Models for Intermodal Depot Selection

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ABSTRACT

The objective of this research is to develop mathematical models to assist intermodal carriers in determining their inland depot network – the location and capacity of each depot and warehouse.

Our modeling framework embraces the fundamental features of the inland depot selection problem – the hierarchical decision-making process, the multi-period structure in multiple levels, and the balancing requirements. A mixed integer programming model for determining the optimal location of inland depots is developed in the context of container liner shipping. The model, called the multi-period model with balancing requirements (MPB), is superior to the models in the literature, because it incorporates the above fundamental features. We propose two solution algorithms, price-directive and resource-directive decomposition algorithms. MPB is successfully applied to a real-world inland depot selection problem facing a leading international shipping company.

The concept of container supply chain management is developed so that the MPB model’s implementation and deployment issues, especially the institutional barriers, can be addressed from the perspective of integrated chain movement.

MPB is further improved in two aspects. First, a procedure, called the selected artificial-depot procedure (SAD), is proposed to handle direct container movement between consignees and shippers, which can significantly reduce transportation cost. Second, the uncertainty in container demand and supply is handled by an MPB-based simulation model, which integrates the MPB optimization model into a statistical simulation model. The MPB-based simulation model is capable of providing decision makers demand satisfaction levels and associated statistical confidence that can be used as feedback to re-run the MPB optimization model. Numerical examples are provided in both cases.

Thesis Supervisor: Ernst G. Frankel
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Chapter 1

Introduction

1.1 Background

Since the 1970’s, freight transportation activities have seen fundamental changes in technology and in organization. In particular, one must note the increasing use of intermodal transportation routes and of unitized transportation practices. A transportation carrier delivers products to its clients using different types of vehicles in various transportation modes: railcars, trucks, containers, etc. After delivery and unloading by the customers, the empty vehicles (railcars, trucks, or containers) are shipped back to a depot or warehouse designated by the carrier for subsequent shipment to other customers for shipment. After loading the new customer’s products, the vehicles are transported directly to their destination or through some intermediate depots.

Under this type of logistics network, vehicles spend a significant amount of time in empty movements. For a major European shipping company with over 300,000 land
container movements and with an estimated total distribution and transportation cost of some US $50,000,000 in 1986, 40% of the movements were empty (Crainic et. al., 1993). In the US rail system, it is estimated that a railcar is empty during 40% of its average car cycle (Mendiratta, 1981).

The importance of empty vehicle movement in carriers' cost structure has inspired a number of studies directed at managing empty vehicle fleets for railroads, liner shipping operators, and truckers. A fundamental question these studies address is: Given an inland logistics network, how should the carrier dispatch empty vehicles to meet shippers' demand, to relocate empty vehicles among depots and warehouses, and to lease on/off vehicles in preparation for future demand.

While these studies have achieved some degree of success in reducing empty vehicle-related operational costs, a more fundamental and strategic question is how to design the underlying logistics network; namely, how to determine the location and capacity of depots and warehouses, in order to achieve a broader and more significant cost reduction.

A distinct characteristic of the depot selection problem is its balancing requirements. Due to regional imbalances in empty vehicle demand and supply over the network at any point in time, some depots are short of empty vehicles, whereas others have too many. This requires the shipment of empty vehicles between depots to rebalance inventory. Due
to periodic changes in supply and demand patterns, optimal rebalancing can only be achieved in a multi-period framework.

In recognition of the lack of research on the depot selection problems with balancing requirements, the objective of this thesis is to develop a decision-support tool to assist transportation carriers in determining their inland depot network – the location and capacity of each depot and warehouse.

The research work is conducted in the context of principal intermodal vehicles – marine containers in liner shipping. Nevertheless, the research methodology, modeling framework, models and solution methods, and insights developed in this thesis are equally applicable to other transportation vehicles, such as railcars, trucks, barges, domestic containers, and any combination of these vehicles – intermodal transportation. These vehicles share some basic characteristics in terms of model development; for example, the balancing requirements, the hierarchical decision making process, etc., as discussed in detail in Chapter 2.

1.2 Problem Context

The inland depot selection (IDS) problem for a liner shipping company can be described in the context of a hierarchical container-related decision-making process,
namely, designing a ship routing network, selecting an inland depot network, and managing empty containers, as depicted in Figure 1.1 below.

A containership calls ports on its routing network according to a predetermined sailing itinerary based on trade pattern, containership fleet availability and many other factors. As an off-shore shipping locational problem, ship routing network selection is one of the most
important strategic decisions for a liner operator, and is usually made and adjusted on a yearly basis by the company’s planning department and senior management. Because of the strategic importance of the ship routing network, both the container inland depot network selection and the container fleet management are subject to, and are greatly influenced by the characteristics of the underlying routing network. For example, the add/drop of a calling port may accordingly require a change in location of inland depots serving the port and a change in the empty container allocation pattern.

On a day-to-day operational basis, empty container management (allocation) means, to a large extent, to dispatch empty containers to meet shippers’ current demand, to relocate empty containers in preparation for future demand, and to lease on/off containers to adjust the deficit/surplus of the operator’s own container inventory. The allocation of empty containers may cross the off-shore ship routing network and the inland depot network, and therefore, involves seaports, inland depots, shippers’ warehouses, etc. Because of their direct and significant impact on container-related costs, empty container allocation problems have recently drawn the attention of researchers, for example, Chu (1995), Gao (1994), Crainic et. al. (1993, 1991), Powell (1992), among others.

In the level between the strategic ship routing network decision and the operational empty container management, as shown in Figure 1.1, there is another important decision issue facing liner operators. This is a land-side locational problem: that is, liner operators need to determine their inland container depot network – the location and size of each
depot. Land movement of empty containers plays a crucial role in container-related activities in liner shipping as stated before.

The inland depot selection decision is made by a shipping or intermodal company based on many factors, for example, empty container demand, supply, and cost structure, and is on the level between the long-term strategic planning of ship routing network and the short-term day-to-day operations of empty containers. We regard the inland depot selection (IDS) as a medium-range strategic/tactical planning decision, which, in container shipping companies, is usually reviewed and updated periodically from every few months to every one or two years by the container fleet management (control) department in conjunction with the planning department and with the involvement of senior management.

1.3 Research Objectives and Scope

The objectives of this thesis research are to establish a framework for liner shipping and intermodal transportation land-side operations, to develop mathematical models to solve the inland depot selection (IDS) problem, and to provide liner shipping and intermodal operators a practically useful decision-support tool to assist them in effectively and efficiently planning and operating inland depots and container (or other intermodal vehicle) fleets.
In particular, the models should be capable of: (a) incorporating empty container demand, supply, and various costs associated with depots and containers; (b) integrating the medium-term strategic/tactical IDS problems with the short-term empty container allocation problems; (c) taking into account the multi-period feature of the problem, characterized by balancing requirements; (d) handling uncertainty in shipping and intermodal operations; (e) providing a least-cost solution for selecting appropriate location and size of depots and for accordingly allocating containers among depots; and (f) addressing the model's implementation issues, especially the institutional barriers to the model's successful deployment.

Therefore, the scope of the research covers the second and third levels in Figure 1.1, with the first level configuration as given. An illustrative diagram depicting the scope of the study is presented in Figure 1.2 below.

In Figure 1.2, the underlying system consists of demand and supply customers, and potential depots. Some depots and demand customers can obtain empty containers from the outside, while some depots and supply customers may return spare containers to the outside.

---

1 In this thesis, we use demand customer and shipper interchangeably, and supply customer and consignee interchangeably.
Figure 1.2 The Scope of the Study of the IDS Problem
1.4 Thesis Structure

The thesis is organized as follows.

In the next chapter, we develop the framework for the IDS problem. First, we present an overview of the intermodal transportation and container liner shipping industry. It is hoped that this overview will deepen our understanding of, and serve as the basis of, the framework for the IDS problem. Next, the IDS problem is formally defined, and a modeling framework, incorporating the fundamental features of the problem is established. The features include the hierarchical decision making processes, the multi-period decision making structure on multiple levels, and the balancing requirements. The establishment of this framework is one of the major contributions of the research. We then review the literature on the IDS models. The only papers addressing the problem (Dejax et al., 1986; Crainic et al., 1989; Crainic et al., 1993; Crainic and Delorme, 1993; and Gendron and Crainic, 1995) use essentially the same model as the classical single-period location-allocation models. Thus, the fundamental feature of IDS – balancing requirements – is unfortunately unaddressed. For this reason, we extensively review the relevant research on facility location models and empty vehicle allocation models.

Chapter 3 is devoted to building mathematical models about the IDS problem. Because of the heterogeneity of locations, including inland and seaport depots, and demand and supply customers, the underlying network is defined and delineated in a 3-dimensional time-depot-customer network. Due to the lack of previous research and the
complexity of the problem, it is necessary to identify and explain the important factors contributing to IDS decisions. A single-period model is then built as the central component of the multi-period model, to gain better understanding of the problem's complexity, and to demonstrate the inadequacy of the single-period modeling approach. The final multi-period model with balancing requirements, called MPB, is developed at the end of the chapter. MPB is a large-scale mixed integer program.

In order to solve MPB efficiently, in Chapter 4 we critically evaluate methods for solving large-scale locational models like MPB, including solution methods for multi-period location models and single-period location-allocation models. We propose two decomposition-based algorithms for solving MPB, the resource-directive (Lagrangian) decomposition algorithm and the price-directive (Benders) decomposition algorithm.

Chapter 5 is concerned with the MPB model’s implementation and deployment in practice. We first apply the MPB model to solve a real-world depot selection problem encountered by a major container shipping company in North America. To address the issues of model implementation and deployment, we develop the concept of container supply chain management so that we are able to examine the sophisticated implementation issues from this perspective. This will help us better understand the institutional barriers behind the IDS problem.
When developing the MPB model in Chapter 3, we assume that there is no direct container movement allowed between shippers and consignees and that the demand for and supply of empty containers are deterministic. These two assumptions are removed in this chapter with the hope of making the model theoretically more sound and practically more useful as a decision-support tool. First, we develop an efficient procedure, called the selected artificial-depot (SAD) procedure, to solve the IDS problem permitting direct movement between shippers and consignees. The uncertainty in container demand and supply is handled by an MPB-based simulation model, which integrates the MPB optimization model into a statistic simulation model. The MPB-based simulation model is capable of providing decision makers the level of customer demand satisfaction and associated statistical confidence that can be used as feedback to re-run the MPB optimization model.

In Chapter 7, the thesis is summarized and concluded, and further research directions are recommended.
Chapter 2
Framework of the IDS Problem

This chapter is devoted to establishing the modeling framework for the IDS problem. First, an overview of intermodal and liner shipping operations is presented. It is hoped that this overview will deepen our understanding of, and serve as the basis of, the framework for the problem. Next, the IDS problem is formally defined, and a modeling framework, incorporating the fundamental features of the problem is established. We then extensively review the relevant research on facility location models and empty vehicle allocation models. Finally, the significance of the research is discussed and the chapter is summarized.

2.1 Overview of Intermodalism and Container Transportation

Liner shipping differs from tramp shipping in many ways. As a common (public) carrier, a liner operator usually provides shipping services with fixed calling ports on a trade route, regular sailing schedules, stable freight rates and uniform bills of lading.
The liner shipping industry pioneered the concept of containerization, which first revolutionized their own business, and subsequently also fundamentally changed the way domestic truckload and carload freight were being handled. Therefore, in many aspects, shipping lines have been the major driving force behind intermodal freight transportation in the United States and the rest of the world. Further, the industry conceived the idea of land-bridge (mini-bridge, micro-bridge) that was the first example of truly intermodal sea-land service. Finally, it was the American President Lines (APL) which first introduced the double-stack service, a breakthrough for intermodal service in the mid-1980’s.

2.1.1 Sea-Land Intermodalism and Container Liner Shipping

Containerization has numerous advantages. It has reduced both the time and the costs of loading and unloading operations at ports and terminals. The turnaround time of ships has been reduced, and the utilization of ship fleets has increased. Losses to cargo due to damage and pilferage has diminished.

Another major innovation in sea-land intermodalism was the launch of so-called bridge services, which refer to a combined ocean-land movement from a foreign port to a domestic port, where the land transportation replaces an ocean leg. It is called a minibridge if the destination of the movement is a domestic port, and a microbridge if the destination is an inland point. An example of a minibridge would be Hong Kong-Los Angeles-New York, with the LA-NY portion provided by rail as opposed to the all-water
route through the Panama Canal. Hong Kong-Los Angeles-Chicago is an example of microbridge service.

Landbridge has been positioned as a premium service for travel-time sensitive freight. It is estimated that landbridge between Asia and the US East Coast can have a 6-12 days shorter trip time than all-water. This shorter trip time not only reduces in-transit inventory cost, it also allows for shorter vessel cycles and therefore higher annual payloads per vessel.

APL took the idea of landbridge to the next level when it pioneered regular double-stack service between Los Angeles and Chicago in 1984. This operation was subsequently copied by most other major Pacific Ocean carriers and railroads.

The Intermodalism brings new entities into the operation of shipping companies, including railroads, trucking companies, freight forwarders, shippers, consignees, regulatory agencies, etc. Coordination of activities among different entities becomes a crucial element of the efficient operation of shipping companies. An important challenge facing shipping operators is the management of intermodal equipment fleets, including containers, chassis, trailers, and their attachments, over a network composed of many ports, inland depots, rail terminals, warehouses, traffic routes of different modes, shippers and receivers.
2.1.2 Intermodal Container Movement

The status of an empty container can be summarized as follows.

- in storage:
  - at depot: ports, inland depots, container freight stations (CFSs), etc.
  - in customer’s hand: either in shippers’ or consignees’ hand
  - in idle: for repair or inspection

- in transit:
  - in ships or barges
  - in railcars or trucks

The task of empty container management is to assign empty containers to shippers, direct returned containers to depots, relocate containers among depots, and to lease on (off) empty containers from (to) lessors. This is to satisfy shippers’ demand for empty containers to load their outbound shipment over a continuous time period, and to minimize overall container-related capital investment and operating costs.

In general, shipper’s empty containers are provided through depots, either inland depots or seaports. There are five ways for an inland depot to obtain empty containers: (1) from a seaport via an inbound ship; (2) from a consignee after stripping the full containers; (3) from a leasing company by leasing on the containers; (4) from recovery of containers after repair or inspection; and (5) from other inland depots.
On the sea-side, empty containers are usually transported using the idle space aboard scheduled ships. Although there is no significant additional cost for this operation, the necessary inland transportation to/from ports, lift on/off costs at inland interchange points, and loading-on/off-ship costs may be great. It is also not unusual that space in other shipping companies' ships has to be purchased to relocate empty containers. On the land-side, empty containers are often transported using external paid transportation services such as trucking companies, railroads, and inland waterway carriers.

Empty containers are usually stored in a depot that is owned and operated by a third party, often a railroad, trucking company, port authority, etc. Depot usage costs usually consist of two parts: fixed charges for rendering the shipping company the right to use the storage facility over a period of time, ranging from several months to a few years, and a daily storage fee for each container stored in the depot.

2.1.3 Container Inland Depot Network

Containerization has been seen as primarily a maritime technology. The progression of container shipping services in the 1970s and early 1980s led to a restructuring of port systems around the world. Within the past several years, however, the most significant innovations in containerization have occurred inland. Not only has there been an important geographical shift in the movement of maritime containers between inland
destinations and ports, but the "boxes" themselves are beginning to replace other cargo systems in domestic freight transportation.

The current container inland depot network, also called the intermodal network, has evolved in a vastly different way from its predecessor, which was oriented towards individual depots, and was characterized by a very large number of ramps in each market center. The current container intermodal network, on the other hand, possesses a distinct hub and spoke structure, with traffic being concentrated at a number of load centers, usually linked by rail. Each hub depot services its own market area by truck. Some of the major players in the system, such as Burlington Northern, have adopted a specific strategy of concentrating rail traffic at a small number of hub depots, each serving an area 250 miles in radius (Slack, 1994). The result has been a concentration of traffic and a significant reduction in the number of intermodal terminals. Thus, whereas in 1978 there were 1176 intermodal depots in the US, there were fewer than 200 in only 106 cities in 1994 (Slack, 1994).

The load center concept is particularly associated with containers in regards to seaports. The maritime load center concept anticipates the development of one or two base ports on each maritime range, from which large capacity cellular ships will sail for other load center ports on other maritime ranges. It is assumed that each load center port will serve its hinterland by small feeder services or rail and highway connections. Thus, a
very concentrated pattern of container flow is predicted, with one or two ports dominating traffic on each maritime range.

A very different picture of load centers is proposed for inland hub depots. The economics of rail transport and the need for inland hubs to be served by trucking suggests that several inland load centers will be established. Thus, although there will be fewer intermodal hub depots than piggyback ramps, the load center concept suggests that there will be a more dispersed pattern of inland depots in comparison to the very small number of hypothesized base seaports.

Slack (1994) points out that the theoretical number of inland load centers that will be established is determined by the effective radius of truck drayage. Evidence from modeling and the actual policies of the railroads indicates that this radius varies between 200 and 300 miles. Taking the land area of the continental USA, 24 hubs could be packed to provide an optimal hub configuration.

The number of actual hubs in this network is considerably larger than 24, even though a remarkable concentration has already taken place. There are 79 non-port cities in the US possessing intermodal terminals with top lift equipment. This could be indicative of the inefficiencies still in the system. For example, Conrail and several other eastern railroads still maintain a dense local network of intermodal hubs (Slack, 1990). However, it must also be recognized that the US market is not uniform and the existence of a particularly
strong market relatively close to other major centers inevitably leads to the establishment of “excess” hubs. Extreme examples are the hubs established to serve automobile plants, such as Fort Wayne, despite their proximity to other hub centers. Furthermore, because the shipping lines, railroads and other intermodal carriers serve overlapping trade areas, there is no assurance that all will select the same center as the regional hub. This latter fact mirrors the situation in the location of maritime load centers. One of the reasons for the absence of a true base port load center system is that not all the shipping lines have chosen the same spatial strategy to serve a particular maritime range (Hayuth, 1988).

