
<tr>
<td>200</td>
<td>10.0</td>
</tr>
<tr>
<td>300</td>
<td>6.67</td>
</tr>
<tr>
<td>400</td>
<td>3.33</td>
</tr>
</tbody>
</table>

#### 3.2.1.3 Elastoplastic beam under an increasing vertical load, \( V \) and a small constant horizontal load, \( H \)

Based on theory, we expect that a graph of applied vertical load versus displacement, \( u \) will show that \( u \) remains constant at a value corresponding to the displacement due to the small load \( H \) as \( V \) increases until the section becomes plastic. After this the displacement \( u \) increases rapidly while \( V \) remains constant. In the ADINA analysis \( H \) was taken as 1GN.
3.2.2 ADINA Results

![Figure 3.4: Applied Load, H vs. Displacement, u for different values of V](image)

**Table 3.2 ADINA Results**

<table>
<thead>
<tr>
<th>Applied constant vertical load, V(GN)</th>
<th>Horizontal load, $H_x$(GN) at which yielding occurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>16.64</td>
</tr>
<tr>
<td>100</td>
<td>13.32</td>
</tr>
<tr>
<td>200</td>
<td>10.0</td>
</tr>
<tr>
<td>300</td>
<td>6.64</td>
</tr>
<tr>
<td>400</td>
<td>3.32</td>
</tr>
</tbody>
</table>
The above results show very good correlation with the theoretical results. This suggests that the mathematical model and iteration methods used here should be suitable for this research. The following is brief description of the element and material types, and the iteration method. More detailed information can be found in the ADINA Modeling Guide.

### 3.3 Details of the ADINA model chosen

**Element type**

The beam element was used in these analyses. This element is based on the Bernoulli-Euler beam theory. The user can request that corrections be made for shear deformation effects; however this option was not selected for the analysis.
The nonlinear elastoplastic beam element matrices are formulated using the Hermitian displacement functions.

In nonlinear analysis only the rectangular or circular cross sections can be used for beam elements.

**Material**

The material is modeled as a plastic bilinear material with isotropic hardening. The material is assumed to be elastoplastic. This means that the gradient of the first part of the curve is equal to the Young’s modulus, \( E \), while the other portion has a zero gradient (i.e. is perfectly plastic).

**Kinematics**

Small displacements and rotations were assumed in the analysis of the beam under lateral load only as well as the analysis of the beam under lateral loading with a constant axial load. For the analysis with an increasing vertical load and small constant horizontal load the large displacement option was chosen.

**Iteration Method**

Of the iteration methods available in ADINA, the BFGS (Broyden-Fletcher-Goldfarb-Shanno) method with line searches was the one which yielded the results above. This method is an alternative to the Newton-Raphson iteration method. It is considered to be an example of a quasi-Newton method, and is a compromise between the full Newton-Raphson method, and the modified Newton-Raphson method. This method is described in detail by Bathe\(^7\).

---

4. The Inelastic One-Degree-of-Freedom System Under Seismic Loading

4.1 Newmark's Inelastic Response Spectrum

In structural dynamics elastoplastic material properties are usually assumed in inelastic design. Design spectra for inelastic design are a series of curves which are the same shape as the corresponding elastic spectra, but the elastic curves are displaced downward by a factor related to the ductility ratio. The ductility ratio is defined as

\[ \mu = \frac{y_{\text{max}}}{y_y} \]  

where \( y_{\text{max}} \) is the maximum displacement of the elastoplastic system under the applied load, and \( y_y \) is the displacement when the material first yields (see Figure 4.1).
4.1.1 Construction of Inelastic Design Spectra

Newmark’s inelastic design spectrum for a specific ductility ratio and damping value can be obtained from the corresponding tripartite elastic design spectrum by the following process which is illustrated in Figure 4.2 below.

For the constant displacement portion of the spectrum (the left hand side), a line is drawn parallel to the constant displacement line of the elastic spectrum. This line is drawn to correspond to a value of spectral displacement equal to the elastic value reduced by the ductility factor.

The constant velocity middle portion of the inelastic spectrum is obtained similarly by drawing a line reduced by the ductility factor parallel to the central horizontal line.

For the acceleration region, however, the recommended reduction factor is $\sqrt{2(\mu - 1)}$. The line is extended to a frequency of about 6 Hz. The spectrum is completed by joining this point to the point on
the elastic spectrum where the transition line between spectral acceleration and maximum ground acceleration intersects the line of maximum ground acceleration.

The spectral acceleration and spectral velocity are directly obtained from the inelastic spectrum described above. The values of displacement read from the graph, however, must be multiplied by the ductility factor in order to obtain the actual maximum displacements.

For the perfect elastoplastic spring the limit of the maximum acceleration is given by

\[ u_{\text{max}} = \frac{F_y}{m} = \frac{k_0y_y}{m} = \omega_0^2 y_y \]  \hspace{1cm} 4.2

where \( k_0 \) is the initial stiffness of the system and \( \omega_0 \) the initial frequency in rad/s.

