Optimization of Stiffener Placement for Thin Plate Buckling

by

Thomas Andrew Hopkins

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Submitted to the Department of Civil and Environmental Engineering
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at the

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June 2008

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Author
Department of Civil and Environmental Engineering
May 20, 2008

Certified by
Jerome J. Connor
Professor of Civil and Environmental Engineering
Thesis Supervisor

Accepted by
Daniele Veneziano
Chairman, Departmental Committee for Graduate Students
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Abstract

Stiffened plates are a common component in many structures, from plate girders and box girder bridges to ships and offshore structures. Plates and stiffened plates buckle into doubly curved surfaces, substantially complicating the mathematics for an analytic solution. Finite element methods allow direct analysis of stiffened plates without requiring the solving of complicated differential equations. This analysis considers stiffened plates from the buoy of a tension leg platform designed to support a 5 megawatt wind turbine. The interior plates were stiffened using conventional, large, triangular, diamond and truss type stiffener arrangements. The truss scheme was designed or “tuned” specifically to the suppress the first buckling mode of the particular plate, and compared to other more arbitrary forms of material placement.

The results showed that the truss stiffener that was tuned to the first mode was the most effective in increasing the critical buckling load. The truss arrangement increased the critical load by 7,269 kips compared to 4,180 for conventional, 2,912 for large, 911 for triangular and 2,562 kips for a diamond stiffener arrangement.

Thesis Supervisor: Jerome J. Conner
Title: Professor of Civil and Environmental Engineering
Acknowledgments

I would like to take this opportunity to thank the Department of Civil and Environmental Engineering faculty and staff for making my five years at MIT unforgettable.

To the MEng. Class of 2008, you are a wonderful group of intelligent and sincere people. I am extremely sad that we will part ways, but I am honored to have shared these memories with you and look forward to our continued network of personal and professional relationships.

To Mark, Jimmy, Donny and Frog, there is no way I would have made it through MIT without being able to spend 4 hours a day at the rink playing hockey. I look forward to joining you on the coaching staff next year and hopefully we can finally bring the cup home where it belongs.

I want to thank Professor Connor for his support and kindness. You are truly a brilliant and insightful person to whom I am extremely grateful. To Lisa O'Donnell, you provided a great amount of real world advice and insight, which was an extremely valuable counterpart to the highly theoretical coursework.

I want to thank Steve, Matt, Walter, Ovid, Jon, Morgan and Dan. I'm glad we all stuck around Boston and really look forward to the next few years.

To my family, I dedicate this thesis to you. To my father, Robert, mother, Kathryn and sister Elizabeth, I am the luckiest person in the world to have such an amazing family. I love you all.
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Chapter 1

Introduction and Scope

1.1 Introduction

The idea of failure by buckling has been around since the days of Euler, who developed the now famous Euler Buckling Equations by testing a fixed-free column loaded in compression. The problem has been studied extensively for columns from both a mathematical and empirical approach in order to generate governing equations. From the governing equations, tables of constants have been generated that allow many different problems to be described by the Euler equation and a constant as shown in the following equation, where $K_c$ is a scaling constant based on boundary conditions.

$$P_c = K_c \frac{\pi^2 EI}{L^2}$$

Modern finite element codes have allowed the problem to be attacked more directly. It has become easier to model a complete system with accurate boundary conditions and multiple load patterns. Finite element codes along with modern computers have allowed researchers to examine increasingly complicated problems such as stiffened plates. The critical load of the stiffened plate can be evaluated directly rather than making a series of assumptions needed to reduce the problem to an easily solvable set of equations. Finite element codes have been used in this analysis to examine the buckling mode shapes, for different stiffener schemes and to evaluate their effectiveness and optimize the position of the stiffener elements.
1.2 Scope

The analysis was broken into two sections, model verification and stiffeners analysis. The model verification used analytical methods to solve simple buckling problems of columns and plates and compared the results to the ADINA finite element model solutions. The ADINA models were validated against the analytic solutions to create a confidence in the modeling approach for the stiffener analysis.

The stiffener analysis considered a specific design case and analyzed different proposed stiffener schemes for effectiveness. The stiffener schemes were compared and optimized based on effectiveness in increasing the critical buckling load and the same effectiveness when normalized to volume of material and length of welds. The effectiveness of the arrangements was compared between schemes which were designed specifically to suppress the first mode, and schemes with uniform material distribution designed without consideration of the first mode.

1.3 Relevant Design Scenarios

Stiffened plate elements are widely used throughout structural design. Three common examples are steel plate girders used in many highway bridges, stiffened plates used in box girder bridges, ship hulls and the interior structure of many offshore structures. The approach taken in this analysis is suitable for design scenarios which have known geometries and boundary conditions, as well as predicable loads. A predictable load is one for which the general magnitude and upper bound are known with confidence and the direction and distribution of the load is relatively consistent.

