High Repetition Rate Mode-locked Erbium-Doped Fiber Lasers with Complete Electric Field Control

by

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Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Electrical Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2008

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Abstract

Recent advances in fully-stabilized mode-locked laser systems are enabling many applications, including optical arbitrary waveform generation (OAWG). In this thesis work, we describe the development of high repetition-rate fiber laser-based systems for the realization of these applications at 1550 nm wavelengths.

To realize these systems, frequency comb sources are needed that are compatible with electric field stabilization techniques, are compatible with integrated arrayed-waveguide grating and modulator technology, and have high repetition rates to allow full use of current modulator bandwidths. Erbium-doped fiber lasers are one of the leading options to fill this role. To that end, fundamentally mode-locked stretched-pulse fiber lasers approaching 250 MHz repetition rate and soliton fiber lasers at over 200 MHz repetition rates are presented, and the limitations of repetition rate scaling in fiber lasers are explored. Using the 200 MHz soliton laser and an external Fabry-Perot cavity, a low-noise, repetition rate multiplied 2 GHz source is demonstrated.

Stabilization systems for high repetition rate sources must also be developed. Carrier-envelope offset locking experiments using self-referencing techniques at 200 MHz repetition rate are described. Initial demonstrations towards repetition rate locking to a methane-stabilized HeNe single-frequency standard using difference-frequency generation are presented.

Thesis Supervisor: Erich P. Ippen
Title: Elihu Thomson Professor of Electrical Engineering and Computer Science, Professor of Physics
Acknowledgments

In some ways, my time at MIT passed very slowly; in others, it passed too quickly. I suspect that I am only beginning to understand what I’ve learned that I will carry into my career and life. For all of the benefits and opportunities I’ve received from my graduate experience, I am extremely grateful.

I could not have asked for a better thesis advisor than Prof. Erich P. Ippen. His experimental intuition and technical knowledge is impressive. Always encouraging, he provided careful guidance both by word and example on scientific and non-scientific matters alike. Thank you for a great experience.

Many professors made my time at MIT fulfilling. I am one of the last students to have known the late Prof. Hermann A. Haus, and still benefit from and look up to him as an example of a great scientist and human being. Prof. Franz X. Kärtner not only served on my dissertation committee, but has served as my informal second research advisor. Getting to know him since his arrival at MIT has been a pleasure, and I admire his honest energy and dedication to science. Thank you to Prof. Rajeev J. Ram for serving on my thesis committee. His knowledge spans many topics, his energy seems limitless, and his teaching is superb. Thank you to Prof. Leslie Kolodziejski for collaborations with her group and her kind support. Thank you to Prof. Ronald Parker for serving as my academic advisor, and for skiing suggestions. Thank you to Prof. Terry Orlando and Prof. Vladimir Bulovic for their excellent teaching.

Big thanks to Donna Gale and Dorothy Fleischer for their wonderful support through the years. Thanks to Dave Foss and Sukru Cinar for keeping our data safe and our lives a little more sane.

Many graduate students have had a huge impact on me. Peter Rakich’s influence started on my first day when he introduced me to the RLE copy room. As a graduate student, post-doc, housemate, officemate (twice), teacher, colleague, collaborator, and friend, my experience at MIT would have been vastly different without him. I would be lucky to have half of his creativity and be half as knowledgeable as he is. Matthew Grein, a patient and smart guy with whom it’s fun to hang out, got me on my feet
with research. Juliet Gopinath showed me what hard, careful work means, and took my joking in stride. Jeff Chen is a great friend, collaborator, and duuuuuude who’s always armed with a “quick question”, and from which I have learned much. Marcus Dahlem is a great friend and officemate (also twice), and always has another business or travel idea. His enthusiasm is boundless, greater than 9, 9, 9, 9, 9, 9, 9, 9, 9, 9,..., 9, 10, 10, 10, 10... Wondering what the reference is? Ask Marcus a couple of questions about it, and he’ll answer immediately. Ali Motamedi has shown me hard work (sprint-style), shown me what it means to really love classes, given good advice, and has a sense of humor such that I know what he’s going to say before he says it. Very dangerous for whoever is standing by... Miloš Popović has shown me hard work (marathon-style), has been a great person off of whom to bounce ideas, and a great partner for class projects. I hope we keep in touch as he pursues his undoubtedly successful academic career.

I’ve also worked with a number of fellow students to get them on their feet; it has been a pleasure working with each of them. David Chao is a fun(ny) guy to be around. There is a 25% chance that someone, somewhere, is more laid-back and deliberate than he is. You’d probably find that person “relaxing” in a public swimming pool, or driving (because it’s safer than flying) to Atlantic City to gamble away a van he/she doesn’t own. Jonathan Morse (Jonny-Mo) was the perfect choice to inherit the group ski trip and group server. He taught me a lot about machining, and most importantly, about pessimism as an art form. Once he comes down from his sugar high, I’m sure he’d be happy to teach you about it, too - as soon as he orders a new [fill in new gadget here] online. Michelle Sander’s focus and endurance are incredible; she always seems on the go, heading to another meeting. She’s the true Renaissance Women, with a talent for every endeavor. Including a talent for finding inexplicable troubles in the lab! To the three of you, I hope I have been a positive influence on you, as you have been on me. I have no doubt that each of you will succeed.

Sheila Tandon was a great partner for the many classes we took together, and a great collaborator for semiconductor saturable absorbers. Andrew Benedick has impressed me with his ability to get things done, and my thesis would be less complete were it not for his help with the DFG experiments. Yu Gu deserves another high-five and
something dangly to wear. Jonathan Birge is a wizard with simulations, and has a satirical, sarcastic sense of humor that is unmatched. The scientific-equipment-named take-out restaurant/pub, our outside-the-box approach to world domination through conquering Antarctica, breathable robots, plans for life as we age...I should stop there before I write something I regret. Thanks to my other officemate, Lester, for guarding the office when Marcus and I weren’t there.

Many visiting scientists and post-docs have been a great influence on me and pleasure to know. Hideyuki Sotobayashi is a great friend, and a smart researcher. His impact on me as a scientist was immense; I will always remember to “...just try.” Kazi Abedin was one of the first to get me going in the lab, and is one of the nicest people I know. Omer Ilday is a sharp guy who knows how to get things done, and from whom I learned much. Prof. Helder Crespo is a very interesting guy, has a great sense of humor, and is a great person with which to have a conversation about virtually anything. Gui-rong “Rona” Zhou was a bit of sunshine in the lab - always fun to joke with, but also great when serious discussions about research or career choices were called for. Carole Deiderichs is a lot of fun to be around, an excellent snowboarder, and a fellow lover of all things chocolate and peanut butter - especially when it is in ice cream...I mean, frozen yogurt. I’m sure she’ll make an great professor. Thomas Schibli provided extremely important circuit help near the end of this thesis work, and is a great combination of joker and engineering genius. Shunichi Matsushita is a great ski partner, and was very helpful by providing nonlinear fibers to us once he returned to Japan. Robert Huber has a wide range of skills and talents that I admire, and was very helpful to me. Edilson Falcão-Filho always seems cheerful, and was great company for the midnight subway ride. Peter Fendel, our visiting scientist from Menlo Systems, is a genuine guy, good friend, and has been wonderfully helpful both technically and otherwise. Jeffrey Moses is a nice guy that is always up for some joking or an interesting conversation. (If you are reading this, Jeff, stop now, so this hour will count.) Miguel “Mmm hmmm” Preciado was a lot of fun, and greatly helped along our understanding of harmonic mode-locking and frequency combs, despite being around for only a few months. Dominik Pudo has brought life to the lab, was great company in Vegas, and is the undisputed champion
of the weekly lab happy hour. For him I have two words: Canada and alcohol. (And
no, Dom, they don’t mean the same thing...)

Outside the research environment, many, many friends in Boston have made my re-
cent years wonderful. Joshua McConnell’s impact was incredible. New vocabulary, bik-
ing, games, jokes, Sloan classes, consulting projects, discussions about the future...this
could fill pages. He is a great friend, is the perfect person to talk about all manner of
things, and was a great housemate. I hope he ends up nearby when he returns from
Australia. Rob LePome was also a great housemate, even if only for one year; he’s one
of the friendliest people I know and I’m glad he’ll be moving nearby to Connecticut.
Trent Yang has been a strong force as well, working on multiple projects with me, and
sharing his post-MIT experiences in the consulting and venture capital world. Pedro
Pinto is a cool guy with whom I anticipate undertaking more than one side project in
the near future. And thanks for helping me get LyX (the program I used to write this
thesis) up and running. Tracy Hammond was one of my first friends at MIT, and I had
a great time playing guitar and taking Sloan classes together. Susan Banki has amazing
energy, a social life beyond compare, and is always a pleasure to see. Elaine Pinheiro
is always welcome company when she stops by to wait for Marcus. Matt, Marissa, and
Sage Hummon (collectively known as Matrissage) have been great friends to Loren and
I along the way. Everyone in the Mazur group at Harvard has been great too, giving
me perspective on graduate life outside of MIT.

The many people who have collaborated or otherwise directly helped me with my
research deserve my thanks. Ingmar Hartl from IMRA has multiple times given fiber
amplifier and locking system advice that saved me much time. Prof. Alfred Leitenstorfer
at the University of Konstanzt provided important amplifier advice. Prof. Fiorenzo
Omenetto at Tufts University provided photonic crystal fibers and advice. Many of
the people at OFS were extremely helpful and deserve thanks, including Lars Grüner-
Nielson (OFS Denmark) for the nonlinear fibers, and Jeff Nicholson and our alumnuus,
John Fini, for their assistance.

There are so many others that deserve my thanks for making my time in graduate
school better. Had I the time and space, I could easily write a paragraph about each of

Thank you to those that proofread this thesis.

My friends before graduate school had much influence on me then, and still do today. From the University of Rochester, Aleksandr Radunsky, Noah Lapidus, and Matthew Heckman are all still great friends, and are a great group of people with which to discuss, among other things, where our lives are going. From my childhood, Greg Swithers, Dana Schneider, Donald Brooks, and Mathew Koski are lifelong friends; there is no summarizing their influence.

My parents, Jeffrey Sickler and Sandra Sickler, deserve infinite thanks. Their love and support run unbroken from as far back as I can remember to today. My sister Alisha, her husband, Zach, and my niece, August, also receive my thanks for their love and support. My extended family...uncles, aunts, cousins, grandparents, brother- and parents-in-law...all have played a supportive role, and all deserve thanks.

Finally, to my wife, Loren Cerami: your patience is endless, your support is complete, and your beauty, intelligence, and sense of humor are worth everything. I wouldn’t have gotten this far without you. Thank you.

As is the nature of acknowledgments, I will certainly fail to include everyone that deserves thanks. To those that I’ve accidentally missed, thank you.

I should also acknowledge the Air Force Office of Scientific Research (AFOSR), Office of Naval Research (ONR), and the Defense Advanced Research Projects Agency (DARPA) for my National Defense Science and Engineering Graduate (NDSEG) fellowship, and research funding, particularly through the Optical Arbitrary Waveform Generation (OAWG) grant.

Bernard of Chartres is attributed as saying, "...we are like dwarfs on the shoulders
of giants, so that we can see more than they, and things at a greater distance, not by virtue of any sharpness of sight on our part, or any physical distinction, but because we are carried high and raised up by their giant size.” Because so much of progress is incremental, not revolutionary, I think it’s more correct that we stand on the shoulders of many other dwarfs with the rare giant in the mix. I also think this idea applies to other aspects of life, not just intellectual pursuits. In all those aspects, I am fortunate and grateful for the abundance of tall people in my life.

- Jason W. Sickler, May 23, 2008
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<tr>
<td>5OD</td>
<td>fifth-order dispersion</td>
</tr>
<tr>
<td>AC</td>
<td>autocorrelation trace</td>
</tr>
<tr>
<td>A/D</td>
<td>analog-to-digital</td>
</tr>
<tr>
<td>AOM</td>
<td>acousto-optic modulator</td>
</tr>
<tr>
<td>APD</td>
<td>avalanche photodiode</td>
</tr>
<tr>
<td>APM</td>
<td>additive-pulse mode-locking</td>
</tr>
<tr>
<td>AR</td>
<td>anti-reflection</td>
</tr>
<tr>
<td>ASE</td>
<td>amplified spontaneous emission</td>
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<tr>
<td>AWG</td>
<td>arrayed waveguide grating</td>
</tr>
<tr>
<td>BiEDF</td>
<td>bismuth oxide erbium doped fiber</td>
</tr>
<tr>
<td>BJT</td>
<td>bipolar junction transistor</td>
</tr>
<tr>
<td>CPA</td>
<td>chirped-pulse amplification</td>
</tr>
<tr>
<td>CW</td>
<td>continuous-wave</td>
</tr>
<tr>
<td>DC</td>
<td>direct current</td>
</tr>
<tr>
<td>DCF</td>
<td>dispersion compensating fiber</td>
</tr>
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<td>DFG</td>
<td>difference frequency generation</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>DOF</td>
<td>degree of freedom</td>
</tr>
<tr>
<td>DSF</td>
<td>dispersion shifted fiber</td>
</tr>
<tr>
<td>DUT</td>
<td>device under test</td>
</tr>
<tr>
<td>EDF</td>
<td>erbium-doped fiber</td>
</tr>
<tr>
<td>EDFA</td>
<td>erbium-doped fiber amplifier</td>
</tr>
<tr>
<td>EDFL</td>
<td>erbium-doped fiber laser</td>
</tr>
<tr>
<td>FDTD</td>
<td>finite-difference time-domain</td>
</tr>
<tr>
<td>FML</td>
<td>fundamentally mode-lock</td>
</tr>
<tr>
<td>FOD</td>
<td>fourth-order dispersion</td>
</tr>
<tr>
<td>FPI</td>
<td>Fabry-Perot interferometer</td>
</tr>
<tr>
<td>FR</td>
<td>Faraday rotator</td>
</tr>
<tr>
<td>FSR</td>
<td>free-spectral range</td>
</tr>
<tr>
<td>FTIR</td>
<td>Fourier transform infrared spectroscopy</td>
</tr>
<tr>
<td>FWHM</td>
<td>full-width half-maximum</td>
</tr>
<tr>
<td>GDD</td>
<td>group delay dispersion</td>
</tr>
<tr>
<td>GVD</td>
<td>group velocity dispersion</td>
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<tr>
<td>GTI</td>
<td>Gires-Tournois interferometer</td>
</tr>
<tr>
<td>HML</td>
<td>harmonically mode-lock</td>
</tr>
<tr>
<td>HNLF</td>
<td>highly nonlinear fiber</td>
</tr>
<tr>
<td>HWHM</td>
<td>half-width half-maximum</td>
</tr>
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<td>HWP</td>
<td>half-wave plate</td>
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<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>IAC</td>
<td>interferometric autocorrelation</td>
</tr>
<tr>
<td>IR</td>
<td>infrared</td>
</tr>
<tr>
<td>ISO</td>
<td>isolator</td>
</tr>
<tr>
<td>KLM</td>
<td>Kerr-lens mode-locking</td>
</tr>
<tr>
<td>MFD</td>
<td>mode-field diameter</td>
</tr>
<tr>
<td>MFL</td>
<td>mode-locked fiber laser</td>
</tr>
<tr>
<td>MZI</td>
<td>Mach-Zehnder interferometer</td>
</tr>
<tr>
<td>NA</td>
<td>numerical aperture</td>
</tr>
<tr>
<td>NPE</td>
<td>nonlinear polarization evolution</td>
</tr>
<tr>
<td>NPR</td>
<td>nonlinear polarization rotation</td>
</tr>
<tr>
<td>OAWG</td>
<td>optical arbitrary waveform generation</td>
</tr>
<tr>
<td>OPO</td>
<td>optical parametric oscillator</td>
</tr>
<tr>
<td>P-APM</td>
<td>polarization additive-pulse mode-locking</td>
</tr>
<tr>
<td>PBS</td>
<td>polarization beam splitter</td>
</tr>
<tr>
<td>PCF</td>
<td>photonic crystal fiber</td>
</tr>
<tr>
<td>PI</td>
<td>proportional-integral</td>
</tr>
<tr>
<td>PID</td>
<td>proportional-integral-derivative</td>
</tr>
<tr>
<td>PLL</td>
<td>phase-locked loop</td>
</tr>
<tr>
<td>PM</td>
<td>polarization-maintaining</td>
</tr>
<tr>
<td>PMD</td>
<td>polarization mode dispersion</td>
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<tr>
<td>PMF</td>
<td>polarization-maintaining fiber</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>-------------</td>
<td>--------------------------------------</td>
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<tr>
<td>PPLN</td>
<td>periodically-poled lithium niobate</td>
</tr>
<tr>
<td>QWP</td>
<td>quarter-wave plate</td>
</tr>
<tr>
<td>RIN</td>
<td>relative intensity noise</td>
</tr>
<tr>
<td>RF</td>
<td>radio frequency</td>
</tr>
<tr>
<td>SA</td>
<td>saturable absorber</td>
</tr>
<tr>
<td>SAM</td>
<td>self-amplitude modulation</td>
</tr>
<tr>
<td>SBR</td>
<td>saturable Bragg reflector</td>
</tr>
<tr>
<td>SC</td>
<td>super-continuum</td>
</tr>
<tr>
<td>SEM</td>
<td>scanning electron microscope</td>
</tr>
<tr>
<td>SHG</td>
<td>second-harmonic generation</td>
</tr>
<tr>
<td>SMF</td>
<td>single-mode fiber</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>SPM</td>
<td>self-phase modulation</td>
</tr>
<tr>
<td>SRS</td>
<td>stimulated Raman scattering</td>
</tr>
<tr>
<td>SS</td>
<td>self-steepening</td>
</tr>
<tr>
<td>TE</td>
<td>transverse electric</td>
</tr>
<tr>
<td>TM</td>
<td>transverse magnetic</td>
</tr>
<tr>
<td>TOD</td>
<td>third-order dispersion</td>
</tr>
<tr>
<td>TPA</td>
<td>two photon absorption</td>
</tr>
<tr>
<td>UV</td>
<td>ultraviolet</td>
</tr>
<tr>
<td>VCO</td>
<td>voltage-controlled oscillator</td>
</tr>
<tr>
<td>WDM</td>
<td>wavelength division multiplex</td>
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Chapter 1

Introduction

1.1 Mode-locked Fiber Lasers

The initial work on the laser [1–3], starting in the mid-1950s, and the following creation of the first (solid-state ruby) laser [4] in 1960 sparked decades of work that quickly resulted in a variety of lasers, including the electrical-discharge-pumped gas [5], semiconductor [6], fiber [7], color-center [8], and dye [9] laser. Throughout the 1960s, various pulsed lasers were developed, including actively [10] and passively [11, 12] Q-switched lasers, and actively [13] and passively [14,15] mode-locked lasers. The 1970s saw further advances in the performance of mode-locked lasers, including shorter pulse durations in the picosecond [16,17] and femtosecond regimes [18].

The 1980s ushered in the first erbium-doped fiber laser (EDFL) [19,20], and the soliton laser [21], which solidified the important role that dispersion and nonlinearity can play in intracavity pulse shaping. In the late 1980s and 1990s new Kerr-based saturable absorber (SA) mode-locking mechanisms were implemented, including additive-pulse mode-locking (APM) [22] and Kerr lens mode-locking (KLM) [23]. The former has been implemented in various ways, including the use of the nonlinear Michelson interferometer [24], the so-called “figure eight” coupled-cavity geometry [25], and the polarization additive-pulse mode-locking (P-APM) scheme [26]; many of these implementations were first demonstrated using mode-locked fiber lasers (MFLs). Also during
that time, designs that exert greater control of dispersion and nonlinearity led to new operating regimes, such as the stretched-pulse/dispersion-managed [27] and the self-similar/“similariton” [28] regimes; these were first demonstrated in the context of MFLs as well. Finally, the 2000s have seen a proliferation of photonic crystal fiber (PCF) [29] and microstructured fiber, which enabled further optimization of MFL designs.

Today’s MFLs show excellent characteristics: insensitivity to misalignment, excellent spatial beam quality and noise performance, modest pulse energies, short pulse durations, modestly high repetition rates, good tunability, and relatively compact size and low cost. However, MFLs could be improved in terms of long-term, turn-key reliability and mass-producibility. As a result, MFLs have been very successful in the laboratory environment, but somewhat less so in a commercial environment, where other types of lasers often offer a better set of trade-offs. A primary example is the optical telecommunications industry, which is best served by externally modulated CW semiconductor lasers due to their reliability, small size, and low cost. Generally, improvements in the performance of MFLs would expand their applicability.

