RAINFALL-RUNOFF AS SPATIAL STOCHASTIC PROCESSES: DATA COLLECTION AND SYNTHESIS

by

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S.B., Massachusetts Institute of Technology (1972)
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ABSTRACT

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Submitted to the Department of Civil Engineering in January 1975, in partial fulfillment of the requirements for the Degree of Doctor of Science

This work recognizes rainfall and runoff as multidimensional stochastic processes. Using the knowledge of such processes, a procedure for designing an optimal network to measure the total precipitation of an event over a fixed area is given. The methodology used in this static problem allows consideration of the following aspects of network design.

1. Spatial Correlation of Process
2. Errors of Measurement Techniques and their Correlation

Optimal networks are given in terms of the number and location of stations together with the resulting cost and mean square error of estimation.

The relation between rainfall and runoff is recognized as a dynamic problem. A statistically non-stationary, multi-dimensional rainfall generator is suggested. The model, capable of simulating historical storm exterior and interiors, assumes the validity of Taylor’s hypothesis of turbulence within a storm interior.

The suggested rainfall model is used together with a runoff model to study the accuracy of discharge prediction as a function of the rainfall sampling network. The runoff model used is a spatially distributed simulation based on a finite difference solution of the Kinematic wave equations. Discharge prediction accuracy at any point can be obtained in terms of the mean square error as a function of the number of rainfall sampling stations, their location, and the errors in sampling devices. Naturally the solution is also a function of the physics of the basin at hand which are described by the rainfall and runoff simulators.

Thesis Supervisor

Ignacio Rodriguez-Iturbe

Title

Associate Professor of Civil Engineering
Acknowledgements

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A fellowship from the Economic Development Administration of Puerto Rico sponsored all of the author's graduate studies for which he remains very grateful.

Particular words of thanks go to Dr. Daniele Veneziano for his contributions to the solution of the static network problem and for his friendship; Dr. Roberto Lenton for the many hours of discussions, comments, and friendly office atmosphere. Messrs. Graeme Dandy, Alonso Rhenals, Juan Valdes, Tomas Facet, and Myron Rosenberg have my sincere appreciation as colleagues, critics and friends. Dr. Stephen F. Moore deserves credit for his time as one of the supervisors of this work and as the person who first introduced the author to the techniques here used.

Dr. Ignacio Rodriguez-Iturbe supervised this work. He has the author's gratitude for his time, his criticism, his enthusiasm, but most important, his friendship.

Thanks to Mr. Walter Hill for his help in programming the rainfall generator. Ms. Erika Babcock did a wonderful typing job.

Finally, for obvious reasons, I would like to dedicate this work to my wife, Pat, and to my parents who really made it all possible.

- 3 -
Table of Contents

Title Page 1
Abstract 2
Acknowledgements 3
Table of Contents 4
List of Figures 8
List of Tables 12
List of Principal Symbols 14

1. Introduction 21
   1.1 Rainfall: Problems in Generation and Measurement 21
   1.2 General Approach 23
   1.3 Literature Review 24
       1.3.1 Network Design 24
       1.3.2 Rainfall Synthesis 29

2. Areal Mean of a Rainfall Event - The Theory 30
   2.1 Problem Formulation 30
       2.1.1 Introduction 30
       2.1.2 Design Criteria 34
       2.1.3 Measure of Accuracy 37
   2.2 Mean Square Model Error 39
       2.2.1 Derivation 39
       2.2.2 Evaluation 40
       2.2.3 Examples 43
   2.3 Mean Square Estimation Error 50
   2.4 The Cross Error Term $\sigma^2_{me}$ 54
   2.5 Optimization Algorithm 57
3. Areal Mean of a Rainfall Event - A Case Study

3.1 Case Study

3.2 Results of the Analysis

3.3 Solution to Example Problem - Estimation

3.4 Sensitivity Analysis

3.4.1 Effects of Varying Point Variance

3.4.2 Sensitivity to Covariance Function Parameter

3.4.3 Sensitivity to Covariance Function Form

3.5 Brief Comments and Comparison to Other Recent Network Design Schemes

3.6 The Estimation of the Covariance Function of Total Storm Depth

3.7 General Comments

3.7.1 Convergence Problems

3.7.2 Computational Requirements

3.7.3 Concluding Remarks

4. Rainfall Generation: A Non-Stationary, Time Varying, Multi-Dimensional Model

4.1 Need for a Model

4.2 Formulation and Assumptions

4.3 Point Rainfall Generation Algorithm

4.4 The Generation of Areal Rainfall

4.5 An Example

4.6 Conclusions
## 5. Rainfall Network Design for Runoff Prediction

166

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Introduction</td>
<td>166</td>
</tr>
<tr>
<td>5.2 State-Space Model of Rainfall</td>
<td>168</td>
</tr>
<tr>
<td>5.3 A State-Space Model of Runoff</td>
<td>173</td>
</tr>
<tr>
<td>5.4 Network Analysis Example</td>
<td>187</td>
</tr>
<tr>
<td>5.4.1 Comments on Implementation</td>
<td>187</td>
</tr>
<tr>
<td>5.4.2 Example Data Description</td>
<td>188</td>
</tr>
<tr>
<td>5.4.3 Results and Analysis</td>
<td>195</td>
</tr>
<tr>
<td>5.5 Concluding Remarks</td>
<td>205</td>
</tr>
</tbody>
</table>

## 6. Conclusions

207

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1 Summary</td>
<td>207</td>
</tr>
<tr>
<td>6.2 Estimation, Filtering and Control in Hydrology</td>
<td>209</td>
</tr>
<tr>
<td>6.3 Further Research</td>
<td>210</td>
</tr>
</tbody>
</table>

Bibliography

213

Appendix 1

222

Appendix 2

224

Analytical Evaluation of Integrals in Mean Square Error Expression, for the Quadratic Exponential Case, in a Square Area, \( A = \ell^2 \)

Appendix 3

227

Derivation of Equation 2-33

Appendix 4

229

Network Design for Mean Areal Total Precipitation Estimation - Computer Program

Appendix 5

262

Rainfall Generator
Appendix 6
The Rainfall-Runoff Model 287

Appendix 7
The Rainfall-Runoff Network Design Program 334

Appendix 8
366

Biography 383
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chapter 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-1</td>
<td>Area A and Superimposed Discretizing Grid</td>
<td>31</td>
</tr>
<tr>
<td>2-2</td>
<td>Plot of Optimal Combinations of Cost and Measure of Accuracy Defining a Transformation Curve</td>
<td>36</td>
</tr>
<tr>
<td>2-3</td>
<td>Mean Square Model Error for Simple Exponential Covariance as a Function of $A\alpha^2$ and $n$</td>
<td>45</td>
</tr>
<tr>
<td>2-4</td>
<td>Mean Square Model Error for Quadratic Exponential Covariance as a Function of $A\alpha^2$ and $n$</td>
<td>46</td>
</tr>
<tr>
<td>2-5</td>
<td>Mean Square Model Error for Bessel Covariance as a Function of $A\alpha^2$ and $n$</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chapter 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-1</td>
<td>Diagram of Test Area Showing Discretization and Other Relevant Information for 9 Point Grid Pattern</td>
<td>65</td>
</tr>
<tr>
<td>3-2</td>
<td>Optimal Solutions for $C_\Delta = 10^{-5}$ Using Full Measure of Accuracy</td>
<td>71</td>
</tr>
<tr>
<td>3-3</td>
<td>Optimal Solutions for $C_\Delta = 10^{-6}$ Using Full Measure of Accuracy</td>
<td>72</td>
</tr>
<tr>
<td>3-4</td>
<td>Diagram of Test Area Showing Discretization and Other Relevant Information for 18 Point Grid Pattern</td>
<td>74</td>
</tr>
<tr>
<td>Figure No.</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3-5</td>
<td>Optimal Solutions for $C_\Delta = 10^{-6}$ Using Full Measure of Accuracy in 18 Point Grid</td>
<td>77</td>
</tr>
<tr>
<td>3-6</td>
<td>Objective Function Versus Number of Stations for $C_\Delta = 10^{-2}$ and $C_\Delta = 10^{-3}$</td>
<td>83</td>
</tr>
<tr>
<td>3-7</td>
<td>Objective Function Versus Number of Stations for $C_\Delta = 10^{-5}$ and $C_\Delta = 10^{-4}$</td>
<td>84</td>
</tr>
<tr>
<td>3-8</td>
<td>Objective Function Versus Number of Stations for $C_\Delta = 10^{-8}$, $C_\Delta = 10^{-7}$ and $C_\Delta = 10^{-6}$</td>
<td>85</td>
</tr>
<tr>
<td>3-9</td>
<td>Transformation Curve of Example Problem</td>
<td>87</td>
</tr>
<tr>
<td>3-10</td>
<td>Variance Reduction Factor Due to Spatial Sampling With Random Design. Used in the Estimation of Areal Mean of Rainfall Event With $r(v) = b\nu K_1(b\nu)$. (After Rodríguez and Mejía, 1974).</td>
<td>100</td>
</tr>
<tr>
<td>3-11</td>
<td>Variance Reduction Factor Due to Spatial Sampling with Stratified Design. Used in the Estimation of Areal Mean of Rainfall Event with $r(v) = b\nu K_1(b\nu)$, (after Rodríguez and Mejía, 1974).</td>
<td>101</td>
</tr>
</tbody>
</table>

Chapter 4

<p>| 4-la      | Undimensional Storm Representation                                   | 117  |
| 4-lb      | Mean Temporal Behavior, $i_a(t)$, Resulting from a 2 inch, 1 hour Duration Storm | 117  |</p>
<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-2</td>
<td>Diagram of 360 sq.mi. Area Showing Discretization, Storm Velocity and Direction</td>
<td>119</td>
</tr>
<tr>
<td>4-3</td>
<td>Hyetograph of Storm at 1,675 days, Areal Mean Intensities</td>
<td>143</td>
</tr>
<tr>
<td>4-4</td>
<td>Hyetograph of Storm at 19.498 days, Areal Mean Intensities</td>
<td>144</td>
</tr>
<tr>
<td>4-5</td>
<td>Areal Distribution of First Storm at 3.238 hrs. into Storm, Areal Mean Intensities</td>
<td>145</td>
</tr>
<tr>
<td>4-6</td>
<td>Areal Distribution of First Storm at 6.476 hrs. into Storm, Areal Mean Intensities</td>
<td>146</td>
</tr>
<tr>
<td>4-7</td>
<td>Areal Distribution of First Storm at 9.719 hrs. into Storm, Areal Mean Intensities</td>
<td>147</td>
</tr>
<tr>
<td>4-8</td>
<td>Areal Distribution of Second Storm at 5.372 hrs. into Storm, Areal Mean Intensities</td>
<td>148</td>
</tr>
<tr>
<td>4-9</td>
<td>Areal Distribution of Second Storm at 10.745 hrs. into Storm, Areal Mean Intensities</td>
<td>149</td>
</tr>
<tr>
<td>4-10</td>
<td>Hyetograph of Storm at 1.675 days, Point Intensities</td>
<td>160</td>
</tr>
<tr>
<td>4-11</td>
<td>Hyetograph of Storm at 19.498 days, Point Intensities</td>
<td>161</td>
</tr>
<tr>
<td>4-12</td>
<td>Areal Distribution of First Storm at 3.238 hrs. into Storm, Point Intensities</td>
<td>162</td>
</tr>
<tr>
<td>4-13</td>
<td>Areal Distribution of First Storm at 6.476 hrs. into Storm, Point Intensities</td>
<td>163</td>
</tr>
<tr>
<td>Figure No.</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4-14</td>
<td>Areal Distribution of Second Storm at 5.372 hrs. into Storm, Point Intensities</td>
<td>164</td>
</tr>
<tr>
<td>4-15</td>
<td>Areal Distribution of Second Storm at 10.745 hrs. into Storm, Point Intensities</td>
<td>165</td>
</tr>
</tbody>
</table>

Chapter 5

| 5-1       | Schematized River Basin                                               | 174  |
| 5-2       | Finite Difference Grid                                                 | 177  |
| 5-3       | State-Space Form of Rainfall-Runoff Model as Applied to Illustration Example | 181  |
| 5-4       | Schematic Representation of Example Basin                              | 189  |
| 5-5       | Time Distribution of Mean Storm and Standard Deviation Around Mean      | 191  |
| 5-6       | Hydrograph at Basin Outlet Resulting from Mean Storm Input             | 194  |
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table No.</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-1</td>
<td>Number of Rainfall Stations Required for Agricultural Watersheds</td>
<td>26</td>
</tr>
<tr>
<td>Chapter 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-1</td>
<td>Measurement Error Variance and Cost Assigned to each Grid in a Point Discretization</td>
<td>67</td>
</tr>
<tr>
<td>3-2</td>
<td>Full Design Results - 9 Grid Points Model</td>
<td>68</td>
</tr>
<tr>
<td>3-3</td>
<td>Measurement Error Variance and Cost Assigned to each Grid in 18 Point Discretization</td>
<td>73</td>
</tr>
<tr>
<td>3-4</td>
<td>Full Design Results - 18 Grid Points Model, $C_A=10^{-6}$</td>
<td>75</td>
</tr>
<tr>
<td>3-5</td>
<td>Example Problem Results - Estimation Error Accuracy Criteria</td>
<td>79</td>
</tr>
<tr>
<td>3-6</td>
<td>Results for Location of Three Stations</td>
<td>82</td>
</tr>
<tr>
<td>3-7</td>
<td>Results for Point Variance Sensitivity Tests, $C_A=10^{-5}$</td>
<td>89</td>
</tr>
<tr>
<td>3-8</td>
<td>Results of Covariance Function Parameter Sensitivity tests, $C_A=10^{-5}$</td>
<td>92</td>
</tr>
<tr>
<td>3-9</td>
<td>Sensitivity to Covariance Function Form, $C_A=10^{-5}$</td>
<td>95</td>
</tr>
</tbody>
</table>
Chapter 4

4-1 Examples of Generated Storms, Areal Average Intensities Given 133

4-2 Examples of Generated Storms, Point Intensities Given 150

Chapter 5

5-1 Basin Elements Data 190

5-2 Measurement Error Variance at Possible Observation Sites 193

5-3 Analysis of Network Alternatives Shown in Parenthesis. Correlation Parameter 0.08. 196

5-4 Analysis of Network Alternatives Shown in Parenthesis. Correlation Parameter 0.15. 200
### LIST OF PRINCIPAL SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area of region ( \alpha ) (Chapter 2)</td>
</tr>
<tr>
<td>( A_i )</td>
<td>Area of each grid (Chapter 2)</td>
</tr>
<tr>
<td>( A(t) )</td>
<td>NxN matrix of Markov rainfall model at time ( t )</td>
</tr>
<tr>
<td></td>
<td>(Chapters 4 and 5)</td>
</tr>
<tr>
<td>A</td>
<td>Cross sectional area of flow in basin stream elements or depth in overland elements (Chapter 5)</td>
</tr>
<tr>
<td>( A_{ij}(t) )</td>
<td>Cross sectional area of flow in stream ( k ), time ( t ), spatial interval ( j ) (Chapter 5)</td>
</tr>
<tr>
<td>( B(t) )</td>
<td>NxN matrix of Markov rainfall model at time ( t )</td>
</tr>
<tr>
<td></td>
<td>(Chapters 4 and 5)</td>
</tr>
<tr>
<td>( C_\delta )</td>
<td>Measure of accuracy equivalent to a unitary change in cost (Chapter 2)</td>
</tr>
<tr>
<td>( C(m, x_i) )</td>
<td>Cost function (Chapter 2)</td>
</tr>
<tr>
<td>( \text{cov}(x_i, x_j) )</td>
<td>Covariance function between points ( x_i ) and ( x_j )</td>
</tr>
<tr>
<td>d</td>
<td>Diagonal of rectangular area (Chapter 2)</td>
</tr>
<tr>
<td>d</td>
<td>Storm total depth</td>
</tr>
<tr>
<td>( D_i )</td>
<td>Total precipitation depth at point ( i ) (Chapter 4)</td>
</tr>
<tr>
<td>( D(t) )</td>
<td>( \chi(t) - \chi_i(t) ) (Chapter 5)</td>
</tr>
<tr>
<td>( e_k(\cdot) )</td>
<td>Functional of overland segment ( k ) (Chapter 5)</td>
</tr>
<tr>
<td>( E_k(\cdot) )</td>
<td>Functional of stream segment ( k ) (Chapter 5)</td>
</tr>
<tr>
<td>E</td>
<td>Expectation operator</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Experiment (network) defined by ( m ) and ( x_{ij}(i=1\ldots m) )</td>
</tr>
<tr>
<td></td>
<td>(Chapter 2)</td>
</tr>
<tr>
<td>( \xi^* )</td>
<td>Optimal experiment</td>
</tr>
</tbody>
</table>
\( f(x,y) \) Function describing total depth of storm event over region \( a \) (Chapter 2)

\( f(x_i) \) Discrete value of \( f(x,y) \) at point \( i \) defined by coordinate vector, \( x_i \) (Chapter 2)

\( \hat{f}(x_i) \) Estimate value of \( f(x_i) \) (Chapter 2)

\( \hat{f} \) \( n \times l \) vector of \( f(x_i) \) values (Chapter 2)

\( \hat{\hat{f}} \) \( n \times l \) vector of estimated rainfall depth, \( \hat{f}(x_i) \) (Chapter 2)

\( f(x'_i, t'; x''_j, t'') \) General covariance function (Chapter 4)

\( f(\tau) \) probability density function of time between storm, \( \tau \) (Chapter 4)

\( f(t_r) \) p.d.f. of storm duration, \( t_r \) (Chapter 4)

\( f(d/t_r) \) Conditional p.d.f. of depth given storm duration (Chapter 4)

\( k^F_2(\cdot) \) Functional of stream element \( k \) (Chapter 5)

\( k^F_2(\cdot) \) Functional of stream element \( k \) (Chapter 5)

\( F \) \( m \times 2 \) complex matrix (Chapter 2)

\( G(r) \) Probability density function of the distance \( r \) between two random points in a 2-dimensional region (Chapter 2)

\( G'(w) \) Radial spectral density (Chapter 4)

\( G(w) \) Radial spectral distribution (Chapter 4)

\( H(r) \) Probability density function of the distance \( r \) between a fixed point \( x_i \) and a random point in the region of interest (Chapter 2)
$H$ (mxn), matrix defining a data collection network

$i = \sqrt{-1}$ (Chapter 2)

$i(x_i, t)$ Rainfall intensity at point with coordinate vector $x_i$ at time $t$ (Chapter 4)

$i_{\mu}(x_i, t)$ Mean rainfall intensity at $x_i$ and $t$ (Chapter 4)

$i_a(t)$ Mean intensity of storm at time $t$ from start of rainfall at point of interest

$i(t)$ Vector of normalized rainfall intensities at time $t$ (Chapter 5)

$\hat{i}(t/t)$ Best linear estimate of vector $i(t)$ given observations up to time $t$ (Chapter 5)

$i_i(t)$ Rainfall intensity at time $t$, overland segment $i$ (Chapter 5)

$\bar{i}_\mu(t)$ Mean rainfall vector at time $t$ (Chapter 5)

$I$ Identity matrix

$K_1$ First order modified Bessel function of the second kind

$\ell_1, \ell_2$ Sides of a rectangle

$\zeta(\cdot)$ Non-linear functional transfer matrix of state-space runoff model (Chapter 5)

$\zeta'(\chi_{\mu}(t))$ First derivative of $\zeta$ evaluated at the mean state vector $\chi_{\mu}(t)$ (Chapter 5)

$m$ number of discrete observation points, $m \leq n$ (Chapter 2)

$m_f$ (nx1) Mean $f$ vector

$m$ Kinematic wave equations parameter (Chapter 5)
$m_k$ Parameter of element $k$ (Chapter 5)

$n$ Number of discrete points where $f(x_i)$ is defined (Chapter 2)

$N$ Number of harmonics (Chapter 4)

$N$ Number of discrete points in space (Chapter 5)

$O(\xi)$ Objective function of experiment $\xi$

$P$ Areal average total depth of storm over region $\alpha$ (Chapter 2)

$\hat{P}$ Approximation of time areal average depth, $P$ (Chapter 2)

$\hat{\hat{P}}$ Estimated value of $\hat{P}$ and final approximation of $P$ (Chapter 2)

$q_L$ Lateral flow into stream segments (discharge per unit length) or rainfall into overland segment (Chapter 5)

$q(t)$ Lateral inflow into stream element at time $t$ (Chapter 5)

$q_k(t)$ Discharge per unit length of overland element $k$ at time $t$ (Chapter 5)

$Q$ Discharge (Chapter 5)

$Q_k(t)$ Discharge of stream element $k$ at time $t$

$r$ Distance between 2 points

$r(x_i, t; x_j, t'')$ General form of normalized covariance function (Chapter 4)

$R$ $E[V V^T]$, $m \times m$ matrix (Chapter 2)

$\bar{R}$ $E[V(t) V^T(t)]$, $m \times m$ matrix (Chapter 5)
$R(x_i, t)$  Mean zero, variance 1 residual at point $x_i$ and time $t$  (Chapter 4)

$s^2$  Point variance

$S_{t,t}(t)$  $E[i(t)x(t)]^T$  (Chapter 5)

$S_{t,t-1}(t)$  $E[i(t)x(t-1)]^T$  (Chapter 5)

$t$  time

$t_D$  Storm duration

$t_r$  Storm duration

$U(\cdot)$  Utility function  (Chapter 2)

$U$  Storm average velocity in x direction  (Chapter 4)

$V$  $mx_1$ vector of white noise related to instrument measurement error

$v$  Distance between two points

$V(t)$  $mx_1$ vector of white instrument noise at time $t$  (Chapter 5)

$w_1$  Random variable with radial spectral distribution $G(w)$  (Chapter 4)

$w(t)$  $mx_1$ vector of white noise with zero mean and unit variance  (Chapter 5)

$x_i$  x coordinate of point $i$

$x$  vector of coordinates

$x$  Longitudinal distance along stream direction  (Chapter 5)

$y_{-1}$  2-dimensional random variable equidistributed on the circle of unit radius  (Chapter 4)

$y_{j}(t)$  Depth at time $t$, element $k$, spatial interval $j$  (Chapter 5)
\( Z \) \( \text{mxl vector of noisy observations (Chapter 2)} \)

\( \mathbf{Z}_0 \) \( \text{mxl vector of observations bias (Chapter 2)} \)

\( \mathbf{Z}(t) \) \( \text{mxl vector of intensity observations at time } t \) (Chapter 5)

\( \alpha \) \( \text{Covariance function parameter} \)

\( \alpha^2 \) \( \text{Covariance function parameter} \)

\( \alpha_k \) \( \text{Kinematic wave equations parameter (Chapter 5)} \)

\( \beta \) \( \text{Parameter of element } k \) (Chapter 5)

\( \mathbf{P} \) \( \text{Matrix of state-space runoff model} \)

\( \mathbf{Y} \) \( \text{nxl vector of weights, } \mathbf{P} \)

\( \delta(m,x) \) \( \text{Measure of accuracy (Chapter 2)} \)

\( \Delta p \) \( \text{Approximation error (Chapter 2)} \)

\( \Delta t \) \( \text{time step} \)

\( \Delta x \) \( \text{Spatial interval} \)

\( \Psi \) \( \mathbb{E}(\mathbf{f} \mathbf{f}^T), (nxn) \text{ matrix (Chapter 2)} \)

\( \Psi \) \( \mathbb{E}(\mathbf{i}(o) \mathbf{i}^T(o)) \) (Chapter 5)

\( \phi \mathbf{2}(\cdot) \) \( \text{Functional related to overland segment } k \) (Chapter 5)

\( \phi \mathbf{1}(\cdot) \) \( \text{Functional related to overland segment } k \) (Chapter 5)

\( \Theta_i \) \( \text{Uniformly distributed random angle betw. 0 and } 2\pi, \text{(Chapt.4)} \)

\( \epsilon(x) \) \( \text{random field in } \mathbb{R}^2 \)

\( \eta(x,t) \) \( \text{Noisy residual obeying a certain covariance function in time and space} \) (Chapter 4)

- 19 -
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{A}(\mu_{A},\gamma_{A})$</td>
<td>Variance reduction coefficient in estimation of areal precipitation (Chapter 4)</td>
</tr>
<tr>
<td>$\rho_{i}$</td>
<td>Weights on spatially discrete storm depth values</td>
</tr>
<tr>
<td>$\sigma_{\Delta}^{2}$</td>
<td>Mean square error, $E[\hat{P} - \hat{P}]^2$ (Chapter 2)</td>
</tr>
<tr>
<td>$\sigma_{e}^{2}$</td>
<td>$E[\hat{P} - \hat{P}]^2$ (Chapter 2)</td>
</tr>
<tr>
<td>$\sigma_{m}^{2}$</td>
<td>$E[\hat{P} - \hat{P}]^2$ (Chapter 2)</td>
</tr>
<tr>
<td>$\sigma_{me}^{2}$</td>
<td>$E[(P - \hat{P})(\hat{P} - \hat{P})]$ (Chapter 2)</td>
</tr>
<tr>
<td>$\sigma(x_{i},t)$</td>
<td>Standard deviation of intensity at point $x_{i}$ and time $t$ (Chapter 4)</td>
</tr>
<tr>
<td>$\sigma_{a}(t)$</td>
<td>Standard deviation at time $t$ after start of rainfall at a given point in space (Chapter 4)</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>$E[(\hat{f} - \bar{f})(\hat{f} - \bar{f})^T]$ mean square error of estimation matrix (Chapter 2)</td>
</tr>
<tr>
<td>$\Sigma(\cdot)$</td>
<td>Mean square error of estimation matrix corresponding to design $\tilde{H}(\cdot)$ (Chapter 2)</td>
</tr>
<tr>
<td>$\Sigma(t/t)$</td>
<td>Mean square error of estimation matrix of rainfall at time $t$ given observations up to time $t$ (Chapter 5)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time between storms</td>
</tr>
<tr>
<td>$x(t)$</td>
<td>State vector of runoff model at time $t$</td>
</tr>
</tbody>
</table>
1.1 Rainfall: Problems in Generation and Measurement

The need of precipitation data in most water resources problems is an uncontested fact. The planner, decision maker, or engineer must deal with the problem of defining what kind of data, where and how to collect, and also with the analysis and synthesis of the obtained information. This is the problem of data management. As remarked by Lenton et al (1974), it is necessary that "all aspects of data management be integrated -- the initial collection of data cannot and should not be treated separately from the later stages of data analysis and synthesis." Furthermore, when possible, the data management procedure should be defined in terms of the final objectives, goals and uses of the collected information.

This work deals mainly with the problems of collection and synthesis of rainfall data.

The problems of data collection networks design has been divided into various levels (Rodda et al, 1969). Levels I and II can be classified as problems in regional estimation; i.e., there is no clearly defined final goal or use for the collected data. The problem of rainfall monitoring for estimating the areal total precipitation average for a storm event and the problem of finding the long term (time) areal mean precipitation fall in these two levels. Level III networks are those designed to collect data for a specific, clearly defined objective which would imply known net benefits or utility on
the data. The problem of rainfall monitoring for use together with a flood forecasting system theoretically fits in this framework.

This work deals with two network design problems, the design of a network for obtaining the areal mean precipitation of an event and the design of a network for obtaining data to be used in a rainfall-runoff model. The first problem is clearly a level I design, as previously discussed. The second problem could possibly be studied as level III, but for reasons of inadequacy of data it was treated as level I. The first problem is discussed in Chapters 2 and 3, the second is presented in Chapter 5.

Since the utility functions related to level I designs are undefined, the design criteria utilized are:

1. A network with fixed budget should be designed to minimize errors in the estimation of the hydrologic variables involved.

2. A network operated with an accuracy constraint should be designed for minimum cost.

The accuracy of the networks is considered a function of the stochasticity of rainfall itself in terms of its spatial and temporal variation as well as the configuration of the network in terms of the number of observations, place of the observations and the errors inherent in the instruments used for making the observations.

Chapter 4 of this work deals with the problem of rainfall synthesis. The generation of synthetic time series has found many uses in the water resources field. Rainfall generation has mostly been developed in terms of univariate storm exterior generators. Very few
models for generating storm interiors (temporal distribution within the event) exist. Moreover, generation in space has been very limited. This work proposes a bidimensional model for the generation of storms exteriors and interiors continuously in space.

1.2 General Approach

Data collection is viewed in this work as an estimation problem. A stochastic event, rainfall, which is continuous in space and time, must be estimated from noisy, discrete and incomplete observations in time and space. Designing a network is then the definition of that system which gives the "best" estimate of the true event (rainfall). "Best" must be defined in terms of some criteria function of a measure of accuracy, cost, or both.

The nature of the problems studied permit the use of linear estimators. In this case, estimators which minimize the mean square error are used. The areal average depth problem requires a static (time invariant) solution. The rainfall-runoff problem uses a dynamic linear estimator, the Kalman-Bucy filter.

The rainfall synthesis problem of Chapter 4 uses a random fields generation technique based on the sampling from the radial spectral density of the process which, in turn, is obtained from the spectrum of the process covariance function. The technique requires the existence of some sort of isotropy in the process. The form of the process covariance function (in time and space) is hypothesized, using as assumption the Taylor hypothesis of turbulence.
1.3 Literature Review

1.3.1 Network Design

Most of the existing literature in network design attempts to define a density of observation; i.e., the number of stations needed. Very few works address the problems of where to locate the observation stations; how to account for instrument errors; and how to take network cost into consideration.

Commonly the design is reduced to a rule of thumb. For example, McKay in Gray's Principles of Hydrology mentions that for standard precipitation gages, a 15 mile separation is adequate for Canadian conditions. The same reference makes a fair review of existing design methods. The following comments derive from that source.

The "index approach" is described in general terms by McKay as being:

1) Based on correlation - Sensors should be exposed so that their measurements have the highest possible correlation with the effects which are being measured.

2) One gage should be located in each "homogeneous" area, which apparently implies that a station should be highly correlated to surroundings but correlation between stations should be minimized. Clearly, these are opposing objectives which will demand some trade-off.

McKay also discusses the common practice of "transposition" which consists of designing by comparison to the "true events" defined as that measured by dense networks in basins of similar characteristics.
For example, McGuinness (1963) suggests the use of the following formula for Coshocton, Ohio:

\[ E = 0.03 P^{0.54} G^{2.4} \]  

(1-1)

where

- \( E \) = absolute difference in inches
- \( P \) = rainfall in inches for "true" (dense) network
- \( G \) = gauging network density in sq.mi. per gage for the reduced network

The above formula was developed from data of watersheds less than 25 sq.mi. but found consistant for larger areas. Its extrapolation to areas with different conditions is speculative.

Hershfield (1965) argues that the average spacing needed to obtain a 0.9 correlation between storm rainfall events can be estimated from the 24 hr and the 1 hr, 2 year return period rainfall. Using this correlation value as a design criteria he then develops graphs of the distance between raingages in miles as a function of the 2 year - 24 hour rainfall (in inches) and the 2 year - 1 hour rainfall (in inches).

McKay also quotes from Holtan et al, (1962), their version of minimum standards of raingage densities for agricultural watersheds. They are given in Table 1-1.

Several studies have been made of densely gaged areas resulting in ideas for network design. McKay refers to a study in Wilson Creek, Manitoba, where the standard error of estimate (rainfall) was found to be in direct proportion to the distance between stations.

Linsley et al, (1958), give results from the United States Wea-
### Table 1-1
(from McKay in Gray's Principles of Hydrology)

<table>
<thead>
<tr>
<th>Size of Drainage Area (acres)</th>
<th>Gauging Ratio (mi²/gage)</th>
<th>Min-No. of Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-30</td>
<td>0.05</td>
<td>1</td>
</tr>
<tr>
<td>30-100</td>
<td>0.08</td>
<td>2</td>
</tr>
<tr>
<td>100-200</td>
<td>0.10</td>
<td>3</td>
</tr>
<tr>
<td>200-500</td>
<td>0.16</td>
<td>1 per 100 acres</td>
</tr>
<tr>
<td>500-2500</td>
<td>0.40</td>
<td>1 per 250 acres</td>
</tr>
<tr>
<td>2500-5000</td>
<td>1.00</td>
<td>1 per sq.mi.</td>
</tr>
<tr>
<td>over 5000</td>
<td>3.00</td>
<td>1 per 3 sq.mi.</td>
</tr>
</tbody>
</table>
ther Bureau where a graphical relation of the standard error of precipitation averages is given as a function of network density and area. That relation was derived for the Muskingum River Basin. Similarly for Wilmington, Ohio, a relation between precipitation averages, network density and "true" precipitation exists.

Neil (1953) used data from the 96 sq.mi. Goose Creek watershed (Illinois). The area had a 50 gage network. Using different but similar ideas to the ones used in this work he considered that the "true" variance of the areal mean rainfall was given by

\[
\sigma^2 = \sigma_{X_n}^2 + \sigma_{X_{48}}^2 + \sigma_0^2
\]  

(1-2)

where

\[
\begin{align*}
\sigma^2 & = \text{true variance} \\
\sigma_{X_n}^2 & = \text{estimation error variance from sample size n} \\
\sigma_{X_{48}}^2 & = \text{error of using 48 raingages instead of infinite observations} \\
\sigma_0^2 & = \text{error variance of observations (instrument)}
\end{align*}
\]

He concluded he could not say much about \(\sigma_{X_{48}}^2\) and \(\sigma_0^2\) but was able to fit an equation for \(\sigma_{X_n}^2\) of the form:

\[
\sigma_{X_n}^2 = A \bar{p}^B \sigma_0^2 n^C
\]  

(1-3)

where \(\bar{p}\) is the population mean of the 48 gages observation.

He produced diagrams of the standard error of estimate vs. n and \(\bar{p}\).
Besides the work in hydrology there is considerable literature in the area of sampling stochastic processes in a plane which is applicable to any field.

Particularly Zubrzycki (1958) discusses and derives the formulas for the standard errors of estimates obtained from random, stratified or systematic (random start) sampling in a plane. No consideration is given to measurement error. Matern (1960) treats the same problems and compares the various sampling techniques.

Gandin (1965) discusses the design of networks for the collection of meteorological data. As in this work he sees the problem as one of estimation and a natural outgrowth of exercises in filtering, estimation, and extrapolation of data which in his work is called "objective analysis".

Zawadski (1972) addressed the design of raingage networks. He limited it to the design of systematic, uniform networks. No consideration to measurement noise was given. Naturally, due to his condition of uniform network grids, no consideration is given to the particular location of observations.

This work, though, uses an approach equivalent to Zawadski's as one part of the static network design problem. Both works use mean square error as an accuracy criteria.

Eagleson (1967) presented some network design criteria for obtaining the long term areal mean precipitation and one of the two works familiar to the author dealing with network design in reference to rainfall-runoff models. Eagleson uses harmonic analysis techniques together with concepts of distributed linear systems to study the sen-
sitivity of peak catchment discharge to the spatial variability of con-
vective and cyclonic storm rainfall. No consideration is given to sta-
tion location nor to measurement errors. Considerable simplifications
of the rainfall-runoff process are made.

Grayman and Eagleson (1971, 1973) studied the problem of
network design in reference to rainfall runoff modeling. Their work
was based on extensive simulation exercises from where ideas of the
statistical distributions of discharge as a function of network design
could be obtained. They treated the problem as a level III design and
attempted to identify the utility of the various network alternatives.

Moore (1971) used Kalman filter techniques for studying
networks for the collection of water quality parameters in a 1-dimen-
sional river system.

More recently, Rodriguez-Iturbe and Mejia (1974) and Lenton
et al (1974) studied network design for the purpose of monitoring rain-
fall total depths. These two works are discussed in more detail in
Chapter 3 where they are compared to one of the solutions suggested
here.

1.3.2 Rainfall Synthesis.

The literature in rainfall synthesis is so vast that it
would be rather difficult to summarize it here in a cohesive way. The
reader is referred to Rhenals et al (1974), Leclerc et al (1973), and
Grayman et al (1971), for good reviews of the literature. Chapter 4
will further discuss the nature of existing rainfall generators and
give other relevant references.
Chapter 2
Areal Mean of a Rainfall Event - The Theory

2.1 Problem Formulation

2.1.1. Introduction

This chapter develops a theory for the design of a precipitation network whose purpose is the estimation of the areal mean depth of rainfall events with particular characteristics over a fixed area.

Consider a region, \(a\), as shown in Figure 2.1. The hydrologist is interested in the estimation of

\[
P = \frac{1}{A} \int_A f(x,y) \, dx \, dy
\]  

(2-1)

where

\(A\) = area of region \(a\)  
\(f(x,y)\) = function describing depth of storm event over region \(a\)  
\(P\) = areal average total depth of storm, \(f(x,y)\), over region \(a\)

Under usual conditions, Equation (2-1) is approximated by discretizing region \(a\) (see Figure 2-1) and then evaluating

\[
\hat{P} = \sum_{i=1}^{n} \rho_i f(x_i)
\]

\[
= \sum_{i=1}^{n} \rho_i f
\]

(2-2)

where

\(\hat{P}\) = approximation of true areal average depth, \(P\)  
\(f(x_i)\) = discrete value of storm, \(f(x,y)\), at point \(i\) defined by coordinates vector, \(x_i\)
Discretized points in area A, n = 30

Possible station locations

Figure 2-1 Area A and super-imposed discretizing grid
\( \rho_i \) = weights on discrete storm depth values  
\( n \) = number of discrete points where \( f(x_i) \) is defined  
\( \gamma^T \) = transpose of \( n \times 1 \) vector of weights, \( \rho_i \)  
\( \mathbf{f} \) = \( n \times 1 \) vector of \( f(x_i) \) values

Uniform, regular, discretization is used in this work (see Figure 2-1). Under such conditions the optimal weights are very nearly given by \( \frac{1}{n} = \frac{A_i}{A} \) where \( A_i \) is the area of each grid. Then Equation (2-2) becomes:

\[
\hat{\mathbf{p}} = \frac{1}{n} \sum_{i=1}^{n} f(x_i)
\]

\[
= \gamma^T \mathbf{f}
\]

The above assumptions are a matter of convenience and normal practice. The same scheme is similarly applicable for irregular discretizations. In such cases the optimal weights \( \rho_i \) no longer correspond to \( \frac{1}{n} \). Optimal weights for general discretization patterns could be obtained with methods of optimal interpolation (see Lenton, et al., 1974) or with traditional weighting schemes like Thiessen coefficients. The author strongly suggests, though, the use of regular grids which makes very simple the problem of weight definition. It will be clear that the use of regular grid does not seriously constraint the final design geometry.
Equation (2-3) together with Figure 2-1 imply the perfect knowledge of rainfall depth at all the discrete points defined by the superposition of a regular grid pattern on region \( G \). Usual conditions, though, are far from perfect and observations are only available in a limited number, \( m < n \), of grid points (see Figure 2-1). These \( m \) points of noisy observations do not necessarily form any recognizable geometrical pattern. Thus, the hydrologist should filter and extrapolate these noisy observations to obtain an approximation of \( \hat{P} \) which is in turn an estimator of \( P \).

This approximation may be written:

\[
\hat{P} = \frac{1}{n} \sum_{i=1}^{n} \hat{f}(x_i)
\]

or

\[
\hat{P} = \hat{\mathbf{f}}^T \hat{\mathbf{Y}}
\]

(2-4)

where

\( \hat{f}(x_i) \) = estimated value of \( f(x_i) \) obtained from noisy observations at a limited number of points, \( m \), and extrapolated to the whole grid of \( n \) points

\( \hat{\mathbf{f}} = (n \times 1) \) vector of estimated rainfall depth values, \( \hat{f}(x_i) \), \( (i=1\ldots n) \)

\( \hat{P} \) = approximated value of \( \hat{P} \) and final estimation of \( P \)
The network design problem at hand then translates to obtaining the number, \( m \), and location of raingages that will give us the best estimate, \( \hat{P} \), of the true areal mean of precipitation, \( P \). "Best" estimate and "best" design are key terms which are defined in the light of the criteria discussed below.

2.1.2 Design Criteria

The "best" network design in terms of the number, \( m \), and locations of stations is defined as that which maximizes a utility function,

\[
U[\delta(m,x_i); C(m,x_i)] \quad i = 1\ldots m
\]  \hspace{1cm} (2-5)

where, \( U \) is the utility function dependent on:

- \( \delta(m,x_i) \) = certain measure of accuracy. It is a function of \( m \), the number of stations and \( x_i \) (i=1\ldots m), the location of stations.
- \( C(m,x_i) \) = cost of \( m \) stations at locations \( x_i \) (i=1\ldots m)

The form of \( U(\delta,C) \) will depend on the uses planned for the data obtained by the network to be designed. It is clear that the structure of the utility function is strongly related to the "levels" of data collection discussed in Chapter 1 and to which level the problem at hand belongs.

Networks for estimating the areal mean of a rainfall event belong in general to the level I or level II types of designs. Future use of obtained data in the planning and managing of hydrologic activities to be undertaken are often unspecified at the time of data collection.
becoming generally impossible to concretely specify the form of the utility function.

An alternative approach, and the one used in this work, is to define an objective function of the type

$$0(\xi) = \delta(\xi) + C_\delta \sum_{i=1}^{m} C(x_i)$$

(2-6)

where

- $\xi$ = experiment (network) defined by $m$, the number of observations, and $x_i$, $(i=1...m)$ the location of the observations
- $\delta(\xi)$ = a measure of the "lack of precision" involved in the experiment. It will be studied in the next section.
- $C(x_i)$ = cost of locating and maintaining a station at location $x_i$; costs are assumed additive
- $C_\delta$ = measure of accuracy equivalent to a unitary change in cost

Equation (2-6) assumes that the measure of accuracy is such that an inverse relation between cost and accuracy exist. That is, as the cost increases $\delta(\xi)$ should decrease.

For each given $C_\delta$ the design process consists of finding the optimal experiment $\xi^*$ which minimizes Equation (2-6).

Solving the design problem for different values of $C_\delta$ allows the construction of a "transformation curve" of the type sketched in Figure 2.2. Every point in this curve represents the values of the
Figure 2-2  Plot of optimal combinations of cost and measure of accuracy defining transformation curve.
cost, $C(\xi^*)$, and accuracy, $\delta(\xi^*)$, associated with a network which minimizes the objective function for a particular value of $C_0$. A parallel table should provide the number and locations of the stations for each optimal experiment, or point, in the transformation curve.

Even without knowing the exact trade-off between cost and accuracy the decision maker can make intelligent choices using a curve like that in Figure 2-2. If, for example, he were operating with a fixed budget the graph (and associated table) would give him the optimal experiment and the corresponding accuracy. Similarly the decision maker could design under accuracy restrictions. These examples are indeed the most common situations in level I and level II designs like the one at hand.

2.1.3 Measure of Accuracy

Accuracy of the design is a measure of how well $\hat{P}$ (Equation 2-4) approximates the true area mean $P$ (Equation 2-1). Define the approximation error as:

$$\Delta_P = P - \hat{P}$$

(2-7)

The chosen measure of accuracy will then be the mean square error

$$\sigma^2_\Delta = E[P-\hat{P}]^2$$

(2-8)

where $E$ stands for the expectation operator.

By adding and subtracting $\hat{P}$ (Equation 2-2), $\sigma^2_\Delta$ becomes:

- 37 -
\[ \sigma^2_\Delta = \text{E}[ (P - \hat{P}) + (\hat{P} - \hat{\hat{P}}) ]^2 \]

\[ = \text{E}[ (P - \hat{P})^2 ] + \text{E}[ (\hat{P} - \hat{\hat{P}})^2 ] + 2\text{E}[ (P - \hat{P})(\hat{P} - \hat{\hat{P}}) ] \]

\[ = \sigma^2_m + \sigma^2_e + 2\sigma^2_{me} \quad (2-9) \]

where

\[ \sigma^2_m = \text{E}[ (P - \hat{P})^2 ] = \text{mean square value of the "model" error}. \]

It represents the error involved in approximating a continuous integral by a discrete summation.

\[ \sigma^2_e = \text{E}[ (\hat{P} - \hat{\hat{P}})^2 ] = \text{mean square value of the "estimation" error}. \]

It represents the error involved in estimating the discrete values \( f(x_i), \ i = 1...n \), from noisy and incomplete observations.

\[ \sigma^2_{me} = \text{E}[ (P - \hat{P})(\hat{P} - \hat{\hat{P}}) ] = \text{term arising from the dependence between the model error and the estimation error}. \]

Having devined \( \sigma^2_\Delta \) as the desired measure of accuracy the network design problem becomes one of minimizing

\[ O(\xi) = \sigma^2_\Delta(\xi) + C_\Delta \sum_{i=1}^{m} C(x_i) \]

over all \( \xi(\text{all m}; \text{all } x_i, i = 1...m) \)

where \( C_\Delta = \text{mean square error equivalent to a unitary change in cost} \)
2.2 Mean Square Model Error

2.2.1 Derivation

The term, \( \sigma_m^2 = E[P - \hat{P}]^2 \) can be expanded as

\[
\sigma_m^2 = E[PP]^2 + E[\hat{P}\hat{P}]^2 - 2E[PP]
\] (2-11)

The evaluation of Equation (2-11) depends on the sampling technique used in selecting the discrete values \( f(x_i) \) of Equation (2-3). The sampling method to be used was described previously as a fixed geometric grid pattern (starting point fixed and known). One may now substitute Equations (2-1) and (2-3) in Equation (2-11) to obtain

\[
\sigma_m^2 = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \text{cov}(x_i, x_j) + \frac{1}{2} \int_{\Lambda}^{\Lambda} \int_{\Lambda}^{\Lambda} \text{cov}(x_1, x_2) dx_1 dx_2
\]

\[- \frac{2}{nA} \sum_{i=1}^{n} \int_{\Lambda} \text{cov}(x, x_i) dx\] (2-12)

In computing the above equations it is assumed that \( f(x,y) \), the rainfall function, is a zero mean process. The estimation of mean square errors in this work will not be affected by this assumption. Complete accounting of a non-zero mean process could be done with no technical difficulties. The covariance function in Equation (2-12) is then defined as

\[
\text{cov}(x_i, x_j) = E[f(x_i) f(x_j)]
\] (2-13)
It is important to point out that the above sampling definition could be called "systematic" but it should not be confused with the traditional "systematic sampling" in space as defined by other authors (Zubrzycki, 1958; Matern, 1960). In those references systematic sampling follows a fix geometric pattern or grid but the starting location of the grid is random. Under these assumptions the expression for $\sigma_m^2$ is different from the one given in Equation (2-12).

Zawadski (1972) has obtained the same results given here and shows an equivalent expression for Equation (2-12), adequate for rectangular grid patterns with observations in the center of gravity of the sub-areas.

2.2.2 Evaluation

The two integral expressions in Equation (2-12) require a functional definition of the covariance of the process. This would restrict the proposed network design method considerably since most of the commonly used functional covariance expressions represent homogeneous and isotropic spatial processes. These restrictions though will be considerably lifted as it will be seen in Chapter 3.

Furthermore, the integrals in Equation (2-12) have no close form solution for most valid covariance functions. Traditional numerical integrations lack the necessary accuracy and most important, they are very expensive computationally, especially for the case of the double integral in space appearing in Equation 2-12. Fortunately integrals of the form at hand can be converted to one dimensional expectation operations which lend themselves to relatively easy and
cheap numerical integration.

Rodriguez-Iturbe and Mejia (1974a) pointed out the following equivalence:

\[
\frac{1}{A^2} \int_A \int_A \text{cov}(x_1, x_2) \, dx_1 \, dx_2 = \int_0^d \text{cov}(r)G(r) \, dr
\]

\[
= E[\text{cov}(r)/A] \quad (2-14)
\]

where

\[
G(r) = \text{probability density function of the distance, } r, \text{ between two randomly chosen points in the region of interest. It depends on the shape and size of the region.}
\]

\[
d = \text{diagonal of rectangular area.}
\]

In the above expression it is inherently assumed that the user is dealing with functional covariances which only depend on the distance between points, i.e., homogeneous. No isotropic assumption is made.

In this work all areas are approximated by rectangles (rectangular grid pattern). For this geometric figure Gosh (1951) and Matern (1960) derive \(G(r)\) as

\[
G(r) = \frac{1}{\sqrt{A}} G(r/\sqrt{A}, \sqrt{x_1/\sqrt{x_2}}) \quad (2-15)
\]

where

\[
G(a,b) = 2a[G_1(a,b) + G_2(ab,b) + G_2(a/b, 1/b)]
\]

with

\[
G_1(a,b) = \begin{cases} 
\frac{\pi + a^2 - 2a(b+1/b)}{2} & 0 \leq a \leq \sqrt{b^2 + 1/b^2} \\
0 & \text{otherwise}
\end{cases}
\]
$$G_2(a,b) = \begin{cases} \frac{2}{a^2 - 1} - 2 \cos^{-1}(1/a) - b^{-2}(a-1)^2 & 1 \leq a \leq \sqrt{1 + b^4} \\ 0 & \text{otherwise} \end{cases}$$

and $l_1$, $l_2$ are the rectangle sides, whose relative magnitudes are not important.

