AN IMPROVED TECHNIQUE TO DETERMINE THE MOUNT EMBEDDING IMPEDANCE OF SIS MIXERS

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Abstract

An improved technique was developed to determine the mount embedding impedance of SIS mixers. The technique uses the normal and photon assisted current versus voltage characteristics of a SIS tunnel junction to determine the mount embedding impedance of the mixer in which it is employed as a diode. The basis for this technique is a model of the mixer which uses the impedance of the junction, mixer mount, and the surrounding environment to determine the power absorbed in the junction. In experimental conditions, the input power is fixed. This technique improves on previous techniques by modeling the input power as a fixed quantity. Given a fixed local oscillator input power, the technique determines the unique mount embedding impedance corresponding to a particular pair of normal and photon assisted current versus voltage characteristics.

Tests were conducted using data from a 630 GHz waveguide mixer with a NbN/MgO/NbN tunnel junction and a tunable mount embedding impedance. The embedding impedance was controlled with a backshort and an E-plane tuner. Tests were also conducted using data from a 230 GHz planar mixer with a Nb/AlO\textsubscript{x}/Nb tunnel junction and a fixed mount embedding impedance.

The results indicate that the improved technique can be used to determine both the mount embedding impedance of SIS mixers and the tuning range of the mixer. The results verified that (1) for good power coupling to occur in waveguide mixers, the waveguide mount must supply an impedance which is close to the conjugate of the impedance of the junction, and (2) there is a periodic relationship between the mount embedding impedance of a waveguide mixer and the electrical length of its tuning elements. It was also shown that heating of the junction due to poor heat sinking, can cause a decrease in the energy gap of the tunnel junction, and a decrease in the gap voltage. Results for the mount embedding impedance of the 630 GHz waveguide mixer are in good agreement with results predicted by low frequency models.

The results for the 230 GHz planar mixer were also in good agreement with low frequency estimates for the fixed mount embedding impedance.
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1. **Introduction**

Superconductor-Insulator-Superconductor (SIS) mixers are the primary components of an important class of low noise, high frequency receivers [1]. Although receivers were first developed for communication at lower frequencies, the development of ultra low noise, high frequency receivers has been driven primarily by applications in radio astronomy where the need for low noise is a priority. Unfortunately for ground based radio astronomy, the atmosphere is opaque in the submillimeter band (300 - 3000 GHz) except for select ranges of frequencies (figure 1.1). These ranges (observing windows) include the strong emission lines of key molecules such as CO. These lines are important to observe, as CO is abundant in many interstellar molecular clouds and therefore can be used as a tracer molecule when studying cloud dynamics and kinematics. Because the absolute intensity of many of the objects observed by astronomers is very low, much information may be lost due to noise added by receivers and other equipment used in observations. The best results are obtained by using receivers which add the least noise, thus getting the best signal to noise ratio possible.

In the case of communication systems, the development of high frequency receiver technology will be driven by the need for increased communication bandwidth. Low noise, high frequency receiver technology is applicable to future space based communication systems because submillimeter transmissions are not as heavily attenuated in space as they are in the atmosphere (figure 1.1). In fact, satellite to satellite communication at submillimeter frequencies would not only have greater bandwidth, but because of atmospheric attenuation, they would also be secure from ground monitoring. Any practical space based communication systems is limited by technology and weight considerations. The weight and volume penalties for high power, high frequency
transmitters are tremendous, but low noise receivers provide a solution to the high frequency communication system. Their high sensitivity makes the use of lower power transmitters feasible, preserving both volume and weight.

Figure 1.1: Atmospheric Transmission Spectrum

1.1 Heterodyne Mixers

Heterodyne mixers are the central components of low noise receivers. These mixers combine two signals, and outputs a third signal at a lower frequency. The output frequency is equal to the difference between the input frequencies, and the intensity of the output signal is proportional to the intensities of the input signals. Since these mixers convert signals from higher frequencies into an output at a lower, intermediate frequency, their output is a direct measure of the higher frequency signals. This output can be amplified by traditional means.
Figure 1.2: Heterodyne Receiver System

The receiver consists of an antenna, local oscillator (LO) signal source, diplexer, mixer, and intermediate frequency (IF) amplification system (figure 1.2). The antennae focuses the radio frequency (RF) input signal. The diplexer combines the RF signal with a LO signal of similar but distinct frequency. The RF signal and LO signal are fed into the mixer, which then converts them into a single IF output. This IF signal is amplified and fed into a spectrum analyzer for detection.

The mixer consists of two parts. A non-linear diode, and a RF embedding circuit. The performance of the mixer is defined by the amount of noise added to input signals, and the fraction of the incident power that is converted into the output signal. The lower the noise added and the higher the fraction of power converted, the better the mixer. The two factors which influence the mixer performance are the quality of the diode non-linearity, and the match of the RF embedding impedance to the diode impedance. Mixers which use high quality SIS tunnel junctions as diodes, add very little noise to the signal being observed. In fact, the noise added by these SIS mixers is comparable to the lowest possible noise predicted by quantum theory (quantum limited noise). Therefore, the match of the RF embedding impedance to the diode impedance is the limiting factor in the performance of SIS mixers.
1.2 SIS Tunnel Junctions

SIS junctions are formed by sandwiching an insulating layer between two superconductors (figure 1.3). The insulator forms a potential barrier to the flow of electrons from one superconductor to the other. The junction can be represented in a circuit by a variable resistor and reactance in parallel with a fixed capacitor. The fixed capacitor is the geometric capacitor formed by the sandwiching of an insulator (dielectric) between two superconductors (plates). The reactance of this geometric capacitor is a function of frequency, but remains constant with changing bias voltage.

![Schematic diagram of SIS tunnel junction.](figure 1.3: (a) Schematic diagram of SIS tunnel junction. (b) Circuit model for SIS junction.)
Figure 1.4: DC current vs. bias voltage for a typical SIS junction with no incident radiation.

Figure 1.4 is a plot of dc current vs. bias voltage (dc I-V) characteristic for a typical SIS junction when it is not exposed to radiation. These junctions exhibit little to no current flow for low values of bias voltage and then experience a sudden rise in current at a certain voltage. This voltage is called the gap voltage and is related to the energy gap (or void) which exists between the filled and unfilled states of a superconductor at absolute zero (See 2.1.2). Junctions of this type are said to have low sub-gap leakage. These junctions make very good low noise mixers because the shot noise added by the mixer is directly related to the current through the barrier. Diodes with low leakage current will have a high signal to noise ratio, because the number of electrons tunneling as a result of the incident signals is much greater than the electrons which comprise the noise from the sub-gap leakage current.
1.3 Match of RF Embedding Impedance to Diode Impedance

![Diagram of SIS waveguide mixer with a backshort and e-plane tuner.]

**Figure 1.5:** SIS waveguide mixer with a backshort and e-plane tuner.

In addition to low sub-gap leakage junctions, quantum limited mixer performance can only be achieved by the correct match of the RF embedding impedance to the diode impedance. Maximum power is absorbed by the mixer, when the reflection of the input signal is minimized. Reflections occur whenever there is a difference impedance. The optimum impedance match occurs when the impedance of the mixer is equal to the impedance of the environment in which it is located. In a typical SIS waveguide mixer, the SIS tunnel junction is the diode and the mounting structure within the waveguide (waveguide mount) is the RF embedding circuit (figure 1.5). For a waveguide mixer, the
impedance of the waveguide mount is the mount embedding impedance. The mounting structure consists of a substrate, backshort and E-plane tuner. The section of waveguide enclosing these components and whose impedance is determined by these components, is also part of the mixer. Any receiver component before this, such as plain waveguide sections and feedhorns, are external to the mixer. It is the impedance match between this external environment and the mixer, which controls the input power reflected by the mixer.