Additionally, new applications for MFLs are emerging where the MFL strengths are well-suited. The rapid progress in the generation of stabilized MFL-based frequency combs over recent years has greatly benefited many applications. For example, frequency metrology can use stable frequency combs as a “frequency ruler” to bridge between different electromagnetic frequencies, allowing the accurate measurement of one frequency using a stable reference at another, very different frequency, as illustrated
on the left in Figure 1.1. The temporal properties of these frequency combs allow
the bridging between very different timescales to make highly accurate optical clock-
works, as illustrated on the right in Figure 1.1. Other example applications include
high-speed optical sampling, timing and frequency distribution systems, and optical
arbitrary waveform generation (OAWG), which will be discussed in Chapter 2. At the
time this text was written, MFLs are among the best laser systems for stable frequency
comb generation, and further research will allow them to reach their full potential in
these applications.

As such, the presented work seeks to improve the performance of MFLs, in terms of
repetition rate, power, and pulse duration. These improvements were pursued through
careful design and the use of new devices and components, including saturable Bragg
reflectors (SBRs), bismuth oxide erbium-doped fiber (BiEDF), highly-doped silica gain
fiber, and new pump lasers. This work was done largely in the context of OAWG ap-
lications, and as such, the applicability of these improvements to OAWG applications
was specifically demonstrated.

1.2 Thesis Contributions and Organization

The main research contributions of this thesis are:

- Demonstration of carrier-envelope offset frequency locking at a 200 MHz repetition rate

- Demonstration of difference frequency generation (DFG) for the locking of the
  MFL repetition rate using a methane-stabilized HeNe laser

- Improvement of EDFL fundamental repetition rates up to 234 MHz (stretched-
  pulse regime with an SBR and polarization additive-pulse mode-locking) and 200
  MHz (soliton regime with polarization additive-pulse mode-locking)

- Demonstration of repetition rate multiplication to 2 GHz using an external Fabry-
  Perot interferometer (FPI), which is compatible with carrier-envelope phase slip
  stabilization
• Progress towards optimized high-power erbium-doped fiber amplifiers (EDFAs) for ultra-short pulse amplification using commercially available components

• Exploration of mode-locked erbium-doped bismuth oxide fiber lasers to create ultrashort pulses

The next three chapters focus on OAWG systems. Chapter 2 outlines OAWG, providing the necessary descriptive tools and highlighting the associated challenges. Chapter 3 describes work on increasing the repetition rate of erbium fiber lasers for use in OAWG. Chapter 4 outlines experiments on the frequency comb stabilization of high repetition rate EDFL sources, again focusing on OAWG applications. Chapter 5 describes the development of fiber lasers using erbium-doped bismuth oxide fiber. Chapter 6 concludes the thesis and suggests directions for future research.
Chapter 2

Optical Arbitrary Waveform Generation

The primary focus of the research described in this thesis is OAWG. OAWG is accomplished by constructing the optical time-domain waveform in the frequency domain using the relationship expressed by the Fourier Transform pairs. In an OAWG system, the frequency components that construct the final waveform are individually adjusted in amplitude and phase to create the desired time-domain waveform. As shown in Figure 2.1, an OAWG system consists of:

- an oscillator, stabilized using an optical frequency standard, to produce the frequency comb
- a demultiplexing system to separate the frequencies in the comb
- a system to control the amplitude and phase of each frequency
- a multiplexer to recombine the frequencies

The OAWG research effort seeks to advance the performance of visible and near-infrared (near-IR) OAWGs. Goals include frequency comb line spacing of 10-40 GHz, pulse-to-pulse reconfiguration of the waveform, operation with optical bandwidths approaching 1 THz and encompassing >100 channels, full flexibility in amplitude (0-
100\%) and phase (0-2\pi) control, and a compact, highly integrated system. The target applications are remote chemical identification and ultra-high resolution imaging.

A number of approaches to OAWG have been demonstrated using mode-locked lasers [30–35]. Many of these demonstrations are limited in the following ways:

- The sources are not fully stabilized.

- The (de)multiplexers used, such as ruled gratings, are bulky and thus not amenable to integration, and may be limited in resolution and/or bandwidth.

- The amplitude and phase control, provided by liquid crystal modulators (10-100 ms, 640 channels) or acousto-optic modulators (10 \mu s, \sim1000 channels), can support a limited number of channels and are too slow for high-speed reconfiguration.

Some demonstrations seek to generate radio frequency (RF) waveforms from the optically generated waveforms, and are not concerned with complete frequency comb stability, insofar that the frequencies remain within the (de)multiplexer channel bandwidth. Other demonstrations do not fully stabilize the laser, relying on the relative stability of the laboratory environment. Target applications for this research seek to directly use

Figure 2.1: A block diagram schematic of an OAWG system.
Figure 2.2: A diagram of an AWG demultiplexer, an array of modulators, and a matching AWG multiplexer.

the generated optical waveform and require operation outside the laboratory environment, making full stabilization of the source critical. At the start of this research effort, fully-stabilized sources based on EDFLs were typically limited to a comb line spacing of 100 MHz.

For the (de)multiplexing and amplitude and phase control systems, collaborators at UC Davis and MIT are working on arrayed waveguide gratings (AWGs) to separate and recombine individual frequency components and on amplitude and phase modulator arrays for frequency component control. This approach was selected for its potential for high resolution, broad optical and RF bandwidth, and integration compatibility. An example of the demultiplexer, modulator array, and multiplexer is shown in Figure 2.2. In order to enable resolution by the AWGs and to keep the modulation bandwidths and number of channels manageable, a frequency comb spacing of 10 GHz is required. With a target optical bandwidth of 1 THz, a challenging 100 channels will need to
be fabricated on a single chip. This also means that stabilized frequency comb sources with 10 GHz comb line spacing must be developed.

The work presented in this thesis is primarily concerned with the development of the stable frequency component source at the telecommunications band. This includes development of a high-repetition rate source and the associated stabilization of the source. In addition to OAWG, other stabilized frequency comb-based applications can benefit from this work. This chapter focuses on the theory and concepts necessary for understanding the experiments in Chapter 3 and Chapter 4. First, a general description of the desired source output is presented. This is followed by an overview of approaches to high repetition rate sources, including their advantages, disadvantages, and important points of interest. Stabilization techniques based on phase-locked loops (PLLs) are then reviewed, followed by a discussion of ultrafast amplifiers.

2.1 The Source

2.1.1 The Pulse Train and Frequency Comb

For the approach to OAWG taken in this work, the source of frequency components is a mode-locked laser. This section describes the fundamentally mode-locked laser output signal, and highlights the number of degrees of freedom (DOFs) in anticipation of the source stabilization discussion that follows.

In the case that the slowly-varying envelope approximation\(^1\) [36] applies, the time domain output of a fundamentally mode-locked laser can be described by the combination of three constituents:

- pulse envelope
- pulse train
- carrier wave

\(^1\)Also known as the carrier-envelope approximation
Figure 2.3: The time domain description of a mode-locked laser output, assuming transform limited pulses.

Figure 2.3 illustrates, pictorially, the way in which these constituents come together to describe the electric field in the time domain.

This pictorial representation can be translated directly to the following equation:

\[
E(t) = \left( v(t) \otimes \sum_{n=-\infty}^{\infty} \delta(t - nT_R) \right) \left( \frac{1}{2} \left( e^{i\left(2\pi \frac{t}{t_0} + \phi_0\right)} + e^{-i\left(2\pi \frac{t}{t_0} + \phi_0\right)} \right) \right)
\]

where \( v(t) \) is the complex functional form of the pulse shape, or pulse envelope, the magnitude-squared of which integrates to unity. By allowing \( v(t) \) to be complex, all higher orders of chirp can be expressed.

Each of these constituents has associated DOFs. The parameters used to describe these DOFs are not unique; a commonly used "basis set" of choices are presented here. For typical pulse shapes, such as a gaussian or hyperbolic secant, there is a full-width half-maximum (FWHM) pulse width, \( \Delta t \), and the complex peak amplitude, \( A_0 \). For the pulse train, there is the pulse spacing, \( T_R \). Finally, the DOFs for the carrier wave are the waveperiod, \( t_0 \), and the phase, \( \phi_0 \).

An equivalent description of the electric field can be given in the frequency domain.

\(^2\)The shift of the pulse train, relative to \( t = 0 \) can also be called a DOF, however, the location of the time axis does not have any physical meaning. The relative positioning of the pulse train and the carrier, however, is important. This is captured by the carrier phase, \( \phi_0 \).
There is a one-to-one correspondence in the frequency domain to each of the three constituents in the time domain description:

- spectral envelope
- frequency comb
- carrier frequency

Figure 2.4 illustrates, pictorially, the frequency domain description. The constituents are positioned to correspond directly with their complementary constituents as shown in Figure 2.3.

Similarly, this pictorial representation can be derived from the Fourier Transform of Equation (2.1) in a straightforward manner:

\[
E(f) = \left( \frac{v(f)}{\text{spectral envelope}} \cdot \sum_{m=-\infty}^{\infty} \delta(f - mF_R) \right) \otimes \frac{1}{2} \left[ \delta(f - f_0)e^{i\phi_0} + \delta(f + f_0)e^{-i\phi_0} \right]
\]

where \(v(f)\) is the Fourier Transform of \(v(t)\) and describes the spectral envelope, and is complex to account for spectral phase. The frequency comb consists of a collection of
comb lines\textsuperscript{3}, spaced at the repetition rate frequency, defined as $F_R = \frac{1}{T_R}$. The carrier frequency is defined as $f_0 = \frac{1}{\Delta \nu}$.

It might appear that the convolved frequency combs centered at $+f_0$ and $-f_0$ could possibly overlap, creating an interleaved frequency comb with unequal spacing; however, the constraints on $v(f)$ prevent this. Specifically, the slowly-varying envelope approximation is only valid when $v(f)$ is much narrower than the carrier frequency (i.e. $\Delta f \ll f_0$); this requirement is equivalent to requiring multiple carrier periods under the envelope\textsuperscript{[36]}. Additionally, any component of the electric field that is constant is not a solution to the wave equation. These two facts ensure that for any valid pulse description, the frequency combs centered at $+f_0$ and $-f_0$ will approach and reach zero amplitude as $f \to 0$, thus preventing the positive and negative frequency combs from overlapping.

\textbf{2.1.2 The Carrier-Envelope Offset Frequency}

In the context of this work, frequency stabilization of the source is of central importance. As indicated in Section 2.1.1, the sources used in this work consist of discrete frequencies, or modes, in the frequency domain. In order to use the source in the context of OAWG, these modes must not fluctuate in frequency. The shape of the spectral envelope and the absolute phase of the carrier frequency do not change the locations of these discrete frequencies, and so do not directly\textsuperscript{4} affect the frequency stability of the source. This can be seen by recognizing that the terms representing the spectral envelope and absolute carrier phase are multiplicative factors of the $\delta$-function train that represents the frequency comb.

This leaves two DOFs that must be controlled: the repetition rate, $F_R$, and the carrier frequency, $f_0$. One can write the absolute frequency location of mode $m$ as:

\textsuperscript{3}Throughout this text, the frequency comb may also be referred to as the “mode comb”, and the individual frequency comb lines may also be referred to as “modes”.

\textsuperscript{4}Note that, while not directly important for stabilization, the spectral envelope does contain relative (to the carrier frequency) phase and amplitude information of the modes in the frequency comb. As a result, the requirements that arise are that a) the fluctuations in the phase and amplitude of the modes must be small enough to allow frequency stabilization, and that b) the amplitudes of the modes in the final, stabilized OAWG signal must be sufficient for the application.
\[ f_m = mF_R + f_0 \]  

(2.3)

where \( m \) is an integer, and generally, \( f_0 \gg F_R \).

The locations of the frequency comb lines can be expressed in another form using an equally valid set of parameters. This uses the carrier-envelope offset frequency, \( f_{ceo} \), in place of the carrier frequency, \( f_0 \):

\[ f_r = rF_R + f_{ceo} \]  

(2.4)

where \( r \) is a non-negative integer. This description is depicted in Figure 2.5. The carrier-envelope offset frequency represents the periodicity of the relative phase of the carrier wave and the pulse envelope. One can consider the pulse train to be sampling the phase of the underlying carrier wave. As a result of aliasing, in the latter formulation, the periodicity of the sampled phase must be greater than the temporal pulse spacing, which requires \( f_{ceo} < F_R \). The latter description given in Equation 2.4 is more typically used in the context of optical frequency combs.

With the necessary description of the source output, and an understanding that repetition rates up to 10 GHz are desired, the challenges faced when creating high repetition rate sources can be explored.

### 2.1.3 High Repetition Rate Sources

Generally, there are three approaches to achieving high repetition rates: reduce the cavity length, increase the number of intracavity pulses, or externally (to the laser oscillator) split the each pulse into multiple pulses. Cavity size reduction decreases the roundtrip time of the cavity; the intracavity pulse thus passes the output coupler more frequently, and the output pulse train is at a correspondingly higher frequency, defined by the cavity round-trip time. Increasing the number of intracavity pulses leads to a shorter duration between pulses which increases the repetition rate; this is known as harmonic mode-locking (HMLing). External cavity methods for generating
Figure 2.5: The slip of carrier phase, relative to the envelope in the time domain (top) is represented by the carrier-envelope offset frequency in the frequency domain (bottom).
high repetition rate pulse trains can be done in various ways by redistributing and attenuating the laser’s output power. Generating high-repetition rate, stabilized combs using any of these approaches presents challenges.

2.1.3.1 Challenges of High Repetition Rate Small Cavity Sources

Based on conceptual understanding, a number of factors can be identified that will set minima on the cavity sizes with which desired mode-locked operation is possible. These factors include:

1. **Cavity Loss**: Assuming that the gain medium in the laser is highly saturated - a valid assumption for EDFLs - then amplified spontaneous emission (ASE) will be minimal and the absorbed pump photons will largely be converted to signal
photons. Therefore, the total power added to the signal per pass is determined by the available pump power and properties of the gain medium, and depends weakly on the input signal power. If all cavity losses are discretely located in the cavity and are of fixed value, then the quality factor\(^5\), \(Q\), of the cavity will decrease as cavity size decreases. Therefore, as the \(Q\) is decreased and the pump power and gain medium are fixed, the intracavity power will decrease. This will result in limitations:

(a) The intracavity power will be too small to saturate the nonlinear saturable absorber, and mode-locking will fail.

(b) The intracavity power will lead to an output power that is too small for amplification without degradation of the signal-to-noise ratio (SNR).

2. **Gain Medium Length**: Given an input signal power, pump power, and gain medium, an optimal length of gain fiber exists - too long, and signal will be absorbed “downstream” in the non-inverted portion of the gain medium - too short, and pump will exit the gain medium that could have inverted more gain medium and thus contributed to the signal power. Thus an optimally-pumped cavity is limited in size to the optimal gain fiber length. A shorter, non-optimal gain fiber length can be used, but the reduced signal power will reduce the sustainable pulse energy, which will hasten the onset of limitations arising from cavity loss.

3. **SA Mechanisms**: As the cavity size is reduced, the options for physically realizing a SA becomes limited. For fiber lasers, the primary example is P-APM, which requires nonlinear polarization rotation (NPR) to accumulate over a length of fiber. For a given fiber nonlinearity, there is a cavity length below which insufficient NPR will accumulate for P-APM to operate, thus setting a minimum cavity size. Any distributed SA mechanism such as this will set a minimum on the cavity size.

4. **Mode-locking Regimes**: As the cavity length drops, the average and local values of group velocity dispersion (GVD) and nonlinearity converge, becoming the same

\(^5\) The quality factor of a resonator quantifies the time rate of decay of energy stored in the cavity.
Table 2.1: A summary of the limitations encountered as cavity size is reduced.

as the cavity length approaches zero. As a result, techniques for dispersion and nonlinearity management, such as those that lead to stretched-pulse and self-similar operation, become less accessible, thereby limiting the operating regimes available.

5. Instability against CW Q-switching and Q-switched mode-locking: In Section D.3 and Section D.4, it is shown that stability against Q-switching declines as the cavity roundtrip time is decreased. In erbium-doped fiber and waveguides, the gain recovery is very slow, which makes the laser particularly susceptible to the growth of relaxation oscillations that occur on timescales much greater than the roundtrip time. Fast SA mechanisms provide better stability against the growth of these oscillations, and thus shorter cavities should be achievable with fast SAs.

Table 2.1 provides a list of the limitations, and the steps that can be taken to mitigate the limitations. Additional application-specific constraints on operation parameters, such as pulse duration or energy, will further limit the achievable repetition rates.

Understanding which limitations are the most constraining is important. Because of the reduced ability to use distributed SA mechanisms and manage the GVD and nonlinearity, a very simple model for a linear cavity soliton laser with a slow SA is considered in order to provide insights into the scaling limitations of erbium fiber and waveguide laser cavities.
A Simple Model: Soliton Laser with Slow SA  It is useful to consider a simple mode-locked laser model in order to gain some quantitative insight into the limitations and scaling rules for achieving high repetition rate fundamentally mode-locked operation. Anomalous GVD fiber is used which will result in soliton-like operation; maintaining a high peak power throughout the cavity will ensure optimal saturation of the SBR. An all-fiber cavity made of only gain fiber is chosen to minimize cavity losses and maximize gain. Uniform pumping along the fiber length is assumed. Finally, small changes per roundtrip are assumed. The model cavity is shown in Figure 2.7.

With these assumptions, the requirement that gain equals loss can be expressed as:

$$e^{2\left(g(2L_{cavity})+l_o+l_p+q\right)} = 1$$  \hspace{1cm} (2.5)

The output coupling loss is represented by the mirror with reflectivity $R_o = e^{2l_o}$. All other non-saturable losses that do not represent useful output, i.e. parasitic losses, are represented by the mirror with reflectivity $R_p = e^{2l_p}$. Saturable losses are represented by $R_s = e^{2q}$. For a linear cavity, the signal passes through the cavity twice per roundtrip, so $2^* = 2$; for a unidirectional cavity\(^6\) where the signal passes through once, $2^* = 1$.

Assuming very slow recovery of the saturable gain, which is appropriate for erbium fiber lasers, the gain saturates on the average cavity power, and thus the saturable gain takes the form:

$$g = \frac{g_0}{1 + \frac{2P_{avg}}{P_{sat}}}$$  \hspace{1cm} (2.6)

---

\(^{6}\)The factor of two arising from the double-pass cavity is called in order to facilitate a similar analysis by the reader of a unidirectional cavity.
The intracavity power traveling in one direction is $P_{\text{avg}}$. For a linear cavity, the gain saturates on both counter-propagating waves, or $2P_{\text{avg}}$, and for a unidirectional cavity, it saturates on $P_{\text{avg}}$, hence the $2\ast$ factor.

To treat a slow SA, such as semiconductor SAs, the following assumptions are made:

1. The SA recovery time is slow compared to the steady-state pulse duration ($\tau_A \gg \tau_p$).

2. The SA recovers completely between pulses, that is, the SA recovery time is much less than the cavity roundtrip time ($\tau_A \ll T_R$).

With these assumptions, the saturable absorber losses are given by Equation D.11, which is given here in it’s Taylor expanded form:

$$q = q_0 \frac{1 - \exp \left( -\frac{E_p}{E_A} \right)}{\frac{E_p}{E_A}} = q_0 \left[ 1 - \frac{1}{2!} \frac{E_p}{E_A} + \frac{1}{3!} \left( \frac{E_p}{E_A} \right)^2 - \frac{1}{4!} \left( \frac{E_p}{E_A} \right)^3 + ... \right]$$  \hspace{1cm} (2.7)

where $E_p$ is the pulse energy, $E_A$ is the saturation energy, and $q_0$ is the small signal loss. To facilitate analytic solution, Equation 2.8 is instead used for the saturable losses:

$$q = \frac{q_0}{1 + \frac{E_p}{E_A}} = q_0 \left[ 1 - \frac{E_p}{E_A} + \left( \frac{E_p}{E_A} \right)^2 - \left( \frac{E_p}{E_A} \right)^3 + ... \right]$$  \hspace{1cm} (2.8)

A comparison of the Taylor expansions of each form shows that for pulse energies much less than the saturation energy, $E_p \ll E_A$, the substitution is sufficient for an order-of-magnitude calculation.

For the case of semiconductor saturable absorbers, the spot radius, $W_{\text{spot}}$, and saturation fluence, $F_A$, determine the saturation energy as follows:

$$E_A = F_A \left( \pi W_{\text{spot}}^2 \right)$$  \hspace{1cm} (2.9)

Finally, the physical cavity length can be written in terms of the repetition rate:

$$L_{\text{cavity}} = \frac{c}{2\ast n_g F_R}$$  \hspace{1cm} (2.10)
where $c$ is the speed of light, $n_g$ is the group index of refraction, and $F_R$ is the repetition rate, corresponding to the inverse of the roundtrip time.