For the last term in the right hand side of Equation 2-12, Lenton et al (1974) has shown it can be written as

$$\int_A \text{cov}(x, x_1) dx = \sum_{i=1}^{4} \int_0^{D_i} \text{cov}(r) H(r) dr$$

(2-16)

where again a rectangular grid area is assumed and $H(r)$ stands for the probability density function of the distance, $r$, between a fix point, $x_1$, and a randomly chosen point in the region of interest.

Through Equation (2-16) the two dimensional integral over the area is transformed to the summation of four one dimensional integrals. The integrals cover the four rectangular regions formed by cutting the area of interest with two perpendicular lines, parallel to the sides of the original area and going through the point $x_1$. The upper limits of integration are the diagonal, $D_i$, of each of the created subregions.

The form of $H(r)$ is the following:

Define: $d_1 = \text{length of subregion along x axis}$

$$d_2 = \text{length of subregion along y axis}$$
For $d_1 > d_2$

$$H(r) = \begin{cases} \frac{\pi}{2} & 0 \leq r \leq d_2 \\ \cos^{-1}\left(\frac{d_2}{r}\right) & d_2 < r < d_1 \\ \frac{\pi}{2} - \frac{1}{2} \cos^{-1}\left(\frac{d_1}{r}\right) - \frac{1}{2} \cos^{-1}\left(\frac{d_2}{r}\right) & d_1 < r < d_2 \end{cases}$$

For $d_2 > d_1$

$$H(r) = \begin{cases} \frac{\pi}{2} & 0 \leq r \leq d_1 \\ \sin^{-1}\left(\frac{d_1}{r}\right) & d_1 < r < d_2 \\ \sin^{-1}\left(\frac{d_1}{r}\right) - \cos^{-1}\left(\frac{d_2}{r}\right) & d_2 < r < D_1 \end{cases}$$

(2-17)

2.2.3 Examples

The mean square model error, Equation (2-12), was evaluated for three types of covariance functions, all of them representative of homogeneous and isotropic processes, which have been used in hydrologic analysis (Rodriguez and Mejia, 1974a). They are:

- **Exponential Covariance;**
  $$\text{cov}(r) = s^2 e^{-\alpha r}$$
  (2-18a)

- **Quadratic Exponential;**
  $$\text{cov}(r) = s^2 e^{-\alpha^2 r^2}$$
  (2-18b)

- **Bessel**
  $$\text{cov}(r) = s^2 b r K_1(br)$$
  (2-18c)

where

- $s^2$ = point variance of the process
- $\alpha, \alpha^2, b$ = parameters of functions
- $K_1$ = a first order modified Bessel function of the second kind
It can be shown (see Appendix 1) that the mean square model error (Equation 2-12) when evaluated for the above given covariance functions in geometrically similar rectangular areas is only a function of the number of rectangular grids (level of discretization) and a non-dimensional area given by $Aa^2$, $Ab^2$, $Ac^2$ respectively for each functional covariance structure.

Figures 2-3, 2-4, and 2-5 show the non-dimensional mean square model error ($\sigma_m^2$ divided by point variance $s^2$) as a function of $n$, the discretization level, and the non dimensional area. Evaluation was done for square areas and square grid pattern over a reasonable range of non-dimensional areas and $n$ values. Calculation of the mean square model error for the single exponential and the Bessel covariance functions was done utilizing Equations 2-14 and 2-16 together with a numerical integration method (trapezoidal and 48 point Gaussian Quadrature respectively). The integrals in Equation (2-12) were analytically solved for the quadratic exponential covariance functions and the resulting expressions are in Appendix 2.

It should be noted from Figures 2-3, 2-4, and 2-5 that as the non-dimensional area increases the value of the normalized model mean square error, $E[\hat{P} - \hat{P}]^2/s^2$, asymptotically approaches that of statistically independent samples, i.e., $1/n$.

Another characteristic is that for any given nondimensional area the exponential covariance function gives smaller mean square error than the Bessel type covariance for all values of $n$. In other words the obtained curves fall faster for the Bessel covariance function.
Figure 2-3  Mean square model error for single exponential covariance as a function of $\alpha^2$ and $n$.
Figure 2-4 Mean square model error for quadratic exponential covariance as a function of $A\alpha^2$ and $n$. 

The graph shows the relationship between $E\left[\frac{(p-p)^2}{S^2}\right]$ and $A\alpha^2$ for different values of $n$, where $n = 4, 9, 16, 25, 36, 49, 69, 81, 100$. The x-axis represents $A\alpha^2$ and the y-axis represents $E\left[\frac{(p-p)^2}{S^2}\right]$. The curves indicate how the mean square model error changes with varying $A\alpha^2$ and $n$. 

The graph is a visual representation of the mean square model error for quadratic exponential covariance as a function of $A\alpha^2$ and $n$.
Figure 2-5  Mean square model error for Bessel
Covariance as a function of $A_b^2$ and $n$
Figure 2-4 illustrates the considerable difference between the quadratic exponential (Equation 2-18b) results and the other covariance functions. The obtained curves (for the range of areas plotted) have three different regions. Moving down in area size we obtain a region of very small changes in model mean square error (shallow slope). This is followed by a very steep region where changes in area produce large changes in model mean square error. Finally there is a region of small areas where the curves for each \( n \) value taper off and appear to approach zero asymptotically, which should be the correct behavior in the small areas region for any covariance function. In fact it appears then that the quadratic exponential function produces the expected qualitative behavior but much more pronounced than that of the other two functions. In the bulk of the curves (the steep regions) comparatively much smaller mean square errors are obtained. This is seemingly paradoxical in the sense that the user would think that since the quadratic exponential function mathematically decreases much faster in value than the single exponential it would imply less correlation in space and so higher errors (and mean square errors) in the estimation of a continuous areal process for a given \( n \).

The above "paradox" led to the investigation of Equation 2-18b as an adequate correlation (covariance) function. Matern (1960) proves the validity of the quadratic exponential as a correlation function, in a two dimensional space, by showing it as an always positive spectra. Nevertheless he points out some peculiar characteristics that do not
make it very attractive. Quoting from Matern:

"The process with cov.f. exp(-v^2) is computationally easy. However, it was thought to be "too continuous" to be realistic. In fact, it is deterministic along any straight line in R_2, see Karhunen (1952)." (meaning that with limited knowledge of the past on a straight line the future can be predicted with probability one along that line).

Interestingly enough it was found that in the very few other places where the behavior of \( e^{-\alpha^2 r^2} \) has been studied, its undesirable peculiarities are mentioned. Yaglom (1952) says with respect to the one dimensional equivalent of Equation 2-18b:

"We emphasize that the correlation function \( (e^{-\tau^2/4}) \) falls off much faster than the function \( e^{-\alpha|\tau|} \), so that at first one might think that in this case the value of \( \varepsilon(t+\tau) \) for large \( \tau \) would be less dependent on the "past" of the process than in \( [e^{-\alpha|\tau|}] \). However it turns out that the situation is just the opposite"

Yaglom goes on to prove this paradox and further to show that for a process obeying \( e^{-\alpha^2 r^2} \) the value of the process in the future can be approximated arbitrarily closely by a linear combination of past values. Whittle (1954) goes as far as saying that the processes, in two dimensions, that have exponential correlation functions are "very artificial". He goes on to show that such functions (even \( e^{-\alpha r} \)) have "no divine rights" in two dimensions and that the function \( br \xi_1(br) \) (Equation 2-18c) can be considered as the "elementary"
correlation in two dimensions, similarly as \( e^{-\alpha |r|} \) is in one dimension.

This brief discussion has been included here because in spite of all these arguments the quadratic correlation function is sometimes proposed to describe two dimensional geophysical processes.

In conclusion then it seems that the advantages in the sense of possible close form analytical solutions, offered by the quadratic exponential correlation function, are hindered by its many undesirable peculiarities. Most importantly, the obtained low mean space errors due to the information extrapolation ability of Equation 2-18b are an undesirable characteristic.

Chapter 3 will discuss another interesting result due to the unusual behavior of this correlation function.

2.3 Mean Square Estimation Error

In Section 2.1.2 the mean square estimation error was defined as:

\[
\sigma^2_e = \text{E}[(\hat{\mathbf{f}} - \hat{\mathbf{f}})^2]
\]

substituting Equations 2-3 and 2-4,

\[
\sigma^2_e = \text{E}[\mathbf{y}^T \hat{\mathbf{f}} - \hat{\mathbf{f}}^T \mathbf{y}]^2
\]

\[= \text{E}[\hat{\mathbf{f}}^T \mathbf{y} - \hat{\mathbf{f}}^T \mathbf{y}]^2
\]

\[= \mathbf{y}^T \text{E}[(\hat{\mathbf{f}} - \hat{\mathbf{f}}) (\hat{\mathbf{f}} - \hat{\mathbf{f}})^T] \mathbf{y} \tag{2-19}
\]

where \( \hat{\mathbf{f}} \) is an \((n \times 1)\) vector estimate of \( \mathbf{f} \). The estimate is obtained from a series of incomplete and noisy observations. These observations
define the data collection network and are mathematically described as

\[ Z = H f + V \]  \hspace{1cm} (2-20) \]

where

- \( Z = (m \times 1) \) vector of noisy observations at each discrete point in space, \( m \leq n \)
- \( f = (n \times 1) \) vector of true values of rainfall at \( n \) points, \( f(x_i) \), \( i = 1...n \) as defined in Section 2.1.1
- \( V = (m \times 1) \) vector of white noise related and representing instrument error
- \( H = (m \times n) \) matrix defining the data collection network
  \( = \{h_{ij}\} = \begin{cases} h_{ij} & \text{if } x_j \text{ is a measurement location} \\ 0 & \text{otherwise} \end{cases} \)

The following terms will also be needed in the analysis:

- \( \Psi = E[ff^T] = (n \times n) \) covariance matrix of true process, called from now on the "prior" covariance matrix
- \( \Sigma = E[VV^T] = (m \times m) \) covariance matrix of white noise vector

It is assumed that \( f \) and \( V \) are zero-mean uncorrelated random variables.

One should note that the above definition of the matrix \( H \) implies a very sparse matrix of \( m \) rows, non-zero elements will only appear in the column location where there is a measurement.

The form of the matrix \( \Sigma \) should be diagonal since \( V \) was defined as an uncorrelated vector. In fact the proposed network design scheme will allow one other form of \( \Sigma \) as it will be seen in a latter section.
Define \( \hat{f} \) as the best linear estimate of \( f \) where "best" means the linear estimate yielding minimum mean square error matrix, in the sense that \( B - E[(f - \hat{f})(f - \hat{f})^T] \) where \( B \) is any other mean square error matrix of a linear estimator, is positive definite. The resulting expression for \( \hat{f} \), a function of \( Z \) (Equation 2-20), is a well known result of estimation theory (Schwepee, 1973; Deutsch, 1965) and is given as:

\[
\hat{f} = \sum H R^{-1} (Z - Z_o) + \sum m_f
\]  

(2-21)

where

\[
\sum = E[(f - \hat{f})(f - \hat{f})^T] = \text{mean square error of estimation}
\]

\[
m_f = (n \times 1) \text{ mean vector, assumed to be zero in this work}
\]

\[
Z_o = (m \times 1) \text{ vector of observations bias assumed zero}
\]

The mean square error of estimation matrix \( \sum = E[(f - \hat{f})(f - \hat{f})^T] \), (many times referred to as the posterior covariance matrix) is given by:

\[
E[(f - \hat{f})(f - \hat{f})^T] = \sum = [\Psi^{-1} + H^T R^{-1} H]^{-1}
\]

\[
= \Psi - \Psi H^T (H \Psi H^T + R)^{-1} H \Psi
\]  

(2-22)

The mean square error of estimation, \( \sigma_e^2 \), is then obtained by substituting 2-22 into 2-19 to obtain

\[
E[(P - \hat{P})^2] = \Psi^T [\Psi^{-1} + H^T R^{-1} H]^{-1} \Psi
\]

\[
= \Psi^T [\Psi - \Psi H^T (H \Psi H^T + R)^{-1} H \Psi] \Psi
\]  

(2-23)
Some comments are due with respect to the linear estimator given in Equation (2-21) (see Bras, et al., 1975).

If $f$ and $V$ have multivariate normal distributions, Equation (2-21) is the absolute minimum mean square error estimator. Interpreted in Baysian terms where $\psi$ is then a prior covariance matrix of $\hat{f}$, the posterior distribution (after one observation) of $\hat{f}$ would also be normal with covariance matrix, $\sum$ (Equation 2-22) and mean vector $\hat{f}$. If $f$ and $V$ are not normally distributed then the posterior distribution of $\hat{f}$ is not necessarily normal and $\hat{f}$ is the best linear estimator of $f$ but there may be a non linear estimator with lower mean square error. The closer the normality condition is approached the better are the chances of having the absolute "best" estimator.

Equation (2-21) can be put in the following forms:

$$\hat{f} = m_f + \sum H^T R^{-1} (Z - Z_o - H m_f)$$

(2-24a)

$$= m_f + \psi H^T (\psi H^T + R)^{-1} (Z - Z_o - H m_f)$$

(2-24b)

It is easy to see (see Bras, et al. 1975) that the best estimate is a linear function of the "innovation vector" $(Z - Z_o - H m_f)$; i.e., the difference $\hat{f} - m_f$ is proportional to the deviations of the observations from the predicted value $(Z_o + H m_f)$. The difference $(\hat{f} - m_f)$ further depends on the matrix product $\sum H^T R^{-1}$ being larger for smaller error covariance, $R$. In fact more belief is attached to measurements statistically affected by small errors, than to unreliable measurements. The difference $(\hat{f} - m_f)$ is also proportional to the
signal to noise strength measured by matrix $H$, the so-called experiment technological transfer matrix. Finally, the presence of $\sum$ in Equation (2-24a) implies that more weight is given to observations when uncertainty in the value of $f$ (large $\sum$) is relatively large. The critical comparison is between $\psi^{-1}$ and $H^T R^{-1} H$ in Equation (2-22). If error variance is small relative to variable uncertainty; i.e., $H^T R^{-1} H$ larger than $\psi^{-1}$; then actual observations are given a lot of weight, vice-versa otherwise.

From Equation 2-22 it is obvious that if $H$ is positive definite then $[\psi - \sum]$ is at least positive semi-definite, a statement that just implies that processing more information can not increase the uncertainty of the process.

To conclude this section the nicest feature about Equation (2-23) is that even though the expression of the best estimate (Equation 2-21) depends on the observations (on the experiment results) the expression for $\sigma_e^2$, the mean square error of the estimation, is independent of the observations. This feature will allow "a priori" estimation of the desire measure of accuracy thereby permitting a rational network design. Another characteristic of Equation (2-23) is that the prior covariance matrix, $\psi$, of the vector $f$ is not in any way restricted to functional definitions nor to homogeneity and/or isotropic assumptions.

2.4 The Cross Error Term, $\sigma_{me}^2$

The cross error term, $\sigma_{me}^2$, in Equation 2-9, has been defined as,
\[
\sigma_m^2 = E[(\hat{P} - \bar{P})(\hat{P} - \bar{P})]
\]

\[
= E(\hat{P}\hat{P}) - E(\hat{P}\hat{P}) - E(\hat{P}\hat{P}) + E(\hat{P}\hat{P})
\]

Using Equations 2-1 and 2-3 we can write

\[
E(\hat{P}\hat{P}) = E[\left(\frac{1}{A} \int_A f(x) dx\right) \frac{1}{n} \sum_{i=1}^{n} f(x_i)]
\]

\[
= \frac{1}{nA} \sum_{i=1}^{n} \int_A \text{cov}(x, x_i) dx
\]

which is a summation of integrals similar to the ones discussed in Section 2.2.2, Equation (2-16).

Substitution of Equations 2-1 and 2-4 together with Equation 2-24b (with \(m_f = 0\)) gives the second term in Equation 2-25 as

\[
E[\hat{P}\hat{P}] = Y^T E[P \Psi H^T (H \Psi H^T + R)^{-1} Z]
\]

\[
= Y^T \Psi H^T (H \Psi H^T + R)^{-1} E(PZ)
\]

Using now the expression for \(Z\), Equation 2-20 and the fact that \(f\) and \(Z\) are uncorrelated gives

\[
E[\hat{P}\hat{P}] = Y^T \Psi H^T (H \Psi H^T + R)^{-1} [HE(P_f) + E(PV)]
\]

\[
= Y^T \Psi H^T (H \Psi H^T + R)^{-1} HE(P_f)
\]
where

\[ E(P_f) = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \]

\[ a_1 = a_2 \ldots a_n = \frac{1}{A} \int_{A} \text{cov}(x, x_i) dx \text{ for } i = 1 \ldots n \]

Again the form and the evaluation of the elements in the \((n \times 1)\) vector \(E(P_f)\) has been discussed in Section 2.2.2.

The third term in 2-25 is simply

\[ E(\hat{P}P) = E[\gamma_T f \tilde{y} f_T] \]

\[ = \gamma_T E[f f_T] \gamma \]

\[ = \gamma_T \psi \gamma \]  \hspace{1cm} (2-29)

The last term, \(E[\hat{P}^T]\) can be written

\[ E[\hat{P}^T] = \gamma_T E[f f_T] \gamma \]

\[ = \gamma_T E[f \{\psi_H T (H \psi_H T + R)^{-1} Z\} T] \gamma \]

\[ = \gamma_T E[f Z T \{\psi_H T (H \psi_H T + R)^{-1} T\} \gamma \]

Substituting Equation 2-20 and again using the independence between \(f\) and \(\psi\),

\[ E[\hat{P}^T] = \gamma_T (\psi_H T) \{\psi_H T (H \psi_H T + R)^{-1}\} T \gamma \]

\[ = \gamma_T \psi_H T (H \psi_H T + R)^{-1} [\psi_H T] \gamma \]

\[ = \gamma_T [\psi_H T (H \psi_H T + R)^{-1} H \psi_T] \gamma \]

\[ - 56 - \]
and due to the symmetry of the covariance matrices we then have

$$E[\hat{p}p] = \gamma^T \left[ \psi H^T (H H^T + R)^{-1} \psi \right] \gamma$$  \quad (2-30)

In summary, the cross error term, $\sigma^2_{me}$, becomes, using Equations 2-26, 2-28, 2-29, 2-30,

$$\sigma^2_{me} = \frac{1}{nA} \sum_{i=1}^{n} \int_{A} \text{cov}(x, x_i) \, dx - \gamma^T \psi H^T (H H^T + R)^{-1} H E(Pf)$$

$$- \gamma^T \psi + \gamma^T \left[ \psi H^T (H H^T + R)^{-1} \psi \right] \gamma$$  \quad (2-31)

where

$$E(Pf) = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \frac{1}{A} \int_{A} \begin{bmatrix} \text{cov}(x, x_1) \\ \vdots \\ \text{cov}(x, x_n) \end{bmatrix} \, dx$$

2.5 **Optimization Algorithm**

In the previous sections of this chapter it was established that optimization of the network design would be done by minimizing Equation 2-10

$$0(\xi) = \sigma^2_{\Delta}(\xi) + C_{\Delta} \sum_{i=1}^{m} C(x_i)$$  \quad (2-10)

over all experiments (networks) $\xi$. This implies a search around the possible combinations of the number of observations points, $m$, and the location of these stations, $x_i$, $i = 1, \ldots, m$.

The costs obviously depend on the location and number of stations. The components of the measure of accuracy $\sigma^2_{\Delta}(\xi)$ were derived in the previous three sections and the final expression is: (combining terms
of 2-12, 2-23, and 2-31)

\[ \sigma^2_A(\xi) = \sigma^2_m(\xi) + \sigma^2_e(\xi) + 2\sigma^2_{me}(\xi) \]

\[ = \gamma^T \left[ \psi - \psi \, H^T \left( H \psi H^T + R \right)^{-1} \, H \, \psi \right] \gamma - \]

\[ 2\gamma^T \psi \left[ \psi \, H^T \left( H \psi H^T + R \right)^{-1} \, H \, \psi \right] \gamma - \]

\[ 2\gamma^T \psi \psi \left[ \psi \, H^T \left( H \psi H^T + R \right)^{-1} \, H \, \psi \right] \gamma - \]

\[ \gamma^T \psi + \frac{1}{\Lambda^2} \int_A \int_A \text{cov}(x_1, x_2) \, dx_1 \, dx_2 \]  \quad (2-32)

The above equation is dependent on \( \xi \), the particular network, through the matrix \( H \) as defined in Section 2-3. The optimization procedure should be a search through the various possible forms of \( H \) and select that one which minimizes Equation (2-16).

An exhaustive search would be possible with continuous reevaluation of Equation 2-32. Such a search would imply not only considerable iterations for large values of \( m \) but also mathematical and computational difficulties in the continuous need of inverting the \( m \times m \) matrix, \( (H \psi H^T + R)^{-1} \), appearing in Equation 2-32.

Federov (1972) shows that for consecutive different networks \( H \), the values of the first term in Equation 2-32 can be obtained iteratively by (see Appendix 3 for derivation)
\[ \sum_{(2)} = \sum_{(1)} - \sum_{(1)} \begin{bmatrix} 1 + F^T \sum_{(1)} F \end{bmatrix}^{-1} F^T \sum_{(1)} \]  

(2-33)

where

\[ \sum_{(1)} = \text{previous mean square estimation error with design } H_{(1)} \]

\[ \sum_{(2)} = \text{mean square estimation error with design } H_{(2)} \]

\[ H_{(2)} = \text{experiment definition matrix which differs from } H_{(1)} \]

by a change of location of one station; i.e., one of

the elements in one of the rows is different

\[ I = 2 \times 2 \text{ identity matrix} \]

The matrix \( F \) is a complex \((m \times 2)\) matrix whose structure depends

on the form of the error covariance matrix \( R \). When \( R \) is diagonal

(uncorrelated errors), \( F \) becomes

\[ F = \begin{bmatrix} 1 h^T_{(1)} : 1 h^T_{(2)} \\ \sigma_{(1)} : \sigma_{(2)} \end{bmatrix} \]  

(2-34)

where

\[ i^2 = -1 \]

\[ h^T_{(1)} = \text{transpose of row in matrix } H_{(1)} \text{ where change is desired} \]

\[ h^T_{(2)} = \text{transpose of changed row} \]

\[ \sigma_{(1)} = \text{standard deviation of element of error vector } V \text{ corresponding} \]

\[ \text{to the station to be changed} \]

\[ \sigma_{(2)} = \text{standard deviation of element of error vector } V \text{ which} \]

\[ \text{corresponds to the new station} \]

Veneziano (see Bras, et al. 1975) obtained an equivalent definition
of $F$ if $R$ corresponds to "colored" noise of the following particular form:

$$ R = \begin{cases} \sigma^2_{ij} = \text{constant for all } i = j \\ \sigma^2_{ij} = \text{constant for all } i \neq j \end{cases} $$

Under these conditions $F$ becomes

$$ F_j = \begin{cases} \frac{h}{(1)}_{j} (R^{-1})_{k,j} \left[ (R^{-1})_{j,j} \right]^{1/2} & \text{if the initial design has a station at location } j \text{ and the new design is at } k \\ 0 & \text{otherwise} \end{cases} $$

$$ F_{j2} = \begin{cases} \frac{h}{(2)}_{j} (R^{-1})_{k,j} \left[ (R^{-1})_{j,j} \right]^{1/2} & \text{if the new design has a station at location } j \text{ and the old design was at } k \\ 0 & \text{otherwise} \end{cases} $$

The advantage of using Equation 2-33 is that it only involves an inversion of the $(2 \times 2)$ matrix $(I + F^T \Sigma(1) F)$. The second and third terms of Equation 2-32 can be obtained by using the results of Equation 2-33. Thus, from Equation 2-22 is known that

$$ \Psi - \Sigma_2 = \Psi H^{T}(H(2) \Psi H^{T}(2) + R)^{-1} H(2) \Psi $$

which is basically the third term in 2-32. Similarly with only one initial inversion of the "prior" covariance matrix, $\Psi$, the second term in 2-32, is obtained by:

$$ \Psi H^{T}(H(2) \Psi H^{T}(2) + R)^{-1} H E(P_F) = (\Psi - \Sigma_2) \Psi^{-1} E(P_F) $$

(2-36)

- 60 -
The vector term $E[Pf]$ in the above and the double integral appearing last in (2-32) do not change with $H$ and are evaluated only once using the ideas of Section 2.2.2.

The term, $\gamma^T \psi y$, does not depend on $H$ either and needs to be evaluated only once.

The optimization scheme to minimize equation 2-10 proceeds as follows:

1. A certain discretization of the area (fix $n$) at hand is defined. Non $H$ dependent terms in 2-32 can be computed.
2. Fix $m$, the number of stations to be located.
3. Set an initial design, i.e., define an initial matrix, $H_I$, ($m \times n$).
4. Compute $\sigma^2 (\xi)$ for the initial design and the corresponding initial cost.
5. Compute initial objective function value (Equation 2-10).
6. Going row by row ($i = 1$ to $m$) in $H$ make changes in the position of the non-zero element (station location). Changes are one element at a time and changes of elements which are non zero in other rows are not necessary since they correspond to the same location.
7. For each variation compute the change in objective function value. Select the design $H$, that gives the maximum positive $(O(\xi_1) - O(\xi_2))$ change.
(8) Stop, optimal experiment is at hand if none of the designs
gives a smaller objective function value than the one started
with or if positive change obtained is small enough for user
satisfaction. If none of the above is the case use the $\frac{\sigma^2}{\Delta}$, and objective function value found in step 7 as initial
design and go back to step 6. This time do not change any
element in the row changed in the last pass through the
process and that gave the maximum positive change in objective
function.

(9) If optimal for $m$ stations was found, vary $m$, if desired, and
go back to step 2.

The above optimization algorithm is in summary a search moving
in the direction of highest partial gradient. Due to the dis-
continuous, discrete, form of the problem the gradient is always
defined by a difference and is limited in scope to immediate surround-
ings thus only one variable (station) is changed at a time (partial
gradient).

Fedorov's (1972) experience with this type of guided search was
favorable. Even though limited to one station location change at a
time he experienced very good convergence to global optima. The
author experienced some limited problems where convergence was to
local optima only. Chapter 3 will discuss these instances and offer
a satisfactory solution to them.
Chapter 3

Areal Mean of a Rainfall Event. — A Case Study

The network design approach proposed in Chapter 2 and the corresponding optimization algorithm have been implemented in a Fortran computer program capable of carrying out the design process. As programmed, the network design solution offers various alternatives not specifically mentioned in Chapter 2. Among these various options it is relevant to mention the following:

1) The solution allows constraints on the location of observation stations. The number of allowable station locations, \( l \), may be less than the discretization level, \( n \).

2) In Chapter 2 it was always assumed that the region of observations was coincident with the region over which the areal average is desired. This is not always necessarily the case. It is possible to define a separate or overlapping regions of observations.

3) If desired, the option of allowing more than one station at any particular location (grid subarea) is available. This implies that the matrix \( H \) defining the experiment will be allowed to have equal rows.

4) As implied in the optimization algorithm, the user has the option of obtaining satisfactory but not optimal solutions by defining the minimum change in objective functions he would be satisfied with. Otherwise, the program will iterate...
until a negative change, implying a global optimum (assuming function is uni-modal) is found.

Appendix 4 contains a users manual and copy of the program listing.

3.1 Case Study.

The network design approach described in Chapter 2 was tested in a realistic, made-up, sample problem. The objective was to design a raingage network for estimating the areal mean, of a certain type of convective storm, over a 360 square mile area. The area is a rectangle with 24 miles and 15 miles to its sides.

In an initial approach, the area was divided in a 3x3 grid pattern as shown in Figure 3-1. (Corresponding identification numbers for each grid are given in the figure.) All nine discrete locations are considered possible observation sites. The design was performed for a level I or level II type of network, meaning that the optimization is performed with respect to a loosely defined objective function of the form given in Equation (2-10). The tradeoff coefficient $C_\Delta$ was not known with accuracy, so the generation of a transformation curve is necessary.

The "prior" correlation matrix was assumed to be homogeneous and isotropic and given by Equation 2-18c,

$$\text{cov} (r) = rb K_1 (rb)$$

(2-18c)
Total area = 360 sq. miles

\[ = \text{idemification number corresponding to discrete point defined as the center of the grid} \]

Figure 3-1 Diagram of test area showing discretization and other relevant information for 9 point grid pattern
where, as mentioned before,

\[ r = \text{distance between any two points in space} \]

\[ K_1 = \text{modified Bessel function of the second kind} \]

The Bessel parameter, \( b \), was given a value of 0.13. This value corresponds to that calculated by Rodríguez and Mejia (1974a) using data from Fogel and Duckstein (1969) corresponding to convective storms in Arizona with a maximum center depth of 5 inches and adequate for an area of the order of 400 sq. mi. The point variance corresponding to the above data was 1.32 square inches (Rodriguez and Mejia, 1974a) (diagonal error covariance matrix is used).

The measurement error variance and data collection costs associated with each discrete point are given in Table 3-1. The error variances were assigned such as to resemble variances obtained from data given by Morgan and Lourence (1969) for errors observed in readings of Fisher and Porter, Standard 8-inches and USSR-GGI3000 raingages, as compared to lysimeter measurements. In doing these calculations to obtain the desired ball-park figures it was assumed that the lysimeter readings represented the true mean value, that rainfall processes were stationary and that raingages behave similarly for different storms. The absolute values actually assigned to each point (variations around calculated variances) were chosen to simulate the fact that similar raingages behave differently (have different errors) depending on location, altitude, wind conditions, etc. (Grayman and Eagleson, 1971, Larson and Peck, 1974, Sevruk, 1974).

The costs given vary around the values given by Grayman and Eagleson (1971) for telemetric raingage installations (around $2000.00) and maintenance (from $1000.00 to $2000.00 per year). Different values
of cost were assigned to represent different location conditions.

Table 3-1

Measurement Error Variance and Cost Assigned to Each Grid in 9 Point Discretization

<table>
<thead>
<tr>
<th>Grid Pt. ID</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Variance (x10^{-3})</td>
<td>5.0</td>
<td>5.0</td>
<td>7.0</td>
<td>4.0</td>
<td>5.5</td>
<td>7.0</td>
<td>4.0</td>
<td>5.0</td>
<td>5.5</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000</td>
<td>2500</td>
<td>1500</td>
<td>4000</td>
<td>4000</td>
<td>2500</td>
<td>4100</td>
<td>2500</td>
<td>4000</td>
</tr>
</tbody>
</table>

3.2 Results of the Analysis

The first results to be presented were obtained using the full expression for the measure of accuracy, Equation 2-32, in the objective function defined by 2-10. Table 3-2 summarizes the obtained results for values of $C_A$, the tradeoff coefficient, of $10^{-5}$ and $10^{-6}$. The table shows the breakdown of the measure of accuracy in the three types of mean square errors, namely: estimation, model and cross-error term.

Throughout this work tradeoff coefficients, $C_A$, ranging from $10^{-2}$ to $10^{-8}$ were used. The tradeoff coefficient is the equivalent mean square error per unit change of costs. With mean square error values around 0 to 1 and individual station costs of the order of $2000, the given range of coefficients is found to be adequate in making pos-
Table 3-2
Full Design Results - 9 Grid Points Model

<table>
<thead>
<tr>
<th>Number of Stations</th>
<th>C = 10^-5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function</td>
<td>.210088</td>
<td>.1648847</td>
<td>.138452</td>
<td>.137553</td>
<td>.148998</td>
<td>.184069</td>
<td>.220427</td>
<td>.256327</td>
<td></td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000</td>
<td>6500</td>
<td>7500</td>
<td>11000</td>
<td>12500</td>
<td>16500</td>
<td>20500</td>
<td>24600</td>
<td></td>
</tr>
<tr>
<td>Optimal Design (ID's)</td>
<td>5</td>
<td>4,6</td>
<td>4,3,8</td>
<td>2,4,6,8</td>
<td>2,3,4,6,8</td>
<td>2,3,4,6,8,9</td>
<td>1,2,3,4,6,8,9</td>
<td>1,2,3,5,6,7,8,9</td>
<td></td>
</tr>
<tr>
<td>Mean Square Error: Estimation</td>
<td>.1738550</td>
<td>.0969951</td>
<td>.0602530</td>
<td>.0220466</td>
<td>.0183342</td>
<td>.0131858</td>
<td>.0093836</td>
<td>.0042084</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td></td>
</tr>
<tr>
<td>Cross Term</td>
<td>-.0103395</td>
<td>-.0036824</td>
<td>-.0033733</td>
<td>-.0010658</td>
<td>-.0009079</td>
<td>-.0006890</td>
<td>-.0005243</td>
<td>-.0004530</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>.1700880</td>
<td>.0998847</td>
<td>.0634516</td>
<td>.0275528</td>
<td>.0239982</td>
<td>.0190687</td>
<td>.0154266</td>
<td>.0103274</td>
<td></td>
</tr>
</tbody>
</table>

(Continued)
Continuation Table 3-2

<table>
<thead>
<tr>
<th>C = 10^-6</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function</td>
<td>.174088</td>
<td>.1063847</td>
<td>.0592322</td>
<td>.0385528</td>
<td>.0364982</td>
<td>.0355687</td>
<td>.0354568</td>
<td>.0349274</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000</td>
<td>6500</td>
<td>10500</td>
<td>11000</td>
<td>12500</td>
<td>16500</td>
<td>20600</td>
<td>24600</td>
</tr>
<tr>
<td>Optimal Design (ID's)</td>
<td>5</td>
<td>4,6</td>
<td>4,5,6</td>
<td>2,4,6,8</td>
<td>2,3,4,6,8</td>
<td>2,3,4,6,8</td>
<td>1,2,3,6,7,8,9</td>
<td>1,2,3,5,6,7,8,9</td>
</tr>
<tr>
<td>Estimation</td>
<td>.1738550</td>
<td>.0969951</td>
<td>.0435945</td>
<td>.0220466</td>
<td>.0183342</td>
<td>.0131858</td>
<td>.00914973</td>
<td>.00420837</td>
</tr>
<tr>
<td>Model</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
<td>.0065720</td>
</tr>
<tr>
<td>Cross Term</td>
<td>-.0103395</td>
<td>-.0036824</td>
<td>-.0014342</td>
<td>-.00010658</td>
<td>-.0009079</td>
<td>-.0006891</td>
<td>-.00086490</td>
<td>-.00045295</td>
</tr>
<tr>
<td>Total</td>
<td>.170088</td>
<td>.0998847</td>
<td>.0487322</td>
<td>.0275528</td>
<td>.0239982</td>
<td>.0190687</td>
<td>.0148568</td>
<td>.0103274</td>
</tr>
</tbody>
</table>
sible network costs and variances compatible in magnitudes.

From Table 3-2, one can observe differences in design for the
two different tradeoff coefficients. Different optimal designs are ob-
tained in locating 3 stations and in locating 7 stations. With \( C_\Delta=10^{-6} \)
costs are weighted less and the solution favors minimizing mean square
error with more expensive alternatives. Using the optimal designs given
in Table 3-2 together with Figure 3-1, it is clear that solutions are not
obvious but are consistent and logical. For example, the solution for
3 stations with \( C_\Delta=10^{-6} \) is to place stations in grid points 4, 5, 6.
With \( C_\Delta=10^{-6} \), as was mentioned before, accuracy is given more attention
and the above solution is the one that minimizes the distance between
observed and unobserved locations, a solution which is consistent with the
idea of maximizing accuracy (minimizing mean square error). Arguments
like the above one are dangerous, though, in that they ignore instrument
noise and costs that the program solution takes into account.

Figures 3-2 and 3-3 show graphically part of the results in
Table 3-2. Figure 3-2 is a plot of objective function value vs. number
of stations (it also gives optimal station location) for \( C_\Delta=10^{-5} \). For
that trade-off coefficient, a minimum objective function was achieved
with four stations at locations 2, 4, 6, and 8.

In Figure 3-3 for \( C_\Delta=10^{-6} \), no minimum was achieved since the
plot of objective function vs. number of station was seemingly still
slowly going down at the last point for 8 stations. Such results imply
that the users discretization (in this case 9 points) is not adequate.
A finer discretization would allow more stations and would lead to an
optimal solution in a plot of objective function vs. number of stations.
Figure 3-2 Optimal solutions for $C_{\Delta} = 10^{-5}$ using full measure of accuracy
Figure 3-3 Optimal solutions for $C_{\Delta} = 10^{-6}$ using full measure of accuracy
Following the above idea, the area of interest was divided into an 18 point rectangular grid as shown in Figure 3-4. Again dimensions and identification numbers are shown. The data of this new model corresponds spatially to that of the 9 point grid as shown in Table 3-3.

<table>
<thead>
<tr>
<th>Grid Pt. ID</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Variance (x10^{-3})</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
<td>7.0</td>
<td>7.0</td>
<td>4.0</td>
<td>4.0</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000</td>
<td>4000</td>
<td>2500</td>
<td>2500</td>
<td>1500</td>
<td>1500</td>
<td>4000</td>
<td>4000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grid Pt. ID</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Variance (x10^{-3})</td>
<td>5.5</td>
<td>7.0</td>
<td>7.0</td>
<td>4.0</td>
<td>4.0</td>
<td>5.0</td>
<td>5.0</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000</td>
<td>2500</td>
<td>2500</td>
<td>4100</td>
<td>4100</td>
<td>2500</td>
<td>2500</td>
<td>4000</td>
<td>4000</td>
</tr>
</tbody>
</table>

Table 3-4 shows the network design results with $C_\Delta=10^{-6}$ and the 18 point grid model. Again the total mean square error is broken down to its different components.

The reader should first note the similarity of solutions (same general pattern) between the 9 point and 18 point grid models. Variations only occur in the solutions for 3 and 7 stations. Notice too the similarity of the objective function values.
Figure 3-4 Diagram of test area showing discretization and other relevant information for 16 point grid pattern.

Total area = 360 sq.mi

Identification number corresponding to discrete point defined as the center of the grid.
### Table 3-4
Full Design Results - 18 Grid Points Model, $C_A = 10^{-6}$

<table>
<thead>
<tr>
<th>Number of Stations</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function</td>
<td>.2193950</td>
<td>.0773306</td>
<td>.0590817</td>
<td>.0429734</td>
<td>.0335077</td>
<td>.0343790</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000</td>
<td>6500</td>
<td>10600</td>
<td>11000</td>
<td>20500</td>
<td>24600</td>
</tr>
<tr>
<td>Optimal Des.</td>
<td>9</td>
<td>8</td>
<td>11</td>
<td>2</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>Estimation Model</td>
<td>.1816210</td>
<td>.0699075</td>
<td>.0477573</td>
<td>.0369776</td>
<td>.0117859</td>
<td>.0069176</td>
</tr>
<tr>
<td>Cross Term</td>
<td>-.0036677</td>
<td>-.005186</td>
<td>-.0007173</td>
<td>-.000449</td>
<td>-.0002199</td>
<td>-.0003545</td>
</tr>
<tr>
<td>Total</td>
<td>.1793950</td>
<td>.0708306</td>
<td>.0484817</td>
<td>.0319734</td>
<td>.0130077</td>
<td>.0097790</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Stations</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function</td>
<td>.0348293</td>
<td>.0347238</td>
<td>.0383833</td>
<td>.0447665</td>
<td>.0518020</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>26100</td>
<td>28100</td>
<td>33100</td>
<td>41100</td>
<td>49200</td>
</tr>
<tr>
<td>Optimal Des.</td>
<td>1 3 5 6 8</td>
<td>1 3 5 6 8 11</td>
<td>1 3 4 5 6 8</td>
<td>1 3 4 5 6 8</td>
<td>1 2 3 4 5 6</td>
</tr>
<tr>
<td></td>
<td>11 13 15 17</td>
<td>13 15 16 18</td>
<td>11 12 13 15</td>
<td>10 11 12 13</td>
<td>7 9 11 12 13</td>
</tr>
<tr>
<td></td>
<td>16 18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimation Model</td>
<td>.0074766</td>
<td>.0053941</td>
<td>.0040311</td>
<td>.0023201</td>
<td>.0012462</td>
</tr>
<tr>
<td>Cross Term</td>
<td>-.0001890</td>
<td>-.0002120</td>
<td>-.0001895</td>
<td>-.0000953</td>
<td>-.0000859</td>
</tr>
<tr>
<td>Total</td>
<td>.0087293</td>
<td>.0066238</td>
<td>.0052833</td>
<td>.0036665</td>
<td>.0026020</td>
</tr>
</tbody>
</table>
Figure 3-5 is a plot of the results given in Table 3-4, again objective function value vs. number of stations. This time, the curve corresponding to $C_{\Delta}=10^{-6}$, achieves a minimum at 7 stations.

The results given in Tables 3-2 and 3-4 give considerable insight with respect to the relative importance of the various terms in the total mean square error. For a covariance function of the Bessel type, the cross term component of the mean square error is always negative. This implies a reduction in the sum of estimation and model mean square errors which, in turn, implies that a straight sum of the latter two will produce some double counting of error. As apparent from Equation 2-31, the cross term depends strongly on the degree of discretization and the magnitudes of the event covariance matrix, $\psi$, and the error covariance matrix $R$. The obtained results show that indeed the cross term of the 18 point grid sample is consistently smaller than that of the 9 point grid sample. It can also be concluded, that for the problem at hand, this term is smaller than the estimation error term, the difference being close to an order of magnitude.

The mean square model error behaves as discussed in Chapter 2, it is smaller for finer discretization and it is constant once the discretization is fixed. It remains smaller than the estimation error term except for large numbers of stations when they become of the same order. Relative to the cross term, the model error term is larger (in absolute value) except in the case of very small number of stations; i.e. one station in the case of the problem at hand. Actually, there is a region where the cross and model terms are of the same order and therefore tend to cancel their effect on the total mean square error.
$C_\Delta = 10^{-6}$

(•,•) Station location

Figure 3-5. Optimal solutions for $C_\Delta = 10^{-6}$ using full measure of accuracy in 18 points grid.
From the above comments, the author concludes that the estimation term of the total mean square error dominates the design procedure for most cases of interest, i.e. cases where the number of stations to be allotted is small relative to the area to be monitored. For large number of stations, the cross term remains small but the model term approaches or may exceed the value of the estimation term. Even in this case, the designing with only the estimation error criteria may be sufficient if the grid pattern over the area is fixed (then the model error term is constant). The validity of this assumption not only reduces computation effort but eliminates any restrictions of isotropy or homogeneity in the process covariance. As it was said in Chapter 2, the estimation error term accepts any matrix covariance formulation.

With the above comments in mind, the next section will present the full network design exercise for the area defined in 3.1 using the mean square estimation error as the sole measure of accuracy.

3.3 Solution to Example Problem—Estimation Error Criteria

In the previous section it was concluded that network design based only on the mean square estimation error, $E[\hat{p} - p]^2$, seemed adequate for most cases of interest. This section presents the complete network design exercise for the area given in Figure 3-1 and described in Section 3-1. The correlation matrix of the storm event was generated with a Bessel function of the type given in Equation 2-18c. The function parameter, $b$, again had a value of 0.13. Table 3-5 presents the design results and gives values of mean square estimation error (M.S.E.E.), cost and objective function for the various designs.

- 78 -
<table>
<thead>
<tr>
<th>Number of Stations</th>
<th>Tradeoff Coefficient $C_\Delta$</th>
<th>(10^{-8})</th>
<th>(10^{-7})</th>
<th>(10^{-6})</th>
<th>(10^{-5})</th>
<th>(10^{-4})</th>
<th>(10^{-3})</th>
<th>(10^{-2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M.S.E.E.</td>
<td>0.170395</td>
<td>0.170395</td>
<td>0.170395</td>
<td>0.170395</td>
<td>0.268429</td>
<td>0.385661</td>
<td>0.385661</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>4000.00</td>
<td>4000.00</td>
<td>4000.00</td>
<td>4000.00</td>
<td>2000.00</td>
<td>1500.00</td>
<td>1500.00</td>
</tr>
<tr>
<td></td>
<td>Obj.Fct.</td>
<td>0.170435</td>
<td>0.170795</td>
<td>0.174395</td>
<td>0.210395</td>
<td>0.468429</td>
<td>1.88566</td>
<td>15.3857</td>
</tr>
<tr>
<td>2</td>
<td>M.S.E.E.</td>
<td>0.0950643</td>
<td>0.0950643</td>
<td>0.0950643</td>
<td>0.0950643</td>
<td>0.152369</td>
<td>0.152369</td>
<td>0.152369</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>6500.00</td>
<td>6500.00</td>
<td>6500.00</td>
<td>6500.00</td>
<td>3500.00</td>
<td>3500.00</td>
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</tr>
<tr>
<td></td>
<td>Obj.Fct.</td>
<td>0.0951293</td>
<td>0.0957143</td>
<td>0.105164</td>
<td>0.1600643</td>
<td>0.502369</td>
<td>3.65237</td>
<td>35.1224</td>
</tr>
<tr>
<td>3</td>
<td>M.S.E.E.</td>
<td>0.0427269</td>
<td>0.0427269</td>
<td>0.0427269</td>
<td>0.0590538</td>
<td>0.090689</td>
<td>0.090689</td>
<td>0.090689</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>10500.00</td>
<td>10500.00</td>
<td>10500.00</td>
<td>7500.00</td>
<td>6000.00</td>
<td>6000.00</td>
<td>6000.00</td>
</tr>
<tr>
<td></td>
<td>Obj.Fct.</td>
<td>0.0437769</td>
<td>0.0532269</td>
<td>0.134054</td>
<td>0.690689</td>
<td>6.09069</td>
<td>6.09069</td>
<td>6.09069</td>
</tr>
<tr>
<td></td>
<td>Design</td>
<td>(4,6)</td>
<td>(4,6)</td>
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<td>(1,2,3,4, 6,7,8,9)</td>
<td>(1,2,3,4, 6,7,8,9)</td>
</tr>
</tbody>
</table>
Since the value of the tradeoff coefficient is not exactly known, results are presented for \( C_\Delta = 10^{-8}, 10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2} \). The reason for this range of weights was explained at the beginning of Section 3-2. Notice that weights in the lower range \((10^{-8})\) give very little importance to cost while in the higher range \((10^{-2})\) cost becomes the dominant factor. Additionally, Table 3-6 shows results obtained in locating 3 stations using trade-off coefficients varying from \(1 \times 10^{-5}\) to \(9 \times 10^{-5}\).

Figures 3-6, 3-7 and 3-8 show the results in plots of objective function value versus number of stations. It must be emphasized that these are plots of optimal solutions, for each number of stations the plotted point represents the design giving the minimum objective function value. The observation locations are given in parenthesis.

It is clear that for trade-off coefficients, \( C_\Delta \), between \(10^{-2}\) to \(10^{-4}\), the optimal solution is to locate only one station; for \( C_\Delta \)'s of \(10^{-2}\) and \(10^{-3}\), a station at location 3, for \( C_\Delta = 10^{-4} \), a station at location 8. A decision maker who can define his \( C_\Delta \) in this range of values has definite design guidelines.

For a trade-off, \( C_\Delta \), of \(10^{-5}\) Figure 3-7 shows that the minimum objective function value is achieved with four stations at locations 2, 4, 6 and 8.

With tradeoff coefficients of \(10^{-6}, 10^{-7}\) and \(10^{-8}\), the decision is unclear since no definite minimum objective function is achieved. As discussed in Section 3-2, this implies that the area of interest should be subdivided into a finer grid. This was done in the previous section (using the full measure of accuracy) and the results...
Table 3-6

Results for the Location of Three Stations

<table>
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<tr>
<th>Tradeoff Coefficient</th>
<th>Optimal Design</th>
<th>Cost ($)</th>
<th>M.S.E.E.</th>
<th>Object. Function</th>
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<td>2.0x10^{-5}</td>
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</tbody>
</table>
Figure 3-6  Objective function versus number of stations for $C_\Delta = 10^{-2}$ and $C_\Delta = 10^{-3}$
Figure 3-7 Objective function versus number of stations for $C_\Delta = 10^{-5}$ and $C_\Delta = 10^{-4}$.
Figure 3-8 Objective function versus number of stations for $C_\Delta = 10^{-8}$, $C_\Delta = 10^{-7}$ and $C_\Delta = 10^{-6}$.
have been shown in Figures 3-3 and 3-5. A minimum of 7 stations at locations 3, 5, 7, 9, 12, 15, and 17 (see Figure 3-5) was obtained for $C_A = 10^{-6}$.

An alternative presentation of the results in Table 3-5 is a plot of mean square error of estimation vs. cost. This corresponds to the previously mentioned transformation curve. Figure 3-9 is such a curve for the example problem at hand. It is constructed by plotting the optimal solutions obtained for each number of stations. Thus, this curve eliminates non-optimal points (points to the northeast relative to others). It must be kept in mind that even though Figure 3-9 is referred to as a curve it consists of a step function of discrete points. The decision maker with completely undefined utility function can approach the design problem by entering the given aggregate curve with a budget or accuracy constraint. Given one, the curve will provide the optimal design for that constraint and the corresponding value of the alternate objectives. Used together with Table 3-5, the decision maker can obtain the inherent tradeoff he is giving to cost and accuracy and the value of the linear objective function with that tradeoff. His relative position relative to other possible solutions is then also available. Since Figure 3-9 is a step function, the user must enter the graph through the smaller plotted point closest to his constraints.

