ENGINEERING DESIGN OF ELECTRON ACCELERATOR TARGETS FOR THE COMMERCIAL PRODUCTION OF ISOTOPES

by

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B.S., Nuclear Engineering (1995)

Pennsylvania State University

Submitted to the Department of Nuclear Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Science in Nuclear Engineering at the

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Abstract

Alternative methods for radioisotope production must be developed to meet the growing demand for radioisotopes in industry and medicine. One method under development by the Massachusetts Institute of Technology and the INEL University Research Consortium is the application of Giant Dipole Resonance reactions using electron-beam generated bremsstrahlung photons. This project is chiefly focused on the commercial production of Tc-99m via a photoneutronic reaction in a Mo-100 slug. This study examines the feasibility of this process by addressing the most pertinent engineering issues for a preliminary target design. A design methodology is presented for the nominal case of an electron beam operating at 40 MeV at a current of 250 μA. MCNP is used to predict yields and energy deposition, and ALGOR, a finite-element analysis code, is used to perform heat transfer analyses. The optimum design includes a tungsten converter plate, aluminum backing plate, and/or a magnetic electron sweep. Enhanced heat transfer is provided by configuring the converter plate as a porous metal heat exchanger and by encasing the slugs in cooled aluminum holders. The thermal-hydraulics for gas flow in porous media is modeled with Mathcad Plus 6.0. Based on the simulations, these systems are capable of maintaining steady-state temperatures well below the melting point for all materials in the target assembly.

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Chapter 1

Introduction

1.1 Rationale

Radioactive isotopes (radioisotopes) are an indispensable tool in both industry and medicine. Their applications in these disciplines are widespread, from medical diagnostic procedures to industrial process control. The demand for radioisotopes is large and continues to grow as new applications are identified and new isotopes are created.

Radioisotopes are typically created in nuclear reactors or with heavy charged particle (ion) accelerators [8]; however, electron accelerators in the 25-40 MeV range may create radioisotopes more conveniently and economically than the existing production methods or may produce radioisotopes that can not be made practically at the present time. One promising application of modern electron accelerators is the creation of radioisotopes by a photon-induced reaction driven by electron-generated bremsstrahlung. Although many interesting isotopes may be created via photon-induced nuclear reactions, our interest in this method has been chiefly focused on the photoneutronic creation of the medical radioisotope technetium-99m (Tc-99m).
1.2  Technetium-99m

Technetium-99m is the most widely used radionuclide in nuclear medicine. In fact, it is used in approximately 80% of all radioimaging procedures; some 30,000 per day, making it a very valuable commodity in the medical community [3]. This nuclide has been incorporated into many radiopharmaceuticals to tag specific biochemical functions \textit{in vivo} in nearly every organ of the human body. For instance, doctors use the nuclide to search for bone cancer and to visualize the metabolic functions of the thyroid, liver, spleen, lungs, and kidneys.

The decay scheme of Tc-99m is what makes it particularly attractive for nuclear medicine. Tc-99m decays to its ground state with a 6.03h half-life primarily by emitting a 140.5 keV gamma ray. These photons pose no serious risk to a patient, yet are sufficiently energetic to escape the body in significant numbers. In addition, the half-life is long enough for short-range transportation and imaging procedures, but the \textit{in vivo} activity drops to minimal levels in a relatively short time, minimizing the dose to the patient.

Tc-99m is not a naturally-occurring isotope, so it must be artificially created. This is accomplished by the creation of Mo-99, which decays primarily by $\beta^-$ emission with a 66h half-life to the metastable radionuclide Tc-99m. Mo-99 was first created by bombarding Mo-98 with thermal neutrons. Besides the $(n,\gamma)$ reaction, there are several other nuclear reactions whose reaction products include Mo-99, including uranium fission. The Mo-99 yield for the fission of U-235 is 6.1%.
Neutron-activation and fission are the two commercial methods for creating Mo-99, and each has its advantages and disadvantages. Neutron activated Mo-99 requires little postirradiation processing and produces minimal radioactive waste, but the product is limited to low specific activities (< 10 Ci/g). On the other hand, fission produced Mo-99 requires sophisticated and expensive processing facilities and quality control measures, and very toxic and radioactive fission products and transuranics are involved in this process. The advantage is that very high specific activities of Mo-99 are attainable, approximately 1000 times higher than by neutron activation. Despite the high costs associated with the disadvantages of producing Mo-99 by fission, this method has displaced neutron activation as the most widely used method for the commercial production of Mo-99 because of the high specific activities which may be achieved.

At this time, the only source of Mo-99 in North America is the 37-year-old NRU reactor in Chalk River, Ontario, which receives large subsidies from the Canadian government to offset the enormous costs involved with postirradiation processing of highly radioactive spent fuel to separate out Mo-99. The uninterrupted operation of this reactor is vital to the flow of Mo-99 in this country. Recently, the reliability of Chalk River has come into question as a result of unexpected shutdowns and labor disputes. Although the flow of Mo-99 was not interrupted, these incidents alarmed doctors and brought attention to the fact that an alternative supply of Mo-99 is not readily available in the event of a shutdown.

In any event, a permanent shutdown for Chalk River is scheduled for the year 2000 when it is expected to be replaced by the new Maple-10 isotope reactors. Shutdowns in the interim period accompanied by problems with the startup of the new
reactors will halt the critical supply of Tc-99m and pose a serious threat to diagnostic medicine. Therefore, alternate sources must be identified and developed to avert a possible crisis.

1.3 Photo-Production Method

One such potential source is the aforementioned electron accelerator method, which has the capability to produce Mo-99 more conveniently, cheaply, and with much less radioactive waste than the existing fission-product derived source, albeit with lower specific activities. The Mo-99 is created primarily via the following photoneutronic reaction:

$$^{100}\text{Mo}(\gamma,\text{n})^{99}\text{Mo}$$

This reaction occurs due to a large resonance region in the gamma absorption cross section, known as the Giant Dipole Resonance (GDR). This resonance exists for all isotopes but the peak varies in magnitude and with photon energy for different isotopes. For Mo-100, the resonance exists for photons with energies between 8 and 20 MeV. A photon flux in this energy band may "strip" a neutron from the nucleus. This approach is attractive compared to neutron addition reactions which require that the incident neutron beams are thermalized. Another advantage to the single-stage photoneutron approach is the creation of relatively high specific activities of Mo-99; however, very small target material volumes must be used in conjunction with focused photon beams.

The gamma flux which drives the photoneutronic reaction is obtained by converting high-energy (25-40 MeV) accelerator electrons into bremsstrahlung photons.
in a high-Z material target. The nominal beam conditions we have specified are 40 MeV electrons at a current of 250 µA. This beam represents 10 kW of power, the majority of which is deposited into the accelerator target as electrons and photons are absorbed and deposit their kinetic energy.

Although Tc-99m is indirectly produced via a photoneutronic reaction, a variety of other photon-induced reactions may be utilized to create a wide range of other radioisotopes. These reactions include (γ,p), (γ,np), (γ,2n), (γ,2p), and (γ,γ'), but cross section data is still lacking for these reactions for many nuclei. The data which is currently available comes mainly from experimentation in the former Soviet Union. A plethora of isotopes may in fact be created via photo-reactions, but it is difficult to find stable parent nuclei which may be transformed into desirable radioisotopes. Some other radioisotopes which may eventually be created with a high-energy electron accelerator include Co-57 and short-lived PET isotopes such as F-18.

This study is mainly concerned with the engineering issues for the preliminary general design of an electron accelerator target, regardless of which isotope is being created. There are still some nuclear physics question to be addressed in this endeavor, such as matching up the bremsstrahlung spectrum with the resonance energy band of the chosen parent nucleus. It is for this reason that the production of Mo-99 is taken as an example, but the design remains general enough to be easily adapted for the commercial production of other isotopes as well.
1.4 Power Dissipation

The accelerator target assembly must be designed to optimize radioisotope production, but to do so it must also effectively dissipate the beam power. A practical design must strike a balance between these two fundamental requirements since the induced radioisotope yields are proportional to the incident electron beam power and are only limited by the amount of energy which can be effectively dissipated by the target.

To create appreciable specific activities of a radioisotope, the target volume must be minimized which results in high power densities in the target. This necessitates a target design with enhanced heat transfer capabilities. In the interests of simplicity and radiological concerns, the design should also contain a minimum number of moving parts, perhaps by implementing passive heat transfer mechanisms.

The major engineering issues to be addressed in the preliminary design process are:

- Target materials
- Target geometry
- Energy deposition
- Product yields
- Thermal hydraulics (Heat Transfer / Fluid Flow)
- Experimental setups

1.5 Research

The need for alternate sources of Tc-99m and the potential use of electron accelerator technology to create a variety of other important radioisotopes has prompted research at the Massachusetts Institute of Technology (MIT) to design commercial electron beam
targets for the creation of radioisotopes. This study describes our design methodology
and the development of several computer models to analyze potential commercial target
designs. The models are based on nuclear and mechanical engineering fundamentals,
using assumptions where necessary to facilitate the modeling process. The results of
several experiments may then serve as validation of the models and their associated
assumptions.

Chapter 2 describes the methodology of the design process and Chapters 3-5
present the preliminary engineering analysis of the target assembly to address the issues
mentioned above. Chapter 6 summarizes the pertinent results and proposes a feasible
commercial design based on them.
Chapter 2

Target Design Methodology

2.1 Design Considerations

The key design parameter is radioisotope production, or yield, which is limited by the amount of energy which can be safely deposited in the target materials. The yield depends on the number of Giant Dipole Resonance (GDR) photons available for interaction and the number of the target nuclei (enrichment) in a material. Enrichment is really only an economic consideration, but the GDR photon flux is a product of the target design.

The overall photon flux depends on the electron beam current and the percentage of electrons converted to bremsstrahlung photons. The bremsstrahlung spectrum depends on the incident electron energy and the subsequent scattering and attenuation of the photons in the target. The optimal condition is to produce photons whose energies are within the GDR region. Clearly, a high-Z material should be present to optimize conversion, but not to the extent along the beam direction that a large portion of GDR photons are absorbed or downscattered to lower energies.

We are technically free to arbitrarily boost the current to any value depending on the limits of the accelerator, thus scaling up the photon flux as high as we desire but again there are practical limits of energy deposition. Even currents on the order of milliamps of high energy electrons possess enough energy to melt a small, uncooled target. In this
regard, yields are limited by the heat capacity of the target system. Commercial yields will necessitate the implementation of a cooling system with enhanced heat transfer mechanisms to boost the heat capacity. Therefore, for a given electron energy, the maximum yield is limited to the maximum beam current for which the cooling system can maintain reasonable temperatures throughout the target.

For a given beam current, the GDR photon flux may be optimized by careful choice of electron energy, target materials, and target geometry. The process to determine these parameters depends on a series of sensitivity studies and is an important consideration to provide the maximize possible yield. The electron energy is chosen based on producing the ideal bremsstrahlung spectrum.

Besides these primary design considerations, a practical target should by definition be as simple as possible for both economic reasons and to facilitate easy and remote handling in an industrial setting. The irradiated materials will be very radioactive and will be subjected to chemical separation procedures and refabrication.

2.2 Simple Model

With a simple model we can design a target that converts a high percentage of the incident electrons into GDR photons and allows these photons to interact with the appropriate parent nuclei, such as Mo-100. Guided by these criteria, a basic design may now proceed using the Mo-99 production process as an example.

The most simple target necessary to create appreciable amounts of Mo-99 by a photoneutronic reaction consists of a slug of the target material itself, (enriched)
molybdenum. If product yields were not the main design parameter, electrons could be converted in the slug itself, providing the atomic number is high enough to allow significant bremsstrahlung production. In the case of molybdenum, in fact the atomic number is sufficiently high to actually achieve this (Z=42), but conversion may be optimized using a much higher-Z material. Yields may be improved if the electrons are first allowed to strike a high-Z converter plate before entering the slug.