An expression for the intracavity pulse energy as a function of repetition rate and cavity parameters can be derived by taking the natural log of Equation 2.5 and substituting for the saturable gain, saturable loss, and cavity length using Equation 2.6, Equation 2.8, and Equation 2.10, respectively:

$$0 = \left( \frac{g_0 c}{n_g F_R} + l_o + l_p + q_0 \right) + \left( \frac{g_0 c}{n_g F_R E_A} + \frac{l_o + l_p}{E_A} + \frac{2 F_R (l_o + l_p + q_0)}{P_G} \right) E_p$$

$$+ \left( \frac{2 F_R (l_o + l_p)}{E_A P_G} \right) E_p^2$$

(2.11)

Assuming the laser operates in the soliton regime, the Area Theorem from Equation D.1 relates the pulse duration to the pulse energy, where $L_{\text{roundtrip}} = 2 L_{\text{cavity}}$ in this context. An equation for the pulse duration results:

$$0 = \left( \frac{g_0 c}{n_g F_R} + l_o + l_p + q_0 \right) \tau_p^2 + \frac{2 |D_2|}{\eta} \left( \frac{g_0 c}{n_g F_R E_A} + \frac{l_o + l_p}{E_A} + \frac{2 F_R (l_o + l_p + q_0)}{P_G} \right) \tau_p$$

$$+ \left( \frac{2 |D_2|}{\eta} \right)^2 \left( \frac{2 F_R (l_o + l_p)}{E_A P_G} \right)$$

(2.12)

where $D_2$ is the net roundtrip GVD, and $\eta$ is the net roundtrip nonlinearity, as defined in Section D.1.1.

This model is, of course, limited in its applicability. Gain filtering is left out of the model; when the pulse bandwidth approaches that of the gain medium, the model becomes invalid. As the pulse duration approaches the SA recovery time, the assumption of a slow absorber becomes invalid. Also, the SA model does not incorporate the roll-off of the saturable response with energy, which arises from two photon absorption (TPA) when using SBRs and the interferometric response when using APM mechanisms. Finally, the effects responsible for Q-switching phenomena are not included in this model,
so any corresponding limitations are left out.

Additionally, it should be noted that any application-specific limits on the maximum acceptable pulse duration will always become relevant at lower repetition rates than limitations set by the lasing requirement of net gain. As the net gain decreases, the intracavity power drops, thus reducing the pulse energy that can be maintained, which increases the resulting pulse duration via the Area Theorem. That is, the pulse energy will approach infinity as the net gain approaches zero, and hence the maximum acceptable pulse duration will become a limitation before lasing fails. As a result, as long as maximum acceptable pulse duration limitations are considered, the net gain lasing condition need not be.

Using this model, a representative case is chosen that will serve as a reference design for estimating the achievable repetition rates. Parameters that achieve a high repetition rate, while still constrained to available technologies, are chosen:

- $R_s = 99\%$, (equivalently, $q_0 = -0.00503$)
- $F_A = 30 \ \frac{\mu J}{cm^2}$
- $W_{spot} = 5.4 \ \mu m$
- $n_g = 1.46$
- $R_o = 99\%$, allowing for 1% output coupling (equivalently, $l_o = -0.00503$)
- $R_p = 99.99\%$, (equivalently, $l_p = -0.00005$)
- $G_{dB/cm} = 3 \ \frac{dB}{cm}$, (equivalently $g_0 = 0.3454 \ \text{cm}^{-1}$)
- $P_{gain}^{sat} = 0.8 \ \text{mW}$, typical for anomalous silica-glass erbium fiber
- $\beta_2 = -22.7 \ \frac{ps^2}{km}$
- $\gamma = 2 \ \frac{1}{W \ km}$
- $\tau_A = 2 \ \text{ps}$
This analysis is primarily concerned with sub-picosecond pulses, the SA recovery time is chosen to be consistent with the shortest SBR recovery times available. Therefore, the range of pulse durations over which this model is valid, based on a gain bandwidth of erbium at 40 nm and SA recovery time, is between 200 fs and 2 ps.

Rough estimates for the limitation that pulse duration and pulse energy requirements impose can be found. Figure 2.8 shows the pulse energy and duration for the base case. For acceptable pulse durations limited to a range of 200 fs to 2 ps, repetition rates between 2 GHz and 5.5 GHz should be achievable.

Changes to the laser parameters from the base case can increase the achievable repetition rate. Variations in the small signal gain, parasitic loss, saturable loss, SA saturation fluence, output coupling, and GVD/nonlinearity ratio are considered.

Figure 2.9 shows how the output pulse energy and pulse duration vary with changes in small signal gain and parasitic losses. The achievable repetition rates within the valid pulse duration range increase with increasing gain and decreasing parasitic loss.

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8 Corresponds to 5 THz of bandwidth, which gives a bandwidth-limited pulse duration of 63 fs for a hyperbolic secant pulse shape.
Waveguides with up to 13.7 dB/cm of small signal gain have been reported [37], which suggests that 10 GHz might be achievable. With the parasitic losses being small compared to other losses, complete elimination of the parasitic losses would provide a small improvement at best.

Variations in the SA saturable loss and saturation energy are shown in Figure 2.10. Reducing the saturable loss provides a modest increase in the achievable repetition rate as well as an increase in the output pulse energy. Additionally, reducing the saturable loss increases the stability against Q-switching [38], making this adjustment beneficial overall, insofar that the pulse train remains stable. Decreasing the saturation fluence provides an increased achievable repetition rate as well, with an accompanying increase in output pulse energy that can be attributed to reduced losses.

Finally, variation of the output coupler and GVD-nonlinearity ratio, $\frac{D_{2}}{n}$, are shown in Figure 2.11. As with reductions in parasitic losses, reducing the output coupling losses improves the laser performance; however, it decreases the power that is coupled from the cavity for use. As a result, for a given repetition rate there is an optimal output coupling value beyond which the output pulse energy begins to decrease.

By decreasing the GVD-nonlinearity ratio, much higher repetition rates with sufficiently short pulse durations can be reached with little-to-no cost to the pulse energy.
Figure 2.10: Pulse energy and duration as a function of repetition rate for variations in saturable loss (left) and SA saturation energy (right).

Figure 2.11: Pulse energy and duration as a function of repetition rate for variations in output coupling loss (left) and the GVD-nonlinearity ratio (right).
Table 2.2: Minimum average output powers for the adjustments to the cavity parameters over the validity range of the model.

However, this method of improvement is limited to one’s ability to reduce the GVD (while keeping it anomalous) and increasing the nonlinearity. Fine control of GVD is most likely achieved through waveguide dispersion, and will be determined by fabrication tolerances. Increasing the nonlinearity can be done through reduction of the mode size, thereby increasing the mode intensity for a given power, or through the use of nonlinear materials. Because of the interplay between mode size, waveguide dispersion, and modal overlap with the gain region, there is a set of trade-offs to be made. The use of higher nonlinearity materials can provide some improvement, but the correlation between nonlinearity and material GVD (through the material resonances), and the erbium (and ytterbium) doping limitations presented by the material will also result in trade-offs. Nonetheless, assuming a minimum controllable GVD of $1 \, \text{ps}^2 \, \text{km}^{-1}$ and a maximum nonlinearity of $20 \, \text{W}^{-1} \, \text{km}^{-1}$, and leaving all other parameters unchanged corresponds to a ratio of $-0.05 \, \text{ps}^2 \cdot \text{W}$ and a maximum achievable repetition rate of more than 50 $\text{GHz}$.

Finally, the laser output power levels and SNR must be sufficient for the intended application. That is, the power levels and SNR must be suitable for direct use, or the SNR must be sufficiently high to permit amplification to the needed power levels without unacceptable degradation to the SNR. Empirically, power levels down to the microwatt range can be amplified successfully [39]. For the validity ranges of the plots, the average powers drop to the minimum values shown in Table 2.2. For most applications, all power levels given by the model should not be too low for amplification.

Based on these results, it appears that single- to sub-picosecond fundamentally
mode-locked (FMLed) lasers operating in the soliton regime should be able to reach 10 GHz repetition rates. This conclusion is consistent with the literature [40-42]. Reaching up to 50 GHz repetition rates may be possible with careful engineering, but would push fabrication limits and leave little margin for error.

Q-Switching As stated, the model does not include the effects that are responsible for Q-switching. The calculation of stability against continuous wave (CW) Q-switching in Section D.3 and Q-switched mode-locking in Section D.4 make assumptions that are consistent with the laser model presented here, and thus are applicable. The conclusions concerning the dependence of stability on various laser parameters provide a qualitative guide for avoiding Q-switching. It appears likely from the literature [42-44] that Q-switching will play a significant role in limiting the achievable repetition rate.

2.1.3.2 Challenges of Harmonically Mode-locked Sources

Harmonically mode-locked EDFLs have long been able to reach repetition rates of many gigahertz [45-47]. However, the fact that the output pulse train is generated by multiple intracavity pulses presents a challenge for applications where the carrier-envelope phase slip must be controlled. To clarify the situation, this section provides a full qualitative and mathematical description of the HMLed laser output, and methods to stabilize a HMLed laser are discussed.

The Uncorrelated Harmonic Pulse Train and Frequency Comb The action of HMLing, illustrated in Figure 2.12, can be described in the frequency domain as one of injection locking. Power resides in the fundamental harmonics of the laser, which are spaced by \( F_R \). The modulation generates sidebands on both sides of every fundamental mode at \( \pm F_H = \pm N F_R \). This injects power from each mode into the mode \( \pm N \) modes away. The result is an optical comb composed of \( N \) “supermodes”.

Clearly, \( f_{ceo} \) is well-defined in the injection locking description; however, the carrier-envelope phase slip is not completely described by \( f_{ceo} \). This can be seen by considering the time domain description. Assuming that \( f_{ceo} \) is fixed, the carrier-envelope phase
slip between pulses that originate from the same intracavity pulse is constant, as is the case for a FMLed laser. However, the carrier-envelope phase slip between two pulses that do not originate from the same intracavity pulse remains as a free parameter; for $N$ intracavity pulses there are $N - 1$ additional free parameters. As a result, $f_{ceo}$ does not completely represent the pulse-to-pulse carrier-envelope phase slip. Generally, the evolution of the carrier-envelope phase slip will have periodicities corresponding to the fundamental repetition rate harmonics. In the frequency domain, this corresponds to frequency comb lines spaced at the fundamental repetition rate, as expected from the injection locking description.

The following example will look at a specific case of the time domain description in order to build intuition. Consider a mode-locked laser with two intracavity pulses, as shown in Figure 2.13; the laser is harmonically mode-locked at the second harmonic of the fundamental repetition rate, and thus its harmonic number, $N$, is 2. One can consider the electric field of the output pulse train to consist of two independent, superpositioned pulse trains, both at the fundamental repetition rate, and each originating solely from a different intracavity pulse. When each fundamental pulse train is considered in isolation, the carrier-envelope slip, $\Delta \phi$, is spectrally represented by $f_{ceo}$. When $f_{ceo}$ does not vary, the carrier-envelope slip of that sub-pulse train is constant. However, the carrier-envelope slip of the entire harmonically mode-locked pulse train will not be
constant from pulse to pulse because \( \phi_1 \) and \( \phi_2 \) are independent; the phase relationship between intracavity pulses is unconstrained.

Based on this example, full expressions for the time domain FMLed laser output pulse train are provided, where the intracavity pulses have uncorrelated phase. The corresponding frequency domain descriptions are subsequently given. It is shown that the expressions reduce to those of a FMLed laser output pulse train with the harmonic repetition rate when the intracavity pulses are forced to have the same absolute carrier phase. Section E provides a detailed derivation of this description.

The following is assumed:

- There are \( N \) intracavity pulses.
- \( v(t) \) is the same for all pulses, and has all of the properties given in Section 2.1.1.

In the time domain, the portion of the output pulse train electric field generated by intracavity pulse \( k \) is given by:

\[
E_k(t) = \frac{E_0}{2\sqrt{N}} \left( e^{i(2\pi f_0 t + \phi_k)} + \text{c.c.} \right) \sum_{n=-\infty}^{\infty} \delta \left( t - nT_R - \frac{k-1}{N}T_R \right)
\]

(2.13)

where \( \frac{E_0}{\sqrt{N}} \) is the electric field amplitude, \( n \) is a number that labels each pulse consecutively in time, \( T_R = \frac{1}{F_R} \) is the cavity round-trip time and equals the inverse fundamental repetition rate, \( f_0 \) corresponds to the carrier frequency, \( k \) is a natural number ranging from 1 to \( N \) that labels each fundamental pulse train that is a subset of the full harmonic pulse train, and \( \phi_k \) is the absolute carrier phase of the fundamental subset pulse train, \( k \). The electric field amplitude is normalized such that \( \frac{E_0^2}{N} \) is the pulse energy, and is divided by the harmonic number, \( N \), in order to keep the average power independent of \( N \). The shape of the pulse, being the same for all pulses, has been left out of the analysis. The amplitude spectrum of this electric field is found to be:

\[
E_k(f) = \frac{E_0}{2\sqrt{N}} \left[ \sum_{m=-\infty}^{\infty} e^{-i \left( 2\pi \frac{f-f_0}{F_R} \frac{k-1}{N} - \phi_k \right) \delta \left( f - f_0 - mF_R \right) \right]
\]
Figure 2.13: Example of the carrier-envelope phase of a harmonically mode-locked laser pulse train, where example values were chosen such that $\phi_1 \neq \phi_2$. The carrier-envelope phase of each pulse is noted next to the pulse.
where the integer, \( m \), labels each mode consecutively in frequency and is zero for modes located at the carrier frequency.

The full electric field of the output from a harmonically mode-locked laser is the superposition of the individual pulse trains from each intracavity pulse, thus is can be written in the time and frequency domains as:

\[
E(t) = \sum_{k=1}^{N} \left( \frac{E_0}{2\sqrt{N}} \left( e^{i(2\pi f_0 t + \phi_k)} + c.c. \right) \sum_{n=-\infty}^{\infty} \delta \left( t - nT_R - \frac{k-1}{N} T_R \right) \right) \quad (2.15)
\]

\[
E(f) = \sum_{k=1}^{N} \frac{E_0}{2\sqrt{N}} \left[ \sum_{m=-\infty}^{\infty} e^{-i \left( \frac{2\pi f_0 k - f_0 + N \phi_k}{N} \right)} \delta (f - f_0 - mF_R) + e^{-i \left( \frac{2\pi f_0 k - f_0 + N \phi_k}{N} \right)} \delta (f + f_0 - mF_R) \right] \quad (2.16)
\]

The time-average power of the full harmonic pulse train in the time domain is:

\[
\langle P(t) \rangle_t = \frac{E_0^2}{2N} \sum_{k=1}^{N} \sum_{n=-\infty}^{\infty} \delta \left( t - nT_R - \frac{k-1}{N} T_R \right) \quad (2.17)
\]

which, as expected, represents an interleaved set of \( N \) pulse trains, each consecutive pair having a relative delay of \( \frac{T_R}{N} \).

The power spectrum for the full harmonic output is:

\[
P(f) = \left\{ \frac{|E_0|^2}{2} \left\{ 1 + \frac{1}{N} \sum_{k=1}^{N-1} \sum_{\bar{k}=k+1}^{N} \sum_{m=-\infty}^{\infty} \left[ \cos \left( \frac{2\pi f - f_0 \bar{k} - k}{F_R} - (\phi_{\bar{k}} - \phi_k) \right) \delta (f - f_0 - mF_R) + \cos \left( \frac{2\pi f + f_0 \bar{k} - k}{F_R} + (\phi_{\bar{k}} - \phi_k) \right) \delta (f + f_0 - mF_R) \right] \right\} \right\} \quad (2.18)
\]

The \( \delta \)-function coefficient represents the frequency comb with the fundamental rep-
etition rate spacing of modes. The amount of power in the mode will be governed by
the coefficient with summations over \( k \) and \( \bar{k} \), which depends on the relative phases
of the independent intracavity pulses. It shows that, in general, power can and will
reside at multiples of the \textit{fundamental} repetition rate, commensurate with the carrier
frequency, \( f_0 \), i.e. \( f_m = mF_R + f_0 \). This can also be written, as in Section 2.1.2,
as \( f_r = f_{ceo} + rF_R \). Therefore, \( f_{ceo} \) as defined cannot be larger than the fundamental
repetition rate, and thus cannot represent periodicities with greater frequency than the
fundamental repetition rate. Correspondingly, carrier-envelope phase slip changes in
the harmonic pulse train that occur on time scales less than the cavity roundtrip time
are not expressed by \( f_{ceo} \).

The Correlated Harmonic Pulse Train and Frequency Comb In order for the
harmonically mode-locked laser output to be indistinguishable from that of a funda-
mentally mode-locked laser, the carrier-envelope phase slip must be constant from pulse
to pulse. This requires that the absolute carrier phase of each subset pulse train be the
same, that is, \( \phi_k \) are equal for all \( k \). In this case, power should only reside at modes
corresponding to the harmonic repetition rate, and none will reside at the remaining
fundamental repetition rate modes. This special case will now be considered. First, at
mode \( m \) located at \( f_m \), the power is:

\[
P(f_m) = \frac{|E_0|^2}{2} \left\{ 1 + \frac{1}{N} \sum_{k=1}^{N-1} \sum_{\bar{k}=k+1}^{N} \cos \left( 2\pi m \left( \frac{\bar{k}-k}{N} \right) - (\phi_{\bar{k}} - \phi_k) \right) \right\}
\]

By setting \( \phi_k \) equal for all \( k \), the power at mode \( f_m \) becomes:

\[
P(f_m) = +\cos \left( 2\pi m \left( \frac{\bar{k}-k}{N} \right) + (\phi_{\bar{k}} - \phi_k) \right) \left\} \right\}
\]

\[
P(f_m) = \frac{|E_0|^2}{2} \left\{ 1 + \frac{1}{N} \sum_{k=1}^{N-1} \sum_{\bar{k}=k+1}^{N} 2 \cos \left( 2\pi m \left( \frac{\bar{k}-k}{N} \right) \right) \right\}
\]

This can be rewritten, after simplification, as:
\[ P(f_m) = |E_0|^2 \left( \begin{array}{c} N \text{ when } (m = rN) \\ 0 \text{ otherwise} \end{array} \right) \]  \hspace{1cm} (2.21)

When mode \( m \) corresponds to a harmonic of the harmonic repetition rate, \( \frac{m}{N} \) will be an integer, \( r \), and power will reside in that mode. When \( \frac{m}{N} \) is not an integer, all terms cancel and no power lies in the mode. As expected, given that this result was formulated to keep the average power independent of \( N \), the power in each mode scales with \( N \). Thus, in this special case, the HMLed laser output is indistinguishable from the output of a FMLed laser.

### Correlating the HMLL Frequency Comb with an Intracavity Filter

One can envision using an intracavity filter to correlate the carrier-envelope phases of the intracavity pulses. It is clear that this can be accomplished by an intracavity amplitude filter that provides greater loss to the fundamental harmonics that do not correspond to the desired harmonic repetition rate. The harmonic repetition rate modes experience less loss, and through the mode competition and the SA action, mode-locked lasing on only the harmonic modes results. A FPI is one such amplitude filter, and has been shown to stabilize the pulse train when placed in the laser cavity and stabilized [48].

An intracavity phase-only ("all-pass") filter’s ability to correlate the pulses is less clear. A pulse train with correlated carrier-envelope phase cannot have power in the fundamental harmonics that do not correspond to harmonic repetition rate harmonics; alone, a phase-only linear filter cannot remove the power from those undesired fundamental harmonics. However, the addition of nonlinearity enables frequency conversion, which provides a mechanism for removing the power from the undesirable fundamental mode harmonics. When a phase filter, such as a Gires-Tournois interferometer (GTI), is combined with the nonlinear elements (i.e. SA, self-phase modulation) of a mode-locked laser cavity, the addition of an appropriate phase-only filter may be able to produce the desired results. Some recent experiments are suggestive; they demonstrate that the use of phase-only filters allows some control over the spectral distribution of power in spectral generation experiments based on self-phase modulation (SPM) [49, 50].
2.1.3.3 Challenges of External Repetition Rate Multiplication

A number of linear techniques for generating high repetition rate pulse trains external to the oscillator cavity have been explored. These include the use of amplitude filters, such as coupled-cavity FPIs [51] (of which a standard FPI [52], thin film mirror, and fiber Bragg grating filter [53]-based approaches are special cases), and phase filters, such as AWGs [54] (of which an imbalanced Mach-Zehnder interferometer, Michelson, and GTIs, and ring resonators are special cases). Also of note are demonstrations of repetition rate multiplication using nonlinearities, particularly the use of modulation instability [55]; this discussion is limited to linear techniques.