3.4 Sensitivity Analysis

The network design method previously presented depends on two basic inputs; the prior covariance of the process to be monitored and the costs of observations. In usual situations, most input uncertainties
Figure 3-9  Transformation curve of example problem
are on the definition of the covariance structure.

Sensitivity of the network design method to the covariance definition was studied with the previously discussed example problem by varying the following:

1) value of the point variance
2) covariance function parameter
3) form of covariance function.

3.4.1 Effects of Varying Point Variance

With a fixed tradeoff coefficient of $C_\Delta=10^{-5}$, the example problem previously described was solved with a Bessel covariance function but with point variance values of 0.5, 2.5 and 5.0 (instead of 1.32) square inches. Results are given in Table 3-7.

For point variances of 2.5 and 5.0, the results show that the objective function achieves a minimum with four stations at locations 2, 4, 6 and 8. This is the same solution as was obtained with $\sigma_p^2 = 1.32$ as seen in Table 3-5. In fact, the only difference produced in the network by larger point variances occurs in the three station case. With a larger point variance, the method opted for a more expensive but more accurate solution.

The results obtained with a point variance of $\sigma_p^2 = 0.5$ differed considerably from those with $\sigma_p^2 = 1.32$. The minimum objective function value was 0.0914 obtained with two stations at locations 2 and 8. Designs equal to the solution of the 1.32 point variance problem were obtained in locating 1, 5, 6 and 7 stations. When dealing with low point variance, optimization is dominated by costs, resulting in general in
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(Continued.)
(Continuation)

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</table>
network schemes which heavily weight the low cost principle, since the
natural variability of the process is not very large to start with.

Generalization from the above limited sensitivity analysis
is difficult but several concepts can be stated with a fair amount of
confidence at least for the example at hand.

1) For a tradeoff coefficient, $C = 10^{-5}$, accuracy considerations
are very important. While more than doubling point variance caused no
major design changes, implying existing accurate solutions, halving it caused
serious modifications. At this point costs became critical.

2) When costs are weighted more, i.e. $C = 10^{-4} \ldots 10^{-2}$, solutions
are more sensitive to $\sigma_p^2$ increases than to point variance reductions.

3) When costs are weighted less, i.e. $C = 10^{-6} \ldots 10^{-8}$, solutions
are fairly insensitive to $\sigma_p^2$ variations unless dealing with
very large decreases.

4) From the above comments it seems that $C = 10^{-5}$ is the trade-
off coefficient which gives the most sensitive solutions to changes in
the point variance of the precipitation process. As discussed before,
point variance decreases of more than 100% can cause noticeable changes
in results. The author speculates though that minor changes will occur
with more reasonable variations.

3.4.2 Sensitivity to Covariance Function Parameter

The original example problem had a Bessel covariance func-
tion with a parameter $b = 0.13$. Sensitivity tests were performed
with $b = 0.06, 0.1, 0.2$ and $0.26$. Results appear in Table 3-8.
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<td></td>
</tr>
<tr>
<td></td>
<td>b=0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>M.S.E.</td>
<td>0.204691</td>
<td>0.135345</td>
<td>0.080954</td>
<td>0.0392072</td>
<td>0.0322473</td>
<td>0.023486</td>
<td>0.0162927</td>
<td>0.0071895</td>
<td></td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000.00</td>
<td>6500.00</td>
<td>8500.00</td>
<td>11000.00</td>
<td>12500.00</td>
<td>16500.00</td>
<td>20500.00</td>
<td>24600.00</td>
<td></td>
</tr>
<tr>
<td>Obj.Funct.</td>
<td>0.244691</td>
<td>0.200345</td>
<td>0.165954</td>
<td>0.149207</td>
<td>0.1572473</td>
<td>0.188486</td>
<td>0.221293</td>
<td>0.253219</td>
<td></td>
</tr>
<tr>
<td>Design</td>
<td>(5)</td>
<td>(4,6)</td>
<td>(4,6,8)</td>
<td>(2,4,6,8)</td>
<td>(2,3,4,6)</td>
<td>(2,3,4,6,8)</td>
<td>(1,2,3,4,6,8)</td>
<td>(1,2,3,4,6,7,8,9)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b=0.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>M.S.E.</td>
<td>0.20465</td>
<td>0.144439</td>
<td>0.0946131</td>
<td>0.0526997</td>
<td>0.0429700</td>
<td>0.0315463</td>
<td>0.0215734</td>
<td>0.00985107</td>
<td></td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000.00</td>
<td>6500.00</td>
<td>8500.00</td>
<td>11000.00</td>
<td>12500.00</td>
<td>16500.00</td>
<td>20500.00</td>
<td>24600.00</td>
<td></td>
</tr>
<tr>
<td>Obj.Funct.</td>
<td>0.24465</td>
<td>0.209439</td>
<td>0.179613</td>
<td>0.162699</td>
<td>0.167970</td>
<td>0.196546</td>
<td>0.226573</td>
<td>0.255851</td>
<td></td>
</tr>
<tr>
<td>Design</td>
<td>(5)</td>
<td>(5,6)</td>
<td>(2,4,6,8)</td>
<td>(2,3,4,6,8)</td>
<td>(1,2,3,4,6,8)</td>
<td>(1,2,3,4,6,7,8,9)</td>
<td>(1,2,3,4,6,7,8,9)</td>
<td>(1,2,3,4,6,7,8,9)</td>
<td></td>
</tr>
</tbody>
</table>
The tradeoff coefficient in this analysis is $C_A = 10^{-5}$.

The results of Table 3-8 can be discussed nearly in the same context as those in the previous section. Small parameters $b$ imply high spatial correlations which in turn imply good extrapolation ability (less M.S.E.E.), effectively reducing variance. The inverse is similarly true.

Major changes in design are observed when the value of $b$ is reduced to 0.06 from the previously used 0.13. A minimum objective function is achieved with two stations at sites 2 and 8. A different solution is also found for four stations. As for the low point variance case, the optimal results are relatively less costly implying a dominating cost factor due to the increased extrapolation ability.

With $b = 0.1$, individual solutions for the various numbers of stations do not differ from the original example. The minimum objective function, though, is achieved with three instead of four stations at locations 3, 4 and 8.

For larger values of $b = 0.2$ and 0.26, minimum objective function values are achieved with four stations at 2, 4, 6 and 8 as originally with $b = 0.13$. Individual differences occur, though, with 3 stations and 2 and 3 stations respectively with solutions favoring higher cost - more accurate alternatives.

As in Section 3.4.1, it can be concluded that most of the appreciable changes occur with reductions in $b$ implying strong accuracy considerations when $C_A = 10^{-5}$. Variations in $C_A$ should result in the same situations discussed in the previous section. With close to 25% reduction in $b' (=0.1)$, no individual optimization changes occurred.
and the minimum objective function only shifted from four to three stations.

### 3.4.3 Sensitivity to Covariance Function Form

All the previously discussed tests were based on a covariance described by a Bessel function of the form given by 2-18c. It was seen in Chapter 2 that the covariance function form implied considerable differences in the mean square model error. Sensitivity of the design procedure to this different covariance function form warrants study.

The design procedure of the 9 point grid example problem was repeated using an exponential and quadratic exponential expressions (see Equations 2-18a and 2-18b) for the covariance function. The functions have parameters 0.08 and 0.009 respectively.

These values give the same correlation radius, i.e. that distance at which the function becomes 0.5, as the Bessel function with a 0.13 parameter. Results of the design with the new covariance forms (point variance is kept at 1.32) and both with mean square estimation error and full accuracy criteria are given in Table 3-9. The tradeoff coefficient, $C_{\Lambda'}$, was assumed to be $10^{-5}$.

Interestingly enough, the results obtained using full measure of accuracy and the limited estimation error criteria were the same, differing only very slightly for the design for 8 stations with a single exponential covariance function. These results further strengthen the comments in Section 3.2 with respect to the use of the mean square estimation error as measure of accuracy.

Furthermore, the form of the covariance function seems to
Table 3-9
Sensitivity to Correlation Function Form, $C_o=10^{-5}$

**Single Exponential $\alpha=0.08$ - Full Measure of Accuracy**

<table>
<thead>
<tr>
<th>Number of Stations</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function</td>
<td>0.253891</td>
<td>0.193850</td>
<td>0.169204</td>
<td>0.165089</td>
<td>0.172103</td>
<td>0.204729</td>
<td>0.237521</td>
<td>0.270848</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000.00</td>
<td>4500.00</td>
<td>7500.00</td>
<td>11000.00</td>
<td>12500.00</td>
<td>16500.00</td>
<td>20500.00</td>
<td>24600.00</td>
</tr>
<tr>
<td>Optimal Design</td>
<td>(5)</td>
<td>(2,8)</td>
<td>(3,4,8)</td>
<td>(2,4,6,8)</td>
<td>(2,3,4,6,8)</td>
<td>(1,2,3,4,6,8)</td>
<td>(1,2,3,5,6,7,8,9)</td>
<td></td>
</tr>
<tr>
<td>Mean Sq.Error:</td>
<td>0.0188939</td>
<td>0.0188939</td>
<td>0.0188939</td>
<td>0.0188939</td>
<td>0.0188939</td>
<td>0.0188939</td>
<td>0.0188939</td>
<td>0.0188939</td>
</tr>
<tr>
<td>Model Estimation</td>
<td>0.218001</td>
<td>0.146091</td>
<td>0.0827027</td>
<td>0.0397896</td>
<td>0.0315609</td>
<td>0.0231323</td>
<td>0.0148909</td>
<td>0.00664084</td>
</tr>
<tr>
<td>Cross Term</td>
<td>-0.0230048</td>
<td>-0.0161356</td>
<td>-0.0073926</td>
<td>-0.00438272</td>
<td>-0.0033523</td>
<td>-0.0022968</td>
<td>-0.0012642</td>
<td>-0.00068653</td>
</tr>
<tr>
<td>Total</td>
<td>0.213891</td>
<td>0.148854</td>
<td>0.0972041</td>
<td>0.0543009</td>
<td>0.0599629</td>
<td>0.0397293</td>
<td>0.0325721</td>
<td>0.0248482</td>
</tr>
</tbody>
</table>

**Quadratic Exponential $=0.009$ - Full Measure of Accuracy**

<table>
<thead>
<tr>
<th>Number of Stations</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective Function</td>
<td>0.166281</td>
<td>0.1367833</td>
<td>0.117889</td>
<td>0.117824</td>
<td>0.132077</td>
<td>0.169774</td>
<td>0.209076</td>
<td>0.248072</td>
</tr>
<tr>
<td>Cost ($)</td>
<td>4000.00</td>
<td>6500.00</td>
<td>7500.00</td>
<td>11000.00</td>
<td>12500.00</td>
<td>16500.00</td>
<td>20500.00</td>
<td>24600.00</td>
</tr>
<tr>
<td>Optimal Des.</td>
<td>(5)</td>
<td>(4,6)</td>
<td>(3,4,8)</td>
<td>(2,4,6,8)</td>
<td>(2,3,4,6,8)</td>
<td>(1,2,3,4,6,8)</td>
<td>(1,2,3,5,6,7,8,9)</td>
<td></td>
</tr>
<tr>
<td>Mean Sq.Error:</td>
<td>0.128034</td>
<td>0.0721541</td>
<td>0.0445916</td>
<td>0.0068341</td>
<td>0.0054705</td>
<td>0.0035816</td>
<td>0.0030158</td>
<td>0.0013903</td>
</tr>
<tr>
<td>Model Estimation</td>
<td>0.000945</td>
<td>0.0009459</td>
<td>0.0009459</td>
<td>0.0009459</td>
<td>0.0009459</td>
<td>0.0009459</td>
<td>0.0009459</td>
<td>0.0009459</td>
</tr>
<tr>
<td>Cross Term</td>
<td>-0.002698</td>
<td>-0.0025728</td>
<td>-0.0026482</td>
<td>+0.0007949</td>
<td>+0.0006605</td>
<td>+0.0002463</td>
<td>+0.0001145</td>
<td>-0.0002640</td>
</tr>
<tr>
<td>Total</td>
<td>0.126281</td>
<td>0.0705272</td>
<td>0.0428894</td>
<td>0.0078242</td>
<td>0.0070767</td>
<td>0.0047777</td>
<td>0.0040763</td>
<td>0.0020723</td>
</tr>
</tbody>
</table>
Table 3-9 (Cont.)

Single Exponential $a=0.08$ - Estimation Error Accuracy Criteria

<table>
<thead>
<tr>
<th>Number of Stations</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.S.E.E. Cost</td>
<td>0.213663</td>
<td>0.143184</td>
<td>0.081057</td>
<td>0.0389978</td>
<td>0.0309328</td>
<td>0.0226719</td>
<td>0.0145945</td>
<td>0.00630513</td>
</tr>
<tr>
<td>Obj.Funct. Optimal Des.</td>
<td>(5)</td>
<td>(2,8)</td>
<td>(3,4,8)</td>
<td>(2,4,6,8)</td>
<td>(2,3,4,6,8)</td>
<td>(2,3,4,6,8,9)</td>
<td>(1,2,3,4,6,8,9)</td>
<td>(1,2,3,4,6,7,8,9)</td>
</tr>
</tbody>
</table>

Quadratic Exponential $a^2=0.009$ - Estimation Error Accuracy Criteria

<table>
<thead>
<tr>
<th>Number of Stations</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.S.E.E. Cost</td>
<td>0.125486</td>
<td>0.0707183</td>
<td>0.0437043</td>
<td>0.00596235</td>
<td>0.00536162</td>
<td>0.00351031</td>
<td>0.00136270</td>
<td>0.00136270</td>
</tr>
<tr>
<td>Obj.Funct. Optimal Des.</td>
<td>(5)</td>
<td>(2,6)</td>
<td>(3,4,8)</td>
<td>(2,4,6,8)</td>
<td>(2,3,4,6,8)</td>
<td>(2,3,4,6,8,9)</td>
<td>(1,2,3,4,6,8,9)</td>
<td>(1,2,3,4,6,7,8,9)</td>
</tr>
</tbody>
</table>
have relatively minor effects in this example. Comparing the full measure of accuracy results of Table 3-9 with those of Table 3-2, it is observed that the single exponential differed from the Bessel solution only in the location of 2 stations, the former gave a 2,8 solution, the latter a 4,6 solution. The quadratic exponential only differed in locating 8 stations; i.e., solution 1, 2, 3, 4, 6, 7, 8, 9, instead of 1, 2, 3, 5, 6, 7, 8, 9. Most important though is that for all covariance functions a minimum objective function was achieved with 4 observations at sites 2,4,6 and 8.

Similarly, the comparison of the estimation error criteria results with those of Table 3-5 yielded the same differences as above in the location of 2 stations for the single exponential covariance function and no differences between the quadratic exponential and Bessel solutions. Again an objective function minimum was achieved with 4 stations in sites 2,4,6 and 8.

The above comments have considerable importance in light of comments by Rhenals et al (1974), who argues that there is considerable difficulty in defining a unique covariance function for a given set of data.

A curious result of Table 3-9 is the fact that the cross term in the mean square error expression for the quadratic exponential covariance function takes positive values in some cases. As mentioned in Section 3.2, the cross term in the mean square error expression is negative when using Bessel or single exponential representation for covariance. This implies a negative correlation between estimation error and model error and therefore a reduction from the total variance if
they were independent. Using a quadratic exponential covariance function, small positive cross terms are observed. These imply a positive correlation between model and estimation errors. This unusual behavior adds to the list of those discussed in Chapter 2 with respect to the quadratic exponential covariance function. A simple logical explanation is not apparent. It should be related, though, to the fact that due to the unusual extrapolation ability of this covariance function the error in spatial estimation (extrapolation) becomes very similar to the model error where all points are assumed perfectly known.

Before ending this section, a brief comment is appropriate to explain the seemingly inconsistent difference between the mean square estimation error of the full and partial measure of accuracy solutions in Table 3-9 (also between Tables 3-2 and 3-5). The small differences are due to different degree of accuracy in the input data used.

3.5 Brief Comments and Comparison to Other Recent Network Design Schemes.

Rodriguez-Iturbe and Mejia (1974a) attacked the problem of finding the necessary number of stations to obtain a certain level of accuracy. Their solutions involved the following conditions:

1) Sampling techniques are random or stratified random. Random implies complete aliatory location of stations over the whole region. Stratified sampling implies random location of observations within fixed specified strata (see Rodriguez-Iturbe and Mejia, 1974a).

2) Their measure of accuracy is the mean square error defined
M.S.E. = \[ E \left[ \frac{1}{A} \int_A f(x) dx - \frac{1}{n} \sum_{i=1}^n f(x_i) \right]^2 \] (3-1)

3) The covariance of the process must be a functional form and the process is isotropic and homogeneous.

4) No consideration is given to instrument error or cost.

Under the above conditions they derived expressions for the mean square error of the two mentioned sampling schemes. Figures 3-10 and 3-11 give the graphical representation of the obtained solutions for a covariance function of the Bessel type. The M.S.E. is given as a function of the undimensional area \( Ab^2 \) and the number of stations, \( n \). A network designer would fix his desired measure of accuracy and just read-off the needed number of stations.

The network design procedure proposed in this work has some advantages over the above in that it gives a spatial arrangement of stations besides the optimal number and it does so considering measurement noise and costs. Also in giving a specific network geometric design, the herein proposed method obtains smaller mean square error than with a random or stratified random criteria. This can be easily seen by comparing results of Table 3-2 with those in Figures 3-10 or 3-11. For example, with 3 stations and a \( 10^{-5} \) weight, the proposed method obtains a total mean square error of about 0.06 including instrument error. This would require a variance reduction factor of about 0.043(0.06/1.32) which from Figures 3-10 and 3-11 would require on the order of 12 stations with a random sampling approach and 5 stations with a stratified sampling system (\( Ab^2 \approx 6 \)).

In his more recent work, Lenton et al (1974) suggest
Figure 3-10 Variance reduction factor due to spatial sampling with random design. Used in the estimation of areal mean of rainfall event with 
\[ r(v) = bv K_1(bv). \] (Rodriguez and Mejia, 1974)
Figure 3-11 Variance reduction factor due to spatial sampling with stratified design. Used in the estimation of areal mean of rainfall event with $r(v) = bv K(v)$. (Rodriguez and Mejia, 1974)
that Rodríguez-Iturbe and Mejía's work could be extended by distributing the obtained number of stations, n, as to minimize cost. Nevertheless it is necessary to point out that if this is done, the resulting spatially defined network will have a corresponding mean square error differing from the one given by Rodríguez and Mejía. The designer will again find himself with no knowledge of the accuracy of his final result. It may be argued that the resulting mean square error will be smaller than the previously predicted. This will be generally true unless the cost minimization yields solutions involving large, unaccounted measurement errors.

This work by Lenton et al (1974) approaches the network design problem in two stages: a data collection stage and a data analysis stage.

The first stage is a multi-dimensional, spatially continuous, non-linear mathematical optimization of an objective function of the form of Equation (2-6) where the measure of accuracy is given by a mean square error, given by Equation (2-12), modified by an added linear term of measurement error variance. Equation (2-12) then looks like:

\[
\text{M.S.E.} = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij} \text{cov} (x_i, x_j) + \frac{1}{A^2} \int_A \int_A \text{cov} (x_1, x_2) \, dx_1 \, dx_2 \\
- \frac{2}{nA} \sum_{i=1}^{n} \int_A \text{cov}(x_1, x_i) \, dx + \sum_{i=1}^{n} \rho_i \sigma_v^2 (x_i)
\]

(3-2)

where

\[
\sigma_v^2 (x_i) = \text{variance of measurement error at point } x_i.
\]
The optimization of the objective function is done, together with Equation 3-2, continuously in space to find for a given \( n \) and the optimal locations, \( x_i \), \( i = 1 \ldots n \). The value of \( n \) is varied and for each optimization the weights \( \rho_i \)'s are fixed, usually at \( 1/n \).

The second stage, the data analysis stage, consists of optimizing the weights, \( \rho_i \), for the obtained network. This is done by one of the available interpolation methods (see Lenton et al, 1974, Chapter 5).

This second stage is necessary in Lenton's approach because of the mathematical difficulties in optimizing, in one path, with respect to \( n, x_i \), and \( \rho_i \) (\( i = 1 \ldots n \)). The two stage approach, although realistic and simple in practice, will not necessarily lead to a global optimum in the design.

Lenton's methodology requires a functional definition of the covariance. Conditions of isotropy and homogeneity are theoretically not required but no attempt has yet been made to develop the full mathematical formulation.

A comparison of the design method proposed in this work with that of Lenton can be summarized in the following comments:

1) The proposed work, using the estimation error measure of accuracy, does not require homogeneous and isotropic assumptions in the rainfall process. Also only a matrix (discrete) covariance form is needed instead of a continuous functional form.

2) The spatial continuity of Lenton's approach requires continuous definition of costs and measurement errors. The
highly discrete, discontinuous nature of these variables makes such a "surface fitting" difficult, limiting and maybe undesirable. The hereby suggested formulation allows considerable flexibility in costs and measurement errors. Even correlated measurement errors could be considered with minor technique modification.

3) As pointed out by Lenton (1974), there are two aspects to network design: (1) the design of a new network, and (2) the modification of an existing network. "The theoretically correct way to treat the problem [of modification] would be to reflect the existence" of raingages in the cost function, i.e., existing raingages could have a negative cost to take into account - the cost of dismantling. The methodology presented here, maybe with some constraints, would permit the simultaneous addition or subtraction of raingages. This is a feasible approach in the discrete multivariate approach.

4) Another difference lies in the fact that optimization is done in one step yielding global optima for the problem at hand in terms of the number of stations, location and appropriate weights.

Combining Equations 2-21 and 2-4 (for zero mean process) results in:

$$\hat{P} = \gamma^T \Sigma H^T R^{-1} Z$$

(3-3)

where $\gamma$ and $R$ are given and previously defined in Chapter 2, and $\Sigma$ and
result from the optimization. It is clear that the weights on the observations, vector $Z$, are given by:

$$
\sum H^T R^{-1}
$$

(3-4)

These weights are those yielding the minimum mean square error (for a linear estimate).

The reader may correctly argue that in reality the storm is a non-zero mean process and so the correct weights are given by Equation 2-24b, which is repeated here:

$$
\hat{f} = m_f + \psi H^T (H \psi H^T + R)^{-1} [Z - H m_f]
$$

(3-5)

Equation 3-5 is a function of $m_f$ which is not generally known. Fortunately for relatively small error covariance matrix $R$, which is the usual case in network design, Equation 3-5 simplifies to:

$$
\hat{f} = m_f + \psi H^T (H \psi H^T + R)^{-1} Z - \psi H^T (H \psi H^T + R)^{-1} H m_f
$$

for small $R$

$$
\approx m_f + \psi H^T (H \psi H^T + R)^{-1} Z - \psi H^T (H \psi H^T)^{-1} H m_f
$$

$$
\approx \psi H^T (H \psi H^T + R)^{-1} Z
$$

(3-6)

Since $\hat{P} = \gamma^T \hat{f}$ it can be shown that 3-6 is equal to Equation 3-3.
3.6. The Estimation of the Covariance Function of Total Storm Depth

The definition of the process covariance matrix, \( \psi \), (Equation 2-2), and the corresponding covariance function, \( \text{cov}(x_1,x_2) \), is a necessary and difficult task in the implementation of this work. Although acknowledging the importance of this exercise, the author will limit his comments to a brief review of works on the topic by Lenton et al, (1974) and especially the very complete work by Rhenals et al, (1974).

The covariance function,

\[
\text{cov}(x_1,x_2) = E \left[ f(x_1) f(x_2) \right] - m(x_1)m(x_2)
\]

must be defined for the storms of interest to the network designer. Such definition can be obtained from either theoretical knowledge of the process at hand or from existing data.

An example of the first approach is given by Whittle (1954). He derives that a process with a correlation function of the form \( e^{-\alpha r} \) obeys the following expression:

\[
\left[ \frac{\partial^2}{\partial x^2} \right]^2 + \left[ \frac{\partial^2}{\partial y^2} \right]^2 - a^2 \gamma_4 \xi(x,y) = \epsilon(x,y)
\]

where \( \xi(x,y) \) is the process of interest and \( \epsilon(x,y) \) is a known stochastic process which obeys certain measurability conditions.

Eagleson (1967) used a similar idea to develop correlation functions for convective and cyclonic storms. He defined a model for the average areal distribution of rainfall (there are several of these available for various locations and storm types, see Eagleson (1970))
and obtained the correlation based on the depth values given by the model.

The covariance matrix or function can alternatively be derived directly from available data in the area of interest or data from similar areas. Lenton et al, (1974), discusses the advantages and disadvantages in the estimation process of the correlation function when using one storm, assuming ergodicity, or various storms, assuming they come from the same process. It is apparent that the multirealization alternative is advantageous if enough events are available. In many situations, though, the decision maker is forced to use the one-realization approach. In such situations the various estimators are given by:

\[ \hat{m} = \frac{1}{n} \sum_{i=1}^{n} f(x_i) \]  
(3-9)

\[ s^2 = \frac{1}{n} \sum_{i=1}^{n} \left[ f(x_i) - \hat{m} \right]^2 \]  
(3-10)

and

\[ \text{cov}(\Delta x) = \frac{1}{n} \sum_{i=1}^{n} \left( f(x_i) - \hat{m} \right) \left( f(x_i + \Delta x) - \hat{m} \right) \]  
(3-11)

where

- \( f(x_i) \) = storm depth recorded at point \( x_i \)
- \( n_\Delta \) = number of pairs of raingages separated a distance \( x \)
- \( S^2 \) = estimated point variance
- \( \hat{m} \) = estimated mean

Unfortunately, the above estimator for the covariance re-
quires regularly spaced data which is seldom the case. Faced with such situations, Rhenals et al. (1974), assumed that total rainfall depths follow a certain pattern that can be represented by a mathematical surface; this is essentially the same idea followed by Eagleson (1967). Rhenals et al. (1974), used a surface fitting technique based on multiquadric equations and estimated the rainfall depth on a uniform square grid. Instead of assuming a homogeneous process with a mean given by Equation (3-5), Rhenals calculated a regional periodic mean by using double fourier analysis on the fitted multiquadric surface. The covariance is then calculated using the easily obtained residuals.

One of the dangers of the above approach is that the obtained covariance may be strongly influenced by the type of surface fitting utilized (Lenton et al., 1974; Rhenals et al., 1974).

If needed, a covariance function can be fitted to the data based covariance estimated by the previously discussed methods. The questions to be answered in this step are the form and parameters of the needed function.

Parameters are usually estimated by least square fitting (Rhenals et al., 1974) or by preserving correlation at a particular distance (Rodriguez-Iturbe and Mejia, 1974a).

Discrimination between functional forms based on data is extremely difficult except on few exceptionally clear cut cases (Rodriguez-Iturbe and Mejia, 1974a; Rhenals et al., 1974b). In such cases, where data based form discrimination is not possible, it is suggested to define a form based on theoretical arguments and knowledge of the process at hand. On such basis Rodriguez-Iturbe and Mejia
propose the use of the Bessel type correlation function to statistically represent storm depths. Their conclusion agrees with that of Whittle (1954) which favors the Bessel expression over any other exponential type alternative. Fortunately it seems that covariance function form is not a major factor in network design as was argued in a previous section of this work.

3.7 General Comments

3.7.1 Convergence Problems

As discussed in Section 2-5, the optimization algorithm suggested by Federov (1972) and used in this work is based on first order differences, that is, only one station at a time is varied in calculating the largest gradient. Due to this limitation, the author experienced several instances of failure to converge to a global optimum. In these cases, a local optimum was achieved because the method failed to realize that varying more than two stations at a time would lead to a better solution. Fortunately, most of the times these points are easily identified as non-optimal points in the transformation curve; i.e. they would lie to the northeast of adjacent solutions.

The easiest and cheapest method, relative to the few occurrences and easy recognition, to avoid this problem can be described as a rule of thumb. The author suggests solving the optimization problem starting with two different initial designs. One of the initial designs should always be the easily identified minimum cost solution, the other any variation of the first with as many different terms as possible. Obviously, in designing for one or one less than the maximum allowable
number of stations, this procedure is not necessary.

As could be expected, failure to converge is more common in problems where large number of possible designs are possible (given by \( \frac{n!}{(n-m)!m!} \)). The larger the differences in stations characteristics, the less is the chance of convergence failure.

The above suggested rule of thumb proved to be very satisfactory in easily achieving global optima in the examples used in this work.

3.7.2 Computational Requirements

The computer program used obtaining the results of this chapter was prepared in Fortran IV, and implemented in an I.B.M. 370/165 of the M.I.T. Information Center. As programmed, it requires double precision capability and the use of IBM's Subroutine Library-Mathematics (1971), jointly with M.I.T.'s Information Center Mathematical Library. In its present condition (as given in Appendix 4), it can handle up to a 20 grid point discretization and requires around 250 K.

As expected, the speed and cost of execution varies considerably with the number of allowable station locations since the size of the covariance matrix is the square of that number. Similarly, speed and cost are functions of the necessary number of iterations to achieve an optimum. Generally, though, iterations rarely exceeded 5 and were usually below that number.

On the average, for the problem of 9 possible station locations, the prize of designing per given number of stations was about $0.50 using the full measure of accuracy. With 18 possible locations,
the prize per optimization (per given number of stations) increased to
the order of $4.00 using the full measure of accuracy. With the estima-
tion error accuracy criteria significant cost reductions could be achie-
ved. The above prizes are based on M.I.T. Information Center prizing
policies of basically $6.18 per c.p.u. minute and $0.55 per K-byte hour.
Handling, reading, printing and other processing costs are also included
but are less important on the average when a large number of optimization
problems are submitted together. The given costs are based on execution
step only. Initial compilation costs are not included but certainly are
not significant on a production operation.

3.7.3 Concluding Remarks.

Measuring the average precipitation depth over an area is
an important problem in hydrology. Chapter 2 presented the theory be-
hind a proposed complete network design procedure for the above problem.
Chapter 3 showed its feasibility technically and economically in realis-
tic network design situations. Section 3.5 discussed what the author
views as the advantages of the proposed procedure over other available
network design techniques.

The presented procedure combines accuracy (taking into ac-
count the process and instrument uncertainties) and cost considerations
in a flexible enough way to include characteristics and constraints
particular to the problem at hand and the interested designer. The
usual warning flag must be raised, though, to remind the user that no
system design approach is meant to completely substitute the decision
maker's designer's) good sense and intuition. Even though the proposed
method can consider many of "his or hers" ideas, the final decision is still the decision maker's responsibility. This could be extremely important, for example, in situations where a mathematically optimal design is found which in fact differs very little from non-optimal solutions. Since a perfect utility function definition will never be possible, the designer will have to use his or her good judgement in accepting or rejecting the given solution. Lenton et al (1974) discusses briefly such dilemma in the use of mean square error as a measure of accuracy.

Finally, it is necessary to mention that the herein given ideas of network design could be applied to similar problems in other disciplines, for example, the design of soil exploration experiments (see Bras et al, 1975).
CHAPTER 4

Rainfall Generation: A Non-Stationary, Time Varying, Multi-Dimensional Model

4.1 Need for a Model

In the last two chapters, a procedure for designing networks to measure total rainfall depth of an event was presented. In doing so the event to be monitored was characterized by a spatial covariance function. Inherently, then, a generator of spatially distributed rainfall depths was implied.

Besides its utility in network design, the synthesis of rainfall data plays many other very important roles in water resources studies. As is often the case, water resources works usually face the fact that historical records are insufficient for analysis and decision making. It is under these conditions that the ability to synthesize and simulate historical series becomes extremely important.

Rhenals et al (1974) suggest that existing rainfall models can be classified as:

a) Point Rainfall Models. Those that generate time-sequences of rainfall depth at a single point

b) Multivariate Rainfall Models. These models consider several raingages simultaneously and are intended to preserve the covariance structure of the historical rainfall data existing in those points.

c) Areal or Multidimensional Rainfall Models. These models
characterize the rainfall phenomenon at every point over the area of interest.

In this work all of the above classifications are subdivided in:

1) Rainfall exterior models. Those that generate storm exterior characteristics like total depth, duration of event and time between events

2) Rainfall exterior-interior models. These models generate the time distribution of the total rainfall depth within each event.

Rhenals et al (1974) and Leclerc and Schaake (1973) give fairly complete reviews of the existing rainfall generators. The most recent model, to the author's knowledge, is that of Mejia and Rodriguez Iturbe (1974) which is of the areal-multiparameter type, and only for storm exteriors in the sense that generates total depth of any point in the area. Lenton et al (1974) extends Rodriguez' model to generate areal average of total rainfall depth instead of point values. Most of the existing models concentrate on storm exterior characteristics. Those which generate storm interiors do so generally at only one point.

Very few models attempt to generate exterior and interior characteristics everywhere in space. Grayman and Eagleson (1971) suggested a statistically stationary model capable of generating storm interiors in finite points in space. The main constraint of this model is the assumption of stationarity behavior at all levels of storm activity, including rain-cells, mesoscale and synoptic level.
The following sections will suggest a multidimensional model, non-stationary, of rainfall exterior and interiors. It would also be very useful in designing rainfall networks with the idea of an accurate prediction of the runoff produced by a storm. This problem is tackled in Chapter 5 of this work. The model has been built to theoretically preserve first order statistics of storm exteriors as well as the correlation in time and space of the storm interior.

4.2 Formulation and Assumptions

The development of the model was based on the following basic knowledge describing the behavior of storms.

1) Each storm moves with an average velocity, \( U \), over the area of interest and follows certain trajectory. Individual disturbances within the storm move about the same velocity (Zawadski, (1973b), Houze, (1969), Grayman and Eagleson (1971)).

2) Water falling at any instant is correlated to what happened at previous times. (Zawadski, (1973a), Leclerc and Schaake (1973))


4) Spatial and Time Correlation are neither separable nor independent (Zawadski, (1973b), Lenton et al, (1974)).

5) Rainfall is a non-stationary process. The mean and variance vary with time at all points in space (Zawadski, 1973b),
At this point it is assumed that the storm interior of an event with given depth and duration can be modeled as:

\[ i(x_i, t) = i(\mu(x_i), t) + \eta(x_i, t) \]  

(4-1)

where \( i(x_i, t) \) is the rainfall intensity at point with coordinate vector \( x_i \) at time \( t \), \( i(\mu(x_i), t) \) is the mean intensity at \( x_i \) and \( t \), where the mean value is taken over all possible storms of the same characteristics, \( \eta(x_i, t) \) is the noisy residual obeying a certain covariance function in time and space.

It is generally accepted that storms of a given type, i.e.: frontal storms, in an area can be represented in an undimensional form of the type shown in Figure 4-1a (Eagleson, (1970), Pilgrim et al, (1969)). That figure gives the average temporal rainfall distribution in terms of the percentage of precipitation versus percentage of duration. Usually, the above information is obtained from data of rainfall mass accumulation in a single raingage. Here it is assumed that every point in space will have the same average mass distribution. By multiplying the undimensional mass curve of Figure 4-1a by a given depth and duration, the mean temporal behavior, \( i_a(t) \) of the storm at all points is obtained, relative to the storm starting time at each point. It is assumed that storm duration is the same total everywhere. In order to represent all stations at the same time, an absolute time
Figure 4-1a: Undimensional storm representation

Figure 4-1b: Mean temporal behavior, $i_a(t)$, resulting for a 2 inch, 1 hour duration, storm
scale is defined within the area. Starting time is the moment the moving storm hits the first point in the area.

Thus, data of the form of Figure 4-1b is, for every point, translated to absolute time by the distance from the origin to the point in the direction of storm movement. Assuming for simplicity that the storm moves parallel to the x axis (see Figure 4-2) it is clear that:

\[ i_{\mu}(x_{i},t) = i_{a}(t - \frac{x_{i}}{U}) \]  \hspace{1cm} (4-2)

where

- \( i_{\mu}(x_{i},t) \) = mean intensity at \( x_{i} \) and time \( t \) after storm reaches the boundaries of the area of interest.
- \( i_{a}(t) \) = average precipitation at time \( t \) where \( t \) is now the time it has been raining at a given point (see Figure 4-1b)
- \( x_{i} \) = x coordinate of point \( i \)
- \( U \) = storm average velocity in x direction

Note that due to the hyetograph form of the input data (Figure 4-1), a discretization in time is unavoidable. Rain accumulation is lumped within finite time intervals.

The next step is to hypothesize the form of the covariance function of the noisy residuals. In its most general form:

\[ E \left[ \eta(x_{i},t') \eta(x_{j},t'') \right] = f(x_{i},t' ; x_{j},t'') \]  \hspace{1cm} (4-3)

In this work the above is written as
Figure 4-2 Diagram of 360 sq. mi. area showing discretization, storm velocity and direction
\[ E[\eta(x_i', t') \eta(x_j', t'')] = \sigma(x_i', t') \sigma(x_j', t'') r(x_i', t'; x_j', t'') \quad (4-4) \]

where

\[ E = \text{expectation operator} \]
\[ f(\cdot) = \text{functional covariance} \]
\[ \sigma(x_i', t) = \text{standard deviation of rainfall intensity at point } x_i \text{ and time } t \]
\[ r(x_i', t'; x_j', t'') = \text{general functional form of normalized covariance} \]

\( \sigma(x_i', t) \) corresponds to the variation around \( \mu(x_i', t) \) and is similarly obtained from data (see Figure 4-1). The same time translation is applicable, so

\[ \sigma(x_i', t) = \sigma_a (t - \frac{x_i'}{U}) \quad (4-5) \]

Equation 4-1 can then be expressed as:

\[ i(x_i', t) = i_a (t - \frac{x_i'}{U}) + R(x_i', t) \sigma_a (t - \frac{x_i'}{U}) \quad (4-6) \]

where

\[ R(x_i', t) = \text{standardized residual at point } x_i \text{ and time } t \text{ with zero mean and unit variance.} \]

\[ E[R(x_i', t') R(x_j', t'')] = r(x_i', t'; x_j', t'') \text{ as defined previously.} \]

The statistical behavior of the residual \( h(x_i', t) \) embodies the spatial and time correlation of the rainfall process. The function
At this point, a basic assumption about the behavior of rainfall is introduced. It is assumed that Taylor's Hypothesis (Taylor, 1935), of turbulence is valid within a storm. The above implies that correlation in time is equivalent to that in space if time is transformed to space in the mean direction of storm movement. If the storm moves in the x direction, Taylor's Hypothesis implies:

\[ r(x_i, t' ; x_j, t'') = r(y_i, x_i, t' ; y_j, x_j, t'') = r(y_i, x_i + Ut' ; y_j, x_j + Ut'') \]  

(4-7)

The reasoning behind Taylor's Hypothesis is that it is applicable for translating processes with relatively weak time dependence within their moving coordinate system such that time dependence in a fixed coordinate system is dominated by the average motion. In other words, it is assumed that "noise" or "turbulence" is connected along the mean flow velocity without evolving appreciably in a "reasonable" distance. Such reasoning has been found to be applicable in general fluid turbulence (Hinze, 1959); wind and gusts studies (Harris, 1971); and large scale atmospheric processes (Gandin, 1965; Panchev, 1971). To the author's knowledge it has been tested and corroborated once for particular rain storms in the work of Zawadski (1973b). Note again that in the present work it is the unit variance residuals that obey Taylor's Hypothesis.

The second assumption used is that rainfall intensities (or
depth per time interval) have isotropic spatial correlation functions at any instant of time. Such assumption is justified by the works of Huff (1970) and Zawadski (1973b).

Therefore,

\[ r(x_i', t'; x_j', t'') = r(s, t) \]

\[ = r\left[ \sqrt{(y_j - y_1)^2 + ((x_j + U_t') - (x_i + U_t''))^2} \right] \]

(4-8)

Notice that the above equation implies that the standardized residuals (mean zero, unit variance) are stationary and isotropic in the variables \( x_1 = y \) and \( x_2 = x + U_t \). The above does not imply stationarity of the "noise" element \( \eta(x_i, t) \) in Equation 4-1 nor does it imply isotropy with respect to \( x, y \) and \( t \) separately.

It is necessary now to have a scheme to synthetize random fields of a prescribed correlation structure. This scheme will then be used to generate rainfall events over an area.

A two-dimensional random field \( \varepsilon(x) \) can be represented through the following equation (Mejia and Rodriguez-Iturbe, 1974):

\[ \varepsilon(x) = \sqrt{\frac{2}{N}} \sum_{i=1}^{N} \cos \left[ (x \cdot y_{i1}) w_{i1} + \theta_{i1} \right] \]

(4-9)

where \( x \) represents a vector of coordinates \((x_1, x_2)\) in \( \mathbb{R}^2 \); \( y_{i1} \) is a two dimensional random variable \((y_{i11}, y_{i12})\) equidistributed on the circle of unit radius; \( w_{i1} \) is a random variable whose distribution is the radial spectral distribution function \( G(w) \) corresponding to
the isotropic correlation function of $\epsilon(x)$; and $\Theta_1$ is a uniformly distributed random angle between 0 and $2\pi$. All random variables are mutually independent with $(x \cdot y_i)$ denoting the vector inner product and $N$ the finite number of harmonics.

Mejia and Rodriguez-Iturbe (1974) have shown that the above process is homogeneous, isotropic, and asymptotically ($N \to \infty$) ergodic and multinormal. It also has zero mean, unit variance and, as $N \to \infty$, correlation function with radial spectral distribution corresponding to $G(w)$. (See also Rhenals et al, 1974, and Lenton et al, 1974).

Since $y_i$ is equidistributed on the unit circle,

$$y_i = (\cos \alpha_i, \sin \alpha_i)$$  \hspace{1cm} (4-10)

where $\alpha_i$ is uniformly distributed between 0 and $2\pi$.

Equation 4-9 then becomes,

$$\epsilon(x_1, x_2) = \sqrt{\frac{2}{N}} \sum_{i=1}^{N} \cos \left( w_i (x_1 \cos \alpha_i + x_2 \sin \alpha_i + \Theta_i) \right)$$  \hspace{1cm} (4-11)

It is then clear that Equation (4-11) can be used to generate a random process in the $R^2$ space defined by $x_1 = y$ and $x_2 = x + Ut$. To do so, the random variables $w_i$ must be sampled from the radial spectral distribution function corresponding to Equation 4-8. Generated values of $\epsilon(x_1, x_2)$ are then, due to the simple relation between $x_2$, $x$ and $t$, samples of the non-stationary random field describing rainfall events $R(x_1, t) = R(x, y, t)$. The spatial and time correlation of the generated values approaches Equation 4-8 as the number
of harmonics, $N$, goes to infinity. The spatial correlation of total depths over the area are also preserved since

$$r(D_i D_j) = \frac{1}{t_D} \int_0^{t_D} \int_0^{t_D} r\left(\sqrt{(y_j - y_i)^2 + ((x_j + Ut') - (x_i + Ut''))^2} \right) dt' dt''$$

(4-12)

where

- $D_i$ = total depth at point $i$
- $t_D$ = storm duration

The functional form of correlation function, Equation 4-8, would naturally depend on the hydrologic conditions of the area at hand. A set of commonly used isotropic correlation functions was given in Chapter 2 and is repeated here for the reader's convenience.

Single Exponential:

$$r(v) = e^{-\alpha |v|} \quad (4-13)$$

Quadratic Exponential:

$$r(v) = e^{-\alpha^2 v^2} \quad (4-14)$$

Bessel Form:

$$r(v) = |v| bK_1(|v|b) \quad (4-15)$$

(See Mejía and Rodríguez-Iturbe (1974) for more discussion in the use of this correlation function.)

The corresponding radial spectral densities and distributions are:
Single Exponential

\[ G'(w) = \frac{w}{\alpha^2 \left[ 1 + \frac{w^2}{\alpha^2} \right]^{3/2}} \quad 0 < w < \infty \quad (4-16) \]

\[ G(w) = 1 - \frac{1}{\sqrt{1 + \frac{w^2}{\alpha^2}}} \quad 0 < w < \infty \quad (4-17) \]

Quadratic Exponential

\[ G'(w) = w \exp\left(-\frac{1}{4}w^2 \alpha^2\right) \quad 0 < w < \infty \quad (4-18) \]

\[ G(w) = \frac{\alpha^2}{8} \left[ 1 - e^{-\frac{1}{2}w^2} \right] \quad 0 < w < \infty \quad (4-19) \]

Bessel Form

\[ G'(w) = \frac{2w}{b^2(1 + \frac{w^2}{b^2})^2} \quad 0 < w < \infty \quad (4-20) \]

\[ G(w) = \frac{w^2}{w^2 + b^2} \quad 0 < w < \infty \quad (4-21) \]

The simplicity of the radial spectral distribution corresponding to the given correlation functions allows the sampling of characteristic \( w \) values by the inverse method.

For example, consider the single exponential spectral distribution, Equation (4-17).

Inverting the equation results in:

\[ w = \alpha \sqrt{\left(\frac{1}{1-G(w)}\right)^2 - 1} \quad (4-22) \]
Generation of uniformly distributed values between 0 and 1 and their substitution for \( G(w) \) in 4-22 results in a series of \( w \) values belonging to the population defined by 4-16 and 4-17.

The works of Huff (1970) and Zawadski (1973b) suggest the use of the single exponential correlation function for representing rainfall intensities. The merits of that assumption will not be discussed here (a related discussion appeared in Chapter 2), but the suggested function will be used in the examples given in the last section of this chapter.

4.3 Point Rainfall Generation Algorithm

The suggested rainfall model consists of an algorithm uniting existing techniques of rainfall exterior generation and the techniques of rainfall interior simulation given in the previous section.

Required inputs are basically the following:

1) Marginal distribution (form and parameters) of time between events, \( \tau \).
2) Marginal distribution of storm duration, \( t_d \). It is assumed that rainfall accumulates during the same amount of time at each point.
3) Conditional distribution of areal average total depth, \( D \), given the storm duration.
4) Distribution of storm average velocities and directions.
5) Probability of occurrence of various types of events;
6) Undimensional time distribution of rainfall (average over all points in space) and corresponding standard deviations of rainfall accumulation in each time step (Figure 4-1) for
each event type.

7) Form of time and space correlation given in Equation 4-8, for each event type.

The generation algorithm is then the following:

1) Set total time of generation desired
2) Sample $T$, $t_d$ and $D$ from corresponding distributions
3) Sample storm velocity, direction and type
4) Construct average time distribution, $i_a(t)$ of generated storm by scaling the undimensional hyetograph by the generated depth and duration
5) Construct corresponding standard deviation, $\sigma_a(t)$
6) Specify points in space where generation is desired
7) Generate zero mean, unit variance residuals with Equation 4-11
8) Create storm interior at desired points with Equation 4-6
9) Go back to step 2, to generate a consecutive event and repeat for desired period of generation

The generation of interiors using Equation (4-6) extends for the storm duration plus the time the storm takes to transverse the area. The storm duration at each point, though, is $t_d$ since $i_a(t - \frac{x_i}{U})$ and $\sigma_a(t - \frac{x_i}{U})$ are zero or near zero before the storm reaches point $x_i$ and become zero again after the storm passes the point in question.

4.4 The Generation of Areal Rainfall

It is sometimes desirable, especially in rainfall-runoff modeling, to obtain estimates of rainfall averaged over particular areas or subareas.
Lenton et al (1974) studied and derived a procedure to obtain an areally averaged stochastic process from a multidimensional process as given in Equation 4-9. He shows that the areal process can be defined through the relationship:

$$
\varepsilon_A(x) = \sqrt{\frac{2}{N}} \sum_{i=1}^{N} \lambda_A(w_i, y_i) \cos \left[ (x \cdot y_i) w_i + \theta_i \right] \tag{4-23}
$$

where $x$ now imply a point in the area of interest (i.e. the center of gravity) used as a parameter vector and $\lambda_A(w_i, y_i)$ is a variance reduction coefficient whose form is dependent on the geometry of the area of interest.

The variance of the areal process is then

$$
\text{Var} (\varepsilon_A(x)) = \sigma_p^2 E\left\{ \lambda_A^2(wy) \right\} \tag{4-24}
$$

where $\sigma_p^2$ is the point variance (see Lenton, et al, 1974).

As previously mentioned, the form of the coefficient $\lambda_A(wy)$ depends on the geometry of the area. For rectangular area it becomes:

$$
\lambda_A(wy) = \frac{4}{\ell_1 \ell_2} \frac{y_1 y_2}{y_1 y_2} \sin \left( \frac{w y_1 \ell_1}{2} \right) \sin \left( \frac{w y_2 \ell_2}{2} \right) \tag{4-25}
$$

where

- $\ell_1$, $\ell_2$ = dimensions of rectangle
- $y_1$, $y_2$ = coordinates of random point in unit circle in the direction of $\ell_1$ and $\ell_2$ respectively.

Lenton et al (1974) offer equivalent expressions for the ellipse and the circle. When using rainfall-runoff models, basins are usually
characterized by combinations of rectangular overland segments (see Bras, 1972; Harley et al, 1970), therefore Equation 4-25 together with 4-23 will allow the generation of rainfall intensities averaged over the individual subareas of a schematized basin.