A converter plate not only may improve electron conversion, but it may absorb a large portion of the incident electron energy that would otherwise be deposited in the slug. By imposing a higher heat load on the converter plate, we can employ very simple, even passive cooling systems for the slug. A complex cooling system would unnecessarily impede the remote handling and reprocessing of the slugs. As part of the chemical separation process, irradiated slugs will be crushed and eventually repressed to undergo subsequent irradiations. The slug will be radioactive, especially following its irradiation, so the recycling process must be hands-off. A simple cooling system will permit remote handling to minimize the radiological hazard to personnel and make the slug refabrication process relatively easy.

### 2.2.1 Converter Plate

The primary roles for the converter plate are to create bremsstrahlung photons by stopping high-energy incident electrons, and to dissipate the heat load imposed by the electrons. The goal is to maximize photon production while sparing the later stages of the target from excessive heating. Ideally, the conversion photons emerge from the back of the converter plate and enter the slug at energies within the GDR energy band of a
parent nuclide. In reality, a significant number of electrons and low-energy photons will also enter the slug as unwanted by-products, since they heat the slug and do not contribute to the yield.

The design considerations for the converter plate are material and thickness. The thickness is chosen as a compromise between photon production and slug heating. It should be made of a high-Z material with excellent heat transfer characteristics since it will absorb a large amount of the beam power. Typically, tungsten (W) is used in these types of applications because of its high atomic number (Z=74), high density (19.3g/cm³), and very high melting point (3660 K). Both the number of electrons which are converted and the Mo-99 yield are very sensitive to the converter plate thickness. The plate should be thick enough along the beam direction to absorb most of the electrons but not so thick that too many GDR photons are attenuated or downscattered.

Since the converter plate will bear a large heat load, its design must include a cooling system that can effectively dissipate the absorbed energy. The beam currents under consideration are on the order of several hundred microamps at electron energies of 30-40 MeV, which translates into maximum beam powers on the order of 10 kilowatts. The resulting power densities make it a difficult engineering challenge to prevent material failure due to melting.

2.2.2 Backing Plate/Magnetic Sweep

Slug heating can be further reduced by locating a backing plate between the converter plate and the slug which is referred to as the low-yield model. This plate absorbs many of
the electrons and low energy photons previously entering the slug, with minimal attenuation of GDR photons. The high-yield model uses an alternative method for reducing electron heating by "sweeping" out those electrons exiting the converter plate with a bending magnetic field of 1-2 Tesla. This method obviates the use of a backing plate and actually permits higher product yields since fewer photons are attenuated without the backing plate. On the other hand, secondary photons generated by electrons in the slug are also eliminated, some of which will be in the productive range.

In contrast to the converter plate, the backing plate should be made of a low-Z material to limit the attenuation of GDR photons, yet still be able to effectively absorb electrons and low energy photons. As with the converter plate, the thickness of the backing plate is also an important design consideration. The backing should be thick enough to permit sufficient absorption of electrons, but not too thick as to significantly attenuate the GDR photon flux.

2.2.3 Slugs

The target material in which the transformation reactions take place is known as the slug, a small cylindrical pellet formed in a pressing operation. An important design goal is to limit the heat burden in the slug from "waste heating". One reason is that although molybdenum has a high melting temperature (2894 K), other possible slug materials may not. Furthermore, a light heat burden will alleviate or even eliminate the implementation of a complex slug cooling system which would make it difficult to recycle slugs for
chemical processing. The design of the converter plate, backing plate, and/or magnetic sweep will determine the amount of slug heating.

Waste heating refers to energy deposition from particles which do not contribute to the yield, such as electrons, and non-GDR photons. Since none of these groups contribute to the yield, we would like to either minimize their numbers, or for the high-energy photons, scatter them down into the GDR energy band. Of course, energy is deposited by GDR photons, but this contribution is not considered as a component of waste heating in light of the fact that these photons are contributing to the yield.

2.3 Heat Transfer Systems

2.3.1 Porous Metal Heat Exchanger (PMHX)

Cooling of the converter plate provides a significant challenge due to the high heat load imposed by the electron beam. One type of device which can effectively remove high heat loads is a pumped single-phase porous metal heat exchanger (PMHX). High-pressure gas coolants are pumped through a packed bed of spheroidal particles or sintered wires. Packed bed-type heat exchangers are known to be an effective device for dissipating very high heat loads, and are used in high-power optical structures, laser diode cooling and high energy beam dumps [9]. In particular, the PMHX design has been in existence for several decades; therefore, it is believed that the heat loads imposed on the converter plate may be effectively removed if the plate itself is composed of sintered tungsten powder pressed into a solid “frit”.
The main advantage to using a packed bed of porous metal powder is its surface area. Under forced convection conditions, energy which is deposited into the metal may be effectively removed to a gas coolant through the large internal surface area. Helium is a good choice for a coolant based on its high heat capacity and nuclear properties. Heat transfer coefficients necessary to determine the solid-to-gas heat fluxes have been determined empirically and are available from several different studies [6], [7], [9], [13]. A PMHX of sintered tungsten powder at 40% average porosity and 100µm mean particle diameter is the initial design which has been chosen for the converter plate. The capability of this design to absorbed the imposed heat loads must be clearly demonstrated by computer simulation and experimental verification.

2.3.2 Holder Assembly

A cooling system is also necessary for the slugs and backing plate and should also permit easy loading and removal of the slugs. Therefore, the components of the system should be kept relatively simple and small in number. These conditions have culminated in a simple design which utilizes semi-passive conduction cooling.

The design being considered is a holder assembly, which consists of multiple thin slugs encased in aluminum holders. The slug is "passively" cooled as heat conducts from it into the surrounding aluminum. Each holder is fabricated from a solid aluminum block and may be machined as shown in Figure 2-1. The slug fits loosely in the central hole to facilitate remote handling. The holes at either end of the holder and grooves along the
back face are for gas or water flow. These flows facilitate "active" convection cooling and will further cool the slug by maintaining lower temperatures in the holders.

A single stage holder assembly consists of a slug encased on either side by two holders (Figure 2-2). The material of choice for the holders is aluminum since it is an effective backing material with short-lived activation products and is relatively inexpensive; however, its heat-transfer capabilities are limited by its low melting temperature (933 K). Other materials should be considered for the holder in light of these facts, such as copper and/or silver, but the atomic numbers of these elements are much higher and will degrade the yield.

The primary heat transfer mechanism for semi-passive cooling is heat conduction. At equilibrium beam conditions, a large power source will exist within the slug during irradiation relative to the surrounding holder material, resulting in a high heat flux from the slug to the holders. For a given heat flux, the resulting temperature distribution in the holder assembly will depend primarily on the amount of thermal contact resistance between the slug-holder interfaces. The higher this resistance is, the larger the temperature drop will be from the slug to the holders. This in fact is desirable considering that the melting temperature of molybdenum is much higher than aluminum. Although there are several methods which are available to improve heat transfer between an interface to lower contact resistance, their use will add unwanted complexity to this simple system; therefore contact resistance is a fundamental limitation of the design. Computer simulation of this cooling system will address this issue and determine the cooling capabilities of the holder assembly.
Figure 2-1: Holder Design
Based on practical engineering arguments, the proposed target design includes a PMHX converter plate, backing plate and/or bending magnet, and multiple stages of holder assemblies including the slugs. All together, these components make up what is referred to as the target assembly. Figures 2-3 and 2-4 are shown at the end of this chapter and illustrate the conceptual low-yield and high-yield target assemblies, respectively, including electron and photon beam stops.
2.4 Computer Modeling

To simulate the engineering aspects of the target assembly, three major software packages were utilized, MCNP, ALGOR, and Mathcad. MCNP is the code of choice for modeling radiation interactions with matter to determine isotopic yields and particle energy deposition, while ALGOR, a state-of-the-art finite-element engineering analysis code, performs heat transfer calculations in complex three-dimensional geometries using solid meshing techniques. The differential equation solvers of Mathcad are well suited to model in one dimension the thermal-hydraulic aspects of a PMHX converter plate.

MCNP is used to determine an initial target design which optimizes yields and minimizes slug heating, without any \textit{a priori} knowledge of the cooling capability of an external heat-transfer system. The corresponding power distribution in the target assembly is determined and incorporated into corresponding ALGOR and Mathcad heat transfer models. ALGOR determines the steady-state temperature distribution in the slug and holders, and Mathcad calculates temperature distributions in the porous converter plate including the coolant pressure and density distributions.

2.5 Experimental Verification

It is necessary to invoke many assumptions when developing computer models, for a variety of practical reasons. As such, the results may be inaccurate and biased. By generating computer results which can be verified by experiments, the models may be largely verified and used with confidence for later stages of the design process. It is this
initial development and verification which is the main subject of this study. The two major areas of uncertainty are the capability of MCNP to model photoneutronic phenomena and the ability to predict the heat transfer capability of a PMHX converter plate. An experiment to verify the predicted Mo-99 yields has been conducted and is mentioned in Chapter 3. Two other experiments are described in Chapter 5 and will serve to verify the methodology used in this study to model thermal-hydraulics in porous media.
Figure 2-3: Commercial Low-Yield Target Assembly

Figure 2-4: Commercial High-Yield Target Assembly
Chapter 3

MCNP Analysis

Monte Carlo N - Particle or MCNP simulates the interaction of radiation with matter using Monte Carlo probabilistic calculations [1]. The interactions of interest in this study are the scattering and absorption of photons and electrons in several low and high-Z materials. Complicated structures such as the target assembly may be easily constructed by defining the intersection of cylindrical and plane surfaces. Tallies are used to determine the particle fluxes and energy deposition in the cells, and the number of particles and amount of energy crossing a surface.

MCNP was first used to create a simple model of the target assembly to examine energy deposition and yields. This simple model only gave rough estimates of these parameters, but more importantly allowed for comparison between different materials and different configurations. In this manner, possible designs were evaluated based on yield versus slug heating.

3.1 Simple Model

The geometry of the simple model was largely determined by the cylindrical slug geometry. A side profile of the standard low-yield target assembly is illustrated in Figure 3-1, where all dimensions are given in cm. In this model, the slugs were 2 cm in length and 4 cm in diameter. For simplicity, the diameters of the converter and backing plates
were also set to 4 cm. The thickness of each plate was based on the range of 40 MeV electrons in each material. 40 MeV was chosen as the baseline electron energy because it gave the best ratio of Mo-99 yield to waste heating at the optimum converter plate thickness.

Figure 3-1: Standard MCNP Simple Model Geometry

The following MCNP input file, ALIN02, models the target shown in Figure 3-1, with the Z-axis arbitrarily chosen as the axis along the beam direction. The format for naming a specific model corresponding to the input file was AL2n, where n=A,B,C, etc.

Standard Model

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<td>-8</td>
<td>3</td>
<td>-4</td>
<td>$\text{molybdenum density}$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>7</td>
<td>2</td>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>8</td>
<td>3</td>
<td>-4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c</th>
<th>Surfaces</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>pz</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>pz</td>
<td>0.450</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>pz</td>
<td>1.950</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>pz</td>
<td>3.950</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>cz</td>
<td>2.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>cz</td>
<td>2.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>cz</td>
<td>2.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To limit run times, energy cutoffs were utilized to stop the tracking of any particle whose energy fell below the cutoff. In this manner, the remaining energy carried by a particle was deposited locally where it was "killed". These cutoffs were primarily necessary to prevent the tracking of low-energy electrons. Energy cutoffs also result in conservative estimates for energy deposition, since particle energy which may eventually escape the boundaries of the target is instead retained. The effect of energy cutoffs on run times is shown in Table 3.1.

<table>
<thead>
<tr>
<th>Cutoffs (MeV)</th>
<th>0.5</th>
<th>0.4</th>
<th>0.3</th>
<th>0.2</th>
<th>0.1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run Time (min)</td>
<td>5.56</td>
<td>7.04</td>
<td>9.28</td>
<td>10.93</td>
<td>19.5</td>
<td>850</td>
</tr>
</tbody>
</table>

These runs were performed with MCNP 4A on a Pentium-120 PC with NPS=5000. It is obvious that cutoffs were necessary upon consideration of the run time

30
with no cutoffs. This table illustrates that for practical run times of 5 min., a cutoff value of .5 MeV was necessary.