The mixer impedance is defined as the combined impedance of the diode and the waveguide mount. The impedance of the waveguide mount can be varied by changing the position of the backshort and E-plane tuner. The backshort terminates the waveguide, and acts as a tuning element by varying the electrical length of the waveguide. In addition to the backshort, the waveguide mount impedance is also adjusted with an E-plane tuner. The impedance that the junction sees because of the position of the backshort and E-plane tuner is the mount embedding impedance of the mixer. These two tuning elements therefore determine the mount embedding impedance of the mixer. By varying the position of the tuning elements, the mount embedding impedance changes, and so does the total impedance of the mixer.

The impedance of the environment (plain waveguide section) is real. This is because a metal waveguide is a resistance only. To minimize the reflection between the mixer and its environment, the mixer impedance should also be real. The junction however, has a complex impedance due to its reactive components. If the waveguide mount can provide the correct inductance in parallel to the junction's reactance, then the mixer will also have a purely real impedance, and the best possible match between mixer and environment will be achieved. The waveguide mount embedding impedance is therefore a critical parameter in the design of a mixer, as it controls the amount of power
that is coupled into the mixer. A bad impedance match can reduce or even cancel the performance of well fabricated low sub-gap leakage junctions.

1.4 Measuring the Waveguide Mount Embedding Impedance

At frequencies above 100 GHz, network analyzers do not exist, so direct measurement of the waveguide mount embedding impedance is impossible. The waveguide mount embedding impedance does however have a distinct effect on the dc I-V characteristics of the SIS junction. When current is passed through the SIS junction that is exposed to radiation, it undergoes photon assisted quasi-particle tunneling. In this process, energy from incoming photons provide an extra boost for electrons to tunnel at voltages below which they would normally tunnel through the insulating barrier. Photon assisted tunneling changes the shape of the dc I-V characteristic from a single sharp rise at the gap voltage. Since the mount embedding impedance controls the impedance match between the mixer and the environment (and this controls the power coupling into the mixer), the number of photons which are available for photon assisted tunneling is a function of the embedding impedance. The shape of the dc I-V characteristic for a junction exposed to radiation (pumped dc I-V characteristic) is therefore a function of the dc I-V characteristic when the junction is not exposed to radiation (unpumped dc I-V characteristic) and the waveguide mount embedding impedance. The relationship between the unpumped dc I-V characteristic, mount embedding impedance and pumped dc I-V characteristic is described by mixer theory. If the unpumped and pumped dc I-V characteristics are known, then mixer theory can be used to infer the waveguide mount embedding impedance.

If mixer theory is applied to an appropriate model for the interface between the junction and the waveguide mount, the combination provides a technique for deducing
the mount embedding impedance. A prior model [2,3] to determine the mount embedding impedance treats the interface between the junction and waveguide mount as complex impedances driven by a constant current source. This approach has been demonstrated to work well at frequencies up to 100 GHz [4], but has never been tested at frequencies as high as 630 GHz. It has been found to be unsuitable for high frequency mixers because it assumes the mixer impedance is much smaller than the environment impedance. This assumption results in a linear relationship between the mixer impedance and the power absorbed in the mixer. The development of an improved model for the interface between the junction and the waveguide mount forms the basis of this thesis. The improved model treats the input signals as sources of constant power, carefully accounting for the power absorbed and reflected due to impedance mismatch.
2. Mixer Theory

Mixer theory predicts the pumped dc I-V characteristic based on the unpumped dc I-V characteristic and the mount embedding impedance. It models the photon assisted quasi-particle tunneling of the SIS junction. It does not include the effect of the geometric capacitance, but since the effect of this capacitance is constant with changing bias voltage, it can be subtracted from the results obtained using theory. This theory is outlined below, beginning with normal quasi-particle tunneling and photon assisted quasi-particle tunneling. The pumped dc I-V characteristic is explained, including the effects of the mount embedding impedance on the shape of the pumped dc I-V characteristic. The appropriate model for the interface between the junction and the RF embedding circuit is determined. The technique used to determine the mount embedding impedance from the shape of these I-V characteristics is then derived.

2.1 Normal Quasi-Particle Tunneling

The tunneling of particles across the potential barrier which exists in a SIS tunnel junction is a quantum mechanical process. The quantum mechanical nature of tunneling is simply and best demonstrated in the following example by Solymar [5].
When an electron with kinetic energy $E$ and no potential energy is incident on a potential barrier of greater energy, there is a probability that the electron may appear on the other side of the barrier. This probability may be derived from the time-independent Schrödinger equation

$$\left( -\frac{\hbar^2}{2m} \nabla^2 + U \right) \psi = E \psi \quad (2-1)$$

along with the wave function for each region and the correct boundary conditions. For figure 2.1, the appropriate wave functions are

$$\psi_1 = A \exp \left( i k_1 x \right) + B \exp \left( -i k_1 x \right)$$
$$\psi_2 = C \exp \left( -\kappa x \right) + D \exp \kappa x \quad (2-2)$$
$$\psi_3 = F \exp \left( i k_3 x \right)$$

where $A, B, C, D, F$ are constants and $k_1, \kappa, k_3$ are given by:
\[ k_1 = \frac{1}{\hbar} (2mE)^{1/2} \]

\[ \kappa = \frac{1}{\hbar} [2m(U_2 - E)]^{1/2} \]

\[ k_3 = \frac{1}{\hbar} [2m(E - U_3)]^{1/2} \] (2-3)

The boundary conditions require that both \( \psi \) and \( \frac{d\psi}{dx} \) are continuous. The result is the following expression for the ratio of wave function amplitudes, or the average probability of quasi-particles tunneling through the barrier.

\[ \left| \frac{E}{A} \right| = \frac{4k_1\kappa}{(k_1^2 + \kappa^2)^{1/2}} \left( \frac{k_3^2 + \kappa^2}{(k_3^2 + \kappa^2)^{1/2}} \right)^{1/2} \exp(-\kappa w) \] (2-4)

In steady state, this expression gives the fraction of the incident electrons which tunnel through to the other side of the barrier. The important thing to note about this expression is the strong dependence of the fraction of electrons that make it through the barrier on the barrier thickness \( w \), and the barrier height \( \kappa \).

2.1.1 Normal Tunneling in Metal-Insulator-Metal Junctions

![Diagram](image)

**Figure 2.2:** Density of states of a metal-insulator-metal junction
(a) at thermal equilibrium, (b) at a potential difference \( V \).
Solymar argues that the number of electrons that tunnel in a particular direction across a junction is proportional to both the number of filled states on the side that the electron are moving from, and the number of empty states on the side that the electron are moving to. The current is determined by multiplying these two numbers together with the appropriate probability of tunneling through the barrier. In figure 2.2(b), the number of filled states on the left side in an energy interval dE at an energy E is given by

$$N_1(E-eV) f(E-eV) \, dE$$  \hspace{1cm} (2-5)

where $N_1$ is the density of states of the metal on the left and $f$ is the Fermi function which describes the distribution of its filled states. Similarly, the number of empty states on the right side is given by

$$N_2(E) \, [1-f(E)]$$  \hspace{1cm} (2-6)

where $N_2$ is the density of states for the metal on the right and $[1 - f(E)]$ is the Fermi distribution for empty states (one minus the Fermi function for filled states). Assuming that the probability of a quasi-particle tunneling is equal in both directions and has a value $P(E)$, the current through the junction will be

$$I \approx \int P(E) \, N_1(E-eV) \, N_2(E) \, [f(E-eV) - f(E)] \, dE$$  \hspace{1cm} (2-7)

2.1.2 Normal Tunneling in SIS Junctions

The mathematics of SIS tunneling is the same as in the metal-insulator-metal junctions, except that the density of states of a superconductor is not the same as that of a normal metal.
Figure 2.3: Density of states of a SIS tunnel junction at 0\(^0\)K.