It should be made clear that the generation algorithm for the areal average of rainfall is the same as described previously in Section 4.3 for point rainfall intensities. The only difference is the use of Equation 4-23 instead of Equation 4-9.

4.5 An Example

For illustration purposes, a simple numerical example of rainfall generation is given. It is assumed that only one type of storm is generated (no seasonal variations nor different storm classifications.) Due to lack of easily available data, storm speed is given as a deterministic input. For convenience storm direction is fixed so as to coincide with the definition of the x coordinate axis.

The area of generation is a 360 sq.mi. rectangular area. Its length is 20 mi in the x direction and 18 mi in y direction.

The residual correlation function is of the single exponential type (Eqn. 4-13) with parameter 0.15. It takes the form:

\[
r(x_j^t, t''; x_i^t, t') = e^{-0.15 \sqrt{(y_j^t - y_i^t)^2 + \left[ (x_j^t + Ut'') - (x_i^t + Ut') \right]^2}}
\]  

(4-26)

Rainfall exterior parameters are assumed to obey simple exponential distributions. Similar distributions have been used previously by Leclerc and Schaake (1972) and Eagleson (1971). The distribution and used parameters are:
1) Time between Storms (τ)
\[ f(\tau) = \lambda_1 \exp(-\lambda_1 \tau) \quad \tau > 0 \quad (4-27) \]
\[ \lambda_1 = 1.0 / 70.4 \text{ hrs} \]

2) Storm Duration (t_r)
\[ f(t_r) = \lambda_2 \exp(-\lambda_2 t_r) \quad t_r > 0 \quad (4-28) \]
\[ \lambda_2 = 1.0/7.692 \text{ hrs.} \]

3) Total average depth over the area conditional on duration
\[ f(d/t_r) = \beta \exp(-\beta d/t_r) \quad d > 0 \quad (4-29) \]
\[ \beta = 15 \]

The undimensional rainfall distribution used was shown in Figure 4-1b together with one standard deviation values. The hyetograph was discretized in eight intervals as shown in the figure.

Note that the correlation function parameter was kept constant at 0.15 even though, as the generator stands, different storms would have different time intervals which should have different correlation parameters. The value of 0.15 seems reasonable in the light of Huff's (1970) work (see Chapter 5 for discussion).

Appendix 5 explains the use of the computer program that carries on the generation.

Using a storm velocity of 12.0 m.p.h. and the data described in previous paragraphs, several storms were generated. In these examples only 50 harmonics were used in the generation procedure.
Table 4-1 gives information on two of the generated storms. Storms are given in terms of average areal intensity over individual grids of the area, for that reason the table identifies the subareas of generation with the coordinates of their center point.

Figures 4-3, and 4-4 show the storm hyetograph of the two storms at 6 of the subareas. Remember these intensities are areal averages over the subareas. Figures 4-5 to 4-7 give the rainfall areal distribution at some specific times of the first storm. Similarly, Figures 4-8 and 4-9 are the areal distribution of the second storm at two different times.

Table 4-2 gives the point intensity values of the same storms. The generation points are the centers of the individual subareas. The larger degree of spatial variation is clear from the generated intensities and from the total generated depths at the various points.

Figures 4-10 and 4-11 give the hyetograph at 6 points of the two storms.

Figures 4-12 to 4-15 show the spatial distribution of point intensities at various times for the two storms.

4.6 Conclusions

A rainfall generator capable of generating storm interiors at different points in space has been suggested. Example results seem to verify the adequacy and correct behavior of the model.

It is felt that the model incorporates rational and realistic assumptions regarding the character of rainfall events. It is non-stationary, dynamic, and multi-dimensional instead of simply multivariate.
Data analyses are needed to extend the available knowledge regarding the possible values of the parameter which characterizes the correlation function. The dependence of the latter with storm type, seasons, sampling time interval, and storm velocities must be defined. The probabilistic nature of storm velocities and directions is another area of needed research.

Even in the face of these needs the suggested model has the advantages of being based on concepts easy to understand, being multidimensional and computationally easy to implement. Moreover, the previously described research needs will be applicable for any kind of rational description of the rainfall process.
Table 4-1: Examples of generated storms, areal mean intensities given

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Duration = 14.33 hrs  
Total areal mean depth = 1.079 in.  
Date of occurrence from initial time(days) = 19.498

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Figure 4-3: Hyetograph of storm at 1.675 days, areal mean intensities
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Figure 4-4: Hyetograph of storm at 19.498 days, areal mean intensities
Figure 4-5: Areal distribution of first storm at 3.23E hrs into storm, areal mean intensities
Figure 4-6: Areal distribution of first storm at 6.476 hrs. into storm, areal mean intensities
Figure 4-7: Areal distribution of first storm at 5.372 hrs. into storm, areal mean intensities.
Figure 4-8: Areal distribution of second storm at 3,372 hrs. into storm, areal mean intensities
Figure 4-9: Areal distribution of second storm at 10,745 hrs. into storm, areal mean intensities

**Time of Storm**: 19.498 Days
**Time Identification**: 7 # 0.745 Hours into Storm
Table 4-2: Examples of generated storms, point intensities given

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Total mean depth = 2.0052
Date of occurrence from initial time(days) = 1.675
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- 158 -
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### Depth (Inches)

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<tr>
<td>0.71300E 00</td>
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</tr>
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<td>0.67367E 00</td>
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<td>0.63404E 00</td>
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</table>

### Storm Hyetograph: Time of Occurrence

1.675

---

**Figure 4-10:** Hyetograph of storm at 1.675 days, point intensities
**Figure 4-11: Hyetograph of storm at 19.498 days, point intensities**

<table>
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<td>0.14240E</td>
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<tr>
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</tbody>
</table>

**Legend:**

1 -> STATION NUMBER 1
2 -> STATION NUMBER 4
3 -> STATION NUMBER 6
4 -> STATION NUMBER 11
5 -> STATION NUMBER 12
6 -> STATION NUMBER 15
Figure 4-12: Areal distribution of first storm at 3.238 hrs. into storm, point intensities
Figure 4-14: Areal distribution of second storm at 5.372 hrs. into storm, point intensities
<table>
<thead>
<tr>
<th>2-D Graph (area)</th>
<th>Time of Storm</th>
<th>19.414 Days</th>
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</thead>
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<table>
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<td>0.168E</td>
<td>0.00</td>
</tr>
<tr>
<td>0.162E</td>
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</tr>
<tr>
<td>0.001E</td>
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</table>

**Figure 4-15:** Area distribution of second storm at 10.745 hrs. into storm, point intensities.
Chapter 5

Rainfall Network Design for Runoff Prediction

5.1 Introduction

Rainfall-Runoff models are commonly used for flood forecasting. In such models, catchment behavior as a low pass filter attenuating high frequency and high wave number input fluctuations, is commonly accepted. Such apparent behavior obviously leads to the question of how detailed temporally and spatially is rainfall needed to be described to keep the accuracy of flood forecasting models within a given range.

At this point the reader may justifiably argue that flood forecasting has fairly well defined related benefits and losses (see Sniedovich et al., 1974, and Robinson, 1970). Therefore, the design of data collection networks for use with forecasting models should be attempted with the objective of maximizing net benefits accrued and not simply with a forecasting accuracy criteria. Lost functions, related to flood forecasting, are inherently non-symmetric, i.e., the lost expected from over-prediction is not the same as under-prediction, and function of forecast response. Maximizing net benefits under such conditions would require knowledge of the conditional distribution of the predicted discharge given the true discharge. Similarly statistical description of the forecasting system and population response to flood warnings would be needed.

In this work, although acknowledging the ideal system design approach described previously, the author chose to deal only with the
forecasting accuracy criteria in designing networks. There are mainly three reasons for that decision. First, the ideal approach briefly described previously, requires information and data that is simply not available. The only way the author sees such approach could be attempted is through the use of extensive simulation together with considerable historical information on discharges and system response to forecasts. Grayman and Eagleson (1971) used the simulation tool as suggested above.

The second reason is the author's opinion, that faced with such lack of data, the problem should be decomposed in various stages, the first optimizing forecasting accuracy. The other stages would be to iteratively solve the first problem with accuracy constraints. The accuracy constraint would be imposed by information on the dissemination and forecast response systems which would become available once the forecasting procedure is in operation.

The third, but important, reason for choosing the accuracy criteria is that the suggested method of analysis is based on first and second order moments, mean and variance (mean square error of estimation). Such approach has the analytical advantages of relative simplicity, minimum data requirements and no simulation needs as can be realized from a similar approach in the first chapter of this work. Unfortunately, the method yields no information with respect to the shape of the probability distributions nor of the joint distributions of the parameters needed for the full system approach.

Eagleson and Shack, (1966), and Harley et al (1970), have studied the problem of determining the sampling time interval with due consideration to rainfall and catchment response characteristics. The first attempt to the author's knowledge, to determine the sensitivity of
catchment peak discharge to sampling density was made by Eagleson, (1967), using harmonic analysis concepts with a distributed, one-dimensional, linear catchment system. Eagleson's (1967) results yielded relations between raingage density and forecast peak "errors". His approach prevented any statement as to the station locations.

In this work, a method will be suggested that will provide the mean square error (as a measure of accuracy) of the estimated discharge, as a function of not only the number of raingages but also their location, inherent measurement error, and naturally, the spatial variability of the rainfall process. The solution is dynamic (time varying) and so gives a measure of accuracy at every time step of the hydrograph not only the peak value. Rainfall is acknowledged to be a non-stationary, dynamic, multidimensional, stochastic process which is a much more realistic representation than the static, stationary one used by Eagleson (1967).

The runoff model used in the analysis is non-linear, spatially distributed, and based on solutions to the kinematic wave equations. It allows for non-homogeneous descriptions of the overland and stream segments comprising the basin being studied.

5.2 A State-Space Model of Rainfall

The network analysis's method to be suggested in this chapter is similar to the estimation methodology used in Chapter 2 of this work where the problem was the estimation of rainfall depth from precipitation events. To use this powerful technique in the dynamic problem at hand it is necessary to express the rainfall input in the form of a state-space, first-order Markov model. This, nevertheless, will be done accounting
for the non-stationary, dynamic, multidimensional character of rainfall and will be based on the rainfall generator developed in Chapter 4.

The rainfall input will be written as

\[ i(t) = A(t-1)i(t-1) + B(t-1)w(t-1) \]  

Equation 5-1 is a discretization in space and time of the continuous model suggested in Chapter 4. The discretization naturally should be made considering issues of homogeneity of input and basin characteristics. It will also need to be compatible with the basin schematization to be used for runoff modeling which will be explained in following sections.

The model suggested above would be valid for the generation of rainfall interiors for a specified family of storms with a given duration, average speed, direction and statistical description of intensities (time varying mean, and multidimensional correlation structure).
Valencia and Schaake (1972) show that the parameters of stationary models similar to Equation 5.1 can be estimated as:

\[
A = \frac{S_{t,t-1}}{S_{t-1,t-1}} \quad (5-2)
\]

\[
B B^T = \frac{S_{t,t} - S_{t,t-1} S_{t-1,t-1}}{S_{t-1,t}} \quad (5-3)
\]

where

\[
S_{t,t} = E[i(t_i)]^T \quad (5-4)
\]

\[
S_{t,t-1} = E[i(t_i) i(t_{i-1})]^T \quad (5-5)
\]

= lag one covariance matrix

For the suggested non-stationary model (Equation 5-1) the parameter estimation equations are generalized to:

\[
A(t) = \frac{S(t)}{S(t-1)} \quad (5-6)
\]

\[
B(t) B^T = \frac{S(t) S(t)}{S(t,t) S(t,t-1) S(t-1,t-1) S(t-1,t)} \quad (5-7)
\]

Chapter 4 suggested a non-stationary, multidimensional rainfall generator based on an assumption which allowed the author to hypothesize a covariance function of the form

\[
f(x_i, t'; x_j, t'') = \sigma(x_i, t') \sigma(x_j, t'') \rho(y_j - y_i)^2 + ((x_j + \tilde{u}_t - x_i + \tilde{u}_{t'})^2
\]

(5-8)

The evaluation of the \( \sigma(x_i, t') \) terms and the possible forms of \( \rho(\cdot) \) were discussed in that chapter.

Clearly, Equation 5-8 can be evaluated at discrete points in space and at given times to form the necessary matrices in Equations
(5-6) and (5-7). The evaluation of \( A(t) \) and \( B(t)^T B(t) \) is then possible once Equation 5-8 is defined.

The following paragraphs will make clear that, for the network design problem, interest lies in obtaining the matrix \( B(t)^T B(t) \) and not \( B(t) \). This is important because the sometimes messy decomposition of \( B(t)^T B(t) \) is avoided (see Valencia and Schaake, 1972).

Similarly to what was done in Chapter 2 assume the rainfall process (storm) is being observed. Describe the observations (or experiment) by

\[
Z(t) = H i(t) + V(t) \quad (5-9)
\]

where

- \( Z(t) = mx1 \) vector of observations at time \( t \)
  - Note that observations are not necessarily made at every discrete point, defining the area of interest, i.e. \( m < N \).
- \( H = mxN \) matrix defining the measurement network. Its definition and form is the same as that given in Chapter 2.
- \( i(t) = N \times 1 \) vector representing the true rainfall intensity values at the \( N \) discrete points in space during time \( t \)
- \( V(t) = mx1 \) vector of white measurement error attributed to the instruments being used.
- \( R = E \left[ V(t)^T V(t) \right] \) in this example assumed stationary in time. As in Chapter 2 the matrix \( R \) will be con-

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- 171 -
considered a diagonal matrix of measurement error vari-
ances. No correlation between measurement errors at
different points is allowed.

In the statement of 5-9, no correlation between
\( i(t) \) and \( V(t) \) is allowed. The existence of such correlation is
manageable but requires more complicated solutions.

Equations (5-1) and (5-9) form an observable discrete dynamic
system and as such estimates of the system state, \( \hat{i}(t) \), can be made from
incomplete, noisy observations, \( Z(t) \).

The best linear dynamic estimator (minimum mean square error)
for such systems is the well known Kalman-Bucy filter which can be ex-
pressed as (see Schwepp, 1973, and Jazwinski, 1970):

\[
\hat{i}(t+1/t+1) = \Sigma(t+1/t+1) \{ H^T \Sigma^{-1} Z(t+1) + \Sigma^{-1} (t+1/t) A(t) \hat{i}(t/t) \}
\]

(5-10)

where

\[
\Sigma(t+1/t) = A(t) \Sigma(t/t) A^T(t) + B(t) R(t) B^T(t)
\]

(5-11)

\[
\Sigma(t+1/t+1) = \Sigma(t+1/t) - \Sigma(t+1/t) H^T \{ R + H \Sigma(t+1/t) H^T \} H \Sigma(t+1/t)
\]

(5-12)

\[
\Sigma(t/t) = E[(i(t) - \hat{i}(t))(i(t) - \hat{i}(t))^T]
\]

(5-13)

\[
\hat{i}(0/0) = 0
\]

(5-14)

\[
\Sigma(0/0) = \Psi = E[i(0) i^T(0)]
\]

(5-15)
The notation \( \Sigma(t+1/t) \) stands for the error covariance matrix at interval \( t+1 \) given observations up to time \( t \). \( \hat{i}(t/t) \) stands for the best linear estimate of the vector \( i(t) \) given incomplete and noisy observations, \( Z(t) \), up to time \( t \).

Notice the similarity of Equations 5-10 to 5-13 with Equations 2-21 and 2-22, which were valid for a static, time-invariant problem. Again, as in Chapter 2, the value of the Kalman filter is that it allows the evaluation of the accuracy of an experiment (network \( H \)) in terms of the mean square error of estimation, \( \Sigma \). Most important, the solution is such that the mean square error can be obtained and is independent of the actual observations (see Equations 5-11 and 5-12), which allows a-priori knowledge of the network accuracy and behavior.

### 5-3 A State-Space Model of Runoff

Similar to the rainfall model of the previous section, it is necessary to define a state-space model of a runoff. A finite difference solution of the Kinematic wave equations was chosen as a distributed runoff model. The model used corresponds to one used by the author in previous work (Bras, 1974).

The formulation in state-space form will be illustrated by use of an example in order to avoid the considerable notational complication of a general development.

Assume the schematized basin shown in Figure 5-1, consisting of 3 overland segments and 2 stream segments for a total of 5 elements (for discussion on basin schematization, see Harley et al, 1970;
Figure 5-1  Schematized river basin
The Kinematic wave equations have the form:

Continuity equation:

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_{\phi}
\]  

(5-16)

Momentum equation:

\[Q = \alpha A^m\]  

(5-17)

where

- \(A\) = cross sectional area of flow in element
  (depth in overland segments)
- \(t\) = time
- \(x\) = longitudinal distance
- \(Q\) = discharge
- \(\alpha, m\) = Kinematic wave parameters
- \(q_{\phi}\) = lateral flow into stream segment or rainfall into overland segment

In a distributed rainfall-runoff model Equations 5-16 and 5-17 can be solved for individual elements with different parameters. One of the possible solutions is a finite difference scheme that takes the following form (see Bras, 1974) for an overland segment:

\[k \cdot y_j(t) = \left[ k \cdot y_j(t-1) - \alpha_k \frac{\Delta t}{\Delta x} m_k \cdot k \cdot y_j(t-1) \right] + \alpha_k \frac{\Delta t}{\Delta x} m_k \cdot k \cdot y_{j-1}(t-1) + i_k(t-1) \Delta t\]  

(5-18)
Equation 5-18 is valid in the time-space diagram (finite difference grid) shown in Figure 5-2. Such diagram is the representation of a discretization of space and time for element k. The levels of discretization, $\Delta x$ and $\Delta t$ are constrained by stability and convergence criteria of the numerical solution (see Bras 1972, 1974.)

The subscript $k$ in Equation 5-18 refers to the element being analyzed. The subscript $j$ refers to the space interval within the spatial discretization. $k y_j(t)$ stands for the depth of flow at time $t$ in segment $j$ of element $k$. Rainfall input to element $k$ is $i_k(t)$. Kinematic wave parameters of the element are $\alpha_k$ and $m_k$.

Time and space discretization are represented by the time step $\Delta t$ and the interval $\Delta x$.

For simplicity refer to the first term on the right of Equation 5-18 as:

$$k_2(y_j(t-1)) = k y_j(t-1) - \alpha_k \frac{\Delta t}{\Delta x} k y_j(t-1)$$

(5-19)

The second term will be redefined as:

$$k_1(y_{j-1}(t-1)) = \alpha_k \frac{\Delta t}{\Delta x} y_{j-1}(t-1)$$

(5-20)

Thus, $k_2(\cdot)$ and $k_1(\cdot)$ are non-linear functionals on $k y_j(t-1)$ and $k y_{j-1}(t-1)$ respectively.

Referring now to the sample basin shown in Figure 5-1: assume each element, overland and stream, is (for simplicity's sake) discretized in two, therefore $j = 1, 2$.

Evaluation of Equation 5-19 for overland element 1 over both
Figure 5-2 Finite difference grid
space intervals results in:

\[ y_1(t) = y_1(t-1) + i_1(t-1) \Delta t \quad (5-21) \]

\[ y_2(t) = y_2(t-1) + i_1(t-1) + i_1(t-1) \Delta t \quad (5-22) \]

Overland segments 3 and 4 of Figure 5-1 have Equations similar to 5-21 and 5-22.

Stream segments have general solutions similar to Equation 5-18. They take the form:

\[
A_j^k(t) = A_j^k(t-1) - \alpha_k \frac{\Delta t}{\Delta x} m_k^{\ell} + \alpha \frac{\Delta t}{\Delta x} m_k^{j-1} + q(t-1) \Delta t \quad (5-23)
\]

Most of the notation in 5-23 have the same meaning as that in 5-18. \( A_j^k(t) \) stands for the cross sectional area of element \( k \) at time \( t \) in space interval \( j \). \( q(t) \) stands for the sum of lateral inflow (\( \text{ft}^3/\text{sec}/\text{ft} \)) to the stream element from adjacent overland segments.

An important feature of Equation 5-23 is that stream elements may have non-zero upstream boundary conditions. In such cases \( A_{j-1}^k \) takes the value of the cross sectional area necessary to accommodate the sum of outflows from the upstream segments.

Again, Equation 5-23 can be expressed as:

\[
A_j^k(t) = F_2(A_j^k(t-1)) + F_1(A_{j-1}^k(t-1)) + q(t-1) \Delta t \quad (5-24)
\]

where \( F_2(\cdot) \) and \( F_1(\cdot) \) are non-linear functions of \( A_j^k(t-1) \) and ...
$k^{A_{j-1}(t-1)}$ respectively.

At this point, recall the kinematic wave momentum Equation 5-17.

It is clear that

$$q_k(t) = \alpha_k k y_2(t) = e_k(k y_2(t)) \quad (5-25)$$

and

$$Q_k(t) = \alpha_k k A_2(t) = E_k(k A_2(t)) \quad (5-26)$$

where $q_k(t)$ is discharge per unit width of overland element $k$ and $Q_k(t)$ is discharge of stream element $k$, both at time $t$. $k y_2(t)$ and $k A_2(t)$ are the depth and cross sectional area of flow at time $t$ in interval 2 of overland segment $k$ and stream segment $k$ respectively.

Now Equation 5-24 can be applied to stream elements 2 and 5 respectively.

For stream element 2:

$$2A_1(t) = 2F_2(2A_1(t-1)) + q_1(t-1) \Delta t + q_3(t-1) \Delta t \quad (5-27)$$

$$2A_2(t) = 2F_2(2A_2(t-1)) + 2F_1(2A_1(t-1)) + q_1(t-1) \Delta t + q_3(t-1) \Delta t \quad (5-28)$$

For stream element 5:

$$5A_1(t) = 5F_2(5A_1(t-1)) + 5F_1(A^*(t-1)) + q_4(t-1) \Delta t \quad (5-29)$$

$$5A_2(t) = 5F_2(5A_2(t-1)) + 5F_1(5A_1(5A_1(t-1))) + q_4(t-1) \Delta t \quad (5-30)$$
where

\[ A^*(t-1) = \left( \frac{Q_2(t-1)}{a_5} \right)^{\frac{1}{m_5}} \]  

(5-31)

One may now represent the total state of the system by means of an aggregate state vector

\[ \chi(t) = \begin{bmatrix} 1y_1(t) \\ 1y_2(t) \\ 3y_1(t) \\ 3y_2(t) \\ 4y_1(t) \\ 4y_2(t) \\ 2A_1(t) \\ 2A_2(t) \\ 5A_1(t) \\ 5A_2(t) \\ U(t) \end{bmatrix} \]  

(5-32)

where \( U(t) \) represents the sum of upstream inflow to element 5, in the example equal to \( Q_2(t) \).

Combining Equations 5-21, 5-22 and all other overland segment equations with Equations 5-27 to 5-30 and using 5-25 and 5-26, it is possible to write a non-linear state space form of the Kinematic wave-finite differences solution. The form of the model for the illustration example is shown in Figure 5-3. Remember that the model is non-linear, all matrices elements are functions.

At this time it is necessary to point out that in the present
Figure 5-3

State-Space Form of Rainfall-Runoff Model as Applied to Illustration Example

\[ \dot{x}(t) = A \cdot x(t-1) + B \cdot i(t-1) \]  \hspace{1cm} (5-33)

\[ x(t) = \begin{bmatrix}
    y_1(t) \\
    y_2(t) \\
    y_1(t) \\
    y_2(t) \\
    y_1(t) \\
    y_2(t) \\
    A_1(t) \\
    A_2(t) \\
    A_1(t) \\
    A_2(t) \\
    U(t)
\end{bmatrix} \]

\[ i(t) = \begin{bmatrix}
    i_1(t) \\
    i_3(t) \\
    i_4(t)
\end{bmatrix} \]

where \( i_k(t) \) is rainfall at element \( k \)
(Figure 5-3, continued)

\[
\zeta(\cdot) = \begin{bmatrix}
1\theta_2(\cdot) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1\theta_1(\cdot) & 1\theta_2(\cdot) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3\theta_2(\cdot) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3\theta_1(\cdot) & 3\theta_2(\cdot) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \theta_2(\cdot) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \theta_1(\cdot) & \theta_2(\cdot) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \Delta \theta_1(\cdot) & 0 & \Delta \theta_2(\cdot) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \Delta \theta_1(\cdot) & 0 & \Delta \theta_2(\cdot) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \Delta \theta_4(\cdot) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \Delta \theta_4(\cdot) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & E_2(\cdot) & 0 & 0 & 0 & 0
\end{bmatrix}
\]

where \( e(\cdot) \) and \( E(\cdot) \) are the functionals defined in Equations 5-25 and 5-26.
\[ \beta = \begin{bmatrix} \Delta t & 0 & 0 \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \\ 0 & 0 & \Delta t \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \] (5-35)
work infiltration is not considered within the rainfall-runoff model statement. In order to directly apply the proposed methodology, the approach would be to form an effective rainfall input vector by subtracting infiltration. Due to the uncertainty inherent in specifying the infiltration rate at any one time we have that the effective rainfall is the differing of two random variables which will effectively increase the variance of the input in the present analysis. The author acknowledges the degree of importance of this extra variance and suggests its study as possible extensions to this work.

Equation 5-33 is, as previously mentioned, a non-linear equation. As it stands, it is very difficult to handle for the purpose of analysis of variance. Proceed then by expanding 5-33 in a Taylor series and ignoring second order terms:

\[
\begin{align*}
\chi(t) &= \zeta(\chi_{\mu}(t-1)) + \zeta'(\chi_{\mu}(t-1)) \left[\chi(t-1) - \chi_{\mu}(t-1)\right] \\
&\quad + \beta \bar{I}(t-1) + \beta \bar{i}(t-1) - \bar{I}_{\mu}(t-1) \\
&= (5-36)
\end{align*}
\]

where
\[
\begin{align*}
\zeta(\chi_{\mu}(t-1)) &= \text{functional matrix} \quad \zeta(\cdot) \text{ evaluated at the mean solution } \chi_{\mu}(t-1) \\
\zeta'(\chi_{\mu}(t-1)) &= \text{first derivative of } \zeta(\cdot) \text{ evaluated at } \chi_{\mu}(t-1) \\
\bar{I}(t-1) &= \text{mean of input vector.}
\end{align*}
\]

The expected value of Equation 5-36 is simply:

\[
\begin{align*}
\chi_{\mu}(t) &= \zeta(\chi_{\mu}(t-1)) + \beta \bar{i}(t-1) \\
&= (5-37)
\end{align*}
\]
Subtracting this mean from Equation 5-36 results in a zero mean process defined as:

\[
X(t) - \mu_X(t) = \zeta'(X(\mu(t-1)) \left[ X(t-1) - \mu_X(t-1) \right] + \beta \left[ i(t-1) - \mu_i(t-1) \right]
\]

(5-38)

where the zero mean input, \(i(t-1) - \mu_i(t-1)\), is that defined by Equation 5-1 at the beginning of this chapter.

Redefining

\[
i(t) = \hat{i}(t) - \mu_i(t)
\]

(5-39)

\[
D(t) = X(t) - \mu_X(t)
\]

(5-40)

Equation 5-38 becomes:

\[
D(t) = \zeta'(X(\mu(t-1))D(t-1) + \beta \left[ \hat{i}(t-1) \right]
\]

(5-41)

Unfortunately, \( \hat{i}(t) \) is not known with certainty due to incomplete, noisy, observations as discussed in Section 5-2. Equations 5-10 to 5-15 describe an estimation procedure and give the mean square error matrix of that estimation. If the estimate \( \hat{i}(t) \) is used in Equation 5-41, then the mean square error matrix of estimating the (zero mean) state vector, \( D(t) \), becomes:

\[
M.S.E. D(t) = \zeta'(X(\mu(t-1)) \left[ M.S.E. D(t-1) \right] \zeta'^T(\mu(t-1)) + \beta \Sigma(t-1/t-1) \beta^T
\]

(5-42)

where \( \Sigma(\cdot) \) is defined by Equations 5-11 to 5-15 and \( \zeta'(X(\mu(t-1)) \) can be obtained by evaluating the rainfall runoff model defined by 5-33.
using $\mu(t)$ as input. The M.S.E. of $D(0)$ is assumedly 0. (In this work this assumption is trivial, since, for simplicity, dry bed initial conditions are assumed).

Notice that the M.S.E. matrix of $D(t)$ is a measure of the error in estimating vector $\mathbf{x}(t)$. With the elements of $\mathbf{x}(t)$ given by 5-32 it is then clear that the mean square error matrix of $\mathbf{x}(t)$ gives the M.S.E. of depths and flow cross sections in all the conceptual basin elements. For example, the next to last diagonal element in M.S.E.$\mathbf{D}$ ($t$) matrix would clearly be the M.S.E. of the cross section at the outlet of the sample basin (Figure 5-1). Linearizing Equation 5-26, again by Taylor's expansion results in:

$$M.S.E. Q_5(t) = \left[E_5'(5A_2(t)) \mu \right] M.S.E.(5A_2(t)) \tag{5-43}$$

where $E_5'(5A_2(t)) \mu$ = first derivative of $E_5$ (Eq.5-26) evaluated at mean value of $5A_2(t)$.

Equations 5-42 and 5-43 are then the available tools for evaluating data collection networks in reference to discharge prediction accuracy. This is so since they are dependent on the input Mean Square Error covariance matrix, $\Sigma$ which is a function of the network design as shown in Section 5-2.

From the above exercise it is clear that the suggested approach is based on a perfect runoff model. No uncertainty in the model or its parameters is allowed. Uncertainty is introduced only by the stochastic nature of the input. Chapter 6 will briefly discuss the implications of introducing uncertainty in the runoff model.

- 186 -
5.4 Network Analysis Example

5.4.1 Comments on Implementation

Implementation and execution of the analysis procedure described in Sections 5-2 and 5-3 is carried on in various steps.

The first step consists of generating the elements of the matrix $\zeta(\cdot)$ evaluated at a mean solution. This is done by using a rainfall-runoff model based on Equation 5-18 and 5-23 to which a mean storm characterizing the family of storms to be described by Equation 5-1 is input. This mean storm has a given depth duration, and time distribution at each discrete point in space. Each overland element of the schematic basin must be given this mean input. The model, in the form of a computer program, will output all the relevant information for the generation of $\zeta(\cdot)$. Appendix 6 briefly explains the use of this program and provides the relevant listing.

The second step consists of another computer program. This program works in various stages. First, it uses statistical information of the storm family being characterized by the mean storm previously described to estimate the parameters $A(t)$ and $B(t)B(t)^T$ of a first order multivariate Markov model of the form given in Equation 5-1. The parameter estimation is done following the ideas given in Section 5-2. Both, $A(t)$ and $B(t)B(t)^T$ are produced and printed for each time step.

The second stage is to obtain the mean square error matrix of rainfall estimation, $\Sigma(t/t)$, using Equations 5-11 to 5-15, for a given network design, matrix $H$. The third stage takes advantage of the sparsity of matrix $\zeta(\cdot)$ and forms this matrix at each time step using data obtained from the rainfall-runoff program, saving considerable computer
memory. Matrix $\beta$ is also internally formed at this stage. Finally, Equations 5-42 and 5-43 are evaluated for each time step. The program produces as output the mean square matrix of rainfall estimation and the mean square error (and its square root) of discharge at the basin outlet at every time step. Stages 2 and 3 are repeated for any number of alternative network designs desired.

Appendix 7 briefly explains the use of this program, describes its data requirements and gives the relevant listing.

5.4.2 Example Data Description

The analysis of various network alternatives was done for a basin that schematically looks as shown in Figure 5-4. It has a total area of 82 square miles. Possible raingage locations are marked in the figure and coordinate locations are given relative to the coordinate system shown.

Data for the rainfall runoff model as well as for the description and parameters of the overland and stream elements are given in Table 5-1. As mentioned before, no infiltration is considered.

The mean storm used to characterize the family of interest is shown in Figure 5-5 along with one standard deviation around that storm. The storm moved with a velocity of 20 mph from left to right relative to Figure 5-4 in the direction of the x axis.

The time step used in the analysis was 1 hr and the discharge was studied up to the 15th hour.

Rainfall intensities (1 hr accumulation) was assumed to obey a
Legend:
C = overland segments
S = stream segments
→ direction of flow
X = point of observation

Storm Direction

Coordinates of observation points (in miles)

<table>
<thead>
<tr>
<th>segment</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>C2</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>C3</td>
<td>2</td>
<td>6.5</td>
</tr>
<tr>
<td>C4</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>C5</td>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>C6</td>
<td>12</td>
<td>8.5</td>
</tr>
<tr>
<td>C7</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>C8</td>
<td>7.5</td>
<td>11</td>
</tr>
</tbody>
</table>

Figure 5-4 Schematic representation of example basin
### Table 5-1

Basin Elements Data

<table>
<thead>
<tr>
<th>Segment</th>
<th>Slope</th>
<th>Manning's N</th>
<th>Alpha (( \alpha ))</th>
<th>( m )</th>
<th>Length of Flow (ft)</th>
<th>Width (ft)</th>
<th>area (miles) (sq.)</th>
<th>Number of Space Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>-</td>
<td>-</td>
<td>0.500</td>
<td>1.67</td>
<td>4817</td>
<td>40000</td>
<td>6.9</td>
<td>2</td>
</tr>
<tr>
<td>C2</td>
<td>-</td>
<td>-</td>
<td>0.700</td>
<td>1.67</td>
<td>6745</td>
<td>40000</td>
<td>9.7</td>
<td>2</td>
</tr>
<tr>
<td>C3</td>
<td>-</td>
<td>-</td>
<td>0.900</td>
<td>1.67</td>
<td>6804</td>
<td>33000</td>
<td>8.1</td>
<td>2</td>
</tr>
<tr>
<td>C4</td>
<td>-</td>
<td>-</td>
<td>0.700</td>
<td>1.67</td>
<td>5292</td>
<td>33000</td>
<td>6.3</td>
<td>2</td>
</tr>
<tr>
<td>C5</td>
<td>-</td>
<td>-</td>
<td>1.400</td>
<td>1.67</td>
<td>9503</td>
<td>40000</td>
<td>13.6</td>
<td>2</td>
</tr>
<tr>
<td>C6</td>
<td>-</td>
<td>-</td>
<td>1.200</td>
<td>1.67</td>
<td>8146</td>
<td>40000</td>
<td>11.7</td>
<td>2</td>
</tr>
<tr>
<td>C7</td>
<td>-</td>
<td>-</td>
<td>1.500</td>
<td>1.67</td>
<td>7509</td>
<td>42000</td>
<td>11.3</td>
<td>2</td>
</tr>
<tr>
<td>C8</td>
<td>-</td>
<td>-</td>
<td>2.000</td>
<td>1.67</td>
<td>10000</td>
<td>42000</td>
<td>14.3</td>
<td>2</td>
</tr>
<tr>
<td>S4</td>
<td>0.005</td>
<td>0.1</td>
<td>0.410</td>
<td>1.33</td>
<td>42000</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>S3</td>
<td>0.003</td>
<td>0.1</td>
<td>0.318</td>
<td>1.33</td>
<td>33000</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>S2</td>
<td>0.002</td>
<td>0.08</td>
<td>0.324</td>
<td>1.33</td>
<td>40000</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>S1</td>
<td>0.0008</td>
<td>0.07</td>
<td>0.219</td>
<td>1.33</td>
<td>40000</td>
<td>-</td>
<td>-</td>
<td>2</td>
</tr>
</tbody>
</table>
Figure 5-5 Time distribution of mean storm and standard deviation around mean

<table>
<thead>
<tr>
<th>Time</th>
<th>Precipitation (in)</th>
<th>Standard deviation (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>2</td>
<td>0.40</td>
<td>0.24</td>
</tr>
<tr>
<td>3</td>
<td>0.56</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>1.68</td>
<td>0.56</td>
</tr>
<tr>
<td>5</td>
<td>0.92</td>
<td>0.36</td>
</tr>
<tr>
<td>6</td>
<td>0.28</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Total depth = 4.0 inches
correlation function of the single exponential type.

\[ r(x_i^T, t'; x_j^T, t'') = e^{-c \sqrt{\sum (y'_j - y'_i)^2 + \left( (x_j^{t' + UT} - x_i^{t' + UT}) \right)^2}} \]  

(5-44)

The network analysis was done with two different values for the parameter \( c \). The parameter was first given a value of \( c = 0.15 \) which implies "correlation distance" (defined when \( r(\cdot) \) takes the value \( 1/e \)) of close to 7 miles. This value agrees with Huff (1970) results which (fitting an exponential correlation) imply a correlation distance of about 5 miles for 1 minute rainfall rates and about 45 miles for total rainfall depth. The value used, 7 miles, for 1 hour rainfall rates, correctly fall between these values. For sensitivity analysis, a value of \( c = 0.08 \), implying a 12.5 mile correlation distance, was also used.

Table 5-2 gives the assumed measurement error variance at each possible site. Appendix 8 gives a listing of all the data used in the network design program (including data cards punched by the runoff model).

Figure 5-6 shows the mean discharge hydrograph obtained with the mean input shown in Figure 5-5.

It is important to emphasize that this mean discharge is obtained by routing the mean storm given in Figure 5-5 through the deterministic rainfall-runoff model given in Figure 5-3. This mean solution is only used for the linearization given in Equation 5-36 which requires the evaluation of \( \beta'(x_j(t)) \). Once \( \beta'(x_j(t)) \) is evaluated using the mean solution it is only necessary to define \( \bar{\Sigma} \) and \( \Sigma(t/t) \) to find the mean square error of discharge estimation (see Equation 5-42). \( \bar{\Sigma} \) is a function...
Table 5.2

Measurement Error Variance at Possible Station Locations

<table>
<thead>
<tr>
<th>Element ID</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.01</td>
</tr>
<tr>
<td>C2</td>
<td>0.005</td>
</tr>
<tr>
<td>C3</td>
<td>0.02</td>
</tr>
<tr>
<td>C4</td>
<td>0.007</td>
</tr>
<tr>
<td>C5</td>
<td>0.008</td>
</tr>
<tr>
<td>C6</td>
<td>0.005</td>
</tr>
<tr>
<td>C7</td>
<td>0.015</td>
</tr>
<tr>
<td>C8</td>
<td>0.017</td>
</tr>
</tbody>
</table>
Table of discharge values

<table>
<thead>
<tr>
<th>Time (hrs)</th>
<th>Discharge (cfs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.09</td>
</tr>
<tr>
<td>2</td>
<td>3.02</td>
</tr>
<tr>
<td>3</td>
<td>28.48</td>
</tr>
<tr>
<td>4</td>
<td>202.71</td>
</tr>
<tr>
<td>5</td>
<td>931.53</td>
</tr>
<tr>
<td>6</td>
<td>3012.10</td>
</tr>
<tr>
<td>7</td>
<td>9774.38</td>
</tr>
<tr>
<td>8</td>
<td>25718.91</td>
</tr>
<tr>
<td>9</td>
<td>33400.65</td>
</tr>
<tr>
<td>10</td>
<td>25508.41</td>
</tr>
<tr>
<td>11</td>
<td>18963.85</td>
</tr>
<tr>
<td>12</td>
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</tr>
<tr>
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</tr>
<tr>
<td>14</td>
<td>8892.06</td>
</tr>
<tr>
<td>15</td>
<td>7190.36</td>
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Figure 5-6 Hydrograph at basin outlet resulting from mean storm input
of the particular basin and the time interval $\Delta t$ and takes the form shown in Figure 5-3. $\gamma(t/t)$ is evaluated by Equations 5-11 and 5-12 and is a function of the rainfall model parameters, $A(t)$ and $B(t)$, and of the network given by the matrix $H$.

5.4.3 Results and Analysis

The results of network analysis for various network alternatives are shown in Table 5-3 and Table 5-4 in terms of the mean square error of discharge (M.S.E.) and the square root of this parameter.

Tables 5-3 and 5-4 lead to several interesting observations.

With respect to station location it can first be said that for the example given location seems to play a fairly important role. In Table 5-3, for example, there is a noticeable reduction in peak estimate mean square errors when stations are located in upstream reaches. Notice that 1 station in Segment 7 produces considerably less error in peak prediction. This behavior is also observed in networks of more than 1 station. At the same time, though, it can be concluded that concentrating stations in upstream reaches deteriorates the quality of the rising stages of the hydrograph. The rising limb of the hydrograph is dominated by overland areas close to the basin outlet as was expected.

Station location plays an even more important role than station number in some of the cases. For example, in Table 5-3, it is more accurate for peak estimation to have one station at location 7 than two at 1 and 3. Similarly, two stations at 7 and 8 are somewhat better in the peak than 3 at 1, 3, 7 or at 2, 3, 6. Stations at 1, 5 and 7 give

\[ -195 - \]
Table 5-3

Analysis of Network Alternatives Shown in Parenthesis

Correlation Parameter 0.08

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Analysis of Network Alternatives Shown in Parenthesis

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better estimates than stations at 3, 4, 5, 6 everywhere except in the receding portion of the hydrograph.

One of the most important results from this example is the large decrease in mean square error obtained when using 8 stations, one at each possible observation site. The square root of the peak mean square error is reduced by a factor of 5 when going from 1 to 8 stations. Between the most accurate (peak estimation) 4 station alternative to the 8 station option, accuracy in terms of the inverse square root of the peak mean square error increased by a factor of two. These results are contrary to the beliefs of other investigators which say that the "filter" characteristics of the basin are such that only small number of stations are needed for discharge forecasting. It seems that for the example studied in this work that is not the case.

By comparing the square root of mean square error with the mean discharge given in Figure 5-6 it is clear that for most network alternatives the bulk of the uncertainty is in the peak and rising limb of the hydrograph. The coefficient of variation (root mean square over mean) of the receding hydrograph portions is less than 1 for all network alternatives. This is rarely the case in the rising portion except for the 8 station network which gives a coefficient of variation of less than 1 at all times, with the exception of the discharge at 2 hours.

The results in Table 5-3 and 5-4 show very little sensitivity to the rainfall correlation parameter. Varying it from 0.08 to 0.15 (implying correlation distances of 12.5 and 7 miles) made little difference in the discharge mean square error.
5.5 Concluding Remarks

This chapter proposed a method of studying the adequacy of rainfall sampling networks when used together with a rainfall-runoff model. The method is presented and developed in reference to a finite difference solution of the Kinematic wave equations as runoff model. It should be applicable though, with any other distributed runoff model which could be expressed in a state-space form with finite lags.

As it stands, the suggested solution also depends on the users ability in defining a multivariate rainfall model of the type given in Equation 5-1. The parameters of such model could be derived directly from available data or fitted to a multidimensional model of the type suggested in Chapter 4 of this work. The latter approach was the one followed by the author under the assumption that the suggested multidimensional model is a more realistic one.

One limitation of the approach, as well as that of the static problem discussed in Chapters 2 and 3, is that the rainfall model (Eqn. 5-1) is only valid for a specific family of storms. Sensitivity of the network design to various types of storms must be required in any network design exercise.

Some of the advantages of the network design approach discussed here are the following:

1) Allows the user to study accuracy of discharge prediction at any point in the basin, not only its outlet

2) If it were necessary to include a model error, i.e. uncertainty in the runoff model, the approach would allow it.

The need of considering such a stochastic runoff model will
be briefly discussed in Chapter 6.

3) Networks are analyzed in terms of their effect on the whole discharge hydrograph, not only its peak.

All of the above advantages come naturally as a trade-off with the relative numerical complexity of the method.

Many readers would suggest that the planner's interest lies mostly on hydrograph peak and so the network design procedure should be simplified to look at that value only. With a runoff model of the form:

\[ Q_{\text{peak}} = \sum_{j=1}^{N} \rho_j i_j \]  \hspace{1cm} (5-45)

where

\[ i_j = \text{rainfall accumulation (to a given time)} \]
\[ \text{at point (area) } j \]
\[ \rho_j = \text{weighting factor} \]
\[ i = \text{vector of } i_j \text{'s} \]
\[ E[ii^T] = \Psi \]

the peak discharge sensitivity to network configuration could be studied as a static problem with the framework discussed in Chapters 2 and 3 of this work. Nevertheless, the author considers it impossible to suggest such a distributed simplified runoff model with any sort of reliability. At the same time he believes that the peak discharge is not the only parameter of interest of an entire hydrograph.
6.1 Summary

This work accomplished three basic objectives. First, a technique for designing a data collection network for sampling a static, spatial process and obtaining its areal mean was suggested. The process studied was the areal distribution of total depth of a rainfall event. Chapter 2 described a technique based on linear estimation theory which led to an accuracy criteria (mean square error) which accounted for sampling density, location, and instrument noise. Consideration of cost and network accuracy led to an "optimal" network design. Chapter 3 discussed implementation and a detailed example.

The method proved to be consistent in its results; thorough in its consideration of all the aspects of network design; and simple and economically implemented in an electronic computer. Comparison with other available solutions was highly favorable.

The development stages led to careful consideration of error involved in approximating an integral average in space by a discrete procedure. The nature of this error and its interesting dependence on the correlation function definition was discussed in Chapter 2.

Chapter 4 discussed the second objective of this work. There a rainfall generator capable of reproducing storms interior time history and spatial distribution was discussed.

The suggested model used the Taylor hypothesis of turbulence and an isotropy assumption to suggest a correlation function, in time
and space, of the rainfall process. The model is multidimensional and non stationary with time dependent mean and variance everywhere in space. The interior generation technique was based on sampling from the spectrum of the suggested correlation. Examples of generated storms are given showing expected behavior.

Chapter 4 also discussed the limitations of the presented rainfall generation. Suggestions as to possible future solutions of these flaws were given.

The final objective of this work was to develop a theory for analyzing a rainfall data collection network in reference to its use in providing the input to a runoff model when accuracy in the forecast of the output is the objective in the network scheme.

With the use of

1) a multivariate non-stationary version of the rainfall model of Chapter 4,
2) a state-space formulation of a non-linear, Kinematic wave, distributed rainfall-runoff model and
3) a linear dynamic estimator (Kalman-Bucy filter), Chapter 5 showed how the mean square error of basin discharge, at any time, can be obtained as a function of data network configuration. The network is defined by the number of stations, their location and their measurement error. A numerical example was presented showing the effect on discharge of various network alternatives. From the example it was concluded that the accuracy of the discharge prediction was more sensitive to the number of observations and their location than expected by other hydrologists. It was also shown that the given example was
relatively insensitive to changes in the parameter of the rainfall corre-
lation function. The above example implied the strong influence of the ba-
sin's physical characteristics on the network design process.

6.2 Estimation, Filtering, and Control in Hydrology

The techniques of estimation, filtering and control, fre-
quently used in the fields of aeronautics, chemical engineering, and
electrical engineering, have not been widely used in water resources or,
for that matter, civil engineering problems.

Moore (1971) used the Kalman filter to study the sampling of
water quality parameters in a 1-dimensional river system. Hino (1970,
1973) has used the Kalman filter and the Wiener-Hoff theory of stationary
processes for identifying unit hydrograph model parameters.

Young et al (1971, 1974) and Whitehead and Young (1974) also
have worked in the model and parameter identification issues in water
quality issues. Veneziano (1974) and Bras et al (1975) have suggested
applications of estimation theory to other sampling problems in civil
engineering fields.

It is the author's opinion that the use of these techniques
in water resources (and civil engineering in general) deserve consider-
ably more investigation. There are many problems, in the field that fits
the desired framework, problems not only of model identification (or
estimation) and monitoring, but of control as well. Following is a brief
list of some water resources problems that, in the author's opinion
should be studied with estimation, filtering, or control theory.

1) The calibration and analysis of radar rainfall data together
with observed raingage behavior can be studied as a filtering estimation problem.

2) Identification of rainfall and runoff problems as well as the study of "model error" may possibly be viewed as a problem in model identification using linear filtering techniques (see Schweppe, 1973).

3) Forecasting problems has long been recognized as an estimation-prediction system. On-line discharge forecasting systems should be "updated" in real time as new historical observations become available. Such updating of forecast and model should be possible with a Kalman filter approach.

4) The determination of reservoir operating rules with the purpose of satisfying a given objective fits within the framework of control theory, although the author foresees serious difficulties in handling non-symmetrical loss functions.

6.3 Further Research

As with most research, this work generated as many (or more) questions than it attempted to answer. Following is a partial list of some areas that deserve investigation in future work:

1.) The rainfall generator suggested in Chapter 4 should be validated with data. This implies a considerable search or collection of historical rainfall interiors. Data should be analyzed not only to see the validity of Taylor's...
and isotropy assumptions but also to study the form and parameters of possible rainfall correlation functions. The parameters' behavior with sampling intervals, $\Delta t$, and type of storms, should be a source of considerable work. Also related is the study and classification of storm types. The investigation of the probability density functions of storm velocities and directions together with their relation to the correlation functions is another open area of study.

2.) The design of networks for determining long term average rainfall characteristics like the time averaged areal mean rainfall can be studied with the methodology used in this work. Both, Rodriguez and Mejia (1974a) and Lenton (1974) have studied this problem with a different kind of framework.