Table 2.1 provides estimated intermodal container traffic in major US inland intermodal centers (Manalytics, 1990). The table reveals that major inland centers handle large numbers of containers, and are comparable to the major seaports of North America. Indeed, Chicago handled more containers than any seaport in North America in 1987. Several of the major historical regional centers of the US, Atlanta, St. Louis, Salt Lake City, etc., are among the largest inland hubs.
Table 2.1 Intermodal Container * Traffic at Major Inland Centers

<table>
<thead>
<tr>
<th>City</th>
<th>Traffic in TEU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago</td>
<td>2,660,493</td>
</tr>
<tr>
<td>Dallas</td>
<td>800,277</td>
</tr>
<tr>
<td>Kansas City</td>
<td>400,771</td>
</tr>
<tr>
<td>St. Louis</td>
<td>362,198</td>
</tr>
<tr>
<td>Stockton</td>
<td>344,721</td>
</tr>
<tr>
<td>Memphis</td>
<td>343,797</td>
</tr>
<tr>
<td>Phoenix</td>
<td>338,197</td>
</tr>
<tr>
<td>Atlanta</td>
<td>263,756</td>
</tr>
<tr>
<td>Detroit</td>
<td>246,806</td>
</tr>
<tr>
<td>St. Paul</td>
<td>225,548</td>
</tr>
<tr>
<td>Fresno</td>
<td>213,101</td>
</tr>
<tr>
<td>Salt Lake City</td>
<td>181,884</td>
</tr>
<tr>
<td>Denver</td>
<td>141,245</td>
</tr>
<tr>
<td>Columbus</td>
<td>75,979</td>
</tr>
<tr>
<td>Albuquerque</td>
<td>66,492</td>
</tr>
</tbody>
</table>

Note: Including maritime and domestic containers together.

2.2 Problem Definition

The problem this research is concerned with can be formally defined as follows.

A container liner shipping company usually regards its inland depot network selection problem as a medium-term strategic/tactical planning issue, and needs to determine and adjust the network periodically, ranging from every several months to every few years so
that (1) it can supply a sufficient amount of empty containers to shippers for their outbound shipment at the right time and right location; and (2) its container-related capital and operating costs can be minimized. The given conditions prior to making the decision comprise:

- Demand for and supply of empty containers in a set of inland shippers and consignees over a time period;
- Locations and sizes of a set of candidate inland depots;
- Container storage cost and capacity in candidate inland depots;
- Transportation cost and capacity between any feasible pair of depot-depot and depot-customer; and
- Container inventory costs.

We want to know how to choose depots from the set of potential ones to minimize the total costs of depots, transportation, and container inventory, while satisfying the customer demand for empty containers, over the entire planning time period. Subsequently, a suggested pattern of empty container allocation among depots, and demand and supply customers, over the time period, can be provided to guide the day-to-day empty container allocation operations.

At first, this problem appears to be similar to those classical location-allocation ones. However, we will argue in detail in the next two sections that our IDS problem has distinct
characteristics from classical location-allocation models, and is mathematically more involved.

2.3 Modeling Framework

The purpose of this section is to construct the modeling framework to guide the subsequent model development. The framework should reflect the decision-making process and its timing structure for solving the IDS problem in a liner shipping company, capture the fundamental feature of the problem – balancing requirements – and other crucial characteristics.

2.3.1 Hierarchical Decision Making Process

As a location-allocation problem, depot selection is a longer-term decision than pure container allocation. It is a strategic/tactical decision, often involving the company’s planning department and senior management. Once a depot selection decision is made and the physical depot layout is deployed, container day-to-day allocation and management has to be based upon this deployment. On the other hand, the container allocation pattern determines container allocation and storage costs, and subsequently influences the choice of depot locations. Therefore, this location-allocation decision is a hierarchical decision-making process, where the depot location decision is made first while considering the subsequent container allocation problem. The container allocation decision is then made
based upon the given depot location decision. This hierarchical decision-making process is depicted in Figure 2.1 below.

![Hierarchical Structure of Location-Allocation Decision-Making for Depot Selection Problem](image)

**Figure 2.1: Hierarchical Structure of Location-Allocation Decision-Making for Depot Selection Problem**

### 2.3.2 Balancing Requirements

Balancing requirements in the IDS problem means to re-allocate empty containers among depots to meet anticipated demand in the subsequent periods. Balancing
requirements are essential in container depot location-allocation problems. The reasons are as follows.

First, regional container demand/supply imbalance is common in liner shipping, and requires the number of containers stored at depots to be adjusted periodically in response to anticipated future demand/supply imbalance. For example, consider an inland depot system only having two depots, A and B. If, due to previous trade patterns, the number of existing empty containers stored in A and B are 80 and 20 respectively, while the expected demand for empty containers in A and B are 30 and 70 respectively in the next period, then there is a need to transport at least 50 containers from A to B to make up the deficit of 50 in B before the beginning of the next period.

Second, empty containers repositioned from one depot (A) to another (B) may not be used at depot B in the next immediate period. The reason for this is that the operator may anticipate the container deficit at depot B two periods from now. However, in the next immediate period, there will not be appropriate capacity to transport these empty containers to depot B. So these required containers have to be shipped to depot B from depot A in the current period.

Third, because depot storage costs vary from depot to depot, sometimes container reallocation arises purely to reduce depot storage costs. For example, if the storage cost difference between two depots is higher than the transportation cost between them, there is
certainly a cost-reduction incentive for the operator to reposition some containers from the higher storage-cost depot to the lower storage-cost depot.

Balancing requirements complicate the problem in several ways: (a) In the underlying network, in addition to customer-facility (depot) movement, there is inter-depot movement to reposition containers among depots. This inter-depot movement does not exist in classical location-allocation models; and (b) Rebalancing can only be reflected in a dynamic environment, because within a single-period static model, one cannot express the sequential behavior of the containers' reallocation from depot A to depot B and their use at depot B. This can only be captured by multi-period models.

2.3.3 Two Levels of Multiple Periods

It is natural now to consider multi-period requirements in the modeling framework. The hierarchical decision-making process makes it necessary to make a distinction between the medium-term strategic/tactical decisions of depot location and the short-term day-to-day container allocation operations. The time framework for these two kinds of decisions is different and needs to be modeled on two different levels accordingly, one for locational decision, the other for allocation decisions.

Locational decisions are regarded as medium-range strategic/tactical planning decisions, which, in a container shipping company, are usually reviewed and updated from
every several months to every few years by the container fleet management department, in conjunction with the planning department and with the involvement of senior management. For instance, suppose the total length of the planning horizon is three years, and each time period is six months long. Then there are five time periods within the planning time horizon. This indicates that by using the rolling-horizon planning approach, each time the planners consider the depot location problem for the next six months, they need to look ahead up to three years. However, they will only need to implement the decision for the next immediate period, i.e., the next six months.

After determining the depot network layout, the operator needs to allocate empty containers over the depot network within the planning period, which is six months long in the above example. The operator then needs to divide the six months into several shorter sub-periods, to capture the dynamics of container demand and supply, and more importantly, to reflect the balancing requirements of periodically adjusting containers among depots. Suppose the length of each sub-period is one month. Then there are six sub-periods within the planning period of six months.

Therefore, there are two levels of multiple periods in this study. The first level is comprises the planning time periods, each corresponding to a locational decision for that period. These periods compose the whole planning horizon. Each time period is further divided into several sub-periods, each accommodating a container allocation decision for the purpose of balancing container inventory among depots.
2.3.4 Container Inventory Reduction

The essence of container management is to minimize container-related costs, while still meeting customer demand for empty containers. One way to minimize container costs is to reduce the container transportation and storage costs of the company’s existing fleet. Another more fundamental way is to cut down the number of containers in the company’s fleet. This can reduce container movement and storage costs to a larger extent as well as container capital and insurance costs. In the IDS problem, if container inventory can be cut down, then intuitively we require less depot capacity, therefore less depot fixed and storage costs, less container movement, and needless to say, less capital investment. Therefore, the model we will build should be capable of reflecting the principle that whenever possible, the level of container inventory in the system should be kept as low as possible.

2.3.5 The System’s Interaction with the Outside

As described in Section 2.2, Problem Definition, the system being studied needs to find some ways to communicate with the outside world. In practice, a shipping company’s inland depot network usually covers a geographically well-defined region, ranging from several counties and states (provinces), to an entire continent (North America, for example). Most of the depots in the system only move containers to/from other depots and/or customers within the system, whereas a few depots, composed of mainly seaports and certain inland depots, may exchange containers with seaports, inland
depots, and/or customers outside of the system. We call these depots as source depots. They can obtain empty containers from outside sources, or in many cases, move spare containers out of the system for the sake of cutting down container inventory (in this case, the net container supply into the system is negative).

Allowing the existence of source depots makes the system possess a sufficient number of containers to meet system-wide demand whenever a demand arises, while keeping that number as low as possible to minimize total container costs. In fact, the system’s interaction with the outside is the only way to adjust the container inventory in the IDS problem.

Therefore, depots in the problem need to be divided into two groups. The first group includes those depots only moving containers to/from other depots and/or customers within the system. The second group is composed of most seaports and a few inland depots, which exchange containers with seaports, inland depots, and/or customers outside the system. This distinction should be reflected in our models.

2.3.6 Mixed Depot Ownership

Shipping companies usually do not build their own inland depots. Instead, they lease a depot or part of a depot owned and even operated by a third party, often a railroad, trucking company, port authority, or warehouse, etc. It is this feature that makes our
research meaningful and important for shipping companies, because if all depots used by a shipping operator were actually owned by him, he would not have much choice about whether or not to use these facilities. Mixed ownership is also the feature that makes the IDS problem different from other location-allocation problems. Therefore, our models should be able to choose depots from a set of candidate depots not owned by the shipping company, and at the same time take the openness of the depots owned by the company as given. For instance, an shipping operator owns two depots A and B, and wants to decide which ones to choose from a set of potential depots owned by a third party, whose depots are C, D, E. The depot selection problem facing the operator is to solve the IDS problem over the network comprises nodes A, B, C, D, and E, while taking A and B open as given.

In summary, the major ingredients of the modeling framework for the IDS problem are as follows.

- Hierarchical decision-making process
- Two levels of multiple periods
- Balancing requirements
- Container inventory reduction
- The system’s interaction with the outside
- Mixed depot ownership.
2.4 Literature Review on Location and Allocation Problems

In this section, we review relevant literature on empty vehicle depot selection problems. Viewing the lack of existing studies in modeling this complex problem, we extend the review to advanced locational modeling approaches and evaluate their solution methods, including capacitated facility location problems, and dynamic and stochastic location models. Because of the importance of balancing requirements in the problem, empty vehicle allocation models with balancing requirements are also reviewed.

2.4.1 Empty Vehicle Inland Depot Selection Problem

The inland depot container selection problem is first studied by Dejax et al. (1986), who treat this problem as part of a global study of the logistics system of a large European container transportation and distribution company. Their approach is to sequentially solve a classical depot location problem, followed by a minimum cost flow problem to allocate containers among depots.

Crainic et al. (1989) then integrate the depot location and container allocation problems, and call their model a location-allocation model with balancing requirements. The model is subsequently solved using a branch-and-bound algorithm (Crainic et al., 1989). Because of the size of the problem, the branch-and-bound procedure cannot
determine the optimal solution in a reasonable computing time. Gendron and Crainic (1992) propose a parallel implementation of the branch-and-bound algorithm. The model is later solved by Tabu search procedure (Crainic et al., 1993) and by a dual-ascent-based branch-and-bound algorithm (Crainic and Delorme. 1993; Gendron and Crainic, 1995).

As pointed out by these papers, the most important distinction of the IDS problem from general location-allocation models is the balancing requirements, which require the container inventory at depots to be adjusted periodically to account for demand and supply imbalances among regions. However, these models basically use the same model formulation and fail to address the issue of balancing requirements. For example, the balancing constraints (4) in Gendron and Crainic (1995, p. 41) are:

\[
\text{total inflow to } i \text{ from (all supply customers + all other depots)} = \text{total outflow from } i \text{ to (all demand customers + all other depots)}
\]

for any depot \( i \)

which are actually flow conservation constraints required for transshipment nodes in any network flow problem.

In essence, balancing requirements imply repositioning empty containers from one location to another in preparation for expected demand in the latter depot in the subsequent time periods. A crucial pre-condition for permitting the modeling of balancing activities is a multiple-period modeling framework, because in a single-period model one cannot express the sequential behavior of the reallocation of containers from an origin depot to a
destination depot and then the use of the containers in the destination depot. This can only be captured by multi-period models. Unfortunately, the previous models are all single-period ones and therefore fail to capture the very nature of balancing requirements.

As we argued in Section 2.3.1, Modeling Framework, another fundamental issue in the empty vehicle depot selection problem is the relationship between the depot location selection decision and the decisions of the empty container allocation among depots and customers, which, in reality, is a hierarchical relationship. The empty container allocation decisions take as given of and is guided by the depot location selection decisions; and the depot selection decisions must take the container allocation decisions into account. None of the previous models reflects this hierarchical relationship. Dejax et al. (1986) considers the two as sequential decisions, and therefore there is no feedback from the allocation decisions to the location decisions. All other models make the location and allocation decisions in the same manner and at the same time. This is not consistent with the real-life decision-making process.

In addition to missing the balancing requirements and the hierarchical decision-making process, the previous models also fail to address some other critical issues inherent to the empty vehicle location selection problem, such as the vehicle inventory reduction, the system's connection with the outside, the mixed depot ownership, and the vehicle storage costs, as we described in Section 2.3.
2.4.2 Facility Location Problems

One widely studied problem in location theory is the so-called uncapacitated facility location problem (UFLP). This problem consists of locating a number of facilities among a finite set of potential sites, in order to minimize the linear combination of fixed facility costs and of variable production and transportation costs to customers. The objective is to achieve the best trade-off between fixed and variable costs: opening a larger number of facilities results in lower transportation costs but higher fixed costs, and vice versa, when fewer facilities are opened. UFLP was originally studied by Kuehn and Hamburger (1963), and is known to be NP-hard (Garey and Johnson, 1979). The most efficient exact methods to solve this problem are dual-based approaches (Erlenkotter, 1978; Körkel, 1989). Several UFLP extensions have been proposed along the lines of capacity constraints, network dynamics and stochastic. In the following, we discuss these extensions in order.

One natural extension is called capacitated facility location problems (CFLP), in which the production capacity of any facility is bounded above by a preset value. CFLP has been studied by Geoffrion and McBridge (1978), and Van Roy (1986), among others. Hansen et al. (1987) provide an overview of these problems and their relationships to other location problems. In our IDS problem, the storage capacity of a depot is usually limited, as is the transportation capacity between any pair of depot-depot and any pair of depot-customer. Therefore, it is expected that our models will fall into the category of capacitated facility location problems (CFLP).
In the dynamic context, the time-phasing of the decisions becomes important. It reflects a multi-period situation where demand, supply, facility and transportation capacity, and cost structures, vary between time periods. In addition, in the context of our problem, multi-period models become necessary to capture the balancing activities of moving containers among depots.

The theoretical sophistication of the state-of-the-art solution methods for multi-period location problems lags behind that of the single-period location models. Only the problems of small dynamic location models can be solved exactly; in most cases one must resort to heuristics (Jacobsen, 1990). With the exception of Van Roy and Erlenkotter’s special model (1982), no computationally feasible exact method is available for solving medium- or large-scale problems.

In recent years, a number of studies have addressed the stochastic dimension present in some facility location problems. Typical elements which may be random in a locational problem are the location of customers, the presence or absence of each customer, the level of demand and supply, the price of product or service, the travel costs, and the transportation capacity. Louveaux (1993) gives a comprehensive survey of the stochastic location problem. As usual in stochastic programming, one makes the distinction between decisions made before the random variables can be observed (these are called first-stage decisions) and decisions made after the random variables are observed (they are called second-stage or recourse decisions). Most stochastic location models consider the location
and the size of facilities as first-stage decisions, which are integer variables, and regard the allocation of customers to facilities as second-stage decisions, which may be integer or continuous variables. Therefore stochastic location models are usually stochastic integer programs, which have a well-deserved reputation for being computationally intractable. In fact, they combine two types of programs which by themselves are often difficult to solve. So far, the most promising result is with the problems involving up to 40 customers and 10 potential facility locations, by using the branch-and-cut method (Laporte et al., 1994). Therefore, it is far from practical for stochastic programming methods to be applicable in any meaningful facility location problems.

2.4.3 Empty Vehicle Allocation Models with Balancing Requirements

Empty vehicle allocation problems arise when a set of customers in different geographical locations need empty vehicles to carry their shipments, and another set of customers return their empty vehicles to carriers. Balancing requirements are a necessity in this problem, because regional trade imbalance is common in most of the transportation modes. The current state-of-the-art is that the problem has been formulated as a dynamic and stochastic programming model with network recourse (Sheffi et al., 1984; Powell, 1987; Crainic et al., 1993; and Chu, 1995). The random factors include vehicle demand and supply, and random transportation link costs and capacity. Efficient approximation algorithms are proposed by Powell (1987), Powell and Cheung (1994), and Chu (1995).
A critical difference between the empty vehicle depot selection problem and the empty vehicle allocation problem is that the former is a mixed integer program, which is generally very mathematically involved by itself. Any dynamic and/or stochastic formulation of the depot selection problem may easily become mathematically intractable.

A different approach adopted by Gao (1994) to solve the empty container allocation problem is to emphasize the impact of empty container return behavior on the allocation of containers, and to allow changes of fleet size, by leasing on and off containers to simultaneously solve the fleet sizing and the allocation problems.

2.5 Chapter Summary

This chapter was designated to lay a foundation for the subsequent research. We first gave an overview of the intermodal transportation and liner shipping industries. After formally defining the problem we want to study, we identified and discussed six fundamental ingredients of the framework for modeling the intermodal inland depot selection problem. Finally we critically reviewed the relevant literature on the IDS problem against our modeling framework, and concluded that there are no existing models and methods for adequately solving the IDS problem.
In the next chapter, we will develop mathematical models for solving the IDS problem. A multi-period model with balancing requirements, called MPB, will be developed in Chapter 3 and solved in Chapter 4.
Chapter 3
Model Development

In this chapter, we develop mathematical models for solving the IDS problem facing intermodal transportation and liner shipping carriers. We first draw a delineation of the container inland depot network in a multi-period context. Because of the sophisticated heterogeneity of locations, which comprise inland and seaport depots, and demand and supply customers, the underlying network is defined and delineated in a 3-dimensional time-depot-customer network. This differs from the common 2-dimensional time-space network for modeling dynamic transportation problems (Chu, 1995). Because of the lack of previous research and the complexity of the problem, it is necessary to identify and explain the determinants of the depot location-allocation problem, such as demand and supply, cost structures, etc.

A single-period model is then built to better understand the problem’s complexity and to demonstrate the inadequacy of single-period modeling approach in reflecting the major ingredients of the modeling framework. These ingredients were identified in Section 2.3, for example, the hierarchical decision-making process, the balancing requirements, etc. The final multi-period model with balancing requirements, called MPB, is developed at the end of the chapter. MPB is a large-scale mixed integer program.
3.1 Three-dimensional Delineation of the Container Depot Movement Network

Dynamic transportation problems are generally represented by a 2-dimensional time-space network. A container shipping service network with three depots and a planning horizon of four time periods is shown in Figure 3.1 below (Chu, 1995).

