Hybrid Lattice and Decision Analysis of Real Options: Application to a Supply Chain Strategy

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by

Nestor Quispez-Asin

Submitted to the Engineering Systems Division on December 13, 2007 in partial fulfillment of the requirements for the Degree of Master of Science in Engineering Systems

Abstract

In many real world systems, two types of uncertainties exist: those that evolve in small, continuous increments and those that may create large, discrete changes in the system. The field of engineering real options posits that flexible system designs can improve system performance in the face of such uncertainties. However, up to now, most analyses of engineering real options deal with one type of uncertainty at a time. One common analysis method for the incremental uncertainty is done by using binomial lattices, while the discrete changes are typically analyzed using traditional decision analysis.

This thesis develops a new hybrid method which combines the lattice and decision analyses for the evaluation of real options. This method makes it possible to account for and display both types of uncertainties at the same time while drawing on the strengths of the two traditional methods. The main advantage is that decision makers are able to compare distributions resulting from strategies rather than only comparing single value evaluations such as expected net present value. The description of the distributions is made via Value at Risk and Gain (VARG) graphs. Also, risk preferences of decision makers are considered directly, rather than by the use of artificial utility functions or by evading the issue entirely. The main disadvantage of the method is that its complexity grows exponentially if many time periods, decision, and chance events are introduced. Therefore, the procedure is outlined for two stages of analysis step by step, and it has been programmed in Excel.

To illustrate the method, an application to a supply chain strategy is developed for a computer wholesale company. The situation facing the company is whether to set up a local distribution mode (LDM) in a region experiencing increasing demand. The competition may also decide to establish local distribution in the region. In this light, the incremental uncertainty is the growth of demand while the discrete uncertainty is the competition's decision to enter the market locally.

Thesis Supervisor: Richard de Neufville Title: Professor of Engineering Systems

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Chapter 1. Introduction

The growing field of real options offers great possibilities to evaluate projects in being able to analyze diverse uncertainties in a system and posit how to build for flexibility. This flexibility may provide the system the capability to avoid a bad result, to take advantage of better than expected outcomes, or do both. However, in trying to account for uncertainties, the relatively new field also has some challenges yet to be resolved.

One such challenge is the ability to account for both a small, incremental uncertainty and a large, discrete uncertainty simultaneously in one model. In many real world cases, situations exist where both types of uncertainties are considered critical for system design. For example, consider an internet service provider laying down a fiber optics network for future customer use. On the one hand, one concern is the evolution of natural demand for internet service, which is best modeled as an incremental evolution over time. On the other hand, there may be a concern about a specific regulatory change which would change potential customers' willingness to pay for the service. Countless other similar examples can be thought of in the fields of energy generation, urban planning, and supply chain strategy, to name a few. Thus, it is clear that both uncertainty types are important to consider for such system designs, and there is the need for one model to consider both types of uncertainties.

Traditional Lattice and Decision Analysis and a Hybrid Model

Within the field of the engineering real options, there are two specific models that have traditionally dealt with each type of uncertainty separately. Lattice analysis, which is a modified method adapted from financial option valuation, is able to analyze incremental uncertainties alone. Decision analysis is a more intuitive method that is better suited to consider large, discrete changes.

The principal aim of this thesis is to illustrate each of these two methods independently in the context of a case study and then develop the theory and application of a new hybrid

lattice and decision analysis method. This new hybrid method, which has computational limitations inherited from traditional decision analysis, combines the two previously separate models. Its main strength lies in its ability to combine elements of the two methods so that one model can consider both types of uncertainties at once. Also, it has the advantages that results are presented in terms of distributions of possible results instead of single values and that risk aversion of decision makers is taken into account directly.

Basis of Case Study

The development of this new hybrid method was inspired by a supply chain problem from the computer wholesale distribution industry in Peru. It is within this sector that the largest distributor in Peru, Grupo Deltron S.A., has an opportunity to expand to a city, Huancayo, which holds a vast potential and relatively untapped market in the interior of the country.

Although many industry experts agree Huancayo and the provinces surrounding it hold great growth potential today, there is considerable uncertainty when it comes to how much growth is expected to materialize, from the most general numbers down to the demand of specific computer parts. This natural demand uncertainty is of the type that evolves in small increments. In addition, there is considerable uncertainty as to if and when the competition might expand to Huancayo. This uncertainty is of the type that has the potential to create large step changes in actual, effective demand. Thus, the challenge is to analyze a flexible plan of expansion considering both the natural demand uncertainty and the competition's decision to enter the market.

Background of Computer Wholesale Distribution Industry

To provide a wider context for the application of the aforementioned methods in this thesis, some details about the specific industry are discussed here. The computer wholesale distribution industry is as large as it is diverse. Most global reports and

industry outsiders typically focus on the manufacturers of computer and IT parts and accessories, but it is the complex distribution channel, differing from region to region worldwide, that ultimately presents the end customer with the products.

The different business and distribution modes that exist range from large multinational firms that supply end user retail stores to small, independent distributors catering to a niche of specialized customers in one specific region of one country. The variety a distributor carries also varies substantially, as industry leaders may be officially authorized by hundreds of brands and product types compared to the small specialized distributor who may focus on only a few brands in one product type. For example, the top two distributors globally, Ingram Micro Inc. and Tech Data Corporation, had revenues of \$28 billion USD and \$20 billion USD respectively in the 2006 fiscal year. Both are headquartered in the United States and have operations in Europe, Asia, and Latin America. They supply retail stores such as Wal-Mart and CompUSA in North America, and their range of products spans from basic computer hardware to highly specialized servers and routers for large corporations (Datamonitor 2007a, Datamonitor 2007b). In contrast, it is fairly common for a small, local, and independent distributor to identify a niche market such as printer consumables and make a living purely from this.

Despite the different distribution and business modes in this industry globally, there are certain characteristics that are similar for all computer parts distributors, large and small. First, all are subject to demand uncertainties caused by the rapid technology changes in all product lines. The processor and memory, for example, are particularly volatile, highcost product lines where constant change in technology wreaks havoc for those distributors who fail to match price and demand at the right times. Second, when the volumes are large enough, most established distributors work a line of credit with a limited amount and time to pay back suppliers. This requires efficient management of inventory levels, as what is bought today must be received, sold for a slight profit, and the original cost paid back to the supplier at the end of the time period. Failure to rotate sufficient inventory levels and pay back at the end of the time period creates a backlog in the line of credit, which can lead to temporary or permanent closure of the credit, aside

from adverse effects of lost confidence from other creditors and suppliers. Third, wholesale computer distributors benefit from the concept called risk pooling in the realm of supply chain management, which counteracts to some extent the volatility of short life computer products. Risk pooling posits that variability of demand can be greatly reduced by aggregating across a variety of fields, such as customers and items (Simchi-Levi, Kaminsky & Simchi-Levi 2003). The idea is that by wholesalers having a variety of customer types to sell to, such as large corporations and small resellers, if one customer segment does not buy an item, the other one might, and so the risk of not being able to sell the item within a designated time period is reduced.

In South America, the distribution of computer products is not as direct to the end user as in North America and Europe, where there are several well established computer and IT specific retail stores such as CompUSA or Micro Center in the United States, or even Dell, which tries to avoid the retail store channel entirely. The sales channel in South America over the past 15 years for this equipment has been more fragmented, needing large wholesalers to resell to smaller local retailers who then market to the local end user. In addition to this reseller market, there are also the niches of growing retailers who do not specialize in IT, corporate entities in need of IT infrastructure, and government programs that are typically organized through bids. Despite the fragmented nature of the client base, a recent study of Latin America shows that the number of personal computers in use has increased since 2001 by 15 to 20 percent each year (Euromonitor International 2007).

Computer Wholesale Distribution Industry in Peru

In Peru specifically, in 2006, Deltron had a majority market share of about 30 to 35 percent of the market. Two foreign companies, Intcomex, an American distributor, and Tech Data Peru, a subsidiary of the global leader Tech Data, are Deltron's main competitors (Avance Economico 2007).

The development of the Peruvian market has typically been centered around Lima and relied on smaller resellers who deal with very price sensitive final customers. There are no specialized large retail IT end user stores in the country. The distribution channel relies mostly on small local computer assemblers, on the order of 65 percent in value of all computers sold in Peru over the span of 2006. Due to the prevalent lower personal income levels in the country, a PC at home for most people has been widely considered a luxury item since the 1990's. This has changed over the past five years, as the economy has improved and the cost of a PC has declined. Thus, the market for cheaper, more accessible PC's has especially grown. Over 60 percent of total PC sales in 2006 in Peru were of computers priced just under \$600 (Avance Economico 2007).

Within the country, most of the market has been centered in the capital city of Lima, but has been recently growing with an improving economy and infrastructure. Prior to the mid-1990's the telecommunications infrastructure was poorly maintained in the provinces outside of Lima, and most demand for computers was from large businesses such as mining companies (Business Monitor International 2004). Peru's geography historically has complicated development, as the northern and southern flat coastal regions have been accessible by highways, but the interior from Lima inwards are the mountainous, hard to reach, and poorer areas of Peru. On the other side of this mountainous region, the provinces of Ucayali and Loreto are only accessible by air since this region is mainly jungle (see Figure 1-1). As infrastructure has improved over the past ten years in the provinces and commerce increased, the market in the provinces has grown, and the main distributors have set up subsidiaries to distribute locally mainly in the north and south of the country. Currently, between the three major players, there are subsidiaries in the north in Chiclayo and Trujillo, and in the south in Arequipa and Cusco. The growing untapped market which concerns this thesis is the central interior region centered around Huancayo, which is currently served by all directly from Lima. Recent highway improvements however have made the provinces of San Martin, Huanuco, Pasco, Junin, and Huancavelica more accessible for commerce, and so Huancayo looks to be a strategic hub to serve that entire region more closely.

Figure **1-1:** Map of Peru.

Finally, there is also an issue as to the different local distribution modes (LDMs) that Deltron can implement if going to Huancayo. Two extreme LDMs considered are distribution centers and full subsidiaries. In a general sense, the distribution center mode seeks to reduce costs by serving as simple pickup points of merchandise for customers. The object of reducing transportation risks for clients is obtained, while reducing personnel and inventory costs. The full subsidiary mode tries to improve service level and capitalize on potential sales at the expense of carrying regular inventory which also implies higher personnel requirements. Obviously, there could also be a compromise in the middle of these two distribution modes.

Case Study Caveat

Although the development of the hybrid method was inspired in trying to solve Deltron's supply chain problem, the real life, actual parameters and results of the models presented in this thesis have been altered. This is done in the interest of protecting proprietary information while still providing a case study with a very realistic flavor.

Thesis Structure

The general thesis structure first defines current real options theory with an emphasis upon the traditional lattice and decision analysis methods. Deltron's supply chain problem is then analyzed in the traditional fashion looking only at one uncertainty type at a time. Next, the theory of the new hybrid lattice and decision analysis method is developed. Finally, the method is applied once more to Deltron's expansion problem, but this time considering both uncertainty types simultaneously.

• Chapter 2. Theory of Real Options Evaluation Methods - This chapter explains the basics of real options with specific step by step instructions for the traditional lattice and decision analysis methods.

- * Chapter 3. Lattice Method and Demand Uncertainty I: Lima versus a Local Distribution Mode **-** Only considering demand uncertainty, Deltron's flexibility to switch from Lima to a local mode of distribution (LDM) is analyzed with a lattice method.
- * Chapter 4. Lattice Method and Demand Uncertainty II: Middle Mode versus Other Distribution Modes **-** Only considering demand uncertainty, Deltron's flexibility to switch from one specific type of LDM, described as a middle mode, to another mode is analyzed with a lattice method.
- * Chapter **5.** Decision Analysis: Effect of Competition **-** Only considering the uncertainty of the competition entering the local market, this chapter employs traditional decision analysis to Deltron's supply chain problem.
- * Chapter 6. Theory of Hybrid Lattice and Decision Analysis **-** The chapter develops a step **by** step general procedure for the new method of analysis.
- * Chapter **7.** Hybrid Lattice and Decision Analysis: Demand Growth and Competition **-** The new hybrid method is illustrated in the case study of Deltron to consider both natural demand growth and competition uncertainties.
- * Chapter **8.** Conclusion **-** General reflections upon the hypothetical results and the advantages and drawbacks of the new hybrid lattice and decision analysis method are summarized.

Chapter 2. Theory of Real Options Evaluation Methods

Uncertainty is everywhere. Fortunately, in most cases so is the flexibility necessary to react in the face of uncertainty. Any project undertaken must consider both of these elements in order to guard against adverse effects, but also to benefit from better than expected scenarios. In the business and engineering world, real options is the technical term for these flexible actions that provide managers with the ability to react to uncertainty, and these real options must be valued appropriately. This chapter explores the origins of real options valuation, from traditional valuation methods which ignore uncertainty and flexibility to real options valuations, which differ in the financial and engineering realms.

Traditional Valuation Methods

When faced with capital budgeting decisions, managers have usually resorted to different traditional techniques to determine which project to undertake among a set of alternatives. Each traditional technique has its advantages and disadvantages, focusing on a certain aspect of the decision. In this section a few of the more common techniques of payback period, return on investment, net present value, and internal rate of return are discussed. In a world of complete certainty, net present value is the most suitable valuation technique; however, since the world is not at all certain and involves nonlinearities, more advanced techniques are necessary for proper project evaluation.

Payback Period

Payback period measures how long it takes to recuperate an initial investment made in a project. This method looks for the project with the shortest payback time and is insensitive to the time value of money. As such it ignores the fact that money in hand today is worth more than money in hand tomorrow, as having money today means it can be presumably invested in another project to earn additional interest. It also does not

consider future cash flows to be gained after the money is paid back and is in essence blind to the entire stream of future cash flows coming from a project. Although it has these numerous disadvantages, the method has appeal due to its simplicity and underlying conservative-minded assumption that long term forecasts of cash flows are inaccurate (de Neufville 2006).

Return on Investment

Return on investment is the total revenue at the end of the project period divided by the initial outlay of the project. It is a percentage of the initial cost of the project, and thus the idea is to take the project with the greatest ROI percentage. Its main appeal lies in its ease of computation and understanding. Unlike payback period, ROI does consider all cash flows over the project lifespan. However, it does ignore the time value of money (i.e., does not discount cash flows) and thus is also unsuitable to value a project correctly (de Neufville 2006).

$$
ROI = \frac{C_f - I_0}{I_0} \tag{1}
$$

Net Present Value

Net present value, or NPV, of a project is the discounted value of all cash inflows minus the discounted value of all cash outflows. The rate at which cash flows are discounted to take into account the time value of money is frequently the weighted average cost of capital, or WACC. The WACC is the opportunity cost of the money being used to finance the company and is determined by the combination of the cost of money coming from equity and debt holders. It reflects the risk adjusted rate of return that could be earned elsewhere by investing in projects of similar risk (Copeland 2007).

$$
NPV = \sum_{t=0}^{T} \frac{C_t}{(1+r)^t}
$$
 (2)

In the above formula, t is the time period at which the cash flow C_t occurs. The cash flow C_t is the net revenue, or profit, in that period. Typically, a large negative cash flow occurs at the beginning of a project at **t=0** when there is the large initial investment. When choosing among a set of projects, the NPV criterion dictates that managers opt for the project with the highest NPV. In a world of complete certainty where the cash flows will not vary from the forecast, NPV is the adequate capital budgeting technique since it takes into account all cash flows and takes into account the time value of money (Copeland, Weston & Shastri 2005).