3.) Infiltration should be included as added stochasticity in studying network design in conjunction with runoff models.

4.) The effect of utilizing other runoff models in the network design should be studied.

5.) Extensive simulation studies should be done to corroborate the analytical "mean square error" results obtained in this work.

6.) Finally, the assumption of a perfect runoff model should
be released. The presented techniques will allow a similar analysis with parallel observations on a stochastic input (rainfall) and output (discharge). Related to this is a necessity of studying runoff model errors. This is a critical issue in determining the degree of accuracy needed in the input since the quality of the output will strongly depend on the inherent uncertainty of the runoff model itself.
BIBLIOGRAPHY


Chadwick, D.G., "Telemetry System Modifications and 1968-69 Operation," Utah Water Research Laboratory, College of Engineering, Utah, State University, Logan, Utah, June 1969


Hershfield, D., "On the Spacing of Rain Gages," IASH and WMO Symposium on Design of Hydrometeorological Networks, Quebec City, 1965


Israelsen, C.E., Griffin, D.L., "USU Telemetering Precipitation Gage Network," Utah Water Research Laboratory, College of Engineering, Utah State University, Logan, Utah, April 1969


Sevruk, Boris F., "Comparison of Mean Rain Catch of Various Gauge Networks," Nordic Hydrolog 5, 1974 50-54 Denmark


Veneziano, D., Personal Communications, M.I.T., 1974

Wei, T.C., and C.L. Larson, "Effects of Areal and Time Distribution of Rainfall on Small Watershed Runoff Hydrographs," Water Resources Research Center, Univ. of Minnesota, Minneapolis, Minnesota, March 1971


Appendix 1

The last two terms in Equation (2-12) can be evaluated (see Section 2.2.2) with integrals of the form

\[
\int_0^{d'} r(r') f(r') \, dr'
\]

Define the correlation function, \( r(r) \), with parameter \( h' \) and on area \( A' \).

Say interest shifts to another area, \( A'' \), defined as

\[
A'' = \frac{A' h'^2}{h''} \quad \therefore \quad A'' h''^2 = A' h'^2
\]

This change in area implies that all distances are scaled by \( \frac{h'}{h''} \), assuming areas are similar.

Therefore, for the new area, Equation Al-1 becomes

\[
\int_0^{d''} r(r'') f(r'') \, dr'' = \int_0^{d'h'/h''} r(r' \frac{h'}{h''}) f(r' \frac{h'}{h''}) \frac{h'}{h''} \, dr'
\]

where

\[
r(r' \frac{h'}{h''}) = e^{-h''} \frac{r'h'}{h''} \cdot e^{-h'r'} = r(r')
\]

\[
f(r' \frac{h'}{h''}) = \frac{h''}{h'} f(r')
\]

Therefore, Equation Al-2 becomes
\[
\int_0^{d''} r(r'') \, f(r'') \, dr'' = \int_0^{d' h'/h''} r(r') \, f(r') \, dr' \quad \text{Al-3}
\]

Equation Al-3 is the same as Al-1 only that the limit of integration changes and the new limit is only a function of \( Ah^2 \).

The first term in 2-12 then obviously depends only on the undimensional area, \( Ah^2 \).
Appendix 2

Analytical Evaluation of Integrals in Mean Square Model Error Expression, for the Quadratic Exponential Case, in a Square Area, $A = \lambda^2$

Double Integral Evaluation:

Changing variables,

\[
\frac{S^2}{A^2} \int_A \int_A e^{-\frac{\partial^2 r^2}{\lambda^2}} dr'dr = \frac{1}{\lambda^4} \int_{-\lambda}^\lambda \int_{-\lambda}^\lambda (\lambda - |x|) (\lambda - |y|) e^{-\alpha^2(x^2+y^2)} dx dy
\]

\[
= \frac{1}{\lambda^4} \int_{-\lambda}^\lambda (\lambda - |y|) e^{-\alpha^2 y^2} dy \left[ \int_{-\lambda}^\lambda (\lambda - |x|) e^{-\alpha^2 x^2} dx \right] dy
\]

(A2-1)

The second integral in A2-1 is:

\[
\int_{-\lambda}^\lambda (\lambda - |x|) e^{-\alpha^2 x^2} dx = 2 \int_{-\lambda}^\lambda (\lambda - x) e^{-\alpha^2 x^2} dx
\]

\[
= 2 \left[ \int_0^\lambda \alpha x e^{-\alpha^2 x^2} dx - \int_0^\lambda x e^{-\alpha^2 x^2} dx \right]
\]

letting $u = \alpha x$

\[
= 2 \left[ \frac{\sqrt{\pi}}{2} \frac{1}{\alpha} \hat{\phi}(\alpha) - \frac{1}{\alpha^2} \int_0^\lambda u e^{-u^2} du \right]
\]

(A2-2)

where

\[
\phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt
\]

\[
= \text{error function}
\]
Finally,
\[
\int_{-\infty}^{\infty} (2 - |x|) e^{-\alpha^2 x^2} = 2 \left[ \int_{-\infty}^{0} \frac{e^{\alpha^2 u^2}}{\sqrt{2\alpha}} \phi (\alpha u) - \frac{1}{\alpha^2} \left( \frac{1}{2} e^{-u^2} \right) \right]_0 \]
\[
= \frac{\sqrt{\pi}}{\alpha} \phi (\alpha l) + \frac{1}{\alpha^2} \left( e^{-\alpha^2 l^2} - 1 \right) \tag{A2-3}
\]

The first integral in A2-1 is the same as the one given by A2-3.

Therefore,
\[
\frac{S^2}{A^2} \int_{A} e^{-\alpha^2 r^2} dr' dr = \frac{S^2}{\sqrt{\pi}} \phi (\alpha l) + \frac{1}{\alpha^2} \left( e^{-\alpha^2 l^2} - 1 \right)^2
\]
\[
= \frac{\pi S^2}{\sqrt{\pi} \alpha^4} \left[ \alpha l \phi (\alpha l) + \frac{\left( e^{-\alpha^2 l^2} - 1 \right)}{\sqrt{\pi}} \right]^2 \tag{A2-4}
\]

Equation A2-4 is only a function of the non-dimensional area $A\alpha^2$.

Now, terms of the form appearing in the last element of 2-12 can be evaluated as follows:
\[
\int_{\mathcal{A}} e^{-\alpha^2 (x - x_1)^2 + (y - y_1)^2} dxdy = \int_{\mathcal{A}} e^{-\alpha^2 \left( (x - x_1)^2 + (y - y_1)^2 \right)} dxdy
\]

Let $u = x - x_1 \quad v = y - y_1$

then,

- 225 -
\[ \int_{-x_1}^{x_1} \int_{-y_1}^{y_1} e^{-\alpha^2 [(x-x_1)^2 + (y-y_1)^2]} \, dx \, dy = \int_{-y_1}^{y_1} \int_{-x_1}^{x_1} e^{-\alpha^2 (u^2 + v^2)} \, du \, dv = \int_{-y_1}^{y_1} e^{-\alpha^2 v^2} \, dv \int_{-x_1}^{x_1} e^{-\alpha^2 u^2} \, du \]

Taking the right-most integral and changing variables,

\[ \int_{-x_1}^{x_1} e^{-\alpha^2 u^2} \, du = \frac{1}{\alpha} \int_{-\alpha x_1}^{\alpha x_1} e^{-\omega^2} \, d\omega = \frac{1}{\alpha} \frac{\sqrt{\pi}}{2} \left[ \varnothing (\alpha (\ell - x_1)) + \varnothing (\alpha x_1) \right] \]

By similarity then,

\[ \int_{A} e^{-\alpha^2 ((x-x_1)^2 + (y-y_1)^2)} \, dx \, dy = \frac{1}{\alpha^2} \frac{\pi}{2} \left[ \varnothing (\alpha (\ell - x_1)) + \varnothing (\alpha x_1) \right] \]

\[ \left[ \varnothing (\alpha (\ell - y_1)) + \varnothing (\alpha y_1) \right] (A2-5) \]
Appendix 3

Derivation of Equation (2-33)

Equation (2-22) defines the relation between the "prior", $\Sigma$, covariance matrix and the "posterior" mean square error matrix as,

$$
\Sigma = \left[ \psi^{-1} + H^T R^{-1} H \right]^{-1}
$$

Therefore,

$$
\Sigma^{-1} = \psi^{-1} + H^T R^{-1} H \quad \text{A3-1}
$$

Let the subscripts (1) and (2) define two consecutive experiments, differing by the location of one station.

The difference between "posterior" covariance matrices for the two experiments is then given by:

$$
\Sigma^{-1} (2) - \Sigma^{-1} (1) = H (2) R^{-1} H (2) - H (1) R^{-1} H (1) \quad \text{A3-2}
$$

Define $\tilde{H} = H (2) - H (1)$, then

$$
\Sigma^{-1} (2) - \Sigma^{-1} (1) = H (1) R^{-1} \tilde{H} + \tilde{H}^T R^{-1} H (2) \quad \text{A3-3}
$$

By direct substitution it is easy to show that

$$
H (1) R^{-1} \tilde{H} + \tilde{H}^T R^{-1} H (2) = F F^T \quad \text{A3-4}
$$

where $F$ is as defined in Section 2.5.

- 227 -
From A3-3 and A3-4 we then have

$$\Sigma^{-1}_{(2)} = \Sigma^{-1}_{(1)} + F^T I F$$  \hspace{1cm} A3-5

which by matrix manipulation becomes,

$$\Sigma_2 = \Sigma_{(1)} - \Sigma_{(1)} F (F^T \Sigma_{(1)} F + I)^{-1} F^T \Sigma_{(1)}$$  \hspace{1cm} A3-6

Equation A3-6 is Equation (2-33) whose derivation was the purpose of this Appendix.
Appendix 4

Network Design for the Estimation of Mean and Total Precipitation - Computer Program

Following is a format and data needs description of the program which performs the network design problem discussed in Chapters 2 and 3 of this work. Also included is a listing of the relevant coding.

As it stands, the program allows maximum array dimensions of 20. All real and complex variables are double precision.

Matrix manipulations are mostly done with I.B.M.'s scientific subroutines belonging to the SLMATH library. Some complex and double precision matrix inversions are made with subroutines available from the M.I.T. Information Processing Center.

The objective function, on which the network design is based, is evaluated within a subroutine named OBJECT, thus easily allowing variations in the definition of the objective function. As it stands, the subroutine evaluates a linear objective function of the form:

\[ 0 = \sigma^2 + C_\lambda \sum_{\ell=1}^{m} C_{\lambda} \]  

as described in the first chapters of this work.

As mentioned in the text, the program can evaluate functional covariances of the Bessel or Exponential type.
DATA CARD DESCRIPTION FOR STATIC NETWORK DESIGN PROBLEMS

Card 1
This card gives the number of designs, IPROB, which correspond to the number of stations, NS, to be allocated. For each value of NS an optimization is done for the number of weights, NW, given in Card 3. Therefore, IPROB x NW is the total number of optimization problems solved in a given run (I3).

Card 2
Reads in NSN(I), I=1,IPROB.
NSN(I) corresponds to the values of NS, the number of stations, to be allocated in different problems (I3).

Card 3
This card must always be in and includes the following variables:

NW  number of problems (number of weights in objective function) to be solved (I4)
ND  dimension of space or network. It can take values 1, 2 or 3 (I4)
NG  number of points in network grid. (I4)
NPS number of points where observations can be made (NPS<NG) (I4, 4x)
IOPT1 takes values of 0,1 or 2; 0 implies covariance matrix is read directly; 1 implies reading co-
ordinates and calculating functional covariance matrix, subjective probabilities are read in and added; 2 implies the grid is rectangular and only starting points and characteristic distances are read in to calculate functional covariance matrix. (II)

IOPT2 takes values 1 or not 1 for diagonal or non-diagonal observation error matrix (II)

ERRVAR variance of model error (F6.0)

PRECI precision to which optimization wants to be carried. It should be the minimum change in objective function desired (F6.0).

PVAR point variance to be used if a functional covariance matrix is calculated. (F6.0)

BE Bessel function parameter to calculate a Bessel type covariance function of form \( BE^2 V K_1(2BEV) \). It should be zero if the functional form is not used (F6.0)

EX parameter in an exponential type covariance function \( \exp(-EXV) \). It should be zero if this functional form is not used (F6.0)

LIMIT maximum number of iterations to be allowed in optimization procedure (I5)

Card 4 WEIGHT (I), I=1 to NW, values of different weights
(or problems) to be used (D16.9)

Card 5

ONLY IF IOPT1 = 1

COORD(I,J) J=1 to ND; I=1 to NG. The order of coordinates should be by columns, that is the subscript J moves faster (F4.2)

Card 6

ONLY IF IOPT1 = 1

SUBJ(I,J) J=1 to NG; I=1 to NG values of subjective covariances to be added to calculated functional covariance matrix. The subscript J (columns) moves faster so input should be given accordingly (F6.0)

Card 7

ONLY IF IOPT1 = 2

XLEN1 length of rectangular grids in the x direction (F6.0)

XLEN2 length of rectangular grids in the y direction (F6.0)

ND1 number of grids in x direction (I3)

ND2 number of grids in y direction (I3)

Card 8

ONLY IF IOPT1 - 2

Same as Card 4
Card 9

ONLY IF IOPT1 = 0

PCOVA(I,J)

J=1 to NG, I=1 to NG, values of covariance matrix moving faster columnwise (D16.9)

Card 10

MEAN(I),

I=1, NG mean rainfall depth values (not necessary) (D16.9)

Card 11

ID(J),

J=1 NPS Identification numbers of grids in given area where observations may be made. Numbering should be in ascending order from left to right and moving up in space grid (I3)

Card 12

ONLY IF IOPT2=1

ER(I),

I=1,NPS variance of measurement error at each of the possible locations. (D16.9)

Card 13

ONLY IF IOPT2=1

EO EE
diagonal terms of measurement error covariance matrix will be EO+EE. Off diagonal terms are given by \( \frac{EO}{EO+EE} \) (D16.9)

Card 14

Hl(I),

I=1 to NPS possible values of elements in
matrix H for allowed grid points (D10.3)

Card 15

C(I) \( I=1, \text{to NPS} \) costs associated with stations in possible observation points (F6.0)

Card 16

\text{GAMMA}(I,1) \quad I=1,NG \quad \text{vector of weights used in calculating areal average from individual points (D10.3)}

Card 17

\text{INIT}(I) \quad I=1,NS \quad \text{identification number of initial design, places where stations are going to be located at start of process. There must be IPROB of these cards, one for each possible value of NS (I3)}
C THIS PROGRAM OPTIMIZES A DATA COLLECTION NETWORK FOR MEASURING A
C SPATIALLY STOCHASTIC EVENT
C PREPARED BY RAFAEL L. BRAS
C VERSION AS OF 12/3/74

INTEGER SAVEI(20)
COMPLEX*16 F(20,2), R6(20,2), R7(2,2), R8(2,20), R9(20,20), ECMPLXCDET
IER
REAL*8 MAXDEL, MAXF, MINF, MEAN(20), NEWVAR, NEWCCS, DREAL, DSCRT, CIMAG
REAL*8 PCCVA(20,20), P(20,20), P1(20,20), GAMMA(20,1), F(20,20), ERRCRK(20,20)
1, R1(2,2), R2(2,2), R3(2,2), R4(2,20), R5(20,20), R10(1,20), R11(21,1), R12(1,1), R13(2,2), SAVE2(20,20), SAVE3(20,20), HEIGHT(20), S, CE
3TERM, CCST, EC, EE, C, CLCH, DIV1, DIV2, C1, REAL 5, OLCR, ARG11, ARG12, ARG22, A
4RG21, DELTA, GVALLE, DOUBL, D3, D4, SSUM, CRCS(20,1), CRCS1, CRCS2, CROS11
5, CROS21, MODERR, XREM, ESTOLO, ESTVAR, RR12, NEWEST, NEWZ, XCLC, RR11, INV
60V(20,20), SCROSS, XNC
DIMENSION COORD(20,3), ID(20), INIT(20), IWORK(2,2), IWORK(20,2),
1SUBJ(20,2), C(20), NSN(20)
COMMON C3, C4, XLEN1, XLEN2, AREA3, X, Y, BE, NG, ND1, ND2
READ(5,28) IPRCB, AREA3
28 FORMAT(13,F10.6)
1 READ(5,31)(NSN(I), I=1, IPRCB)
2 READ(5,11) NW, NC, NS, T, IOPT1, IOPT2, ERRVAR, PRECI, PVAR, BE, EX, EX2, LIMI
1 IT
1 FORMAT(4I4, 4X, 2I1, 6F6.6, 5)
C I OPT1 TAKES VALUES 0 OR 1 OR 2 - 0 IMPLIED COVARIANCE MATRIX READ DIRECTLY,
C 1 IMPLIES READING COORDINATES AND CALCULATING FUNCTIONAL MATRIX, SUBJECTIVE
C PROBABILITIES ARE READ AND ADDED 2 IMPLIES THE GRID IS SQUARE AND ONLY
C STARTING POINTS AND CHARACTERISTIC DISTANCES ARE READ IN, TOGETHER WITH
C SUBJECTIVE PROBABILITIES
C READ(5,6000) (WEIGHT(I), I=1,NW)
600 FORMAT(5D16.9)
1 READ(5,145) XLEN1, XLEN2, ND1, ND2
145 FORMAT(2F6.6, 2I3)
1 IF (ICPT1-1) 20, 21, 140
21 READ(5,2)((COORD(I,J), J=1,NC), I=1,NC)
REAL(5,4)((SUBJ(I,J),J=1,NG),I=1,NG)
2 FORMAT(20F4.2)
   CALL CCVAR(COORD,ND,PVAR,SUBJ,PCCVA)
   IF (ND.FC.0) GOTO 100C
C IMPORTANT *************
C COORDINATES SHOULD BE GIVEN MOVING ROW BY ROW, LEFT TO RIGHT FROM BOTTOM
C LEFT CORNER OF AREA
GO TO 25
140 READ(5,4)((SUBJ(I,J),J=1,NG),I=1,NG)
   CALL CCVAR1(XLEN1,XLEN2,ND1,ND2,PVAR,PCCVA,SUBJ,ND)
   IF (ND.FC.0) GOTO 100C
GO TO 25
20 READ(5,600)((CCVA(I,J),J=1,NG),I=1,NG)
GO TO 25
25 READ(5,6CC)(MEAN(I),I=1,NG)
   READ(5,3)(ID(J),J=1,NPS)
C IMPORTANT *************
C READ INDICATORS OF POSSIBLE STATION LOCATIONS IN ASCENDING ORDER, REMEMBER
C NUMBERING IS LEFT TO RIGHT, BOTTOM UP
3 FORMAT(26I3,2X)
   IF (1OPT2=1) 32,31,32
C 1OPT2 CAN TAKE VALUES 1 CR NOT-1 FCR DIAGONAL CR NACIAAL ERROR
C COVARIANCE MATRIX
31 READ(5,6CC)(CC(I),I=1,NPS)
GO TO 33
32 READ(5,600) EE,EE
7 FORMAT(13F6.0,2X)
33 READ(5,8)(HI(I),I=1,NPS)
8 FORMAT(8C10.3)
   READ(5,7)(C(I),I=1,NPS)
   READ(5,8)(GAMMA(I,1),I=1,NG)
   XNG=NG
   DO 8051 1=1,NG
8051 GAMMA(I,1)=1.CC/XNG
C COMPUTE DOUBLE INTEGRAL OF CCOVARIANCE

CALL CINTEG(DOUBLE)
WRITE (6,7013) CCULE

7013 FORMAT (1HC,13X,'***DOUBLE ',D14.6)
DOBUE=DOUBLE*PVAR

C COMPUTE DOUBLE SUMATION OF CCVARIANCE
IER=C
CALL DMGG(GAMMA,-2CPCOVA,2C,1,NG,NG,RIC,1,IER)
CALL DMGG(R10,1,GAMMA,20,1,NG,1,R12,1,IER)
SLLM=R12(1,1)
WRITE (6,7017) SSUM

7017 FFORMAT(1HO,13X,'**SUM ',D14.6)
C COMPLETE SUM OF CCV(XI,X)
CALL CINTEG(SCROSS,CCROSS,PVAR)
DO 6051 I=1,NG
DO 6051 J=1,NG

6051 INVCV(I,J)=PCOVA(I,J)
CALL DMINV(20,NG,INVCV,DETERM,IIWORK,IERR)
DO 998 KLL=1,IFPCE
NS=NSN(KLL)

C IMPORTANT 4444444
C READ IN INITIAL DESIGN, IF POSSIBLE IN ASCENDING ORDER. REMEMBER NUMBERING
C IS FROM LEFT TO RIGHT, BOTTOM UP
READ(5,3)(INIT(I),I=1,NS)
C FORM H AND ERROR MATRICES, ALSO SUM INITIAL DESIGN COSTS
IER=C
COST=0.0
DO 40 I=1,NS
IF(INIT(I)=IC(L))41,51,41
CONTINUE

41 CONTINUE
40 CONTINUE

51 LL=L
COST=COST+C(LL)
DO 40 J=1,NG
H(I,J)=0.0
H(I,INIT(I))=H(I,LL)
CONTINUE
WRITE (*,999) XCCST=SNGL(CCST)

C FORM ERROR CCVARIANCE MATRICES
IF(IOPT2-1)52,53,52
53 DEC 55 I=1,NS
   DEC 55 J=1,NS
   IF(I-J)59,56,59
56 CC 57 L=1,NPS
   IF(INIT(I)-IC(L))57,58,57
57 CONTINUE
58 L=L
   ERRCR(I,J)=ER(LL)
   GO TO 55
59 ERROR(I,J)=0.0C0
55 CCNTINUE
GO TO 5CC
D=EO/(EO+EE)
DEC 54 I=1,NS
DEC 54 J=1,NS
IF(I-J)60,61,6C
61 ERROR(I,J)=EC4EE
1 GC TC 54
60 ERROR(I,J)=D
54 CONTINUE
9C0 ICCLNT=O
   NNW=C
   IFIRST=1
C COMPLETE POSTERIOR CCVARIANCE MATRIX
C
   CALL CMPPCC(PCOVA,20,H,-20,NG,NG,NS,R1,2C,IER)
   CALL DMPCC(H,2C,R1,20,NS,NC,NS,R2,20,IER)
   CALL DMAGG(R2,2C,ERROR,2C,NS,NS,3,R2,20,IER)
   IF(NS .NE. 1) GOTO 997
      R2(1,1)=1./R2(1,1)
   GO TO 956
997 CALL CMFINV(20,NS,R2,DETERM,IIWORK,IERR)
C COMPUTE CRCSS TERMS FOR INITIAL DESIGN
CALL DMMGG(R1,2C,R2,2C,NG,NS,NS,R4,20,IER)
CALL DMMGG(R4,20,1,2C,NG,NS,NG,3,2C,IER)
CALL DMMGG(R3,20,CRSS,20,NG,NG,1,R4,2C,IER)
CALL DMMGG(GAMMA,-2C,R4,2C,1,NG,1,R11,1,IER)
CROS21=R11(1,1)
CALL DMMC(R3,20,PCVCA,20,NG,NG,NC,R5,2C,IER)
CROS1=TERM
CALL CMCC(R3,20,PCVCA,20,NC,NGN(,R5,2C,IER)
CALL OMM(R10,1,GAMMA,20,1,NG,1,R12,1,IER)
CRCSII=R12(1,1)
WRITE(6,7C21) CROS1,CROS21
7021 FORMAT(1,0,13X,'CROSII ',14.6,CROS2I *,D14.6)
CALL DMAGG()
CALL DMMGC
CALL DPGG
ESTCLD=R12(R1C,1,IER)
MOCERR=SSUM
XCLC=SCRCS2-2,CO*CRCS2I-2,CO*SSUM+2,CC*CRCS1
OLDR=MODERR+SCRCSS
SAVE RELEVANT DATA IN CASE PROBLEM IS REPETEATED WITH DIFFERENT CONSTANTS
IN OBJECTIVE FUNCTION
DO 170 I=1,NS
170 SAVE1(I)=INIT1(I)
DO 171 J=1,NG
171 SAVE2(I,J)=R5(I,J)
DO 172 J=1,NS
172 SAVE3(I,J)=H(I,J)
WRITE(6,200) NS,NG
2C0 FORMAT(1H1,25X,39H THIS PROBLEM OPTIMIZES THE LOCATION OF,13,7TH POINTS)
WRITE(6,201)
2C1 FORMAT(1H,25X,39H CORR CCVARVANCE MATRIX IS GIVEN BY:)
DO 2C2 I=1,NG
202 WRITE(6,203) (PCOVA(I,J),J=1,NG)
2C3 FORMAT(1H10,10E12.5)
WRITE(6,2C4) PRECI
204 FORMAT(1H0,25X,80H OPTIMIZATION PROCEEDED UNTIL MAXIMUM CHANGE IN
1 OBJECTIVE FUNCTION WAS LESS THAN: ,F6.3)
WRITE(6,2C5)
205 FORMAT(1H0,25X,27H INITIAL DESIGN IS GIVEN BY:)
WRITE(6,206) (INIT(I), I=1,NS)
2C6 FORMAT(1H10,3913,13/)
WRITE(6,207) XCOST, CLDR
2C7 FORMAT(1H0,25X,26H COST OF INITIAL DESIGN IS ,FS,2,2X,22H INITIAL
1 ROR VARIANCE, D13.6)
WRITE(6,2C8) 2C9)
5000 FORMAT(1H0,25X,12H VARIANCES: ,11H ESTIMATED, D13.6, 7H MODEL, D13.
16, 7H CROSST, D13.6)
WRITE(6,2C9)
208 FORMAT(1H0,25X,20H INITIAL F MATRIX IS:)
DO 209 I=1,NS
2C9 WRITE(6,210) (H(I,J), J=1,NG)
210 FORMAT(1H10,12D11.3)
WRITE(6,211)
211 FORMAT(1H0,25X,42H INITIAL A POSTERIORI C VARIANCE MATRIX IS:)
DO 212 I=1,NG
212 WRITE(6,203) (R5(I,J), J=1,NG)
C CHECK TYPE OF ERROR MATRIX AND INVERT IF NON-DIAGONAL
IF (IOPT2.EQ.1) I1C,4CCC, I1C
110 CALL DMINV(20,NS,ERROR,DETERM,IIWORK,IER)
4CC0 CONTINUE
270 NNK=NNK+1
INCIC5=1
INCFAK=0
INC=0
IND2=0
C INITIALIZE SEARCH PARAMETERS
111 MAXCEL=C.CDC
INC1=0
IND2=0
MAXI=C.CDC
INCUTC=INCUTC+1
DO EC I=1,NS
C CHECK IF THIS RCW HAS THE CNE CHANGED IN PREVIOUS PASS
IF (I-ICTAM) 81,80,81
81 IOLD=INIT(I)
OLDH=H(I,IOLD)
H(I,IOLD)=0.00C
IND=1
C SEARCH ALL ELEMENTS IN A RCW
DO 84 J=1,NG
C CHECK IF ELEMENT J IS AN ALLOWED SITE
IF (J-IO(T(I))) 84,83,85
C CHECK THAT ELEMENT IS NOT A NON-ZERO CNE IN ANY RCW
83 DO 3CO NN=1,NS
IF (J-INIT(NN)) 300,85,300
3CO CONTINUE
C SET ELEMENT J TO CORRESPONDING F VALUE
86 H(I,J)=H(I,IND)
C FORM F MATRIX ACCORDING TO TYPE OF ERROR
IF (IOPT2=1) 91,90,91
90 DO 1CO II=1,NG
DIVI=H(I,II)/DSQRT(ER(I))
F(I,II)=CCMPLX(DIVI,0.00C)
1CO F(I,II)=CCMPLX(0.00C,0.00C)
DO 6C1 KK=1,NS
IF (IOLD .EQ. ID(KK)) GOTO 6C1
6000 CONTINUE
6C1 DIV2=CLCH/DSQRTER(KK))
F(IOLD,1)=CCMPLX(C.CDC,DIv2)
GOTO 105
91 D1=ERRCR(I,J)/DSQRT(FERRCR(I,J))
DO 1C1 II=1,NG
DIVI=H(I,II)*D1
F(I,II)=CCMPLX(DIVI,0.00C)
1C1 F(I,II)=CCMPLX(C.CDC,C.CCC)
DIV2=F(IOLC)*C1
F(IOLC,1)=DCMPLX(0,0,0,CIV2)
C
COMPLETE CHANGES IN CCVARIANCE
105 DO 3CC0 I1=1,NG
DO 3000 J2=1,NC
REALS=R5(I1,J2)
3CC0 R9(I1,J2)=DCMPLX(REALS,C.CDC)
CALL CMMEC(R9,F,R6,NG,NG,2,20,20,2,2C,2)
CALL CMMEG(F,R6,R7,-2,NG,2,20,2,2,2,2C)
R7(1,1)=R7(1,1)+CMPLX(REALS,C.CDC)
R7(2,2)=R7(2,2)+CMPLX(1.0,0.0)
S=1.0 CD-6
CALL CMME(M(2,2),R7,CDETER,IWORK,S,IERR)
CALL CMMEC(R7,F,R8,2,-2,NG,2,20,2,2,2C)
CALL CMMEG(F,R8,R9,NG,2,NG,2,20,2,2,2C)
DO 115 I=1,NG
DO 115 J2=1,NC
115 R3(I1,J2)=CREAL(R9(I1,J2))
222 CALL DMMGG(R5,2C,R3,2C,NG,NG,NG,R4,20,IERR)
1 CALL DMMGG(R4,20,R5,2C,NG,NG,NG,R3,2C,IERR)
1 CALL DMMGG(R5,20,R3,20,NG,NC,1,R3,20,IERR)
C
COMPLETE ELEMENTS CF CROSS TERM CF TRUE ERPCH VARIANCE
CALL DMMGG(PCOVA,20,R3,20,NG,NG,1,R4,2C,IERR)
CALL CMMEC(GAMMA,-20,R4,20,1,NG,NG,R1C,1,IERR)
CALL DMMGG(R10,1,GAMMA,20,1,NG,1,R11,1,IERR)
CROS1=R11(1,1)
C
CALL DMMGG(R4,20,INVCCV,20,NG,NG,NG,R2,20,IERR)
CALL DMMGG(R2,2C,CROSS,2C,NG,NG,1,R4,2C,IERR)
CALL CMMEC(GAMMA,-20,R4,20,1,NG,NG,R1C,1,IERR)
CROS2=R11(1,1)
CALL DMMGG(GAMMA,-20,R3,2C,1,NG,NG,R1C,1,IERR)
CALL CMMEC(R10,1,GAMMA,20,1,NG,1,R11,1,IERR)
IF (FIRST .NE. 1) GCTC 350
IF (INDICES .NE. 0) GCTC 35C
CALL CMMEC(GAMMA,-20,R5,2C,1,NG,NG,R1C,1,IERR)
CALL DNAMEC(R10,1,CAPMA,20,1,NC,1,R12,1,IER)

INDC5=C
ESTOLD=R12(1,1)
OLCR=ESTCLC+MCCLERR+XCLC
RR12=OLDR

C COMPUTE OBJECTIVE FUNCTION ARGUMENTS

350 ARG11=ERRVAR+RR12
ESTVAR=R11(1,1)
XTERM=SCROSS-2.DO*CROS2-2.DO*SSLM+2.DO*CRCS1
RR11=ESTVAR+MCCL+XTERM
ARG12=CCST
ARG21=ERRVAR+RR11
DO 4001 KK=1,NPS
IF (ICLC.EQ. IC(KK)) GCTC 4002

4001 CONTINUE

4002 ARG22=COST-C(KK)+C(IND)
CALL DEJECT(ARG11,ARG12,ARG21,ARG22,WEIGHT,DELTA,GVALLE,NNW)

C CHECK AND STORE MAXIMUM DELTA AND PLACE WHERE IT OCCURS

IF (DELTA.LE.MAXDEL) GCTC 85

MAXCEL=DELTA
MING=GVALUE
IND1=I
IND2=J
MAXI=H(I,J)
NEWVAR=RR11
NEWEST=ESTVAR
NEWX=XTERM
NEWCCS=ARG22
DO 120 LI=1,NC
DO 120 LJ=1,NC
120 R13(LI,LJ)=R3(LI,LJ)
85 IF (IND.LT. NPS) IND=IND+1
84 H(I,J)=0.0DO
H(I,ICLC)=CLDH
80 CONTINUE

C MAKE CHANGES IN H MATRIX
IF (INCl .EQ. 0) GOTO 7000
H(INC1,INIT(INC1))=0.0
H(INC1,IND2)=MAXH
ICF=INC1
INIT(INC1)=INC2
C

STORE NEW COVARIANCE MATRIX IN R5
DC 125 LI=1,NG
DC 125 LJ=1,NG
125 R5(LI,LJ)=R13(LI,LJ)
C

COST=NEWCOS
RR12=NEWVAR
C

IF STARTING NEW PROBLEM WRITE PERTINENT FEASING
IF (IFIRST .NE. 1) GOTO 225
WRITE(6,220) NNW,WEIGHT(NNW),CLDR
220 FORMAT('IH15H THE IS PROBLEM, I3, 36H WITH OBJECTIVE FUNCTION COEFFICIENT, O16.6, 27H AND INITIAL ERROR VARIANCE, D13.6')
IFIRST=0
225 WRITE(6,215) MAXCEL,MING
215 FORMAT('ICH,25X,26H MAXIMUM DELTA Obtained WAS, C13.4, 28H OBJECTIVE FUNCTION VALUE IS, D15.6')
XNEW=SAGL(NEWCCS)
WRITE(6,216) NEWVAR,XNEW,NEWEST,MCERR,NEWX
216 FORMAT('ICH, 25X, 34H NEW DESIGN GIVES ERROR VARIANCE CF, D15.6, 12H AND 1 COST CF, F12.2/26X, 21H VARIANCES: ESTIMATION, D14.6, 7H MCCEL, D14.6, 17H CROSS, D14.6')
WRITE(6,217)
217 FORMAT('ICH, 25X, 49H THE NEW DESIGN IS GIVEN BY THE FOLLOWING STATICS')
WRITE(6,2C6)(INIT(I), I=1,NS)
XMAX=SAGL(MAXCEL)
IF (XMAX .LE. PRECI) GOTO 250
IF (ICOUNT .LE. LIMIT) GOTO 111
WRITE (6,230)
230 FORMAT('ICH, 25X, 46H THIS PROBLEM EXCEEDS ITERATION LIMITS*****)
7CC0 WRITE(6,7CC1) NNW,WEIGHT(NNW)
7CC1 FORMAT('ICH, 2G10.6')
7CC2 WRITE(6,7CC3) NNW,WEIGHT(NNW)
7CC3 FORMAT('ICH, 2G10.6')
7CC4 WRITE(6,7CC5) NNW,WEIGHT(NNW)
7CC5 Format('ICH, 2G10.6')
FOLLOWED OBJECTIVE FUNCTION COEFFICIENT WAS, E16.8)
WRITE(E,7C02) VALLE, DELTA
7002 FORMAT(1H, 25X, 'LAST OBJECTIVE FUNCTION VALUE', D15.6, ' LAST DELTA'
, D15.6)
250 IF(NNW .GE. NW) GOTO 26C
   251 I=1, NS
   DO 252 I=1, NG
   DO 252 J=1, NG
252 R5(I, J)=SAVE2(I, J)
   DO 253 I=1, NS
   DO 253 J=1, NG
253 H(I, J)=SAVE3(I, J)
   IFIRST=1
   ICONT=C
   COST=XCCST
   GOTO 27C
1 260 CONTINUE
2998 CONTINUE
51C0 CONTINUE
1 CALL EXIT
ENC
SUBROUTINE COVAR(CCCRC, NC, FVAR, SUEJ, PCCVA)
REAL*8 PCOVA(2C, 2C), 03, 04
DIMENSION CCCRC(2C, 3), SUBJ(2C, 2C)
COMMON 03, 04, XLEN1, XLEN2, AREA3, EX, EX2, BE, NC, ND1, ND2
C  GIVEN COORDINATES EVALUATES COVARIANCE MATRIX IF THE EXPONENTIAL
C  OR BESSEL TYPE, ADDS SUBJECTIVE VARIANCE TO ELEMENTS IF ANY
IER=0
C=C.
B=C.
IF (AC.GE.3) C=1.
IF (AD.GE.2) B=1.
DO 440 I=1, NG
DO 440 J=1, NG
XDIST=SQRT((CCCRC(J, I) - CCCRC(1, 1))**2 + 8*(COORD(J, 2) - COORD(I, 2))**2)
1+C*(COORD(J, 3) - COORD(I, 3))**2)
IF (BE.NE.0.) GOTO 402
IF (EX2.NE.0.) GOTO 401
PCOVA(I, J) = EXP(-EX2*XDIST)*FVAR+SUEJ(I, J)
GOTO 440
401 ARCE=BE*XDIST
CALL BESK(ARGB, 1, RK, IER)
IF (IER.GT.1) ND=0
PCOVA(I, J) = BE*XDIST*RK*FVAR+SUEJ(I, J)
GOTO 440
402 PCOVA(I, J) = EXP(-EX2*(XDIST**2.))*FVAR+SUEJ(I, J)
440 CONTINUE
RETURN
END
SUBROUTINE OBJECT(ARG11, ARG12, ARG21, ARG22, WEIGHT, DELTA, GVALUE, NNW)
REAL*8 WEIGHT(2C), ARG11, ARG12, ARG21, ARG22, DELTA, GVALUE
GVALUE = ARG11 + WEIGHT(2C) * ARG12
DELTA = GVALUE - GVALUE
RETURN
END
SUBRCUTINE COVAR1(DX, CY, NX, NY, VAR, PCOVA, SUBJ, ND)
DIMENSION X(20), Y(20)
REAL*8 PCOVA(2C, 2C), D3, D4
DIMENSION C(20, 20), COORDX(20), COORDY(20),
ISUBJ(2C, 2C)
COMMON D3, D4, XLEN1, XLEN2, AREA3, EX, EX2, BE, NC, ND1, NC2
IER=0
ICUT=6

C FIX COORDINATES
C
Y(1)=DY/2.
X(1)=DX/2.
DO 10 I=2, NX
10 X(I)=X(I-1)+DX
DO 20 J=2, NY
20 Y(J)=Y(J-1)+DY
DO 25 J=1, NY
K=(J-1)*NX+I
CONTINUE

NT=NX*NY
DO 30 I=1, NT
30 J=1, NT
IF(I.EQ.J) GO TO 40
D(I,J)=SORT((CCCRDY(I)-CCCRDY(J))**2+(CCCREX(I)-CCCREX(J))**2)
IF(BE.LE.C.) GO TO 32
V=BE*C(I,J)
CALL RESK(V, 1, RK, IER)
PCOVA(I,J)=V*RK
IF(IER.GT.1) ND=0
GO TC 35
32 CONTINUE
   IF (EX .LE. C.) GOTO 33
   PCCVA(I,J)=EXP(-EX*C(I,J))
   GOTO 35
33 PCOVA(I,J) = EXP(-EX2*(D(I,J)**2.))
   GOTO 35
40 PCCVA(I,J)=1.000
25 PCOVA(I,J)=VAR*PCVVA(I,J)+SUBJ(I,J)
30 CONTINUE
RETURN
END
COMMON CMGG(A,B,C,NA,NB,NC,M1,M2,M3,M4,M5,M6)

C THIS SUBROUTINE MULTIPLIES TWO COMPLEX MATRICES A*B AND PUTS THE RESULT
C IN COMPLEX C. NA IS THE ROWS YOU WANT TO MULTIPLY IN A, NB ARE THE ROWS OF B,
C EQUAL TO THE COLUMNS OF A, NC ARE THE COLUMNS OF C. IF A IS REALLY THE
C TRANSPOSE OF THE TRANSFERRED MATRIX MAKE NA NEGATIVE.
C SIMILARLY FOR B MAKE NB NEGATIVE

COMPLEX*16 A(M1,M2),B(M3,M4),C(M5,M6)
N1=1ABS(NA)
N2=1ABS(NB)
N3=1ABS(NC)
DO 2CCC I=1,N1
DO 2CCC K=1,N3
C(I,K)=(0.0D0,0.0D0)
2CCC CONTINUE
DO 2CCC J=1,N2
2CCC CONTINUE
2C01 IF(AA.GT.0.) CCTC 2C02
2C02 C(I,K)=C(I,K)+A(I,J)*B(J,K)
2C00 CONTINUE
RETURN
END
FUNCTION CREAL(X)
REAL*8 DREAL, DIMAG, RES(2)
COMPLEX*16 X, XX
EQUIVALENCE (XX, RES(1))
XX=X
DREAL=RES(1)
RETURN
ENTRY CIMAG(X)
XX=X
DIMAG=RES(2)
RETURN
END
SUBROUTINE WIEP(IER,NC)
RETURN
END
SUBROUTINE CINTEC(SCROSS,CROSS,PVAR)
REAL*8 SUM,SCROSS,CROSS(20,1),C SQRT, XN, XI, XJ, XINT,C3,C4,XXX
COMMON D3,D4,XLEN1,XLEN2,AREA3,EX,EX2,EX,NC,ND1,NC2
SUM=0.00
XN=NC
DO 7 I=1,ND2
DO 7 J=1,ND1
XI=((I-1)+.5)*XLEN2
XJ=((J-1)+.5)*XLEN1
XXX=XINT(XJ,XI)
SUM=SUM+XXX
WRITE(6,7015) XXX,SUM
7015 FORMAT(IHC,13X,'**CROSS ELEMENT ',D14.6,7X,C14.6)
CROSS((I-1)*NC1+J,1)=XXX*PVAR
7 CONTINUE
SCROSS=(SUM*2.0C)/XN*PVAR
WRITE(6,7016) SCROSS
7016 FORMAT(IHC,13X,'***SCROSS TERM ',C14.6)
RETURN
END
REAL FUNCTION XINT*8(XJ, XI)
REAL*8 S1, S2, S3, S4, C3, C4, AR, AR1, XI, XJ
COMMON C3, C4, XLEN1, XLEN2, AREA1, EX, EX2, RE, NC, ND1, ND2
AR1=C*C0
S1=XLEN2*ND2-XI
S2=XLEN1*ND1-XJ
S3=XI
S4=XJ
D3=S1
D4=S2
CALL AREA(AR)
AR1=AR1+AR
C
D3=S1
D4=S4
CALL AREA(AR)
AR1=AR1+AR
C
D3=S3
D4=S4
CALL AREA(AR)
AR1=AR1+AR
C
D3=S3
D4=S2
CALL AREA(AR)
AR1=AR1+AR
C
XINT=AR1/AREA3
RETURN
END
SUERCUTINE AREA(AR)

EXTERNAL FR

REAL*8 SMAX,DSCRT,AR,DMIN,CMAX,C3,C4,VAL,FR

COMMON C3,D4,XLEN1,XLEN2,AREA3,EX,EX2,BE,NC,ND1,ND2

AR=C

SMAX=DSCRT(0.3*C3+0.4*D4)

IER=C

IF(C3 .LE. C4) GOTO 15

DMIN=D4

DMAX=D3

GOTO 16

15 DMIN=D3

DMAX=D4

16 CALL DGGF(FR,0.3,DMIN,48,VAL,IER)

AR=AR+VAL

CALL DGGF(FR,DMIN,DMAX,48,VAL,IER)

AR=AR+VAL

CALL CCGF(FR,CMAX,SMAX,48,VAL,IER)

AR=AR+VAL

RETURN

END
REAL FUNCTION FR*8(V)
REAL*8 F1,R,V
FR=F1(V)*R(V)
RETURN
END
REAL FUNCTION R*8(V)
REAL*8 V,DEX,DEX2,CEX,CEX2,CBE,CBS
COMMON D3,D4,XLEN1,XLEN2,AREA3,EX,EX2,CEX,CEX2,CBE,CBS
IF(V .LT. 5.0)
WRITE (6,50)
RETURN
10IER=0
DIST=CBE*V
IF(R. LT. 0.1E-70) R=0.0
RETURN
20R=CEX(-CEX*V)
IF(R. LT. 0.1E-70) R=0.0
RETURN
40R=1.0
RETURN
END
REAL FUNCTION F1*E(V)
REAL*8 C3,C4,P1,THET12,CARSIN,V,CARCCS
COMMON D3,D4,XLEN1,XLEN2,AFAA3,EX,EX2,PE,AC,AC1,AC2
PI=3.14159265DC
IF (C3 .GE. C4) CCTC 10
C CASE WHEN C4 IS GREATER THAN C3
C
IF (V .LE. D3) THET12=PI/2.C0
IF (V .GT. D3 .AND. V .LE. C4) THET12= CARSIN(D3/V)
IF (V .GT. C4) THET12=CARSIN(D3/V)-CARCCS(D4/V)
GCTC 2C
10 CONTINUE
C CASE WHEN C3 IS GREATER THAN C4
C
IF (V .LE. D4) THET12=PI/2.DC
IF (V .GT. D4 .AND. V .LE. D3) THET12=PI/2.DC-CARCCS(D4/V)
IF (V .GT. D3) THET12=CARSIN(D3/V)-CARCCS(D4/V)
20 CONTINUE
F1=V*THET12/(C3*C4)
RETURN
END
SUBROUTINE DINTEC(DOUBLE)
REAL*8 C,Y,FCT,X,AREA,CSCRT,CEXP,CARCOS,X5,YLEN1,YLEN2,DOUBLe,D3,
COMMON C3,D4,XLEN1,XLEN2,AREA3,EX,EX2,NE,ND1,ND2
EXTERNAL FCT
YLEN1=NC1*XLEN1
YLEN2=NC2*XLEN2
D=CSCRT(YLEN1**2 + YLEN2**2)
Y=FAREA(C,CC,C,FCT,150)
DOUBLE=Y
RETURN
END
REAL FUNCTION FC1*8(X)
REAL*8 X,F1,F2,FC,F,P,XL,S2,DSQRT,DEXP,CARCCS,RK2,XLL,S3,S4,F3,C
13,C4,DEX,DEX2,DEE,DIS
CALL D3,D4,XLEN1,XLEN2,AREA3,EX,EX2,BE,NG,ND1,ND2
DEX=EX
DEX2=EX2
DEE=EE
XL=SQR(I(AREA3))
IF(XLEN1*ND1 .GE. XLEN2*ND2) GOTO 6111
XLL=SQR((XLEN2*ND2)/(XLEN1*ND1))
GOTO 6112
6111 XLL=SQR((XLEN1*ND1)/(XLEN2*ND2))
6112 P=X/XL
IER=0
IF (X .LT. 0.CC5DC) GC TC 2000
IF (EX .NE. 0.) FC=DEXP(-DEY*X)
IF (EX2 .NE. 0.) FO=DEXP(-DEX2*(X**2.))
IF(BE .EQ. C.) GC IC 2CC
IER=C
DIST=X*EE
CALL DEKS(DIST,1,RK,IER)
GOTO 500
2000 F0=1.00
GOTO 600
500 F0=DEBE*X*RK2(2)
600 S2=DSQRT((XLL**2.+(1.DC/XLL**2.)))
IF (P .LE. S2 .AND. P .GE. 0.0) GOTO 2001
F1=C.DC
GOTO 201
2001 F1=3.14159265C0+(P**2.)-2.00*P*(XLL+1.00/XLL)
2001 F2=DSQRT(1.DC+XLL**4.)
IF (P*XLL .LE. S3 .AND. P *XLL .GE. 1.00) GOTO 202
F2=0.00
GOTO 203
202 F2=2.00*DSQRT((P*XLL)**2.-(1.DC)-2.00*CARCCS(1.00/(P*XLL))-(1.00/(X
1LL**2.))*(P*XLL-1.00)**2.)

REAL FUNCTION FC1*8(X)
REAL*8 X,F1,F2,FC,F,P,XL,S2,DSQRT,DEXP,CARCCS,RK2,XLL,S3,S4,F3,C
13,C4,DEX,DEX2,DEE,DIS
CALL D3,D4,XLEN1,XLEN2,AREA3,EX,EX2,BE,NG,ND1,ND2
DEX=EX
DEX2=EX2
DEE=EE
XL=SQR(I(AREA3))
IF(XLEN1*ND1 .GE. XLEN2*ND2) GOTO 6111
XLL=SQR((XLEN2*ND2)/(XLEN1*ND1))
GOTO 6112
6111 XLL=SQR((XLEN1*ND1)/(XLEN2*ND2))
6112 P=X/XL
IER=0
IF (X .LT. 0.CC5DC) GC TC 2000
IF (EX .NE. 0.) FC=DEXP(-DEY*X)
IF (EX2 .NE. 0.) FO=DEXP(-DEX2*(X**2.))
IF(BE .EQ. C.) GC IC 2CC
IER=C
DIST=X*EE
CALL DEKS(DIST,1,RK,IER)
GOTO 500
2000 F0=1.00
GOTO 600
500 F0=DEBE*X*RK2(2)
600 S2=DSQRT((XLL**2.+(1.DC/XLL**2.)))
IF (P .LE. S2 .AND. P .GE. 0.0) GOTO 2001
F1=C.DC
GOTO 201
2001 F1=3.14159265C0+(P**2.)-2.00*P*(XLL+1.00/XLL)
2001 F2=DSQRT(1.DC+XLL**4.)
IF (P*XLL .LE. S3 .AND. P *XLL .GE. 1.00) GOTO 202
F2=0.00
GOTO 203
202 F2=2.00*DSQRT((P*XLL)**2.-(1.DC)-2.00*CARCCS(1.00/(P*XLL))-(1.00/(X
1LL**2.))*(P*XLL-1.00)**2.)

