Computer Science and Artificial Intelligence Laboratory
Technical Report

MIT-CSAIL-TR-2008-073 December 2, 2008

Resilient Provision of a Public and/or Private Good, or: Resilient Auctions of One Good in Unlimited Supply

Jing Chen and Silvio Micali

massachusetts institute of technology, cambridge, ma 02139 usa — www.csail.mit.edu

Resilient Provision of a Public and/or Private Good
or
Resilient Auctions of One Good in Unlimited Supply

Jing Chen and Silvio Micali

CSAIL, MIT

December 16, 2008

Abstract. We present two resilient mechanisms: the first one is for the provision of a public good, and the second is for the provision of a private good. Both mechanisms adopt a knowledge-based benchmark.

Notations. Let $N = \{1, \ldots, n\}$ be a set of players, and $\gamma \in \mathbb{R}^+$ the cost (to the "potential provider") of provisioning the good. A player i's valuation of the good is a non-negative real. The profile of all possible valuations of the players is denoted by $\mathbb{V}$. The profile of the players' true valuations is denoted by TV. The set of independent players is denoted by I.

When the good is publicly accessible, an outcome is a pair (x, P), where x is a bit indicating whether the good will be provided ($x = 1$) or not ($x = 0$), and P is a profile of prices (real numbers). A player i's utility is $TV_i \cdot x - P_i$. When the good is privately accessible, an outcome is a triple (x, A, P), where x and P are as above, and A is a bit profile indicating that when $x = 1$, whether each player i is allowed to access the good ($A_i = 1$) or not ($A_i = 0$). A player i's utility is $TV_i \cdot x \cdot A_i - P_i$.

In both cases, a player i's *general external* knowledge, denoted by GK_i, is i's information about TV_{-i}. A player i's *relevant external knowledge*, denoted by RK^i, is a subprofile in $\mathbb{V}_{-i}$ such that, for each $j \neq i$, RK^i_j is the maximum integer consistent with GK_i and less than TV_j. All knowledge of a player is private to him. Also in both cases, collusion is considered as illegal, and the mechanism treats every player as independent.

In the two mechanisms below, "numbered steps are performed by players, and bullet ones by the mechanism."

1 Our First Mechanism (for Provision of a Public Good)

Our first mechanism relates to the provision of a public good. Its benchmark is particularly attractive when the players —i.e., the potential beneficiaries of the good— are few in number and/or know each other quite well.

Mechanism $\mathcal{M}_1$

- Set $x = 0$ and $P_i = 0$ for each player i.

1. Each player i simultaneously and publicly announces a valuation subprofile V^i for players in $-i$.

- Set: $\gamma_i = \sum_{j \neq i} V^i_j$ for each player i, and $\star = \arg\max_i \gamma_i$.
  (We shall refer to player $\star$ as the "star player".)
- If $\gamma_\star < \gamma$, HALT.

2. (If $\gamma_\star \geq \gamma$) Each player i such that $V_i^\star > 0$ publicly and simultaneously announces YES or NO.

- If some player announces NO, reset $P_\star = \gamma_\star$, and HALT.
- (If all players announce YES) Reset: (1) $x = 1$; (2) $P_\star = \gamma - \gamma_\star$; and (3) $P_i = V_i^\star$ for each player $i \neq \star$.

Variant. In the last mechanism step replace instruction 2 with the following instruction: (2′) $P_\star = \alpha \cdot (\gamma - \gamma_\star)$, where the coefficient α is a constant between 0 and 1 (so as to generate a "surplus" for the lab).

Benchmark. Our mechanism $\mathcal{M}_1$ adopts the same benchmark as in [CM'08] —the relevant external knowledge of the best informed independent player, that is, $\max_{i \in I} \sum_{j \neq i} RK_j^i$. When the benchmark is at least γ, the sum of the social welfare and revenue of $\mathcal{M}_1$ is at least this benchmark.

Lemmas (with Proofs Coming Later).

1. for each independent player i, for any strategy in Σ_i^1, if $i \neq \star$, then i announces YES if $V_i^\star < TV_i$ and NO if $V_i^\star > TV_i$.
2. for each collusive set C, for any strategy in Σ_C^1, if $\star \notin C$, then all players $i \in C$ announce YES if $\sum_{i \in C} V_i^\star < \sum_{i \in C} TV_i$, and at least one player $i \in C$ announces NO if $\sum_{i \in C} V_i^\star > \sum_{i \in C} TV_i$.
3. for each independent player i such that $\sum_{j \neq i} RK_j^i \geq \gamma$, i won't underbid in Σ_i^2, i.e., $\sum_{j \neq i} V_j^i \geq \sum_{j \neq i} RK_j^i$.

2 Our Second Mechanism (for Provision of a Private Good)

Our second mechanism relates to the provision of a private good, that is to the case in which some players can be denied access to the good. Its benchmark *aggregates* the external knowledge of the independent players, and works particularly well when the players are quite numerous and/or may have only *local knowledge*, that is, when each player only knows a few of the other players. (In this case, for concreteness, we envision the provisioning to occur in a city, and refer to the good as a *private park*, to the players as citizens, and to the potential provisioner as the *builder*.)

Remark. Notice that, provisioning a private good with cost γ is equivalent to a single parameter auction [NRTV'06], where there is a single good for sale, but in unlimited supply, each player would like to buy one copy of the good, and the (total) reserved price is γ. Thus the following mechanism $\mathcal{M}_2$ applies to this other setting too.

Mechanism $\mathcal{M}_2$

- Set $x = 0$, $A_i = 0$, and $P_i = 0$ for each player i.

1. Each player i simultaneously and publicly announces (A) a subset of players $S_i \subseteq -i$ and (B) a valuation subprofile V^i for the players in S_i.

- $\forall j$: If $j \notin S_i$ for all $i \neq j$, then set $EV_j = 0$; else, $bip_j = \arg\max_{i: S_i \ni j} V_j^i$, and set $EV_j = V_j^{bip_j}$. Set $K = \sum_j EV_j$. (The terms "bip_j" can be interpreted as "best informed player about j".)
- If $K < \gamma$, HALT.
- ($K \geq \gamma$) Set $x = 1$, $A_i = 1$, and $P_i = EV_i$ for each player i.

2. Each player i such that $EV_i > 0$ publicly and simultaneously announces YES or NO.

- $\forall j$ such that player j announces NO, reset $P_j = P_j - EV_j$, $A_j = 0$, and $P_{bip_j} = P_{bip_j} + EV_j$.

Benchmark. The benchmark is the aggregation of the independent players' external knowledge, that is, $\sum_i \max_{j \in I \setminus \{i\}} RK_i^j$. When the benchmark is at least γ, the revenue generated by this mechanism is at least the benchmark.

Lemmas (with Proofs Coming Later). Let $\mathbb{C}$ be the partition of the players into collusive sets, C_i the collusive set including i for each player i, I the set of independent players (that is, all players i such that $\{i\} \in \mathbb{C}$), we have:

1. For each player $i \in I$, in Σ_i^1, i announces YES if $EV_i < TV_i$ and NO if $EV_i > TV_i$.
2. For each collusive player i, in $\Sigma_{C_i}^1$, if $bip_i \notin C_i$, then i announces YES if $EV_i < TV_i$, NO if $EV_i > TV_i$; if $bip_i \in C_i$, then i always announces YES.
3. For each player $i \in I$, in Σ_i^2, i doesn't underbid, that is, $\sum_{j \neq i} V_j^i \geq \sum_{j \neq i} RK_j^i$.

Variant. As stated the above mechanism achieves the benchmark stated for the independent players only. It is possible for $\mathcal{M}_2$ to achieve the benchmark extended to more players, even all players (i.e., the benchmark can be extended to all collusive sets as well). Assume that a player i's external knowledge about player j, RK_j^i, not only is the highest guaranteed price known to i that j is willing to pay, but also that such knowledge is not improvable, that is, that (e.g., relative to a proper Bayesian setting) i knows that there exists a positive probability ϵ such that j's true valuation for the good is RK_j^i. Then, one easy way to see that $\mathcal{M}_2$ can achieve the benchmark $\sum_i \max_{j \notin C_i} RK_i^j$ is to change the last mechanism step by replacing "$P_{bip_j} = P_{bip_j} + EV_j$" with "$P_{bip_j} = +\infty$".

References

[CM'08] J. Chen and S. Micali. Resilient Knowledge-Based Mechanisms For Truly Combinatorial Auctions (And Implementation in Surviving Strategies). Submitted to STOC'09.

[NRTV'06] N. Nisan, T. Rougharden, E. Tardos, and V. Vazirani (Eds.). Algorithmic Game Theory. Cambridge University Press, Sep. 2007. (Chapter 13, Hartline and Karlin, Profit Maximization in Mechanism Design)