From the definition of ductility ratio

\[ y_{\text{max}} = \mu y_y \]  \hspace{1cm} 4.3

If the maximum acceleration for the elastoplastic system is defined as the pseudo-acceleration, to obtain the maximum displacement from the same plot the value read from the displacement scale must be multiplied by the ductility ratio. The pseudo-velocity for the system, \( \omega_0 y_y \) has no real significance but is a convenient way of relating the elastic and inelastic spectra and of obtaining the values of pseudo-acceleration and pseudo-displacement from a tripartite plot.
4.2 Analysis of an inelastic one degree of freedom system using ADINA

In a dynamic analysis the frequency, \( \omega \), is a function of the mass and the stiffness of a system. When the system is modeled as a cantilever beam, therefore

\[
\omega = f(E, I, m, L)
\]

where

\( E \) = Young's modulus of the material
\( I \) = Moment of inertia of section
\( m \) = Mass
\( L \) = Length

The applied force is also proportional to the mass, \( m \). The relationship between frequency and response thus depends on all these parameters.

In this study, it was decided to vary only the mass of the system. The stiffness of the member therefore remains constant and the yield force and displacement at first yield also remain unchanged. Varying the mass changes the frequency of the system as well as the magnitude of the applied load, since the load is mass proportional.

The model shown in Figure 4.3 was subject to a transient dynamic analysis in ADINA. The solution of the finite element equation in nonlinear dynamic analysis is obtained by direct integration procedures. Implicit integration using the Newmark method was used in all nonlinear dynamic analysis in this study. The element in Figure 4.3 is the same element described in section 3.2 but with a lumped mass at the top. The yield load, yield displacement, ultimate load and ultimate displacement are thus the same as calculated in the static analysis of the previous section. Again, the bilinear material option with isotropic hardening was chosen.
In ADINA the mass proportional loading option is used to model ground acceleration. The time function used was the accelerogram for the first 53.74 seconds of the SE component of the 1940 El Centro earthquake with a peak ground acceleration of 1g. A time step magnitude of 0.02 seconds was used. The maximum relative displacement, relative velocity, and absolute acceleration were recorded. This analysis was repeated for several values of m while keeping the other parameters constant.

\[ L = 10m \]
\[ d = 2m \]
\[ b = 1m \]
\[ E = 2e11Pa \]
\[ \sigma_y = 250e6Pa \]

**Figure 4.3**

4.2.1 Theoretical results

From the static analysis described in section 3.2.1.1, the ultimate load for the cantilever beam is 25GN.

The displacement at the yield load is 0.04167m. Now

\[ F_u = \frac{\sigma_y L}{L} = ma = \frac{m}{k} ka = \frac{ka}{\omega^2} \]

The ultimate acceleration, \( a_u \) can thus be written as
\[ a_u = \frac{F_u \omega^2}{k} \]  \[ 4.6 \]

Since the ultimate load \( F_u \) cannot be exceeded it follows that the maximum acceleration cannot exceed the value in equation 4.6 above.

Figure 4.4 Inelastic response of one d-o-f system (yield stress=250MPa)
<table>
<thead>
<tr>
<th>Ω (rad/s)</th>
<th>f (cps)</th>
<th>Max. Displacement (m)</th>
<th>Ductility Ratio μ</th>
<th>Max. Acceleration (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.31831</td>
<td>0.866</td>
<td>20.78941</td>
<td>0.025723</td>
</tr>
<tr>
<td>2.390457</td>
<td>0.380453</td>
<td>0.816</td>
<td>19.57537</td>
<td>0.035725</td>
</tr>
<tr>
<td>3.651484</td>
<td>0.581152</td>
<td>0.685</td>
<td>16.44973</td>
<td>0.080414</td>
</tr>
<tr>
<td>4.472136</td>
<td>0.711763</td>
<td>0.318</td>
<td>7.639842</td>
<td>0.119719</td>
</tr>
<tr>
<td>6.324555</td>
<td>1.006584</td>
<td>0.384</td>
<td>9.215153</td>
<td>0.236527</td>
</tr>
<tr>
<td>7.071068</td>
<td>1.125395</td>
<td>0.305</td>
<td>7.32321</td>
<td>0.295483</td>
</tr>
<tr>
<td>8.164966</td>
<td>1.299495</td>
<td>0.248</td>
<td>5.946643</td>
<td>0.393866</td>
</tr>
<tr>
<td>10</td>
<td>1.591549</td>
<td>0.198</td>
<td>4.742492</td>
<td>0.589093</td>
</tr>
<tr>
<td>14.14214</td>
<td>2.250791</td>
<td>0.108</td>
<td>2.599534</td>
<td>1.133491</td>
</tr>
<tr>
<td>20</td>
<td>3.183099</td>
<td>0.0764</td>
<td>1.834731</td>
<td>2.197591</td>
</tr>
<tr>
<td>22.36068</td>
<td>3.558813</td>
<td>0.0504</td>
<td>1.209772</td>
<td>2.459009</td>
</tr>
<tr>
<td>23.90457</td>
<td>3.804531</td>
<td>0.0581</td>
<td>1.393607</td>
<td>3.035772</td>
</tr>
<tr>
<td>28.28427</td>
<td>4.501582</td>
<td>0.0547</td>
<td>1.312093</td>
<td>4.195508</td>
</tr>
<tr>
<td>31.62278</td>
<td>5.032921</td>
<td>0.0505</td>
<td>1.211761</td>
<td>4.924534</td>
</tr>
<tr>
<td>36.51484</td>
<td>5.811517</td>
<td>0.0406</td>
<td>0.97379</td>
<td>5.52045</td>
</tr>
<tr>
<td>44.72136</td>
<td>7.117625</td>
<td>0.0237</td>
<td>0.568708</td>
<td>4.836416</td>
</tr>
<tr>
<td>63.24555</td>
<td>10.06584</td>
<td>0.0111</td>
<td>0.266208</td>
<td>4.527979</td>
</tr>
<tr>
<td>70.71068</td>
<td>11.25395</td>
<td>0.00993</td>
<td>0.238408</td>
<td>5.069201</td>
</tr>
</tbody>
</table>
Figure 4.4 shows that the maximum accelerations obtained from ADINA agree with the theoretical results for most of the frequency range considered. For frequencies below 0.2Hz the values of acceleration obtained were found to be greater than the ultimate acceleration. This suggests that for the design assumptions used the results are not accurate for small frequencies.