All-pass phase only filters will not suffice for applications that require constant carrier-envelope phase slip from pulse to pulse. As shown in Section 2.1.3.2, a pulse train with constant carrier-envelope slip from pulse to pulse will correspond to a mode spacing that is the inverse pulse period. Thus, to multiply a low repetition rate pulse train with constant carrier-envelope phase slip from pulse to pulse to a high repetition rate pulse train with constant carrier-envelope phase slip from pulse to pulse without using nonlinearities, power must be removed to generate the mode comb with greater mode spacing. A phase-only filter obviously cannot accomplish this; linear methods of external multiplication for applications that require a constant carrier-envelope phase slip from pulse to pulse must be done with amplitude filters.

For the amplitude filters mentioned above, the following challenges arise to varying degrees:

- **Mode Suppression**: The qualities of the filter will determine the degree to which the modes incommensurate with the harmonic repetition rate are attenuated. Incomplete suppression represents a non-ideal variation in the intensity from pulse to pulse.

- **Loss**: Because the undesirable modes are being attenuated, the overall power of the signal will be reduced. This limits the increase in repetition rate that is achievable.
• **Stability:** The laser mode comb and the filter must be stably locked together. Drift of either will lead to intensity and/or timing noise in the repetition rate multiplied pulse train.

In designing the system, these considerations trade off. For example, an improved FPI finesse will lead to better mode suppression but will place tighter constraints on the locking and may lead to more noise in the resulting pulse train.

### 2.1.4 Summary

This section provided an overview of high repetition rate sources. A general description of a mode-locked pulse train was given, with emphasis on the DOFs and the carrier-envelope offset frequency. Paths to high repetition rates - small cavity FMLed lasers, HMLed lasers, and external repetition rate multiplication techniques - were then discussed in detail, with a focus on the challenges each method presents.

Once a high repetition rate laser source is developed, stabilization of the frequency comb lines must be implemented. The next section will review some of the standard techniques for frequency comb stabilization.

### 2.2 Stabilization Techniques

A number of techniques exist to control and stabilize various DOFs of the frequency comb. All involve general phase-locking techniques [56], which will be summarized below, followed by summaries of locking techniques applied to mode-locked lasers.

Figure 2.14 shows the basic elements of a generic PLL. Using Laplace notation and starting with the voltage-controlled oscillator (VCO), the transfer function relationships can be written. The VCO is any frequency source whose center frequency can be shifted by an input signal, typically a voltage, $v_c$:

$$\theta_{VCO}(s) = \frac{K_{VCO}v_c(s)}{s} \quad (2.22)$$
where $K_{VCO}$ is a proportionality constant. The reference signal, with phase, $\theta_{\text{ref}}$, is any signal to which the VCO signal, filtered with response functions, $M(s)$ and $N(s)^9$, is locked in phase. The phase detector provides a signal proportional to the difference in phase between the filtered VCO and reference signal:

$$V_{PD}(s) = K_{PD} (\theta_{\text{ref}}(s) - M(s) \cdot N(s) \cdot \theta_{VCO}(s))$$  \hspace{1cm} (2.23)

The loop filter, with response, $F(s)$, converts the phase detector output to the control voltage:

$$V_c(s) = F(s) \cdot V_{PD}(s)$$  \hspace{1cm} (2.24)

Putting these equations together, one arrives at the general loop equations that describe the loop functioning:

$$\frac{\theta_{VCO}(s)}{\theta_{\text{ref}}(s)} = H(s) = \frac{K_{VCO}K_{PD}F(s)}{K_{VCO}K_{PD}F(s) \cdot M(s) \cdot N(s) + s}$$  \hspace{1cm} (2.25)

---

9 The general PLL does not require $M(s)$ or $N(s)$ to be explicitly shown, as their behavior can be included as part of the VCO. These two response functions are explicitly identified here to facilitate discussion of specific examples relating to mode-locked laser locking. The benefit of this representation is that the VCO need only represent the mode-locked laser.
Figure 2.15: A schematic showing the basic elements of a PLL for stabilizing a mode-locked laser.

\[
\frac{\theta_{\text{ref}}(s) - \theta_{\text{VCO}}(s)}{\theta_{\text{ref}}(s)} = \frac{\theta_{e}(s)}{\theta_{\text{ref}}(s)} = 1 - H(s) = \frac{K_{\text{VCO}}K_{\text{PD}}F(s)[M(s) \cdot N(s) - 1] + s}{K_{\text{VCO}}K_{\text{PD}}F(s) \cdot M(s) \cdot N(s) + s}
\]

(2.26)

\[
V_c(s) = \frac{s \theta_{\text{ref}}(s)}{K_{\text{VCO}}} H(s) = \frac{s K_{\text{PD}} \cdot F(s) \cdot \theta_{\text{ref}}(s)}{K_{\text{VCO}} K_{\text{PD}} F(s) \cdot M(s) \cdot N(s) + s}
\]

(2.27)

where \(H(s)\) is the closed-loop transfer function. When working properly, the PLL will lock the phase (and hence frequency) of the VCO output to that of the reference frequency.

A number of PLL-based schemes are used to stabilize mode-locked lasers. A schematic of a PLL is shown in Figure 2.15 that identifies specific elements common to mode-locked laser PLL locking systems. In these systems:

- The VCO is the mode-locked laser, which includes the feedback actuator.

- The loop filter, \(M(s)\), is the response function of the optical portion of the system that occurs between the laser and phase detector. This will vary significantly from system to system.
The loop filter, \( N(s) \), is the response function of the RF portion of the system that occurs between the laser and phase detector. In all cases considered here, this will consist of a photodetector, and may subsequently include RF filters, amplifiers, and phase delays.

The phase detector is either an analog RF mixer, or an RF digital phase detector.

The loop filter, \( F(s) \), consists of a proportional-integral (PI) or proportional-integral-derivative (PID) circuit, and possibly RF filters and amplifiers.

The reference signal can, in general, be any RF frequency source. Often an independent oscillator is used.

These locking systems reviewed below include RF line (repetition rate) locking, optical line locking, DFG repetition rate locking, and second-harmonic generation (SHG) carrier-envelope offset frequency locking.

2.2.1 RF Line (Repetition Rate) Locking

This type of locking system is the simplest of those considered, and is commonly used to lock the laser repetition rate. In this type of system, the laser output is directly detected \((M(s) = 1)\) to generate an RF frequency comb. Because the optical carrier information is lost in direct detection, the heterodyne detection of the beating of the optical modes results in RF comb lines residing in absolute frequency at exact multiples of the laser repetition rate. Filters and amplifiers then select and amplify, respectively, one of the frequency comb lines. This frequency is locked to the reference RF frequency, typically an independent low-noise oscillator, using the remaining standard PLL components. The actuator in the laser that adjusts the repetition rate may be a piezo-driven mirror, fiber stretcher, or a motorized stage; bandwidth requirements of the application will greatly influence the actuator used.
2.2.2 Optical Line Locking

A slightly more complex system than the RF line lock, the optical line lock allows an optical frequency comb line to be locked to an optical reference frequency. This enables locking to a very stable optical frequency source, such as a optical atomic transition.

As described in Section 2.1.1, there are two DOFs in the frequency comb. In previous sections the choice of parameters to represent those DOFs were $f_{ceo}$ and $F_R$. One can also chose the location of two distinct comb lines, at frequency $f_m$ and $f_n$, as the free parameters. Thus, locking one comb line to a stable frequency source reduces the DOFs by one, and results in a correlation between $f_{ceo}$ and $F_R$.

Obviously, locking two distinct frequency comb lines will reduce the DOFs to zero, and hence completely stabilize the comb. For example, this could be implemented by stabilizing two different comb lines to two different atomic transitions. This is extremely restrictive, however, because it requires two usable optical atomic transitions to lie within the bandwidth of the frequency comb. More practically, the comb can be stabilized by locking one comb line to an atomic transition, and then locking either $f_{ceo}$ or $F_R$. In this way, the frequency comb can be more conveniently referenced to well-known atomic transitions.

Figure 2.16 shows a typical loop filter, $M(s)$, used to lock a comb line to an atomic transition. The loop filter consists of a stable single-frequency optical reference beating (i.e. mixing) with the laser output frequency comb. The baseband mixing product then serves as the frequency that is subsequently locked to the reference RF frequency, as done in the RF line lock. Again, the actuators used in this feedback can be any of those given in Section 2.2.1, as well as modulation of the optical pump power via an acousto-optic modulator (AOM) or direct current modulation of the pump laser.

2.2.3 DFG Repetition Rate Locking

Frequency shifting by a DFG process and appropriate optical filtering in the loop filter, $M(s)$, can result in a system that allows the repetition rate to be locked to an optical reference frequency, as shown in Figure 2.16.
To lock the repetition rate using DFG, spectral components on the high frequency end of the spectrum at frequency $k f_0 + f_{c eo}$ are mixed in a nonlinear crystal with low frequency components of the spectrum at $p f_0 + f_{c eo}$. A SC generation process may be needed to produce the required spectral bandwidth. Note that, in general, measures must be taken to achieve correct phasing of the temporal envelopes to ensure efficient frequency conversion by the DFG process. This generates spectral components at $(k - p) f_0$ as illustrated in Figure 2.17, which is a frequency comb with $f_{c eo} = 0$, and hence only has one DOF. This resulting comb is then detected, and the RF filter in loop filter, $N(s)$, filters out one of the frequency lines. By locking this line to the RF reference frequency in the standard way, all DOFs are eliminated, and thus the repetition rate of the comb is locked. The repetition rate can be actuated as described in Section 2.2.1.

2.2.4 SHG ("1f-2f") Carrier-Envelope Offset Frequency Locking

Replacing the DFG process with an SHG process, and ensuring that an octave-spanning spectrum is available for the SHG process, results in a method for locking the carrier-envelope offset frequency. This technique, often referred to as "1f-2f" locking, was used in the first demonstration of carrier-envelope offset locking using a Ti:Al₂O₃ laser [57], and has been demonstrated using fiber lasers [58].
Figure 2.17: An illustration showing the way in which the DFG process generates an $f_{ceo}$-free frequency comb.

Figure 2.18: A typical schematic of the loop filter, $M(s)$, for a DFG repetition rate locking system.
Figure 2.19: An illustration showing the way in which the SHG process can be used to generate a beat at $f_{ceo}$.

The schematic of the loop filter, $M(s)$, is shown in Figure 2.20. A portion of the frequency comb at the low frequency portion of the octave-spanning spectrum at frequency $rF_R + f_{ceo}$, where $r$ is an integer, is doubled via SHG, resulting in a frequency comb at $2rF_R + 2f_{ceo}$. As in Section 2.2.3, correct temporal phasing of the envelopes is necessary; this can be achieved using an interferometer in a “two-beam” configuration, or a single-beam configuration where the temporal phasing of the envelopes is achieved with group-delay [59]. This frequency comb is then beat (i.e. mixed) with a portion of the original mode comb at $2rF_R + f_{ceo}$ which spectrally overlaps with the SHG-generated frequency comb. The difference frequency between the two optical frequency combs, among other mixing products, is generated; the lowest frequency mixing product is precisely the carrier-envelope offset frequency, $f_{ceo}$, as illustrated in Figure 2.19. The RF filter in loop filter, $N(s)$, then isolates the base-band frequency at $f_{ceo}$. This is then, in the standard way, locked to an RF reference frequency, source such as an external oscillator or a sub-multiple of the repetition rate generated by direct detection of the laser output. Actuation of $f_{ceo}$ is typically accomplished by modulation of the optical pump power via an AOM or direct current modulation of the pump laser.
Figure 2.20: A schematic of a typical 1f-to-2f locking scheme.

<table>
<thead>
<tr>
<th>Method</th>
<th>Locks</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF Line</td>
<td>Repetition Rate</td>
</tr>
<tr>
<td>Optical Line</td>
<td>Single Comb Line</td>
</tr>
<tr>
<td>DFG Repetition Rate</td>
<td>Repetition Rate</td>
</tr>
<tr>
<td>SHG Carrier-Envelope Offset</td>
<td>Carrier-Envelope Offset Frequency</td>
</tr>
</tbody>
</table>

Table 2.3: A table summarizing the locking schemes and their affect on various DOFs.

2.2.5 DFG Carrier-Envelope Offset-Free Frequency Comb Generation

In Section 2.2.3 it was pointed out that DFGs can generate an $f_{\text{ceo}}$-free comb from an unstabilized mode-locked laser; in some cases, the resulting $f_{\text{ceo}}$-free frequency comb may be directly used. Attractive features of this approach are that it does not require an octave of spectrum, and does not require a stable external oscillator. However, this approach is limited by the the bandwidth and power requirements of the application as it relates to the bandwidth and efficiency of the DFG process [60, 61]. With this approach, bandwidths on the order of 10 nm and powers on the order of 10 $\mu$W should be achievable with reasonable effort.

2.2.6 Summary of Stabilization Techniques

A summary of the stabilization techniques and the DOFs that are controlled by them is given in Table 2.3. The implementation at high repetition rates of those techniques that require broad optical spectra to be generated presents additional challenges that
must be addressed. The next section discusses those challenges in detail.

### 2.3 Amplification of High Repetition Rate Sub-picosecond Pulse Trains with Erbium

A number of the locking schemes presented in Section 2.2 requires a broad spectrum. For fiber and waveguide lasers in the 1550 nm band, achieving a broad spectrum requires external spectral generation, typically using nonlinear fibers. The resulting spectrum depends on the energy and duration of the pulses; shorter pulses and higher energy pulses tend to produce broader supercontinuum (SC) spectra [62–65]. Given a minimum required pulse energy for spectral generation, the corresponding minimum average power will scale up proportionally with the repetition rate. For fiber and waveguide lasers, this means that amplification will be required at high repetition rates.\(^\text{10}\)

#### 2.3.1 Pulse Energy

The lack of single-mode high-power pumps\(^\text{11}\) for erbium make amplification to average powers approaching 1 W challenging, particularly when limited to single-mode, commercially available components. As such, an estimation of the attainable power is useful. The following rough calculation, using the model shown in Figure 2.21, provides

\(^\text{10}\)Specific applications of OAWG systems also benefit from high-power ultrafast pulse train amplifier development. For example, laser radar applications can require multi-Watt power levels.

\(^\text{11}\)Currently, the highest powers available are 660 mW at 980 nm, and 360 mW at 1480 nm.
such an estimate. Let:

- $T_{\text{before}}$ be the total power transmission (loss mechanisms) before the gain
- $T_{\text{after}}$ be the total power transmission (loss mechanisms) after the gain
- $P_{\text{stage}}$ be the power added by the amplifier stage
- $P_{\text{pump}}$ be the pump power available for each stage
- $\eta$ be the quantum efficiency when the gain is heavily saturated
- $E_r$ be the energy of a photon described by $r$, where $r$ is either signal or pump
- $P^m_{\text{in}}$ be the input power
- $P^m_{\text{out}}$ be the output power

With deep saturation of the gain, which is accurate for power EDFAs, and a fixed amount of available pump power, $P_{\text{pump}}$, the amplifier will add approximately a fixed amount of power to the signal, $P_{\text{stage}}$:

\[
P_{\text{stage}} = P_{\text{pump}} \frac{E_{\text{signal}}}{E_{\text{pump}}} \eta = P_{\text{pump}} \frac{\lambda_{\text{pump}}}{\lambda_{\text{signal}}} \eta
\]  

\[
P^m_{\text{out}} = T_{\text{after}} \left( P^m_{\text{in}} T_{\text{before}} + P_{\text{stage}} \right)
\]  

Multiple amplifiers of the same type can be placed in series to get further amplification; the superscript $n$ indicates the $n^{th}$ amplifier in the series. Because the total power lost per stage scales with input signal power at the stage, but the added power per stage is constant, the output power will asymptotically approach a power value as the number of stages is increased. This value is reached when power lost is equal to the power gained at a given stage, i.e. $P^n_{\text{out}} = P^n_{\text{in}}$ as $n \to \infty$:

\[
P^\infty_{\text{out}} = T_{\text{after}} \left( P^\infty_{\text{in}} T_{\text{before}} + P_{\text{pump}} \frac{E_{\text{signal}}}{E_{\text{pump}}} \eta \right)
\]  

\[
P^\infty_{\text{out}} = \frac{T_{\text{after}} P_{\text{pump}} \lambda_{\text{pump}} \eta}{1 - T_{\text{after}} T_{\text{before}}}
\]
This assumes that each stage is identical. As mentioned, the total power cost of a given amount of loss increases at later stages because the power there is greater. Depending on the trade-offs between the location and magnitude of the losses, the pump power losses, etc., it may be optimal to have greater pump power at the expense of greater loss in earlier stages, and accept lower pump power in exchange for decreased signal loss at later stages.

2.3.2 Pulse Duration

In addition to achieving high pulse energies, the sub-picosecond duration of the pulses must be maintained. This means that gain filtering, GVD and nonlinearity become important considerations.

One approach to preserving the pulses is to avoid nonlinearity. Large amounts of GVD will chirp the pulses and reduce the peak powers to nearly eliminate nonlinearity; this approach is known as chirped-pulse amplification (CPA) [66]. This can only work if the gain bandwidth is large enough to support the target pulse duration.

Another approach is to manage the GVD and nonlinearity so that the chirp of the amplified output pulse is compressible. This can be done by using fibers with normal GVD, which in combination with SPM, can produce a linear, compressible chirp [67]. If gain filtering will lengthen the transform-limited pulses beyond the target pulse duration, this approach can also be used to recover spectrum.

2.4 Summary

This chapter has reviewed the concepts and mathematics necessary to understand and describe the laser source. Three general approaches to high repetition rate sources were discussed, and the challenges they present were reviewed in detail. Approaches to the stabilization of those sources were reviewed. Lastly, ultrafast pulse amplification were discussed. Based on the understanding provided in these chapters, experiments in high repetition rate sources and in source stabilization are presented in Chapter 3 and Chapter 4.
Chapter 3

Experiments with High Repetition Rate Fiber Laser Sources

Research into stable frequency sources has progressed rapidly since the first demonstrations of full control of the pulse electric field in 2000 [57]. Only recently have 1550 nm fiber lasers in the multi-100 MHz regime been demonstrated [68, 69]. The work outlined in this section seeks to develop stabilized fiber-based frequency comb sources in the >1 GHz regime at 1550 nm.

As outlined in Chapter 2.1.3, achieving high repetition rates presents a number of challenges. In this chapter, an experiment is described that takes advantage of recent improvements in high-power pump sources and higher gain-per-length erbium fibers to increase the repetition rates achieved, and thoroughly understand the system’s operation. In the second experiment, the pulse train repetition rate of a laser is increased using an external interferometer.

3.1 High Repetition Rate Fundamentally Mode-locked Fiber Lasers

This section describes experiments constructing and testing high repetition rate stretched-pulse fiber lasers combining an SBR and P-APM. These experiments sought to:
• test the scaling limitations of P-APM as the cavity size is reduced

• reduce the minimum cavity sizes at which P-APM operates by using an SBR

• explore the operation of various SBRs in short cavities

• achieve the highest repetition rate in an EDF-based fiber laser mode-locked with an SBR and P-APM

• reduce the minimum cavity size at which the stretched-pulse regime is accessible by using bulk silicon elements

The first three goals are concerned with the SA mechanism limitations discussed in Section 2.1.3.1. The use of SBRs, which are slow and discrete, and P-APM, which is fast and distributed, allows both the speed and localization aspects of the SA mechanism to be explored, and offers the possibility of using the best characteristics of each mechanism to achieve the fourth goal.

The fifth goal seeks to mitigate the operating regime limitations, also discussed in Section 2.1.3.1. Stretched pulse operation requires sections of net anomalous and normal GVD in the cavity with comparable magnitudes. Because the GVD per unit length of bulk silicon is high compared to normal GVD fibers, normal GVD fiber can be replaced with bulk silicon, which reduces the cavity roundtrip time while keeping the linear evolution of the pulse chirp the same. The figure of merit for obtaining the normal GVD is thus:

\[
FOM \propto \frac{\text{Total GVD}}{\text{Group Delay}} = \frac{\beta_2 L}{L/v_g} \propto v_g \beta_2 = c \frac{\beta_2}{n_g}
\]

where \( \beta_2 \) is the GVD per unit length, \( L \) is the physical path length, \( v_g \) is the group velocity, \( n_g \) is the group index of refraction, and \( c \) is the speed of light. For bulk silicon, \( FOM = 0.093 \text{ fs} \), whereas for silica-based dispersion compensating fibers (DCF)s with the largest GVD\(^1\) that was available, \( FOM = 0.03 \text{ fs} \), which shows that using bulk

\(^1\)Avanex Corporation, Model: PowerForm\(^TM\) DCF, \( \beta_2 = 154.7 \text{ ps}^2/\text{km}, \beta_3 = -0.91 \text{ ps}^3/\text{km}, \eta = 5.3 \text{ W}^{-1}/\text{km}, \)
silicon introduces more than three times the normal GVD in a given optical path length than DCF.