A design that fits this criteria is the internal structure of a large offshore buoy. The dimensions and boundary conditions are known from the specified design sizes and the presence of fully welded plates. The loads are hydrostatic and as such can be assumed to be acting in pure compression with no bending. The design case to be evaluated is discussed in more depth in section 4.2 on page 30.
Chapter 2

Buckling Background

Buckling is an undesirable failure mode characterized by sudden failure of a structural member loaded in compression. The failure generally occurs at stress levels well below the failure stress of the material. A buckled failure mechanism is characterized by an out-of-plane deflection caused by compression loading. Figure 2-1 shows three common buckling mode shapes.

2.1 Column Buckling

Buckling of columns is controlled by a fourth order differential equation relating the moment and the transverse displacement, \( w \). When the column is simply supported the equation reduces to the second order equation given by equation 2.1a, using the dimensions defined in figure 2-2

\[
EIw'' + Pw = 0 \quad (2.1a)
\]

\[
K^2 = \frac{P}{EI} \quad (2.1b)
\]

Equation 2.2 reduces to

\[
w'' + K^2w = 0 \quad (2.2)
\]

Equation 2.2 can be further generalized by including the eccentricity of the applied load, as shown in figure 2-2 and thus resulting in equation 2.3.
Figure 2-1: Three Examples of a Buckled Structural Member

(a) First Buckling Mode of a Pinned-Pinned Column

(b) Second Buckling Mode of a Pinned-Pinned Column

(c) First Buckling Mode of a Clamped Column
Figure 2-2: Diagram of Column Buckling Dimensions

\[ w'' + K^2 w = -K^2 e \]  
(2.3)

The general solution for the governing equation is given by equation 2.4.

\[ w(x) = C_1 \sin(Kx) + C_2 \cos(Kx) - e \]  
(2.4)

The constants \( C_1 \) and \( C_2 \) are found by applying the boundary conditions of the problem, \( w(0) = 0 \) and \( w(L) = 0 \) and \( e = 0 \), for a purely axially loaded column. The resulting equation, and its two solutions (the first a trivial solution of \( C_1 = 0 \) and second non-trivial solution) are given by equation 2.5.

\[ C_1 \sin(KL) = 0 \]

\[ C_1 = 0 \]  
(2.5)

or

\[ KL = n\pi \]

The critical buckling load is given by the solution where \( n = 1 \) and when equation 2.1b is substituted for \( K^2 \). The resulting Euler critical buckling load is given by
equation 2.6.

\[ P_c = \frac{\pi^2 EI}{L^2} \]  

(2.6)

Material Buckling is highly sensitive to slight eccentricities and material imperfections in the structural member. Types of imperfections can include initial transverse deflections, initial material strains and residual stresses and material imperfections. All these imperfections lead to buckling at lower loads than those predicted mathematically. The mathematically predicted buckling loads are accurate for very slender members, such as rebar in concrete, however they are not sufficiently accurate to use for designing members in the normal slenderness range. Because of this lack of safety, LRFD codes have used numerous laboratory tests to generate a body of empirical data and draw representative empirical solutions for critical buckling loads. The LRFD Design Equations are given below by equation 2.7 through 2.9. [1]

\[ F_{cr} = \text{Critical Buckling Stress} \]

\[ F_{cr} = \left[ 0.658 \frac{F_y}{F_c} \right] F_y \text{ when } \frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}} \]  

(2.7)

and

\[ F_{cr} = 0.877 F_e \text{ when } \frac{KL}{r} \geq 4.71 \sqrt{\frac{E}{F_y}} \]  

(2.8)

where

\[ F_c = \text{Elastic Yield Stress} \]

\[ F_e = \frac{\pi^2 E}{(KL/r)^2} \]

\[ K = \text{Constant Based on Boundary Conditions} \]  

(2.9)

Sample values for K are given in figure 2-3 from the LRFD Steel Design Code [2]
### TABLE C-C2.2
Approximate Values of Effective Length Factor, $K$

<table>
<thead>
<tr>
<th>Buckled shape of column is shown by dashed line.</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical $K$ value</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Recommended design value when ideal conditions are approximated</td>
<td>0.65</td>
<td>0.80</td>
<td>1.2</td>
<td>1.0</td>
<td>2.10</td>
<td>2.0</td>
</tr>
<tr>
<td>End condition code</td>
<td><img src="image" alt="Rotation fixed and translation fixed" /></td>
<td><img src="image" alt="Rotation free and translation fixed" /></td>
<td><img src="image" alt="Rotation fixed and translation free" /></td>
<td><img src="image" alt="Rotation free and translation free" /></td>
<td><img src="image" alt="Rotation free and translation free" /></td>
<td><img src="image" alt="Rotation free and translation free" /></td>
</tr>
</tbody>
</table>

Figure 2-3: K Values Used for Design with LRFD Equations
2.2 Plate Buckling

Plate buckling is similar to buckling of columns with the noticeable exception that plates are two dimensional elements as opposed to one dimensional elements. This means that where a column would buckle in only one plane, a plate would buckle in two planes creating a doubly curved surface. Figure 2-4 shows the deformed buckled modes of two different plates.

Because plate buckling occurs in 2 dimensions, the geometric properties and stiffnesses must be defined differently than those for a beam. In plate buckling the geometry of the plate, analogous to the slenderness ratio, is defined by the aspect ratio $\frac{a}{b}$ where $a$ is the dimension in the direction of primary loading and $b$ is the dimension perpendicular to the primary loading, as shown in figure 2-5. The bending rigidity of the plate, the equivalent of $EI$ in a beam is given by the term $D$ which is given by equation 2.10.

$$D = \frac{Et^3}{12(1 - \nu^2)}$$

where

$t =$ Thickness of the Plate

$\nu =$ Poisson’s Ratio of the Material

The governing differential equation for plate buckling is given by equation 2.11 [5]

$$D\nabla^4 w + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0$$

The solution for this equation reduces to a simple form similar to column buckling. The solution utilizes a scaling factor $K_c$ based on boundary conditions, aspect ratio and the plate flexural rigidity $D$ given by 2.10. The solution is given by equation 2.12 where $P_c$ is the total critical load and $N_c$ is distributed load in the form [load/length]

$$P_c = K_c \frac{\pi^2 D}{b} = N_c b$$

The value for the scaling constant, $K_c$, varies with the aspect ratio of the plate, the boundary conditions of all four edges of the plate and the applied load cases. For
(a) First Buckling Mode of a Clamped-Clamped Plate with Aspect Ratio 1

(b) First Buckling Mode of a Pinned-Pinned Plate with Aspect Ratio 3

(c) Second Buckling Mode of a Pinned-Pinned Plate with Aspect Ratio 3

Figure 2-4: Three Examples of a Buckled Plate
combinations of loads the critical buckling coefficient has been solved using different boundary conditions and aspect ratios and then compiled into simple charts. The charts can be used to quickly and easily locate the critical buckling coefficient and compute the critical buckling load for a given plate. Two such charts are shown in figures 2-6a and 2-6b.

Plates can also be stiffened with additional components or grids of components. This is common on large plate girders, box girders and nautical components. The addition of stiffeners to the plate considerably complicates the analytical solution for the buckling load and buckling mode shapes. The advancement of modern finite element analysis codes, such as ADINA, have made the analysis of stiffened plates more straightforward and accessible.
(a) $K_c$ for Uni-axially Uniformly Loaded Plates [3]

(b) $K_c$ for Uni-axially Loaded Plates with One Free Edge and Eccentricity [4]

Figure 2-6: 2 Design Charts for Differently Loaded and Supported Plates
Chapter 3

Model Validation

The ADINA model, used to evaluate the stiffened buckling plates, must first be validated against simpler examples where the analytical solution is known. This chapter will outline the procedure used and the results of the model validation.

3.1 Validation Strategy

The models will be validated by comparing the solution for critical buckling load from the ADINA finite element model with the analytical solution. The ADINA model predicted buckling mode will also be compared against the expected buckling mode for the simple analytical models. The solutions will be compared over a range of slenderness ratios for columns and aspect ratios for plates. Different meshes and mesh elements will be compared to demonstrate the effect on refining the critical load and buckling mode shape.

3.2 Column Buckling

The ADINA model considered a 20 foot column with square cross section and with pinned boundary conditions on both ends. The slenderness ratio $\lambda$ was varied by adjusting the cross section of the beam and thereby modifying the radius of gyration $r$ where:
Critical Buckling Load Comparison

\[ \lambda = \frac{L}{r} \]  
\[ r = \sqrt{\frac{I}{A}} \]  

The model was composed of 2 node isobeam elements and subdivided into 64 elements.

### 3.2.1 Column Buckling Validation

The critical buckling loads calculated with ADINA are shown in comparison to the critical loads calculated using both Euler (Equation 2.6) and LRFD (Equations 2.7 and 2.8) in figure 3-1. Example hand calculations for the analytic solutions can be found in Appendix A.

Figures 3-1 shows that the ADINA model agrees very closely with the Euler model as expected and converges with the LRFD model at higher slenderness ratios. The ADINA model produces slightly lower critical buckling value because of the inclusion of shear effects in the beam that are neglected in the Eulerian formulation. The AD-
INA model converges with the LRFD at higher slenderness ratios when the geometry of the member becomes the limiting factor in buckling as opposed to imperfections. Based on this evidence the model is verified.