The straight line in Figure 4.4 represents the ultimate acceleration, $a_u$, which is obtained by taking the logarithm of the maximum acceleration.

$$\log a_u = \log \frac{F_u}{k} + 2\log \omega \quad 4.7$$

The analysis was repeated with the material yield stress increased to 250e7N/m$^2$. The results of this analysis are shown in Figure 4.5 and Table 4.2.

![Figure 4.5 Inelastic response of one d-o-f system (yield stress=2500MPa)](image)

36
The force displacement curve for an elastoplastic beam in bending is not perfectly bilinear. As can be seen in Figure 4.1 there is a transition between the two straight portions of the curve. The yield force and ultimate force are two distinct values.

<table>
<thead>
<tr>
<th>$\omega$(rad/s)</th>
<th>$f$(cps)</th>
<th>Max. Displacement(m)</th>
<th>Ductility Ratio $\mu$</th>
<th>Max. Acceleration(g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.365148</td>
<td>0.058115</td>
<td>0.928</td>
<td>2.226037</td>
<td>0.008313</td>
</tr>
<tr>
<td>0.447214</td>
<td>0.071176</td>
<td>1.03</td>
<td>2.478096</td>
<td>0.011914</td>
</tr>
<tr>
<td>0.632456</td>
<td>0.100658</td>
<td>1.23</td>
<td>2.959416</td>
<td>0.022948</td>
</tr>
<tr>
<td>1.154701</td>
<td>0.183776</td>
<td>0.686</td>
<td>1.647247</td>
<td>0.07214</td>
</tr>
<tr>
<td>1.414214</td>
<td>0.225079</td>
<td>0.536</td>
<td>1.286702</td>
<td>0.103287</td>
</tr>
<tr>
<td>2</td>
<td>0.31831</td>
<td>0.880</td>
<td>2.111868</td>
<td>0.22345</td>
</tr>
<tr>
<td>2.390457</td>
<td>0.380453</td>
<td>0.789</td>
<td>1.8927</td>
<td>0.316597</td>
</tr>
<tr>
<td>3.651484</td>
<td>0.581152</td>
<td>0.573</td>
<td>1.376129</td>
<td>0.706742</td>
</tr>
<tr>
<td>4.472136</td>
<td>0.711763</td>
<td>0.486</td>
<td>1.16688</td>
<td>0.956521</td>
</tr>
<tr>
<td>6.324555</td>
<td>1.006584</td>
<td>0.559</td>
<td>1.342543</td>
<td>2.115304</td>
</tr>
<tr>
<td>7.071068</td>
<td>1.125395</td>
<td>0.521</td>
<td>1.251418</td>
<td>2.525204</td>
</tr>
<tr>
<td>8.164966</td>
<td>1.299495</td>
<td>0.409</td>
<td>0.981869</td>
<td>2.783113</td>
</tr>
<tr>
<td>10</td>
<td>1.591549</td>
<td>0.290</td>
<td>0.69661</td>
<td>2.961797</td>
</tr>
<tr>
<td>14.14214</td>
<td>2.250791</td>
<td>0.296</td>
<td>0.709798</td>
<td>6.035381</td>
</tr>
<tr>
<td>20</td>
<td>3.183099</td>
<td>0.127</td>
<td>0.305731</td>
<td>5.199987</td>
</tr>
<tr>
<td>22.36068</td>
<td>3.558813</td>
<td>0.0915</td>
<td>0.219604</td>
<td>4.668743</td>
</tr>
</tbody>
</table>
Research conducted at MIT by Sehayek\textsuperscript{8} evaluated the Newmark inelastic spectrum as a method of estimating the response of inelastic systems to earthquakes. He concluded that Newmark's method was satisfactorily accurate for the analyses he performed.