In the experiments described below, the bulk silicon used was anti-reflection (AR) coated for 1550 nm. The silicon provided the additional benefit of protecting the SBR from absorbing pump power, when appropriately placed in the cavity.

### 3.1.1 Stretched-Pulse Laser Using an SBR and P-APM: In-situ SBR Behavior Study

The first set of experiments explored the combined use of an SBR and P-APM as SA mechanisms. The study looks in detail at the role of the SBR in the pulse formation and stabilization process at moderately high repetition rates. Specifically, SBRs were systematically changed and placed in the laser cavity without changing other cavity components, and the operating states were fully characterized. The stretched-pulse regime, as opposed to the soliton regime, was focused on because pulses with energies and durations approaching 1 nJ and 100 fs or less, respectively, are possible.

#### 3.1.1.1 Setup

The schematic for this laser as used in this study is shown in Figure 3.1. The cavity is in a sigma configuration, which allows for placement of an SBR. The 64.35 cm of gain fiber\(^2\) is pumped with two polarization multiplexed\(^3\) 974 nm pump diodes\(^4\) (not shown) through coupling lenses\(^5\), a short-wave-pass 980 nm/1550 nm dichroic mirror\(^6\), and into receptacle-type collimators\(^7\). The second dichroic\(^8\) allows remaining pump to exit the cavity, and the AR-coated silicon\(^9\) ensures that remaining pump power will not interfere with SBR operation. The half-wave plate (HWP) controls the output

\(^2\)Liekki Corporation, Model: ER80-8/125
\(^3\)SIFAM Fibre Optics (via Optimark Fiber Optics), Model: FFP-5M3280G10
\(^4\)Bookham, Inc., Model: LC96UF74-20R, Serials: SO224692.001 0746 (“Pump 1” or “Slot 3”) and SO224693.001 0746 (“Pump 2” or “Slot 2”)
\(^5\)Thorlabs, Inc., Model: LA1708-B (1st) and LC1120-B (2nd)
\(^6\)CVI Melles-Griot, Model: SWP-45-RU1550-TU775-PW-1025
\(^7\)Thorlabs, Inc., Model: F240APC-1550
\(^8\)CVI Melles-Griot, Model: SWP-45-RU1550-TU775-PW-1025
\(^9\)Unknown origin, 12.5 mm thick
Figure 3.1: A schematic showing the high repetition rate fundamentally mode-locked laser design used in the SBR study. Metal mirrors are not labeled. Mirrors with beams passing through them were mounted on removable mounts.
coupling at the polarization beam splitter (PBS)\textsuperscript{10}. The vertically polarized (relative to the optical table) portion of the beam enters the sigma arm of the cavity, and is focused on the SBR with an aspheric lens\textsuperscript{11}. The linear path includes a quarter-wave plate (QWP) oriented so that the vertically polarized beam returns to the PBS horizontally polarized. After the PBS, the polarizing isolator\textsuperscript{12} ensures unidirectional operation. Finally, a HWP and QWP allow control of the polarization state that enters the gain fiber.

To better understand the intracavity dynamics, the two dichroic filters were used as taps for the intracavity power. By using an uncoated silicon window\textsuperscript{13}, any remaining 980 nm pump power was absorbed, leaving behind the 1550 nm power that leaked through the dichroics. This was then coupled to the diagnostics beam path using various silver or gold mirrors, some of which were on flip mounts\textsuperscript{14}. Combined with a loss measurement of the components in the cavity which are found in Section F.1.1, the output and tap powers enabled the calculation of the intracavity powers, and ultimately the fluence incident on the SBR during operation.

The resulting GVD map is shown in Figure 3.2. Accounting for only the GVD of the gain fiber and the silicon, the net GVD is nearly zero at -634 fs\textsuperscript{2}. Given the behavior of lasers operating in the stretched-pulse regime, sub-100 fs pulses with pulse energies ranging from 0.1-0.5 nJ should be expected \cite{70}.

As stated before, as few cavity parameters as possible were changed between SBR characterizations. The only physical changes in the components were to the SBR focusing lens position (which was realigned each time for best coupling) and SBR position (for best coupling) and model were changed. Great care was taken to not disturb the gain fiber, nor change the waveplate orientations.

Table 3.1 outlines the four SBR models\textsuperscript{15} that were characterized in the laser. These

\begin{itemize}
\item Unknown origin
\item Unknown origin, Model: C220-TM, $f = 11$ mm, AR coated with “C” coating, which covers approximately 1100 nm -1700 nm.
\item Isowave, Model: I-16-JM-3.5-4, Single Stage Round
\item Korth Kristalle GmbH, 2 mm thick
\item New Focus, Inc., Model: 9891
\item BATOP GmbH
\end{itemize}
Figure 3.2: The GVD map of the high repetition rate fundamentally mode-locked stretched-pulse laser.

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Linear Loss</th>
<th>Modulation Depth</th>
<th>Saturation Fluence</th>
<th>Recovery Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAM-1550-09-25.4s</td>
<td>9%</td>
<td>6%</td>
<td>50 (\text{\textmu J/cm}^2)</td>
<td>2 ps</td>
</tr>
<tr>
<td>SAM-1550-20-25.4s</td>
<td>20%</td>
<td>12%</td>
<td>50 (\text{\textmu J/cm}^2)</td>
<td>12 ps</td>
</tr>
<tr>
<td>SAM-1550-23-25.4s</td>
<td>23%</td>
<td>14%</td>
<td>25 (\text{\textmu J/cm}^2)</td>
<td>2 ps</td>
</tr>
<tr>
<td>SAM-1550-35-25.4s</td>
<td>35%</td>
<td>21%</td>
<td>20 (\text{\textmu J/cm}^2)</td>
<td>2 ps</td>
</tr>
</tbody>
</table>

Table 3.1: A summary of the characteristics of SBRs that were tested in this study.
Figure 3.3: The mode-locked optical spectra obtained using SAM-1550-23-25.4s by varying the pump, and hence the intracavity, power. The legend indicates the output power associated with the spectrum. Because the output coupling was unchanged, the output power is proportional to the intracavity power.

SBRs were 4\textit{mm} x 4\textit{mm} in area, 400 \textmu\textit{m} thick, and solder-mounted to gold-coated copper bases that were 25.4\textit{mm} in diameter, 5\textit{mm} thick in the machined recession for the SBR, and 6\textit{mm} thick elsewhere.

3.1.1.2 Results

For each SBR, the pump power was varied by changing the pump diode driving currents. As the pump powers were varied, the following data was collected:

1. Full characterization of the laser operation at maximum spectral width

2. Full characterization of the laser operation at minimum spectral width

3. Partial characterization at various pump powers

The complete sets of data for the full characterizations can be seen in Section F.1.2.
Figure 3.4: The mode-locked optical spectra with the maximum bandwidth obtained using each SBR. The SBRs are indicated by their linear loss and modulation depth. These were obtained at maximum pump power.

Overall, the laser operated in a manner consistent with stretched pulse operation. Variations in the intracavity power, which was done by changing the pump diode current, led to variations in the optical bandwidth. Figure 3.3 shows the mode-locked optical power spectrum for various output power levels.

From the maximum spectral width achievable with each SBR model, shown in Figure 3.4, it was found that the results varied significantly. The lowest modulation depth SBR provided a wide spectrum. The moderate modulation depth models provided even wider spectra and, despite their significantly different recovery times, produced nearly the same spectral shape; the latter point suggests the the SBR temporal recovery is not playing a role in pulse shaping. The spectrum generated from the highest modulation depth SBR was markedly different, both in spectral width and in shape.

Also of note, the spectral feature near 1530 nm shows a shift towards the middle of the spectrum as an increasingly higher modulation depth SBR is used. This indicates a change in the cavity GVD, and hence intracavity pulse chirp. With little chirp, the
Figure 3.5: The bandwidth-limited pulse duration as a function of output pulse energy. “Single” and “Double” refer to single-pulse and double-pulse states. The SBRs are indicated by their linear loss and modulation depth.

The spectral feature’s power will lie near the temporal peak of the pulse where $\frac{\partial f}{\partial t}$ is small, and thus SPM will impart very little frequency shift on the spectral feature. As the cavity GVD is shifted, the spectral feature will move away from the temporal peak of the pulse, and SPM will then impart a frequency shift on that feature. For a spectral feature on the high frequency (low wavelength) side of the spectrum, the feature will temporally be at the trailing edge of the pulse for normal chirp and will be shift down in frequency (up in wavelength), and will lie in the leading edge of the pulse for anomalous chirp and receive a shift up in frequency (down in wavelength). This indicates that the cavity GVD becomes more normal with higher modulation depth SBRs.

Figure 3.5 provides more insight into the differences. All but the high modulation depth SBR provide pulse energy-duration combinations that lie nearly on the same contour. The high modulation depth SBR provides much higher energy pulses, with much longer durations.
The data can be understood by considering the fact that the SBR resonance has an associated phase response that goes as the frequency derivative of the resonance, as described by the Kramers-Kronig relation. This means that the SBR will change the cavity GVD in a way that depends on the modulation depth of the absorber. Specifically, the SBR will exhibit anomalous GVD on the short wavelength side of the resonance, and normal GVD on the long wavelength side of the resonance. Therefore, the lowest modulation depth SBR leads to a cavity with nearly zero GVD, and hence provides pulses with moderate pulse energies and fairly short lengths. The moderate modulation depth SBRs lead to a slightly more net normal GVD cavity, resulting in even shorter pulses, still of moderate energy. The highest modulation depth absorber makes the net cavity GVD decidedly normal, and thus higher pulse energies, longer bandwidth-limited pulse durations, and the characteristic square spectral shapes are observed [70].
Figure 3.7: The output, pre-gain tap, and residual pump powers as a function of the orientation angle of the HWP that varies the output coupling, for a) mode-locked operation, and b) continuous-wave operation. Both data sets are plotted on the same scale. The hollow data markers indicate mode-locked states.

The tap and output power, and the components' losses were used to calculate the fluence incident on the SBR. Figure 3.6 shows the bandwidth-limited pulse duration plotted as a function of fluence. Again, the fluences do not appear to be dependent on the SBR saturation fluence, which suggests that the fluence levels are clamped by the P-APM saturation, and not the SBR. This is further evidence that the P-APM mechanism is shaping the pulse and stabilizing the pulse train.

Finally, the fact that mode-locking depends on the waveplate orientations indicates that P-APM must be playing a role. To explore this in detail, the HWP that controls the output coupling was varied through its range while the output power, pre-gain tap power (which is proportional to intracavity power), and residual pump power were recorded.

This measurement was first done with the input polarization state to the gain fiber biased for mode-locked operation. The results are shown in Figure 3.7a. The lowest output power (50°) occurs with low residual pump and high pre-gain tap power, indicating low output coupling, high intracavity power, and strong gain saturation. As the output coupling is increased, the output power increases, the intracavity power drops, and the residual pump increases as the gain becomes less saturated. As this continues,
at some point (80°) the increased output coupling is overtaken by reduced saturation of, and hence poorer power extraction from, the gain, and the output power begins to decrease. At maximum output coupling (95°), the cavity losses are maximized and the gain is minimized due to poor saturation, and so the laser is oscillating closest to threshold and the output contains significant amounts of ASE.

For comparison, this measurement was also done when the waveplates were biased to launch a linear polarization state into the gain fiber. The results are shown in Figure 3.7b. Because the polarization state launched into the gain fiber is an eigenstate, NPR will not occur. Eventually, residual birefringence (due to fabrication imperfections and mechanical stresses) transforms the polarization into an elliptical state. Because the fiber strain and bends were carefully minimized, the birefringence and therefore NPR should be minimized. As a result, P-APM should be suppressed, and the polarization exiting the gain fiber should be in a nearly linear state. Adjustment of the HWP should then allow the fullest range of output coupling ratios, approaching 0%-100%. At 47° and 137°, the pre-gain tap power shows that the intracavity power is maximized and the output power is essentially zero, indicating near-zero output coupling, and therefore a linear polarization state at the gain fiber output. Note that mode-locking was not observed in any case when the polarization state launched into the gain fiber was a linear eigenstate, showing that the SBR alone was not able to mode-lock the laser.

Coming back to Figure 3.7a, it is clear from the comparison that the polarization state exiting the gain fiber is elliptical. That is, the state cannot be linear because the output power is significantly above zero at maximum output coupling, and the state cannot be circular because the output power varies with HWP orientation angle. The location of the mode-locked state is found over the range of 86° to 90°, which corresponds to significant output coupling and slightly incomplete saturation of the entire length of gain fiber.

3.1.1.3 Summary

These experiments explored the laser dynamics and operation of a combined SBR and P-APM mode-locked fiber laser. The following key results emerge:
1. The SBR GVD becomes a more significant part of the overall cavity GVD as the cavity size is reduced. This was demonstrated in a stretched-pulse laser, but will also be true in any laser where GVD significantly influences the pulse evolution.

2. The SBR enables self-starting.

3. P-APM is still sufficient to dominate the pulse shaping and stabilization against noise.

4. The SBR failed to mode-lock the laser when P-APM was suppressed.

5. This laser is the highest repetition rate EDFL using an SBR and P-APM at 234 MHz, as of the writing of this text.

The second and third points show that the laser used in these experiments combines the best features of SBRs and P-APM mechanisms. The slower, “lumped element” nature of the SBR allows for self-starting even as the cavity length is reduced; P-APM, once mode-locking is started by the SBR, enables sub-100 fs pulses to be generated. Together, these mechanisms allowed the generation of sub-100 fs pulses at higher repetition rates.

3.1.2 Maximizing the Repetition Rate

With the laser cavity presented in Section 3.1.1 serving as a reference, experiments are also being pursued to scale the down the stretched-pulse laser cavity size in both sigma and ring geometries. Both efforts seek to minimize the cavity size by reducing the length of the free-space portion of the cavity through careful opto-mechanical design, and by incorporating the silicon as the primary normal GVD element to allow for inclusion of more gain fiber. The sigma cavity will use P-APM and an SBR, and the ring cavity will rely on P-APM alone. The goals of these experiments are twofold:

- the achievement of >300 MHz fundamental repetition rates
- a greater understanding, through careful characterization, of the limitation(s) that prevents further scaling
This work is ongoing at the time of the writing of this text. Figure 3.8 shows the current miniaturized mount setup that will be used for the sigma and ring cavities.

3.2 200 MHz Fundamentally Mode-locked Fiber Laser with External Repetition Rate Multiplication

To achieve a high repetition rate pulse source, the direct laser output need not be high repetition rate; external multiplication with an interferometer can be used. In these experiments, done in collaboration with Jeff Chen, a 200 MHz fundamentally mode-locked soliton fiber laser was designed and constructed. The output from this laser was then passed through a high-finesse FPI in order to multiply the repetition rate to 2 GHz [71].

3.2.1 The Laser

The soliton laser, at a fundamental repetition rate of 200 MHz, was the highest repetition rate fundamentally mode-locked soliton laser at the time it was reported [68].
The schematic and GVD map for this laser are shown in Figure 3.9.

The laser is pumped with a 974 nm pump diode\textsuperscript{16}. The PBS\textsuperscript{17} provides the output port. Intracavity light goes through a polarizing isolator\textsuperscript{18} and a QWP\textsuperscript{19}, before coupling into the fiber. After a short length of anomalous GVD fiber\textsuperscript{20}, the signal is amplified by a backward-pumped length of highly-doped erbium fiber\textsuperscript{21}. The signal then passes through the wavelength division multiplexer (WDMer)\textsuperscript{22} fiber\textsuperscript{23} and more anomalous GVD fiber\textsuperscript{24} before exiting through another collimator\textsuperscript{25}. Finally, the signal passes through a QWP\textsuperscript{26} and HWP\textsuperscript{27} before reaching the PBS output coupler.

The laser mode-locks by P-APM; this is created by the PBS and waveplates. More anomalous fiber is placed after, rather than before, the gain fiber to increase the effective

\begin{footnotesize}
\begin{itemize}
\item\textsuperscript{16}Bookham, Model: LC96UF74-20R, Serial: SO224651.001 0745
\item\textsuperscript{17}Newport Corporation, Model: 05FC16PB.9, broadband polarizing cube beamsplitter
\item\textsuperscript{18}Isowave, I-15-UHP4, Serial: 24558
\item\textsuperscript{19}Meadowlark Optics, Inc., Model: NQM-50-1550
\item\textsuperscript{20}Corning, Inc., Model: SMF-28e
\item\textsuperscript{21}Liekki Corporation, Model: ER80-8/125
\item\textsuperscript{22}Gould Technology, LLC, Model: 40-22798-55-11601, 980 nm / 1550 nm WDM, 1 x 2, Fiber: Corning, Inc., Model: HI 1060
\item\textsuperscript{23}Corning, Inc., Model: H1060
\item\textsuperscript{24}Corning, Inc., Model: SMF-28e
\item\textsuperscript{25}OZ Optics, Model: LPC-02-1300/1500-9/125-S-1.4-6.2AS-60-X-1-1
\item\textsuperscript{26}Meadowlark Optics, Inc., Model: NQM-50-1550
\item\textsuperscript{27}Meadowlark Optics, Inc., Model: NHM-50-1550
\end{itemize}
\end{footnotesize}
nonlinearity; this keeps the pulse energies smaller and eases the demands on the pump and gain fiber.

The optical power spectrum of the laser output has a FWHM bandwidth of 19.3 nm, and can be seen on the left in Figure 3.10. The resonant sidebands [72] can be clearly seen, indicating soliton operation. The interferometric autocorrelation measurement is shown on the right in Figure 3.10 and corresponds to a pulse intensity FWHM duration of 130 fs. The time-bandwidth product of 0.31 indicates a bandwidth-limited hyperbolic secant pulse. The output power was measured to be 33.6 mW, or 173 pJ per pulse. Figure 3.11 shows the RF spectrum and oscilloscope trace of the detected pulse train. The bandwidth of the former measurement is determined by the detector’s 15 GHz bandwidth 28, and the latter is determined by the oscilloscope 29 bandwidth of 300 MHz.

To arrive at the 194 MHz operating point, the laser cavity was systematically reduced in size by removing fiber 30 from the cavity. As shown in Figure 3.12, the Kelly sidebands move further away from the center of the spectrum as the length is reduced; the length reduction reduces the nonlinear phase shift per roundtrip, and thus shorter

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28 Discovery Semiconductor, Model: DSC-40S, Serial: 402307
29 Tektronix, Inc., Model: TDS 3032B, Serial: B027127
30 Corning, Inc., Model: SMF-28e
Figure 3.11: RF spectrum (left) and oscilloscope trace (right) of the detected pulse train.

Figure 3.12: Changes in the optical power spectrum as the cavity size was reduced.
Figure 3.13: The external repetition rate multiplication setup.

pulses can be sustained. Ultimately, the available pump power limited reductions in the cavity size; the final cavity required the single pump to be running at 95% of it’s maximum drive current.

3.2.2 External Multiplication

A copy of the 200 MHz laser was constructed in order to do external repetition rate multiplication to 2 GHz using a FPI. The setup is shown in Figure 3.13. The laser was built in a sigma configuration to allow for repetition rate control using a piezo-driven mirror. The output from this laser was put through a FPI to multiply the repetition rate. The laser output passes through a lens\textsuperscript{31} to match the transverse mode to the FPI mode. A Brewster plate\textsuperscript{32} is placed inside the FPI to enable the use of the Hänsch-Couillaud technique \cite{73} to lock the laser repetition rate to the FPI free-spectral range (FSR). The FPI output power is then used to control the pump current and reduce fluctuations\textsuperscript{33} in the carrier-envelope offset frequency by keeping the laser modes aligned with the FPI resonances.

\textsuperscript{31}Thorlabs, Inc., Model: LA1172, BK7 uncoated plano-convex, \( f = 40 \text{ cm} \)

\textsuperscript{32}Unknown origin, 100 \( \mu \text{m} \) thick fused silica glass

\textsuperscript{33}This feedback system does not provide absolute stabilization of the carrier-envelope offset frequency.
Figure 3.14: The optical power spectrum of the laser, and multiplied pulse trains for (measured) finesses of 200 and 2000.

The FPI structure was constructed from Super-Invar\textsuperscript{34} metal for improved stability, and the steel mirror mounts were attached to the structure such that thermal expansion common to both mirror mounts would cancel and thus reduce fluctuations in the FPI FSR. Dielectric mirrors, dispersion-flattened at 1560 nm and with reflectivities of 99\%\textsuperscript{35} and 99.9\%\textsuperscript{36} were used, which corresponds to a calculated cavity finesse of 300 and 3000, respectively.

When the laser repetition rate slightly detunes from the FSR of the FPI, the laser modes at the center of the spectrum may remain lined up with FPI resonances, while the modes at the edge of the optical spectrum will detune and be attenuated, ultimately narrowing the optical spectrum and lengthening the transform-limited pulse duration. Figure 3.15 shows the optical power spectrum of the laser, and the optical spectra of the multiplied pulse train at two different finesses. The Kelly sidebands are significantly attenuated, but the pulse spectrum is largely intact, only narrowing slightly.