The value of NPS may also greatly affect the run time, but it also will affect the statistical accuracy of the results. By increasing NPS by a factor of N, the statistical uncertainty of a result is only lowered by a factor of $1/\sqrt{N}$. For example, to decrease uncertainty by a factor of 10, NPS must be increased by a factor of 100. 5000 source particles was found to give acceptable uncertainties of $\leq5\%$.

### 3.2 Energy Balance

The first application of this model was to verify if the 40 MeV of electron energy entering the system could be accounted for, within statistical variation, both in deposited and escaping energy. ALIN02 contained the necessary tallies to calculate the net energy crossing the external surfaces of the target: 1, 4, 6, 7, and 8. The cosine bins were necessary to distinguish between outgoing and incoming energy at each surface. Table 3.2 contains the tally results for the escaping photon and electron energy at each external surface, where $X_P$ and $X_E$ is the estimated tally mean value (MeV) for photons and electrons, respectively. For each tally, R is the relative uncertainty calculated directly by MCNP. SP and SE are the estimated errors or uncertainties within a $1\sigma$ confidence interval and are simply determined by multiplying $X_P$ or $X_E$ by the corresponding R value.
Table 3.2: Surface Current Energy Tally Results

<table>
<thead>
<tr>
<th>Escaping Energy</th>
<th>XP</th>
<th>R</th>
<th>SP</th>
<th>XE</th>
<th>R</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface 1</td>
<td>0.66877</td>
<td>0.038</td>
<td>0.025413</td>
<td>0.157955</td>
<td>0.142</td>
<td>0.02243</td>
</tr>
<tr>
<td>Surface 4</td>
<td>5.72551</td>
<td>0.0172</td>
<td>0.098479</td>
<td>0.266373</td>
<td>0.0956</td>
<td>0.025465</td>
</tr>
<tr>
<td>Surface 6</td>
<td>0.167621</td>
<td>0.0849</td>
<td>0.014231</td>
<td>0.000838</td>
<td>0.707</td>
<td>0.000592</td>
</tr>
<tr>
<td>Surface 7</td>
<td>1.97832</td>
<td>0.0261</td>
<td>0.051634</td>
<td>0.298372</td>
<td>0.0741</td>
<td>0.022109</td>
</tr>
<tr>
<td>Surface 8</td>
<td>2.71008</td>
<td>0.0229</td>
<td>0.062061</td>
<td>0.141992</td>
<td>0.1206</td>
<td>0.017124</td>
</tr>
</tbody>
</table>

The mean value for the total amount of escaping energy was obtained by summing XE and XP over all surfaces. The associated error was determined by taking the sum of the squares of each individual error.

The rounded results:

\[ \bar{X} = 12.116 \text{ MeV} \]
\[ S = 0.138 \text{ MeV} \]

To demonstrate if MCNP was conserving energy, the net energy deposited in the entire target was calculated by subtracting the total escaping energy from the incoming energy. This result should then match the MCNP value for total energy deposition within the target from the \(*f8\) tally.

Incoming Energy = 40. ± 0.0 MeV

\[ \text{XNET} = 40. - 12.116 = 27.884 \text{ MeV} \]
\[ \text{SNET} = 0.138 \text{ MeV} \]

From the \(*f8\) tally:

\[ \bar{X} = 27.873 \text{ MeV} \]
\[ S = 0.103 \text{ MeV} \]
Within their respective errors, the results matched and all energy was accounted for; therefore, it was reasonable to assume that the MCNP deposition values were consistent and limited only by the quality of the internal database.

### 3.3 Modeling Yields with MCNP

After verifying the accuracy of the energy deposition, a Mo-99 production tally was added to the model. MCNP can not model a photoneutronic reaction directly, since it does not have a library of photonuclear cross sections. A \((\gamma,n)\) cross-section set for Mo-100 was developed using existing experimental data which combines contributions from both \((\gamma,n)\) and \((\gamma,np)\) reactions. We reduced the set by averaging over 1 MeV intervals between 8-20 MeV and used these values in MCNP as energy-dependent multipliers of the cell-averaged photon flux in the slug. A sum of the weighted photon flux tally over all energy bins then gave the photoneutronic reaction rate per unit volume since only those photons whose energies fall within the resonance energy band have non-zero multipliers and contributed to the tally. The Mo-99 production tally we used is as follows:

\[
f4:p \quad 4 \\
c4 \quad 8 \ 111 \ 20 \\
em4 \quad 0 \ 15 \ 25 \ 25 \ 58 \ 97 \ 135 \ 166 \ 128 \ 85 \ 50 \ 12 \ 0 \\
f4c \quad (\gamma,n) \ weighted \ photon \ flux \ in \ slug
\]

The ability to predict accurate yields with this tally has been verified by the irradiation of natural molybdenum foils by the 40 MeV electron linac at the Rensselaer
Polytechnic Institute. Subsequent measurement of the induced Mo-99 activity were within 5% of the predicted values.

3.4 Energy Deposition

The energy deposition tally of MCNP was utilized to obtain the distribution of energy deposition from both electron and photons in the target assembly. Each electron enters the system with 40 MeV of kinetic energy. In the standard low-yield model, approximately 70% of the 40 MeV of incident energy was deposited into the target assembly. A large portion of the absorbed energy was deposited into the converter plate as most of the electron's kinetic energy was deposited. The following table indicates the distribution of deposited energy among the three regions of the standard target assembly by a single 40 MeV electron. All table values are accurate to within 2%.

<table>
<thead>
<tr>
<th></th>
<th>Converter</th>
<th>Backing</th>
<th>Slug</th>
<th>Escape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (MeV)</td>
<td>17.1</td>
<td>4.07</td>
<td>6.71</td>
<td>12.1</td>
</tr>
<tr>
<td>Percentage</td>
<td>42.7</td>
<td>10.2</td>
<td>16.8</td>
<td>30.3</td>
</tr>
</tbody>
</table>

It is desirable for particles which do not directly contribute to the yield to escape the target to limit heating. Approximately 9 MeV of the escaping energy was due to photons and electrons escaping from the slug. Only about 500 keV of escaping electron energy contributed to this 9 MeV. The remainder was due to escaping photons as given in the following table:
Table 3.4: Distribution of Photon Energy Escaping Slug

<table>
<thead>
<tr>
<th>Back Face</th>
<th>MeV</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5 to 8 MeV</td>
<td>2.74 ± .05</td>
<td>1.17 ± .02</td>
</tr>
<tr>
<td>8 to 20 MeV</td>
<td>2.12 ± .07</td>
<td>.173 ± .005</td>
</tr>
<tr>
<td>20 to 40 MeV</td>
<td>.865 ± .07</td>
<td>.0336 ± .0025</td>
</tr>
</tbody>
</table>

Cylindrical Surface

<table>
<thead>
<tr>
<th>MeV</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5 to 8 MeV</td>
<td>1.74 ± .04</td>
</tr>
<tr>
<td>8 to 20 MeV</td>
<td>.796 ± .04</td>
</tr>
<tr>
<td>20 to 40 MeV</td>
<td>.174 ± .03</td>
</tr>
</tbody>
</table>

where photon weight is simply the number of photons per source electron.

3.4.1 Slug

In the standard model, the total energy deposition from electrons and photons in the slug was approximately 6.71 MeV. The following table contains a breakdown of this heating over the three major energy bins from .5 to 40 MeV. The bin spacing was chosen using the lower and upper energy bounds considered in the problem, .5 and 40 MeV, respectively, while 8-20 MeV is the location of the GDR region for Mo-100.

Table 3.5: Distribution of Slug Heating

<table>
<thead>
<tr>
<th>Bin (MeV)</th>
<th>MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5 to 8</td>
<td>1.62 ± .03</td>
</tr>
<tr>
<td>8 to 20</td>
<td>4.08 ± .09</td>
</tr>
<tr>
<td>20 to 40</td>
<td>1.01 ± .07</td>
</tr>
<tr>
<td>Total</td>
<td>6.71 ± .09</td>
</tr>
</tbody>
</table>

We can infer the distribution of heating within these coarse bins by determining the net energy entering the slug surfaces as in the energy balance calculation. Energy
deposition was still preserved, but the prediction of heating within each bin did not agree with the results in Table 3.5 which combined the deposition from both particles in each energy bin. This was due to the conversion of electron into photons, and vice versa, which occurred within the slug volume before either particle could escape to the surface. Nevertheless, it was reasonable to assume that electrons contributed about 1 MeV of energy to the slug, and that GDR photons contributed the most photon energy.

Table 3.6: Surface Energy Tally Estimate of Slug Heating Distribution

<table>
<thead>
<tr>
<th>Bin (MeV)</th>
<th>Electrons</th>
<th>Photons</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5 to 8</td>
<td>.281 ± .030</td>
<td>1.60 ± .09</td>
</tr>
<tr>
<td>8 to 20</td>
<td>.814 ± .056</td>
<td>2.49 ± .14</td>
</tr>
<tr>
<td>20 to 40</td>
<td>.058 ± .020</td>
<td>1.47 ± .13</td>
</tr>
<tr>
<td>Total</td>
<td>1.15 ± .066</td>
<td>5.56 ± .21</td>
</tr>
</tbody>
</table>

However, the majority of photons which entered the slug were in the low energy range, as seen in Figure 3-2. Approximately 6 low-energy photons entered the slug for every GDR photon. Although the majority of photons had low energy, their contribution to slug heating was much less than the heating from GDR photons. To lower the slug heating while preserving the yield, we may only influence heating from electrons and low-energy photons.
Figure 3-2: Photon Energy Distribution at Slug Surfaces

(Note: error bars are too small to be seen on figure)

The axial distribution of slug heating from both photons and electrons in the slug is shown in Figure 3-3. More energy was deposited in the front portion of the slug as short-range particles are stopped. The shorter slug (1 cm) which is used in a holder assembly will have a more uniform distribution across its thickness.

Figure 3-3: Axial Distribution of Slug Heating
3.4.2 Electron Heating of Slug

The effect of either a backing plate or magnetic sweep is explained by the following table, which compares slug heating and yields for different designs. The vacuum and sweep models still had a 1.5 cm distance between the converter plate and slug. The value of the electron importance in the gap determined, for MCNP calculations, whether an electron exiting the converter is swept. The value was 1 for the low-yield model with aluminum, 1 for the vacuum, and 0 for the high-yield, which represented a "sweep".

<table>
<thead>
<tr>
<th>Backing Plate</th>
<th>Slug Heat (MeV)</th>
<th>Yield (mb/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>6.71 ± .09</td>
<td>1.43 ± .04</td>
</tr>
<tr>
<td>Vacuum</td>
<td>8.88 ± .11</td>
<td>1.55 ± .04</td>
</tr>
<tr>
<td>Sweep</td>
<td>5.84 ± .09</td>
<td>1.48 ± .04</td>
</tr>
</tbody>
</table>

As aforementioned, the sweep resulted in the least amount of slug heating since the contribution from electrons was removed, but the yield was somewhat decreased since electrons were prevented from entering the slug and generating secondary photons which may contribute to the yield. Even though the yield may not be increased directly by utilizing a sweep, the low heating may allow the electron current to be increased which will boost production. We might also expect the optimum beam energy to be slightly higher than 40 MeV.
3.5 Optimum Converter Plate Thickness

Slug heating and yield were sensitive to the converter plate thickness. The thickness should take into consideration the range of 40 MeV electrons in the plate material, which is about 1 cm for tungsten. The range of fast electrons in dense material may be roughly estimated by the following simple formula [5].

\[
\text{Range (cm)} = \frac{\text{Energy (MeV)}}{2 \cdot \text{Density (g/cc)}} = \frac{40}{2 \cdot 19.3} = 1.03\text{cm}
\]

Based on the range, we have considered converter thicknesses from 0.1 cm to 1.0 cm. The optimum thickness will lie between 30% and 45% of the electron range. Below 3 mm, there was low electron conversion and excessive slug heating. Above 4.5 mm, yields fell due to GDR photon degradation. The effect of four different plates on the slug are show in Table 3.8.