\textsuperscript{8} Sehayek, S. "Effect of Ductility on Response Spectra for Elasto-plastic Systems"
5. Elastic Analysis of a Typical Multistory Building Under Seismic Loading

5.1 Choosing a mathematical model

Multistory buildings are actually continuous systems with an infinite number of degrees of freedom. Most continuous systems cannot be solved so before a dynamic analysis of a multistory building can be performed it must first be idealized as a discrete system.

Multistory structures are usually idealized as multiple-degree-of-freedom systems. For simplicity, the columns are usually considered to be massless shear or bending beams, or a combination of the two. The mass of the structure is assumed to be concentrated at the floor levels. Figure 5.1 illustrates the levels of abstraction in arriving at the discrete model for a multistory building.

![Figure 5.1](image)

In the above figure
(a) The actual structure
(b) The structure is simplified to include only its main beams and columns but is still a continuous system.
The mass of the building is concentrated at the floor levels. Rotational inertia of the masses is neglected. The columns of this two dimensional model are idealized such that they have the same properties in shear and bending in the direction of loading as the actual structure.

Axial deformation of the members is also neglected. The girders are typically considered to be infinitely rigid.

The above method of arriving at a suitable mathematical model will be applied to a fifteen story building. The building will be idealized as a three degree of freedom system. The response spectrum method for multiple degree of freedom system dynamic problems will be illustrated using this model.

Figure 5.2 gives the general dimensions of the building which will be used in the analysis. The structure is fifteen stories tall. The average height between floors is 3m. The average mass density, $\rho$ is 250kg/m$^3$. All the floors are considered to have the same column layout. This is shown in the typical floor plan view in Figure 5.3. All the columns of the first floor are 350cm$^2$ in cross-section. The cross-sectional area of the columns will be assumed to decrease linearly with height by 20cm$^2$ per floor. The top floor columns therefore are 70cm$^2$ in cross-section. The moments of inertia will also be assumed to decrease linearly in the same proportion. The radii of gyration thus are constant for all the columns in the structure and are given in Figure 5.3. We will assume that the $y$ axis runs in an east-west direction.
The building will be modeled as a three degree of freedom system with lumped masses at elevations 15, 30 and 45m. The girders are assumed to be infinitely rigid. Rotational inertia (but not rotation) is also neglected.

The building is considered to be a continuous cantilever bending/shear beam with linearly varying properties which are equivalent to the properties of the actual structure.

The 3x3 flexibility matrix can be found by virtual work.
5.2 The Building Under Seismic Loading

The structure is assumed to suffer an earthquake in the East-West direction. The mass matrix, \( \mathbf{M} \) is given by

\[
\mathbf{M} = \begin{bmatrix}
3.375 & 0 & 0 \\
0 & 6.75 & 0 \\
0 & 0 & 6.75
\end{bmatrix} \times 10^6 \text{kg}
\]

From virtual work the stiffness matrix in the EW direction, \( \mathbf{K} \) is

\[
\mathbf{K} = \frac{1.457 \times 10^{10}}{45} \begin{bmatrix}
0.8660 & -0.9418 & 0.0460 \\
-0.9418 & 2.6624 & -1.7500 \\
0.0460 & -1.7500 & 4.2790
\end{bmatrix} \text{Nm}^{-1}
\]

The equation of motion for free vibration of undamped multiple-degree-of-freedom systems is

\[
\mathbf{M} \ddot{\mathbf{U}} + \mathbf{KU} = 0
\]

Where \( \mathbf{M} \) and \( \mathbf{K} \) are the mass and stiffness matrices respectively, and \( \mathbf{U} \) is the displacement matrix.

Substituting a solution of the form

\[
\mathbf{U} = \Phi \sin \omega t
\]

gives

\[
-\omega^2 \mathbf{M} \Phi \sin \omega t + \mathbf{K} \Phi \sin \omega t = 0
\]
\[ \mathbf{K}\Phi = \omega^2 \mathbf{M}\Phi \]

The frequencies and mode shapes are found by solving the above eigenvalue problem.