![Time-Space Network Diagram](image)

**Figure 3.1 A 2-D Time-Space Network for Dynamic Transportation Problems**

Node 1 represents location 1 at time period 1, node 2 represents location 2 at time period 1. Node 4 represents location 1 at time period 2. Arc (1,8) represents an available traffic link leaving location 1 at time period 1 and arriving at location 2 at time period 3.
The horizontal dotted arcs model the inventory activities at each location across the time periods.

In our IDS problem, we have two types of locations: depots and customers. They are heterogeneous with regard to the locations' functionality, cost and capacity structures. Moreover, unlike classical location-allocation problems, here there are bi-directional movements between depots. That is, we allow container movement between two depots in both directions, of course, at different time periods. Another dimension of the problem is time. Because a container supply customer (consignee) may become a demand customer (shipper) in the next period, and vice versa, the movements between a depot-customer pair may change direction from one period to another. In order to depict the complicated relationships of depots and customers in the context of multiple periods, we delineate a 3-dimensional time-space-space network as shown in Figure 3.2 below.

The network consists of three depots, two customers, and four time periods. All transportation arcs are directed with arrows pointing to the destination. Nodes 1, 2 and 3 stand for depots 1, 2 and 3 at time period 0. Nodes 4 and 5 represent customers 4 and 5 at time period 0. At time period 1, these depots and customers are represented by nodes 6, 7, 8, 9, and 10 respectively. The same holds for the time periods 2 and 3.
Figure 3.2 A 3-D Time-Space-Space Network for Multi-period Depot Selection Problems

In the depot-time plane, the vertical dotted arcs with arrows model the inventory activities in each depot across time periods, and the diagonal arcs with arrows represent inter-depot movement across time periods. For example, arc (2,11) represents a movement leaving depot 2 at time period 0 and arriving at depot 1 at time period 2, arc (11,17) respectively a movement leaving depot 1 at time period 2 and arriving at depot 3 at time period 3. Note that if the time taken to travel between two depots is less than the
length of the time period, depot-depot movements may take place at the same time period and are represented by horizontal dotted arcs with arrows. For example, arc (7,8) stands for a shipment from depot 2 to depot 3 within time period 1.

Depot-customer movements are depicted by arcs between the depot-time plane and the customer-time plane. For instance, arc (5,1) represents a depot to customer shipment at time period 0. It is noted that customer 5 is a supply customer at time period 0, and becomes a demand customer at time period 2, because there is a shipment, arc (8,15), leaving depot 3 at time period 1 and arriving at customer 5 at time period 2. Arc (3,9) is a shipment leaving depot 3 at period 0 and arriving at customer 4 at period 1.

Because no direct movements are allowed between customers, there is no arc in the time-customer plane.

Our goal is to select appropriate depots from a set of candidate depots over this 3-D time-depot-customer network to meet customer demand for containers and to minimize total cost over a planning horizon. Our modeling work will be based on this 3-D network.
3.2 Determinants of Container Inland Depot Selection

The major determinants of IDS include container demand and supply from customers, depot and transportation link costs, depot storage and transportation link capacity, and container inventory cost. We discuss them in detail as follows.

3.2.1 Empty Container Demand and Supply

Customers demand empty containers when they have a shipment to be loaded in their sites – warehouses, factories, etc., or in the shipping carrier’s sites – depots, container freight stations (CFSs), etc. For the sake of modeling, we aggregate a certain number of demand customers as a single customer, depending on the scale of the study area and the scope of the modeling requirements. For example, if the study area is an entire country, a customer in our model may represent an aggregation of as many as 100 actual customers in a province (state); if the study area covers only several counties, each customer in the model may consist of as few as several customers in a county.

The same aggregation principle is also applied to shipment receivers, i.e., consignees. After stripping their shipment, consignees need to return empty containers to a depot designated by the shipping company within certain days pre-stipulated in the shipment contract.
In general, empty container demand and supply are difficult to predict because of inherent uncertainties. Because our model is a long- and medium-term planning model, day-to-day demand and supply uncertainty can be approximately assumed to be deterministic in a longer-run, say, one month as a time period in our model. In addition, aggregation of actual individual customers can offset, to a certain degree, the uncertainty. Therefore, we do not consider stochastic factors in our optimization models. However, we will, in Chapter 6, develop a simulation model to address demand and supply uncertainty.

3.2.2 Depot Costs and Capacity

Container shipping companies usually do not own inland depots. Instead they lease depots from railroads, trucking companies, port authorities, warehouses, and others.

Depot costs generally consist of fixed and variable costs. The fixed cost pays for the right to use the depot. The amount of the fixed cost is fixed for the period specified in the contract, say, one year. A contract usually also specifies the maximum number of containers the shipping company can store in the depot. This is the depot storage capacity for the shipping company.

In addition to the fixed depot cost, depots also charge for storage each time a container is stored in the depots. This variable charge takes the form of $ per unit per day. Fixed and variable costs vary from depot to depot, depending upon the services provided by the
depots (such as lift equipment efficiency, record-keeping accuracy), competition among depots, and the local real estate market.

3.2.3 Transportation Link Costs and Capacity

Transportation cost of depot-depot and depot-customer movement is an important factor in determining depot selection. For the shipment of empty containers, the transportation costs are usually linear to the number of containers transported, although in certain circumstances, such as union trains, batch movement of containers may show some economies of scale by obtaining discount from railroads. In fact, transportation costs can be further divided into in-transit cost and lift-on-off cost. For short distance shipment, lift-on-off cost may account for more than half of the total transportation cost. Transportation cost is represented as $ per container per shipment. Table 3.1 gives transportation tariffs for empty container/trailer movements for selected rail services.

Transportation link capacity is a critical constraint for empty container relocation. This is particularly true if the relocation is carried out by a shipping company’s own ships. The priority of ship space is always given to loaded containers, because the revenue brought in by a loaded container is much higher than that by an empty container. This is why in practice, some empty containers are relocated from depot A to depot B not for immediate use in depot B. Instead they are used in depot B some time in the future, because the operator anticipates the unavailability of spare ship space for empty containers.
at the future time when the containers are needed in depot B. So, it is wise to ship these empty containers to depot B at current time when there is ship space available.

Table 3.1 Rail Tariffs for Empty Container/Trailer Movements

<table>
<thead>
<tr>
<th>origin-destination</th>
<th>rail mileage</th>
<th>40' cntr. &amp; 20' trailer</th>
<th>20' container</th>
</tr>
</thead>
<tbody>
<tr>
<td>from Chicago to:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Orleans</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ramp-to-ramp</td>
<td>931</td>
<td>395</td>
<td>395</td>
</tr>
<tr>
<td>ramp-to-pier</td>
<td>1,033</td>
<td>415</td>
<td>405</td>
</tr>
<tr>
<td>Savannah</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>ramp-to-ramp</td>
<td>1,495</td>
<td>588</td>
<td>574</td>
</tr>
<tr>
<td>ramp-to-pier</td>
<td></td>
<td>636</td>
<td>622</td>
</tr>
<tr>
<td>Miami</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>ramp-to-ramp</td>
<td></td>
<td>505</td>
<td>288</td>
</tr>
<tr>
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<td>Houston</td>
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<td></td>
<td></td>
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<td>ramp-to-ramp</td>
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<td>701</td>
<td>188</td>
</tr>
<tr>
<td>ramp-to-pier</td>
<td></td>
<td>423</td>
<td>289</td>
</tr>
<tr>
<td>from St. Louis to:</td>
<td></td>
<td></td>
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</tr>
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<td>ramp-to-pier</td>
<td></td>
<td>400</td>
<td>303</td>
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<td>from Memphis to:</td>
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<tr>
<td>ramp-to-pier</td>
<td></td>
<td>326</td>
<td>241</td>
</tr>
</tbody>
</table>

Source: Marcus (1993),

Note that there is another important service measurement – transit time – for any mode of transportation, which, for our problem, means the travel time for depot-depot and depot-customer movement. We intentionally omit it because the IDS problem is a planning issue. The time interval even for the lower level of operational allocation decision is
usually no shorter than a month, which is usually sufficient long for any shipment to complete its journey in an inland network. If a shorter time interval is needed, however, our model is capable of incorporating transit time.

3.2.4 Container Inventory Costs

A new container usually costs several thousand US dollars, and the daily rent per TEU is several US dollars. Therefore, the daily cost of the container inventory for a container shipping company with a fleet of 50,000 TEU is at least hundreds of thousands of US dollars. Thus, reducing the container fleet inventory is one of the primary goals for container management in a shipping company. In addition, smaller container inventory can also reduce container transportation and storage costs.

3.3 Single-Period Model

In this section, we present a single-period model formulation of the IDS problem. The purpose of formulating a single-period model is twofold. First, this simplified model enables us to gain better understanding of the problem's complexity. Second, it shows that this model is not capable of conveying the important ingredients of our modeling framework, for example, the balancing requirements and the dynamics.
A “depot” in the model is referred to as any location used to store containers, including seaports, rail terminals, trucking terminals, warehouses, and others.

### 3.3.1 Assumptions, Notation, Input Data and Decision Variables

**Assumptions**

1. No direct movements between demand and supply customers are allowed. Although shipping companies usually do not want customers to adjust empty containers among themselves because of the risk of losing track of those containers, the shipping companies and their agencies often directly move containers among customers (consignees and shippers). Therefore, this assumption will be relaxed in Chapter 6.

2. The container inventory (capital) cost for a period is the average cost of the container inventories at the initial and the end period in all depots.

3. The storage cost for a period is the average storage cost at the initial and the end period in that depot.

4. The number of containers initially stored in each depot is given, and the number stored at the end of the planning horizon is optimally determined by the model. Note that the end number could also be pre-set by the shipping company if so it desires. The model is capable of handling this.

**Notation**

\( O = \) the set of customers supplying empty containers.
\[ D = \text{the set of customers demanding empty containers.} \]

\[ I = \text{the set of depots not allowing container exchange with the outside.} \]

\[ E = \text{the set of depots allowing container exchange with the outside.} \]

\[ \text{SP} = \text{the set of depots owned by the shipping company.} \]

\[ F = I \cup E, \text{ the set of all depots.} \text{ F } \supset \text{ SP.} \]

\[ G = (N, A), \text{ the network concerned in the study, where } N = O \cup D \cup F, \text{ and } A = \text{ the set of transportation links among } N. \]

\[ F^+(i) = \{ j \in F \cup D: (i, j) \in A \}, \text{ for } \forall i \in N. \text{ This includes all depots and demand customers receiving containers from depot } i. \]

\[ F^-(i) = \{ j \in F \cup O: (j, i) \in A \}, \text{ for } \forall i \in N. \text{ This consists of all depots and supply customers providing containers to depot } i. \]

**Input Data**

\[ s_i = \text{units of containers supplied from supply customer } i, i \in O. \]

\[ d_i = \text{units of containers demanded by demand customer } i, i \in D. \]

\[ c_{ij} = \text{transportation cost on arc } (i, j) \text{ in terms of } \$ \text{ per container, } (i, j) \in A. \]

\[ u_{ij} = \text{transportation capacity on arc } (i, j) \text{ in terms of units of containers, } (i, j) \in A. \]

\[ f_j = \text{fixed cost at depot } j \text{ if it is open, } j \in F. \]

\[ c_j = \text{half of the storage cost at depot } j \text{ in terms of } \$ \text{ per container for the period, } j \in F. \]

\[ v_j = \text{storage capacity at depot } j \text{ in terms of units of containers, } j \in F. \]
\( r_j \) = cost for shipping in/out one container from/to depots allowing exchange with the outside, \( j \in E \).

\( w_j \) = units of containers initially stored at depot \( j \), \( j \in F \).

\( k \) = half of the inventory cost in terms of $ per container for the period.

**Decision Variables**

\( y_j \) = 0-1 variable, equals 1 if depot \( j \) is open; otherwise, 0. \( j \in F \).

\( x_{ij} \) = units of containers transported from \( i \) to \( j \), \( (i, j) \in A \).

\( z_j \) = units of containers stored at depot \( j \) at the end of the period, \( j \in F \).

\( q_j \) = units of containers supplied to depot \( j \) from an outside source, \( j \in E \). Negative \( q_j \) means there is net outflow of containers from depot \( j \) to the outside.

### 3.3.2 Single-period Model Formulation

The single-period model for solving the IDS problem is formulated as a mixed integer program as below:
\[
\min_{x_{ij}, y_j, z_j, q_j} \left[ \sum_{j \in F} f_{ij} y_j + \sum_{(i,j) \in A} c_{ij} x_{ij} + \sum_{j \in F} c_j z_j + \sum_{j \in F} r_j |q_j| + \sum_{j \in F} k_j \right]
\]
\hspace{1cm} (1.1)

subject to:

\[
\sum_{j \in F^-(i)} x_{ij} = s_i \quad \forall i \in O
\]
\hspace{1cm} (1.2)

\[
\sum_{j \in F^+(i)} x_{ij} = d_i \quad \forall i \in D
\]
\hspace{1cm} (1.3)

\[
z_j + \sum_{k \in F^+(j)} x_{jk} - w_j - \sum_{k \in F^-(j)} x_{kj} = 0 \quad \forall j \in I
\]
\hspace{1cm} (1.4)

\[
z_j + \sum_{k \in F^+(j)} x_{jk} - w_j - \sum_{k \in F^-(j)} x_{kj} - q_j = 0 \quad \forall j \in E
\]
\hspace{1cm} (1.5)

\[
u_{ij} \geq x_{ij} \geq 0 \quad \forall (i,j) \in A
\]
\hspace{1cm} (1.6)

\[
x_{ij} \leq v_j y_j \quad \forall (i,j) \in A
\]
\hspace{1cm} (1.7)

\[
|q_j| \leq v_j y_j \quad \forall j \in E
\]
\hspace{1cm} (1.8)

\[
v_j y_j \geq z_j \geq 0 \quad \forall j \in F
\]
\hspace{1cm} (1.9)

\[
y_j \in \{0, 1\} \quad \forall j \in F
\]
\hspace{1cm} (1.10)

\[
y_j = 1 \quad \forall j \in SP
\]
\hspace{1cm} (1.11)
3.3.3 Discussion of the Model and Its Inability to Incorporate Balancing Requirements

In the above single-period model, the objective function (1.1) states that the total container-related costs considered in the model include: (a) depot opening cost; (b) transportation costs from supply customers to depots, between depots, and from depots to demand customers; (c) container depot storage cost; (d) handling cost for shipping containers into or out of the system; and (e) container inventory (capital) cost.

Constraints (1.2) and (1.3) require that customer demand for and supply of containers be fully satisfied. Transportation link capacity is met by constraints (1.6). Constraints (1.7) and (1.8) are to ensure that no shipment leaves or arrives at a depot if the depot is determined not to be open, and that the shipment size is less than a depot’s maximum storage capacity if the depot is determined to be open. Shipment includes depot-customer, depot-depot, and in/out of the system movements. Constraints (1.9) are a straightforward expression of depot storage limitations. The depot opening variable $y_j$ is a binary variable, if $y_j$ equals 1, depot $j$ is open; and if $y_j$ equals 0, depot $j$ is closed, as shown in constraints (1.10). As an ingredient of our modeling framework, mixed depot ownership is reflected by constraints (1.11), where all depots owned by the shipping company are designated to be open. Some depots not owned by the shipping company may also need to be open, due to some strategic considerations or simply because of previous contract obligations. These scenarios are modeled in constraints (1.11).
Two sets of constraints – constraints (1.4) and (1.5) – are very important and interesting. They are called flow conservation in general network flow problems. They simply say that the total flow into a depot should be equal to the total flow out of the depot. Constraints (1.4) are for depots without interactions with the outside, and (1.5) for depots having interactions with the outside.

It is important to note that these constraints have nothing to do with balancing container inventory among depots. A balancing activity takes place after the imbalance of container distribution among depots occurs. In practice, shipping operators rebalance container distribution periodically to meet expected future demand and supply patterns among their depots. In this single-period model, however, there is no information available about the imbalance status of container distribution in the previous periods or the expected demand and supply pattern in the subsequent periods. Therefore, we conclude that a single-period model is not able to model balancing requirements, the fundamental characteristics of the IDS problem, and we have to turn to multi-period models to address this issue. Unfortunately, in Gendron and Crainic (1995) and a series of early papers mentioned in Section 2.4.1, a similar set of constraints to constraints (1.4) and (1.5) are claimed to be capable of modeling balancing requirements. The above analysis demonstrates that that claim does not hold.

The single-period model is a mixed integer program, which is NP-hard and difficult to solve for large-scale applications. The basic objective is to achieve the best tradeoff
between fixed depot cost and various transportation, container inventory and storage costs. Because of the capacity constraints on the depot storage and transportation as explained in Section 2.4.2, the model falls into the category of capacitated facility location problems (CFLP). An overview of these problems can be found in Hansen et al. (1987).

3.4 Multi-period Model with Balancing Requirements (MPB)

In this section, we present our multi-period model with balancing requirements (MPB) for the IDS problem. MPB is an extension of the above single-period model to multiple periods. This extension enables us to model balancing requirements. MPB is a large-scale mixed integer program.

To be concise and consistent with the above single-period model, we will state the assumptions, notation, input data and decision variables for the MPB formulation with the minimum overlap with those for the single-period model.

3.4.1 Assumption, Notation, Input Data and Decision Variables

Assumption

(1) No direct movements between demand and supply customers are allowed, as discussed in Section 3.3.1.
The overall planning horizon comprises several time periods, each of which consists of several time sub-periods. A location decision is made for each time period, and an allocation decision for each time sub-period.

The length of the travel time for any of the depot-depot and the depot-customer movements is less than a time sub-period. This is usually true because as a planning issue, a container allocation decision covers a time sub-period of one month or longer, which is sufficiently long for any movement within an inland network.

Notation

$T =$ the number of time periods over the planning horizon. $t$ represents the $t^{th}$ time period, which is a basic time unit for making depot location decisions. For example, if the overall planning horizon is two years long, and the length of each time period is equal to six months, then $T$ equals 4, and $t$ ranges from 1 through 4.

$P =$ the number of time sub-periods within a time period. $\tau$ represents the $\tau^{th}$ time sub-period, which is a basic time unit for making operational container allocation decisions. For example, if the length of each time period is six months, and the length of each time sub-period is one month, then $P$ equals 6, and $\tau$ ranges from 1 through 6.

