# Alternative Values for $\operatorname{Sin}(2 B e t a)$ Measured from Electron/ Positron Collisions at Babar 

by

Susama Agarwala
Submitted to the Physics in partial fulfillment of the requirements for the degree of Bachelor of Science in Physics at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2001
(C) Susama Agarwala, MMI. All rights reserved.

The author hereby grants to MIT permission to reproduce and distribute publicly paper and electronic copies of this thesis document in whole or in part.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

AUG 312001 LIBRARIES

Physics
May 24, 2001

Certified by


Professor Richard K. Yamamoto Senior Thesis Supervisor, Department of Physics Thesis Supervisor

Accepted by
Professor David E. Pritchard Senior Thesis Coordinator, Department of Physics

# Alternative Values for Sin(2Beta) Measured from 

# Electron/ Positron Collisions at Babar 

Susama Agarwala

May 24, 2001

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of Bachelors of Science in Physics.


#### Abstract

Babar is measuring the value for $\sin (2 \beta)$ in the unitary triangle of neutral $B_{d}$ mesons produced in $e^{+} e^{-}$collision. This thesis explores a model of the $\Upsilon(4 S)$ resonance created in this collision that is composed of two one-state systems instead of one twostate system. Considering only neutral mesons, I write a Monte Carlo simulation to determine an adjusted value for $\Delta m$ and use this value to fit the data that Babar published. Based on this analysis, I find $\sin (2 \beta)=.75 \pm .27$, about double the value that Babar measures.


Thesis Supervisor: Richard K. Yamamoto
Title: Professor

## Contents

1 Introduction ..... 4
1.1 BaBar Experiment ..... 5
1.2 My Simulation ..... 6
2 Theory and Background ..... 7
2.1 CP Violation ..... 7
2.1.1 The Dirac Equation ..... 8
2.1.2 Charge and Parity Symmetries ..... 9
2.1.3 CKM matrix ..... 10
2.2 The specific case of neutral B-mesons ..... 14
2.2.1 Quantum mechanical treatment of B-meson mixing ..... 14
2.2.2 CP violation view of B-meson mixing ..... 18
3 My simulation ..... 22
3.1 Physics of resonances ..... 22
3.2 Obtaining $\Delta m^{\prime}$ ..... 24
3.2.1 Algorithm for the Monte Carlo ..... 25
3.2.2 Fitting sinusoids ..... 27
3.3 Determining $\sin (2 \beta)$ ..... 31
4 Conclusion ..... 33
A Appendix: Code for the Monte Carlo Simulation ..... 36

## 1 Introduction

The standard model in particle physics allows for CP violation, or charge parity conjugation asymmetry, in decays under the weak force. However, this effect was not initially predicted. It was first discovered in 1964 by Christenson, Cronin, Fitch and Turlay in Kaon decay. [4] Since then, much theoretical and experimental effort has been put forth to understand and explain this phenomenon better.

The currently accepted explanation comes from the quark mixing. In 1973, Kobaybashi and Maskawa developed a unitary representation of quark mixing, by introducing a third quark family, following Cabbibo's earlier theory based on two families. This $3 \times 3$ CKM matrix, named after Cabbibo, Kobaybashi and Maskawa, introduced into the Standard Model a possibility for CP violation to enter.[9] Several elements of this matrix have been measured experimentally.[5] There is ongoing research to measure the others, getting a better understanding of quark interactions under the weak force, and probing origins of CP violation.

The CKM matrix and the theory of CP violation is being tested in another way as well. The unitarity constraint on the CKM matrix can be represented in the complex plane as triangles, whose sides are directly related to the magnitudes of the entries of the matrix, and whose angles are related to the amount of CP asymmetry found in neutral meson decays. The current experimental effort lies in measuring the angles of the triangle for the B meson system. Babar, at SLAC (Stanford Linear Accelerator), is one of several experiments involved in this effort, looking in particular at the decays of $B_{d}^{0}$ and $\bar{B}_{d}^{0}$. Babar has measured $\sin (2 \beta)=.34 \pm .20$, where $\beta$ is one of the angles of a particular unitary triangle.

In this thesis I explore what the consequences are if different quantum mechanical as-
sumptions are made from the traditional ones in describing the $B^{0}$ and its anti-particle. I write a Monte Carlo experiment to simulate the measurement of the B-mesons, and then compare the traditional assumptions with a different set of assumptions about the physical picture immediately after collision. Based on the results from my simulations, I propose other values for $\sin (2 \beta)$.

### 1.1 BaBar Experiment

The Babar experiment produces B-mesons from electron-positron collisions. The center of mass energy of the beams is such that the collisions result in a $\Upsilon(4 S)$ resonance. This resonance decays quickly and predominantly into B , anti-B pairs (about $50 \%$ into $B^{0} \bar{B}^{0}$ pairs and $50 \%$ into $B^{+} B^{-}$pairs). Because the decay times of B-mesons are so small, particles produced nearly at rest in the lab frame would be extremely difficult to differentiate through their decay products, and thereby determine their decay times.

To circumvent this problem, the beam energies at BaBar are asymmetric. The positron beam is $3[\mathrm{Gev}]$ and the electron beam is 9 [Gev]. The resulting $\Upsilon(4 S)$ resonance moves at a significant fraction of the speed of light in the lab frame. This decays into a $B$ and a $\bar{B}$ that are almost at rest in the center of mass frame, but are moving in the laboratory. As a result, there is a measurable distance between the $B$ 's and their production point, as well as between them. This allows us to differentiate between the particles and enables the measurement of decay times as well as decay modes.[3]

Since it is not possible to directly measure neutral particles, the identity of neutral Bmesons must be established from their decay products. Determining the difference in decay
times of the $B$ s is important, since either the $B$ can decay as itself or its anti-particle depending on a time-dependent probability, due to mixing. Being able to relate a particle with its decay mode is important for measuring CP asymmetry.

### 1.2 My Simulation

Under the traditional picture of mixing, it is assumed that the instant one particle from a particle anti-particle pair decays, the other one is its anti-particle. When the $\Upsilon(4 S)$ resonance decays into a neutral B meson pair, the identity of the first decaying particle may be identified from its decay products at time $t_{\text {decay1 }}$. The identity of the other particle is then known to be the anti-particle at time $=t_{\text {decay } 1}$. The period for the oscillating wave function due to mixing is $2 \pi / \Delta m=1.33 \times 10^{-11}$ where $\Delta m=.472 \times 10^{12}$, (based on the world average value for the mass difference).

If this is not what happens in the $\Upsilon(4 S)$ resonance, but instead at the time of $e^{+} e^{-}$ collision a complete $\mathrm{B} /$ anti- B pair is created rather than a number of quarks that later hadronize, then mixing starts at $t_{\text {collision }}$ instead of at $t_{\text {decay } 1}$. In this scenario my simulation shows that the apparent period for mixing increased (i.e. as $\Delta m$ decreases), the apparent value for $\sin 2 \beta$ increases within the error bars of the BaBar result. This thesis contains two simulations that explore this interpretation of the $\Upsilon(4 S)$ resonance. It utilizes Monte Carlo methods for determining the mass oscillation, then compares these to the measured results.

## 2 Theory and Background

This section is divided into two parts. The first gives the background for a general understanding of CP violation. The second section looks specifically at the case of B-mesons, and applying the underlying physics from the first section to relevant areas.