Solving equation 5.6 using the \( \mathbf{K} \) and \( \mathbf{M} \) matrices for our problem gives the matrix of eigenvalues, \( \Lambda \) as

\[
\Lambda = \begin{bmatrix}
23.4643 & 0 & 0 \\
0 & 125.5296 & 0 \\
0 & 0 & 267.0283
\end{bmatrix}
\]

The matrix of eigenvectors, \( \Phi \) is given by

\[
\Phi = \begin{bmatrix}
0.6919 & 0.6899 & 0.2128 \\
0.4666 & -0.3428 & -0.4059 \\
0.2071 & -0.3801 & 0.5591
\end{bmatrix}
\]

The frequencies and periods corresponding to the three modes of the model are given in Table 5.1 below.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency ( \omega ) (rad/s)</th>
<th>Frequency ( f ) (Hz)</th>
<th>Period (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.844</td>
<td>0.771</td>
<td>1.297</td>
</tr>
<tr>
<td>2</td>
<td>11.204</td>
<td>1.783</td>
<td>0.561</td>
</tr>
<tr>
<td>3</td>
<td>16.341</td>
<td>2.601</td>
<td>0.384</td>
</tr>
</tbody>
</table>
For seismic loading

\[ M \ddot{Y} + C \dot{Y} + K Y = -M E \ddot{U}_g \]

where

\( Y \) = Relative displacement matrix

\( \ddot{U}_g \) = Ground acceleration matrix

and

\[ U = EU_g + Y \]

Most frequently, only the degrees of freedom in one direction are considered active in earthquake analysis.

If only the x degrees of freedom are active then

\[
U = \begin{bmatrix}
    u_{x1} \\
    u_{x2} \\
    . \\
    . \\
    u_{xn}
\end{bmatrix}, \quad V = \begin{bmatrix}
    v_{x1} \\
    v_{x2} \\
    . \\
    . \\
    v_{xn}
\end{bmatrix}, \quad E = \begin{bmatrix}
    1 \\
    1 \\
    . \\
    . \\
    1
\end{bmatrix}
\]

The methods most commonly used in solving the equilibrium equation for seismic analysis are:

1. Modal superposition
2. Response spectrum method
5.3 Modal Superposition Method

We define

\[ Y = \Phi X \]  \hspace{1cm} (5.11)

Substituting for \( Y \) in equation 5.9 gives

\[ M\ddot{\Phi}X + \Phi^T C\dot{\Phi}X + K\Phi X = ME\ddot{U} \]  \hspace{1cm} (5.12)

Pre-multiplying by \( \Phi^T \) we get

\[ \Phi^T M\ddot{\Phi}X + \Phi^T C\dot{\Phi}X + \Phi^T K\Phi X = -\Phi^T ME\ddot{U} \]  \hspace{1cm} (5.13)

For ground motion in one direction only and assuming proportional damping we write for the \( j^{th} \) mode

\[ \mu_j \dddot{X}_j + 2\xi_j \omega_j \mu_j \dot{X}_j + \kappa_j X_j = -\gamma_j \mu_j \ddot{U} \]  \hspace{1cm} (5.14)

where \( \gamma_j \) is the participation factor, and \( \mu_j \) and \( \kappa_j \) are components of the modal mass and modal stiffness respectively and are given by

\[ \mu = \Phi^T M\Phi \]  \hspace{1cm} (5.15)

\[ \kappa = \Phi^T K\Phi \]  \hspace{1cm} (5.16)
Solving the differential equation 5.14 gives

\[ q_j = e^{-\xi_j \omega_j t} \left( q_{j0} \cos \omega_{Dj} t + \left( q_{j0} + \frac{\omega_j \xi_j q_{j0}}{\omega_{Dj}} \right) \sin \omega_{Dj} t \right) - \mu_j \gamma_j \tilde{U}_g * h_j \]

where the damped frequency, \( \omega_{Dj} \) is defined as

\[ \omega_{Dj} = \omega_j \sqrt{1 - \xi_j^2} \]

and the subscript 0 denotes the initial conditions.

5.4 Response Spectrum Method

The maximum modal response (relative displacement, say) is

\[ \bar{X}_j = |\gamma_j| S_{\delta j} \]

The contribution of this mode to the total structural response is

\[ \bar{Y}_j = |\gamma_j| |\phi_j| S_{\delta j} \]

The effects of all the modes can be superimposed to obtain the total response of the structure. The square root of the sum of the squares method (SRSS) method is one method whereby this is done.
In the SRSS method

\[ \bar{Y} = \sqrt{\sum_{j} Y_j^2} \]  \hspace{1cm} 5.22

For our problem, in the calculation of the participation factors we will use

\[ E = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \]  \hspace{1cm} 5.23

Solving equation 4.16 gives the following participation factors

\[ \Gamma = \begin{bmatrix} 2.0393 \\ -0.7560 \\ 0.5193 \end{bmatrix} \]  \hspace{1cm} 5.24

The spectral quantities corresponding to the frequencies of the structure are read from the response spectrum. Using the undamped response spectrum for the El Centro earthquake (Figure 2.3), the maximum relative displacements of the structure can be found. These results are summarized in the Table 5.2.
Using the SRSS method: The maximum relative displacements $\bar{Y}_i$, where $i$ is the elevation being considered, are given by equations 4.24 to 4.26.