To verify that the pulses exiting the FPI remained transform-limited and short, an

\[ \Delta L = 3.5 \times 10^{-7} \frac{1}{T_0} \text{ over } -55^\circ C \text{ to } 95^\circ C, \text{ crossing zero near } 20^\circ C. \]

\textsuperscript{34} Advanced Thin Films, Model: Custom, 50 cm concave curvature, 1560 nm HR coating, low dispersion over 100 nm; flat, 1560 nm AR coating; fused silica substrate, 0.165” thick, 0.5” diameter

\textsuperscript{35} Advanced Thin Films, Model: Custom, 50 cm concave curvature, 1560 nm HR coating, low dispersion over 100 nm; flat, 1560 nm AR coating; fused silica substrate, 0.165” thick, 0.5” diameter
IAC of the direct laser output and the low-finesse FPI output were done. Because the FPI attenuates all but every $10^{th}$ mode, the FPI output average power is approximately ten times less than the laser output. Additionally, the pulse energy is further reduced by a factor of ten from the increased repetition rate. As a result, the FPI output needed to be amplified using a GVD-compensated erbium amplifier for the IAC measurement. Care was taken to ensure that the nonlinearity in the amplifier was negligible. Because the pulse spectrum exceeds the erbium gain bandwidth, some spectral narrowing is expected in the amplification process. The results of the measurement, shown in Figure 3.15, show that the 130 $fs$ pulse lengthens to 150 $fs$. This is largely attributed to the spectral narrowing of the amplifier, and thus the pulse is essentially bandwidth limited.

The detected RF spectrum of the pulse train exiting the FPI is shown in Figure 3.16. For the low finesse case, the suppression of the unwanted modes is approximately 24 $dB$, which corresponds to 30 $dB$ of suppression in the optical comb. For the high finesse case, the suppression is below the noise floor of the RF spectrum analyzer, which means the suppression is greater than 40 $dB$ corresponding to greater than 46 $dB$ of suppression of the optical modes. The measurements correspond to cavities with fineses of approximately 200 and 2000, respectively. Timing jitter was measured using
Figure 3.16: The RF spectrum of the detected FPI-multiplied pulse train for the low finesse (top) and high finesse (bottom) FPI.
a signal source analyzer\textsuperscript{37} and resulted in 27 $fs$ of jitter over 1 $kHz$ to 10 $MHz$. RIN measurements using a vector signal analyzer\textsuperscript{38} show a RIN of 0.23\% over 1 $kHz$ to 10 $MHz$ for the high finesse case.

\textbf{3.2.3 Summary}

This section presented a 200 $MHz$ fundamentally mode-locked EDFL, which at the time this text was written was the highest repetition rate soliton EDFL mode-locked with P-APM. With this laser, external multiplication of the repetition rate was accomplished using a FPI, resulting in a 150 $fs$ pulse source at 2 $GHz$ with low timing jitter and RIN. Amplification was demonstrated without degradation to the pulses. As a result, this high repetition-rate pulse source is compatible with carrier-envelope phase stabilization techniques.

\textsuperscript{37}Agilent Technologies, Inc., Model: 5052A, Signal Source Analyzer

\textsuperscript{38}Agilent Technologies, Inc., Model: 89410A, Serial: US40514647, Options: AY7AYAAAYB, “Vector Signal Analyzer”, DC-10 $MHz$
Chapter 4

Experiments with Frequency Comb Stabilization at High Repetition Rates

This chapter describes experiments for the stabilization of high repetition rate EDFLs. These experiments include femtosecond pulse amplifier development and the generation of octave-spanning SC spectra [74], which were subsequently used in SHG carrier-envelope offset frequency and DFG repetition rate stabilization experiments.

4.1 Octave-spanning Supercontinuum Generation

Generally, mode-locked fiber lasers do not directly generate the spectral widths necessary for $f_{\text{ceo}}$ stabilization via SHG, nor for repetition rate stabilization via DFG. Therefore, in order to use these stabilization methods, spectral generation outside of the cavity must be done [58, 62, 63, 75].

External spectral generation is done by taking advantage of optical nonlinearities, which in turn require sufficiently high pulse peak powers to generate the needed spectral components. At low ($< 100 \, MHz$) repetition rates, this can be done using widely available highly nonlinear fibers (HNLFs) [76]. As the repetition rate is increased and the pulse energies are kept constant, the average power increases. Because of doping
and pump power limitations, the average powers achievable with EDFAs are limited. As a result, reaching sufficiently high pulse energies and peak powers at high repetition rates using erbium-doped fiber (EDF) is challenging.

As such, it is important to optimize the external spectral generation process. To do this, the amplification process must be optimized for both average power as well as pulse quality, and the choice of HNLF must provide the best conversion of power to the desired wavelength ranges.

### 4.1.1 Amplifier Development

The goal of the amplifier development effort was to design an amplifier that enables nonlinear spectral generation suitable for 1f-2f and DFG locking. This means seeking maximum pulse energy while maintaining pulse duration and quality. Based on various results in the literature [62–65], the target pulse duration was chosen to be below 100 fs.

To keep the resulting system design more accessible, the following design constraints were followed:

- When possible, use standard components for compatibility with conventional fibers, and cleaving and splicing technologies.
- Limit the design to single-mode fibers (to avoid modal beating issues and avoid less-standard multi-clad fibers).

The following lists the important factors that needed to be considered in the amplifier development:

- **Gain and Loss**: Obviously, maximizing gain and minimizing loss is desired.
- **ASE**: Erbium’s long excited-state lifetime\(^1\) ensures strong gain saturation and hence nearly complete extraction of the power stored in the gain medium by the signal. As a result, ASE issues were assumed negligible.

\(^1\)Approximately 10 ms.
• **Gain Filtering:** The bandwidth associated with the target pulse duration of 100 fs is greater than the gain bandwidth of erbium, and thus gain filtering would tend to lengthen the amplified pulses to more than 200 fs. Therefore, some form of *compressible* spectral generation is required to maintain the 100 fs pulse durations. This rules out CPA approaches, outlined in Section 2.3.2.

• **Nonlinearity and Dispersion:** To generate spectrum, nonlinearity is required. Dispersion and chirp will need to be adjusted to optimize the spectral width and phase.

Because of the need for compressible spectral generation, spectral generation using anomalous fiber that takes advantage of SPM, higher-order nonlinearities, and soliton effects must be avoided due to the nonlinear chirp that tends to result. Because spectral generation done solely by SPM in normal GVD fiber can result in a linear, compressible normal chirp [67], the approach taken was to amplify in normal GVD gain fiber.

Another practical challenge lies in low-loss post-amplification compensation of the resulting normal chirp. At best, a four diffraction grating compressor will lead to unacceptable losses of at least 20%, and anomalous fibers will lead to higher-order nonlinearities that will destroy the pulse. Brewster-cut prisms remain as the only option. Better still would be to provide the appropriate pre-chirp on the fiber so that the pulse would emerge from the amplifier as close to bandwidth-limited as possible.

Because of the complexity that arises from the coupling of GVD, nonlinearity, gain, loss, and higher-order effects, numerical simulations of the pulse evolution through the amplifier is useful. To that end, split-step Fourier simulations of the amplification process were conducted.

A set of generic simulations were done to build and verify a qualitative understanding of the pulse evolution. Figure 4.1 shows a plot of the evolution of the 100 fs, 20 pJ pulse as it passes through 25 cm of anomalous GVD fiber, followed by 1 m of normal GVD gain fiber. Two stages of evolution can be seen. First, the pulse chirps anomalously due to GVD, and the spectrum narrows slightly due to SPM acting on the anomalously chirped pulse. Upon entering the gain fiber, the pulse begins to compress, both due
Figure 4.1: Results of split-step simulations. A 100 fs, 20 pJ pulse is propagated through 25 cm of Corning SMF-28e, followed by 1 m of normal GVD gain fiber. The first and second plots, respectively, show the pulse duration and spectral bandwidth, and the pulse peak power and energy, as a function of propagation distance.

to the normal GVD and SPM, and the spectrum begins to narrow significantly as the pulse energy increases. Eventually, the pulse is optimally compressed, after which the pulse received a normal chirp from GVD and SPM and the spectrum broadens through the SPM acting on the normal chirp.

With a general approach to pulse amplification, and a qualitative understanding of the expected pulse evolution, a first generation amplifier design was studied.

4.1.1.1 First Generation Design - Multi-stage

For the first generation amplifier, a multi-stage design was pursued. Through construction and testing of the amplifier design, the following advantages and disadvantages were identified:

Positives:

- low risk of pump damage
- no need for pump isolators
- scalable
Negatives:

- splice and isolator losses limit power performance
- NPR scrambles polarization
- polarization independent isolators induce polarization mode dispersion (PMD), which scrambles polarization

To explore the powers that could be realistically achieved with this approach, a three-stage version was built. Figure 4.2 shows a schematic of the amplifier. The input pulses are pre-chirped with anomalous fiber$^2$. Three stages are then used for amplification, each optimized based on the input power to the stage. Each consists of a 500 $mW$, 977 nm pump laser$^3$, an evanescent 980 nm / 1550 nm WDM coupler$^4$, normal GVD erbium gain fiber$^5$, and a polarization-insensitive isolator$^6$. The amplified pulses are then collimated into free-space, and pass twice through a pair of silicon Brewster-cut prisms$^7$ to implement a 4-prism compressor.

Using the 42 MHz source referenced in Appendix G.1, an amplified pulse train was obtained. The input source power used was 1.0 $mW$, resulting in a maximum achieved average power of 241.1 $mW$ at the output of the amplifier. A pre-chirp fiber$^8$ was placed at the input of the amplifier, and was adjusted in length to optimized the

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$^2$Corning, Inc., Model: SMF-28e  
$^4$Lightel Technologies, Inc., Fiber: OFS, Model: BF05635-02; Serial #1: 01120394, Serial #2: A0-02807, Serial #3: 01116650  
$^5$Liekki Corporation, Model: ER110-4/125  
$^6$Oplink, Fiber: Corning, Inc., Model: SMF-28e, Serial #1: 155175, Serial #2: Z1094863  
$^7$Janos Technology, Inc., Model: 901040-01, 38.1 mm x 25.4 mm x 21.057 mm  
$^8$Corning, Inc., Model: SMF-28e
Figure 4.3: Autocorrelation of the pulses exiting the “1st Generation” amplifier. A gaussian intensity FWHM fit of the central peak corresponds to a 110 fs pulse.

autocorrelation trace. At a length of 248 cm, the autocorrelation shown in Figure 4.3 was obtained. The trace suggests a 110 fs pulse; however, the large pedestal suggests a poor pulse shape. The pedestal’s square shape and amplitude of roughly 33% of the trace peak suggests three closely-spaced pulses with approximately the same energy. The structure and width of the corresponding spectrum, shown in Figure 4.4, also suggests higher-order nonlinear effects that would lead to pulse break-up. One can clearly see that the prism compressor bandwidth of operation is filtering the spectrum.

The results indicate three problems. First, the spectral and temporal quality of the pulse train is degraded. Based on the asymmetry towards longer wavelengths, Raman shift appears to be playing a significant role. The spectral distortion and fine structure suggest that soliton break-up may be occurring. It was necessary, given available components, to have approximately 10-cm leads of anomalous GVD fiber on the isolators; these short fiber lengths may be responsible for the accumulation of Raman and higher-order nonlinear effects that lead to the shift and fine structure.

Second, the amplifier losses limited the achievable output power. Splice loss is heavily dependent on the selection of WDM fibers, gain fibers, and splicing capabilities.
Figure 4.4: Optical power spectrum of the pulses (black) at the output of the “1st Generation” amplifier, and (red) at the output of the prism compressor.

Additionally, even though the splice loss may be a few percent per splice, the fact that the total number of loss elements scales with the number of stages means that loss quickly limits the power performance of the amplifier.

Using Equation 2.31 and making the following assumptions, based on component specifications and measurements:

- 0.132 dB of loss per splice ($T = 0.970$)
- 1.9 dB of isolator loss ($T = 0.646$)
- Two splices before the gain ($T_{before} = 0.941$), and one splice and the isolator after the gain ($T_{after} = 0.626$)
- 500 $mW$ pump power (980 nm), 55% quantum efficiency$^9$

the scaling of the current design with the components available at the time show that the maximum output power obtainable is 337 $mW$.

$^9$The highest reported values are approximately 89% [77]. Typical values are significantly lower.
Finally, polarization-scrambling mechanisms mentioned above prevent the output pulses from being TM polarized, meaning that the Brewster prisms will induce significant losses. Even when optimized, at least 33\% of the power was lost to reflections at the surfaces of the prisms. Taken together, the inherent disadvantages of this approach led to a single-stage design.

4.1.1.2 Commercial Amplifier

To provide an initial evaluation of a single-stage amplifier, and to allow other experiments to be done, a commercial high-power single-mode amplifier\textsuperscript{10} designed for sub-100 fs pulse amplification was purchased. A schematic of the amplifier is shown in Figure 4.5.

To test the operation of the amplifier, the 200 MHz laser used in Section 3.2 was provided as the input. A total of 5.0 mW of power from the laser was coupled into the amplifier input. The post-amplifier fiber\textsuperscript{11} connected to the amplifier’s fiber output port was 27.0 cm long. This fiber was a proxy for the length of fiber\textsuperscript{12} spliced at the input of the HNLFs used for SC generation, and ensured that the following optimization of the pre-chirp fiber would give an optimized amplified pulse at the nonlinear fiber input.

\textsuperscript{10}Menlo Systems GmbH, Model: AC-1550, Serial: 014
\textsuperscript{11}Corning, Inc., Model: SMF-28e
\textsuperscript{12}OFS fiber, Model: unknown (verified with manufacturer that fiber is equivalent to SMF-28e)
Figure 4.6: Optical power spectrum of the amplified pulse train from the commercial amplifier.

The experimentally determined pre-chirp fiber\textsuperscript{13} length was 156.75 cm, which provides a total GVD of -36,000 $fs^2$.

The resulting output optical power spectrum and autocorrelation trace of the pulse are given in Figure 4.6 and Figure 4.7, respectively. Both traces show characteristics similar to the first generation amplifier performance, presented in Section 4.1.1.1. The output power of the amplifier was 274.5 mW, also similar to the first generation amplifier. The main advantage of this amplifier is the fact that the output is linearly polarized and compressed, thus removing the need for a prism compressor. As a result, the effective output power of this amplifier was roughly 75\% greater than the first generation design.

\textbf{4.1.1.3 Second Generation Design - Single-stage}

Given the lessons learned from the first generation and commercial amplifiers, a second generation single-stage, multi-pump design was considered. This design addresses issues that limited the performance of the first generation design by using:

\textsuperscript{13}Corning, Inc., Model: SMF-28e
Figure 4.7: Autocorrelation trace (black) and hyperbolic secant fit (red) of the amplified pulses from the Menlo Systems amplifier. The fit gives a pulse intensity FWHM of 50 fs.

- a single-stage design to minimize number of splices and isolators, and hence minimize their loss
- polarization-maintaining fiber, to avoid NPR problems
- polarization sensitive (i.e. polarizing) isolators. Polarization insensitive isolators typically have 10-20 fs of PMD, which corresponds to a polarization mode phase delay of up to 10π at 1550 nm; with polarization maintaining fibers (PMFs), polarization sensitive isolators can be used, and thus additional PMD can be avoided.

The signal path will consist of PMF and have polarization sensitive isolators at both ends of the amplifier. WDMs will be placed for pumping in both the forward and backward direction with polarization-multiplexed 980 nm and 1480 nm laser pump diodes. The multi-wavelength pumping allows for greater effective pump powers, and hence more signal power out of the amplifier. Because of the bi-directional pumping,

\[14\] The multi-wavelength pumping will, in general, lead to a different gain spectrum; these differences are not of concern here. The goal is to provide ways to incoherently multiplex together pump power.
polarization insensitive isolators will be needed to protect the pumps from damage. The current design plan is shown in Figure 4.8.

With six splices in the signal path, and four splices in each pump path, splice loss is still important. This will be largely determined by fibers selected. This design calls for PMF only, and only normal GVD fibers in the signal path. The choice of WDM coupler mechanism also affects fiber selection - evanescent-coupling-based WDMs show lower loss and handle higher power than micro-optic-based WDMs, whereas micro-optic-based WDMs better accommodate different fibers\textsuperscript{15} on the same component and can typically be made with sharper spectral features. The inclusion of the backward 1480 nm pump will likely require a micro-optic WDM to minimize signal loss due to the WDM filter function, and may not be included in the final system. The fiber choices are further limited by the fact that the pump laser diodes are often only available with specific fibers as pigtails.

\textsuperscript{15}Evanescent-coupling-based devices require the propagation constants (eigenvalues) to be similar in both waveguides for coupling to occur.
Based on some reasonable assumptions and component loss specifications provided by vendors\textsuperscript{16}, an estimate of the achievable powers was done. Various loss specifications were collected from a number of vendors; the components that led to the highest amplifier output power were used in the analysis. The input signal power is taken to be 5 $mW$ in order to ensure gain saturation, and hence efficient power extraction, while keeping the source power requirement modest. Pump diode output powers of 360 $mW$ at 1480 $nm$\textsuperscript{17} and 660 $mW$ at 976 $nm$\textsuperscript{18} were assumed, and the highest powers available when this work was done. A quantum efficiency of 20%-40% is assumed, based on the data from the multi-stage design. All other values used are shown in Table 4.1.

The model for the power estimate is shown in Figure 4.8. The resulting output powers of this model gives a range of 229.2 $mW$ to 341.6 $mW$ for a range of 20% to 40% quantum efficiency, respectively, using the worst case component losses. For the typical component losses, the output powers vary from 341.6 $mW$ to 456.4 $mW$ for a range of 20% to 40% quantum efficiency, respectively.

It is clear that the amplifier performance still depends significantly on the losses, as well as on the achievable quantum efficiency. An improvement over the commercial system on the order of 50% to 75% more power can be expected. To achieve significantly higher powers approaching 1 $W$, normal GVD cladding-pumped gain fibers\textsuperscript{19}, which can take advantage of larger pumps, should be considered.

4.1.1.4 Summary

Octave-spanning spectral generation requires more average power as repetition rates are increased, thus amplification becomes necessary. Efforts to develop ultrashort pulse amplifiers for erbium-based sources using commercially available components were pre-

\textsuperscript{16}Agiltron, Inc.; AOC Technologies, Inc.; Lightel Technologies, Inc; Micro-Optics, Inc.; Novawave Technologies, Inc.; SIFAM Fibre Optics

\textsuperscript{17}Furukawa America, Inc., Model: FOL1425RUX-617-1480

\textsuperscript{18}JDS Uniphase Corporation, Model: 30-7602-660. Bookham offers 750 $mW$ pump diodes at 974 $nm$, but these were measured by the author to consistently, from module to module, put out a maximum of 675 $mW$. JDS Uniphase pump diode performance typically exceeds specifications by 10-20% in the author’s experience.

\textsuperscript{19}These were considered, but as of the time this thesis was written, normal GVD cladding-pumped erbium fibers were not commercially available and could not be located.
<table>
<thead>
<tr>
<th>Component</th>
<th>Loss - Worst Case</th>
<th>Loss - Typical Case</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarization Sensitive Isolator (1550 nm)</td>
<td>see WDM (1480 nm/1550 nm)</td>
<td>see WDM (1480 nm/1550 nm)</td>
<td>Lightel Technologies, Inc.; integrated with WDM (1480 nm/1550 nm)</td>
</tr>
<tr>
<td>Polarization Insensitive Isolator (980 nm)</td>
<td>-1.0 dB</td>
<td>-0.7 dB</td>
<td>Lightel Technologies, Inc.</td>
</tr>
<tr>
<td>Polarization Insensitive Isolator (1480 nm)</td>
<td>-0.65 dB</td>
<td>-0.65 dB</td>
<td>Lightel Technologies, Inc.</td>
</tr>
<tr>
<td>WDM (980 nm/1550 nm)</td>
<td>-0.6 dB (960-1000 nm); -0.8 dB (1480 nm); -0.8 dB (1500-1600 nm)</td>
<td>specification not provided, assume worst case</td>
<td>Lightel Technologies, Inc.; micro-optic type; 1480 nm loss assumed from 1550 nm loss</td>
</tr>
<tr>
<td>WDM (1480 nm/1550 nm)</td>
<td>-0.6 dB (1450-1480 nm); -1.1 dB (1550-1600 nm)</td>
<td>-0.4 dB (1450-1480 nm); -0.8 dB (1550-1600 nm)</td>
<td>Lightel Technologies, Inc.; micro-optic type;</td>
</tr>
<tr>
<td>Polarization Beam Combiner (1480 nm)</td>
<td>-0.4 dB</td>
<td>-0.4 dB</td>
<td>evanescently coupled mode-type, assumed from 980 nm combiner specifications</td>
</tr>
<tr>
<td>Polarization Beam Combiner (980 nm)</td>
<td>-0.4 dB</td>
<td>specification not provided, assume worst case</td>
<td>SIFAM Fibre Optics (via Optimark Fiber Optics); evanescently coupled mode-type</td>
</tr>
<tr>
<td>Passive Fiber Splices</td>
<td>n/a</td>
<td>-0.1 dB</td>
<td>assumes fiber modes are well-matched, number based on author’s experience</td>
</tr>
<tr>
<td>Passive-to-Gain Fiber Splicers</td>
<td>n/a</td>
<td>-0.45 dB</td>
<td>assumes fiber modes are not well-matched, number based on author’s experience</td>
</tr>
</tbody>
</table>

Table 4.1: Worst and typical loss values used in the power calculation. The source of the estimate is given as well.
sented. Issues of soliton formation and higher-order nonlinearities in anomalous GVD fibers, PMD, and otherwise small loss contributions were found to limit achievable powers. Amplifier topologies were discussed, and an amplifier design that mitigates these limitations was presented. Achieving average powers exceeding 500 mW are expected to require non-standard components, such as PMFs with normal GVD, and cladding-pumped erbium or erbium-ytterbium gain fibers.

