Resilient Provision of a Public Good

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Abstract. We present two resilient mechanisms for the provision of a public good. Both mechanisms adopt a knowledge-based benchmark.

Introduction Our first mechanism is appropriate to the case when the players —i.e., the potential beneficiaries of the good— are few in number and/or know each each other quite well. (In this case, for concreteness, we envision the provisioning to occur in a laboratory, and refer to the good as —a new piece of— equipment, to the players as members of the lab, and to the potential provisioner as the lab director.)

Our second mechanism is more appropriate when the players are quite numerous and/or may have only local knowledge, that is, each player only knows a few of the other players. (In this case, for concreteness, we envision the provisioning to occur in a city, and refer to the good as a public park, to the players as citizens, and to the potential provisioner as the mayor.)

Notation. Let \( N = \{1, \ldots, n\} \) be a set of players, and \( \gamma \in \mathbb{R}^+ \) the cost (to the "potential provider") of provisioning the good. A player \( i \)'s valuation of the good is a non-negative real. The profile of all possible valuations of the players is denoted by \( V \). The profile of the players’ true valuations is denoted by \( TV \). An outcome is a pair \((x, P)\), where \( x \) is a bit indicating whether the good will be provided \((x = 1)\) or not \((x = 0)\), and \( P \) is a profile of prices (real numbers). A player’s utility is \( TV_i \cdot x - P_i \). A player \( i \)'s general external knowledge, denoted by \( GK_i \), is \( i \)'s information about \( TV_\cdot -i \). A player \( i \)'s relevant external knowledge, denoted by \( RK_i \), is a subprofile in \( V_\cdot -i \) such that, for each \( j \neq i \), \( RK_j^i \) is the maximum value with \( GK_i \) and less than \( TV_j \). All knowledge of a player is private to him.

In the two mechanisms below, ”numbered steps are performed by players, and bullet ones by the mechanism.”

1 Our First Mechanism

Mechanism \( M_1 \)

- Set \( x = 0 \) and \( P_i = 0 \) for each player \( i \).

1. Each player \( i \) simultaneously and publicly announces a valuation subprofile \( V_i \) for players in \(-i\).
   - Set: \( \gamma_i = \sum_{j \in -i} V_j^i \) for each player \( i \), and \( * = \arg \max_i \gamma_i \).
     (We shall refer to player \( * \) as the “star player”.)
   - If \( \gamma_* < \gamma \), HALT.

2. (If \( \gamma_* \geq \gamma \)) Each player \( i \) such that \( V_i^* > 0 \) publicly and simultaneously announce YES or NO.
   - If some player announces NO, reset \( P_* = P_* + V_i^* \) for each player \( i \) who announces NO, and HALT.
   - (If all players announce YES) Reset: (1) \( x = 1 \); (2) \( P_* = \gamma - \gamma_* \); and (3) \( P_i = V_i^* \) for each player \( i \neq * \).
Variant. In the last mechanism step replace instruction 2 with the following instruction (2′) $P_\star = \alpha \cdot (\gamma - \gamma_\star)$, where the coefficient $\alpha$ is a constant between 0 and 1 (so as to generate a “surplus” for the lab).

2 Our Second Mechanism

Mechanism $\mathcal{M}_2$

- Set $x = 0$ and $P_i = 0$ for each player $i$.

1. Each player $i$ simultaneously and publicly announces (A) a subset of players $S_i \subset -i$ and (B) a valuation subprofile $V^i$ for the players in $S_i$.

- $\forall j$: If $j \notin S_i$ for all $i \neq j$, then set $EV_j = 0$; else, $n_j = \arg \max_{i \neq j} V^i_j$, and set $EV_j = V^n_j$. Set $K = \sum_j EV_j$.

- If $K < \gamma$, HALT.

2. $(K \geq \gamma) \forall j$ such that $EV_j > 0$ publicly and simultaneously announces YES or NO.

- If some player announces NO: $\forall k$ such that player $k$ announces NO, reset $P_{n_k} = EV_k$. HALT.

- (All players announce YES) (1) reset $x = 1$ and $P_j = EV_j$ for all $j$; (2) $\forall j$, reset $P_{n_j} = P_{n_j} - (K - \gamma) \frac{EV_j}{K}$.

Variant. In the last mechanism step replace instruction 2 with the following instruction

$$(2') P_{n_j} = P_{n_j} - \alpha \cdot (K - \gamma) \frac{EV_j}{K}$$

where the coefficient $\alpha$ is a constant between 0 and 1 (so as to generate a “surplus” for the city).