REAL FUNCTION FC1*8(X)
REAL*8 X,F1,F2,FC,F,P,XL,S2,DSQRT,DEXP,CARCCS,RK2,XLL,S3,S4,F3,C
13,C4,DEX,DEX2,DEE,DIS
CALL D3,D4,XLEN1,XLEN2,AREA3,EX,EX2,BE,NG,ND1,ND2
DEX=EX
DEX2=EX2
DEE=EE
XL=SQR(I(AREA3))
IF(XLEN1*ND1 .GE. XLEN2*ND2) GOTO 6111
XLL=SQR((XLEN2*ND2)/(XLEN1*ND1))
GOTO 6112
6111 XLL=SQR((XLEN1*ND1)/(XLEN2*ND2))
6112 P=X/XL
IER=0
IF (X .LT. 0.CC5DC) GC TC 2000
IF (EX .NE. 0.) FC=DEXP(-DEY*X)
IF (EX2 .NE. 0.) FO=DEXP(-DEX2*(X**2.))
IF(BE .EQ. C.) GC IC 2CC
IER=C
DIST=X*EE
CALL DEKS(DIST,1,RK,IER)
GOTO 500
2000 F0=1.00
GOTO 600
500 F0=DEBE*X*RK2(2)
600 S2=DSQRT((XLL**2.+(1.DC/XLL**2.)))
IF (P .LE. S2 .AND. P .GE. 0.0) GOTO 2001
F1=C.DC
GOTO 201
2001 F1=3.14159265C0+(P**2.)-2.00*P*(XLL+1.00/XLL)
2001 F2=DSQRT(1.DC+XLL**4.)
IF (P*XLL .LE. S3 .AND. P *XLL .GE. 1.00) GOTO 202
F2=0.00
GOTO 203
202 F2=2.00*DSQRT((P*XLL)**2.-(1.DC)-2.00*CARCCS(1.00/(P*XLL))-(1.00/(X
1LL**2.))*(P*XLL-1.00)**2.)
2C3 \[ S4 = \sqrt{1.0 + \left(\frac{1.0}{XLL}\right)^4} \]

IF (P/XLL \leq S4 AND P/XLL \geq 1.0) GCTC 204

\[ F3 = 0.0 \]

GCTC 205

\[ F3 = 2.0 \times \text{DSQRT} \left( \left(\frac{P}{XLL}\right)^2 - 1.0 \right) - 2.0 \times \text{CARCCS} \left(\frac{1.0}{(P/XLL)}\right) - (XLL^2) \]

\[ F = F0 \times \left(\frac{1.0}{XLL}\right) \times (2.0 \times P \times (F1 + F2 + F3)) \]

FCT=F

RETURN

ENC
REAL FUNCTION FAREA*(A,B,FCT,N)
REAL*8 A,B,H,S,FCT,D3,D4
COMMON D3,D4,XLEN1,XLEN2,AREA3,EX,EX2,BE,NG,ND1,ND2
H=(B-A)/N
IF(N .EQ. 1) GC1C 10
S=0.00
NM=N-1
DO 1 I=1,NM
  1 S=S+FCT(A+I*H)
FAREA=*((FCT(A)+FCT(B))/2.+S)
RETURN
10 FAREA=C.5D0*H*(FCT(A)+FCT(B))
RETURN
END
Appendix 5

Rainfall Generator

Following is a data card description and listing of the rainfall generator described in Chapter 4.

The program generates exterior characteristics from exponential distributions. Interior generation is done by sampling from the spectrum of a single exponential correlation function.

There are two options of graphical output representation. One is a linear hyetograph, plot of rainfall distribution in time at any given station. The other is 2-dimensional areal plots of the storm at a given time.

Both point and mean areal intensity values can be generated.

As it stands, the program uses a constant correlation parameter for all generated storms. Similarly it is presently programmed for a given storm velocity moving in the direction of the defined x axis.

The generator is programmed in Fortran IVG and was tested in an IBM 370/168. Core requirements are about 220K.
## Data Cards for Rainfall Generator

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Variable</th>
<th>Description</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ILOPTST(N), N=1,25</td>
<td>Id's of stations whose hyetograph is desired</td>
<td>2512</td>
</tr>
<tr>
<td>2</td>
<td>ISKIP</td>
<td>time intervals at which 2-dimensional storm graphs are desired</td>
<td>I3</td>
</tr>
<tr>
<td></td>
<td>MAXTIM</td>
<td>maximum number of 2-dimensional plots, per storm, desired. Default value is 5</td>
<td>I3</td>
</tr>
<tr>
<td>3</td>
<td>XLEN</td>
<td>length in miles in the x direction of area (if rectangular)</td>
<td>F6.0</td>
</tr>
<tr>
<td></td>
<td>YLEN</td>
<td>length in miles in the y direction of area (if rectangular)</td>
<td>F6.0</td>
</tr>
<tr>
<td></td>
<td>NX</td>
<td>grid dimension in x direction</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>NY</td>
<td>grid dimension in y direction</td>
<td>I5</td>
</tr>
<tr>
<td>Card No.</td>
<td>Variable</td>
<td>Description</td>
<td>Format</td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>-------------</td>
<td>--------</td>
</tr>
<tr>
<td>NPTS</td>
<td>number of points in area</td>
<td>I5</td>
<td></td>
</tr>
<tr>
<td>ISEED</td>
<td>odd integer less or equal to 5 digits, seed for random number generator</td>
<td>I5</td>
<td></td>
</tr>
<tr>
<td>IND1</td>
<td>0 generate point coordinates in regular grid</td>
<td>I2</td>
<td></td>
</tr>
<tr>
<td>IHARM</td>
<td>number of harmonics used in generation</td>
<td>I10</td>
<td></td>
</tr>
<tr>
<td>IPLOT</td>
<td>1 graph output hyetograph</td>
<td>I3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 do 2 dimensional plots of storm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 do both types of plots</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MTAU</td>
<td>mean time between storms</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>VTAU</td>
<td>variance of time between storms</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>MDEPTH</td>
<td>mean storm depth</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>VDEPTH</td>
<td>variance of storm depth</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>MTDUR</td>
<td>mean of storm duration</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>VTDUR</td>
<td>variance of storm duration</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>storm speed (mph)</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>correlation function parameter</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>Card No.</td>
<td>Variable</td>
<td>Description</td>
<td>Format</td>
</tr>
<tr>
<td>----------</td>
<td>----------</td>
<td>-------------</td>
<td>--------</td>
</tr>
<tr>
<td>DELPER</td>
<td>percent time interval of undimensional hyetograph</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>VMIN</td>
<td>synoptic standard deviation at a point, noise or std. dev. related to rainfall before storm reaches point</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>YEARS</td>
<td>number of years of generation desired</td>
<td>F10.0</td>
<td></td>
</tr>
<tr>
<td>AREAL</td>
<td>&quot;Areal&quot; for average areal intensities, &quot;point&quot; for point intensities</td>
<td>A4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>PINT(I)</td>
<td>ordinates of I=1,...1/DELPER undimensional hyetograph</td>
<td>10F8.4</td>
</tr>
<tr>
<td>7</td>
<td>VINT(I)</td>
<td>std.dev.(in pct.) of I=1,...1/DELPER undimensional hyetograph</td>
<td>10F8.4</td>
</tr>
<tr>
<td>8</td>
<td>IX10</td>
<td>These variables are odd</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>IX20</td>
<td>integers</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>IX30</td>
<td>less or equal to 5 digits in length</td>
<td>I5</td>
</tr>
<tr>
<td>Card No.</td>
<td>Variable</td>
<td>Description</td>
<td>Format</td>
</tr>
<tr>
<td>---------</td>
<td>--------------</td>
<td>-----------------------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>9</td>
<td>COORDX(I)</td>
<td>(only if I = 1, NPTS) x coordinate of points</td>
<td>13F6.2,2x</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(only if IND1=1)</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>COORDY(I)</td>
<td>(only if I=1,NPTS) y coordinate of points</td>
<td>13F6.2,2x</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(only if IND1=1)</td>
<td></td>
</tr>
</tbody>
</table>
C THIS PROGRAM GENERATES STORMS EXTERIORS AND INTERIORS

C-----AUTHORS R. L. BRAS AND W. HILL

INTEGER LTC
RFPL INT(50,60),MTAU,MTLUR,MDEPTH
COMMON /A/ RES(10,60),TCTDEP(50),X(50),Y(50),COORDX(50),INT,
1COORDY(50),VINT(50),AINT(50),PINT(50),VPINT(50),MTAU,TDUR,MTDUR,
2DEPTH,DELPER,DTC,MDEPTH,VTAU,VEUR,VDEPTH,TAU,TOTAL,DATE,TTIME,
3VMIN,DT,C,U,XLEN,YLEN,NX,NY,NPTS,NDT,NPER,ISEED,IX10,IX20,IX30,
4IOPTST(25),IND1,IHARM,IFLOT,ISKIP,AREAL
COMMON /F/ MAXTIM

CALL INPUT
READ(5,999) (TOPTST(N),N=1,25)
999 FORMAT(25I2)
C ISKIP THE NUMBER OF TIME INTERVALS SKIPED IN THE 2D PLOT
READ(5,991) ISKIP,MAXTIM
991 FORMAT(213)
IF (ISKIP.LT.0) ISKIP=1
IF (IND1.EQ.0) GOTO 9
READ(5,27) (COORDX(I),I=1,NPTS)
READ(5,27) (COORDY(I),I=1,NPTS)
27 FORMAT(13F6.2,2X)

GOTO 6
6 CALL COORT
DATE=DATE+TDUR/24.
6 CALL EXTER
CALL MFAN
CALL RAIN
CALL STORM
CALL PRINT1
IF (IPLT.NE.1.AND.IFLOT.NE.3) GOTO 100
CALL PLOT1
100 CONTINUE
IF (IPLT.NE.2.AND.IFLOT.NE.3) GOTO 110
CALL PLOT2
110 CONTINUE
IF (DATE.LE.TTIME) GOTO 6
CALL EXIT
END

MAIN0037
MAIN0038
SUBROUTINE INPUT
C INPUT ALL NECESSARY DATA FOR SUBROUTINES
INTEGER DT
REAL INT(50,60),MTAU,MTDUR,MDEPTH
COMMON /A/ RES(10,60),TCTDWP(50),X(50),Y(50),COORDY(50),INT,
1COORDX(50),AINI(50),PINT(50),VPINT(50),MTAU,TDUR,MTDUR,
2DEPTH,DELPER,DT,C,MDEPTH,VTU,VDPD,VDPR,MTDUR,TTOTAL,DATE,TTIME,
3VMIN,DT,C,VTLEN,YLEN,NX,NY,NPTS,NPER,ISEED,IX10,IX20,IX30,
4TOPTST(25),IND1,ITHRM,ILOT,ISKIP,AREAL
C READ DIMENSIONS OF AREA, NUMBER OF POINTS AND DISTANCE BETWEEN THEM
---ILOT=1->CALL PLOT1: ILOT=2->CALL PLOT2: ILOT=3->CALL BOTH.
READ(5,2) XLEN,YLEN,NX,NY,NPTS,ISEED,IND1,ILOT
WRITE(6,2001) ITHRM
2001 FORMAT('IHM=",',I12)
TF (IHM.GE.600) ITHRM=50
2 FORMAT(2F6.0,4I5,12I10,13)
AREA=XLEN*YLEN
C READ VELOCITY, C, TIME OF STORM DURATION, ITS MEAN, MEAN OF TIME BETWEEN
C STORMS, DEPTH OF STORM AND PERCENT INTERVAL
READ(5,30) MTAU,CTAU,MDEPTH,VDEPTH,MTDUR,VTDUR
30 FORMAT(6F10.0)
READ(5,3) U,C,DELPER,VMIN,YEARS,AREAL
3 FORMAT(5F10.0,6X,A4)
TTIME=YEARS*365.
NPER=1./DELPER
C READ INTENSITY AND ITS VARIANCE
READ(5,1) (PINT(I),I=1,NPER)
READ(5,1) (VPINT(I),I=1,NPER)
1 FORMAT(10F8.4)
DO 6 J=1,10
X(J)=0.
6 X(J)=0.
READ(5,4) IX10,IX20,IX30
4 FORMAT(315)
WRITE(6,27)
27 FORMAT(1H1,30X,'INPUT INFORMATION')
WRITE(6,28)
28 FORMAT('AREA DESCRIPTION: ', 20X)
WRITE(6,29) XLEN,YLEN,NX,NY,NPTS,C,AREA
29 FORMAT('XLEN(MI) = ', I4,' YLEN(MI) = ', F6.2,' NX = ', I4,' NY = ', I4,' NPTS = ', I4,' COV. PAR. = ', F6.2,' AREA(SQMI) = ', F7.3)
WRITE(6,35)
35 FORMAT('STORMS EXTERIOR STATISTICS: ', 20X)
WRITE(6,36) MTAU,VTAU
36 FORMAT('MEAN', 6X,'VARIANCE')
WRITE(6,37) MTDUR,VTDUR
37 FORMAT('TIME PET. EVENTS', 6X,'F8.3')
WRITE(6,38) MDEPTH,VDEPTH
38 FORMAT('DURATION', 6X,'F8.3')
WRITE(6,39) DEPT,HTH,FRH
39 FORMAT('DEPTH', 6X,'F8.3')
WRITE(6,40) UNDIMENSIONAL MEAN STORM:
40 FORMAT('1H', 20X) DO 90 L=1,NPER
41 FORMAT('INTENSITY(IN)', 5X,'STD.DEV.')
41 FORMAT('INTENSITY(IN)', 5X,'STD.DEV.')
42 FORMAT('D = ', 25X,'F8.4',5X,'F8.4')
43 FORMAT('TOTAL PERIOD OF GENERATION WILL BE ',F7.2,'1X','YEARS')
C INITIALIZE VARIABLES
DATE=0.0
TAU=0.0
TDUR=0.0
TOTAL=0.
RETURN
END
SUBROUTINE EXTER
C GENERATES STORM EXTERIOR
INTEGER DII
REAL INT (50, 60), MTAU, MTDUR, MDEPTH
COMMON /A/ RES (10, 60), IGTDEP (50), X (50), Y (50), COORDY (50), INT,
1COORDY (50), VINT (50), AINT (50), PINT (50), VPINT (50), MTAU, TDEP, MTDUR,
2DEPTE, DELLER, DTC, MDEPTH, VTAU, VDUR, VDEPTH, TAU, TTOTAL, DATE, TTIME,
3VMIN, DT, C, U, XLEN, YLEN, NX, NY, NPTS, NPER, ISEED, IX10, IX20, IX30,
4IPTST (25), IND1, IHARM, IELOT, ISKTP, AREAL
C GENERATE STORM PARAMETERS
ZIAM1 = 1.0 / MTAU
ZIAM2 = 1.0 / MTDUR
BETA = MDEPTH
C CALCULATE TIME BETWEEN STORMS FOR NEXT STORM
CALL GEN (IX10, ZIAM1, TD)
TAU = TD
DATE = DATE + TAU / 24.
C CALCULATE TIME OF DURATION OF ACTUAL STORM
CALL GEN (IX20, ZIAM2, TD)
TDUR = TD
C CALCULATE DEPTH OF ACTUAL STORM
CALL GEN (IX30, BETA, TD)
DEPTH = TD * TDUR
RETURN
END
SUBROUTINE GEN(IX,ZLAM,ID)
C THIS SUBROUTINE FINDS THE VALUE OF TIME OR DEPTH PARAMETERS THAT
C DESCRIBE A STORM
CALL SNUF(IX,IX,X)
IX=IX
TD=-(ALOG(1.0-X))/ZLAM
RETURN
END
SUBROUTINE MEAN
C CREATES MEAN AERIAL STORM AND ITS VARIANCE
INTEGER DTC
REAL INT(50,60),MTAU,MTLUR,MDEPTH
COMMON /A/ RES(10,60),ICIDEP(50),X(50),Y(50),COORDX(50),INT,
COORDY(50),VINT(50),AINI(50),PINT(50),VPINT(50),MTAU,TDUL,MTDUR,
2DEPTH,DELPFR,DTC,MDEPTH,MTAU,VDUR,VDEPTH,TAU,TOTAL,DATE,TIME,
3VMIN,DT,C,U,XLEN,YLEN,NX,NY,NPTS,NPTF,ISEFD,IX10,IX20,IX30,
4IOPST(25),IND1,THARM,IFLOT,ISKIP,AREAL
C CALCULATE TIME INTERVAL
DT=DELPFR*TDUR
DTC=(XLEN/U)/DT+1
NPT=NPF*DT+1
C CALCULATE TIME INTERVAL
DC 1 J=1,NPER
AINT(J)=DEPTH*PINT(J)
VINT(J)=DEPTH*VPINT(J)
RETURN
END
SUBROUTINE STORM

C FORM STORM INTERIOR BY MULTIPLYING RESIDUALS BY VINT

INTEGER DTC
REAL INT (50, 60), MTAU, MTEUR, MDEPTH
COMMON /A/ RES (10, 60), ICTDEP (50), X (50), Y (50), COORDX (50), INT,
1 COORDY (50), VINT (50), AINT (50), PINT (50), VPINT (50), MTAU, TDUP, MTDUR,
2 DEBT, DELFER, DTC, MDEPTH, VTAU, VDUR, VDEPTH, TAU, TTOTAL, DATE, TTIME,
3 VMIN, DT, C, U, XLEN, YLEN, NX, NY, NPTS, NPER, ISEED, IX10, IX20, IX30,
4 ICPTST (25), INDI, IHARM, IELOT, ISKIP, AREAL
DC 60 L=1, NPTS
60 TOTDEP (L) = 0.
DO I = 1, NDT
DO 1 K = 1, NPTS
L1 = (COORDX (K) / U) / DT
IF (J-1-L1) 2, 3
AVG = AINT (J-1-L1)
VAR = VINT (J-1-L1)
GO TO 4
2 AVG = 0.
VAR = VMIN
3 IF ((J-1-L1) .GT. NPER) GOTO 2
1 AVG = AINT (J-1-L1)
VAR = VINT (J-1-L1)
GO TO 4
2 AVG = 0.
VAR = VMIN
4 INT (K, J) = AVG + RES (K, J) * VAR
IF (INT (K, J) .LT. 0.) INT (K, J) = 0.
TOTDEP (K) = TOTDEP (K) + INT (K, J)
1 CONTINUE
RETURN
END
SUBROUTINE PRINT1
    INTEGER ITC
    REAL INT(50,60),MTAU,MTURI,MDPTCH
    COMMON /A/ RES(10,60),TCTD2P(50),X(50),Y(50),COORDX(50),INT,
    1COORDY(50),VINT(50),AINT(50),VINT(50),MTAU,TDUR,MDUR,
    2DEPTimations,DT,VTU,MDU,VTU,MDU,DEPTimations,TAU,TOTAL,DATE,TIME,
    3VINT,VT,UN,XYL,NE,SN,NTS,NTN,MDT,IFED,IX10,IX20,IX30,
    4IOPTST(25),IND1,THARM,ILFLOT,SKIP,AREAL
    DATA XM/'M'/
    WRITE(6,1)
      1 FORMAT(1H1,20X,'GENERATED STORM INFORMATION')
    WRITE(6,45) U,TU,DEPH
      45 FORMAT(1H0,20X,'VELOCITY = ',F6.2,3X,'DURATION (HRS) = ',F6.2,3X,'TOTAL MEAN
    1AL DEPTH (IN)',F8.4)
    WRITE(6,46) DATE
      46 FORMAT(1H0,20X,'DATE OF OCCURRENCE FROM INITIAL TIME (DAYS)',F8.3)
      IF (AREAL.LT.XM) GOIC 100
        WRITE(6,2) 100
      WRITE(6,20)
        GOTC 101
    101 CONTINUE
      20 FORMAT(1H0,3X,'TIME',3X,'AREA ID',3X,'COORDX (MI)',3X,'COORDY (MI)'
    1,3X,'RESIDUAL',3X,'DEPTH/DT (IN)')
    2 FORMAT(1H0,3X,'TIME',3X,'POINT ID',3X,'COORDX (MI)',3X,'COORDY (MI)'
    1,3X,'RESIDUAL',3X,'DEPTH/DT (IN)')
      DO 4 J=1,NDT
        TIME=(J-1)*DT
        WRITE(6,9) TIME,T,COORDX(I),COORDY(I),RES(I J),INT(I J)
      DO 4 I=1,NPTS
        WRITE(6,13)
          13 FORMAT(1H0,12X,'STATION',5X,'TOTAL DEPTH')
      DO 17 M=1,NPTS
        WRITE(6,15) M,TOTDEP(M)
      15 FORMAT(1H0,'',15X,I4,9X,F7.3)
      RETURN
SUBROUTINE TO CALCULATE COORDINATES IN UNIFORM GRID

POINTS ARE ASSUMED IN CENTER OF EACH GRID

SUBROUTINE COORD

REAL INT(50,60),MTAU,MTLUP,MDEPTH
INTEGER DTC

COMMON /A/ RES(10,60),ICTD(DP(50),X(50),Y(50),COORDX(50),INT,
1 COORDY(50),VINT(50),AINT(50),PINT(50),VPIINT(50),MTAU,TDUR,MTDUR,
2 DEPTH,DELPER,DTC,MDEPTH,VTAU,VDUR,VDEPT,TDTOTAL,DATET,TTIME,
3 VMIN,DTC,UXLEN,YLEN,NX,NY,NPTS,NDT,NPPR,ISEED,IX10,IX20,IX30,
4 IOPTST(25),IND1,ICHARM,ITLOT,ISKIP,AREA

DX=XLEN/NX
DY=YLEN/NY

C SET FIRST COORDINATE POINTS

Y(1)=DY/2.
X(1)=DX/2.

DO 10 I=2,NX
10 X(I)=X(I-1)+DX

DO 20 I=2,NY
20 Y(I)=Y(I-1)+DY

DO 25 J=1,NY
25 CONTINUE

RETURN

END
SUBROUTINE RAIN
INTEGER LDC
REAL LAMDA
REAL IN (50, 60), MTAU, MTDUR, MDEPTH
COMMON /A/ RES (10, 60), ICTDEP (50), X (50), Y (50), COORDX (50), INT,
1COORDY (50), VINT (50), AINT (50), VPINT (50), MTAU, TDUR, MTDUR,
2DEPT, DELPER, DTC, MDEPTH, VTAU, VDUR, VDEPTH, TAU, TTOTAL, DATE, TTIME,
3VIN, DT, C, U, XLEN, YLEN, NX, NY, NPTS, NDT, NPER, TSFED, IX10, IX20, IX30,
4OPTST (25), IND1, IHARM, IELOT, ISKIP, AREAL
* DATA XM/'M'/
1 DO 10 K=1, NPTS
DO 10 J=1, NDT
10 RES (K, J) = 0.
C------ TO REDUCE HARMONICS, ALTER EQUATION (") BELOW
C- IHARM=5C
  DO 20 T=1, IHARM
  CALL RGEN (X1, X2, X3, X4)
  L1=XLEN/NX
  L2=YLEN/NY
C....... AREAL='AREA'|'POINT': IT IS INPUTED WITH OTHER DATA.
IF (AREAL.GT.XM) GOTO 45
  LAMDA= (4./(L1*L2*COS (X3) *SIN (X3))) *SIN ((L1*COS (X3) *X1)/2.*
  * SIN (L2*SIN (X3) *X1/2.))
GO TO 60
45 LAMDA=1.
60 CONTINUE
DO 25 L=1, NDT
DO 25 K=1, NPTS
  P=COOPDX (K) + (U*(L-1) * DT)
25 RES (K, L) = RES (K, L) + LAMDA*COS ((P*COS (X3) + COORDY (K) *SIN (X3)) *X1+X2)
20 CONTINUE
DO 30 L=1, NDT
DO 30 K=1, NPTS
30 RES (K, L) = RES (K, L) / (IHARM**.5)
RETURN
END
SUBROUTINE RGEN(X1, X2, X3, X4)

INTEGER DTC
RGEN INT(50,60), MTAU, MTDUR, MDEPTH
COMMON /A/ RES(10,60), TCTDEP (50), X(50), Y (50), COORDX(50), INT,
1COORDY (50), VINT(50), AINT (50), PINT(50), VPINT(50), MTAU, TDUR, MTDUR,
2DEPTH, DELPER, DTC, MDEPTH, VTAU, VDUR, VDEPTH, TAU, TTOTAL, DATE, TTIME,
3VMIN, DT, C, U, XLEN, YLEN, NX, NY, NPTS, NDT, NPER, ISEED, TX10, IX20, IX30,
4IOPST(25), IND1, IHARM, IFLAT, ISKIP, AREAL
PI=3. 1416
CALL RANDU(ISEED, ISEED, R)
X1= ((1. - R)**2 - 1.)**.5) * C
CALL RANDU(ISEED, ISEED, R)
X2=2.*PI*R
CALL RANEU(ISEED, ISEED, R)
X3=2.*PI*R
CALL RANDU(ISEED, ISEED, R)
X4=2.*PI*R
RETURN
END
SUBROUTINE PLOT1

/** THIS PLOTS DEPTH OF RAIN IN A TIME VS. DEPTH GRAPH. (6 STATIONS). **/

INTEGER DTC
INTEGER CPTSTA
REAL INT(50,60),MTAU,MTDUR,MDEPTH
COMMON /A/ RES(10,60),TCTDEP(50),X(50),Y(50),COORDX(50),INT,
1COORDY(50),VINT(50),PINT(50),VPINT(50),MTAU,TDUR,MTDUR,
2DEPTH,DELPER,DTC,MDEPTH,VTAU,VDUR,VDEPTH,TAU,ITOTAL,DATE,TTME,
3VMIN,DT,C,U,XLEN,YLEN,NX,NY,NPTS,NRT,NPFP,ISEED,IX10,IX20,IX30,
4ICPTST(25),IND1,INHARM,IPLOT,ISKIP,AREAL,
5DIMENSION OPTST(6)
6DIMENSION STORE(9)
7DIMENSION TSCALE(5)
8DIMENSION CHAR(5)
9DIMENSION PLOT(110,20),CODE(7)
10DATA CODE/'1','2','3','41, 5#,'6'/

C...... CPTST->OPTIONS OF WHICH STATIONS SHOULD BE PRINTED
C READ THE STATIONS TO BE PRINTED IN A LINEAR PLOT.

IIOPT=0
77 CONTINUE
777 CONTINUE
100 IF (OPTSTA(N).LE.0) GOTO 78
200 IF (OPTSTA(N).LE.0) CPTSTA(N)=N DEFAULT
201 WRITE(6,104) DATE
GOTO 751
750 WRITE(6,108) DATE
751 CONTINUE
MAXX=100
MAXY=20
MAXXP=110
DO 10 I=1,MAXIXP
DO 10 J=1,MAXIY
PLOT(I,J)=CHAR(1)
IF ((I-1)*EQ.10*(I/10)) PLOT(I,J)=CHAR(3)
IF ((J-1)*EQ.5*(J/5)) PLOT(I,J)=CHAR(2)
IF (J.EQ.MAXIY) PLOT(I,J)=CHAR(2)
10 CONTINUE
C...FIND LARGEST VALUE
DMAX=0
DO 4 I=1,NPTS
DO 4 J=1,NDT
4 IF (INT(I,J).GT.DMAX) DMAX=INT(I,J)
SCALEY=3.*DMAX/2.
SCALEX=DT*NDT
C
INTENSITY->INT(STATION,TIME)
ISIZE=100/NDT
IGRAPH=ISIZE*NDT
DO 21 IX=1,IGRAPH,ISIZE
DO 22 ISTAT=1,6
IY=MAXIY+4.4999-MAXIY*INT(OPTSTA(ISTAT),IX/ISIZE+1)/SCALEY
IF (IY.GT.MAXIY) IY=MAXIY
IX9=IX+ISIZE-1
IF (PLOT(IX,IY).GE.CODE(1) AND PLT(IX,IY).LE.CODE(6)) GO TO 60
IF (PLOT(IX,IY).NE.CHAR(1)) AND
A PLOT(IX,IY).NE.CHAR(2) AND
B PLOT(IX,IY).NE.CHAR(3)) GOTO 60
DO 50 K=IX,IX9
50 PLOT(K,IY)=CODE(ISTAT)
GOTO 70
60 STORE(1)=PLOT(IX,IY)
IX=IX
MULT=1
IX1=IX+1
DC 61 IX1=IX1,IX9
MULT=MULT+1
IF (STORE(1).EQ.PLOT(IX1,IY)) GOTO 65
IF (STORE(1).LT.CODE(1)) GOTO 65
C----- >>>>IF THE FOLLOWING WORKS, YOU NEED NOT DIMENTIONALIZE STORE.
61 CONTINUE
GOTO 70
65 PLOT(IX, IY) = CODE(IISTAT)
     IX1=IX+1
     IF (IX1.GT.IX9) GOTO 70
     DO 66 K=IX1,IX9
86 PLOT(K, IY) = PLOT(K-MULT, IY)
70 CONTINUE
22 CONTINUE
21 CONTINUE
DO 100 IY=1,MAXIY
     IF (IY.NE.(10*IY/10)) GOTO 110
     TEMP= SCALEY*(MAXIY-IY)/MAXIY
     WRITE(6,102) TEMP, (PLOT(IX, IY), IX=1, MAXIX)
     GO TO 100
110 CONTINUE
     WRITE(6,101) (PLOT(IX, IY), IX=1, MAXIX)
100 CONTINUE
DO 780 I=1,10,2
     J=(I+1)/2
870 TSCALE(J) = SCALEX*(I-1)*.1
     WRITE(6,103) TSCALE
101 FORMAT(' ',20X,100A1)
102 FORMAT(' ',F15.5X,100A1)
103 FORMAT(' ',10X,5(F15.5X),' TIME(HOURS) ')
104 FORMAT('1 DEPTH (INCHES) ' STORM HYETOGRAPH: TIME OF OCCURANCE
6 =',F10.3, ' POINT VALUES')
105 WRITE(6,106)
     WRITE(6,105) (CODE(N), OFSTA(N), N=1,6)
106 FORMAT(' ','A1', ' -> STATION NUMBER',I5)
107 FORMAT(' ','/','/','/','/')
108 FORMAT('1 DEPTH (INCHES) ' STORM HYETOGRAPH: TIME OF OCCURANCE
6 =',F10.3, ' AREAL VALUES')
78 CONTINUE
SUBROUTINE PLOT2

C---->THIS IS A 2D (LENGTH VS. LENGTH) GRAPH.

INTEGER DTC
REAL INT(50,60),MTAU,MDLUR,MDEPTH
COMMON /A/ RES(10,50),ICTDEP(50),X(50),Y(50),COORDX(50),INT,
1CCORDY(50),AINT(50),PINT(50),VPINT(50),MTAU,TDUR,MDLUR,
2DDEPTH,DELFPR,DTC,MDEPTH,MTAU,TDUR,VDUR,VDEPTH,TAU,TTOTAL,DATE,TTIME,
3VMIN,DR,C,U,XLEN,YLEN,NX,NY,NPTS,NDT,NPER,ISFED,IX10,IX20,IX30,
4IOPST(25),IN1,IFLAM,IFLOT,ISKIP,APREAL

COMMON /B/ MAXTAM
DIMENSION TTT(6)
DIMENSION CODE(8),PLOT(90,50)
DIMENSION CHAR(5)
C /'X','X','*','*','*','*','*','/'
DATA CHAR/'X','X','*','*','*','*','*','/'
DATA CODE/'X','X','*','*','*','*','*','/
C NVALUE->THE NUMBER OF POSSIBLE CHARACTERS GRAFED
NVALUE=6
NVALUE=NVALUE+1
C----M IS THE SIZE OF THE PLOTTED SQUARE (ORIGINALLY 3 X 3).
C----RECALL THE OLDEN DAYS WHEN M1X=M1Y=M1=M+1
M=4
M1=M+1
M1X=13
M1Y=9
M1Y=44/NY
M1X=88/NX
IF (MAXTIM.LE.0) MAXTIM=5
MAXT=90
MAXJ=50
MAXJ1=MAXJ+1
DO 30 J=1,MAXJ
DO 30 I=1,MAXI
30 PLOT(I,J)=CHAR(1)
NX31=NX*M+1
NX33=NX*M+3
NY31 = NY * M + 1
NX31 = NX * M1 + 1
NX31 = NX * M1X + 1
NX31 = NX * M1 + 3
NX31 = NX * M1X + 1
NX31 = NX * M1 + 1
NX31 = NX * M1Y + 1

DC 31 I = 1, NX31
31 PLOT(I, MAXJ + 1 - NY31) = CHAR(2)
   DO 32 J = 1, NY31
32 PLOT(NX31, MAXJ + 1 - J) = CHAR(3)

DC 20 IDTIM = 1, MAXTIM
   IDTIME = 1 + (IDTIM - 1) * ISKIP
   DMAX = 10. ** (-7.)
   DMIN = 10. ** 8

C
   DO 1 J = 1, MAXTIM, ACTIVATING THIS STATEMENT-> UNITS IN GLOBAL TIME (DMA
      J = IDTIM
         IF (J .GT. NDT) GOTO 70
   1   DO 1 I = 1, NPTS
      IF (INT(I, J) < DMIN .AND. INT(I, J) > (10. ** (-4))) DMIN = INT(I, J)
      IF (INT(I, J) > DMAX) DMAX = INT(I, J)
      IF (DMIN < 10. ** (-4)) DMAX = 10. ** (-4)
      IF (DMIN .LE. DMIN) DMIN = 0.9999 * DMAX
      IF (AREAL < GT .AND. CHAR(5)) GOTO 531
   1
      WRITE(6, 997) DATE
      GOTO 532

531 WRITE(6, 998) DATE
      CONTINUE
      TEMP = DT * (IDTIME - 1)
      WRITE(6, 998) IDTIME, TEMP
      IF (INT = 0
         NX3 = NX * M1X
         NY3 = NY * M1Y
         NY4 = NY * M1Y
         NX4 = NX * M1X
         DC 10 J = 1, NY4, M1Y
DO 10 I=1,NX4,M1X
IPOINT=IPOINT+1
TTI=NVALUE* (INT(IPOINT,IDTIME)-DMIN)/(DMAX-DMIN)
IF (TTI.LT.0) TTI=1
DO 10 II=1,M1X
DO 10 JJ=1,M1Y
PLOT(I+II-1,MAXJ1-(J+JJ-1))=CODE(TTI+2)
IF (II.EQ.1) PLOT(I+II-1,MAXJ1-(J+JJ-1))=CHAR(2)
IF (II.EQ.1) PLOT(I+II-1,MAXJ1-(J+JJ-1))=CHAR(3)
CONTINUE
10 K=MAXJ-M1Y*NY
K=MAXJ-M1Y*NY
DO 50 J=K,MAXJ
IF (PLOT(2,J).EQ.CHAR(2)) GOTO 51
JHAT=MAXJ-J
TEMP=YLEN*(JHAT-JHAT/M1Y)/((M1Y-1)*NY)
WRITE(6,995) TEMP,(PLOT(I,J),I=1,MAXI)
GOTO 50
51 WRITE(6,990) (PLOT(I,J),I=1,MAXI)
50 CONTINUE
DO 65 N=1,MAXI,15
PLOT(N,1)=CHAR(4)
CONTINUE
WRITE(6,989) (PLOT(N,1),N=1,MAXI)
989 FORMAT(' ',15X,110A1)
DC 2001 T=1,MAXI,15
J=(I/15)+1
TTT(J)=XLFN*(I-1)/NX3
2001 CONTINUE
WRITE(6,994) TTT
WRITE(6,992) CODE(1)
DO 60 N=2,NVALUE
TEMP=DMIN+(N-1)*(DMAX-DMIN)/NVALUE
TEMP=DMIN+(N-2)*(DMAX-DMIN)/NVALUE
WRITE(6,993) CODE(N),TEMP,
(TEMP)
**WARNING, TIME PARAMETER EXCEEDED IN PLOT2**

```
FORMAT('15X,100A1')
FORMAT('25X,A1,' = NO PRECIPITATION')
FORMAT('25X,A1,' = '1E14.4,' TO 'E12.4,' INCHES')
FORMAT('10X,6(E15.5)')
FORMAT('E9.3,' MILES', 100A1)
FORMAT('1 2-D GRAPH (AREA) . TIME OF STORM=' ,F10.3,' DAYS .')
FORMAT('1 2-D GRAPH (POINT) . TIME OF STORM=' ,F10.3,' DAYS .')
FORMAT(' TIME IDENTIFICATION ',I7,' = ',F6.3,' HOURS INTO STORM')
CONTINUE
RETURN
WRITE(6,982)
RETURN
END
```
Appendix 6

The Rainfall-Runoff Model

Following is a data card description of the finite difference rainfall-runoff model described in Chapter 5. The model should be used to obtain the mean solution resulting from a mean rainfall input. It will give discharge hydrographs at any desired element from which values for $e_k'(y(t))$ or $E_k'(A(t))$ can be evaluated. It would also punch out in cards all the elements of the matrix $\xi'(\cdot)$ evaluated at the mean solution.

Punched cards will be in the format and order necessary for use by the network design program (Appendix 7) after the following simple rearrangements.

Without altering general output order:

1) Group all cards with format $18X, D10.3, 2X, D10.3$

2) Follow by group with format $10X, I3, 2X, I3, 2X, D10.3$.

3) Follow by group with format $13X, I3, 2X, I3, 2X, D10.3$.

The program is programmed in Fortran IV G and operational in an IBM 360 or 370 or equivalent. It utilizes approximately 250 K.

The program is adequate for use as a general rainfall runoff model (see Bras, 1974), if the card punch in subroutines FLOW, QUPS and QLAT are suppressed. For use together with the network
design procedure suggested in this work the condition that

$$\alpha_k \frac{m_k A}{\Delta x} \frac{\Delta t}{\Delta x} < 1.3$$

A5-1

for all elements $k$ at all times is required. This condition assures
the stability of the finite difference scheme discussed in Chapter 5
(see Bras, 1972, and Bras, 1974, for further discussion).

For general rainfall runoff use, the model is always stable
and condition A5-1 is not a limitation. The reader is again referred
to Bras, 1972, and Bras, 1974, for general discussion of the rainfall
runoff model.

A listing of the model follows the data and description.

Present array dimensions limits the number of elements
in the basin model to 50. For use with the network design program,
more stringent limitations apply, see Appendix 7 for further infor-
mation.
# Data Cards Description

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Variable</th>
<th>Description</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NSTRM</td>
<td>No. of storms to be run</td>
<td>I5</td>
</tr>
<tr>
<td>2</td>
<td>TITITLE</td>
<td>General title of run</td>
<td>20A4</td>
</tr>
<tr>
<td>3</td>
<td>Must be blank</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>TITTRE</td>
<td>Title of problem</td>
<td>18A4</td>
</tr>
</tbody>
</table>

(each storm is a problem)

ICA       | 1 read catchment data | I2     |
|          | 0 do not, use previous |        |

ICA       | index defining type of storm of storm |

ICA       | 1 rectangular pulse  |
|          | 2 triangular         |
|          | 3 block shape        |
|          | 4 use previous rain data |

INDX      | 1 read NDX(I)        | I2     |
|          | 0 do not, reuse values of preceding problem |

IND11     | 1 or 0, punch QOBS or not | I1     |

IND12     | 1 or 0, punch QSYNT or not | I1     |

(note: for first storm ICA and INDX must be 1)
<table>
<thead>
<tr>
<th>Card</th>
<th>Variable</th>
<th>Description</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>(always needed if ICA = 1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>NSEG</td>
<td>no. of segments</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>NRG</td>
<td>no. of raingages (max. 50)</td>
<td>F10.0</td>
</tr>
<tr>
<td></td>
<td>AROOF</td>
<td>area of typical roof in sq.ft.</td>
<td>F10.0</td>
</tr>
<tr>
<td></td>
<td>NDISCH</td>
<td>no. of discharge points</td>
<td>I2</td>
</tr>
<tr>
<td></td>
<td>DROOF</td>
<td>linear roof discharge</td>
<td>F5.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(note: a pair of cards 6 and 6B are needed for each element)</td>
<td></td>
</tr>
<tr>
<td>6a</td>
<td>ISEG(I)</td>
<td>segment descriptor</td>
<td>A4</td>
</tr>
<tr>
<td></td>
<td>IUP(I,J) (J=1,3)</td>
<td>segments directly upstream of segment I</td>
<td>3A4</td>
</tr>
<tr>
<td></td>
<td>ILAT(I,J) (J=1,4)</td>
<td>lateral segments to I</td>
<td>4A4</td>
</tr>
<tr>
<td></td>
<td>ITYPE(I)</td>
<td>type of segment</td>
<td>I1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 rectangular channel</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 pipe</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 triangular channel</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 $a$ and $m$ are given, $Q_{max} = \infty$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 overland flow - turbulent</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 overland flow - laminar</td>
<td></td>
</tr>
</tbody>
</table>