The following symbols are same as in Section 3.3.1:

$O, D, I, E, SP, F, N, A, F^+(I), F^-(I)$. 
Input Data

\( s_i(t, \tau) = \text{units of containers supplied from customer } i \text{ in the } \tau^{th} \text{ time sub-period of the } t^{th} \text{ time period}, \ i \in O. \)

\( d_i(t, \tau) = \text{units of containers demanded by customer } i \text{ in the } \tau^{th} \text{ time sub-period of the } t^{th} \text{ time period}, \ i \in D. \)

\( c_{ij}(t, \tau) = \text{transportation cost on arc } (i, j) \text{ in terms of $ per container in the } \tau^{th} \text{ time sub-period of the } t^{th} \text{ time period}. \ (i, j) \in A. \)

\( u_{ij}(t, \tau) = \text{transportation capacity on arc } (i, j) \text{ in terms of units of containers in the } \tau^{th} \text{ time sub-period of the } t^{th} \text{ time period}. \ (i, j) \in A. \)

\( f_j(t) = \text{fixed cost at depot } j \text{ if it is open in the } t^{th} \text{ time period}, \ j \in F. \)

\( c_j(t) = \text{variable container storage cost at depot } j \text{ in terms of $ per unit in the } t^{th} \text{ time period, } j \in F, \text{ except for the last period } t = T. \ c_j(T) \text{ is half the amount of the storage cost for that period.} \)

\( v_j(t) = \text{container storage capacity at depot } j \text{ in terms of units of containers in the } t^{th} \text{ time period, } j \in F. \)

\( r_j(t, \tau) = \text{cost for shipping in/out one container from/to a depot allowing exchange with the outside, in terms of $ per container in the } \tau^{th} \text{ time sub-period of the } t^{th} \text{ time period, } j \in E. \)

\( w_j(0, P) \text{ and } w_j(T, P) = \text{units of containers stored at depot } j \text{ at the beginning and the end respectively, of the planning horizon, } j \in F. \)
k(t, τ) = container inventory (capital) cost in terms of $ per unit for the τth time sub-period of the tth time period, except for the last sub-period of the last period, t = T and τ = P. k(T, P) is half the amount of the inventory cost for that interval.

Decision Variables

y_j (t) = 0-1 variable, equal to 1, if depot j is open in the tth time period; equal to 0, otherwise. j ∈ F.

x_{ij} (t, τ) = units of containers transported from i to j in the τth time sub-period of the tth time period. (i, j) ∈ A.

z_j (t, τ) = units of containers stored at depot j at the end of the τth time sub-period of the tth time period except for z_j (T, P), which is half the number of the containers stored at the end of the Pth time sub-period of the Tth time period. j ∈ F.

q_j (t, τ) = units of containers supplied to depot j from an outside source in the τth time sub-period of the tth time period, j ∈ E. Negative q_j means a net outflow of containers from depot j to the outside.

3.4.2 MPB formulation.
\[
\text{Min } \sum_{i,j} \sum_{t=1}^{T} \left\{ \sum_{i \in I} f_i(t) y_i(t) + \sum_{i \in I} \left[ \sum_{(j) \in A} c_{ij}(t, \tau) x_{ij}(t, \tau) + \sum_{j \in F} c_{j}(t) z_{j}(t, \tau) + \sum_{(j) \in A} r_{ji}(t, \tau) |q_{ij}(t, \tau)| + \sum_{j \in F} k(t, \tau) z_{j}(t, \tau) \right] \right\} 
\]

subject to:

\[
\sum_{j \in F} x_{ij}(t, \tau) = s_i(t, \tau) \quad \forall i \in O \quad t \in [1, T] \quad \tau \in [1, P] \tag{2.2}
\]

\[
\sum_{j \in F} x_{ij}(t, \tau) = d_i(t, \tau) \quad \forall i \in D \quad t \in [1, T] \quad \tau \in [1, P] \tag{2.3}
\]

\[
z_j(t, \tau) = \sum_{i \in F} x_{ij}(t, \tau) - z_j(t, \tau - 1) - \sum_{i \in F} x_{ij}(t, \tau) = 0 \quad \forall j \in \mathcal{I} \quad t \in [1, T] \quad \tau \in [1, P] \tag{2.4}
\]

\[
z_j(t, \tau) = \sum_{i \in F} x_{ij}(t, \tau) - z_j(t, \tau - 1) - \sum_{i \in F} x_{ij}(t, \tau) - q_j(t, \tau) = 0 \quad \forall j \in \mathcal{E} \quad t \in [1, T] \quad \tau \in [1, P] \tag{2.5}
\]

\[
u_{ij}(t, \tau) \geq x_{ij}(t, \tau) \geq 0 \quad \forall (i, j) \in A \quad t \in [1, T] \quad \tau \in [1, P] \tag{2.6}
\]

\[
x_i(t, \tau) \leq v_j(t) y_j(t) \quad \forall (i, j) \in A \quad t \in [1, T] \quad \tau \in [1, P] \tag{2.7}
\]

\[
|q_{ij}(t, \tau)| \leq v_j(t) y_j(t) \quad \forall j \in \mathcal{E} \quad t \in [1, T] \quad \tau \in [1, P] \tag{2.8}
\]

\[
v_j(t) y_j(t) \geq z_j(t, \tau) \geq 0 \quad \forall j \in \mathcal{F} \quad t \in [1, T] \quad \tau \in [1, P] \tag{2.9}
\]

\[
z_j(0, P) = w_j(0, P) \quad \forall j \in F \tag{2.10}
\]

\[
y_j(t) \in \{0,1\} \quad \forall j \in \mathcal{F} \quad t \in [1, T] \tag{2.11}
\]

\[
y_j(t) = 1 \quad \forall j \in \mathcal{SP} \quad t \in [1, T] \tag{2.12}
\]
3.4.3 MPB’s ability to Satisfy Balancing Requirements

MPB is an extension of the previous single-period model to two levels of multiple periods, one level for depot location decisions and the other for container allocation decisions.

We want to highlight the important role of the constraints (2.4) and (2.5) play. (2.4) says that the number of containers stored in a depot at the end of a sub-period is equal to the total inflow into the depot during that sub-period plus the initial storage in the sub-period minus the outflow from the depot during the sub-period. This container leftover at the end of a sub-period acts as a bridge between two sub-periods so that shipping companies can take action to correct the imbalance of container distribution among depots from one sub-period to another to meet expected future demand and supply patterns. It is the multi-period model, MPB, that makes information available about the existing imbalance status of container inventory and the expected demand and supply pattern in the future. Therefore, we conclude that a multi-period model is able to model balancing requirements, the fundamental characteristic of the IDS problem. The capability to model balancing requirements is a crucial difference between MPB and the single-period model.
3.5 Chapter Summary

In Chapter 3, we focused on developing mathematical models for solving the IDS problem. We started from drawing a delineation of the container inland depot network in the context of a multi-period framework. Because of the complicated heterogeneity of locations, the underlying network is defined and delineated in a 3-dimensional time-depot-customer network, which differs from the common 2-dimensional time-space network for modeling dynamic transportation problems. We then identified and explained the determinants of the depot location-allocation problem, such as demand/supply, cost structures, etc. Next, a single-period model was built to better appreciate the problem's complexity and to demonstrate the inadequacy of a single-period modeling approach in reflecting the major ingredients of the modeling framework. The final multi-period model with balancing requirements, called MPB, was developed at the end of the chapter.

MPB is a dynamic capacitated facility location model, and is NP-hard. For example, for a medium-sized problem with 50 potential depots to choose from, 24 time sub-periods within 6 time periods, there are 1200 0-1 variables, plus more than 10,000 columns and rows. This is a challenging problem to solve under current OR techniques. We will spend the next chapter discussing appropriate solution algorithms to efficiently solve MPB for real-world problems in the intermodal and liner shipping industries.
Chapter 4
Solution Methods and Algorithms

MPB is a multi-period (dynamic) mixed integer program. Thus far the theoretical sophistication of the state-of-the-art solution methods for multi-period location problems lags behind that of the single-period location models. Only small size problems can be solved exactly; in most cases, one must resort to heuristics. In order to solve MPB efficiently, in this chapter we critically evaluate methods for solving large-scale locational models, and propose two decomposition-based algorithms.

4.1 Evaluation of Methods for Solving Large-Scale Locational Models

Throughout the years, the closely related notions of bounding techniques, duality, and decomposition have been central to the advances in large-scale mixed integer programming. In the light of this, the scope of the mathematical techniques evaluated in this section includes methods for solving multi-period (dynamic) location models and algorithms for solving single-period location-allocation models.
4.1.1 Solution Methods for Multi-period location Models

Jacobsen (1990) provides an excellent survey of multi-period location models and solution methods. We use some of his ideas to study possible solution method for MPB.

From the perspective of dynamic programming, constraints (2.4) and (2.5) in MPB are the system's "equations of motion," which describes the trajectory of the state variables $z_j(t, \tau)$ from the initial value $z_j(0, 0)$. The state variables may be eliminated by substituting constraints (2.4) and (2.5) into (2.1) and (2.9). The decision variables are $x_{ij}^*(t, \tau)$, $y_j^*(t)$, and $q_j^*(t, \tau)$. For given location opening decision $y_j(t)$, it is straightforward to find the flows $x_{ij}(t, \tau)$ and $q_j(t, \tau)$ using an allocation algorithm.

The backward recursion for the dynamic programming (DP) derived from formulation (2.1) through (2.12) becomes:

\[
B[z(t, \tau), y(t-1)] = \min_{y} \left\{ \sum_{j=1}^{T} \left[ \sum_{j \in F} f_j(t) y_j(t) + A[z(t, \tau), y(t)] + B[z(t+1, \tau), y(t)] \right] \right\}
\]

\[
B[w(T, \tau), \cdot] = 0 \quad t \in [1, T] \quad \tau \in [1, P]
\]

where $z(t, \tau)$ is a vector of $z_j(t, \tau)$, and $y(t)$ is a vector of $y_j(t)$, for $j \in F$, $t \in [1, T]$, $\tau \in [1, P]$. $A[\alpha, \beta]$ denotes the optimal objective function value of the following allocation model with inventory $\alpha$ (representing $z(t, \tau)$) and depot opening state $\beta$ (representing $y(t)$):
Min $\sum_{(ij) \in A} \left[ \sum_{\tau=1}^{P} c_{ij}(t,\tau)x_{ij}(t,\tau) + \sum_{j \in F} c_j(t)z_j(t,\tau) + \sum_{j \in E} r_j(t,\tau)q_j(t,\tau) + \sum_{j \in F} k(t,\tau)z_j(t,\tau) \right]$

subject to

$\sum_{j \in F^+(i)} x_{ij}(t,\tau) = s_i(t,\tau) \quad \forall i \in O \quad \tau \in [1, P]$ (3.3)

$\sum_{j \in F^-(i)} x_{ji}(t,\tau) = d_i(t,\tau) \quad \forall i \in D \quad \tau \in [1, P]$ (3.4)

$\alpha_i(t,\tau) \cdot \sum_{i \in F^+(j)} x_{ij}(t,\tau) - z_j(t,\tau) - \sum_{i \in F^-(j)} x_{ij}(t,\tau) = 0 \quad \forall j \in I \quad \tau \in [1, P]$ (3.5)

$\alpha_i(t,\tau) \cdot \sum_{i \in F^+(j)} x_{ij}(t,\tau) - z_j(t,\tau) - \sum_{i \in F^-(j)} x_{ij}(t,\tau) - q_j(t,\tau) = 0 \quad \forall j \in E \quad \tau \in [1, P]$ (3.6)

$u_{ij}(t,\tau) \geq x_{ij}(t,\tau) \geq 0 \quad \forall (i, j) \in A \quad \tau \in [1, P]$ (3.7)

$x_{ij}(t,\tau) \leq v_{ij}(t) \beta_j \quad \forall (i, j) \in A \quad \tau \in [1, P]$ (3.8)

$|q_{ij}(t,\tau)| \leq v_{ij}(t) \beta_j \quad \forall j \in E \quad \tau \in [1, P]$ (3.9)

$B[z(t, \tau), y(t-1)]$ in (3.1) is the minimal cost of getting from state $z$ at time $(t, \tau)$ to the end of the planning horizon $(T, P)$. $B[z(0, 1), y(0)]$ is the optimal value of the objective function (2.1). As stated in Chapter 3, two levels of time intervals $t$ (period) and $\tau$ (sub-period) interact with each other and overlap at the initial and final time intervals. For example, period $(t, P+1)$ is equivalent to period $(t+1, 1)$. 

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The major single obstacle to the use of the DP method is the dimension of the state vector. If the dimension is high, the number of different states to be evaluated at each state \((t, \tau)\) becomes very large and computationally prohibitive. This is often referred to as the "curse of dimensionality." Special considerations or approximations are needed to reduce the number of states evaluated. In (3.1) the dimension of state vector \(z\) equals the number of locations, which may be as large as 100 in our IDS problem. In general, a single location case can be solved directly by the DP method given that the regeneration point theorem reduces the set of states to be considered. Unfortunately, the regeneration point theorem\(^2\) does not hold in the case of multiple locations. However, approximations that reduce the size of the state-space as though the theorem could be generalized create good heuristic procedures. Several heuristics of this type have been proposed. One may refer to Jacobsen (1990) for details.

Van Roy and Erlenkotter (1982) present the following multi-period version of the uncapsiclated facility location model:

\(^2\) See Jacobsen (1990) for details about regeneration point theorem, p. 183.
\[
\begin{align*}
\text{Min} & \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{s=1}^{S} \sum_{j=1}^{J} c_{ji}^{ts} y_{ji}^{ts} + \sum_{s=1}^{S} \sum_{j=1}^{J} f_j^{s} x_j^{s} \\
\text{subject to} & \\
\sum_{j=1}^{J} \sum_{s=1}^{S} y_{ji}^{ts} = 1 & i \in [1,I], \quad t \in [1,T] \\
y_{ji}^{ts} \leq x_j^{s} & \forall (i,j,t,s) \\
y_{ji}^{ts} \geq 0 & \in \{0,1\}
\end{align*}
\]

where, the input data are:

\(c_{ji}^{ts}\): the cost of supplying customer \(i\) in time period \(t\) from capacity established at location \(j\) at the beginning of time period \(s\), with \(c_{ji}^{ts} = +\infty\) for \(t < s\).

\(f_j^{s}\): the fixed cost of establishing capacity at location \(j\) at the beginning of the time period \(s\).

The decision variables are:

\(y_{ji}^{ts}\): the fraction of customer \(i\)'s demand in time period \(t\) delivered from capacity established at the beginning of time period \(s\) at location number \(j\).

\(x_j^{s}\): binary variable indicating whether or not capacity is established at location \(j\) at the beginning of time period \(s\).
Model (4.1) can be interpreted as a single-period model with "locations" \( j, s \) and "customers" \( i, t \). Consequently, (4.1) can be solved by DUALOC (Erlenkotter, 1978).

However, by noting that
\[
\sum_{s=1}^{T} x_{j}^{s} \leq 1 \quad j \in [1, J]
\]

in an optimal solution \( f_{js} > 0 \) is assumed) and by assuming that \( c_{ji}^{ts} \) is independent of \( s \), Van Roy and Erlenkotter (1982) develop a substantially more efficient implementation, which they refer to as DYNALOC. DYNALOC is efficient in solving this kind of simple uncapacitated facility location problem.

**4.1.2 Algorithms for Single-period Location-Allocation Models**

As discussed in Chapter 3, the model developed by Gendron and Crainic (1995) is a single-period container depot selection model, essentially the same as the single-period model (1.1) through (1.11) we presented in Chapter 3. Gendron and Crainic (1995) propose a branch-and-bound algorithm in which bounds are computed by a dual-ascent procedure and the design of efficient branching, fathoming and preprocessing rules is particularly emphasized.
Gendron and Crainic (1995) report that the algorithm outperforms other existing methods, such as the TABU search heuristic (Crainic, et al., 1992), the dual ascent heuristic (Crainic and Delorme, 1993), etc.

The maximum number of potential depots in randomly generated examples in Gendron and Crainic (1995) is 43 depots; the time taken to solve them ranges from 47 to 3975 seconds on a SUN Sparc2 workstation. For a multi-period model with 4 time periods and 12 time sub-periods, the maximum number of potential depots allowed, in order not to exceed the maximum number of candidate depots handled in Gendron and Crainic (1995), is about 4. Thus, for our MPB, the above solution algorithm is far from being useful in practice.

4.2 Price-Directive (Lagrangean) Decomposition Algorithm for MPB

Because discrete facility location problems contain two types of inherently different decisions – where to locate facilities and how best to allocate demands to the resulting facilities – this problem class is an attractive candidate for decomposition. Once the discrete-choice facility location decisions have been made, the continuous allocation problem typically becomes much simpler to solve. Two basic decomposition strategies applicable to location problems are price directive (Lagrangian relaxation) and resource directive (Benders’) decomposition. The reader is referred to Shapiro (1979) for details.
about these two decompositions and to Magnanti and Wong (1990) for decomposition methods for facility location problems. In the following, we discuss two decomposition-based algorithms for solving MPB; one is the price-directive decomposition algorithm, and the other is the resource-directive decomposition algorithm.

A natural way to perform price directive decomposition on our MPB model, (2.1)-(2.12), is to bring the constraints linking locational and allocation decisions up to the objective function (2.1). The linking constraints turn out to be constraints (2.7)-(2.9), which tie the locational variables $y$ together with allocation variables $x$, $q$ and $z$. The obtained Lagrangean relaxation is as follows:

$$L(\lambda) = \min \left\{ \text{OBJ} + \sum_{(ij) \in A} \lambda_{ij}(t,\tau) \left[ x_{ij}(t,\tau) + \theta_{ij}(t,\tau) - v_j(t)y_j(t) \right] \right\}$$ (5.1)

subject to

$$\begin{align*}
(2.2) - (2.6), (2.10) - (2.12) \\
\theta_{ij}(t,\tau) \geq 0 & \quad (ij) \in A \quad t \in [1,T] \quad \tau \in [1,P] \\
\lambda_{ij}(t,\tau) \geq 0 & \quad (ij) \in A \quad t \in [1,T] \quad \tau \in [1,P] 
\end{align*}$$ (5.2)

Where OBJ in (5.1) is the objective function (2.1). $\theta$ vector in (5.3) is the slack variables in constraints (2.7) through (2.9). $x$ stands for the continuous variables of $x$, $q$, or $z$ in MPB.
The dual problem to the original problem (2.1) – (2.12) is:

\[(\text{DU}) \quad \text{Maximize } L(\lambda) \quad (6.1)\]

To solve the dual problem (6.1), note that the Lagrangean relaxation as given in (5.1)-(5.4) may be separated into the following two independent sub-problems:

\[(\text{LC}) \quad \text{Min} \left\{ \sum_{t=1}^{T} \sum_{j \in F} f_j(t)y_j(t) - \sum_{(ij) \in A} \sum_{t, \tau} \lambda_{ij}(t, \tau)v_{ij}(t)y_j(t) \right\} \quad (7.1)\]

subject to

\[ (2.11) \text{ and } (2.12) \quad (7.2)\]

and

\[(\text{AL}) \quad \text{Min} \left\{ SCD + \sum_{(ij) \in A} \sum_{t, \tau} \lambda_{ij}(t, \tau) \left[ x_{ij}(t, \tau) + \theta_{ij}(t, \tau) \right] \right\} \quad (8.1)\]

subject to

\[ (2.2) - (2.6), (2.10) \quad (8.2)\]

\[ \theta_{ij}(t, \tau) \geq 0 \quad (ij) \in A \quad t \in [1, T] \quad \tau \in [1, P] \quad (8.3)\]
where SCD in (8.1) stands for the second through the fifth items in the objective function of formulation (2.1).

