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Abstract. We present various resilient auction mechanisms for a good in limited supply. Our mechanisms achieve both player-knowledge and aggregated player-knowledge benchmarks.

Notation. By $N = \{1, \ldots, n\}$ we denote the set of players, and by $\gamma \in \mathbb{R}^+$ the cost to the seller of provisioning the good(s), or the total reserve price, that is the minimum revenue needed for the sale of any good to take place. A player i's valuation of the good is a non-negative real. The profile of all possible valuations of the players is denoted by $\mathbb{V}$. The profile of the players' true valuations is denoted by TV. The set of independent players is denoted by I. There may be collusive players, possibly partitioned into different collusive sets.

We consider the goods to be equivalent to each other as the players are concerned, and denote their number by m. (Equivalently, we may assume that there are m copies of the same good for sale). For simplicity of analysis the goods can be considered untransferable once sold.

An outcome is a triple (x, A, P), where x is a bit indicating whether the sale occurs ($x = 1$) or not ($x = 0$); P is the profile of prices; and A is a profile of bits, indicting whether player i has won one of the goods ($A_i = 1$) or not ($A_i = 0$). A player i's utility is $TV_i \cdot x \cdot A_i - P_i$.

A player i's *general external* knowledge, denoted by GK_i, can be considered to be i's information about TV_{-i}. A player i's *relevant external knowledge*, denoted by RK^i, consists of a subprofile in $\mathbb{V}_{-i}$ such that, for each $j \neq i$, RK^i_j is the maximum integer known to i, consistent with GK_i, and guaranteed to be less than TV_j.[1] All knowledge of a player is private to him.

In our mechanisms "numbered steps are performed by players, and bullet ones by the mechanism."

1 All-or-None Auctions of a Single Good in Limited Supply and Unit Demand

In the following mechanism each player desires to acquire only one good. Either all goods are sold, or none are. For simplicity, we assume $m < n$ (else we can adopt an unlimited supply solution).

Possible Alternatives. Possible alternatives include:

1. All copies must be sold at the same price;

2. The copies are transferable; or

3. Each player is allowed to buy more than one of the copies.

[1] RK^i_j can always be 0. Considering RK^i as a subprofile for $n-1$ players is without loss of generality. RK^i_j is a fixed integer guaranteed to be less than TV_j for simplicity only. It could also be a distribution with proper "tails".

Mechanism $\mathcal{M}_3$

- Set $x = 0$. For all player i, set $A_i = 0$ and $P_i = 0$.

1. Each player i simultaneously and publicly announces (a) S_i a subset of $-i$ of cardinality $m-1$ and (b) a valuation subprofile V^i indexed by the players in S_i.[2]

- For each player i, let $\gamma_i = \sum_{j \in S_i} V^i_j$. Set $\star = \arg\max_i \gamma_i$.
- If $\gamma_\star < \gamma$, HALT.

2. (If $\gamma_\star \geq \gamma$) Each player j such that $j \in S_\star$ and $V^\star_j > 0$ simultaneously and publicly announces YES or NO.

- If some player announces NO, set $P_\star = \gamma_\star$, and HALT.[3]
- (If all players announce YES) Set $x = 1$, $A_i = 1$, and $P_i = V^\star_i$ for each player $i \in S_\star$; set $A_\star = 1$.[4]

Benchmark. For each player i, let N_i be the set of $m-1$ players having the highest valuation according to RK^i. Then our benchmark is $\max_{i \in I} \sum_{j \in N_i} RK^i_j$.

2 Auctions of a Single Good in Limited Supply and Unit Demand

The following mechanism envisages also outcomes in which some but not all of the goods are sold. It aggregate the external knowledge of the players.

Mechanism $\mathcal{M}_4$

- Set $x = 0$. For each player i, set $A_i = 0$ and $P_i = 0$.

1. Each player i simultaneously and publicly announces (a) a subset of players $S_i \subseteq -i$ and (b) a valuation subprofile V^i indexed by the players in S_i.

- $\forall j \in N$: if $j \notin S_i$ for all $i \neq j$, then set $EV_j = 0$; else, let $bip_j = \arg\max_{i: S_i \ni j} V^i_j$ and set $EV_j = V^{bip_j}_j$. (We refer to each EV_i as the *external valuation* of player i.)
- Rename the players so that $EV_1 \geq EV_2 \geq \cdots \geq EV_n$, set $W = \emptyset$, $K = 0$, and $t = 1$.
- Repeat until $|W| \geq m-1$: reset $W := W \cup \{t\}$ and $K := K + EV_t$; if $bip_t \notin W$, then reset $W := W \cup \{bip_t\}$ and $K := K + EV_{bip_t}$; reset $t := \min_{i \notin W} i$.
- If $|W| = m-1$: if $bip_t \in W$, reset $W := W \cup \{t\}$ and $K := K + EV_t$; else, reset $W := W \cup \{bip_t\}$ and $K := K + EV_{bip_t}$.
- If $K < \gamma$, HALT.

[2] Each V^i_j can be 0. Letting the cardinality of each S_i be $m-1$ is without loss of generality.

[3] That is, in this variant the star player is punished with a fine of $\gamma_\star$. In other variants, the star player may be punished differently, in particular with different fines. Also, other players could also be punished, in particular those players k for which $V^k_j \geq V^\star_j$ where j answered NO. In addition, rather than halting outright, the mechanism can continue in different manners, in particular by choosing a next star player, etc.

[4] In this variant the star player is encouraged not to "underbid" in reporting his valuations about the other players by a reward: namely, assigning him for free one of the goods. Other variants may consider different rewards for the star player. Possibly, a positive price may be obtained for each good.

- (If $K \geq \gamma$) Reset $x := 1$, and $A_i := 1$ and $P_i := EV_i$ for each player $i \in W$.

2. Each player $i \in W$ such that $EV_i > 0$ publicly and simultaneously announces YES or NO.

- $\forall j$ such that j announces NO, reset $P_j := P_j - EV_j$, $A_j := 0$, and $P_{bip_j} := P_{bip_j} + EV_j$.

Benchmark. The benchmark that $\mathcal{M}_4$ achieves is sum of the highest $\lfloor m/2 \rfloor$ external valuations and the lowest $\lceil m/2 \rceil$ external valuations.

References

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