### 2.1 CP Violation

In relativistic quantum mechanics, there are three interesting Lorenz symmetries, charge conjugation (which exchanges particles with its anti-particles), parity conjugation (which changes the sign of spatial coordinates) and time reversal (which changes the sign of the temporal coordinate). Each of these operators act alone, or in combinations on bilinear wave functions. For instance, note that the CP operator maps $\mathbf{x}$ to $-\mathbf{x}$, changes the charge, and then changes the spatial dimensions back again, so the net change is just a change in charge. One important relation stemming from relativistic quantum field theory is the CPT theorem, which states that the operation CPT on any physical system leaves the system invariant. That is, CPT is a universal symmetry for all physical processes.

Moreover, until 1956, C, P and T were separately thought to be universal symmetries. At this time T. D. Lee and C. N. Yang showed that there was not evidence for the conservation of parity in the weak force. Soon thereafter, C. S. Wu and E. Ambler discovered that the beta decay from ${ }^{60} \mathrm{Co}$ violated parity. This interaction also violated the charge operator in such a way that the interaction did not break the combined CP symmetry. For the next few years, the theory evolved to state that while charge and parity are violated separately in weak interactions, the combined symmetry of CP is preserved. However, in 1964 experiments
showed that CP was violated in the weak decay of neutral Kaons.[4]

### 2.1.1 The Dirac Equation

To get an understanding of CP violation, we will briefly examine the Dirac equation. This is the relativistic version of the Schrödinger's equation for fermions. Using $E^{2}=p^{2} c^{2}+m^{2} c^{4}$ and $i \hbar \psi=H \psi$, we can find

$$
\begin{equation*}
i \hbar \frac{d}{d t} \psi=\left(m c^{2} \beta+\alpha^{j} c \frac{\hbar}{i} \partial x_{j}\right) \psi \tag{1}
\end{equation*}
$$

where $\beta$ and $\alpha^{j}$ are unknown matrices. Squaring both sides gives

$$
\begin{equation*}
-\hbar^{2} \frac{d^{2}}{d t^{2}} \psi=\left(m^{2} c^{4} \beta^{2}+\frac{\hbar}{i} m c^{2}\left(\beta \alpha^{j}+\alpha^{j} \beta\right)-\hbar^{2} c^{2} \frac{\alpha^{i} \alpha^{j}+\alpha^{j} \alpha^{i}}{2} \partial x_{j} \partial x_{i}\right) \psi \tag{2}
\end{equation*}
$$

The time dependent part of $\psi$ in the form of a plane wave provides a condition for finding $\alpha$ and $\beta$. In this case, the left hand side of the equation equals $E^{2}$, which dictates $\beta^{2}=$ $1,\left(\alpha^{j}\right)^{2}=1$, and $\alpha^{j}, \alpha^{i}$ anti-commute, as does $\beta \alpha^{i}$. Because the Hamiltonian is always hermitian, we know that $\beta \alpha^{i}$ is also hermitian. From the constraints on $\beta$ and $\alpha$ we see that the eigenvalues of $\beta \alpha^{j}$ have to be $\pm 1$ and that the trace of of $\alpha^{j}=0$. Given these restrictions, the simplest non-trivial matrices for $\beta$ and $\alpha$ can be represented in the form

$$
\beta=\left[\begin{array}{cc}
0 & 1  \tag{3}\\
1 & 0
\end{array}\right] ; \alpha=\left[\begin{array}{cc}
\sigma^{j} & 0 \\
0 & -\sigma^{j}
\end{array}\right]
$$

where the $\sigma^{j}$ 's are the three Pauli matrices. This giver the equation for spin $1 / 2$ fermions. Matrices of Higher dimension are needed for the Dirac equation for fermions with larger spin numbers. Furthermore, define $\gamma^{0}=\beta$ and $\gamma^{j}=\alpha^{j}$. These $\gamma^{\mu}$ 's satisfy the property that $\psi^{\dagger} \gamma^{0} \gamma^{\mu} \psi$ is Lorenz invariant. A general convention is to define $\psi^{\dagger} \gamma^{0}=\bar{\psi}$. This develops a means of dealing with relativistic wave functions. Substituting into equation (1) gives the Dirac equation. [6]

$$
\begin{equation*}
\left(i \gamma^{\mu} \partial_{\mu}-m\right)=0 \tag{4}
\end{equation*}
$$

### 2.1.2 Charge and Parity Symmetries

The Dirac equation gives rise to two types of solutions, $\psi^{+}=u(p) e^{-i p x}$ and $\psi^{-}=v(p) e^{i p x}$, where $u(p)$ and $v(p)$ are positive and negative energy spinors, and $\psi^{+}$and $\psi^{-}$are the positive and negative energy solutions respectively. General convention assigns particles to the positive energy solutions and anti-particles to the negative energy solutions. Since charge conjugation interchanges particles with their anti-particles, the positive and negative energy solutions to the Dirac equations (given below) need to be conjugates of each other.

$$
\begin{equation*}
\left[\left(i \partial_{\mu}+e A_{\mu}\right) \gamma^{\mu}-m\right] C \psi^{+} C=0\left[\left(i \partial_{\mu}-e A_{\mu}\right) \gamma^{\mu}-m\right] \psi^{-}=0 \tag{5}
\end{equation*}
$$

Conjugate the negative energy solution to fix the first sign difference.

$$
\begin{equation*}
\left[\left(i \partial_{\mu}+e A_{\mu}\right)\left(\gamma^{\mu}\right)^{*}+m\right]\left(\psi^{+}\right)^{*}=0 \tag{6}
\end{equation*}
$$

But this introduces another sign error for the mass term. The easiest way to resolve this is to find a unitary matrix $\alpha$, such that $\alpha \gamma^{\mu} \alpha^{-1}=-\gamma^{\mu}$. It turns out that $\alpha=i \gamma^{2}$ satisfies this condition.

$$
\begin{equation*}
C \psi^{+} C=i \gamma^{2} \psi^{*}=i\left(\bar{\psi} \gamma^{2}\right)^{T} \tag{7}
\end{equation*}
$$

This is the charge conjugation relationship. [7]
The parity transform is much more easily defined. The parity operator changes the momentum from $p=\left(p^{0}, \mathbf{p}\right)$ to $p^{\prime}=\left(p^{0},-\mathbf{p}\right)$, so that $p \cdot x=p^{\prime} \cdot(t,-\mathbf{x})$. We need a $4 \times 4$ matrix that enacts these changes. Doing the matrix algebra, we see that $P \psi P=\gamma^{0} \psi \cdot[10]$

Each term in the Lagrangian for the weak interaction in the standard model is hermitian. It can be written as a term plus its hermitian conjugate. Using the definitions of C and P defined above, it is possible to show that each term, up to the coefficients, transforms to its hermitian conjugate. This means that the Lagrangian is invariant under the CP operation. However, if there is a coefficient that has an imaginary component, the entire term (coefficient times bilinear) is no longer taken to its Hermitian conjugate, and the Lagrangian is no longer CP invariant. This is the theoretical basis for CP violation. The coefficients depend on mass terms and coupling terms. These are not well determined.[2]