\[
\bar{Y}_{15} = \sqrt{0.8381^2 + 0.2493^2 + 0.0249^2} = 0.8747 \text{m}
\]

5.25

\[
\bar{Y}_{30} = \sqrt{0.5652^2 + 0.1239^2 + 0.0474^2} = 0.5806 \text{m}
\]

5.26

\[
\bar{Y}_{45} = \sqrt{0.2509^2 + 0.1374^2 + 0.0653^2} = 0.2934 \text{m}
\]

5.27

Table 5.2

<table>
<thead>
<tr>
<th>Mode j</th>
<th>Frequency, $f_j$ (Hz)</th>
<th>$S_{ij}$</th>
<th>Participation Factor</th>
<th>$S_{ij}\phi_{ij}$</th>
<th>$S_{ij}\phi_{ij}$</th>
<th>$S_{ij}\phi_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.771</td>
<td>0.594</td>
<td>2.0393</td>
<td>0.8381</td>
<td>0.5652</td>
<td>0.2509</td>
</tr>
<tr>
<td>2</td>
<td>1.783</td>
<td>0.478</td>
<td>-0.7560</td>
<td>-0.2493</td>
<td>0.1239</td>
<td>-0.1374</td>
</tr>
<tr>
<td>3</td>
<td>2.601</td>
<td>0.225</td>
<td>0.5193</td>
<td>0.0249</td>
<td>-0.0474</td>
<td>0.0653</td>
</tr>
</tbody>
</table>
6. The Inelastic Analysis of A Multistory Building

6.1 The model

The same fifteen story structure used in the previous section will be used to investigate the inelastic behavior of a typical multistory structure.

The column properties are as before, with the each first floor column having a cross-sectional area of 350cm² and the cross-sectional area of the columns decreasing by 20cm² per floor such that each top floor column has a cross-sectional area of 70cm².

The building is modeled as a fifteen story, five bay frame as shown in Figure 6.1. Each column on any floor of the model has cross-sectional area and moment of inertia nine times greater than that of an actual column.

![Image of Frame analyzed using ADINA](image)

In the elastic analysis of chapter 5 the flexibility of the girders was neglected. In this analysis, however, axial shortening, and girder flexibility will be included. For simplicity, all the girders are assumed to be of uniform cross-section. The girder cross section chosen for the model is one that yields approximately
the same results as were obtained in the previous elastic analyses. The girders are also modeled as rectangular in cross-section.

**Material Properties**

In practice columns are designed to be stronger than girders. If yielding does occur, it is desirable that plastic hinges form in the girders rather than the columns. To incorporate this philosophy in the analysis the girders are modeled so that they will yield while the columns will not. This is achieved by assigning 'elastic' material properties to the columns while using an elastoplastic material of the same Young’s modulus but low yield stress to the beams. Elastic and inelastic material types cannot be used together in the same analysis in ADINA. The columns and the beams were thus both modeled as elastoplastic, with the column yield stress being so high that the columns would remain elastic throughout the analysis. Isotropic hardening was chosen for both materials.

**Degrees of Freedom**

The base of the structure is fully fixed while all other nodes have three degrees of freedom: y-translation, z-translation, and x-rotation. The mass of the building is distributed throughout the model with lumped masses at every node. These masses are assigned only in the direction of the earthquake (y direction). Each mass therefore has only one dynamic degree of freedom.

**Analysis details**

Mass proportional loading is used to model the ground motion. The earthquake used is again the SE component of the 1940 El Centro earthquake, and is applied in the y direction. Each analysis is performed using four different values of peak acceleration: 0.1g, 0.5g, 1.0g, and 2.0g.

Let us first consider the effect of increasing the peak ground acceleration for an undamped inelastic one-degree-of-freedom system.
In Figure 6.2 above systems (1) and (2) have the same yield force, $f_y$ while the yield force of system (3) is reduced by a factor $\lambda$.

The equations of dynamic equilibrium for the three systems can be written as

\[ m\ddot{y}_1 + f(y_1, y_y) = -m\ddot{u}_g \]  \hspace{1cm} 6.1

\[ m\ddot{y}_2 + f(y_2, y_y) = -m(\lambda\ddot{u}_g) \]  \hspace{1cm} 6.2

\[ m\ddot{y}_3 + f\left(y_3, \frac{y_y}{\lambda}\right) = -m\ddot{u}_g \]  \hspace{1cm} 6.3

where $y_1$, $y_2$, and $y_3$ are the relative displacements of the three systems, and $y_y$ is the yield displacement associated with a yield force of $f_y$.

