Secondary Flow, Total Pressure Loss and the Effect of Circumferential Distortions in Axial Turbine Cascades

by

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Abstract

An investigation of secondary flow formation and the generation of total pressure loss in turbine cascades is presented. Part I presents an investigation into the effects of circumferential distortions on turbine rotor secondary flow structure. Experimental observations of time dependent rotor exit flowfields are presented, showing cyclic variations in rotor secondary flow structure. These variations occur at the vane passing frequency, which indicates that some vane-blade interaction is responsible. Two possible interactions are described, that of a quasi-steady wake interaction, and that of an unsteady interaction due to the relative motion of incoming vane wakes. A computational investigation based on a multi-domain spectral simulation of the three-dimensional viscous incompressible laminar flow through a planar turbine cascade is used to show that the quasi-steady interaction can not be responsible for the observed variations. It is thus deduced that the unsteady interaction must be the source of the time dependent changes in rotor secondary flow structure.

Part II describes the investigation of the generation of total pressure loss in turbine cascade flow. The simulated total pressure field from Part I is analyzed to show that the use of conventional techniques for classifying loss contributions can lead to an underprediction of the losses due to secondary flow within the cascade passage. This underprediction of the secondary losses causes difficulty in determining the source of the increase in loss due to secondary flow. To avoid this difficulty, an expression is derived that relates the increase in loss in the cascade passage to the distribution of the vorticity, expressed in intrinsic coordinates. This allows the generation of secondary flow loss to be related to the presence of streamwise vorticity in the cascade passage. Computational results confirm that there is a positive loss associated with the presence of streamwise vorticity in the simulated flowfield.

Thesis Supervisor: Dr. C.S. Tan
Title: Principal Research Engineer
Acknowledgments

Quos deus vult perdere prius dementat.

In the pursuit of the Doctorate, one has the chance to interact with many individuals. I would like to take this opportunity to express my thanks to those individuals with whom the interaction was a positive one.

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Je me souviens.
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Chapter 1

Introduction and History

1.1 Background

To achieve high power densities and minimize part counts, modern gas turbines are designed with low aspect ratios, high blade loadings, and small axial gap to chord ratios [41]. This can result in the development of strong secondary flows and an associated drop in stage efficiency. Detailed experimental studies in stationary and rotating cascades have shown that the secondary vorticity in blade passages is concentrated in discrete vortex cores, which are not unlike the horseshoe vortices formed around cylinders immersed in endwall shear layers [23]. This concentrated vortical region can result in the presence of high shear and hence high total pressure loss. In addition, since the secondary vorticity forms in a discrete core, large local variations in flow angle occur near the location of the vortex core. For this reason, analytic methods based on linearized theory or axisymmetric approximations inadequately predict the spatial variations in the passage flow. While such methods do yield satisfactory estimates of the integrated value of the secondary circulation and average flow angle, they do not give accurate values for the total pressure loss due to secondary flow [37]. Accurate predictions of secondary flow loss must then make use of either detailed descriptions of the passage flowfield, or must be based on empirical methods. Most turbine design systems currently make use of such an integrated or axisymmetric-inlet method for gas angle prediction, with the addition of a semi-empirical correlation for the resulting loss.
Such a system generally yields satisfactory results for time averaged flows.

Since the generation of secondary flow can be attributed to the tilting and stretching of vorticity due to flow turning, prediction techniques can be based on a kinematic formulation of generation of streamwise vorticity [15]. Examination of such relations suggests that gradients in the pitchwise direction have little influence on the generation of secondary flow, since spanwise vortex filaments associated with such gradients are unaffected by the flow turning. For this reason, pitchwise gradients are usually averaged out, yielding a steady axisymmetric inflow condition for the cascade flow. As the unsteadiness due to the relative motion of the incoming vane wakes has been averaged out, such methods cannot be expected to predict the effects of this unsteadiness on secondary flow generation nor the resulting loss. Until recently, due to a lack of evidence indicating otherwise, such effects were assumed to be small.

Recently reported experimental evidence [40] indicates that the secondary flow vortices in multi-row turbine stages are neither uniform nor steady in time. Spatially detailed, high response measurements of turbine rotor exit flowfields have shown that the secondary flow vortices periodically change in strength, size and location, at the vane passing frequency. Analysis of the total pressure loss within rotor blade passages indicates that the level of loss associated with the secondary flow vortices also varies periodically, in phase with the variation of the vortices. Since these periodic changes occur at the vane passing frequency, some vane-rotor interaction must be responsible. Several possible interactions could be responsible for the observed variations. These include:

1. the effect of the radial component of vorticity in the incoming wake, which is related to the deficit in total velocity in the vane viscous wake region,
2. concentrated incoming axial vorticity from the vane secondary flow vortices,

3. variations in the distributed axial vorticity due to the relative skew of the incoming endwall boundary layer,

4. relative velocity of the vane wake fluid toward the rotor suction surface, and,

5. variations in the rotor loading due to vane-rotor potential interaction.

At the time the phenomena were first observed, it was not possible to rule out any of the above listed possible interactions as the cause of the measured variations.

1.2 Thesis Objectives

This thesis presents a study of the formation of the secondary flow vortex system and the source of the observed variations in secondary vortex structure. The objectives of the investigation are as follows.

1. Investigate the detailed formation of the discrete secondary flow vortex system, so that the source of the vortical structures that have been observed at the exits of turbine cascade passages, and their relationship to the horseshoe vortex structure formed at the blade leading edge, can be appropriately determined.

2. Identify the source of the variations in size and strength of secondary flow vortices observed in rotating turbine stages and research cascades, and attempt to explain how that effect results in the observed changes on vortex structure.

3. Investigate the relationship between the formation of the secondary flow vortex system and the measured increase in total pressure loss caused by the presence
of secondary flow, and to show the significance of streamwise vorticity in the generation of total pressure loss.

1.3 Organization

The thesis is divided into two parts. Part I details the investigation of the source of the observed variations in the secondary flow vortex system. Part II investigates the relationship between the loss of total pressure in the blade passage and the secondary flow formation in that passage.

The first chapter in Part I contains a description of the experimental evidence for the variation in the rotor passage secondary flow, and includes a discussion of the possible mechanisms that could cause such a variation. The next chapter begins with a brief review of classical secondary flow theory. This is then followed by a discussion on the difficulties of using this classical theory for examining the observed variations in secondary flow. This material is presented to provide the background and motivation for the current study.

The remainder of the thesis presents the methodology and results of the study of secondary flow and loss. A modified secondary flow theory, based on the kinematics of deformation of the incoming filament vorticity is proposed. It is then shown how this hypothetical flow model could possibly explain the cause of the observed variation in secondary flow. Arguments are then put forward as to what should constitute a suitable method for evaluating the validity of the hypothesized flow model, from which the choice of a numerical technique follows. Following this is a chapter describing the procedure, developed for this investigation, for simulating the flowfield. We also present comparisons of simulated flowfields with known analytic and experimental results to
demonstrate the validity of the method. The final chapter in Part I describes the results of the computational investigation into a possible source of the observed variations, and hence the validity of the hypothetical flow model presented in Chapter 3.

The development of the numeric technique outlined in Part I yielded a computational tool ideally suited for answering basic questions on the sources of loss in three dimensional viscous flow through turbine blade passages. The second part of the thesis presents a computational study of the change in total pressure loss through the blade passage, in light of the development of the secondary vorticity. The first chapter defines loss generation in turbine cascades, and describes accepted techniques for assigning portions of the gross loss to various sources in the flow. An explanation is then presented to show how such a classification may lead to difficulties in the classification of the sources of the increase in loss due to secondary flow. Such a difficulty may be overcome by characterizing the change in total pressure along a streamline in terms of the vorticity field defined in an intrinsic coordinate system (streamwise, normal and binormal directions). An attempt is made to explain the physical significance of each of the terms in the resulting equation, which are shown to consist of streamwise vorticity, and a combination of normal and binormal vorticity gradients. Suggestions for future work are given at the end of the thesis.
Part I

Effect of Inlet Distortion on Turbine Secondary Flow
Chapter 2
Experimental Observation of Temporal Variations in Secondary Flow

2.1 Background

Recent experiments [40] indicate that the structure of the rotor passage secondary flow vortices in axial flow turbine stages undergo periodic temporal variations driven by the passing of the upstream vanes. This chapter presents key results from these experiments, showing the variation in the secondary flow pattern. Comparisons of the results from different experiments are used to indicate the likely cause or source of the changes in the vortical structure.

2.2 UTRC Large Scale Rotating Turbine

The first experiments to indicate the presence of temporal variations in rotor secondary flows were conducted at the United Technologies Research Center (UTRC) in their large scale rotating turbine rig (LSRR). Results from this experiment have been analyzed and presented in [40], [18] and [41]. We shall give a brief review of the work in [40] to motivate the undertaking of the present investigation.
2.2.1 Experimental Apparatus and Flow Parameters

The experiment was conducted in a large scale rotating turbine rig at the United Technologies Research Center. The LSRR is a one and one half stage axial flow air turbine, having 22 first vanes, 28 first blades, and 28 second vanes, each with a 0.8 hub to tip ratio and a unit aspect ratio; representative of modern, high pressure turbine stage geometries. The blade chords average approximately 6 inches, a value roughly five times that of true engine scale. Table 2.1 summarizes the airfoil geometry and the nominal operating conditions for the experiment. Figure 2.1 shows a through flow diagram of the apparatus. High frequency data consisting of the instantaneous velocity components, total pressure and total temperature were acquired at the four axial locations shown. As the experiment was implemented to study the unsteadiness due to the relative motion between the vanes and the blades, the data was averaged using a phase lock average (PLA) technique[18] to separate the periodic and random velocity components. The resulting data set thus consists of a spatially resolved, time dependent flowfield definition at each of the axial locations indicated in Figure 2.1. These flowfields could then be analyzed to determine the effect of vane-blade interactions on the evolution of the flowfield through the turbine stage. The results of this analysis are examined in the following section, with emphasis placed on the examination of the time dependent variation of the rotor exit flowfield.

2.2.2 Rotor Exit Flowfield and Unsteady Secondary Flow Structure

Examination of the time-dependent flowfield downstream of the turbine rotor indicates that the structure of the secondary flow changes periodically as the rotor passages move relative to the upstream vanes. Pertinent data from these experiments are pre-
Throughflow velocity, \( C_x \) = 19 m/s (75 ft/s)
Inlet total pressure = ambient, 1 atm
Inlet total temperature = ambient, -294 K (530°R)
Flow coefficient, \( C_x/U_y \) = 0.78

<table>
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<th>Guide vane</th>
<th>Rotor</th>
<th>Second vane</th>
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<td>Axial chord, ( b_x )</td>
<td>0.151 m (5.93 in.)</td>
<td>0.161 m (6.34 in.)</td>
<td>0.164 m (6.45 in.)</td>
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<td>Number of airfoils</td>
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<td>28</td>
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<tr>
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<td>0.930</td>
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<td>Interairfoil gap</td>
<td>0.65(( b_x ))</td>
<td></td>
<td>0.56(( b_x ))</td>
</tr>
<tr>
<td>Tip clearance</td>
<td>0.01(( b_x ))</td>
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<td>Exit velocity, absolute frame</td>
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<td></td>
<td>(210 ft/s)</td>
<td>(112 ft/s)</td>
<td>(165 ft/s)</td>
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<tr>
<td>Exit ( Re ) No./in</td>
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<td>1.04 \times 10^5</td>
<td>0.86 \times 10^5</td>
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Table 2.1: Aerodynamic and geometric parameters for LSRR. (from [40])

sented to elucidate this time-dependent variation, and to highlight those aspects of the variation involving the secondary flow structure.

The plane of measurement at the rotor exit covers a circumferential distance of 2.55 rotor blade pitches, a distance corresponding to two upstream vane pitches, which can be inferred from the ratio of the number of vanes to the number of rotor blades.

Regions of low total pressure are associated with the vortical structures at the rotor exit. It is therefore appropriate to examine the measured total pressure distribution at the rotor exit to deduce the structure of the secondary flow vortices within the rotor passages. Countour plots total pressure are presented to illustrate the variations in the rotor secondary flow as the rotor moves relative to the upstream vanes. The data are presented in the absolute reference frame, with the rotor moving from left to right, in the clockwise direction, as shown in Figure 2.2. Arrows at the top of each plot show the projection of the rotor blade trailing edges, with the blade suction surface being to the right and pressure surface being to the left of each arrow. We first examine the flow in the passage between the airfoil trailing edges marked 1 and 2 to show the structure of
the secondary flow.

Figure 2.2 shows contour plots of the total pressure coefficient at the rotor exit plane. The total pressure coefficient has been computed according to

\[ C_{PTU} = \frac{(P_{T2} - P_T)}{\frac{1}{2} U_m^2} \]  

where \( U_M \) is the midspan rotor wheel speed. With a flow coefficient of \( \phi = 0.78 \), a loss of one upstream axial dynamic head would be indicated by a total pressure loss coefficient of \( C_{PTU} = 0.6084 \). The figure shows three distinct circular regions of low total pressure near the rotor suction surface, two in the mid-span region of the blade, and one partially visible near the tip. Although not presented here, yaw and pitch data[40] do indeed confirm the existence of two counter-rotating vortices coinciding with the nearly circular regions of low total pressure near midspan. The vortical region near the rotor tip (on suction surface) is most likely the blade tip leakage vortex. As we are concerned here with the secondary vortical regions other than that due to tip leakage, we shall not devote further attention to the tip vortical region. The region of low total pressure near the blade midspan trailing edge is associated with the migration of the blade suction surface boundary layer to midspan, thus although a result of the secondary flow, this region should not be viewed as an organized vortex. The mid-passage region between rotor airfoils 1 and 2 shows a higher and nearly uniform absolute total pressure suggesting the presence of irrotational flow in this region. Based on this observation it may be deduced that any upstream vane wake fluid is confined to the boundary layer on the blade suction surface.

Figure 2.3 shows the contours of absolute total pressure coefficient corresponding to an instant one half cycle later. The passage between blades 1 and 2 has thus moved a distance of approximately four tenths of a vane pitch from its location in the previous figure. The flow in the midgap region is very different from that seen in Figure 2.2. The
midgap flow now has gradients in total pressure in the midgap region that are nearly equal to those in the blade wake, suggesting the existence of vortical motion in the passage center. The well defined secondary flow vortex at the root is absent, having been replaced by a more distributed region of low total pressure near the hub endwall. The tip secondary flow vortex, while still in evidence, is weaker, while the region of low pressure fluid at the wake centerline appears to have intensified. Yaw and pitch data indicate that this midgap loss fluid has little axial vorticity associated with it, indicating that it may be due to either a blade boundary layer separation or an accumulation of the upstream vane wake fluid. The passage to the immediate left, between blades 2 and 3, represents the flow three quarters of the way through the vane passing cycle. The tip secondary flow vortex has increased in strength while the gradients in the midgap region are diminished. The low total pressure region previously visible at the root section is now absent, suggesting an absence or breakdown of the root secondary flow vortex.

2.2.3 Rotor Inlet Flowfield

As the variation in the rotor secondary flow pattern is observed to occur at a frequency equal to that of the passing of the upstream vanes, it may be inferred that the cause of this effect is vane-blade interaction. Thus it is instructive to first examine the flowfield at the exit of the first vane row, and to inquire whether such a flowfield, when presented to the rotor inlet, could result in the observed variation in the secondary flow pattern.

Temporal variation in the rotor inlet flowfield may be responsible for the observed unsteadiness at the rotor exit. This rotor inlet flowfield unsteadiness can have as its source the time dependent flow in the vane, or in the relative motion of an essentially steady vane flowfield. Examination of the magnitudes of each of the possible sources of
rotor inflow unsteadiness should indicate the relative importance of each to the vane-blade interaction alluded to above.

Unsteadiness in the flowfield can be divided into two components, one being a periodic unsteady value that varies at some multiple of the rotor rotational frequency, and the other component that is aperiodic in rotor passing frequencies, or truly random. We will refer to the former as the deterministic unsteadiness, while the later will be referred to as turbulence, even though they may include organized periodic effects not linked to rotor passing, such as trailing edge vortex shedding. Both of these sources of unsteadiness in the vane exit flowfield may affect the flow in the downstream rotor passage. However, the data indicates that it is the deterministic unsteadiness that has a significant influence, and could be the cause of the variation in the rotor passage flowfield.

Deterministic unsteadiness at the rotor inflow consists of the deterministic unsteadiness in the vane exit flowfield, and that due to the relative motion of the circumferentially non-uniform vane exit flowfield. A thorough examination of the experimental data was used to determine the relative importance of these two types of deterministic unsteadiness.

The time-dependent vane exit flowfield data showed that the magnitude of the deterministic unsteadiness varied across the vane pitch, reaching a maximum value of 2% in the vane wake regions, and a minimum of less than 0.5% in the mid-passage region. However, the data shows that the unsteadiness associated with the relative motion is far more significant. In the vane wake regions, the circumferential variations in the flowfield are of the same order as the time-mean values. The relative motion of these distortions will thus produce a deterministic unsteadiness having a magnitude of the same order
as the time mean flow quantities. As this unsteadiness is an order of magnitude larger than that associated with the unsteady flow in vane exit flowfield, the unsteadiness in the vane exit flowfield can be considered as negligibly small when compared with the unsteadiness due to the motion of the rotor passages through a circumferentially steady nonuniform vane exit flowfield. Thus the vane flowfield may be treated as a steady flow, and can be examined to determine which flow structures are the possible sources of the rotor secondary flow temporal variation.

The time average of the vane exit flow at axial location 1 (Fig 2.1) is shown in Figure 2.4. More specifically, the figure shows contours of the time averaged total pressure loss coefficient, over a region of the flow extending from 8% to 90% span and over a circumferential distance equal to two vane pitches. The two first vane wakes can be identified as regions of low total pressure. The vane wakes (and hence the vane surface boundary layers) are seen to be almost invariant in the spanwise direction over a region extending from 20% to 60% span. The regions of higher loss near the hub and tip can be identified as being associated with the vane endwall secondary flow vortices. The vane endwall boundary layer is not visible in the figure; one may conclude that the thickness of the boundary layer on that endwall at vane exit cannot be more than than 8% span. Thus, the vane exit flowfield can be described in terms of a viscous wake due to nearly uniform vane surface boundary layers, small organized secondary flow vortices near the hub and tip, and at most a thin vane endwall boundary layer region. These flow non-uniformities, in combination with the relative motion of the blade rows provide the means for the generation of the responsible mechanism for the observed secondary flow variation at the rotor exit. Unfortunately, the data set is not exhaustive enough that it alone can be used to determine the sequence of events leading to the observed temporal variation. It is therefore appropriate to examine other available experimental work in an effort to enhance our understanding of the variations in secondary flow structure and
the possible sources of the observed flowfield interaction.

2.3 Purdue University Planar Water Tunnel

We now describe the results of an experimental investigation conducted at Purdue University that may clarify the cause of the observed flow variations. Herbert and Tiederman [17] conducted a study in which straight rods were traversed upstream of a planar cascade in a water tunnel, with the rods mounted through a slot in the endwall of the cascade. Downstream LDV data were obtained at three axial planes to allow evaluation of all three velocity and vorticity components at the cascade exit. Vortex variations observed in this experiment were very similar to those reported by Sharma et. al. [37], with the variations occurring at a frequency equal to the passing of the upstream rods. This experiment used a flow distortion upstream of the cascade different from that in [37], yet it reproduced the same variations in the secondary flow pattern as were observed in the UTRC experiments. Thus it can be deduced that the distortions responsible for the flow variation in both experiments must bear a strong similarity. As the distortions at the cascade inflow were generated using small unloaded circular rods, potential interactions with the blade row can be considered to be negligible. Furthermore, the use of unloaded circular bars eliminated flow distortions associated with secondary flows (which are present in the UTRC experiment). This experiment implies that it is the interaction associated with the incoming viscous wake and/or the relative motion of that upstream flow distortion with respect to the downstream blade row that results in the observed temporal variation in the secondary flow pattern at the blade passage exit.
2.3.1 Summary of Experimental Observations

To summarize, the strength and structure of the rotor passage secondary flow exhibits periodic variations as the rotor passage moves through the vane flowfield. This variation can be best described by following the sequence of changes in the flow pattern. Both the hub and tip secondary flow vortices first decrease in strength, with the larger decrease occurring at the hub vortex. While the strength of the hub secondary vortex decreases, a region of distributed low total pressure appears at the endwall, replacing the hub vortex. This low total pressure region then convects out of the blade passage while the tip secondary flow vortex regains strength. Finally, the hub secondary flow vortex reappears with increasing strength, until it again forms an organized uniform vortex structure similar to the tip secondary vortex. The entire sequence of events is observed to repeat at a frequency corresponding to the upstream wake passing frequency.