### 2.1.3 CKM matrix

As of yet, the Cabibbo Kobaybashi Maskawa (CKM) matrix provides the only mechanism for introducing CP violation in the Standard Model. This is a unitary matrix that describes the mixing between different quark flavors. The quarks are paired into families in which
they primarily interact. This means that up interacts with down, charm with strange, and top with bottom. There is also some small amount of cross family interaction that occurs. So while the up quark interacts primarily with the down quark, it also interacts with the strange and bottom quarks, under the weak force. In this manner, the states that the $u$, $c$, and $t$ quarks interact with are linear combinations of the intrinsic $d, s$, and $b$ states, call them $d^{\prime}, s^{\prime}$, and $b^{\prime}$. The coupling between the primed and unprimed quarks is expressed by the CKM matrix:

$$
\left[\begin{array}{c}
d^{\prime}  \tag{8}\\
s^{\prime} \\
b^{\prime}
\end{array}\right]=\left[\begin{array}{ccc}
V_{u d} & V_{u s} & V_{u b} \\
V_{c d} & V_{c s} & V_{c b} \\
V_{t d} & V_{t s} & V_{t b}
\end{array}\right]\left[\begin{array}{c}
d \\
s \\
b
\end{array}\right]
$$

The $V_{i j}$ 's give the mixing constants. $\left|V_{i j}\right|^{2}$ is the probability of the $j^{\text {th }}$ down type quark interacting with the $i^{t h}$ up type quark.[9]

Mathematically speaking, an $N \times N$ unitary matrix has to have ${ }_{N} C_{2}$ Euler angles, or real parameters, and at most $(N-1)(N-2)$ non-zero phases. It is easy to check that this makes sense, without proving the theorem behind this. With three different types of quarks that the mixing occurs between, there are three $\left({ }_{3} C_{2}\right)$ different mixing angles needed to connect all three.

Introducing the phase $\delta$ changes the time evolution factor from $e^{i w t}$ to $e^{i(w t+\delta)}$ for the wave function of the quarks. In the general case, this factor does not have any symmetry under the Toperator. Therefore, assuming validity of the CPT theorem, and that the wave equation without this extra factor is invariant under time reversal, the total wave function


Figure 1: Unitary triangle for b and d quark mixing
(including phase) is not symmetric under the CP operator. The magnitude $\delta$ is the CP violation parameter.[9]

One way of depicting CP violation is by imposing unitarity on the CKM matrix, and thus deriving a "unitary triangle" from this constraint. Constraining $V$ to be unitary means that $V^{\dagger} V=1$. This leads to nine equations, of which, only six are independent.

$$
\begin{align*}
& V_{u d}^{*} V_{u d}+V_{c d}^{*} V_{c d}+V_{t d}^{*} V_{t d}=1 \\
& V_{u s}^{*} V_{u s}+V_{c s}^{*} V_{c s}+V_{t s}^{*} V_{t s}=1 \\
& V_{u b}^{*} V_{u b}+V_{c b}^{*} V_{c b}+V_{t b}^{*} V_{t b}=1 \\
& V_{u s}^{*} V_{u d}+V_{c s}^{*} V_{c d}+V_{t s}^{*} V_{t d}=0 \\
& V_{u b}^{*} V_{u s}+V_{c b}^{*} V_{c s}+V_{t b}^{*} V_{t s}=0 \\
& V_{u b}^{*} V_{u d}+V_{c b}^{*} V_{c d}+V_{t b}^{*} V_{t d}=0 \tag{9}
\end{align*}
$$

The last three of these equations each provide information about the sides and angles of triangles, as shown in figure (1). To see how this is done, consider each addend $V_{i j}^{*} V_{i k}$ to be a vector in the complex plane. The vectors need to add to 0 . With three vectors, this forms
a triangle. It is an interesting fact that the areas of all of these unitary triangle are equal. If the area of the triangles is 0 , there is no CP violation in any interaction. The first three of these equations simply states that the sums of the probabilities of mixing for any down type quark is equal to one. [5]

Figure (2) shows the unitary triangle that involve $B_{d}$ mesons, except this time, it has been normalized. This is the usual convention used for the triangle. From this, we can read off the angles of the triangle with respect to the entries of the CKM matrix. [8]

$$
\begin{equation*}
\alpha=\arg \left(-\frac{V_{t b}^{*} V_{t d}}{V_{u b}^{*} V_{u d}}\right) ; \beta=\arg \left(-\frac{V_{c b}^{*} V_{c d}}{V_{t b}^{*} V_{t d}}\right) ; \gamma=\arg \left(-\frac{V_{u b}^{*} V_{u d}}{V_{c b}^{*} V_{c d}}\right) \tag{10}
\end{equation*}
$$

Each of the first three equations in (9) describe the triangle for a different interaction. For instance, for the neutral $B_{d}$ system, $(b \bar{d})$ and $(d \bar{b})$ the last equation is desired because that relates all the quark interchanges for a particle consisting of a bottom and a down quark/antiquark. For similar reasons, in the $K^{0} / \bar{K}^{0}$ system, which consists of strange and down quarks, the appropriate equation is the fourth one. However, while the areas of all these triangles are the same, because of the sizes of different angles, the easiest triangle to measure is that for $B_{d}^{0}$. Most of the sides of for this have been measured experimentally to various degrees of accuracy. The current efforts are focused on measuring the angles of the triangle to measure the amount of CP violation in $B_{d}$ decays and to test the consistency of the CKM matrix. This is the easiest to study because all the angles of this triangle are not near $0^{\circ}$ or $180^{\circ}$.


Figure 2: Normalized unitary triangle for b and d quark mixing

### 2.2 The specific case of neutral B-mesons

The next two sections discuss CP violation in B-mesons. Because of charge conservations, this is a phenomenon that appears only in the case of neutral mesons. Because mixing is the means through which CP violation was first discovered in the Kaon system, I first introduce the physics behind meson mixing as if there were no CP asymmetries, and then extend the discussion to include them.

### 2.2.1 Quantum mechanical treatment of $B$-meson mixing

In the discussion of neutral B-meson mixing, we only have to work with the strong and weak forces. While the electromagnetic force does play a role in the quark/ anti-quark binding to create the mesons, it has minimal effects on the meson after creation. The problem breaks down into a two state problem, with the particle and anti-particle being the two basis vectors of the strong interaction Hamiltonian.

There are two types of neutral B-mesons: $B_{d}^{0}$, which is a combination of bottom and down quarks, and ${ }^{i} B_{s}^{0}$, which has bottom and strange constituent quarks. The following discussion is valid for either case. Mesons are created under the strong interaction and are eigenstates
of the strong interaction Hamiltonian. For a simplified explanation of their creation, consider the bottom and down quarks coming together under the strong force only. This is only a first order approximation, but the weak force is small compared to the strong force and can be treated as a perturbation. Thus, disregarding it for a moment, and working in the rest frame of the meson, the energy eigenstates are $\mid B^{0}>$ and $\left|\bar{B}^{0}\right\rangle$. They have the same eigenvalues, $H\left|B^{0}>=H\right| \overline{B^{0}}>=m_{o} \mid B>$ because quarks and their anti-quarks have the same mass.