(LC) is a pure location problem, and (AL) a pure allocation problem. Hence, the Lagrangean relaxation (5.1) – (5.4) is decomposed into two sub-problems, each of which is relatively easily solved. It is important to note how the multipliers \{\lambda\} act to split the various costs between the (LC) and (AL) models. In fact, the determination of the appropriate multipliers (or equivalently, the appropriate allocation of the total cost between the (LC) and (AL) models) can be interpreted as a feedback process in the hierarchical framework of our IDS problem.

The dual problem is to find \{\lambda\} to maximize the Lagrangean. One procedure to solve the dual would be iterative, where at each step the Lagrangean is solved for a given \lambda, and based on this solution a new set of multipliers is picked.

The solution of the dual problem given in (6.1) need not, and usually will not, identify a primal feasible solution to the original problem. In such instances, a duality gap is said to exist and the dual solution is just a lower bound for the optimal value of the original problem. Two procedures have been suggested for resolving these duality gaps. The first approach is to use the dual problem for generating bounds in a branch-and-bound or implicit enumeration procedure; the feasibility of such an approach would depend on the tightness of the bounds from the dual problem, and on the number of integer variables.
The number of integer variables in our IDS problem is equal to the number of locations times the number of time periods, which is very large.

The second approach is to incorporate the solution of the dual problem into a heuristic procedure. Here, at each iteration of the dual solution process, a feasible solution to IDS, corresponding to the current dual solution, is generated. The lowest cost of these feasible solutions can be compared with the value of the dual problem, which is a lower bound on the primal problem. The procedure stops either when the best feasible solution is sufficiently close to the lower bound generated from the dual problem, or after a preset number of iterations, whichever occurs first. We adopt this approach in our algorithm.

Thus, our price-directive decomposition algorithm for solving MPB is proposed as follows.

**Step 0**: form sub-problems (LC), (AL), and the dual (DU) of the original MPB problem

**Step 1**: set \( k = 0 \), choose \( \lambda_0 = 0 \)

**Step 2**: solve (AL) and (LC) separately for \( \lambda = \lambda_k \)

**Step 3**: generate a feasible solution to MPB:
- based on the (AL) solution in Step 2, solve constraints (2.7) – (2.9) in MPB to obtain a feasible solution to MPB

**Step 4**: if \( y_k < \delta \), then stop

**Step 5**: update \( \lambda_k \): \( \lambda_{k+1} = \lambda_k + \alpha_k y_k \), set \( k = k + 1 \), and go back to Step 1
\( \gamma_k \) is equal to the difference between the objective value of (2.1) for the feasible solution derived in Step 3 and \( L(\lambda_k) \) obtained from the solution in Step 2. \( \delta \) is a preset stopping criterion. Another stopping criterion is to preset the number of iterations allowed. Scalar \( \alpha_k \) is the step size. Its choice is critical to the convergence behavior of the procedure. \( \alpha_k \) is usually set between 0.25 and 1.

The economic interpretation of this price-directive algorithm is as follows. At any arbitrary iteration \( k \), the planner responsible for the depot location decision chooses an optimal set of depots, given the Lagrangean multipliers (prices) \( \lambda_k \), by solving (LC). Then a lower-level container operator tries to allocate containers, given the above depot-opening decision and the prices \( \lambda_k \). Because of the arbitrary setting of \( \lambda_k \), (AL) may not have a feasible solution. Thus, in the next round of iteration, \( \lambda_k \) is readjusted and (LC) and (AL) are resolved according to the new prices. In this way, the overall performance of the system can be improved over time.

This algorithm can also be embedded into a branch-and-bound algorithm to find a lower bound of the original problem to speed up the branch-and-bound algorithm.
4.3 Resource-Directive (Benders’) Decomposition Algorithm for MPB

For our MPB formulation of the IDS problem, Benders’ decomposition consists of an integer programming master problem involving the variables \( y \), and a linear programming sub-problem involving the variables \( x \) (including \( x, q \) and \( z \) variables in our MPB model).

The approach is resource directive, because for \( y \) fixed at values 0-1, (2.1) reduces to the linear programming sub-problem:

\[
 u(\hat{y}) = \text{Min} \left\{ \sum_{i=1}^{r} \left[ \sum_{t=1}^{p} c_i(t,\tau)x_i(t,\tau) + \sum_{j=1}^{q} c_j(t,\tau)z_j(t,\tau) \right] + \sum_{j=1}^{q} \sum_{\tau=1}^{T} q_j(t,\tau) \right\}
\]

subject to

(2.1) - (2.10)

The dual for (9.1) is as follows:

\[
 u(\hat{y}) = \text{Max}_\lambda \left\{ \lambda^1 y + \lambda^2 s + \lambda^3 d + \lambda^4 u \right\}
\]

subject to

\[
\lambda^1 + \lambda^2 + \lambda^3 + \lambda^4 \leq 0
\]

\[
\lambda^1, \lambda^4 \leq 0
\]
where \( y, s, d, \) and \( u \) stand for the value of depot-opening vector \( \hat{y} \), container supply vector \( s \), demand vector \( d \), and transportation link capacity vector \( u \) respectively.

Denote \( u^n \) for \( n=1,...,N \) and \( u^m \) for \( m=1,...,M \) to be the respective extreme points and extreme rays of the dual feasible region in (10.1). At any arbitrary iteration of the algorithm, we use subsets of \( u^n \) and \( u^m \) to construct the integer programming master problem:

\[
v' = M \quad i \in \nu
\]

subject to

\[
v \geq \sum_{i=1}^{r} \sum_{j \in F} f_j(t) y_j(t) + u^n[\lambda^s y + \lambda^d s + \lambda^d d + \lambda^u u] \quad n \in [1,N] \tag{11.1}
\]

\[
u^n[\lambda^s y + \lambda^d s + \lambda^d d + \lambda^u u] \leq 0 \quad m \in [1,M] \tag{11.2}
\]

\[
y_j(t) \in \{0,1\} \quad \forall j, t \tag{11.4}
\]

The solution \( \hat{y} \) is used in the pair of linear programming problems (9.1) and (10.1), which can be optimized by the simplex method. If (9.1) is infeasible, then a new dual extreme ray is discovered and a constraint is added to the constraint set in (11.3). If (9.1) is feasible, then it has an optimal \( \hat{x} \) and \((\hat{x}, \hat{y})\) is a feasible solution to the mixed integer
programming (2.1). Let \( \hat{\lambda} \) denote the optimal solution to (10.1), found by the simplex method. The solution \((\hat{x}, \hat{y})\) is optimal in (2.1) if

\[
y' \geq \sum_{t=1}^{T} \sum_{j \in F} f_j(t) \hat{y}_j(t) + u^T [\hat{\lambda}_1 \hat{y} + \hat{\lambda}_2 s + \hat{\lambda}_3 d + \hat{\lambda}_4 u]
\]  

(12.1)

holds. If this optimality test fails, then condition (12.1) is added to the constraints in (11.2).

Thus our resource-directive (Benders') decomposition algorithm for MPB can be stated as follows.

**Step 0:** form linear sub-problem (9.1), its dual (10.1), and integer programming master program (11.1).

**Step 1:** set \( k = 1 \); and provide an initial set of binary values for location variables \( y_k \).

**Step 2:** use simplex method to solve linear sub-problem (9.1) and its dual (10.1), given \( y_k \).

**Step 3:** if (9.1) is infeasible, then a new dual extreme ray is discovered and an additional constraint is added to the constraint set in (11.3); and go to Step 5; if feasible, then \((x_k, y_k)\) is a feasible solution to MPB.

**Step 4:** if \((x_k, y_k, u_k)\) satisfies condition (12.1), then \((x_k, y_k)\) is optimal in MPB, stop; otherwise, condition (12.1) is added to the first constraint set in (11.2).
Step 5: set $k = k + 1$, solve the integer master problem (11.1), and go back to Step 2.

The resource-directive decomposition algorithm for MPB converges in a finite number of iterations to an optimal solution because each time the integer programming problem (11.1) is solved, there is a new constraint added to the constraint set of (11.1) or (11.2), and there are only a finite number of such constraints possible. The algorithm has the desirable feature of producing a feasible solution to (2.1) at each iteration that (9.1) is feasible, and the lower bound $v'$ is the cost of an optimal solution in (2.1). Moreover, the lower bounds increase monotonically with iterative solutions of the master problem.

Note that the multi-period container allocation problem (9.1) can be further decomposed into single-period transportation problems, if doing so is computationally more efficient.

The Benders' decomposition algorithm has the following economic justification. At any arbitrary iteration $k$, the depot location decision maker wishes to select a set of depots so that the IDS problem, formulated as MPB in (2.1), is solved when individual linear programming problems, (9.1), corresponding to each time sub-period's container allocation problem can be solved using some simpler techniques. Because of the arbitrary selection of the depot set, container allocation problems for individual time sub-periods may not have a feasible solution. Thus, in the next iteration, the chosen depot set is readjusted so that container allocation problems of the individual time sub-periods are resolved
accordingly. In this way, the overall performance of the IDS problem can be improved over time.

4.4 Chapter Summary

MPB is a multi-period (dynamic) mixed integer program. In order to solve MPB efficiently, in this chapter we critically evaluated methods for solving large-scale locational models, including solution methods for multi-period location models and for single-period location-allocation models. We proposed two decomposition-based algorithms for solving MPB. The first one is a price-directive (Lagrangian) decomposition algorithm and is a heuristic. The second one is a resource-directive (Benders) decomposition algorithm and can be solved to optimality.
Chapter 5

MPB Model Implementation

We devote this and the next chapters to dealing with the application and implementation issues of the MPB model. In this chapter, we first apply the MPB model to solve a real-world depot selection problem encountered by a major container shipping company in North America.

To address the issues of the model implementation and deployment, we develop the concept of container supply chain so that we are able to examine the sophisticated implementation issues from the perspective of container supply chain movement. This will help us better understand the institutional issues behind the container depot selection problems.

5.1 MPB Model Application

In this section, we apply the model to solve an inland depot selection problem for a major liner shipping company in North America. Some of the information is disguised to protect the confidentiality of the company. However, the changes in numerical values do not distort the fundamental relationships of the factors involved.
5.1.1 Problem Description

The problem is about the company's inland depot operation in the hinterland of the Port of New York, including the northeast of the US and the Lake Ontario area in Canada. Currently, the inland depot network in the region consists of 10 inland depots.

On a weekly basis, the number of 40-foot containers into/out of each depot is presented in Table 5.1. The number of containers into/out of each depot changes with time, and the size of the depots varies. The New York depot and the Chicago depot are much larger than the others. This indicates the important role of New York as the sea access of the northeastern region in the US to the Atlantic Ocean and to the Pacific through an all-water way, and of Chicago as the intermodal hub between the West coast and the northeast.

We aggregate customers according to their geographical location, so that each depot in the network serves three to four aggregated customers (shippers and/or consignees). In some cases, a shipper may later become a consignee, or vice versa. Table 5.2 shows the distance and the number of 40-foot containers transported between depots and their major customers during a week. Altogether, these volumes represent about 80% of the company's total container movement in the region.
## Table 5.1  Number of 40’ Containers Into and Out of Each Depot

<table>
<thead>
<tr>
<th>depots</th>
<th>into depot (containers / week)</th>
<th>out of depot (containers/week)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>average</td>
<td>minimum</td>
</tr>
<tr>
<td>Depot 1: Randolph, MA</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Depot 2: Worcester, MA</td>
<td>62</td>
<td>42</td>
</tr>
<tr>
<td>Depot 3: Portland, MN</td>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>Depot 4: Chicago, IL</td>
<td>320</td>
<td>260</td>
</tr>
<tr>
<td>Depot 5: Pittsburgh, PA</td>
<td>82</td>
<td>70</td>
</tr>
<tr>
<td>Depot 6: Stanton, NJ</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>Depot 7: Cincinnati, OH</td>
<td>58</td>
<td>44</td>
</tr>
<tr>
<td>Depot 8: Buffalo, NY</td>
<td>92</td>
<td>70</td>
</tr>
<tr>
<td>Depot 9: Toronto, ON</td>
<td>70</td>
<td>60</td>
</tr>
<tr>
<td>Depot 10: New York, NY</td>
<td>280</td>
<td>202</td>
</tr>
</tbody>
</table>

Note:  The data is for 40' containers only in a typical week.
Source: Provided by the shipping company.

## Table 5.2  Distance and Number of 40’ Containers to and from Customers

<table>
<thead>
<tr>
<th>depots</th>
<th>customers</th>
<th>distance (miles)</th>
<th>containers/week</th>
<th>to shippers</th>
<th>from consignees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depot 1: Randolph, MA</td>
<td>1</td>
<td>28</td>
<td>18</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>42</td>
<td>12</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>64</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depot 2: Worcester, MA</td>
<td>5</td>
<td>8</td>
<td>38</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>15</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depot 3: Portland, MN</td>
<td>8</td>
<td>82</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>14</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>60</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depot 4: Chicago, IL</td>
<td>12</td>
<td>20</td>
<td>180</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>38</td>
<td>64</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>112</td>
<td>40</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>140</td>
<td>86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depot 5: Pittsburgh, PA</td>
<td>16</td>
<td>20</td>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>40</td>
<td>18</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>60</td>
<td>29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depot 6: Stanton, NJ</td>
<td>19</td>
<td>8</td>
<td>15</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>48</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>108</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depot 7: Cincinnati, OH</td>
<td>22</td>
<td>18</td>
<td>26</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>42</td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depot 8: Buffalo, NY</td>
<td>25</td>
<td>10</td>
<td>46</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>15</td>
<td>28</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depot 9: Toronto, ON</td>
<td>28</td>
<td>40</td>
<td>40</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>60</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depot 10: New York, NY</td>
<td>31</td>
<td>22</td>
<td>160</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>28</td>
<td>80</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>33</td>
<td>40</td>
<td>28</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>34</td>
<td>62</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note:  The data is only for 40' containers in a typical week.
Source: Provided by the shipping company.
The question facing the company is that, given the forecast in one year about the container demand and supply, the depot and transportation costs and capacity, and the container inventory cost, how should it choose its inland depot network from the set of 10 currently operated depots to meet customer demand for empty containers and to minimize the overall container-related costs. There are 17 consignees (empty container suppliers) and 25 shippers (empty container demanders), as shown in Table 5.2. The consignees are numbered from 1 through 17, and shippers from 1 through 25. The total planning horizon of one year is divided into two planning periods, each is six months long. The inland depot location decision is made for each six-month planning period, each of which is further divided into three two-month-long sub-periods. The container allocation decision (rebalancing action) is made for each two-month sub-period.

5.1.2 Model Result and Interpretation

The MPB model of the problem is solved by a Branch-and-Bound algorithm using the OSL solver (Optimization Subroutine Library) embedded in the GAMS optimization language (General Algebraic Modeling System, Brooke et al., 1992). The model is run on a PC-486 computer with 24 MB memory.

To compare the effectiveness of the MPB model, we run the model under several scenarios, as shown in Table 5.3. “CURRENT” in Table 5.3 stands for the current

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3 The GAMS code of the MPB model is available upon request.
situation of the shipping company's inland depot operation in the region. "SINGLE" means the single-period model developed in Section 3.3. MPB1 through MPB4 are the variants of the MPB model using different parameters. One of the major differences among the testing scenarios is in source depots. In contrast to a general depot, a source depot is allowed to move containers into or out of the system through this depot. The existence of source depots enables the MPB model to move containers into or out of the system to minimize the system's overall container-related costs. In general, seaport depots and inland hub depots can be regarded as source depots. MPB1 and MPB4 are two extreme scenarios. MPB2 and MPB3 can be regarded as closer to real-world operation, where some of the depots are designated by the shipping company as source depots. The results of the depot-opening decisions determined by the MPB model are reported in Figure 5.1. The corresponding container allocation decisions in the depot network over the planning horizon are simultaneously determined\(^4\).

\(^4\) The complete running result for the problems are available upon request.
Table 5.3 Parameters of Testing Scenarios

<table>
<thead>
<tr>
<th>scenario</th>
<th>periods</th>
<th>depot fixed cost</th>
<th>source depots</th>
<th>storage cost</th>
<th>inventory cost</th>
<th>transport cost</th>
<th>handling cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>CURRENT</td>
<td>1</td>
<td>$600,000</td>
<td>none</td>
<td>$1.2/cntr.day</td>
<td>$0.9/cntr.day</td>
<td>$75+0.34*miles</td>
<td>$56/cntr.</td>
</tr>
<tr>
<td>SINGLE</td>
<td>1</td>
<td>$600,000</td>
<td>none</td>
<td>$1.2/cntr.day</td>
<td>$0.9/cntr.day</td>
<td>$75+0.34*miles</td>
<td>$56/cntr.</td>
</tr>
<tr>
<td>MPB1</td>
<td>6</td>
<td>$600,000</td>
<td>none</td>
<td>$1.2/cntr.day</td>
<td>$0.9/cntr.day</td>
<td>$75+0.34*miles</td>
<td>$56/cntr.</td>
</tr>
<tr>
<td>MPB2</td>
<td>6</td>
<td>$600,000</td>
<td>4, 10</td>
<td>$1.2/cntr.day</td>
<td>$0.9/cntr.day</td>
<td>$75+0.34*miles</td>
<td>$56/cntr.</td>
</tr>
<tr>
<td>MPB3</td>
<td>6</td>
<td>$600,000</td>
<td>1, 4, 5, 7, 8, 10</td>
<td>$1.2/cntr.day</td>
<td>$0.9/cntr.day</td>
<td>$75+0.34*miles</td>
<td>$56/cntr.</td>
</tr>
<tr>
<td>MPB4</td>
<td>6</td>
<td>$600,000</td>
<td>1–10</td>
<td>$1.2/cntr.day</td>
<td>$0.9/cntr.day</td>
<td>$75+0.34*miles</td>
<td>$56/cntr.</td>
</tr>
</tbody>
</table>

* Note: The handling cost means the cost for moving a container into/out of the system.
Source: The data is compiled by the author based on the information provided by the shipping company.

Because there is no data available on the depot shut-down and re-opening cost, we assume that a depot will stay open or shut-down through the planning horizon. For the 10 depots currently operated by the company, depots 1, 3 and 4 are determined not to be open.
by the MPB model to minimize the total costs. When there are more depots allowed to interchange containers with the outside, as shown from scenarios MPB1 to MPB4, depot 7 is dropped from the opening list. The total container-related costs of these scenarios are depicted in Figure 5.2.