However, there are two main difficulties with the use of NPV. First, the choice of the discount rate to be used can be quite critical and somewhat subjective. A low discount rate favors capital intensive projects whose strong positive cash flows are to come in the distant future. High discount rates on the other hand favor projects whose benefits will be received relatively soon. Second, the real world can be very uncertain, and the assumption that cash flows are completely predictable is severely flawed. Thus, the simple NPV methodology must be supplemented with a recognition of uncertainty that does not simply look at the most likely scenarios (Spinler 2007).

Looking at only the most likely scenarios can have disastrous consequences, as Sam Savage posits in the "The Flaw of Averages". "The Flaw of Averages states that: Plans based on the assumption that average conditions will occur are usually wrong" (Savage 2000). The critical insight here is that many times when managers are faced with uncertain future states of a system, they tend to make decisions and plans based on average conditions. Clearly, this presents a problem, if for example, one planned the capacity of a computer distribution center based on the fact that the average throughput is 1,000 PC's per month. In periods of high demand however, if the demand in a month is 1,500 PC's then the system design will not be able to accommodate the extra 500 PC's. More rigorously, Jensen's inequality, shown below in formula 3, states that the expected value of a function whose input is a random variable is not necessarily equal to the function of the expected value of the random variable, especially when the functions involved are nonlinear (de Neufville 1990). Thus, NPV calculations made based on the expected values of inputs cannot be satisfactory to provide a complete picture for a capital budgeting decision.

$$
E[g(x)] \neq g(E[x])
$$
 (3)

Internal Rate of Return

The internal rate of return, or IRR, is the discount rate in the NPV formula at which the NPV of a project is zero. The significance of this rate is that the rate of return of the project should be greater than the opportunity cost of capital. Though related to NPV directly, IRR is not a good metric for capital budgeting since the solution is not always unique. That is, there can be multiple rates of return for the same project depending on the order of cash flows. Also, the IRR does not always correlate with the NPV, meaning that an increasing NPV does not necessarily translate into a decreasing IRR (de Neufville 1990). Finally, IRR, like NPV, also suffers from the flaw of averages as the most likely cash flows are used to determine its value.

Financial Real Options

The financial real options were the first developed since they stem from financial options valuations in stocks and securities markets. To understand them, one must first know the basics of financial options valuations. From there, the analogy to the financial real options can be made.

Origins

Real options in general have origins in financial options, which are rights, but not obligations, to buy or sell an underlying asset for a specific price, called the strike price. The underlying asset can be a share of stock, commodity, or currency. As such, financial options apply in the realm of stock markets, meaning that they are tradable assets. An option allowing one to buy the asset for the strike is a call option, and one that allows the investor to sell the asset for the strike is a put option.

Consider a share of a company's stock currently valued at \$90. If one holds a call option on the underlying with a strike price of \$95, then if the share's price rises to **\$97,** one can exercise the call option **by** buying for **\$95** what is worth **\$97** and have a payoff of \$2. Similarly, if one owns a put option with a strike price of \$88 and the share's price drops to **\$86,** then one can sell for \$88 what is worth **\$86** and also have a payoff of \$2. The question, however, is how to value what an option is worth.

Fischer Black, Robert Merton, and Myron Scholes solved a great part of this issue in 1973 **by** developing the closed form Black-Scholes formula, which values European call options on non-dividend paying assets. A financial option is European if it can only be exercised at the time of maturity, and not before. (American options can be exercised at any time up to the maturity date.) The put-call parity formula in finance theory implies that the Black-Scholes formula can also be used to value European put options. As such the formula is very powerful, but its derivation is complicated and proceeds from solving stochastic differential equations (Mun 2002).

Due to its complexity, the Black-Scholes formula is not of direct interest to this thesis, but its inputs which determine the value of a European call or put option on a nondividend paying asset are. The five inputs are as follows (Mun 2002):

- * **S,** the value of the underlying risky asset. This is the value of the share price or commodity being traded.
- \bullet X, the strike price. This is the price at which an underlying can be bought with a call or sold with a put option.
- *** t,** the time to maturity of the option. This is the maximum time one can wait to exercise the option.
- σ , the standard deviation of the underlying asset. This describes how volatile the stock is.
- r_f , the risk-free rate. This is the rate of return one can earn without investing in a risky asset.

The Black-Scholes derivation is based on the assumption of arbitrage enforced pricing, which also gives rise to the method of using replicating portfolios and risk-neutral "probabilities" to value options. Arbitrage is a situation in which one makes a profit by identifying mismatches in an underlying asset's prices across markets. To the extent that all assets can be traded without a problem, arbitrage enforced pricing assumes that there are no arbitrage opportunities (i.e., that all markets price correctly). The driving force behind the no arbitrage principle is that one can always form a replicating portfolio, or a set of assets that has the same payoffs of the options by borrowing or lending at the riskfree rate and buying or selling the option (or asset) at the same time. With such replication available, the objective probabilities of the underlying asset's value rising or falling become irrelevant, as one can obtain the same payoff in any state of nature by forming a hedged position. Thus, besides the Black-Scholes formula, one can also obtain the value of an option in a more intuitive way by either constructing a replicating portfolio or using what is known as the risk-neutral "probabilities" instead of objective probabilities (Antikarov, Copeland 2003).

Real Options

Financial options provide investors with the ability to take advantage of an upside in an asset's rise or to get out of a bad situation in case of the asset's fall. The analogy can be made to non-traded assets, such as a private company's project. This analogy of applying options type thinking to real world, non-marketable projects is what is known as real options. A real option is thus "a right, but not an obligation to take some action now, or in the future for a predetermined price (i.e., the strike price)" in a real world project (de Neufville 2006). The term was coined by Professor Stewart Myers of MIT in 1977.

Real options exist everywhere in the business world, and their analogies to financial options can be easily made. For example, a computer assembly factory can choose to build an extra wing to its plant that is opened only during the Christmas season when demand exceeds capacity at the rest of the factory. In this example, the underlying asset is the present value of the cash flows of the company over the period being analyzed, and

the strike price is the value of making the extra wing operational for the season. The time to maturity is until the end of the Christmas season when demand is higher than normal. The volatility of the asset can be obtained from historical data of the company's periodic cash flows, and the risk-free rate is easily observable from the market.

The example described above falls within what is termed in this thesis as the "financial real options" point of view, since it inherently assumes that the underlying in the analogy can be treated as a traded asset. Several practitioners such as Copeland and Antikarov adhere to this view. Copeland and Antikarov call this the Marketed Asset Disclaimer, or MAD assumption (Antikarov, Copeland 2003). They suggest that in the absence of a twin security, to apply arbitrage enforced pricing for real projects, it is best to use the present value of the project itself without flexibility. This assumes that the value of the project itself is the best estimate of the project were it a traded asset.

The MAD assumption may be acceptable in some cases, but in general poses serious difficulties. The assumption is not as problematic to accept when the company whose project is being valued is publicly traded and produces assets that are also traded. Copeland, Antikarov, and Spinler have developed such case applications which include the valuation of an oil company's decision to acquire land to develop a field in the future or a telecommunications firm's decision to invest in technology that can be further developed in upcoming years if the market demand grows. In these cases, there is substantial knowledge of a firm's cash flows to determine the asset's value and volatility, and since the firms in these examples are publicly traded, it is not a far off extension that an individual project within that firm could be a viewed as a traded asset, since it has a direct correlation to the traded shares. However, in cases where the company is privately owned, and there is little data to determine the present value of cash flows and the corresponding volatility, the use of this assumption is unrealistic. In the context of the case study in this thesis, use of the MAD assumption is out of place, as the case deals with a privately owned company whose potential local distribution outpost cannot be traded in an open market.

There are several different types of financial real options. Most are real options "on" projects, which are actions that can be taken upon the system without the need for knowledge of detailed technical specifications. Many times it is difficult to classify a real option as being only one type, but here are a few of the most basic and common options (Schwartz, Trigeorgis 2001):

- Option to Defer: A firm makes an investment to acquire the right to develop a certain resource. Usually the time to develop the resource has a time of expiration, such as mining ore rights. The firm can choose to take advantage of upside potential by exercising its option, depending on market conditions.
- Option to Abandon: A company can choose to leave a project entirely if market conditions are not right.
- Option to Stage Development: A firm chooses to make its investments step by step in order to be more flexible to market conditions, instead of investing in the complete project in the beginning and risking being locked into narrower market possibilities.
- Option to Expand: A company makes an investment to grow its operations. The first operations create subsequent growth options, such as an online bookseller subsequently expanding to selling CD's.
- Option to Switch: A firm invests in having the infrastructure that can alternate between different modes of operations, such as a power plant having the ability to use either gas or oil to generate electricity, depending on the commodities prices. As stated before, a firm can choose to have different flexibilities that can be combinations

of these basic option types, resulting in simultaneous options. Also, there are situations where firms exercise options that create further options, and these are known as compound options.

Binomial Lattice Option Valuation

Some financial real options valuations make use of a recombining binomial lattice to evolve uncertainties of the underlying asset in a relatively simple and concise manner. This model was first introduced by Cox, Ross, and Rubinstein as a more mathematically

intuitive approximation to the Black-Scholes formula (Cox, Ross & Rubinstein 1979). This method uses the same inputs as the Black-Scholes formulas to form a lattice where the underlying evolves either to an "up" state (multiplication by u) or to a down state (multiplication by d). The factors u and d are reciprocals of each other and are calculated from the time step (Δt) used and the volatility of the underlying (σ). The method involves evolving the tree over several time periods and then finding the payoffs of exercising the option at each state of nature. The method calls for finding the option value at a future time and then working back to the option value at the present time, using the risk-neutral "probability", q, and the risk-free rate to discount. In this way, it is intrinsically assumed that the underlying can be traded and that there are no arbitrage opportunities. Though Cox, Ross, and Rubinstein use this approach, the lattice format can be used independently of these assumptions, as is shown later in the chapter. The formulas are as follows:

$$
u = e^{\sigma \sqrt{\Delta t}}
$$
(4)

$$
d = \frac{1}{u}
$$
(5)

$$
q = \frac{1 + r_f - d}{u - d}
$$
(6)

The main advantage of using this method is that from each time step to the next, the number of possible outcomes increases linearly (at time **0** there is one possibility, at time 1 two possibilities, and so on). This linear increase from each period to the next allows for a wide array of possibilities to be handled in manageable fashion (Chambers 2007). Also, the value of the underlying is projected following a lognormal distribution, with only non-negative values shown. Though the model was originally developed for valuation of financial options, financial real options simply apply the aforementioned analogy of inputs in order to use the model in the real options world. The following example illustrates the valuation a European call-like financial real option.

Figure 2-1 shows the valuation of an expansion option which one can exercise only at maturity in year 2 using the financial real options binomial lattice methodology. The

Notation:

 $V_{i,t}$ is option value at node (i,t) where i is row number and t is time period.

Figure 2-1: Financial Real Options Binomial Lattice Valuation Example. First, the underlying asset is evolved forward in time. Then using the risk-free rate to discount and the risk-neutral probabilities to weigh the option payoffs, the option value is traced backwards.

underlying in this case is the present value of future cash flows of a project with no flexibility to expand. This is valued at \$100 million. At year 2, one can choose to add extra capacity to expand the operation for a strike price of \$110 million. The methodology first projects forward the present value of the cash flows from now until year 2. Next, at year 2 when the option can be exercised, the payoffs for each of the three states of nature are calculated. Since the no arbitrage principle is in place and the underlying asset is being treated as a traded asset, the payoffs are discounted using the risk-free rate and weighted using the risk-neutral probabilities. The result is that the managers should be willing to invest up to \$10.77 million to have the flexibility to expand in year 2 should growth occur in the first two years of operation.

Although this method provides ease and simplicity for valuation, the binomial lattice assumes path independence. This means that the valuation does not distinguish between an up-down movement or a down-up movement, since both arrive at the same state of nature (de Neufville 2006). Clearly, in the financial options arena this does not present a problem, since the action one takes in a period due to there being an up state does not affect the value of the underlying. The financial real options approach also accepts this assumption, but it may prove to be problematic if, for example, a construction firm increases capacity for a toll highway. This in turn might cause the demand for highway use to increase, causing the underlying asset that is the present value of the cash flows to change.

Another assumption that may be problematic with this method is assuming that the present value of future cash flows is the direct underlying asset, since this implies that this value can never be negative. Clearly, it is possible that while a factor such as the demand for oil or PC's must be positive, a firm can lose money over a period of time. If one were to use another factor that can never be negative as the underlying instead of the present value of the cash values, this potential problem could be avoided. Indeed, as is explained later in this chapter, the engineering real options approach to binomial lattice valuation changes the underlying in order to account for this.

Engineering Real Options

The engineering real options approach breaks away from financial real options valuation. The approach does not accept the MAD assumption and can use different valuation methods. The two traditional methods relevant to the thesis and described here are the modified binomial lattice valuation and decision analysis.

Basics

The real options concept of investing in having the right, but not the obligation, to take some action now or in the future for a price can also be developed away from its original financial realm. This alternative perspective has developed in the field of engineering, where some of the underlying assumptions made in the financial realm cannot be accepted. This school of thought is referred to as the engineering real options in this thesis.

The most problematic assumption made in the financial real options approach according to the engineering real options view is the Marketed Asset Disclaimer. Most engineering projects onto which one can apply flexible options are not common and traded assets. Therefore, the underlying concept of enforcing a no arbitrage condition to perform a valuation falls apart. For example, if managers of a pharmaceutical company are considering developing a cutting-edge technology which produces a common drug but can also be used to produce a new drug if it gets FDA approval, it is difficult to think that this special technology can be treated as a traded asset. In the case study of this thesis, it is difficult to think that any flexible option built into the expansion strategy can be thought of as a traded asset as well. The only other interested parties who could value the flexibility would be other competitors, and since they do not possess the same distribution chain as Deltron, it is illogical to assume that this flexibility would have the same value to them as anyone else. Thus, the asset cannot be treated as if it were a traded asset.

On the other hand, one main criticism of the engineering real options approach is that it is difficult to establish the objective probabilities and an appropriate utility valuation of an expectation. That is, by probability weighting back and choosing the highest expected NPV, one is assuming that investors in general are risk-neutral, and not risk-averse as is more common. For example, faced with a decision of having \$100 with the toss of heads on a coin flip and nothing for tails or taking an assured \$40, this methodology assumes that the investor would take the coin flip since its expected value is \$50. One can try to modify this by using a utility function which translates the expected values into units called utiles, but then the problem which arises is the correct identification of a utility function (Spinler 2007).

Real Options In vs. On a System

The description of the types of options in the financial real options section also applies in the engineering real options realm. In addition to these types, the engineering real options also tend to focus on the distinction between options "on" a system versus those "in" a system.