- 290 -
<table>
<thead>
<tr>
<th>Card</th>
<th>Variable</th>
<th>Description</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>junction</td>
<td>junction (dummy segment)</td>
<td>I1</td>
</tr>
<tr>
<td>8</td>
<td>gutter</td>
<td>gutter</td>
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</tr>
<tr>
<td>IPR(I)</td>
<td>1</td>
<td>print discharge from segment I</td>
<td>F6.0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>do not</td>
<td></td>
</tr>
<tr>
<td>FLGTH(I)</td>
<td>length of segment I</td>
<td></td>
<td>F5.0</td>
</tr>
<tr>
<td>SLOPE(I)</td>
<td>slope of segment I</td>
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<td>F5.0</td>
</tr>
<tr>
<td>FRN(I)</td>
<td>Manning's n of segment I</td>
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<td></td>
</tr>
<tr>
<td>PARAM(I,J) (J=1,2)</td>
<td>parameters which depend on segment type</td>
<td>2F5.0</td>
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</tr>
<tr>
<td>Type</td>
<td>PARAM(I,1)</td>
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<td>PARAM(I,2)</td>
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<tr>
<td></td>
<td>width of segment</td>
<td></td>
<td>depth</td>
</tr>
<tr>
<td></td>
<td>pipe diameter (ft)</td>
<td></td>
<td>cross-section. area of manhole (ft²)</td>
</tr>
<tr>
<td></td>
<td>width at 1 ft. depth</td>
<td></td>
<td>cross section. area</td>
</tr>
<tr>
<td></td>
<td>a</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>width of segment</td>
<td></td>
<td>% impervious</td>
</tr>
<tr>
<td></td>
<td>width of segment</td>
<td></td>
<td>% impervious</td>
</tr>
<tr>
<td></td>
<td>none</td>
<td></td>
<td>none</td>
</tr>
<tr>
<td></td>
<td>cross sectional area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Card</td>
<td>Variable</td>
<td>Description</td>
<td>Format</td>
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<tr>
<td>------</td>
<td>----------</td>
<td>--------------------------------------------------</td>
<td>--------</td>
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<tr>
<td></td>
<td>LGIV(I)</td>
<td>1 read parameters and ( m ) directly</td>
<td>(10X,I2)</td>
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<td></td>
<td></td>
<td>0 do not read</td>
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<tr>
<td></td>
<td></td>
<td>(note: this feature is only used to read ( \alpha ) and ( m ) for an overland segment with ( \text{ITYYPE}(I) = 5 ) or 6)</td>
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</tr>
<tr>
<td></td>
<td>NROOF(I)</td>
<td>number of houses in segment ( I )</td>
<td>I2</td>
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<td>GROOF(I)</td>
<td>% of roof discharging directly into gutters</td>
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<td>MAELEV(I)</td>
<td>elevation of manhole corresponding to pipe segment ( I ) (ft)</td>
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<td>6B</td>
<td>RCOEFF(I,J)</td>
<td>weighting coefficients for raingages</td>
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<td>J=1,NRG</td>
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<td>ALPHA(I)</td>
<td>( \alpha )</td>
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<td></td>
<td>EM(I)</td>
<td>( m )</td>
<td>F10.0</td>
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<tr>
<td>Card</td>
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<tr>
<td>------</td>
<td>--------------</td>
<td>------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>8</td>
<td>(if IPLUI= 1)</td>
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<td>rainfall intensity (in/hr)</td>
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<td>DT</td>
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<td>time increment ((\Delta t) in min.)</td>
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<td>OSI</td>
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<td>output sampling interval (min)</td>
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<td>9</td>
<td>(if IPLUI = 2)</td>
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<td>storm duration (hrs)</td>
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<td>EXPRE</td>
<td>F10.0</td>
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<td>maximum rainfall</td>
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<td></td>
<td>EXPRE</td>
<td>F10.0</td>
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<td></td>
<td>intensity (in/hr)</td>
<td></td>
</tr>
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<td>F10.0</td>
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<td>ECOMP</td>
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<td>time to maximum rainfall</td>
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<td>intensity (hrs)</td>
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<td>10</td>
<td>(if IPLUI = 3)</td>
<td>NDP</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. of rainfall data points</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>OSI</td>
<td>F10.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>output sampling interval (min)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>ECOMP</td>
<td>F10.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>end of computations (min)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>DT</td>
<td>F10.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>time increment (min)</td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td>293</td>
</tr>
<tr>
<td>Card</td>
<td>Variable</td>
<td>Description</td>
<td>Format</td>
</tr>
<tr>
<td>------</td>
<td>----------</td>
<td>--------------------------------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>11</td>
<td>CAFCS</td>
<td>optional scaling factor for output, usually 1</td>
<td>F10.0</td>
</tr>
<tr>
<td>11a</td>
<td>(if IPLUI = 3 and along with card 10, one for each hyetograph time)</td>
<td>TRAIN(J)</td>
<td>time since beginning of rainfall (min)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>QOBS(I)</td>
<td>observed historical discharge (cfs)</td>
</tr>
<tr>
<td>11b</td>
<td>(together with each 11)</td>
<td>P(I,J)</td>
<td>Rainfall intensity (in/hrs)</td>
</tr>
<tr>
<td>12</td>
<td>(if IPLUI = 4)</td>
<td>NDP</td>
<td>No. of rainfall data points</td>
</tr>
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<td></td>
<td>OSI</td>
<td>output sampling interval (min)</td>
</tr>
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<td>ECOMP</td>
<td>end of computations (min)</td>
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<td>DT</td>
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<td>CAFCS</td>
<td>optional scaling factor for output, usually 1</td>
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<tr>
<td>13</td>
<td>(if INDX = 1)</td>
<td>NDX(I)</td>
<td>I = 1,25</td>
</tr>
<tr>
<td>Card</td>
<td>Variable</td>
<td>Description</td>
<td>Format</td>
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<td>------</td>
<td>----------</td>
<td>-----------------------------------------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>14</td>
<td>NDX(I), I = 1,25</td>
<td>no. of sections in segment I, spatial intervals</td>
<td>25I3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(if INDX=1, and only if there are more than 25 segments in model)</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>(note: remember that the index I corresponds to the order element data was entered)</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>ILOSS</td>
<td>initial lump loss of rainfall if desired (in)</td>
<td>F5.0</td>
</tr>
<tr>
<td></td>
<td>F0</td>
<td>initial loss rate in Horton's equation (in/hr)</td>
<td>F5.0</td>
</tr>
<tr>
<td></td>
<td>FC</td>
<td>steady lose rate in Horton's equation (in/hr)</td>
<td>F5.0</td>
</tr>
<tr>
<td></td>
<td>PK</td>
<td>decay rate (min(^{-1}))</td>
<td>F5.0</td>
</tr>
<tr>
<td></td>
<td>DPERV</td>
<td>detention depth (in) in pervious areas</td>
<td>F5.0</td>
</tr>
<tr>
<td></td>
<td>DIMP</td>
<td>detention depth (in) in impervious areas</td>
<td>F5.0</td>
</tr>
</tbody>
</table>
REAL ISEG, IUP, ILAT, ILCSS
COMMON /BLOC01/ IOUT, INPT, NL, MAXL, NSEG, NRC, KSEG(5C)
COMMON /BLOC02/ TITLE(2C), TITRE(18), NPAGE
COMMON /BLOC03/ ILCSS, FC, FK, DETER(5C), TINFIL(5C), CPERV, CIMP
COMMON /BLOC05/ IPR(50), FLGTH(50), SLOPE(50), FRM(5C)
COMMON /BLOC17/ LGIV(5C), AMCCF, NCISCH, CRCCF, GROCF(5C), NROOF(5C)
COMMON /BLOC3C/ QDHS(4CC), QSYNT(4CC), TCUTFIL(400)
INPT=5
ICUT=6
NPAGE=1
MAXL=55
READ(INPT,5103) ASTRM
51C3 FORMAT(I5)
READ(INPT,5CC0) TITLE
5CC0 FORMAT(20A4)
READ(INPT,5104) TITRE
5104 FORMAT(18A4)
CALL PAGE
15 WRITE(ICUT,210)
210 FORMAT(/26X,15HFLOW ROUTING : 43HKINEMATIC THEORY - FINITE DIFFE
ENCE METHOD//)
NL = NL + 3
19 CONTINUE
DO 1028 JMJ=1,NSTRM
READ(INPT,5302) TITRE, ICA, IPLUI, INCX, INC11, IND12
53C2 FORMAT(18A4,3I2,2I1)
91 CALL PAGE
IF(ICA.EQ.0) GOTO 5305
CALL CATCH
5305 CONTINUE
CALL RAIN(IPLUI, INCX)
20 WRITE(ICUT,218)
218 FORMAT(/26X,21HINFILTRATION MODEL : 12H FERTON'S LAW//)
NL = NL + 3
READ(INPT,140) ILCSS, FC, FK, CIMP, CPERV
140 FORMAT(6F5.0)
90 NL=NL + 13
WRITE(ICUT,6005) ILCSS,FO,FC,FK,CPERV,CIMP
6005 FORMAT('///48X,23HINfiltration Parameters///
1 5CX,14Hinitial lcss =,F5.2,7H inches/
2 43X,19Hinitial loss rate =,F5.2,1CH inches/hr/
3 43X,18Hsteady lcss rate =,F5.2,10F inches/hr/
4 4CX,24Hexponential decay rate =,F5.3,12H minutes**-1/
5 4CX,26Hpervicious detention depth =,F5.3,7H inches/
6 4CX,28Himpervious detention depth =,F5.3,7H inches)
NL = NL + 3
CALL ROUTE(IPLUI,INCI1,IND12)
1C28 CONTINUE
END
SUBROUTINE AM
REAL ISECIUPILATIS
COMMON /BLOC01/ LCL1, INPT, NL, MAXL, NSEG, NRC, KSEC(50)
COMMON /BLOC05/ LUP(5C, 3), ILAT(5C, 4), ISEG(5C)
COMMON /BLOC07/ LPR(5C), FLGTP(5C), SLOPE(5C), FRN(5C), TOTRAI, TDEP
COMMON /BLOC08/ MAX(5C), STC(50), ELEV(50)
COMMON /BLOC09/ ALPHA(5C), EM(50)
COMMON /BLOC14/ ITYPE(50), NCD(50), CX(5C), ELEMAX(5C)
COMMON /BLOC15/ RCCF(50, 5C, 50), PAR(50, 2), CRCCF(50), FRCCF(5C)
COMMON /BLOC17/ LGII(5C), ARCCF(50), NDISCH, CRCCF, CRCCF(50), ARCCF(50)
DO 10 I = 1, NSEG
N = ITYPE(1)
GO TO (20, 30, 40, 50, 60, 60, 70, 80, 90, 90)
90 Z1 = SQRT(PAR(1, 1))
Z2 = 1. + (PAR(1, 1)**2.)
ALPHA(I) = 1.182/FRN(I)*SQRT(SLCPE(I))**((Z1/(1. + SQRT(Z2)))**(2./3.))
EM(I) = 1.33
QMAX(I) = ALPHA(I)*PAR(1, 1)**2.*EM(I)
GCTC 1C
20 ALPHA(I) = 1.49/FRN(I)*SQRT(SLOPE(I))/PAR(1, 1)**(2./3.)
AMAX = PAR(1, 1)*PAR(1, 2)**(2./3.)
EM(I) = 1.67
QMAX(I) = ALPHA(I)*AMAX**2.*EM(I)
GO TO 10
30 AMAX = 3.14*PAR(1, 1)**2./4.
QFULLL = 1.49/FRN(I)*AMAX**(PAR(1, 1)/4.)**(2./3.)*SQRT(SLCPE(I))
ALPHA(I) = QFULL/(AMAX**1.25)
EM(I) = 1.25
QMAX(I) = QFULL
GO TO 1C
40 Z1 = PAR(1, 1)
Z2 = 1. + (PAR(1, 1)**2.)
Z3 = Z1/Z2
ALPHA(I) = .94/FRN(I)*SQRT(SLOPE(I))*Z3**((1./3.))
EM(I) = 1.33
QMAX(I) = ALPHA(I) * (PARAM(I,2) ** EM(I))
GO TO 10
50 ALPHA(I) = PARAM(I,1)
EM(I) = PARAM(I,2)
QMAX(I) = 10. ** 10
GO TO 10
60 IF (LGIV(I).EQ.1) GC TC 63
   ALPHA(I) = 1.49 / FRN(I) * SQRT(SLPE(I))
   EM(I) = 1.67
   GO TC 64
63 READ(INPT,91) ALPHA(I), EM(I)
91 FORMAT (2F10.0)
64 QMAX(I) = 10. ** 10
SCE IF (NRCOF(I).EQ.C) GC TC 1C
   ROOF = NRCOF(I) * ARCOF
   TOTARE = PARAM(I,1) * FRTA(I)
   AREDF = TOTARE - RCFA
   WL = FRTA(I)
   FLTA(I) = AREDF / PARAM(I,1)
   ALPHA(I) = ALPHA(I) * FLTA(I) / WL
GO TO 1C
80 QMAX(I) = 10. ** 10
   ALPHA(I) = 0.
   EM(I) = C.
   GO TO 1C
70 IF (LGIV(I).EQ.1) GC TC 73
   EM(I) = 3.
   ALPHA(I) = 64.4 * SLPE(I) / (FRN(I) ** 0.333141)
   GO TC 74
73 READ(INPT,91) ALPHA(I), EM(I)
74 QMAX(I) = 10. ** 1C
   IF (NRCCF(I).NE.C) GC TC 9C
GO TC 10
10 CONTINUE
   IF (NL - MAXL + 1C) 1CC, 1CC, 1C1
1C1 CALL PAGE
1CO WRITE (IOMT,6000)
6COO FORMAT ('/46X,28H KINEMATIC CHANNEL PARAMETERS/ '
   1 42X,7HSEGMENT,3X,5HALPHA,6X,1HM)
   NL = NL + 5
   DO 120 I=1,NSEG
      IF (NL - MAXL + 1) 121,121,122
122 CALL PAGE
   WRITE (ICUT,6001)
   NL = NL + 1
   WRITE (ICUT,6001) ISEG(I),ALPHA(I),EM(I)
6C1 FORMAT (43X,A4,3X,F9.3,3X,F9.3)
120 CONTINUE
RETURN
END
SUBROUTINE CATCH
REAL ISEG, IUP, ILAT, ILOSS
COMM /PLCC01/ICUT, NL, MAXL, NSEG, RNC, KSEG(50)
COMM /PLCC04/ILP(50,3), ILAT(50,4), ISEG(50)
COMM /PLCC05/PR(5C), FLGTH(5C), SLOPE(5C), FRN(5C)
COMM /PLCC06/ACUT, TCUT, CCUT(50), SEC(5C), JOLT(5C)
COMM /PLCC14/IYPE(5C), ACX(5C), EX(5C), ELEM(50)
COMM /PLCC15/RCCF(5C,5C), PARAM(5C,2), CRCCF(50), FRCCF(50)
COMM /PLCC17/LGIV(5C), ARCCF, NDISH, CROOF, CROOF(5C), NRCOF(5C)
READ(INPT,5001) NSEG, ARG, ARCCF, NDISH, CROOF, CROOF(5C), NRCOF(5C)
COMMON /PLCC18/PLCC19/PRCCF, NRCOF(5C)
READ(INPT,5001) NSEG, ARG, ARCCF, NDISH, CROOF, CROOF(5C), NRCOF(5C)
5C01 FORMAT(215, F1C.C, I2, F5.C)
C READ CATCHMENT DATA
WRITE(ICUT,7500) AVCCF, CROOF
75CC FORMAT(/13X, 12HROOF AREA = ,F8.2, 8H (SC FT), 4X, 23HF CISC ARG
1E PTS = ,I3, 4X, 28H DISCHARGE (CFS/IN/OUTLET) = ,F6.4)
5 WRITE(ICUT,6001)
6CC1 FORMAT(/36X, ELENGTH, I1X, 9HRCGLHNESS, 21X, 6HNUMER, 2X, 3HFCT/ 1 8H SEGMENT, 2X, 17HUPSTREAM SEGMENTS, 3X,
2 17HADJACENT SEGMENTS, 2X, 4H TYPE, 2X, 3HIPR,
3 4X, 6H FEET), 3X, 5H SLOPE, 2X,
4 9H PARAMETER, 3X, 16HF CHER PARAMETERS, 3X, 6H HOUSES, 2X, 3HGUT)
NL = NL + 7
DO 10 I=1, NSEG
READ(INPT,5002) ISEG(I), (IUP(I,J), J=1,3), (ILAT(I,J), J=1,4),
1 (IYPE(I,J), IPR(I,J), FLGTH(I,J), SLCPE(I), FRN(I)),
2 PARAM(I,J), J=1,2), LGIV(I), NRCCF(I),
3 CRCCF(I), ELEM(50)
5C02 FORMAT(2A4, 2I1, F6.0, 4F5.0, 10X, 2I2, F3.0, F3.0)
IF(IYPE(I,J), FG. 5) GCTC 3E
IF(IYPE(I), NE. 6) GCTO 3G
3B READ(INPT,5005)(RCCF(I,J), J=1, NRC)
5C05 FORMAT(4CF2.C)
39 NL=NL+5
10 WRITE(TCUT,6002) ISEG(I), (IUP(I,J), J=1,3), (ILAT(I,J), J=1,4),
1 IYPE(I), IPR(I), FLGTH(I), SLCPE(I), FRN(I),
(PARAM(I,J), J=1, 2), NR0CF(I), G04CF(I)
6C02 FORMAT (2X, A4, 4X, 3(1X, A4), 3X, 4(1X, A4), I5, I5,
   IF (NL - MAXL + 1C) 14, 14, 15
15 CALL PAGE
   WRITE (ICUT, 60C1)
14 NL = NL + 4
C SET-UP COMPUTATION SEQUENCE
   CALL SEC
C SET-UP OUTPUT SEQUENCE
   NOUT = C
   DO 50 I=1, NSEC
      IF (IPR(I)) 50, 50, 51
51 NOUT = NOUT + 1
   JOUT(NCUT) = I
   SEG(NCUT) = ISEC(I)
50 CONTINUE
C SET-UP KINETIC MODEL PARAMETERS
   CALL AP
   RETURN
   END
SUBROUTINE FLCH
REAL ISEG, IUP, ILAT, ILCSS
REAL INWAT
COMMON /BLOC01/ICUT, INFT, NL, MAXL, NSEG, NRC, KSEG(50)
COMMON /BLOC04/ IUP(5C, 3), ILAT(5C, 4), ISEG(50)
COMMON /BLOC05/ IPR(5C), FLGTH(5C), SLOPE(5C), FRN(5C)
COMMON /BLOC09/ALPHA(50), EP(50)
COMMON /BLC04/ IUP(5C, 3), ILAT(5C, 4), ISEC(50)
COMMON /BLC05/ IP(50), FLGTH(5C), SLOPE(5C), FRN(5C)
COMMON /BLC10/DT, ECMP, CSI, T, IPMT, IPMT, EP(50), INCRA
COMMON /BLC11/ Q1(50), Q2(5C), QSLM(5C), QSLM(5C)
COMMON /PLCC09/A[LPI-4(50), EV(50)
COMMON /PLCC10/INWAT(5C), OUTWAT(5C)
COMMON /PLCC11/LGIV(5C), ARCCF, NDISCH, ERCCF, EROCF(5C), NRCOF(5C)
COMMON /PLCC12/THETM(5C)

C COMPUTE FLCHS AT T + DT
T = T + DT
DO 1 I = 1, NSEG
  K = KSEG(I)
  IF (ITYPE(K) - 7) 5, 30, 303
  IF (AROCF .EQ. 0.) GOTO 101
  N = NDX(K) + 1
  I = ITYPE(K) .EQ. 5 GOTO 1CC
  I = ITYPE(K) .EQ. 6 GOTO 101
  GO TO 1CC
101 CALL RCCF(K)
1CC CONTINUE
CALL UPI(K, I, QUP)
CALL LAT(K, GLAT, EP, AP)
CALL WATERI(K, CLAT, GLAT, AP, EP)
ALP = ALPHA(K)
XEM = EM(K) - 1.
YEM = EM(K)
50 QI(K) = C2(K)
   ALAT = GLAT * DTS
A(K,1) = (CUP/ALP)**(1./YEM)
DO 20 J=2,N
IF(E(K),NE.1.) GC TC 35
THETA=ALP* DTS/DX(K)
IF(THETA.GT.1.0) GC TC 80
A(K,J)=ALAT+AA(K,J-1)*THETA+AA(K,J)*(1.-THETA) -
GO TO 40
80 A(K,J)=(ALAT+AA(K,J-1)*(THETA-1.0)*AA(K,J-1))/THETA
GO TO 40
A9=(AA(K,J)+A(K,J-1))/2.C
THETA=ALP*YEM*CTS/CX(K)*(A9**XEM)
IF(THETA.GT.1.3) GC TC 38
A1=(ALP*OTS)/DX(K)
PHI1=(1.-A1*YEM*(AA(K,J)**XEM))
PHI2=A1*YEM*(AA(K,J-1)**XEM)
WRITE(IOUT,15C) T,K,J,PHI1,PHI2
150 FORMAT(1H ,***,F8.2,2X,13,2*X,13,2*(F8.3))
WRITE(7,152) T,K,J,PHI1,PHI2
152 FORMAT(F8.2,2*X,13,2*X,13,F8.3,*,DO*,2*X,F8.3,*,CO*)
A(K,J)=ALAT+AA(K,J)*(1.-A1*(AA(K,J)**XEM))+A1*(AA(K,J-1)**XEM)
GO TC 40
A1=DX(K)/(ALP*OTS)
A(K,J)=A(K,J-1)*(AA(K,J-1)**XEM)-A1+ALAT*A1+AA(K,J-1)*A1
IF(A(K,J)) 90,91,91
90 WRITE(6,83) A(K,J),ALAT,A1,K,J
83 FORMAT(//4X,*,A,ALAT,A1,K,J*,3(E17.7,2X),2(I2,2X))
A(K,J)=0.
91 A(K,J)=A(K,J)**(1. /YEM)
40 IF(A(K,J)) 42,42,2C
42 A(K,J)=0.
20 CONTINUE
DO 45 J=1,N
45 AA(K,J)=A(K,J)
G2(K)=ALP*(A(K,N)**YEM)
CALL WATERC(K)
IF(TETM(K).GT.THERA) GC TC 69
THETM(K)=THETA

69 CONTINUE
GO TO 10

30 CALL UP(K,1,QUP)
CALL WATERI(K,CUP,0.0,0.0,0.0)
Q1(K)=Q2(K)
Q2(K)=QUP
CALL WATERC(K)
THETM(K)=C.

10 CONTINUE
RETURN
END
SUBROUTINE INFIL(K,EP,AP)
REAL ISEG, IUP, ILAT
COMMON /BLCO01/ICUT,INPT,NL,MAXL,NSEG,NRC,KSEG(50)
COMMON /BLCO03/ILCSS,FC,FC,FK,CETEN(50),TINFIL(50),CEPERV,CIMP
COMMON /BLCO05/IPR(50),FLGTH(50),SLOPE(50),FRN(50),CTREI,TDID
COMMON /BLCC10/CT,ECCMP,CS1,T,IPRNT,IRAIN,CT5,TO(50),INDRA
COMMON /BLCC12/A(50,50),AA(50,50)
COMMON /BLCC13/TRAIN(250),P(250,5),NDP,IND5(50),IND6(5C),INC7(50)
COMMON /BLCC14/ITYPE(50),NCX(50),CX(5C),ELMAX(5C)
COMMON /BLCC15/RCCEF(50,50),PARAM(50,2),CRCF(50),PRCCF(50)
COMMON /BLCC17/LGIV(5C),ARCDF,NDISCH,CRCCF,CRCCF(50),ARCCF(50)

C PREPARE BY RAFAEL BRAS JAN 1973
C COMPLEX EXCESS PRECIPITATION AT SEGMENT K
C BY SUBTRATING INFILTRATION AND RETENTION
5 IF (T-TRAIN(IRA')) 10,10,20
20 IF (IRA'-NDP) 25,30,20
25 IRA' = IRA' + 1
GOTO 5
30 IF (NRCCEF(K).EQ.0) GOTO 200
TAREA = PARAM(K,1)*FLGTH(K)
AP = ((QROOF(K)*NRCCEF(K))/TAREA)*(1.-GRCCEF(K))*4320.
1 215 IF (FC.EQ.0.) GOTO 150
EP = AP-(FC+(FO-FC)*EXP(-FK*(T-TO(K))))
IF (EP.GE.C.) GOTO 400
N = NCX(K) + 1
PRCFU = 0.
DO 401 J = 2, N
Z1 = ((AA(K,J-1) + AA(K,J))/2.)*12.
Z2 = EP*(DT/60.)
Z3 = Z2
401 CONTINUE
302 PRCFU = PRCFU + Z1
GOTO 401
402 CONTINUE
PRCFL = PRCFU / (N - 1)

TINFIL(K) = TINFIL(K) + PRCFU

GOTC 205

400 F = (FC + (FC - FC) * EXP(-FK * (T - TC(K))))

TINFIL(K) = TINFIL(K) + F * (DT/6C.)

GOTG 205

150 EP = AP

GOTO 2C5

200 AP = C.

GOTO 215

205 RETURN

10 AVG = 0.

DO 4C I = 1,NRG

4C AVG = AVG + RCOEF(K,I) * P(IRA, N, I)

TCTRAI = TCTRAI + ((AVG * DT) / 72C.) * (PARAM(K, I) * FLGTH(K) + NRCCF(K) * ARCCF)

AP = AVG

IF (NROCF(K) .EQ. C) GOTC 21C

TAREA = PARAM(K, I) * FLGTH(K)

AVG = AVG + (((CRCCF(K) * NRCCF(K)) / TAREA) * (1 - CROOF(K)) * 432C.)

AP = AVG

210 DETEN(K) = DETEN(K) + AVG * DT/6C.

IF (FC .EQ. C) GOTO 35C

IF (FK .NE. 0.) GOTO 500

F = FC

IF (AVG - F) 600, 6C1, 6C1

600 EP = 0.

AP = AVG

TINFIL(K) = TINFIL(K) + AVG * DT/60.

F = AVG

GOTC 7C0

6C1 TINFIL(K) = TINFIL(K) + FC * DT/6C.

GOTG 760

5C0 IF (INC5(K) - 1) 78, 79, 79

78 F = (FC + (FC - FC) * EXP(-FK * T))

IF (AVG - F) 3CC, 3C1, 3C1

300 EP = 0.
F=C.
RETURN
90 AP=AVG*(1.-EXP(-CETEN(K)/DIMP))
TDEP=TCEP+((AVG-AP)*ET)/760.*PARAM(K,1)*FLCTF(K)
EP=AP
RETURN
ENC
SUBROUTINE INIT
REAL INWAT
COMMON /BLOC01/ICLT,INPT,AL,MAXL,ASEG,NPC,KSEG(50)
COMMON /BLOC03/ILOS5,FC,FC,FK,DETEM(5C),TINFIL(5C),CPERV,CIMP
COMMON /BLOC05/ IPR(50),FLGT(50),SLOPE(5C),FRN(5C),TOTRAI,TDEP
COMMON /BLOC07/JLP(50,3),JLAT(50,4),ICNEXT(50)
COMMON /BLOC08/QMAX(5C),STC(5C),ELEV(50)
COMMON /BLOC10/DT,ECCMP,OSI,T,IPRNT,IRAIN,UTS,TO(5C),INDRA
COMMON /BLOC11/C1(50),C2(50),CSUM(50),CSUML(50)
COMMON /BLOC12/A(5C,5C),AA(50,50)
COMMON /BLOC13/TRAIN(250),P(25C,5),NDP,IND5(5C),IN5C,INC7(5C)
COMMON /BLOC14/ITYPE(50),NCX(50),EX(50),ELEM(5C)
COMMON /BLOC15/RCCF5C,5C),PARAM(5C,2),CRCF(50),FRCCF(50)
COMMON /BLOC16/INWAT(5C),OUTWAT(5C)
COMMON /BLOC17/LCIV(50),RCOFF,NCISCH,CRCOF,GOOF(5C),NRCCF(5C)

C INITIALIZE CATCHMENT

T = 0.
INCRA = 1
TRAIN = 1
IPRNT = 1
TOTRAI = 0.
TDEP = 0.
DO IC I = 1, NSEG
   QSUM(I) = 0.
   QSUM(I) = 0.
   IDNEXT(I) = 0.
   STO(I) = 0.
   DETEN(I) = 0.
   INWAT(I) = 0.
   TO(I) = 0.
   INC5(I) = 0.
   TINFIL(I) = 0.
   ELEV(I) = 0.
   INC6(I) = 0.
   IND7(I) = 0.
QRCOF(I)=C.
HRCOF(I)=0.
OUTWAT(I)=C.
K = KSEG(I)
N = NDX(K) + 1
DO 2C J=1,N
   AA(K,J)=0.
20 A(K,J) = 0.
Q2(I) = 0.
10 Q1(I) = C.
RETURN
END
FUNCTION ITRAN(X)
REAL ISEG, IUP, ILAT, ILCSS
COMMON /BLOC01/ICLT, INPT, NL, MAXL, NSEG, NRC, KSEG(5C)
COMMON /BLOC04/IUP(5C,3), ILAT(5C,4), ISEG(50)
I = 1
3 IF (X - ISEG(I)) 10,5,10
5 ITRAN = I
RETURN
10 I = I + 1
20 IF (I - NSEG) 3,3,20
20 ITRAN = C
RETURN
END
SUBROUTINE LAT(K, CLAT, EP, AP)
REAL ISEG, IUP, ILAT, ILCSS
COMMON /BLOC01/IOUT, INPT, NL, MAXL, NSEG, NRG, KSEG(5C)
COMMON /BLOC05/IPR(50), FLGTH(50), SLCEP(50), FRN(50)
COMMON /BLOC07/JUP(50,3), JLAT(50,4), ICNEXT(5C)
COMMON /BLOC09/ALPHA(50), EM(50)
COMMON /BLOC10/DY, EC, PSI, T, IPRNT, IRAIN, DTS, TC(5C), INCRA
COMMON /BLOC11/Q1(5C), Q2(5C), QSUM(5C), QSUML(50)
COMMON /BLOC14/ITYPE(5C), NDX(50), D(5C), ELEMAX(5C)
COMMON /BLOC15/RCCF(50,50), PARAM(50,2), CRCF(50), HRCCF(50)
COMMON /BLOC17/LGIV(5C), ARCCF, NDISCH, CRCCF, GRCF(50), HRCCF(50)

C COMPUTE LATERAL INFLOW RATE TO SEGMENT K
K = K
QLAT = C.
IF (ITYPE(K) - 5) 5,30,5
5 IF (ITYPE(K) - 6) 10,30,10
10 QPR = QSUML(K)
313 DO 15 J=1,4
15 IF (JLAT(K,J)) 15,15,2C
2C JJ = JLAT(K,J)
10 IF (ITYPE(JJ) - 5) 1C,1C1,1CC
100 IF (ITYPE(JJ) - 6) 102,101,102
101 QLAT = QLAT + Q2(JJ) + (RCCF(JJ) * CRCF(JJ) * HRCCF(JJ)) / FLGTH(K)
GAM = (Q2(JJ) / ALPHA(JJ))**(1./EM(JJ))
TEMP = ALPHA(JJ) * DTS * EM(JJ)
GAM = TEMP * (GAM ** (EM(JJ) - 1.))
WRITE(IOUT,200) T,K,JJ,GAM
200 FORMAT(1H, '****', F8.2, 2(2X,I3), 2X,F8.3)  
WRITE(7,210) T,K,JJ,GAM
210 FORMAT(F8.2, 2(2X,I3), 2X,F8.3,'DO')
GOTO 15
102 QLAT = QLAT + Q2(JJ)
15 CONTINUE
QSUML(K) = QLAT
QLAT = (QLAT + QPR) / 2.
RETURN
30 CALL INFIL (K,EP,AP).
QPR=QSUML(K)
QLAT=(1./43200.)*(PARAM(K,2)*AP+(1.-PARAM(K,2))*EP)
QSUML(K) = QLAT
QLAT=(QLAT+QPR)/2.
RETURN
END
SUBROUTINE PAGE
COMMON /BLOC01/ICLT,INPT,NL,MAXL,KSEG,NRC,KSEG(50)
COMMON /BLOC02/TITLE(2),TITRE(10),NPAGE
NPAGE = NPAGE + 1
WRITE (IOUT,6000) NPAGE,TITLE
6000 FORMAT (1H1,11X,4HPAGE,I4//'2X,2CA4)
NL = 3
WRITE (ICLT,6100) TITRE
6100 FORMAT (//'2X,18A4/)
NL=NL + 4
RETURN
END
SUBROUTINE RAIN(IPLUI, INCX)
COMMON /BLC01/IOUT, INPT, NL, MAXL, NSEG, NRC, KSEG(50)
COMMON /BLC04/IUP(50, 3), ILAT(50, 4), ISEG(50)
COMMON /BLC05/ IPR(50), FLGTH(50), SLOPE(50), FRN(50)
COMMON /BLC10/ DT, ECMP, CSI, T, IPTN, IRAIN, EIF, TC(50), INCRA
COMMON /BLC13/ TRAIN(250), P(250, 5), NDP, IND5(50), INC6(50), IND7(50)
COMMON /BLC14/ ITYPE(50), NDX(50), DX(50), ELEMAX(50)
COMMON /BLC30/ QBS(400), CSYNT(400), TCUTFL(400)
IF (IPLUI.EQ.4) GO TO 60
DO 12345 J=1,400
12345 QBS(J)=C.
CONTINUE
C
RAINFALL DATA
GO TO (10, 20, 30, 35), IPLUI
10 READ(INPT, 11) TR, EXPRE, DT, CSI, ECMP
11 FORMAT(5F10.0)
1 NDP=2
315 P(1,1)=EXPRE
315 P(2,1)=C.
1 TRAIN(1)=TR*60.
1 TRAIN(2)=1000.
GO TO 1000
C
TRIANGULAR RAINFALL PATTERN
20 READ(INPT, 21) TR, EXPRE, DT, CSI, ECMP, TMAX
21 FORMAT(6F10.0)
EX IN=EXPRE
TMAX=TMAX*60.
T1=C.
T3=(TR*60.)-TMAX
T2=(TR*60./CSI)
NDP=1T2
DO 110 J=1, IT2
110 TRAIN(J)=T1+OSI
IF (TRAIN(J).GT.TMAX) GO TO 120
X2=EXIN*TRAIN(J)/TMAX
GO TO 1CC0
120 X2=(((TR*60.-TRAIN(J))/T3)*EXIN
130 P(J,I)=X2
TI=TRAIN(J)
110 CONTINUE
GO TO 1CC0
C BLOCK RAINFALL OBSERVATION TYPE (CNE TO FIVE RAINGAGES)
30 READ(INPT,150) NCP,OSI,ECCOMP,DT,CACFS
150 FORMAT(5,4F10.0)
35 IF(IPLUI.EQ.4) WRITE(IOUT,6543) DT
6543 FORMAT(1H0,15X,RAINFALL INTENSITY AS FOR PRECEEDING STORM'/
120X,*DT (MINUTES) =',F10.3/
35 IF(IPLUI.EQ.4) GO TO 1CC0
DO 155 I=1,NCP
READ(INPT,157) TRAIN(I),QCES(I)
157 FORMAT(7F8.0)
READ(INPT,177) (P(I,J),J=1,NRG)
177 FORMAT(20F4.0)
155 QOBS(I)=QOBS(I)*CACFS
1CC0 CONTINUE
WRITE(IOUT,900) DT,OSI,ECCOMP
900 FORMAT(///20X,TIME INCREMENT (MINUTES) =',F8.2/
121X,*OUTPUT SAMPLING INTERVAL (MINUTES) =',F8.2/
221X,*END OF COMPUTATIONS (MINUTES) =',F8.2/)
316 IF(NL-MAXL+10) ECC,ECC,8C1
800 CALL PAGE
NL=NL+7
8C1 CONTINUE
11CC0 DTS=DT*EC.
1 IF(INDX.EQ.0) GO TO 1CC0
C INPUT OF THE SPACE INCREMENTS FOR THIS RUN
READ(INPT,173) (NDX(I),I=1,25)
IF(INDX.LE.25) GO TO 1CC0
READ(INPT,173) (NDX(I),I=26,50)
173 FORMAT(25I3)
1003 CONTINUE
   IF (NL - MAXL + 10) 191,191,192
192  CALL PAGE
191  WRITE (ICUT, 179)
179  FORMAT (1HO, 30X, 'DESCRIPTION OF THE SPACE AXIS'//)
    WRITE (ICUT, 193)
193  FORMAT (1HO, 20X, 'ISEG', 10X, 'LENGTH', 1CX, 'NDX', 1CX, 'DX'//37X, 'FEET'//1, 21X, 'FEET'//)
       NL = NL + 9
       DO 810 I = 1, NSEG
           IF (ITYPE(I) .NE. 7) GOTO 160
           DX(I) = C.
       GOTO 161
160  DX(I) = FLGTH(I) / NDX(I)
161  WRITE (ICUT, 811) ISEG(I), FLGTH(I), NDX(I), CX(I)
811  FORMAT (1H0, 21X, A4, 8X, F10.2, 8X, 13, 8X, F10.4)
       NL = NL + 1
       IF (NL - MAXL) 810, 810, 812
812  CALL PAGE
       WRITE (IOUT, 193)
       NL = NL + 5
810  CONTINUE
       GO TO 1CC2
1001 WRITE (ICUT, 813)
813  FORMAT (1HO, 30X, 'THE SPACE DESCRIPTION IS IDENTICAL TO THE DESCRIPTION USED FOR THE PRECEDING PROBLEM'//)
       NL = NL + 6
1002 CONTINUE
       RETURN
       END
SUBROUTINE ROOF(K)
COMMON /BLOC01/IOUT,INPT,NL,MAXL,ASEG,NRG,KSEG(5C)
COMMON /BLOC02/DT,ECCMP,CSI,T,IPRINT,TRAIN,CTS,T0(5C),INDRA
COMMON /BLOC13/TRAIN(250),P(250,5),NCP,IND5(50),IND6(5C),IND7(50)
COMMON /BLOC15/RCCF(5C,5C),PARAM(5C,2),RCCF(50),HRCCF(5C)
COMMON /BLOC17/LGIV(50),AROOF,NDISCH,DROOF,ROOF(5C),NROOF(5C)
!
11 IF(1-TRAIN(INCPA))7,T78
8    IF(INDRA-NDP) 5,11,10
9    INDRA=INDRA+1
GOTC 11
7    AVEG=0.
DO 20 I=1,NRG
20    AVEG=AVEG+RCCF(K,I)*F(INCPA,I)
25    HRCCF(K)=HRCCF(K)+AVEG*DT/60.
    IF(HRCCF(K).LE.6) GOTC 70
WRITE(ICUT,69)
69 FORMAT(/40X,'******RCCF CVERFLCWE EXECUTICN TERMINATING******')
CALL EXIT
30
70 IF (HRCCF(K).GT..0625) GOTO 71
1     QRCOF(K)=C.
1     DELH=0.
GOTC 23
71  QRCOF(K)=QRCOF*(HRCCF(K)-.0625)*NEISCH
    DELH=QRCOF(K)*DTS/ARCCF*12.
    IF(HRCCF(K)-.0625-DELH)22,23,23
22  QRCOF(K)=((HRCCF(K)-.0625)/12.)*ARCCF/CTS
    HRCCF(K)=.0625
    RETURN
23  HRCCF(K)=HRCCF(K)-CEL-
    RETURN
10  AVEG=C.
GOTC 25
END
C

SUERCUTINE ROUTE(IPLUI,IND11,IND12)
REAL ISEG,IUP,ILAT,ILCSS
COMMON /BLOC01/IOUT,INPT,NL,MAXL,KSEG,ARC,KSEG(50)
COMMON /BLOC03/ILOSS,FC,FC,FK,DETEN(5C),TINFIL(5C),CPRV,DIMP
COMMON /BLOC04/IUP(50,3),ILAT(50,4),ISEG(50)
COMMON /BLOC06/NCUT,TCUT,QCUT(50),SEG(50),JCUT(50)
COMMON /BLOC05/IPR(50),FLGTH(50),SLOPE(5C),FRN(5C),TCTRAI,TDEP
COMMON /BLOC08/CMAX(50),STC(50),ELEV(50)
COMMON /BLOC09/ALPHA(50),EP(50)
COMMON /BLOC10/DTECOMP,OSTIP,RAINtESTC(5C),EPRA
COMMON /BLOC11/ITYPE(50),NDX(50),DX(50),ELEMAX(5C)
COMMON /BLOC15/RCCEF(50,50),PARAM(50,2),CROCF(50),HRCEOF(5C)
COMMON /BLOC16/INWAT(50),OUTWAT(50)
COMMON /BLOC17/LGIV(50),AROF,NDISCH,GROCF,GROOF(5C),ARCCF(50)
COMMON /BLOC18/THETM(50)
COMMON /BLOC30/QSUMS(4CC),QSYNT(400),TCUTFL(400)

2 CALL INIT
  DO 1168 JJ=1,NSEG
  1168 THEIM(JJ)=0.
C COMPUTE OUTFLOW HYDROGRAPHS FCR EACH SEGMENT
  KKI=NCUT/10+1
  CALL PAGE
  WRITE(IOUT,600E) (SEG(J),J=1,NOUT)
  CALL FLNW
  65 CALL FLNW
  66 IF(IPRNT*CSI-T) 60,60,70
  70 IF(T-ECOMP) 65,65,100
  60 TOUT=IPRNT*OSI
          B = (TCUT - (T - CT))/CT
  DO 61 J=1,NOUT
          K = JOUT(J)
  61 QOUT(J) = Q1(K) + (Q2(K) - CI(K))*B
          IF (NL - MAXL + 3) 75,75,76
76 CALL PAGE
   WRITE (ICUT,6008) (SEG(J),J=1,NOUT)
6CC8 FORMAT (8X,4HTIME,7X,32HOUTFLOW HYDRGRAPHS FOR SEGMENTS/
   1 7X,5H(MIN),1X,10(6XA4)/19X,A4,6X,A4,6X,A4,6X,A4,6X,A4,6X,A4,6X,A4,6X,A4)
   NL=NL+KK1
75 NL=NL+KK1
   WRITE (ICUT,6009) TOUT,(QOUT(J),J=1,NOUT)
   WRITE (ICUT,6009) TOUT,(QOUT(J),J=1,NOUT)
6C09 FORMAT (6X,F7.2,2X,10F10.4/(I5X,10F10.4))
C C THE ONLY SEGMENT FOR WHICH QSYNT(I) IS SAVED IS THE CL1E1 CF THE SITE
C C TO SAVE OTHER HYDRCGRAPHS ,NEEC TC ADD ALPHANUMERIC COMP... 
C
   K1=KSEG(NSEG)
   QSYNT(IPRNT)=Q1(K1)+(C2(K1)-Q1(K1))*B
   TOUTFL(IPRNT)=ICUT
   IPRNT = IPRNT + 1
   DO 80 I=1,NSEG
   IF(IND6(I).EQ.C) GOTO 80
   NL=NL+1
   IF(IND7(I).EQ.C) GOTO 900
   WRITE(IOUT,6080) ISEG(I),TALPHA(I),STC(I)
   6C80 FORMAT(2CX,'PRESSURE FLOW-SEGMENT ','A4,' TIME = ',',F10.1,' ALPHA = '
   1,F12.5,' STORAGE = ',',F10.2,' CF'
   GOTO 9C1
   WRITE(IOUT,9917)ISEG(I),T,ALPHA(I),STC(I)
9C0 WRITE(IOUT,9917)ISEG(I),T,ALPHA(I),STC(I)
9917 FORMAT(20X,'PRESSURE-FLOW-SEGMENT ','A4',' TIME = ',',F10.1,' ALPHA = '
   1,F12.5,' STORAGE = ',',F10.2,' CF',' GRAC. NEGATIVE')
9C1 IND6(I)=C
   IND7(I)=C
80 CONTINUE 
   GO TO 70
100 CONTINUE
   CALL PAGE
   WRITE(ICUT,2080)
2080 FORMAT(IHC, 20X, 'MASS BALANCE AT THE END OF THE COMPUTATIONS' /
    120X, 'SEGMENT', 1C6, 'IN', 1C5, 'IN', 1C5, 'OUT', 1C5, 'SURFACE', 1C5, 'RG
    NL = NL + 7
    TOTIN = C.
    TOTT = C.
    TOTSUR = 0.
    DO 25 I = 1, NSEG
    TOTIN = TOTIN + (TINFIL(I) / 12.) * (PARA(I,1) * FLCTF(I) * (1. - FARAJ(I,2))
    IF(I.EQ.KSEG(NSEG)) TOTOUT = OUTWAT(I)
    VSUR = 0.
    TT = NDX(I)
    VR = (HROOF(I) / 12.) * ARCOF * NRCOF(I)
    TOTCC = TCC + VROOF
    DO 26 J = 1, N
    IF(ITYPE(I) = 5) 312, 313, 312
    IF(ITYPE(I) = 6) 314, 313, 314
    VSUR = VSUR + (((A(I,J) + A(I,J+1)) / 2.) * CX(I))
    GOTO 26
    312 VSUR = VSUR + (((A(I,J) + A(I,J+1)) / 2.) * DX(I)) * PARAM(I,1)
    26 CONTINUE
    TOTSUR = TOTSLR + VSLR
    IF(NL + MAXL + 1) 2C85, 2C85, 2C86
    2086 CALL PAGE
    WRITE(ICUT, 2080)
    2C85 WRITE(IOUT, 2082) ISEG(I), INMAT(I), CLMAT(I), VSUR, VROOF
    2082 FORMAT(IH , 19X, A4, 8X, F15.5, 4X, F15.5, 7X, F15.5, 4X, F1C.5)
    NL = NL + 1
    25 CONTINUE
    TOTAL = TOTOUT + TOTSLR + TCC + TOTIN + TDEP
    PCT = ((TOTAL - TOTTRAI) / TOTTRAI) * 100.
    WRITE(ICUT, 3086) TCR, TCC, TCSUR, TCC
    3086 FORMAT(//10X, 'TOTALS TOTAL RAIN', F17.5, 4X, F15.5, 7X, F15.5, 3X, F11
    1.5)
    WRITE(ICUT, 3087) TCTIN, TCEF, PCT
    3087 FORMAT(1H , 11X, 'TCTAL INTEGRATION', 2X, F15.5, 10X, 'TCTAL DEPRESSION

RCLT0109
RCLTC110
ROUTC111
RCLTC112
RCLTC113
RCLTC114
RCUT0115
RCUT0116
RCUT0117
RCUT0118
RCUT0119
RCUT0120
RCUT0121
RCUT0122
RCUT0123
RCUT0124
RCUT0125
RCUT0126
RCUT0127
RCUT0128
RCUT0129
RCUT0130
RCUT0131
RCUT0132
RCUT0133
RCUT0134
RCUT0135
RCUT0136
RCUT0137
RCUT0138
SUBROUTINE SEQ
REAL ISEG, IUP, ILAT, ILCSS
DIMENSION ITEST(50)
COMMON /BLOC01/ IOUT, INPT, NL, MSEG, NSEG, NRG, KSEG(50)
COMMON /BLOC04/ IUP(50, 3), ILAT(5C, 4), ISEG(5C)
COMMON /BLOC07/ JUP(50, 3), JLAT(50, 4), JNEXT(50)
COMMON /BLOC14/ ITYPE(5C), NCX(5C), CX(5C), ELEMAX(50)
DO 60 I=1, NSEG
ITEST(I) = 0
DO 61 J=1, 3
X = IUP(I, J)
JUP(I, J) = ITRAN(X)
JI=JLP(I, J)
IF (ITYPE(JI)-2) 61, 3C1, 61
301 IDNEXT(JI) = I
61 CONTINUE
DO 6C J=1, 4
X = ILAT(I, J)
JLAT(I, J) = ITRAN(X)
60 CONTINUE
II = 0
DO 70 I=1, NSEG
IF (ITYPE(I) - 5) 71, 72, 71
71 IF (ITYPE(I) - 6) 70, 72, 70
72 N = 0
DO 73 J=1, 4
IF (JUP(I, J)) 73, 73, 74
74 N = 1
73 CONTINUE
IF (N) 75, 75, 7C
75 II = II + 1
KSEG(II) = I
ITEST(I) = 1
70 CONTINUE
I = 1
NIT = C
82 IF (ITEST(I)) 81,81,8C
80 I = I + 1
81 I = 1
83 NIT = NIT + 1
84 IF (NIT - 3*NSEG) 20,20,149
20 IF (II - NSEG) 82,120,120
81 N = 0
DO 90 J=1,3
82 IF (JU(J,J)) 90,90,91
91 K = JU(J,J)
92 IF (ITEST(K)) 92,92,9C
91 N = 1
90 CONTINUE
DO 95 J=1,4
92 IF (JL(J,J)) 95,95,96
93 K = JL(J,J)
94 IF (ITEST(K)) 97,97,95
95 CONTINUE
96 N = 1
97 CONTINUE
110 II = II + 1
111 KSEG(II) = I
112 ITEST(I) = 1
113 IF (II - NSEG) 8C,120,120
120 IF (NL - MAXL + 10) 121,121,122
122 CALL PAGE
C OUTPUT COMPUTATION SEQUENCE
123 N = 0
WRITE (ICUT,6008)
6008 FORMAT (/5CX,2CHC,COMPUTATION SEQUENCE//52X,5HINDEX,
1 3X,7HSEGMENT)
NL = NL + 5
DO 130 I=1,NSEG
K = KSEG(I)
IF (ITYPE(K) - 4) 131,131,140
131 NN = 0
DO 135 J=1,3
IF (JLAT(K,J)) 136,136,134
136 IF (JUP(K,J)) 135,135,134
134 NN = 1
135 CONTINUE
133 N = 1
IF (NL - MAXL + 1) 143,143,142
142 CALL PAGE
WRITE (IOUT,6008)
143 NL = NL + 1
WRITE (ICUT,6009) K,ISEG(K)
6009 FORMAT (52X,10HMISSING INFLOW SEGMENT)
GO TO 13C
140 IF (NL - MAXL + 1) 144,144,145
145 CALL PAGE
WRITE (IOUT,6009)
144 NL = NL + 1
WRITE (ICUT,6010) K,ISEG(K)
6010 FORMAT (52X,10HMISSING INFLOW SEGMENT)
GO TO 13C
130 CONTINUE
149 WRITE (ICUT,6011) II,(ITEST(I),I=1,NSEG)
6011 FORMAT (52X,35HINPUT DATA ERROR, EXECUTION STCFPEC/
1 5X,15/(5X,10I5))
WRITE (ICUT,6020) (I,(JUP(I,J),J=1,3),
1 (JLAT(I,J),J=1,4),I=1,NSEG)
6020 FORMAT (5X,10I5)
CALL EXIT
150 CONTINUE
RETURN
ENC
SUBROUTINE UP(K, I, QUP)
REAL ISEG, IQUP, ILAT, ILCSS
COMMON /BLOC01/ILCT, INPT, NL, MAXL, NSEG, NRC, KSEG(50)
COMMON /BLOC05/FLGTH(50), SLCE(50), FRN(50)
COMMON /BLOC07/JUP(50, 3), JLAT(50, 4), ICNEXT(50)
COMMON /BLOC08/QMAX(50), STC(50), ELEV(50)
COMMON /BLOC09/ALPHA(50), EM(50)
COMMON /BLOC10/DT, ECOMP, OSI, T, IPRTN, IRAIN, DTS, TC(50), INDRA
COMMON /BLOC11/C1(50), C2(50), CSUM(50), CSML(50)
COMMON /BLOC13/RAIN(250), P(250, 5), IND5(50), INC6(50), INC7(50)
COMMON /BLOC14/ITYPE(50), NDX(50), DX(50), ELEMAX(50)
COMMON /BLOC15/RCEF(50, 50), PARAM(50, 2), CR0OF(50), FCROOF(50)
COMMON /BLOC17/IC1(50), ARCCF, ADISCH, CRCCF, CRCCF(50), ARCCF(50)

COMPLETED UPSTREAM INFLOW TO SEGMENT K
QUP=C.
QPR=QSUM(K)
DO 10 J=1,3
  IF(JUP(K, J)) 1C, 10, 11
11 JU=JUP(K, J)
  GAMI=(C2(JJ)/ALPHA(JJ))**(1./EM(JJ))
  TEMP2=ALPHA(JJ)*EM(JJ)
  GAM2=TEMP2*(GAM2**(EM(JJ)-1.))
WRITE(IOUT,6) T, K, JJ, GAM1
  FORMAT(2X,1H5,2X,13F8.3)
WRITE(7,250) T, K, JJ, GAM1
  FORMAT(3X,1H3,2X,13F8.3, 'DC')
QUP=QUP+Q2(JJ)
1C CONTINUE
QSUM(K)=QUP
IF(ITYPE(K).EQ.2) GOTO 11
IF(QUP-EQMAX(K)) 30, 30, 15
15 IF(QPR-EQMAX(K)) 21, 25, 25
21 PDT=(QUP-EQMAX(K))/IQUP-QPR
  STC(K)=STC(K)-CTS*(EQMAX(K)-QPR)/2.*(1.-PCT)

IF (STO(K)) 22,23,23
22 STO(K) = 0.
23 STC(K) = STO(K) - CTS*(QUP - QMAX(K))/2.*PCT
    QUP = QMAX(K)
    RETURN
25 STO(K) = STO(K) + ((QPR + QUP)/2. - QMAX(K))*DTS
    QUP = QMAX(K)
    RETURN
30 IF (QPR - QMAX(K)) 31,31,32
31 IF (STC(K)) 40,40,41
40 RETURN
32 PDT = (QPR - QMAX(K))/(QPR - QUP)
    STC(K) = STO(K) - PDT*CTS*(QPR - QMAX(K))/2.
    IF (STC(K)) 33,35,35
33 STO(K) = C.
35 QUP = QMAX(K)
    RETURN
41 STO(K) = STO(K) - (QMAX(K) - (QPR + QUP)/2.)*DTS
    IF (STO(K)) 42,43,43
42 STO(K) = 0.
43 QUP = QMAX(K)
    RETURN
111 AMAX=3.14*(PARAM(K,1)**2.)/4.
    IDTS=DTS
    IF(ELEV(K).NE.C.) GOTO 130
    C CHECK FOR LAST SEGMENT
    IF(I.EC.NSEG) GOTO 135
    C CHECK IF NEXT SEGMENT IS PIPE
    KNEXT=IDNEXT(K)
    IF(I.TYPE(KNEXT).NE.2) GOTO 135
    IF(ELEV(KNEXT).NE.0.) GOTO 140
    C CHECK IF NEXT SEGMENT IS PIPE
    IF(FN=QPR-QMAX(K)) 200,200,202
    C CHECK IF NEXT SEGMENT IS PIPE
    ELEV(K)=C.
    STC(K)=C.
RETURN

202  S=SLOPE(K)
    INC6(K)=1
18  QSMAX=0.
7  DO 4CO LL=1,NTS
    STO(K)=QUP-QMAX(K)+STC(K)
    IF (STC(K)) 401,401,402
4CI  Q=STO(K)+QMAX(K)
    STO(K)=0.
    ELEV(K)=0.
    QSMAX=QSMAX+Q
    QMAX(K)=1.49/FRN(K)*AAMAX*(PARAM(K,1)/4.)**(2./3.)*SQRT(SLCEPE(K))
    GOTO 4CO
4CO  ELEV(K)=STC(K)/PARAM(K,2)
    IF (ELEV(K).LE.ELEMXAX(K)) GOTO 500
    ELEV(K)=ELEMAX(K)
500  GRAD=ELEV(K)/FLC(TH(K)+S
    QM1=QMAX(K)
    IF (GRAD) 4IC,4IC,4IC
4IC  GRAD=000001
411  QMAX(K)=1.49/FRN(K)*AAMAX*(PARAM(K,1)/4.)**(2./3.)*SQRT(GRAD)
    QM2=QM1+STO(K)
    IF (QM2-QMAX(K)) 415,415,415
415  ELEV(K)=0.
    O=QM1+STO(K)
    QSMAX=QSMAX+Q
    STC(K)=0.
    QMAX(K)=1.49/FRN(K)*AAMAX*(PARAM(K,1)/4.)**(2./3.)*SQRT(SLCEPE(K))
    GOTO 4CC
416  Q=QMAX(K)
    QSMAX=QSMAX+Q
    STO(K)=QM1+STO(K)-QMAX(K)
    ELEV(K)=STO(K)/PARAM(K,2)
    IF (ELEV(K).LE.ELEMAX(K)) GOTO 600
    ELEV(K)=ELEMAX(K)
600  GRAD=ELEV(K)/FLC(TH(K)+S
IF (GRAD) 430, 430, 431
430 GRAD = .CCCC01
431 QMAX(K) = 1.49/FRN(K)*AMAX*(PARAM(K,1)/4.)***(2./3.)*SCRT (GRAD)
400 CONTINUE
   QUP = CSMAX/ICTS
   ALPHA(K) = QUP/(AMAX**1.25)
   RETURN
140 INC6(K) = 1
   GRAD = (SLCPE(K)*FLCTH(K)-ELEV(KNEXT))/FLCTH(K)
   IF (GRAD) 450, 450, 451
450 S = GRAD
451 QMAX(K) = 1.49/FRN(K)*AMAX*(PARAM(K,1)/4.)***(2./3.)*SCRT (GRAD)
   IF (QUP-QMAX(K)) 2CC, 2CC, 18
130 INC6(K) = 1
   IF (I.EQ.NSEG) CCTC 460
   KNEXT = ICNEXT(K)
   IF (ITYPE(KNEXT).EQ.2) COTO 465
460 GRAD = ELEV(K)/FLGTH(K)+SLCPE(K)
   QMAX(K) = 1.49/FRN(K)*AMAX*(PARAM(K,1)/4.)***(2./3.)*SCRT (GRAD)
   S = SLOPE(K)
   QM3 = QUP+STC(K)/CTS
   IF (QM3-QMAX(K)) 200, 2CC, 18
465 GRAD = (SLOPE(K)*FLGTH(K)-ELEV(KNEXT))/FLGTH(K)
   IF (GRAD) 470, 470, 471
471 S = GRAD
   GRAD = ELEV(K)/FLGTH(K)+S
   QMAX(K) = 1.49/FRN(K)*AMAX*(PARAM(K,1)/4.)***(2./3.)*SCRT (GRAD)
   QM3 = QUP+STC(K)/CTS
   IF (QM3-QMAX(K)) 2CC, 2CC, 18
470 S = GRAD
   GRAD = (ELEV(K)-ELEV(KNEXT))/FLGTH(K)+SLOPE(K)
   IF (GRAD) 475, 475, 476
476  QMAX(K) = 1.49/FRN(K)*AMAX*(PARAM(K,1)/4.)**(2./3.)*SQRT(Grad)
QM3=CUP*STC(K)/CTS
IF(QM3-QMAX(K)) 200,2CC,18
   END
SUBROUTINE WATERI(K, QUP, QLAT, AP, EP)
REAL INWAT
COMMON /BLOC05/ IPR(50), FLGTH(50), SLCPE(5C), FRN(50)
COMMON /BLOC10/ DT, ECCMP, CSI, T, IPRAT, IRAIN, LTS, TC(5C), ICREA
COMMON /FLCC17/ AA(50,50), AA(50,50)
COMMON /BLOC14/ IYPE(5C), NCX(50), EX(50), ELEMAX(5C)
COMMON /BLOC15/ RCOEF(5C,5C), PARAM(5C,2), GREEF(5C), FCCEF(50)
COMMON /FLCC16/ INWAT(50), OUTWAT(5C)

C COMPILE THE WATER INPT TC EACH SEGMENT
VUP=C.
VLAT=0.
IF (ITYPE(K) = 5) 5, 10, 5
5 IF (ITYPE(K) = 6) 15, 1C, 15
15 IF (ITYPE(K) = 7) 20, 3C, 2C
20 VLAT=QLAT*FLGTH(K)*CTS
VLP=QLP*DIS
INWAT(K)=INWAT(K) + VLAT + VUP
GO TC 50
10 IF (EP.GE.C.) GC TC 69
N=NCX(K)+1
DO 75 J=2, N
Z1=((AA(K,J-1)+AA(K,J))/2.)*EX(K)
Z2=QLAT*DX(K)*CTS
Z3=-Z2
IF(Z1.GE.Z3) GC TC 71
VLAT=-Z1
GO TO 75
71 VLAT=Z2
75 INWAT(K)=INWAT(K)+VLAT*PARAM(K,1)
GO TO 5C
69 VLAT=QLAT*CTS*FLGTH(K)*PARAM(K,1)
INWAT(K)=INWAT(K) + VLAT
GO TO 5C
30 VUP=QUP*CTS
INWAT(K)=INWAT(K) + VUP
50 RETURN
END
SUBCUTINE WATERO(K)
REAL INWAT
COMMON /BLOC1C/ DT,ECMP,CSI,T,IPRT,IRAIN,ETS,TC(50),INDRA
COMMON /BLOC11/Q1(50),Q2(50),QSUM(50),QSLML(50)
COMMON /BLOC14/ITYPE(50),NEX(50),EX(50),ELEM(50)
COMMON /BLOC15/RCCEF(50,50),PARAM(50,2),CRCCF(50),CRCC(50)
COMMON /BLOC16/INWAT(5C),OUTWAT(5C)
IF(ITYPE(K).EQ.7) GO TO 25
Q=(Q1(K)+Q2(K))/2.
   IF(ITYPE(K)-5) 10,11,10
  10 Q=Q*PARAM(K,1)
   11 OUTWAT(K)=OUTWAT(K)+(Q*DTS)
   GO TO 40
  25 OUTWAT(K)=INWAT(K)
   GO TO 40
END
Appendix 7

The Rainfall-Runoff Network Design Program

The rainfall-runoff network design program is the second step in the network design algorithm described in Section 5.4.1. It performs the following functions:

1) Estimates rainfall model parameters
2) Performs filtering algorithm on incomplete, noisy rainfall observations
3) Propagates uncertainty through state-space runoff model

A description of the format and data needs of the program follows together with a code listing.