If the strong Hamiltonian commuted with the total Hamiltonian, then the perturbative forces would only add a small $\delta m$ to $m_{0}$. But, as mentioned above, these eigenstates aren't necessarily also eigenstates of the other forces involved. If the operators of the weak and the strong forces do not commute, then the perturbation from the weak force causes a small difference in the mass between $B^{0}$ and $\overline{B^{0}}$, and there are two different mass eigenstates of the combined Hamiltonian, $\left|B_{L}\right\rangle$ and $\left|B_{H}\right\rangle$, that are linear combinations of these original states. The states are related by a unitary matrix. (When CP violation is included in the discussion, this will change.)

$$
\begin{align*}
& \left|B_{L}>=p\right| B^{0}>+q \mid \overline{B^{0}}> \\
& \left|B_{H}>=p\right| B^{0}>-q \mid \overline{B^{0}}> \tag{11}
\end{align*}
$$

Orthonormality of $\mid B_{H}>$ and $\mid B_{L}>$ applies conditions on $q$ and $p$, such that

$$
\begin{align*}
& |p|^{2}+|q|^{2}=1 \\
& |p|^{2}-|q|^{2}=0 \tag{12}
\end{align*}
$$

Without CP violation, the probability amplitudes for the particles and anti- particles are the same, so $p=q=1 / \sqrt{2} .[2]$

To see the time evolution, it is easiest to work in the Heisenberg picture.

$$
\begin{equation*}
\left|\psi(t)>_{s}=\right| \psi(0)>_{s} e^{-i H(0) t} \tag{13}
\end{equation*}
$$

Here, $H$ is the total Hamiltonian, that is, it includes all forces. For a simple system, the Hamiltonian in the rest frame of $B_{H}$ and $B_{L}$ has mass eigenvalues. For example $H \psi_{H}=$ $E \psi_{H}=m_{H}$. However, the Hamiltonian for the strong, and weak forces are far from simple. Most obviously, this is shown by the fact that mesons decay in time. This means that probability is not conserved. This would not be the case for the solutions of the entire Hamiltonian, which contains the interaction terms of the $B$ mesons with the other mesons, photons and leptons they decay into, then probability would be conserved. But the total Hamiltonian is as yet unknown, and these terms lie outside of the two dimensional subspaces of $B^{0}$ and $\bar{B}^{0}$ in Hilbert space so they are not taken into account in the simplified version of the Hamiltonian, and probability is not conserved.

A simple way to account for the decay is to construct an effective Hamiltonian, with a real and an imaginary part.

$$
\begin{equation*}
H_{e f f}=H+i \Gamma \tag{14}
\end{equation*}
$$

Both matrices are hermitian, $H$ is as introduced above, and $\Gamma$ is the Hamiltonian for the decay. In this construction, $H_{e f f}$ is not hermitian, and the eigenvalues have imaginary parts.

In the rest frame of the meson, these are

$$
\begin{gather*}
\left(m_{H}+i / 2 \tau_{H}\right) \\
\left(m_{L}+i / 2 \tau_{L}\right) \tag{15}
\end{gather*}
$$

where $\tau_{H}$ and $\tau_{L}$ are the lifetimes of $B_{H}$ and $B_{L}$ respectively. Therefore, we can rewrite equation (13) as

$$
\begin{align*}
\mid B_{H}(t)> & =\left\lvert\, B_{H}(0)>e^{-\left(\frac{1}{2 \tau_{H}}+i m_{H}\right) t}\right. \\
\mid B_{L}(t)> & =\left\lvert\, B_{L}(0)>e^{-\left(\frac{1}{2 \tau_{L}}+i m_{L}^{2}\right) t}\right. \tag{16}
\end{align*}
$$

The imaginary exponent will yield the oscillations, while the negative real exponent is the term for the decay. To see exactly how this works for the $\mid B^{0}>$ and $\mid \overline{B^{0}}>$ states, rewrite them in terms of $\mid B_{H}>$ and $\left|B_{L}\right\rangle$. Using equation (13) we find the new states to be

$$
\begin{align*}
& \left\lvert\, B^{0}(t)>=\frac{1}{\sqrt{2}}\left(\left|B_{L}(t)>+\right| B_{H}(t)>\right)\right. \\
& \left\lvert\, \bar{B}^{0}(t)>=\frac{1}{\sqrt{2}}\left(\left|B_{L}(t)>-\right| B_{H}(t)>\right)\right. \tag{17}
\end{align*}
$$

These two states can be rewritten in terms of the original $\mid B_{0}(0)>$ and $\mid \bar{B}(0)>$. Substituting
equations (13) and (16) into (17),

$$
\begin{align*}
& \left\lvert\, B^{0}(t)>=\frac{1}{\sqrt{2}}\left(\left|B_{0}(0)>+\right| \bar{B}_{0}(0)>\right) e^{-\left(\frac{1}{2 \tau_{L}}+i m_{L}^{2}\right) t}+\left(\left|B_{0}(0)>-\right| \bar{B}_{0}(0)>\right) e^{-\left(\frac{1}{2 \tau_{H}}+i m_{H}^{2}\right) t}\right. \\
& \left\lvert\, \vec{B}^{0}(t)>=\frac{1}{\sqrt{2}}\left(\left|B_{0}(0)>+\right| \bar{B}_{0}(0)>\right) e^{-\left(\frac{1}{2 \tau_{L}}+i m_{L}^{2}\right) t}+\left(\left|B_{0}(0)>-\right| \bar{B}_{0}(0)>\right) e^{-\left(\frac{1}{2 \tau_{H}}+i m_{H}^{2}\right) t}\right. \tag{18}
\end{align*}
$$

To find the probability of a particle being in the state $\mid B^{0}>$ at time $t$, tag a particle as $\mid B^{0}>$ at time $t=0$. The probability amplitude of the particle being detected in the same state at time $t$ is

$$
\begin{equation*}
<B_{0}(0) \left\lvert\, B_{0}(t)>=\frac{1}{4}\left(e^{-t / \tau_{H}}+e^{-t / \tau_{L}}+e^{-\left(1 / 2 \tau_{H}+1 / 2 \tau_{L}\right) t} 2 \cos \left(\left(m_{L}-m_{H}\right) t\right)\right)\right. \tag{19}
\end{equation*}
$$

Similarly, an expression for the beam becoming a $\bar{B}^{0}$ at time $t$ can be derived. This is

$$
\begin{equation*}
<B_{0}(0) \left\lvert\, \bar{B}_{0}(t)>=\frac{1}{4}\left(e^{-t / \tau_{H}}+e^{-t / \tau_{L}}-e^{-\left(1 / 2 \tau_{H}+1 / 2 \tau_{L}\right) t} 2 \cos \left(\left(m_{L}-m_{H}\right) t\right)\right)\right. \tag{20}
\end{equation*}
$$

This is a general derivation for meson mixing. There is a simplifying assumption that can be made in the case of the B-meson, and that is that the decay times are similar for $B_{H}$ and $B_{L}$. Then the oscillations simplify to a decaying $\cos ^{2}(\Delta m t / 2)$ function. [9] [2]

### 2.2.2 CP violation view of B-meson mixing

If CP were truly a symmetry of the universe, then $\mid B_{H}>$ and $\mid \bar{B}_{L}>$ would be eigenstates of the CP operator, as they are defined above. But the Hamiltonian relating the $B_{0}$ 's and the $\bar{B}_{0}$ 's with these states is not unitary. There is a term of $\mathcal{O}(\epsilon)$ in the off diagonal entries.