Options "on" a system are those that do not concern themselves with the intricacies of the system design itself. The most basic examples of such options are those to open or close an operation depending on the state of nature. For example, Deltron could set up an outpost in Huancayo, and, seeing that demand is not as high as was foreseen, shut down the operation and revert to satisfying the region's demand directly from Lima. Most financial real options cases deal with these types of options (Wang 2006).

Options "in" a system are more interesting to the engineering community, as they require detailed knowledge of the system design. For example, a satellite system to be deployed for telecommunications can also be outfitted with dual technology to serve as a GPS satellite if demand for telecommunications is not as high as was expected (Wang 2006). In the case of Deltron, designing the interior space of the local outpost in Huancayo so it

can serve either as storage or a customer service area is also an option "in" the system. Both examples require specific knowledge about how the system operates.

Modified Binomial Lattice Valuation

The binomial lattice structure presented in the financial real options section can be modified under the engineering real options view. The objective is to take advantage of the recombining structure of the lattice and its lognormal distribution to project forward many distinct outcomes and keep the analysis neat by recombination.

There can be some variations on the exact implementation of the method described below, but there are two important differences from the binomial lattice method used in the financial real options. First, the underlying asset is a demand value, or some other physical factor which can never be negative, unlike the present value of cash flows in the financial view. A benefit function transforms this underlying into the units one wishes to measure the system in. In general, this can be anything, such as lives saved or, more commonly, currency. For the purposes of this thesis, the benefit function is called the free cash flow function, which measures the profit net of taxes.

Assuming that the benefit function measures financial profitability, the second main modification is that no arbitrage enforced pricing argument is used. Hence, no risk-free rate or risk-neutral "probabilities" are used. Instead, the firm's discount rate, typically the WACC, is used to discount values back to the present.

The underlying asset, or outcome, which is not a dollar value but some other factor such as demand, evolves based upon an average growth factor (v) expressed in terms of percentage growth from period to period. A periodic volatility (σ) factor, also expressed as a percent per period, determines how sparse the distribution is. At each time step, the possible value of the underlying is multiplied by the up or down factor, also as before. In this way, a continuous lognormal distribution is approximated by a discrete random walk. The analogous formulas are (Luenberger 1998):

$$
u = e^{\sigma \sqrt{\Delta t}} \tag{7}
$$

$$
d = \frac{1}{u} \tag{8}
$$

$$
p = .5 + .5\left(\frac{v}{\sigma}\right)\sqrt{\Delta t} \tag{9}
$$

There can be different implementations of modified binomial lattice valuation. In this thesis, a one-time option is to be valued. This means that over the evaluation period, only one option can be exercised, and once it is exercised it is irreversible. The method used in this thesis is to begin by developing the results of the "option never exercised" (ONE) and the "option exercised at the beginning" (OEB) scenarios independently. The OEB scenario performs calculations assuming that the option is exercised at *t=0* and is intended to be an artifact to arrive at the final valuation rather than as a final product. The results of the two scenarios are compared to determine when flexibility should be exercised and how much this flexibility is worth. The following list of steps describes in detail the valuation method used in this thesis:

- 1. The outcome lattice, which is the analogy of the underlying asset, is evolved over the periods of analysis using the starting outcome value, growth, and volatility parameters. It is assumed here that these parameters are the same for each scenario. Having them change would cause the lattices not to recombine. The notation used for each node in the outcome lattice is $OL_{i,t}$ where i is the row number and t is the time period.
- 2. A probabilities lattice is evolved over time for each scenario to determine for each particular outcome what the likelihood is of realizing that outcome. Although this lattice is not used explicitly in the rest of the analysis, it serves to provide an idea of how the underlying will evolve over time as a discrete approximation to a lognormal distribution.
- 3. For each scenario, the outcome lattice translates directly into an instant value lattice (IVL) through a free cash flow function. Each IVL contains the undiscounted cash flows of each node of the lattice given the outcome at that node. This free cash flow function depends entirely on the scenario parameters, of course, as it uses information such as fixed costs, variable costs, and profit margins to calculate the undiscounted

Figure 2-2: Outcome, Probabilities, and Generic Instant and State Value Lattice Construction for either the ONE or OEB Scenario.

cash flows. Also, the OEB scenario IVL excludes the initial investment, or strike price, that it costs to exercise the option. Note that the analysis here assumes that cash flows are realized at the end of a period. The notation used for each node in a generic instant value lattice is $IV_{i,t}$, where i is the row number and t is the time period.

- 4. The state value lattices (SVL) for both the ONE and OEB scenarios are constructed. Each node in these lattices provides the present value of being at that node looking forward in time. It takes into account the cash flow received at that period and adds the discounted expected future cash flows. The initial node of the ONE SVL provides the expected value of the fixed design with no option considering uncertainty. The OEB SVL does not consider the strike price at the initial node. The notation used for each node in a generic state value lattice is $SV_{i,t}$, where i is the row number and t is the time period. Figure 2-2 shows how the outcome and probabilities lattices are constructed and how a SVL is constructed from its corresponding IVL for either the ONE or OEB scenario.
- 5. Another SVL is constructed, but now for the flexibility scenario where the option may or may not be exercised. This is the most crucial and intricate step of the analysis where the results of the ONE and the OEB scenarios are compared using a dynamic program. The flexibility SVL provides the state value for each node assuming that the option has not been exercised at or before the beginning of that period. At each node is the value of the present cash flow plus the maximum discounted expected future cash flows of either not exercising the option now but possibly in the future or exercising now with the strike price considered. In less technical terms, this amounts to assuming at each node that one has not yet changed the original setup, but is looking forward in time to choose whether changing the setup now would be best. The initial node of this lattice provides the expected value of the flexible design considering uncertainty of the outcome. As with the outcome lattice parameters, it is assumed that the discount rate is the same in all scenarios, so as to make the lattice recombine. The notation used for each node in the flexibility state value lattice is $FSV_{i,t}$, where i is the row number and t is the time period. Figure 2-3 shows the equations of the dynamic algorithm used to construct the flexibility SVL along with other notational definitions similar to those in Figure 2-2.

Figure 2-3: State Value Lattice Formation for the Flexibility Scenario.

- 6. An optimal strategy lattice (OSL) can then be constructed to qualitatively answer when it is best to exercise the one time option. More specifically, the following question can be answered for each node: if the option has not yet been exercised, should it be exercised now? At each node, if the maximum function in the flexibility SVL algorithm chooses the second expression in which it is best to exercise the option at that instant, the answer is yes.
- 7. The option value is obtained by calculating the difference between the beginning nodes of the ONE and flexibility SVLs. If it is better to exercise the option at any node, then the flexibility SVL initial node is greater than the ONE initial node. Thus, the option value must be greater than or equal to zero. Note that this valuation of the option assumes that the investors are risk neutral, since the method is based on maximizing expected value. In reality, option value may be less given risk aversion of individuals.

The flow of information shown in Figure 2-4 is one general and common implementation of the binomial lattice structure in the engineering real options methodology for determining the value of a one-time option. It is the outline of the method employed in this thesis. The exact formulas and where the option is available clearly depend on the case study, which gives the decision maker more flexibility to alter the model as necessary. As has been mentioned, free cash flows can now be negative in the instant and state value lattices, and there is no use of the MAD assumption, which can be more realistic.

Furthermore, the results of several outcomes are concisely summarized due to the assumption of recombination. While this model may not entirely suitable in situations where path dependency is greatly consequential, the lattice can be decomposed easily into each individual path if necessary. For example, a Value at Risk and Gain (VARG) graph which shows the range of possible NPVs with their corresponding cumulative probabilities can be built for any scenario. However, when the problem in the system is considered heavily path dependent, another method such as decision analysis can be used in the engineering real options approach instead of the modified binomial lattice method.

Figure 2-4: Flow of Information Among Lattices as applied in this thesis.

Traditional Decision Analysis

Decision analysis is a more intuitive approach to valuing options where the main possibilities of system states are evolved over time, discounted to a NPV, and then probability weighted back to the present to identify the best strategy. The method is a natural extension of NPV analysis, but which incorporates uncertainty. A tree consisting of alternating decision and chance nodes serves as a visual guide. The procedure involves identifying the key uncertainties and obtaining the NPV for each possible scenario. Then, working backwards through the decision tree, one chooses the strategy at each decision node that maximizes expected NPV (Ramirez 2002).

The key advantages of decision analysis include high flexibility of modeling and an ability to value path dependent systems with large step changes. For the simulations ran for each possible scenario, any type of free cash flow function can be used to obtain the expected NPV for that scenario. Path dependency can now be modeled since the branches of the tree do not necessarily have to recombine. However, the analysis brings challenges in the form of identifying the correct objective probabilities and discount rate, and the previously manageable tree (with the binomial lattice) now increases branches exponentially from stage to stage.

Figure 2-5 shows an example decision tree for a company which is considering two strategies. The fixed strategy is to establish a store and stay with that same store format over two periods. The flexible strategy is to invest so that if demand is high, it can expand its store and better fulfill customer demand. Each period has two possibilities at the chance nodes of high or low demand. In the flexible strategy, in stage 2 one has the option to decide to expand or stagnate. Taking expected values at each chance node, one uses these values to make a decision at the decision nodes. The resulting strategy, as shown, is to be flexible so that in the second stage one can expand and benefit from increased demand. The option value in this case is the difference of expected NPVs of the fixed and flexible strategies.

As mentioned, the NPV values for each scenario are obtained by applying the free cash flow function to the system parameters. By implementing the strategy suggested by the decision tree analysis, one can derive a cumulative distribution function (CDF) of the possible NPVs. Also known as a Value at Risk and Gain (VARG) graph, this CDF provides an idea of the shape of the distribution of NPVs one can expect by implementing each strategy. Figure **2-6** is an example VARG graph for the preceding store expansion example. Indeed, the benefit of the flexible strategy is considerable, and the VARG graph demonstrates the dominance of the flexible strategy over the fixed strategy over most of the distribution. The flexible strategy performs worse than the fixed strategy only in the most unlikely results of lowest demand over the two periods.

For this example of traditional decision analysis, the choices made at the decision nodes are made upon simply taking the maximum of the expected values. Therefore, the analysis assumes risk neutrality of management. However, using utility theory, one can convert the NPV numbers to utiles for comparison and account for risk aversion. Software programs such as TreeAge **C** used to analyze decision trees incorporate such options. However, one criticism of this method is that the utility functions can seem somewhat artificial, abstruse, and difficult to construct.

Figure **2-5:** Example Decision Tree for Decision Analysis.

Figure **2-6:** Example Value at Risk and Gain (VARG) **Values.** graph, plotted with Expected

Conclusion

This chapter has explained the basic parts to real options valuation, from its origins in modeling financial options origins to the financial and engineering real options schools of thought. The financial real options approach to valuation draws a close analogy to financial options valuation, where underlying assets are traded. The engineering real options approach which will be used in this thesis, breaks from the analogy. A modified binomial lattice method and traditional decision analysis are two methods to be used in the case study for this thesis. Each approach and method has its advantages and drawbacks, but they are all an improvement over other valuation methodologies which do not take into account uncertainty or flexibility in systems.

Chapter 3. Lattice Method and Demand Uncertainty I: Lima versus a Local Distribution Mode

Over the next few years, the computer market is perceived to be growing in the provinces around Huancayo, which is a hub for the interior of the country. Grupo Deltron sees a clear opportunity to establish a local outpost to better fulfill demand and remain the leader in the region. However, there is considerable uncertainty as to the actual growth of demand. Given the demand growth uncertainty, the company is concerned with the foreseeable mid to long term future on aggregate basis, if and when it would be a good idea to establish such an outpost.

Currently the plan is to fulfill demand in the region directly from Lima. With a local outpost under a certain local distribution mode (LDM), demand would be fulfilled mainly from that outpost. Up to now, the rationale for fulfilling demand for the region from Lima has been that the volume sold to the region does not merit the extra fixed costs and organizational structure needed to operate a local outpost. With increasing demand, this rationale is under scrutiny, and the advantage of having a local outpost is becoming clearer as most estimates of future demand forecast growth over the next three years.

The analysis in this chapter will be structured logically in three parts that evolve from the traditional no uncertainty and no flexibility approach to a more dynamic approach that uses a lattice analysis to account for the incremental demand uncertainty. First, a traditional NPV analysis with no uncertainty and no flexibility to switch distribution modes is performed. Second, uncertainty is considered to see how varying demand changes the results in the fixed strategy that still does not allow for distribution mode changes. Third and lastly, the complete modified binomial lattice method is implemented in order to consider uncertainty and flexibility. The end result is a dynamic strategy, which depends on how demand evolves, and it must be better than or equal to a traditional fixed strategy.

Characteristics of Analysis

The three LDMs considered are designated as distribution center, full subsidiary, and middle mode. A distribution center (DC) mode seeks to reduce costs by making the locale a pickup point of merchandise for customers. The objective of reducing transportation risks for customers is obtained, while reducing personnel and inventory costs. At the other end of the inventory and personnel spectrum, the full subsidiary (FS) mode tries to improve service level (percentage of demand fulfilled) for most products and therefore capitalize on potential sales at the expense of carrying inventory and having more fixed costs, including higher personnel requirements. A middle mode is a compromise between the DC and FS modes. The exact definitions of the three modes are explained in detail in terms of inputs later in the chapter. Also, in this rapidly evolving industry, the foreseeable mid to long time span is on the order of three years, and therefore this will be the time span used for analysis in this chapter.

In general these inputs are for aggregate analysis. As has been noted, the main uncertainty is demand in terms of a generic PC. Although Deltron sells mainly disassembled computer parts, the industry has the advantage that since the computer is the prototypical modular product, for the most part, Deltron's sales in terms of units can be described in terms of matching ratios. For example, a computer typically needs one motherboard, one CPU, one hard drive, one memory, video and sound cards, one mouse, one monitor, and one set of speakers. Thus, the sales in dollar terms can be divided by the units sold in the right ratio and the result would be a good approximation for the generic value of a PC sold. This aggregation simplifies the analysis. Other aggregate level inputs include the shipping cost per PC, profit margin, locale cost, and labor cost.

The output of this aggregate level analysis will not be at all exact due to the various simplifying assumptions made. However, the result will give a sense of the scale of difference achievable by implementing the correct expansion strategy.

Rationale for Using a Modified Binomial Lattice Method

This chapter uses a modified binomial lattice method within the engineering real options approach to solve the option valuation problem that Deltron faces. The reasons for using a method with the engineering real approach in this case has been discussed at length in the previous chapter, but the use of the modified binomial lattice method for this analysis merits further explanation.

First and foremost, since the raw demand uncertainty evolves incrementally, it is best described by a continuous walk rather than by large step changes. While a drastic change is more suitable for decision analysis, continuous step changes are more suited to the binomial lattice, which evolves by a factor from period to period.