4.1.2 Nonlinear Spectral Generation

Using the ultra-short pulse amplifiers available, a variety of nonlinear fibers were tested in order to find those most suitable for the spectral generation that is needed for carrier-envelope offset frequency and DFG locking systems. These fibers included two (2) samples from Furukawa, three (3) samples from OFS, and two (2) PCF samples from Prof. Fiorenzo Omenetto at Tufts University.

Depending on the amplifier and nonlinear fiber being tested, either the free-space setup or fiber-coupled setup, both shown in Figure 4.9, was used. Both systems isolate the laser output, and optionally pass the output through pre-chirping fiber. The signal is then amplified and optionally passed through post-amplification fiber.
For the free-space configuration, the signal is then coupled into free-space and passed through a QWP and HWP to adjust the polarization. The signal is then focused into and out of the nonlinear fiber using microscope objectives and a combination of mechanical and piezo-driven stages. Finally the light is coupled into the optical spectrum analyzer and/or spectrometer\textsuperscript{20}. For the fiber-coupled configuration, the light passes through a strain-type polarization controller to adjust the polarization. The nonlinear fiber, spliced to connectors at both ends, is connected at one end to the amplifier output, and at the other end to the optical spectrum analyzer and/or spectrometer. Note that the spectrometer measurements are not corrected for the detector responsivity; the high frequency side of the spectra are actually higher power than the data indicates.

4.1.2.1 Furukawa PM-HNLF

Two variations of PM-HNLF were obtained from Furukawa through Dr. Shunichi Matsushita. Sample \#2\textsuperscript{21} consistently produced results that were slightly inferior to Sample \#1\textsuperscript{22}, and thus only results using Sample \#1 are provided.

**First Generation** Using the stretched-pulse laser described in Section G.1 and the first generation amplifier from Section 4.1.1.1, SC was generated with the first generation amplifier. The beam exiting the prism compressor, whose parameters are described in Section 4.1.1.1, was coupled into an approximately 50 cm piece of Furukawa HNLF \#1, which was then spliced to a short FC/APC-type fiber\textsuperscript{23} connector. The resulting SC spectrum is shown in Figure 4.10.

**Menlo Systems** The 200 MHz laser outlined in Section G.2 and the commercial amplifier were used to generate the SC shown in Figure 4.11. The spectrum spans an octave between 15 dB and 20 dB down from the peak.

\textsuperscript{20}Acton Research Corporation, Model: SpectraPro-275; Detector: Thorlabs, Inc., Model: DET10D, IR-enhanced InGaAs

\textsuperscript{21}\beta_2=, \beta_3 =, \eta =

\textsuperscript{22}\beta_2=, \beta_3 =, \eta =

\textsuperscript{23}Corning, Inc., Model: HI1060
Figure 4.10: SC spectrum generated using the first generation amplifier and Furukawa HNLF #1 (red) and the amplified source spectrum (blue).

Figure 4.11: Supercontinuum spectrum generated using the commercial amplifier and Furukawa HNLF #1 (red) and the amplified source spectrum (blue).
4.1.2.2 OFS HNLFs

Three highly nonlinear fibers were obtained from OFS, through Dr. Lars Grünern-Nielson. These fibers came out of work on optimizing spectral generation using ultraviolet (UV) exposure [76].

Two of the samples were connected to the amplifier output using fiber connectors. Approximately 10 cm of nonlinear fiber was used, with approximately 28 cm of SMF-28e-equivalent fiber at the input, and nearly 1 m of SMF-28e equivalent fiber at the output.

Type B Figure 4.12 shows the spectrum generated using the 200 MHz laser in Section 3.2, the commercial amplifier in Section 4.1.1.2, and OFS HNLF for SC Type B fiber. This spectrum provided the best spectrum for both SHG carrier-envelope offset frequency and DFG repetition rate locking. For SHG carrier-envelope offset frequency locking, the optimum choice of frequencies was approximately 296.4 THz (1012 nm) and 148.2 THz (2024 nm). DFG repetition rate locking could then be achieved simultaneously with 277.3 THz (1082 nm) and 188.7 THz (1590 nm).
Figure 4.13: Supercontinuum spectrum generated using the commercial amplifier and OFS HNLF for SC Type C (red) and the amplified source spectrum (blue)

**Type C** Figure 4.12 shows the spectrum generated using the 200 MHz laser in Section 3.2, the commercial amplifier in Section 4.1.1.2, and OFS HNLF for SC Type C fiber. While this fiber generated higher-frequency spectral components, the structure of the spectrum was considered less-suitable for both SHG carrier-envelope offset frequency and DFG locking.

**4.1.2.3 SF6 PCF**

PCFs made of tellurite and SF6 glass and provided by Prof. Omenetto from Tufts University were also tested. Well-behaved spectral generation over many octaves using a sub-centimeter length of fiber and an optical parametric oscillator (OPO) for the input signal was demonstrated using the SF6 PCF [78]. The structure of the tested SF6 PCF fiber is shown in Figure 4.14.

To test the fiber’s performance, the free-space coupling setup shown in Figure 4.9 was used. The laser described in Section G.2 and the commercial amplifier described in Section 4.1.1.2 were used. Microscope objectives with 0.65 NA were used for optimal power coupling. A total of 205 mW of power was incident on the coupling system; a
Figure 4.14: Image of the end of a cleaved piece of SF6 PCF.

Figure 4.15: Supercontinuum spectrum generated using the commercial amplifier, as measured by the optical spectrum analyzer (black) and spectrometer (gray).
total of 137 $mW$ is estimated to have been coupled into the SF6 PCF. The resulting spectral measurements are shown in Figure 4.15. The spectrometer$^{24}$ data was taken using a lock-in amplifier$^{25}$, for which the internal transimpedance gain was set high in order to reduce the noise floor; the higher power portion of the spectrum, therefore, caused the transimpedance amplifier to reach its rails, thereby clipping the spectral data. The two traces indicate spectrum generated from 1250 $nm$ to 2000 $nm$ at 30 $dB$ down from the peak.

The tellurite PCF produced narrow SC at best, and so results are not presented.

4.1.2.4 Summary

With the amplified pulse energies and duration available, a variety of HNLFs and PCFs were tested for their ability to generate broad SC at high repetition rates. The OFS HNLF B fiber resulted in the broadest spectral width, reaching an octave at 10-15 $dB$ down from the peak. Frequency comb stabilization using SHG and DFG techniques is possible at these power levels.

4.2 SHG Carrier-Envelope Offset Frequency Locking

An SHG carrier-envelope offset frequency locking system was constructed to lock the carrier-envelope offset frequency of the laser presented in Section 3.2. For this system, a laser source, a spectral generation system, a 1f-2f interferometer, and locking electronics were required. In order to provide a carrier-envelope offset frequency that is stable enough to enable locking, the laser was:

- placed on a stack of interleaved foam$^{26}$ and lead foil layers$^{27}$

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$^{24}$Acton Research Corporation, Model: SpectraPro-275; Detector: Thorlabs, Inc., Model: DET10D, IR-enhanced InGaAs

$^{25}$Stanford Research Systems, Model: SR510, Serial: 1251

$^{26}$McMaster-Carr, Part: 8722K922, polyethylene foam sheet, 36"×24"×0.25"

$^{27}$Unknown origin
Figure 4.16: The 1f-2f interferometer schematic (left) and photograph (right) for the carrier-envelope locking system. On the former, the red line indicates a low frequency beam, the blue line indicates a high frequency beam, and the rainbow-colored line indicates a beam composed of the full SC spectrum.

- placed inside a plastic box\textsuperscript{28}

- stabilized by a temperature control system\textsuperscript{29}

Based on qualitative observations, the reduction of air currents provided by the box seemed to have the greatest effect on stability. Ambient vibrations of the optical table had a lesser effect on the unlocked beat stability. The octave-spanning spectrum that the laser generated is shown in Figure 4.12, and its generation is described in Section 4.1.2.2.

\textsuperscript{28}Home-built, machined aluminum edges, with 0.25" thick polycarbonate sheeting from Modern Plastics. Box was not air-tight, but care was taken to minimize the size of any openings.

\textsuperscript{29}Minco, Inc.: Heating Element Model: HK5184R11.7L36A, Kapton foil heating element, No backing, 10"x10", 36" leads, $R = 11.7\Omega$; Sensor Model: TS665 TFY36A, Thermistor ribbon, 36" leads, no adhesive; Controller Model: CT325-TF-1-A-1, 4.75-10 VDC across heating element
A schematic of the 1f-2f interferometer is shown in Figure 4.16. In this system, \( \lambda_{1f} = 2024 \text{ nm} \) and \( \lambda_{2f} = 1012 \text{ nm} \) were chosen for locking, based on the spectral power distribution of the supercontinuum. The octave-spanning spectrum passes in fiber through a strain-type polarization controller\(^{30}\), which is used to adjust the polarization of the \( \lambda_{1f} \) light, and is collimated\(^{31}\) in free-space. The beam then passes through a dichroic\(^{32}\) that reflects \( \lambda_{1f} \) and passes \( \lambda_{2f} \). In the \( \lambda_{2f} \) path, the polarization of \( \lambda_{2f} \) is adjusted with waveplates\(^{33}\), and the path length is adjusted using a manual linear translation stage. In the \( \lambda_{1f} \) path, two silver mirrors are used to steer the \( \lambda_{1f} \) beam. Both beams are recombined using another identical dichroic. This interferometer allows the temporal and spatial overlap of the \( \lambda_{1f} \) and \( \lambda_{2f} \) light to be adjusted to maximize the beat signal. The recombined collinear beam then is focused into and out of the periodically-poled lithium niobate (PPLN) grating\(^{34}\) using 60 mm focal length plano-convex lenses\(^{35}\). To achieve the quasi-phase matching necessary for generating the second harmonic of \( \lambda_{1f} \), the 29.5 \( \mu \text{m} \) grating is used at a temperature of 160\( ^\circ \text{C} \). The collimated light exiting the PPLN is finally filtered with an optical bandpass filter\(^{36}\) angle tuned to 1012 nm, focused with a lens\(^{37}\) to ensure that all of the power is incident on the detector’s active area, and detected with an InGaAs avalanche photodiode (APD)\(^{38}\). Because of the square-law nature of the detector, the two optical signals are effectively mixed, and their mixing products show up in the generated RF electrical signal.

In order to observe the beat signal on an RF spectrum analyzer, RF amplification is required because the noise floor of the RF spectrum analyzer is typically 40-50 \( \text{dB} \) above the detection noise floor in this case. In this system, the generated RF beat signal was amplified with built-in amplification in the detector unit, and with a subsequent

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\(^{30}\)General Photonics Corporation, Model: PLC-003-S-25

\(^{31}\)Thorlabs, Inc., Model: F240APC-1550

\(^{32}\)CVI Melles-Griot, Model: SWP-45-Rs2000-Tp1300-PW-1012-C

\(^{33}\)Unknown origin

\(^{34}\)Stratophase (via Thorlabs, Inc.), Model: SHG7-10, Serial: JP240505-R64-02

\(^{35}\)Thorlabs, Inc., Model: LA1134-C

\(^{36}\)Ealing Catalog, Model: 42-5900-000. "1050.0 IF 10", Lot: GFRF, 1050 nm center wavelength at normal incidence, 15 nm bandwidth, 50.3% measured peak transmission

\(^{37}\)Thorlabs, Inc., Model: LA1074-C, \( f = 2 \text{ cm}, \) BK7 glass

\(^{38}\)Menlo Systems GmbH, Model: APD310, Serial: 0215, 35 \( \mu \text{m} \) active diameter with built-in amplification
+35 dB RF amplifier\textsuperscript{39}. With the amplifier on, the noise floor that appeared on the RF spectrum analyzer\textsuperscript{40} increased, verifying that the noise floor was that of the detection system and not that of the RF spectrum analyzer. Once a beat note was observed, the following controls were adjusted to maximize the beat SNR:

- **Spatial overlap**
  - mirrors in the $\lambda_{2f}$ path

- **Polarization alignment**
  - polarization controller
  - waveplates in the $\lambda_{2f}$ path

- **Temporal overlap**
  - stage delay in the $\lambda_{1f}$ path

- **Power**
  - PPLN position traverse to the input beam
  - amplifier output power controls

The beat signal could be observed over a delay stage range of approximately 600 $\mu m$, corresponding to an interferometer delay range of 4 $ps$. The measured interferometer path difference was 7 $mm$; the calculated required delay between $\lambda_{2f}$ and the frequency-doubled $\lambda_{1f}$ as a result of the $\sim70$ $cm$ HNLF input pigtail\textsuperscript{41} resulted in a 10.6 $mm$ path difference, which is in good agreement with the measured results. The RF spectrum of the maximized beat signal, out to the 1$^{st}$ harmonic of the repetition rate, is shown in Figure 4.17.

Figure 4.18 shows the RF locking electronics that were used to lock the carrier-envelope offset frequency. The reader is referred to Section 2.2, particularly Section

\textsuperscript{39}MITEQ, Model: AM-3A-000110, Serial: 1330923
\textsuperscript{40}Agilent Technologies, Inc., Model: 8565EC, Serial: 07363
\textsuperscript{41}OFS-equivalent of Corning, Inc., Model: SMF-28e
Figure 4.17: RF spectrum showing the fundamental and first repetition rate harmonic. The carrier-envelope offset frequency beats are the four sideband frequencies.

2.2.4 and associated references, for the principles of operation of the lock. The optical elements of the loop filter, $M(s)$, consist of the erbium power amplifier and SC generation fiber covered in Section 4.1.2.2, and the 1f-2f interferometer. The subsequent RF loop filter, $N(s)$, consists of an amplifier\textsuperscript{42}, two low-pass filters\textsuperscript{43}, and a directional coupler\textsuperscript{44} for monitoring the beat signal. A portion of the laser output was tapped off before the EDFA, attenuated\textsuperscript{45}, detected\textsuperscript{46}, filtered\textsuperscript{47}, amplified\textsuperscript{48}, and then frequency divided\textsuperscript{49} to 12.1 $MHz$ to serve as the RF reference frequency. These two signals were

\textsuperscript{42}MITEQ, Model: AM-3A-000110, Serial: 1330923
\textsuperscript{43}Mini-Circuits, Model: SLP-100+, 100 $MHz$ low-pass filter; Mini-Circuits, Model: SLP-30+ 30 $MHz$ low-pass filter
\textsuperscript{44}Mini-Circuits, Model: ZDC-10-1
\textsuperscript{45}Unknown origin, absorption-type ND 2.0 filter
\textsuperscript{46}Thorlabs, Inc., Model: DET10C, InGaAs detector, $\sim$125 $MHz$ bandwidth
\textsuperscript{47}Arra, Model: 1-9572E, DC block; Mini-Circuits, Model: SHP-150+, 150 $MHz$ high-pass filter; Mini-Circuits, Model: SLP-250, 250 $MHz$ low-pass filter
\textsuperscript{48}MITEQ, Model: AU-1014, Serial: 1342147
\textsuperscript{49}Menlo Systems GmbH, Model: FDT200, frequency divider module for custom mainframe, Model: XPS800, Serial: 9112-2
then fed to a digital phase detector\(^{50}\). A PI lockbox circuit\(^{51}\) was used as the loop filter, \(F(s)\), and provides an output voltage, \(V_{\text{mod}}\), that is used to modulate the laser’s pump current and thus control \(f_{\text{CEO}}\).

Initially, a diode driver\(^{52}\) with an analog modulation input was used to drive the pump laser and allow modulation of the current. It was found that, when the RF input for modulation was connected to anything, be it to a BNC cable or open, short, or 50 \(\Omega\) termination, the beat frequency noise increased, manifesting as spurious jumps in the beat. This setup also prevented the use of low-pass filters to improve the noise on the direct current (DC) drive current, as it would filter out the feedback modulation.

The laser’s original current driver\(^{53}\) with its DC low-pass noise filter\(^{54}\) was then used again, and the control voltage was fed to the pump diode current modulation circuit\(^{55}\), shown in Figure 4.19. The bipolar junction transistor (BJT)\(^{56}\) was modulated by the control voltage, which varied the current drawn away from the pump diode by the transistor, resulting in a current modulation of the pump diode. The pump diode

\(^{50}\)Menlo Systems GmbH, Model: DXD200, digital phase detector module for custom mainframe, Model: XPS800, Serial: 9112-2


\(^{52}\)Thorlabs, Inc., Model: ITC510, Serial: M00226832

\(^{53}\)ILX Lightwave, Mainframe Model: LDC-3900 Serial: 39001510, Module: LCM-39440 Serial: 394401707

\(^{54}\)ILX Lightwave, Model: 320, “Low noise CW Filter”

\(^{55}\)The circuit design is modeled after a design by Thomas Schibli.

\(^{56}\)Model: 2N2222A
current driver is represented by current source, $I_{DC}$. The forward-biased pump diode behavior is modeled in the diagram as a voltage source and resistor. The resistor values used in the circuit were $R_1 = 500 \ \Omega$ and $R_{shunt} = 2.2 \ \Omega$. A high-power Schottky diode, $D_1$, with its fast response time and low forward-bias voltage, was placed in parallel with the pump diode as a protection against inadvertent reverse biasing, which can easily destroy the pump diode. When the laser diode is forward biased, the Schottky diode is reverse biased and essentially serves as an open; if the circuit tries to reverse-bias the laser diode, the Schottky diode quickly opens, drawing the current away from the laser diode and limiting the reverse bias voltage placed on the laser diode to the low forward bias voltage of the Schottky diode.

To obtain a measure of the lock quality, the RF spectrum analyzer was set to retain the highest value measured at each frequency over the measurement time. Figure 4.20 shows the resulting trace after ten minutes when $f_{ceo}$ is locked and when

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57 Model: 1N5818, Forward Bias Voltage: 0.55 V at 1.0 A
58 Agilent Technologies, Inc., Model: 8565EC, Serial: 07363
59 This is done using the “Max Hold” trace setting.
Figure 4.20: Variation of the beat signal position over ten minutes when locked and unlocked.

it is unlocked. Despite efforts to disturb the lock\textsuperscript{60}, the laser remained locked. The drift of the locked signal is due, in part, to the fact that the reference frequency is a fraction of the repetition rate, which is not locked. The lock was observed to be stable for more than an hour, at which point it was turned off. The unlocked beat location was manually adjusted to be offset from the locking frequency and could be reliably locked by merely turning on the feedback circuit, which demonstrates the locking range of the feedback phase detector.

4.2.1 Summary

Using a mode-locked 200 MHz soliton EDFL, a carrier-envelope offset frequency stabilization system was demonstrated using the self-referencing SHG-based technique. A wide locking range and stable locking on a timescale of hours was observed. Continuing efforts should seek to improve the quality of the lock, with particular focus on the suppression of high frequency fluctuations. The 30 dB of SNR achieved approached the minimum required for a lock; SNR improvement should lead to better high frequency

\textsuperscript{60}These included forceful tapping of the table, and jumping up and down by the author.
fluctuation suppression. Higher SNR can be achieved through improved power performance of the EDFA outlined in Section 4.1.1.3. An improved SHG system can also improve the SNR and reduce the power requirements [79]. Improvements may also be achievable using different combinations of HNLF, which may provide more power at the desired wavelengths or lead to less noise in the generated supercontinuum [80].

4.3 DFG Beat for Repetition Rate Locking

Along with the SHG carrier-envelope offset frequency locking system, a repetition rate locking system using DFG was pursued.