Equation 6.2 can be rewritten in dimensionless form as
\[ m\ddot{y}_2 + f_y F \left( \frac{y_2}{y_y} \right) = -m\ddot{\mu}_g \]  

6.4

with \( F = F \left( \frac{y_2}{y_y} \right) \) being the dimensionless inelastic force. Dividing by \( \lambda \) we get

\[ m\frac{\ddot{y}_2}{\lambda} + f_y F \left( \frac{y_2}{y_y} \right) = -m\ddot{\mu}_g \]  

6.5

which can be written as

\[ m\frac{\ddot{y}_2}{\lambda} + f_y F \left( \frac{y_2}{y_y} / \lambda \right) = -m\ddot{\mu}_g \]  

6.6

Let us define

\[ y_3 = \frac{y_2}{\lambda} \]  

6.7

Substituting in equation 6.6 above gives

\[ m\ddot{y}_3 + f_y F \left( \frac{y_3}{y_y / \lambda} \right) = -m\ddot{\mu}_g \]  

6.8

Equation 6.8 is equivalent to equation 6.3. This shows that scaling up the ground acceleration while keeping all other parameters constant produces the same effect as scaling down the yield stress and keeping the ground acceleration constant.
A similar reasoning can be applied to multiple degree of freedom systems. Thus, the results of analyses performed at various levels of peak ground acceleration can be used to investigate the behavior of a structure at different values of yield stress by dividing these results by the peak ground acceleration.

### 6.2 Analysis with all beams elastoplastic

A transient dynamic analysis of the model with all the girders elastoplastic was performed for four different values of peak ground acceleration. The absolute acceleration, relative displacement, and relative velocity time histories were recorded for each analysis at nodes 2, 10 and 17 (see Figure 6.3). The absolute acceleration time histories were used to compute the floor response spectra at these nodes. These spectra were normalized by the peak ground acceleration.

**Figure 6.3** Frame showing node numbers
6.2.1 Results

Figure 6.4 Floor response spectra at node 2-All beams elastoplastic

Figure 6.5 Floor response spectra at node 10-All beams elastoplastic
The normalized results are equivalent to the results we would obtain if a constant peak ground acceleration of $1g$ had been used and the yield stress of the beams was reduced for each analysis. In the normalized response spectra, therefore, the curves correspond to different levels of yield stress, with the curve of the highest value of peak ground acceleration representing the lowest yield level in the normalized system.

The floor response spectrum at any point in the structure gives the response of an oscillator positioned at that node. If the floor response spectrum is plotted using tripartite logarithmic scales the curve will tend asymptotically to the maximum absolute acceleration of the floor at high frequencies. At low frequencies the response spectrum tends asymptotically to the maximum absolute floor displacement.
Figure 6.4 shows the floor response spectra at node 2. For most frequencies a decrease in yield level does not produce any significant change in the results. Figure 6.4 suggests that neither the acceleration nor the total displacement of the first floor is significantly affected as the yield level of the beams is changed.

At node 10 (see Figure 6.5) as the yield stress decreases the shape of the floor response spectrum remains essentially unchanged but is displaced downwards. At high frequencies the acceleration of the oscillator decreases with yield stress, indicating that the acceleration of the floor also decreases with yielding level.

At low frequencies the spectral accelerations also decrease with yield level which suggests that the absolute floor displacement also decreases.

Figure 6.6 shows that the same is true for the floor response spectra of node 17.

6.2.2 Interpretation of results

As the floors of the building are allowed to yield, the stiffness of the system decreases. The building becomes increasingly more flexible and the displacements and accelerations decrease. Since yielding has been restricted to the girders only, the behavior of the structure from elevation 0m to 3m is not significantly affected by a change in yield level. This explains why the floor response spectra at node 2 remain essentially unchanged as the yield level is decreased.

6.3 Analysis with first two floors elastoplastic

The analysis was repeated with only the first two floors elastoplastic. These results (Figure 6.7 to Figure 6.9) are similar to those of the previous analysis. Comparison of the two sets of results indicate that the spectral accelerations with all the beams elastoplastic are slightly lower than those with just the first two floors elastoplastic.

This is more evident in Figure 6.10 to Figure 6.13. Figure 6.10 and Figure 6.12 show the normalized relative displacement (or the deformed shape) of the columns along line A for the cases of all the beams elastoplastic, and the first two floors elastoplastic respectively. These figures show that displacement
decreases with yield level, and that this effect is more significant when all the beams are allowed to yield.

Figure 6.11 and Figure 6.13 again show the deformed shape of the columns along line A but in both figures the displacement at node 17 is kept constant for all yield levels. Comparison of these two figures show that the deflected shapes corresponding to a peak ground acceleration of 1g are almost identical. At a peak ground acceleration of 2g the differences are more pronounced. These results suggest that the beams of the lower floors yield first. As the yield level is decreased the behavior of the system with all beams elastoplastic appears to be significantly affected by the yielding of the upper floors.

6.3.1 Results

![Floor Response Spectra Normalized to 1g](image)

Figure 6.7 Floor response spectra at node 2-First two floors elastoplastic
Figure 6.8 Floor response spectra at node 10-First two floors elastoplastic

Figure 6.9 Floor response spectra at node 17-First two floors elastoplastic
Figure 6.10 Normalized relative displacements along gridline A-All beams elastoplastic

Figure 6.11 Normalized relative displacements along gridline A (with top displacement constant)-All beams elastoplastic
Figure 6.12 Normalized relative displacements along gridline A-
First two floors elastoplastic

Figure 6.13 Normalized relative displacements along gridline A
(with top displacement constant)-First two floors elastoplastic
6.4 P-Δ Effects

When gravity loading is included in an analysis involving lateral loads additional moments are produced. This effect is known as the P-Δ effect.

6.4.1 Single-degree-of-freedom system with P-Δ effects

Consider the single degree of freedom system. For a cantilever beam under axial load only, the critical vertical load, \( P_{\text{crit}} \), is defined as

\[
P_{\text{crit}} = \frac{\pi^2 EI}{4L^2}
\]

The applied vertical load, \( P \), is given by

\[
P = mg
\]

When vertical loading is considered in the seismic analysis of one degree of freedom systems, the effective frequency of the system changes. It has been found that the relationship between this new frequency, \( \omega' \), and the true frequency can be approximated as

\[
\omega' \approx \omega \sqrt{1 - \frac{P}{P_{\text{crit}}}}
\]

For the cantilever beam

\[
\frac{P}{P_{\text{crit}}} = \frac{4mgL^2}{\pi^2 EI} = \frac{4mL^3}{3EI} \frac{3g}{\pi^2 L}
\]
We recall that for the cantilever beam

\[ k = \frac{3EI}{L^3} \]  \hspace{1cm} 6.13

and

\[ \omega^2 = \frac{k}{m} \]  \hspace{1cm} 6.14

Substituting equations 6.13 and 6.14 in equation 6.12 gives

\[ \frac{P}{P_{\text{crit}}} = \frac{12g}{\pi^2 L k} = \frac{12g}{\pi^2 L \omega^2} \]  \hspace{1cm} 6.15

### 6.4.2 Multiple-degree-of-freedom system with P-\( \Delta \) effects

The building is now analyzed with all the beams and columns elastic and a peak ground acceleration of 1g. The analysis was repeated including gravity loading. The acceleration time histories at nodes 2 and 17 were once again used to obtain the floor response spectra at these floor levels.

Figure 6.15 shows that the inclusion of P-\( \Delta \) effects causes a reduction in the maximum displacement and acceleration of the top floor.

Figure 6.16 and Figure 6.17 show the results obtained when the building floors are modeled as elastoplastic and P-\( \Delta \) effects are included. When P-\( \Delta \) effects are considered, equations 6.1 to 6.10 no longer hold and scaling the normalizing the results is not applicable. However, comparison of the results with and without P-\( \Delta \) effects implies that the P-\( \Delta \) effect exaggerates the effect of yielding.
Figure 6.14 Floor response spectra at node 2 - Elastic analysis

Figure 6.15 Floor Response spectra at node 17 - Elastic analysis
Figure 6.16 Floor response spectra at node 2 with all floors elastoplastic and including P-Δ effects

Figure 6.17 Floor response spectra at node 17 with all floors elastoplastic and including P-Δ effects
6.5 Conclusion

The performance of a structure in an earthquake is influenced by several factors including the construction material, framing system, and connection details. We cannot hope to simulate the behavior of an actual structure to perfect accuracy since it is impossible to predict and duplicate the exact site conditions, ground motion and difficult to model every structural detail. However, research and computer modeling does serve to teach us about the response of buildings to ground shaking.

We would expect that changing the material properties of the structure, the ratio of the stiffnesses of the columns and the beams, the size of the interior and exterior columns, and the layout of the columns and beams would result in a change in the inelastic behavior of the building. How this behavior will be affected cannot be predicted through the analysis of one structural system. Indeed, even if several system were to be analyzed and tested, the results would not necessarily be applicable to the inelastic design of all multistory buildings.

Thus, it is impossible to make general statements on the inelastic behavior of multiple-degree-of-freedom systems based on the results of the analyses performed. These results can only be considered to reflect the actual behavior of this particular model.

While finite element programs like ADINA and other structural analysis software are widely used in industry today, their use in the analysis of structures is not always economically feasible. In elastic analysis, the response spectrum and modal analysis methods can be used to predict the behavior of multi-degree-of-freedom systems. The response spectrum method for inelastic systems attempts to provide a similar alternative to finite element analysis for inelastic multi-degree-of-freedom systems. This research does not prove or refute the accuracy of this method. Further studies of the behavior of several different types of inelastic multiple degree of freedom systems would have to be performed if the response spectrum method for inelastic systems is to be evaluated.
This research does confirm that a decrease in yield level results in a reduction of the absolute accelerations, and displacements of this system as it does for the one-degree-of-freedom-system.

Similarly, including the effects of gravity also reduce the accelerations and displacements of the multi-degree-of-freedom-system considered.
7. Bibliography


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