Also, there is no path dependence in the system. On an aggregate level, one overall assumption is that Deltron's expansion as a local presence in the region will not drastically elevate the growth rate of demand. The underlying market reason for this is that most empirical evidence suggests that the main driver of demand for computers in the provinces outside of Lima is the overall economic growth and the population's general income level. Thus, a wholesaler's presence in the region could in theory mean increased demand due to the proximity of the product and the clients not having to assume the risk of transportation anymore. However, the increase in demand due to this should be considered very slight in comparison to the more general economic underlying factors (Avance Economico 2007). Therefore, Deltron's actions will not affect the raw demand uncertainty directly, which means that path dependency is not an issue and that use of the recombining structure of the lattice makes sense.

The assumption that the demand will grow following a lognormal distribution is also valid on an aggregate level. It is very difficult in practice to perform a detailed analysis of the probabilistic demand distribution of the computer market growth in the provinces of Peru for various reasons. The data is sparse coming from various formal and informal sources and is incomplete many times due to the existence of the gray and black markets.

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Thus, a generic lognormal distribution as used in many other case studies seems perfectly reasonable if one is to account for uncertainty in some manner.

Definition of Distribution Mode Parameters

The different distribution modes to be analyzed in this section may contrast across the inventory and personnel setup, the PC cost and margins, the fixed costs of operation, and the tax and discount rates used. Figure 3-1 shows the parameters assumed for each of the distribution modes. The costs and rates shown are for six month time periods, since this will be the granularity of time increments used in the modified binomial lattice model.

Note **on Accuracy of Data :** Actual data has been changed considerably to protect proprietary information of costs, rates, personnel, and inventory setup.

Figure 3-1: Definition of Distribution Mode Parameters. Money values are in USD.

The parameters as shown in Figure **3-1** describe the fixed parameters that are assumed for the modified binomial lattice valuation in contrasting Lima with each of the different LDMs. When comparing Lima to these potential local outposts, only the factors extraneous to the current setup which would be used to serve the Huancayo region are considered. That is, whatever the cost of the facilities already installed in Lima are not considered in the analysis, since one must match expenses with the revenues generated from a certain project. The analysis is done in this way for only the portion of Deltron which is dedicated to the demand of this region.

The inventory and personnel setup reflects this type of designation but needs some qualification. The setup that would be used to serve the Huancayo directly from Lima would be able to capture about 80% of the demand. In order to do this, one estimate is that Lima would be required to hold about 5.5 days worth of inventory more than is currently done. As all the numbers in this thesis are purely hypothetical, this designation is a very rough estimate without much justification. In a more accurate application with real data, the determination of the inventory levels versus service levels would require a separate study, but for the purposes of the thesis, this analysis is not explicitly performed here. This trade-off between inventory needed for a certain service level depends on demand volatility, and a probabilistic model for the demand would be used. The parameters used for each LDM adhere to this same word of caution, but the general essence of the problem is captured. In order to satisfy a greater level of demand, one needs to carry a higher level of average inventory so as to guard against demand and lead time variations. A greater number of personnel would also have to be used in each of the LDMs. For simplicity, the analysis has broken up the types of personnel into two types: commercial and warehouse personnel, which reflects the mix needed in each mode. Naturally, the FS mode requires the greatest level of personnel since more inventory is handled and more clients are served while the DC mode is at the other end of the LDM personnel spectrum.

The PC cost and margin parameters used also demonstrate some basic trade-offs of the situation facing Deltron. By selling directly from Lima to the Huancayo customers, the

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company avoids shipping charges (and the cost of the risk associated with shipping), which would be assumed by the clients. In contrast, by setting up a local outpost, whatever the distribution mode, there is a shipping cost added to the PC cost. However, the company can now justify an additional markup on the cost by having assumed the responsibility of shipping.

The fixed costs of operation vary as one would expect for each distribution mode. For Lima, it is assumed that no additional capital investment, including personnel and utilities, needs to be made in order to support the movement of the extra inventory for the region. For each LDM, the fixed costs depend on the size of the outpost and on the number and type of personnel. Special note should be made of the locale cost, since in the modified binomial lattice valuation, this is considered the strike price needed to upgrade operation from Lima to one of the LDMs.

Finally, the corporate tax and discount rates used are assumed to be the same for whichever distribution mode used. Tax rates can vary if the laws require it, based on where the operation takes place, and the model as applied in this thesis can easily accommodate such a situation. However, although different discount rates could be possible theoretically due to different funding and risks across the distribution modes, this would cause the analysis to become too complex, as the lattices would no longer recombine. In this application, there is no reason to think that the different modes require different discount rates, and thus the recombining lattice analysis can be used without a problem.

Steps of Analysis

The question facing Deltron in general is two-fold. First, if Deltron had to make a choice now about which of the four distribution modes (Lima and the three LDMs) to establish to run for the next three years, which mode would be best? The first and second steps of analysis will answer this question, the first with no uncertainty considered and the second taking into account variations in demand. Second, if one views switching from Lima to a

LDM as a one-time option, when is it optimal to exercise it and how much is it worth? The third step of analysis which considers both flexibility and uncertainty answers this question.

No Uncertainty and No Flexibility

The most basic question to ask is: if Deltron were to choose a distribution mode now among the four alternatives, which one would be best, given a fixed demand increase projection? To answer this question, the most simple and direct method is traditional NPV analysis.

The analysis begins by defining the beginning demand, expected demand growth, and time period of analysis. It is assumed that demand for Deltron PC's is currently 25,000 units per six month period, and demand will grow at an expected rate (v) of 15% per year over the next three years, which is the period of analysis for the system life. The granularity of time is six months. Figure 3-2 summarizes these parameters.

Figure 3-2: Expected Demand Parameters and Evolution over 3 Years.

The next step in the analysis is to construct the free cash flows for each distribution mode based on the parameters identified in Figure 3-1. That is, the cash flow function must be defined, which is done below in Figure 3-3 with an example and the subsequent formulas.

Free Cash Flow Calculation:						
DC Mode, at Period 6, Expected Demand						
Revenue	S	18,130,731				
-Variable Cost	\$	16,482,483				
-Fixed Cost	S	75,000				
Earnings Before Taxes	S	1,573,248				
-Taxes	S	220,531				
Net Income	S	1,352,718				
-Cost of Inventory	S	63,928				
Free Cash Flow		1,288,790				

Figure 3-3: Example Calculation of Free Cash Flow.

The equations **for** each of the free cash flow components are listed below:

```
Revenue = Demand * Service Level * (PC Cost + Ship Cost) * (1 + Gross Profit Rate)
```
Variable Cost = Demand * Service Level * (PC Cost + Ship Cost)

Fixed Cost = Utilities Cost + Personnel Cost

Earnings Before Taxes = Revenue - Variable Cost - Fixed Cost

Taxes = Earnings Before Taxes * Corporate Tax Rate

Net Income = Earnings Before Taxes - Taxes

Demand 1 month Cost of Inventory = ays * Extra Days Inventory* (PC Cost + Ship Cost) * Discount Rate **(6** months)(**28** days)

Free Cash Flow = Net Income - Cost of Inventory

Using this function and the expected demand, the net present value of each of the four modes can be easily calculated. The results are shown in Figure 3-4. The results are shown down to very detailed significant figures only to give an idea of how close the three LDMs are. In reality, due to the broad nature of the analysis only 3 significant figures should be considered. Thus, the result is that Lima would render \$4.71 M over three years, while any of the LDMs would represent a 10% gain over Lima.

	6 Month Periods							% Gain over l	
								NPV	Lima
Lima	\$0l	\$804.268	\$862,482	\$924,908		\$991.854 \$1.063.645 \$1.140.632 \$4.714.152			--
$_{\text{DC}}$	-\$100.0001	\$889.719				\$958,785 \$1,032,850 \$1,112,275 \$1,197,450 \$1,288,790 \$5,174,194			9.76%
Middle Mode	-\$125.000l	\$889,347				\$961,809 \$1,039,516 \$1,122,847 \$1,212,210 \$1,308,041 \$5,190,418			10.10%
FS	$-$150.000$	\$882,305				\$957,991 \$1,039,155 \$1,126,195 \$1,219,534 \$1,319,629 \$5,171,482			9.70%

Figure 3-4: Net Present Value for each Distribution Mode, with No Uncertainty and No Flexibility to Switch Modes.

The parity among the three LDMs can be attributed to two factors. First, relative to the discounted sum of cash flows, the strike prices are relatively low and about the same. This is due to the low estimated locale costs in a developing region of the country. Second, the trade-off between more sales due to an increased service level and more personnel and inventory cost balances over the three years among the three modes.

In conclusion, the NPV analysis which does not consider demand uncertainty or flexibility suggests that Deltron should opt for a LDM. Furthermore, from an NPV standpoint, it does not matter which mode is used. Of course, if Deltron were to make a decision purely on this result, there are other strategic and real world factors such as market share and ease of implementation that would make a difference in determining which LDM to employ.

Uncertainty and No Flexibility

The next step in the analysis is to ask the question of how the demand uncertainty affects the results. To do this, binomial lattices are used to evolve each of the four fixed scenarios forward in time and obtain the expected value for each. At this point, there is still no option to switch distribution modes. As an added feature, one can now also compare NPV distributions via VARG graphs instead of looking simply at the expected NPVs.

To do this, first the outcome lattice evolved in six month periods over three years is the number of PC's demanded from Deltron. It is assumed that the demand is independent of the company's decision to have a local outpost or not, and so the same parameters are

used for each of the four scenarios. Figure 3-5 shows the relevant outcome, or demand, lattice parameters along with the resulting up and down factors and objective probability resulting. These last parameters are obtained by applying formulas 7, 8, and 9 in Chapter 2.

Demand Lattice Parameters	
6 Month Demand at Period 0	25000
σ, per year	25.0%
v, per year	15.0%
Δt , in years	0.5
	1.19
$\frac{1}{\mathsf{d}}$	0.84

Figure 3-5: Demand Lattice Parameters.

The parameters in Figure **3-5** are then used to construct the demand lattice and a probabilities lattice, as shown in Figure 2-2 and explained in Chapter 2. The demand lattice is the more relevant lattice, as it is this lattice that contains the main uncertain variable used to calculate the free cash flows in each node. Figure 3-6 shows the outcome lattice.

	OUTCOME LATTICE						
	6 Month Periods						
		2			5	6	
25000	29834	35603	42487	50703	60507	72207	
	20949	25000	29834	35603	42487	50703	
		17555	20949	25000	29834	35603	
			14710	17555	20949	25000	
				12327	14710	17555	
					10329	12327	
						8656	

Figure 3-6: Outcome, or Demand, Lattice.

The probability lattice serves to give an idea of the likelihood of each state of nature based on the volatility and growth parameters (v and σ , respectively), as shown in Figure 3-7. These are the probabilities of the demand as seen from the point of view of period 0.

	PROBABILITIES LATTICE						
	6 Month Periods						
O		2	З		5	6	
1.00	0.71	0.51	0.36	0.26	0.18	0.13	
	0.29	0.41	0.44	0.42	0.37	0.32	
		0.08	0.18	0.25	0.30	0.32	
			0.02	0.07	0.12	0.17	
				0.01	0.02	0.05	
					0.00	0.01	
						0.00	

Figure 3-7: Probabilities Lattice.

It is important to remember that the binomial lattice is a discrete approximation to a continuous lognormal distribution. As time increments are shortened, the probability density function (PDF) of the discrete distribution starts to resemble closely the continuous lognormal PDF. To illustrate this point, Figure **3-8** shows the discrete PDFs of the binomial lattice as used in this analysis and the corresponding continuous distributions, as seen from period **0.** As can be observed, **by** the end of period 6, the demand can vary from about **15,000** to 140,000 PC's demanded per six month period. This spread is a result of the volatility parameter. However, due to the growth parameter, the probabilities are skewed to the upper end of the distribution, where **50,000** PC's is much more likely than 20,000 PC's.

The next step in comparing the four fixed strategies is to construct their corresponding instant value lattices (IVLs) from the outcome lattice, as described in Chapter 2. The IVLs show the undiscounted free cash flows at the end of the six month time period at each node for implementing each LDM. The values of these lattices do not include the locale cost needed to be paid in order to obtain that value. In this case, this is what will correspond to the strike price later in the chapter when valuing the option of switching from Lima to an LDM.

In order to obtain the IVLs, the free cash flow function as defined earlier in this chapter is applied to the outcome at each node. Figure **3-9** shows the IVLs for each scenario.

Figure **3-8:** Comparison **of** Demand Probability Density Functions, Discrete and Continuous, as seen from Period **0.**

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Figure 3-9: Instant Value Lattices.

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Figure **3-10:** State Value Lattices.

With the IVLs in hand, it is simple to obtain the expected NPV for each fixed distribution mode by applying the state value lattice calculation algorithm shown in Figure 2-2. Thus, the SVLs for each fixed scenario are shown in Figure 3-10. Since in the fixed scenarios the locale costs are paid in period 0, one must subtract the respective locale cost for each distribution mode to obtain the expected NPV of that mode, as shown in Figure 3-11.

	SVL Initial Node	Locale Cost	Expected NPV
I Lima	\$5,013,327	\$0	\$5,013,327
Distribution Center	\$5,629,143	\$100,000	\$5,529,143
Middle Mode	\$5,687,820	\$125,000	\$5,562,820
Full Subsidiary	\$5,710,456	\$150,000	\$5,560,456

Figure 3-11: Expected NPV Calculation from SVL initial nodes.

Aside from the expected NPV values for each mode, the corresponding VARG graphs can be plotted to provide an idea of how spread out the possible results from implementing each mode from the beginning can be. This is done by decomposing the paths in the lattice. In this case, with **6** periods in which the NPV from period to period has two subsequent possibilities (up or down), there are 2^6 (64) possible paths. Each path has between 0 and **6** cumulative number of "up's" (U) by the end of the path, and this means that the probability of that path occurring is $p^{U}(1-p)^{6-U}$. By combining the NPV of each path with its corresponding probability, the VARG graph as shown in Figure 3-12 is constructed. This figure also shows the relevant maximum, minimum, and variability results for each fixed strategy. As before, the results are shown to detailed numbers to show how close the results turn out to be for the three LDMs, but the results can only be taken to three significant figures.

In this case, the uncertainty does not drastically change the results obtained by the simple NPV methodology. The following observations can be made based on Figure 3-12:

The expected NPV for Lima rose from about \$4.71M to \$5.01M. The expected NPVs for the 3 LDMs rose from \$5.18M to \$5.55M. This is due to the fact that the probability distribution is skewed toward the upper end of the outcomes.

Figure 3-12: Expected NPV Comparison for each Scenario, with Uncertainty but No Flexibility to Switch Mode, along with the Corresponding VARG graph.

- All three LDMs look almost exactly alike. The expected NPVs for all three now represent about an 11% gain compared to Lima, and this is slightly higher than the 10% result from before. The VARG graphs line up almost identically as well.
- The only noticeable differences between the three LDMs are the ends of the distribution. The FS mode, although providing the greatest possible gain for the highest possible demand, also results in the lowest possible NPV in the very unlikely event demand is very low. In contrast, the DC mode has the lowest spread among the LDMs.
- More than 90% of the time it is better to go to a LDM now than to stay at Lima over the three years.

Uncertainty and Flexibility

From the preceding analysis, one can establish that if Deltron had to choose now among the four distribution modes and stay with that same mode over the next three years, it should choose the middle mode local outpost, or if not, another LDM since they all provide roughly a 10% improvement over Lima. However, although current market forecasts point to great growth over the next three years, this does not mean it is necessarily best to switch from a Lima distribution mode to a LDM now. To answer this question, the switch from Lima to a LDM is viewed as a one-time option, and by performing the option valuation as explained in the lattice step procedure of Chapter 2, this answer follows as a natural extension.

Referring back to the option lattice valuation procedure of Chapter 2, three options are to be valued separately: switching from Lima to a DC, from Lima to a middle mode, and from Lima to a FS. Steps 1 through 4 have already been shown in the preceding section. The ONE scenario is Lima, and the three OEB scenarios are going to each LDM now. The only step left in valuing each option is to construct the corresponding flexibility state value lattices as depicted in Figure 2-3. In this implementation, the strike price is the locale cost for each LDM.

As a reminder, the flexibility SVL assumes at each node that the one-time option has not yet been exercised at or before the beginning of that period. At each node it adds the corresponding ONE instant value lattice node (in this case, this is the Lima scenario) and the maximum of the discounted expected scenarios where the option is either not exercised now but possibly in the future or exercised now considering the strike price. As an example, consider the period **5** flexibility SVL node at the highest possible demand for the DC mode. The locale cost (strike price) is **\$100,000.** The necessary IVL and SVL lattices have already been calculated in Figures **3-9** and **3-10.** Therefore, the calculation is as follows:

$$
$3,874,463 = $2,089,082 + \text{Max}\left(\frac{.71 * $2,166,162 + .29 * $1,521,054}{1 + .058},\frac{.71 * $2,505,506 + .29 * $1,740,132}{1 + .058} - $100,000\right)
$$

Figure **3-13** shows the complete set of flexibility SVLs for each scenario. The initial node values for each distribution mode correspond exactly to the expected NPV values derived in the preceding section where no option was considered. This suggests that for each LDM option, exercising the option at the beginning node is indeed the best possible implementation.

Another set of lattices, the optimal strategy lattices, can be constructed to qualitatively answer if it is best to exercise at that node. This is step **6** as outlined in Chapter 2. The question posed is: if the option has not yet been exercised, would it be best to exercise it now? From the example calculation above, if the second expression in the maximum function is chosen, the answer is yes. Otherwise, the answer is no. Figure 3-14 shows these lattices, and each in case it is verified that it is indeed best to exercise the option now.

Since each of the three options are optimally exercised now, the analysis with uncertainty and flexibility in this case does not give different answers from the preceding stage of analysis where uncertainty is considered but there is no flexibility in the future periods. Thus, the value of each of the three options is the difference between the ONE (Lima)

Figure 3-14: Optimal Strategy Lattices.

SVL and the OEB (for each LDM) flexibility SVLs. It is the same result as obtained before, shown here in terms of option value for completion of the analysis in Figure 3-15.

E[NPV] for Flexibility with DC	\$5,529,143
E[NPV] for Lima	\$5,013,327
Value of Option to Establish DC	\$515,816
% Gain over Lima	10.29%
E[NPV] for Flexibility with DC	\$5,562,820
E[NPV] for Lima	\$5,013,327
Value of Option to Establish MM	\$549,493
% Gain over Lima	10.96%
E[NPV] for Flexibility with FS	\$5,560,456
E[NPV] for Lima	\$5,013,327
Value of Option to Establish FS	\$547,129
% Gain over Lima	10.91%

Value to Switch from Lima to a LDM. Figure 3-15: Option

In addition, the corresponding VARG graphs for the flexible strategy are the same as the fixed strategy of choosing a LDM from the beginning since it is optimal to exercise each option now.

Conclusion

Although the option to defer the decision of expansion is not particularly interesting in this case, with this analysis it has been verified that it is indeed a good idea to expand to the Huancayo region now. It is clear that switching to a LDM has the potential to generate gains on the scale of 10%. Moreover, the analysis shows that the three LDMs are very similar due to the relatively low locale costs and the trade-offs between inventory and personnel versus service level. However, now that it has been determined that establishing a LDM now is best, the question arises of how to value the option of switching the type of LDM or possibly switching back to Lima in the future given the demand uncertainty. It is this issue that will be analyzed in the next chapter.

Chapter 4. Lattice Method and Demand Uncertainty II: Middle Mode versus Other Distribution Modes

The preceding chapter shows that if one takes the Lima scenario as the **ONE** scenario and values the option to switch to a LDM, the result is to switch to a LDM now. Furthermore, the three LDMs are very close to each other in terms of NPV results. However, what has not been determined is how much value there is in having the option to start out with a LDM and be able to switch either to another type of LDM or back to Lima. It is conceivable that there could be considerable value in switching to a FS when demand is high, for example. Conversely, the put option to close the LDM and return to Lima could be worthwhile in cases of low demand.

In this chapter, it is assumed that Deltron establishes a middle mode of distribution now. Three options are valued. They are the options to switch to a FS, DC, or leave the LDM setup entirely and fall back to Lima. Since the main uncertainty is still demand, the same procedure as outlined in Chapter 2 for the modified binomial lattice method is used. However, now the middle mode is taken to be the ONE scenario and each of the other three modes are the OEB scenarios.

Determination of Option Values and Optimal Times of Exercise

As before, the first two steps of the analysis are to evolve the outcome and probabilities lattices. Since this is assumed to be independent of Deltron's distribution mode, this has already been done in Figures 3-6 and 3-7 in Chapter 3.

The next step is to construct the IVLs for the ONE and OEB scenarios. The same free cash flow functions as before are used for each node, depending on the distribution mode. The same distribution mode parameters as in Chapter 3 are used. These lattices are shown in Figure 4-1.

Figure 4-1: Instant Value Lattices.

Figure 4-2: State Value Lattices.

Once the IVLs are constructed, the SVLs for each scenario are put together. Following the algorithm of Figure 2-2, the ONE scenario includes the locale cost since this is not a strike price. The OEB scenarios do not include the strike price, as this is to be included in the corresponding flexibility SVLs. The SVLs are shown in Figure 4-2.

The strike prices in this case are derived from the locale costs for simplicity. Thus, since the middle mode is the ONE scenario, it is assumed that \$125,000 has already been paid. For the FS mode, the strike price to switch from MM to FS is the difference between the locale cost of the FS mode (\$150,000) and that of the MM mode, which results in a strike of \$25,000. To switch from MM to a DC or to Lima, it is assumed that the strike is zero since the locale costs for the latter are lower that what has already been paid for the MM. In an application with real data, the strikes could easily be modified to take into account closing costs and salvage costs, depending on the situation.

Next, the flexibility SVLs for each of the three options are built given the strike prices. The algorithm as described in Figure 2-3 is once again applied. As a reminder, at each node, the maximum of the exercising the option now or not exercising the option now but possibly in the future is being taken at each node. Figure 4-3 shows the flexibility SVLs for this analysis.

Having worked back through the flexibility SVLs using its dynamic algorithm, the optimal strategy follows from each, based on which expression the maximum function chooses at each node. At each node, the question is asked: if the option to switch from MM to DC/FS/Lima has not yet been exercised, should it be exercised now? Figure 4-4 shows the optimal strategy lattices for each option.

Finally, the option values for switching to a **DC, FS,** or Lima are obtained **by** taking the difference between the initial nodes of the flexibility SVLs and the ONE SVL. The option values are shown in Figure 4-5.

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Figure 4-4: Optimal Strategy Lattices.

Figure 4-5: Option Value to Switch from MM for Second Tier Analysis.

As an added feature, with the optimal strategy lattices in hand, the lattices can be decomposed into their individual paths. The VARG graphs are constructed along with the relevant statistics of the distributions. These are in Figure 4-6.

Figure 4-6: Expected NPV Comparison for each Scenario and the VARG graphs.

Analysis of Option Values and Optimal Times of Exercise

The most obvious result is that the options to switch distribution modes relative to a MM are almost worthless, considering that the analysis cannot be taken beyond three significant figures in the NPV numbers. The following are some of the reasons for the lack of substantial option value in this analysis.

- The trade-offs between service level and inventory and personnel make the gains of switching very small, even in the more likely scenario of very high demand for the **FS** mode. Also, in the case of the **FS,** a strike price must be paid.
- For the DC and Lima switching options, the result of the trade-off is similar even though there is no strike to be paid. Also, these put options are very unlikely to be exercised, based on comparing the optimal strategy lattices with the probabilities lattice.

The model and theory are correct, however, as it must be the case that flexibility render results that are greater than or equal to the fixed design. Also, from the optimal strategies lattices it is indeed the case that the call option of expanding to a FS is exercised in cases of very high demand, while the put options of downsizing the operation to a DC or Lima are exercised in cases of very low demand.

Conclusion

Although Chapters 3 and 4 have given the system every opportunity to benefit from flexibility in the face of the incremental demand uncertainty, it is clear that based on the system parameters that flexibility does not provide much value for this system. Thus, based on this analysis alone the recommendation would be that the company set up a LDM, since this does represent a **10%** gain compared to Lima.

From a methodological point of view, the modified binomial lattice has allowed for analysis of a system where the main uncertainty is of an incremental, continuous nature. Moreover, the recombining structure of the lattices and the path independence of the

system allow for analysis over many time periods so that many possibilities over a long time horizon can be concisely summarized. The result is a description of an optimal strategy to be followed and the value of a real option that would provide the system with the flexibility necessary to react to the evolution of the uncertainty. However, this method is not well suited to evaluate a system where the main uncertainty is of a large, discrete nature. In Deltron's case, this may be the uncertainty that the competition might enter the market locally and thereby reduce demand by some substantial factor. Chapter 5 analyzes this situation independently of the intrinsic demand uncertainty using traditional decision analysis, which is better suited for this type of uncertainty.

Chapter 5. Decision Analysis: Effect of Competition

The previous two chapters have analyzed Deltron's decision to switch distribution modes based purely on the small, incremental uncertainty which is the demand uncertainty. A lattice model was used and it was determined that the company should indeed switch to a LDM now and that each LDM is very similar to the next due to trade-offs in the system.

However, the effect of the competition also deciding to enter the market locally has not been considered. This chapter uses traditional decision analysis to consider the effect of the competition, which is a large step uncertainty. The varying demand is ignored for now. Also, decisions are made to maximize expected **NPV,** and no utility functions are used.

Model Assumptions and Parameters

The most limiting aspect of using decision analysis is the due to the curse of dimensionality, meaning that as one considers many time periods, chance events, and options, the tree size grows exponentially. No longer is there a recombining structure as with the lattices to solve this issue. Therefore, the following simplifying assumptions are made in this section in order to analyze the system in a clear way. First, the system life is reduced from three years to two years, so that the problem is now a two stage situation. Second, instead of analyzing the option to switch from Lima to each of the three different LDMs defined earlier **(DC,** MM, and **FS),** it is assumed that if Deltron changes to a LDM it will be to a MM. This is a perfectly reasonable simplification because as has been seen in previous chapters, the results of the three LDMs are actually quite similar due to the trade-offs. Third, it is assumed that both the option and chance node event in the tree (i.e., the decision of either Deltron or the competition to establish a LDM) are one-time occurrences. This reduces the possible tree scenarios considerably and is entirely reasonable over the short to mid-term period of two years.

The model parameters used to calculate the cash flows for the NPVs are taken from the previous lattice analysis. The relevant columns for this model would be those for Lima and the local MM. A similar cash flow function as before is used, only modified to take into account for decrease in demand if the competition enters the market locally.

Thus, two additional and new types of critical parameters are the percent decrease of demand in the event that the competition also decides to establish have a local outpost (z) and the objective probabilities in the tree chance nodes.

Decision Analysis Tree

As hinted before, the tree chance nodes are the chances that the competition will establish a local outpost if they have not already done so. This is the only uncertainty considered in this chapter. The decision nodes describe Deltron's dilemma which is to either exercise the option to go to a LDM if it has not already done so. Over two years with each year as each period, Figure 5-1 shows the scenario descriptions and the decision tree. Note that the nine scenarios are the specific paths through the tree that terminate at the endpoints labeled A through I.

As can be seen, the provision is made for different chance node probabilities across years and across scenarios. This is done since the probability of the competition establishing a local outpost may be correlated with Deltron's decisions, this being in essence a gaming situation. This is the type of increased modeling flexibility that decision analysis can provide but at the expense of complexity. Therefore, the probabilities of the competition establishing a local outpost are denoted as p_{1i} , p_{1ii} , p_{2ii} , p_{2v} , or p_{2vi} depending on the chance node. Note that the probabilities that would correspond to p_{2i} , p_{2ii} , or p_{2iv} are set to 1 automatically since it is assumed that if the competition goes to a LDM in year **1,** the action is irreversible over two years.

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Figure 5-1: Scenarios and Decision Tree for Traditional Decision Analysis

Scenario Net Present Values

In order to apply the decision tree methodology, the NPV for each of the nine scenarios A through I must be obtained. This is done by applying the free cash flow function to a deterministic demand growth projection. The demand projection, which is 15% annual growth over the next two years, is shown in Figure 5-2.

Figure 5-2: Deterministic Demand Growth for Traditional Decision Analysis.

Six month time increments are used to obtain the cash flows for the **NPV** of each scenario. The cash flows depend on the free cash flow function, current distribution mode, and model parameters. The following example for scenario **D** shows how to obtain a **NPV** for a scenario.

In scenario D, the competition goes to a LDM now but Deltron waits until the end of year 1 to do the same. The results of the free cash flow calculations for each time period are shown in Figure **5-3.**
	6 Month Periods				
			2		
Revenue	\$0	\$8,766,711		\$9,401,250 \$11,674,906 \$12,519,942	
-Variable Cost	\$0	\$8,042,854		\$8,625,000 \$10,613,551 \$11,381,766	
-Fixed Cost	\$0	\$0	\$125,000	\$130,000	\$130,000
Earnings Before Taxes	\$0	\$723,857	\$651,250	\$931,355	\$1,008,177
-Taxes	\$0	\$101,467	\$91,289	\$130,553	\$141,322
Net Income	\$0	\$622,390	\$559,961	\$800,802	\$866,855
-Cost of Inventory	\$0	\$19,189	\$20,578	\$49,109	\$52,664
Free Cash Flow	\$0	\$603,201	\$539,383	\$751,693	\$814,191

Figure 5-3: Free Cash Flow Calculation for Scenario D.

To build Figure 5-3, the appropriate parameters must be used at each time period. The available parameters are shown in Figure 5-4. These differ based on whether the company is operating from Lima or from a MM. Also, there is the percent decrease in demand parameter z, which is applied to the demand if the competition goes to a LDM.

Figure 5-4: **Free Cash Flow Calculation Parameters.**

The formulas for the modified free cash flow function are as follows:

Effective Demand **=** If Competition has a LDM, then : Original Demand * (1 **-** z). Otherwise, Original Demand

Revenue = Effective Demand * Service Level * (PC Cost + Ship Cost) * (1 **+** Gross Profit Rate)

Variable Cost **=** Effective Demand * Service Level ***** (PC Cost **+** Ship Cost)

Fixed Cost **=** If going to a LDM next period, then: Utilities Cost **+** Personnel Cost **+** MM Locale Cost Otherwise, Utilities Cost **+** Personnel Cost

Earnings Before Taxes **=** Revenue **-** Variable Cost **-** Fixed Cost

Taxes **=** Earnings Before Taxes * Corporate Tax Rate

Net Income **=** Earnings Before Taxes **-** Taxes

 ℓ month \mathcal{N} $Cost of Inventory = \frac{1}{2}$ **6** months **) 28** days)

Free Cash Flow **=** Net Income **-** Cost of Inventory

For example, in Figure 5-3, the period 1 free cash flow is \$603,201. This is obtained with the Lima parameters, since in this period Deltron does not yet have a LDM. Also, having assumed that the percent decrease in demand z is 25%, at this point the competition has established a LDM, and the calculation takes this into account.

Effective Demand **=** 26810* (1 **-** 25%) **=** 20108

Revenue **=** 20108 * 80%* *(\$500 +* \$0) * (1 **+ 9%) =** \$8,766,711

Variable Cost **=** 20108 * 80% * (\$500 **+** \$0) **=** \$8,042,854

Fixed Cost = $$0 + $0 = 0

Earnings Before Taxes **=** \$8,766,711 **-** \$8,042,854 **-** \$0 **=** \$723,857

Taxes **=** \$723857 * 14.0% **=** \$101,467

Net Income **=** \$723,857 **-** \$101,467 **=** \$622,390

Cost of Inventory =
$$
\left(\frac{20108}{6 \text{ months}}\right) \left(\frac{1 \text{ month}}{28 \text{ days}}\right) * 5.5 * ($500 + $0) * 5.8\% = $19,189
$$

Free Cash Flow **=** \$622,390 **-** \$19,989 = \$603,201

In similar fashion, depending on the situation at that period in scenario D, the free cash flows are obtained. For example, in period 2, the fixed cost rises from \$0 to \$125,000 because Deltron is going to a LDM in the next period and so the MM locale cost is paid. This also means that the MM parameters are used in periods 3 and 4 for the cash flow calculations.

In this way, the free cash flows for each scenario are obtained, depending on Deltron's and the competition's distribution mode. Figure 5-5 summarizes the free cash flows for each scenario and also calculates the NPV of each scenario using the 5.8% six month discount rate. Notice how the row for scenario D corresponds exactly to the free cash flows calculated in Figure 5-3.

	6 Month Periods					
Scenario	0		2	3	4	NPV
A	$-$0.13$	\$0.64	\$0.69	\$0.75	\$0.81	\$2.38
в	$-$0.13$	\$0.89	\$0.96	\$0.75	\$0.81	\$2.86
C	$-$0.13$	\$0.89	\$0.96	\$1.04	\$1.12	\$3.35
D	\$0.00	\$0.60	\$0.54	\$0.75	\$0.81	\$2.33
Е	\$0.00	\$0.60	\$0.65	\$0.69	\$0.74	\$2.33
F	\$0.00	\$0.80	\$0.76	\$0.75	\$0.81	\$2.72
G	\$0.00	\$0.80	\$0.76	\$1.04	\$1.12	\$3.21
н	\$0.00	\$0.80	\$0.86	\$0.69	\$0.74	\$2.71
	\$0.00	\$0.80	\$0.86	\$0.92	\$0.99	\$3.10

Figure 5-5: NPV Calculations for each Scenario. Values are in **USD** \$M.

Application to the Decision Tree

With the NPV for each scenario obtained, the standard decision tree process can begin. At chance nodes, the NPVs are probability weighted to obtain expected values. At decision nodes, the maximum expected NPV is chosen. This is done for year 2 and then year 1, until one reaches the initial node, which is Deltron's decision to go to a LDM now. No utility functions are used.

The objective probabilities as shown in the tree of Figure 5-1 are p_{1i} , p_{1ii} , p_{2i} , p_{2ii} , p_{2iii} , p_{2iv} , p_{2v} , and p_{2vi} . For simplicity, it is assumed that: $p_1 = p_{1i} = p_{1ii} = 25\%$. This is the

probability of the competition establishing a LDM now. Also, it is assumed that: $p_2 = p_{2i} = p_{2ii} = p_{2iv} = p_{2v} = p_{2vi} = 75\%$. This is the probability of the competition going to a LDM at the end of year 1 if it has not already done so. In essence, these two assumptions say that the probability of the competition going to a LDM is independent of Deltron's decision over the two stage analysis. This could be valid given that it takes time to reconfigure one's supply chain, so instant reaction may not be possible.

With the objective probabilities defined and the NPVs for each scenario obtained, Figure 5-6 shows the solved decision tree. Based on maximizing expected NPV, the analysis suggests to go to a LDM now. Furthermore, the optimal strategy is defined, which is to go now. In this case, this is trivial since we are dealing with a one-time option and there is no decision to be made at year 2. If, however, Deltron would stay now, the tree also provides an optimal strategy to be followed in the lower part of the tree. The optimal strategy in this case would be to go to a LDM, regardless of what the competition has done at the beginning of year 1.

Figure 5-6: Solved Decision Tree Assuming $z=25\%$, $p_1=25\%$, and $p_2=75\%$.

Comparison of VARG for Going Now and Staying Now

From the solved tree in Figure 5-6, the VARG graphs for going to a LDM now or staying in Lima now can be obtained. For going to a LDM now, the three possible resulting scenarios are A, B, and C. Each of these has a NPV and corresponding tree path probability, which is shown in Figure 5-6. By sorting these NPVs, one can construct the VARG graph. For staying in Lima now, the three possible resulting scenarios and D, F, and G because in year 2 we would choose to go to a LDM. Like before, each of these scenarios has its NPV and corresponding path probability, so the VARG graph can be constructed. Figure 5-7 shows the VARG graphs for these two cases. It can be seen that the decision to go to a LDM now clearly dominates that of staying now given the parameters used.

Figure 5-7: VARG graph for Going Now to a LDM and for Staying in Lima Now.

Sensitivity Analysis

Having gone through the entire traditional decision analysis, the question arises of how the decision to go to a LDM now changes with different values for the objective probabilities and the percent decrease in demand parameter. To perform this sensitivity analysis concisely, similar assumptions about the objective probabilities as before hold: $p_1=p_{1i}=p_{1i}$ and $p_2=p_{2i}=p_{2i}=p_{2i}=p_{2i}=p_{2v}=p_{2v}$. Therefore, the sensitivity analysis can be performed given a fixed value for z, the percent decrease in demand parameter and varying p_1 and p_2 at the same time.

Although not critically necessary, TreeAge © software was used for this part of the analysis. The tree in the program is linked to an Excel sheet that performs the NPV calculations for each scenario given a value for z. The program then performs a sensitivity analysis across the two probabilities, asking the question of when it would be better to go to a LDM now. The results, shown if Figure 5-8, are unanimous. For the ranges of p_1 and p_2 specified, it is always better to go to a LDM, based on the criterion of maximizing expected NPV.

Conclusion

This chapter neglected the small, incremental demand uncertainty and concentrated on the large step uncertainty associated with the entrance of the competition into the local market. For this type of uncertainty, a traditional decision analysis method is applied which maximizes expected NPV and uses no utility functions. The end result for Deltron in the example application is to go to a LDM now, which coincides with the previous result obtained in the lattice evaluation which only considered demand uncertainty.

From a methodological point of view, several assumptions had to be made in order to apply this traditional decision analysis. These assumptions, such as reducing the total period of analysis from 3 years to 2 years and setting chance node probabilities for the same year equal, were necessary mainly to simplify the dimensionality that the decision tree brings with it. The analysis can easily become too complex without such simplifications. This is the main drawback of the method, and making the necessary assumptions can be too limiting for evaluation of certain systems. Also, risk preferences are not quite adequately accounted for with traditional decision analysis. In this implementation, it is assumed that managers are risk neutral because the standard criterion of maximizing expected value is used. An alternative could have been to apply subjective utility functions which convert the NPV values to dimensionless utiles. However, these utility functions, though attempting to account for risk preferences, can be abstruse and artificial.

Finally, having presented applications of the lattice and decision analysis methods separately, the challenge arises as to how to combine the two methods. In this case study, the parameters are such that it is best to go to a LDM now no matter what the method used. Nevertheless, in general it is quite possible that the result may not be as straightforward. Therefore, the following two chapters present the theory and an example application of a method that combines the lattice and decision analysis methods. This is done so as to account for both small and large step uncertainties to be considered in a system at the same time.

Chapter 6. Theory of Hybrid Lattice and Decision Analysis

The previous chapters covered the basic theory of lattice and decision analysis and provided a case application for the methods. The lattice method serves better to model systems where the uncertainty evolves in relatively small increments from period to period where path dependency is not a great issue. In contrast, decision analysis is less useful in covering small changes but works better for situations where uncertainty evolves in large step changes.

This chapter defines a new, hybrid method of analysis which combines in one model the better attributes of lattice and decision analysis to account for both small incremental and large step changes of uncertainties. Although the model inherits these positive aspects of the two methods, it also inherits the limitations of complexity issues from decision analysis. Thus, the theory of this new method is developed for a two stage analysis.

Key Insights behind Hybrid Method

The main insight which fuels the need for a hybrid model is the aforementioned presence of different types of uncertainties which can be present in a system at the same time. In real world systems, combinations of these uncertainty types are present everywhere. For example, the demand for air travel in a city from year to year may be best modeled **by** incremental changes according to a lattice evolution. However, the decision to build an extra airport would probably cause a large change in the actual air traffic going through the city, and therefore decision analysis would suit this chance event best. This hybrid analysis acknowledges the contrast between these uncertainty types and seeks to integrate two previously disparate methods of analysis within the engineering real options approach.

Another key insight which helped originate this method is that when it is time to make a decision to exercise a real option, both lattice and decision analysis tend to influence the manager at looking at single values to describe strategies, be it in terms of expected **NPV**

or utiles. In reality, however, when one chooses between strategies, one is really choosing among a set of possible **NPV** distributions. Thus, looking only at single values to describe strategies can be misleading since a strategy with the highest expected **NPV** may have an unacceptable downside, for example. The VARG **(CDF)** graphs become central to analysis in this view.

A third key insight is that both the lattice and decision analysis come under criticism due to improper and/or indirect handling of risk aversion. The basic argument is that use of expected values is completely inexact because most people are risk averse. In addition, even if the methods implement utility functions to compare dollar values, the functions themselves can be complicated, artificial, and indirect. The hybrid method seeks to resolve this issue **by** presenting the decision to be made **by** managers in terms of arguments for and against possible distributions of strategies. In the end, to make a subjective decision, managers must make a direct comparison among the possibilities, therefore intrinsically accounting for risk aversion and not using utility functions obscure to most managers.

Steps of the Hybrid Method

The following is a list of the main steps of the hybrid method. In following the logic of these steps, it is important to remember that both the lattice and decision analysis uncertainties evolve in parallel simultaneously and that all **NPV** values are in terms of the point of view of the beginning node of the decision tree. Later in this section each step is discussed in more detail.

- *1. Build the Decision Analysis tree.* Identify the scenarios, which are the unique paths through the tree. The decision nodes separate the scenarios of exercising an option or not and the chance nodes represent the large step uncertainties.
- 2. Construct the lattice VARG graphs for each scenario. Evolve the incremental uncertainty using individual paths through lattices for each scenario.
- *3. Combine VARG graphs at the chance nodes.* Starting from the end of the tree, use objective probability weighted sums of the individual tree path VARG graphs.
- *4. Decide among a set of combined VARG graphs at the decision nodes.* It is recommended to do a sensitivity analysis of the chance node probabilities and other model parameters in order to make sure the preferred decision is being made.
- *5. Repeat steps 3 and 4 over the first stage.* The final result is a decision to exercise or not at the beginning of period **1.**
- *6. Repeat steps I through 5 after the first time period in the tree elapses.* Since the lattice uncertainty has evolved, it may provide new information that changes the VARG graphs and the subjective preferences at decision nodes.

Step 1: Build the Decision Analysis tree

The scenarios, which are individual paths through the tree, are first identified. For simplicity in the example shown in Figure **6-1,** it is assumed that there is a one-time option and a one-time chance event (i.e., once the option is exercised or the chance event occurs they cannot be reversed). The decision tree is built with the large step changes in mind at the chance nodes. Exercising the option or not is the decision that has to be made at the decision nodes. The most likely objective probabilities for the chance nodes are identified in this step.

It is critical for understanding to recall that parallel to the tree chance event development, there is also the outcome lattice uncertainty. Although not explicitly shown in the tree, this uncertainty forms the basis for the next step in the analysis. This is the point emphasized in Figure **6-1.**

In parallel to the development of the tree chance event uncertainty, the original outcome lattice uncertainty also unfolds although not explicitly shown in the tree.

Figure 6-1: Example Scenario Definition, Decision Tree, and Original Outcome Lattice for Hybrid Analysis Method.

Step 2: Construct the lattice VARG graphs for each scenario

The VARG graphs for each scenario are constructed from the point of view of $t=0$. Each scenario, which is a unique path along the tree, is evolved by using lattices from the beginning of period 1 to the end of period 2. First, the original outcome lattice is evolved forward in time over the time period corresponding to the decision tree. This original outcome lattice is the same as described in Chapter 2 and describes the lattice uncertainty evolution under a lognormal distribution.

Next, where the decision tree chance event occurs, the lattice uncertain variable needs to be modified to form an effective outcome lattice. This is necessary in order to reflect the large step change which can occur in the lattice uncertainty. This can be accomplished in two main ways: either the lattice parameters (growth and volatility) are changed or a multiplicative factor (z) is applied to represent a percent increase or reduction in the original outcome lattice at each node. Changing the growth and volatility parameters, however, would cause the lattice not to recombine and make the analysis more complicated. Therefore, in this method the multiplicative factor z is applied where if the chance event in the tree occurs, the corresponding original outcome lattice nodes are modified. This is an easier way to represent large step changes in the lattice uncertainty itself while still conserving the important recombination property of binomial lattices.

Third, with the effective outcome lattice, an effective instant value lattice is constructed for each scenario. This effective IVL is similar to the IVL discussed in Chapter 2. The difference is that these effective IVLs do include the strike price at each node if in the scenario the option has been exercised. The effective IVL thus shows the net undiscounted cash flows for each node, and the free cash flow function used at each node in the effective IVL obviously depends on if and when the option has been exercised.

The following step is to decompose the effective IVL for each scenario into its individual paths. For example, a lattice with time periods 0 through 2 has 2^{2} (4) individual paths: up then up, up then down, down then up, or down then down. Each of these paths can have

Figure 6-2: Example of Constructing the Lattice VARG Graph for a Scenario. The example builds on scenario F from Figure 6-1.

unique NPVs and probabilities. If **U** represents the cumulative numbers of "up" movements for the path, then the probability for an individual path to occur (and therefore, for that NPV to occur) is $p^{U}(1-p)^{T-U}$, where T is the number of time periods in the lattice and p is simply the binomial probability. Since each lattice path NPV has its corresponding probability, the VARG graph for each tree scenario from the point of view of the initial tree node can be easily obtained. Figure 6-2 shows an example development of Step 2 for scenario F from figure 6-1. Note that although the total time described in the lattices must match that of the decision tree, the lattice can have finer granularity so as to provide a more precise picture.

Step 3: Combine VARG graphs at the chance nodes

Under traditional decision analysis, at the chance nodes one multiplies the expected NPVs by the corresponding objective probabilities to obtain an expected value for the chance node. In this hybrid method, instead of multiplying one single value (i.e., the expected NPV for each scenario), one multiplies the entire VARG distributions of NPVs by the corresponding objective probability and combines the distributions into one VARG that describes the chance node.

Mathematically, this is entirely valid because the area of each probability density function that makes up each VARG for each scenario is equal to one. Moreover, the sum of the objective probabilities at each chance node is also equal to one. Therefore, the weighted sum of each probability density function is the new combined probability density function that makes up the new combined VARG graph, and its area is also equal to one. The proof is as follows:

Let A_i be the area of a probability density function for a scenario i.

By definition, $A_i = 1$.

Let p_i be the objective probability of a scenario i.

By definition
$$
\sum_{i=1}^{n} p_i = 1
$$
.
\n $\therefore \sum_{i=1}^{n} p_i A_i = \sum_{i=1}^{n} p_i (1) = \sum_{i=1}^{n} p_i = 1$.

Figure 6-3: Example of Combining VARG Graphs at Chance Nodes. The example builds **on** scenarios F and G from Figure 6-1.

And so, each probability for each lattice path NPV is multiplied by the corresponding tree objective probability of that scenario. This is done for all scenarios of the chance node. The NPVs of all lattice paths of all scenarios are sorted in ascending order and listed with their corresponding new weighted probabilities. The cumulative probabilities are then calculated. This list of sorted NPVs and their corresponding cumulative probabilities is used to produce the new combined VARG graph. Figure 6-3 shows an example of this Step 3 procedure which builds on scenarios F and G from figure 6-1.

Step 4: Decide among a set of combined VARG graphs at the decision nodes

In traditional decision analysis, at decision nodes one makes a decision based on maximizing expected NPV or utiles if an artificial utility function is used. Here, the decision whether to exercise an option or not is subjective and is made by comparing the combined VARG graphs and their statistics. In this way, risk aversion is accounted for directly.

The one caveat is that the combined VARG graphs are subject to the objective probabilities used at the chance nodes and other model parameters such as z, the percent increase or decrease of the original outcome lattice node values. Therefore, this step should include comparing VARG graphs by performing a fairly robust sensitivity analysis to see how the combined VARG graphs change with different parameters. This step can be somewhat cumbersome since there are infinitely many combinations of parameters to test against. The recommendation is to choose a few combinations of parameters that will render both intermediate and extreme results at opposite ends (i.e., combinations that favor either exercising the option or not exercising the option). In the end, it is a manager's risk aversion and feeling about the most likely values of the sensitive parameters that will produce the decision of which combined VARG graph to accept and which to discard.

Figure 6-4 builds upon the example illustration of this chapter. In this figure, the combined VARG graph of F and G is compared to the combined VARG graph of H and I. Under traditional decision analysis, one would look to maximize expected NPV, and therefore decide not to expand which leads to the chance outcomes of either H or I. However, for whatever reason, it may be the case that managers want to do well in low probability outcomes. As can be seen from the figure, in the lower 35% of outcomes the F and G combination dominates the H and **I** combination. Therefore, it may be the case that due to risk aversion the extra upside that the H and **I** combination carries is not sufficient for managers to choose not to expand.

This is the type of important extra information that this hybrid analysis provides managers by illustrating the subtleties and trade-offs of the decisions available. With sensitivity analysis for the objective probabilities and other parameters, the same graphs would be compared, but their shapes will shift and potentially drive management to change decisions.

Figure 6-4: Example of Deciding Among a Set of Combined VARG Graphs. The example builds on scenarios **F, G, H and I** from Figure **6-1.**

Step 5: Repeat steps 3 and 4 over the first stage

As with traditional decision analysis, the concept is simple. After going through the first chance node/ decision node evaluation cycle, the chosen combined VARG graphs are carried back to the next chance node to the left in the tree. The combined VARG graphs from the last period of analysis are recombined using the next set of objective probabilities. New sensitivity analyses are performed, and now at the next decision node, managers choose between the newly recombined VARG graphs. Although easily explained, this process can get cumbersome quickly as the range of sensitivity analysis to be performed grows at the next period that the method is carried back.

Step 6: Repeat steps I through 5 after the first time period in the tree elapses

In traditional decision analysis, by working backwards in the tree, one establishes an optimal strategy to be followed until the end of the system life. This works because the only uncertainties are those explicitly shown at the chance nodes. However, in this hybrid analysis, aside from the large step uncertainties in the chance nodes explicitly shown in the tree, there is also the evolution of implicit incremental lattice uncertainty. Therefore, the VARG graphs looking forward from an intermediate time in the tree are changing over time as the lattice uncertainty evolves. In addition, values of objective probabilities for chance nodes that were once far in the future may change based upon new information. This has the potential to change a manager's subjective choice among a new set of distributions. Thus, the entire analysis should be redone after each period in the tree. The final result is that by the end of the system life, the manager has made the best informed subjective decision at each time period based on future contingencies given the evolution of both the large step and small incremental uncertainties.

Following the example evolved in the figures of this chapter, suppose that management decides not to expand in period 1 and that the tree chance event does not occur. This means that management must now decide between expanding, which will result in scenarios F or G, or not expanding again, which will result in scenarios H and I. Assume

that the chance node probabilities as used for Figures 6-3 and 6-4 are almost certain and that the outcome lattice uncertainty is yielding very high values. When the analysis is performed again, the H and I combination may dominate the F and G combination entirely since there may be no more chance that the outcome will be so low that the F and G combination will be better, even in lower echelon low probability cases. In other words, it may be the case that the outcome lattice uncertainty has evolved in a beneficial fashion to expand and take advantage of high outcome values.

Critiques of Hybrid Lattice and Decision Analysis

The greatest limitation of this method is that, as with traditional decision analysis, the decision tree can become a "messy bush" if it is carried out over very many periods. This may be a key limitation to practitioners if it is absolutely necessary to look at more that two periods with possibly multiple options each time period.

Another main critique of this hybrid method is that the objective probabilities in the chance nodes cannot be determined very accurately. This critique is not unique to this method, as traditional decision analysis is subject to this criticism as well. Proponents of the financial real options approach argue that this is a main reason to accept the MAD assumption and treat assets that are not publicly traded as if they were so as to be able to use risk neutral probabilities instead. However, for reasons explained in Chapter 2, this is not an acceptable solution to proponents of the engineering real options approach. Thus, objective probabilities must be used within this approach, and the most sensible way to deal with the uncertainty of the objective probabilities is to perform robust sensitivity analyses. Even though these can get complicated, these tests provide very comprehensive views of system development in the face of various unknown variables.

Also, the possibility is left open to managers to make "wrong" decisions due to the inherent subjectivity. Despite this, as argued earlier in the chapter, this can be precisely one of the strongest points of the method since it accounts directly for risk aversion rather than using an artificial utility function. Depending on the model results, in some cases, an

argument for exercising or not exercising an option may be very weak or very strong. Of course, in an agency situation with multiple stakeholders where results are not so clear, the method leaves the door open for possible manipulation of arguments in order to benefit a subset of the stakeholders. However, this is real life, and in the end any mathematical model can be skewed **by** a subversive minority. The model thus requires intelligent interpretation of results so that many different stakeholders can make subjective arguments based on objective data.

Conclusion

This chapter introduced a hybrid lattice and decision analysis method, based on the fact that in many systems two types of uncertainties $exist - those$ that change in small increments and those that are revolutionary in nature. In addition, this hybrid method draws from the insight that when one chooses to exercise a real option, one is really choosing among a set of distributions and not just among single expected values, as traditional engineering real options methods do. Moreover, the method takes care of the risk aversion of managers directly without the need for disregarding the issue entirely or introducing abstruse utility functions. Although the method has its drawbacks such as exponentially increasing complexity over multiple time periods and options, it has great benefits **by** combining the better attributes of the previous disparate lattice and decision analysis methods. The following chapter develops an example application using the same case that has been developed in this thesis.

Chapter 7. Hybrid Lattice and Decision Analysis: Demand Growth and Competition

In chapters **3** and 4 Deltron's decision to establish a LDM is analyzed using lattice analysis with the main uncertain variable as the demand which is expected to grow in small increments over the next few years. In Chapter **5,** the decision to establish a LDM is reviewed concentrating only on the uncertainty of the competition's action.

In this chapter, both forms of uncertainty are analyzed together using the new hybrid lattice and decision analysis method as outlined in Chapter 6. The step **by** step methodology as outlined is followed in order to provide a coherent picture of the situation so that the option of switching to a LDM can be viewed in a wider and more complete context. The great advantage is only one model is now necessary to consider both uncertainties at once. However, this methodology brings computational limitations of how many periods and options can be analyzed in a manageable fashion due to its inherited traits from traditional decision analysis.

Model Assumptions and Parameters

Since the hybrid model has certain limitations, some key assumptions are modified as in the analysis that applied pure decision analysis in Chapter 5. These simplifications are summarized as follows:

- * The analysis life is shortened from three years to two years, so that the problem facing Deltron is a two stage situation.
- * Instead of analyzing the option to switch from Lima to each of the three different LDMs defined earlier (DC, MM, and FS), it is assumed that if Deltron changes to a LDM it will be to a middle mode (MM).
- It is assumed that both the option and chance node event in the tree (i.e., the decision of either Deltron or the competition to establish a LDM) are one-time occurrences.

The model parameters and free cash flow function used to calculate the cash flows for the NPVs are taken from Chapter 5 as well. As a reminder, two additional and new types of critical parameters for this hybrid model are the percent decrease of demand in the event that the competition also decides to establish have a local outpost (z) and the objective probabilities in the tree chance nodes.

Step 1: Build the Decision Tree

The scenarios, which are the individual unique paths through the tree are first identified. It is here that the assumption of a one-time option and chance event helps since this reduces the number of possible scenarios to nine, labeled A through I. As hinted before, the tree chance nodes are the chances that the competition will establish a local outpost if they have not already done so. In parallel, there is the underlying demand uncertainty which is also evolving. The decision nodes describe Deltron's dilemma which is to either exercise the option to go to a LDM if it has not already done so. Over two years with each year as each period, Figure 7-1 shows the description of each of the nine scenarios, decision tree, and the original outcome lattice, which is used in step 2.

As can be seen from Figure 7-1, the provision is made for different chance node probabilities across years and across scenarios. This is realistic since the probability of the competition establishing a local outpost may be correlated with Deltron's decisions this being in essence a gaming situation. Therefore, the probabilities of the competition establishing a local outpost are denoted as p_{1i} , p_{1ii} , p_{2ii} , p_{2v} , or p_{2vi} depending on the chance node. Note that the probabilities that would correspond to p_{2i} , p_{2iii} , or p_{2iv} are set to 1 automatically since it is assumed that if the competition goes to a LDM in year 1, the action is irreversible over two years.

Figure 7-1: Scenario Definition, Decision Tree, and Original Outcome Lattice for Application of Hybrid Analysis Method.

Step 2: Construct the lattice VARG graphs for each scenario

Each of the nine scenarios, A through I, as described in Figure 7-1 is evolved by using lattices. Depending on if and when the competition goes to a LDM, the effective outcome lattice for each scenario reflects this change via a percent reduction in demand at each node of the outcome lattice. The parameter z is applied where appropriate. The effective IVL is then constructed for each scenario in order to obtain the NPV for each node, including the strike price (i.e., locale cost) if Deltron moves to a MM at that node. Next, each effective IVL is decomposed into its individual paths with their corresponding probabilities. Lastly, the VARG graphs along with their relevant statistics are obtained.

It is important to note that all NPV values are calculated from the point of view of the beginning of year 1, or now.

To demonstrate this step in the context of this case study, Figure 7-2 evolves the lattice for scenario B. This is the scenario in which Deltron goes to a LDM now but the competition waits until the end of year **1** to do the same. The granularity of time in the lattices is six month periods, since this is adopted from the similar lattice analysis of chapters 3 and 4. The growth and volatility lattice parameters (v and σ) are kept at 15% and 25% per year, respectively. The total time of the lattice evolution now however is two years, which corresponds to time periods 0 through 4. As seen in the effective outcome lattice, it is only for periods 3 and 4 that the percent decrease of demand (z) is applied since this corresponds to year 2 when the competition has decided to establish a local outpost. The parameter chosen for the analysis shown is a 25% decrease in demand $(z=25%)$. In the effective IVL, in period 0 the strike price of the MM locale cost is paid while in the rest of the periods the free cash flow function is applied taking the values from the effective outcome lattice as inputs. The $2⁴(16)$ paths through the effective IVL are decomposed into the resulting NPVs and corresponding probabilities. Once sorted, these become the VARG graph for scenario B.

Figure 7-2: Illustration of Constructing the Lattice VARG Graph for Scenario B.

A similar procedure is applied to the other eight scenarios to build their VARG graphs and charts of their relevant statistics. All NPV results are from the point of view of the beginning of year 1. Figure 7-3 summarizes these statistics for all nine scenarios, assuming $z=25\%$. As one would imagine, the highest expected NPV results from scenario C, where Deltron moves to a LDM now and the competition never follows. The lowest expected NPV occurs when the opposite happens in scenario E. However, scenario C has the largest spread in possible NPVs, while its counterpart E has the smallest. These are the types of initial observations one can make. In subsequent steps, the entire VARG graphs will be compared to provide a richer picture.

				Coefficient	
Scenario	Minimum	Maximum	Spread	of Variation	EINPVI
А	\$1.10	\$3.30	\$2.20	25%	\$2.51
в	\$1.43	\$3.86	\$2.43	23%	\$3.00
	\$1.64	\$4.57	\$2.93	24%	\$3.52
D	\$1.13	\$3.19	\$2.06	24%	\$2.44
Е	\$1.30	\$3.07	\$1.77	21%	\$2.43
F	\$1.39	\$3.64	\$2.25	23%	\$2.83
G	\$1.60	\$4.35	\$2.75	23%	\$3.35
н	\$1.56	\$3.52	\$1.95	20%	\$2.82
	\$1.73	\$4.09	\$2.36	21%	\$3.24

Figure 7-3: Summary of NPV Statistics after Constructing Lattice VARG Graphs for each Scenario. Values are in **USD** \$M.

Step 3: Combine VARG graphs at the chance nodes

This step in the procedure is trivial at the year 2 tree chance nodes for scenarios A, D, and E since their probabilities of occurring are equal to 1. Therefore, the VARG graphs for these three scenarios are immediately carried back to the year 2 tree decision nodes.