<table>
<thead>
<tr>
<th>Thickness (cm)</th>
<th>Slug Heat (MeV)</th>
<th>Yield (mb/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.45</td>
<td>6.71 ± .09</td>
<td>1.43 ± .04</td>
</tr>
<tr>
<td>.40</td>
<td>7.76 ± .10</td>
<td>1.45 ± .04</td>
</tr>
<tr>
<td>.35</td>
<td>8.68 ± .10</td>
<td>1.50 ± .04</td>
</tr>
<tr>
<td>.30</td>
<td>10.1 ± .11</td>
<td>1.58 ± .04</td>
</tr>
</tbody>
</table>

As expected, decreasing the thickness increased the yield but also increased the slug heating in favor of smaller heat loads in the converter. The optimum thickness was between .30 and .45 cm, but the increase in yield obtained by thinner plates may also be achieved with thicker plates by boosting the current. The following exercise demonstrates this effect. Since the specific activity scales directly with current, boosting
the current from the nominal value of 250μA to 300μA will increase the yield by a factor of 20%:

The slug heating will also increase proportionally by a factor of 20%. Therefore, when using the mean values for the .35 cm plate, a 20% increase in current gives:

Yield = 1.5 * 1.2 = 1.80

Heating = 8.68 * 1.2 = 10.4

For about the same amount of slug heating with the .30 cm plate, the yield was much less. A .35 cm plate was a reasonable compromise and was chosen to be the baseline for the remainder of the study.

The cell-averaged heating of the four converter plates is seen in Table 3.9. Since the converter design will allow it to dissipate much more heat than the slug, the converter design should minimize slug heating in favor of converter heating.

Table 3.9: Energy Deposition in Converter Plates

<table>
<thead>
<tr>
<th>Thickness (cm)</th>
<th>Heat (MeV)</th>
<th>Power (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.45</td>
<td>17.1</td>
<td>4.28</td>
</tr>
<tr>
<td>.40</td>
<td>15.1</td>
<td>3.78</td>
</tr>
<tr>
<td>.35</td>
<td>13.2</td>
<td>3.30</td>
</tr>
<tr>
<td>.30</td>
<td>10.9</td>
<td>2.73</td>
</tr>
</tbody>
</table>

where all errors are within 1% of the mean

The power was determined using the fraction of the total energy (40 MeV) deposited in a cell and the beam power, which for the nominal case was 10 kW. For the .30 cm plate:

\[
\frac{10.9}{40} \times 10\text{kW} = 2.73\text{kW}
\]
As one would expect, the same of amount of energy should be deposited in a porous converter plate as in the solid model in MCNP, given that the density is reduced and that an equivalent thickness of material is used. The equivalent thickness of a porous plate was determined using the solid material density and porosity while holding the diameter fixed.

\[
v_p = \frac{V}{1 - \varepsilon}
\]

\[
t_p = \frac{4V_p}{\pi D^2} = \frac{4V}{(1 - \varepsilon) \pi D^2} = \frac{t}{1 - \varepsilon}
\]

where: subscript p indicates porous model
\[\varepsilon = \text{porosity}\]
\[t = \text{thickness}\]
\[V = \text{volume}\]

The "porous" density was also simply determined:

\[
\rho_p = (1 - \varepsilon) \rho
\]

The axial distribution of energy deposition in the converter is shown in the following figure for the solid .45 cm plate and a corresponding 60% porous plate at 7.72 g/cm³ and t = 1.125 cm. Even though 60% porosity is too high to be achieved in practice, it was used here as an extreme value. The distribution was almost the same in both cases, with the difference attributable to beam spreading in the longer porous plate.
There was a small but appreciable effect on the slug heating and yield. Table 3.10 shows the effect of using porous versions of .45 and .35 cm solid plates.

Table 3.10: Effect of Porous Converter Plates

<table>
<thead>
<tr>
<th>c</th>
<th>t (cm)</th>
<th>Slug Heat (MeV)</th>
<th>Yield (mb/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.45</td>
<td>6.71 ± .09</td>
<td>1.43 ± .04</td>
</tr>
<tr>
<td>.40</td>
<td>.75</td>
<td>6.59 ± .09</td>
<td>1.35 ± .04</td>
</tr>
<tr>
<td>.60</td>
<td>1.125</td>
<td>6.35 ± .09</td>
<td>1.33 ± .04</td>
</tr>
<tr>
<td>0</td>
<td>.35</td>
<td>8.68 ± .10</td>
<td>1.50 ± .04</td>
</tr>
<tr>
<td>.40</td>
<td>.583</td>
<td>8.38 ± .10</td>
<td>1.49 ± .04</td>
</tr>
<tr>
<td>.60</td>
<td>.875</td>
<td>8.78 ± .10</td>
<td>1.36 ± .04</td>
</tr>
</tbody>
</table>

Equivalent thicknesses of the .45 cm plate at 40% and 60% had significantly degraded yields, but yield fell off more slowly when starting with the .35 cm plate. The slug heating was only marginally affected for each case. In fact, the 40% porous version of the .35 cm plate had practically the same results as the equivalent solid plate. This was encouraging since we can implement a PMHX cooling system for the converter plate without affecting the nuclear processes in the target assembly.
3.6 Yield Optimization

After determining the Mo-99 production and heating values for the standard model, several other similar models were devised to determine if the standard results could be improved. In other words, we attempted to improve the yield for the same amount of slug heating. Given the results of the standard model, improvements may be made by reducing the electron and low-energy photon population entering the slug, while either preserving or even increasing the GDR photon population in the slug. The different converter/backing plate designs that were examined included:

- Different W, Al Thickness
- Higher Z-Material Converter Plates (Pb, Pt)
- Lower Z-Material Backing Plates (Be, B)
- Higher Z-Material Backing Plates (Ti, Cu)
- Composite Backing Plates (Al-Ti,Cu,B)

Tables 3.11 through 3.20 list the results for the alternative models that were considered; grouped according to similarities in design. Each model is identified and grouped by the name of the output file, which relates the model to the proper input file. For instance, AL2HB and AL2KB belong to the ALIN02 model group which includes all models containing aluminum-only backing plates. Table 3.11 first gives the results of varying the thickness of the converter and backing plates in the standard model.

Table 3.11: AL2 Model Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Config</th>
<th>W (cm)</th>
<th>Al (cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL2HB</td>
<td>W-Al-Mo</td>
<td>0.45</td>
<td>1.5</td>
<td>1.43 ± 0.04</td>
<td>6.71 ± 0.09</td>
</tr>
<tr>
<td>AL2KB</td>
<td>W-Al-Mo</td>
<td>0.3</td>
<td>1.65</td>
<td>1.49 ± 0.04</td>
<td>9.55 ± 0.11</td>
</tr>
<tr>
<td>AL2LB</td>
<td>W-Al-Mo</td>
<td>4.5</td>
<td>2</td>
<td>1.29 ± 0.04</td>
<td>5.72 ± 0.09</td>
</tr>
<tr>
<td>AL2MB</td>
<td>W-Al-Mo</td>
<td>0.3</td>
<td>2</td>
<td>1.39 ± 0.04</td>
<td>8.38 ± 0.10</td>
</tr>
</tbody>
</table>
Tables 3.12 and 3.13 contain results for introducing Ti and Cu into the model; either as a partial or total replacement of the aluminum backing plate. In both cases, equivalent thicknesses of Ti and Cu were calculated by preserving the original number of aluminum atoms in the standard backing plate. By calculating an equivalent thickness, the number of atoms is conserved, which allows one to make an atom-for-atom comparison of the results for different elements. In all four cases, the standard thickness of W was used.

### Table 3.12: AL3 Model Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Config</th>
<th>Al (cm)</th>
<th>Ti (cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL3AA</td>
<td>W-Ti-Mo</td>
<td>0</td>
<td>1.594</td>
<td>1.22 ± .04</td>
<td>5.70 ± .09</td>
</tr>
<tr>
<td>AL3BB</td>
<td>W-Al-Ti-Mo</td>
<td>1.3</td>
<td>0.212</td>
<td>1.40 ± .04</td>
<td>6.65 ± .09</td>
</tr>
</tbody>
</table>

### Table 3.13: AL4 Model Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Config</th>
<th>Al (cm)</th>
<th>Ti (cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL4AA</td>
<td>W-Cu-Mo</td>
<td>0</td>
<td>1.064</td>
<td>1.25 ± .04</td>
<td>5.54 ± .09</td>
</tr>
<tr>
<td>AL4BB</td>
<td>W-Al-Cu-Mo</td>
<td>1.3</td>
<td>0.142</td>
<td>1.36 ± .04</td>
<td>6.28 ± .09</td>
</tr>
</tbody>
</table>

Table 3.14 shows the effect of replacing W with higher-Z materials; namely, Pb and Pt. In both cases, the equivalent thickness of material was used to replace the .45cm of W.

### Table 3.14: Higher-Z Converter Plates

<table>
<thead>
<tr>
<th>Model</th>
<th>Config</th>
<th>Pb/Pt (cm)</th>
<th>Al (cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL5AA</td>
<td>Pb-Al-Mo</td>
<td>0.859</td>
<td>1.5</td>
<td>1.26 ± .04</td>
<td>5.88 ± .09</td>
</tr>
<tr>
<td>AL6AA</td>
<td>Pt-Al-Mo</td>
<td>0.427</td>
<td>1.5</td>
<td>1.39 ± .04</td>
<td>6.46 ± .09</td>
</tr>
</tbody>
</table>
Tables 3.15 and 3.16 contain the results for Be and B backing plates. For each material two models were created; one with an equivalent thickness of Be and B and one with the standard thickness.

Table 3.15: Beryllium Backing Plate Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Config</th>
<th>Be (cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL7A</td>
<td>W-Be-Mo</td>
<td>0.731</td>
<td>1.67 ± .05</td>
<td>9.50 ± .10</td>
</tr>
<tr>
<td>AL7B</td>
<td>W-Be-Mo</td>
<td>1.5</td>
<td>1.50 ± .04</td>
<td>7.65 ± .10</td>
</tr>
</tbody>
</table>

Table 3.16: Boron Backing Plate Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Config</th>
<th>B (cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL8A</td>
<td>W-B-Mo</td>
<td>0.705</td>
<td>1.70 ± .04</td>
<td>9.35 ± .10</td>
</tr>
<tr>
<td>AL8B</td>
<td>W-B-Mo</td>
<td>1.5</td>
<td>1.46 ± .04</td>
<td>7.26 ± .10</td>
</tr>
</tbody>
</table>

The results of tables 3.17-3.19 are for backing plates composed of both B and Al. The results are grouped into three tables according to increasing thickness of B.