From the above results and discussion, it is evident that the structure and strength of the rotor secondary flow, particularly at the hub, are undergoing periodic variations. Since this variation occurs at the vane passing frequency, it can be deduced that vane-blade interaction can be responsible. The experiments reported have identified two possible interactions as the likely source of the observed variation. The first is the presence of the upstream viscous wake, and the effect of the associated velocity deficit as it interacts with the cascade flowfield. The second possible interaction is the relative motion of the upstream wake relative to the cascade blade row. Since each of the experiments reported involved both effects, these investigations could not conclusively identify which effect is responsible for the variations.
2.4 Possible Sources of Wake Generated Interaction

Recapping the essence of the experimental results from the previous sections, we have: (1) the rotor secondary flow vortex system undergoes periodic variations in phase with the upstream vane passing cycle, (2) this variation is generated by the interaction of the secondary flow vortex formation and the circumferential distortion associated with the upstream wake structure, (3) the variation alluded to above was observed to occur in two experimental investigations, even though the upstream flow distortions were generated differently in each experiment, and (4) it can thus be deduced that the cause for the observed event has to be similar in both experiments. From these we may identify two possible interactions as the likely cause of the observed variation in secondary flow pattern. We will next proceed to describe these two possible interaction and to differentiate the differences between them, involving the relative motion between the incoming wake and the blade passage.

2.4.1 Deficit in Velocity Magnitude - Quasi-Steady Interaction

The first interaction mechanism that is present in both experimental investigations described above is the effect of the deficit in velocity in the incoming vane or rod wake. This deficit in velocity results in a circumferential variation in both the magnitude of the velocity and the related total pressure distribution at the cascade passage inlet. The relative motion between the cascade blades and the vanes (or rods) generating the distortions means that the position of the incoming distortions relative to a given blade passage will vary in time. If this circumferential distortion affects the formation of the vortex, the relative motion may result in a temporal variation in the vortex structure as the distortion moves relative to the cascade inflow. As this interaction does not
change character with increasing reduced frequency, this mechanism can be considered as quasi-steady.

2.4.2 Variation in Wake Flow Angle - Unsteady Interaction

The second possible interaction mechanism that is present in both experiments is the variation in relative flow angle in the wake fluid due to the relative motion of the cascade and the upstream vane (or rod) generating that distortion. Figure 2.5 shows the velocity triangles in the wake flow region and describes the transformation from the vane or rod frame of reference to the frame of reference of the cascade blade passage. The relative motion of the blades and the upstream vanes affects the structure of the wake in the frame relative to the blade passage [19]. The vane wake fluid has a lower velocity in the vane relative frame than that of the free stream flow. The vector addition of the relative rotor velocity to the free stream fluid and to the wake flow alters the angle and magnitude of the wake fluid relative to that of the mean flow. In the blade passage frame of reference, the wake fluid thus has both an increase in velocity magnitude and skew angle with respect to the mean flow. This additional pitchwise velocity of the wake fluid causes a migration of the vane wake fluid toward the suction surface of the rotor passage. The vane wake fluid may thus impinge on the blade suction surface and flow toward the endwalls, influencing the formation of the secondary vortex system. As the relative flow angle between the wake and the undistorted flow depends on the velocity between the blade rows and hence on the reduced frequency of the interaction, this mechanism can be considered as unsteady.

Since both of the above listed phenomena were present in the experiments described, further investigation is required to identify which of these two interaction mechanisms is responsible for the observed variation in secondary flow vortex structure.
Figure 2.1: UTRC Large Scale Rotating Rig. (from [40])
Figure 2.2: UTRC LSRR rotor exit absolute total pressure coefficient and schematic flow interpretation, initial time (from [40])
Figure 2.3: UTRC LSRR rotor exit absolute total pressure coefficient and schematic flow interpretation, initial plus one half cycle (from [40])
Figure 2.4: UTRC LSRR first vane exit, time-averaged total pressure loss coefficient (from [40])
Figure 2.5: Velocity triangles defining transformation of vane relative to rotor-relative frame of reference.
Chapter 3
Hypothesized Model and Method of Investigation

This chapter first gives a brief review of classical secondary flow theory, then demonstrates how such a theory may not be able to provide an adequate explanation of the observed flow changes due to vane-blade interaction, as alluded to in the last chapter. It then introduces a modified secondary flow model based on the convection of incoming filament vorticity; a model that entails the inclusion of the effect of the quasi-steady vane-blade interaction, also described in the last chapter. It then shows how this model can possibly account for the observed variation in the rotor secondary flow. Finally a method is proposed for investigating the validity of applying the hypothesized model to explain the link between the observed changes and the vane-blade interaction.

3.1 Generation of Secondary Vorticity

The presence of upstream boundary layers on the cascade endwalls leads to the development of secondary flows in blade passages. As was first explained by Hawthorne [16] for flow in curved channels, and later for turbomachinery flows by Squire and Winter [44], these endwall boundary layers lead to the generation of streamwise vorticity in the blade passage, and variations from the desired flow angles. Following the analysis of Hawthorne, one can derive an expression for the growth of streamwise vorticity along a
streamline using the Helmholtz vorticity equation for a steady incompressible flow,

\[ (\mathbf{u} \cdot \nabla) \mathbf{\omega} = (\mathbf{\omega} \cdot \nabla) \mathbf{u} - \mathbf{\omega} (\nabla \cdot \mathbf{u}) \quad (3.1) \]

with \( \mathbf{u} \) as the velocity and \( \mathbf{\omega} \) the vorticity. Making use of an intrinsic coordinate system \((s, n, b)\) as shown in Figure 3.1, the expression for the growth of the streamwise component of vorticity in the streamwise direction can be shown to be

\[ \frac{\partial}{\partial s} \left( \frac{\omega_z}{pq} \right) = \frac{2}{(pq)^2 R} \frac{\partial P_t}{\partial b} \quad (3.2) \]

where \( q \) is the magnitude of velocity, \( R \) is the radius of curvature of the streamline, \( s \) denotes the streamwise direction, \( n \) the normal to the streamline, and \( b \) the binormal.

The left hand side of this equation gives the rate of change in the streamwise direction of the streamwise circulation associated with the mass flow per unit area of each streamtube. The term on the right hand side is the binormal gradient in total pressure, which upon using Crocco's theorem can be shown to be \( \frac{2\omega_n}{pqR} \). Thus, eqn. 3.2 can be rewritten as

\[ \frac{\partial}{\partial s} \left( \frac{\omega_z}{pq} \right) = \frac{2\omega_n}{pqR} \quad (3.3) \]

Since the normal is defined as the direction perpendicular to the streamwise direction, in the plane of curvature, the above expression indicates that streamwise vorticity is produced by turning a vortical flow that consists of a normal vorticity component.

The above expression for the generation of streamwise vorticity has been applied to the analysis of secondary flow in turbine cascades. In cascade flow, the normal vorticity at the cascade inlet can be computed from the boundary layer profile on the cascade endwall, having components in either the circumferential or axial directions. However, classical secondary flow theory implies that spanwise vorticity associated with circumferential variation of the flow field do not contribute to the generation of streamwise vorticity within the blade passage. Thus the secondary vorticity can be computed.
using the circumferentially averaged inflow to the cascade, i.e. the pitchwise averaged flowfield is used to compute the initial vorticity distribution $\omega_n$. This together with the knowledge of the streamline geometry in the blade passage enables the determination of the distribution of streamwise vorticity within the blade passage, including the blade exit plane.

A useful approximation for secondary flow analysis is usually adopted to simplify the calculation procedure. This approximation involves assuming that there is a uniform turning through the cascade, and that the resulting distortion of the averaged inlet vorticity distribution may be calculated using these assumed streamlines. The calculated streamwise vorticity is then added as a perturbation to the assumed flowfield to yield the predicted vortical flow.

3.2 Modified Model for Quasi-Steady Vane-Blade Interaction

In the application of classical secondary flow theory to turbomachinery, it is assumed that the inlet boundary layer undergoes a uniform turning in the cascade passage. This results in a distortion of the normal vorticity in the boundary layer and the subsequent production of streamwise vorticity. This assumption is justified on the basis of viewing the flow through airfoil cascades as a flow through curved channels of rectangular cross section. A consequence of this assumption is that any interaction of the incoming vorticity field with the leading edge region of the airfoils is considered to be negligible. However, one cannot categorically rule out the associated change in the structure of the flowfield in the cascade passage due to the interaction of the leading edge with the incoming vorticity field. It is therefore not at all inappropriate to propose a modification
to the secondary flow model that include the interaction of the leading edge with the incoming vorticity field. This section proposes such a modification to the classical secondary flow theory described above, and shows how such a modified theory indicates a possible link between the quasi-steady vane-blade interaction and the observed changes in cascade exit secondary flow patterns.

3.2.1 Cascade Secondary Flow Structure

Figure 3.2 shows a schematic diagram of the formation of the secondary passage vortex system is a turbine cascade. The figure (from [37]) was generated after a literature review of the available experimental investigations into secondary flow, and additional wind tunnel and water tunnel experiments. As shown in the diagram, the incoming boundary layer separates upstream of the leading edge region of the blade, forming a horseshoe vortex, similar to that formed around circular cylinders immersed in an endwall boundary layer [3]. The flow around the suction surface of the cascade is quite similar in structure to that formed about a cylinder. This leg of the vortex remains close to the suction surface endwall corner, gradually weakening as it moves downstream, and may continue downstream as a distinct vortex to the exit or may have diffused before reaching the exit, dependent on Reynolds number [43]. The formation of the pressure side leg of the vortex however, differs from that formed around a circular cylinder. The flow on this side is influenced by the presence of the blade to blade pressure gradient and the flow features associated with the separation in three dimensional flow. The separation line extends across the passage inlet, with the upstream boundary layer fluid separating from the endwall along this line. The entrainment of vortical fluid from upstream into this vortex core could in turn lead to an increase in the strength of the vortex. As the vortex crosses the passage above the separation line, it entrains more
of the incoming boundary layer fluid, and as this fluid enters the vortex, it is turned in the direction parallel to the separation line, generating streamwise vorticity, which increases the strength and size of the passage vortex. Thus, it appears that the presence of a separation line in the three dimensional flow is an essential feature of the formation of the secondary flow vortex system.

3.2.2 Kinematics of the Passage Vortex Formation

In incompressible flow, often it is sufficient to consider only the kinematics of the flow. Since kinematic descriptions are frequently easier to visualize, such considerations may yield insight into sources of three dimensionality in cascade flows. For this reason we will consider the kinematics of vortex formation in an attempt to gain such insight. The boundary layer at the inlet of a planar cascade consists of distributed normal vorticity, the strength of which is related to the local slope of the velocity profile. If $q$ is the magnitude of the velocity, we can define the vorticity in the direction normal to the streamline as the gradient of $q$ in the binormal direction (see equation 3.2 above) as

$$\omega_n = \frac{\partial q}{\partial b}. \quad (3.4)$$

The incoming boundary layer may thus be viewed as a vortical region with a continuous distribution of vortex filaments that are normal to the velocity and in plane, parallel to the endwall. Such a flowfield is depicted graphically in Figure 3.3. The figure shows the vortex filaments associated with the incoming boundary layer being convected toward the leading edge of the airfoil. In inviscid flow, the vortex line is always associated with the same set of fluid elements, so that as these fluid elements move about the flowfield, they carry the vortex filaments along with them. At the leading edge, there will be a point on each filament where adjacent fluid elements will move downstream on opposite sides of the blade. In the flow situation considered here, these vortex filaments neither
break nor end on the stationary blade surface. Thus as the fluid elements move past the blade leading edge, the vortex filaments wrap around the blade leading edge, leading to the formation of a horseshoe vortex there. The formation of this vortex and its growth into the passage vortex can thus be viewed as an essentially inviscid phenomenon, with its core size determined by the value of the flow Reynolds number.

3.2.3 Vane Wake Kinematic Structure

In order to evaluate the kinematics of the effect of an incoming vane wake on the blade secondary flow it is necessary to first view the vane wake as a vortical region with a continuous distribution of vortex filaments, and then to consider the configuration of these vortex filaments in relation to the geometry of the flow path and the blade passage. The vanes are spanned by solid endwalls, so that the vane passage flowfield is similar to that in a short curved channel of rectangular cross section. Thus, the boundary layer at the exit plane consists of vortex filaments that form closed loops, extending across the bottom endwall, up one vane surface, back across the top endwall, and down the opposite vane surface to close the loop. This description of the flowfield at the vane exit is consistent with the data shown in Figure 2.4, and the normal vorticity should be confined to the boundary layer in the endwall regions. This flowfield is shown schematically in Figure 3.4. As the flow moves downstream, any diffusion would tend to reduce the velocity deficit at the wake centerline. We shall attempt to use this kinematic description of the flowfield at the vane exit to arrive at a hypothetical flow model. As shall be seen in the following section, such a flow model indicates a possible link between the quasi-steady vane-blade interaction and changes in the formation of a discrete passage vortex core, and hence may provide an explanation for the observed temporal variation in the rotor exit flowfield.
3.2.4 Hypothesized Quasi-Steady Vane-Rotor Interaction

The structure of the passage vortex is a result of blade leading edge flow and the formation of the horseshoe vortex, which is dependant on the presence of normal vorticity at the blade leading edge. Thus, one might expect that any decrease in normal vorticity at the blade leading edge would have a direct effect on the size and strength of the resulting vortex core. In the limiting case where there is an absence of normal vorticity (or a zero wake centerline velocity), with a location coinciding with the blade leading edge, a horseshoe vortex formation would not be expected. Consideration of the flow scenario shown in Figure 3.5 serves to illustrate this situation. In the absence of normal vorticity at the leading edge, the resulting flow evolution will be different from that with no circumferential distortion. Any normal vorticity in the passage will be tilted and stretched due to the flow turning, but no discrete vortex will be formed. The exit flowfield will instead consist of distributed axial vorticity, similar to that predicted by the linearized classical secondary flow theory. Thus, if there are upstream vanes or rods moving relative to the cascade, the periodic impingement of the wake fluid on the leading edge stagnation line can cause a periodic decrease in the size and strength of the leading edge horseshoe vortex. Based on these observations, we now propose the following hypothesis for a possible explanation of the temporal variation in the rotor exit flowfield secondary flow pattern observed in the experiments reported in [37] and [17]. As circumferential variations in the distribution of normal vorticity on the endwall at the rotor passage inlet may be expected to result in the variations of the size and the strength of the leading edge horseshoe vortex, and as the existence of a discrete secondary vortex core appears to be related to the formation of the leading edge horseshoe vortex, the observed variations in the cascade exit secondary flow pattern may be the result of changes in the strength of the leading edge vortex caused by the interaction of
incoming vane wakes with the blade leading edge region.

3.3 Method of Investigation

A model has been proposed to explain the observed variation in rotor secondary flow structure. The proposed model attempts to relate this variation in structure to temporal variations in the strength of the normal vorticity at the blade leading edge. An investigation has been carried out to evaluate the applicability of this model to the observed phenomenon, and to establish whether the hypothesized interaction is the source of the variation in vortex structure.

To determine the source of the variation in vortex structure, it was necessary to determine if the above model correctly describes the formation and variation in the leading edge horseshoe vortex, and its subsequent growth into the passage secondary flow vortex. To accomplish this, an investigation was undertaken to examine two different cascade flowfields, one resulting from a circumferentially uniform inflow, and one with a circumferential distortion at the cascade inflow, representative of an upstream vane (or rod) wake. The objectives of this examination were first to investigate the detailed formation of the horseshoe vortex and its relationship to the passage vortex system, and second to determine if an incoming wake profile could significantly alter the horseshoe vortex and hence the passage vortex structure. In order to be able to draw the desired conclusions from such an examination, these flowfields should be simple enough to allow analysis of the effects of the distortion without interference from other unintended phenomena, yet should not be simplified to the point of excluding the relevant physical effects.
3.3.1 Dominant Flow Features and Choice of Flowfield Type

Flows through gas turbine passages generally occur at high speeds, with Reynolds numbers that are very large compared to unity, strong secondary flows, high heat transfer rates, turbulent boundary layers, and Mach numbers of order unity. These flowfields exhibit unsteadiness of widely varied time scales, ranging from low frequency variations resulting from blade passings, to the high frequency variations associated with boundary layer turbulence. This complexity makes such flowfields difficult to analyze without some simplifying assumptions. Such simplifications may include restrictions to steady flow, incompressible flow, flow at adiabatic conditions, two-dimensional planar flow, or restrictions to laminar flow regimes. Each of these assumptions simplifies the analysis of the flowfield by neglecting a particular class of physical phenomenon. For an analysis that yields insight into a physical situation, simplifications must be chosen that will give meaningful results without neglecting the processes that are essential to the phenomena of interest.

Although the flow through axial flow turbines often occurs at high mach numbers, the experimental evidence for the periodic variation of the secondary flow vortices has been obtained at low Mach number conditions. The experiments reported by Sharma et.al. [40] were obtained in a large scale, low speed rotating turbine stage, with axial Mach numbers less than one tenth. The investigations of Herbert and Tiederman [17] were carried out in a water tunnel cascade, and thus correspond to incompressible or very low Mach number flow. Thus investigations into this phenomenon can be limited to incompressible flow conditions, while still yielding useful results.

Turbine blade flows generally occur with Reynolds numbers that are very large compared to unity. Thus, the viscous effects are confined to the boundary layers and free
shear layers with essentially inviscid core regions. Since the time averaged loss in total pressure and the diffusion of vorticity are of viscous origin, the behavior of the viscous boundary layers is an essential component of the generation of secondary flow and the resulting loss in total pressure. Correct treatment of the turbulent boundary layers is thus necessary to accurately predict the absolute level of loss and the correct behavior of the viscous boundary layers in turbine cascades. However the goals of the current investigation are to relate the the passage secondary flow structure to the formation of the leading edge horeshoe vortex, and the distortion generated temporal variations in that formation. Experimental investigations [2] have determined that the level of turbulent stresses within the passage vortex core in turbine cascades are insignificant when compared with the viscous stress, suggesting that investigations of laminar flow may still provide a reasonably good prediction of the secondary flow structure away from the wall [43]. This suggestion can be justified as follows. The generation of secondary flow in turbine cascades is a kinematic process involving the deformation of incoming filament vorticity. Such a process occurs with characteristic length scales of the same order as the overall cascade geometry. Small scale turbulent motions occur at length scales many times smaller than that of the cascade geometry, typically of the order of the viscous diffusion length. Such small scale motions may thus be viewed as a sub-length scale phenomena, altering only the rate of diffusion. Turbulence thus should not be responsible for the formation of large scale flow structures such as secondary flow vortices.

Although the convection and the distortion of vorticity may be viewed as an essentially invicid process, the presence of viscosity is important to the development of several cascade flowfield structures. These include any vortical region that has its origin within the cascade passage, either on the blade or endwall surface, such as the blade surface boundary layer, endwall boundary layer behind the separation, and any counter vortices
associated with the larger leading edge horseshoe vortex. As inviscid calculation pro-
cedures would not include the effect of such structures, and as the importance of such
structures was not known a priori, it was decided that a calculation that included the
effect of viscosity on the blade and endwall surfaces might yield insight not obtainable
with inviscid techniques.