Because the scenarios of CURRENT, SINGLE, and MPB1 are closed system and have no container interchange with the outside, the number of containers within the system is fixed. Thus, the container storage and inventory costs in these three scenarios are the same. The depot-opening and container allocation decisions in SINGLE and MPB1 are obtained by our optimization model MPB. Therefore, their costs are lower than that of CURRENT. Notice that, because the parameters used in the problems, including depot fixed cost, storage cost, inventory cost, transportation cost, and handling cost, are constant over the planning horizon, the optimal solutions for SINGLE and MPB1 are identical. However, the problem with a single-period model is not because it cannot yield least-cost solution; rather, a single-period model is not able to reflect the actual operations in the real world. It would not be realistic that the container inventory imbalance over an inland depot network be adjusted only once a year as implied in the SINGLE scenario.

For scenario MPB2, depots 4 and 10 are allowed to interchange containers with the outside world. Compared with MPB1, MPB2 reduces the system’s storage, inventory, and overall costs, while managing to meet customer demand, even though the transportation cost is higher. When more depots are designated as source depots to allow interaction
Figure 5.2 Costs of the Inland Depot Operation
with the outside, as shown in scenarios MPB3 and MPB4. Cost reduction mainly comes from container storage and inventory.

Container inland transportation and handling charges are generally open to the public and stable in the intermodal industry; container inventory cost can be easily derived from the capital cost of new containers. Hence, given container demand and supply, depot fixed cost and storage cost are the most sensitive parameters in Table 5.3 for the problem. In addition, depot fixed cost and storage cost are the focus of our IDS problem (intermodal depot selection problem). Therefore, we perform a sensitivity analysis to test the robustness of the MPB model, that is, whether MPB can outperform the CURRENT scenario (actual operation) and the SINGLE scenario (the single-period model) under various depot fixed and storage costs. The container inventory cost, transportation cost, and handling cost, remain the same as the above; the depot fixed and storage cost vary as described in Table 5.4 below. MPB4 is the representative of the MPB model. CURRENT stands for the current operations. SINGLE represents the single-period model.

<table>
<thead>
<tr>
<th>Table 5.4 Parameters of the Sensitivity-Testing Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>depot fixed cost change</td>
</tr>
<tr>
<td>P2</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>0%</td>
</tr>
<tr>
<td>storage cost change</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>0%</td>
</tr>
</tbody>
</table>
The model results for these sensitivity-testing problems are reported in Figure 5.3. The cost of the MPB4 result is lower than those of the CURRENT and MPB1 results. This shows that the MPB model is capable of providing lower-cost solution than the current operation or the single-period model under various cost settings.

To compare and contrast the patterns of the container allocation in the current operation and the MPB model result, Figure 5.4 reports the container flow between the New York depot (depot 10) and its shippers and consignees in the current operation and in the MPB1 model result (called Current and Optimal respectively in Figure 5.4). The resulting flow between the depot and consignees of the current operation and the MPB1 solution are very close. This implies that the current operation between the New York depot and its consignees is close to optimum. On the other hand, there is a much larger number of containers transported to shippers from the depot in the current operation than it should do as recommended by the model. Instead, some of the containers in the New York depot should be moved to other depots to optimize the overall inland depot network-wide performance.

Container inter-depot movement plays a crucial role in rebalancing container inventory among depots. Our MPB model enables the planners to “look ahead” to adjust container inventory among depots according to the future demand and supply pattern, and the cost and capacity structure. Our multi-period modeling framework provides an adequate framework for the “look ahead” planning and rebalancing operations.
Figure 5.3  Sensitivity of Total Inland Depot Cost
Figure 5.4 Container Flows between New York Depot and Its Consignees and Shippers
5.2 Container Supply Chain Management

In order to address the implementation and deployment issues of the MPB model, in this section, we develop the concept of container supply chain so that we can examine the sophisticated implementation issues from the viewpoint of the container supply chain. This will help us better understand the institutional issues behind the container depot selection problems. After introducing the supply chain management in manufacturing and service industries, we develop the concept of container supply chain. Finally, we address the implementation issues of the MPB model from this perspective.

5.2.1 Introduction to Supply Chain Management in Manufacturing and Service Industries

Christopher (1994) defines a supply chain as a network of organizations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services in the hands of ultimate consumer. Most of the studies about supply chain thus far have focused on manufacturing, because physical material flow in manufacturing industries is more tangible and visible than chain channels in service industries.

The focus of supply chain management is given to the following areas (Franciose, 1995).
(1) Strategic issues: strategic drivers of supply chain management to achieve competitive advantage through focus on:

- **strategic goals**: increased efficiency, customer focus, reduced costs
- reduced time to market, and adapting to market changes
- **strategic choices**: core competency, inter-company ties, and supply chain choice

(2) Process restructuring: process restructuring techniques to improve the flow of materials, information and cash:

- within companies
- between two companies
- within a multi-company supply chain

(3) Organizational issues:

- supply chain organizational forms
- changing roles, need for new skill sets, human resource issues
- behavior incentives
- monitoring mechanisms for supply chain relationships

(4) Integration enablers: supporting systems and tools for supply chain management:

- information systems and technology
- measurement systems and metrics (including costing methods)
- supply chain analysis tools and models
One of the first systematic studies about service supply chain management was conducted by Smith and Barry (1991), who describe a service supply chain as a process of coordinating non-material activities necessary to the fulfillment of the service in a cost-and customer service-effective way.

Smith and Barry (1991) compare and contrast the characteristics of the manufacturing supply chain and service supply chain activities as shown in Table 5.5.

Table 5.5 Characteristics of Manufacturing and Service Supply Chain Activities

<table>
<thead>
<tr>
<th>Manufacturing Supply Chain</th>
<th>Service Supply Chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>sales forecasting</td>
<td>service request forecasting</td>
</tr>
<tr>
<td>sourcing/purchasing</td>
<td>partnership development, staff hiring, data acquisition</td>
</tr>
<tr>
<td>production planning</td>
<td>staff and equipment scheduling, distribution</td>
</tr>
<tr>
<td>inbound transportation</td>
<td>channel selection, capacity planning</td>
</tr>
<tr>
<td>inventory management</td>
<td>data collection, customer pick-up, repair part pick-up</td>
</tr>
<tr>
<td>warehousing</td>
<td>capacity management, database management</td>
</tr>
<tr>
<td>customer service</td>
<td>customer record management, personnel training</td>
</tr>
<tr>
<td>order processing</td>
<td>data/information storage, retrieval and management</td>
</tr>
<tr>
<td>distribution systems</td>
<td>quality measurement and management, expediting, billing</td>
</tr>
<tr>
<td>field warehousing</td>
<td>interacting, assessing need, negotiating, and committing to customer, monitoring delivery</td>
</tr>
<tr>
<td>distribution control</td>
<td>network layout, network planning, systems planning, channel planning</td>
</tr>
<tr>
<td>intra-company transportation</td>
<td>data/information storage, retrieval and control</td>
</tr>
<tr>
<td>distribution administration</td>
<td>network control, communications control</td>
</tr>
<tr>
<td>outbound transportation</td>
<td>personnel/customer movement, data/information management</td>
</tr>
<tr>
<td></td>
<td>network administration</td>
</tr>
<tr>
<td></td>
<td>customer reporting, service engineering routing and scheduling to customer transportation sites</td>
</tr>
</tbody>
</table>
5.2.2 Conceptual Development of Container Supply Chain

The focus of the shipping companies’ container management is on their empty fleet, which is the product delivered by the container management department to serve the customers’ shipping requirements. The movement of empty containers ends in the hands of shippers, which load their cargo for the subsequent shipment. There are several sources supplying empty containers: seaports after unloading containers from inbound ships; inland depots with containers transported from seaports, consignees or other inland depots; and consignees after containers stripped for a shipment; and lessors and repair shops.

A container supply chain possesses the characteristics of both manufacturing and service supply chains. Container shipping, as a mode of transportation services, falls into the category of service industries. Therefore, its supply chain should have the characteristics of service supply chains. The actual flow along a container supply chain, however, is tangible containers. Thus container supply chains share some features with manufacturing supply chains.

A empty container supply chain is developed and presented in Figure 5.5 below. There are two kinds of flow along the chain. The first is about physical containers as depicted by the real lines in Figure 5.5. A shipper receives containers from seaports, inland depots, or consignees. A consignee returns stripped empty containers, as instructed by the shipping company, to seaports, inland depots, or directly to shippers. A seaport obtains empty containers from inbound ships, inland depots, or consignees, and sends
containers to other seaports, inland depots, or shippers. An inland depot receives empty containers from seaports, other inland depots, or consignees, and dispatches containers to seaports, other inland depots, or shippers. To simplify the presentation, the repair shops and lessors are not included in the figure as sources of empty containers. The second kind of flow is about information as shown by the dotted arcs in Figure 5.5, which involves four major departments in a shipping company. The inbound freight department is in charge of the import operations of full and empty containers and deals with consignees, seaports, and inland depots. The outbound freight department is responsible for the export operations of full and empty containers and is involved with shippers, seaports, and inland depots. The responsibility of the marketing/sales department is for marketing outbound freight shipment and maintaining a close relationship with shippers. Therefore, the inbound and outbound freight departments and the marketing/sales department possess a large amount of information about containers, in particular, the data about the container demand and supply, although it is the container fleet management department that is actually responsible for planning and allocating containers over the company’s geographical network. The inbound and outbound freight departments and the marketing/sales department influence the container management through the information exchange with it as shown in the upper part of Figure 5.5.
Figure 5.5 Container Supply Chain
5.3 MPB Model Implementation: the Perspective of Supply Chain Management

In order to make the MPB model implementable and deployable for shipping and intermodal carriers as an effective decision-support tool, in this section, we address the implementation and deployment issues of the MPB model using the container supply chain concept developed in the previous sector.

The most distinct characteristic of container supply chains is that containers are usually under the control of the fleet management department for their entire movement through the supply chain, as shown in the lower part of Figure 5.5. Other functional departments, such as the inbound and outbound freight departments, and the marketing/sales department, do not directly involve containers' operations. This feature suggests that the complexity of the container supply chains mainly lies on the information flow instead of on the physical flow of containers.

In order to solve the MPB model, there is a need to collect and forecast a large amount of reliable data about the future container demand and supply, transportation cost and capacity, depot cost and capacity, and container inventory cost in the planning horizon. The data about the future transportation cost and capacity, depot cost and capacity, and container inventory cost are generally available within the fleet management department.
The future container supply consists of the containers recovered after being used by shippers and is usually estimated based on the projected accumulation of inbound container flow at each point of the planning horizon in the future. Empty containers can be recovered after being used by shippers. After loading a shipper’s cargo, a container is transported to a seaport and boards a ship to its consignee’s location, where the container is stripped by the consignee and transported to a designated seaport, inland depot or shipper. The container then rejoins the empty container supply chain. Therefore, in order to manage empty containers, the fleet management department needs to obtain information about the inbound shipment from the inbound freight department to track the full containers that will become empty at a later time. The inbound freight department can provide the inbound container flow data, comprising volume of traffic, origin and destination (including inland shippers and consignees, and loading and unloading ports) at each point of the planning horizon in the future.

The above data are generally reliable and sufficiently good for the use in a planning model like MPB. The most troublesome data are container demand, which come from the marketing/sales and the outbound freight departments. In order to maximize their primary objective, customer satisfaction, the marketing/sales and the outbound freight departments tend to require more containers than are actually needed in order to protect the shippers (and their own departments) from running into the stockout of empty. Therefore the fleet department has to prepare for more containers than needed. This is a cost inefficiency for the entire company.
The principles of the container supply chains can be used to reduce the container-related inefficiency, including the unnecessarily high level of container storage, etc., as discussed below.

(1) Set the appropriate strategic goals of the company so that the performance of individual departments can be evaluated from global optimum instead of local functional optimum. For example, the satisfaction maximization of customer demand for empty containers and the minimization of total container-related costs can be used as the global objectives to evaluate the performance of each individual department. This indicates that when managing its containers, the fleet management department should consider its own objective of minimizing total container-related costs. It should also meet other departments’ objectives, for example, the customer demand satisfaction maximization of the marketing/sales department.

(2) Restructure the process of information flow among the functional departments. The marketing/sales department and the inbound and outbound freight departments should provide the necessary data as accurately as possible, to assist the fleet department in making proper container management decisions, in order to realize the objective of global optimum for the entire shipping company.
(3) Improve the supply chain enablers. One of the tasks is to strengthen our MPB model's capability to handle the uncertainty in the container demand and supply. This will be discussed in detail in the next chapter.

There are some other important organizational issues for the successful implementation of the MPB model in shipping companies. The MPB model is generally run by the fleet management department to optimize its decisions about inland depot selection and container management. In today's highly competitive shipping markets, however, customer demand satisfaction is the top priority in any shipping companies. The container fleet management is a "service and assurance" activity in the shipping business. Under pressure from customer-related departments and from senior management, it may be difficult for the fleet department to manage containers to achieve both the goals of the satisfaction maximization of customer demand and the minimization of container-related costs. Therefore, to implement MPB, the top management needs to establish an efficient and effective organizational form, to pay attention to the changing roles of individual functional departments, to provide proper behavior incentives, and to develop monitoring mechanisms for supply chain relationships. For example, the fleet management department should be given the full responsibility to coordinate the container-related activities of obtaining relevant information and implementing necessary operations. In other words, the fleet management activities should be regarded as an integrated part of the company's strategic supply chain development.
5.4 Chapter Summary

This chapter was concerned with the MPB model’s application in practice. The MPB model was successfully applied to solve a real-world depot selection problem facing a major container shipping company for its North American operations. To address the issues of the model’s implementation and deployment, we developed the concept of container supply chains, and examined the sophisticated implementation issues from this perspective.

Chapter 6 will go on to solve two technical issues for the effective and efficient implementation of the model – direct container movement between consignees and shippers, and stochastic container demand and supply.
As demonstrated in Chapter 5, the multi-period model with balancing requirements (MPB), developed in the previous chapters, successfully determines the optimal location and size of inland depots for intermodal transportation companies. MPB is superior to the models in the literature, because it incorporates the fundamental features of the IDS problem, including the balancing requirements. On the other hand, however, MPB needs some improvements. When developing the models in Chapter 3, we assumed that direct movement is not allowed between shippers and consignees and that the demand for and supply of empty containers are deterministic. These two assumptions do not always hold in real world, and therefore limit its application scope and value as an effective decision-support tool in practice, although both assumptions can be justified on the grounds that they simplify the model development and make the solution methods computationally tractable.

In this chapter, we remove these two assumptions with the hope of making it more practically useful. First, we develop an efficient procedure based on the MPB solution to solve the IDS problem, permitting direct movement between shippers and consignees. Second, we develop a MPB-based simulation model, which integrates the MPB
optimization model into a statistical simulation model. It is capable of providing decision-makers customer satisfaction level and associated statistical confidence that can be used as feedback to re-run the MPB optimization model. Numerical examples are given in both cases.

6.1 Direct Movement between Suppliers and Demanders

Recall from Chapter 1 that we did not explicitly draw direct links between suppliers and demanders in Figure 1.2. In Figure 6.1, we re-draw most of Figure 1.2 with the explicit reflection (in thicker lines) of direct movement between suppliers and demanders.

Figure 6.1 Direct Movement of Containers between Suppliers and Demanders
In this section, we first show the necessity of adding movement between container suppliers and demanders to the MPB modeling framework. We then propose and evaluate several procedures to handle the movement. Finally, we present a numerical example to demonstrate the effectiveness of the proposed procedure.

6.1.1 Movement between Suppliers and Demanders

Consider the following example.

The simple network in Figure 6.2 has one depot, one empty container supplier and one demander respectively. The depot stores 100 empty containers, the supplier can provide 150, and the demander requires 250. The unit transportation cost is $50 on depot-demander link, $30 on supplier-demander link, and $40 on supplier-depot link.

The assumption made in the MPB modeling framework does not allow the direct movement from suppliers to demanders. Thus, the supplier has to first move 150 containers to the depot, and then transship them together with the 100 units stored in the depot to the demander. Using this route, the total transportation cost to meet the demand is:

\[ 150 \times 40 + (150 + 100) \times 50 = 18,500 \]

If we remove the assumption and allow direct movement from the supplier to the demander, the optimal route and cost are:
Then, the total cost is $9500, which is much lower than $18,500 in the previous situation. The significant cost saving results from the fact that the supplier and demander in this example is so close that moving containers between them is much more cost efficient than any supplier-depot-demander triangle movement.

![Diagram of direct movement between supplier, depot, and demander](image)

*Figure 6.2 Direct Movement between Supplier and Demander*

This example mirrors the real operations of liner shipping and other intermodal transportation. For example, a shipping liner may have a depot in Boston, Massachusetts,
and an empty container supplier (consignee) as well as a demander (shipper) in Providence, Rhode Island. If the time of an empty container released from the supplier is just several days ahead of the time required by the demander to load his shipment, the shipping company may require the supplier in Providence to transport the empty containers to the nearby demander’s site in the same town instead of first returning to the depot in Boston and then moving back to the demander in Providence several days later.

Container shipping companies and other intermodal transportation carriers operate inland depot networks consisting of many depots, suppliers and demanders. Some suppliers and demanders may be located close to one another. Intuitively, it may be more cost efficient to directly adjust container supply and demand between them without using a depot as an intermediary.

Our MPB modeling framework should reflect this reality and provides efficient solution methods to solve the resulting models.

6.1.2 Straightforward Procedures

The most straightforward way to deal with direct supplier-demander movement is to simply remove the restriction in the MPB modeling framework and add all supplier-demander links to the MPB model.
This procedure complicates the model structure and accordingly its solution in several ways.

First, link variables $x$ in MPB, standing for the number of containers transported between two locations (including supplier-depot, depots-depot and depot-demander), represent the largest number of continuous variables in MPB. Because the number of suppliers and demanders is usually larger than the number of depots, the number of link variables $x$ will increase significantly by adding all supplier-demander links to MPB, so too the total number of continuous variables. For example, suppose the network has $N$ candidate depots, $1.3\times N$ suppliers and demanders respectively. If supplier-demander links are not permitted, the total number of link variables $x$ is:

$$1.3\times N^2 + 1.3\times N^2 + N\times (N-1) = 3.6\times N^2 - N \quad (13.1)$$

Permitting supplier-demander movement will increase the number of link variables $x$ by $1.69\times N^2$, an increase of above 47%. Because the number of link variables $x$ dominates the number of depot storage variables $z$ or depot inflow variables $q$, the increase of the overall number of continuous variables due to the introduction of direct supplier-demander movement will be in the same order as the increase of the dominant variables $x$.

Second, in the constraint set of the MPB model – constraints (2.2) through (2.12) – the number of constraints in (2.6) and (2.7) is dominant. In the example of (13.1), permitting
direct supplier-demander movement will increase the number of overall constraints by above 40%.