The positive and negative eigenstates are

$$
\begin{align*}
& \left\lvert\, B_{\text {phys }}(t)>=\frac{1}{\sqrt{2\left(1+\epsilon^{2}\right)}} e^{-\left(i m_{L}-\Gamma_{L} / 2\right) t}\left((1+\epsilon)\left|B^{0}>+(1-\epsilon)\right| \bar{B}^{0}>\right)\right. \\
& \left\lvert\, \bar{B}_{p h y s}(t)>=\frac{1}{\sqrt{2\left(1+\epsilon^{2}\right)}} e^{-\left(i m_{H}-\Gamma_{H} / 2\right) t}\left((1+\epsilon)\left|B^{0}>-(1-\epsilon)\right| \bar{B}^{0}>\right)\right. \tag{21}
\end{align*}
$$

where $\epsilon$ is the asymmetry factor differentiating $\mid B_{H}>$ and $\mid B_{L}>$ from the eigenstate of the CP operator. These equations are the similar to the definitions of $\mid B^{0}>$ and $\mid \bar{B}^{0}>$ if the definitions $p \equiv \frac{1+\epsilon}{\sqrt{2\left(1+\epsilon^{2}\right)}}$ and $q \equiv \frac{1-\epsilon}{\sqrt{2\left(1+\epsilon^{2}\right)}}$ are made. Using this identity to simply rewrite equation (21)

$$
\begin{align*}
& \mid B_{\text {phys }}>=e^{-(i M / 2+\Gamma) t / 4}\left(p \cos (\Delta m t / 2)\left|B^{0}>+i q \sin (\Delta m t / 2)\right| \bar{B}^{0}>\right) \\
& \mid \bar{B}_{p h y s}>=e^{-(i M / 2+\Gamma) t / 4}\left(i q \sin (\Delta m t / 2)\left|B^{0}>+p \cos (\Delta m t / 2)\right| \bar{B}^{0}>\right) \tag{22}
\end{align*}
$$

Here $M \equiv m_{H}+m_{L}$ and $\Gamma \equiv \Gamma_{H}+\Gamma_{L}$. The state $\mid B_{p h y s}>$ is simply the $\mid B_{0}>$ state evolving in time, written with respect to stationary $\mid B_{0}>$ and $\mid \operatorname{bar} B_{0}>$ states.

The quantity $p / q=((1+\epsilon) /(1-\epsilon)) \neq 1$, can be shown to be,

$$
\begin{equation*}
\frac{q}{p}=-\frac{2 H_{12}^{*}-i \Gamma_{12}^{*}}{\Delta m-\frac{i}{2} \Delta \Gamma} \tag{23}
\end{equation*}
$$

from the definition of $H_{\text {eff }}$, equation (14). To study CP violation, we need to examine the decay of the neutral B mesons into CP eigenstates $\left(f_{C P}^{ \pm}\right)$. [2] The decay amplitudes are
defined

$$
\begin{align*}
& A \equiv<f_{C P}^{ \pm}\left|H_{e f f}\right| B^{0}> \\
& \bar{A} \equiv<f_{C P}^{ \pm}\left|H_{e f f}\right| \bar{B}^{0}> \tag{24}
\end{align*}
$$

This can also be thought of as a sum of decay amplitudes, each times a weak and a strong phase.[8]

$$
\begin{equation*}
A=\sum_{j} A_{j} e^{i \delta_{j}^{s}} e^{i \delta_{j}^{w}} ; A=\sum_{j} \bar{A}_{j} e^{i \delta_{j}^{s}} e^{-i \delta_{j}^{w}} \tag{25}
\end{equation*}
$$

If we also define $\lambda \equiv \frac{q}{p} \frac{\bar{A}}{A}$ we can then define the branching ratios of initially pure B-meson beams as

$$
\begin{align*}
& \Gamma\left(B_{\text {phys }}(t) \rightarrow f_{C P}^{ \pm}\right)=|A|^{2} e^{-\Gamma t}\left[\frac{1+|\lambda|^{2}}{2}+\frac{1-|\lambda|^{2}}{2} \cos (\Delta m t)-\mathcal{I} m \lambda \sin (\Delta m t)\right] \\
& \Gamma\left(\bar{B}_{\text {phys }}(t) \rightarrow f_{C P}^{ \pm}\right)=|A|^{2} e^{-\Gamma t}\left[\frac{1+|\lambda|^{2}}{2}-\frac{1-|\lambda|^{2}}{2} \cos (\Delta m t)+\mathcal{I} m \lambda \sin (\Delta m t)\right] \tag{26}
\end{align*}
$$

The asymmetry factor is just the difference of the branching ratios of the meson and it's anti-particle decaying into one CP eigenstate divided by the total probability of decay into a decay amplitude. Since the asymmetry with respect to either eigenstate should be equal to the other, this amplitude can be written as [2]

$$
\begin{equation*}
\left|a_{f_{c p}}(t)\right| \equiv \frac{\Gamma\left(B_{p h y s}(t) \rightarrow f_{C P}\right)-\left(\Gamma \bar{B}_{p h y s}(t) \rightarrow f_{C P}\right)}{\Gamma\left(B_{p h y s}(t) \rightarrow f_{C P}\right)+\left(\Gamma\left(\bar{B}_{p h y s}\right)(t) \rightarrow f_{C P}\right)} \tag{27}
\end{equation*}
$$

Substituting in equations (26), yields

$$
\begin{equation*}
\left|a_{f_{c p}}(t)\right|=\frac{\left(1-|\lambda|^{2}\right) \cos (\Delta m t)-2 \mathcal{I} m \lambda \sin (\Delta m t)}{1+|\lambda|^{2}} \tag{28}
\end{equation*}
$$

It is possible to write these in terms of the elements of the CKM matrix introduced in section 2.3. This way, we can write the asymmetries in terms of the angles of the unitary triangle. In the case of $\Gamma_{12} \ll H_{12}$ he relative phase $p / q$ can be approximated as a ratio of quark mixings. For $B_{d}$ and $B_{s}$, these are

$$
\begin{equation*}
\left(\frac{q}{p}\right)_{B_{d}}=\frac{V_{t b}^{*} V_{t d}}{V_{t b} V_{t d}^{*}} ;\left(\frac{q}{p}\right)_{B_{s}}=\frac{V_{t b}^{*} V_{t s}}{V_{t b} V_{t s}^{*}} \tag{29}
\end{equation*}
$$

In this limit it is obvious that $|p / q|=1$. Furthermore, if all the amplitudes contributing to a CP eigenstate decay mode $(f)$ have the same weak phase, then by equation $(25)$ we see that $A / \bar{A}=e^{i 2 \delta^{w}}$. The phase $\delta^{w}$ is just the phase between the two decay amplitudes, and therefore can also be thought of as an angle in the unitary triangle. Which angle it corresponds to depends on the details of the specific decay, in particular, the quarks involved in the process. Under this representation, $\lambda$ can be rewritten as $\eta_{f} e^{-i 2 \theta}$ where $\eta_{f}$ is the CP eigenvalue for the decay mode being studied, and $\theta$ ranges from 0 to $\pi .[8]$ Substituting this into equation (28), $a_{f_{c p}}(t)$ simplifies to $-\mathcal{I} m \lambda \sin (\Delta m t)$. Combining this expression with the expression for $\lambda$, we get

$$
\begin{equation*}
a_{f_{C P}}=-\eta_{f} \sin (2 \theta) \sin (\Delta m t) \tag{30}
\end{equation*}
$$

The only unknown in this equation is $\sin (2 \theta)$. These values are being determined experimentally. The particular decay that Babar is studying is $B \rightarrow J / \psi K_{S}$ and $B \rightarrow J / \psi K_{L}$ which leads to a measurement of $\sin (2 \beta)$ with $\eta=1$ and $\eta=-1$ respectively.[1]

## 3 My simulation

As I have mentioned before, the Babar experiment measures $\sin (2 \beta)=.34 \pm .20$. Some data for the asymmetry measurements from decays into $J / \psi K_{S}(\eta=-1)$ and $J / \psi K_{L}(\eta=1)$ suggests that a better fit would come from a function with a longer period. In the following sections, I explore one method of lengthening the period relating the asymmetry to the decays, equation (19). I take these results from a Monte Carlo simulation, and compare them to the data obtained at SLAC.