In view of the above discussion, it is thought that steady three-dimensional laminar
cascade flow would give an adequate description of the formation of secondary vorticity
for the purposes of this investigation. The cases chosen for comparison were those
resulting from laminar, incompressible flow in a planar turbine cascade, with either a
circumferentially uniform inflow or with a circumferential distortion representative of
an incoming steady wake. A comparative study based on these two cases would indicate
the validity of the proposed hypothesized theory for explaining the observed variations
in passage vortex structure.

3.3.2 Proposed Flowfield Models for Investigation

To evaluate the effect of an incoming circumferential distortion on the formation
of the secondary flow, it was decided to examine the effect of such a distortion on
the three dimensional, laminar viscous flow through a planar turbine cascade. These
flowfields could be examined with several different methods, including wind or water
tunnel experiments, annular cascade test, or through numerical simulation. The method
chosen for the current investigation was numerical simulation. Obtaining the flowfield
information through numerical simulation offers several advantages. The first is the
ability to specify a given type of inflow condition, without the difficulties associated
with generating those conditions experimentally. This allows for the elimination of
unwanted flow complexities at the inflow, such as turbulence, upstream secondary flow
vortices, or vane trailing edge unsteadiness.

The two flowfields generated with the numerical procedure were that with a circumferentially uniform inflow and one with a wake-like distortion. To generate the secondary flow within the blade passages, the inflow of both simulations consisted of a spanwise velocity profile representing an incoming endwall boundary layer. The chosen distortion for the second flowfield was a simple notch velocity profile, 20% thick at the upstream boundary. Such a distortion is representative of the incoming velocity distortions observed in the two experimental investigations. The pitchwise location of the distortion was chosen so that the centerline of the distortion is aligned with the blade leading edge, such that the centerline of the distortion would convect downstream to the blade leading edge. This inflow would then correspond to the hypothetical vorticity distribution suggested in the above secondary flow model as being responsible for the variation in vortex structure. Variations in the simulated secondary flow structure would then confirm the validity of the hypothesized quasi-steady interaction. Conversely, a lack of variation in the vortex structure with a distorted inflow would indicate that this interaction is not responsible for the measured phenomena, and that the remaining possible interaction mechanism (wake transport) is the source of the vortex variations.

We have now decided on the technique for validating the hypothesized flow model. In the next chapter, we will describe in detail this technique, including the reasons for choosing a particular method from among the various computational tools available for performing “numerical experimentation”.
Figure 3.1: Streamwise, normal and binormal coordinate definition.
Figure 3.2: Schematic diagram of cascade passage secondary flow structure (from [37])
Figure 3.3: Formation of a horseshoe vortex due the convection of filament vorticity.

Figure 3.4: Distribution of filament vorticity at vane trailing edge.
Figure 3.5: Effect of absence of filament vorticity at the blade leading edge.
Chapter 4
Numerical Technique

A numerical simulation was used to simulate the three dimensional viscous flow through a planar turbine cascade. The purpose of the simulation was to generate flowfields that could be used to answer basic questions about the formation of secondary flows and the interaction of that secondary flow with incoming flow distortions.

4.1 Choice of Algorithm

When using a numerical simulation as a tool to investigate fluid phenomena, an investigator must choose between many possible methods, both with respect to the equations used to model the flow, and to the particular method used to solve the equations. These choices must be made on the basis of the phenomenology of the problem of interest, the applicability of a particular technique to the investigation of those phenomena, and the available resources. Quite often there is no particular tool that is best suited for all the criteria, and the selection must involve compromise between the various requirements and constraints.
4.1.1 Flow Phenomena of Interest

As was stated in the previous chapter, the flowfields to be calculated are three dimensional, incompressible, laminar flows through a turbine cascade. Thus, the numerical solution should be based on equations that describe the flow of a viscous fluid through the desired geometry. Such flows are described by the three dimensional Navier-Stokes equations [4], and the incompressible continuity equation. This system of nonlinear, partial differential equations in three spatial dimensions and time is the mathematical description of the motion of a viscous incompressible flow.

4.1.2 Solution Technique

The particular method chosen to approximate the continuous Navier-Stokes equations in a discrete form for numeric solution was the technique of direct simulation. Direct simulation involves substitution of the discretized form of the state variables and their derivatives directly into the full equations of motion. A given spatial discretization is first selected and the full discretized expressions for the state quantities and derivatives are derived. These discretized forms of the state quantities can then be substituted into the full equations of motion to yield a set of discretized equations with the desired number of degrees of freedom. The accuracy of such a simulation is thus directly determined by the accuracy of the chosen spatial and temporal discretizations, which may be determined a posteriori [28].
4.1.3 Multi-Domain Spectral-Spectral Simulation

The discretization chosen for the direct Navier-Stokes simulation was a multi-domain spectral-spectral method, as implemented in [45]. The technique uses a full spectral expansion in Chebyshev polynomials [10] for the properties in the spanwise direction, while the discretization in the axial-pitchwise (X,Y) plane is based on local spectral expansion in Chebyshev polynomials on local subdomains, or spectral elements [32]. Such a discretization offers several advantages. Since the discretization is spectral in nature, truncation error decreases exponentially with increasing spectral order for sufficiently smooth flowfields [10]. The solution technique has minimal numeric dissipation, and since the expansions in the (X,Y) plane are defined in local subdomains, the technique offers a high degree of geometric flexibility, so that the flow about complex turbine geometries can readily be simulated.

4.2 Governing equations and temporal discretization

The equations governing the flow are the incompressible Navier-Stokes equations written in rotational form,

\[
\frac{\partial \vec{V}}{\partial t} = \vec{V} \times \vec{\omega} - \nabla P_t + \frac{1}{Re} \nabla^2 \vec{V} \tag{4.1}
\]

\[
\nabla \cdot \vec{V} = 0 \tag{4.2}
\]

Here, \( \vec{V} \) is the velocity field normalized by upstream axial velocity at midspan, \( \vec{\omega} = \nabla \times \vec{V} \) is the vorticity field, \( P_t \) is the total pressure normalized by twice the upstream axial dynamic head, and \( Re \) is the Reynolds number based on the upstream axial velocity at midspan and blade axial chord.
The solution to Eqs.(1-2) is advanced forward in time using a fractional time stepping scheme [10], consisting of a non-linear convective step, a pressure step imposing continuity, and a viscous correction step imposing the no-slip boundary condition. Here, the non-linear convective step is implemented through an explicit fourth order Runge-Kutta scheme as follows:

\[
\tilde{V}^{1,n+\frac{1}{2}} = \tilde{V}^n + \frac{\Delta t}{2} (\tilde{V} \times \tilde{\omega})^n \tag{4.3}
\]

\[
\tilde{V}^{2,n+\frac{1}{2}} = \tilde{V}^n + \frac{\Delta t}{2} (\tilde{V} \times \tilde{\omega})^{1,n+\frac{1}{2}} \tag{4.4}
\]

\[
\tilde{V}^{3,n+1} = \tilde{V}^n + \Delta t (\tilde{V} \times \tilde{\omega})^{2,n+\frac{1}{2}} \tag{4.5}
\]

\[
\tilde{V}^{n+1} = \tilde{V}^n + \frac{\Delta t}{6} [(\tilde{V} \times \tilde{\omega})^{3,n+1} + 2(\tilde{V} \times \tilde{\omega})^{2,n+\frac{1}{2}} + 2(\tilde{V} \times \tilde{\omega})^{1,n+\frac{1}{2}} + (\tilde{V} \times \tilde{\omega})^n] \tag{4.6}
\]

Furthermore, here we sub-cycle the convective step in order to relieve constraints on the overall time stepping. The convective step is subdivided as necessary, with the sub-cycle ratio chosen dynamically to satisfy the stability criterion, as follows.

\[
\hat{V}^{n,0} = \tilde{V}^n \tag{4.7}
\]

for \( m=1,M \)

\[
\hat{V}^{n,m} - \hat{V}^{n,m-1} = \frac{\Delta t}{M} (\tilde{V} \times \tilde{\omega})^{n,m-1} \tag{4.8}
\]

\[
\hat{V}^{n,M} = \tilde{V}^{n+1} \tag{4.9}
\]

Here each of the steps of Eqn. (4.8) is evaluated by the fourth order Runge-Kutta scheme shown above.

Once \( \hat{V} \) is determined, we are left with an unsteady Stokes problem which can be split in time as follows. First, the pressure correction step is discretized in time by a Backward Euler method, yielding

\[
\frac{\hat{V}^{n+1} - \hat{V}^n}{\Delta t} = -\nabla P_t \tag{4.10}
\]
and
\[ \nabla \cdot \vec{V} = 0 \quad (4.11) \]
subjected to the boundary condition
\[ \frac{\vec{V}}{\Delta t} \cdot \vec{e}_n = 0 \quad (4.12) \]
on the blade surface and endwalls. Computationally, the above step is reformulated as a solution for $P_t$ by taking the divergence of Eqn. (4.10) and applying Eqn. (4.11) to yield
\[ \nabla^2 P_t = \nabla \cdot \left( \frac{\vec{V}^{n+1}}{\Delta t} \right) \quad (4.13) \]
subjected to the boundary condition
\[ \frac{\partial P_t}{\partial n} = \frac{\vec{V}^{n+1} \cdot \vec{e}_n}{\Delta t} \quad (4.14) \]
on the solid walls. The velocity field $\vec{V}^{n+1}$ that satisfies continuity identically is then computed from Eqn. (4.10).

Following the solution of the pressure step is the viscous correction step imposing the non-slip boundary condition on the solid surfaces. The step is discretized using the implicit Crank-Nicolson scheme, giving
\[ (\nabla^2 - \frac{2Re}{\Delta t})(\vec{V}^{n+1} + \vec{V}^n) = -\frac{2Re}{\Delta t} (\vec{V}^{n+1} + \vec{V}^n) \quad (4.15) \]
subject to the appropriate no-slip boundary conditions on the solid surfaces. At the inflow, the velocity is assumed known, while at the outflow a homogeneous Neumann boundary condition is imposed.
4.3 Spatial discretization and elemental mapping

4.3.1 Spatial discretization in $Z$

Since the geometry is invariant in the spanwise dimension (planar cascade), one can choose a direct spectral expansion for the flow variation in the $Z$ direction. The need to account for different boundary conditions on the endwalls (either Neumann or Dirichlet for the velocity, and Neumann for the pressure) leads us to define an eigenfunction expansion

$$F_l(Z) = \sum_m f_{lm} T_m(Z)$$  \hfill (4.16)

satisfying the following Sturm-Liouville problem,

$$\frac{d^2 F_l(Z)}{dZ^2} = \lambda_l^2 F_l(Z),$$  \hfill (4.17)

subject to homogeneous Neumann boundary conditions for the pressure and homogeneous Dirichlet boundary conditions for the velocity. These functions are constructed separately for the viscous velocity step and pressure step in a preprocessing procedure via the tau method [46]. The Chebyshev polynomials $T_m(Z)$ are given as

$$T_m(Z) = \cos(m \cos^{-1}Z)$$  \hfill (4.18)

with the collocation points chosen at

$$Z_k = \cos \frac{\pi k}{L}.$$  \hfill (4.19)

It should be noted that the choice of collocation points can be arbitrary, yet the distribution given in Eqn. (4.19) can be shown to give an error that satisfies the minimax criterion [31]. In addition, such a choice also results in good resolution of the viscous boundary layers near the endwall.
4.3.2 Spatial discretization in the X-Y plane

The complexity of the geometry prohibits a simple global spectral discretization of the flow variables in the \((X,Y)\) plane. The region is instead divided into a number of subdomains, or spectral elements, following the technique developed by Patera [32]. In each \(i^{th}\) subdomain or element we can expand the flow variables as

\[
\begin{pmatrix}
\vec{V} \\
p
\end{pmatrix} = \sum_{j=0}^{N_x} \sum_{k=0}^{N_y} \sum_{l=0}^{N_z} \begin{pmatrix}
\vec{V}_{jkl} \\
p_{jkl}
\end{pmatrix} h_j^i(\xi)^i h_k^i(\eta)^i F_i(Z)
\]

(4.20)

where \(F_i(Z)\) are the interpolants from the direct expansion in the spanwise direction given in Eqn. (4.16) above, and \(h_m(S)\) are high order local Lagrangian interpolants in terms of Chebyshev polynomials. These can be written as

\[
h_m(S) = \frac{2}{M} \sum_{n=0}^{M} \frac{1}{C_m C_n} T_n(S_m) T_n(S)
\]

(4.21)

with

\[
C_m = \begin{cases} 
1 & \text{for } m \neq 0 \text{ or } M \\
2 & \text{for } m = 0 \text{ or } M 
\end{cases}
\]

(4.22)

where the \(S_m\) are the collocation points in the computational space.

4.3.3 Elemental Mapping

The mapping from the physical coordinate space \((X,Y,Z)\) to the local natural coordinate space \((\xi, \eta, \zeta)^i\) is given by an isoparametric tensor-product mapping [20],

\[
(X,Y)^i = \sum_{j=0}^{J} \sum_{k=0}^{K} (X,Y)^i_{jk} h_j^i(\xi)^i h_k^i(\eta)^i,
\]

(4.23)

where we have chosen \(\zeta = Z\), with the collocation points in the elemental computational space defined as

\[
(\xi, \eta, z)^i_{jkl} = \left(\cos \frac{\pi j}{J} \cos \frac{\pi k}{K} \cos \frac{\pi l}{L}\right)^i
\]

(4.24)
where \( j, k, l = 0 \rightarrow J, K, L \).

To complete the definition of the mapping from the physical coordinate space \((X, Y, Z)\) to the elemental computational space \((\xi, \eta, \zeta)\), we need to define the collocation points in the physical space \((X, Y)_{jk}^i\). This can be accomplished by several different methods. The first is through the use of an analytic conformal functional mapping, which can be used whenever the elements are rectangular in some suitable regular curvilinear coordinate system. A second and less restrictive method uses a Laplace equation to generate a linear functional variation in two dimensions over the element in physical space. The functional values are then used to map the points in the computational space to the physical element. The third method is an algebraic method using elements with two linear and two generally curved sides. Due to its relative simplicity, ease of implementation and geometric generality, the algebraic method is used here.

In the algebraic method, we first define a general parametric function \(X(S) = f_X(S)\) and \(Y(S) = f_Y(S)\) on each side of each element. Collocation points are then distributed along each element side in arc length according to the formula

\[
\begin{bmatrix}
X \\
Y
\end{bmatrix} = f_{X,Y}(S_m)
\]

where the collocation arc lengths \(S_m\) are defined as in Eqn. (4.18). Next, by making use of the linearity of two opposing sides, the interior points are defined along straight lines connecting the points on opposing curved sides, distributed according to Eqn. (4.25) as illustrated in Figure 4.1.

The final discretized equations are obtained by substituting Eqn. (4.20) into the relevant temporal discretizations, Eqns. (4.6-4.15). For a complete derivation of the final discretized equations and computational cycle, see Tan [45].
4.4 Validation

Verification of the accuracy or correctness of a three dimensional full Navier-Stokes simulation code can be a difficult task. Closed form analytic solutions to the equations of motions are rare, and usually exist only for simple flows or geometries, such as plane Poiseuille or other essentially one dimensional flows. While there does exist a large body of experimental data, the flowfields measured are usually complex enough that the cost of simulation makes them unsuitable for code validation purposes. Thus, there is a very limited body of analytic or experimental solutions with which to verify the accuracy of a new calculation procedure.

4.4.1 Comparison of Surface Vorticity and Limiting Velocity

To ensure the consistency of the calculated results, a comparison was made of the vorticity on a solid surface and the limiting velocity above that surface. A velocity profile that varies linearly with distance from the solid surface will yield a velocity that is proportional to the surface shear stress. Under this restriction, we may relate the surface vorticity to the limiting velocity above the surface. The vorticity and the shear stress on the surface are related according to the relation

\[ \tau_w = \vec{\omega} \times \hat{n} \]  \hspace{1cm} (4.26)

where \( \tau_w \) is the surface shear stress, and \( \hat{n} \) is the surface unit normal. Thus, with the assumption of a linear velocity variation, the limiting velocity and the vorticity on the surface will be perpendicular to each other, and proportional. Figure 4.2 shows the results of a comparison between the calculated surface vorticity and the limiting velocity, defined as the velocity at a normalized distance of 0.01 above the surface, near a saddle point in a simulated three dimensional flow. The figure shows both sets of vectors, with
the limiting velocity being represented by thin vectors, while the vorticity is represented by bold arrows. As the figure shows, the surface vorticity and the limiting velocity are orthogonal and proportional, indicating that the three dimensional treatment of the vorticity and velocity is consistent in the numerical procedure.

4.4.2 Comparison to Analytic Solutions

There does exist a class of solutions to the three dimensional Navier-Stokes equations that yield analytic solutions. These are for steady fully developed pipe flow, which are solvable using Fourier series, making them suitable as test cases for Navier-Stokes computational procedures. Of these solutions, the solution for flow through rectangular channels most closely models the geometry of a planar cascade spanned by parallel endwalls, and it is this solution that has been chosen as a test of the described algorithm.

For fully developed flow in the \( z \)-direction, in a rectangular section defined by \( y = [-a, a] \) and \( z = [-b, b] \), the velocity distribution in the \( y, z \) plane can be written [47] as

\[
\frac{u}{u_{\text{max}}} = \hat{A} \sum_{i=1,3,5,...}^{\infty} (-1)^{(i-1)/2} \left[ 1 - \frac{\cosh(i \pi x/a)}{\cosh(i \pi z/b)} \right] \frac{\cos(i \pi y/2a)}{i^3}.
\]  

(4.27)

A rectangular channel with a height to width ratio of two yields \( a = 1, b = 2 \), has been chosen as the geometry for the validation case. A numerical simulation consisting of flow through a channel of length twelve has been run for comparison with the analytic expression. A flow Reynolds number of one, based on channel half width, ensures that the flow becomes fully developed within the length of the computational domain. The geometry consists of 18 spectral elements, in a 6 by 3 array, with each element representing a 7 by 7 by 33 order spectral expansion. Figure 4.3 shows a plot of the spatial discretization on the \( x, y \) plane. Boundary conditions for the simulation consist of viscous no-slip conditions at \( y = -1, +1 \) and \( z = -1 \). A symmetry boundary condition
is imposed at \( z = +1 \), taking advantage of the inherent symmetry of the flow. This gives a channel computational height to width ratio of 1, corresponding to a physical height to width ratio of two. The simulation was run with a time step size of \( \Delta t = 0.05 \) for 100 time steps, sufficient for the calculation to reach a steady state condition. Figure 4.4 shows a plot of axial velocity on the \( z, y \) plane, at the symmetry point, \( z = 1 \). The plot shows the rapid development of the flow away from the wall, reaching a fully developed condition in a distance of 1.0. The existence of a fully developed profile is confirmed by the static pressure, shown in figure 4.4, indicating an \( z \) pressure gradient independent of \( y \) and varying linearly with \( z \). Variations from parallel flow near the outflow are due to necessary flow adjustment to the outflow extrapolation boundary condition. Calculated results are compared at a location upstream of this outflow boundary layer, at \( z = 8.0 \). Figure 4.5 shows contours of velocity in the lower half of the \( y, z \) plane, with a maximum value of 1.0 occurring on the \( z \) symmetry plane, mid-channel, while Figure 4.5 shows the values given by the analytic expression, evaluated at the collocation points of the numerical simulation. The figures show no differences that can be seen on the scale of the plots. A direct numerical comparison between the two solutions shows that the average absolute error between the simulation and the analytic solution to be \( 9.94 \times 10^{-5} \).

### 4.4.3 Comparison to Experimental Data

Since the numerical procedure is being used to investigate the formation of secondary vortical structures in turbine cascades, it is important that the vortex structure simulated by the numerical procedure agrees qualitatively with the vortex structure formed in turbine cascades operating at realistic machine conditions. Although experimental results cited in the previous chapter [2][43] suggest that a laminar calculation
should yield realistic secondary flow structures, direct comparison to experimental results would further indicate the validity of the numerical approach. Thus, to verify that the simulation does result in a representative vortex structure, we compare the simulated flowfield to experimental results obtained with similar geometry.

A comparison is made to the benchmark cascade investigation of Langston [24]. The experiment involved a survey of the flow through a large scale planar cascade, including measurements of the fluid velocity, total pressure and surface static pressure, and visualization of the endwall limiting streamlines. The cascade was operated with an 11.8% thick inlet boundary layer, with an inlet Reynolds number based on axial chord of $5.9 \times 10^5$. The cascade blade aspect ratio was chosen as 1.0, with a pitch-to-chord ratio of .9888.

The numerical simulation used the cascade blade shape from the above experiment, with a blade aspect ratio and pitch-to-chord ratio of 1.0. The cascade was operated with a 45 deg inflow angle, and a 21.8% thick, laminar collateral boundary layer at the cascade inlet. The inlet Reynolds number based on axial chord was $Re = 1000$, or two orders of magnitude lower than the experimental investigation. This would ensure stability of all boundary and free shear layers in the flow, resulting in laminar flow throughout the cascade.

Figure 4.6a and 4.6b show a comparison of the blade surface static pressure coefficient at various spanwise locations from experiment and numerical calculation. Major differences in the flow are the lower trailing edge pressure in the numerical case, and the larger decrease in the loading with span. The increased blockage of the laminar simulation results in a higher free stream exit velocity, and hence, a lower static pressure. The lower endwall-region loading can be attributed to the thicker inlet boundary layer.
used in the numerical calculation. Although it does not yield the experimental values, the numerical calculation does give qualitative agreement with the measured variations. The locations of the saddle points from the experiment and the simulation are shown in Figure 4.7a. The location of the saddle point, and the trajectories of the surface topological features is well predicted, as shown.

Comparisons of total pressure distributions on several axial planes are shown in Figure 4.8. The figure shows contours of the total pressure coefficient, normalized by upstream dynamic head, from the experiment, and those calculated by the numerical procedure. The 15% plane comparison shows the thicker incoming boundary layer in the simulation, and hence the increase in boundary layer thicknesses on the cascade side and endwall. At increased axial chords, the behavior of the simulated flowfield is similar to the experimental results. The simulation correctly predicts the pitchwise migration and separation of the inlet boundary layer. The extent of the separated region matches the experimental values, as does the location of the blade surface three-dimensional separation at 95% span. The major differences between the two distributions is the thicker blade and endwall boundary layers, and the much more severe blade surface separation. However the spatial relationships between the high loss regions, blade and endwall separations, and the rollup of the inlet boundary layer are correctly predicted by the numerical procedure. This indicates that the simulation is calculating a secondary flow vortex structure representative of those in cascades at realistic engine conditions.

4.5 Outflow Boundary Treatment

As was stated above, the boundary condition used at the outflow is a homogenous Neumann, or natural boundary condition. This has the effect that the flow development
normal to the outflow is zero at that boundary. Since this is a non-physical boundary condition, its application in the numerical procedure may influence the physical relevance of the solution. Thus the influence of the non-physical extrapolation outflow boundary condition needs to be evaluated.

4.5.1 Natural Boundary Condition for Outflow Boundaries

Using a natural outflow boundary condition for solution of a convective equation has the effect of enforcing the condition that the flow does not evolve any further in the direction normal to the outflow boundary. In the case of the current calculation, that direction is the axial or $X$ direction. Such a boundary condition may or may not represent a physically realistic situation. For a convection-diffusion equation such as the Navier-Stokes equations, such situations are only realizable in fully developed flow. Thus, the imposition of a natural boundary condition may influence the results of the simulation.

The imposition of an improper outflow boundary condition in a convection-diffusion problem for incompressible flow leads to a spurious boundary layer phenomena at that boundary. The solution adjusts to the outflow condition through a boundary layer, whose thickness is determined by the type of adjustment required. In a steady constant area flow, the only streamwise gradients are those associated with viscous diffusion, and the relevant length scale is the only physical length scale in the flow at the boundary, the viscous length. Thus, for steady flows, a spurious boundary layer may form at the outflow boundary, of thickness that scales as

$$
\delta_v = \frac{1}{Re}.
$$

Since the adjustment may result in strong gradients normal to the outflow, as in any
boundary layer phenomenon, higher resolution may be require near the outflow than immediately upstream of the boundary layer.

The imposition of an extrapolation boundary condition at the outflow of an unsteady flow problem can cause more serious difficulties. Since the normal derivative now involves more than viscous diffusion, there will be additional length scales in the flow. In particular, each unsteady time scale will involve a convective length scale, depending on the frequency of the motion \( \omega \) and the average convective velocity \( U_c \),

\[
l_\omega = \frac{U_c}{\omega}
\]  

(4.29)

If the average convective velocity is at a relative angle \( \beta \) to the outflow boundary normal \( \hat{n} \),

\[
\beta = \cos^{-1}(\vec{U}_c \cdot \hat{n})
\]

(4.30)

then the motion will have a length scale in the direction normal to the outflow boundary of

\[
l_{\omega n} = \frac{U_c}{\omega} \cos \beta.
\]

(4.31)

The inclusion of these additional length scales in the flow at the outflow boundary will affect the spatial and temporal development of the flow in a region within the characteristic length of that motion of the outflow boundary. Since the extrapolation boundary condition specifies no further development of the flow in the direction normal to the outflow, disturbances convecting toward that boundary will be suppressed for a characteristic length upstream. This can have the effect of supressing unsteadiness over the region of influence of the outflow boundary. This suppression of unsteadiness can influence such laminar unsteady motions as wake vortex rollup, and if close enough to the blade trailing edge, unsteady separation and vortex shedding.
4.5.2 Influence of Cascade Outflow Boundary Condition

Since an artificial boundary condition has been applied to the cascade simulation, the possibility exists that this local condition may affect the global solution of the equations of motion in the cascade. To determine the effect of the outflow boundary condition on the solution, two two-dimensional simulations were completed, with the outflow boundary at different downstream locations. The results of the two simulations were then compared to determine the effect of the boundary location.

Figure 4.9 shows the computational domain decomposition for the first simulation, referred to as the long outflow case. The outflow boundary in this simulation is at an axial location of \( x = 2.0 \), or approximately two chord lengths downstream of the trailing edge. The grid outflow H-mesh region has a resolution decreasing with increasing axial distance from the trailing edge, with two columns of high resolution elements to resolve any possible outflow boundary layers. Any unsteadiness generated at the blade having length scales less than the distance to the outflow should not be affected by the boundary condition. Since blade generated unsteadiness should have length scales no larger than the cascade dimensions, the effect of the outflow on the cascade flow should thus be minimal.

The domain discretization for the second simulation, referred to as the short outflow case, is shown in Figure 4.10. This simulation has the same number of spectral elements as the long outflow case, but with the outflow boundary at an axial location of \( x = 1.5 \), or less than one chord downstream of the trailing edge. This has the effect of putting the trailing edge of the cascade airfoils within the region of influence of the outflow boundary.

Both simulations used the same initial condition, with the simulations run to pe-
periodic or steady state conditions. Figures 4.11b and 4.11a show time histories of the midgap trailing edge plane axial velocity for the two simulations. The figure shows that the solution for the long outflow case reaches a periodic unsteady condition, having a dominant temporal period of \( r = 0.453 \), corresponding to an unsteady frequency of \( f = 2.2075 \). The length scale in the axial direction, normal to the outflow boundary, for this motion, based on the average axial velocity of 1.0 is thus

\[
l_x = 0.453.
\]  

(4.32)

It should be noted that this distance is less than the distance from the outflow to the trailing edge plane in the short outflow case, suggesting that the outflow boundary condition can affect this unsteady behavior in the short outflow simulation. The solution for the short outflow case reaches a steady state solution, without the unsteadiness exhibited in the long outflow solution. These computed results illustrate that the outflow boundary condition in the short outflow case is suppressing the unsteadiness normally exhibited in the solution.

It has been shown that the extrapolation condition at the outflow boundary affects the temporal development of the solution through suppression of unsteadiness within a characteristic length of the boundary. However since the steady flow has a dominant length scale dependent only on the Reynolds number, the location of the boundary should not affect the time average or zeroth eigenmode of the flow. Figure 4.12 compares the airfoil surface static pressure coefficient distributions for the long and short outflow cases, defined as

\[
C_p = \frac{P - P_{\infty}}{\frac{1}{2} U_{\infty}^2}
\]  

(4.33)

The time average of the unsteady long outflow case, along with the envelope of maximum and minimum values is shown on the graph with solid lines. Also shown is the final steady state distribution for the short outflow case. As can be seen from the figure,
the steady short outflow solution is very close to the time average of the long outflow solution. Numeric comparison shows that the maximum difference between the two distributions is

\[ \| C_{Pshort} - C_{Plong} \|_\infty = .0021 \]  \hspace{1cm} (4.34)

while the error in the two norm is

\[ \| C_{Pshort} - C_{Plong} \|_2 = .03 \]  \hspace{1cm} (4.35)

Such close numeric agreement indicates that the suppression of the unsteady modes in the short outflow solution does not affect the time averaged value. This suggests that the unsteady temporal modes in the wake are orthogonal, and that suppression of higher order temporal modes does not affect the lower order modes in the solution [7]. Thus it is felt that solutions using the short outflow type of geometric discretization may be used to yield reliable calculation of the time averaged value of the flow quantities.
Figure 4.1: Element collocation grid in physical and computational space.
Figure 4.2: Comparison of surface vorticity and endwall shear stress in the vicinity of a saddle point.

Figure 4.3: Elemental grid for validation simulation, $x,y$ plane.
Figure 4.4: Contours of a) $x$ velocity and b) static pressure coefficient at the symmetry plane.
Figure 4.5: Comparison of x-velocity in the $y,z$ plane for a) spectral element solution, and b) analytic expression. (Contour increment .05)
Figure 4.6a: Calculated surface static pressure distribution.

Figure 4.6b: Experimental surface static pressure distribution (from [23])
Figure 4.7a: Comparison of measured and simulated endwall flow topology (from [23])
Figure 4.7b: Comparison of measured and simulated endwall flow topology (cont.).
Figure 4.8: Comparison of measured and simulated total pressure distributions.
Figure 4.9: Computational domain, long outflow case.

Figure 4.10: Computational domain, short outflow case.
Figure 4.11a: Mid-gap trailing edge plane velocity, long outflow case.

Figure 4.11b: Mid-gap trailing edge plane velocity, short outflow case.
Figure 4.12: Comparison of surface static pressure distributions for the long and short outflow simulations.
Chapter 5

Results of Numerical Simulation

This chapter examines the effect of a steady circumferential distortion on the formation and subsequent growth of the secondary flow passage vortex. A comparison is made between two simulated cascade flowfields having different inflow boundary conditions: (1) circumferentially uniform flow with a spanwise velocity profile representing an incoming endwall boundary layer, and (2) the same spanwise profile with the addition of a circumferential distortion representing an incoming wake. The first part of the chapter presents the results of a study of the flowfield through a blade row having a circumferentially uniform inflow. Results are presented to elucidate the formation of the passage vortex and the associated secondary flow patterns in that flowfield. This computed flowfield forms a baseline case to which other computed flowfields with different inflow conditions can be compared. The second section investigates the results of a computed flowfield that has a circumferential distortion imposed along the inflow boundary upstream of the blade leading edge. Both cases have the same average inflow velocity and thus the same average inlet Reynolds number. A comparative study of the two calculated solutions is then used to draw conclusions about the validity of the secondary flow interaction model proposed in Chapter 3.
5.1 Flow in a Turbine Cascade with Circumferentially Uniform Inflow

The direct numerical simulation technique described in Chapter 4 is used to compute the flowfield through a turbine cascade passage. At the inflow boundary the flow is taken to be circumferentially uniform, with a spanwise velocity profile near the endwall. The spanwise variation in the velocity profile is chosen such that it realistically represents the presence of an incoming endwall boundary layer, which is a necessary ingredient for the formation of a horseshoe vortex system. In what follows, the computation of flowfield and the computed results are described to bring out the details of the flow features associated with the formation of a horseshoe vortex.

5.1.1 Cascade Geometry and Inflow Velocity Profile

Figure 5.1 shows the geometry of the turbine cascade. The blade profile is that used by Langston [24] in his benchmark cascade investigation. The profile is also the midspan blade shape used in the turbine rotor in the UTRC Large Scale Rotating Turbine. This geometry was chosen for several reasons. The first is that the profile has been the subject of a number of published investigations, hence the blade coordinates are available in the open literature. Second, the profile is representative of modern high pressure turbine rotors, with high enough loading to ensure strong secondary flows. Lastly it was this cascade that the phenomena of interest were observed in and thus provides a link with those experiments. Cascade geometrical parameters are listed in Table 5.1. Figure 5.2 shows the elemental discretization used in the solution of the equations of motion. The discretization uses 164 seventh order elements, with a direct 33rd order expansion in the spanwise direction over one half of the blade span.
The inflow condition to the cascade consisted of a uniform flow in the pitchwise direction, with a spanwise velocity profile representative of an incoming boundary layer.

The profile is an $8^{th}$ order polynomial velocity profile

\[
\begin{align*}
  u_0(y, z) &= 1.0 - z^8 \
  v_0(y, z) &= u_0(y, z) \tan(\alpha_0).
\end{align*}
\]  

(5.1)

(5.2)

Calculated boundary layer parameters for the specified profile are shown in Table 5.2.
5.1.2 Comparison of Midspan Results to Two Dimensional Flow.

A comparison is made between the midspan three dimensional results and those calculated for a true two dimensional flow with the same geometry and inflow condition. Figure 5.3 show the blade surface static pressure distribution for the two and three dimensional cases. The figure shows that the suction surface static pressure at the midspan of the three dimensional flow is higher than that of the 2-D simulation. In addition, the pressure surface shows a decrease in static pressure relative to the three dimensional case. This has the net effect of decreasing the midspan loading by 12% from that predicted by the 2-D calculation. The source of the calculated decrease in loading is the pressure relief that occurs along the blade midspan line. The spanwise pressure gradient on the blade surface due to the inlet profile induced loading variation results in an increase in the spanwise size of the streamtubes at midspan. This reduces the massflow at midspan from that of the constrained 2-D solution, and causes a corresponding decrease in the loading.

The second major difference between the two dimensional prediction and the calculated midspan solution is in the boundary layer thickness on the suction surface. Figure 5.4 shows contours of total pressure coefficient for the 2-D simulation and the midspan values for the 3-D case. The plot shows that the suction surface boundary layer separates from the blade at \( x = .78 \), while the 3-D results show no evidence of a mid-span boundary layer separation. The mid-span suction surface boundary layer is approximately 80% thicker than that calculated for the 2-D flow. This is a result of the spanwise secondary velocities on the blade suction surface. This flow toward midspan results in the blade boundary layer fluid being driven toward midspan, where it accumulates as shown in the figure. Since this thickening is related to the amount of boundary layer fluid affected by the endwall secondary flow, it will thus be sensitive
to blade aspect ratio, and the effect would be less at higher aspect ratios. The calculated increase shown here compares well to Langston's results of the measured midspan boundary layer being twice as thick as that predicted by a two dimensional calculation [24].

5.1.3 Endwall Surface Values and Flow Topology

The structure of the secondary flow in the leading edge region can be deduced by examining the pattern of shear stress on the endwall surface in the leading edge region. A careful examination of the wall shear stress vectors and their corresponding magnitude will allow one to identify (i) flow separations, (ii) reverse flow regions, and (iii) the location of discrete vortex cores. Figure 5.5a shows the distribution of shear on the endwall surface. There is a strong gradient in the shear across a line that extends from upstream of the leading edge across the passage. The sudden change in shear stress across this line suggests the presence of an organized flow structure at this location. Upstream of this line, at a location near the leading edge, there is a local minimum in the shear stress magnitude; from the computed results, this local minimum appears to be located at $(-.25, .35)$. This minimum in the magnitude of the endwall shear stress indicates the possibility of the existence of a saddle point at that location. To confirm this and to determine the type of flow structure associated with the strong gradient in shear mentioned above requires a further examination of the shear stress vectors on the endwall surface. The distribution of shear stress vectors is shown in Figure 5.5b. The existence of a saddle point flow can be deduced through an examination of the stress vectors at the location of the minimum in stress magnitude. As shown in Fig. 5.5b, there is a set of shear stress vectors that approach this location from two opposite directions, with a corresponding set of vectors that leave from it in two opposing directions which
are orthogonal to the former set of vectors. This observation clearly indicates that this particular flow feature is associated with a saddle point flow.

The existence of a saddle point flow upstream of the blade leading edge suggests that there is a horseshoe vortex about the leading edge, as this feature is often seen in conjunction with horseshoe vortex flows [3]. A further examination of the shear stress vectors show that there is a strong reverse flow between the saddle point and the blade leading edge. If we proceed along a line coinciding with the region of high gradient in shear, we note that this reverse flow decreases in magnitude and changes direction. However, the results in Fig. 5.5b show no evidence of the existence of a saddle point along this line, implying that this line cannot be a line of separation of the boundary layer [35]. This feature is commonly seen in horseshoe vortex flows [23,46], and is often mistakenly identified as a separation line. The strong gradient in shear magnitude behind the separation line is related to the formation of the horseshoe vortex about the leading edge. The swirling flow in the horseshoe vortex core implies the existence of a radial pressure gradient within the vortical region and hence a correspondingly decreased static pressure in the vortex center. Thus the vortical motion can affect the static pressure distribution on the endwall. This effect on the endwall static pressure can be deduced from Figure 5.6a, which shows the comparison of the pressure distribution on the endwall to that distribution at the midspan. Beneath the location of the vortex there is a decrease in the static pressure, and a corresponding increase in the flow velocity near the wall. Thus the fluid that passes beneath the vortex in the reverse flow region first accelerates, then upon moving past the location of the vortex center it encounters an adverse pressure gradient. This adverse pressure gradient results in the observed deceleration and subsequent turning of the flow along the line observed above. Thus this line gives the appearance of a separation line, but since it does not originate in a singular point (either a saddle or stagnation point [35] it cannot be classed as a
topological feature of the flow.

5.1.4 Leading Edge Horseshoe Vortex Formation

Figures 5.7b through 5.7d show the detailed structure of the horseshoe vortex formation about the leading edge of the airfoil. Shown are the distributions of streamwise vorticity, given by

$$\omega_s = \frac{\vec{o} \cdot \vec{u}}{q}$$

where $q$ is the velocity magnitude. Distributions are plotted on a number of planes normal to the leading edge, as shown in figure 5.7a. The figures show that the horseshoe vortex has three vortical regions associated with it. The distribution indicates the presence of a single separation line that is very similar to that reported by Langston and described by Sieverding [43] in his review of secondary flow theory. The figures show that there is a primary vortex located above and behind the separation line, of the same sign as that of the vorticity associated with the upstream boundary layer. The spanwise flow along the blade surface results in vorticity of opposite sign being generated on the blade surface, which then convects toward the endwall to collect in the blade endwall corner region. It is this flow that is commonly identified as the endwall corner vortex [24,43]. The flow beneath the vortex, in a direction opposite to that of the external flow, results in the formation of vorticity of opposite sign to that associated with the horseshoe vortex flow. This vorticity is first convected along the endwall, and then accumulates between the separation line and the core of the horseshoe vortex. This accumulation of vorticity of different sign along the endwall contributes to the gradient in wall shear stress shown in the previous section.

The vortex structure at the leading edge can be summarized as follows. A horseshoe
vortex forms about the blade leading edge, between the leading edge and the endwall separation line, of vorticity of the same sense as that of the incoming boundary layer. Between this horseshoe vortex and the blade leading edge-endwall junction an endwall corner vortex forms, with vorticity of opposite sign to that of the incoming boundary layer. Finally, between the horseshoe vortex and the separation line a third vortex forms, again with a vorticity of opposite sense to that in the incoming boundary layer. As we shall see below, it is this three vortex structure that enters the cascade passage and develops into the passage vortex observed at the cascade exit.

5.1.5 Development of Passage Vortex

The growth of the horseshoe vortex pressure side leg and its development into the passage vortex are shown in Figure 5.8b through 5.8f. These figures show contours of the value of the streamwise vorticity on planes of constant axial location, indicated in 5.8a. At the leading edge plane the two legs of the horseshoe vortex are nearly identical in strength. The accumulation of counter vorticity is seen to the left (upstream) of the pressure side leg of the vortex. The counter vortex is also visible about each side of the leading edge circle.

At 20% axial chord, the strength of the pressure side leg of the vortex has increased, while the suction side leg has decreased, as shown in Figure 5.8c. The increase in the pressure side leg can be attributed to entrainment of the upstream boundary layer fluid into the recirculating region. Along the separation line the incoming boundary layer fluid separates from the endwall. As this fluid enters the core flow region it is swept downstream of the separation line, where it rolls into the vortex core under the influence of the pressure gradient described above. It is this entrainment of increasing amounts of endwall boundary layer fluid that accounts for the growth in the pressure side leg
of the vortex. In addition to the increase in the pressure side leg of the horseshoe vortex there has been a corresponding increase in both the endwall-counter and corner vortices. The location of the pressure side leg of the vortex has moved to the middle of the passage, maintaining its position relative to the location of the separation line. At this axial location (Fig. 5.8c), the suction side leg of the vortex is much weaker, having a maximum vorticity magnitude of less than 15% of that of the pressure side leg. As the pressure side leg of the vortex moves closer to the suction surface of the passage it begins to influence the suction side leg of the vortex, which begins to lift away from the endwall. This is the beginning of a pattern of rotation of the suction surface vortex about the (much stronger) pressure side leg of the vortex, a pattern which will continue until the suction side leg is no longer discernible as a discrete vortical structure.

By the midchord location (50% axial chord), the pressure side leg has moved across the passage to a position close to the suction surface. To avoid confusion, this structure will be referred to as the passage vortex downstream of the midchord location. As can be seen in Figure 5.8d, there is no longer evidence of the suction side leg of the horseshoe vortex that was observed upstream. The endwall counter vortex structure is now located at the suction surface and endwall corner. The vortical structure that had been in the corner position has been lifted away from the endwall under the influence of that vortical motion associated with the passage vortex, as described above.

At the 75% chord position (Fig. 5.8e), the flow pattern has evolved further. The leading edge vortical structure has lifted away from the blade surface under the influence of the passage vortex, and due to diffusion has begun to merge with the larger passage vortex structure. The endwall counter vortex, still at blade-endwall corner, appears to have been stretched out as the flow structures rotate around the passage vortex. Above the passage vortex, the vorticity associated with the separated boundary layer fluid
from upstream is intensifying.

The last plane shown is the passage exit, 100% axial chord plane. Here the flow structures have evolved considerably from that at \( \frac{3}{4} \) chord. The upstream endwall fluid is now directly above the passage vortex structure, and has intensified. Both structures have moved away from the blade surface, due to the rapid thickening and separation of the laminar boundary layer on the blade surface.

The development of the horseshoe passage vortex in this simulation may be summarized as follows, and shown schematically in Figure 5.9. The incoming boundary layer first separates upstream of the blade leading edge, forming a saddle point flow at that location. The separation line from that saddle point extends across the passage to the suction surface of the next blade. The upstream boundary layer fluid lifts away from the wall at the separation, and rolls up to form a horseshoe vortex structure above and behind the separation line. High momentum fluid from outside the boundary layer flows around under the vortex and results in an increase in the shear stress and a decrease in the endwall static pressure immediately under the vortex. As this high momentum fluid approaches the rear of the separation line, it decelerates and is turned in a direction parallel to the separation line. It is this deceleration and turning that has occasionally been mistakenly identified as a separation line. This turning of the shear layer beneath the vortex results in a smaller counter vortex forming between the horseshoe vortex and the separation line. This combined two-vortex structure wraps around the blade leading edge, with the suction surface leg following the blade-endwall corner and the pressure side leg extending across the blade passage. As the vortex crosses the passage, it intensifies, as a result of the entrainment of the upstream boundary layer fluid by the vortex. Once the vortex reaches the suction surface of the passage, further development of the flow is dominated by the motion of the various structures about the passage.
5.2 Flow with Leading Edge Centered Circumferential Distortion

The effect of a circumferential inlet distortion on the formation of the passage vortex structure will be examined in this section. The structure of the incoming distortion is described, the resulting formation of vortex structure is examined, and a comparison is made to the structure resulting from that with a circumferentially uniform inflow.

5.2.1 Structure and Choice of Inflow Distortion

As was described in Chapter 4, the numerical solution technique involves the use of a Dirichlet boundary condition on the velocities and a homogeneous Neumann boundary condition for the solution of the pressure step. Thus, the total pressure may vary in the pitchwise direction. Since the blade loading results in a static pressure variation throughout the flow (blade potential field), the imposition of a fixed velocity at the inflow will yield a pitchwise total pressure gradient proportional to the strength of the loading-induced static pressure gradient at the inflow plane. To minimize this loading produced distortion, it is necessary to locate the inflow boundary at a location far upstream of the blade leading edge.

The objective of this investigation is to determine the effect of a circumferential distortion on the passage vortex formation. Thus the inflow location was chosen at a location that would be close enough to the blade to provide a 50% velocity deficit at the blade leading edge, yet would be far enough upstream to ensure that the total
pressure profile was uniform outside the specified distortion. To ensure that the loading-
induced variation in total pressure was less than 1% of the upstream dynamic head, it
was determined that the upstream boundary should be one axial chord upstream of the
blade leading edge.

The distortion profile imposed at the inflow boundary consists of a notched velocity
deficit, of unit magnitude, such that the imposed velocity at the distortion centerline is
zero, i.e.

\[
\begin{align*}
    u_0' &= u_0(y, z)[1 - \cos\left(\frac{2(y - y_c)}{\Delta y}\right)] \\
    v_0' &= v_0(y, z)[1 - \cos\left(\frac{2(y - y_c)}{\Delta y}\right)]
\end{align*}
\]  

(5.4) (5.5)

where \(\Delta y\) is the wake width and \(y_c\) is the location of the wake centerline. Figure 5.10
shows the contours of the midspan total pressure distortion upstream of the leading
edge. Diffusion of the initial profile results in an increase in the centerline total pressure
and a corresponding spreading of the profile. Figure 5.11 shows a cross section of the
inflow profile at .25 axial chord upstream of the leading edge. At this location the
profile has a 30% velocity deficit at midspan, corresponding to a total pressure deficit
\(C_{pt} = .82\). In addition to the pitchwise diffusion of the profile, significant diffusion
has also occurred above the blade endwall. Figure 5.12 shows the distribution in total
pressure across the distortion at the same location. Near the endwall the strength of
the distortion decreased from the 30% midspan value, to 8% at a distance of .2 from
the wall, to 1% at a distance of .05 above the wall. Since the strength of the horseshoe
vortex is related to the strength of the vorticity in the incoming endwall boundary layer,
it is the vortical region in the endwall that must change in order to have a significant
effect on the formation of the leading edge vortex. Since the distortion is less near
the wall than in the free stream region, the decrease in vorticity between this case and
that with circumferentially uniform inflow is less than was at first expected. Since this
decreases the effect the distortion can have on the leading edge flow, it may decrease the effect of the distortion on the passage vortex formation.

5.2.2 Leading Edge Horseshoe Vortex Formation

Figure 5.13 shows the contours of streamwise vorticity on a plane normal to the blade surface at $z = 0$. The figure shows vortical structures very similar to the corresponding structures in the uniform inflow solution. The horseshoe vortex core has a lower maximum vorticity, 12% less than that of the undistorted formation. In addition, the core is slightly more diffuse, and the centroid of the structure has moved forward from an axial location of $z = -1.18$ to $z = -1.15$, or 3% axial chord. The other two vortical structures have also weakened and become more diffuse. However, the net effect of the distortion on the formation of the horseshoe vortex is much less than was anticipated.

5.2.3 Passage Vortex Structure

The structure of the passage vortex is shown in Figure 5.14a through 5.14e. The figure shows contours of the streamwise vorticity on the same planes as were used to present the vorticity distributions in the computed solution with a circumferentially uniform inflow. At the leading edge plane, the distribution of vorticity is similar to that in the undistorted solution. The suction side leg of the vortex shows little change from the previous solution, with identical maximum vorticity and a slightly more diffuse structure. The pressure side leg of the vortex shows a similar structure to that of the Case 1, with a 11% increase in the maximum value of the vorticity in the vortex core.

At 20% axial chord the major difference in the two cases is the increase in vorticity along the passage pressure surface. This reflects the generation of streamwise vorticity
in that portion of the low momentum distortion fluid that convects down the pressure side of the blade. The passage distribution is again very similar to Case 1, with a small (10%) increase in the vortex strength. This increase in the maximum value of streamwise vorticity, and a corresponding increase in the net streamwise circulation associated with each structure, is the result of increase blockage in the passage. As the wake fluid convects down the blade surfaces, the combined blockage of the passage due to the blade surface boundary layer and the incoming wake fluid increases the velocity in the passage free stream. This increase in passage velocity results in increased stretching of the streamwise vorticity filaments, and hence in an increase in streamwise circulation.

The vortex structure through the remainder of the passage follows the pattern established in the first two axial locations shown. The structure of the vortex system remains unchanged from that given in Case 1, with a small increase in the maximum value of the vorticity in the passage and endwall vortex cores. The other flow structures, such as the suction surface corner vortex and blade separation are unaffected by the introduction of the distortion.

5.3 Effect of Inflow Distortion and Validity of Hypothesized Model

Two simulations have been presented showing the effect of the presence of a circumferential velocity deficit profile on the formation of the secondary flow and passage vortex system. The results presented indicate that the effect of the velocity deficit is to decrease the strength of the horseshoe vortex core within the low momentum distortion. Outside of the distortion, the vortex structure remains unchanged, with a small increase in the vortex strength, of the order of the increase in free stream velocity due to the
distortion induced blockage.

The hypothesized model described in Chapter 3 predicted a decrease in the strength of the secondary flow vortex proportional to the change in the net amount of normal vorticity at the blade leading edge. The model overpredicts the decrease in strength of the horseshoe vortex at the leading edge, and fails to describe the computed increase in the strength of the passage vortex system. The failure of the model to predict the observed behavior is related to the differences between the actual conditions leading to the formation of the vortex and those assumed in the model. As can be seen in Figure 5.13, the formation of the horseshoe occurs within one tenth span of the cascade endwall. To affect the formation of the vortex, the strength of the normal vorticity in this region must vary. The model assumes that the proportional decrease in the velocity will not vary with span as the wake diffuses, and that a given decrease in the midspan velocity will correspond to a similar decrease in the strength of the vorticity at the endwall. However the results show that this assumption does not hold. Figure 5.15 shows the velocity profiles at $x = -.3$, at the pitchwise location of the distortion centerline. The figure shows that at midspan the distortion maintains a 42% reduction in velocity from the undistorted case. This decrease in midspan velocity does not correspond to a decrease in vorticity in the endwall region. Within one tenth span of the endwall, the velocity profiles are nearly identical, with equal slopes. Since normal vorticity is proportional to the slope of this profile, it can be seen that the vorticity in this region is unchanged between the two cases. Since this is the region where the horseshoe vortex forms, similar vorticity distributions in this region will produce similar vortex strengths and structures. Thus, the persistence of normal vorticity near the endwall results in the formation of a horseshoe vortex at the leading edge, and the associated saddle point separation.
The presence of a separation line across the passage ensures a similar structure for the passage vortex. Although the strength of the horseshoe vortex is proportional to the value of the vorticity at the leading edge, the subsequent growth of that vortex as it crosses the passage is dependent only on the entrainment of the upstream endwall fluid. Since there is still a separation line, the enwall boundary layer still separates, and is entrained into the vortex core. Since the distortion raises the free stream velocity, the net circulation in that boundary layer is increased, and the strength of the passage vortex increases. Thus, to eliminate the passage vortex requires the elimination of the separation line across the passage. Since the separation line will form if there is any normal vorticity present at the leading edge, the elimination of the separation line requires the elimination of all endwall normal vorticity (or wake centerline velocity). Since such a velocity deficit cannot persist for a finite time in a viscous flow, the elimination of the separation line is not possible.

5.3.1 Source of Observed Variation in Vortex Structure

As was described in Chapter 3, previous investigations had identified two possible interactions as the source of the observed variation in secondary flow structure. These two interactions were the effect of a circumferential velocity deficit at the leading edge on the formation of the horseshoe vortex formation, and the effect of the movement of the wake fluid toward the blade suction surface due to the relative motion of the blade rows. The investigation described above has shown that the quasi-steady velocity deficit is insufficient to result in the observed variation in vortex structure. Thus, it can be deduced that the only remaining effect, that of the relative motion of the wake fluid, must be the cause of the observed variation in secondary flow structure.
Figure 5.1: Cascade geometry for flow simulation.

Figure 5.2: Computational domain discretization, using 164, 7th order spectral elements.
Figure 5.3: Comparison of two dimensional static pressure distribution with three dimensional mid span values.
Figure 5.4: Comparison of two dimensional total pressure distribution with three dimensional mid span values.
Figure 5.5a: Distribution of shear on the endwall surface, showing saddle point, attachment and separation lines.
Figure 5.5b: Distribution of shear on the endwall surface, showing saddle point, attachment and separation lines (cont.).
Figure 5.6a: Comparison of mid-span and endwall static pressure distributions showing characteristic endwall vortex trough.
Figure 5.6b: Comparison of mid-span and endwall static pressure distributions showing characteristic endwall vortex trough (cont.).
Figure 5.7a: Definition of leading edge normal planes.
Figure 5.7b: Distribution of streamwise vorticity about blade leading edge showing horseshoe vortex formation.
Figure 5.7c: Distribution of streamwise vorticity about blade leading edge showing horseshoe vortex formation (cont.).
Figure 5.7d: Distribution of streamwise vorticity about blade leading edge showing horseshoe vortex formation (cont.).
Figure 5.8a: Definition of axial planes for presentation of streamwise vorticity distributions.

Figure 5.8b: Streamwise vorticity distribution, 0% axial chord (leading edge plane).
Figure 5.8c: Streamwise vorticity distribution, 25% axial chord.

Figure 5.8d: Streamwise vorticity distribution, 50% axial chord.
Figure 5.8e: Streamwise vorticity distribution, 75% axial chord.

Figure 5.8f: Streamwise vorticity distribution, 100% axial chord (trailing edge plane).
Figure 5.9: Formation of the passage and endwall counter vortex system, and relationship to leading edge flow structures.
Figure 5.10: Total pressure distortion at mid-span, showing convection and diffusion of inflow profile.

Figure 5.11: Cross section of the distortion in velocity magnitude, on an axial plane upstream of leading edge saddle point, $z = -0.25$. 
Figure 5.12: Cross section of the distortion in total pressure, on an axial plane upstream of leading edge saddle point, $x = -0.25$.
Figure 5.13: Distribution of streamwise vorticity normal to leading edge, with distorted inflow.
Figure 5.14a: Streamwise vorticity distribution with inlet distortion, 0% axial chord (leading edge plane).

Figure 5.14b: Streamwise vorticity distribution with inlet distortion, 25% axial chord.
Figure 5.14c: Streamwise vorticity distribution with inlet distortion, 50% axial chord.

Figure 5.14d: Streamwise vorticity distribution with inlet distortion, 75% axial chord.
Figure 5.14e: Streamwise vorticity distribution with inlet distortion, 100% axial chord (trailing edge plane).

Figure 5.15: Velocity profiles with clean and distorted inflow, upstream of the saddle point location $x = -0.3$.
Part II

Generation of Total Pressure Loss in Cascade Flows
Introduction to Part II

The presence of viscous diffusion in the flow through turbine cascades results in a decrease in the integrated flux of total pressure through the cascade. Since this decrease in total pressure flux is related to the amount of kinetic and potential energy lost by the flow, it is commonly referred to as total pressure loss, or simply “loss”. This decrease in total pressure has a detrimental effect on the efficiency of the blade row [12], hence it is of interest to minimize the total pressure loss. In order to reduce this loss however, it is necessary to understand the relationship between the generation of loss and the structure of the flow through the blade row.

Part II of this thesis examines the generation of loss in a planar turbine cascade. The purpose of this examination is twofold. First, in order for one to reduce secondary losses in turbines, one has to understand how those losses are related to the vortical structure of the flow. Numerical results of the calculation of loss are presented, in an effort to relate the generation of total pressure loss in the flowfield to the vortical structures associated with the secondary flow. Second, different published techniques for estimating secondary losses are based on widely different assumptions and physical phenomenology, but do predict secondary losses to the same order of accuracy [41]. It is thus unclear what are the dominant mechanisms in the generation of secondary loss in cascades. The relative contribution of each of the kinematic phenomena involved in loss generation are examined in an effort to determine which mechanisms are important in the generation of secondary loss.

The first chapter in Part II presents the results of the numerical simulation of total pressure loss in a planar turbine cascade in conventional form, i.e. we adopt procedures outlined in the open literature. The gross loss in the cascade is decomposed into profile
and secondary loss components following accepted procedures [37],[24] for analyzing those losses. Results are presented showing the spatial distribution of total pressure throughout the cascade, profiles of net loss and secondary loss at various axial locations, and the axial variation in profile and secondary loss through the cascade. Attempts are then made to relate the calculated secondary loss to the flow structures described in Part I.

The second chapter in Part II presents the generation of total pressure loss in an unconventional form, i.e. we propose techniques not previously applied to the study of loss generation in three dimensional viscous flows. A new formulation of the equation governing the generation of loss in a viscous flow is presented, based on the decomposition of the viscous diffusion into kinematic components consisting of streamwise, normal and binormal vorticity. This formulation is then used to evaluate the source of loss in the simulated flowfield in an effort to relate the generation of secondary loss to specific vortical structures in the flow, and to determine the relative contribution of the different terms in the governing equation.
Chapter 6

Secondary Total Pressure Loss in Cascades

This chapter examines the distribution of loss in the cascade passage. Following the methods and nomenclature for secondary loss generation from the open literature[37],[24], the net loss in total pressure is divided into two components, profile or blade surface loss, and endwall or secondary loss. Such a division aids in the identification of that portion of the loss due to the presence of secondary flows in the passage. Understanding the source of this loss is the first step in the reduction of the effect of secondary flows on turbine cascade efficiency. Thus, this chapter attempts to relate the formation of secondary loss in the cascade passage to the observed flow structures that were described in Chapter 5.

6.1 Components of Total Pressure Loss

The sources of loss in a cascade are usually divided into two groups. The first is the loss due to the boundary layer on the blade surface, termed profile loss due to its relationship to the shape of the blade profile [39]. The remainder of the loss is that due to the effect of a finite aspect ratio and the presence of solid endwalls, usually termed the secondary loss. The secondary loss will thus include the loss from boundary layers on the endwall wetted surfaces, losses due to separated free shear flows, diffusion of the passage secondary vorticity, and additional loss due to changes in the blade surface
boundary layers caused by secondary flows. To minimize the loss, it is important to understand the magnitude of the relative contributions of each of these components, and to be able to relate those contributions to the flow structures that are present in the cascade passage.

### 6.1.1 Definition of Total Pressure Loss

To evaluate losses in turbomachinery cascades, it is necessary to assign a numerical value to that loss. Thus loss is usually expressed as the ratio of the average total pressure loss to some reference dynamic pressure. To eliminate changes in reference total pressure with changing inflow angle, the definition used here is based on the axial dynamic pressure. Defining $Y$ as the average total pressure loss in the cascade, then

\[
Y = \frac{\text{Average Stagnation Pressure Deficit}}{\text{Upstream Axial Dynamic Head}}
\]  

(6.1)

The deficit in stagnation pressure is the difference between the local total pressure and some reference total pressure. By taking the reference pressure as the free stream total pressure far upstream we can define a coefficient of total pressure loss,

\[
C_{pt} = \frac{P_{\infty} - P_t}{\frac{1}{2} U_{x_{\infty}}^2}.
\]  

(6.2)

To find the average loss at a given axial location, we must define averaging operators. First, to reduce the field quantities to spanwise profiles at each axial location, we can define a mass weighted gap average,

\[
f(x, z)_{\text{gap ave}} = \frac{\int_{s_{\infty}}^{p_{\infty}} u(x, y, z) f(x, y, z) dy}{\int_{s_{\infty}}^{p_{\infty}} u(x, y, z) dy}
\]  

(6.3)

where the integral is taken from the suction surface of one blade to the pressure surface of the adjacent blade. This averaged quantity will be referred to as the gap-averaged quantity, since the average is calculated across the blade gap. To reduce the field
quantifies one dimension further, we can define a mass weighted passage average, such
that
\[ f(x)_{\text{pass. ave.}} = \frac{\int_0^h \int_{p_s}^{p_u} u(x, y, z) f(x, y, z) \, dy \, dz}{\int_0^h \int_{p_s}^{p_u} u(x, y, z) \, dy \, dz} \] (6.4)

The spanwise \((z)\) integration is performed from the endwall to midspan, taking advan-
tage of the symmetry in the planar cascade field. Upstream and downstream of the
blade the pitchwise average \((y)\) is taken over one blade pitch. This averaged quantity
will be referred to as the passage averaged quantity, since the average is calculated over
the passage cross section. The passage average gives the value of the average flux of the
quantity \(f\) as a function of axial position through the cascade passage.

The averaging process used to calculate the passage average involves two potential
sources of error. The first is the spectral interpolation onto planes of constant axial loca-
tion. The second approximation is the numerical integration of the values interpolated
onto that plane. Following the method of Langston [24], the value of the integrated
mass flux on the interpolated planes is used to evaluate the accuracy of the numerical
integrations of equation (6.10). Figure 6.1 shows the mass flux fraction \(\eta\), as a function
of axial location, defined as
\[ \eta(x) = \frac{\int_0^h \int_{p_s}^{p_u} u(x, y, z) \, dy \, dz}{\int_0^h \int_{p_s}^{p_u} u(0, y, z) \, dy \, dz} \] (6.5)
calculated using the same procedure used to evaluate the loss. The figure indicates
that the maximum error in mass flux is .13\%, hence the interpolation and integration
procedure can be expected to introduce errors of this magnitude.

Using the above definition of the passage average, the net total pressure loss can
thus be written as the passage average of the total pressure loss coefficient at a given
axial location, or
\[ Y(x) = \frac{\int_0^h \int_{p_s}^{p_u} u(x, y, z) C_p(x, y, z) \, dy \, dz}{\int_0^h \int_{p_s}^{p_u} u(x, y, z) \, dy \, dz} \] (6.6)
The loss as defined above will not only include the losses that occur within the cascade up to the point of integration, but will also include that loss associated with the incoming endwall shear layer. To yield useful results, some way of determining what percentage of the total loss has occurred within the passage must be developed.

6.1.2 Division of Passage and Incoming Loss

To determine the effect of passage secondary flow on the level of loss produced in a cascade, it is necessary to first determine how much of that loss is due to the incoming boundary layer. Sharma [37] compared the measured cascade losses from various experiments, where each was conducted with both thick and thin inlet boundary layers. In all the cases examined, the difference between the passage averaged loss for the flow with thick and thin boundary layers was constant through the passage, as is shown in Figure 6.2. The plot shows the loss as a function of chord for three different experiments, and the loss versus span for a fourth experiment. In each case, loss was measured both with thin and thick inlet boundary layers, corresponding to two different inlet loss levels for each experiment. As each of the first three graphs show, the difference in loss between the two inlet conditions is entirely accounted for by the difference in inlet loss level. The fourth graph shows that increasing the inlet loss increases the exit loss in the endwall regions, but has no effect on the loss over the midspan region. Sharma thus concluded that the loss within the cascade passage is independent of the amount of loss in the incoming boundary layer, and that the effect of the inlet total pressure deficit was simply additive. Thus the loss at a given axial location can be split between the inlet losses and the cascade passage loss. Following Sharma, if we let $Y_i$ represent the loss contained within the cascade inlet boundary layer, and $Y_{pass}$ represent the loss
due to flow in the cascade passage, then

$$Y = Y_{pass} + Y_i.$$ \hspace{1cm} (6.7)

Such a split allows for the evaluation of the loss occurring in the blade passage independent of the loss associated with the incoming boundary layer.

6.1.3 Division of Profile and Endwall Loss

The loss in the cascade passage can further be split into two loss components, the profile or blade surface loss, and the endwall or secondary loss, as

$$Y_{pass} = Y_{sec} + Y_p.$$ \hspace{1cm} (6.8)

The profile loss is that portion of the loss due to the boundary layers on the blade wetted surface. This profile loss can be defined as the limit of the gap averaged midspan loss as the aspect ratio approaches infinity \[37\], or

$$Y_p = \lim_{l \rightarrow \infty} Y_{gap \ ave. \ \frac{h}{l}}.$$ \hspace{1cm} (6.9)

Thus the profile loss can be thought of as that portion of the loss that is independent of aspect ratio, and may be evaluated by using the loss calculated for an infinite aspect ratio (or 2-D) geometry.

It should be emphasized that it is incorrect to set the profile loss level equal to the mid-span loss in finite aspect ratio cascades. As was seen in Chapter 4, the mid span boundary layer in low aspect ratio cascades may be significantly thicker than that given in the limiting case of infinite aspect ratio. As pointed out by Sharma, \[37\] using mid span loss as the profile loss in the above relation may result in the calculation of negative endwall loss. The correct method should make use of either measurement of loss in a
very high aspect ratio cascade or through the use of a two dimensional simulation. As we will describe in the next section, this investigation makes use of such a mass weighted two dimensional calculation to evaluate the profile loss level.

6.1.4 Evaluation of Profile Loss

Profile loss has been defined as that portion of the loss independent of blade aspect ratio. For the present investigation the profile loss is evaluated using a two-dimensional simulation, with the same geometry as the mid-span section of the three-dimensional flow. Defining \( Y_{2D} \) as the gap averaged loss for the two dimensional flow, we write

\[
Y_{2D}(z, y) = \frac{\int_{z_1}^{z_2} u(z, y) C_{pt,2D}(z, y) \, dy}{\int_{z_1}^{z_2} u(z, y) \, dy}.
\]

At each axial station, the profile loss can be subtracted from the net passage loss to yield the value of the secondary loss at that location.

6.2 Evaluation of Loss in Planar Cascade Flow

The numeric procedure developed in Chapter 4 was used to calculate the distribution of total pressure in the planar cascade geometry used in Part I. Results presented in this section include evaluation of the spatial distribution of total pressure in the cascade passage, gap averaged profiles of total pressure loss at various axial locations, and the variation of passage averaged loss as a function of axial chord.
6.2.1 Profile Loss

As was described above, profile loss is calculated using a two dimensional simulation of flow about the cascade midspan profile. To evaluate the profile loss, the distribution of the total pressure loss coefficient was mass averaged at various axial locations to get the variation in 2-D loss as a function of axial chord.

Since the two dimensional loss thus calculated is for a unit inflow velocity, it would be incorrect to assign this level of profile loss to the spanwise locations with a lower mass flow. Hence the profile loss is calculated by mass weighting $Y_{2D}$ with the integrated mass flow in the three dimensional simulation divided by the effective flux in the 2D simulation, or

$$Y_p = Y_{2D} \frac{\int_0^{\frac{h}{2}} \int_{p_{\text{in}}}^p u(x, y, z) \, dy \, dz}{\frac{h}{2} \int_{p_{\text{in}}}^p u(x, y)_{2D} \, dy}.$$  \hspace{1cm} (6.11)

As was described in Chapter 5, the upstream boundary layer is represented by an eighth order velocity profile, hence, the integrated mass flux is the integral of the axial component of that profile, i.e.

$$\dot{m} = \int_0^{\frac{h}{2}} \int_{-\frac{5}{2}}^{\frac{5}{2}} u(y, z)_{\infty} \, dy \, dz.$$  \hspace{1cm} (6.12)

where

$$u(y, z)_{\infty} = 1 - (2z)^8$$  \hspace{1cm} (6.13)

hence,

$$\frac{\dot{m}_{3D}}{h\dot{m}_{2D}} = .8888.$$  \hspace{1cm} (6.14)

Thus the profile loss $Y_p = .8888Y_{2D}$. The profile loss thus calculated is shown in Figure 6.3. It can be seen from the figures that the value at the leading edge plane is non-zero, indicating that some diffusion of the boundary layer has occurred "upstream" of this plane as the boundary layer fluid flows around the blade leading edge. The figure shows
that the loss increases at a slower rate after the initial increase near the leading edge, then much more rapidly after the separation point. The value of the 2-D loss at the trailing edge was $Y_{2D} = .7625$ with 39% of that loss having been generated downstream of the separation point. This profile loss can now be used to evaluate the amount of loss due to the secondary flow.

6.2.2 Spatial Distribution of Total Pressure

Figure 6.4a through 6.4e show the axial evolution of the total pressure field through the cascade. Plotted are contours of the total pressure loss coefficient, as defined above, at several axial locations. At the leading edge plane of Figure 6.4a, the blade boundary layers are shown on the passage side surfaces. The term pressure surface boundary layer is not applicable at this location, as the fluid on that surface of the blade is the beginning of the suction surface boundary layer. The pressure surface boundary layer begins at the midspan stagnation point, located at an axial location of .023. The figure (6.4a) indicates that at the leading edge plane there is little variation from a two dimensional boundary layer over much of the span, as is shown by the existence of straight parallel contours over those portions of the boundary layers. However, near the endwall the blade surface boundary layer is beginning to distort under the influence of the horseshoe vortex about the blade leading edge. On the endwall of the passage we have the low total pressure fluid associated with the upstream inflow profile. The deflection of this incoming boundary layer fluid under the action of the leading edge vortical flow is evident near the two blade surfaces.

The 25% chord location, shown in Figure 6.4b, still indicates a two dimensional blade boundary layer behavior over much of the span. The endwall boundary layer however is being affected by the separation and vortex rollup. To the right of the separation the
level of total pressure loss is closer to that of the free stream, reflecting the convection of high momentum fluid around the passage vortex. The higher shear beneath the vortex results in a stronger total pressure gradient beneath the vortex location. The loading produced acceleration of the flow to the left (upstream) of the separation causes an increase in the endwall total pressure gradient in that region. Near the endwall, the suction surface boundary layer is beginning to thin out (by comparison with that at the midspan) under the action of the secondary vortical motion.

The total pressure contours in Figure 6.4c for the mid-chord plane show that the boundary layer retains a two dimensional character above 25% span, although the thickness is greater than that calculated for a two dimensional flow. The suction surface boundary layer below 25% span, which can be identified as the region of straight parallel contours near the surface, is much thinner than that at mid-span, and much of the upstream endwall boundary layer has been entrained into the passage vortex. Beneath the passage vortex, low total pressure fluid is being lifted from the wall, under the combined influence of the passage and counter vortices. Between this low total pressure fluid and the passage vortex center, there is an increase in total pressure, which is related to the convection of higher total pressure free stream fluid around the passage vortex.

At the three quarter chord location, shown in Figure 6.4d, the blade surface boundary layer has begun to separate along the separation line at \( z = -.23 \). This is the \( S_2 \) separation line identified by Langston [24] and Sieverding [43] as the separation line extending from the saddle point near the leading edge. In addition to this separation, the boundary layer fluid on the blade surface has been distorted by the impingement of the higher total pressure fluid from beneath the passage vortex. Near the passage bottom, the passage endwall boundary layer is thicker than was observed at the upstream locations.
The trailing edge plane total pressure distribution is dominated by the three dimensional separation of the blade boundary layer, as is shown in Figure 6.4e. This separation moves the low total pressure fluid from the blade surface out into the cascade passage, above the passage vortex structure. This separation also influences the flow at midspan, pulling fluid away from the midspan surface. Although this motion eliminates the suction surface separation seen in the two dimensional simulation, it does result in a thickening of the midspan suction surface boundary layer, giving a net thickness greater than that of the separated two dimensional boundary layer.

In summary, the passage vortex and three dimensional separation have a pronounced effect on the axial evolution of the total pressure loss in the cascade. The spatial distribution of total pressure shows that the three dimensional separation of the blade boundary layer moves low total pressure fluid from the blade surface out into the mid-passage region. The presence of the horseshoe vortex increases the shear on the endwall and lower portion of the endwall, resulting in an increase in total pressure loss in those regions. The total pressure in the vortex core is lower than the free stream fluid, but is of the same order as that of the upstream endwall boundary layer fluid, suggesting that the loss within the vortex may be due to the entrainment of the inlet boundary layer fluid only, rather than being the result of additional loss due to the presence of the passage vortex.

The total pressure distributions above provide some clues as to the nature of the sources of loss due to secondary flow in the passage. It remains difficult however to determine what physical mechanisms are responsible for the distribution of loss in total pressure shown. It is thus necessary to perform additional reduction of the total pressure field values before definite conclusions can be drawn about the relationship between the generation of the observed loss and the secondary flow structures. The next step in the
reduction is the calculation of gap averaged profiles of loss at each station. Comparison of these profiles will show the growth of the gap averaged loss through the cascade, and will indicate the spanwise distribution of the change in loss.

6.2.3 Spanwise Total Pressure Profiles

Gap averaged profiles of the secondary loss as described above are shown in Figure 6.5. Profiles are given for 11 different axial location, from cascade inlet to exit. Comparison of these profiles shows the growth in the loss versus span at each axial location.

The profile for the leading edge plane shows the loss due to the incoming boundary layer below 25% span, along with the small amount of loss at midspan due to the blade boundary layer flow around the leading edge. The next two profiles are similar to that at the leading edge, showing the loss due to the inlet boundary layer, with increasing loss due to the diffusion of the blade boundary layers, as indicated in the figure. Yet the profiles show that the value of the gap averaged total pressure between 5 and 10% span is not increasing, suggesting minimal loss generation in this region. Examination of the profiles for 30, 40 and 50% axial chord show that the average total pressure loss in this spanwise region actually decreases with increasing axial distance. This indicates that the low total pressure fluid that had been at this spanwise location is being convected toward midspan, increasing the loss toward midspan of this local minimum.

Downstream of 50% percent chord the loss profiles begin to take on a different shape. While the loss at 10% span location continues to decrease, the value of the gap averaged loss between 10 an 20% span is increasing, indicating convection of the low total pressure endwall fluid toward the midspan. In addition, the profile is beginning to show a second
local maximum near 30% span. This local maximum is associated with the thickening of the boundary layer due to the spanwise migration of fluid between the blade surface and the passage vortex, as was shown above.

The profiles at 80, 90, and 100% axial chord show that the loss at the 10% span is increasing, having reached a minimum near the 70% axial chord position. Downstream of this the loss increases at the same rate as that close to the endwall, indicating that the upstream boundary layer fluid has convected out of the endwall region. Above the 10% span location there is also a change in form from the upstream profiles. An additional local maximum forms at the 80% chord location, and increases more rapidly than the surrounding flow structures. This local maximum is associated with the three dimensional separation of the blade surface boundary layer along the surface separation line, as identified by Langston [24] and Sharma [38].

The profiles presented show that both the thickening of the boundary layer due to the spanwise migration of the upstream endwall fluid and the three dimensional separation of the blade surface flow are major contributors to the secondary loss. Yet from these profiles it is still difficult to determine how much of the loss is related to the upstream boundary layer and what portion is directly attributable to the presence of the passage vortex. Although the vortex results in a redirection of the flow, no numerical value can be assigned to that portion of the loss due to the diffusion of that secondary flow vortex, or to the portion of the flow that has been affected by the presence of the vortex. In order to assign a specific value to this it is first necessary to evaluate the profile loss, then to determine how the loss in the passage varies from that of the two dimensional calculation.
6.2.4 Passage Averaged Net and Secondary Loss

The passage averaged loss is shown in Figure 6.7 as a function of axial chord. The plot shows the variation in the net loss through the cascade. Also shown on the figure are the relative contributions of the inlet loss, profile loss, and secondary loss. The numerical results used to produce this figure are also shown in Table 6.1. The results show that the secondary loss contribution as calculated by this process is negligible up to 35% chord. Beyond this, the secondary loss becomes a larger portion of the passage loss, accounting for 24% of the passage loss at the exit plane. The results that show negligible secondary loss in the first third of the passage should be viewed with some doubt, since there exists strong secondary vorticity in this region. This may point to

<table>
<thead>
<tr>
<th>$\bar{\xi}$</th>
<th>$Y_{in}$</th>
<th>$Y_p$</th>
<th>$Y_{sec}$</th>
<th>$Y_{pass}$</th>
<th>$Y_{net}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.122</td>
<td>0.041</td>
<td>0.006</td>
<td>0.047</td>
<td>0.169</td>
</tr>
<tr>
<td>0.1</td>
<td>0.122</td>
<td>0.110</td>
<td>-0.000</td>
<td>0.109</td>
<td>0.232</td>
</tr>
<tr>
<td>0.2</td>
<td>0.122</td>
<td>0.158</td>
<td>-0.001</td>
<td>0.156</td>
<td>0.279</td>
</tr>
<tr>
<td>0.3</td>
<td>0.122</td>
<td>0.200</td>
<td>-0.000</td>
<td>0.200</td>
<td>0.322</td>
</tr>
<tr>
<td>0.4</td>
<td>0.122</td>
<td>0.240</td>
<td>0.005</td>
<td>0.245</td>
<td>0.368</td>
</tr>
<tr>
<td>0.5</td>
<td>0.122</td>
<td>0.283</td>
<td>0.019</td>
<td>0.301</td>
<td>0.424</td>
</tr>
<tr>
<td>0.6</td>
<td>0.122</td>
<td>0.331</td>
<td>0.044</td>
<td>0.374</td>
<td>0.497</td>
</tr>
<tr>
<td>0.7</td>
<td>0.122</td>
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<td>0.076</td>
<td>0.465</td>
<td>0.588</td>
</tr>
<tr>
<td>0.8</td>
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<td>0.461</td>
<td>0.123</td>
<td>0.585</td>
<td>0.707</td>
</tr>
<tr>
<td>0.9</td>
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<td>0.569</td>
<td>0.185</td>
<td>0.754</td>
<td>0.876</td>
</tr>
<tr>
<td>1.0</td>
<td>0.122</td>
<td>0.759</td>
<td>0.243</td>
<td>1.001</td>
<td>1.124</td>
</tr>
</tbody>
</table>

Table 6.1: Passage averaged inlet, profile, secondary, passage, and net loss.
flaws in the logic of using a two dimensional simulation to estimate profile loss. Hence in order to correctly calculate the loss due to the presence of secondary vorticity, it may be necessary to reformulate the assignment of loss in the passage.

6.3 Relationship Between Secondary Loss and Secondary Flow Structures

We have attempted to use the computed results to evaluate the various individual sources of loss in total pressure. The gross loss can then simply be deduced from the sum of the various components of the loss. Results have been presented of the division of the gross loss in the passage into components relating to different physical mechanisms. This division of the loss suggests that the secondary flow results in negligible loss in the first third of the cascade passage. From this point to the cascade exit the secondary loss increases, accounting for a quarter of the passage loss at the exit plane.

Gap averaged loss profiles show that the largest source of loss in the passage can be attributed to the presence of the three dimensional separation above the passage vortex. This separation brings low total pressure fluid out into the mid passage region, resulting in an increased flow blockage and an increased passage loss. This separation is caused by the presence of the passage vortex structure, and is thus included in the secondary loss thus calculated. It is not however, directly due to the diffusion of the vortex itself. Additional sources of loss are the thickening of the midspan boundary layer under spanwise secondary flow, and the spanwise migration of low total pressure endwall fluid.

The presented results have shown that the presence of secondary flow increases the
loss in the cascade, and have suggested which flow structures are responsible for that increase in loss. Yet the results do not allow quantitative evaluation of the amount of secondary loss generated by each flow structure. To accomplish such a quantification we have found it necessary to reformulate the classification of loss. Such a reformulation may allow one to have a better appreciation of the relative contributions of the sources of loss discussed in the above.
Figure 6.1: Integrated mass flux as a function of axial location.
Figure 6.2: Comparison of measured cascade losses with thick and thin inlet boundary layers (from [37])
Figure 6.3: Axial variation of two dimensional integrated profile loss, $Y_p$. 
Figure 6.4a: Distribution of total pressure loss coefficient, 0% axial chord (leading edge plane).

Figure 6.4b: Distribution of total pressure loss coefficient, 25% axial chord.
Figure 6.4c: Distribution of total pressure loss coefficient, 50% axial chord.

Figure 6.4d: Distribution of total pressure loss coefficient, 75% axial chord.
Figure 6.4e: Distribution of total pressure loss coefficient, 100% axial chord (trailing edge plane).

Figure 6.5: Spanwise profiles of gap averaged total pressure.
Figure 6.6: Axial variation in individual loss components.
Figure 6.7: Axial variation in integrated net passage loss, with contributions from inlet, profile, and secondary components shown.
Chapter 7

Loss Evaluation in an Intrinsic Coordinate System

The previous chapter presented the calculation of the generation of loss in a planar cascade. This loss was broken down into component losses in an attempt to determine quantitatively the contribution of each of the observed flow structures to the overall loss generation. This division was accomplished following accepted procedures for the division of loss in turbine cascades, and was based on the use of a two dimensional simulation to evaluate the blade surface loss. Yet the results presented did not allow definite conclusions to be drawn about the relative contributions of each flow structure. Since the loss was divided into inlet, profile and secondary losses, the only conclusion that could be drawn was about the relative magnitude of those contributions. Since secondary loss as calculated includes both increases in wetted surface losses due to secondary flow, and the additional diffusion of the free shear layers and streamwise vortices, both effects are included in the value of the secondary loss. To evaluate the relative contributions of these components of secondary loss, an expression relating the generation of loss to those flow structures is required.

This chapter presents a reformulation of the equation describing the evolution of loss in an intrinsic coordinate system. An equation is derived relating the convective derivative of total pressure to the diffusion. The diffusion is then rewritten in terms of intrinsic coordinates. The resulting expression allows the loss in total pressure to be
related to those components involving surface shear and free shear, and that involving streamwise vorticity or streamline skew. The magnitudes of each of the component terms in the resulting expression are then evaluated in the simulated flowfield. This allows a quantitative evaluation of the relative contribution of each of the components to the overall generation of total pressure loss.

7.1 Governing Equations

In this section the equations governing the loss in total pressure in a viscous fluid are derived. The diffusion term is then rewritten in terms of intrinsic coordinates to elucidate the role of the streamwise component of vorticity in the generation of loss. A simple model flowfield is then examined to show the physical significance of the various terms in the resulting expression.

7.1.1 Convective Derivative of Total Pressure

Consider the incompressible Navier-Stokes equations

\[
\frac{\partial \bar{u}}{\partial t} = \bar{u} \times \bar{\omega} - \nabla P_t - \frac{1}{Re}(\nabla \times \bar{\omega})
\]  

(7.1)

For a steady flow the time derivative is equal to zero, and the equation reduces to

\[
\nabla P_t = \bar{u} \times \bar{\omega} - \frac{1}{Re}(\nabla \times \bar{\omega})
\]  

(7.2)

If we dot the above vector equation with the velocity the first term on the right hand side is identically zero,

\[
\bar{u} \cdot (\bar{u} \times \bar{\omega}) = 0
\]  

(7.3)
and the resulting equation becomes

\[ \bar{u} \cdot \nabla P_t = -\frac{1}{Re} \bar{u} \cdot (\nabla \times \bar{\omega}) \]  \hspace{1cm} (7.4)

In a steady flow the left side of the above equation is the convective derivative of total pressure, which expresses the rate of change of total pressure following the fluid particles. The equation thus relates the volumetric rate of change of total pressure in the flowfield to the local kinematics of the vorticity field. Integrating this quantity will then yield the net rate of change and hence the flux of total pressure into or out of the volume of integration. Using the divergence theorem we may write

\[ \int \int \int \bar{u} \cdot \nabla P_t \, dV = \int_S \int P_t (\bar{u} \cdot \hat{n}) \, dS \]  \hspace{1cm} (7.5)

which relates the volumetric integral of the convective derivative of total pressure to the flux of total pressure through the surface surrounding that volume. It we replace the convective derivative by the equivalent expression from equation (7.4) we may write

\[ -\int \int \int \frac{1}{Re} \bar{u} \cdot (\nabla \times \bar{\omega}) \, dV = \int_S \int P_t (\bar{u} \cdot \hat{n}) \, dS \]  \hspace{1cm} (7.6)

which relates the flux of total pressure through the surface to the kinematics of the vorticity field within the volume enclosed by that surface.

In the previous chapter an expression was given defining the passage loss as the difference between the loss at a given location and the inlet loss. Thus, if we define a control volume within a surface coinciding with the blade surfaces, the endwall and mid-span surfaces, the blade leading edge plane, and an axial plane at any location within the blade passage, then the surface integral in the above equation becomes

\[ \int_S \int P_t (\bar{u} \cdot \hat{n}) \, dS = \int_0^{\frac{h}{2}} \int_{ss} u(x, y, z) P_t(x, y, z) \, dy \, dz - \int_0^{\frac{h}{2}} \int_{ss} u(0, y, z) P_t(0, y, z) \, dy \, dz \]  \hspace{1cm} (7.7)

The right hand side of this equation is proportional to the passage loss defined in the previous chapter. Thus, we can relate the passage loss at any axial location to the
volume integral involving the diffusion of vorticity in the passage upstream of that location

$$\int_0^P \int_0^{1/2} \frac{1}{Re} \bar{u} \cdot (\nabla \times \bar{\omega}) \, dy \, d\zeta = c_1 Y_{pass}$$

(7.8)

Such an equation provides a link between the loss of total pressure and the distribution of vorticity in the cascade passage. In order to be useful it is necessary to be able to relate the integrand in the above expression to flow structures that are observable in the cascade flowfield. To accomplish this it is helpful to express the integrand in terms of an intrinsic coordinate system, which will yield an expression in terms of the streamwise, normal and binormal components of vorticity and their spatial gradients.

### 7.1.2 Flowfield Description in Intrinsic Coordinates

Before we can begin to describe a flow in an intrinsic coordinate system we must first learn how to evaluate the various operators in terms of the intrinsic coordinates $(s,n,b)$, where $s$ denotes the streamwise direction, $n$ denotes the normal direction, and $b$ the binormal direction. To be more specific, we are interested in obtaining an equation that describes the rate of change of total pressure following a fluid element in terms of the streamwise, normal and binormal vorticity $(\omega_s, \omega_n, \omega_b)$. Since we have defined secondary vorticity as vorticity in the streamwise direction, such an equation will provide a link between the existence of secondary flow and loss of total pressure in the cascade passage. If we let $\hat{s}$ be the local unit vector in the streamwise direction, then the local velocity can be written as

$$\bar{u} = q \hat{s}$$

(7.9)

so that

$$\hat{s} = \frac{\bar{u}}{q}$$

(7.10)
Likewise, we will let $\hat{n}$ denote a unit vector in the plane of curvature of the streamline, the plane which contains both the streamline and the convective acceleration vectors. The convective acceleration in a steady flow is given as

$$\vec{a} = \vec{u} \cdot \nabla \vec{u} \quad (7.11)$$

Using the gradient operator

$$\nabla = (\hat{s} \frac{\partial}{\partial s}, \hat{n} \frac{\partial}{\partial n}, \hat{b} \frac{\partial}{\partial b}) \quad (7.12)$$

equation (7.11) can be rewritten as

$$\vec{u} \cdot \nabla \vec{u} = \frac{\partial q}{\partial s} \hat{s} + q \frac{\partial \hat{s}}{\partial s} = \vec{a}_s + \vec{a}_n \quad (7.13)$$

where we have used $\hat{b}$ to denote the unit vector in the binormal direction. Upon subtracting the streamwise component of the acceleration we obtain the equation for the normal component of acceleration as

$$\vec{a}_n = q \frac{\partial \hat{s}}{\partial s} = \vec{u} \cdot \nabla \vec{u} - \frac{\partial q}{\partial s} \hat{s}. \quad (7.14)$$

We can then use equation (7.14) to compute the unit normal vector as

$$\hat{n} = \frac{\vec{a}_n}{|\vec{a}_n|} \quad (7.15)$$

Finally, the unit vector $\hat{b}$ in the binormal direction is simply given as

$$\hat{b} = \hat{s} \times \hat{n}. \quad (7.16)$$

With the above definition of an intrinsic coordinate system, we can now derive the desired expression for the rate of change in total pressure following a fluid element. In the intrinsic coordinate system, the vorticity field can be computed from

$$\vec{\omega} = \nabla \times \vec{u} \quad (7.17)$$

$$= \omega_s \hat{s} + \omega_n \hat{n} + \omega_b \hat{b} \quad (7.18)$$

$$= -q(\phi + \sigma) \hat{s} + \frac{\partial q}{\partial b} \hat{n} + (q \kappa - \frac{\partial q}{\partial n}) \hat{b} \quad (7.19)$$
where

\[ \kappa = \ \text{curvature of the streamline} \]
\[ = \hat{n} \cdot \frac{\partial \hat{s}}{\partial s} \]  \hspace{1cm} (7.20)
\[ = -\hat{s} \cdot \frac{\partial \hat{n}}{\partial s} \]  \hspace{1cm} (7.21)

\[ \phi = \ \text{normal rate of change of} \ \hat{s} \ \text{in the binormal direction} \]
\[ = -\hat{b} \cdot \frac{\partial \hat{s}}{\partial n} \]  \hspace{1cm} (7.22)

and

\[ \sigma = \ \text{binormal rate of change of} \ \hat{s} \ \text{in the normal direction} \]
\[ = -\hat{n} \cdot \frac{\partial \hat{s}}{\partial b}. \]  \hspace{1cm} (7.23)

Thus the quantity \((\phi + \sigma)\) represents the rate of streamline skew in the directions perpendicular to the streamline. Note that the streamwise component of vorticity involves only derivatives of direction, and no derivatives of velocity magnitude. If we define "shear" as a gradient in velocity magnitude, then the normal and binormal vorticity relate to the shear, while the streamwise vorticity relates only to the streamline skew.

Finally, dotting velocity with the curl of the vorticity gives

\[ \bar{u} \cdot \nabla \times \bar{\omega} = q\left(\frac{\partial \omega_b}{\partial n} - \frac{\partial \omega_n}{\partial b}\right) + \omega_s^2 + q(\omega_n \eta + \omega_b \lambda) \]  \hspace{1cm} (7.24)

and, using equation (7.4) we arrive at

\[ \bar{u} \cdot \nabla P_T = -\frac{1}{Re} \left[q\left(\frac{\partial \omega_b}{\partial n} - \frac{\partial \omega_n}{\partial b}\right) + \omega_s^2 + q(\omega_n \eta + \omega_b \lambda)\right] \]  \hspace{1cm} (7.25)

where \(\eta\) is the binormal curvature of the normal direction

\[ \eta = \hat{b} \cdot \frac{\partial \hat{n}}{\partial n} \]  \hspace{1cm} (7.26)
and \( \lambda \) is the normal curvature of the binormal direction

\[
\lambda = -\mathbf{n} \cdot \frac{\partial \mathbf{b}}{\partial b}.
\]

(7.27)

This expression thus relates the local rate of change of total pressure following a fluid element to the kinematics of the vorticity field.

Upon substituting the above expression into the equation for the passage loss yields an integral equation relating the passage loss to the kinematics of the vorticity field as

\[
\int_0^z \int_0^y \int_0^{\frac{z}{2}} \frac{1}{Re} \left[ q \left( \frac{\partial \omega_b}{\partial n} - \frac{\partial \omega_n}{\partial b} \right) + \omega_x^2 + q(\omega_n \eta + \omega_b \lambda) \right] dy \, dz = c_1 Y_{pass}.
\]

(7.28)

This equation is referred to as the Intrinsic Dissipation Equation, and contains three groupings of terms. The first grouping involves spatial derivatives of the normal and binormal vorticity in the cross stream directions, the second involves only the streamwise component of vorticity, and the third grouping involves the curvature of the vortex filaments on a plane perpendicular to the streamline.

### 7.1.3 Significance of Intrinsic Dissipation Equation

An equation has been derived that relates the loss to vorticity components in an intrinsic coordinate system. The vortical terms in equation 7.28 can be usefully divided into three grouping, with the physical significance of each grouping being described as follows.

The first grouping in equation 7.28 involves the gradients of the vorticity in the plane perpendicular to the velocity. These two terms are the binormal derivative of the normal vorticity and the normal derivative of the binormal vorticity, and both are related to the gradients in the velocity magnitude. The following example will serve to illustrate the relevant physics. Consider a planar two dimensional flow. All curvatures are confined
to the plane of the velocity, hence the normal vector is also in the plane. The vorticity in such a flow will thus be in the third, or binormal direction. A normal gradient in the vorticity thus corresponds to a velocity profile in the plane with a non-constant slope. As is well known, such a profile does not balance the shear on the adjacent streamlines [36] and the total pressure will change along the streamlines. For the total pressure to remain constant along a streamline, the shear on adjacent streamlines must balance, which corresponds to no normal gradient in the vorticity. The analogous situation is true for the binormal derivative of the normal vorticity. Hence, these two terms relate to quasi-two dimensional planar diffusion in a non-constant vorticity profile. Thus this grouping of terms will be referred to as the planar shear.

The third grouping of terms in equation 7.28 involves the product of the vorticity perpendicular to the velocity, and the curvature of the coordinate direction of that velocity. Again, a simple example that serves to illustrate the relevant effects is described here. Consider the unidirectional fully developed flow of a viscous fluid between concentric cylinders. If the inner cylinder has a constant velocity $U$ in the axial direction, the Navier-Stokes equations in cylindrical coordinates reduce to

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} = 0 \quad (7.29)$$

with the boundary conditions $u(r = r_i) = U$, and $u(r = r_o) = 0$. The solution is thus given by

$$u(r) = \frac{U}{\ln\left(\frac{r}{r_0}\right)} \ln\left(\frac{r}{r_0}\right). \quad (7.30)$$

To evaluate the terms in the intrinsic dissipation equation it is necessary to define the intrinsic coordinate direction. Since the streamline curvature is zero, we may arbitrarily set $\hat{n} = \hat{e}_r$, and $\hat{b} = \hat{e}_\theta$. With this choice of coordinates, the vorticity is again confined
to the binormal direction, and may be written as

\[
\vec{\omega}(r) = \omega_b \hat{b} = -\frac{Ur_0}{\ln(\frac{r}{r_o})} \frac{1}{r}
\]  

(7.31)

which is a nonconstant vorticity field. Thus the derivative in the normal direction will be nonzero, or

\[
\frac{\partial \omega_b}{\partial n} = \frac{\partial \omega_b}{\partial r} = \frac{Ur_0}{\ln(\frac{r}{r_o})} \frac{1}{r^2}.
\]  

(7.32)

(7.33)

The curvature of the binormal direction is simply the curvature of the cylindrical coordinate system, the \( \lambda = \frac{1}{r} \), and the term \( \omega_b \lambda \) is

\[
\omega_b \lambda = -\frac{Ur_0}{\ln(\frac{r}{r_o})} \frac{1}{r^2}.
\]  

(7.34)

The combination of the two terms yields zero, indicating the streamwise rate of change of the total pressure is zero, as expected. Thus, in such a flow the balance of shear on adjacent streamlines is dependent not only on the derivative of the vorticity perpendicular to the velocity, but also on the curvature of the vorticity filaments at that location.

As we have seen from the above, the third grouping of terms in equation (7.28) clearly relates the shear to the curvature of the perpendicular vorticity components, and thus it would be appropriate to denote it as the circular shear.

The remaining term in equation 7.28 involves the square of the streamwise vorticity. Since the streamwise vorticity is dependent only on the derivative of the flow direction, and not on the magnitude, this term is referred to as the streamline skew. It should be noted that unlike the other two terms, the streamwise vorticity is not multiplied by the velocity magnitude. In addition, since this term is squared, the sign is always positive. Thus, the presence of streamwise vorticity in a flowfield is inherently dissipative, with
a rate of dissipation that is independent of the magnitude of the local velocity. This suggests that streamwise vorticity existing in low velocity regions may play a significant role in the generation of loss.

### 7.2 Evaluation of Loss Generation in Intrinsic Coordinates

The previous chapter presented calculations of the variation of the passage and secondary loss using accepted technique for the calculation of the individual components. The results of the loss calculation as presented showed that in the first third of the cascade, secondary flow had a negligible (less than 1%) contribution to the gross passage loss, even though the first third of the cascade corresponds to the region of formation of the horseshoe vortex and the endwall boundary layer separation. Thus the fact that these results indicate negligible secondary loss in this region should be viewed with doubt. It was then suggested that these results indicate that the conventional procedure used to assign profile and secondary loss may be inappropriate. In particular, we noted that the use of a two-dimensional calculation to compute the profile loss may lead to an inconsistency.

This section presents an evaluation of the generation of loss using the intrinsic dissipation equation derived above. This equation is used to classify the integrated convective derivative of total pressure into two components based on different kinematic processes. The aim of this classification is to assess the relative contribution of streamwise vorticity in the flow to the net integrated total pressure loss observed in the cascade passage.
7.2.1 Classification of Loss Components

The kinematic terms in the intrinsic dissipation equation can be divided into two types. The first type is related to gradients in the velocity magnitude, while the second type is related to gradients in the velocity direction. As was described above, the grouping termed planar shear and circular shear are of the first type, while the streamline skew is of the second type. Thus the generation of loss will here be divided into components based on these two types of phenomena; one a quasi-two dimensional term involving shear, and the other an inherently three dimensional term involving only streamline skew.

The intrinsic dissipation equation is divided into two components for evaluation of the loss generation, with the first component referred to as loss due to shear, and the second referred to as loss due to skew, or

\[ Y_{\text{pass}} = Y_{\text{shear}} + Y_{\text{skew}}. \]  
(7.35)

The left side of the intrinsic dissipation equation is then broken into two components, resulting in separate expressions for the component losses,

\[ \int_0^z \int_{ss} \int_0^{\frac{z}{2}} \frac{1}{Re} [q(\frac{\partial^2 \nu}{\partial \eta^2} - \frac{\partial^2 \eta}{\partial \nu^2}) + q(\nu \eta + \omega \lambda)] \, dz \, dy \, dx = c_1 Y_{\text{shear}} \]  
(7.36)

\[ \int_0^z \int_{ss} \int_0^{\frac{z}{2}} \frac{1}{Re} \omega^2 dz \, dy \, dx = c_1 Y_{\text{skew}} \]  
(7.37)

These expressions will be used to evaluate the generation of loss in the cascade passage, and to relate that loss to the vortical structure in the flowfield.

7.2.2 Spanwise Profiles of Loss Generation

In this section, we shall examine the loss computed in accordance with equations (7.36) and (7.37). Such a procedure may offer the advantage of relating the generation
of total pressure loss to the various sources that exist in the flowfield, including blade surface and endwall boundary layers, free shear layers, and secondary vorticity. Spanwise profiles of the integrand of the intrinsic loss equation, integrated in the pitchwise direction \((y)\) across the passage (suction surface to pressure surface) are presented in Figure 7.1a through 7.1e. The plots show the generation of loss due to shear, and the gross generation of loss due to both shear and skew. The integrated area between the curves thus represents the axial rate of change of total pressure loss due to streamline skew.

The spanwise variation in the calculated generation of total pressure loss at the leading edge plane is shown in Figure 7.1a. The figure shows the two profiles described above, representing the generation of loss integrated over the blade gap versus span. The curves show nearly uniform values of loss generation between quarter- and mid-span, due to the loss generated in the boundary layer about the leading edge, outside of the endwall region. The figure also shows the peak in shear loss generation near the endwall representative of the loss due to the diffusion in the thin endwall boundary layer behind the leading edge separation line. The small scale structures between 10\% and 25\% span are a result of insufficient spanwise resolution in this region to allow for the evaluation of the gradients of vorticity in the intrinsic coordinate system.

At 25\% chord, Figure 7.1b shows that the generation of total pressure loss from \(\frac{1}{4}\) to \(\frac{1}{2}\) span is due mostly to the planar and circular shear, with very little contribution from the streamline skew. Between \(\frac{1}{4}\) and the endwall the contribution due to skew between the streamlines is more significant. To be more specific, about 14\% of the loss generation arises from the skew between the streamlines. This change in loss can be related to the streamwise vortex structures centered at 8\% span, as was seen in the vorticity distributions in Chapter 5.
The profile of the integrated dissipation at midchord is shown in Figure 7.1c, which shows that the midspan loss generation has no significant streamwise skew component. The region of maximum loss generation due to skew is now between 5% and 20% span, with 20% of the loss generation at this axial location being entirely due to this vortical region dominated by the presence of streamwise vorticity. The movement of the peak in skew loss generation toward midspan is merely a reflection of the growth of the secondary passage vortex. One might recall Figure 5.8b (Chapter 5) that the vortex is centered at about 13% span. The profile shows a large peak in the shear loss generation near 4% span, associated with the high shear between the passage vortex and the smaller secondary vortex beneath.

The three-quarter chord profile in Figure 7.1d shows a similar behavior to that at midchord. There is a large peak in the shear loss generation near 5% span, again reflecting the diffusion of the shear layer beneath the passage vortex. There has also been an increase in the magnitude of this peak, as well as an increase in the loss generation between 5% and 25% span. The loss in this region appears to be generated between the blade suction surface and the passage vortex, where there exists a high gradient in velocity magnitude. The loss associated with streamline skew has migrated closer to the midspan, with a corresponding increase in extent, which merely reflects the diffusion of the passage vortex along the blade suction surface.

The trailing edge profile shows a change in structure from the previous axial locations. As shown in Figure 7.1e, between 25% and 35% span there is an “s” shaped structure in the profile of the loss generation due to shear. This distribution is the result of the existence of a spanwise free shear profile centered at 27% span. This shear layer is the result of the three dimensional separation of the blade suction surface boundary layer above the passage vortex, as was seen in Figure 6.4e. This separation brings low
momentum fluid from the blade surface out into the passage above the vortex. Thus, the fluid in this separated region has a lower total velocity than the fluid in the vortex structure below. The low momentum fluid is accelerated under the action of the viscous stress of the higher momentum fluid, thus causing an increase in the total pressure of this fluid along the streamlines. This same viscous stress decelerates the higher momentum flow, lowering its total pressure along the streamlines. Hence, above the shear layer centerline the convective derivative of total pressure should exhibit a local minimum, and below the center the derivative should exhibit a local maximum. The loss generation distribution at the trailing edge shown in Figure 7.1e shows this type of structure, related to the existence of the free shear layer above the passage vortex.

These profiles show that the evaluation of the loss through the use of the intrinsic dissipation relation allows the generation of loss to be related to specific flow structures in the cascade passage. The role of streamwise vorticity in the generation of loss can clearly be seen as an increase in the pitchwise integrated loss generation in that portion of the span where the passage vortex exists. In addition, free shear layers and the loss generation associated with them may be identified by their characteristic s-shaped generation profile, as described above. Thus, the use of these profiles to examine the distribution of loss generation allows the rate of loss generation to be related to flow structures that exist at a particular spanwise and chordwise location. This technique may also be used to resolve confusion between total pressure loss generated at that location, and the convection of low total pressure fluid from other regions.

7.2.3 Passage Integrated Loss Generation

The pitchwise integrated profiles of loss generation presented above delineates the relative contribution of the loss components at each spanwise location. The level of con-
tribution of each component to the total passage loss, however, depends on the integral of that loss generation over the passage cross section. This section presents the passage integration of the loss generation as a function of axial location, and the integration of that distribution in the axial direction. The passage integration shows relative contribution of the components at a given axial location to the net chordwise rate of change of loss, while the axial integral of these distributions will show the contributions to the integrated loss level.

The chordwise distribution of the passage integration of the loss generation components is shown in Figure 7.2. The plot shows the value of the shear loss generation and gross loss generation integrated over the passage cross section. The distribution shows a smooth variation of both the shear loss generation and skew loss term from 5% to 95% axial chord. The values at the leading and trailing edges however show marked change from the values adjacent to them. This may indicate that the values of loss generation at the leading and trailing edges, as calculated here, are subject to resolution related errors. Re-examination of the profiles at the leading and trailing edges, and of the distribution of vorticity at the trailing edge, suggest that the flow structures are not fully resolved in these regions. Figures 7.6a and 7.6b shows spanwise profiles of the mid-gap streamwise vorticity at two axial locations, midchord and the trailing edge plane. It can be seen that the distribution at the trailing edge has small scale oscillations, indicating a possible lack of spectral resolution. Examination of the amplitude of the spectral coefficients, also shown in Figure 7.6b, confirms the existence of insufficient resolution at the exit plane. The amplitudes of the spectral coefficients are non-zero at the upper limit, hence the truncation of the series representation of the flow may have affected the solution in this region. Numerical values of the loss generation in this region thus may be less accurate than in the remainder of the passage.
The axial variation in the integrated loss generation is shown in Figure 7.3 and 7.4. The figures show the value of the components of the passage loss due to shear and the sum of that due to shear and skew. The relative amount of the passage loss due to skew increases from 8% at the leading edge to 20% at the 60% axial chord location. From here the relative amount of skew loss remains constant up to the 80% chord location. The results indicate that the relative amount of skew loss decreases from there to the passage exit, but as was seen above, these results may not be as accurate as we would like it to be. Figure 7.5 shows a comparison between the integration of the intrinsic dissipation equation 7.28 and the passage loss, evaluated by integrating the flux of total pressure. The plot shows that the loss is accounted for by the integration of the two loss generation terms up to 75% axial chord. The divergence of the two results downstream of the three-quarter chord location is a result of the lack of resolution for evaluating the loss generation terms in this region.

A mixing analysis has been conducted to evaluate the relative importance of the loss due to streamline skew within the passage. This analysis compared the loss generated within the passage due to skew with the additional amount of loss that would be generated downstream if the net secondary kinetic energy of the flow is lost. The analysis indicates that the loss due to the mixing of the total secondary kinetic energy, normalized by upstream dynamic head, yields an additional passage average loss of .051. This compares to passage averaged loss due to streamline skew of .033 at the cascade exit. Hence, if the net secondary kinetic energy were allowed to mix out such that no energy was recovered, the loss value due to secondary flow could be expected to increase by an approximate factor of three. Such an increase in secondary loss downstream of the cascade trailing edge plane has been observed in experimental investigations [39], increasing confidence that the classification of losses described above gives a meaningful estimate of the contribution of the loss due to secondary vorticity to the total loss.
in the cascade passage.

### 7.3 Summary of Natural Coordinate Loss Evaluation

An expression has been derived that showed that the passage averaged loss defined in the previous chapter could be related to the volume integral of the the diffusive terms in the Navier-Stokes equations. The evaluation of the resulting expression in intrinsic coordinates allows the loss generation so described to be split into components based on the kinematics of the vorticity field. The generation of loss in a cascade passage evaluated using this intrinsic dissipation equation showed that the contribution of streamwise vorticity (or streamline skew) to the total loss varied from 8% to 20% of the total passage loss through the cascade. Profiles of the spanwise integration of the dissipation terms also showed that this additional loss could be related to the vortical structures in the flowfield. It is thus concluded that the presence of streamwise circulation plays a significant role in the generation of loss in turbine cascades, and that this loss can be evaluated using the derived intrinsic dissipation equation. Furthermore, we have shown that a laminar calculation, such as the one presented here, yields flow structures which are qualitatively similar to those that exist in flows at higher Reynolds numbers. As such, examination of the generation of loss in simulated laminar flowfields may yield insight into the generation of loss due to secondary flow in more complex higher Reynolds number flows.
Figure 7.1a: Spanwise variation of y-integrated generation of total pressure loss, 0% axial chord (leading edge plane).

Figure 7.1b: Spanwise variation of y-integrated generation of total pressure loss, 25% axial chord.
Figure 7.1c: Spanwise variation of y-integrated generation of total pressure loss, 50% axial chord.

Figure 7.1d: Spanwise variation of y-integrated generation of total pressure loss, 75% axial chord.
Figure 7.1e: Spanwise variation of y-integrated generation of total pressure loss, 100% axial chord (trailing edge plane).
Figure 7.2: Axial variation in passage-integrated dissipation.
Figure 7.3: Axial variation in passage loss components.
Figure 7.4: Axial variation in passage loss, from dissipation expression, showing component contributions.
Figure 7.5: Comparison of passage loss calculated from dissipation expression, with integration of total pressure flux.
Figure 7.6a: Spanwise profile and spectral coefficients for mid-passage streamwise vorticity, 50% axial chord.
Figure 7.6b: Spanwise profile and spectral coefficients for mid-passage streamwise vorticity, 100% axial chord.
Chapter 8

Conclusions and Suggestions for Future Work

This chapter presents a summary of Parts I & II and the conclusions that can be drawn from this thesis. Suggestions are then offered for areas that warrant further investigation.

8.1 Summary of Part I

A secondary flow theory was proposed to explain experimentally observed variations in the structure of the secondary flow in turbine stages. The proposed theory was based on kinematics of the interaction of incoming vorticity filaments with the blade leading edges. The hypothetical model suggested that the effect of a steady velocity deficit at the cascade inflow, aligned with the blade leading edges, was sufficient to cause the observed variations. To investigate the applicability of the proposed theory, a numerical procedure was developed to simulate the flow through a turbine cascade. The procedure allowed two flowfields to be compared, one with an inflow uniform in the pitchwise direction, and one with an inflow distortion representative of an incoming viscous wake. Examination of these flowfields yielded the following results.

First, it was determined that the presence of the saddle point flow and horseshoe vortex system is directly related to the appearance and subsequent growth of the secondary flow vortex in the cascade passage. The passage vortex has its origin in the
pressure surface leg of the horseshoe vortex at the blade leading edge. As the incoming boundary layer separates along the separation line in the three-dimensional flow, the fluid is entrained into that leg of the horseshoe vortex. Thus, the amount of upstream boundary layer fluid in the pressure side leg of the horseshoe vortex increases as the vortex crosses the inlet plane of the cascade passage. As the boundary layer fluid is entrained into the vortex core, it is turned in a direction parallel to the separation line, generating an additional streamwise component of vorticity. Thus, the net amount of streamwise vorticity associated with this leg of the vortex increases as it proceeds across the blade passage in the downstream direction. Since the suction side leg of the vortex does not entrain an increasing amount of upstream boundary layer fluid, it becomes more diffuse as it wraps around the blade suction surface. Hence, the entrainment and subsequent turning of the upstream boundary layer fluid by the pressure side leg of the horseshoe vortex as it crosses the passage is the source of the preferential growth of that leg of the horseshoe vortex.

Since the growth of the pressure side leg of the vortex is related to the entrainment of the upstream boundary layer fluid, the formation of the passage vortex is related only to the existence of a horseshoe vortex at the leading edge, and not to the strength of the vortex in that region. Thus, the existence of a horseshoe vortex and its related saddle point separation is sufficient to cause the separation of the inlet boundary layer, and the growth of the pressure side leg of the vortex. The elimination of the formation of a discrete passage vortex would thus require the elimination of the saddle point separation and the associated separation line. Such an elimination of the leading edge saddle point is only possible in the absence of normal vorticity at that location. Thus, the originally proposed flow model is applicable only to those distortions having zero velocity in a region centered at the wake-centerline at the blade leading edge.
The simulation showed that the proposed theory is applicable only for flow distortions involving a total absence of normal vorticity at the leading edge. Any vorticity present at the leading edge will result in the separation described above, and the formation of a discrete passage vortex, and thus will not yield the distributed secondary vorticity predicted by the proposed model. Since the upstream distortions in the original experimental investigations did not result in a complete absence of normal vorticity at the leading edge, it must be concluded that the hypothesized interaction is not responsible for the observed secondary flow variations, as was confirmed by the numerical investigation. This eliminates one of the two hypothesized mechanisms for the observed variation of the secondary vortex, indicating that the other mechanism, the effect of the relative motion of the incoming viscous wake, is responsible for the experimental observations. This brings the investigation of the variation of the secondary vortex a step closer to identification of the physical mechanism responsible for the observed effects.

A number of additional secondary results were presented in Part I, involving both the structure of secondary flow in cascades, and the technique used to investigate that flow. It was shown that the multi-domain spectral element technique may be used to simulate the three dimensional flow through complex geometries such as turbomachinery blading. The investigation also showed that the examination of the laminar generation of secondary flow yields insight into the corresponding formation of secondary flows at high Reynolds number. Lastly, the investigation of the formation of the secondary flow vortex showed that the counter vortex frequently identified as a corner vortex at the cascade exit has its origin in the endwall counter vortex at the leading edge, between the horseshoe vortex and the three dimensional separation line.
8.2 Conclusions from Part I

Examination of the results summarized above led to the following conclusions:

1. The existence of a discrete passage vortex core is the result of the separation of the incoming endwall boundary layer; thus it is only related to the presence of a leading edge horseshoe vortex and the accompanying saddle point flow.

2. The strength of the passage vortex is related to the net vorticity in the incoming endwall boundary layer and the amount of turning experienced by that vorticity after separation and entrainment into the vortex.

3. Since the strength of the passage vortex is not related to the strength of the leading edge horseshoe vortex, changes in the leading edge vortex caused by the incoming quasi-steady wakes are insufficient to account for the observed variations in the cascade secondary flow pattern.

4. The remaining interaction mechanism, that of an unsteady wake-blade interaction, must be responsible for the secondary flow variations observed in the experimental investigations.

8.3 Summary of Part II

The loss in total pressure in the cascade passage was examined in Part II. The distribution of total pressure was calculated using the numerical procedure developed in Part I. This calculated total pressure field has been used to address issues associated with the nature of secondary loss in turbines. In an effort to determine the source of
secondary losses, the calculated loss was divided into contributions due to the blade profile loss and endwall secondary loss. This investigation yielded the following results.

The classification of the gross loss into profile and secondary loss using currently accepted procedures resulted in negligible loss being attributed to secondary flow over much of the passage. This result is related to the use of a two dimensional simulation to evaluate the blade profile loss. Since much of the blade surface in low aspect ratio cascades is influenced by the presence of secondary flow, the amount of loss generated on the blade surface may be very different from that predicted using a two dimensional method. Thus, even though such techniques may yield satisfactory estimates of the relative contribution of secondary loss to the gross passage loss at the cascade exit, their use to evaluate the source of loss within the passage may give rise to inconsistencies which in turn may affect design decisions based on such analysis.

The conventional technique for evaluating the loss components gives a result with the implication of no loss attributed to secondary flow over portions of the blade passage. Furthermore, this technique cannot be used to relate the formation of secondary total pressure loss to the structure of the secondary passage vortex. To identify the relationship between the generation of secondary passage loss and the formation of the secondary flow vortex a different method of classifying the components of loss is required.

To better understand the relationship between the generation of loss and the presence of secondary flow, an alternate method was used to classify the various contributions to the generation of total pressure loss in the cascade passage. An expression was derived that relates the generation of loss in the passage to the kinematics of the vorticity distribution, using a description of flow in intrinsic coordinates. The use of intrinsic
coordinates is based on the definition of coordinate directions according to the local flow topology, and thus yields useful expressions for various kinematic components of the flow. This expression was then used to evaluate the total pressure field calculated in Part I. The relative contributions of the different physical mechanism represented by the terms in the derived equation were used to classify the sources of loss in the cascade passage. The derived expression for the generation of loss within the passage shows that the presence of streamwise vorticity contributes to the generation of loss in a positive definite sense.

Evaluation of the generation of loss in the simulated flowfield from Part I using the different components in the derived dissipation expression indicates that streamwise vorticity was responsible for approximately twenty percent of the gross loss generated in the passage. This indicates that the streamline skew associated with streamwise vorticity is an important source of total pressure loss in the cascade passage, suggesting that secondary loss prediction techniques based on changes in wetted surface loss only could be misleading. The classification of loss using the terms in the derived dissipation equation yielded a value of the loss attributed to secondary (streamwise) vorticity that increases with increasing axial distance. The use of a conventional technique gives negligible values of secondary losses over much of the blade chord. In contrast the present method for classification of the loss gives a direct interpretation of the generation of total pressure loss in terms of the secondary flow structures in cascade passages. Such an interpretation could be useful when attempting to reduce losses associated with turbine secondary flows.
8.4 Conclusions from Part II

Examination of the results from Part II, as summarized above, led to the following conclusions:

1. The use of conventional techniques for classification of total pressure loss contributions may underpredict the loss due to secondary flow within the blade passage.

2. The expression for the generation of total pressure loss may be rewritten in intrinsic coordinates, relating the gross generation of loss to the normal, binormal and streamwise components of vorticity.

3. The derived loss generation expression indicates that streamwise vorticity contributes in a positive definite sense to the generation of total pressure loss in cascade passages.

8.5 Suggestions for Future Work

From the investigation presented in Part I, we have concluded that the quasi-steady vane-blade interaction is not responsible for the observed variation in secondary flow pattern. It was then concluded that the remaining possible mechanism, that of the unsteady vane-blade interaction, must be responsible. However, the physical mechanism by which the unsteady interaction results in the observed variation is not well understood. The unsteady vane-blade interaction and its effect on the secondary flow pattern should be investigated. To determine the nature of the interaction mechanism, the investigation should include the following steps:
1. The spatially resolved, time dependent flowfield within the blade passage needs to be simulated, either by experiment or numerical calculation. Numeric simulation has the advantages of ease of control of the inflow parameters, available high spatial resolution, and available computational tools for simulation and data reduction. However, the high computational resources required may prohibit the simulation of the unsteady three dimensional viscous flow. Experimental investigation has the advantage of lower cost yet requires the design and assembly of an experimental facility. In addition, data reduction procedures used in the current investigation would require modification before use with experimental data sets.

2. Simulated time-dependent flowfield data should be analyzed to determine the physical mechanism responsible for the observed variation. Since it has been determined that the time-dependent motion of the incoming distortions is responsible for the observed variations, the analysis should examine an unsteady interaction having the same reduced frequency as that of the UTRC Large Scale Rig investigation.

3. Once the physical mechanism for the specified interaction has been determined, the effect of reduced frequency should be explored. To accomplish this, several different time-dependent flowfields having various reduced frequencies should be examined to determine the relationship between the variations in secondary flow pattern and the reduced frequency of the interaction.

Although considerable effort was spent on the development of the numerical technique used in the current investigation, the computational tool requires additional work before it can be made for wider range of geometries and flow parameter. To further generalize the code, the following work is suggested:
1. The specification of the computational geometry requires a detailed knowledge of the numerical algorithm. For the code to be useful for a wider range of flows, the specification of geometry and boundary conditions should be generalized and built into a menu-driven, window oriented, interactive preprocessor package.

2. Any "hard-wired" geometric dependencies in the code, including the specification for fixed order conformal polynomial expansions, should be removed.

3. The wide variety of post-processing programs and tools developed for the current investigation should be assembled into a single menu-driven, window oriented, interactive postprocessor package.

The modifications suggested above would result in a computational tool that could easily be used in wide range of investigations, without requiring a detailed understanding of algorithms involved. As such, the program could then be used by students and others interested in examining basic phenomena of viscous fluid mechanics in a user oriented interactive environment.
Bibliography