Therefore, the size of the constraint matrix for the MPB model will increase by about 40% for the above example, if we adopt the straightforward procedure of simply adding all supplier-demander links into the MPB modeling framework. This is a huge computational burden.

A variant of the straightforward procedure is to add an artificial depot to the network for each connected supplier-demander pair. This artificial-depot procedure works in the following way.

**Step 1:** create an artificial depot h for each supplier-demander link i-j

**Step 2:** add artificial depot h, and links i-h and h-j to the network for each i-j link, where supplier i ∈ O and demander j ∈ D

**Step 3:** let $c_h(t) = M$ and $v_h(t) = M$, where $M$ is a sufficient large number;

let $f_h(t) = 0$, and $w_h(0,P) = 0$; and

let $c_{ih}(t,t) = c_{hj}(t,t) = 1/2 c_{ij}(t,t)$, and $u_{ih}(t,t) = u_{hj}(t,t) = u_{ij}(t,t)$

**Step 4:** run MPB on the expanded network.
**Proposition 6.1:**

The artificial-depot procedure is equivalent to the straightforward procedure in terms of the solution for decision variables.

**Proof:**

Because the storage cost $M$ is sufficiently high, there will be no containers stored at an artificial depot $AD$ in an optimal MPB solution. Then, if $AD$ is determined to be open by MPB, containers entering $AD$ need to leave $AD$ at the same sub-period, i.e., they are through-flow at $AD$ and there are no storage costs associated with $AD$.

Thus, path supplier-$AD$-demander on the expanded network is equivalent to supplier-$AD$ link plus $AD$-demander link in terms of cost and container flow. Therefore, the artificial-depot procedure is equivalent to the straightforward procedure.

Because the artificial-depot procedure introduces many new 0-1 variables, one for each supply-demander link, and a number of continuous variables – link flow variables $x$, depot storage variables $z$, and depot outsourcing variables $q$, it is not computationally superior to the straightforward procedure in any aspect. However, the artificial-depot procedure is instrumental in developing a computationally efficient and applicable procedure as discussed below.
6.1.3 Selected Artificial-Depot (SAD) Procedure

Notice that MPB on the original network may fail only if

$$\exists i \in O, j \in D. \text{ and } k \in F, \text{ such that}$$

$$c_{ik}(t,t) + c_{kj}(t,t) > c_{ij}(t,t), \text{ and } x_{ik}(t,t) \ast x_{kj}(t,t) > 0$$  \hspace{1cm} (14.1)

provided that i-j movement is allowed. This implies that condition (14.1) is a key in developing an improved solution procedure. We may need only to consider those i-j movements, where condition (14.1) is satisfied.

Based on this observation, we develop the following improved procedure, called the selected artificial-depot (SAD) procedure.

**Step 1:** run MPB on the original network

**Step 2:** find depots h \(\in\) F, suppliers i \(\in\) O and demanders j \(\in\) D satisfying condition (14.1)

**Step 3:** add the artificial depot h, and links i-h and h-j to the network for each (i,j) pair identified in Step 2. Remove the original i-j link

**Step 4:** let \(c_h(t) = M\) and \(v_h(t) = M\), where M is a sufficiently large number; let \(f_h(t) = 0\), and \(w_h(0,P) = 0\); and let \(c_{ih}(t,t) = c_{hj}(t,t) = 1/2 c_{ij}(t,t)\) and \(u_{ih}(t,t) = u_{hj}(t,t) = u_{ij}(t,t)\)

**Step 5:** run MPB again on the expanded network
Proposition 6.2:

The Selected Artificial-Depot procedure (SAD) is equivalent to the straightforward procedure in terms of the solution for decision variables.

Proof:

Because we have proved in Proposition 6.1 that the artificial-depot procedure is equivalent to the straightforward procedure, it is sufficient to show here that the SAD procedure is equivalent to the artificial-depot procedure.

Suppose (P) is an optimal solution of the artificial-depot procedure, (P') is an optimal solution of the SAD procedure.

Note that the artificial-depot procedure introduces all supplier-demander links to the network, whereas the SAD procedure only chooses those hopefully-carry-flow supplier-demander links into the network. Thus, the two procedures differ only in the way of selecting supplier-demander links into the expanded network. That is:

\[
\{(i,j) : i \in O \text{ and } j \in D \text{ in (P') } \subseteq \{(i,j) : i \in O \text{ and } j \in D \text{ in (P)}\}
\]  

(15.1)

Now consider a link l-m, l \in O and m \in D, l-m \in \{(i,j) : i \in O \text{ and } j \in D \text{ in (P)}\}, but l-m \notin \{(i,j) : i \in O \text{ and } j \in D \text{ in (P')}\}. By condition (14.1), if

\[c_{lk}(t,t) + c_{km}(t.t) > c_{lm}(t,t), \text{ for } \forall k \in F\]

then, either \(x_{lk}(t,t) = 0\) or \(x_{km}(t,t) = 0\) on the original network, viz., either supplier l does not supply any containers to depot k, or demander m does not receive any
containers from depot k, viz., demander m does not receive any containers directly from supplier l. Thus

\[ x_{lm}(t,t) = 0 \text{ for } l-m \in \{(i,j): i \in O \text{ and } j \in D \text{ in } (P)\} \]  (15.2)

i.e., there is no flow directly from supplier l to demander m on (P)'s expanded network.

Therefore, we conclude from observations (15.1) and (15.2) that (P') is the same as (P), i.e., the SAD procedure is equivalent to the straightforward procedure in terms of the final solutions for the decision variables.

The SAD procedure has less link flow variables \(x\) than the straightforward procedure and less depot opening variables (0-1 variables) than the artificial-depot procedure. This makes it computationally more tractable. Moreover, the SAD procedure can be implemented within the existing MPB modeling framework.

### 6.1.4 Numerical Example

We present the following example to illustrate step-by-step the SAD procedure in dealing with direct movement between suppliers and demanders.
The network consists of two container suppliers, two demanders, and four candidate depots, as shown in Figure 6.3

![Diagram of network showing two suppliers, four depots, and two demanders.]

Figure 6.3 Network for the Numerical Example in 6.1.4

The problem has two time periods corresponding to the depot selection decision. Each time period includes three sub-periods for the container allocation decision. The
transportation cost of supplier-depot, depot-demander, and direct supplier-demander are presented in Table 6.1.

Table 6.1 Transportation Costs of Supplier-Depot, Depot-Demander and Direct Supplier-Demander Movement

<table>
<thead>
<tr>
<th>Movement</th>
<th>Time-frame</th>
<th>Period 1</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sub-period</td>
<td>sub-period 2</td>
<td>sub-period 3</td>
<td>sub-period</td>
</tr>
<tr>
<td>supplier-depot</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>supplier 1-depot</td>
<td>1</td>
<td>140</td>
<td>140</td>
<td>140</td>
</tr>
<tr>
<td>supplier 1-demander</td>
<td>2</td>
<td>210</td>
<td>210</td>
<td>210</td>
</tr>
<tr>
<td>supplier 2-depot</td>
<td>2</td>
<td>170</td>
<td>170</td>
<td>170</td>
</tr>
<tr>
<td>supplier 2-demander</td>
<td>3</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>supplier 2-depot</td>
<td>4</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>depot-depot</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>depot 1-demander</td>
<td>1</td>
<td>110</td>
<td>110</td>
<td>110</td>
</tr>
<tr>
<td>depot 2-demander</td>
<td>1</td>
<td>110</td>
<td>110</td>
<td>110</td>
</tr>
<tr>
<td>depot 2-demander</td>
<td>2</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>depot 3-demander</td>
<td>1</td>
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<td>160</td>
<td>160</td>
</tr>
<tr>
<td>depot 3-demander</td>
<td>2</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>depot 4-demander</td>
<td>2</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>

The SAD procedure for this example works in the following way. After running MPB assuming no direct movement, we find that in sub-period 1, 50 containers are transported from supplier 1 to demander 1 through depot 1, while in Table 6.1 the unit transportation cost for the path of supplier 1-depot 1-demander 1 is 250, and the unit cost for the direct link of supplier 1-demander 1 is 200. This suggests the possibility of directly
moving containers from supplier 1 to demander 1 to reduce transportation cost. The same observation holds for the path of supplier 2-depot 2-demander 2 in sub-periods 5 and 6.

We therefore re-run the model on the extended network according to the SAD procedure. The container flow result of the model is reported in Table 6.2.

The model result shows that during the first sub-period, there are 200 containers moving directly from supplier 1 (via artificial depot AD1) to demander 1 without passing through any of the four candidate depots. During the fifth and sixth sub-periods, there are respectively, 150 and 50 containers directly transported from supplier 2 (via AD2) to demander 2. This would not have occurred if we had not removed the restriction on direct movement between suppliers and demanders.

The overall cost for the problem during the planning horizon has been reduced by over 12% ($3,066,450 vs. $2,680,150) after direct supplier-demander movement is allowed.
6.2 Uncertainty in Container Demand and Supply

There are many uncertainties within the existing MPB modeling framework, for example, container demand and supply, transportation cost and capacity, and depot cost and capacity. Among them, the most bothersome uncertainty for shipping companies is container demand and supply.
The goal of this section is to handle container demand and supply uncertainty so that shipping companies can evaluate the level of customer demand satisfaction provided by the MPB solution, and accordingly adjust their container allocation decisions to achieve a desired satisfaction level under stochastic container demand and supply.

An MPB-based simulation model is developed. The simulation model is capable of providing decision makers customer satisfaction level and associated statistical confidence that can be used as feedback to re-run the MPB optimization model.

The simulation model is composed of three parts: (a) the analytical derivation of the statistical distribution of container supply and demand, and inventory in depots; (b) the evaluation of demand satisfaction and statistical testing; and (c) the adjustment of container flow and re-run of the simulation model.

### 6.2.1 Statistical Distribution of Container Movement

First, we justify our assumptions for the distribution of container supply and demand, and then derive the distribution of depots’ container inventory. Finally, we arrive at the distribution of containers received by demanders.
(1) Container Supply Distribution

Empty containers are primarily supplied by shipping consignees, who receive loaded containers and return the empty ones to the shipping carrier after stripping them. From the perspective of a shipping carrier, the number of recovered empty containers from a consignee in a period (day, week, or month) is unknown in advance and follows a certain statistical distribution, as assumed below.

Assumption 6.1 The number of empty containers supplied from a supplier (consignee) CS to a depot D during a certain sub-period follows a Poisson distribution, $\phi(\lambda)$ with mean $\lambda$.

The justification for Assumption 6.1 is as follows. First, from the viewpoint of the depot receiving empty containers, the arrival of containers from a CS can be regarded in the same way as arriving vehicles in transportation, or incoming phone calls in telecommunications, which are widely assumed to follow a Poisson distribution. Second, a CS’s release of empty containers after receiving and stripping them is a random process and takes time ranging from several hours to several weeks, depending on the stripping and transportation link conditions. The number of containers returned in any two time sub-periods is usually independent. Finally, in any two equal-length-time sub-periods, the possibility of a container being returned within either of the periods is the same. That is,

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5 We use the term “sub-period” as explained in the MPB modeling framework. See Sections 2.3.3 and 3.4.1 for the discussion and definition of sub-period.
the probability of a container being returned during any particular very short sub-period of
time is approximately proportional to the length of the sub-period.

(2) Container Demand Distribution

Demand for empty containers is derived from the demand for container transportation
services. Shippers request empty containers to load their shipment. A shipper’s demand
for transportation services varies from time to time and is determined by many factors.
The number of empty containers required is a random variable. We have the following
assumption.

Assumption 6.2 The number of empty containers demanded from a depot D by a
demander (shipper) CD during a certain sub-period follows a normal distribution, \( \varphi(x) \sim N(\mu, \sigma^2) \).

This assumption reflects the following observations. First, demand for empty
containers is predictable from the perspective of both shipping carriers and shippers. In
fact, econometric methods are among the most widely used for forecasting transportation
service demand. \( \varphi(x) \) obtained from econometric models generally follows a normal
distribution. Second, if we regard the number of containers demanded by a CD as the
mean of a large sample drawn from the shipper’s demand distribution, and each sample is
approximately a realization of the actual demand in the previous years for the same time
period, then by the Central Limit theorem, \( \varphi (x) \) follows a normal distribution.
(3) Distribution of Container Depot Storage

A depot $D_i$ receives containers from four sources: (a) other depots $D_j$ ($i \neq j$); (b) supply customers $CS$; (c) inventory in $D_i$ from the previous period; and (d) the outside if $D_i$ is designated as a source depot $^6$. We know from the MPB model that there exists at least one depot which does not receive any containers from other depots. Otherwise we could always find a container flow along a cycle of depots in that sub-period. This would contradict the objective of cost minimization in MPB.

Thus, the derivation of the distribution of container inventory at depots can have the following sequence, according to the ways depots receive containers from other depots: (a) outflow-only depots; (b) depots only receiving containers from outflow-only depots; and (c) all other depots.

Shipping consignees (container suppliers) are generally independent of each other with respect to the times of receiving loaded containers and returning stripped empty containers. Therefore, we have the following assumption.

**Assumption 6.3** If the container supply of consignees $CS_1, CS_2, \ldots, CS_n$ to depot $D$, follows Poisson distribution $\phi(x_i/\lambda_1), \phi(x_2/\lambda_2), \ldots, \phi(x_n/\lambda_n)$ respectively, then $\phi(x_i/\lambda_1), \phi(x_2/\lambda_2), \ldots, \phi(x_n/\lambda_n)$ are independent.

$^6$ See Sections 2.3.5 and 3.3.1 for the discussion of the depot interaction with the outside.
The total number of containers from all suppliers to a depot $D_i$ in group (a) is $\sum_j^n x_j$, which is a summation of $n$ independent Poisson variables with respective mean $\lambda_1, \lambda_2, \ldots, \lambda_n$. Thus, $\sum_j^n x_j$ follows a Poisson distribution with mean $\sum_j^n \lambda_j$.

Denote $q_i$ to be the number of containers sent to depot $D_i$ from the outside sources, $e_i$ to be the container inventory from the previous sub-period at $D_i$, then the deterministic number of containers at $D_i$ in the current sub-period is $(q_i + e_i)$.

Therefore, the total number of containers at depot $D_i$ is $\sum_j^n x_j + (q_i + e_i)$; the first part is stochastic and the second is deterministic. We denote this as $(\delta_i + d_i)$, where $\delta_i$ stands for the stochastic part and $d_i$ for the deterministic part respectively.

Now we can derive the container inventory distribution in group (b) depots. Although we make a distinction between the stochastic and deterministic parts of the container inventory in a depot for the purpose of the study, the depot operators are not able and do not need to track whether a container belongs to stochastic or deterministic part. Thus, it is reasonable to make the following assumption.

**Assumption 6.4** *A depot $D$ sends its containers to other depots and shippers (demanders) both the stochastic and deterministic parts of its container inventory proportionally.*

---

7 This is meaningful only if $D_i$ is a depot allowing container exchange with the outside.
Therefore, the container inventory distribution of a group (b) depot is \((\delta_1 + d_1 + \delta_2 + d_2)\), where \(\delta_1\) is the number of containers sent to the depot from all suppliers. \(\delta_1\) is stochastic. \(d_1\) is the total number of containers from the outside and from the previous sub-period at the depot. \(d_1\) is deterministic. \(\delta_2\) and \(d_2\) stand for the respective stochastic and deterministic parts of total container inventory supplied by all other depots only receiving containers from outflow-only depots.

Finally, we can derive the container inventory distribution of group (c) depots. This has the form: \((\delta_1 + d_1 + \delta_2 + d_2 + \delta_3 + d_3)\). The meaning of \(\delta_1\), \(d_1\), \(\delta_2\), and \(d_2\) is the same as in the above. \(\delta_3\) and \(d_3\) are the stochastic and deterministic parts of the container inventory received from all other depots in group (c).

(4) Distribution of Containers Received by Demanders

Containers arriving at demander CD come exclusively from depots, which send the stochastic and deterministic parts of their container inventory to the demander proportionally as assumed in Assumption 6.4. Suppose there are \(m\) depots. Each depot sends \((y_i + d_i)\) containers to demander CD; \(y_i\) is its stochastic part and \(d_i\) is its deterministic part. Then the total number of containers supplied to CD is \(\sum_{i}^{m}(y_i + d_i)\). The stochastic part is a summation of \(m\) independent Poisson variables and therefore also follows a Poisson distribution, whose mean is the summation of the means of the \(m\) Poisson distribution.
6.2.2 MPB-Based Simulation Model

After deriving the statistical distribution of containers supplied to each demander, we now can answer the following questions: under the stochastic demand and supply, can the customer demand be met using the MPB model result? And how can shipping companies adjust the container inflow (outflow) from (to) the outside to obtain a statistically desired level of demand satisfaction?

The number of containers demanded by a shipper is given and assumed to follow a normal distribution. The number of containers supplied to the shipper has been derived, and has stochastic and deterministic parts. The stochastic part follows a Poisson distribution. Then, we can define the demand satisfaction factor for demander i at the jth sub-period as:

\[
r_{ij} = \frac{(\phi(x/\lambda) + d)}{\varphi(x)}
\]

where, \(\phi(x/\lambda) + d\) is the total number of containers received by demander i at the jth sub-period, \(\phi(x/\lambda)\) is its stochastic part with mean \(\lambda\). d is its deterministic part; \(\varphi(x)\) is the number of containers requested by demander i at the jth sub-period, and follows a normal distribution \(N(\mu, \sigma^2)\).

\(r = 1\) means that the customer demand for containers is fully satisfied. Neither \(r > 1\) nor \(r < 1\) is desirable in practice; \(r > 1\) means unnecessary extra container storage and associated costs, and \(r < 1\) results in unmet customer demand. If \(r \neq 1\), the shipping
company needs to increase or reduce the supply of containers to the demander by sending
more containers into (out of) the system through the designated source depots.

Because \( r \) is a random variable, we need to take the mean \( \bar{r} \) as the measurement of
satisfaction factor. Thus, \( \bar{r} \) is the mean of a random sample taken from the distribution \( r \)
\( = \left( \phi(x/ \lambda) + d \right) / \varphi(x) \). By the Central Limit theorem, \( \bar{r} \) has a distribution, which is
approximately normal with mean \( \mu \) and variance \( \sigma^2 / n \). \( n \) is the sample size.

In fact, the sample mean and variance of the normal distribution \( \bar{r} \) can be computed
from the simulation. Shipping companies can conduct statistical tests about the true value
of \( \bar{r} \) to achieve a desired level of demand satisfaction. There are two types of tests
relevant to this goal, the mean hypothesis test and the mean confidence intervals.