### 3.1 Physics of resonances

Resonances are short lived objects that are created at certain energies. They can be thought of as intermediate states in collisions. At sufficient energies, the cross-section of collisions show peaks. These are the energies at which resonances are formed. $\Upsilon(4 S)$ is the lowest energy resonance produced by electron-positron collisions that can decay into B-mesons.[4]

Meson resonances can generally be described as complex superpositions of the wave functions of the constituent quarks that they eventually decay into. In this case, when the resonance decays into a particle/ anti-particle meson pair, it behaves as a two state system. The wave function of neither particle is known until one is measured. (Since B-mesons cannot directly be detected, the measurement of the wave function occurs when the meson
decays. Its state immediately before decay is reconstructed from the decay mode. Thus, the objects of interest for this study are the particles from which the wave functions decay, not the decay products.) At the time of the first decay, the other particle is constrained to be the anti-particle of the first. From this time onward, other the state of the remaining particle continues to oscillate between it and its anti-particle as in equation (19). I will refer to this picture as "scenario 1." Since the decay lifetimes of $B_{H}$ and $B_{L}$ are so similar, we can make the approximation $\tau_{H} \approx \tau_{L} \equiv \tau$. This simplifies the mixing equation to

$$
\begin{equation*}
<B^{0}(0) \left\lvert\, B^{0}(t)>=e^{-t / \tau}\left(\cos ^{2}\left(\frac{\Delta m}{2} t\right)\right)\right. \tag{31}
\end{equation*}
$$

Simply substituting the known values for $\tau$ and $\Delta m$ yields the probability that the remaining particle decays as either a $B^{0}$ or $\overline{B^{0}}$.

It is possible that this is actually not how resonances behave, and that somehow the $\Upsilon(4 S)$ resonance is simply the $B^{0}$ and $\bar{B}^{0}$ particles in a bound state as two distinct particle systems, the pure particle state required by equation (19) occurs at the time of the collision, not at the time of the first decay. Suppose that at time $t_{0}$ one particle decays with probability $\sin ^{2}\left(\frac{\Delta m}{2} t_{0}\right)$ as $B^{0}$ and probability $\cos ^{2}\left(\frac{\Delta m}{2} t_{0}\right)$ as $\overline{B^{0}}$. Then at this time the other particle has the same probabilities of decaying into the opposite particles. That is it has a probability of $\sin ^{2}\left(\frac{\Delta m}{2} t_{0}\right)$ to decay as $B^{0}$ and a probability of $\cos ^{2}\left(\frac{\Delta m}{2} t_{0}\right)$ to decay as $\overline{B^{0}}$. Treating these decay probabilities as amplitudes for the mixing equation, we can derive the superposition of states for the second particle at a later time $t_{\text {meas }}$. A little algebra shows $\mathcal{P}\left(\right.$ Particle $\left._{2} \rightarrow \bar{B}^{0}\right)$
is

$$
\begin{equation*}
e^{-t_{\text {meas }} / \tau}\left(\sin ^{2}\left(\frac{\Delta m}{2} t_{0}\right) \cos ^{2}\left(\frac{\Delta m}{2} \Delta t\right)+\cos ^{2}\left(\frac{\Delta m}{2} t_{0}\right) \sin ^{2}\left(\frac{\Delta m}{2} \Delta t\right)\right) \tag{32}
\end{equation*}
$$

Here $\Delta t$ is defined as $t_{\text {meas }}-t_{0}$. The first term in this equation comes from the probability of the particle decaying as $\overline{B^{0}}$ at time $t_{0}$ and the second term comes from the probability of decay as $B^{0}$ at $t_{0}$. Similarly, $\mathcal{P}\left(\right.$ Particle $\left._{2} \rightarrow B^{0}\right)$ is

$$
\begin{equation*}
e^{-t_{\text {meas }} / \tau}\left(\sin ^{2}\left(\frac{\Delta m}{2} t_{0}\right) \sin ^{2}\left(\frac{\Delta m}{2} \Delta t\right)+\cos ^{2}\left(\frac{\Delta m}{2} t_{0}\right) \cos ^{2}\left(\frac{\Delta m}{2} \Delta t\right)\right) \tag{33}
\end{equation*}
$$

I will refer to this picture as "scenario 2" for the rest of this paper. This complicated function can be approximated by a single sinusoid of frequency $\Delta m^{\prime}$ in the case where only one peak is visible. Since the decay lifetime of B-mesons is of the same order as the frequency of oscillations, this is a valid approximation in the case where the statistics aren't good enough to measure secondary peaks.

### 3.2 Obtaining $\Delta m^{\prime}$

To measure the adjusted frequency $\Delta m^{\prime}$, I simulated the process of decay after the formation of the $\Upsilon(4 S)$ via a Monte Carlo process. I ran this process for 500,000 virtual collisions and plotted the results in a histogram. I was then able to extract the lengthened period from these graphs. The next few sections deal with the algorithms involved in this process. The actual code is included in the Appendix.


Figure 3: Histograms of decays into similar and different particles of the two wave functions produced in the $\Upsilon(4 S)$ decay in Scenario 1 and Scenario 2

### 3.2.1 Algorithm for the Monte Carlo

This code is written based on a simplified view of what happens in the accelerator. The two necessary constants, $\Delta m$ and $\tau$ for neutral B-meson oscillations are taken to be $.472 \times$ $10^{12} \hbar s^{-1}$ and $1.548 \times 10^{-12} s$ respectively. As mentioned above, the decay times for $B_{p h y s}$ and $\bar{B}_{\text {phys }}$ are taken to be the same, so the mixing equation for purposes of the program is

$$
\begin{equation*}
\ll B^{0}(0) \left\lvert\, B^{0}(t)>=e^{-t / \tau} \frac{1+\cos (\Delta m t)}{2}=e^{-t / 1.548 \times 10^{-12}} \cos ^{2}\left(\frac{.472 \times 10^{12}}{2} t\right)\right. \tag{34}
\end{equation*}
$$

The simulation of decay is done by a random number generator according to the equation $r=e^{t_{\text {decay }} / \tau}$, where $0 \leq r \leq 1$ is a random number. I am setting $t=0$ to be the time of
collision, and $t_{\text {decay }}$ is the time at which the particles decays as a $B^{0}$ or a $\bar{B}^{0}$. From this, I determine the decay times for the two beams. In a detector, the exact time of the collision is not determinable, but the time difference between the two decays is. Therefore, the variable of interest is "delta_t", the difference between the two decays.

At this point, the program performs two different "experiments." The first "experiment" simulates the collisions under the assumption that the wave functions of the two beams collapse at the time of the first decay (Scenario 1). The second particle has been oscillating for a time $\Delta t$ when the second decay occurs. This value is entered into the mixing equation, and whether or not the particle decays as a $B^{0}$ or a $\bar{B}^{0}$ is determined by generating another random number and comparing it's size to the probability of the particle staying in it's initial state. If the second particles decays as the same state it was in at the time of the first decay, then the two particles decayed as anti-particles, and "delta_t" is written to the file "oppositeexp." Otherwise, the two beams decay as similar particle, and "delta_t" is written to "similarexp."