The program is coded (as all other programs in this work) to be compatible with a Fortran IV G compiler and has been operated in an IBM 370/168. As it stands, it requires access to I.B.M's Scientific Packages, SL MATH and S.S.P. Similarly, it makes use (in subroutine filter) of a double precision matrix inversion subroutine, DMINV, available from the M.I.T. Information Processing Center.

Present array dimensions limit the number of overland segments, (possible station locations) to 10. Time steps are limited to 15 and the basin model should have 50 or less spatial elements, including all elements and their spatial subdivisions. No more than 15 different network alternatives can be tried in a single computer run.

Memory requirements of the program is about 325 K, varying with compiler and machine needs.
Once compiled, the program analyzes a network alternative of the type shown in Chapter 5 for about $1.00 based on M.I.T.'s Information Processing Center charges as of 11/30/74. C.P.U. charges at that time were $12/minute.
DATA CARD DESCRIPTION FOR RAINFALL-RUNOFF NETWORK

DESIGN PROBLEM

<table>
<thead>
<tr>
<th>Card No.</th>
<th>Variable</th>
<th>Description</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>IPROB</td>
<td>no. of problems to be solved (alternative forms of matrix H)</td>
<td>(13)</td>
</tr>
<tr>
<td>2</td>
<td>NSN(I), I = 1,IPROB = number of stations to be located in each problem</td>
<td>(26I3,2x)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>NW</td>
<td>No. of weights - not used</td>
<td>I4</td>
</tr>
<tr>
<td></td>
<td>PRECI</td>
<td>precision of optimization, not used</td>
<td>F6.0</td>
</tr>
<tr>
<td></td>
<td>LIMIT</td>
<td>No. of optimization iterations, not used</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>NPTS</td>
<td>No. of station locations - same as no. of overland segments in basin description</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>NPS</td>
<td>No. of possible station locations (&lt; NPTS)</td>
<td>I5</td>
</tr>
<tr>
<td></td>
<td>NDTDT</td>
<td>No. of time steps in runoff model output</td>
<td>I5</td>
</tr>
<tr>
<td>4</td>
<td>WEIGHT (I), I = 1,NW = weights - not used (blank card needed)</td>
<td>8D10.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>ID(J), J = 1,NPS = id's of possible station locations, given in ascending order and corresponding to order in runoff input</td>
<td>(26I3,2x)</td>
<td></td>
</tr>
</tbody>
</table>
6 $\text{ER}(I), I = 1, NPS$ - variance of instrument measurement error $8D10.5$

7 $\text{Hl}(I), I = 1, NPS$ - values of $H$ matrix at possible points $8D10.5$

8 $\text{C0}(I), I = 1, NPS$ - costs of stations at possible locations, not used $(11F7.0, 3x)$

9 $\text{NOVER}$ - No. of overland segments $I4$

9 $\text{NRIVER}$ - No. of stream segments $I4$

9 $\text{NELEM}$ - Total no. of elements $I4$

9 $\text{NUPS}$ - No. of streams flowing into another $(\text{NRIVER}-1)$

9 $\text{NLAT}$ - No. of overland segments flowing into another segment $(\text{~NOVER})$ $I4$

9 $\text{NODES}$ Number of stream confluence points $I4$

10 $\text{NODX}(I), I = 1, \text{NODES}$

10 ID of element coming out of node $15I5, 5x$

11 $\text{NDX}(I), I = 1, 1, \text{NELEM}$

11 Spatial discretization of all elements $20I4$

12 $\text{PHI}(I,J,K)$ $J = 1, 2$

12 $I = 1, \text{NDIM} - \text{NODES}$

12 $K = 1, \text{NDTDT}$

12 $(18x, D10.3, 2x, D10.3)$

--- 337 ---
<table>
<thead>
<tr>
<th>Card No.</th>
<th>Variable Description</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elements of runoff transfer matrix, ( \xi' )</td>
<td>obtained from mean runoff solution (see Appendix 5)</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>JLAT(F), JLAT(I), XLAT(I,K)</td>
<td>( K = 1, NDTDT ) &lt;br&gt; ( I = 1, NLAT ) (10x, I3, 2x, I3, 2x, D10.3)</td>
</tr>
<tr>
<td>JLAT(I)</td>
<td>Id of segment being laterally fed by JJLAT(I). XLAT(I,K), corresponding element of transfer matrix. Cards obtained from mean runoff solution (see Appendix 5)</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>JUP(I), JJUP(I), XUP(I,K)</td>
<td>( I = 1, NUPS, K = 1, NDTDT ) (13x, I3, 2x, I3, 2x, D10.3)</td>
</tr>
<tr>
<td>JUP(I)</td>
<td>id's of element fed upstream by JJUP. XUP(I,K) element of transfer matrix given by mean solution (see Appendix 5)</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>DIS(K), K = 1, NDTDT</td>
<td>derivative of basin outlet discharge, ( m^m - 1 ) ( m ) calculated from mean solution (8D10.3) (See Appendix 5)</td>
</tr>
<tr>
<td>16</td>
<td>TDUR - storm duration (hrs)</td>
<td>F6.0</td>
</tr>
<tr>
<td>U - storm velocity (mi/hr)</td>
<td>F6.0</td>
<td></td>
</tr>
<tr>
<td>Direct - storm direction (not used)</td>
<td>F6.0</td>
<td></td>
</tr>
<tr>
<td>Card No.</td>
<td>Variable Description</td>
<td>Format</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------</td>
<td>--------</td>
</tr>
<tr>
<td>XLEN</td>
<td>length in x direction of area, optional if coordinates are to be generated uniformly (mi)</td>
<td>F6.0</td>
</tr>
<tr>
<td>YLEN</td>
<td>length in y direction (mi)</td>
<td>F6.0</td>
</tr>
<tr>
<td>C</td>
<td>coefficient of spatial correlation</td>
<td>F6.0</td>
</tr>
<tr>
<td>NX</td>
<td>grid dimension in x direction (optional)</td>
<td>I3</td>
</tr>
<tr>
<td>NY</td>
<td>grid dimension in y direction</td>
<td>I3</td>
</tr>
<tr>
<td>NPTS</td>
<td>number of points (overland segments) (optional)</td>
<td>I3</td>
</tr>
<tr>
<td>NDT</td>
<td>No. of time steps in which storm is divided</td>
<td>I3</td>
</tr>
<tr>
<td>OPT1</td>
<td>1, 2 or 3 for single exponential quadratic or Bessel covariance</td>
<td>I2</td>
</tr>
<tr>
<td>OPT2</td>
<td>0 or 1 for reading or generating coordinates</td>
<td>I2</td>
</tr>
</tbody>
</table>

17 If OPT2=1
COORDX(I), I=1, NPTS - x coordinates 16F5.0

18 If OPT2=1
COORDY(I), I=1, NPTS - y coordinates 16F5.0

19 VINT(J) J=1, NDT - standard deviation of mean storm 16F5.0
<table>
<thead>
<tr>
<th>Card No.</th>
<th>Variable Description</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>AINT(J), J=1,NDT - time distribution of mean storm</td>
<td>16F5.0</td>
</tr>
<tr>
<td>21</td>
<td>VMIN - minimum synoptic standard deviation of precipitation. This is the noise before storm arrives. Must be very small but non-zero for rainfall parameter estimation</td>
<td>F5.0</td>
</tr>
<tr>
<td>22</td>
<td>INIT(J), J=1,NS</td>
<td>(2613,2x)</td>
</tr>
</tbody>
</table>
THIS PROGRAM ANALYSES A DATA COLLECTION NETWORK IN REFERENCE TO
A FINITE DIFFERENCE SOLUTION OF THE KINEMATIC WAVE EQUATIONS.
PROGRAM PREPARED BY RAFAEL L. BRAS
LAST VERSION AS OF 12/3/74
SUBROUTINE GAMMA WAS PREPARED BY DARIO VALENCIA
SEE VALENCIA AND SCHAAKE 1972
DIMENSION NOX(15),NSN(15),IUPS(15,3),ILAT(15,2)
REAL*8 XUP(15,15)
REAL*8 DIS(15),CH1(50,50),CH2(50,50),BETA(50,10),DUMMY(50,50)
REAL*8 AA(10,10,15),BB(10,10,15),ERROR(10,10),H(10,10),H1(10),COST
1,ER(10),WEIGHT(15),XLAT(15,15),PHI(50,2,15),SIG1(10,10),SIG2(10,10)
COMMON /A1/ AA,BB
COMMON /A2/ ERROR,H,COST,H1,ER,CO(10),INIT(10),ID(10)
COMMON /A4/ X(10),Y(10),CCORDX(10),COORDY(10),VINT(10),AINT(10),XL
LEN,YLEN,VMIN,C,U,DT,NX,NY,NPTS,NDT,N,NS,NPS
COMMON /A5/ PHI,WEIGHT,D1S,ILAT,IUPS,NSN,NDIM,NELEM,NOTDT
COMMON /A6/ SIG1,CH1,CH2,BETA,DUMMY
COMMON /A7/ XLAT,XUP,ILAT(15),JJLAT(15),JUP(15),JJUP(15),NOD(15),N
10VER,NRIVER,NUPS,NLAT,NCDES,IFROB
READ NETWORK DESIGN DATA
CALL INPT
READ RAINFALL DATA AND COMPUTE MARKOV RAINFALL MODEL
CALL RAINFA
ITERATION ON NUMBER OF PROBLEMS
DO 11 I=1,IPROB
NS=NSN(I)
INITIALIZE FIRST SOLUTION
CALL INIT
FIND ERROR COVARIANCE OF RAINFALL AND PROPAGATE THROUGH RUNOFF MODEL
CALL ROUTINES DESIGN, FILTER, RUNOFF
OBTAIN VARIANCES OF DISCHARGE FOR EACH TIME
CALL DESIGN
11 CONTINUE
CALL EXIT
SUBROUTINE INPT1

REAL*8 XUP(15,15),SIG1(10,10),SIG2(10,10)
REAL*8 CHI1(50,50),CHI2(50,50),BETA(50,10),DUMMY(50,50)
REAL*8 PHI(50,2,15),XLAT(15,15),WEIGHT(15),DIS(15)
REAL*8 ERROR(10,10),H(10,10),COST(10),ER(10)

DIMENSION NOX(15),NSN(15),IUPS(15,3),ILAT(15,2)
COMMON /A2/ ERRORHCOSTH1,ERCO(10),INIT(10),ID(10)
COMMON /A4/ X(10),Y(10),COORDX(10),COORDY(10),VINT(10),AINT(10),XL
COMMON /A5/ PHI,WEIGHT,DIS,ILAT,IUPS,NDX,NSN,NL,NELEM,NDT
COMMON /A6/ SIG1SIG2,CHI1,CHI2,BETA,DUMMY
COMMON /A7/ XLATXUPJLAT(15),JJLAT(15),JUP(15),JJUP(15),N

C SUBROUTINE INPUT1 READS IN ALL DATA RELEVANT TO NETWORK DESIGN FOR
C A RAINFALL RUNOFF SYSTEM
C
C READ SERIES OF NUMBER OF STATIONS FOR WHICH DESIGN WILL BE TRIED
READ(5,28) IPROB
28 FORMAT(13)
READ(5,3) (NSN(I),I=1,IPROB)

C READ NUMBER OF OBJECTIVE FUNCTION WEIGHTS; PRECITION; ITERATION LIMITS
C TOTAL NUMBER OF POINTS; NUMBER OF POSSIBLE STATIONS LOCATIONS; NUMBER
C ER OF TIME INTERVALS
READ(5,1) NW,PRECILIMIT,NPTS,NPS,NDT
1 FORMAT(14,F6.0,4I5)

C READ WEIGHTS
READ(5,4) (WEIGHT(I),I=1,NW)
4 FORMAT(8C10.5)

C READ ID'S OF POSSIBLE STATION LOCATIONS
C ***** IMPORTANT ***** READ INDICATORS IN ASCENDING ORDER
C NUMBERING IS CONTINUOUS FROM LEFT TO RIGHT BOTTOM UP
READ(5,3) (ID(J),J=1,NPS)
3 FORMAT(26I3,2X)

C READ MEASUREMENT ERROR VARIANCE AT POSSIBLE STATION LOCATIONS
READ(5,4) (ER(I),I=1,NPS)
4 FORMAT(8C10.5)
C READ POSSIBLE ELEMENTS OF MATRIX H
READ(5,4) (HI(I),I=1,NPS)
C READ COSTS AT POSSIBLE POINTS
READ(5,7) (CI(I),I=1,NPS)
7 FORMAT(11F7.0,3X)
C READ INFORMATION ON HYPOTHETICAL CATCHMENT MODEL
READ(5,30) NOVER,NRIVER,NELEM,NUPS,NLAT,NOEC
C NUPS IS THE NUMBER OF STREAMS FLOWING INTO ANOTHER
C NLAT IS THE NUMBER OF OVERLAND SEGMENTS FLOWING INTO ANOTHER
30 FORMAT(614)
READ(5,90) (NODE(I),I=1,NOEC)
90 FORMAT(15I5,5X)
READ(5,31) (NOE(I),I=1,NELEM)
31 FORMAT(20I4)
NDIM=0
DO 35 I=1,NELEM
   35 NDIM=NDIM+NOE(I)
C READ PHI MATRIX ELEMENTS
   NDIM=NDIM+NOES
   ND2=NDIM-NOES
   DO 50 K=1,NDTCT
      DO 50 I=1,ND2
         50 READ(5,51) (PHI(I,J,K),J=1,2)
   51 FORMAT(18X,D10.3,2X,D10.3)
C READ UPSTREAM CONNECTIVITY
C
   DO 54 K=1,NDTCT
      DO 54 I=1,NLAT
         54 READ(5,55) JLAT(I),JJLAT(I),XLAT(I,K)
   55 FORMAT(10X,I3,2X,I3,2X,D10.3)
C READ UPSTREAM CONDITIONS
   DO 57 K=1,NDTCT
      DO 57 I=1,NUPS
         57 READ(5,58) JUP(I),JJUP(I),XUP(I,K)
   58 FORMAT (13X,I3,2X,I3,2X,D10.3)
C READ DISCHARGE LINEARIZATION
READ(5,105) (DIS(K),K=1,NDTDT)
105 FORMAT(8D10.3)
C INITIALIZE DUMMY
DO 70 I=1,NDIM
DO 70 J=1,NDIM
70 DUMMY(I,J)=0.00
C INITIALIZE BETA
DO 74 I=1,NDIM
DO 74 J=1,NPTS
74 BETA(I,J)=0.00
IPREV=0
DO 73 I=1,NELEM
131 KK=NDX(I)
DO 72 J=1,KK
132 IF(I.GT.NPTS) GOTO 72
133 L=IPREV+J
134 BETA(L,I)=1.00/12.00
72 CONTINUE
73 CONTINUE
RETURN
END
SUBROUTINE RAINFA
INTEGER OPT1, OPT2
REAL*8 AA(10,10,15), BB(10,10,15), ERROR(10,10), HI(10), H1(10), COST
1, ER(10)
REAL*8 WEIGHT(15), XLAT(15,15), PHI(50,2,15), SIG1(10,10), SIG2(10,10)
1, CHI1(50,50), CHI2(50,50), BETA(50,10), DUMMY(50,50), DIS(15)
DIMENSION A(10,10), BBT(10,10), COV(10,10), COR(10,10), SYX(10,10), SXX
1(10,10), SY(10,10), SXY(10,10)
DIMENSION ILAT(15,2), IUPS(15,3), NSN(15), NDX(15)
DIMENSION T1(100), T2(100), T3(10)
COMMON /A1/ AA, BB
COMMON /A2/ ERROR, H, COST, H1, ER, CO(10), INIT(10), ID(10)
COMMON /A4/ X(10), Y(10), COORDX(10), COORDY(10), VINT(10), AINT(10), XL
1EN, YLEN, VMIN, C, U, DT, NX, NY, NPTS, NDT, N, NS, NPS
COMMON /A5/ PHI, WEIGHT, DIS, ILAT, IUPS, NDX, NSN, NDIM, NELEM, NDTDT
COMMON /A6/ SIG1, SIG2, CHI1, CHI2, BETA, DUMMY
READ INPUT DATA
READ(5,1) TDUR, U, DIREC, XLEN, YLEN, C, NX, NY, NPTS, NDT, OPT1, OPT2
1 FORMAT (6F6.0,4I3,2I2)
C DIREC CORRESPONDS TO ANGLE FROM FIXED X AXIS IN DEGREES
C OPT1 CAN TAKE VALUES 1, 2 OR 3 FOR SINGLE EXP, QUADRATIC EXP OR BESSEL
C CORRELATION
C OPT2 IS 0 FOR READING COORDINATES, 1 FOR GENERATING UNIFORM GRID
IF (OPT2) 3, 4, 3
4 READ(5,2) (COORDX(I), I=1, NPTS)
2 FORMAT (16F5.0)
READ(5,2) (COORDY(I), I=1, NPTS)
GOTO 7
3 CALL COORD
7 DT = TDUR/NDT
READ(5,2) (VINT(J), J=1, NDT)
READ(5,2) (AINT(J), J=1, NDT)
C READ SYNOPTIC MINIMUM VARIANCE
READ(5,2) VMIN
C EVALUATE NORMALIZED COVARIANCE
CALL CORR(COV, COR, OPT1)
C  CALCULATE PARAMETERS A(T), BBT(T)
CALL PARAM(CG,COR,AA,BB)
WRITE(6,100)
100 FORMAT(1H1,25X,'MARKOV MODEL RAINFALL APPROXIMATION - PARAMETER ESTIMATION')
WRITE(6,101) TCUR,U
101 FORMAT(1H0,12X,'STORM PARAMETERS: ',DURATION(HRS), F5.2, VELOCITY(MPH), F5.2)
WRITE(6,102) XLEN, YLEN
102 FORMAT(1H0,12X,'AREA INFORMATION: ', LENGTH X DIRECTION(MI), F5.2,
1,3X,'Y DIRECTION(MI)', F5.2)
WRITE(6,103) NPTS, DT
103 FORMAT(1H0,12X,'NUMBER OF SPATIAL POINTS ', I4, 3X,'TIME INTERVAL(HRS)', F5.3)
WRITE(6,501) C
501 FORMAT(1H0,12X,'CORRELATION COEFFICIENT ', F6.3)
WRITE(6,104)
104 FORMAT(1H0,25X,'POINTS COORDINATES(MI)')
WRITE(6,105)
105 FORMAT(1H0,25X,'X COORD.', 2X,'Y COORD.')
DO 300 I=1,NPTS
WRITE(6,106) CCCRDX(I), CCCRCY(I)
300 CONTINUE
WRITE(6,107)
107 FORMAT(1H0,25X,'MEAN AREAL STORM INFORMATION')
WRITE(6,108)
108 FORMAT(1H0,15X,'TIME(HRS)', 4X,'INTENSITY(IN)', 4X,'STD.DEV.')
DO 320 I=1,NDT
TIME=I*DT
WRITE(6,109) TIME, AINT(I), VINT(I)
320 CONTINUE
WRITE(6,115)
115 FORMAT(1H0,25X,'MARKOV MODEL PARAMETERS')
WRITE(6,116)
FORMAT(1H0,32X,'ALPHA MATRIX')
DO 400 I=1,N
RTIME=I*DT
WRITE(6,117) RTIME
117 FORMAT(1H0,32X,'REAL TIME(HRS)',F6.2)
DO 400 M=1,NPTS
WRITE(6,118) (AA(M,L,I),L=1,NPTS)
118 FORMAT(1H0,10(F7.3,3X))
CONTINUE
WRITE(6,120)
120 FORMAT(1H0,32X,'BETA MATRIX')
DO 420 I=1,N
RTIME=I*DT
WRITE(6,121) RTIME
121 FORMAT(1H0,32X,'REAL TIME(HRS)',F6.2)
DO 420 M=1,NPTS
WRITE(6,122) (BB(M,L,I),L=1,NPTS)
122 FORMAT(1H0,10(F7.3,3X))
CONTINUE
RETURN
END
SUBROUTINE PARAM(CCV, COR, AA, BB)
THIS SUBROUTINE CALCULATES MARKOV MODEL PARAMETERS AT EACH TIME
REAL*8 AA(10, 10, 15), BB(10, 10, 15), WEIGHT(15), XLAT(15, 15), PHI(50, 2, 1)
COMMON /A4/ X(10), Y(10), COORDX(10), COORDY(10), VINT(10), AINT(10), XL
COMMON /A5/ PHI, WEIGHT, DIS, X, Y, COR(10, 10)
DO 100 K = 1, N
DO 40 I = 1, NPTS
L1 = (COORDX(I) / U) / DT
IF (K - L1) 10, 11, 11
IF (K - L1) .GT. NDT GOTO 10
V11 = VINT(K - L1)
GOTO 12
10 V11 = VMIN
11 IF ((K - L1) .GT. NDT) GOTO 11
V11 = VMIN
GOTO 12
40 CONTINUE
DO 10 J = 1, NPTS
L2 = (COORDX(J) / U) / DT
IF (K - L2) 13, 14, 15
IF ((K - L2) .GT. NDT) GOTO 13
V12 = VINT(K - L2)
GOTO 25
12 V12 = VMIN
25 IF ((K - L2) .GT. NDT) GOTO 25
N = N + NDT
IF (N .LE. NDT) N = NDT
DO 100 K = 1, N
DO 40 I = 1, NPTS
L1 = (COORDX(I) / U) / DT
IF (K - L1) 10, 11, 11
IF (K - L1) .GT. NDT GOTO 10
V11 = VINT(K - L1)
GOTO 12
10 V11 = VMIN
11 IF ((K - L1) .GT. NDT) GOTO 11
V11 = VMIN
GOTO 12
40 CONTINUE
DO 10 J = 1, NPTS
L2 = (COORDX(J) / U) / DT
IF (K - L2) 13, 14, 15
IF ((K - L2) .GT. NDT) GOTO 13
V12 = VINT(K - L2)
GOTO 25
13 V12 = VMIN
25 IF ((K - L2) .GT. NDT) GOTO 25
27 IF((K-1-L2).GT. ADT) GOTO 26
   V22=VINT(K-1-L2)
   GOTO 35
26  V22=VMIN
35  SXX(I,J)=V21*V22*COV(I,J)
    SYY(I,J)=V11*V12*COV(I,J)
    SXY(I,J)=V21*V12*COR(I,J)
40  CONTINUE
    CALL TRANS(SXY,NPTS,NPTS,SYY)
    CALL GAMMA(SYY,SXY,SXX,NPTS,NPTS,10,10,A,BBT,T1,T2,T3)
    DO 60 M=1,NPTS
    DO 60 L=1,NPTS
       AAA(M,L,K)=A(M,L)
60   BB(M,L,K)=BBT(M,L)
100  CONTINUE
    RETURN
END
SUBROUTINE TRANS(SXY,N1,N2,SYX)
C SUBROUTINE TO TRANSPOSE A GENERAL MATRIX
DIMENSION SXY(10,10),SYX(10,10)
COMMON /A4/ X(10),Y(10),COORDX(10),COORDY(10),VINT(10),AINT(10),XL
LEN,YLEN,VMIN,C,UT,NX,NY,NPTS,NDT,N,NS,NPS
DO 20 I=1,N1
DO 20 J=1,N2
20 SYX(J,I)=SXY(I,J)
RETURN
END
SUBROUTINE CORR(COV, COR, OPT1)
C  SUBROUTINE TO FORM NORMALIZED COVARIANCE MATRIX AND NORMALIZED C  LAG ONE CORRELATION AT EACH POINT
INTEGER OPT1
DIMENSION COV(10,10), COR(10,10)
COMMON /A4/ X(10), Y(10), COORDX(10), COORDY(10), VINT(10), AINT(10), XL
LEN, YLEN, VMIN, C, U, DT, NX, NY, NPTS, NDT, N, NS, NPS
GOTO (1, 2, 3), CFTL
C  IF OPT1 IS 1 EVALUATE SINGLE EXP COVARIANCE
1 CONTINUE
Do 15 I=1, NPTS
Do 15 J=1, NPTS
D=(COORDY(I)-COORDY(J))**2 + (COORDX(I)-COORDX(J))**2
D=SQRT(D)
COV(I,J)=EXP(-C*D)
D1=(COORDY(I)-COORDY(J))**2 + (COORDX(I)-COORDX(J))**2 + U*DT)**2
D1=SQRT(D1)
COR(I,J)=EXP(-C*D1)
15 CONTINUE
GOTO 18
C  IF OPT1 IS 2 EVALUATE FOR QUADRATIC EXPONENTIAL
2 CONTINUE
Do 16 I=1, NPTS
Do 16 J=1, NPTS
D=(COORDY(I)-COORDY(J))**2 + (COORDX(I)-COORDX(J))**2
COV(I,J)=EXP(-C*D)
D1=(COORDY(I)-COORDY(J))**2 + (COORDX(I)-COORDX(J))**2 + U*DT)**2
COR(I,J)=EXP(-C*D1)
16 CONTINUE
GOTO 18
C  IF OPT1 IS 3 EVALUATE BESSEL FORM
3 CONTINUE
Do 17 I=1, NPTS
Do 17 J=1, NPTS
D=(COORDY(I)-COORDY(J))**2 + (COORDX(I)-COORDX(J))**2
D=SQRT(D)
V = C * D
CALL BESK(V, I, RK, IER)
COV(I, J) = V * RK
D1 = (COORDY(I) - COORDY(J))^2 + (COORDX(I) - COORDX(J) + U * DT)^2
D1 = SQRT(D1)
V = C * D1
CALL BESK(V, I, RK, IER)
COR(I, J) = V * RK
17 CONTINUE
18 CONTINUE
RETURN
END
SUBROUTINE COORD
C SUBROUTINE TO CALCULATE COORDINATES IN UNIFORM GRID
C POINTS ARE ASSUMED IN CENTER OF EACH GRID
COMMON /A4/ X(10), Y(10), COORDX(10), COORDY(10), VINT(10), AINT(10), XL
1EN, YLEN, VMIN, C, U, DT, NX, NY, NPTS, NDT, N, NS, NPS
DX=XLEN/NX
DY=YLEN/NY
C SET FIRST COORDINATE POINTS
Y(1)=DY/2.
X(1)=DX/2.
DO 10 I=2, NX
10 X(I)=X(I-1)+DX
DO 20 I=2, NY
20 Y(I)=Y(I-1)+DY
DO 25 J=1, NY
25 CONTINUE
COORDX(K)=X(I)
COORDY(K)=Y(J)
RETURN
END
SUBROUTINE FOR COMPUTING THE MATRICES A AND BBT

DESCRIPTION OF PARAMETERS
S11, S12, S21, S22: COVARIANCE MATRICES TO BE USED BY THE SUBROUTINE
M: ACTUAL NUMBER OF ROWS, AND ACTUAL NUMBER OF COLUMNS OF THE MATRIX S11; ACTUAL NUMBER OF ROWS OF THE MATRIX S12; ACTUAL NUMBER OF COLUMNS OF THE MATRIX S21
MM: ACTUAL NUMBER OF COLUMNS OF THE MATRIX S12; ACTUAL NUMBER OF ROWS OF THE MATRIX S21; ACTUAL NUMBER OF ROWS, AND ACTUAL NUMBER OF COLUMNS OF THE MATRIX S22
IM: SAME AS M BUT DIMENSIONED (IN THE CALLING PROGRAM) INSTEAD OF ACTUAL
IMM: SAME AS MM BUT DIMENSIONED (IN THE CALLING PROGRAM) INSTEAD OF ACTUAL
A: THE RESULTANT MATRIX EQUAL TO S12*INVERSE OF S22 (ACTUAL SIZE: M*MM; DIMENSIONED SIZE IN THE CALLING PROGRAM: IM*IMM)
BBT: THE RESULTANT MATRIX EQUAL TO S11-S12*INVERSE OF S22*S21 (ACTUAL SIZE: M*M; DIMENSIONED SIZE IN THE CALLING PROGRAM: IM*IM)
T1: AUXILIARY VECTOR FOR THE COMPUTATION (DIMENSIONED SIZE IN THE CALLING PROGRAM: IMM TIMES IMM)
T2, T3: AUXILIARY VECTORS FOR THE COMPUTATION (DIMENSIONED SIZE OF EACH ONE IN THE CALLING PROGRAM: IMM)

REMARK
THIS SUBROUTINE REQUIRES THE SUBROUTINE MINV (FOR MATRIX INVERSION), AVAILABLE IN THE SCIENTIFIC SUBROUTINE PACKAGE.

SUBROUTINE GAMMA (S11,S12,S21,S22,M,MM,IM,IMM,A,BBT,T1,T2,T3) DIMENSION S11(IM,IM),S12(IM,IMM),S21(IMM,IM),S22(IMM,IMM),
A(IM,IMM),BBT(IM,IM),T1(1),T2(1),T3(1)
L=0
DO 20 J=1,MM
DO 20 I=1,MM
L=L+1  
T1(L)=S22(I,J)  
CALL MINV(T1,MM,D,T2,T3)  
IF(D) 60,60,61  
60 WRITE(6,97) D  
97 FORMAT(1H0,'***ERROR *** NEAR ZERO OR NEGATIVE DETERMINANT D= ',F16.10)  
16,10)  
CALL EXIT  
61 CONTINUE  
L=0  
DO 30 J=1,MM  
DO 30 I=1,MM  
L=L+1  
30 S22(I,J)=T1(L)  
DO 40 I=1,M  
DO 40 J=1,MM  
SUM=0  
DO 35 K=1,MM  
35 SUM=SUM+S12(I,K)*S22(K,J)  
40 A(I,J)=SUM  
DO 50 I=1,M  
DO 50 J=1,M  
SUM=0  
DO 45 K=1,MM  
45 SUM=SUM+A(I,K)*S21(K,J)  
50 BBT(I,J)=S11(I,J)-SUM  
RETURN  
END
SUBROUTINE IN111
REAL*8 ERROR(10,10),H(10,10),HI(10),ER(10),COST
COMMON /A2/ ERROR,H,COST,H1,ER,CO(10),INIT(10),ID(10)
COMMON /A4/ X(10),Y(10),COORDX(10),COORDY(10),VINT(10),AINT(10),XL
LEN,YLEN,VMIN,C,U,DT,NX,NY,NPTS,NDT,N,NS,NPS
C THIS SUBROUTINE SETS INITIAL MATRIX H AND CORRESPONDING
C MEASUREMENT ERROR COVARIANCE MATRIX. INITIAL DESIGN COST IS
C ALSO CALCULATED
C READ ID'S OF INITIAL DESIGN
READ(5,12) (INIT(I),J=1,NS)
12 FORMAT(26I3,2X)
C FORM H MATRIX AND INITIAL COST
COST=0.DO
DO 40 I=1,NS
DO 41 L=1,NPS
IF(INIT(I)-ID(L)) 41,51,41
41 CONTINUE
51 LL=L
COST=COST+CO(LL)
DO 40 J=1,NPTS
HI(I,J)=0.DO
HI(I,INIT(I))=HI(LL)
40 CONTINUE
C FORM ERROR COVARIANCE MATRIX
DO 55 I=1,NS
DO 55 J=1,NS
IF(I-J) 59,56,59
56 DO 57 L=1,NPS
IF(INIT(I)-ID(L)) 57,58,57
57 CONTINUE
58 LL=L
ERROR(I,J)=ER(LL)
GOTO 55
59 ERROR(I,J)=0.DO
55 CONTINUE
WRITE(6,100)
100 FORMAT(1H1,20X,'*** H MATRIX ***')
DO 70 I=1,NS
70 WRITE(6,102) (H(I,J),J=1,NPTS)
102 FORMAT(1H ,10(C9.1,3X))
WRITE(6,107)
107 FORMAT(1HO,20X,'*** ERROR MATRIX ***')
DO 72 I=1,NS
72 WRITE(6,103) (ERROR(I,J),J=1,NS)
103 FORMAT(1H ,10(C10.2,2X))
RETURN
END
SUBROUTINE DESIGN
REAL*8 XUP(15,15)
REAL*8 AA(10,10,15),BB(10,10,15),CHI1(50,50),CHI2(50,50),BETA(50,10),DUMMY(50,50),DIS(15)
REAL*8 SIG1(10,10),SIG2(10,10),AAA(10,10),BBB(10,10),ERROR(10,10),
1H(10,10)
REAL*8 WEIGHT(15),XLAT(15,15),PHI(50,2,15)
REAL*8 COST,H1(10),ER(10)
DIMENSION ILAT(15,2),IUPS(15,3),NSN(15),NDX(15)
COMMON /A1/ AA,BB
COMMON /A2/ ERROR,H,COST,H1,ER,CO(10),INIT(10),ID(10)
COMMON /A4/ X(10),Y(10),CCORDX(10),COORDY(10),VINT(10),AINT(10),XL
1EN,YLEN,VMIN,C,U,DT,NX,NY,NPTS,NNDT,N,NLAP,N
COMMON /A5/ PHI,WEIGHT,DIS,ILAT,IUPS,NDX,NSN,NDIM,NELEM,NDTDT
COMMON /A6/ SIG1,SIG2,CHI1,CHI2,BETA,DUMMY
COMMON /A7/ XLATXUP,JLAT(15),JJLAT(15),JUP(15),JJUP(15),NOD(15),N
1OVER,NRIVER,NUPS,NLAT,NODES,IFPROB
DO 30 I=1,NPTS
DO 30 J=1,NPTS
SIG1(I,J)=0.D0
30 INITIALIZE CHI1 AND CHI2
DO 47 I=1,NDIM
DO 47 J=1,NDIM
CHI1(I,J)=0.D0
47 CHI2(I,J)=0.D0
DO 32 K=1,N
DO 33 I=1,NPTS
DO 33 J=1,NPTS
AAA(I,J)=AA(I,J,K)
33 BBB(I,J)=BB(I,J,K)
CALL FILTER(AAA,BBB,SIG2,SIG1)
RTIME=K*DT
WRITE(6,75)
75 FORMAT(1HO,20X,'ERROR COVARIANCE MATRIX - RAINFALL')
WRITE(6,55) RTIME
55 FORMAT(1HO,5X,'REAL TIME(HRS)',F6.2)
DO 35 L=1,NPTS
35 WRITE(6,40) (SIG2(L,M),M=1,NPTS)
40 FORMAT(10(D10.3,2X))
   CALL RUNOFF(K)
   DO 45 I=1,NPTS
   DO 45 J=1,NFTS
45 SIG1(I,J)=SIG2(I,J)
32 CONTINUE
   RETURN
END
SUBROUTINE FILTER(AAA, BBB, SIG2, SIG1)
C THIS SUBROUTINE FINDS THE MEAN SQUARE ERROR INVOLVED IN ESTIMATING
C POINT RAINFALL INTENSITY FROM A GIVEN NETWORK AT TIME T
REAL*8 AAA(10,10), BBB(10,10), SIG1(10,10), SIG2(10,10)
REAL*8 R1(10,10), R2(10,10), ERROR(10,10), H(10,10), H1(10), COSTER(10,10)
REAL*8 DETERM
COMMON /A2/ ERROR, H, COST, H1, ER, CO(10), INIT(10), ID(10)
COMMON /A4/ X(10), Y(10), COORDX(10), COORDY(10), VINT(10), AINT(10), XLEN, YLEN, VMIN, C, U, DT, NX, NY, NPTS, NDT, N, NS, NPS
DIMENSION IWORK(10,2)
C OBTAIN SIGMA(T+1/T)
IER=0
CALL DMMGG(AAA,10,SIG1,10,NPTS,NPTS,NPTS,SIG2,10,IER)
CALL DMMGG(SIG2,10,AAA,-10,NPTS,NPTS,NPTS,SIG1,10,IER)
CALL DMAGG(SIG1,10, BBB,10,NPTS,NPTS,3,SIG2,10,IER)
C OBTAIN SIGMA(T+1/T+1)
CALL DMMGG(H1S SIG2,10,NS,NPTS,NPTS,R1,10,IER)
CALL DMMGG(R1,10,H,-10,NS,NPTS,NS,SIG1,10,IER)
CALL DMAGG(ERROR,10,SIG1,10,NS,NPTS,3,SIG1,10,IER)
IF(NS .NE. 1) GOTO 997
SIG(1,1)=1./SIG(1,1)
GOTO 996
997 CALL DMINV(10,NS,SIG1,DETERM,IIWORK,IERR)
996 CALL DMMGG(SIG1,10,H,10,NS,NS,NPTS,R1,10,IER)
CALL DMMGG(R1,10,SIG2,10,NS,NPTS,NPTS,SIG1,10,IER)
CALL DMMGG(SIG2,10,H,-10,NPTS,NPTS,NS,R1,10,IER)
CALL DMAGG(SIG1,10, R1,10,NPTS,NPTS,R2,10,IER)
CALL DMAGG(SIG2,10,R2,10,NPTS,NPTS,1,SIG2,10,IER)
RETURN
END
SUBROUTINE RUNCFF(K)

DIMENSION LAT(15,2),NSN(15),NDX(15),IUPS(15,3)
REAL*8 DSQRT
REAL*8 STDEV
REAL*8 XUP(15,15)
REAL*8 SIG1(10,10),SIG2(10,10),CHI1(50,50),CHI2(50,50),DUMMY(50,50)
1,BETA(50,10),R8(50,50),R9(50,50)
REAL*8 WEIGHT(15),XLAT(15,15),PHI(50,2,15),DIS(15),DISVAR
COMMON /A4/ X(10),Y(10),COORDX(10),COORDY(10),VINT(10),AINT(10),XL
1EN,YLEN,VMIN,C,UT,NX,NY,NPTS,NOT,N,NS,NPS
COMMON /A5/ PHI,WEIGHT,DIS,ILAT,IUPS,NDX,NSN,NDIM,NELEM,NDTDT
COMMON /A6/ SIG1,SIG2,CHI1,CHI2,BETA,DUMMY
COMMON /A7/ XLAT,XUP,ILAT(15),JLLAT(15),JU(15),JJUP(15),NOD(15),N
1OVER,NNRIVER,NUPS,NLAT,NODES,IPROB

I-362-

IPREVI=0
DO 43 I=1,NELEM
KK=NDX(I)
DO 42 J=1,KK
I=IPREVI+J
IF(J .NE. 1) GOTO 57
DUMMY(L,L)=PHI(L,2,K)
GOTC 42
57 DUMMY(L,L-1)=PHI(L,1,K)
DUMMY(L,L)= PHI(L,2,K)
42 CONTINUE
IPREVI=L
43 CONTINUE
DO 60 I=1,NLAT
II=JLAT(I)
JJ=0
DO 62 NI=1,II
JJ=JJ+NDX(NI)
IJ=JLLAT(I)
KK=0
DO 63 NI=1,IJ
KK=KK+NDX(NI)
62 CONTINUE
DO 60 I=1,NLAT
II=JLAT(I)
JJ=0
DO 62 NI=1,II
JJ=JJ+NDX(NI)
IJ=JLLAT(I)
KK=0
DO 63 NI=1,IJ
KK=KK+NDX(NI)
62 CONTINUE
DO 60 I=1,NLAT
II=JLAT(I)
JJ=0
DO 62 NI=1,II
JJ=JJ+NDX(NI)
IJ=JLLAT(I)
KK=0
DO 63 NI=1,IJ
KK=KK+NDX(NI)
60 CONTINUE
ND2=NDIM-NCDES
DO 80 I=1,NLPS
II=JUP(I)
DO 82 J=1,NCDES
IF (II .EQ. NCES(J)) GOTO 83
82 CONTINUE
83 JJ=ND2+J
IJ=JJUP(I)
KK=0
DO 84 NI=1,IJ
84 KK=KK+NDX(NI)
DUMMY(JJ,KK)=XLAT(I,K)
IK=II-1
LL=0
DO 85 NI=1,IK
85 LL=LL+NDX(NI)
DUMMY(LL+1,JJ)=PHI(LL+1,1,K)
80 CONTINUE

C COMPUTE COVARIANCE OF DISCHARGE
IER=0
CALL DMMGG(BETA,50,SIG2,10,NDIM,NPTS,NPTS,R8,50,IER)
CALL DMMGG(R8,50,BETA,-50,NDIM,NPTS,NDIM,R9,50,IER)
CALL DMMGG(DUMMY,50,CHI1,50,NDIM,NDIM,NDIM,NDIM,R8,50,IER)
CALL DMMGG(R8,50,DUMMY,-50,NDIM,NDIM,NDIM,CHI2,50,IER)
CALL DMMGG(CHI2,50,R9,50,NDIM,NDIM,NDIM,3,CHI2,50,IER)
DO 100 I=1,NDIM
DO 100 J=1,NDIM
100 CHI1(I,J)=CHI2(I,J)
RTIME=K*DT
WRITE(6,102) RTIME
102 FORMAT(1HC,5X,'REAL TIME(HRS)',F6.2)
ND2=NDIM-NCDES
DISVAR=(DIS(K)**2)*CHI2(NC2,NC2)
WRITE(6,105) DISVAR
105 FORMAT(1H0,'***DISCHARGE VARIANCE IS ',D10.3)
STDEV=DSQRT(DISVAR)
WRITE(6,106) STDEV
106 FORMAT(1H0,'*** DISCHARGE STANDARD DEVIATION IS ',D10.3)
RETURN
END
SUBROUTINE WIER(IER, NO)
RETURN
END
Appendix  8

Following is a listing of the data used with the rainfall-runoff network design program.
<p>| 120.00 | 1 3 | C.C43DC | C.957CC |
| 120.00 | 2 2 | C.C DC | C.957DC |
| 120.00 | 2 3 | 0.043DC | 0.957CC |
| 120.00 | 3 3 | C.0 DC | C.949DC |
| 120.00 | 4 2 | 0.0 DC | C.945DC |
| 120.00 | 4 3 | C.05DC | C.945DC |
| 120.00 | 5 2 | C.C DC | C.938DC |
| 120.00 | 5 3 | 0.065DC | C.938DC |
| 120.00 | 6 2 | C.0 DC | C.938DC |
| 120.00 | 6 3 | C.062DC | C.938DC |
| 120.00 | 7 2 | C.0 DC | C.916DC |
| 120.00 | 7 3 | 0.084DC | C.916DC |
| 120.00 | 8 2 | C.C DC | C.916DC |
| 120.00 | 8 3 | 0.084CC | C.916DC |
| 120.00 | 9 2 | 0.0 CO | 0.894CC |
| 120.00 | 10 3 | C.106DC | 0.894CC |
| 120.00 | 10 3 | 0.081DC | C.919CC |
| 120.00 | 11 2 | C.0 DC | C.920CC |
| 120.00 | 11 3 | C.086DC | C.920CC |
| 120.00 | 12 2 | C.081DC | C.958DC |
| 120.00 | 12 3 | 0.042DC | C.958DC |
| 180.00 | 1 2 | C. C DC | C.881DC |
| 180.00 | 1 3 | 0.119DC | C.881DC |
| 180.00 | 2 2 | 0.0 CO | 0.881CC |
| 180.00 | 2 3 | 0.119DC | C.881DC |
| 180.00 | 3 2 | C.0 DC | C.849DC |
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Biography

Rafael Luis Bras was born on October 28, 1950, in San Juan, Puerto Rico. All his primary and secondary education was received in that city.

He was awarded a Bachelor in Science in Civil Engineering from M.I.T. in February 1972. He continued his graduate studies at M.I.T. after receiving a fellowship from the Economic Development Administration of Puerto Rico. He received the S.M. degree in Civil Engineering in February 1974.

The author is a member of the American Geophysical Union, American Society of Civil Engineers, Boston Society of Civil Engineers, and the American Association for the Advancement of Science. He is also a member of Tau Beta Pi, Chi Epsilon and Sigma Xi honorary associations.

He is a registered, licensed Engineer (Puerto Rico).

His publications include:


