FLEXIBLE PAVEMENT SYSTEMS: AN ANALYSIS OF THE STRUCTURAL SUBSYSTEM'S DETERIORATION

by

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ABSTRACT

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The performance of a large class of public facilities is dependent upon the subjective evaluation of the users and their relative acceptability of these facilities. The serviceability level of any structural system in an operating environment is bound up by uncertainties resulting from randomness in both the physical characteristics of the system, and the surrounding environment. These uncertainties are expressed in terms of the system's reliability, i.e., the probability of providing satisfactory levels of serviceability. The levels of maintenance exercised on the system control its serviceability as well as its reliability and its operational lifetime.

A method of analysis is presented for the prediction of the deterioration of the pavement system's serviceability and reliability due to traffic loads and environment, and the effects of maintenance activities on that deterioration. A limited sensitivity study is presented to demonstrate the capability of this model to predict the serviceability, reliability, expected lifetime, and optimum maintenance strategy, given the initial design configuration, as well as to predict the effects of alternative design configurations.

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Chapter One. Introduction

Highway systems belong to a large class of public (or, in some cases, private) facilities whose specific functions derive from the more general goals of the society using the systems. The performance of these systems is largely dependent on the subjective evaluation and acceptance of their users. It is therefore desirable to evaluate these systems from the levels of service that they provide their users at any time during their operational lives. In this context, failure may be regarded as a threshold that is reached as the performance level deteriorates below some unacceptable limit, as defined by the users of the facility.

The present design practices for highway systems are largely empirical, based on experience and engineering judgments. They are basically expressed in terms of correlations between soil type, base course properties, layer thicknesses, and traffic and environmental factors. Although these practices have met with moderate success in the past, the rapid increases in traffice volumes, in construction and maintenance costs and number of techniques, and the potential of new materials make empiricism obsolescent or even non-existent. Therefore, a design method based on theory and calibrated to empiricism is required. The method must encompass a set of analytical procedures that can effectively predict the behavior of pavement systems and the interactions among their components. Further, this design method must choose as a means of system evaluation such measures of effectiveness as define the specific goals and functions of the system it represents, and which are representative of the desires of the users it serves.

I. 1 Measures of Effectiveness

The analysis and design of pavement systems, just as the analysis and selection of other public investments, require a knowledge of both the supply and demand functions of the public to be served. In this context, the supply functions may be regarded as the set of available techniques to combine a variety of resources to produce highway pavements in the most socially beneficial manner. A set of resources combined in a certain way over a particular interval of time is referred to as a design strategy. Usually, there are many strategies that are acceptable in any given situation. The question becomes which strategy meets the demand requirements most efficiently, where efficiency must be interpreted in societal terms.

The demand functions may be expressed in terms of the three components of performance: serviceability, reliability, and maintainability (S-R-M). (4).* The level of <u>serviceability</u> of any system is bound up by the uncertainties inherent in the physical characteristics of the system and of the surrounding environment.

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* The numbers in parenthesis refer to the list of references.

These uncertainties may be expressed in terms of the <u>reliability</u> of the system, i.e., the probability of providing acceptable levels of serviceability at any point within the operational lifetime of the system. Maintenance efforts exercised throughout the lifetime of the system enhance the level of serviceability of the system and its reliability, as well as its anticipated lifetime. This may be expressed in terms of the <u>maintainability</u> of the system, which is a measure of the effort required to maintain acceptable levels of serviceability throughout the design life of the system.

Economic constraints play an important role in controlling the levels of serviceability throughout the lifetime of the system by dictating the initial costs, maintenance costs, and vehicle operating costs.

The levels of (S-R-M) for each pavement should be commensurate with the anticipated usage of that pavement. The design decision is then to choose that strategy which meets the demand requirements subject to certain societal constraints, which may be economic, environmental, safety, etc.

In order to predict that a certain pavement system will meet the demand requirements, it is essential to have analytic or empirical means to evaluate how that pavement system will perform in the specified environment under the projected loading conditions. Most empirical means attempt to assess the performance capability by simply evaluating a single response of the system, such as maximum stress or deformation. These maxima or limits are often based on field experience and past experience, so that their applicability under a different set of conditions is questionable at best.

I. 2 The Proposed Design Framework

This study presents a methodological framework for the analysis and selection of pavement systems for a given set of goals and constraints. A set of models and algorithms has been developed at two different levels of analysis: analysis of the physical behavior of the system, and analysis for the selection and optimization of a design system. The first involves a set of mechanical and phenomenological models which describe the response of the system in a realistic operating environment. From these models the progression of damage within the system can be evaluated using physical transfer functions. The second level of analysis utilizes the above information to determine the levels of service that the system is providing at any point in time and the reliability of the system in the operational environment. Maintenance policies may be generated and evaluated, and an optimum set of strategies selected over the lifetime of the system, conditioned on the geometrical design configuration. Similarly, alternative design configurations may be generated and evaluated, and their optimal associated maintenances strategies obtained.

The basic features which characterize this study can be summarized as follows:

1. The proposed method of design for structural systems represents a departure from the traditional cook-book style methods generally pursued in the literature. Instead, the design is regarded as a process of sequential evolution of systematic analyses whose ultimate goal is the achievement of an optimal design configuration.
2. The criteria for model selection and evaluation are based on the users' subjective preferences for constructed facilities derived from their particular needs and sets of values. From this standpoint, the highway pavement is viewed as a system which is providing certain services to its users, and the quality of providing these services must be evaluated from the users' demands and preferences.
3. The models cover a wide spectrum of activities encompassing a large body of knowledge ranging from rational and applied mechanics to probability and operations research disciplines. The particular advantage derived from this coverage is that it provides continuity and integrity to the analysis and design.
4. The models possess a causal structuring thereby defining the different interactions between the system and the surrounding environment. Also, the feedback processes resulting from maintenance activities are accounted for.

5. The models recognize and incorporate the elements of uncertainty associated with the natural phenomena and processes represented.

The following chapter deals with the primary response of a three-layer viscoelastic halfspace to static and repeated loadings in a realistic operating environment. Chapter Three presents the ultimate damage to the system over time, using damage indicators similar to those developed by AASHO (5) as the components of damage that the users are generally sensitive to. A serviceability model utilizes the information provided by the structural model to predict stochastically the serviceability, reliability, and life expectancy of the system at desired points in time.

A framework for a decision structure for the choice of optimum maintenance strategies for a given design configuration is presented in Chapter Four. Further, a limited sensitivity analysis is provided in Chapter Five to validate and demonstrate the effectiveness of the developed models, which have been coded into a set of computer programs.

The development of further research activities to complement and calibrate these models in order that they may be effectively used as practical design tools is discussed in Chapter Six.

This study should be understood as an extension and refinement of the pioneering work done in this area by H.K. Findakly (1), familiarity with which is assumed of the reader.

Chapter Two. Closed-Form Probabilistic Solutions for the Response of a Three-Layered System

II. 1 Closed-Form Probabilistic Solutions to the Static Load Response of an Elastic Three-Layer System

Findakly has shown (1) that the mean and variance of a function $g(x_1,x_2,\ldots,x_n)$ of n random variables x_i may be evaluated, approximately, as:

$$E\,[g(x_1,x_2,\ldots,x_n)] \simeq g(\underline{M}) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \left.\frac{\partial^2 g}{\partial x_i \partial x_j}\right|_{\underline{M}} Cov(x_i,x_j) \qquad (2\text{-}1)$$

$$Var\,[g(x_1,x_2,\ldots,x_n)] \simeq \sum_{i=1}^{n} \left(\left.\frac{\partial g}{\partial x_i}\right|_{\underline{M}}\right)^2 \sigma^2_{x_i} \qquad (2\text{-}2)$$

where $\underline{M}$ is the point $(\bar{x}_1,\bar{x}_2,\ldots,\bar{x}_n)$; $E[\cdot]$ is the expectation operator; $Var[\cdot]$ is the variance operator; $Cov(x_i,x_j)$ is the covariance of x_i and x_j; and $\sigma^2_{x_i}$ is the variance of x_i.

If, in addition, the x_i are independent random variables, equation (2-1) above reduces to:

$$E\,[g(x_1,x_2,\ldots,x_n)] \simeq g(\underline{M}) + \frac{1}{2}\sum_{i=1}^{n} \left.\frac{\partial^2 g}{\partial x_i^2}\right|_{\underline{M}} \sigma^2_{x_i} \qquad (2\text{-}3)$$

while equation (2-2) remains as is.

Moavenzadeh and Elliott (2) have shown that the static load system response function Ψ for elastic three-layered deterministic systems can be expressed in the form:

$$\Psi_k = qa \int_0^\infty Bes_k(m,\rho,a) \frac{\sum_{j=1}^{18} \phi_{k,i,j}\beta_{i,j}}{\sum_{j=1}^{9} \Theta_j \alpha_{i,j}} dm \tag{2-4}$$

where k denotes which stress-strain-deflection component is desired; q is the intensity of the applied loading; a is the radius of the applied loading; Bes_k is a product of Bessel terms of order zero or one; ρ is the horizontal offset of the point of interest from the axis of the applied loading; m is a dummy integration variable; $\phi_{k,i,j}$ and Θ_j are functions of the system geometry only; and the $\beta_{i,j}$ and $\alpha_{i,j}$ are products of inverse integral powers of the elastic constants, i.e.:

$$\begin{aligned} \beta_{i,j} &= \prod_{r=1}^{3} E_r^{-\ell_{i,j,r}} \\ \alpha_{i,j} &= \prod_{r=1}^{3} E_r^{-n_{i,j,r}} \end{aligned} \tag{2-5}$$

where i denotes the layer in which the point of interest is located.

Since the random variables being considered are the elastic constants (which are assumed to be independent), only the partials of the $\beta_{i,j}$ and $\alpha_{i,j}$ are necessary, as these are the only terms containing the elastic constants. Upon defining:

$$B_k \triangleq qaBes_k(m,\rho,a) \tag{2-6}$$

$$U_{k,i} \triangleq \sum_{j=1}^{18} \phi_{k,i,j}\beta_{i,j} \tag{2-7}$$

$$D_i \triangleq \sum_{j=1}^{9} \Theta_j \alpha_{i,j} \tag{2-8}$$

and utilizing equations (2-2) and (2-3), we would have for the mean and variance of Ψ_k:

$$E[\Psi_k] \simeq \int_0^\infty B_k U_{k,i}/D_i \Big|_{\underline{M}} dm + \tfrac{1}{2}\int_0^\infty B_k \sum_{p=1}^{3} [\{\frac{\partial^2}{\partial E_p^2} (U_{k,i}/D_i)\}\Big|_{\underline{M}} \sigma^2_{E_p}]\, dm \tag{2-9}$$

$$Var[\Psi_k] \simeq \sum_{p=1}^{3} [\int_0^\infty B_k \{\frac{\partial}{\partial E_p} (U_{k,i}/D_i)\}\Big|_{\underline{M}} dm]^2 \sigma^2_{E_p} \tag{2-10}$$

We can further evaluate, as follows:

$$\frac{\partial}{\partial E_p} (U_{k,i}/D_i) = (D_i \frac{\partial U_{k,i}}{\partial E_p} - U_{k,i} \frac{\partial D_i}{\partial E_p})/D_i^{\,2} \tag{2-11}$$

$$\frac{\partial^2}{\partial E_p^2}(U_{k,i}/D_i) = (D_i\frac{\partial^2 U_{k,i}}{\partial E_p^2} - U_{k,i}\frac{\partial^2 D_i}{\partial E_p^2})/D_i^2 +$$

$$2\,\frac{\partial D_i}{\partial E_p}(U_{k,i}\frac{\partial D_i}{\partial E_p} - D_i\frac{\partial U_{k,i}}{\partial E_p})/D_i^3 \qquad (2\text{-}12)$$

where:

$$\frac{\partial U_{k,i}}{\partial E_p} = \sum_{j=1}^{18} \phi_{k,i,j}\frac{\partial \beta_{i,j}}{\partial E_p}$$

$$\frac{\partial^2 U_{k,i}}{\partial E_p^2} = \sum_{j=1}^{18} \phi_{k,i,j}\frac{\partial^2 \beta_{i,j}}{\partial E_p^2}$$

$$(2\text{-}13)$$

$$\frac{\partial D_i}{\partial E_p} = \sum_{j=1}^{9} \Theta_j\,\frac{\partial \alpha_{i,j}}{\partial E_p}$$

$$\frac{\partial^2 D_{i,j}}{\partial E_p^2} = \sum_{j=1}^{9} \Theta_j\frac{\partial^2 \alpha_{i,j}}{\partial E_p^2}$$

Using equations (2-5) we obtain:

$$\frac{\partial \beta_{i,j}}{\partial E_p} = -\ell_{i,j,p}\, E_p^{-\ell_{i,j,p} - 1} \prod_{\substack{r=1 \\ r\neq p}}^{3} E_r^{-\ell_{i,j,r}}$$

$$\frac{\partial^2 \beta_{i,j}}{\partial E_p^{\,2}} = \ell_{i,j,p}(\ell_{i,j,p} + 1)\, E_p^{-\ell_{i,j,p} - 2} \prod_{\substack{r=1 \\ r\neq p}}^{3} E_r^{-\ell_{i,j,r}}$$

$$(2\text{-}14)$$

$$\frac{\partial \alpha_{i,j}}{\partial E_p} = -n_{i,j,p}\, E_p^{-n_{i,j,p} - 1} \sum_{\substack{r=1 \\ r\neq p}}^{3} E_r^{-n_{i,j,r}}$$

$$\frac{\partial^2 \alpha_{i,j}}{\partial E_p^{\,2}} = n_{i,j,p}\,(n_{i,j,p} + 1)\, E_p^{-n_{i,j,p} - 2} \prod_{\substack{r=1 \\ r\neq p}}^{3} E_r^{-n_{i,j,r}}$$

Back-substitution of these results yields the desired solution. The randomness of the load characteristics are considered in the repeated loading phase. The radius of the applied loading is assumed deterministic. The integral in the above equations is evaluated by the parabolic integration technique discussed in Appendix A, while the Bessel terms may be evaluated by the approximations discussed in Appendix B.

II.2 Closed-Form Probabilistic Solutions to the Static Load Response of a Viscoelastic Three-Layered System

Utilizing the correspondence principle, Moavenzadeh and Elliott (2) obtained the deterministic static load response of a three-layered linear viscoelastic system from the deterministic elastic response. Just as the derivation of the probabilistic elastic solution from the deterministic elastic solution involved essentially only the evaluation of the partial derivatives of the $\alpha_{i,j}$ and $\beta_{i,j}$, so is the probabilistic viscoelastic solution derived from the probabilistic elastic solution by evaluating the partial derivatives of the $\alpha_{i,j}$ and $\beta_{i,j}$ when the E_k are replaced by their viscoelastic operator equivalents.

Utilizing the correspondence principle, i.e.:

$$\frac{1}{E_k}(\cdot) \rightarrow D_k(0)(\cdot) - \int_0^t (\cdot) \frac{\partial D_k(t-\xi)}{\partial \xi} d\xi \tag{2-15}$$

where $D_k(t)$ is the creep compliance function of the k^{th} layer, equations (2-5) become:

$$\begin{aligned} \alpha_{i,j} &\rightarrow \left(\prod_{k=1}^{3} \left\{D_k(0)(\cdot) - \int_0^t (\cdot) \frac{\partial D_k(t-\xi)}{\partial \xi} d\xi\right\}^{n_{i,j,k}}\right) * I(t) \\ \beta_{i,j} &\rightarrow \left(\prod_{k=1}^{3} \left\{D_k(0)(\cdot) - \int_0^t (\cdot) \frac{D_k(t-\xi)}{\partial \xi} d\xi\right\}^{\ell_{i,j,k}}\right) * H(t) \end{aligned} \tag{2-16}$$

where H(t) is the Heaviside step function:

$$\begin{aligned} H(t) &= 0 \qquad t < 0 \\ H(t) &= 1 \qquad t \geq 0 \end{aligned} \tag{2-17}$$

I(t) has the constant value 1, and * and Π denote convolution operations.

Following Moavenzadeh (2), we can represent a random creep compliance function as:

$$D_k(t) = \eta_k \sum_{j=1}^{m} \bar{G}_j^{\,k} \; e^{-\delta_j t} \tag{2-18}$$

where $\bar{G}_j^{\,k}$ and δ_j correspond to the constants G_j and δ_j, respectively, of Appendix C; and η_k is a random variable of mean 1 and variance $\sigma^2_{\eta_k}$ which is not a function of time. Appendix C presents a method for determining the G's and δ's from creep data measurements.

Substituting equation (2-18) for the D_k's into equations (2-16), and upon realizing that the η_k are time-independent and thus will move through the convolutions, yields (after suppressing the i,j in the $\ell_{i,j,k}$ and $n_{i,j,k}$ temporarily).

$$\alpha_{i,j} \rightarrow \left(\prod_{k=1}^{3} \eta_k^{\,n_k}\right)\left(\prod_{k=1}^{3} \left\{\left(\sum_{j=1}^{m} \bar{G}_j^{\,k}\right)(\cdot) - \int_0^t (\cdot) \sum_{j=1}^{m} \delta_j \bar{G}_j^{\,k} e^{-\delta_j (t-\xi)} d\xi\right\}^{n_k}\right) * I(t) \tag{2-19}$$

$$\beta_{i,j} \rightarrow \left(\prod_{k=1}^{3} \eta_k^{\,\ell_k}\right)\left(\prod_{k=1}^{3} \left\{\left(\sum_{j=1}^{m} \bar{G}_j^{\,k}\right)(\cdot) - \int_0^t (\cdot) \sum_{j=1}^{m} \delta_j \bar{G}_j^{\,k} e^{-\delta_j (t-\xi)} d\xi\right\}^{\ell_k}\right) * H(t) \tag{2-20}$$

Defining:

$$A_{i,j}(t) \triangleq \prod_{k=1}^{3}\{(\sum_{j=1}^{m} \overline{G}_j^k)(\cdot) - \int_0^t(\cdot) \sum_{j=1}^{m} \delta_j \overline{G}_j^k e^{-\delta_j(t-\xi)} d\xi\}^{n_k} \tag{2-21}$$

$$B_{i,j}(t) \triangleq \prod_{k=1}^{3}\{(\sum_{j=1}^{m} \overline{G}_j^k)(\cdot) - \int_0^t(\cdot) \sum_{j=1}^{m} \delta_j \overline{G}_j^k e^{-\delta_j(t-\xi)} d\xi\}^{\ell_k} \tag{2-22}$$

equations (2-19, 2-20) acquire the relatively simple form of:

$$\alpha_{i,j} \rightarrow (\prod_{k=1}^{3} \eta_k^{n_k}) A_{i,j}(t) * I(t) \tag{2-23}$$

$$\beta_{i,j} \rightarrow (\prod_{k=1}^{3} \eta_k^{\ell_k}) B_{i,j}(t) * H(t) \tag{2-24}$$

where here and in what follows * denotes convolution. In this form, the partial derivatives with respect to the random variables η_k are easily obtained, as follows:

$$\frac{\partial \alpha_{i,j}}{\partial \eta_p} \rightarrow n_p \eta_p^{n_p-1} (\prod_{\substack{k=1 \\ k \neq p}}^{3} \eta_k^{n_k}) A_{i,j}(t) * I(t) \tag{2-25}$$

$$\frac{\partial^2 \alpha_{i,j}}{\partial \eta_p^2} \rightarrow n_p(n_p-1)\eta_p^{n_p-2} (\prod_{\substack{k=1 \\ k \neq p}}^{3} \eta_k^{n_k}) A_{i,j}(t) * I(t) \tag{2-26}$$

$$\frac{\partial^2 \alpha_{i,j}}{\partial \eta_p \partial \eta_q} \rightarrow n_p n_q \eta_p^{n_p - 1} \eta_q^{n_q - 1} \left(\prod_{\substack{k=1 \\ k \neq p,q}}^{3} \eta_k^{n_k} \right) A_{i,j}(t) * I(t) \tag{2-27}$$

$$\frac{\partial \beta_{i,j}}{\partial \eta_p} \rightarrow \ell_p \eta_p^{\ell_p - 1} \left(\prod_{\substack{k=1 \\ k \neq p}}^{3} \eta_k^{\ell_k} \right) B_{i,j}(t) * H(t) \tag{2-28}$$

$$\frac{\partial^2 \beta_{i,j}}{\partial \eta_p^2} \rightarrow \ell_p (\ell_p - 1) \eta_p^{\ell_p - 2} \left(\prod_{\substack{k=1 \\ k \neq p}}^{3} \eta_k^{\ell_k} \right) B_{i,j}(t) * H(t) \tag{2-29}$$

$$\frac{\partial^2 \beta_{i,j}}{\partial \eta_p \partial \eta_q} \rightarrow \ell_p \ell_q \eta_p^{\ell_p - 1} \eta_q^{\ell_q - 1} \left(\prod_{\substack{k=1 \\ k \neq p,q}}^{3} \eta_k^{\ell_k} \right) B_{i,j}(t) * H(t) \tag{2-30}$$

With these conversions, the elastic probabilistic solution is carried over into the viscoelastic probabilistic form. In the above derivation, it is to be remembered that the ℓ_k and n_k are also functions of i and j; i.e., $\ell_{i,j,k}$ and $n_{i,j,k}$, but these subscripts were suppressed for notational brevity.

The above derivation assumes that the creep functions have a constant coefficient of variation; i.e., the standard deviation is in constant ratio to the mean. It is also possible to formulate the case of constant variance, as follows:

Assume the random creep function has the form:

$$D_k(t) = G_m^{\ k} + \sum_{j=1}^{m-1} \overline{G}_j^{\ k} e^{-\delta_j t} \tag{2-31}$$

where the $\overline{G}_j^{\ k}$ are constants and $G_m^{\ k}$ is a random variable which does not depend on time. Then the variance of $D_k(t)$ is equal to the variance of $G_m^{\ k}$ for all t.

To obtain the viscoelastic probabilistic formulation, we require the partial derivatives of the convolution integrals (the $\alpha_{i,j}$ and $\beta_{i,j}$) with respect to the random varibles $G_m^{\ k}$. Returning to the convolution expression:

$$D_k(0)(\cdot) - \int_{0^+}^{t} (\cdot) \frac{\partial D_k(t-\xi)}{\partial \xi} d\xi \tag{2-32}$$

where $(\cdot)$ denotes an arbitrary function of time; if we take a partial derivative of this expression, after substituting equation (2-31) for D_k, with respect to the random variable $G_m^{\ k}$, we obtain:

$$(\cdot) + D_k(0) \frac{\partial(\cdot)}{\partial G_m^{\ k}} - \int_{0^+}^{t} \frac{\partial(\cdot)}{\partial G_m^{\ k}} \frac{\partial D_k(t-\xi)}{\partial \xi} d\xi \tag{2-33}$$

which is simply:

$$(\cdot) + D_k(t) * \frac{\partial(\cdot)}{\partial G_m^{\ k}} = \frac{\partial}{\partial G_m^{\ k}} [D_k(t) * (\cdot)] \tag{2-34}$$

where again * denotes convolution, as does the "product" of any number of creep function operators.

If we replace $(\cdot)$ by $D_\ell^N(t)$, this becomes:

$$\frac{\partial}{\partial G_m^{\ k}}[\, D_k(t) * D_\ell^N(t)] = D_\ell^N(t) + D_k(t) * \frac{\partial D_\ell^N(t)}{\partial G_m^{\ k}} \tag{2-35}$$

where $\dfrac{\partial D_\ell^N(t)}{\partial G_m^{\ k}}$ may be evaluated by repeated use of relation (2-34), since it is zero if $\ell \neq k$. Hence we have:

$$\frac{\partial D_\ell^N(t)}{\partial G_m^{\ k}} = \delta_{\ell k}\{\, D_k^{N-1}(t) + D_k(t) * \{\, D_k^{N-2}(t) + D_k(t) * \{\ldots \tag{2-36}$$

where $\delta_{\ell k}$ is the Kronecker delta:

$$\delta_{\ell k} = 1,\ \ell = k;\ \ \delta_{\ell k} = 0,\ \ \ell \neq k \tag{2-37}$$

Upon combining terms, this yields:

$$\frac{\partial D_\ell^N(t)}{\partial G_m^{\ k}} = \delta_{\ell k} N D_k^{N-1}(t) \tag{2-38}$$

and hence:

$$\frac{\partial}{\partial G_m{}^k}\left[D_k(t) * D_\ell{}^N(t)\right] = \begin{cases} D_\ell{}^N(t) & \ell \neq k \\ (N+1)\, D_\ell{}^N(t) & \ell = k \end{cases} \tag{2-39}$$

In the analysis of Appendix D, and in equation (D-9) in particular, we can see that:

$$D_k(t) * D_\ell(t) = D_\ell(t) * D_k(t) \tag{2-40}$$

Hence, by repeated application of the above, any multiple convolution of the type we have been describing can be arranged into the form:

$$\alpha_{i,j} \rightarrow \left[\prod_{k=1}^{3} D_k{}^{n_k}(t)\right] * I(t) = D_1{}^{n_1}(t) * D_2{}^{n_2}(t) * D_3{}^{n_3}(t) * I(t) \tag{2-41}$$

$$\beta_{i,j} \rightarrow \left[\prod_{k=1}^{3} D_k{}^{\ell_k}(t)\right] * H(t) = D_1{}^{\ell_1}(t) * D_2{}^{\ell_2}(t) * D_3{}^{\ell_3}(t) * H(t)$$

From these and equations (2-39), we have:

$$\frac{\partial \alpha_{i,j}}{\partial G_m{}^p} \rightarrow n_p D_p{}^{n_p - 1}(t) * \left\{\prod_{\substack{k=1 \\ k\neq p}}^{3} D_k{}^{n_k}(t)\right\} * I(t) \tag{2-42}$$

$$\frac{\partial^2 \alpha_{i,j}}{\partial G_m^{p2}} \rightarrow n_p(n_p - 1)D_p^{n_p-2}(t) * \{\prod_{\substack{k=1 \\ k\neq p}}^{3} D_k^{n_k}(t)\} * I(t) \tag{2-43}$$

$$\frac{\partial^2 \alpha_{i,j}}{\partial G_m^{p} \partial G_m^{q}} \rightarrow n_p n_q D_p^{n_p-1}(t) * D_q^{n_q-1}(t) * \{\prod_{\substack{k=1 \\ k\neq p,q}}^{3} D_k^{n_k}(t)\} * I(t) \tag{2-44}$$

$$\frac{\partial \beta_{i,j}}{\partial G_m^{p}} \rightarrow \ell_p D_p^{\ell_p-1}(t) * \{\prod_{\substack{k=1 \\ k\neq p}}^{3} D_k^{\ell_k}(t)\} * H(t) \tag{2-45}$$

$$\frac{\partial^2 \beta_{i,j}}{\partial G_m^{p2}} \rightarrow \ell_p(\ell_p - 1)D_p^{\ell_p-2}(t) * \{\prod_{\substack{k=1 \\ k=p}}^{3} D_k^{\ell_k}(t)\} * H(t) \tag{2-46}$$

$$\frac{\partial^2 \beta_{i,j}}{\partial G_m^{p} \partial G_m^{p}} \rightarrow \ell_p \ell_q D_p^{\ell_p-1}(t) * D_q^{\ell_q-1}(t) * \{\prod_{\substack{k=1 \\ k\neq p,q}}^{3} D_k^{\ell_k}(t)\} * H(t) \tag{2-47}$$

With these conversions, the probabilistic elastic solution discussed in Section II. 1 is carried over into the viscoelastic probabilistic formulation. In the above derivation, it is to be remembered that the ℓ_k and n_k are also functions of i and j; i.e., $\ell_{i,j,k}$ and $n_{i,j,k}$, these subscripts being suppressed for brevity.

Rather than choosing between the constant variance or constant coefficient of variation formulations, both may be used simultaneously. Equations (2-1, 2-2) then become:

$$E[g(\eta_1,\eta_2,\eta_3 G_m^1,G_m^2,G_m^3)] \simeq E[g(\eta_1,\eta_2,\eta_3)] +$$

$$E[g(G_m^1,G_m^2,G_m^3)] - g(\underline{M}) \qquad (2\text{-}48)$$

$$\mathrm{Var}[g(\eta_1,\eta_2,\eta_3,G_m^1,G_m^2,G_m^3)] \simeq \mathrm{Var}[g(\eta_1,\eta_2,\eta_3)] +$$

$$\mathrm{Var}[g(G_m^1,G_m^2,G_m^3)]$$

where $E[g(\eta_1,\eta_2,\eta_3)]$ is the expected value given the η_k formulation; $E[g(G_m^1,G_m^2,G_m^3)]$ is the expected value given the G_m^k formulation; $g(\underline{M})$ is the value at the mean; $\mathrm{Var}[g(\eta_1,\eta_2,\eta_3)]$ is the variance given the η_k formulation; and $\mathrm{Var}[G_m^1,G_m^2,G_m^3)]$ is the variance in the G_m^k formulation.

The variances $\sigma^2_{\eta_k}$ and $\sigma^2_{G_m^k}$ may be obtained as follows.

Measure N values of the creep function and its standard deviation:

$$D_k(t_i),\ \sigma_k(t_i) \qquad i = 1,2,\ldots N \qquad (2\text{-}49)$$

From the above analysis we will have, as an approximation variance function $\overline{\sigma}_k^2(t_i)$:

$$\overline{\sigma}_k^2(t_i) = \sigma^2_{G_m^k} + \sigma^2_{\eta_k} D_k^2(t_i) \simeq \sigma_k^2(t_i) \quad i = 1,2,\ldots N \tag{2-50}$$

Then the error between the measured variance and the fitted variance will be

$$E_{ik} = \overline{\sigma}_k^2(t_i) - \sigma_k^2(t_i) \qquad i = 1,2,\ldots N \tag{2-51}$$

Then the total squared error S_k will be given by:

$$S_k = \sum_{i=1}^{N} E_{ik}^2 = \sum_{i=1}^{N} \{\sigma^2_{G_m^k} + \sigma^2_{\eta_k} D_k^2(t_i) - \sigma_k^2(t_i)\}^2 \tag{2-52}$$

Minimizing this error with respect to the $\sigma^2_{G_m^k}$ and $\sigma^2_{\eta_k}$ yields:

$$\frac{\partial S_k}{\partial \sigma^2_{G_m^k}} = 0 = 2 \sum_{i=1}^{N} \{\sigma^2_{G_m^k} + \sigma^2_{\eta_k} D_k^2(t_i) - \sigma_k^2(t_i)\}$$

$$\frac{\partial S_k}{\partial \sigma^2_{\eta_k}} = 0 = 2 \sum_{i=1}^{N} \{\sigma^2_{G_m^k} + \sigma^2_{\eta_k} D_k^2(t_i) - \sigma_k^2(t_i)\} D_k^2(t_i) \tag{2-53}$$

which imply:

$$\sigma^2_{G^k_m} = \frac{1}{N} \sum_{i=1}^{N} (\sigma^2_k(t_i) - \sigma^2_{\eta_k} D^2_k (t_i)) = E\,[\sigma^2_k] - \sigma^2_{\eta_k} E\,[D^2_k]$$

$$\sigma^2_{G^k_m} \sum_{i=1}^{N} D^2_k (t_i) = \sum_{i=1}^{N} \sigma^2_k (t_i) D^2_k (t_i) - \sigma^2_{\eta_k} \sum_{i=1}^{N} D^4_k (t_i) \tag{2-54}$$

or:

$$\sigma^2_{G^k_m} = E\,[\sigma^2_k] - \mathrm{Cov}\,[\sigma^2_k, D^2_k] \;\; E\,[D^2_k] \,/\, \mathrm{Var}\,[D^2_k]$$

$$\sigma^2_{\eta_k} = \mathrm{Cov}\,[\sigma^2_k, D^2_k] \,/\, \mathrm{Var}\,[D^2_k] \tag{2-55}$$

From the computational standpoint, however, the η_k formulation is much simpler and less time consuming.

II. 3 Repeated Loading Analysis using Boltzmann's Superposition Principle

The response of a pavement system to a random loading history and variable environment may be represented, through Boltzmann's Superposition Principle, as:

$$R(t) = \int_0^t \Psi(t - \xi, \Theta_\xi^t)\, \dot{P}(\xi)\, d\xi \tag{2-56}$$

where Ψ is the static load system response function, Θ_ξ^t is the environmental history from time ξ to time t, and $\dot{P}$ is the time derivative of the loading function P.

If, however, the system behaves differently in loading and unloading, equation (2-56) becomes:

$$R(t) = \int_0^t \{\Psi_+(t - \xi, \Theta_\xi^t)\, \dot{P}_+(\xi) + \Psi_-(t - \xi, \Theta_\xi^t)\, \dot{P}_-(\xi)\}\, d\xi \tag{2-57}$$

where Ψ_+ and Ψ_- are the loading and unloading static system response functions, respectively, and:

$$\dot{P}_+(\xi) = \begin{cases} \dot{P}(\xi), & \dot{P}(\xi) \geq 0 \\ 0, & \dot{P}(\ ) < 0 \end{cases}$$

$$\dot{P}_-(\xi) = \begin{matrix} 0, & \dot{P}(\xi) \geq 0 \\ \dot{P}(\xi), & \dot{P}(\xi) < 0 \end{matrix} \tag{2-58}$$

Assuming a functional relationship between the loading and unloading functions of the form:

$$\Psi_-(t - \xi, \overset{t}{\underset{\xi}{\Theta}}) = [1 - f(\xi,\lambda,\overset{\xi}{\underset{0}{\Theta}})]\ \Psi_+(t - \xi, \overset{t}{\underset{\xi}{\Theta}}) \tag{2-59}$$

then equation (2-57) becomes:

$$R(t) = \int_0^t \Psi_+(t - \xi, \overset{t}{\underset{\xi}{\Theta}})\ [\dot{P}(\xi) - f(\xi,\lambda,\overset{\xi}{\underset{0}{\Theta}})\ \dot{P}_-(\xi)]\ d\xi \tag{2-60}$$

Assuming a Haversine loading centered at time x and of duration D, amplitude A:

$$P(x + \tau) = A \sin^2(\frac{\pi}{2} + \frac{\pi\tau}{D}) \qquad -\frac{D}{2} \leq \tau \leq \frac{D}{2}$$

$$\dot{P}(x + \tau) = -\frac{A\pi}{D} \sin(\frac{2\pi\tau}{D}) \qquad -\frac{D}{2} \leq \tau \leq \frac{D}{2} \tag{2-61}$$

$$\dot{P}_-(x + \tau) = -\frac{A\pi}{D} \sin(\frac{2\pi\tau}{D}) \qquad 0 \leq \tau \leq \frac{D}{2}$$

We may evaluate the contribution to R(t) from the small time interval $(x - \frac{D}{2}, x + \frac{D}{2})$, with a load cycle centered at time x, as:

$$\begin{aligned} R_x(t) = & \int_{x-\frac{D}{2}}^{x+\frac{D}{2}} -\frac{A\pi}{D} \sin(\frac{2\pi\xi}{D})\ \Psi_+(t - \xi, \overset{t}{\underset{\xi}{\Theta}})\ d\xi \\ & + \int_{x}^{x+\frac{D}{2}} \frac{A\pi}{D} \sin(\frac{2\pi\xi}{D})\ f(\xi,\lambda,\overset{\xi}{\underset{0}{\Theta}})\ \Psi_+(t - \xi, \overset{t}{\underset{\xi}{\Theta}})\ d\xi \end{aligned} \tag{2-62}$$

Assuming a static load system response function of the form:

$$\Psi_+(t - \xi, \overset{t}{\underset{\xi}{\Theta}}) = \eta \sum_{j=1}^{m} G_j e^{-\delta_j t^*(\overset{t}{\underset{\xi}{\Theta}})} \tag{2-63}$$

where t^* is the equivalent reference-temperature time interval between time ξ and time t:

$$t^*(\overset{t}{\underset{\xi}{\Theta}}) = \int_{\xi}^{t} \gamma(\tau)\, d\tau = \int_{t_\ell}^{t} \gamma(\tau) d\tau + \gamma_\ell (t_\ell - \xi) \tag{2-64}$$

where the temperature history has been broken up into intervals of constant temperature have the time-temperature shift factors γ_ℓ, $t_{\ell-1} \leq \xi \leq t_\ell$. Then equation (2-62) becomes:

$$R_{x,\ell}(t) = I^1_{x,\ell}(t) + I^2_{x,\ell}(t) \tag{2-65}$$

where:

$$I^1_{x,\ell}(t) = -\frac{A\pi\eta}{D} \sum_{j=1}^{m} G_j e^{-\delta_j t^*(\overset{t}{\underset{t_\ell}{\phi}})} e^{-\delta_j \gamma_\ell t_\ell} \int_{x-D/2}^{x+D/2} e^{\delta_j \gamma_\ell \xi} \sin(2\pi\xi/D) d\xi$$

$$I^2_{x,\ell}(t) = \frac{A\pi\eta}{D} \sum_{j=1}^{m} G_j e^{-\delta_j t^*(\overset{t}{\underset{t_\ell}{\Theta}})} e^{-\delta_j \gamma_\ell t_\ell} \int_{x}^{x+\frac{D}{2}} e^{\delta_j \gamma_\ell \xi} f(\xi, \lambda, \overset{\xi}{\underset{0}{\Theta}}) \cdot \tag{2-66}$$

$$\sin\left(\frac{2\pi\xi}{D}\right) d\xi$$

Now, since $f(\xi,\lambda,\Theta)\Big|_0^\xi$ represents the fractional difference between the load and unload system response functions and is determined from the past loading and environmental histories prior to time ξ, it goes monotonically to zero as the number of previous loadings $(\lambda\cdot\xi)$ goes to infinity, and thus may be represented as an exponential series:

$$f(\xi,\lambda,\Theta)\Big|_0^\xi = \sum_{r=1}^{N} \mu_r e^{-k_r \xi^*} \tag{2-67}$$

where ξ^* represents the equivalent reference-temperature number of previous loadings:

$$\xi^* = \int_0^\xi \lambda(\tau)\Gamma(\tau)d\tau = \int_0^{t_{\ell-1}} \lambda(\tau)\Gamma(\tau)d\tau + \lambda_\ell \Gamma_\ell (\xi - t_{\ell-1}) \tag{2-68}$$

$\lambda(\tau)$ being the traffic rate at time τ, $\Gamma(\tau)$ being the time-temperature shift factor for the function f, and $t_{\ell-1} \le \xi \le t_\ell$.

The integrals in $I^1_{x,\ell}(t)$ and $I^2_{x,\ell}(t)$ may be evaluated (See Appendix E) to yield:

$$I^1_{x,\ell}(t) = \sum_{j=1}^{m} B_{\ell,j} e^{-\delta_j t^*(\Theta)\Big|_x^t} \tag{2-69}$$

$$I^2_{x,\ell}(t) = \sum_{j=1}^{m} e^{-\delta_j t^*(\Theta)\Big|_x^t} \sum_{r=1}^{N} C_{\ell,j,r} e^{-k_r x^*} \tag{2-70}$$

where:

$$B_{\ell,j} = \frac{-A\eta G_j \sinh(\delta_j \gamma_\ell \frac{D}{2})}{[1 + (\delta_j \gamma_\ell \frac{D}{2\pi})^2]} \tag{2-71}$$

$$C_{\ell,j,r} = \frac{A\eta G_j \mu_r e^{-k_r \Gamma_\ell \lambda_\ell \frac{D}{4}} \cosh[(\delta_j \gamma_\ell - k_r \lambda_\ell \Gamma_\ell)\frac{D}{4}]}{\{1 + [(\delta_j \gamma_\ell - k_r \lambda_\ell \Gamma_\ell)\frac{D}{2\pi}]^2\}} \tag{2-72}$$

Hence, we obtain:

$$R_{x,\ell}(t) = \sum_{j=1}^{m} e^{-\delta_j t^*{}_x^t(\Theta)} \{B_{\ell,j} + \sum_{r=1}^{N} C_{\ell,j,r}\, e^{-k_r x^*}\} \tag{2-73}$$

Parzan has shown (3) that the mean and variance of a compound filtered poisson process, such as the pavement response to poisson loadings in a variable environment, may be written as:

$$E[R(t)] = \int_0^t \lambda(x) E[R_x(t)]dx$$

$$Var[R(t)] = \int_0^t \lambda(x) E[R_x^2(t)]dx \tag{2-74}$$

or, in our case:

$$E[R(t)] = \sum_{\ell=1}^{N_t} \lambda_\ell \int_{t_{\ell-1}}^{t_\ell} E[R_{x,\ell}(t)]dx$$

$$Var[R(t)] = \sum_{\ell=1}^{N_t} \lambda_\ell \int_{t_{\ell-1}}^{t_\ell} E[R_{x,\ell}^2(t)]dx \tag{2-75}$$

Hence, using equation (2-73), we obtain:

$$E[R_{x,\ell}(t)] = \sum_{j=1}^{m} e^{-\delta_j t^*(\Theta)\big|_{x}^{t}} \{E[B_{\ell,x}] + \sum_{r=1}^{N} E[C_{\ell,j,r}]e^{-k_r x^*}\}$$

$$E[R^2_{x,\ell}(t)] = \sum_{j=1}^{m}\sum_{p=1}^{m} e^{-(\delta_j+\delta_p)t^*(\Theta)\big|_{x}^{t}} \{E[B_{\ell,j}B_{\ell,p}] +$$

$$\sum_{r=1}^{N} E[B_{\ell,p}C_{\ell,r}]e^{-k_r x^*} + \sum_{q=1}^{N} E[B_{\ell,j}C_{\ell,p,q}]e^{-k_q x^*} + \tag{2-76}$$

$$\sum_{r=1}^{N}\sum_{q=1}^{N} E[C_{\ell,j,r}C_{\ell,p,q}]e^{-(k_r+k_q)x^*}\}$$

Combining equations (2-75, 2-76) and utilizing the definitions given in equations (2-64, 2-67) we obtain, for the desired solution (See Appendix F for these calculations)

$$E[R(t)] = \sum_{\ell=1}^{N_t} \lambda_\ell \sum_{j=1}^{m} e^{-\delta_j t^*(\Theta)\big|_{t_{\ell-1}}^{t}} \{Y_{\ell,j} + \sum_{r=1}^{N} Z_{\ell,j,r}\} \tag{2-77}$$

where:

$$Y_{\ell,j} = E[B_{\ell,j}](e^{\delta_j \gamma_\ell \Delta_\ell} - 1)/\delta_j\gamma_\ell$$

$$Z_{\ell,j,r} = \frac{e^{-k_r x^*(\Theta_0^{t_{\ell-1}})} E[C_{\ell,j,r}](e^{(\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell)\Delta_\ell} - 1)}{\delta_j\gamma_\ell - k_r\Gamma_\ell\lambda_\ell}$$

$$\Delta_\ell = t_\ell - t_{\ell-1} \tag{2-78}$$

$$x^*(\Theta_0^{t_{\ell-1}}) = \int_0^{t_{\ell-1}} \lambda(\tau)\Gamma(\tau)d\tau$$

and

$$\mathrm{Var}[R(t)] = \sum_{\ell=1}^{N_t} \lambda_\ell \sum_{j=1}^{m} \sum_{p=1}^{m} e^{-(\delta_j+\delta_p)t^*(\Theta_{t_{\ell-1}}^{t})} \cdot$$

$$\{U_{\ell,j,p} + 2\sum_{r=1}^{N} V_{\ell,j,p,r} + \sum_{r=1}^{N}\sum_{q=1}^{N} W_{\ell,j,p,r,q}\} \tag{2-79}$$

where

$$U_{\ell,j,p} = E[B_{\ell,j}B_{\ell,p}](e^{(\delta_j+\delta_p)\gamma_\ell\Delta_\ell} - 1) \;/\; (\delta_j + \delta_p)\gamma_\ell$$

$$V_{\ell,j,p,r} = \frac{e^{-k_r x^*(\Theta_0^{t_{\ell-1}})} E[B_{\ell,j}C_{\ell,p,r}]\{e^{[(\delta_j+\delta_p)\gamma_\ell - k_r\lambda_\ell\Gamma_\ell]\Delta_\ell} - 1\}}{(\delta_j + \delta_p)\gamma_\ell - k_r\lambda_\ell\Gamma_\ell}$$

$$W_{\ell,j,p,r,q} = \frac{e^{-(k_r+k_q)x^*(\Theta_0^{t_{\ell-1}})} E[C_{\ell,j,r}C_{\ell,p,q}]\{e^{[(\delta_j+\delta_p)\gamma_\ell - (k_r+k_q)\lambda_\ell\Gamma_\ell]\Delta_\ell} - 1\}}{(\delta_j + \delta_p)\gamma_\ell - (k_r + k_q)\lambda_\ell\Gamma_\ell} \tag{2-80}$$

The evaluations of the expected values of $B_{\ell,j}$, $C_{\ell,j,r}$, etc., may be found in Appendix F.

Chapter Three. Probabilistic Damage Indicators, Serviceability, Reliability, and State Transition Probabilities.

The damage variables are expressed in terms of two damage manifestations in the pavement structure: deformation and cracking. Deformation accumulates in both the transverse and longitudinal profiles of the pavement. Transversely, it is manifested by the rutting in the heavily travelled paths of the roadway; and longitudinally in the roughness of the pavement profile, measured by the slope variance of the profile. The mechanisms of the development of these damage manifestations are described below.

III. 1 Rutting

The component is assumed to be primarily the result of a channelized system of traffic thereby causing differential surface deformation under the areas of intensive load applications in the wheel-paths. Given the statistical characteristics of the road materials and of the traffic, we can determine the rut depth from the spatial properties of the traffic loads, as follows:

The mean and variance of rut depth are obtained from equations (2-77) and 2-79), respectively, with a reduced traffic rate λ' described as follows:

$$\lambda' = \lambda_c + (\lambda - \lambda_c)/N \qquad (3\text{-}1)$$

where:

λ_c is the channelized traffic rate in one lane

λ is the mean rate of traffic in the lane

N is the number of possible combinations of traffic channels in the lane (degrees of freedom)

If, for example, 70% of the traffic is channelized at the center of the lane (i.e, $\lambda_c = 0.7\lambda$), and there are three other possible paths that the traffic passes through in the lane, then:

$$\lambda' = 0.7\lambda + \frac{\lambda - 0.7\lambda}{3} = 0.8\lambda \qquad (3\text{-}2)$$

The values of $\overline{G}_i$, δ_i, and η are obtained by evaluation of the vertical deflections at the surface of the pavement beneath the center of a static loading.

These values, with λ' substituting for λ in equations (2-77) and (2-79) yield the mean and variance of the rut depth versus time.

III. 2 Slope Variance

This component defines the deformation along the longitudinal profile of the pavement. To obtain some measure of slope variance, information about the spatial correlation of the material properties of the system must be obtained. This can be expressed in terms of the autocorrelation function of the surface deformation, assuming that slope variance is mainly caused by the variation of the material properties and methods of fabrication. We can relate the spatial variations in the materials to those in the surface deformation along the pavement profile.

The spatial autocorrelation function of a system's response $\tilde{R}_t(x)$ is defined as:

$$\tilde{R}_t(x) = E_x[R(t,x_1)R(t,x_2)] \tag{3-3}$$

where $E_x[\ \ ]$ signifies that the expectation operation is taken only over the space variable x, and $R(t,x_j)$ is the surface deflection at time t and location x_j.

The analysis in Appendix G results in the following expression:

$$\tilde{R}_t(x) = [1 + \rho_x\sigma_\eta^2]\ (R(t)|_{\eta=1})^2 \tag{3-4}$$

where:

$$\rho_x = \mathrm{Cov}[\eta_0\eta_x]/\sigma_\eta^2 \tag{3-5}$$

is the spatial correlation coefficient of the surface deflection of the pavement. Expressing:

$$\rho_x = 1 - B(1 - e^{-x^2/c^2}) \tag{3-6}$$

and using the fact that the slope variance is equal to the negative of the second spatial derivative of $\tilde{R}_t(x)$ evaluated at $x = 0$, we have:

$$SV(t) = - \left. \frac{\partial^2 R_t(x)}{\partial x^2} \right|_{x=0} = \frac{2B}{C^2} \sigma_\eta^2 \left[R(t)\Big|_{\eta=1} \right]^2 \tag{3-7}$$

$$E[SV(t)] \simeq \frac{2B}{C^2} \sigma^2_\eta \{ Var[R(t)\Big|_{\eta=1}] + E^2[R(t)\Big|_{\eta=1}] \} \tag{3-8}$$

$$Var[SV(t)] \simeq \{\frac{4B}{C^2} \sigma_\eta^2 \; E[R(t)\Big|_{\eta=1}]\}^2 \; Var[R(t)\Big|_{\eta=1}] \tag{3-9}$$

where $E[R(t)\big|_{\eta=1}]$ is obtained from equation (2-77), and $Var[R(t)\big|_{\eta=1}]$ is obtained from equation (2-79), and $E^2[\ \ ] = E[\ \ ] \cdot E[\ \ ]$.

III. 3 Cracking

Cracking is a phenomenon associated with the brittle behavior of materials. A fatigue mechanism is believed to cause progression of cracks in pavements. In this study, a phenomenological approach has been adopted, namely a modified stochastic Miner's law for progression of damage within materials. It is recognized, however, that a probabilistic microstructural approach based on fracture mechanics may provide a better substitute for the prediction of crack initiation and progression within the pavement structure.

The criterion for cracking used in this study is based on fatigue resulting from the tensile strain at the bottom of the surface layer. This requires the determination of the moments for the radial strain amplitudes at the bottom of the surface layer, using the radial strains obtained from the step loadings of the static load program. These moments for the strain amplitudes may be determined from the following equations, derived in Appendix H:

$$\Delta\varepsilon_{\ell} = \frac{\pi^2 A\eta}{2} \sum_{i=1}^{n} \bar{G}_i (1 + e^{-u_{\ell,i}})/(\pi^2 + u_{\ell,i}^2) \tag{3-10}$$

where $u_{\ell,i} = \gamma_{\ell}\delta_i D/2$, and $\Delta\varepsilon_{\ell}$ represents the radial (tensile) strain amplitude at a temperature represented by γ_{ℓ} and a loading of amplitude A and duration D.

The mean and variance of $\Delta\varepsilon_{\ell}$ can be obtained by the probabilistic analysis of Appendix H and may be written as:

$$E[\Delta\varepsilon_{\ell}] \simeq \frac{1}{2}\pi^2\overline{A\eta} \sum_{i=1}^{n} \bar{G}_i \, (1 + e^{-\bar{u}_{\ell,i}})/(\pi^2 + \bar{u}_{\ell,i}^2)$$

$$+ \frac{1}{4}\pi^2\overline{A\eta}\,\sigma_D^2 \; \{\sum_{i=1}^{n} \bar{G}_i (\frac{\delta_i\gamma_{\ell}}{2})^2 \; [e^{-\bar{u}_{\ell,i}} + (4\bar{u}_{\ell,i}e^{-\bar{u}_{\ell,i}} - 2(1 + e^{-\bar{u}_{\ell,i}}))$$

$$/(\pi^2 + \bar{u}_{\ell,i}^2) + 8\bar{u}_{\ell,i}^2(1 + e^{-\bar{u}_{\ell,i}}) \; / \; (\pi^2 + \bar{u}_{\ell,i}^2)^2]/(\pi^2+\bar{u}_{\ell,i}^2)\} \tag{3-11}$$

$$\mathrm{Var}[\Delta\varepsilon_\ell] \simeq \{\frac{1}{2}\pi^2 \sum_{i=1}^{n} \overline{G}_i \ (1 + e^{-\overline{u}_{\ell,i}})/(\pi^2 + \overline{u}_{\ell,i}^{\,2})\}^2 \ (\overline{\eta}^2\sigma_A^2 + \overline{A}^2\sigma_\eta^2) \quad +$$

$$\sigma_D^2\{\frac{1}{2}\pi^2\overline{A\eta} \sum_{i=1}^{n} \overline{G}_i (\frac{\delta_i\gamma_\ell}{2})[e^{-\overline{u}_{\ell,i}} + 2\overline{u}_{\ell,i}(1 + e^{-\overline{u}_{\ell,i}})/(\pi^2 + \overline{u}_{\ell,i}^2)]/(\pi^2 + \overline{u}_{\ell,i}^2)\}^2$$

(3-12)

Miner's law can be expressed as:

$$D(t) = \sum_{k=1}^{M} n_k/N_k \tag{3-13}$$

where $D(t)$ is the damage at time t resulting from a repetition of Σn_k loads to failure at the k^{th} period, having the same statistical properties as the n_k loads. The ratio n_k/N_k represents the proportion of damage in terms of fatigue cracking the the k^{th} period.

A fatigue law has been used to determine N_k, the number of loads to failure in the k^{th} period in terms of the tensile strain amplitudes obtained above:

$$\frac{1}{N_k} = C(T_k)(\Delta\varepsilon_k)^{a(T_k)} \tag{3-14}$$

where C and a are material characteristics with certain statistical properties, which are temperature dependent.

Appendix H presents the analysis used to obtain the moments of damage D(t) versus time. These are:

$$E[D(t)] \simeq \sum_{k=1}^{M} \overline{n}_k(\overline{\frac{1}{N_k}}) \tag{3-15}$$

$$\mathrm{Var}[D(t)] \simeq \sum_{k=1}^{M} \{ (\overline{\frac{1}{N_k}})^2 \sigma^2_{n_k} + \bar{n}_k^2 \sigma^2_{\frac{1}{N_k}} \} \tag{3-16}$$

where $\bar{n}_k$ is the mean number of Poisson loads in the k^{th} period, $(\overline{\frac{1}{N_k}})$ is the mean of the inverse number of loads to failure in the k^{th} period, $\sigma^2_{\frac{1}{N_k}}$ is the variance of $\frac{1}{N_k}$, and $\sigma^2_{n_k}$ is the variance of traffic loads in the k^{th} period = n_k for a Poisson process.

Assuming that D(t) takes on a normal distribution at all times t with mean of E[D(t)] and variance of Var[D(t)], then the expected area of cracking, as a percentage of total surface area, will be given by the probability of D(t) being greater than 1.

The above damage indicators have been expressed in algorithmic forms and are obtained readily by computer analysis to be used in the next step in the hierarchy of the present analysis: to determine the serviceability of the system with time and the associated reliability and life expectation.

III. 4 Pavement Serviceability

In this study, the serviceability of a pavement system will be restricted to consideration of pavement surface riding quality. The term "performance" will be used to designate the broader concept of serviceability, safety, maintenance and costs.*

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* See Findakly (1) for discussion of serviceability.

Following AASHO, the present serviceability index can be expressed as a function of some objective distress components, as cracking and deformation. Using the same components used by AASHO, we can write a general expression for the present serviceability index as:

$$PSI = f(RD, (C + P), SV) \tag{3-17}$$

where:

RD refers to rut depth

C + P refers to area of cracking and patching

SV refers to slope variance

Any function of these variables may be produced by regression analysis. In the present study, however, AASHO's present serviceability expression has been used without change. AASHO's equation is a special case of equation (3-17) and is written as:

$$PSI = C_0 + C_1 \log_{10}(1 + SV) + C_2 \sqrt{C + P} + C_3 (RD)^2 \tag{3-18}$$

where:

$C_0 = 5.03$ $\quad$ $C_2 = -0.01$

$C_1 = -1.91$ $\quad$ $C_3 = -1.38$

If we regard the components SV, C + P, and RD as random variables, the PSI will be a random variable, and we can determine the moments at any instant t utilizing equations (2-1, 2-2)

$$E[PSI(t)] \simeq C_0 + C_1 \{\log_{10}(1 + \overline{SV}(t)) - \frac{1}{2}\frac{(1 + \overline{SV}(t))^{-2}}{(\ln 10)} \sigma^2_{SV(t)}\} +$$

$$C_2\{\sqrt{\overline{(C + P)}(t)} - \frac{1}{8}((\overline{C + P})(t))^{-\frac{3}{2}} \sigma^2_{(C+P)(t)}\} + C_3\{\overline{RD}^2(t) + \sigma^2_{RD(t)}\}$$

(3-19)

$$Var[PSI(t)] \simeq C_1^2\frac{(1 + \overline{SV}(t))^{-2}\sigma^2_{SV(t)}}{(\ln 10)^2} + \frac{1}{4}C_2^2(\overline{C + P}(t))^{-1} \sigma^2_{(C+P)(t)}$$

$$+ 4C_3^2\overline{RD}(t)^2\sigma^2_{RD(t)} \quad (3\text{-}20)$$

where $\overline{SV}(t)$, $\overline{(C + P)}(t)$, $\overline{RD}(t)$ refer to the expected values of slope variance, cracking plus patching, and rut depth respectively at time t and $\sigma^2_{SV(t)}$, $\sigma^2_{(C+P)(t)}$, $\sigma^2_{RD(t)}$ represent the corresponding variances.

As in most analyses of this type, the probability distribution of the serviceability is assumed gaussian, with a density function of:

$$f_S(s) = \frac{1}{\sqrt{2\pi}\,\sigma_S} \exp[-\frac{1}{2}(\frac{s - \overline{S}}{\sigma_S})^2] \quad (3\text{-}21)$$

and cumulative density function of:

$$F_S(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} \exp(-\frac{u^2}{2})du \quad (3\text{-}22)$$

where $u = (s - S)/\sigma_S$ is the standard normal random variable of mean 0 and variance 1. Appendix I presents an approximate evaluation of a cumulative gaussian variable which is easily implemented on computers.

III. 5 Pavement Reliability, Marginal Probabilities, and Life Expectancy

The reliability of a pavement system's serviceability is given by the probability that the serviceability is above some unacceptable level which has been established beforehand, say S*, i.e.:

$$\text{Reliability} = \text{Prob}[s \geq S^*] = 1 - F_S(S^*) \qquad (3\text{-}23)$$

and, since $s = s(t)$, this yields the value of reliability at any time t for which the serviceability is known. Also, marginal or state probabilities $q_i(t)$ of the serviceability being in a given interval i at time t may be obtained as:

$$q_i(t) = \text{Prob}[S_{i-1} \leq s(t) \leq S_i] = F_S(S_i) - F_S(S_{i-1}) \qquad (3\text{-}24)$$

of which equation (3-23) is of course a special case.

The expected lifetime of acceptable pavement performance is the integral of its reliability over time, i.e.:

$$E[L] = \int_0^{t_N} 1 - F_S(S^*)dt = t_N - \int_0^{t_N} F_S(S^*)\, dt \qquad 3\text{-}25)$$

where t_N is the analysis horizon. However, if we could obtain the transition matrix taking the n marginal probabilities at time t, into the n marginal probabilities at time u, for all t, u, i.e.:

$$\underline{q}(u) = P(t,u)\underline{q}(t) \tag{3-26}$$

then the expected lifetime would be also determinable by:

$$E[L] = \int_0^\infty \{1 - \sum_{i=1}^{n} P_{ni}(u,o)q_i(o)\}du \tag{3-27}$$

where the integrand is the reliability at time u,

$q_i(0)$ are the marginal probabilities at time 0

$(P_{ni}(u,o))$ is the transition matrix from time 0 to time u

n is the number of states into which the serviceability has been divided, and

q_n is the unacceptable state.

Appendix J presents a method for obtaining the transition matrix between two time points t_1 and t_2, but a presentation of the determination of P(t,u) for all t,u is rarely possible, unless total information is assumed.

Chapter Four. Maintenance Activities

Maintenance of a system may be viewed as work performed on the system during its operational life to improve it, assure a "desireable" serviceability level and improve its life expectancy. Quantitatively, maintenance may be viewed as an algebraically negative damage. Maintenance activities exercised at some point in time may be defined as a vector of quantity and the associated cost, discounted to constant value.

In order to introduce maintenance activities into the model and to establish a framework for decision making, the concept of state values is introduced. If the system is in state S_i, it has an associated benefit of B_i as a consequence. This benefit may be the reduction in user operating costs, or annualized construction costs, or safety measures, etc., depending on the objectives of the model user. The value of the pavement system may then be evaluated at any point in time to be:

$$V(t_k) = \sum_{i=1}^{n} P(S_i)B_i/(1 + r)^k \tag{4-1}$$

If no maintenance is ever employed, the total value of the pavement system over its design life would be given by:

$$V_T = \sum_{k=0}^{N} (1 + r)^{-k} \sum_{i=1}^{n} P(S_i(t_k))B_i \tag{4-2}$$

If maintenance M is applied at time t_m at a cost of C_m in constant dollars, then the accrued benefits resulting from that maintenance will be:

$$B_M = \sum_{k=m}^{N} (1 + r)^{-k} \sum_{i=1}^{n} B_i(P(S_i(t_k)|_M) - P(S_i(t_k))) \tag{4-3}$$

Thus, if $B_M > c_m$, the maintenance effort was justified. The problem now is reduced to determining $P(S_i(t_k)|_M)$, i.e., the state probabilities encountered as a result of a maintenance effort.

IV. 1 Maintenance Effects

Of the damage components considered, cracking and deformation, only cracking is directly reduceable by patching activities. However, the patching will tend to reduce the other components, rut depth and slope variance, indirectly, in proportion to the area patched. Any spatially statistical random variable would have its mean and variance altered as:

$$m \rightarrow \alpha m \tag{4-4}$$

$$\sigma^2 \rightarrow \alpha(\sigma^2 + (1 - \alpha)m^2) \tag{4-5}$$

where α is the fraction of area which is not patched. Thus, if $(1 - \alpha)$ of the surface area is patched, the mean and variance of rut depth will be altered by:

$$E[RD] \rightarrow \alpha E[RD] \tag{4-6}$$

$$Var[RD] \rightarrow \alpha\{Var[RD] + (1 - \alpha)(E[RD])^2\} \tag{4-7}$$

while the mean and variance of the slope variance will become:

$$E[SV] \rightarrow \alpha E[SV] \tag{4-8}$$

$$Var[SV] \rightarrow \alpha^3 Var[SV] + \alpha^3(1 - \alpha)\left(\frac{4B}{c^2}\sigma_\eta^2 E[RD|_{\eta=1}]\right)^2 \tag{4-9}$$

the last equations being obtained by substituting equations (4-4), (4-5) into equations (3-8), (3-9). Of course, the cracked area is reduced by the amount patched, and its variance is reduced as in equation (4-5).

Thus, from equations (4-4), (4-9), given a maintenance patching effort, we can evaluate the effects of that effort on the subsequent value history of the pavement, and decide whether such an effort is or is not justifiable economically, through equation (4-3).

Once the increments of damage have been evaluated, these may be added to the damage history for all time points following the maintenance application. However, this maintenance effort will undoubtedly not be a permanent improvement, but will itself begin to deteriorate.

This deterioration is assumed to be exponential, i.e., a damage increment of δ applied at time t_M would be reduced to an increment of only $\delta\exp(-a(t - t_M))$ at time $t > t_M$, where a is a material characteristic. Thus, from equations (4-4) - (4-9) we have:

$$E[RD(t)] \rightarrow E[RD(t)] - (1 - \alpha)E[RD(t_M)]\ e^{-a(t-t_M)} \tag{4-10}$$

$$Var[RD(t)] \rightarrow Var[RD(t)] - (1- \alpha)[Var[RD(t_M)] - \alpha E[RD(t_M)]^2]e^{-a(t-t_M)} \tag{4-11}$$

$$E[SV(t)] \rightarrow E[SV(t)] - (1 - \alpha)E[SV(t_M)]e^{-a(t-t_M)} \tag{4-12}$$

$$Var[SV(t)] \rightarrow Var[SV(t)] - \{(1 - \alpha^3)Var[SV(t_M)]$$

$$-\alpha^3(1 - \alpha)(\frac{4B}{c^2}\sigma_\eta^2 E[RD(t_M)|_{\eta=1})^2\}e^{-a(t-t_M)} \tag{4-13}$$

from which we can evaluate the mean and variance of serviceability after the maintenance effort has been applied. From these we may calculate the marginal state probabilities after the maintenance effort, which in turn may be utilized in equation (4-3) to determine the total benefits incurred from the maintenance. Then, if c_m is the unit cost of patching, and Q units are to be patched, we would wish to maintain at that level if:

$$B_M \geq c_m Q/(1 + r)^{k(t_m)} \qquad (4\text{-}14)$$

and $(1 - \alpha) = Q/(\text{total number of units})$

IV. 2 Maintenance Strategies

The above decision criterion provides a framework for the choice of alternate strategies for highway systems. Each strategy defines a set of rules for experimentation and action to which consequences are defined in terms of a multiple set of attributes. In this context, maintenance strategies are generated in terms of such attributes. The selection of any optimal strategy is effective only to the extent that these measures of effectiveness are relevant and exhaustive.

The design model described in chapters II and III provides information about the marginal state probabilites at different time points. The states are described in terms of the serviceability index, which will be raised by a maintenance effort, how much so depending on the level of maintenance and the state of the system. In any case, maintenance "level" is taken to define the effort (in terms of labor and materials) which is expended to upgrade the system from its present level. An associated cost can be assigned for any maintenance effort, discounted to constant value.

Let us follow through the analysis. At the outset, say time t_0, the system may be in some state S_i, with probability p_{io}, as determined by the analytical model. At this point a decision is encountered: at what level to maintain? (The no-maintenance level is considered an option.) It is assumed that the costs of these levels are known. The conditional future history, conditioned on the present decision and future traffic, may then be determined for each maintenance option, which may be compared on some utility basis against the do-nothing alternative. If the time points at which maintenance decisions are encountered are specified, the optimal maintenance strategy can be determined describing the course of action depending on the outcome of the branches preceding the nodes.

IV. 3 The Decision Variables

If the design horizon is N years, and a maintenance decision is to be made at each year from one of M options, and K traffic levels may be considered, then the feasible set of maintenance sequences will number:

$$N^* = (KM)^N \tag{4-15}$$

Clearly, this set must be reduced dramatically if it is to be at all practical. Since we are interested primarily in the choice among the maintenance options, we should reduce the traffic options. The most obvious way to do this is to assume an initial traffic volume and growth rate, determine the "best" maintenance strategy conditioned on these assumptions, and apply the initial "best" maintenance level. Then, as time progressed, we can compare the encountered traffic volumes with our previous assumptions, compute revised growth rates, and a revised "best" maintenance strategy.

Alternately, one could reduce the design horizon or the number of maintenance options, but these alternatives are relatively self-defeating, as they reduce the scope of the variables we are most interested in. We arrive at the following decision algorithm.

IV. 4 The Algorithm

We assume all necessary inputs are known, except for the traffic history. Then, we compute the "best" maintenance strategy as follows:

1. Given the initial traffic rate λ_1, assume an initial growth rate of r_1 holds for the design horizon. Evaluate the "best" maintenance sequence given these assumptions, as:

$$S_1 = (M_{11}, M_{21}, M_{31} \ldots M_{N1} | \lambda_1, r_1) \tag{4-16}$$

   and implement decision M_{11} at time t_1. Here M_{ij} denotes any possible member of the set of maintenance options.

2. As traffic information becomes known, revise your traffic growth rates accordingly, and compute the "best" maintenance sequence given the revised values. Of course, maintenance decisions

that were previously implemented cannot be altered. Thus, in the second decision period, the decision would be implemented from the "best" sequence given by:

$$S_2 = (M_{11}, M_{22}, M_{32} \ldots M_{N2} \mid \lambda_1, \lambda_2, r_2) \tag{4-17}$$

where:

$$r_2 = (\lambda_2 - \lambda_1)/\lambda_1 \tag{4-18}$$

Clearly, if the revised r_i is equal to r_{i-1}, S_{i-1} is still optimal and S_i need not be evaluated.

3. Iterating over step 2 until the design horizon is reached, we arrive at the "best" sequence given the input traffic stream, as:

$$S_N = (M_{11}, M_{22} \ldots M_{NN} \mid \lambda_1, \lambda_2, \ldots \lambda_N, r_N) \tag{4-19}$$

However, the process outlined above will still contain an enormous number of possible sequences if the number of maintenance options is large, i.e., if there are M options at each of N years,

$$N^* = \frac{M}{M-1} (M^N - 1) \tag{4-20}$$

which grows extrememly fast in both M and N. It is suggested that the number of decision nodes and maintenance levels be kept small until a familiarity with the model enables us to establish some pruning rules for the set of possible sequences.

The salvage value at the design horizon must also be evaluated, which can be done as in the state values, but now not in annualized form; the cost of constructing a new facility, in terms of constant dollars, which would provide the quality that was left at the design horizon, should be a suitable evaluation procedure.

The traffic effects may be studied by varying the traffic stream which has been input. Also, the initial growth rate may be varied, but this is anticipated to have little effect, since only small amounts of maintenance are anticipated in the initial years of service. Environmental effects may be manipulated in the same manner.

The output of the model will consist of a sequence of conditioned sequences of maintenance efforts which will provide "optimal" serviceability given some utility function and traffic stream. In this sense, "Optimal" can only be interpreted as best possible given the assumptions.

Chapter Five. Sensitivity Analysis

In this chapter, several maintenance strategies are examined with a view to identify the sensitivity of the pavement system's behavior to various design parameters under the influence of these maintenance activities. Further, comparisons are drawn among these alternatives to establish the level of agreement between the predicted patterns and those anticipated in real-world situations.

The parameters investigated include the traffic stream, the maintenance budget, the initial design thicknesses, the initial design material quality, and the initial design quality. To establish a common yardstick for comparison, the same temperature history and load characteristics were used throughout this analysis. The relevant data values are given in Table 5.1, while the user benefits, maintenance options, and material characteristics are shown in Figures 5.1, 5.2, and 5.3 respectively. The actual values were arbitrarily chosen for this example.

V. 1 Results Under Zero Maintenance

This section examines the effects of changes of the layer thicknesses, their material quality, and their initial quality to different traffic streams under conditions

Figure 5.1 Dollar Reductions of User Operating Costs As A Function of Serviceability Range

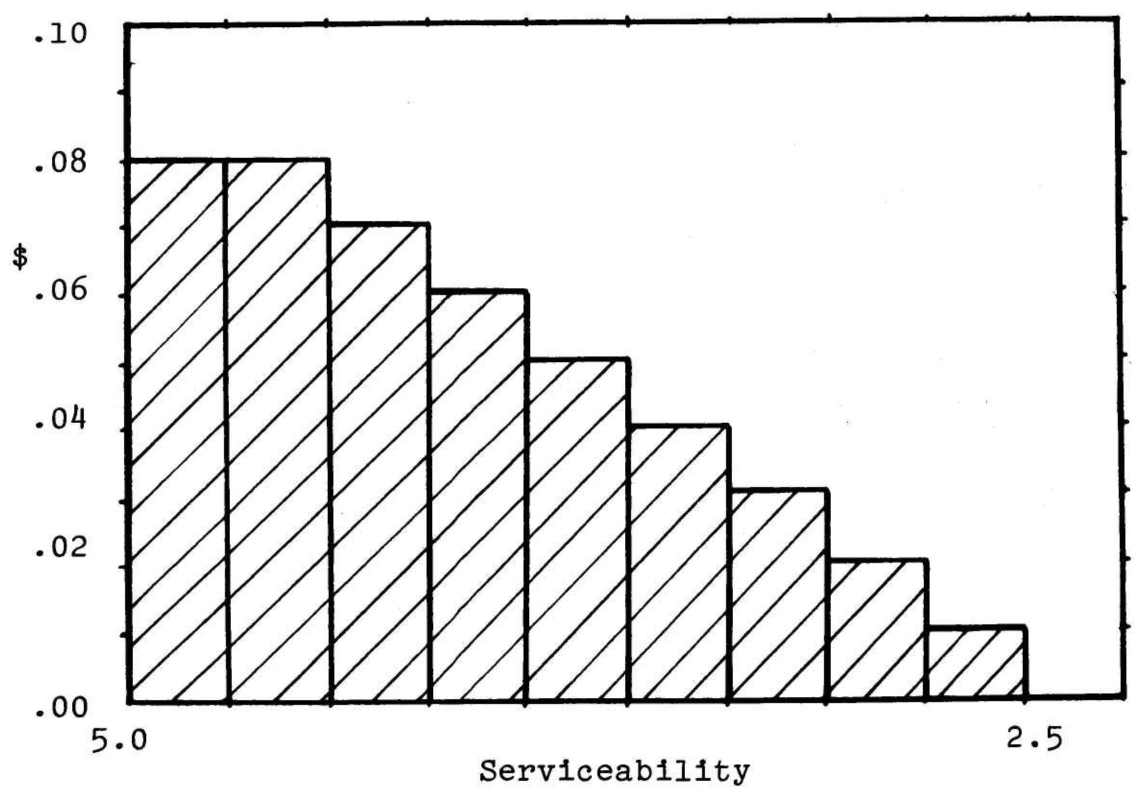


Figure 5.2 Twenty-year Maintenance Strategies - %

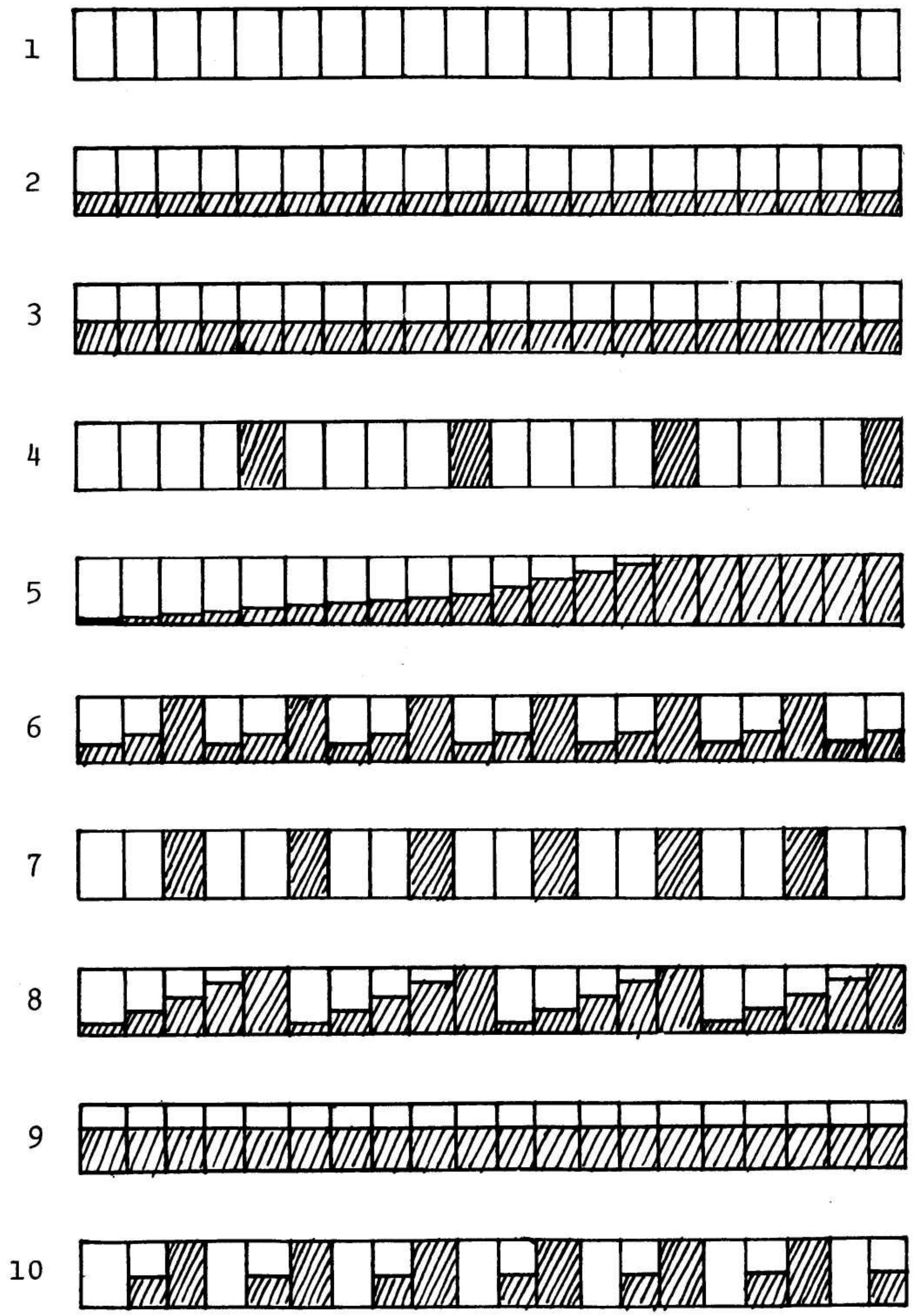


Figure 5.3 Initial Design Configurations

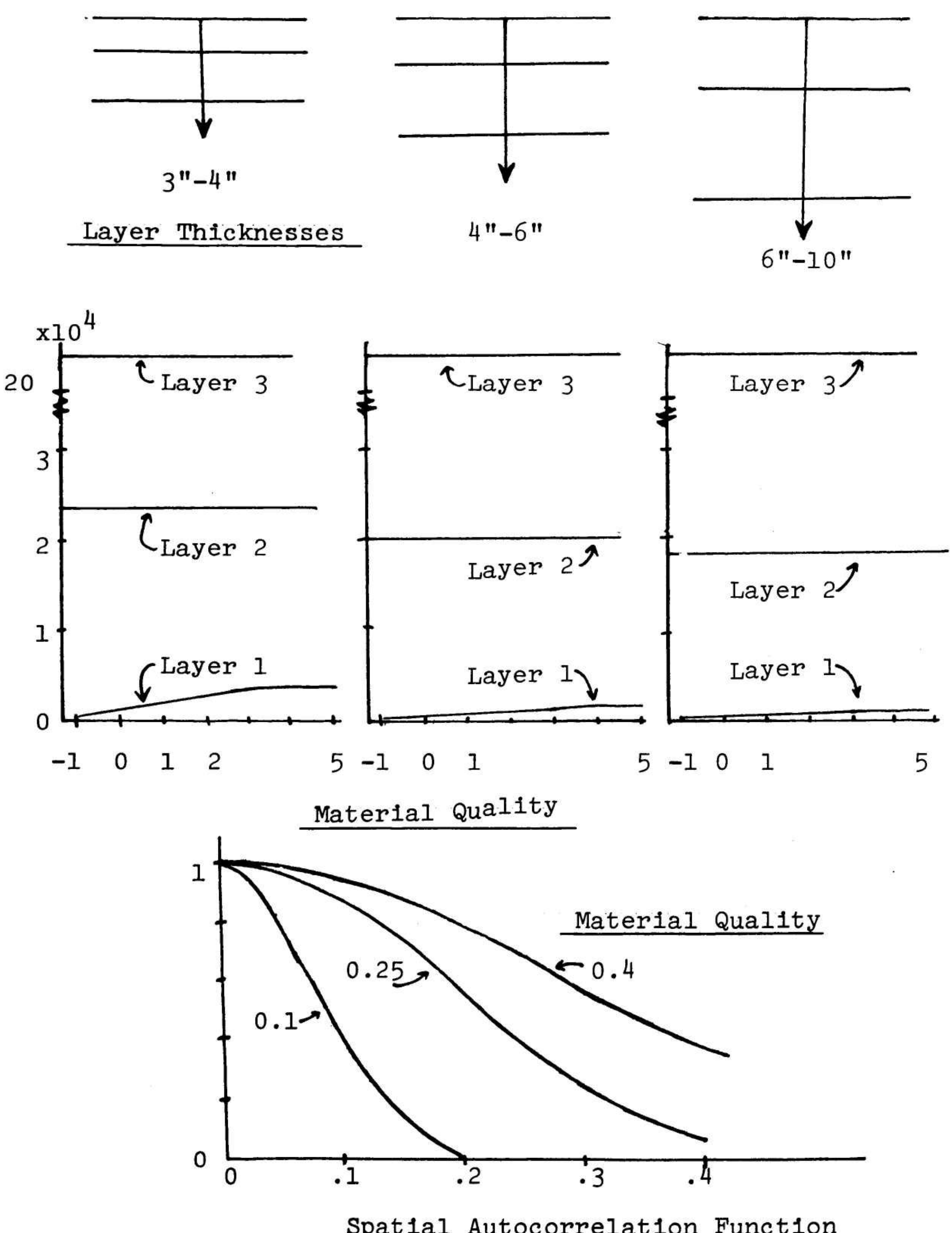


Table 5.1 Variables Held Constant

All Cases

Radius of Applied Loading	6.0 inches
Amplitude of Applied Loading	80.0 psi
Duration of Applied Loading	0.1 seconds
Reference Temperature	70° F
Environmental Temperature	70° F
Miner's Exponent	3.613
Miner's Coefficient	$4.662 \cdot 10^{-7}$
Initial Serviceability	5.0
Initial St. Dev. of Serviceability	0.3
Serviceability Failure Level	2.5

Figures 5.4, 5.5, 5.10, 5.11

Initial Quality	0.1
Material Quality	M

Figures 5.6, 5.7, 5.12, 5.13

Initial Quality	0.1
Layer Thicknesses	3-4

Figures 5.8, 5.9, 5.14, 5.15

Material Quality	M
Layer Thicknesses	3-4

of zero maintenance activity. All of the tendencies predicted by the models under these conditions are intuitively consistent.

V.1. 1 Effects of Layer Thickness

The effects of layer thickness, given zero maintenance are shown most clearly in Figures 5.4 and 5.5, with summaries in Table 5.2. Figure 5.4 shows the present serviceability index as a function of time for several traffic histories. In all cases, the thicker the system, the higher the serviceability index. Quantitatively, from Table 5.2, we can read that the unmaintained lifetime of the system (i.e., Base Life) goes from 3.87 years for the 3"-4" system up to 6.70 years for the 4"-6" system (an increase of 73%) up to 14.68 years for the 6"-10" system (an increase of 279%) for the traffic history of 1000 ADT base and zero growth; similar figures may be obtained for the other traffic histories. These, along with the percentage increases for improvements in initial quality and material quality are summarized in Table 5.3.

Similar shifts in the reliability of the system may be observed in Figure 5.5. The integral of the reliabilty over time yields the expected lifetime, so these values are no tabulated separately.

Figure 5.4 Serviceability .vs. Layer Thickness

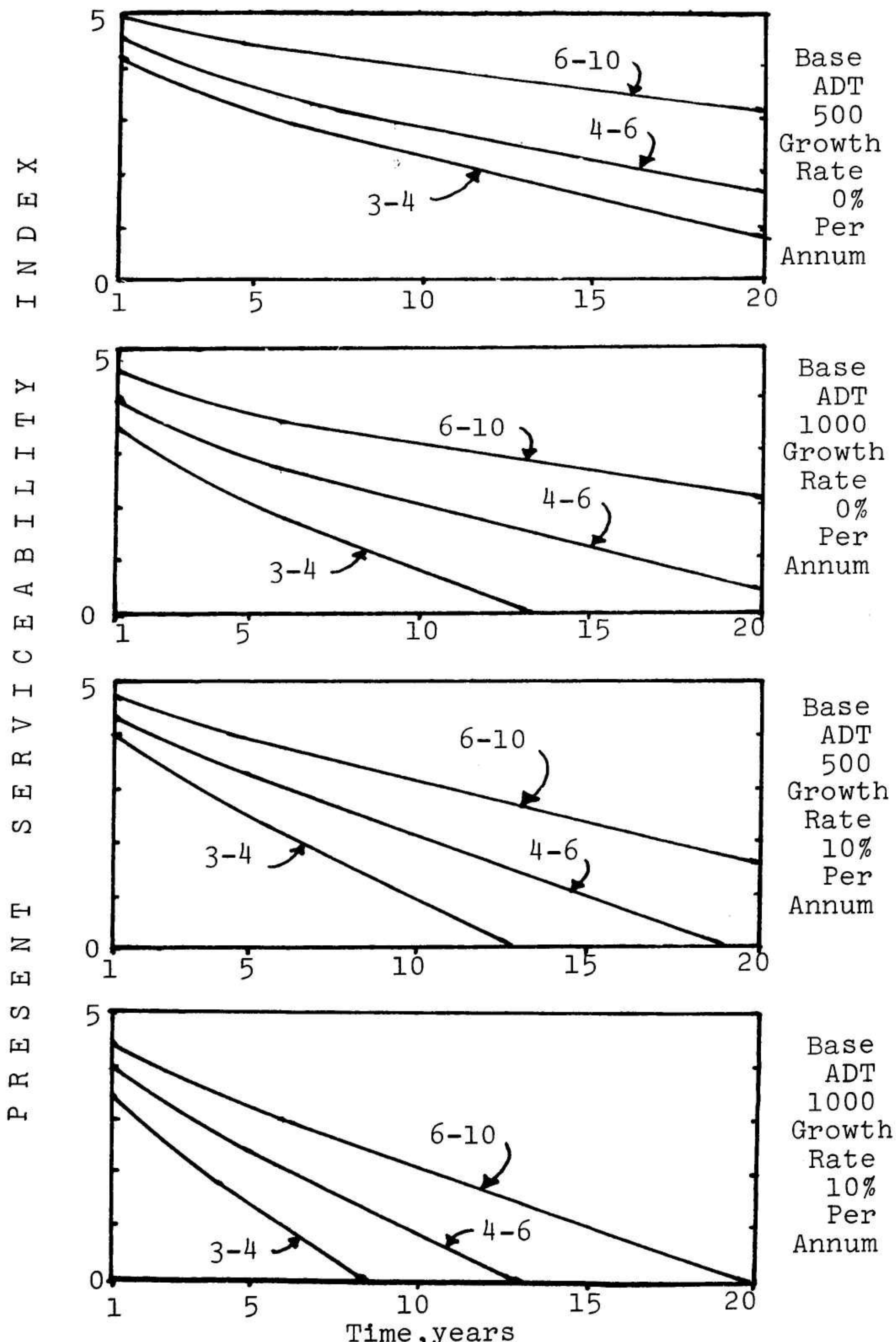
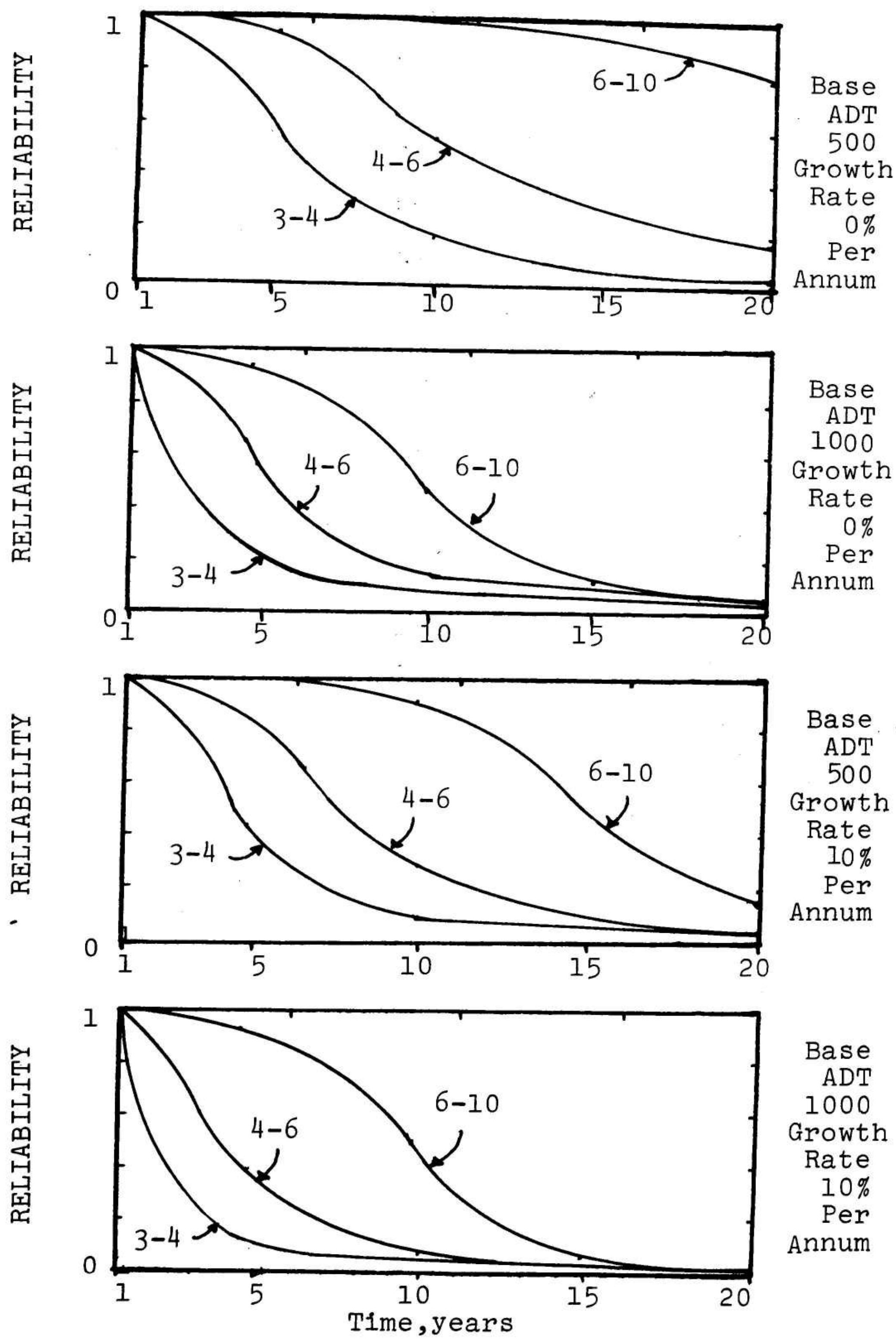


Figure 5.5 Reliability .vs. Layer Thickness



V.1. 2 Effects of Material Quality

The effects of the quality of the material properties, given zero maintenance, are shown most clearly in Figures 5.6 and 5.7, with summaries in Tables 5.2 and 5.3. Figure 5.6 shows the present serviceability index as a function of time for several traffic histories. In all cases, the higher the material quality (i.e., the stiffer the material) the higher the serviceability index. Quantitatively, from Table 5.3, we can read that the unmaintained lifetime of the system (i.e., Base Life) goes from 3.87 years for the M system up to 5.55 years for the M' system (an increase of 43%), up to 8.80 years for the M" system (an increase of 127% over the M system) for the traffic history of 1000 ADT base and zero annual growth; similar figures are given in Table 5.3 for the other traffic histories, along with their percentage increase over the lifetime of the M system.

Similar shifts in the reliability of the system may be observed in Figure 5.7. Since the integral of the reliability over time yields the expected lifetime of the system, the values of reliability increase are not tabulated separately.

Figur 5.6 Serviceability .vs. Material Quality

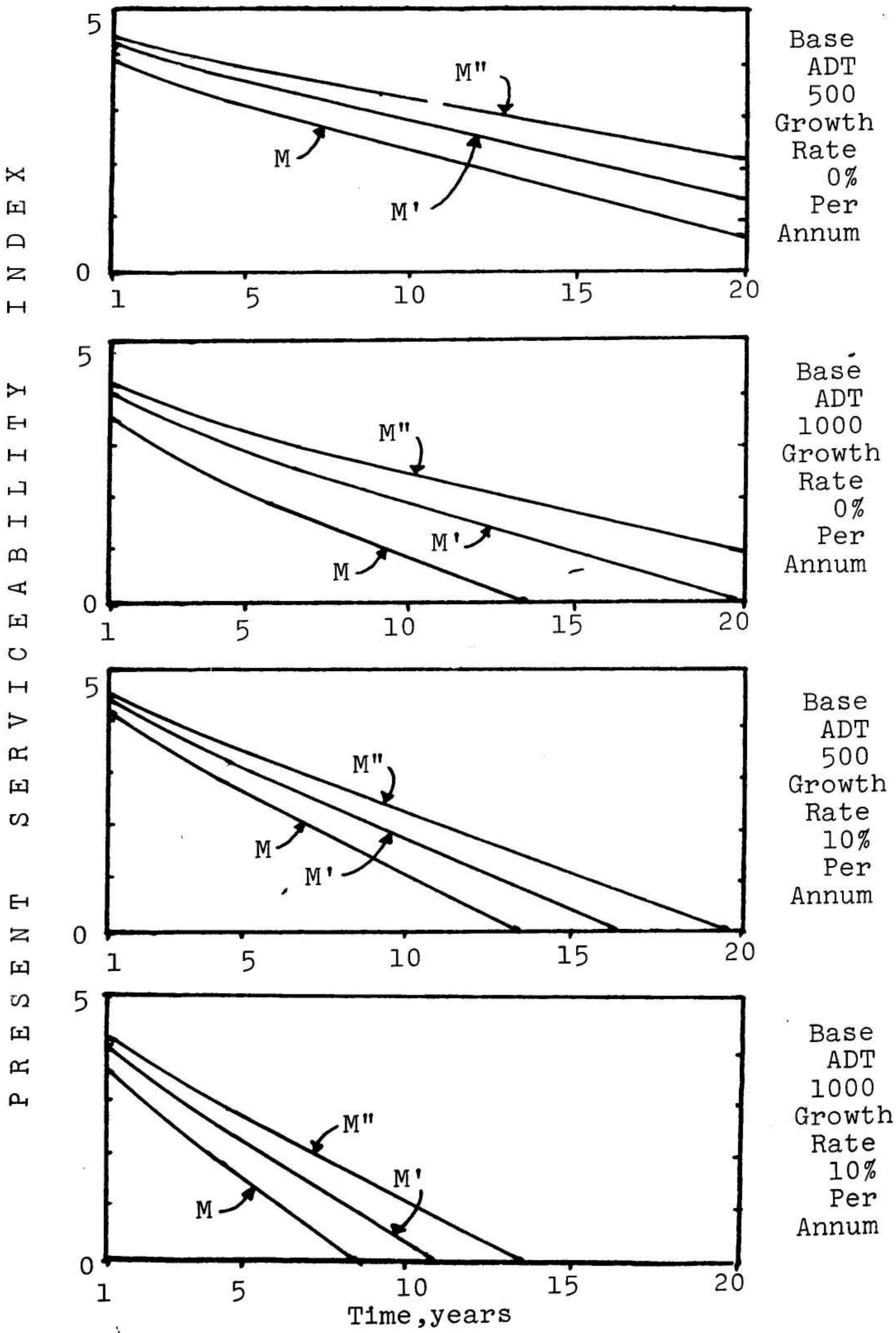
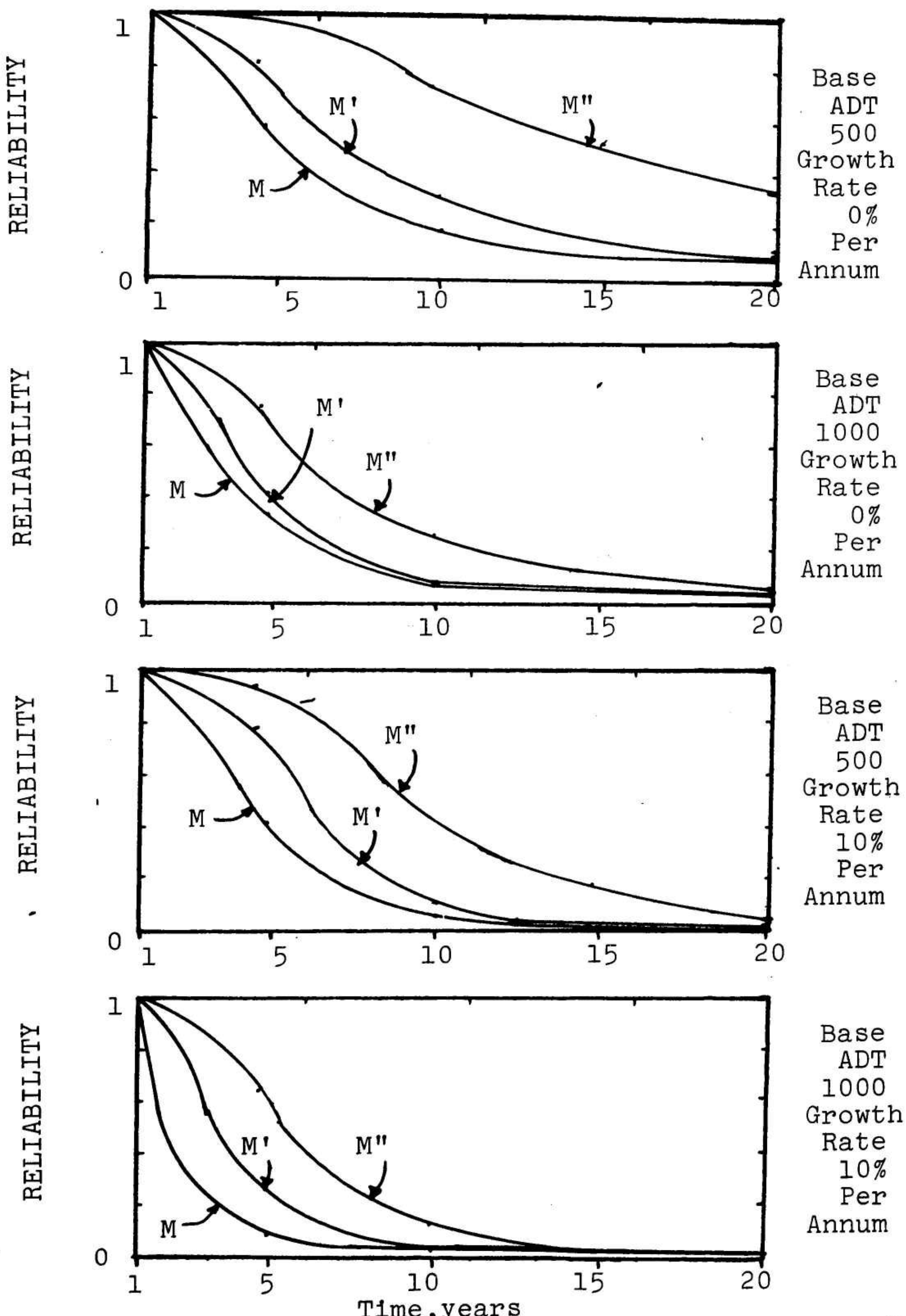


Figure 5.7 Reliability .vs. Material Quality



V.1. 3 Effects of Initial Quality

The effects of the initial quality of the system, in this case taken as the autocorrelation exponent of the system's normal surface deformation, given zero maintenance, are shown most clearly in Figures 5.8 and 5.9, with summaries in Tables 5.2 and 5.3. Figure 5.8 shows the present serviceability index as a function of time for several traffic histories. In all cases, the higher the initial quality (i.e., the less susceptible the system is to longitudinal deformation or slope variance) the higher the serviceablity index. Quantitatively, from Table 5.3, we can read that the unmaintained lifetime of the system (i.e., Base Life) goes from 3.87 years for the 0.1 system up to 9.09 years for the 0.25 system (an increase of 135% over the 0.1 system), up to 11.56 years for the 0.4 system (an increase of 199% over of the 0.1 system) for the traffic history of 1000 ADT base and zero annual growth; similar figures are given in Table 5.3 for the other traffic histories, along with their percentage increase over the lifetime of the 0.1 system.

Similar shifts in the reliability of the system may be observed in Figure 5.9. Since the integral of the reliability over time yields the expected lifetime of the system, the values of reliability increase are not tabulated.

Figure 5.8 Serviceability .vs. Initial Quality

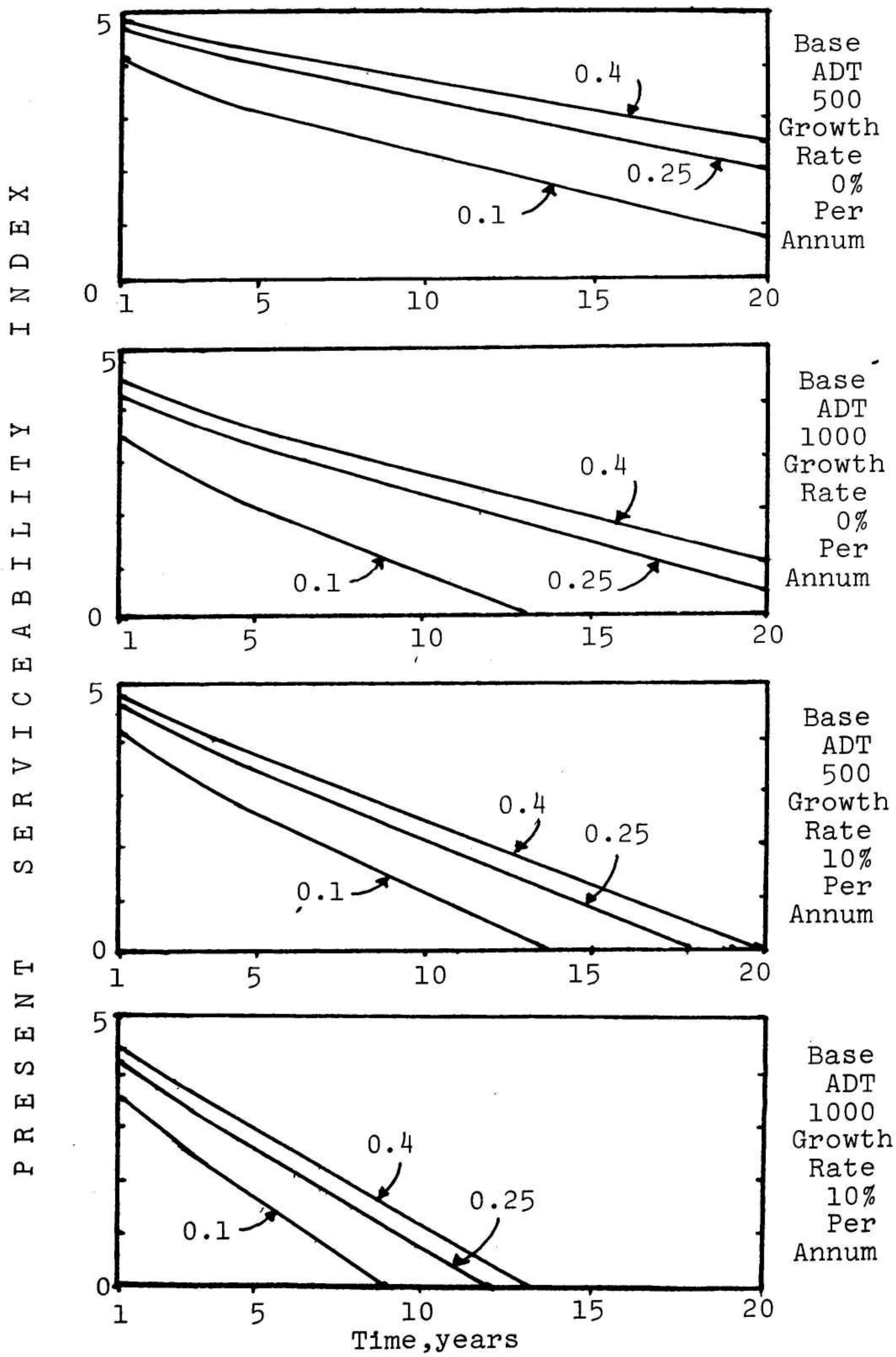
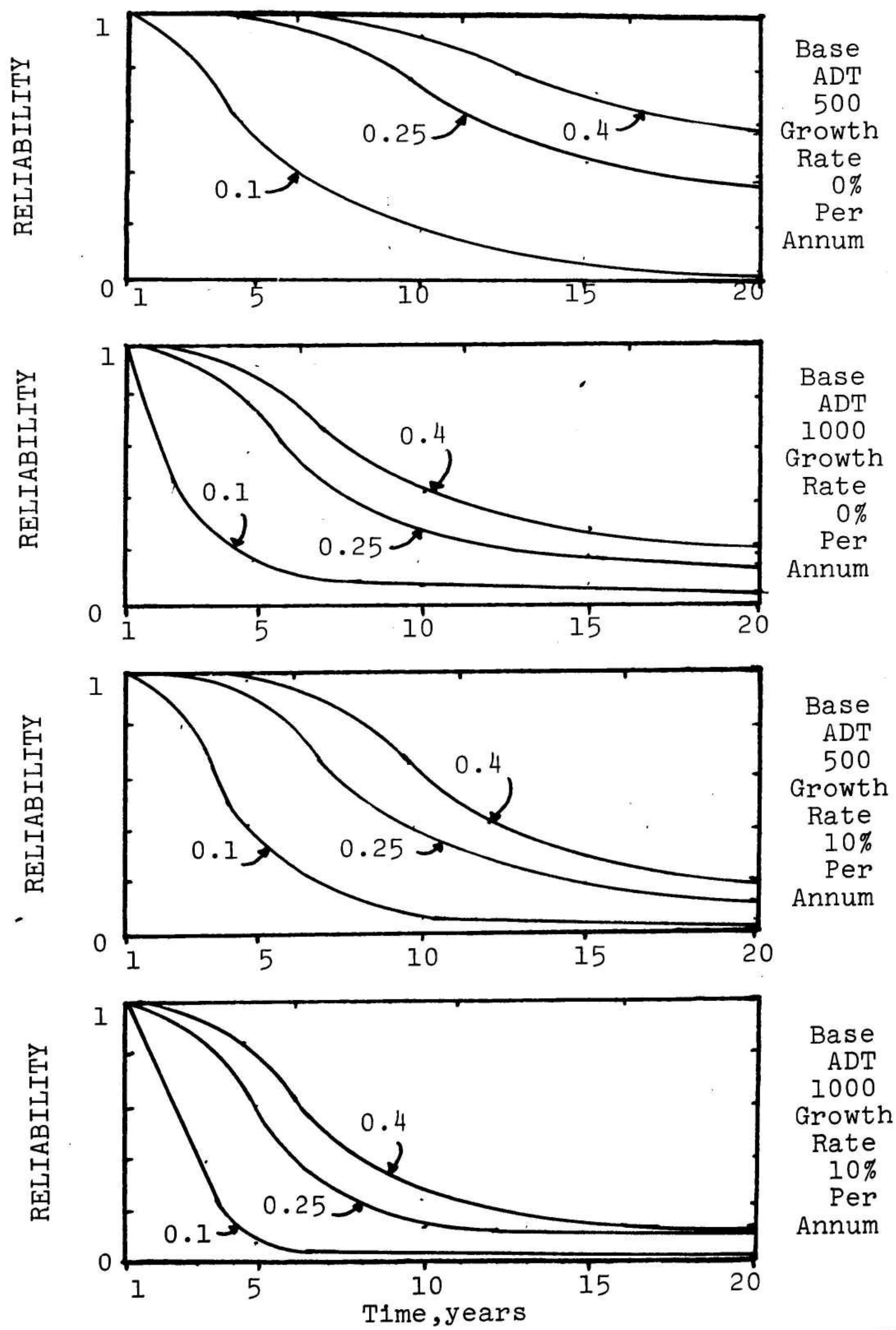


Figure 5.9 Reliability .vs. Initial Quality



V. 2 Results Under Maintenance Activities

This section examines the effects of changes of the layer thicknesses, their material quality, and their initial quality to different traffic streams and different levels of maintenance activity. In all cases, maintenance consists only of patching cracked areas. Although more complicated than the results under zero maintenance, all of the trends predicted by the models under these conditions are intuitively consistent.

V.2. 1 Effects of Layer Thickness

The effects of layer thickness, under low and high maintenance lavels, are shown most clearly in Figures 5.10 and 5.11, with summaries in Table 5.2. Figure 5.10 shows the present serviceability index as a function of time for several traffic histories under these maintenance levels. Some very interesting results may be obseved, particularly in the case of 70% maintenance level. In this case, all of the systems behave pretty much identically; however, the dollar costs of the maintenance required to produce this uniformity vary significantly, as may be seen in the last column of Table 5.2. For the 1000 ADT base and 10% growth case, the maintenance costs

Figure 5.10 Serviceability .vs. Layer Thickness

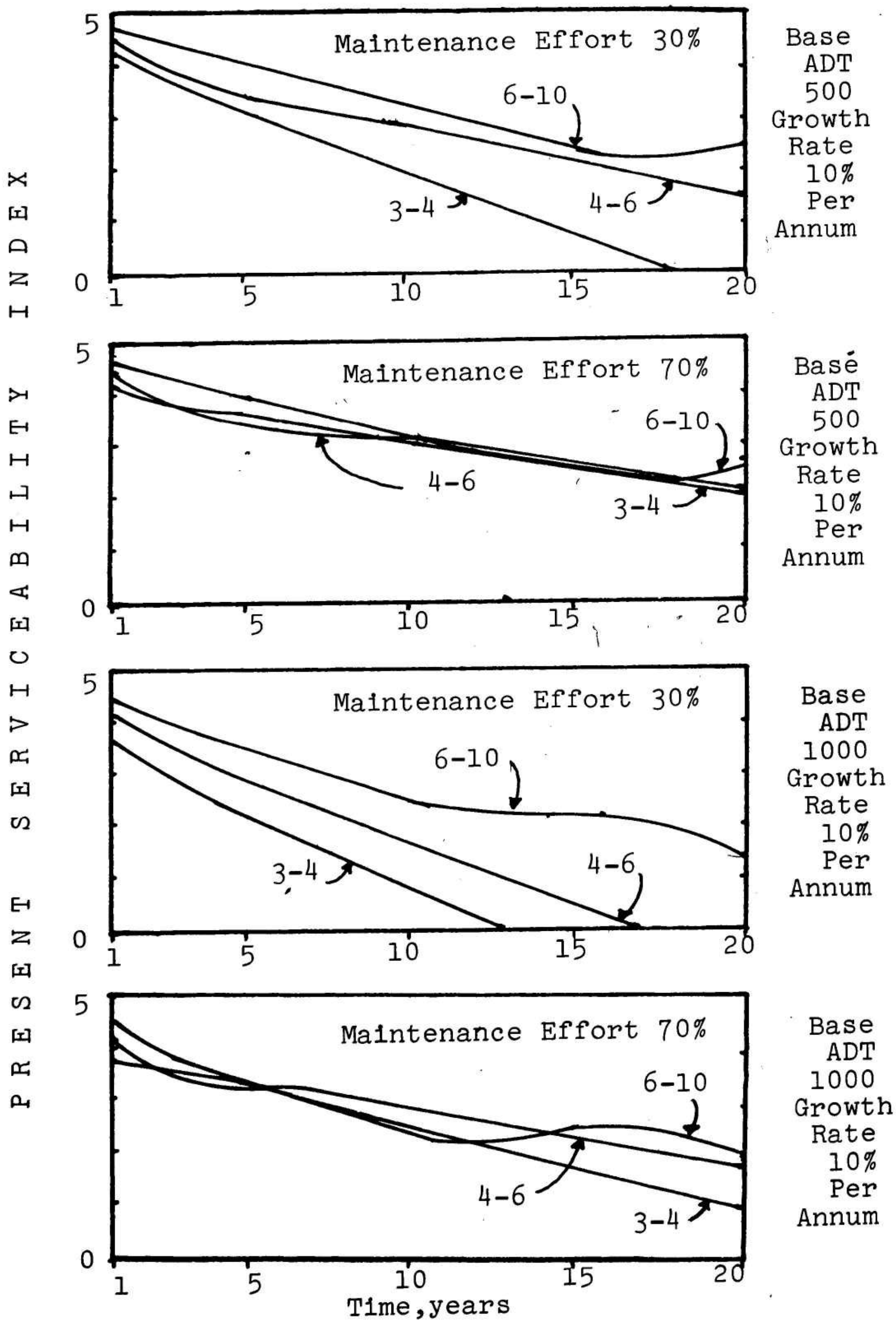
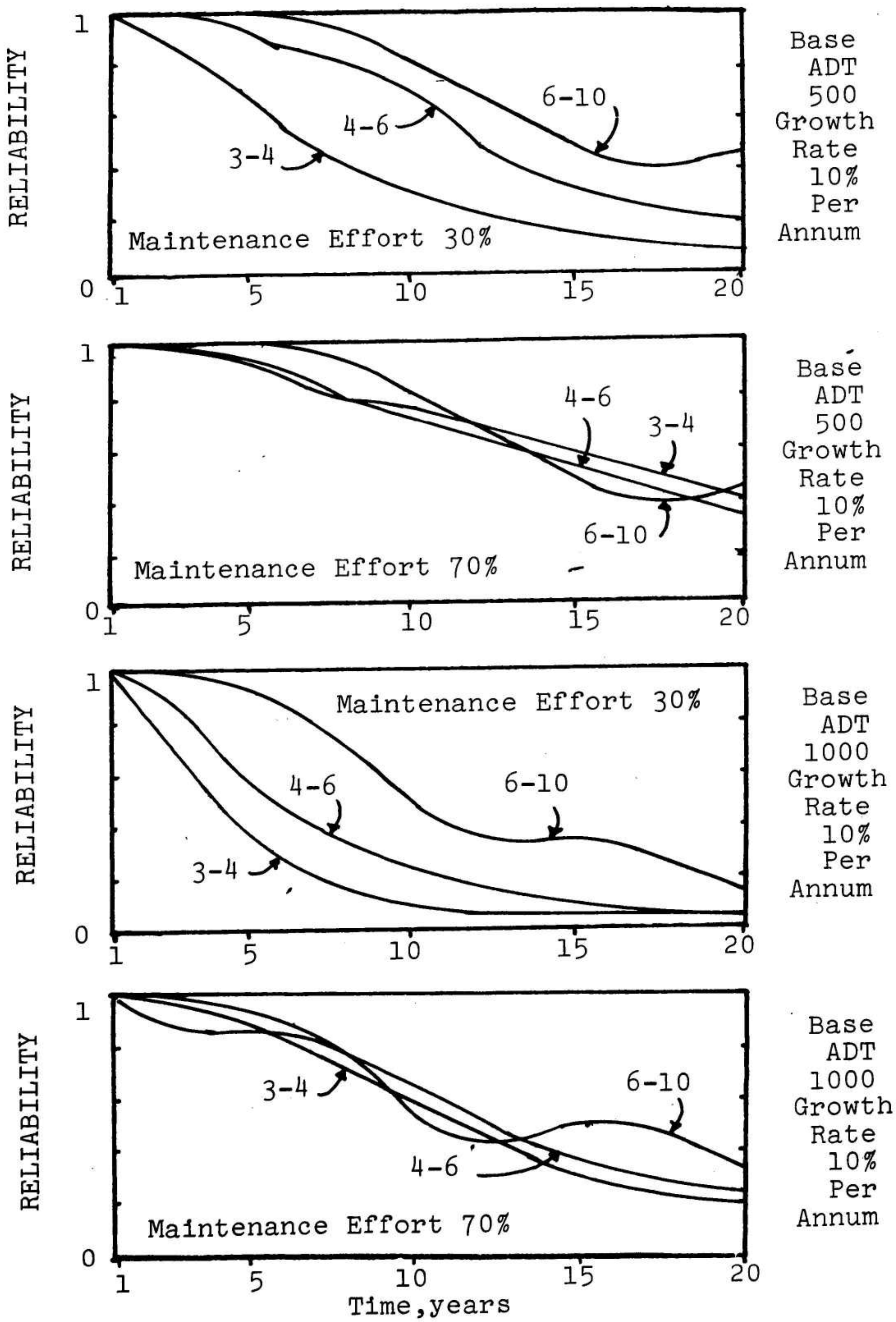


Figure 5.11 Reliability .vs. Layer Thickness



required to keep the 3-4 system at a par with the 6-10 system totalled $24244, as compared to $16489 for the 4-6 system, and only $8292 for the 6-10 system. In this study, the cost of patching was taken as $1/sq.yd.; no economies of scale were considered. Thus, on a percentage basis, the maintenance costs for the 4-6 system were 99% higher than those for the 6-10 system, while those for the 3-4 system were 192% above those for the 6-10 system. Thus, a direct comparison can be made between the added initial costs of the thicker systems and the subsequent savings in maintenance costs. This should be a great convenience in aiding the design process. Similar cost figures can be found in Table 5.2 for the other traffic histories.

Similar shifts in the reliability of the system may be observed in Figure 5.11. The reason these curves are not monotonic is the influence of the maintenance activities, which affect different systems at different times, depending on their rate of cracking. These reliability increments, again, were not tabulated separately, since they are interrelated to the increases in expected lifetimes shown in Table 5.2.

An additional feature of Table 5.2 is the tabulation of the dollar costs of maintenance to add one year to the expected lifetime of the system. For the 1000 ADT base and zero growth case, the cost per year of life added is $3094 for the 3-4 system, $2183 for the 4-6 system, and only $560 for the 6-10 system. This may be used as a measure of the system's maintainability, although it is tied to an economic unit rather than an index.

V.2. 2 Effects of Material Quality

The effects of the quality of the material quality, under low and high maintenance levels, are shown most clearly in Figures 5.12 and 5.13, with summaries in Table 5.2. Figure 5.12 shows the present serviceability index as a function of time for several traffic histories under these maintenance levels. Again, the maintenance effects tend to uniformize all the systems, but the ensuing costs are not at all uniform, as indicated by Table 5.2. The dollar costs for the M system required to keep it at a par with the others was $24244, while those for the M' system were $15383 and those for the M" system were only $10134, all for the 1000 ADT base and 10% annual growth. Thus on a percentage basis, the total maintenance costs for the M and M' systems were 139% and 52% higher than those for the M" system, respectively. Again, a direct comparison between the added initial costs of these higher quality systems and the reduced maintenance costs may

Figure 5.12 Serviceability .vs. Material Quality

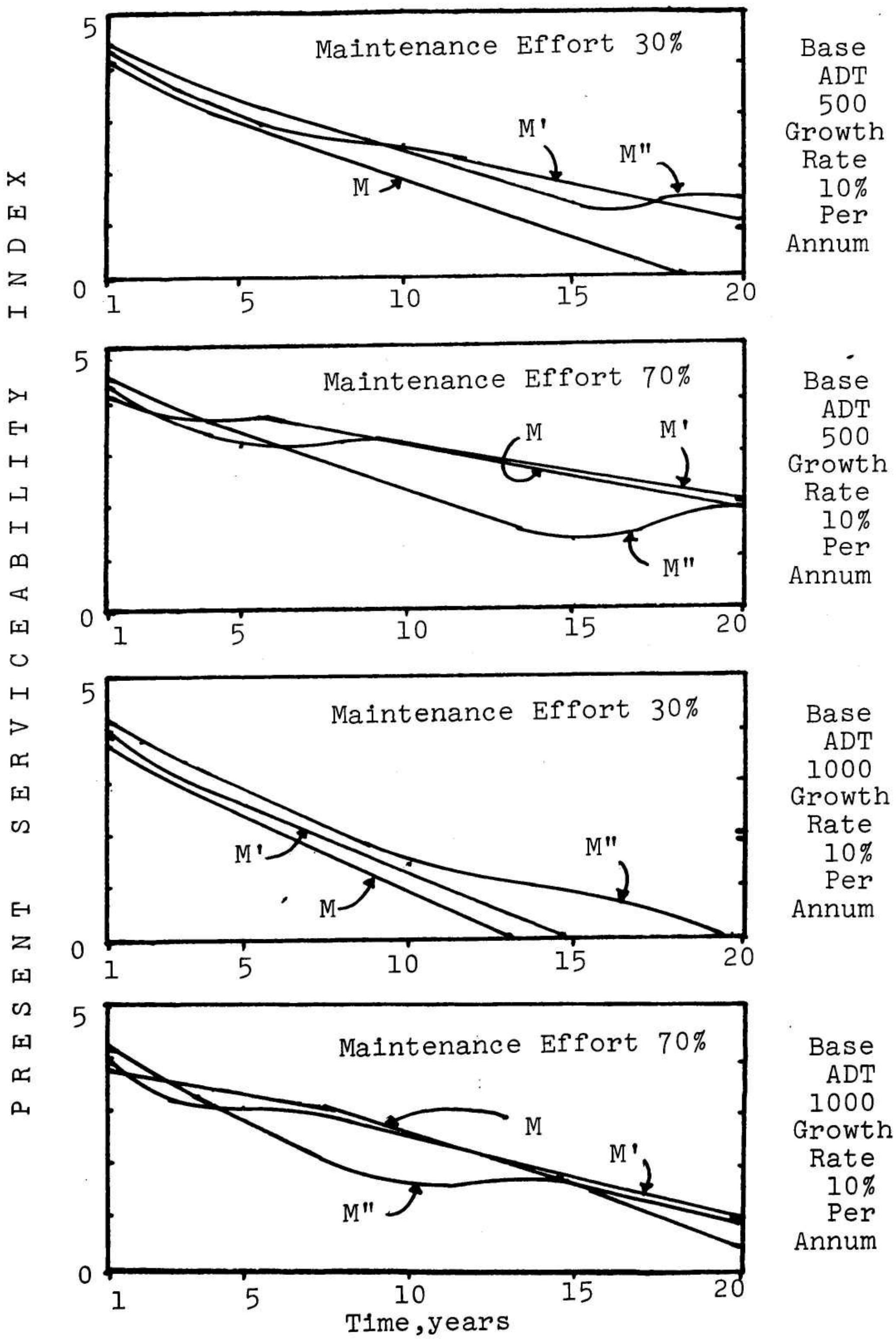
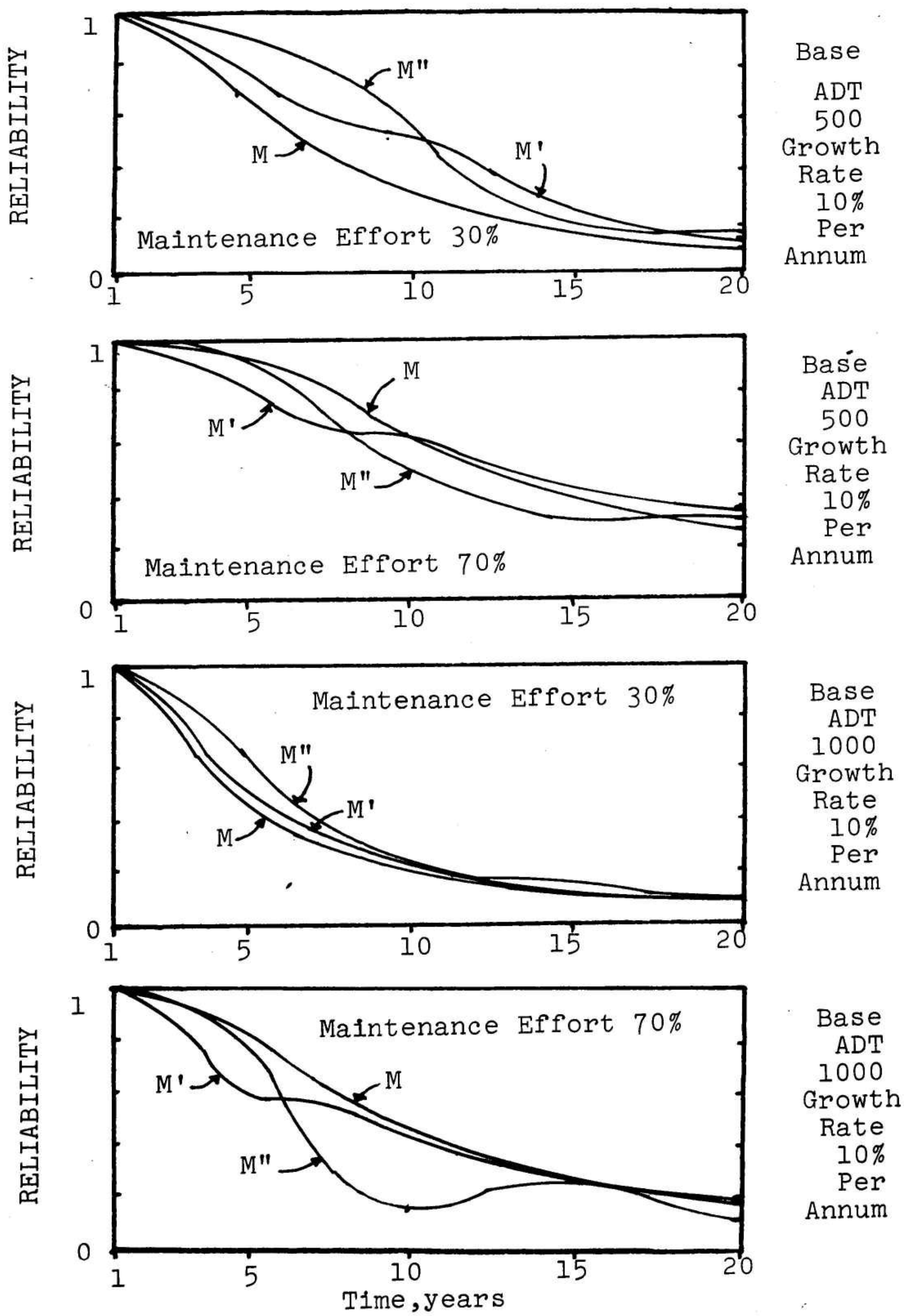


Figure 5.13 Reliability .vs. Material Quality



undertaken. Similar cost figures can be found in Table 5.2 for the other traffic histories.

The dollar costs of maintenance required to add one year to the expected lifetime of the system are found in Table 5.2. For the 1000 ADT base and zero growth case, the cost per year of life added is \$3094, \$2202, and \$1754, respectively, for the M,M', and M" systems. Thus, on a percentage basis, the maintenance costs per year of life added were 76% and 26% higher for the M and M' systems, respectively, than for the M" system, indicating its greater maintainability.

Similar shifts in the reliability of the system may be observed in Figure 5.13. Again, these values were not separately tabulated, for reasons indicated previously.

V.2. 3 Effects of Initial Quality

The effects of the initial quality, or resistance to slope variance, under low and high maintenance levels, are shown most clearly in Figures 5.14 and 5.15, with summaries in Table 5.2. Figure 5.14 shows the present serviceability index as a function of time for several traffic histories under these maintenance levels. The maintenance effects do not tend to uniformize the system as much as the other parameters, due undoubtedly to the negligible effect that

Figure 5.14 Serviceability .vs. Initial Quality

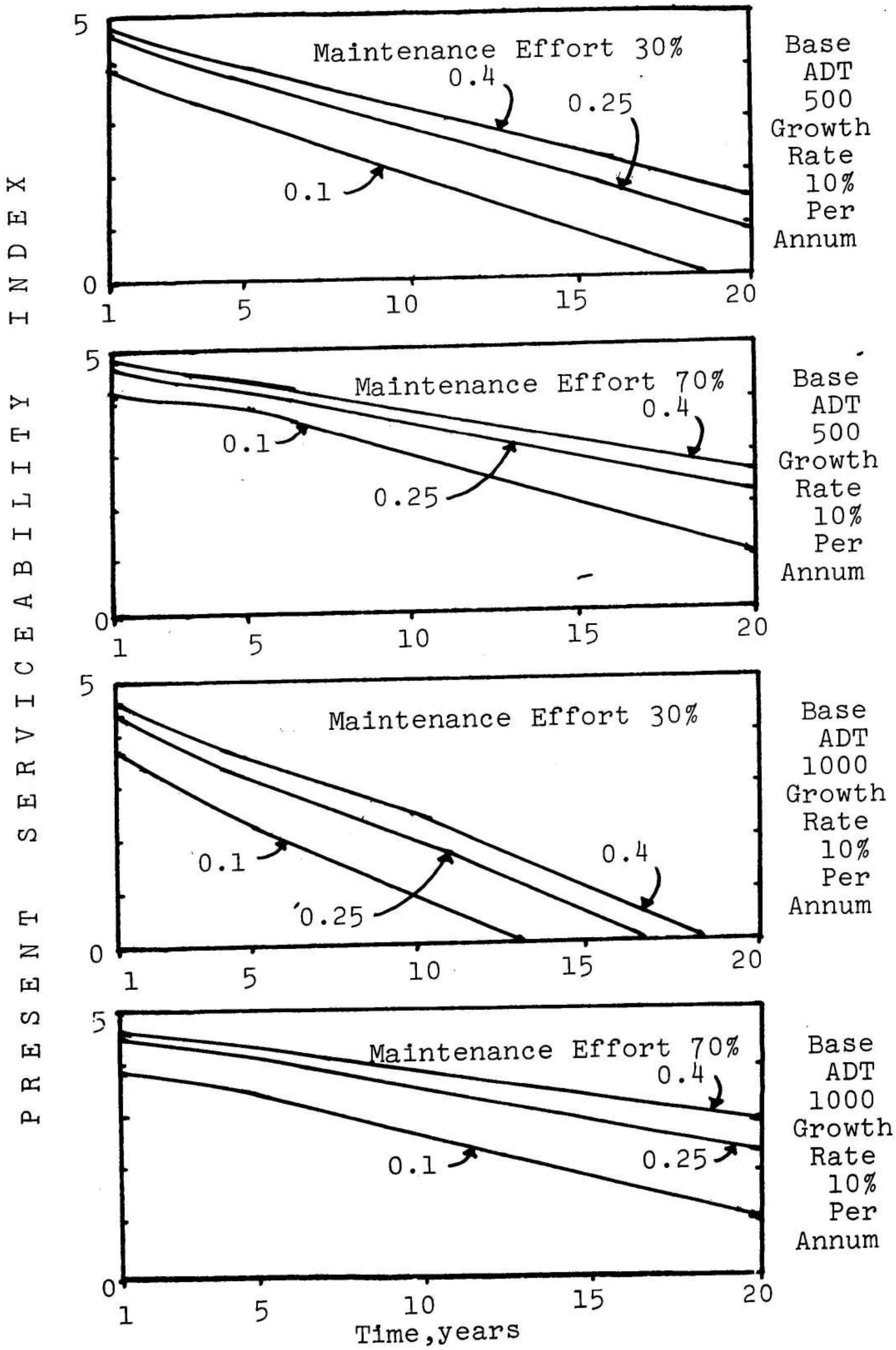
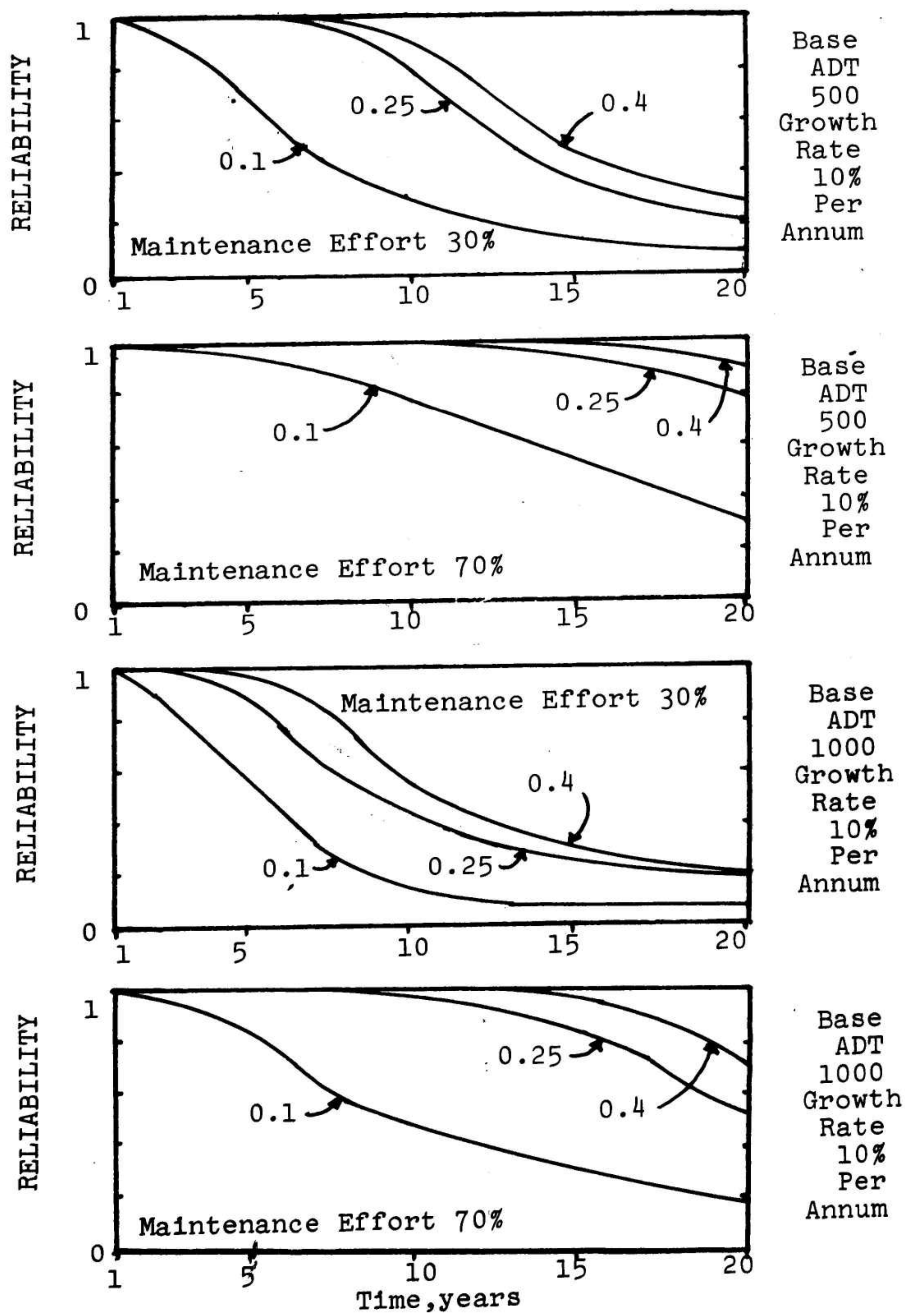


Figure 5.15 Reliability .vs. Initial Quality



the initial quality has on the rut depth of the system. All the tendencies are intuitively consistent. Table 5.2 presents the dollar costs of the maintenance needed to add one year of life to the system, yielding $3094,$3129, and $3701, respectively for the 0.1, 0.25, and 0.4 systems. Although this indicates that the higher the initial quality the higher will be the incremental costs of adding life to the system, this is not the whole story. Although the total maintenance expenditures were nearly constant, as were the costs of incremental life, the serviceability of the system improved as the initial quality improved, implying presumably greater reductions in operating costs to its users. It must also be remembered that only cracking can be repaired, which is little effected by initial quality as the model now stands.

Figure 5.15 shows the reliability of the system over time for the various traffic stresms and initial qualities. Again, this higher reliability for higher initial quality systems would indicate a greater reduction of user costs for that system.

V.3 Effects of the Maintenance Budget

The effects of the maintenance budget may be seen most clearly in the two sections of Table 5.2, one having an

Material Quality	Thickness H1-H2	Initial Quality	Base ADT	Growth Rate %	Base Life	Strategy Chosen	Years Added	Cost/ Year Added $	Mean Cost/ Year Added	Total Cost $
M	3-4	0.1	1000	0	3.87	10	7.30	3094	3368	22590
				10	3.50	10	6.94	3493	4021	24244
			500	0	6.94	10	6.58	2624	2680	17264
				10	5.53	8	7.72	3069	3161	23693
M	4-6	0.1	1000	0	6.70	10	6.12	2183	2321	13363
				10	5.39	10	6.35	2596	2952	16489
			500	0	11.84	5	6.53	1470	1669	9597
				10	8.49	5	9.00	1571	2133	14134
M	6-10	0.1	1000	0	14.68	5	0.09	560	630	53
				10	10.04	5	6.19	1340	1637	8292
			500	0	18.81	10	0.00	0	0	0
				10	14.55	5	1.43	1014	1166	1447
M	3-4	0.25	1000	0	9.09	9	9.38	3129	3459	29357
				10	7.20	9	9.85	3290	3654	32396
			500	0	14.38	5	5.50	2985	4713	16415
				10	10.52	9	8.54	3056	3595	26102
M	3-4	0.4	1000	0	11.56	9	7.93	3701	4131	29357
				10	8.86	9	9.53	3398	3872	32396
			500	0	16.91	5	3.08	5326	8136	16415
				10	12.52	9	7.21	3619	4270	26102
M'	3-4	0.1	1000	0	5.55	10	5.47	2202	2472	12050
				10	4.66	10	5.76	2673	3331	15383
			500	0	10.11	5	6.65	1321	1465	8783
				10	7.43	5	8.28	1605	2243	13285
M'	4-6	0.25	1000	0	17.51	5	1.31	1173	1126	1532
				10	12.49	5	6.65	1524	1635	10134
			500	0	19.89	5	0.00	0	0	0
				10	16.80	5	2.60	1537	1315	3397
M"	3-4	0.1	1000	0	8.80	5	0.47	1754	2075	816
				10	6.59	5	5.97	1625	3258	9700
			500	0	14.44	5	0.24	3	2	0
				10	10.16	5	2.38	1356	2135	3224

Table 5.2 Effects of Maintenance Activities Annual Maintenance Budget Is $7000/mile/lane

Material Quality	Thickness H1-H2	Initial Quality	Base ADT	Growth Rate %	Base Life	Strategy Chosen	Years Added	Cost/ Year Added $	Mean Cost/ Year Added	Total Cost $
M	3-4	0.1	1000	0	3.87	3	4.95	4103	3368	20301
				10	3.50	2	2.00	6946	4021	13873
			500	0	6.94	3	5.32	2627	2680	13984
				10	5.53	2	2.56	4365	3161	11186
M	4-6	0.1	1000	0	6.70	3	4.32	2399	2321	10363
				10	5.39	2	2.09	4384	2952	9178
			500	0	11.84	9	4.95	1469	1669	7278
				10	8.49	3	4.06	2225	2133	9035
M	6-10	0.1	1000	0	14.68	5	0.09	560	630	53
				10	10.04	3	1.64	1943	1637	3189
			500	0	18.81	10	0.00	0	0	0
				10	14.55	9	0.79	1116	1166	881
M	3-4	0.25	1000	0	9.09	3	6.50	3123	3459	20300
				10	7.20	2	3.09	4495	3654	13873
			500	0	14.38	3	4.12	3390	4713	13984
				10	10.52	2	3.32	3368	3595	11186
M	3-4	0.4	1000	0	11.56	3	5.99	3392	4131	20300
				10	8.86	2	3.23	4293	3872	13873
			500	0	16.91	3	2.63	5315	8136	13984
				10	12.52	2	3.18	3517	4270	11186
M'	3-4	0.1	1000	0	5.55	9	5.43	2534	2472	13766
				10	4.66	2	1.47	5759	3331	8473
			500	0	10.11	9	4.33	1354	1465	5857
				10	7.43	3	2.98	2695	2243	8023
M'	4-6	0.25	1000	0	17.51	8	0.81	1086	1126	883
				10	12.49	3	2.94	1534	1635	4515
			500	0	19.89	5	0.00	0	0	0
				10	16.80	3	1.48	1023	1315	1514
M"	3-4	0.1	1000	0	8.80	5	0.47	1754	2075	816
				10	6.59	3	0.78	5282	3258	4134
			500	0	14.44	5	0.00	3	2	0
				10	10.16	3	0.43	2926	2135	1254

Table 5.2 (Cont'd.) Effects of Maintenance Activities Annual Maintenance Budgte Is $3000/mile/lane

Thickness H1-H2	Base Life	Increase %	Material Quality	Base Life	Increase %	Initial Quality	Base Life	Increase %
3-4	3.87	0	M	3.87	0	0.1	3.87	0
4-6	6.70	73	M'	5.55	43	0.25	9.09	135
6-10	14.68	279	M"	8.80	127	0.4	11.56	199
		Base ADT 1000		Growth Rate 0% Per Annum				
3-4	3.50	0	M	3.50	0	0.1	3.50	0
4-6	5.39	54	M'	4.66	33	0.25	7.20	106
6-10	10.04	187	M"	6.59	88	0.4	8.86	153
		Base ADT 1000		Growth Rate 10% Per annum				
3-4	6.94	0	M	6.94	0	0.1	6.94	0
4-6	11.84	71	M'	10.11	46	0.25	14.38	107
6-10	18.81	171	M"	14.44	108	0.4	16.91	144
		Base ADT 500		Growth Rate 0% Per Annum				
3-4	5.53	0	M	5.53	0	0.1	5.53	0
4-6	8.49	54	M'	7.43	34	0.25	10.52	90
6-10	14.55	163	M"	10.16	84	0.4	12.52	126
		Base ADT 500		Growth Rate 10% Per Annum				

Table 5.3 Unmaintained Improvements for Various Design Parameters:

Effects on Life Expectancy

annual maintenance budget of $7000/mile/lane and the other having only $3000/mile/lane. Although, of course, the more money one has to spend the higher level of maintenance can be applied, as can be clearly seen from Table 5.2 and Figure 5.2, it is interesting to observe from Table 5.2 that as your maintenance budget increases, the incremental costs of life (i.e., dollars per year of life added) decrease, even without considering economies of scale. Traffic also effects the choice of optimal maintenance strategy, again indicated by Table 5.2.

Appendix L contains sample inputs and outputs from some selected representative runs from Table 5.2.

Chapter Six. Summary and Conclusion

This study presented a framework for analysis and selection of optimal maintenance policies for a given initial pavement design. The development of this framework is predicated upon the basic philosophy that public facilities are intended to provide certain services to their users. Thus, the functioning of these systems must be evaluated from the standpoint of the users' demands and satisfaction.

To this end, a set of models has been developed to account for the interactions which exist among the materials, environment, traffic, and economic attributes of the system. The analysis and selection process is realized through the implementation of two major phases. One is concerned with the selection of materials and probabilistic evaluation of the physical behavior of the system in an operational environment, utilizing a set of mechanical and phenomonological models. The second phase is aimed at the evaluation of measures of effectiveness for the system at hand in terms of its serviceability, reliability, and maintenance strategies throughout the design lifetime.

In order to demonstrate the capacity of these models to predict maintenance effects, a set of numerical examples were presented. These examples examined the sensitivity of the behavior of the system to various parameters. In this context, the maintenance decision was examined in terms of budget limitations, as well as in terms of the traffic and initial design configuration. These studies have shown that the trends predicted by the model are in reasonable agreement with the anticipated behavior of real-world systems.

The basic features which characterize this study are summarized as follows:

1. The design framework proposed in this study represents a departure from the conventional methods which are generally pursued in the literature of structural design. Instead, design is viewed as a process of sequential evolution of systematic analyses whose ultimate goal is the achievement of an optimal design configuration suitable for the given set of goals and constraints.

2. The criteria for system selection and evaluation are based on the users' subjective preferences for the systems as derived from their particular needs and set of values. From this standpoint, the highway pavement is viewed as a system which is providing certain services to its users. The quality of providing these services at any time must then be evaluated from the users' preferences and satisfaction.

3. The proposed models cover a wide spectrum of activities which encompass a rather large body of knowledge,

ranging from rational mechanics to probability and operations research disciplines. This wide coverage provides a means of continuity and integrity to the design process.

4. The models possess a causal structure which defines the interactions among the system, the operating environment, and the imposed economic constraints. Further, the feedback processes resulting from maintenance activities are accounted for.

5. The models recognize and incorporate the elements of uncertainty which are inherent in both the physical properties of the system and the surrounding environment.

Chapter Seven. Recommendations for Future Work

In view of what has been presented so far, further research activities in this area can proceed along two lines, not necessarily mutually exclusive. The first involves field verification and calibration of the models, while the second includes the extension of the existing models within the established framework. In this chapter, these activities are discussed with emphasis on the relevance and applicability of the models, and their adaptability to a comprehensive design methodology for highway systems.

VII. 1 Field Verification and Model Calibration

In order that the models are used as a meaningful design tool for highway pavements, they must be tested and calibrated against actual field and laboratory measurements. Test tracks and accelerated-life experiments provide some means for these measurements. In this context, the particular values predicted by the models must be compared with the measured values in the field. If significant discrepancies exist, appropriate adjustments both in the particular relationships and the relevant assumptions must be made accordingly.

Both laboratory experiments and field obsevations may be used to examine the validity of several of the assumptions upon which the model development is based, and to provide a proper characterization for the in situ materials. Such tests must include the range of linearity for the characteristics of the system under representative laoding and temperature histories. For these, one may assess the errors involved in the linear approximations, and the significance of these approximations in the prediction of the system response.

Further characterization requirements involve the determination of the coefficients for the time-temperature superposition of the system response within realistic ranges of values for temperatures and material properties.

The fatigue model used in this study is based on a phenomenological approach, namely Miner's criterion. This approach, however, does not provide a quantitative description for crack initiation and propagation, nor does it account for the viscoelastic nature of these processes. A more realistic approach is needed to account for the different stages of crack formation and accumulation in the system based on a micromechanical methodology.

VII. 2 Extension of the Existing Models

The extension of the above models may proceed in parallel along the following lines:

A. Implementation of a large-scale sensitivity analysis to identify the sensitivity of the measures of effectiveness of the system to various maintenance policies. Furthermore, the influence of the particular measures of effectiveness on the choice of the optimal strategies for maintenance can also be examined. This can provide an insight into the selection of relevant measures for maintenance optimization as a part of an overall system optimization framework.

B. Study of potential integration of the Highway Cost Model (7) with the existing models. This model has been developed primarily for low volume roads, but the possibility of using it for normal highway networks has been explored. The cost model generates the cost of construction of a given design configuration, and determines a set of maintenance and vehicle operation costs for various maintenance strategies. These costs may then be used in conjunction with the optimization criteria for the selection of an optimal highway system strategy. The development of this structure is essential to attain an overall design methodology in which the various strategies and design configurations are generated and evaluated, from which an optimal design is provided.

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Appendix A. Parabolic Interpolation and Integration

In this method, the interpolation is performed by fitting a parabola to three consecutive points and then evaluating the ordinates of the parabola at intermediate points as desired. Of course, the parabola is only a valid approximation in the range between the two extreme points. The general equation of the parabola is:

$$y(x) = ax^2 + bx + c \tag{A-1}$$

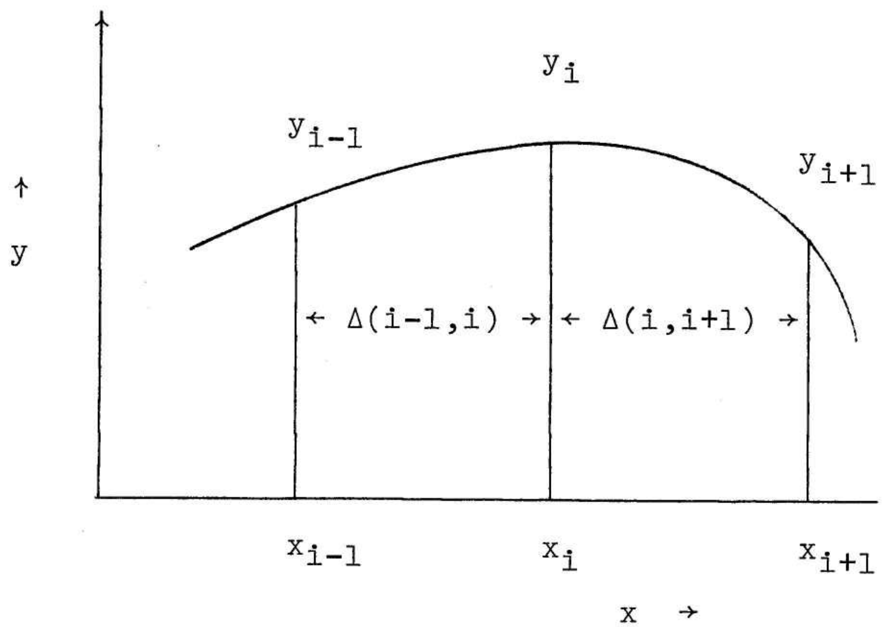

Figure A.1 Curve-fitting Parameters

The fitting process starts at a non-extreme point, say (x_i, y_i), where x_i is taken as the zero of the x-axis. The points (x_k, y_k), $k = 1, 2, \ldots, n$ and the x-intervals between them $\Delta(k, k+1)$ $k = 1, 2, \ldots, n-1$ are assumed to be known. Since, after setting x_i to zero on the x axis, the x values of x_{i-1} and x_{i+1} become $-\Delta(i-1, i)$ and $\Delta(i, i+1)$ respectively, we have the following three equations to solve for a, b, and c:

$$y_{i-1} = a(\Delta(i-1, i))^2 + b(-\Delta(i-1, i)) + c$$

$$y_i = c \tag{A-2}$$

$$y_{i+1} = a(\Delta(i,i+1))^2 + b(\Delta(i,i+1)) + c$$

Setting $\Delta(i,i+1)$ to Δ_i, these are solved by:

$$c = y_i$$

$$a = \frac{y_{i+1} - y_i}{\Delta_i(\Delta_{i-1} + \Delta_i)} + \frac{y_{i-1} - y_i}{\Delta_{i-1}(\Delta_{i-1} + \Delta_i)} \tag{A-3}$$

$$b = \frac{y_{i+1} - y_i}{\Delta_i} - \frac{y_{i+1} - y_i}{\Delta_{i-1} + \Delta_i} + \frac{y_i - y_{i-1}}{\Delta_{i-1}(\Delta_{i-1} + \Delta_i)}$$

If, however, $\Delta_i = \Delta_{i-1} = \Delta$, these reduce to:

$$c = y_i$$

$$a = (y_{i-1} - 2y_i + y_{i+1}) / 2\Delta^2 \tag{A-4}$$

$$b = (y_{i+1} - y_{i-1}) / 2\Delta$$

The representation of the curve between (x_{i-1}, y_{i-1}) and (x_{i+1}, y_{i+1}) is now explicitly given, since a, b, and c are now known. This means that the ordinate y for any value of x between x_{i-1} and x_{i+1}, say, x_k, may be obtained from:

$$y(x_k) = ax_k^2 + bx_k + c \tag{A-5}$$

To evaluate the integral of a function y(x) for which n data values have been measured, where n is odd, we can use the above interpolation technique:

$$\int_{x_1}^{x_n} y(x)\,dx \simeq \sum_{\substack{i=2 \\ i\ \text{even}}}^{n-1} \int_{x_{i-1}}^{x_{i+1}} (a_i x^2 + b_i x + c_i)\,dx \tag{A-6}$$

This integration can easily be evaluated as:

$$\sum_{\substack{i=2 \\ i\ \text{even}}}^{n-1} \left\{\frac{a_i}{3}(x_{i+1}^3 - x_{i-1}^3) + \frac{b_i}{2}(x_{i+1}^2 - x_{i-1}^2) + c_i(x_{i+1} - x_{i-1})\right\} \tag{A-7}$$

where the a_i, b_i, and c_i's are determined from points (x_{i-1}, y_{i-1}), (x_i, y_i), and (x_{i+1}, y_{i+1}) as described above.

Appendix B. Evaluation of the Bessel Terms

The Bessel functions that occur in the solutions are defined by the following infinite series:

$$J_N(x) \equiv \sum_{k=0}^{\infty} (-1)^k \left(\frac{x}{2}\right)^{2k+N} / (k!\,(k+N)!) \tag{B-1}$$

However, the above series converges very slowly for large x. The following approximations were used in this analysis (4), with a resulting maximum error of less than $3.6 \cdot 10^{-7}$.

$$J_0(x) \simeq \sum_{i=0}^{6} A_i (x/3)^{2i} \qquad x \leq 3$$

$$J_0(x) \simeq x^{-\frac{1}{2}} \left(\sum_{i=0}^{6} B_i (3/x)^i\right) \cos\left(x + \sum_{i=0}^{6} C_i (3/x)^i\right) \qquad x > 3$$

$$J_1(x) \simeq x \sum_{i=0}^{6} D_i (x/3)^{2i} \qquad x \leq 3$$

$$J_1(x) \simeq x^{-\frac{1}{2}} \left(\sum_{i=0}^{6} E_i (3/x)^i\right) \cos\left(x + \sum_{i=0}^{6} F_i (3/x)^i\right) \qquad x > 3$$

$$J_1(x)/x \simeq \sum_{i=0}^{6} D_i (x/3)^{2i} \qquad x \leq 3$$

$$J_1(x)/x \simeq x^{-\frac{3}{2}} \left(\sum_{i=0}^{6} E_i (3/x)^i\right) \cos\left(x + \sum_{i=0}^{6} F_i (3/x)^i\right) \qquad x > 3$$

where the values of A_i, B_i, C_i, D_i, E_i, and F_i are given in Table B1.

Table B1. Coefficients for the Bessel Terms

$A_0 = 1.0000000$	$B_0 = 0.79788456$
$A_1 = -2.2499997$	$B_1 = -0.00000077$
$A_2 = 1.2656208$	$B_2 = -0.00552740$
$A_3 = -0.3163866$	$B_3 = -0.00009512$
$A_4 = 0.0444479$	$B_4 = 0.00137237$
$A_5 = -0.0039444$	$B_5 = -0.00072805$
$A_6 = 0.0002100$	$B_6 = 0.00014476$
$C_0 = -0.78539816$	$D_0 = 0.50000000$
$C_1 = -0.04166397$	$D_1 = -0.56249985$
$C_2 = -0.00003954$	$D_2 = 0.21093573$
$C_3 = 0.00262573$	$D_3 = -0.03954289$
$C_4 = -0.00054125$	$D_4 = 0.00443319$
$C_5 = -0.00029333$	$D_5 = -0.00031761$
$C_6 = 0.00013558$	$D_6 = 0.00001109$
$E_0 = 0.79788456$	$F_0 = -2.35619449$
$E_1 = 0.00000156$	$F_1 = 0.12499612$
$E_2 = 0.01659667$	$F_2 = 0.00005650$
$E_3 = 0.00017105$	$F_3 = -0.00637879$
$E_4 = -0.00249511$	$F_4 = 0.00074348$
$E_5 = 0.00113653$	$F_5 = 0.00079824$
$E_6 = -0.00020033$	$F_6 = -0.00029166$

Appendix C. Least Squares Curve-Fitting Method *

An exponential series (Dirichlet series) may be used to approximate a class of functions which behave temporally in a particluar way, i.e., at large times the function tends to a constant value. Such is the behavior of viscoelastic creep functions and system responses to static loadings. If the function to be represented has measured values of $\gamma(t_i)$ at time t_i, i = 1, 2, ...n, let:

$$\bar{\gamma}\ (t) = \sum_{j=1}^{m} G_j e^{-\delta_j t} \tag{C-1}$$

be the approximating series representation.

The error between the measured values and the approximate values will then be:

$$E_i = \gamma(t_i) - \bar{\gamma}(t_i), \quad i=1,\ 2,\ \ldots,\ n \tag{C-2}$$

If n points in time are chosen to perform the curve-fit process, where $n \geq m$, then n such error terms will be obtained. In order to apply the least squares method of curve-fitting to this case, the δ_j's are given specific values (dependent on the orders of magnitude of time being considered).

---

* Following Moavenzadeh and Elliott, "Moving Load on a Viscoelastic Layered System"

The total squared value of the errors is then:

$$S = \sum_{i=1}^{n} E_i^2 = \sum_{i=1}^{n} (\gamma(t_i) - \sum_{j=1}^{m} G_j e^{-\delta_j t_i})^2 \tag{C-3}$$

The coefficients, G_j, are determined by minimizing the total squared error S with respect to the G_k:

$$\frac{\partial S}{\partial G_k} = 0 = \sum_{i=1}^{n} - 2(\gamma(t_i) - \sum_{j=1}^{m} G_j e^{-\delta_j t_i}) e^{-\delta_k t_i} \quad k = 1,2\ldots m \tag{C-4}$$

which can be rewritten as:

$$\sum_{i=1}^{n} \gamma(t_i) e^{-\delta_k t_i} = \sum_{j=1}^{m} \sum_{i=1}^{n} G_j e^{-t_i(\delta_j+\delta_k)} \quad k = 1,2\ldots m \tag{C-5}$$

Equation (C-5) is a system of m linear equations whose unknowns are the m coefficients G_j. Setting:

$$B_k = \sum_{i=1}^{n} \gamma(t_i) e^{-\delta_k t_i}$$

$$A_{kj} = \sum_{i=1}^{n} e^{-(\delta_j+\delta_k) t_i} \tag{C-6}$$

we would have:

$$B_k = \sum_{j=1}^{m} A_{kj} G_j \tag{C-7}$$

or, in matrix form:

$$\underline{G} = A^{-1}\underline{B} \tag{C-8}$$

where $\underline{G}$ and $\underline{B}$ are m-dimensional column vectors and A^{-1} is the m x m inverse matrix of A. Numerous computer techniques are available for matrix inversion, which is the key computational step in this curve-fit technique.

A satisfactory method for determining the δ's is to set:

$$m = \log_{10}(t_n) - \log_{10}(t_1) + 1$$

$$\delta_m = 0$$

$$\delta_k = 5 \cdot 10^{-k-\log_{10}(t_1)} = 5 \cdot 10^{-k}/t_1 \qquad k = 1,2...m-1 \tag{C-9}$$

Appendix D. Convolution Integral Evaluation

The following analysis yields an exact expression for multiple convolution integrals (the $A_{i,j}(t)$ and $B_{i,j}(t)$ from section II. 1) Assume that each $D_k(t)$ can be represented by a Dirichlet series:

$$D_k(t) = \eta_k \sum_{j=1}^{m} G_j^k e^{-\delta_j t} \tag{D-1}$$

as in section II. 1. Since the η_k factor through the convolution integrations, they will be omitted until the end of this analysis, where they can easily be replaced in the derived expressions.

Consider first a single convolution integral with $D_r(t)$ convoluted onto $D_q(t)$. The operator equation of the convolution can be expressed as (2)

$$I_1(t) = D_r(0)D_q(t) + \int_{0^+}^{t} D_q(t-\xi)\frac{\partial D_r(\xi)}{\partial \xi}\, d\xi \tag{D-2}$$

In this and what follows, it is irrelevant whether the $D_q(t)$ and $D_r(t)$ are distinct or identical creep functions. Substituting equation (D-1) (without the η term) for $D_q(t)$ and $D_r(t)$, equation (D-2) becomes:

$$I_1(t) = \sum_{i=1}^{m} G_i^r \sum_{j=1}^{m} G_j^q e^{-\delta_j t} + \int_{0^+}^{t} \{ (\sum_{j=1}^{m} G_j^q e^{-\delta_j (t-\xi)})(- \sum_{i=1}^{m} G_i^r \delta_i e^{-\delta_i \xi}) \, d\xi \tag{D-3}$$

After interchanging summations and integration, this becomes:

$$I_1(t) = \sum_{j=1}^{m} G_j^q e^{-\delta_j t} \{ \sum_{i=1}^{m} G_i^\gamma - \sum_{i=1}^{m} G_i^\gamma \delta_i \int_{0^+}^{t} e^{-\xi(\delta_i - \delta_j)} d\xi \qquad \text{(D-4)}$$

The integrals in (D-4) may easily be evaluated, but the result varies depending on whether i=j:

$$\int_{0^+}^{t} e^{-\xi(\delta_i - \delta_j)} d\xi = \begin{cases} t & i=j \\ \dfrac{-1}{\delta_i - \delta_j} (e^{-t(\delta_i - \delta_j)} - 1) & i \neq j \end{cases} \qquad \text{(D-5)}$$

Substituting these values into equation (D-4) yields:

$$I_1(t) = \sum_{j=1}^{m} \{ G_j^q \sum_{i=1}^{m} G_i^\gamma - \delta_j G_j^q G_j^\gamma t - G_j^q \sum_{i=1}^{m} \frac{G_i^\gamma \delta_i (1-\delta_{ij})}{\delta_i - \delta_j} \} e^{-\delta_j t} +$$

$$\sum_{j=1}^{m} \sum_{i=1}^{m} G_j^q G_i^\gamma \delta_i (1-\delta_{ij}) e^{-\delta_i t} / (\delta_i - \delta_j) \qquad \text{(D-6)}$$

where δ_{ij} is the Kronecker delta and

$$(1 - \delta_{ij})/(\delta_i - \delta_j) = 0 \text{ if } i=j \qquad \text{(D-7)}$$

Interchanging the dummy summation indices in the last term of equation (D-6) yields, for that term:

$$\sum_{j=1}^{m} \sum_{i=1}^{m} \frac{-G_i^q G_j^\gamma \delta_j (1-\delta_{ji})}{\delta_i - \delta_j} e^{-\delta_j t} \qquad \text{(D-8)}$$

Thus, finally, after factoring the $e^{-\delta_j t}$ term and isolating the t term, this becomes:

$$I_1(t) = \sum_{j=1}^{m} \{[G_j^q \sum_{i=1}^{m} G_i^\gamma - \delta_j G_j^\gamma \sum_{i=1}^{m} G_i^q (1-\delta_{ji})/(\delta_i - \delta_j)$$

$$- G_j^q \sum_{i=1}^{m} G_i^\gamma \delta_i (1-\delta_{ij})/(\delta_i - \delta_j)] - \delta_j G_j^q G_j^\gamma t\} e^{-\delta_j t} \qquad \text{(D-9)}$$

Substituting $B_j^{\gamma q} = -\delta_j G_j^\gamma G_j^q$ (D-10)

$$A_j^{\gamma q} = G_j^q \sum_{i=1}^{m} G_i^\gamma - \delta_j G_j^\gamma \sum_{i=1}^{m} G_i^q (1-\delta_{ji})/(\delta_i - \delta_j) - G_j^q \sum_{i=1}^{m} \frac{G_i^\gamma \delta_i (1-\delta_{ij})}{\delta_i - \delta_j}$$

we arrive at the relatively simple expression:

$$I_1(t) = \sum_{j=1}^{m} (A_j^{\gamma q} + B_j^{\gamma q} t) e^{-\delta_j t} \qquad \text{(D-11)}$$

From the preceeding analysis, and the form of equation (D-11), it is readily apparent that after N convolutions, the result will be of the form:

$$I_N(t) = \sum_{j=1}^{m} \{\sum_{\ell=0}^{N} A_{j\ell} t^\ell\} e^{-\delta_j t} \qquad \text{(D-12)}$$

Now, since we know the value of $I_1(t)$, we can evaluate any number of convolutions, if we can determine $I_{N+1}(t)$ from $I_N(t)$ convoluted with, say, $D_k(t)$. We would then have the following, after transposing the convolution equation from that of equation (D-2) into:

$$I(t) = D_k(0)(\cdot) - \int_0^{t^-} (\cdot) \frac{\partial D_k(t-\xi)}{\partial \xi} d\xi \tag{D-13}$$

we have:

$$I_{N+1}(t) = D_k(0) I_N(t) - \int_0^{t^-} I_N(\xi) \frac{\partial D_k(t-\xi)}{\partial \xi} d\xi \tag{D-14}$$

Substituting equations (D-1) and D-12) into (D-14) and again suppressing the η term we obtain:

$$\begin{aligned} I_{N+1}(t) = & \sum_{i=1}^{m} G_i^k \sum_{j=1}^{m} \sum_{\ell=0}^{N} A_{j,\ell} t^\ell e^{-\delta_j t} \\ & - \int_0^{t^-} \sum_{j=1}^{m} \sum_{\ell=0}^{N} A_{j,\ell} \xi^\ell e^{-\delta_j \xi} \sum_{i=1}^{m} \delta_i G_i^k e^{-\delta_i (t-\xi)} d\xi \end{aligned} \tag{D-15}$$

Rearranging the summations and integrations and interchanging the dummy summation indices on the last term, yields, for that term only:

$$- \sum_{j=1}^{m} \sum_{i=1}^{m} \delta_j G_j^k e^{-\delta_j t} \sum_{\ell=0}^{N} A_{i,\ell} \int_0^{t^-} \xi^\ell e^{-\xi(\delta_i - \delta_j)} d\xi \tag{D-16}$$

The integrals in equation (D-16) may be evaluated, but the result varies depending on whether i=j :

$$\int_0^t \xi^{\ell} e^{-\xi(\delta_i-\delta_j)} d\xi = \frac{t^{\ell+1}}{\ell+1} \qquad i=j \tag{D-17}$$

$$\frac{\ell!}{(\delta_i-\delta_j)^{\ell+1}} - \frac{e^{-t(\delta_i-\delta_j)}}{(\delta_i-\delta_j)^{\ell+1}} \{(\delta_i-\delta_j)^{\ell} t^{\ell} + \ell(\delta_i-\delta_j)^{\ell-1} t^{\ell-1} + \ldots + \ell!\} \qquad i \neq j$$

Defining

$$C_{i,j,\ell}(t) = \frac{\{(\delta_i-\delta_j)^{\ell} t^{\ell} + \ell(\delta_i-\delta_j)^{\ell-1} t^{\ell-1} + \ldots + \ell!\}}{(\delta_i-\delta_j)^{\ell+1}}$$

yields, when substituted into equation (D-16)

$$\begin{aligned} &- \sum_{j=1}^{m} \delta_j G_j^k e^{-\delta_j t} \sum_{\ell=0}^{N} A_{j,\ell} t^{\ell+1}/(\ell+1) \\ &- \sum_{i=1}^{m} \sum_{j=1}^{m} \delta_j (1-\delta_{ij}) G_j^k \sum_{\ell=0}^{N} \ell! A_{i,\ell} e^{-\delta_j t}/(\delta_i-\delta_j)^{\ell+1} \\ &+ \sum_{i=1}^{m} \sum_{j=1}^{m} \delta_j (1-\delta_{ij}) G_j^k \sum_{\ell=0}^{N} e^{-\delta_i t} C_{i,j,\ell}(t) A_{i,\ell} \end{aligned} \tag{D-18}$$

Upon interchanging the dummy indices, the last term in equation (D-18) becomes:

$$\sum_{j=1}^{m} \sum_{i=1}^{m} \delta_i (1-\delta_{ji}) G_i^k e^{-\delta_j t} \sum_{\ell=0}^{N} C_{j,i,\ell}(t) A_{i,\ell} \tag{D-19}$$

Thus, conbining equations (D-15) and (D-19), we obtain the desired, iterative solution for evaluating the multiple convolutions (now with the η terms reinstated):

$$I_{N+1}(t) = (\eta_k \prod_{\ell=0}^{N} \eta_\ell) \{ \sum_{j=1}^{m} (\sum_{i=1}^{m} G_i^k)(\sum_{\ell=0}^{N} A_{j,\ell} t^\ell) e^{-\delta_j t}$$

$$- \sum_{j=1}^{m} \delta_j G_j^k (\sum_{\ell=0}^{N} A_{j,\ell} t^{\ell+1} / \ell+1) e^{-\delta_j t} \tag{D-20}$$

$$- \sum_{j=1}^{m} \delta_j G_j^k (\sum_{i=1}^{m} (1-\delta_{ij}) \sum_{\ell=0}^{N} \frac{A_{i,\ell} \ell!}{(\delta_i - \delta_j)^{\ell+1}}) e^{-\delta_j t}$$

$$+ \sum_{j=1}^{m} (\sum_{i=1}^{m} \delta_i (1-\delta_{ji}) G_i^k \sum_{\ell=0}^{N} \frac{A_{j,\ell}}{(\delta_j - \delta_i)^{\ell+1}} [(\delta_j - \delta_i)^\ell t^\ell + \ldots + \ell!]) e^{-\delta_j t} \}$$

Appendix E. Evaluation of Repeated Load Integrals

To integrate the expression for $I^1_x(t)$, i.e.:

$$I = \int_{x-D/2}^{x+D/2} \sin\left(\frac{2\pi\xi}{D}\right) e^{\delta_j \gamma_\ell \xi} d\xi \tag{E-1}$$

we resort to complex integration:

$$I = \mathrm{Im}\left\{\int_{x-D/2}^{x+D/2} e^{z\xi} d\xi\right\} = \mathrm{Im}\left\{ \frac{e^{z\xi}}{z}\Big|_{x-D/2}^{x+D/2}\right\} \tag{E-2}$$

where $z = \delta_j \gamma_\ell + i\frac{2\pi}{D}$; $i^2 = -1$; $\qquad z = \rho e^{i\theta}$; $\tan\theta = \frac{2\pi}{D\delta_j \gamma_\ell}$

$$\rho^2 = \delta_j^2 \gamma_\ell^2 + \frac{4\pi^2}{D^2}$$

Hence:

$$I = \mathrm{Im}\left\{e^{\delta_j \gamma_\ell \xi}\left(\cos\left(\frac{2\pi\xi}{D}\right) + i\sin\left(\frac{2\pi\xi}{D}\right)\right)\rho^{-1}(\cos\theta - i\sin\theta)\right\}\Bigg|_{x-D/2}^{x+D/2} \tag{E-3}$$

or, upon evaluating the angle products:

$$I = e^{\delta_j \gamma_\ell \xi} \rho^{-1} \sin\left(\frac{2\pi\xi}{D} - \theta\right)\Bigg|_{x-D/2}^{x+D/2} \tag{E-4}$$

Since the load is assumed to be centered at time x:

$$x = (N + 1/2)D \quad \text{for some } N$$

and equation (E-4) becomes:

$$I = e^{\delta_j\gamma_\ell x}\{e^{\delta_j\gamma_\ell D/2}\rho^{-1}(-\sin\sigma) - e^{-\delta_j\gamma_\ell D/2}\rho^{-1}(-\sin\sigma)\} \tag{E-5}$$

hence:

$$I = -2e^{\delta_j\gamma_\ell x}\rho^{-1}\sin\sigma\sinh(\delta_j\gamma_\ell D/2) \tag{E-6}$$

But, we also have:

$$2\sin\sigma\,\rho^{-1} = \frac{D}{\pi}\,\frac{1}{\left(1 + \left(\frac{\delta_j\gamma_\ell D}{2\pi}\right)^2\right)} \tag{E-7}$$

and so, we finally arrive at:

$$I = \frac{-D}{\pi}\left\{\frac{e^{\delta_j\gamma_\ell x}\sinh(\delta_j\gamma_\ell D/2)}{\left(1 + \left(\frac{\delta_j\gamma_\ell D}{2\pi}\right)^2\right)}\right\} \tag{E-8}$$

This is the value of the integral in $I_x^1(t)$.

To integrate the expression in $I_x^2(t)$, i.e.:

$$I = \int_x^{x+D/2} \sin\left(\frac{2\pi\xi}{D}\right)e^{(\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell)\xi}\,d\xi \tag{E-9}$$

we observe that the integrand is of the same form as in (E-1), but now with $\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell$ instead of just $\delta_j\gamma_\ell$.

Hence, equation (E-4) becomes:

$$I = e^{(\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell)\xi}\rho^{-1}\sin(\frac{2\pi\xi}{D} - \sigma)\Big|_{x}^{x+D/2} \qquad \text{(E-10)}$$

$$= e^{(\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell)x}\{e^{(\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell)D/2}\rho^{-1}\sin\sigma - \rho^{-1}(\sin(-\sigma))\}$$

yielding:

$$I = e^{(\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell)(x+D/4)}\{\frac{\frac{D}{\pi}\cosh((\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell)D/4)}{(1 + \frac{(\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell)^2 D^2}{4\pi^2})}\} \qquad \text{(E-11)}$$

which is the desired result for $I_x^2(t)$.

Appendix F. Evaluation of Repeated Load Expectations and Integrations

Defining $u = \frac{1}{2}\delta_j \gamma_\ell D_\ell$ $\quad y = \pi^2 + u^2$ equation (2-71) becomes:

$$B_{\ell,j} = \frac{-\pi^2 \eta A_\ell G_j \sinh u}{y} \tag{F-1}$$

and:

$$E[B_{\ell,j}] \simeq B_{\ell,j}\Big|_{\underline{M}} + \frac{1}{2} \frac{\partial^2 B_{\ell,j}}{\partial D_\ell^2}\Big|_{\underline{M}} \sigma^2_{D_\ell} + \frac{\partial^2 B_{\ell,j}}{\partial A_\ell \partial D_\ell}\Big|_{\underline{M}} \mathrm{Cov}(A_\ell, D_\ell) \tag{F-2}$$

where:

$$\frac{\partial^2 B_{\ell,j}}{\partial D_\ell^2} = \left(\frac{\delta_j \gamma_\ell}{2}\right)^2 \frac{\partial^2 B_{\ell,j}}{\partial u^2} \; ; \qquad \underline{M} = (\overline{\eta}, \overline{A}_\ell, \overline{D}_\ell) \tag{F-3}$$

$$\frac{\partial B_{\ell,j}}{\partial u} = \frac{\pi^2 \eta G_j A_\ell}{y} \left\{ \cosh u - \frac{2u}{y} \sinh u \right\} \tag{F-4}$$

$$\frac{\partial^2 B_{\ell,j}}{\partial u^2} = \frac{\pi^2 \eta G_j A_\ell}{y} \left\{ \left(1 - \frac{2}{y} + \frac{8u^2}{y^3}\right) \sinh u - \frac{4u}{y} \cosh u \right\} \tag{F-5}$$

Hence, we have:

$$E[B_{\ell,j}] \simeq \frac{\pi^2\overline{\eta}G_j}{\overline{y}} \{(\sinh \overline{u})[\overline{A}_\ell + \frac{\overline{A}_\ell \overline{u}^2}{2D_\ell^2} \sigma^2_{D_\ell} (1 - \frac{2}{\overline{y}} + \frac{8\overline{u}^2}{\overline{y}^3})$$

$$- \frac{2\overline{u}^2}{\overline{D}_\ell \overline{y}} \mathrm{Cov}(A_\ell, D_\ell)] + (\cosh \overline{u})[\frac{\overline{u}}{\overline{D}_\ell} \mathrm{Cov}(A_\ell, D_\ell) - \frac{2\overline{u}^3}{\overline{y}\overline{D}_\ell} \overline{A}_\ell \sigma^2_{D_\ell}]\} \qquad \text{(F-6)}$$

Defining $u = \frac{1}{4}(\delta_j\gamma_\ell - k_r\lambda_\ell\Gamma_\ell)D_\ell$, $y = \pi^2 + 4u^2$, $Q = G_j\mu_r\pi^2 e^{-k_r\lambda_\ell\Gamma_\ell D/4}$

equation (2-72) becomes:

$$C_{\ell,j,r} = \frac{A_\ell \eta Q \cosh u}{y} \qquad \text{(F-7)}$$

$$E[C_{\ell,j,r}] \simeq C_{\ell,j,r}\Big|_{\underline{M}} + \frac{1}{2} \frac{\partial^2 C_{\ell,j,r}}{\partial D_\ell^2}\Big|_{\underline{M}} \sigma^2_{D_\ell} + \frac{\partial^2 C_{\ell,j,r}}{\partial A_\ell \partial D_\ell}\Big|_{\underline{M}} \mathrm{Cov}(A_\ell, D_\ell,) \qquad \text{(F-8)}$$

and:

$$\frac{\partial C_{\ell,j,r}}{\partial D_\ell} = \frac{A_\ell \eta \dot{Q} \cosh u}{y} + \frac{A_\ell \eta Q \dot{u} \sinh u}{y} - \frac{A_\ell \eta Q \dot{y} \cosh u}{y^2} \qquad \text{(F-9)}$$

where:

$$\dot{u} = \frac{1}{4}(\delta_j\gamma_\ell - k_r\Gamma_\ell\lambda_\ell); \quad \dot{y} = 8u\dot{u}; \quad \dot{Q} = \frac{-k_r\lambda_\ell\Gamma_\ell}{4} Q \qquad \text{(F-10)}$$

$$\frac{\partial^2 C_{\ell,j,r}}{\partial D_\ell^2} = \frac{A_\ell \eta}{y} [(\cosh u)\{\ddot{Q} - \frac{2\dot{Q}\dot{y}}{y} + Q\dot{u}^2 - \frac{Q\ddot{y}}{y} + \frac{2Q\dot{y}^2}{y^2}\} \qquad \text{(F-11)}$$

$$+ (\sinh u)\{2\dot{Q}\dot{u} - 2Q\dot{u}\dot{y}/y\}]$$

where:

$$\ddot{y} = 8\dot{u}^2; \quad \ddot{Q} = \left(\frac{k_r \lambda_\ell \Gamma_\ell}{4}\right)^2 Q \tag{F-12}$$

$$\frac{\partial^2 C_{\ell,j,r}}{\partial A_\ell \partial D_\ell} = \frac{1}{A_\ell} \frac{\partial C_{\ell,j,r}}{\partial D_\ell} \tag{F-13}$$

which yield the desired solution when substituted into (F-8).

Again, utilizing equation (2-1):

$$E[B_{\ell,j}B_{\ell,k}] \simeq \{B_{\ell,j}B_{\ell,k}\Big|_{\underline{M}} + \frac{1}{2}\frac{\partial^2[F]}{\partial D_\ell^2}\Big|_{\underline{M}}\sigma^2_{D_\ell} + \frac{1}{2}\frac{\partial^2[F]}{\partial A_\ell^2}\Big|_{\underline{M}}\sigma^2_{A_\ell}$$

$$+ \frac{\partial^2[F]}{\partial A_\ell \partial D_\ell}\Big|_{\underline{M}} \mathrm{Cov}(A_\ell, D_\ell)\}(1 + \sigma^2_\eta) \tag{F-14}$$

where:

$$F = B_{\ell,j}B_{\ell,k} = \frac{\sinh u_j \sinh u_k}{(\pi^2+u_j^2)(\pi^2+u_k^2)}\eta^2 G_j G_k A_\ell^2; \quad u_j = \frac{1}{2}\delta_j \gamma_\ell D_\ell; \; u_k = \frac{1}{2}\delta_k \gamma_\ell D_\ell \tag{F-15}$$

$$\frac{\partial F}{\partial D_\ell} = \frac{\partial B_{\ell,j}}{\partial D_\ell} B_{\ell,k} + \frac{\partial B_{\ell,k}}{\partial D_\ell} B_{\ell,j} \tag{F-16}$$

$$\frac{\partial^2 F}{\partial D_\ell^2} = \frac{\partial^2 B_{\ell,j}}{\partial D_\ell^2} B_{\ell,k} + 2\frac{\partial B_{\ell,j}}{\partial D_\ell}\frac{\partial B_{\ell,k}}{\partial D_\ell} + \frac{\partial^2 B_{\ell,k}}{\partial D_\ell^2} B_{\ell,j} \tag{F-17}$$

$$\frac{\partial^2 F}{\partial A_\ell^2} = \frac{2F}{A_\ell^2} \tag{F-18}$$

$$\frac{\partial^2 F}{\partial A_\ell \partial D_\ell} = \frac{2}{A_\ell} \frac{\partial F}{\partial D_\ell} \qquad \text{(F-19)}$$

where the partials of B are as before. Hence, equations (F-17), (F-18) and (F-19) yield the desired solution when substituted into equation (F-14).

To evaluate the mean value of $C_{\ell,j,r}C_{\ell,k,q}$ we again expand equation (2-1):

$$E[C_{\ell,j,r}C_{\ell,k,q}] \simeq \{C_{\ell,j,r}C_{\ell,k,q}|_{\underline{M}} + \frac{1}{2}\frac{\partial^2[F]}{\partial D_\ell^2}\Big|_{\underline{M}}\sigma^2_{D_\ell} + \frac{1}{2}\frac{\partial^2[F]}{\partial A_\ell^2}\Big|_{\underline{M}}\sigma^2_{A_\ell}$$

$$+ \frac{\partial^2[F]}{\partial A_\ell \partial D_\ell}\Big|_{\underline{M}} \mathrm{Cov}[A_\ell, D_\ell]\}(1 + \sigma^2_\eta) \qquad \text{(F-20)}$$

where:

$$F = C_{\ell,j,r}C_{\ell,k,q}; \quad \frac{\partial F}{\partial D_\ell} = \frac{\partial C_{\ell,j,r}}{\partial D_\ell}C_{\ell,k,q} + C_{\ell,j,r}\frac{\partial C_{\ell,k,q}}{\partial D} \qquad \text{(F-21)}$$

$$\frac{\partial^2 F}{\partial D_\ell^2} = \frac{\partial^2 C_{\ell,j,r}}{\partial D_\ell^2} C_{\ell,k,q} + 2\frac{\partial C_{\ell,j,r}}{\partial D_\ell}\frac{\partial C_{\ell,k,q}}{\partial D_\ell} + C_{\ell,j,r}\frac{\partial^2 C_{\ell,k,q}}{\partial D_\ell^2} \qquad \text{(F-22)}$$

$$\frac{\partial^2 F}{\partial A_\ell^2} = \frac{2F}{A_\ell^2} \qquad \text{(F-23)}$$

$$\frac{\partial^2 F}{\partial A_\ell \partial D_\ell} = \frac{2}{A_\ell}\frac{\partial F}{\partial D_\ell} \qquad \text{(F-24)}$$

where the partials of C are as before. Hence, equations (F-21) and (F-24) yield the desired solution when substituted into (F-20).

To evaluate the mean value of $B_{\ell,j}C_{\ell,k,r}$ we proceed as before, with:

$$F = B_{\ell,j}C_{\ell,k,r} \tag{F-25}$$

$$E[B_{\ell,j}C_{\ell,k,r}] = E[F] \simeq \{F|_{\underline{M}} + \frac{1}{2}\frac{\partial^2 F}{\partial D_\ell^2}\Big|_{\underline{M}}\sigma^2_{D_\ell} + \frac{1}{2}\frac{\partial^2 F}{\partial A_\ell^2}\Big|_{\underline{M}}\sigma^2_{A_\ell}$$
$$+ \frac{\partial^2 F}{\partial A_\ell \partial D_\ell}\Big|_{\underline{M}}\mathrm{Cov}(A_\ell,D_\ell)\}\ (1 + \sigma^2_\eta) \tag{F-26}$$

where:

$$\frac{\partial F}{\partial D_\ell} = \frac{\partial B_{\ell,j}}{\partial D_\ell} C_{\ell,k,r} + B_{\ell,j}\frac{\partial C_{\ell,k,r}}{\partial D_\ell} \tag{F-27}$$

$$\frac{\partial^2 F}{\partial D_\ell^2} = \frac{\partial^2 B_{\ell,j}}{\partial D_\ell^2} C_{\ell,k,r} + 2\frac{\partial B_{\ell,j}}{\partial D_\ell}\frac{\partial C_{\ell,k,r}}{\partial D_\ell} + B_{\ell,j}\frac{\partial^2 C_{\ell,k,r}}{\partial D_\ell^2} \tag{F-28}$$

$$\frac{\partial^2 F}{\partial A_\ell^2} = \frac{2F}{A_\ell^2} \tag{F-29}$$

$$\frac{\partial^2 F}{\partial A_\ell \partial D_\ell} = \frac{2}{A_\ell}\frac{\partial F}{\partial D_\ell} \tag{F-30}$$

where the partials of B and C are as before. Hence, equations (F-27) and (F-30) yield the desired result when substituted back into equation (F-26).

The repeated load integrations may be evaluated as follows. Upon inspection, we note that all the terms in equations (2-75) and (2-76) may be written in the form:

$$\int_{t_{\ell-1}}^{t_\ell} C\, e^{-at^*(\phi)\Big|_x^t}\, e^{-bx^*}\, dx \qquad \text{(F-31)}$$

and, upon expanding out t* and x*, we obtain:

$$C e^{-at^*(\phi)\Big|_{t_\ell}^t}\, e^{-at_\ell\gamma_\ell}\, e^{-bx^*(\phi)\Big|_0^{t_{\ell-1}}}\, e^{b\lambda_\ell\Gamma_\ell t_{\ell-1}} \int_{t_{\ell-1}}^{t_\ell} e^{(a\gamma_\ell-b\lambda_\ell\Gamma_\ell)x}\, dx \qquad \text{(F-32)}$$

which is easily integrated for non-zero exponent to be:

$$C e^{-at^*(\phi)\Big|_{t_\ell}^t}\, e^{-bx^*(\phi)\Big|_0^{t_{\ell-1}}}\, e^{-at_\ell\gamma_\ell}\, e^{b\lambda_\ell\Gamma_\ell t_{\ell-1}} \left\{ \frac{e^{(a\gamma_\ell-b\lambda_\ell\Gamma_\ell)t_\ell} - e^{(a\gamma_\ell-b\lambda_\ell\Gamma_\ell)t_{\ell-1}}}{(a\gamma_\ell-b\lambda_\ell\Gamma_\ell)} \right\} \qquad \text{(F-33)}$$

or, after combining terms:

$$C e^{-at^*(\phi)\Big|_{t_{\ell-1}}^t}\, e^{-bx^*(\phi)\Big|_0^{t_{\ell-1}}}\, \frac{\{e^{(a\gamma_\ell-b\lambda_\ell\Gamma_\ell)(t_\ell-t_{\ell-1})}-1\}}{a\gamma_\ell-b\lambda_\ell\Gamma_\ell} \qquad \text{(F-34)}$$

which is the desired evaluation. Substitution of the various quantities for C, a, and b then yield the solution to (2-75).

Appendix G. Determination of Slope Variance

The following analysis has been used to obtain the spatial autocorrelation function of the surface deflection, from which the slope variance is fairly easily obtained. The spatial autocorrelation function of the system's response $\tilde{R}_t(x)$ can be expressed as:

$$\tilde{R}_t(x) = E_x[\, R(t_1,x_1)\; R(t_1,x_2)\,] \tag{G-1}$$

where $E_x[\;]$ signifies that the expectation operates on the space variable only, and $R(t,x_j)$ is the response of the system at time t and location x_j. In this case the response is the vertical deflection at the surface of the pavement.

Since the roughness of the road, at least initially, is totally attributed to the spatial variation in the materials' properties, the only space variables in the above expression will be η, which in this case can be written as η_1 and η_2 to be related to points x_1 and x_2, respectively. We then have:

$$\tilde{R}_t(x) = E_x[\eta_1 R(t)\,|_{\eta_1=1}\eta_2 R(t)\,|_{\eta_2=1}\,] \tag{G-2}$$

or:

$$\tilde{R}_t(x) = E_x[\eta_1\eta_2]R^2(t)\,|_{\eta=1} \tag{G-3}$$

Since:

$$E_x[\eta_1\eta_2] = \mathrm{Cov}[\eta_1\eta_2] + E[\eta_1]E[\eta_2] \tag{G-4}$$

and:

$$E[\eta_1] = E[\eta_2] = 1.0 \tag{G-5}$$

$$\mathrm{Cov}[\eta_1\eta_2] = \rho_{x_1x_2}\sigma_{\eta_1}\sigma_{\eta_2} \tag{G-6}$$

and assuming that the spatial correlation of materials is a homoscedastic process (i.e., the variance has no spatial or temporal variation), equation (G-3) above becomes:

$$\tilde{R}_t(x) = (\rho_{x_1x_2}\sigma_\eta^2 + 1)R^2(t)\big|_{\eta=1} \tag{G-7}$$

Now $\rho_{x_1x_2}$ is the spatial correlation coefficient for the surface deflection, which is related to the properties of the materials by solving for the step response of the system. A model which seems to best fit the pavement system (5) can be expressed as:

$$\rho_{x_1x_2} = 1 - B(1 - e^{-|x_1-x_2|^2/C^2}) \tag{G-8}$$

where $|x_1-x_2|$ is the absolute distance between the points x_1 and x_2. B and C are materials properties. Substituting $x = |x_1-x_2|$ and substituting (G-8) into (G-7) yields:

$$\tilde{R}_t(x) = (1 + \sigma_\eta^2(1 - B(1 - e^{-x^2/c^2})))R^2(t)\Big|_{\eta=1} \tag{G-9}$$

We now wish to show that the slope variance is equal to the negative of the second derivative of $\tilde{R}_t(x)$ evaluated at $x = 0$. From the definition of variance, the slope variance is the slope squared minus the mean slope, which is zero. Thus:

$$SV(t) = \lim_{\varepsilon \to 0} (R(t,\varepsilon) - R(t,0))^2/\varepsilon^2 \tag{G-10}$$

or, expanding the above:

$$SV(t) = \lim_{\varepsilon \to 0} \{\tilde{R}_t(\varepsilon,\varepsilon) - 2\tilde{R}_t(\varepsilon,0) + \tilde{R}_t(0,0)\}/\varepsilon^2 \tag{G-11}$$

or:

$$SV(t) = -\frac{\partial^2 \tilde{R}_t(x)}{\partial x^2}\Big|_{x=0} \tag{G-12}$$

which was what we wished to show. Upon substituting equation (G-9) into (G-12) and doing the derivatives, we obtain:

$$SV(t) = \frac{2B}{c^2}\sigma_\eta^2 (R(t)\big|_{\eta=1})^2 \tag{G-13}$$

Taking moments of the above, yields, approximately:

$$E[SV(t)] \simeq \frac{2B}{C^2}\sigma_\eta^2(\mathrm{Var}[R(t)|_{\eta=1}] + (E[R(t)|_{\eta=1})^2) \qquad \text{(G-14)}$$

$$\mathrm{Var}[SV(t)] \simeq (\frac{4B}{C^2}\sigma_\eta^2 E[R(t)|_{\eta=1}])^2 \mathrm{Var}[R(t)|_{\eta=1}] \qquad \text{(G-15)}$$

which were the desired results.

Appendix H. Probabilistic Miner's Law

Miner's Law can be expressed as:

$$D(t) = \sum_{k=1}^{M} n_k \left(\frac{1}{N_k} \right) \qquad \text{(H-1)}$$

$$E[D(t)] = E\left[\sum_{k=1}^{M} n_k \left(\frac{1}{N_k} \right) \right] = \sum_{k=1}^{M} E\left[n_k \left(\frac{1}{N_k} \right) \right] \qquad \text{(H-2)}$$

where n_k represents the number of loads in the kth period and N_k is the number of loads to failure in the kth period, and n_k, N_k are independent random variables. Thus $D(t)$ is the probability density function of cracking damage at time t, and the probability of $D(t)$ being greater than 1 is the probability of cracking. Since n_k is independent of N_k, and thus of $1/N_k$ we have:

$$E[D(t)] = \sum_{k=1}^{M} E[n_k] E\left[\frac{1}{N_k} \right] = \sum_{k=1}^{M} \bar{n}_k \left(\overline{\frac{1}{N_k}} \right) \qquad \text{(H-3)}$$

$$Var[D(t)] = \sum_{k=1}^{M} \left\{ \left(\overline{\frac{1}{N_k}} \right)^2 \sigma^2_{n_k} + \bar{n}_k^{\,2} \sigma^2_{1/N_k} \right\} \qquad \text{(H-4)}$$

where $\overline{n}_k$ and $\sigma^2_{n_k}$ are found from the average rate of traffic loads occurring in a Poisson fashion.

$E[\frac{1}{N_k}]$ and Var $[\frac{1}{N_k}]$ are found from the fatigue relation:

$$\frac{1}{N_k} = c(T_k)(\Delta\varepsilon_k)^{a(T_k)} \tag{H-5}$$

where $\Delta\varepsilon_k$ is the strain amplitude derived in Appendix I, and c(T) and a(T) are material properties. Again, using approximation equations,assuming that only a(T) and c(T) are correlated, we get the second order approximation for the expected value:

$$E[\frac{1}{N_k}] \simeq \overline{c}(\overline{\Delta\varepsilon_k})^{\overline{a}} + \frac{1}{2}[\frac{\partial^2(1/N_k)}{\partial c^2}|_{\underline{M}}\sigma_c^2 + \frac{\partial^2(1/N_k)}{\partial a^2}|_{\underline{M}}\sigma_a^2 + \frac{\partial^2(1/N_k)}{\partial(\Delta\varepsilon_k)^2}|_{\underline{M}}\sigma^2_{\Delta\varepsilon_k}]$$
$$+ \frac{\partial^2(1/N_k)}{\partial c\partial a}|_{\underline{M}} \mathrm{Cov}[c,a] \tag{H-6}$$

where:

$$\frac{\partial^2(1/N_k)}{\partial c^2}|_{\underline{M}} = 0$$

$$\frac{\partial^2(1/N_k)}{\partial a^2}|_{\underline{M}} = \overline{c}(\ln\overline{\Delta\varepsilon_k})^2(\overline{\Delta\varepsilon_k})^{\overline{a}} \tag{H-7}$$

$$\frac{\partial^2(1/N_k)}{\partial(\Delta\varepsilon_k)^2}|_{\underline{M}} = \overline{c}\ \overline{a}(\overline{a}-1)(\overline{\Delta\varepsilon_k})^{\overline{a}-2}$$

$$\mathrm{Cov}[c,a] = \rho_{c,a}\sigma_c\sigma_a$$

where $\rho_{c,a}$ is the correlation coefficient of c and a, and σ_c and σ_a are the standard deviations of c and a respectively.

The first order approximation for the variance is given by:

$$\mathrm{Var}[1/N_k] \simeq \left(\frac{\partial(1/N_k)}{\partial c}\Big|_{\underline{M}}\right)^2 \sigma_c^2 + \left(\frac{\partial(1/N_k)}{\partial a}\Big|_{\underline{M}}\right)^2 \sigma_a^2 + \left(\frac{\partial(1/N_k)}{\partial \Delta\varepsilon_k}\Big|_{\underline{M}}\right)^2 \sigma^2_{\Delta\varepsilon_k}$$

$$+ 2\,\frac{\partial(1/N_k)}{\partial c}\Big|_{\underline{M}}\,\frac{\partial(1/N_k)}{\partial a}\Big|_{\underline{M}}\mathrm{Cov}(c,a) \tag{H-8}$$

where;

$$\frac{\partial(1/N_k)}{\partial c}\Big|_{\underline{M}} = (\overline{\Delta\varepsilon_k})^{\overline{a}}$$

$$\frac{\partial(1/N_k)}{\partial a}\Big|_{\underline{M}} = \overline{c}(\ln\overline{\Delta\varepsilon_k})(\overline{\Delta\varepsilon_k})^{\overline{a}} \tag{H-9}$$

$$\frac{\partial(1/N_k)}{\partial \Delta\varepsilon_k}\Big|_{\underline{M}} = \overline{c}\;\overline{a}(\overline{\Delta\varepsilon_k})^{\overline{a}}$$

Back substituting equations (H-9) and (H-7) into (H-8) and (H-6), respectively, we obtain the expected value and variance of $1/N_k$, which can then be substituted into equations (H-3) and (H-4), respectively, to yield the expected value and variance of the damage function D(t). By further assuming a gaussian distribution for D(t), we can compute the probability of D(t) being greater than unity, which yields the statistical fraction of cracked surface to total surface.

Appendix I. General Response at Peak Loading

To evaluate the system response for any stress-strain-deflection component at the peak of a haversine loading of amplitude A and duration D, we utilize the Boltzmann superposition method described in Chapter II.3, and obtain:

$$\Delta\varepsilon_{\ell} = \int_{-D/2}^{0} - \frac{A\pi}{D} \sin(\frac{2\pi\xi}{D})\eta \sum_{j=1}^{n} \overline{G}_j e^{\delta_j \gamma_{\ell} \xi} d\xi \qquad \text{(I-1)}$$

or equivalently:

$$\Delta\varepsilon_{\ell} = - \frac{A\pi\eta}{D} \sum_{j=1}^{n} \{\overline{G}_j \int_{-D/2}^{0} \sin(\frac{2\pi\xi}{D}) e^{\delta_j \gamma_{\ell} \xi} d\xi\} \qquad \text{(I-2)}$$

where $\Delta\varepsilon_{\ell}$ has been chosen as the response component signifying radial strain for use in Chapter III.3. From Appendix E, we have for the integral in equation (I-2):

$$I = \rho^{-1}(\sin(-\theta) - e^{-\delta_j \gamma_{\ell} D/2} \sin(-\pi - \theta)) = -\rho^{-1}\sin\theta(1 + e^{-\delta_j \gamma_{\ell} D/2})$$

$$= - \frac{D}{2\pi} \frac{(1 + e^{-\delta_j \gamma_{\ell} D/2})}{(1 + (\frac{\delta_j \gamma_{\ell} D}{2\pi})^2)} \qquad \text{(I-3)}$$

Hence:

$$\Delta\varepsilon_{\ell} = \frac{A\eta}{2} \pi^2 \sum_{i=1}^{n} \overline{G}_i (1 + e^{-u_{\ell,i}})/(\pi^2 + u_{\ell,i}^2) \qquad \text{(I-4)}$$

where:

$$u_{\ell,i} = \gamma_\ell \delta_i D/2$$

The mean and variance of $\Delta\varepsilon_\ell$ may be obtained by utilizing equations (2-2), (2-3), assuming that η, D, and A are uncorrelated:

$$E[\Delta\varepsilon_\ell] \simeq \Delta\varepsilon_\ell(\overline{A},\overline{\eta},\overline{D}) + \frac{1}{2}\{\frac{\partial^2\Delta\varepsilon_\ell}{\partial A^2}|_{\underline{M}}\sigma_A^2 + \frac{\partial^2\Delta\varepsilon_\ell}{\partial\eta^2}|_{\underline{M}}\sigma_\eta^2 + \frac{\partial^2\Delta\varepsilon_\ell}{\partial D^2}|_{\underline{M}}\sigma_D^2\} \quad \text{(I-5)}$$

$$\mathrm{Var}[\Delta\varepsilon_\ell] \simeq (\frac{\partial\Delta\varepsilon_\ell}{\partial A}|_{\underline{M}})^2\sigma_A^2 + (\frac{\partial\Delta\varepsilon_\ell}{\partial D}|_{\underline{M}})^2\sigma_D^2 + (\frac{\partial\Delta\varepsilon_\ell}{\partial\eta}|_{\underline{M}})^2\sigma_\eta^2 \quad \text{(I-6)}$$

where $\underline{M}$ is the point $(\overline{A},\overline{\eta},\overline{D})$. Hence we obtain:

$$E[\Delta\varepsilon_\ell] \simeq \frac{1}{2}\pi^2\overline{A\eta}\sum_{i=1}^{n}\overline{G}_i(1 + e^{-\overline{u}_{\ell,i}})/(\pi^2 + \overline{u}_{\ell,i}^2)$$
$$+ \frac{1}{4}\pi^2\overline{A\eta}\sigma_D^2\{\sum_{i=1}^{n}\overline{G}_i(\frac{\delta_i\gamma_\ell}{2})^2[e^{-\overline{u}_{\ell,i}} + \frac{(4\overline{u}_{\ell,i}e^{-\overline{u}_{\ell,i}} - 2(1 + e^{-\overline{u}_{\ell,i}}))}{(\pi^2 + \overline{u}_{\ell,i}^2)}$$
$$+ 8\overline{u}_{\ell,i}^2(1 + e^{-\overline{u}_{\ell,i}})/(\pi^2 + \overline{u}_{\ell,i}^2)^2]/(\pi^2 + \overline{u}_{\ell,i}^2)\} \quad \text{(I-7)}$$

$$\mathrm{Var}[\Delta\varepsilon_\ell] \simeq \{\frac{1}{2}\pi^2\sum_{i=1}^{n}\overline{G}_i(1 + e^{-\overline{u}_{\ell,i}})/(\pi^2 + \overline{u}_{\ell,i}^2)\}^2(\overline{\eta}^2\sigma_A^2 + \overline{A}^2\sigma_\eta^2)$$
$$+ \frac{1}{2}\pi^2\overline{A\eta}\sum_{i=1}^{n}\overline{G}_i(\frac{\delta_i\gamma_\ell}{2})(\frac{e^{-\overline{u}_{\ell,i}} + 2\overline{u}_{\ell,i}(1 + e^{-\overline{u}_{\ell,i}})/(\pi^2 + \overline{u}_{\ell,i}^2))}{(\pi^2 + \overline{u}_{\ell,i}^2)}\}^2\sigma_D^2$$

Appendix J. Transition Matrices

If we know the mean and variance of the serviceability at two points in time t_1 and t_2, then we can construct the state transition matrix between those two time points as follows:

Since the behavior of pavement systems may be characterized by Markovian processes (6), a system which is below (or above) the predicted mean will tend to remain below (or above) the mean over time. Further, if we assume the serviceability streams are well behaved, not tending to cross one another, then the serviceability at time t_2 may be related to that at t_1 by:

$$s(t_2) = a(t_1, t_2)\, s(t_1) + b(t_1, t_2) \tag{J-1}$$

Taking the expected values of both sides of (J-1) yields:

$$m_2 = a(t_1, t_2) m_1 + b(t_1, t_2) \tag{J-2}$$

Taking the variance of both sides of (J-1) yields:

$$\sigma_2^2 = a^2(t_1, t_2)\sigma_1^2 \tag{J-3}$$

which, when solved for $a(t_1, t_2)$ and $b(t_1, t_2)$ and substituted into (J-1) yields:

$$s(t_2) = \frac{\sigma_2}{\sigma_1}(s(t_1) - m_1) + m_2 \qquad \text{(J-4)}$$

Then the state transition probabilities from state S_j to state S_i will be given by:

$$P(s(t_2) \varepsilon\ S_i | s(t_1) \varepsilon\ S_j) = P_{ij}(t_1, t_2) =$$

$$\frac{1}{\frac{\sigma_2}{\sigma_1}(S_{j^+} - S_{j^-})} [S_i \cap (\frac{\sigma_2}{\sigma_1}(S_{j^-} - m_1) + m_2, \frac{\sigma_2}{\sigma_1}(S_{j^+} - m_1) + m_2)]$$

where S_k+ and S_k- are the upper and lower bounds of state S_k, and $\cap$ denotes the usual intersection operation.

Appendix K. Cumulative Gaussian Approximation

The following algorithm gives an approximate expression for the cumulative distribution of a standardized gaussian variable x:

$$F_X(x) = 1-\tfrac{1}{2}\left\{\sum_{i=0}^{6} d_i x^i\right\}^{-16} + \varepsilon(x) \qquad x \geq 0 \qquad \text{(K-1)}$$

where: $F_X(-x) = 1 - F_X(x)$

$\varepsilon(x) < 1.5 \cdot 10^{-7}$

$d_0 = 1.0000000000$

$d_1 = 0.0498673470$

$d_2 = 0.0211410061$

$d_3 = 0.0032776263$

$d_4 = 0.0000380036$

$d_5 = 0.0000488906$

$d_6 = 0.0000053830$

Appendix L. Sample Inputs and Outputs

```
*********** INPUT DATA VALUES FOR RUN          1
  SAMPLE TEST RUNS FOR THESIS - - 1-24-75     ARBITRARY DATA USED
TYPE              1
NOPART
THICK1     0.3000E+01
THICK2     0.4000E+01
LOADING    0.1000E+01
RADIUS     0.6000E+01
ZPOINT     0.4000E+01
NRPOINTS          1
RPOINTS
 0.0
NTSTATIC         12
VARCOEF1   0.5000E+00
VARCOEF2   0.5500E+00
VARCOEF3   0.6000E+00
TSTATIC
 0.3000E-01  0.1000E+00  0.2000E+00  0.5000E+00  0.1000E+01  0.2000E+01  0.5000E+01  0.1000E+02  0.1000E+03  0.1000E+04
 0.1000E+05  0.1000E+06
LAYER1
 0.1709E-05  0.4920E-05  0.6600E-05  0.9550E-05  0.1220E-04  0.1500E-04  0.1820E-04  0.2110E-04  0.2840E-04  0.3640E-04
 0.3950E-04  0.4000E-04
LAYER2
 0.2340E-04  0.2340E-04  0.2340E-04  0.2340E-04  0.2340E-04  0.2340E-04  0.2340E-04  0.2340E-04  0.2340E-04  0.2340E-04
 0.2340E-04  0.2340E-04
LAYER3
 0.2060E-03  0.2270E-03  0.2380E-03  0.2540E-03  0.2670E-03  0.2800E-03  0.2960E-03  0.3080E-03  0.3500E-03  0.3920E-03
 0.4330E-03  0.4500E-03
ITYPES         1000
QUALITYO   0.5000E+01
STDEVO     0.3000E+00
REFTEMP    0.7000E+02
NTRANDOM         20
TRANDOM
 0.1000E+01  0.2000E+01  0.3000E+01  0.4000E+01  0.5000E+01  0.6000E+01  0.7000E+01  0.8000E+01  0.9000E+01  0.1000E+02
 0.1100E+02  0.1200E+02  0.1300E+02  0.1400E+02  0.1500E+02  0.1600E+02  0.1700E+02  0.1800E+02  0.1900E+02  0.2000E+02
LAMBDA
 0.1000E+04  0.1100E+04  0.1210E+04  0.1331E+04  0.1464E+04  0.1611E+04  0.1772E+04  0.1949E+04  0.2143E+04  0.2358E+04
 0.2594E+04  0.2853E+04  0.3138E+04  0.3452E+04  0.3797E+04  0.4177E+04  0.4595E+04  0.5054E+04  0.5560E+04  0.6116E+04
NTEMPS            1
TEMPS
 0.7000E+02
BETA       0.1620E+00
GNU
 0.3000E-02  0.1000E-07  0.6000E-05
AMPLITUD   0.8000E+02
VCAMP      0.0
DURATION   0.1000E+00
VCDUR      0.0
ALPHA
 0.4000E+00  0.9650E+00  0.4000E+00
STRNEXP    0.3613E+01
STRNCOEF   0.4662E-06
CORLCOEF   0.1000E+01
CORLEXP    0.1000E+00
```

INITIAL DATA VALUES ARE:

THICKNESS OF FIRST LAYER IS 3.000 THICKNESS OF SECOND LAYER IS 4.000

CREEP FUNCTIONS - - - LAYER 1, LAYER 2, LAYER 3, DELTAS

-0.7168695E-05	0.0	-0.4912046E-04	0.5000000E+01
-0.9443669E-05	0.0	-0.3758748E-04	0.4999998E+00
-0.6708797E-05	0.0	-0.4475471E-04	0.4999999E-01
-0.1017516E-04	0.0	-0.3707409E-04	0.4999999E-02
-0.4795147E-05	0.0	-0.5255640E-04	0.4999998E-03
-0.7115304E-06	0.0	-0.2727285E-04	0.4999999E-04
0.4000000E-04	0.2340000E-04	0.4499999E-03	0.0

COEFFICIENTS OF VARIATION - - LAYER 1, LAYER 2, LAYER 3 0.500E+00 0.550E+00 0.600E+00

RADIAL POSITION = 0.0 VERTICAL POSITION = 0.0 LAYER = 1

VERTICAL DISPLACEMENT

MEAN VALUE	COEFFICIENT OF VARIATION	MEAN AT ZERO VARIANCE	TIME
0.5717911E-03	0.4597548E+00	0.6453102E-03	0.3000000E-01
0.6480983E-03	0.4598718E+00	0.7256083E-03	0.1000000E+00
0.7006535E-03	0.4593319E+00	0.7792914E-03	0.2000000E+00
0.7790595E-03	0.4585002E+00	0.8589160E-03	0.5000000E+00
0.8363121E-03	0.4581184E+00	0.9172980E-03	0.1000000E+01
0.8853776E-03	0.4580566E+00	0.9673743E-03	0.2000000E+01
0.9507223E-03	0.4583611E+00	0.1034873E-02	0.5000000E+01
0.9964609E-03	0.4586702E+00	0.1082915E-02	0.1000000E+02
0.1134007E-02	0.4600059E+00	0.1229020E-02	0.1000000E+03
0.1264268E-02	0.4616402E+00	0.1368986E-02	0.1000000E+04
0.1373912E-02	0.4620118E+00	0.1489365E-02	0.1000000E+05
0.1414058E-02	0.4618971E+00	0.1533971E-02	0.1000000E+06

RESPONSE	DELTA (I)
-0.2164878E-03	0.5000000E+01
-0.1693154E-03	0.4999998E+00
-0.1477078E-03	0.4999999E-01
-0.1258478E-03	0.4999999E-02
-0.1457408E-03	0.4999998E-03
-0.6395206E-04	0.4999999E-04
0.1414058E-02	0.0

RADIAL POSITION = 0.0 VERTICAL POSITION = 3.000 LAYER = 1

RADIAL STRAIN

MEAN VALUE	COEFFICIENT OF VARIATION	MEAN AT ZERO VARIANCE	TIME
0.4476904E-05	0.3993058E+00	0.5426811E-05	0.2000000E-01
0.5379302E-05	0.4679110E+00	0.6536164E-05	0.1000000E+00
0.5705020E-05	0.5417366E+00	0.6867815E-05	0.2000000E+00
0.5687944E-05	0.6772919E+00	0.6695864E-05	0.5000000E+00
0.5437317E-05	0.7948217E+00	0.6259452E-05	0.1000000E+01
0.5135486E-05	0.9147624E+00	0.5752141E-05	0.2000000E+01
0.4628042E-05	0.1103521E+01	0.4945741E-05	0.5000000E+01
0.4247732E-05	0.1253017E+01	0.4370144E-05	0.1000000E+02
0.3092146E-05	0.1876947E+01	0.2645957E-05	0.1000000E+03
0.1903057E-05	0.3196783E+01	0.9421510E-06	0.1000000E+04
0.1244072E-05	0.5019163E+01	0.4201475E-07	0.1000000E+05
0.1083095E-05	0.5816801E+01	-0.1693445E-06	0.1000000E+06

RESPONSE	DELTA (I)
-0.1822151E-05	0.5000000E+01
0.1672624E-05	0.4999998E+00
0.9184587E-06	0.4999999E-01
0.1480308E-05	0.4999999E-02
0.9132200E-06	0.4999998E-03
0.2594898E-06	0.4999999E-04
0.1083095E-05	0.0

0.1000E+01

RUT DEPTH INCHES	VAR RUT DEPTH INCHES**2	SLOPE VARIANCE RADIANS*10**6	VAR SLOPE VARIANCE (RADIANS*10**6)**2	TIME SECONDS
0.27678E+00	0.16193E-01	0.39231E+01	0.88677E+01	0.31558E+08
0.40109E+00	0.34005E-01	0.82384E+01	0.39106E+02	0.63115E+08
0.50355E+00	0.53598E-01	0.12985E+02	0.97154E+02	0.94673E+08
0.59626E+00	0.75150E-01	0.18207E+02	0.19100E+03	0.12623E+09
0.68387E+00	0.98856E-01	0.23950E+02	0.33050E+03	0.15779E+09
0.76882E+00	0.12494E+00	0.30270E+02	0.52794E+03	0.18935E+09
0.85255E+00	0.15363E+00	0.37222E+02	0.79827E+03	0.22090E+09
0.93602E+00	0.18519E+00	0.44868E+02	0.11599E+04	0.25246E+09
0.10200E+01	0.21989E+00	0.53274E+02	0.16353E+04	0.28402E+09
0.11050E+01	0.25808E+00	0.62525E+02	0.22525E+04	0.31558E+09
0.11915E+01	0.30008E+00	0.72701E+02	0.30454E+04	0.34713E+09
0.12799E+01	0.34628E+00	0.83894E+02	0.40552E+04	0.37869E+09
0.13706E+01	0.39709E+00	0.96204E+02	0.53326E+04	0.41025E+09
0.14639E+01	0.45298E+00	0.10975E+03	0.69395E+04	0.44181E+09
0.15601E+01	0.51446E+00	0.12464E+03	0.89511E+04	0.47336E+09
0.16595E+01	0.58209E+00	0.14103E+03	0.11459E+05	0.50492E+09
0.17623E+01	0.65649E+00	0.15905E+03	0.14576E+05	0.53648E+09
0.18689E+01	0.73832E+00	0.17888E+03	0.18436E+05	0.56804E+09
0.19796E+01	0.82835E+00	0.20069E+03	0.23206E+05	0.59959E+09
0.20946E+01	0.92738E+00	0.22468E+03	0.29086E+05	0.63115E+09

TEMPERATURE DEGREES-F	STRAIN INS./IN.	VAR STRAIN (INS./IN.)**2	NFAIL CYCLES	VAR NFAIL CYCLES**2	K1 CYCLES	K2 DIMENSIONLESS	GAMMA DIMENSIONLESS
0.70000E+02	0.37557E-03	0.22491E-07	0.10597E+07	0.27038E+12	0.46620E-06	0.36130E+01	0.10000E+01

DAMAGE INDEX DIMENSIONLESS	VAR DAMAGE INDEX DIMENSIONLESS	AREA CRACKED SQ.YDS./1000SQ.YDS.	TIME SECONDS
0.57821E+00	0.22653E+00	0.18775E+03	0.31558E+08
0.12142E+01	0.50064E+00	0.61897E+03	0.63115E+08
0.19139E+01	0.83230E+00	0.84176E+03	0.94673E+08
0.26835E+01	0.12336E+01	0.93520E+03	0.12623E+09
0.35300E+01	0.17191E+01	0.97317E+03	0.15779E+09
0.44615E+01	0.23071E+01	0.98866E+03	0.18935E+09
0.54860E+01	0.30184E+01	0.99509E+03	0.22090E+09
0.66130E+01	0.38789E+01	0.99781E+03	0.25246E+09
0.78521E+01	0.49192E+01	0.99900E+03	0.28402E+09
0.92155E+01	0.61788E+01	0.99953E+03	0.31558E+09
0.10715E+02	0.77031E+01	0.99977E+03	0.34713E+09
0.12365E+02	0.95469E+01	0.99988E+03	0.37869E+09
0.14179E+02	0.11778E+02	0.99994E+03	0.41025E+09
0.16175E+02	0.14477E+02	0.99997E+03	0.44181E+09
0.18371E+02	0.17743E+02	0.99998E+03	0.47336E+09
0.20786E+02	0.21695E+02	0.99999E+03	0.50492E+09
0.23443E+02	0.26478E+02	0.99999E+03	0.53648E+09
0.26365E+02	0.32265E+02	0.10000E+04	0.56804E+09
0.29580E+02	0.39268E+02	0.10000E+04	0.59959E+09
0.33116E+02	0.47741E+02	0.10000E+04	0.63115E+09

SERVICEABILITY	VARIANCE OF SERVICEABILITY	TIME
0.356449E+01	0.351201E+00	0.100000E+01
0.282802E+01	0.446942E+00	0.200000E+01
0.230377E+01	0.535311E+00	0.300000E+01
0.186319E+01	0.649772E+00	0.400000E+01
0.145801E+01	0.807492E+00	0.500000E+01
0.106568E+01	0.102407E+01	0.600000E+01
0.673918E+00	0.131661E+01	0.700000E+01
0.274667E+00	0.170535E+01	0.800000E+01
-0.137965E+00	0.221454E+01	0.899999E+01
-0.569303E+00	0.287432E+01	0.999999E+01
-0.102376E+01	0.372091E+01	0.110000E+02
-0.150557E+01	0.479841E+01	0.120000E+02
-0.201905E+01	0.616080E+01	0.130000E+02
-0.256868E+01	0.787403E+01	0.140000E+02
-0.315909E+01	0.100184E+02	0.150000E+02
-0.379522E+01	0.126919E+02	0.160000E+02
-0.448237E+01	0.160134E+02	0.170000E+02
-0.522609E+01	0.201274E+02	0.180000E+02
-0.603279E+01	0.252110E+02	0.190000E+02
-0.690891E+01	0.314772E+02	0.200000E+02

STATE1	STATE2	STATE3	STATE4	STATE5	STATE6	STATE7	STATE8	STATE9	STATE10	TIME	RELIABILITY
0.79872	0.16326	0.03467	0.00322	0.00013	0.00000	0.0	0.0	0.0	0.00000	0.0	1.00000
0.02283	0.04090	0.08316	0.13558	0.17724	0.18579	0.15616	0.10524	0.05687	0.03623	0.1000E+01	0.96377
0.00203	0.00506	0.01398	0.03244	0.06326	0.10364	0.14264	0.16492	0.16019	0.31183	0.2000E+01	0.68817
0.00042	0.00113	0.00348	0.00925	0.02124	0.04215	0.07227	0.10711	0.13721	0.60573	0.3000E+01	0.39427
0.00017	0.00044	0.00136	0.00367	0.00880	0.01872	0.03528	0.05896	0.08736	0.78524	0.4000E+01	0.21476
0.00013	0.00028	0.00079	0.00203	0.00473	0.00999	0.01916	0.03334	0.05266	0.87689	0.5000E+01	0.12311
0.00014	0.00025	0.00063	0.00148	0.00322	0.00647	0.01205	0.02077	0.03316	0.92181	0.6000E+01	0.07819
0.00019	0.00028	0.00063	0.00133	0.00264	0.00493	0.00870	0.01445	0.02261	0.94424	0.7000E+01	0.05576
0.00031	0.00036	0.00071	0.00135	0.00244	0.00423	0.00701	0.01107	0.01671	0.95582	0.8000E+01	0.04418
0.00051	0.00047	0.00085	0.00146	0.00244	0.00393	0.00610	0.00915	0.01324	0.96186	0.9000E+01	0.03814
0.00085	0.00063	0.00103	0.00164	0.00254	0.00382	0.00559	0.00797	0.01105	0.96488	0.1000E+02	0.03512
0.00138	0.00082	0.00125	0.00185	0.00268	0.00381	0.00529	0.00720	0.00959	0.96613	0.1100E+02	0.03387
0.00215	0.00105	0.00148	0.00207	0.00284	0.00384	0.00509	0.00666	0.00855	0.96627	0.1200E+02	0.03373
0.00320	0.00128	0.00172	0.00229	0.00300	0.00388	0.00495	0.00624	0.00777	0.96567	0.1300E+02	0.03433
0.00456	0.00152	0.00196	0.00249	0.00313	0.00391	0.00483	0.00590	0.00714	0.96457	0.1400E+02	0.03543
0.00624	0.00175	0.00216	0.00266	0.00324	0.00392	0.00471	0.00560	0.00662	0.96311	0.1500E+02	0.03689
0.00824	0.00195	0.00234	0.00280	0.00332	0.00391	0.00458	0.00533	0.00616	0.96139	0.1600E+02	0.03861
0.01053	0.00212	0.00249	0.00289	0.00335	0.00387	0.00444	0.00506	0.00575	0.95950	0.1700E+02	0.04050
0.01309	0.00226	0.00259	0.00296	0.00336	0.00380	0.00428	0.00481	0.00538	0.95748	0.1800E+02	0.04252
0.01589	0.00237	0.00266	0.00298	0.00333	0.00371	0.00412	0.00456	0.00503	0.95538	0.1900E+02	0.04462
0.01886	0.00243	0.00269	0.00297	0.00327	0.00360	0.00394	0.00431	0.00470	0.95323	0.2000E+02	0.04677

EXPECTED LIFETIME IS AT LEAST 0.3499E+01 YEARS

STATE UPPER BOUNDS - 0.1E+51 4.74889 4.46778 4.18667 3.90555 3.62444 3.34333 3.06222 2.78111 2.50000

STATE LOWER BOUNDS - 4.74889 4.46778 4.18667 3.90555 3.62444 3.34333 3.06222 2.78111 2.50000 -0.1E+51

SERVICEABILITY FAILURE LEVEL IS 2.50000

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********** INPUT DATA VALUES FOR RUN          2
  SAMPLE TEST RUNS FOR THESIS - - 1-24-75     ARBITRARY DATA USED
TYPE               5
NOPART
INTRATE    0.1000E+00
BUDGET
 0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04
 0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04  0.3000E+04
COSTSQYD  0.1000E+01
BENEFITS
 0.8000E-01  0.8000E-01  0.7000E-01  0.6000E-01  0.5000E-01  0.4000E-01  0.3000E-01  0.2000E-01  0.1000E-01  0.0
TERMINAL
 0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
LAMBDA
 0.1000E+04  0.1100E+04  0.1210E+04  0.1331E+04  0.1464E+04  0.1611E+04  0.1772E+04  0.1949E+04  0.2143E+04  0.2358E+04
 0.2594E+04  0.2853E+04  0.3138E+04  0.3452E+04  0.3797E+04  0.4177E+04  0.4595E+04  0.5054E+04  0.5560E+04  0.6116E+04
STRATEGY           1
 0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
 0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
STRATEGY           2
 0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00
 0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00
STRATEGY           3
 0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00
 0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00
STRATEGY           4
 0.0         0.0         0.0         0.0         0.1000E+01  0.0         0.0         0.0         0.0         0.1000E+01
 0.0         0.0         0.0         0.0         0.1000E+01  0.0         0.0         0.0         0.0         0.1000E+01
STRATEGY           5
 0.5000E-01  0.1000E+00  0.1500E+00  0.2000E+00  0.2500E+00  0.3000E+00  0.3500E+00  0.4000E+00  0.4500E+00  0.5000E+00
 0.6000E+00  0.7000E+00  0.8000E+00  0.9000E+00  0.1000E+01  0.1000E+01  0.1000E+01  0.1000E+01  0.1000E+01  0.1000E+01
STRATEGY           6
 0.3000E+00  0.5000E+00  0.1000E+01  0.3000E+00  0.5000E+00  0.1000E+01  0.3000E+00  0.5000E+00  0.1000E+01  0.3000E+00
 0.5000E+00  0.1000E+01  0.3000E+00  0.5000E+00  0.1000E+01  0.3000E+00  0.5000E+00  0.1000E+01  0.3000E+00  0.5000E+00
STRATEGY           7
 0.0         0.0         0.1000E+01  0.0         0.0         0.1000E+01  0.0         0.0         0.1000E+01  0.0
 0.0         0.1000E+01  0.0         0.0         0.1000E+01  0.0         0.0         0.1000E+01  0.0         0.0
STRATEGY           8
 0.2000E+00  0.4000E+00  0.6000E+00  0.8000E+00  0.1000E+01  0.2000E+00  0.4000E+00  0.6000E+00  0.8000E+00  0.1000E+01
 0.2000E+00  0.4000E+00  0.6000E+00  0.8000E+00  0.1000E+01  0.2000E+00  0.4000E+00  0.6000E+00  0.8000E+00  0.1000E+01
STRATEGY           9
 0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00
 0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00
STRATEGY          10
 0.0         0.5000E+00  0.1000E+01  0.0         0.5000E+00  0.1000E+01  0.0         0.5000E+00  0.1000E+01  0.0
 0.5000E+00  0.1000E+01  0.0         0.5000E+00  0.1000E+01  0.0         0.5000E+00  0.1000E+01  0.0         0.5000E+00
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MAINTENANCE DECISION TIME = 1.00 YEARS MAINTENANCE DECISION = 3 AGING FACTOR = 0.1583E-05

MAINTENANCE EFFORT	COST $/MILE	AREA CRACKED BEFORE	AREA CRACKED AFTER	RUT DEPTH BEFORE	RUT DEPTH AFTER	SLOPE VARIANCE BEFORE	SLOPE VARIANCE AFTER	SERVICEABILITY BEFORE	SERVICEABILITY AFTER	RELIABILITY BEFORE	RELIABILITY AFTER	TIME YEARS
0.50	600.81	187.75	93.88	0.2768	0.2508	3.9231	3.3375	3.56	3.74	0.9638	0.9801	1.00
0.50	1645.24	591.88	282.78	0.3914	0.2807	7.8461	4.5915	2.88	3.48	0.7169	0.9213	2.00
0.50	1902.80	813.57	359.75	0.4794	0.3069	11.7692	5.7705	2.42	3.29	0.4559	0.8580	3.00
0.50	1833.02	916.08	381.21	0.5536	0.3425	15.6922	7.3004	2.06	3.09	0.2842	0.7765	4.00
0.50	1680.99	962.20	384.55	0.6189	0.3809	19.6152	9.0499	1.76	2.89	0.1843	0.6844	5.00
0.50	1523.51	982.91	383.38	0.6780	0.4180	23.5382	10.8917	1.48	2.70	0.1271	0.5936	6.00
0.50	1379.09	992.24	381.74	0.7323	0.4527	27.4612	12.7588	1.23	2.52	0.0936	0.5111	7.00
0.50	1249.79	996.46	380.54	0.7828	0.4849	31.3842	14.6248	1.00	2.36	0.0732	0.4399	8.00
0.50	1134.00	998.38	379.82	0.8303	0.5150	35.3071	16.4825	0.78	2.21	0.0603	0.3800	9.00
0.50	1029.81	999.26	379.41	0.8752	0.5432	39.2300	18.3323	0.57	2.07	0.0518	0.3303	10.00
0.50	935.66	999.66	379.20	0.9180	0.5699	43.1530	20.1762	0.36	1.94	0.0461	0.2893	11.00
0.50	850.35	999.84	379.08	0.9588	0.5953	47.0758	22.0164	0.16	1.82	0.0421	0.2556	12.00
0.50	772.94	999.93	379.03	0.9979	0.6197	50.9987	23.8544	-0.03	1.69	0.0393	0.2278	13.00
0.50	702.62	999.97	379.00	1.0356	0.6431	54.9214	25.6910	-0.22	1.58	0.0374	0.2049	14.00
0.50	638.72	999.98	378.99	1.0719	0.6657	58.8444	27.5270	-0.40	1.46	0.0360	0.1860	15.00
0.50	580.65	999.99	378.98	1.1071	0.6875	62.7673	29.3625	-0.58	1.35	0.0351	0.1703	16.00
0.50	527.85	1000.00	378.98	1.1412	0.7087	66.6899	31.1980	-0.76	1.25	0.0344	0.1571	17.00
0.50	479.87	1000.00	378.97	1.1743	0.7292	70.6128	33.0333	-0.93	1.14	0.0340	0.1461	18.00
0.50	436.24	1000.00	378.97	1.2064	0.7492	74.5357	34.8684	-1.10	1.04	0.0338	0.1368	19.00
0.50	396.58	1000.00	378.97	1.2378	0.7687	78.4585	36.7035	-1.27	0.94	0.0337	0.1289	20.00

SQ.YDS. PATCHED/1000SQ.YDS. = 7197.199 BENEFITS = 37816.64 COST = 20300.52

ALTERNATIVE	NET BENEFITS $/MILE/LANE	NET COSTS $/MILE/LANE	BENEFITS - COSTS	LIFE ADDED	$/UNIT LIFE ADDED	BUDTET SURPLUS
1	0.0	0.0	0.0	0.0	****************	25540.82
2	18977.57	12581.75	6395.83	2.56	4906.76	12959.07
3	37816.64	20300.52	17516.12	4.95	4103.03	5240.30
4	39003.02	9650.08	29352.94	3.21	3005.24	15890.73
5	48550.77	20393.11	28157.66	9.44	2160.66	5147.71
6	65787.69	25598.18	40189.50	7.66	3343.03	-57.37
7	65248.89	16384.38	48864.52	5.24	3124.11	9156.44
8	64982.77	25662.12	39320.64	7.94	3232.94	-121.31
9	68691.25	29357.43	39333.82	8.79	3340.02	-3816.62
10	65623.00	22589.96	43033.04	7.30	3093.55	2950.86

UNMAINTAINED LIFETIME = 3.87 MEAN DOLLARS/YEAR ADDED = 3367.70 ST. DEV. = 3443.59

MAINTENANCE DECISION TIME = 2.00 YEARS MAINTENANCE DECISION = 2 AGING FACTOR = 0.1583E-05

MAINTENANCE EFFORT	COST $/MILE	AREA CRACKED BEFORE	AREA CRACKED AFTER	RUT DEPTH BEFORE	RUT DEPTH AFTER	SLOPE VARIANCE BEFORE	SLOPE VARIANCE AFTER	SERVICEABILITY BEFORE	SERVICEABILITY AFTER	RELIABILITY BEFORE	RELIABILITY AFTER	TIME YEARS
0.50	600.81	187.75	93.88	0.2768	0.2508	3.9231	3.3375	3.56	3.74	0.9638	0.9801	1.00
0.30	1037.02	618.97	415.89	0.4011	0.3296	8.2384	5.9845	2.83	3.21	0.6882	0.8464	2.00
0.30	1217.75	841.76	537.20	0.5036	0.3876	12.9853	8.4975	2.30	2.87	0.3943	0.6909	3.00
0.30	1191.71	935.20	578.29	0.5963	0.4485	18.2068	11.4850	1.86	2.56	0.2148	0.5299	4.00
0.30	1117.28	973.17	596.39	0.6839	0.5091	23.9505	14.8623	1.46	2.26	0.1231	0.3902	5.00
0.30	1037.67	988.66	609.28	0.7688	0.5681	30.2686	18.5630	1.07	1.97	0.0782	0.2843	6.00
0.30	961.33	995.09	620.90	0.8525	0.6257	37.2184	22.5827	0.67	1.69	0.0558	0.2101	7.00
0.30	889.12	997.81	631.69	0.9360	0.6826	44.8632	26.9512	0.27	1.41	0.0442	0.1600	8.00
0.30	820.90	999.00	641.54	1.0199	0.7395	53.2714	31.7110	-0.14	1.13	0.0381	0.1268	9.00
0.30	756.61	999.52	650.43	1.1049	0.7969	62.5215	36.9100	-0.57	0.84	0.0351	0.1049	10.00
0.30	696.22	999.77	658.36	1.1915	0.8553	72.6967	42.5992	-1.02	0.54	0.0339	0.0904	11.00
0.30	639.68	999.88	665.39	1.2799	0.9149	83.8892	48.8341	-1.51	0.22	0.0337	0.0808	12.00
0.30	586.93	999.94	671.57	1.3706	0.9761	96.2018	55.6759	-2.02	-0.11	0.0343	0.0747	13.00
0.30	537.85	999.97	676.96	1.4639	1.0392	109.7455	63.1910	-2.57	-0.45	0.0354	0.0708	14.00
0.30	492.32	999.98	681.61	1.5601	1.1044	124.6424	71.4529	-3.16	-0.82	0.0369	0.0687	15.00
0.30	450.16	999.99	685.57	1.6595	1.1719	141.0309	80.5440	-3.80	-1.22	0.0386	0.0677	16.00
0.30	411.21	999.99	688.87	1.7623	1.2420	159.0509	90.5495	-4.48	-1.64	0.0405	0.0676	17.00
0.30	375.32	1000.00	691.62	1.8690	1.3150	178.8784	101.5713	-5.23	-2.09	0.0425	0.0682	18.00
0.30	342.29	1000.00	693.83	1.9796	1.3910	200.6888	113.7153	-6.03	-2.58	0.0446	0.0693	19.00
0.30	311.96	1000.00	695.57	2.0946	1.4702	224.6798	127.0965	-6.91	-3.11	0.0468	0.0708	20.00

SQ.YDS. PATCHED/1000SQ.YDS. = 5181.820 BENEFITS = 23167.77 COST = 13873.29

ALTERNATIVE	NET BENEFITS $/MILE/LANE	NET COSTS $/MILE/LANE	BENEFITS - COSTS	LIFE ADDED	$/UNIT LIFE ADDED	BUDTET SURPLUS
1	0.0	0.0	0.0	0.0	***************	25540.82
2	23167.77	13873.29	9294.48	2.00	6946.41	11667.53
3	50554.88	22822.12	27732.75	4.09	5576.19	2718.69
4	102678.25	9688.00	92990.25	3.28	2952.33	15852.82
5	193755.62	22719.44	171036.19	9.02	2519.36	2821.38
6	150639.00	28099.79	122539.19	7.48	3757.48	-2558.97
7	149698.44	16489.02	133209.37	5.38	3067.70	9051.79
8	149917.56	28445.87	121471.69	7.56	3760.71	-2905.05
9	105893.94	32371.03	73522.87	7.83	4132.60	-6830.21
10	148752.37	24115.94	124636.37	6.93	3479.58	1424.87

UNMAINTAINED LIFETIME = 3.50 MEAN DOLLARS/YEAR ADDED = 4021.37 ST. DEV. = 4233.05

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*********** INPUT DATA VALUES FOR RUN          2
  SAMPLE TEST RUNS FOR THESIS - - 1-24-75    ARBITRARY DATA USED
TYPE               5
NOPART
INTRATE   0.1000E+00
BUDGET
 0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04
 0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04  0.7000E+04
COSTSCYD  0.1000E+01
BENEFITS
 0.8000E-01  0.8000E-01  0.7000E-01  0.6000E-01  0.5000E-01  0.4000E-01  0.3000E-01  0.2000E-01  0.1000E-01  0.0
TERMINAL
 0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
LAMBDA
 0.1000E+04  0.1100E+04  0.1210E+04  0.1331E+04  0.1464E+04  0.1611E+04  0.1772E+04  0.1949E+04  0.2143E+04  0.2358E+04
 0.2594E+04  0.2853E+04  0.3138E+04  0.3452E+04  0.3797E+04  0.4177E+04  0.4595E+04  0.5054E+04  0.5560E+04  0.6116E+04
STRATEGY           1
 0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
 0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
STRATEGY           2
 0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00
 0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00  0.3000E+00
STRATEGY           3
 0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00
 0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00  0.5000E+00
STRATEGY           4
 0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00
 0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00  0.7000E+00
```

MAINTENANCE DECISION TIME = 1.00 YEARS MAINTENANCE DECISION = 4 AGING FACTOR = 0.1583E-05

MAINTENANCE	COST	AREA CRACKED		RUT DEPTH		SLOPE VARIANCE		SERVICEABILITY		RELIABILITY		TIME
EFFORT	$/MILE	BEFORE	AFTER	BEFORE	AFTER	BEFORE	AFTER	BEFORE	AFTER	BEFORE	AFTER	YEARS
0.70	841.14	187.75	56.33	0.2768	0.2404	3.9231	3.1159	3.56	3.81	0.9638	0.9851	1.00
0.70	2320.49	591.88	170.93	0.3914	0.2353	7.8461	3.4921	2.88	3.73	0.7169	0.9638	2.00
0.70	2717.82	813.57	220.22	0.4794	0.2331	11.7692	3.8077	2.42	3.67	0.4559	0.9486	3.00
0.70	2642.27	916.08	235.50	0.5536	0.2494	15.6922	4.5403	2.06	3.55	0.2842	0.9193	4.00
0.70	2436.37	962.20	238.87	0.6189	0.2740	19.6152	5.5321	1.76	3.40	0.1843	0.8762	5.00
0.70	2214.33	982.91	238.81	0.6780	0.3002	23.5382	6.6415	1.48	3.25	0.1271	0.8244	6.00
0.70	2007.03	992.24	238.10	0.7323	0.3255	27.4612	7.7907	1.23	3.11	0.0936	0.7688	7.00
0.70	1819.83	996.46	237.48	0.7828	0.3491	31.3842	8.9458	1.00	2.98	0.0732	0.7131	8.00
0.70	1651.54	998.38	237.07	0.8303	0.3710	35.3071	10.0953	0.78	2.86	0.0603	0.6591	9.00
0.70	1499.88	999.26	236.83	0.8752	0.3916	39.2300	11.2375	0.57	2.74	0.0518	0.6080	10.00
0.70	1362.75	999.66	236.69	0.9180	0.4110	43.1530	12.3739	0.36	2.64	0.0461	0.5603	11.00
0.70	1238.49	999.84	236.62	0.9588	0.4294	47.0758	13.5060	0.16	2.54	0.0421	0.5162	12.00
0.70	1125.73	999.93	236.58	0.9979	0.4470	50.9987	14.6356	-0.03	2.44	0.0393	0.4758	13.00
0.70	1023.31	999.97	236.56	1.0356	0.4640	54.9214	15.7637	-0.22	2.35	0.0374	0.4390	14.00
0.70	930.24	999.98	236.55	1.0719	0.4803	58.8444	16.8909	-0.40	2.26	0.0360	0.4057	15.00
0.70	845.66	999.99	236.55	1.1071	0.4960	62.7673	18.0175	-0.58	2.18	0.0351	0.3756	16.00
0.70	768.77	1000.00	236.55	1.1412	0.5113	66.6899	19.1441	-0.76	2.10	0.0344	0.3486	17.00
0.70	698.88	1000.00	236.55	1.1743	0.5261	70.6128	20.2703	-0.93	2.02	0.0340	0.3243	18.00
0.70	635.35	1000.00	236.55	1.2064	0.5406	74.5357	21.3965	-1.10	1.94	0.0338	0.3025	19.00
0.70	577.59	1000.00	236.55	1.2378	0.5546	78.4585	22.5226	-1.27	1.86	0.0337	0.2830	20.00

SQ.YDS. PATCHED/1000SQ.YDS. = 10443.641 BENEFITS = 68691.19 COST = 29357.43

ALTERNATIVE	NET BENEFITS $/MILE/LANE	NET COSTS $/MILE/LANE	BENEFITS - COSTS	LIFE ADDED	$/UNIT LIFE ADDED	BUDTET SURPLUS
1	0.0	0.0	0.0	0.0	***************	59595.30
2	18977.57	12581.75	6395.83	2.56	4906.76	47013.55
3	37816.64	20300.52	17516.12	4.95	4103.03	39294.78
4	68691.19	29357.43	39333.75	8.79	3340.00	30237.87

UNMAINTAINED LIFETIME = 3.87 MEAN DOLLARS/YEAR ADDED = 4116.60 ST. DEV. = 4166.00

MAINTENANCE DECISION TIME = 2.00 YEARS MAINTENANCE DECISION = 4 AGING FACTOR = 0.1583E-05

MAINTENANCE EFFORT	COST $/MILE	AREA CRACKED BEFORE	AREA CRACKED AFTER	RUT DEPTH BEFORE	RUT DEPTH AFTER	SLOPE VARIANCE BEFORE	SLOPE VARIANCE AFTER	SERVICEABILITY BEFORE	SERVICEABILITY AFTER	RELIABILITY BEFORE	RELIABILITY AFTER	TIME YEARS
0.70	841.14	187.75	56.33	0.2768	0.2404	3.9231	3.1159	3.56	3.81	0.9638	0.9851	1.00
0.70	2435.90	618.97	179.43	0.4011	0.2332	8.2384	3.4839	2.83	3.73	0.6882	0.9632	2.00
0.70	2847.19	841.76	230.70	0.5036	0.2325	12.9853	3.8944	2.30	3.66	0.3943	0.9445	3.00
0.70	2782.57	935.20	248.01	0.5963	0.2512	18.2068	4.7809	1.86	3.51	0.2148	0.9081	4.00
0.70	2607.57	973.17	255.65	0.6839	0.2759	23.9505	5.9107	1.46	3.34	0.1231	0.8571	5.00
0.70	2421.39	988.66	261.14	0.7688	0.3004	30.2686	7.1345	1.07	3.18	0.0782	0.7983	6.00
0.70	2243.14	995.09	266.11	0.8525	0.3232	37.2184	8.4058	0.67	3.03	0.0558	0.7363	7.00
0.70	2074.63	997.81	270.73	0.9360	0.3447	44.8632	9.7282	0.27	2.89	0.0442	0.6732	8.00
0.70	1915.45	999.00	274.95	1.0199	0.3656	53.2718	11.1201	-0.14	2.75	0.0381	0.6103	9.00
0.70	1765.43	999.52	278.76	1.1049	0.3862	62.5220	12.6011	-0.57	2.61	0.0351	0.5486	10.00
0.70	1624.51	999.77	282.16	1.1915	0.4070	72.6971	14.1913	-1.02	2.47	0.0339	0.4890	11.00
0.70	1492.59	999.88	285.17	1.2799	0.4283	83.8897	15.9110	-1.51	2.33	0.0337	0.4326	12.00
0.70	1369.50	999.94	287.82	1.3706	0.4501	96.2024	17.7820	-2.02	2.19	0.0343	0.3801	13.00
0.70	1254.99	999.97	290.12	1.4639	0.4729	109.7462	19.8282	-2.57	2.04	0.0354	0.3322	14.00
0.70	1148.74	999.98	292.12	1.5601	0.4967	124.6431	22.0758	-3.16	1.88	0.0369	0.2895	15.00
0.70	1050.37	999.99	293.81	1.6595	0.5218	141.0309	24.5534	-3.80	1.72	0.0386	0.2521	16.00
0.70	959.50	999.99	295.23	1.7623	0.5483	159.0525	27.2925	-4.48	1.55	0.0405	0.2200	17.00
0.70	875.74	1000.00	296.41	1.8690	0.5764	178.8802	30.3261	-5.23	1.37	0.0425	0.1929	18.00
0.70	798.67	1000.00	297.35	1.9796	0.6061	200.6908	33.6914	-6.03	1.17	0.0446	0.1705	19.00
0.70	727.90	1000.00	298.10	2.0946	0.6377	224.6820	37.4257	-6.91	0.97	0.0468	0.1523	20.00

SQ.YDS. PATCHED/1000SQ.YDS. = 12095.395 BENEFITS = 105994.06 COST = 32395.74

ALTERNATIVE	NET BENEFITS $/MILE/LANE	NET COSTS $/MILE/LANE	BENEFITS - COSTS	LIFE ADDED	$/UNIT LIFE ADDED	BUDTET SURPLUS
1	0.0	0.0	0.0	0.0	***************	59595.30
2	23204.66	13883.88	9320.79	2.00	6945.34	45711.42
3	50621.28	22839.36	27781.93	4.10	5577.26	36755.95
4	105994.06	32395.74	73598.31	7.84	4134.55	27199.56

UNMAINTAINED LIFETIME = 3.50 MEAN DOLLARS/YEAR ADDED = 5552.38 ST. DEV. = 5669.75

Appendix M. Computer Programs

Available From

Department of Civil Engineering

School of Engineering

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Cambridge 39, Massachusetts