Figure 2.3 is a plot of the number of electrons in each energy state of the superconductors in a SIS junction. The Fermi energy (E\(_F\)), separates the filled and empty states in the superconductor. In the case of a unbiased superconductor at 0\(^0\)K, electrons below the Fermi energy exist in the form of Cooper pairs [6]. The Fermi energy is surrounded by a region \(2\Delta\) wide where electrons are forbidden. This is the gap of the superconductor, and the value \(2\Delta\) is the gap energy. The gap energy is equal to the energy difference between the highest filled state and the lowest empty state. The gap voltage (\(V_{\text{gap}} = 2\Delta/e\)), is proportional to the gap energy of the superconductor. Since a particle must tunnel from a filled to an empty state, and the empty states are above the energy gap, no tunneling of electrons form one superconductor to the other occurs. With
no external bias, no current flows through the junction. If the junction is biased with a voltage that is greater than the gap voltage, then some of the filled states in one superconductor will have the same energy as some of the empty states in the other superconductor, and single electrons or quasi-particles can tunnel through the barrier. In figure 2.4, the bias voltage (V) brings some of the filled states in the superconductor on the left up to the same energy as the empty states in the one on the right, and some quasi-particles tunnel through the barrier. Therefore, there will be a sudden rise in current at the instant where the filled states reach the energy of the unfilled states. In the dc I-V characteristic, this is seen as the sharp rise in current at the gap voltage (figure 1.5), as tunneling begins to occur.
Cooper pairs are part of a special condensed state where electrons pair up and combine into a single quantum state, in defiance of the Pauli exclusion principle. Josephson [7] predicted that these Cooper pairs could tunnel through the insulating barrier in a SIS junction, generating a current when the junction is not biased. Since it is the tunneling of quasi-particles which is used to measure the input power, it is necessary to suppress the Josephson current in favor of the dc quasi-particle current. This is achieved by introducing magnetic flux quanta into the junction to suppress the
undesirable Josephson current. Once the Josephson current is suppressed, quasi-particle tunneling remains as the only source of nonlinear elastic tunneling through the junction.

2.2 Photon Assisted Tunneling

At millimeter and submillimeter wavelengths, the photon energy of incident radiation is comparable to the gap energy of materials such as niobium (Nb) and niobium-nitride (NbN). If the junction is biased with a voltage which is less than the gap voltage, the energy difference between the filled states in one superconductor and empty states in the other is decreased. Since the photon energy is comparable to the gap energy, integer multiples of incoming photons can provide the additional energy necessary for a single electron to reach the energy of the empty states in the other superconductor. For a single photon of frequency \( f \) and energy \( hf \), the energy imparted to an electron is \( hf \). If this happens, the electron may tunnel though the insulating barrier. In figure 2.5, the bias voltage (\( V \)) brings some of the filled states in the superconductor on the left close enough to the energy of the empty states in the one on the right, that photons can provide enough energy to accomplish tunneling. This process of photon assisted quasi-particle tunneling is more fully explained in the Tucker theory of quasi-particle mixing.
2.2.1 Tucker Mixer Theory

The Tucker mixer theory [8] provides a semi-quantum mechanical treatment of the mixing process. The following section is a summary of the steps in Tucker’s derivation. The theory treats the barrier as a weak couple for a pair of many-body systems, and constructs a transfer Hamiltonian [9] function to describe the tunneling across the junction. The transfer Hamiltonian relates the filled states on one side of the junction to all the empty states on the other side. It is a matrix formulation that gives the relationship between a single filled state and all the possible empty states to which the
electron in that state may tunnel. In the Tucker theory, the Hamiltonian for the entire junction is given by:

$$H = H^0_L + H^0_R + eV(t)N_L + H_T$$  \hspace{1cm} (2-8)

where $H^0_L, R$ are the full many-body Hamiltonians for the left and right electrodes when there is no coupling through the barrier. The term $eV(t)N_L$ represents the modulation of the energy of the $N_L$ electrons on the left side of the junction by a time dependent bias voltage $V(t)$, and the $H_T$ term is the transfer Hamiltonian. Since the current through the junction must either go from left to right, or right to left (conservation of charge), the time derivative of the number of electrons $N_R$ on the right side will give the net number of electrons tunneling through the junction. The current will be equal to the transfer of charge caused by these tunneling electrons, and is given by:

$$I = e \frac{dN_R}{dt}$$  \hspace{1cm} (2-9)

One of the key assumptions made, is that the modulation of the electrons on the left side of the junction occurs slowly. If this is true, then the response of the junction will be adiabatic, and the density matrix will retain its equilibrium form

$$\rho = \frac{\exp \left[-\beta \left(H^0 - \mu N \right)\right]}{\text{Tr} \left\{ \exp \left[-\beta \left(H^0 - \mu N\right)\right] \right\}}$$  \hspace{1cm} (2-10)

For the density matrix to retain its equilibrium form, the addition of the potential $V(t)$ to the left side Hamiltonian must be offset by a change in chemical potential:

$$\mu_L(t) = \mu_R + eV(t)$$  \hspace{1cm} (2-11)

The assumption that the density matrix retains its equilibrium form is very critical to Tucker's results, as he calculates the current that tunnels through the barrier by doing a
linear expansion about the equilibrium. At equilibrium, the biasing voltage $V(t)$ is zero, and no electrons tunnel through the barrier. Tucker treats the transfer Hamiltonian as a perturbation, and as the condition of equilibrium dictates, the current goes to zero for the uncoupled system. The expected current is given by:

$$\langle I(t) \rangle = \text{Im} \int \! \! \! \int d\omega' d\omega'' W(\omega') W^*(\omega'') e^{-i(\omega' - \omega'')} t \times j(\omega' + eV_0/\hbar)$$

(2-12)

where the complex response function

$$j(\omega) = \frac{4\pi}{e} \chi_{\text{r.}} (\omega)$$

(2-13)

is proportional to the Fourier transform for the retarded current commutator evaluated in the equilibrium system

$$\chi_{\text{r.}} (t-t') = i \Theta(t-t') \langle [I^0(t), I^0(t')] \rangle_0$$

(2-14)

The retarded current commutator may be written in terms of the nonretarded commutator

$$\chi_{\text{r.}} (t-t') = \frac{1}{2} \langle [I^0(t), I^0(t')] \rangle_0$$

(2-15)

via the Fourier expression

$$\chi_{\text{r.}} (\omega) = \int \! \! \! \int \frac{d\omega'}{\pi} \frac{\chi_{\text{r.}} (\omega')}{\omega - \omega - i\eta}, \quad \eta \to 0^+$$

(2-16)

Once this expression is substituted into the expression for the complex response function, it can be seen that the real and imaginary parts of the complex response function are connected via the Kramers-Kronig transformation and that the imaginary part is equal to the dc I-V characteristic of the junction.
Re \( j(\omega) \) = \( P \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{I_{dc}(\omega \epsilon/e)}{\omega - \omega} \)

Im \( j(\omega) \) = \( I_{dc}(\hbar \omega/e) \) \hspace{1cm} (2-17)

2.2.2 RF Tunneling Current

Let us now suppose that the time dependent potential is sinusoidal and of the form

\[ V(t) = V_0 + V_1 \cos \omega t \] \hspace{1cm} (2-18)

The time dependent current operator induced by this potential may be evaluated using the phase relationship

\[ \exp \left\{ -i \frac{e}{\hbar} \int dt' V_1 \cos \omega t' \right\} = \exp \left\{ -i \left( \frac{e V_1}{\hbar \omega} \right) \sin \omega t' \right\} \]

\[ = \sum_{n=-\infty}^{\infty} J_n \left( e V_1 / \hbar \omega \right) e^{-in\omega t} \] \hspace{1cm} (2-19)

to explicitly remove the bias voltage from the integral. The Fourier transform of the time dependency is therefore given by:

\[ W(\omega') = \sum_{n=-\infty}^{\infty} J_n(\alpha) \delta(\omega' - n\omega) \] \hspace{1cm} (2-20)

where \( \alpha = eV_1/\hbar \omega \). When this transform is substituted into the expression for the expected current, the result is:

\[ \langle I(t) \rangle = \text{Im} \sum_{n,m=-\infty}^{\infty} J_n(\alpha) J_{n+m}(\alpha) e^{im\omega t} j(n\omega + eV_0/\hbar) \]

\[ = a_0 + \sum_{m=1}^{\infty} \left[ 2a_m \cos m\omega t + 2b_m \sin m\omega t \right] \] \hspace{1cm} (2-21)
and the values of the current components at the different harmonic frequencies are given by:

\[ 2a_m = \sum_{n=-\infty}^{\infty} J_n(\alpha) \left[ J_{n+m}(\alpha) + J_{n-m}(\alpha) \right] I_{dc}(V_0 + n \, \hbar \, \omega / e) \]

\[ 2b_m = \sum_{n=-\infty}^{\infty} J_n(\alpha) \left[ J_{n+m}(\alpha) - J_{n-m}(\alpha) \right] \text{Re} \left( n \, \omega + e \, V_0 / \hbar \right) \]  

(2-22)

This complicated sum of Bessel functions for the factors $2a_m$ and $2b_m$, although impossible to solve in closed form, has a very simple physical interpretation. It is a weighted sum which accounts for the probabilities of an integer number of photons ($n$ photons) interacting with a single electron tunneling at a particular harmonic frequency $m\omega$. This sum accounts for the fact that the interaction of photons with specific harmonics will occur at different rates, and this rate depends on the probability of these $n$ photons interacting with a specific electron simultaneously. The weighting of each interaction in the sum is normalized such that the most probable simultaneous interaction, that of a single photon, is given a weighted value of one.

Another important thing to note about this sum, is the existence of out of phase reactive current components. These currents do not appear in classical mixer analysis, and are the result of a virtual tunneling process. The time average of this tunneling process is non zero, and gives rise to a quantum susceptance. This susceptance, as the name suggests, is a direct result of the quantum theory, and is explained below along with the quantum conductance.
2.2.3 Quantum Conductance

Electrons that tunnel across the barrier to states which are integer multiple of photon energies away can absorb the energy of incident photons. Because these photons are absorbed at the same time that the tunneling occurs, the current that results is in phase with the driving potential of the incident RF signal. This current is the dissipative part of the tunneling process, and is the real part of the complex RF current through the junction. The complex RF current can be used to calculate the conductance of the junction, since the current through the junction divided by the potential applied across the junction is equal to the admittance. The quantum conductance is the real part of this admittance.

2.2.4 Quantum Susceptance

Tunneling is a quantum mechanical process. Some of the electrons which tunnel across a barrier may not tunnel to states that are integer multiple of photon energies away. Electrons which tunnel to these states have to tunnel back because they cannot absorb energy to sustain their transition. This can occur as long as the time scale of this tunneling is short compared to the time of the interaction which is driving the tunneling [10,11]. This tunneling back and forth across the junction, or quantum mechanical 'sloshing', does not absorb any of the energy contained in the photons and is therefore non-dissipative, and out of phase. Over a longer period of time, this sloshing is seen as a time averaged susceptance. It is referred to as the quantum susceptance, or the virtual susceptance caused by the non-dissipative sloshing of electrons across the insulating barrier.
2.3 Pumped DC I-V Characteristic

Now that the quantum mechanical processes that occur in SIS junctions have been explained, it is necessary to show how these processes govern the shape of the dc I-V characteristic of the junction when it is irradiated. During photon assisted quasi-particle tunneling, photons provide extra energy to electrons when they interact. A single photon interacting with a single electron will increase the energy of that electron by the energy contained in the photon. If the increased energy of the electron is equal to the energy of an unfilled state in the superconductor on the other side of the barrier, then the electron may tunnel through the barrier. Similarly, an integer multiple of electrons can assist a single electron to tunnel if the energy they give to that electron will increase it’s energy to a value that is equal to the energy of an unfilled state. Since the maximum energy value of the gap is the threshold for unfilled states, the dc I-V characteristic will have current rises at bias voltages where the energy of the interacting photons will provide just enough energy to increase the electron’s energy to an energy which is above the gap (i.e. to the energy of an unfilled state). Therefore, current rises occur at voltages which are less than the gap voltage by integer multiples of the photon voltage $\hbar \omega / e$ (voltage equivalent to the energy of a single photon) since only an integer number of photons can interact with an electron. In the dc I-V characteristic, these rises and subsequent regions of photon assisted tunneling look like steps (figure 2.6). The width of these steps are equal to the photon voltage, and are called photon steps. Steps are also present above the gap voltage since photon to electron interaction can also remove energy causing electrons to tunnel in the opposite direction, decreasing the current flow above the gap voltage.
Figure 2.6: DC I-V characteristic of SIS junction undergoing photon assisted tunneling. Note the steps of width $\hbar \omega / c$ above and below the gap voltage.

2.3.1 Effect of Mount Embedding Impedance

The mount embedding impedance has a distinct effect on the dc I-V characteristics of a SIS junction. Since the mount embedding impedance controls the impedance match between the mixer and the environment (and this controls the power coupling into the mixer), the number of photons which are available for photon assisted tunneling is a function of the embedding impedance. The shape of the pumped dc I-V is therefore a function of the unpumped dc I-V and the mount embedding impedance. Although the width of the photon steps are fixed by the frequency of photons incident on the junction, the slope and shape of the steps are affected by the mount embedding impedance.
2.4 Modeling the Interface Between the Mount and Junction

As explained above, the mount embedding impedance controls the number of photons that are absorbed by the SIS junction, and therefore controls the shape of the pumped dc I-V. Using the unpumped and pumped dc I-V, the mount embedding impedance can be calculated using the Bessel function sums in the Tucker mixer theory. The important thing in performing this calculation is the model used for the interface between the junction and the mixer RF embedding circuit. Below are two models. The first, discussed by Shen [2] and Skalare [3], treats the source of input power as a constant current source in a very simple circuit model. The second model, developed in this thesis, improves on the first by treating the input power source as a source of constant power, and modeling the change in power absorbed by the mixer when the bias voltage across the junction changes.
2.4.1 Constant Current Source Model

\[ Z_M = \frac{Z_{MM} Z_J}{Z_{MM} + Z_J} \]

![Diagram of Constant Current Source Model](image)

Figure 2.7: Constant Current Source Model

This model assumes that the local oscillator source acts as a source of constant current (figure 2.7). The mount and the junction are placed in a parallel combination across the current source, and the RF pumping voltage is defined as the voltage across the mixer (the parallel combination of the mount and the junction). The local oscillator source is represented by a constant current source \( I_{LO} \), the mixer impedance is \( Z_M \), the mount impedance is \( Z_{MM} \), the junction impedance is \( Z_J \), and the RF pumping voltage is \( V_M \).

When the bias voltage across the junction is changed, the impedance of the junction changes. This causes a change in the impedance of the mixer as the mount embedding impedance is not a function of bias voltage, and the impedance of the mixer is the net impedance of the mount and junction in parallel. Implicit in this model is the
linear relationship between the power absorbed in the mixer and the impedance of the mixer. The power absorbed in the mixer is given by

\[ P_A = I_{LO}^2 \text{Re}(Z_M) \]  

(2-23)

where \( P_A \) is the power absorbed, \( I_{LO} \) is the constant current source, and \( Z_M \) is the impedance of the mixer. A change in the impedance of the mixer causes a linearly proportional change in the power absorbed. This is not entirely accurate, since a change in the mixer impedance really causes a change in the mismatch between the mixer and its environment, and this mismatch does not always have a linear relationship with the power absorbed.

### 2.4.2 Limitations of Current Source Model

The true relationship between mixer impedance and power absorbed is not linear. There is however, a region where the relationship is approximately linear. In this region, the constant current source model is valid. It can be shown that the region where the constant current source model is valid is defined by the condition

\[ Z_M << Z_E \]  

(2-24)

where \( Z_E \) is the impedance of the environment external to the mixer.

The magnitude of the geometric and quantum susceptance are a function of frequency. The geometric susceptance is constant with changing bias voltage. The quantum susceptance is not. It is believed that the variation of the quantum susceptance with bias voltage increases with frequency. If the variation in the quantum susceptance is indeed larger at higher frequencies, assuming a small mixer impedance may be an error, and the constant current source model will be invalid. The mismatch between the
impedance of the environment and the impedance of the mixer can no longer be approximated by a linear relationship. The advantage of the power source model is that it is not limited to the region where the mixer impedance is small. At high frequencies, it is a more robust method for calculating the mount embedding impedance.

2.4.3 Constant Power Source Model

\[ Z_M = \frac{Z_{MM} Z_J}{Z_{MM} + Z_J} \]

Figure 2.8: Constant Power Source Model

An improved model is derived by assuming that the local oscillator source acts as a source of constant power (figure 2.8). The local oscillator source is represented by a constant power source \( P_{LO} \), the impedance of the environment is \( Z_E \), the mixer impedance is \( Z_M \), the mount impedance is \( Z_{MM} \), and the junction impedance is \( Z_J \). The RF pumping voltage \( V_M \) is the voltage across the mixer. This is consistent with the physical reality of local oscillator sources which supply radiation at a constant incident power setting. In addition, the mismatch which occurs between the impedance of the environment and the impedance of the mixer can be included. This model thus takes a
step closer to the reality of the interaction between the incoming radiation and the mixer, by modeling the changes in the power absorbed by accounting for the change in the power reflected.

The change in absorbed power arises from a chain of interactions similar to the one outlined in the constant current source model. The change in bias voltage causes a change in the impedance of the highly nonlinear junction, and a subsequent change in mixer impedance. The difference with this model, is the functional dependence of the power absorbed in the mixer on the impedance of the mixer. The power absorbed in the mixer is given by

$$P_A = P_{LO} \left[ 1 - \left| \frac{Z_M - Z_E}{Z_M + Z_E} \right|^2 \right]$$

(2-25)

where $P_A$ is the power absorbed, $P_{LO}$ is the constant power source, $Z_M$ is the impedance of the mixer, and $Z_E$ is the impedance of the environment. A change in the mixer impedance does not cause a linearly proportional change in the power absorbed. In fact, the relationship can be rearranged to give the neater expression:

$$P_A = P_{LO} \frac{2 \left( Z_M^* Z_E + Z_M Z_E^* \right)}{|Z_M + Z_E|^2}$$

(2-26)

where $Z_E^*$ and $Z_M^*$ are the complex conjugates of $Z_E$ and $Z_M$. In the case of environments with only a real impedance, this equation becomes

$$P_A = P_{LO} \frac{4 R_M R_E}{|Z_M + Z_E|^2}$$

(2-27)

where $R_M = \text{Re} \{Z_M\}$, the resistance of the mixer, and $R_E$ is the resistance of the environment.
2.5 Calculating the Mount Embedding Impedance

The only thing that remains is to outline the sequence of calculations which must be followed in order to determine the mount embedding impedance from a pair of unpumped and pumped dc I-V characteristics. The RF voltage match technique used by Skalare with the constant current source model has been employed with the power source model.

The first step is to use the dc I-V characteristics to determine the pumping voltage across the mixer. The actual pumping voltage can be found from the equation:

\[ I_{DC_{pumped}} = \sum_{n=-\infty}^{\infty} J_n^2(\alpha) I_{DC}(V_0 + n \frac{\hbar \omega}{e})_{unpumped} \]  

(2-28)

where \( I_{DC_{pumped}} \) is the pumped current at bias voltage \( V_0 \), \( I_{DC_{unpumped}} \) is the unpumped current at different bias voltages on the unpumped I-V characteristic, \( \alpha = e V_M / \hbar \omega \) is the normalized RF pumping voltage across the mixer, and \( J_n \) is the bessel function. The sum on the right side is the weighted sum of the currents at equally spaced points on the unpumped dc I-V characteristic. The spacing between these points is \( \hbar \omega / e \) (photon voltage), and the points are centered on \( V_0 \). This sum is calculated for successive iteration of the pumping voltage \( V_M \) until the equality is satisfied. When the sum and the current on the pumped characteristic at \( V_0 \) are equal, the assumed \( V_M \) is equal to the actual voltage being seen across the mixer at that bias voltage. This equality holds at every bias voltage, so at every voltage for which there is a value for the pumped current, a pumping voltage is found. A series of pumping voltages at different bias voltages is determined.
With this series of pumping voltages, there exists a corresponding series of junction impedances. The RF current flowing through the junction is related to the unpumped dc I-V characteristic via a relationship similar to the one for the pumped dc I-V characteristic. The RF current through the junction is given by

\[
I_{RF} = \sum_{n=-\infty}^{\infty} J_n(\alpha) \left[ J_{n+m}(\alpha) + J_{n-m}(\alpha) \right] I_{dc}(V_0 + n \hbar \omega / e)_{\text{unpumped}} + j \times \sum_{n=-\infty}^{\infty} J_n(\alpha) \left[ J_{n+m}(\alpha) - J_{n-m}(\alpha) \right] I_{KK}(V_0 + n \hbar \omega / e)_{\text{unpumped}}
\]  
(2-29)

where \( I_{KK} \) is the Kramers-Kronig transform of the unpumped dc I-V characteristic. The junction impedance at a specific bias voltage is calculated using the RF current through the junction and the pumping voltage across the mixer, via the relationship \( Z_j = V_M / I_{RF} \). There now exists a series of junction impedances which can be applied to the power source model.

Using the model at every bias voltage gives a relationship for the power absorbed in terms of the pumping voltage, the power of the source, the mixer impedance, and impedance of the environment. The power absorbed in the mixer is also given by

\[
\langle \text{Re} \left( \frac{V_M^2}{Z_M} \right) \rangle = \frac{1}{2} V_M^2 \text{Re} \left( \frac{Z_M^*}{Z_M Z_M^*} \right) = \frac{1}{2} V_M^2 \frac{R_M}{|Z_M|^2}
\]
(2-30)

where \( V_M \) is the amplitude of the voltage across the mixer. Equating this to eqn. 2-27, the pumping voltage predicted is given by

\[
V_M^2 = P_{LO} \frac{8 |Z_M|^2}{|Z_M + Z_E|^2} \frac{R_E}{|Z_M + Z_E|^2}
\]
(2-31)
The impedance of the environment and junction are known. For the series of bias voltages, there are as many equations as there are bias voltages, but only two unknowns. These are the power of the source and the mount embedding impedance. The solution to this system of equations is a unique pair of mount embedding impedance and source power which minimizes the sum of the squares of the differences (error sum) between the pumping voltages calculated using the model and the pumping voltages determined from the dc I-V characteristics. The error sum for this least squares fit of the pumping voltage is given by

$$\varepsilon = \sum_{V_0} \left( \frac{V_M - \sqrt{P_{LO} \frac{8 |Z_M|^2 R_E}{|Z_M + Z_E|^2}}} {V_0} \right)^2$$  \hspace{3cm} (2-32)

where the mixer impedance $Z_M$ is left in the expression for the sake of clarity. The power of the source is constant, so the power term can be eliminated by setting the derivative of the error sum with respect to source power to zero. Expanding the terms in the error sum

$$\varepsilon = \sum_{V_0} V_M^2 - 2 P_{LO}^{1/2} \sum_{V_0} V_M \sqrt{\frac{8 |Z_M|^2 R_E}{|Z_M + Z_E|^2}} + P_{LO} \sum_{V_0} \frac{8 |Z_M|^2 R_E}{|Z_M + Z_E|^2}$$  \hspace{3cm} (2-33)

and setting the derivative to zero gives the least squares source power as

$$P_{LO} = \left[ \frac{\sum_{V_0} V_M (4 R_E)^{1/2} |Z_M|}{\sum_{V_0} 8 R_E |Z_M + Z_E|^2} \right]^2$$  \hspace{3cm} (2-34)

Resubstituting this power into the expression for the error sum gives the error as a function of mixer impedance only.
\[
\varepsilon = \sum_{V_0} V_M^2 - \left[ \frac{\sum V_M \left( 8 R_E \right)^{1/2} \frac{|Z_M|}{|Z_M + Z_E|}}{\sum 8 R_E \frac{|Z_M|^2}{|Z_M + Z_E|^2}} \right]^2
\]  
(2-35)

where \( R_M \) and \( Z_M \) are functions of the mount embedding impedance. A search of the complex impedance plane will result in a unique mount impedance which minimizes the error in the least squares fit to the pumping voltages.
3. Experiment

To determine the mount embedding impedance of a mixer, it is necessary to measure unpumped and pumped dc I-V characteristics for SIS junctions mounted in that mixer. The mixer used to acquire unpumped and pumped dc I-V characteristics is part of a 630 GHz receiver for the Caltech Submillimeter Observatory. The observatory is located on Mauna Kea in Hawaii and is used primarily for radio astronomy. The mixer and receiver were designed and built by the Advanced Microwave Devices Group at the Jet Propulsion Laboratory. The mixer and its test facility are described below [12]. A discussion of the method used to measure the I-V characteristics, and the numerical reduction required to determine the mount embedding impedance, follows the description.

3.1 Experimental Mixer Facility

As outlined in the introduction, the receiver consists of an antenna, LO signal source, diplexer, mixer, and IF amplification system. The antenna focuses the RF input signal. The diplexer combines the RF signal with a LO signal of similar but distinct frequency. The mixer converts the RF and LO signals into a single IF output. This IF signal is amplified and fed into a spectrum analyzer for detection.

Figure 3.1 is a schematic of the mixer test facility used to measure the I-V characteristics. Since the SIS junction must be cooled well below 100K to become superconducting, the mixer is housed in a cooled vacuum cryostat. The test system in figure 3.1 is cooled with liquid helium and nitrogen. In the test facility, the mixer (figure 3.2) is attached to the surface of a metal plate which is exposed on its other surface to
liquid helium (figure 3.3). The amplifier used in the IF system is attached to a liquid nitrogen cooled stage to minimize its noise.
Figure 3.1: Mixer test system housed in a cooled vacuum cryostat. The mixer is attached to a liquid helium cooled stage, and the IF amplifier is attached to a liquid nitrogen stage.
Figure 3.2: 630 GHz Mixer
Figure 3.3: Mixer attached to liquid helium cooled plate in cryostat.
Figure 3.4: Cutaway of Mixer
Figure 3.5: Junction attached to substrate perpendicular to waveguide.
As seen in figure 3.1, the LO signal is generated at a lower frequency by a Gunn oscillator, and multiplied up to 630 GHz. The beamsplitter acts as a diplexer and combines the RF and LO signals before they enter the cryostat. The unpumped I-V is measured when no radiation is allowed to enter the mixer. The pumped I-V is measured with ambient radiation entering the mixer. The combined signals enter the cryostat via a mylar vacuum window. After passing through black polyethylene and fluorogold filters, the signals are focused with a teflon lens into a feedhorn attached to the SIS waveguide mixer.

In the mixer (figure 3.4), the signals travel via waveguide to the SIS junction. The junction is mounted on a substrate which sits in a channel perpendicular to the direction of the waveguide (figure 3.5). On the substrate, is a high frequency filter which is comprised of alternating areas of high and low impedance. This filter prevents RF power from escaping out the substrate channel. The IF output power, however, easily travels along this filter. The IF output is fed into the cooled HEMT amplifier and then out the cryostat for processing.

3.2 Collection of I-V Characteristics from Test Mixer

DC I-V characteristics are obtained by ramping the bias voltage and measuring the dc current of the junction. The method used in experiment is described below.

The bias voltage on the junction is increased to a voltage above the gap voltage where the resistance of the junction is equal to its resistance in the normal state. From this point, the bias voltage is steadily decreased. The bias voltage and dc current are sampled periodically with an analog to digital conversion board and stored in a computer. The rate of the sweep in bias voltage is manually controllable, so areas of special interest
such as the gap or photon steps may be sampled in more detail. The sweep is completed when the bias voltage reaches zero volts. The data points are averaged in groups, to reduce the number of data pairs to no more than 100. This averaging is a part of the data sampling routine for the mixer test facility. A computer file with data pairs of bias voltage and current is saved for numerical reduction.

3.3 Numerical Reduction

The algorithm described here follows closely the theoretical discussion on how to determine the mount embedding impedance from unpumped and pumped dc I-V characteristics. The calculations were performed on a VAX 750.

Both the digitized unpumped and pumped dc I-V characteristics from the experimental runs are entered into a numerical reduction routine. The pumped dc I-V characteristic is interpolated so that there is a value for the pumped current at each bias voltage for which there is a value for the unpumped current. At each bias point, the pumping voltage is iterated using the Newton-Raphson method [13] (eqn. 3-1). \(I_{dc}\) is the dc current calculated with the Bessel weighted sum in eqn 2-28. Iteration stops when the calculated dc current is equal to the pumped current at that bias voltage. The local gradient is determined by recording the change in the calculated dc current with each iteration of the assumed pumping voltage. The expression used to determine the next pumping voltage to assume is

\[
V_{M_2} = V_{M_1} + \left( \frac{I_{dc, \text{pumped}} - I_{dc_1}}{\frac{dI_{dc}}{dV_M}} \right)
\]

(3-1)

where \(dI_{dc}/dV_M\) is the local gradient.
The routine begins at the lowest bias voltage. At this bias voltage, the first iteration begins with the assumption that the normalized pumping voltage $\alpha = 1$, and that the local gradient is equal to the gradient between $\alpha = 1$ and $\alpha = 0$ (unpumped current at that bias voltage). Once the correct pumping voltage is found, iteration begins at the next bias voltage in the data set.

At all subsequent bias voltages, the first iteration assumes the pumping voltage and gradient of the bias voltage preceding it, as a first guess. This is possible because the pumping voltages are piece wise continuous with changing bias voltage.

Once the entire series of bias voltages has been completed, the corresponding set of junction impedances is determined via eqn 2-29 and the relationship $Z_J = V_M/IR_F$. Using the series of bias voltages and junction admittances, and the known environment impedance, the complex impedance plane is searched for the unique embedding impedance which minimizes the error (eqn. 2-35) in the least squares fit to the pumping voltages.
4. Results

Since both the backshort and E-plane tuner determine the embedding impedance of the mount, the tuning range can be determined by varying the position of each tuning element independently. Therefore, pumped dc I-V characteristics were measured for an NbN/MgO/NbN SIS junction [14] irradiated by 630 GHz radiation at 12 different positions. Six of these positions were measured by varying only the position of the backshort, while the other six were measured by varying only the position of the E-plane tuner. The backshort and E-plane tuner were moved approximately one half the electrical length of the waveguide. These pumped characteristics were combined with the unpumped characteristic for the specific junction used in the mixer, to calculate the mount embedding impedance. The range of bias voltages over which the least squares fit was performed is 2 - 3 mV. This range encompasses the first photon step below the gap, and was chosen because this step is most sensitive to the value of the mount embedding impedance.

4.1 Effect of Backshort Position

To determine the range of the mount embedding impedance versus the position of the backshort, pumped dc I-V characteristics were measured at six backshort positions while the position of the E-plane tuner remained fixed. The first position was chosen by varying the backshort position until the pumped current appeared by eye to reach a maximum value.
4.1.1 Pumped DC I-V Characteristics vs. Backshort Position

Figures 4.1 - 4.6 show the pumped characteristics for the NbN junction at six backshort positions. The unpumped characteristic has been included in each graph to show the level of photon assisted tunneling taking place, and to better illustrate the effect of the backshort position of the pumped current.

![Graph showing pumped and unpumped DC I-V characteristics for NbN junction with backshort positions.](image)

**Figure 4.1:** DC I-V characteristics with backshort in position 1.
Figure 4.2: DC I-V characteristics with backshort in position 2.

Figure 4.3: DC I-V characteristics with backshort in position 3.
Figure 4.4: DC I-V characteristics with backshort in position 4.

Figure 4.5: DC I-V characteristics with backshort in position 5.
The important thing to note about the pumped dc I-V characteristics is the magnitude of the pumped current on the first step below the gap. The currents at positions 1 and 5 are high. The currents at positions 2 and 6 are similar, but less than at positions 1 and 5. The next lowest in magnitude is the current at position 4. The lowest current is at position 3. In fact, the pumped current in position 3 is very close to the unpumped current, suggesting that the coupling is very poor. The current in positions 1 and 5 are high, indicating good coupling and close proximity to the optimum mount impedance.

4.1.2 Mount Embedding Impedance vs. Backshort Position

The mount embedding impedances for the series of six backshort positions are tabulated below.
<table>
<thead>
<tr>
<th>Backshort Position</th>
<th>Resistance (ohms)</th>
<th>Reactance (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>2500</td>
<td>-2500</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
<td>-150</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>6.5</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>65</td>
</tr>
</tbody>
</table>

The mount embedding impedances are also presented on the Smith chart in figure 4.7. This Smith chart has been normalized to 50 ohms. Each position is labeled with its corresponding number, to show the magnitude and direction of the change in embedding impedance when the backshort position is changed.
Figure 4.7: Mount Embedding Impedance for Six Backshort Positions on 50 ohm Smith Chart

Figure 4.7 shows that the mount embedding impedances lie on the arc of a circle which is slightly smaller than the bounds of the smith chart. Following the backshort positions from 1 to 6, the impedance appears to encircle the center of the smith chart in a clockwise direction. In addition, there is a strong correlation between the impedance and the pumped current. Embedding impedance position 3 illustrates this correlation. Position 3 is on the far right of the Smith chart. The typical impedance for the type of junction used in experiment is left half of the Smith chart. Therefore, the mount embedding impedance is not close to the complex conjugate of the junction impedance. As a result, the impedance match is poor. As figure 4.3 indicates, the pumped current is very similar to the unpumped current, consistent with a poor impedance match. By similar reasoning, position 5 gives the best impedance match. The encirclement of the
center of the smith chart illustrates the periodic relationship between backshort position and mount impedance. This is consistent with the fact that the impedance of a shunted section of waveguide varies periodically on the scale of half a wavelength, with the electrical length of the waveguide.

4.2 Effect of E-Plane Tuner Position

To determine the range of the mount embedding impedance versus the position of the E-plane tuner, pumped dc I-V characteristics were measured at six E-plane tuner positions while the position of the backshort remained fixed. The first position is as close by eye as possible to the point used as position 1 in the backshort series.

4.2.1 Pumped DC I-V Characteristics vs. E-Plane Tuner Position

Figures 4.8 - 4.13 show the pumped characteristics for the NbN junction at six E-plane tuner positions. Again, the unpumped characteristic has been included in each graph to show the level of photon assisted tunneling taking place, and to better illustrate the effect of the E-plane tuner position on the pumped current.
Figure 4.8: DC I-V characteristics with E-plane tuner in position 1.

Figure 4.9: DC I-V characteristics with E-plane tuner in position 2.
Figure 4.10: DC I-V characteristics with E-plane tuner in position 3.

Figure 4.11: DC I-V characteristics with E-plane tuner in position 4.
Figure 4.12: DC I-V characteristics with E-plane tuner in position 5.

Figure 4.13: DC I-V characteristics with E-plane tuner in position 6.
Again, the important trend to note about the pumped dc I-V characteristics is the change in the magnitude of the pumped current on the first step below the gap. The current decreases as the E-plane tuner is varied from position 1 to position 4, then increases as it is varied from position 4 to position 6. In this case, the minimum pumped current (position 4) is not as close to the unpumped current as the minimum pumped current achieved using the backshort. The current in positions 1 and 6 are high, indicating good coupling and close proximity to the optimum mount impedance. The magnitude of the pumped current again appears to be periodic.

4.2.2 Mount Embedding Impedance vs. E-Plane Tuner Position

The mount embedding impedances for the series of six E-plane tuner positions are tabulated below.

<table>
<thead>
<tr>
<th>E-Plane Tuner Position</th>
<th>Resistance (ohms)</th>
<th>Reactance (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>5.5</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>9.5</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>56</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>

The mount embedding impedances are also presented on the Smith chart in figure 4.14. This Smith chart has been normalized to 50 ohms. Each position is labeled with its corresponding number, to show the magnitude and direction of the change in embedding impedance when the E-plane tuner position is changed.
The mount embedding impedance follows a circular path, and returns in position 6 to the same mount embedding impedance as position 1. As in the case of the backshort, this circular path and the periodic behavior of the pumped current are consistent with the periodic relationship between the impedance of a waveguide section and the electrical length of its E-plane tuner.
4.3 Mount Tuning Range

Since both the backshort and E-plane tuner combine to determine the mount impedance, any change in the position of either tuning element will move the embedding impedance along the circle for that tuning element. In addition, all circles defined by varying only the position of the backshort must overlap the circles defined by varying only the position of the E-tuner. The region of impedances in which the circles for the tuning elements do not overlap cannot be achieved by the mount. This region is illustrated in figure 4.15, and is called the forbidden region. All impedances outside the forbidden region are within the tuning range of the mount. These results are in good agreement with the low frequency modeling results discussed in section 5.2.
5. Analysis

5.1 Comparison of Calculated and Actual Pumped I-V Characteristics

One way of checking the accuracy of the mount embedding impedance that is calculated, is to compare the measured pumped characteristic to the pumped characteristic calculated using the mixer theory, calculated mount impedance and unpumped characteristic. The degree to which these two overlap in an indication of the accuracy of the value of the mount embedding impedance. Figure 5.1 shows the measured and calculated pumped characteristics for position 3 in the series of backshort tuner positions.

Figure 5.1: Measured and calculated pumped characteristics for backshort position 3.

Over the fitted range of 2-3 mV, the measured and calculated pumped current agree very well. At bias voltages outside this range, the agreement between measured and calculated pumped current is also good. This indicates that the improved technique
works well, and the mount embedding impedance calculated is close to the actual mount embedding impedance of the mixer in backshort position 3.

Figure 5.2: Measured and calculated pumped characteristics for backshort position 1.

Figure 5.2 shows the measured and calculated pumped characteristics for backshort position 1. As in backshort position 3, the measured and calculated pumped current agree very well over the fitted range of 2-3 mV. However, the agreement between measured and calculated pumped current over the entire range of bias voltages is not as good for backshort position 1 as it is for backshort position 3. The gap voltage appears to have decreased, indicating that the size of the energy gap has become smaller. This suggests a secondary effect between the number of photons interacting (pumping level) and the pumped characteristic. One possible explanation is the heating of electrons when the power coupling is good and the heat sinking is poor. Another possibility is the failure of the small perturbation assumption made by Tucker in deriving the tunneling
current through the junction. At high power levels, it may not be possible to use the equilibrium density of states as the actual density of states may be markedly different.

5.1.1 Heating Effects

The heating of the superconductor by the energy from absorbed photons occurs when the junction is poorly heat sunk. Because the heat cannot be properly dissipated, it remains in the junction and raises the energy of the electrons. This can decrease the energy gap, causing a proportionate decrease in the gap voltage.

The magnitude of the incident power irradiating the junctions was on the order of 15 to 30 microwatts. The power absorbed in the junctions at high pumping levels was approximately 1 microwatt. The observed decrease in gap voltage was 0.25 millivolts. Assuming a $T_c$ of 13 °K for the NbN films, this corresponds to a temperature rise of 3 °K. This implies a thermal heat sink resistance of 0.33 $\mu$W/°K, which is larger than expected. Thus heating may be a problem.

5.2 Mount Embedding Impedance Predicted by Low Frequency Mixer Model

When a high frequency mixer is to be designed, a low frequency scale model of the mixer is made, and the mount embedding impedance is determined from the model using a network analyzer. This was done by K. Jacobs and W. R. McGrath [15]. For a mixer with tunable elements such as a backshort and E-plane tuner, a range of mount embedding impedance is determined. Scaling the size of components from the low frequency model to the actual high frequency mixer is by no means exact, and losses such as surface losses in waveguides can not be accurately scaled. However, the scale model gives a rough idea of the tuning range of the mixer. Figure 5.3 gives the tuning range predicted by the scale model for the test mixer.
As figure 5.3 indicates, the tuning range predicted by the scale model is very similar to the range obtained using the constant power source model and voltage matching technique.

5.3 Results at 230 GHz

In order to independently verify the performance of the improved technique for calculating mount embedding impedances, the technique was applied to dc I-V characteristics from a 230 GHz planar mixer designed and built by P. Seigel, P. Stimson, and R. Dengler at JPL [16]. Planar mixers have no tuning elements, so the mount embedding impedance is fixed. If the improved technique is accurate, it will give the
same mount embedding impedance for any pumped and corresponding unpumped dc I-V characteristics measured in a specific planar mixer. Pumped and unpumped dc I-V characteristics were measured for two different Nb/AlOx/Nb junctions [17] A & B. For junction B, the pumped characteristic was measured with the incident radiation at two different intensities. The dc I-V characteristics are presented in figures 5.4 - 5.6.

![Graph showing DC I-V characteristics of junction A.](image)

**Figure 5.4:** DC I-V characteristics of junction A.
The range of bias voltages over which the least squares fit was performed is 2.0 - 2.7 mV.

The mount embedding impedances calculated are presented in figure 5.7.
As figure 5.7 shows, the mount embedding impedance calculated for the 3 incident power settings fall within a very small region of the smith chart. Within the limits of experimental error, the mount embedding impedance for all three is the same, verifying the improved technique. The calculated and measured pumped characteristics for the three cases above are presented below in figures 5.8 - 5.10. These results also agree well with low-frequency model measurements of the planar mixer [18].
Figure 5.8: Measured and calculated pumped characteristics of junction A.

Figure 5.9: Measured and calculated pumped characteristics of junction B at low incident power.
Figure 5.10: Measured and calculated pumped characteristics of junction B at high incident power.

As seen from these figures, the calculated and measured pumped characteristics are in good agreement.

5.4 Accuracy of Mount Embedding Impedance

The accuracy of the mount embedding impedance is sensitive to errors in the I-V characteristics. The I-V is used to calculate the pumping voltage and RF current through the tunnel junction, which are then used to determine the embedding impedance. The relationship between the embedding impedance and the I-V is a bessel sum combined with an absolute value sum of complex numbers. Because of this complicated relationship, it is not possible to develop an analytic expression for the error in the embedding impedance as a function of the error in the measured I-V. Therefore, the functional dependence can only be determined by perturbing the pumped current on the first photon step by a known percentage and calculating the change in the embedding
impedance. Results are tabulated below for the 630 GHz mixer. Position 1 is a point close to the point of optimum pumping, while position 2 is a point that is further away. The pumped current at position 1 (high pumping) has been increased by 5%, while the pumped current at position 2 (low pumping) has been decreased by 5%.

Table 5.1: Impedance of E-plane positions 1 and 3 perturbed by 5%

<table>
<thead>
<tr>
<th>Position</th>
<th>Resistance (ohms)</th>
<th>Reactance (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position 1</td>
<td>1.5</td>
<td>5</td>
</tr>
<tr>
<td>Position 1 + 5%</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Position 2</td>
<td>13</td>
<td>-12</td>
</tr>
<tr>
<td>Position 2 + 5%</td>
<td>2.5</td>
<td>-7.5</td>
</tr>
</tbody>
</table>

The points in the table are presented below on a smith chart normalized to 50 ohms.
As seen in figure 5.11, the mount embedding impedance has a varying sensitivity to changes in the pumping current. When the impedance is close to the optimum, the sensitivity is lower than when the impedance is far from optimum.

### 5.4.1 Sensitivity of Pumping Voltage to Errors in I-V Characteristics

Pumping voltage errors were determined by perturbing the current at a nominal point on the first photon step by a known percentage and calculating the change in the pumping voltage at that particular bias voltage. This gives an approximation for the error in pumping voltage as a function of current error. The error in the pumping voltage for known errors in pumped current are tabulated below.
Table 5.2: Error in pumping voltage vs. error in pumped current

<table>
<thead>
<tr>
<th>Error in Current (%)</th>
<th>Error in Pumping Voltage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+5</td>
<td>+3.9076</td>
</tr>
<tr>
<td>+2</td>
<td>+1.5679</td>
</tr>
<tr>
<td>+1</td>
<td>+0.7848</td>
</tr>
<tr>
<td>-1</td>
<td>-0.7866</td>
</tr>
<tr>
<td>-2</td>
<td>-1.5751</td>
</tr>
<tr>
<td>-5</td>
<td>-3.9131</td>
</tr>
</tbody>
</table>

The results are graphed in figure 5.11. The linear relationship between the error in the pumped current and the error in pumping voltage is evident.

Figure 5.11: Pumping voltage error vs. pumped current error
It was found that the measured values of current had a maximum error of 2%. For such an error in pumped current, the pumping voltage can have a maximum error of 1.6%, which is close to one to one response of the pumping voltage error to pumped current error.
Conclusion

The constant power source model developed in this thesis can be used to determine the mount embedding impedance of SIS mixers at high frequencies. The results indicate that the position of the backshort and E-plane tuner in the 630 GHz waveguide mixer developed by the Microwave Advanced Devices Group at JPL, does determine the mount embedding impedance. Furthermore, the effect that these tuning elements have on the embedding impedance is periodic on the scale of one half the electrical length of the waveguide. By varying the position of either tuning element it is possible to move the embedding impedance along two contiguous circles. These results verify the results of the low frequency model used to build the mixer. They show that the mixer mount has a tuning range similar to the range determined in the low frequency model.

It was found that the best power coupling occurs when the mount embedding impedance was close to the complex conjugate of the impedance of the SIS tunnel junction. The sensitivity of the calculated value of the embedding impedance varies with the level of pumping. At high pumping levels, the mount embedding impedance is relatively insensitive to changes in the pumped current. At low pumping levels, the sensitivity of the mount embedding impedance to changes in the pumped current is much greater.

Analysis was also done on results from a 230 GHz planar mixer with fixed mount embedding impedance. For this mixer, with different frequency and type of mount, the technique agreed with low frequency modeling data, and gave the same mount embedding impedance for different pumping levels, thus demonstrating its versatility.
The results also indicate that there is a pronounced decrease in the energy gap of the superconductor in the tunnel junction at high pumping levels. The decrease in gap energy at the highest observed pumping level is consistent with a temperature rise of 3 °K in the superconductor, and a thermal heat sink resistance of 0.33 μW/°K. This resistance is higher than expected for the type of junction used in the mixer, indicating that heating of the junction may be the cause of the decrease in gap energy.
References


4. C. A. Mears, Qing Hu, P. L. Richards, A. H. Worsham, D. E. Prober, and A. V. Räisänen, “Quantum limited quasiparticle mixers at 100 GHz,”