3.2.2 Column Buckling Modes

The buckling modes for the columns agree well with the predicted modes. The buckled shapes conform to the boundary conditions and match the correct number of sine waves along the buckled geometry. The first mode of the pinned-pinned column shown in figure 3-2, displays the classic half sine wave expected in a buckled column.

The second buckling mode of the pinned-pinned column, shown in figure 3-3 on page 23, displays a full sine wave shape corresponding to the solution $KL = 2 \pi$.

The first mode of a clamped-clamped column shown in figure 3-4 on page 23, shows no deflection or rotation at the ends, and a half sine wave displacement with inflection points at the $\frac{L}{4}$ and $\frac{3L}{4}$ positions of the beam. The loads and mode shapes match the analytically predicted values and therefore validate the ADINA model for
Figure 3-3: Second Buckling Mode of a Pinned-Pinned Column

Figure 3-4: First Buckling Mode of a Clamped-Clamped Column
column buckling. The ADINA model must also be validated for plate buckling before it can be used in analysis.

### 3.3 Plate Buckling

The ADINA model for plate buckling consisted of 1 inch thick, 4 node shell elements. The loaded edges of the plates were assumed to be either clamped or simply-supported and the unloaded edges were assumed to be either clamped, simply-supported or free. The buckling modes were examined with a rough mesh and then refined by doubling the mesh density along both edges and in some cases, further refined by again doubling the mesh density. Aspect ratios of 3 (36in x 12in) and 1 (24in x 24in) were considered. The results of the analysis are shown in tables 3.1, 3.2 and 3.3.

#### 3.3.1 Plate Buckling Validation

The analytic solutions for the various buckling plates were generated using equation 2.12 and a $K_c$ values from figure 2-6. The analytic solutions for the different boundary conditions are given in the following tables along with the critical load as given by the ADINA models with different mesh densities.

As with column buckling, the ADINA model generates solutions that agree very closely with the analytic solutions. The values fluctuate slightly based on mesh density, going from low with a very rough mesh, to an optimum mesh size and then decreasing as the mesh density increases. This implies that for more complicated

<table>
<thead>
<tr>
<th>AS-3 Simply Supported Edges: $K_c = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic Solution</td>
</tr>
<tr>
<td>Normal Mesh</td>
</tr>
<tr>
<td>Refined Mesh</td>
</tr>
</tbody>
</table>

Table 3.1: Predicted Buckling Loads based on Mesh Size for AS-3 Simply Supported Plate
Table 3.2: Predicted Buckling Loads based on Mesh Size for AS-3 Simply Supported Plate with Clamped Edges

<table>
<thead>
<tr>
<th>AS-3 Pinned Ends with Clamped Edges: $K_c = 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic Solution</td>
</tr>
<tr>
<td>Rough Mesh</td>
</tr>
<tr>
<td>Refined Mesh</td>
</tr>
<tr>
<td>Finest Mesh</td>
</tr>
</tbody>
</table>

Table 3.3: Predicted Buckling Loads based on Mesh Size for AS-1 Fully Clamped Plate

<table>
<thead>
<tr>
<th>AS-1 Fully Clamped Plate: $K_c = 11$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic Solution</td>
</tr>
<tr>
<td>Rough Mesh</td>
</tr>
<tr>
<td>Refined Mesh</td>
</tr>
</tbody>
</table>

problems a reasonable overall mesh size should be considered with very fine meshes only in areas critical to mode shape or expected to see stress concentrations, specifically boundary conditions and connections between plate and stiffener elements.

### 3.3.2 Plate Buckling Modes

The buckling modes of the plates match the expected buckling modes. They display an increasing number of halfwaves along the length of the plate, with increasing mode number, and generally obey the boundary conditions. As can be seen in figure 3-5, the number of halfwaves increases with the buckling modes and in all cases the simply-supported edges show no displacement but are free to rotate.

The mesh density around the boundaries does, however, have an effect on the mode shape. As can be seen in figure 3-6, refining the mesh leads to a smoother more accurate plot of the buckling mode. In this example refining the mesh leads to a buckling mode that more closely follows the non-rotational boundary condition for
the unloaded edges.

Again, in the case of the square fully clamped plate, the refined mesh leads to a smoother plot that more closely obeys the boundary conditions as shown in figure 3-7 on page 29.

3.4 Results

Through the analysis of simple plate and beam elements, the ADINA finite element buckling model has been sufficiently verified as accurate. This instills confidence in the analysis method that it can be used to examine more complicated buckling problems with more complicated boundary conditions and loadings. It allows the combination of beam and plate components in order to examine the behavior of stiffened plates in compression.
Figure 3-5: First Three Modes of a Buckled Plate
(a) First Buckling Mode of AS-3 Plate with Clamped Edges and Rough Mesh

(b) First Buckling Mode of AS-3 Plate with Clamped Edges and Refined Mesh

(c) First Buckling Mode of AS-3 Plate with Clamped Edges and Finest Mesh

Figure 3-6: Three Examples of the Effect of Mesh Density on Buckled Mode Shape
Figure 3-7: Two Examples of the Effect of Mesh Density on Buckled Mode Shape in Square Clamped Plates
Chapter 4

Stiffened Plate Model

4.1 Stiffened Plates

Stiffening elements are used on thin plates in order to increase the critical buckling load of the plate and thus increase the safety. Stiffeners add moment of inertia to a section of the plate and force the plate into a higher buckling mode, which corresponds to an increased load carrying capacity. Stiffeners frequently are standard rolled sections laid in a grid pattern and welded to the plate. A majority of the welding is done using automatic submerged arc welding in order to save on labor costs although stiffener intersections and other difficult welds must still be hand welded. The regular grid pattern is effective and efficient for mass produced sections with variable or unknown load conditions. If a section has given boundary conditions and reliable load conditions it is possible to study the buckled mode shapes and tailor the reinforcement pattern to match that buckled shape and thus save on material and weld length.

4.2 Model Geometry and Loading

The problem of optimizing stiffener placement for a given geometry and loading using a plate element from a large submerged buoy. The buoy was designed to support a 5 megawatt wind turbine anchored on a tension leg platform in deep water. The buoy
Figure 4-1: Cutaway Showing Interior Buoy Structure

is an octagon 11 meters [36.1 ft] in radius and 10 meters [32.8 ft] in height. The buoy is broken into 8 triangular buoyancy tanks each of which is further subdivided into 4 triangular sections. The subdivision are made to ensure that if the buoy is breached only a small portion of the total volume will be flooded and the buoy will not sink. The walls of the buoyancy tank are created with steel plate which also serves as the primary structural system of the buoy. As the structural system the steel plates are placed into compression by hydrostatic pressure and are therefore susceptible to buckling.

Figure 4-1 shows a cutaway of the buoy interior structure shown without stiffeners. The unrestrained length of each triangular subsection is 4.18 meters [13.7 feet] while retaining the original height of 10 meters [32.8 ft]. The radial plates that extend from the core to the exterior are being considered in this analysis and are assumed to be loaded uniformly in uniaxial compression. The vertical hydrostatic forces are assumed to be taken by the interior plates in order to simplify the analysis. The boundary
conditions are all assumed to be fully clamped meaning fixed in all translational and rotational degrees of freedom. The clamped condition comes from the fully welded condition of the plates as they must be watertight.

4.3 Finite Element Model

The plate was analyzed using the ADINA finite element code version 8.4 and a linearized buckling analysis. The model was composed of 4 node shell elements and fully clamped boundary conditions on all four edges. The material was modeled as structural steel with $E = 29,000$ ksi and $\nu = 0.3$. The stiffener elements were modeled as 6 inches deep in all cases except the large stiffeners which were 12 inches. The stiffeners were attached to the plate using rigid link constraints to model the welds and create a composite system. The program output consists of the critical load and mode shape for the first four buckling modes.

Six models were run, the base unstiffened case and the 5 different stiffener schemes, conventional stiffeners, large stiffeners, triangular stiffeners, diamond stiffeners and truss stiffeners.

4.3.1 Conventional Stiffener Model

The conventional stiffener model takes a rectangular grid of stiffeners at the quarter points of the plate. The stiffeners are 6 inches deep and rigidly attached to the plate at corresponding nodes. The conventional stiffener arrangement serves as a baseline comparison for the other stiffener arrangements in the optimization procedure. The conventional stiffener model with generated mesh as input to the ADINA code is shown in figure 4-2 on page 33.

4.3.2 Large Stiffeners

The large stiffener model is similar to the conventional model in that it features a grid of stiffeners. The large stiffener model substitutes material in the place of weld
Figure 4-2: ADINA Input Model Showing Conventional Stiffeners

length with stiffeners that are twice the size of the conventional stiffeners, 12 inches to 6 inches. The stiffeners intersect at the middle of the plate, where the primary deflection is in the first mode and should help to force the plate away from the first mode. The concern with the large stiffener configuration is the relatively large unbraced area that is still susceptible to buckling. The ADINA input model for the large stiffener configuration is shown in figure 4-3.

4.3.3 Triangular Stiffeners Arrangement

The triangular stiffener arrangement tries to reduce the amount of material and weld length by using a combination of equilateral triangles on the edges with a center stiffener connecting the two triangles. The equilateral triangles are formed by two stiffeners running from the corners to a point vertically in the middle of the plate. The stiffeners are the same length as the edge of the plate [13.7 ft]. The central stiffener is meant to resist the first buckling mode by engaging the stiffness of the
triangular edges. The ADINA input model for the triangular stiffeners is shown in figure 4-4 on page 35.

4.3.4 Diamond Stiffener Arrangement

The diamond stiffener arrangement tries to reduce the unbraced length of the plate by using an inscribed diamond shape that intersects at the midpoints of the edges. A vertical stiffener spans the midpoints of the loaded edge in an attempt to further reduce the unbraced area of the plate and eliminate the first buckling mode. The ADINA input model for the triangular stiffeners is shown in figure 4-5 on page 36.

4.3.5 Truss Stiffener Arrangement

The truss stiffener arrangement takes a direct approach at eliminating the first buckling mode and forcing the plate into a higher mode. The stiffeners are concentrated
Figure 4-4: ADINA Input Model Showing Triangular Stiffener Arrangement around the middle of the plate taking advantage of the clamped boundary conditions and the added stiffness from the corner intersection of two clamped conditions. The truss stiffeners scheme has three vertical stiffeners at the quarter points of the loaded edges spanned by intersecting diagonal stiffener elements. The concentration of material at the middle of the plate should dramatically increase the stiffness in the first mode and thereby increase the critical load of the plate. The ADINA model for the truss stiffener arrangement is shown in figure 4-6 on page 37.

### 4.4 Optimization Strategy

In order to optimize the stiffener placement the results were compared in several areas. The critical buckling load $P_c$ for each arrangement was compared to evaluate the overall effectiveness. The increase in stiffness $\Delta P_c$ was calculated to display the stiffness increase over the unstiffened plate and serve as the basis for the normalization
The stiffness increase $\Delta P_c$ was normalized to two parameters, material volume to represent total material costs and total length of stiffeners to represent the required weld length. The optimum arrangement was then taken as the arrangement with the greatest normalized strength increase in the two categories.

The optimization method is limited because it currently lacks a weighting function to weight the cost of welding (labor) versus volume (material). A full optimization would take these into account and also take a larger number of models and vary the depth of stiffeners and hence the volume in more arrangements than just the large stiffeners.
Figure 4-6: ADINA Input Model Showing Truss Stiffener Arrangement
Chapter 5

Results

The results of the analysis are broken down into four sections, critical buckling load, increase in critical buckling load $\Delta P_c$, $\Delta P_c$ normalized to weld length and $\Delta P_c$ normalized to material volume. The buckled mode shapes for each configuration are shown in Appendix B through Appendix G.

5.1 Comparison of Critical Buckling Loads

The critical buckling loads of the six models, unstiffened, conventional stiffeners, large stiffeners, triangular stiffeners, diamond stiffeners and truss stiffeners are shown in figure 5-1 on page 39 and in table 5.1.

The truss stiffeners provided the greatest stiffness against the first mode and hence

<table>
<thead>
<tr>
<th>Mode</th>
<th>Unstiffened</th>
<th>Large</th>
<th>Conventional</th>
<th>Triangular</th>
<th>Diamond</th>
<th>Truss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,905</td>
<td>4,817</td>
<td>6,085</td>
<td>2,816</td>
<td>4,467</td>
<td>9,174</td>
</tr>
<tr>
<td>2</td>
<td>2,653</td>
<td>5,022</td>
<td>9,210</td>
<td>4,979</td>
<td>4,766</td>
<td>9,403</td>
</tr>
<tr>
<td>3</td>
<td>3,809</td>
<td>7,846</td>
<td>10,206</td>
<td>7,132</td>
<td>7,246</td>
<td>10,788</td>
</tr>
<tr>
<td>4</td>
<td>4,058</td>
<td>8,492</td>
<td>10,709</td>
<td>7,411</td>
<td>7,462</td>
<td>11,331</td>
</tr>
</tbody>
</table>
the critical load. This was expected because the truss configuration was designed specifically to eliminate the first buckling mode. The triangular arrangement had a poor performance of only 2,800 kips critical load. The buckling loads increased with a rough relation to the amount of stiffener material located in the center of the plate corresponding to the first mode which was also to be expected.

**5.2 Comparison of $\Delta P_c$**

The increase in buckling load from the unstiffened to each of the stiffener arrangements is shown in figure 5-2 and table 5.2.

The data show the conventional stiffeners being effective over all the modes with increases of 4,000 kips in the first mode and around 6,500 kips in the second, third and fourth modes. An interesting case is the triangular stiffeners which perform very poorly in the first mode, increasing the capacity only 900 kips and performing in the range of the other arrangements in the second third and fourth modes. This is due to the intersection of the stiffener elements being located close to the point of
Figure 5-2: $\Delta P_c$ by Mode for Each Stiffener Arrangement

Table 5.2: $\Delta P_c$ by Mode for Each Stiffener Arrangement

<table>
<thead>
<tr>
<th>Mode</th>
<th>Large</th>
<th>Conventional</th>
<th>Triangular</th>
<th>Diamond</th>
<th>Truss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,912</td>
<td>4,180</td>
<td>911</td>
<td>2,562</td>
<td>7,269</td>
</tr>
<tr>
<td>2</td>
<td>2,369</td>
<td>6,557</td>
<td>2,326</td>
<td>2,113</td>
<td>6,750</td>
</tr>
<tr>
<td>3</td>
<td>4,037</td>
<td>6,396</td>
<td>3,322</td>
<td>3,437</td>
<td>6,979</td>
</tr>
<tr>
<td>4</td>
<td>4,434</td>
<td>6,651</td>
<td>3,353</td>
<td>3,405</td>
<td>7,274</td>
</tr>
</tbody>
</table>
maximum displacement in the higher modes and thus being much more effective in preventing higher modes than the primary mode. The truss stiffeners show a much greater increase in buckling capacity in the first mode of 7,300 kips and staying fairly constant around 7,000 kips in the other three modes. Again this is expected because the truss stiffeners were tuned to the primary buckling mode of the plate.

5.3 $\Delta P_c$ Normalized to Weld Length

The length of welds was taken as a normalization parameter representing labor cost. A large percentage of the cost in fabricating a stiffened plate comes from welding the stiffener element to the plate. The weld length is taken as the full length in linear-inches of the stiffener elements. The approximation does not account for the option of stitched or staggered welds of the stiffener or the requirement of hand welding but provides an adequate approximation assuming the amount of hand welding would be fairly equal across all schemes. The results are shown in figure 5-3 and table 5.3 on page 42.
Table 5.3: $\Delta P_c$ Normalized to Weld Length $[\text{kip}/L_{\text{Weld}}$

<table>
<thead>
<tr>
<th>Mode</th>
<th>Large</th>
<th>Conventional</th>
<th>Triangular</th>
<th>Diamond</th>
<th>Truss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.22</td>
<td>2.50</td>
<td>1.21</td>
<td>2.52</td>
<td>7.22</td>
</tr>
<tr>
<td>2</td>
<td>4.25</td>
<td>3.92</td>
<td>3.10</td>
<td>2.08</td>
<td>6.71</td>
</tr>
<tr>
<td>3</td>
<td>7.24</td>
<td>3.82</td>
<td>4.43</td>
<td>3.38</td>
<td>6.94</td>
</tr>
<tr>
<td>4</td>
<td>7.95</td>
<td>3.97</td>
<td>4.47</td>
<td>3.35</td>
<td>7.23</td>
</tr>
</tbody>
</table>

Table 5.4: $\Delta P_c$ Normalized to Volume of Material $[\text{kip}/\text{m}^3$

<table>
<thead>
<tr>
<th>Mode</th>
<th>Large</th>
<th>Conventional</th>
<th>Triangular</th>
<th>Diamond</th>
<th>Truss</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.43</td>
<td>0.42</td>
<td>0.20</td>
<td>0.42</td>
<td>1.20</td>
</tr>
<tr>
<td>2</td>
<td>0.35</td>
<td>0.65</td>
<td>0.51</td>
<td>0.35</td>
<td>1.12</td>
</tr>
<tr>
<td>3</td>
<td>0.60</td>
<td>0.64</td>
<td>0.73</td>
<td>0.56</td>
<td>1.17</td>
</tr>
<tr>
<td>4</td>
<td>0.66</td>
<td>0.66</td>
<td>0.74</td>
<td>0.55</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Again the truss stiffener arrangement outperformed the other options because its efficient placement of material. The large stiffener arrangement was also very successful. This is due to the low number of stiffeners and the attempt to minimize welding by minimizing the number of elements.

5.4 $\Delta P_c$ Normalized to Volume of Material

Normalizing the incremental critical load to the volume of material helps to optimize the stiffeners based on cost of material. A high ratio of $\Delta P_c$ to volume indicates an efficient use and placement of material and a low ratio indicates that material has been wasted. The results are shown in figure 5-4 on page 43 and in table 5.4.

The majority of the stiffener arrangements have a volume normalized $\Delta P_c$ in the 0.4 to 0.6 range. The truss arrangement is double that at 1.20, confirming that
Figure 5-4: $\Delta P_c$ Normalized to Volume of Material $[\frac{\text{kins}}{\text{in}^3}]$

the truss arrangement was a much more efficient use of material than any of the other arrangements. This result serves to support the method of tuning the stiffener placement to the fundamental buckling mode in a known and understood design scenario.
Chapter 6

Summary

6.1 Conclusion

The analysis has shown that a particular problem with known boundary conditions and predictable loading, can be solved for an optimum stiffener configuration by examining the unstiffened buckling modes. The analysis showed that for the design case examined a truss like structure located at the middle of the plate was the most effective in increasing the critical buckling load. The truss configuration had the highest overall critical buckling load, as was also the most effective when normalized to weld length and material volume.

The analysis showed that by tuning the stiffeners to the buckling mode of the unstiffened plate a configuration of stiffeners could be be designed that will be very effective in eliminating the first buckling mode and forcing the plate to buckle into variations of higher modes. Examining the buckling conditions and tuning the stiffeners can go a long way toward reducing the cost of material and labor. The truss arrangement was designed in this way proved to be very successful.

The limitation of the truss arrangement is in the fabrication and constructibility. The point where three stiffeners cross at the middle of the plate requires smaller tolerances than a normal grid and also makes machine welding difficult. The truss configuration would require a more labor intensive hands on approach to assembly. One possible construction sequence would involve prefabricating the stiffener truss in
a jig and using automatic submerged arc welding to make a majority of the welds with a limited amount of specialized manual welding at the center and edges.

6.2 Future Work

The analysis can be improved and extended in a variety of ways, many of which involve re-running the models using different parameters and improving the optimization procedure. As the analysis stands, the cost of material (volume) and labor (weld length) do not incorporate any actual cost data. Very accurate cost estimates are highly regional and difficult to determine but sensible estimates could be incorporated into weighting functions that more accurately reflect true costs.

The models could be re-run with any number of different parameters to validate the assumptions and results of this analysis over a more broad set of design cases. Some possible parameters are using a variety of WT rolled sections as stiffener elements instead of simple plate elements. Re-running the same models and varying the width and depth of the stiffeners for each model would give a better idea of how the volume of a stiffener changes the effectiveness. Another valuable re-run option would involve experimenting with different boundary conditions, geometries and loading cases, such as biaxial, uneven or incorporating applied moments or other imperfections.

This particular design case proved very valuable in examining the behavior of stiffened plates and opens up many other aspects of the problem that can be approached in the future using finite element methods.
Appendix A

Buckling Validation Hand Calculations
Test Column

4.31 x 4.31 in Square

Steel: $E = 29,000$ kpsi
$\sigma_y = 50$ kpsi

20 ft long
- 240 inches

Pinned - Pinned Connections.

Euler Buckling Load

$P_c = \frac{\pi^2 E I}{L^2}$

$= \frac{\pi^2 \times 29,000 \text{ kpsi} \times 28.756 \text{ in}^4}{240^2 \text{ in}^2}$

$P_c = 1412.9$ kips
LRFD Buckling Load

\[ \frac{K_L}{R} = \frac{240}{1.24} = 193.5 > 4.71 \]

\[ F_{cr} = 0.877 F_c \]

\[ F_c = \frac{\pi^2 E}{(kL)^2} \]

\[ = \frac{\pi^2 \times 39,000}{193.5^2} = 7.64 \text{ kips} \]

\[ F_{cr} = 0.877 F_c \]

\[ P_c = A F_{cr}^2 \]

\[ = 18.57 \times 0.877 \times 7.64 \]

\[ P_c = 124.45 \text{ kips} \]
Appendix B

Unstiffened Plate Buckling Modes
Figure B-1: The First and Second Buckling Modes of an Unstiffened Plate
Figure B-1: The Third and Fourth Buckling Modes of an Unstiffened Plate

(c) Third Buckling Mode of an Unstiffened Plate

(d) Fourth Buckling Mode of an Unstiffened Plate
Appendix C

Large Stiffener Arrangement

Buckling Modes
Figure C-1: The First and Second Buckling Modes of Large Stiffener Arrangement
Figure C-1: The Third and Fourth Buckling Modes of Large Stiffener Arrangement
Appendix D

Conventional Stiffener
Arrangement Buckling Modes
Figure D-1: The First and Second Buckling Modes of Conventional Stiffener Arrangement
Figure D-1: The Third and Fourth Buckling Modes of Conventional Stiffener Arrangement
Appendix E

Triangular Stiffener Arrangement

Buckling Modes
Figure E-1: The First and Second Buckling Modes of Triangular Stiffener Arrangement
Figure E-1: The Third and Fourth Buckling Modes of Triangular Stiffener Arrangement
Appendix F

Diamond Stiffener Arrangement
Buckling Modes
Figure F-1: The First and Second Buckling Modes of Diamond Stiffener Arrangement
Figure F-1: The Third and Fourth Buckling Modes of Diamond Stiffener Arrangement
Appendix G

Truss Stiffener Arrangement

Buckling Modes
Figure G-1: The First and Second Buckling Modes of Truss Stiffener Arrangement
Figure G-1: The Third and Fourth Buckling Modes of Truss Stiffener Arrangement
Bibliography