Suppose variables \( \hat{r}_1, \hat{r}_2, \ldots, \hat{r}_n \) form a random sample from the normal distribution \( \bar{r} \),
whose mean and variance are unknown. However, by simulation we can obtain the
sample mean and variance of \( \hat{r}_1, \hat{r}_2, \ldots, \hat{r}_n \), denoted as \( \bar{\mu} \) and \( \bar{\sigma}^2 \) respectively. In order to
test a hypothesis about the true mean of the normal distribution \( \bar{r} \), we can construct the
following statistic:

\[
U = \frac{\sqrt{n}(\bar{\mu} - \mu_0)}{\sigma}
\]

(16.1)
where \( \mu_0 \) is the hypothesized threshold value we want to test about the true mean, and \( n \) is the sample size. Now we can test the hypothesis:

\[
H_0 : \mu \geq \mu_0 \\
H_1 : \mu < \mu_0
\]

under a certain level of significance \( \alpha_0 \). For \( \Pr(U \geq c) = \alpha_0 \), If \( U > c \), then we can accept the hypothesis \( H_0 : \mu \geq \mu_0 \).

For example, the shipping company may want to know whether the true value of the mean is no less than 1 (\( \mu_0 \geq 1 \)) with a 1% significance level, given sample mean \( \bar{\mu} = 1.006 \), sample variance \( \bar{\sigma}^2 = 0.009 \), and sample size \( n = 1000 \). By statistic (16.1), we compute \( U = 2.000 \). From a t-distribution table, \( c(0.01, 1000) = 2.326 \). Then \( U < c(0.01, 1000) \). Thus, we reject the hypothesis that the true mean is no less than 1 at the 1% significance level. However, if the significance level is 5% instead of 1%, then \( c(0.05, 1000) = 1.645 \), and \( U > c(0.05, 1000) \). Thus, we are able to accept the hypothesis at the 5% significance level.

Based on the acceptance or rejection of the hypothesis about the desired level of demand satisfaction, shipping companies can decide whether to adjust the container supply to achieve a desired level of customer satisfaction.

Another related method to grasp stochastic demand satisfaction factor \( \bar{r} \) is to calculate the confidence interval that the true mean \( \mu \) falls into, given sample mean \( \bar{\mu} \), sample
variance $\sigma^2$, sample size $n$, confidence coefficient $\alpha_0$, and $c$ satisfying $\Pr(U \geq c) = \alpha_0$.

Let

$$\Pr\left( \bar{\mu} - \frac{c\sigma}{\sqrt{n}} < \mu < \bar{\mu} + \frac{c\sigma}{\sqrt{n}} \right) = \alpha_0$$

and $a = \bar{\mu} - \frac{c\sigma}{\sqrt{n}}$, $b = \bar{\mu} + \frac{c\sigma}{\sqrt{n}}$, then $(a, b)$ is called a confidence interval for $\mu$ with confidence coefficient $\alpha_0$. We can then make the statement that the unknown value of $\mu$ lies in the interval $(a, b)$ with confidence $\alpha_0$. The confidence interval method is particularly useful when shipping companies want the satisfaction factor $r$ to be within a certain range, for example, $(0.97, 1.02)$ on the justification that if $r > 1.02$, the costs associated to achieve the goal is too high and unnecessary; if $r < 0.97$, the customer satisfaction level is too low to be acceptable.

In general, $r$ should be close to 1. $r$ greater than 1 indicates there are a more than sufficient number of containers at the demander’s hand in the sub-period. The surplus of containers contradicts the objective of cost minimization. However, in certain circumstances, some shipments are so important that the carrier prefers to provide the demander some spare containers to prepare for unpredictable events, for instance, transportation delay, more-than-expected demand, etc. $r$ less than one means that the demand is not fully satisfied for the demander in the time period. The acceptance of $r < 1$ depends on the shipping company’s relationship with the shipper and the market
competition. \( r < 1 \) is usually not acceptable in the highly competitive and high-value-added shipping markets.

We now combine the major components developed above into a complete MPB-based simulation model, as shown in Figure 6.4, which is explained in detail as follows.

**Step 1** Run the MPB model:

Run MPB, assuming all the input data are deterministic, using the mean of the Poisson distribution of CSs and the normal distribution of CDs as the deterministic container supply and demand for each supplier CS and demander CD, respectively, at each sub-period.

**Step 2** Derive depot’s distribution

Derive the distribution of container inventory for each depot at each sub-period, using the procedure developed in (3) of Section 6.2.1

**Step 3** Derive demanders’ distribution

Derive the distribution of containers received by each demander during each sub-period, using the procedure developed in (4) of Section 6.2.1
run MPB

derive depot’s distribution of container inventory

derive demander’s distribution of containers received

compute \( r = \frac{\text{received}}{\text{demanded}} \) as demand satisfaction

hypothesis test and confidence interval for true value of \( r \)

r is satisfied?

stop

adjust depot container flow

Figure 6.4 The MPB-Based Simulation Model
**Step 4** Compute demand satisfaction factor \( r = \frac{\text{received}}{\text{demanded}} \)

Containers received by each demander are composed of stochastic and deterministic parts, while containers demanded follow a normal distribution. We can use a statistical program to calculate \( r \) for each demander at each sub-period.

**Step 5** Mean hypothesis test and confidence interval

Conduct mean hypothesis test and calculate confidence interval for each \( r \), as detailed in the earlier part of this section.

**Step 6** Satisfaction factor \( r \) is desired?

Now the decision maker needs to make a judgment whether \( r \) is desirable based on the result from Step 5: whether the hypothesis that \( r \) is larger than a certain level is accepted and/or whether \( r \) falls into a desired confidence interval under a given significance level?

**Step 7** Container depot inflow adjustment

If \( r \) is not accepted in Step 6, the decision maker needs to adjust the inflow (outflow) of containers from (to) the outside to (from) the depots that are designated to have interaction with the outside under the MPB modeling framework.

After the adjustment, we need to re-compute depots’ container distribution and start the algorithm from there all over again.
Note that each step of the above algorithm is usually performed for all depots or shippers before moving to the next step. However, there is the case where the decision maker knows the demand satisfaction factors for some shippers at some sub-periods are so important that if these factors do not meet certain criteria, the adjustment is necessary. Therefore, these factors may be preferred to obtain and tested prior to other factors. We accommodate this observation by drawing a returning path from Steps 3 to 2 and from Steps 5 to 4.

6.2.3 Numerical Experiment and Insights

A numerical experiment is conducted to illustrate the procedure of the MPB-simulation model and to gain some insights of the effects of uncertainty on the intermodal depot selection (IDS) problem for liner shipping and intermodal transportation.

To simplify the presentation, the problem concerned is the same as the one presented in section 6.1.4 except that: (a) direct supplier-demander movement is not permitted; and (b) container demand and supply is assumed to be stochastic instead of deterministic.

In order to make the presentation more concise, we consider only the first three sub-periods out of the entire 6 sub-periods. The deterministic supply and demand is used as the mean of the respective stochastic supply and demand as given in Table 6.3.
Table 6.3  Stochastic Supply and Demand

<table>
<thead>
<tr>
<th>time-frame</th>
<th>customer</th>
<th>Poisson distribution</th>
<th>Normal distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>supplier 1</td>
<td>supplier 2</td>
<td>demander 1</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>mean</td>
<td>mean</td>
</tr>
<tr>
<td>sub-period 1</td>
<td>200</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>sub-period 2</td>
<td>400</td>
<td>150</td>
<td>600</td>
</tr>
<tr>
<td>sub-period 3</td>
<td>300</td>
<td>400</td>
<td>450</td>
</tr>
</tbody>
</table>

Following Steps 1 through 3 in the above simulation model, we obtain the container distribution for each depot and demander at each sub-period, and report them in Table 6.4. In Step 4, a statistical simulation is performed to calculate the sample mean and variance of the demand satisfaction factor r for each demander at each sub-period. The result is shown in Table 6.5.

Table 6.4  Container Inventory Distribution for Depots and Demanders

<table>
<thead>
<tr>
<th>location</th>
<th>time-frame</th>
<th>sub-period 1</th>
<th>sub-period 2</th>
<th>sub-period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sub-period</td>
<td>stochastic</td>
<td>deterministic</td>
<td>stochastic</td>
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<tr>
<td>depots</td>
<td>sub-period 1</td>
<td>40</td>
<td>710</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>sub-period 2</td>
<td>1</td>
<td>499</td>
<td>33</td>
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<td></td>
<td>sub-period 3</td>
<td>158</td>
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<td></td>
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<td>1,449</td>
<td>150</td>
</tr>
<tr>
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<td>demander 1</td>
<td>11</td>
<td>389</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>demander 2</td>
<td>11</td>
<td>299</td>
<td>500</td>
</tr>
</tbody>
</table>
Table 6.5  Sample Mean and Variance of Demand Satisfaction Factor r

<table>
<thead>
<tr>
<th>time-frame</th>
<th>sub-period 1</th>
<th>sub-period 2</th>
<th>sub-period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>variance</td>
<td>mean</td>
</tr>
<tr>
<td>demander</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.006447</td>
<td>0.005953</td>
<td>1.003469</td>
</tr>
<tr>
<td>2</td>
<td>1.004216</td>
<td>0.004460</td>
<td>1.004649</td>
</tr>
</tbody>
</table>

Having obtained these information, the shipping company is able to assess the acceptability of each demand satisfaction factor r using mean hypothesis test and/or confidence interval as discussed in Step 5. The calculation result is presented in Table 6.6 for various significance levels.

Table 6.6  Mean Hypothesis Test and Confidence Interval

<table>
<thead>
<tr>
<th>time-frame</th>
<th>period 1</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sub-period 1</td>
<td>sub-period 2</td>
<td>sub-period 3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>hypothesis</td>
<td>confi. interval</td>
<td>hypothesis</td>
<td>confi. interval</td>
</tr>
<tr>
<td>demander</td>
<td>sig.</td>
<td>sig. interval</td>
<td>sig.</td>
<td>sig. interval</td>
</tr>
<tr>
<td>1</td>
<td>99%</td>
<td>y:0.990</td>
<td>99%</td>
<td>1.0008-1.0121</td>
</tr>
<tr>
<td>2</td>
<td>90%</td>
<td>n:1.002</td>
<td>99%</td>
<td>0.9952-1.0053</td>
</tr>
</tbody>
</table>

Note: H₀(r) stands for hypothesis H₀: true mean > r; H₁: true mean < r; "y" means accepted, "n" not accepted
It is observed in Table 6.6 that the values of satisfaction factor $r$ for demander 1 at sub-period 2 and for demander 2 at all three sub-periods 1, 2 and 3 do not achieve the desired level. Another way to analyze the satisfaction factor $r$ is to consider maximum $u_0^{\text{max}}$, which is the maximum value of $u_0$ that the hypothesis test $H_0: u > u_0$ can be accepted for a given significance level. $u_0^{\text{max}}$ is the lower bound of the corresponding confidence level for each demander at each sub-period. For example, $u_0^{\text{max}}$ for demander 1 at sub-period 1 is 1.0008 at 99% significance level. This means that at 99% confidence level, the true mean of the demand satisfaction level $r$ will be no less than 1.0008.

To achieve a desired level of customer satisfaction, the shipping company can adjust the inflow (outflow) of containers through depots 3 and 4 as illustrated in Step 7. For instance, the company may think it is costly to maintain the satisfaction factor at a level greater than 1.0008 for shipper 1 at sub-period 1. Accordingly, he can move some containers in depot 3 and/or depot 4 out of the system to lower the $r$ level. On the other hand, one may believe shipper 2 at sub-period 1 is very important so that the satisfaction factor should be higher than 0.9952. Then additional containers should be sent into shipper 2 from the outside through depot 3 and/or depot 4 at sub-period 2.

From the numerical example, we can obtain the following insights of the intermodal depot selection (IDS) problem.
(1) The larger the number of deterministic containers supplied to a demander, the lower the variance of the satisfaction factor, i.e., the more certain the demand satisfaction level is. In the context of the MPB modeling framework, shipping companies can adjust the supply of deterministic containers using the inflow/outflow of the outside source through designated source depots.

(2) Even in the presence of stochastic demand/supply, the values of customer satisfaction factors obtained using deterministic MPB are usually fairly close to 1. However, in many circumstances, the shipping companies may have to use the MPB-based simulation model developed in this section to achieve an appropriate level of customer satisfaction.

(3) In contrast to common sense, the equal mean of number of containers received and demanded may result in a greater-than-1 satisfaction level. The reason is that the number of containers supplied follows a Poisson distribution and the number of containers demanded follows a normal distribution, therefore the supplied variance is generally greater than the demanded variance. Given the equal mean, it is possible for the satisfaction factor $r$ to be greater than 1.

The MPB-based statistical simulation model developed in this section handles the uncertainty of demand/supply in the MPB optimization model, however, it is not a
stochastic version of the MPB optimization model, which is mathematically intractable under the current OR techniques.

6.3 Chapter Summary

This chapter is a refinement of the MPB model developed in the previous chapters. The improvements were done based on the principles of relaxing unrealistic assumptions and enhancing the MPB model’s strength.

First, we proposed an efficient procedure to solve the IDS problem, permitting direct movement between shippers and consignees. Second, we developed a simulation model, which is capable of providing decision makers with customer satisfaction level and associated statistical confidence that can be used as feedback to re-run the MPB optimization model. Numerical examples are given in both cases.

Both improvements were done on the basis of the MPB model so that we can provide shipping and intermodal carriers a unified decision-support tool for their intermodal inland depot selection and container management problems.

Note that for the purpose of simplifying the presentation, the handling of uncertainty in Section 6.2 still assumed no direct movement between customers. The MPB-based simulation model can be equally applied to the situations, where there is direct movement.
Chapter 7
Conclusions and Future Research

7.1 Summary and Conclusion

This research developed mathematical models to assist intermodal carriers in determining their inland depot network – the location and capacity of each depot and warehouse.

A mixed integer programming model for determining the optimal location of inland depots is developed in the context of container liner shipping. The model, called the multi-period model with balancing requirements (MPB), embraces the fundamental features of the inland depot selection problem – the container balancing requirements, the hierarchical decision-making process, the multi-period structure in multiple levels, the container inventory reduction, the system’s interaction with the outside, and the mixed depot ownership.

Two solution algorithms were proposed: the price-directive decomposition algorithm and the resource-directive decomposition algorithm. The MPB model was applied to solve
an inland depot selection problem facing a major international liner shipping company in its North American operations.

Given the fact that many sound OR models failed in their implementation and deployment stage, both Chapters 5 and 6 on discuss how to effectively apply the MPB model in practice. The concept of container supply chain management was developed so that the MPB model’s implementation and deployment issues, especially the institutional barriers to deploying the model, was addressed from the perspective of integrated container chain movement.

The MPB model was enhanced further in two aspects. Our interviews with liner operators suggested that the restriction on direct container movement between consignees and shippers is not a realistic assumption. Therefore, an efficient procedure, called the selected artificial-depot procedure (SAD), was proposed to incorporate direct movement into the MPB modeling framework. As one of the most difficult issues concerning the container management, the uncertainty in container demand and supply was handled by an MPB-based simulation model, which integrates the MPB optimization model into a statistical simulation model. It provides decision makers demand satisfaction levels and associated statistical confidence to be used as feedback to re-run the MPB optimization model. The work of the model’s enhancement was done based on the MPB model so that we can provide shipping and intermodal carriers a unified decision-support tool for their intermodal inland depot selection and container management problems.
7.2 Research Contributions

(1) Establishing the Modeling Framework

This research systematically establishes the first modeling framework for the intermodal inland depot selection problem. The following fundamental components of our framework were not included in the literature. They are composed of:

- Hierarchical decision-making process
- Two levels of multiple periods
- Container balancing requirements
- Container inventory reduction
- System’s interaction with the outside
- Mixed depot ownership.

(2) Developing the Multi-period Model with Balancing Requirements

The multi-period model with balancing requirements (MPB) is developed to solve the inland depot selection problem. MPB incorporates the fundamental features of the modeling framework. In particular, MPB embodies the very nature of the problem – container rebalancing activities, which is not included in the literature.

(3) Proposing Two Decomposition-Based Solution Algorithm

These two algorithms are the price-directive (Lagrangean) decomposition algorithm and the resource-directive (Benders) decomposition algorithm. In addition, MPB is
successfully applied to a real-world inland depot selection problem facing a leading international shipping company.

(4) Developing the Concept of Container Supply Chain Management

To effectively address the issues of model implementation and deployment, we develop the concept of container supply chain management to be able to examine the sophisticated implementation issues from this perspective.

(5) Integrating MPB into a Statistical Simulation Model to Handle Uncertainty

The uncertainty in container demand and supply is handled by an MPB-based simulation model, which integrates the MPB optimization model into a statistical simulation model. The MPB-based simulation model is capable of providing decision makers the level of customer demand satisfaction and associated statistical confidence that can be used as feedback to re-run the MPB optimization model.

7.3 Future Research

Future research work could be carried out along three directions: enhancing the existing MPB’s efficiency and effectiveness, further addressing organizational issues, and expanding the current modeling framework.
The MPB model is tested against a real-world problem with 10 candidate depots in the Northeast region of the US. However, a large-scale application may cover all of North America and has as many as 80 potential depots. The efficiency of the proposed two decomposition algorithms need to be tested. There may be a need to develop more efficient algorithms (Shapiro, 1979).

The current simulation model can provide decision makers only with the information about customer satisfaction level after adjusting container inflow (outflow) into (from) the system, but they may also want to know the costs associated with the adjustment. The simulation model should be able to reflect this cost change. In addition, the simulation model may be extended to cover uncertainty in transportation costs and capacity, and depot costs and capacity.

We have discussed institutional issues for successfully implementing and deploying the MPB model. In particular, we recommended three important principles: setting appropriate strategic goals, restructuring the information flow process, and improving the supply chain enablers. Further research work is needed to examine organizational issues. For example, what kind of organizational changes will be necessary to accommodate the model’s deployment? Which might be the major organizational barriers to the deployment? What might be the major costs and risks associated with the deployment? The model can be better deployed when these important organizational questions are appropriately answered. We need to develop models to study these issues in greater detail.
The reader is referred to Sussman (1994) for a discussion of the implementation and deployment issues of transportation systems. Intermodal industry-wide regulatory and institutional issues are addressed in National Research Council (1992).

With respect to expanding the modeling framework, a first step could be to incorporate the off-shore locational problem, i.e., selection of seaport network into the framework so that seaport network, inland depot network and container allocation problems can be addressed in a systematic manner to achieve global optimum, as depicted in Figure 1.1. The relationship among these three levels may be found in Marcus (1993) and Dejax and Servant (1986).

Ideally the uncertainty issues should be addressed in the MPB optimization model, viz., formulating a dynamic and stochastic optimization model to solve the problem. However, considering the limit of current OR techniques in solving large-scale dynamic and stochastic locational optimization models, there would be long way to go before any meaningful attempt can be accomplished (Louveaux, 1993).
Bibliography