The second scenario (Scenario 2) is conducted in a similar manner. In this case, since the decay of one particle does not give any information about the decay of the other particle, the mixing functions and the decay modes for each particle are recorded separately, as well as which particle decayed first, and which decayed second. Then, if the first and second particles decay in the same manner, "delta_t" is written to the file "similar," otherwise, it is written to "opposite."

After 500,000 events, the data is organized in the form of histograms are as shown in figure (3). Each file contains about 250,000 points, and each histogram is divided into 300
bins. Figure (3) shows that only the primary peak of any data set is visible, and in the graphs, the peaks from scenario 2, also look wider than the peaks from scenario 1 , as is expected.

This is not exactly how the procedure works at Babar. There, they keep track of all four types of decays, $\left(B^{0} \rightarrow B^{0}\right),\left(\bar{B}^{0} \rightarrow \bar{B}^{0}\right),\left(B^{0} \rightarrow \bar{B}^{0}\right)$ and $\left(\bar{B}^{0} \rightarrow B^{0}\right)$ because they need to be able to identify each particle to measure any CP asymmetry. However, this simulation is not concerned with the CP asymmetry. It only tries to measure the effective $\Delta m$ from the two scenarios. Adding the first two decays together and the last two decays together does not affect this number at all. However, it simplifies the code, so I chose to write it this way.

### 3.2.2 Fitting sinusoids

Getting mathematical programs to fit exponentially decaying sinusoids is not a trivial task. However, it is greatly simplified if one separates the mixing equation into two parts, the decay and the sinusoid. I am only interested in the period of the sinusoid, and the decay term only effects the amplitude, so it can be ignored. Rerunning the program without the decay time proportional to the random number yields graphs that look similar to figure (4). The main difference is that in this figure, the random numbers are in a smaller range, $0 \leq r \leq 2.322 \times 10^{-11}$, so that only a few periods are recorded, instead of several billion. In this range, a full period extends over about one hundred bins, and thus statistics for calculating the period are adequate.

Finding the periods in scenario 1 is easy after the decay is separated out. Only the original mixing functions $\sin \left(\frac{\Delta m t}{2}\right)$ and $\cos \left(\frac{\Delta m t}{2}\right)$ remain. The value for $\Delta m$ is still $.472 \times 1012 \hbar$.


Figure 4: Histograms of the sinusoids for Scenario 1 and Scenario 2 without the decay term present

Finding $\Delta m^{\prime}$ requires further analysis of figure (4). One obvious item of note in these graphs is that they do not show a pure sinusoid. This is because the bins are not registering the decay time, but the difference between the two decay times. The probability of two events being a distance $D$ apart in $N$ bins is

$$
\begin{equation*}
\frac{N-D}{N^{2}} \tag{35}
\end{equation*}
$$

Here, $D$ ranges from 0 to $N-1$. The probabilities in this case are more complicated than just this because there is a sinusoid convoluted into the function. However, the amplitude of the sinusoid still follows a similar type of linear relationship. The equation that describes the histogram is

$$
\begin{align*}
& x_{n}=A\left(t_{n}\right) \sin ^{2}\left(\frac{\Delta m^{\prime} t_{n}}{2}\right) \\
& x_{n}=A\left(t_{n}\right) \cos ^{2}\left(\frac{\Delta m^{\prime} t_{n}}{2}\right) \tag{36}
\end{align*}
$$

(The variables $t_{n}$ and $x_{n}$ are the time and the counts in the $n^{t h}$ bin.) The first equation is for the case of different types of decays and the second is for similar decays.

Some algebra shows that

$$
\begin{align*}
\frac{\Delta m^{\prime} t_{n}}{2} & =\sin ^{-1}\left(\sqrt{\frac{x_{n}}{A\left(t_{n}\right)}}\right) \\
\frac{\Delta m^{\prime} t_{n}}{2} & =\cos ^{-1}\left(\sqrt{\frac{x_{n}}{A\left(t_{n}\right)}}\right) \tag{37}
\end{align*}
$$



Figure 5: Plots to determine $\Delta m^{\prime}$. The first $\approx 100$ points of the two histograms from scenario two are plotted here to determine the period of this oscillation, which is simply the magnitude of the slope.' The slopes of the two lines in each graph should simply be the negatives of each other.
in the two cases. The amplitudes as a function of time can be read off the graphs. They are

$$
\begin{equation*}
A_{d i f}\left(t_{n}\right)=2531-1.072 \times 10^{14} t_{n} ; A_{s i m}\left(t_{n}\right)=2430-1.031 \times 14 t_{n} \tag{38}
\end{equation*}
$$

Substituting this into equation (41), gives the plots in figure (5). The slope of the two lines are negatives of each other. Regressing time against the value of the arcsin gives $\Delta m^{\prime} / 2=1.52 \times 10^{11} \pm .08 \times 10^{11}$ for similar decays and $\Delta m^{\prime} / 2=1.50 \times 10^{11} \pm .09 \times 10^{11}$. Averaging the two and multiplying by two gives $\Delta m^{\prime}=3.02 \times 10^{11} \pm .18 \times 10^{11}$. This gives about 1.56 times the period for the mixing as in scenario 1 .

### 3.3 Determining $\sin (2 \beta)$

Equation (34) shows that $\sin (2 \beta)$ is just an amplitude for the part of the asymmetry that varies with time. Fitting the elongated sinusoid to the data published by the Babar Collaboration in March of 2001 gives a different value for $\sin (2 \beta)$ than what they published. However, trying to fit just the data points to find the amplitude for $\sin \left(\Delta m^{\prime} t / 2\right)$ gives very small values for the amplitude. Therefore, this analysis is done on a more qualitative basis, taking the large error bars of the data points into account. Figure (6) shows the raw Babar data with four different amplitudes drawn in. Figure (7) shows the fit to the raw data that Babar published. The amplitudes included in the former are $.2, .25, .3$, and .35 in that order in the figure. From a purely visual approximation, $.25<\sin (2 \beta)<.3$. [1]

An item of note is that in performing the experiment there is a certain fraction, $w$, of mistagging that causes the raw data to have an amplitude of $\sin (2 \beta)$ that is lower than the actual amount by $(1-2 w)$. [1] While this error is unknown, it linearly effects the value for


Figure 6: Comparisons of different values of $\sin (2 \beta)$ for the positive (left column) and negative (right column) eigenstates of the CP operator. Each graph compares the fit proportional to $\sin (\Delta m t / 2)$ with the original $\sin \left(\Delta m^{\prime} t / 3\right)$ CP asymmetry is plotted against $\Delta t$. The best fit value occurs between the third and fourth graphs, where $.25 \leq \sin (2 \beta) \leq .3$ seems to fit the data the best with the lengthened neriod. [11


Figure 7: The curves for $\sin (2 \beta)$ fitted to the raw data from Babar, as published in March, 2001. Graph a) is for eta $a_{f}=-1$ and b) is for et $a_{f}=+1$. This uses $\Delta m=.472 \times 10^{12}$. [1]
$\sin (2 \beta)$ regardless of the period of the function. Again visually judging from the graph, the apparent $\sin (2 \beta)$ for the Babar data is between .10 and .15 . Comparing these two ranges, I find that for $\Delta m^{\prime}, \sin (2 \beta)=.75 \pm .27$. The error on this is not including any of the error bars that Babar measured.

## 4 Conclusion

The Babar experiment at SLAC collides electron and positron beams together to create Bmesons and anti-B mesons, looking in particular at $B_{d}$ mesons. It studies the asymmetries
of these decays of these particles to determine CP violation. These asymmetries are simply the difference in probability of a particle and its anti particle decaying into the same CP eigenstate, divided by the sum of the probabilities. While the experiment studies the decays of both charged and neutral mesons, I restrict the discussion to neutral mesons in this thesis. The decay $\left(B_{d}^{0} \rightarrow J / \psi K_{S}\right)$ is a positive eigenstate of the CP operator, while the decays $\left(\bar{B}_{d}^{0} \rightarrow J / \psi K_{L}\right)$ are the negative eigenstates. The asymmetry in these decays is proportional to the CP eigenvalue, $\sin (2 \beta)$ and $\sin (\Delta m t)$.

The electron-positron collision at Babar produces a resonance that decays into two Bmesons. I explore the implications of two different interpretations of this resonance on the measurements of $\sin (2 \beta)$. The two scenarios I studied treat the resonance and its decay as a two particle system, as is usually done, and then as two one particle systems. In the former case, the decay of one particle determines the wave function of the other particle, and in effect, it starts oscillating from that point in time, until it decays. In the latter case, both particles have well defined wave functions at the time of collision, and both mix until they decay.

I conducted a Monte Carlo simulation of both scenarios and found that if experimental analysis is done on the second case, treating it like the first, then the period of oscillation would be found to be about 1.56 times larger than the expected period. This causes different values for $\sin (2 \beta)$ as well. Because of the large error bars on the asymmetries, if the period of the the oscillation term changes then the amplitude changes as well. I did not run a simulation on the decays into different Kaon modes. To make an estimate on the value for $\sin (2 \beta)$, I took estimates off of the data that Babar published earlier this year. From this, I
found that a rough estimate for $\sin (2 \beta)$ to be $.75 \pm .27$, compared to Babar's $.34 \pm .20$

## A Appendix: Code for the Monte Carlo Simulation

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
// #include <sys/time.h>
// #include <unistd.h>
#include <math.h>
#define B_LIFE (1.548E-12) /* lifetime and mass as published */
#define MIX_FREQ (.472E12) /* 2000 Review of Particle Physics */
#define B_ (0)
#define BO (1)
#define boolean int
FILE *similar, *similarexp, *opposite, *oppositeexp;
void seed_rand();
double my_rand();
void main() {
long i;
double tnot, tbar, delta_t; /*time variables*/
double mixnot, mixbar, mixexp; /*mixing variables*/
boolean first, second, decaynot, decaybar; /*bookkeeping*/
seed_rand();
similar = fopen("similar", "a");
similarexp = fopen("similarexp", "a");
opposite = fopen("opposite","a");
oppositeexp = fopen("oppositeexp","a");
for (i = 1; i<=500000L; i++)
    {
        tnot = -1*B_LIFE*log(my_rand()); /*time when initially Bnot beam decays*/
        tbar = -1*B_LIFE*log(my_rand()); /*time when initially Bnot beam decays*/
        delta_t = fabs(tnot-tbar); /*time delay between decays*/
/*determine the mixing functions for both scenarios*/
    mixnot = pow(cos(.5*MIX_FREQ*tnot),2); /*Bnot --> Bnot : scenario 2*/
```

```
    mixbar = pow(sin(.5*MIX_FREQ*tbar),2); /*Bbar --> Bbar : scenario 2*/
    mixexp = pow(cos(.5*MIX_FREQ*delta_t),2); /*accepted probability of decaying
    into same probability as initial
    beam : scenario 1*/
/* write scenario 1 data to file */
    if (my_rand() < mixexp) {fprintf(oppositeexp, "%e \n", delta_t);}
        else {fprintf(similarexp, "%e \n", delta_t);}
/*Determine whether or not the particle decayes as Bnot */
    if (my_rand() > mixnot) {decaynot = B_;} /* TRUE if initially Bnot beam */
        else {decaynot = BO;} /* decays as Bnot*/
    if (my_rand() <= mixbar) {decaybar = B_;} /* TRUE if initially Bbar beam */
        else {decaybar = BO;} /* decays as Bnot*/
/*Ordering */
if (tnot < tbar) {first= decaynot; second=decaybar;}
        else {first = decaybar; second = decaynot;} /* determine which beam*/
            /*decayed how when*/
/* write scenario 2 data regards to delta t to file*/
    if ((first && second) || (!first && !second))
                                    {fprintf(similar, "%e \n", delta_t);}
        else {fprintf(opposite, "%e \n", delta_t);}
    }
```

```
fclose(similar);
```

fclose(similar);
fclose(similarexp);
fclose(similarexp);
fclose(opposite);
fclose(opposite);
fclose(oppositeexp);
fclose(oppositeexp);
}
}
double my_rand()
{ return (double) rand()/RAND_MAX;} /*normalizes random number range*/
void seed_rand() { /*seed the random number generator off of time*/
// timeval my_tv;
// timezone my_tz;
time_t my_time;
time(\&my_time);
srandom((unsigned int) my_time);
}

```

\section*{Acknowledgments}

I would like to thank Prof. Yamamoto for supervising this thesis and guiding me through the research. I would also like to thank Michael Spitznagel for helping me gain access to materials from the Babar experiment and for the time he spent discussing ideas with me, and proofreading my work. I also extend my gratitude to Professor Yamamoto, and Professor Williamson and everyone else who aided by reading my thesis and giving me feedback. Finally, thanks to Daniel Berger, Arcell Fraizer and Ira Cooper for emotional and technical support through this process.

\section*{References}
[1] Aubert, B. et al. "Measurement of CP-Violating Asymmetries on \(B^{0}\) Decays to \(C P\) Eigenstates" Physical Review Letters Vol. 86. 19, March, 2001. pp. 211-250.
[2] Boutigny, D. et al. The BaBar Physics Book (SLAC-R-504 http://www.slac.stanford.edu/ubs/confproc/babar504/babar504-001.htm accessed 3/24/01) pp. 1-25
[3] Boutigny, D et al. The BaBar Physics Book (SLAC-R-504 http://www.slac.stanford.edu/ubs/confproc/babar504/babar504-001.htm accessed 3/24/01) pp. 73-80
[4] Coughlan, G. D. and Dodd, J. E. Ideas of Particle Physics: An Introduction for Scientists, \(2^{\text {nd }}\) ed. (Cambridge University Press, New York, NY, 1994)
[5] Kile, Jennifer Erin "Measuring CP Violation in B Meson Decays" (bachelor's thesis, Massachusetts Institute of Technology, 1998)
[6] Negele, John W.Lecture for Quantum Field Theory I (01/03/01)
[7] Negele, John W.Lecture for Quantum Field Theory I (13/03/01)
[8] Nir, Yosef and Quinn, Helen R. "CP Violation in B Physics" Annual Review of Nuclear and Particle Physics Vol. 42, 1992. pp. 211-250.
[9] Perkins, Donald H Introduction to High Energy Physics, \(4^{\text {th }}\) ed. (Cambridge University Press, United Kingdom, 2000)
[10] Peskin, Micheal E. and Schroeder, Daniel V.Introduction to Quantum Field Theory (Addison-Wesley Pub. Co., Redding, Mass, 1995)```