Table 3.17: Composite B-Al Backing Plate Configurations

<table>
<thead>
<tr>
<th>Model</th>
<th>Config.</th>
<th>B (cm)</th>
<th>Al (cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL8F</td>
<td>W-B-Al-Mo</td>
<td>0.605</td>
<td>0.1</td>
<td>1.63 ± .04</td>
<td>9.25 ± .10</td>
</tr>
<tr>
<td>AL8P</td>
<td>W-B-Al-Mo</td>
<td>0.605</td>
<td>0.2</td>
<td>1.60 ± .04</td>
<td>8.88 ± .10</td>
</tr>
<tr>
<td>AL8E</td>
<td>W-B-Al-Mo</td>
<td>0.705</td>
<td>0.1</td>
<td>1.64 ± .04</td>
<td>9.00 ± .10</td>
</tr>
<tr>
<td>AL8J</td>
<td>W-B-Al-Mo</td>
<td>0.705</td>
<td>0.2</td>
<td>1.60 ± .04</td>
<td>8.62 ± .10</td>
</tr>
<tr>
<td>AL8C</td>
<td>W-B-Al-Mo</td>
<td>0.705</td>
<td>0.795</td>
<td>1.42 ± .04</td>
<td>6.92 ± .09</td>
</tr>
</tbody>
</table>

Table 3.18: Composite B-Al Backing Plate Configurations

<table>
<thead>
<tr>
<th>Model</th>
<th>Config.</th>
<th>B (cm)</th>
<th>Al (cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL8G</td>
<td>W-B-Al-Mo</td>
<td>0.805</td>
<td>0.1</td>
<td>1.61 ± .04</td>
<td>8.63 ± .10</td>
</tr>
<tr>
<td>AL8H</td>
<td>W-B-Al-Mo</td>
<td>0.805</td>
<td>0.2</td>
<td>1.62 ± .04</td>
<td>8.31 ± .10</td>
</tr>
<tr>
<td>AL8I</td>
<td>W-B-Al-Mo</td>
<td>0.805</td>
<td>0.3</td>
<td>1.55 ± .04</td>
<td>8.06 ± .10</td>
</tr>
<tr>
<td>AL8V</td>
<td>W-B-Al-Mo</td>
<td>0.805</td>
<td>0.4</td>
<td>1.54 ± .04</td>
<td>7.81 ± .10</td>
</tr>
<tr>
<td>AL8W</td>
<td>W-B-Al-Mo</td>
<td>0.805</td>
<td>0.5</td>
<td>1.49 ± .04</td>
<td>7.57 ± .10</td>
</tr>
<tr>
<td>AL8X</td>
<td>W-B-Al-Mo</td>
<td>0.805</td>
<td>0.6</td>
<td>1.44 ± .04</td>
<td>7.28 ± .09</td>
</tr>
</tbody>
</table>
Table 3.19: Composite B-Al Backing Plate Configurations

<table>
<thead>
<tr>
<th>Model</th>
<th>Config.</th>
<th>B (cm)</th>
<th>Al (cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL8L</td>
<td>W-B-Al-Mo</td>
<td>0.905</td>
<td>0.1</td>
<td>1.60 ± .04</td>
<td>8.45 ± .10</td>
</tr>
<tr>
<td>AL8M</td>
<td>W-B-Al-Mo</td>
<td>0.905</td>
<td>0.2</td>
<td>1.56 ± .04</td>
<td>8.09 ± .10</td>
</tr>
<tr>
<td>AL8N</td>
<td>W-B-Al-Mo</td>
<td>0.905</td>
<td>0.3</td>
<td>1.51 ± .04</td>
<td>7.90 ± .10</td>
</tr>
<tr>
<td>AL8O</td>
<td>W-B-Al-Mo</td>
<td>1.005</td>
<td>0.1</td>
<td>1.54 ± .04</td>
<td>8.20 ± .10</td>
</tr>
<tr>
<td>AL8U</td>
<td>W-B-Al-Mo</td>
<td>1.005</td>
<td>0.2</td>
<td>1.51 ± .04</td>
<td>7.93 ± .10</td>
</tr>
<tr>
<td>AL8S</td>
<td>W-B-Al-Mo</td>
<td>1.405</td>
<td>0.1</td>
<td>1.40 ± .04</td>
<td>7.31 ± .10</td>
</tr>
</tbody>
</table>

The final table examines more complicated composite B-Al backing plates.

Table 3.20: 3-Cell Composite B-Al Backing Plate Configurations

<table>
<thead>
<tr>
<th>Model</th>
<th>W-x-y-z-Mo</th>
<th>x(cm)</th>
<th>y(cm)</th>
<th>z(cm)</th>
<th>Yield</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL8CA</td>
<td>W-B-Al-B-Mo</td>
<td>0.7</td>
<td>0.1</td>
<td>0.7</td>
<td>1.47 ± .04</td>
<td>7.35 ± .10</td>
</tr>
<tr>
<td>AL8CB</td>
<td>W-B-B-Al-Mo</td>
<td>0.7</td>
<td>0.7</td>
<td>0.1</td>
<td>1.45 ± .04</td>
<td>7.17 ± .10</td>
</tr>
<tr>
<td>AL8CC</td>
<td>W-Al-B-B-Mo</td>
<td>0.1</td>
<td>0.7</td>
<td>0.7</td>
<td>1.44 ± .04</td>
<td>7.25 ± .10</td>
</tr>
<tr>
<td>AL8CD</td>
<td>W-B-Al-B-Mo</td>
<td>0.6</td>
<td>0.2</td>
<td>0.6</td>
<td>1.48 ± .04</td>
<td>7.57 ± .10</td>
</tr>
<tr>
<td>AL8CE</td>
<td>W-B-Al-B-Mo</td>
<td>0.7</td>
<td>0.2</td>
<td>0.7</td>
<td>1.42 ± .04</td>
<td>7.11 ± .10</td>
</tr>
<tr>
<td>AL8CF</td>
<td>W-B-Al-B-Mo</td>
<td>0.7</td>
<td>0.1</td>
<td>0.8</td>
<td>1.45 ± .04</td>
<td>7.14 ± .10</td>
</tr>
<tr>
<td>AL8CG</td>
<td>W-B-Al-B-Mo</td>
<td>0.8</td>
<td>0.1</td>
<td>0.7</td>
<td>1.42 ± .04</td>
<td>7.12 ± .10</td>
</tr>
<tr>
<td>AL8CH</td>
<td>W-B-Al-B-Mo</td>
<td>0.6</td>
<td>0.1</td>
<td>0.9</td>
<td>1.43 ± .04</td>
<td>7.10 ± .10</td>
</tr>
<tr>
<td>AL8CI</td>
<td>W-B-Al-Mo</td>
<td>0.6</td>
<td>0.6</td>
<td>0.2</td>
<td>1.46 ± .04</td>
<td>7.69 ± .10</td>
</tr>
</tbody>
</table>

Of the several different target designs which were analyzed, none presented a clear advantage over the results of the standard model in both categories of production and heating. The basic reason was that the standard model is a good design and did not represent a wild "guess" as far as materials and geometry were concerned. Alternate target designs did not improve results; i.e. production was not increased for the same amount of heating present in the standard model; therefore, W is a satisfactory material for the converter plate and Al is a satisfactory material for backing plate.

In general, MCNP made consistent predictions of energy deposition in the target. A backing plate was necessary to reduce slug heating, but a magnetic sweep was superior.
Yields and energy deposition were very sensitive to converter and backing plate material and thickness. Having established this, we next developed a realistic MCNP model of the target assembly.

### 3.7 Target Assembly Model

A more realistic MCNP model of the full target assembly, with a PMHX converter plate and a single-stage holder assembly was created to give a better estimate of the distribution of energy throughout the target. Only a single-stage of slug and holders was considered, namely the stage that is initially struck by the beam and will absorb the most energy. If the first stage can safely dissipate the beam, then others stages behind it will also surely be able to dissipate the lower powers which they will absorb. It is the stacking of successive stages that will create the circular cross section for the small central grooves.

The geometry for the low and high-yield models was taken directly from Figures 2-3 and 2-4, where the beam stops were not considered. The converter plate was the 40% porous version of the equivalent .35 cm solid converter plate with a diameter of 2 cm and thickness of .583 cm. The slug was 1 cm in diameter and thickness. The gap between the converter plate and bottom holder (the holder which is closest to the converter plate) was 1.5 cm and can be used for a 2 cm diameter backing plate or to permit the bending of the electron beam.

The detailed model, ASSEM, provided a realistic distribution of energy deposition in the target assembly which would be necessary to predict a realistic
temperature distribution. The MCNP input set for ASSEM is given in Appendix A.

Each holder was comprised of six cells. Tables 3.21 and 3.22 give the results of ASSEM for 250,000 source particles for low- and high-yield models. AS1 was the low-yield model containing a 1.5 cm thick aluminum backing plate, while AS2 was the high-yield model with a vacuum gap and electron sweep. The errors were small enough to be neglected in cells where the most heat was deposited. Otherwise, only the cells which received at least 1 keV of energy were considered, for which the errors were < 10%.

Table 3.21: Power Density Distribution in AS1

<table>
<thead>
<tr>
<th></th>
<th>MeV</th>
<th>Vol (cm³)</th>
<th>W/cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converter</td>
<td>13.0</td>
<td>1.83</td>
<td>1780.</td>
</tr>
<tr>
<td>Backing</td>
<td>4.24</td>
<td>4.71</td>
<td>225.</td>
</tr>
<tr>
<td>Holder (Btm.)</td>
<td>.971</td>
<td>15.6</td>
<td>15.5</td>
</tr>
<tr>
<td>Slug</td>
<td>1.46</td>
<td>.785</td>
<td>465.</td>
</tr>
<tr>
<td>Holder (Top)</td>
<td>.684</td>
<td>15.6</td>
<td>11.0</td>
</tr>
</tbody>
</table>

Table 3.22: Power Density Distribution in AS2

<table>
<thead>
<tr>
<th></th>
<th>MeV</th>
<th>Vol (cm³)</th>
<th>W/cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converter</td>
<td>13.0</td>
<td>1.83</td>
<td>1770.</td>
</tr>
<tr>
<td>Backing</td>
<td>-</td>
<td>4.71</td>
<td>-</td>
</tr>
<tr>
<td>Holder (Btm.)</td>
<td>.174</td>
<td>15.6</td>
<td>2.79</td>
</tr>
<tr>
<td>Slug</td>
<td>1.08</td>
<td>.785</td>
<td>343.</td>
</tr>
<tr>
<td>Holder (Top)</td>
<td>.304</td>
<td>15.6</td>
<td>4.87</td>
</tr>
</tbody>
</table>

The powers were determined using the nominal current of 250μA and the heating of all six holder cells was added and divided by the total holder volume to estimate a uniform holder power density.
Chapter 4

ALGOR Analysis

ALGOR is a combined CAD and finite-element analysis (FEA) code which can be used to perform the preliminary heat transfer studies of the holder assembly. A three-dimensional model with complex geometry may be easily divided into solid finite element "bricks" using the built-in automatic meshing features of ALGOR. A preprocessor program is used to define boundary conditions and material properties and to create a processor input file. A processor then performs the heat transfer analysis and calculates the temperature and heat flux distributions throughout the model. The particulars of the modeling process are outlined in the following section.

4.1 Solution Method

In general, an ALGOR model is built in a series of steps. A wireframe model is first constructed using SUPERDRAW II. This basic model is then transferred to SUPERSURF which creates a "hollow" surface mesh using the wireframe. Next, the corresponding solid mesh is built with XGEN using the surface mesh; thus, the solid volume of the holder assembly is split into many small finite volumes, or bricks. At this point, boundary conditions are assigned. Constant temperature and adiabatic boundary conditions may be directly added with SUPERDRAW, while convection and radiation boundary conditions and volumetric heat generation conditions are added by specifying a
unique color for each region of the model which is subject to any combination of these conditions. The values associated with each condition are assigned with the preprocessor, or decoder.

The decoder, DECODT, is also used to define material properties such as thermal conductivity and prepares the processor input file which actually performs the finite-element calculations. With this file, the steady-state heat transfer processor, SSAP10, calculates the temperature and heat flux distributions in the model using the specified material properties and boundary conditions.

Another important consideration for heat transfer in composite systems is thermal contact resistance which causes a temperature drop across an interface between materials. This resistance is primarily due to surface roughness effects and can be minimized by polishing mating surfaces, increasing the joint pressure, or adding an interfacial fluid of high thermal conductivity. The most reliable prediction for contact resistance values comes from experimental studies.

The interfaces where contact resistance was deemed to be significant in the holder assembly was between the ends of the slug and the holder since a large amount of heat passes across these surfaces. ALGOR can not directly model the effect of contact resistance; therefore, to account for this, very thin volumes of holder material on either end of the slug were assigned reduced thermal conductivities to represent a resistance to heat transfer from the slug to the surrounding holder material. Thermal resistance will also be present, though not at a mating interface, due to the air gap between the cylindrical surface of the slug and the surrounding holder. Different from the thin disks at the slug end, which must be explicitly modeled but are not physically real, the annular
gap will be necessary for easy slug loading and removal and is physically present. It represents a major resistance to heat transfer due to the low thermal conductivity of air.

To model the air gap in ALGOR, a very thin annulus of the holder material in contact with the slug was assigned the thermal conductivity of air. This "insulating sheath" is .5 mm in thickness. The thermal conductivities of the thin disks simulating contact resistance at the ends of the slug were determined using the disk thickness, t, and an experimental contact resistance value $(R''_{t,c})$ according to the following expression.

$$R''_{t,c} = \frac{t}{k}$$

The remaining interface is between the aluminum holders, but the effect of contact resistance is negligible since there will be very low heat fluxes across this interface.