However, the VARG graphs of the following scenarios have to be combined according to their corresponding probabilities: B with C, F with G, and H with I. The respective objective probabilities are denoted as p_{2ii} (probability of B), p_{2v} (probability of F), and p_{2vi} (probability of H). Each is the probability of the competition going to a LDM at the end of year 1. Figure 7-1 shows these graphically.

Figure 7-4: Illustration of Combining the VARG Graphs of Scenarios B and **C.**

For the illustration purposes here, the example of combining scenarios B and C is shown in Figure 7-4. The method used is as described in Figure 6-3, where each lattice path probability is multiplied by the tree chance objective probability. The lattice path NPVs are sorted together and the new VARG graph is constructed. In Figure 7-4, the objective probability of B is p_{2ii} . At $p_{2ii}=100\%$, this means that the combined B and C VARG graph is simply that of B. Conversely, at $p_{2ii}=0\%$, this graph is simply that of C. The figure shows combined B and C VARG graphs for intermediate values of p_{2ii} . As is evident, scenario C dominates B in all states of nature since in scenario C the competition has not gone to a LDM, leaving a higher demand value for Deltron.

In similar fashion, scenario F is combined with G and H is combined with I assuming a values for z and the objective probabilities. The appropriate sensitivity analyses are carried out in the next step when a decision at the year 2 nodes has to be made.

Step 4: Decide among a set of combined VARG graphs at the decision nodes

This step in the hybrid analysis is the most interesting because it is here that sets of distributions must be compared by performing sensitivity analyses. This section will perform the comparisons for the year 2 decision nodes. As seen in Figure 7-1, there are only two choices to be made at these nodes in the lower part of the tree, choosing between D or E and between the F and G combination or H and I combination. In the upper part of the tree, there is no decision to be made at year 2 because of the one-time decision and chance events simplifications.

D vs. E

For this decision, it is assumed that Deltron did not go to a LDM in year 1 but the competition did. Therefore, Deltron must now decide in year 2 to go (scenario D) or to stay (scenario E). Since the competition already went to a LDM in year 1, the corresponding chance nodes in year 2 are trivial, and so $p_{2iii} = p_{2iv} = 1$. This means that the relevant

parameter to test for sensitivity is only the percent decrease in demand due to the competition switching to a LDM (z).

Suppose that according to past experience and managerial intuition, the potential range of this parameter is **10%** to 40%. Figure 7-5 shows the contrasting D and E VARG graphs for values of z of 10%, 25%, and 40%.

Intuitively, one expects that the lesser the effect of the competition on Deltron's demand, the greater the potential benefit in following the competition to a LDM because there will be a greater part of demand for Deltron to obtain and thereby offset the extra costs. Indeed, for values of z at **10%** and 25%, the expected NPV of scenario D is greater than that of E. This is due to the greater upside potential available by switching to a LDM. However, this information could have been easily obtained by only calculating expected NPVs and VARG graphs would not have been necessary. The extra information that the VARG analysis provides is that, although the expected NPVs are greater for z at **10%** and 25%, the NPVs for D are worse in cases of low demand and the minimum NPVs are also lower than those of E. If one now considers z at 40%, it is difficult to make an argument in favor of D since its VARG is better than that of E about only 25% of the time in cases of very high demand.

Therefore, the decision whether to choose scenario D or E at this decision node is subjective and depends on managers' perceptions about the distribution of the parameter z. If managers feel that it is more likely that z will fall between **10%** and 25% the arguments favor going to a LDM (scenario D), while if they feel that z is more likely to fall between 25% and 40% the argument for E is stronger. However, due to individual preferences, such as wanting to go for the highest possible NPV despite the possible downside, choosing D even in the case of z at 40% is a possibility.

Figure 7-5: Decision between D and E at the End of Year 1 Considering Different Values of z.

F and G Combination vs. H and I Combination

For scenarios F, G, H, and I the decision between going and staying in year 2 for Deltron is slightly more complex to analyze because now the competition can go or stay in year 2. In the case that Deltron goes to a LDM, the probability that the competition will follow is p_{2v} , and in the case that Deltron stays in Lima, the probability that the competition will go to a LDM is p_{2vi} . In addition, there is also the uncertainty of z, the percent decrease in demand that Deltron will experience if the competition goes to a LDM.

To simplify the dimensionality of the sensitivity analysis for illustration purposes, it is assumed that p_{2v} is the same as p_{2vi} . That is, the competition has the same probability of going to a LDM in year 2 whether Deltron does or not. This may be a realistic assumption if one considers that it takes time to reconfigure one's supply chain and that the action to follow is not executable immediately. With this simplification, Figure 7-6 shows the qualitative situation at hand in terms of parameters. When the competition's effect on demand is low and the probability of them going to a LDM is low, the option to switch to a LDM looks most attractive. Conversely, if the effect on demand is high and there is a very strong probability that the competition will go to a LDM, the option to go to a LDM looks less attractive. The three combinations of parameters that will be analyzed here will be the combinations along the left to right diagonal of Figure 7-6. That is, the most extreme combinations along with one "middle of the road" combination will be looked at in detail.

		$p_{2v} = p_{2vi}$				
		50%	75%	99%		
	10%	Most Attractive to Go				
z	25%		7			
	40%		?	Least Attractive to Go		

Figure 7-6: Qualitative Description for Sensitivity Analysis to Choose between F and G or H and I.

Different Values of z and the Objective Probabilities.

Figure 7-7 shows the results for the three different simulations. Arguments in favor of either the F and G combination or the H and I combination can now be made. In favor of establishing a LDM, F and G always have the highest upside in cases of very high demand. Also, the expected NPV is only worse for the very extreme parameters of the objective probabilities at 99% and z at 40% and even then only very slightly by \$0.01M. Also, even in this worst case, the F and G VARG graph dominates that of H and I about 50% of the time. However, if one is more concerned about the possible downside, the H and I combination always provides a higher minimum value than its counterpart. However, in the best case of z at 10% and the probability at 50%, it is better than F and G only 25% of the time. The same is true for the middle of the road simulation. Therefore, as before, coherent arguments can be made on both sides of opinion. In the end, the decision depends upon managers' feelings about the distributions of the parameters and their risk aversion.

Step 5: Repeat Steps 3 and 4 over the first stage

At this point the analysis has been reduced to one time period. Assume that management has decided to choose scenario **D** over E and the combined F and G scenario over H and I. Figure 7-8 shows the remaining tree to be analyzed. Scenario A must be combined with the combined B and C, and scenario **D** with the combined F and G.

Figure 7-8: Remaining Decision Tree after Analyzing Year 2.

The combined scenario of A plus B and C, denoted as $A+(B+C)$, must now be compared to that of D+(F+G). As before, the simplifying assumption that p_{1i} is the same as p_{1ii} so that the range of the sensitivity analysis is narrowed down. In addition, the values for p_{2ii} and p_{2v} are assumed to be 75%. The qualitative question is asked about when it is best to go to a LDM given a range of z between 10% and 40% and a range of the year 1 objective probability between 1% and 50%. Figure 7-9 shows these ranges.

		$p_{1i}=p_{1ii}$			
		1%	25%	50%	
z	10%	Most Attractive to Go	2		
	25%			7	
	40%		?	Least Attractive to Go	

Figure 7-9: Qualitative Description for Sensitivity Analysis to Choose between A+(B+C) and D+(F+G).

Figure **7-10** shows the results for the parameters corresponding to the left to right diagonal of Figure **7-9.** In this case, it is difficult to make an argument for not going to a LDM now. As can be seen, in all three simulations, the expected **NPV** of **A+(B+C)** is better than that of **D+(F+G).** Furthermore, even in the case of z at 40% and the objective probability at **50%,** the **A+(B+C)** dominates its counterpart about **70%** of the time. The only argument that could be made in favor of not going now to a LDM would be **by** very risk averse individuals who wish to protect against very low probability events of low demand scenarios. Therefore, the decision is made at the initial node to set up a MM LDM now. In the very worst result, the competition follows behind now and has a very negative effect on very low demand. The best possible scenario to hope for having taken this course of action is very high demand and a choice **by** the competition not to follow behind now or at the end of year **1.** Thus, one must be prepared for very variable results, as the resulting **NPV** can vary between less than \$1.OM and more than \$4.5M over the first two years of implementation.

Figure 7-10: Decision between A+(B+C) and D+(F+G) Now Considering Different Values of z and the Objective Probabilities.

Step 6: Repeat steps 1 through 5 after the first time period in the tree elapses

Given the result to exercise the one-time option to go to a LDM now, this last step does not have to be done because it will not make a difference on the decision making. However, for illustration purposes, let us suppose that for whatever reason, Deltron decided to stay now and so did the competition. Furthermore, demand has progressed along a very low possible path in the outcome lattice such that the demand over a 6 month period is down 30% from what it was at the beginning of year 1. At the end of year 1, the decision must now be made whether to go to a LDM given the resolution of the year **1** uncertainties of the competition's decision and the evolution of demand. Figure 7-11 shows the tree situation at hand.

Figure 7-11: Hypothetical New Decision Tree at End of Year 1.

Assuming the same lattice parameters as before but with the new starting value of demand **30%** than what it was a year before, the effective IVLs are evolved for scenarios F, **G,** H, and I. The VARG graphs of **F** and **G** are combined using the updated objective probabilities, and the same is done for scenarios H and I. **A** decision must then be made between going or staying at the end year **1.**

In order to make this decision, assumptions must be updated about the parameter z and the objective probabilities. Based on these, new sensitivity analyses are performed. The simplifying assumption that p_{2v} is the same as p_{2vi} is kept, but the new estimated range is between *25%* and **75%.** The percent decrease in demand due to competition is estimated

Ifigure 7-12: Decision between F+G and H+I at End of Year 1 Considering Different Values of z and the Objective Probabilities.

between **10%** and 40% as before. As before, sensitivity analysis is performed for three cases, two at the extremes and one in the middle. These are: z at 10% with the probability at 25%, z at 25% with the probability at 50%, and z at 40% with the probability at 75%.

The results are completely in favor of not going to a LDM. In every case shown in Figure 7-12, the H and I combination dominates the F and G combination for any demand. Note that earlier in the chapter, however, it was decided to go to a LDM. What has changed is that the volume of demand now does not justify the extra costs of this mode of distribution.

From a methodological point of view, it is thus very important to perform the complete analysis after each time period since there are both explicit and implicit uncertainties evolving along in the tree. The explicit tree uncertainty is the competition's decision about whether to go to a LDM, while the implicit uncertainty is the lattice demand uncertainty. In addition, the opportunity can be used to update the objective probabilities and other model parameters.

Conclusion

Following the hybrid lattice and decision analysis procedure outlined in Chapter 6, it is shown that, given the range of parameters considered, going to a LDM now would be in the best interests of Deltron in this hypothetical case study. This result makes perfect sense given the results of the individual lattice and traditional decision analysis models which both point to the same answer. However, in general, one can imagine a case where the individual models do not agree, and thus the combined hybrid model which incorporates two types of uncertainties simultaneously would be of considerable value to make a decision.

Thus, as far as the methodology is concerned, the hybrid method considers two types of uncertainties, large step uncertainties and small incremental uncertainties. Moreover, the method allows for a complete analysis of possibilities to be analyzed rather than single value through the use of VARG graphs, and individual risk preferences are directly

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accommodated since there is no single rule for optimal decision making. The results simply serve as bases upon which to make arguments in favor of or against a decision, which is a more realistic and complete way to evaluate a decision of this magnitude. As a final word of caution, it must be noted that the method does have computational limitations if it is to be developed over more than two stages. Thus, if simplifying assumptions that would alter the problem formulation substantially must be made, this model may prove to be of little value.

Chapter 8. Conclusion

This thesis has defined a new way to combine two traditional methods of analysis within the engineering real options approach. A supply chain expansion application has illustrated the benefits and limitations of the methods discussed. Based on the results of the case study, specific recommendations emerge for the Deltron inspired problem given the hypothetical data used. More importantly, key lessons about both the traditional and new methods have emerged from developing the theory and performing the case study.

Hypothetical Case Study Recommendations

Although real data was not used for this case study, for completion of the example case, the results are summarized. First, the lattice analysis which analyzed only demand uncertainty showed that it would be best to switch to a LDM now, and that over three years, this action could generate an increase on the order of 10% over a central, Lima distribution mode. Moreover, despite the system was given the opportunity to benefit from the flexibility of switching between types of LDMs, due to inherent trade-offs in the system, this flexibility proved almost valueless.

Second, a traditional decision analysis was applied to the same case to analyze only the effect of competition, but with some key simplifications in order to make analysis manageable. This was necessary due to the decision tree's proclivity to expand exponentially, unlike the lattice analysis. Despite derived by looking at a distinct uncertainty, the recommendation to establish a LDM now was the same as in the lattice analysis.

Finally, the new hybrid lattice and decision analysis was used to consider both uncertainties at the same time. With this novel method, both the incremental demand uncertainty and the large, discrete competition uncertainty were developed. As would be expected, the method results again in the decision to go to a LDM now.

Methodological Lessons

In defining the hybrid lattice and decision analysis method, different aspects of each individual method have been combined, and this has led to an interesting model that inherits some positive and negative aspects of the separate models. However, the model also adds some new key elements of its own, rendering the entire model more than the exact sum of its parts.

The greatest advantage of the hybrid model is obviously the ability to integrate two types of uncertainties in one model that previously were usually analyzed separately. Although not the case in the example developed in this thesis, it is quite possible that the two individual analyses differ in their recommendations about the exercise of a particular flexible strategy. Thus, the integrated method becomes even more valuable in this scenario. In order to achieve this unification however, the hybrid model must also inherit traditional decision analysis' main drawback, which is the necessity to make key simplifying assumptions to make the tree manageable. This includes limiting the analysis to two stages in the tree.

Another main challenge which the individual models pass onto the hybrid model is the determination of critical parameters. This challenge is for mathematical models in general, and this is typically resolved by performing other system studies and sensitivity analyses. Fortunately, the hybrid method as defined in this thesis outlines how to carry out these important sensitivity tests to evaluate how robust the model results are.

Two new elements in the hybrid model that make it more than the sum of its underlying parent models are its criterion for deciding based on comparing distributions rather than single value descriptors and its direct accounting for risk preferences. In both the traditional and lattice analyses, decisions are made upon maximizing single expected values. Even if these values were converted to non-dimensionless figures using utility theory, these would still be single values describing scenarios that in reality have a wide array of outcomes. The hybrid model shows the description of a decision more realistically as a VARG graph,

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which shows both upsides and downsides to a decision rather than single values. Related to this point, it now falls upon decision makers to make direct comparison between a few distributions of possibilities in order to determine what course of action to take. Subjective arguments are made in favor of or against decisions based on the objective data. This brings the tremendous advantage of making the decision process very realistic without having to resort to expected values or abstruse utility functions. Thus, risk preferences are accounted for as directly as possible.

In closing, the general hope in developing this new method is that it will offer a fresh point of view for the evaluation of projects whose main uncertainties are a small, incremental type uncertainty and the possibility of a large, discrete type change. By integrating two previously disparate methods, a more realistic portrayal of real systems can be analyzed to carry forth in the spirit of the growing real options field and determine how to best build with flexibility in an uncertain world.

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