The four coolant holes at the end of the holders and the two small grooves along the back face of each holder were assigned constant temperature and convection boundary conditions, respectively. The remainder of the boundary conditions on the exposed holder surfaces were left unspecified which invokes an adiabatic condition. This is somewhat conservative, but these surfaces will only be subject to cooling via natural convection, and the temperatures will be too low to consider radiation heat transfer. Specific boundary conditions and material properties are described for two different models in the following sections.
4.2 Simple Model

The simple model was constructed as described in the previous section, but only considered a power source within the slug and the effect of the insulating sheath, without considering the contact resistance at the slug ends. A fixed low temperature boundary conditions was fixed at the inner surface of the outer coolant holes and was the only mechanism which may draw heat out of the slug where it was being generated. The goal was to draw out enough heat to maintain a reasonable steady-state temperature within the slug.

The backing plate, which must also be cooled, is not modeled since the most likely commercial design will include an electron sweep and because it will be difficult to cool based only on its contact with the bottom holder. As a result, the heating results of AS2 were considered, where the slug power density was 343 W/cm³ at an electron current of 250 µA. The thermal conductivities were taken for aluminum, molybdenum, and air based on the conservative estimates of the average temperatures which were expected to occur in practice [4].

\[
k_{Mo} = 1.18 \text{ W/cm-K (800 K)}
\]
\[
k_{Al} = 2.38 \text{ W/cm-K (300 K)}
\]
\[
k_{air} = 4.69e-4 \text{ W/cm-K (600 K)}
\]

The inner surface of the coolant holes were fixed at 293 K such as may be attained with the flow of water through the holes.

Figure 4-1 shows the temperature distribution of a cross section of the holder assembly (in K) for the simple ALGOR model AS2S. This view is in the Y-Z plane and
was generated by slicing the model by a Y-Z plane situated at X=0. Figure 4-2 is the corresponding heat flux distribution (in W/cm²) and clearly shows that heat flowed through the ends of the slug and not through the cylindrical surface.

Figure 4-1: AS2S Temperature Distribution

Figure 4-2: AS2S Heat Flux Distribution
The maximum temperature in the holder assembly was 417 K at the center of the slug and the maximum temperature in the aluminum was about 375 K near the slug ends and the coolant grooves. The heat flux was highest at the perimeter of slug ends as heat preferentially flowed to the ends of the holder where the low-temperature boundary condition existed. The relatively low temperatures in this model were due to the lack of resistance at the slug ends which did not provide a significant temperature drop across the aluminum-molybdenum interface. We conclude that a more realistic model should include this contact resistance which will provide a larger temperature drop from the slug to the holders.

4.3 Detailed Model

The detailed model (AS2D) may predict more realistic temperature and heat flux distributions by considering several modifications to AS2S. The first modification was the inclusion of the energy deposition in the holders predicted by ASSEM. Next, was the consideration of the aluminum-molybdenum contact resistance at the ends of the slug. Finally, the cooling capability of the small coolant grooves was added.

The holder power densities were smeared out over the entire holder volume in Tables 3.21 and 3.22; however, much more of the heat is deposited in the "cube" immediately surrounding the slug and in direct line with the beam (Cells 3&4, Cells 5&6). For AS2D, the more accurate power density distribution in the holder was used according to the MCNP cell layout of ASSEM. The energy deposition in cells 3 and 4 were combined in the bottom holder, and in cells 5 and 6 in the top holder. Cells 7, 8, 9,
and 10 were also modeled separately. The heating in cells 11-14 was negligible. Table 4.1 gives the resulting power densities at 250 μA:

Table 4.1: AS2D Power Density Distribution

<table>
<thead>
<tr>
<th>Cell(s)</th>
<th>MeV</th>
<th>Vol (cm³)</th>
<th>W/cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-4</td>
<td>1.71e-1</td>
<td>5.47</td>
<td>7.81</td>
</tr>
<tr>
<td>5-6</td>
<td>2.94e-1</td>
<td>5.47</td>
<td>13.4</td>
</tr>
<tr>
<td>7</td>
<td>1.55e-3</td>
<td>4.2</td>
<td>0.092</td>
</tr>
<tr>
<td>8</td>
<td>4.45e-3</td>
<td>4.2</td>
<td>0.267</td>
</tr>
<tr>
<td>9</td>
<td>1.67e-3</td>
<td>4.2</td>
<td>0.099</td>
</tr>
<tr>
<td>10</td>
<td>5.23e-3</td>
<td>4.2</td>
<td>0.311</td>
</tr>
</tbody>
</table>

A heat transfer coefficient was also calculated for the inner surface of the grooves based on laminar gas flow. To calculate this coefficient, we treated the groove as a small pipe that is .2 cm in diameter and 3 cm in length. For helium entering the grooves at 5 atmospheres, we estimated a 2 atmosphere pressure drop. At the mean pressure of 4 atmospheres and 300 K, the density of helium was found by the equation of state to be 0.650 kg/m³. Using this density with the fixed inlet and outlet pressures, the corresponding inlet and outlet temperatures must be 375 K and 225 K respectively.

Next we invoked Bernoulli's Law, (even though gas flow is compressible), to relate the pressure drop to the gas velocity and form losses.

\[
\frac{p_i - p_o}{\rho} = \frac{v_o^2 - v_i^2}{2} + k_i \frac{v_i^2}{2} + k_o \frac{v_o^2}{2}
\]

For a sharp entrance, \(k_i = 0.5\) and for a sharp exit into a reservoir, \(k_o = 1.0\) [12]. We may determined a rough estimate for the velocities involved by setting \(v_i = v_o = v\):

\[
\frac{2(101325) Pa}{0.650 \text{kg/m}^3} \approx \frac{15v^2}{2}
\]

55
Solving for \( V \),

\[
V = 645 \, \text{m/s}
\]

The corresponding mass flow rate was now determined:

\[
\dot{m} = \rho VA = (0.650)(645)\pi \frac{(0.002)^2}{4} = 0.0013 \, \text{kg/s}
\]

Using the viscosity of He at 300 K, we may estimated the Reynolds number.

\[
\text{Re} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.0013)}{\pi(0.002)(199 \cdot 10^{-7})} = 42000
\]

This value was actually in the turbulent regime, so we used the Dittus-Boelter correlation for heated flow to calculate the Nusselt number.

\[
\text{Nu} = 0.023(\text{Re})^0.4(\text{Pr})^4 = 98
\]

where the Prandtl number for He at 300 K = 0.67

The heat transfer coefficient between the gas flow and the pipe wall was now determined using the pipe diameter and thermal conductivity of helium at 300 K.

\[
h = \frac{k(\text{Nu})}{D} = \frac{0.152(98)}{0.002} = 7450 \, \text{W/m}^2\text{K} = 745 \, \text{W/cm}^2\text{K}
\]

This was the value assigned in the decoder to the inner surface of the groove which allowed for convection heat transfer to the helium coolant at an "ambient" temperature of 300 K.

The insulating disks on the ends of the slug used to model contact resistance between the slug and the holder were .01 cm in thickness. For an aluminum-aluminum interface with air as the interfacial fluid, a surface roughness of 10-µm, and an interface
pressure of $10^5$ Pa, the contact resistance is approximately $2.75 \times 10^{-4}$ m$^2$-K/W [4]. This value was used as a reasonable, conservative value for the resistance of an aluminum-molybdenum interface. The thermal conductivity given to the disks was therefore:

$$k = \frac{.0001}{2.75 \times 10^{-4}} = .364 \text{ W/m} \cdot \text{K} = .00364 \text{ W/cm} \cdot \text{K}$$

The following figures show the results of the construction of AS2D from AS2S by adding the three modifications described in the previous section.
Figure 4-3: AS2D Temperature Distribution

Figure 4-4: AS2D Heat Flux Distribution
The inclusion of the slug end resistance had the most dramatic effect on the results, raising the maximum temperature in the slug to 881 K, and lowering the maximum heat flux by 40%. The holder heating was added for completeness, but had little effect (~20K) on the overall results. The groove cooling also had a small effect (~20K), but it helps to maintain a more uniform and lower temperature in the hottest part of the holder (400 K); the aluminum closest to the slug ends. A significant amount of heat which before was deposited into this region was drawn out to the helium gas flowing in the grooves by convection.

The effect of the increased slug end resistance was that a larger temperature difference must exist between the slug end and the holder to drive the same amount of heat out of the slug as in the case without resistance. The result was a higher maximum temperature and a more uniform temperature distribution in the slug. In fact, since the melting point of molybdenum is three times that of aluminum, it is desirable to impose a higher heat burden on the slug than on the holders. This condition was actually provided by the large temperature drop across both the air gap and the slug end interface.

The proposed passive cooling system for the slug is promising since the temperature distribution is well within the melting temperature of both materials for the nominal 10 kW case with an electron sweep.
Chapter 5

Thermal Hydraulics in Porous Media

To evaluate the heat-removal capability of a PMHX converter plate, a computer model of the combined porous metal matrix and gas coolant was created using the basic thermal-hydraulic governing equations and empirical heat transfer coefficient correlations. The remaining sections outline the development of this model.

5.1 Fundamentals

There are a large number of studies in the literature which attempt to model thermal-hydraulics in porous media. These studies are either too rudimentary or too specialized for the present study. The current analysis was based on a two-dimensional explicit finite difference scheme which models the unsteady flow of a hot gas into a packed bed without assuming local thermal equilibrium [13]. This analysis provided the framework for the steady-state, one-dimensional model with heat generation that was appropriate for the current study.

The challenge is to begin with the basic governing equations for the system of interest, invoke all necessary assumptions, yet allow for a practical numerical solution. We begin with an analytical study of transient gas flow in porous media. The basic goal is to determine the pressure drop across the bed which results in an appreciable coolant mass flow rate and the temperature distributions of both the gas and solid phase. The
basic relations governing the system are the gas continuity equation, the gas momentum equation, the gas equation of state, and the first law of thermodynamics for both phases.

The continuity equation must by definition allow for the compressibility of the gas. By performing a mass balance on a differential volume element, we obtain:

\[
\varepsilon \frac{\partial \rho_g}{\partial t} + \nabla \cdot (\rho_g \mathbf{u}) = 0
\]  

(1)

where: 
- \( \varepsilon \) = bulk frit porosity
- \( \mathbf{u} \) = gas velocity (m/s)
- \( \rho_g \) = gas density (kg/m³)

The vapor phase momentum equation is a constitutive equation characteristic of flow in porous media, and is known as Darcy’s Law. This equation differs from the conventional fluid momentum equation in that it accounts only for microscopic viscous and inertial effects due to form and friction resistance of the porous matrix [13].

\[
\nabla P = -\mu \frac{\mathbf{u}}{K} - \rho_g \frac{F}{\sqrt{K}} \mathbf{u} \cdot |\mathbf{u}|
\]  

(2)

where: 
- \( K \) = permeability of porous matrix (cm²)
- \( P \) = gas pressure (Pa)
- \( F \) = empirical inertial correction term
- \( \mu \) = gas viscosity (Pa-s)

The non-linear term represents a correction to the basic form of Darcy's Law which is necessary to account for microscopic inertial effects which arise for higher Reynolds number flows. The flow regimes in porous media may be roughly identified by the following ranges of Reynolds numbers [7]:

- \( \text{Re} < 1 \): Darcy or creeping flow regime. Viscous effects dominate
- \( 10 < \text{Re} < 50 \): Inertial-flow regime. Steady nonlinear laminar flow as inertial force begins to dominate.
- \( 150 < \text{Re} < 300 \): Unsteady laminar-flow regime.
- \( 300 < \text{Re} \): Unsteady, turbulent-flow regime.
The Reynolds number is based on the average pore velocity, \( u \), and average pore length, \( d_p \):

\[
Re = \frac{\rho g |u| d_p}{\mu}
\]

The permeability is a measure of the flow conductance of the matrix and may be expressed empirically in terms of the porosity and average particle diameter [13].

\[
K = \frac{\varepsilon^3 d_p^2}{150(1 - \varepsilon)^2}
\] (3)

In addition, \( F \) has been empirically determined as [13]:

\[
F = \frac{1.75}{\sqrt{150\varepsilon^{1.5}}}
\] (4)

The gas density, temperature and pressure are related by the ideal gas equation of state.

\[
P = \rho_g RT_g
\] (5)

where: \( R = \) gas constant = 2077.03 J/kg-K (Helium)

The energy equations are also determined by volume-averaging over a differential element and consider heat storage terms, advection, conduction, and convection.

Gas:

\[
\varepsilon \rho_g C_v \frac{\partial T_g}{\partial t} + C_p g \rho_g \bar{u} \cdot \nabla T_g = \nabla \cdot \left( k_{\text{eff}} \cdot \nabla T_g \right) + hA_s \left( T_s - T_g \right)
\] (6)

Solid:

\[
(1 - \varepsilon) \rho_s C_p \frac{\partial T_s}{\partial t} = \nabla \cdot \left( k_{\text{eff}} \cdot \nabla T_s \right) - hA_s \left( T_s - T_g \right) + \dot{q}
\] (7)

where: \( h = \) local particle-to-coolant heat transfer coefficient

\( C_p = \) heat capacity

\( k_{\text{eff}} = \varepsilon \cdot k_g \)

\( k_{\text{eff}} = (1 - \varepsilon) \cdot k_s \)

\( \dot{q} = \) volumetric heat generation term
The interfacial surface area, \( A_s \), may be determined in terms of the average frit porosity and mean particle diameter [2]:

\[
A_s = \frac{6(1 - \varepsilon)}{d_p}
\]  

(8)

Empirical correlations for the interfacial heat transfer coefficient are available from several sources and depend on the coolant flow regime. The following correlation in the form of a Nusselt number is suggested for non-Darcian gas flow in a packed bed of sintered spheres [6]:

\[
Nu = 0.093 \cdot (Re)^{1.04} \cdot (Pr)^{0.33}
\]

(9)

where:

\[
h = \frac{Nu \cdot k_g}{d_p}
\]

and:

\( Pr = \) Prandtl number
\( k_g = \) fluid thermal conductivity

Equations (1), (2), (5), (6), and (7) are the basis for modeling the transient forced convection flow of a gas through a packed bed. These equations comprise a system of five highly-coupled nonlinear partial differential equations which describe the five variables of interest: \( P_g, u_g, \rho_g, T_g, \) and \( T_v \).

### 5.2 Solution Method

These solutions have been investigated analytically for the case of isothermal flow [11] and numerically for non-isothermal, local thermal non-equilibrium conditions by explicit
finite difference methods [13]. The usual assumptions which are invoked in these cases include:

- No natural convection or buoyancy effects
- No intra-particle radiation heat transfer
- Negligible boundary and entrance effects
- Uniform porosity, permeability

It was necessary in this study to invoke the above assumptions, but clearly isothermal flow and local thermal equilibrium are inappropriate assumptions since heat is generated in the solid. A frit model should also consider variable temperature effects in both the coolant and solid. The parameters of interest for an effective PMHX design include the pressure drop across the length of the packed bed and the temperature distributions in the gas and solid. The pressure drop basically determines the mass flow rate which in turn determines the amount of heat which may be effectively carried away by the coolant and its effectiveness in maintaining reasonable temperatures in the solid.

The coupled set of partial differential equations was transformed into ordinary differential equations by dropping all time-dependent terms and only considering a steady-state solution. The transient behavior occurs over a very short time interval whose modeling is unnecessary for this analysis [13]. Furthermore, only one spatial dimension was considered; the centerline of the frit's axial length. Solving this simplified system is still a challenge since the equations are still a highly-coupled stiff nonlinear set of ordinary differential equations which tend to be unstable in typical numerical schemes such as the Runge-Kutta technique. The stiff equation solver available with Mathcad Plus 6.0 was therefore used to solve the equations.
5.3 Mathcad Model

By considering conduction in each phase, the equations are second order in temperature and are first order in pressure and velocity. The differential equation solvers of Mathcad require that each variable be expressed as a function of its highest derivative; therefore, we expressed the system of equations as functions of \( \frac{dP}{dz}, \frac{du}{dz}, \frac{dT_g}{dz^2}, \) and \( \frac{d^2T_s}{dz^2} \) [10]. The density does not have to be solved for directly and was found using the calculated gas pressure and temperature. Substituting for the density in terms of the pressure and temperature and expanding the spatial derivative in (1) by the chain rule:

\[
\frac{dP}{dz} = -\frac{\mu}{K} u - \frac{P\varepsilon}{RT_g \sqrt{K}} u^2 \tag{10}
\]

\[
\frac{du}{dz} = -\frac{u}{P \cdot T_g} \left[ T_g \frac{dP}{dz} - P \frac{dT_g}{dz} \right] = \frac{u}{T_g} \frac{dT_g}{dz} + \mu \frac{u^2}{PK} + \varepsilon \frac{u^3}{RT_g \sqrt{K}} \tag{11}
\]

\[
\frac{d^2T_g}{dz^2} = \frac{\rho C_p u}{k_g} \frac{dT_g}{dz} - \frac{hA_s}{k_g} \left( T_s - T_g \right) \tag{12}
\]

\[
\frac{d^2T_s}{dz^2} = -\frac{q}{k_s} + \frac{hA_s}{k_s} \left( T_s - T_g \right) \tag{13}
\]

To add more stability to the system, we substituted for the solid temperature equation by introducing a new variable:

\[
\Delta T = T_s - T_g \tag{14}
\]

Differentiating twice with respect to \( z \):
\[ \frac{d^2(\Delta T)}{dz^2} = \frac{d^2T_s}{dz^2} - \frac{d^2T_g}{dz^2} \]  

(15)

Using equations (15), (12), and (13):

\[ \frac{d^2(\Delta T)}{dz^2} = -\frac{\dot{q}}{k_s} - \frac{\rho C_p u dT_g}{k_g dz} + (h A_s \Delta T) \left( \frac{1}{k_s} + \frac{1}{k_g} \right) \]  

(16)

(16) was now solved instead of (13).

To completely solve the system with Mathcad, we needed to know the inlet values for P, u, T_g, and ΔT. In addition we needed to specify the first derivatives at the inlet for T_g and ΔT. The only conditions that were known were those that may be physically controlled in practice, P_i, P_o, and T_{g_i}.

Since we needed inlet conditions, we used the two outlet conditions that were known to specify the inlet conditions. First, we replaced P_o with \( \dot{\alpha} \) which sets the mass flow rate. The corresponding outlet pressure may then be set to achieve the desired mass flow rate in practice. Furthermore, the outlet gas temperature may be determined by the observation that in the steady-state it must obey the energy balance.

\[ q = \dot{m} C_p (T_{g_0} - T_{g_i}) \]  

(17)

q is simply the power that is dumped into the converter plate which must completely enter the gas at equilibrium. Using the known frit length, and gas temperatures at the inlet and outlet, the linear gas temperature gradient was easily determined. We also adjusted the inlet value of \( d(\Delta T)/dz \) to match the value of \( T_{g_0} \) from the energy balance. The remaining boundary condition for ΔT was not known, so it was determined by engineering judgment and trial and error. The equilibrium value of ΔT should be positive and be relatively constant across the frit.
Appendix B contains the Mathcad model FRIT, where the temperature
dependence of all material parameters was considered. The one exception is the heat
capacity of helium which is practically constant with temperature. As a result, the
Reynolds and Prandtl numbers and heat transfer coefficient were also temperature
dependent. The frit geometry was taken directly from ASSEM, where the cross-sectional
area was only used in defining the power density. The variables of interest were solved
along the frit length by: (1) defining the power (q), (2) defining an inlet velocity (mass
flow rate), (3) adjusting the inlet ΔT gradient to match the outlet gas temperature, (4)
adjusting the inlet values of ΔT and d(ΔT)/dz such that ΔT is constant over the frit. Step
(4) involved a certain degree of guesswork, but was justified since the system of
equations is underdetermined.

5.3.1 Isothermal Flow Model

One case for which the boundary conditions were completely specified is the
isothermal flow case where the frit power is zero. In solving this case, we examined the
fluid mechanics of gas flow in the frit. The following figures illustrate the results of
FRIT for the isothermal flow of helium arbitrarily entering the frit at 10 atmospheres and
300 K. The inlet flow velocity is 9.787 m/s which results in a mass flow rate of 5 g/s.
Figure 5-1: Isothermal Gas Pressure Distribution in Frit

Figure 5-2: Isothermal Gas Velocity Distribution in Frit

Figure 5-3: Isothermal Gas Density Distribution in Frit
The pressure dropped to 8.5 atmospheres over the short length of the frit. This result is encouraging as far as heat transfer is concerned since high mass flow rates may be achieved by fixing a much lower exhaust pressure. The Reynolds number was 84, well below the turbulent regime.

5.3.2 Heated Frit Model

We next considered the case where heat was generated in the frit, according to the heat load imposed by a 30 kW beam which is the worst-case scenario. The concern was whether the solid temperature can be maintained well below the melting temperature of tungsten. MCNP predicted a frit power of 9750 W for 40 MeV electrons at 750 µA. It was found by trial-and-error that to achieve convergence for an inlet helium flow at 10 atmospheres and 5 g/s, the required inlet values of ΔT and d(ΔT)/dz were 24.0 K and -0.1 K/m, respectively.
Figure 5-5: Gas Pressure Distribution in Heated Frit

Figure 5-6: Gas Velocity Distribution in Heated Frit

Figure 5-7: Gas Temperature in Heated Frit
Figure 5-8: Gas Density Distribution in Heated Frit

Figure 5-9: Reynolds Number of Heated Gas Flow

Figure 5-10: Temperature-Dependent Heat Transfer Coefficient
There was a 3.5 atmosphere drop in the heated case. Also shown is the variation of the heat transfer coefficient, given in W/cm²-K, and the volume flow rate in L/min. The resulting temperature distribution in the solid was the same as the gas, but was shifted upwards by 24 K; therefore, the maximum temperature in the solid was 699 K. The Reynolds number was also low, which lends support to the form of the momentum equation we have used, and is an important concern for flow stability. Whether these results are realistic depends on the validity of the assumptions that were used to develop FRIT, especially if two-dimensional effects will be significant in reality.
5.4 Flow Experiments

To determine the level of reality in FRIT, according to the assumptions which have been invoked, two experiments will be independently conducted as benchmark studies. The actual setups for these experiments are still pending at the time of this writing, so only a qualitative description is offered herein.

The first experiment will provide realistic data to compare with the isothermal flow model, where helium gas will be forced through an unheated experimental frit by imposing a pressure differential across its length. Subsequent measurements of the entrance and exit flow rate, pressure, and temperature will serve to verify the predicted results from FRIT, independent of any temperature effects and associated assumptions.

A more involved heated flow experiment is also possible using electrical resistance heating of the frit. The amount of power deposited by an electric current into the frit depends on the available potential difference and the effective frit resistance. This resistance is governed by the effective frit resistivity and the frit dimensions. Of course, the resistivity of the porous matrix will be reduced in relation to that of solid tungsten. For the purposes of this experiment, the commercial frit dimensions, namely its length, will probably be too small to achieve an appreciable amount of heating; thus, the experimental frit must have a greater length on the order of 1 cm. Measurement of the flow conditions as before, and possibly of the frit surface temperatures with thermocouples, will give additional insight into the accuracy of FRIT.
Chapter 6

Conclusions

This study has proposed a preliminary design for a high-energy electron accelerator target appropriate for commercial radioisotope production by photon-induced reactions. The methodology was focused on a design for the production of Mo-99 by a photoneutronic reaction, but may be extended for other radioisotopes which may be created via an incident photon. Many advanced engineering issues were left for further investigation, such as structural mechanics, radiation shielding, implementation of magnetic fields, effects of accelerator duty cycles on beam power, coolant flow recirculation systems, and two-dimensional effects of flow in porous media which prompts the question of flow stability. These issues were deemed unlikely to substantially modify our conclusions.

A converter plate was necessary to optimize yields and minimize slug heating, and cooling of this component was provided by configuring the plate as a porous frit of sintered tungsten powder. The nuclear properties of the plate were unaffected by this modification, and the heat load imposed by the electron beam was effectively removed to a helium gas coolant. It was shown that the heat load imposed by a beam of 30 kW may be dissipated by a frit of 40% porosity cooled by a helium flow with inlet conditions of 10 atmospheres and 300 K. The maximum temperature in the tungsten was only about 700 K. The amount of heat which may be removed primarily depends on the coolant conditions, i.e. the inlet pressure and mass flow rates which may be achieved in practice.
A sweep of the electrons exiting the converter plate was the desired method for minimizing waste heating of the slug. The sweep prevents the deposition of about 250 W into the slug with only a small effect on the yield. The alternative to the sweep is an aluminum backing plate, but this method results in lower yields and increased slug heating, along with the added concern of cooling the extra plate.

The passive cooling of the slugs provided by externally-cooled aluminum holders for the nominal case was sufficient to maintain a low temperature in the slug relative to its melting temperature and about 40-50% of the melting temperature in the holders. Increasing the power beyond 10 kW may not be justified based on the uncertainty involved with the slug end resistance and the sensitivity of the results to its value. Nevertheless, the proposed holder assembly design was definitely capable of dissipating the slug heating for the nominal, high-yield case and satisfied the conditions of simplicity. If higher beam powers are desired, this is the system which must be reconsidered, and other designs are definitely possible which may be more effective in slug heat dissipation. The results of this study may clearly serve as a starting point for a more advanced design.

In this preliminary feasibility study, the following target assembly design has been shown to satisfy the most important engineering issues of product yields, energy deposition, and thermal-hydraulics.

1) Converter plate as primary beam target
   • Configure as a 40% porous sintered tungsten powder frit; 2cm dia.; .6 cm thick
   • Helium Coolant at 5-10 atmospheres, 300 K
2) Electron Sweep with Magnetic Field
3) Multiple stages of thin slugs encased in aluminum holders
   • Holders cooled at ends by gas/water flow
   • Cooled at center by helium gas at 5 atmospheres, 300-400 K

The temperatures in all materials and pressures of all gas coolants were well within engineering limits, thus the feasibility of the design has been demonstrated. Furthermore, the costs involved with the development and operation of the target will be small in comparison with the capital costs associated with a high-energy, high-current electron accelerator. This method for radioisotope production may create commercial quantities of radioisotopes, such as Tc-99m, with minimal high-level radioactive waste and with much lower capital costs than the fission production method, government subsidies notwithstanding. Future research activities will improve the credibility of the proposed design, including the described flow experiments and the actual setup and demonstration of a full-scale commercial setup.
Appendix A

ASSEM MCNP Model

Detailed Target Assembly

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| 2 | 0 | | -1 | 50 | -4 | 51 |
| 3 | 2 | -2.699 | 4 | -14 | -18 | 19 | -22 | 23 | 51 |
| 4 | 2 | -2.699 | 14 | -15 | -18 | 19 | -22 | 23 | 2 |
| 5 | 2 | -2.699 | 15 | -16 | -18 | 19 | -22 | 23 | 2 |
| 6 | 2 | -2.699 | 16 | -17 | -18 | 19 | -22 | 23 | 52 |
| 7 | 2 | -2.699 | 4 | -15 | 18 | -20 | -22 | 23 |
| 8 | 2 | -2.699 | 15 | -17 | 18 | -20 | -22 | 23 |
| 9 | 2 | -2.699 | 4 | -15 | -19 | 21 | -22 | 23 |
| 10 | 2 | -2.699 | 15 | -17 | 19 | 21 | -22 | 23 |
| 11 | 2 | -2.699 | 15 | -17 | 20 | -24 | -22 | 23 | 53 |
| 12 | 2 | -2.699 | 15 | -17 | 20 | -24 | -22 | 23 | 54 |
| 13 | 2 | -2.699 | 4 | -15 | -21 | 25 | -22 | 23 | 55 |
| 14 | 2 | -2.699 | 15 | -17 | -21 | 25 | -22 | 23 | 56 |
| 15 | 3 | -10.2 | | 14 | -16 | -2 |
| 16 | 0 | 1 | 3 | -4 | -24 | 25 | -22 | 23 | 51 |
| 17 | 0 | | -3 | -22 | 23 |
| 18 | 0 | 24 | 3 | -17 | -22 | 23 |
| 19 | 0 | -25 | 3 | -17 | -22 | 23 |
| 20 | 0 | | 22 |
| 21 | 0 | | -23 |
| 22 | 0 | 17 | 52 | -22 | 23 |
| 23 | 0 | | -53 | -22 | 23 |
| 24 | 0 | | -54 | -22 | 23 |
| 25 | 0 | | -55 | -22 | 23 |
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| 28 | 0 | | -52 | -22 | 23 |

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</tr>
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<td>pz</td>
<td>0.000</td>
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52  c/x  0.000 3.483 .100
53  c/x  3.850 2.433 .250
54  c/x  3.850 3.133 .250
55  c/x -3.850 2.433 .250
56  c/x -3.850 3.133 .250
4   pz  2.083
14  pz  2.283
15  pz  2.783
16  pz  3.283
17  pz  3.483
18  py  1.500
19  py  -1.500
20  py  3.500
21  py  -3.500
22  px  1.500
23  px  -1.500
24  py  4.200
25  py  -4.200

c  Data
mode  p e
imp:p  1 14r 0 6r 1 5r
imp:e  1 1 12r 0 6r 1 5r  $ change 2nd 1 to 0 for high yield
cut:p  j .5
cut:e  j .5
sdef  vec 0 0 1 dir=1 pos 0 0 0 erg=40 par=3

c  Tallies
*f8:p,e 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 t
f4:p  15
e4   8 11i 20
em4  0 15 25 25 58 97 135 166 128 85 50 12 0
m1   74000  -1
m2   13000  -1
m3   42000  -1
nps  250000
ctme  2000
print

78
Appendix B

FRIT Mathcad Model

1.0 Definition of all empirical relations and constants.

\[ e := 0.40 \quad \text{Porosity} \]
\[ L := 0.00583 \quad \text{Frit Length (m)} \]
\[ dp := 105 \times 10^{-6} \quad \text{Mean Particle Diameter (m)} \]
\[ \text{Area} := \frac{3.14159.02^2}{4} \quad \text{Frit Area (m}^2\text{)} \]
\[ K := \frac{e^3 dp^2}{150 (1 - e)^2} \quad \text{Permeability (m}^2\text{)} \]
\[ R := 2077.03 \quad \text{Gas Constant (J/kg-K)} \]
\[ F := \frac{1.75}{\sqrt{150 e^{1.5}}} \quad \text{Inertial Correction Coefficient} \]
\[ q := \frac{0}{\text{Area}} \quad \text{Linear Power Density (0 W or 9500 W)} \]
\[ \text{Pin} := 10 - 101325 \quad \text{Inlet Pressure (Pa)} \]
\[ \text{Tin} := 300 \quad \text{Inlet Temp (K)} \]
\[ \text{As} := \frac{6 (1 - e)}{dp} \quad \text{Specific Surface Area (m}^2/\text{m}^3\text{)} \]
\[ \text{uin} := 9.787 \quad \text{Inlet Velocity (m/s)} \]
\[ \text{Cp} := 5193 \quad \text{Heat Capacity (J/kg-K)} \]
\[
\mu(T) := 199 \times 10^{-7} \left( \frac{T}{300} \right)^{6.5464}
\]
Viscosity (Pa-s)

\[
k_g(T) := 0.153 \left( \frac{T}{300} \right)^{0.7024} e
\]
Effective Gas Thermal Conductivity (W/m-K)

\[
k_s(T) := 186.27 \left( \frac{T}{200} \right)^{-0.2736} (1 - e)
\]
Effective Solid Thermal Conductivity (W/m-K)

The temperature dependence for the viscosity and thermal conductivities is taken from data from [4]. The data was fitted using the power trendline in Excel.

\[
Re(P, u, T) := \frac{P \cdot u \cdot dp}{R \cdot T \cdot \mu(T)}
\]
Reynolds Number

\[
h(P, u, T) := 0.93 \left( \frac{P \cdot u \cdot dp}{R \cdot T \cdot \mu(T)} \right)^{1.04} \left( \frac{C_p \cdot \mu(T)}{kg(T)} \right)^{33} \frac{kg(T)}{dp}
\]
Heat Transfer Coefficient (W/m²-k)

\[
\text{mdot} := \frac{P \cdot u \cdot \text{Area} \cdot 1000}{R \cdot \text{Tin}}
\]
Mass-Flow (g/s)

\[
\text{To} := \frac{q \cdot \text{Area} \cdot 1000}{\text{mdot} \cdot C_p} + \text{Tin}
\]
Outlet gas temperature based on energy balance

\[
\text{grad} := \frac{\text{To} - \text{Tin}}{L}
\]
Gas Temperature Gradient (K/m)

\[
A(T) := \frac{\mu(T)}{K}
\]

\[
B := \frac{F \cdot e}{R \cdot V \cdot K}
\]

\[
C := \frac{q \cdot R}{L \cdot C_p}
\]

Constants
2.0 Define the matrix containing the first and second derivatives.

$$D(x,y) := -A(y_2) y_1 - \frac{y_0 y_1^2}{y_2} B$$

$$y_1 y_3 + \frac{A(y_2) y_1^2}{y_2} + \frac{B(y_1)^3}{(y_2)^2}$$

$$y_3$$

$$y_0 y_1 y_3 C_p - h(y_0, y_1, y_2) A_s y_4$$

$$y_2 k g(y_2) R$$

$$y_5$$

$$h(y_0, y_1, y_2) A_s y_4 \left( \frac{1}{y_2} + \frac{1}{k g(y_2)} \right) - \frac{q}{k s(y_2) L} - \frac{y_0 y_1 y_3 C_p}{y_2 k g(y_2) R}$$

3.0 Define the Jacobian Matrix used by the stiff equation solver.

$$J(x,y) := \begin{bmatrix}
0 & -A(y_2) - \frac{y_0 y_1^2}{y_2} B & \frac{y_0 y_1^2}{(y_2)^2} & 0 & 0 & 0 \\
0 & -A(y_2) y_3 - \frac{A(y_2) y_1}{y_2} + \frac{B y_0 (y_1)^2}{(y_2)^3} & y_1 & 0 & 0 & 0 \\
0 & y_1 y_3 C_p - \frac{y_0 y_2 C_p}{y_2 k g(y_2) R} & -C_p y_0 y_1 y_3 & -h(y_0, y_1, y_2) A_s y_4 & \frac{y_0 y_1 C_p}{k g(y_2) R} & 0 \\
0 & y_0 y_3 C_p - \frac{y_0 y_2 C_p}{y_2 k g(y_2) R} & \frac{y_0 y_1 y_3 C_p}{k g(y_2) R} & \frac{y_0 y_1 C_p}{y_2 k g(y_2) R} & \frac{h(y_0, y_1, y_2) A_s y_4}{k g(y_2)} & y_2 k g(y_2) R \\
0 & \frac{y_0 y_3 C_p}{y_2 k g(y_2) R} & \frac{y_0 y_1 y_3 C_p}{k g(y_2) R} & \frac{y_0 y_1 C_p}{y_2 k g(y_2) R} & \frac{h(y_0, y_1, y_2) A_s y_4}{k g(y_2)} & y_2 k g(y_2) R \\
0 & \frac{y_0 y_3 C_p}{y_2 k g(y_2) R} & \frac{y_0 y_1 y_3 C_p}{k g(y_2) R} & \frac{y_0 y_1 C_p}{y_2 k g(y_2) R} & \frac{h(y_0, y_1, y_2) A_s y_4}{k g(y_2)} & y_2 k g(y_2) R \\
\end{bmatrix}$$

4.0 Define Inlet Conditions

$$\begin{bmatrix}
Pin \\
\text{uin} \\
Tin \\
\text{grad} \\
0.0 \\
0.0
\end{bmatrix}$$

Inlet Vector for Isothermal Case
Inlet Vector for $q = 9500$ W

$$\begin{bmatrix}
\text{Pin} \\
\text{uin} \\
\text{Tin} \\
\text{grad} \\
24.0 \\
-0.1
\end{bmatrix}$$

5.0 Solve

$$Z := \text{stifb}(y, 0, L, 1, D, J, 100, .1)$$

$$n := 0..100$$

Plots in Chapter 5
References