Towards this goal, an interferometer setup for DFG was constructed. Figure 4.21 shows the schematic and a photograph of the system. A comparison to Figure 4.16 shows that the design is very similar to the 1f-2f interferometer. In this system, the same octave-spanning supercontinuum shown in Figure 4.12 used for SHG carrier-envelope offset frequency locking was used. For the DFG system, \( \lambda_{\text{low}} = 1590 \text{ nm} \) and \( \lambda_{\text{high}} = 1082 \text{ nm} \) were chosen based on the spectral power distribution of the SC and the chosen frequencies for \( f_{ceo} \) locking.

In the final form, the DFG system input would be the SC reflected from the 1f-2f interferometer’s 1012 nm optical bandpass filter, as shown in Figure 4.16. Waveplates at 1550 nm and 1060 nm would then be needed to adjust the spectral component polarizations. For this demonstration, the octave-spanning spectrum passes in fiber through a strain-type polarization controller\(^{61}\) at the start of the 1f-2f interferometer, where the polarization of the \( \lambda_{\text{high}} \) light is adjusted. After being collimated\(^{62}\) in free-space, the beam is then diverted from the 1f-2f interferometer to the DFG system with two silver steering mirrors. The beam then passes through a dichroic\(^{63}\) that reflects \( \lambda_{\text{high}} \) and passes \( \lambda_{\text{low}} \). In the \( \lambda_{\text{low}} \) path, the polarization of \( \lambda_{\text{low}} \) is adjusted with waveplates\(^{64}\), and the path length is adjusted using a manual linear translation stage. In the \( \lambda_{\text{high}} \) path,

\(^{61}\)General Photonics, Model: PLC-003-S-25
\(^{62}\)ThorLabs, Model: F240APC-1550
\(^{63}\)CVI Melles-Griot, Model: LWP-45-Rs-1064-Tp-1570-PW-1025-UV
\(^{64}\)Unknown origin
Figure 4.21: Schematic (left) and photograph (right) of the DFG interferometer for repetition rate locking.
path, two silver mirrors are used to steer the $\lambda_{\text{high}}$ beam. Both beams are recombined using another dichroic\textsuperscript{65}. This interferometer allows the temporal and spatial overlap of the $\lambda_{\text{low}}$ and $\lambda_{\text{high}}$ beams to be adjusted to maximize the DFG signal. The recombined collinear beam is then focused into and out of the PPLN grating\textsuperscript{66} using 40 mm focal length plano-convex lenses\textsuperscript{67}, where the output lens is placed on a linear translation stage to allow control of the output beam focusing. To achieve the quasi-phase matching necessary for generating the difference frequency of $\lambda_{\text{high}}$ and $\lambda_{\text{low}}$, the 30.0 $\mu m$ PPLN grating is used at a temperature of 160$^\circ$C. The light exiting the PPLN is then filtered with an optical bandpass filter\textsuperscript{68}, resulting in a difference frequency signal at 3.39 $\mu m$.

In order to find the DFG signal, the beam was chopped with a chopper\textsuperscript{69}, and detected with an InSb liquid-nitrogen-cooled detector\textsuperscript{70} that had a responsivity of 2.56 $\frac{A}{W}$ at $\lambda = 3.39$ $\mu m$. The generated current signal was then passed through a transimpedance amplifier\textsuperscript{71} which provided 2 $\frac{mA}{V}$ of gain. This signal, now voltage-proportional, was put into a lock-in amplifier\textsuperscript{72} which provided a tunable filter roll-off of 24 $\frac{dB}{\text{decade}}$. In total, this lead to a power-voltage conversion of 1.28 $\frac{\mu W}{V}$.

Once the signal was found, the chopper was removed, and the transimpedance amplifier output was viewed on an oscilloscope\textsuperscript{73}. Optimization of the DFG signal depended on the same variables discussed for the 1f-2f interferometer, presented in Section 4.2. After optimization, the signal as a function of stage delay was observed, shown in Figure 4.22. The signal could be observed over a stage delay range of approximately 400 $\mu m$, corresponding to interferometer delay range of 2.67 ps. The maximum DFG power observed was 3 $\mu W$. The bandwidth of the 3.39 $\mu m$ bandpass filter was 40 nm.

\textsuperscript{65}CVI Melles-Griot, Model: LWP-45-Rp-1064-Tp-1319-PW-1012-C
\textsuperscript{66}Stratophase (via Thorlabs, Inc.), Model: SHG7-10, Serial: KM121207-R127-01, no AR coating, model intended for SHG
\textsuperscript{67}Thorlabs, Inc., Model: LA5370
\textsuperscript{68}ISP Optics, Model: NBP-3390-25-N, PO: 4500934623, 3.39 $\mu m$ center wavelength at normal incidence, 50 $nm$ bandwidth at 50% transmission, 88% measured peak transmission, 1" diameter
\textsuperscript{69}Stanford Research Systems, Model: SR540, Serial: none, "chopper controller"
\textsuperscript{70}Judson Technologies, Model: J10D-M204-R250U-20, Serial: 07-10-32150, 20$^\circ$ field-of-view, 250 $\mu m$ active area
\textsuperscript{71}Stanford Research Systems, Model: SR570, Serial: 71078
\textsuperscript{72}Stanford Research Systems, Model: SR830, Serial: 54524
\textsuperscript{73}Tektronix, Inc., Model: TDS 3032B, Serial: B027127. A voltmeter would work as well - the oscilloscope was used out of convenience.
(1.04 THz) and therefore approximately 5000 modes are represented by the total power measured, giving a maximum power per mode of approximately 0.6 nW.

The DFG spectrum was then beat on the detector with a methane-stabilized HeNe laser operating at 3.39 μm. Figure 4.23 shows the schematic of the system used to generate and measure the DFG beat. The HeNe output power was approximately 34 μW; the DFG interferometer was further adjusted, giving approximately 720 pW per mode in the DFG comb. The beams were combined with a beamsplitter\(^7\), with the DFG comb in transmission (82.5%, 0.594 nW) and the HeNe reflected from the first surface (3.5%, 1156 nW). Both beams were incident at approximately 15° from normal; this provided as close to (but still far from) even amounts of DFG and HeNe power on the detector as was possible with the beamsplitter, and hence as close to optimal SNR as was possible, assuming shot noise-limited beat detection, as described in Section H.2.

The resulting RF signal was amplified\(^7\) and put into an RF spectrum analyzer\(^7\). Figure 4.24 shows a composite of various data traces that were taken. The black (no

---
\(^7\)Unknown origin. Measurements suggest it is likely to be an uncoated silica substrate.
\(^7\)MITEQ, Model: AU-1014, Serial: 1362592, with +60 dB of amplification and 1.4-1.6 dB noise figure
\(^7\)Agilent Technologies, Inc., Model: 8565EC, Serial: 07363
Figure 4.23: Schematic of the system used to beat the DFG comb with the methane-stabilized HeNe Laser.

Figure 4.24: RF spectrum of the DFG-HeNe beat. The black and gray curves are video-averaged over a few minutes with the beat signal and no signal, respectively. The red trace shows a snapshot of the beat spectrum.
signal) and gray (beat signal) curves are the maximum values observed at each frequency over the measurement time\textsuperscript{77}. These curves show the SNR and range of frequencies over which the beat could be observed. The red curve shows a single, standard trace of the beat. The beat was able to be measured with up to 15 dB SNR, and is expected to be shot noise-limited. For a good lock, at least 30 dB SNR is required. HeNe discharge tubes can be used to amplify 3.39 \textmu m light; due to the lack of availability of those tubes, attempts to lock the repetition rate were left for future work.

4.3.1 Summary

Using a 200 MHz soliton mode-locked EDFL, a DFG-based comb at 3.39 \textmu m was demonstrated. This comb was then successfully beat against a methane-stabilized single-mode HeNe laser, producing a shot noise-limited beat frequency with 15 dB SNR without amplification. Continuing efforts should add a HeNe discharge tube to amplify the DFG signal, thereby increasing the beat SNR to at least 35 dB to enable repetition rate locking.

\textsuperscript{77}This is done using the “Max Hold” trace setting.
Chapter 5

Short Pulse Generation from Bismuth Oxide Erbium-Doped Fiber Lasers

Advances in fiber processing has increased the variety of optical fibers, including gain fibers, and fiber-compatible optical components that are available. This includes gain fibers with broader gain bandwidths, such as tellurite fibers [81], Raman fiber amplifiers [82], and bismuth oxide-based erbium-doped fibers [83-85]. In addition, improvements in material growth processes have led to broader bandwidth saturable absorbers, particularly those of the SBR type [86-89].

In this chapter, results from a self-starting, stable femtosecond fiber sigma laser and ring laser are discussed. Both lasers utilized BiEDF which provides a broad gain bandwidth. Self-starting of the sigma laser was obtained using an ultra-broadband, large-area, oxidized SBR. The BiEDF and SBR are discussed, respectively, in Section 5.1 and Section 5.2. Section 5.3 reviews experiments that explore the short pulse generation potential of these elements. Section 5.4 concludes and discusses future work.
5.1 Bismuth Oxide Fiber Characteristics

BiEDF displays two characteristics that make it attractive for fiber lasers. First, very high erbium doping levels are achievable. Dopant levels of 13,000 ppm have been shown, as compared to the 10-100 ppm doping levels of standard silica-based erbium doped fibers. Such high levels of doping are possible as a result of the host bismuth ions, as well as boron and lanthanum co-dopants, which “screen” erbium atoms from each other and prevent the gain-quenching effects of clustering.

These high dopant levels lead to high gain per length. A 22.7 cm piece of BiEDF used as a broadband amplifier was shown to provide between +12 dB and +22 dB of gain for 1 ps pulses whose center wavelengths were tuned between 1520-1600 nm [84]. By virtue of the high gain per length, less fiber is needed, making shorter cavity fiber lasers possible. For mode-locked fiber lasers that rely on P-APM, BiEDF’s high Kerr nonlinearity further enables shorter cavity designs [70].

The second characteristic of note is the broad gain bandwidth of BiEDF. Figure 5.1 shows the ASE spectra of BiEDF and conventional silica-based EDFs as an indication of the emission spectrum. The BiEDF displays a broader gain bandwidth than conventional silica EDF, covering both the C- and L-bands. Bismuth is responsible for the broader gain bandwidth, as it causes greater local field variations at the erbium sites than in silica glass, and therefore is inhomogeneous [90].

The high gain per length, high Kerr nonlinearity, and broad gain bandwidth offer potential for building ultra-short pulse, high repetition rate laser systems.

5.2 Ultra-Broadband Semiconductor Saturable Bragg Reflectors

The short-pulse potential of BiEDF lasers can be more easily realized using a semiconductor SBR, a critical component which enables self-starting of mode-locked operation. The use of semiconductor SBRs for self-starting operation has been demonstrated in a number of ultrafast mode-locked lasers [86], and work continues to improve their
Figure 5.1: Amplified spontaneous emission spectrum of BiEDF and standard silica EDF.

effectiveness [89].

A typical SBR consists of a saturable absorbing layer on top of a Bragg mirror, an example of which is shown in Figure 5.2. The saturable absorbing layer may be fabricated from a semiconductor, where the semiconductor bandgap energy determines the bandedge of the absorption. However, the bandedge in such systems is set by the material that, coupled with fabrication constraints, limits design flexibility. One may alternatively use quantum well structures. By tailoring the quantum well width, and hence the allowed energy states of the quantum wells, the bandedge can be adjusted. The quantum wells are placed at a peak of the standing wave electric field pattern, as can be seen in Figure 5.2, to maximize the effect of the quantum well saturable absorption.

The saturable absorbing layer lies on a Bragg mirror, which is typically constructed of two epitaxially-grown, lattice-matched semiconductors that are alternated. An example of material choice is the pair AlAs and Al$_{0.3}$Ga$_{0.7}$As. The index of refraction contrast provided by these materials results in reasonably broadband reflectivity. For a 30-pair stack of these layers, >99.5% reflectivity over 1400-1550 nm can be expected [91].

The broadest bandwidth ultrashort pulse laser systems require broader reflectivity bandwidths than standard SBRs can provide. Improvements in fabrication have enabled
Figure 5.2: A schematic of the SBR structure, and the corresponding standing wave electric field amplitude.

the creation of high index contrast Al\textsubscript{x}O\textsubscript{y}/Al\textsubscript{0.3}Ga\textsubscript{0.7}As Bragg mirrors that offer this ultra-broadband performance (R > 99.5% from 1200 nm to 1800 nm) [88]. A normalized reflectivity measurement of one such structure, including the saturable layers, is shown in Figure 5.3. Such mirrors are fabricated by producing an AlAs/Al\textsubscript{0.3}Ga\textsubscript{0.7}As Bragg mirror that is oxidized to convert AlAs to Al\textsubscript{x}O\textsubscript{y}.

Initial attempts to fabricate such devices met with difficulty due to the oxidation process and the polycrystalline nature of Al\textsubscript{x}O\textsubscript{y}. The oxidation process involves high temperature processing of the SBR; the resulting thermal shock leads to delamination of the Al\textsubscript{x}O\textsubscript{y}/Al\textsubscript{0.3}Ga\textsubscript{0.7}As layers. It was discovered, however, that by ramping up the temperature slowly during oxidation, thermal shock to the Bragg stack is reduced, delamination is prevented, and high quality crystalline polycrystalline Al\textsubscript{x}O\textsubscript{y}/Al\textsubscript{0.3}Ga\textsubscript{0.7}As Bragg mirrors are produced [88]. Figure 5.4 shows a scanning electron microscope (SEM) image of such a structure.

These ultra-broadband SBRs facilitate self-starting and improve the stability of mode-locked fiber lasers.

5.3 Short Pulse Fiber Laser Experiments

In light of soliton and stretched-pulse laser mode-locking theory, the first set of experiments sought to utilize the broad bandwidth of the BiEDF and the SBR to produce
Figure 5.3: Fourier transform infrared spectroscopy (FTIR) normalized reflectivity measurement of the ultra-broadband SBR. The drop in reflectivity at 1550 nm corresponds to the bandedge of the quantum wells.

Figure 5.4: SEM image of the ultra-broadband SBR. The poly-crystalline nature of the Al$_x$O$_y$ is clear. SEM and device growth was done by Sheila Tandon.
an ultrashort pulse fiber laser. In these experiments, a ring configuration laser and a sigma configuration laser incorporating an SBR were constructed and tested.

Initially, a basic ring configuration was pursued. Figure 5.5 shows a schematic of the laser. Beginning at the PBS output coupler, the polarization state is set by the HWP and QWP, which determines the NPR in the ring. This state is then launched into anomalous GVD fiber\(^1\), where the pulse chirps anomalously and experiences NPR and soliton effects. The pulse then enters the BiEDF\(^2\), compresses and then chirps normally, and experiences more NPR. Next, the pulse passes through more fiber\(^3\), and is partially recompressed. Finally, the second HWP and QWP set biases the polarization state for passage through the PBS, which doubles as the output coupler and polarizing element for P-APM. Figure 5.6 shows the GVD map as a function of length around the cavity for one round trip, starting at the PBS output coupler.

A 55.6 cm piece of BiEDF was used as the gain medium. The BiEDF was pumped co-directionally with 975 nm laser diodes. The gain fiber’s normal GVD was compensated by the anomalous GVD fiber. Lengths of anomalous GVD fiber were chosen to adjust the net cavity GVD. For the results shown here, the average cavity GVD was -4.13

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\(^1\)Corning, Inc., Model: SMF-28e
\(^2\)Asahi Glass Co., Ltd., Model: T1M or T2M (unsure); 6,500 ppm. erbium ion concentration, NA: 0.2, \(\lambda_{cutoff} < 1450 \) nm, MFD = 6.0 ± 0.1 \(\mu m\)
\(^3\)Corning, Inc., Model: SMF-28e
The GVD variation around the cavity led to periodic chirping of the intracavity pulse over a round trip. Based on experimental results, a stretching factor of 1.33 was calculated, indicating operation in the soliton regime.

Laser operation resulted in slightly normally chirped output pulses. Using a combination of Brewster-cut silicon prisms and silicon slabs for external chirp compensation, autocorrelations of the pulses were obtained and are shown in Figure 5.7. The black trace shows the autocorrelation data and the gray dashed trace shows the fit assuming a hyperbolic secant pulse. The curve fit corresponds to pulse durations of 171 fs. The optical spectrum was centered at approximately 1562 nm with a 3-dB bandwidth of 16.5 nm, shown in Figure 5.8. The repetition rate was 27.3 MHz, the fundamental harmonic of which is shown in Figure 5.9. The average output power was 1.11 mW, corresponding to pulse energies of 40.7 pJ. When initially mode-locked, multiple-pulse operation resulted. Single-pulse operation, demonstrated by the oscilloscope trace in Figure 5.10, required a reduction of the pump power after initial mode-locking. The laser was observed to remain stably mode-locked for several hours.

To encourage self-starting, provide more stable operation, and obtain shorter pulses, a modification of the ring cavity design was employed to allow for the use of an SBR. Figure 5.11 shows a schematic of this configuration, known as a sigma configuration. The cavity operated in a way similar to the ring cavity, with the following exception:
Figure 5.7: Autocorrelation trace (solid) and hyperbolic secant pulse fit (dashed).

Figure 5.8: The optical power spectrum of the ring cavity output.
Figure 5.9: The RF spectrum of the fundamental beat at 27.3 MHz.

Figure 5.10: Oscilloscope trace of pulse train during single-pulse operation.

Figure 5.11: Diagram of the sigma configuration laser system.
Figure 5.12: GVD profile of cavity, starting at the PBS as the beam path enters the ring portion of the cavity.

Pulses that entered the linear section of the laser (from the left through the PBS) passed through a Faraday rotator (FR), were focused by an aspheric lens, reflected off of the SBR, passed back through the aspheric lens and FR, and exited the linear section (down out of the PBS). The two passes through the FR ensured that pulses exit the linear section in a linear polarization state orthogonal to that in which they entered. Figure 5.12 shows the lengths and GVD values of the fiber that corresponds to one round trip, starting at the PBS going down through the half- and quarter-wave plates.

To provide broad gain bandwidth, 22.7 cm of BiEDF, pumped bi-directionally with 975 nm laser diodes, served as the gain medium. The emission spectrum shown in Figure 5.1 was taken from the same piece of BiEDF using the same pump configuration as this laser, and thus is indicative of the gain spectrum of the laser system. The gain fiber’s strong normal GVD was compensated by single-mode fiber (SMF). Additional lengths of SMF and DCF were used to adjust the cavity GVD. For the results shown here, the average cavity GVD was $-2.82 \text{ ps}^2/\text{km}$. The GVD variation around the cavity led to some periodic chirping of the intracavity pulse over a round trip. Based on experimental results, a stretching factor of 2.5 was calculated, indicating stretched-pulse operation.

The broadband SBR was placed in the linear section of the cavity. This SBR consisted of an epitaxially-grown Bragg stack of seven high-index contrast $\text{Al}_x\text{O}_y/\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ layer pairs that acted as a broadband mirror ($R > 99.5\%$ from 1200 nm to 1800 nm), a $\frac{3}{2}$-layer of InP containing six quantum wells with a bandedge of 1550 nm, and a $\frac{3}{4}$-layer
of Al₂O₃ which acted as an anti-reflective coating. The SBR was proton-bombarded to reduce the quantum-well recovery time to 6 ps. The normalized reflectivity data shown in Figure 5.3 is that of the SBR used in this laser.

After external frequency chirp compensation using SMF, the pulse duration was measured with an autocorrelator, the trace of which is shown in Figure 5.13. The black trace shows the autocorrelation data and the gray dashed trace shows the fit assuming a hyperbolic secant pulse. The curve fit corresponds to pulse durations of 155 fs. The optical spectrum was centered at approximately 1555 nm with a 3-dB bandwidth of 22.2 nm, shown in Figure 5.14. The repetition rate was 19.9 MHz repetition rate, shown in Figure 5.15. The average output power was 573 µW, corresponding to pulse energies of 28.8 pJ. When initially mode-locked, multiple-pulse operation resulted. Single-pulse operation, demonstrated by the oscilloscope trace shown in Figure 5.16, required a reduction of the pump power after initial mode-locking. The laser was observed to remain stably mode-locked for several hours.

5.4 Summary

The characteristics of a new type of gain fiber, BiEDF, and ultra-broadband saturable absorbers were discussed, and investigations into the potential of BiEDFs and ultra-
Figure 5.14: The optical power spectrum of the sigma cavity output.

Figure 5.15: The RF spectrum of the fundamental beat at 19.9 MHz.
broadband SBRs to produce ultrashort pulse lasers were reviewed. A ring and sigma configuration laser using BiEDF gain fiber, and in the latter case an ultra-broadband SBR, were found to produce stable 171 fs and 155 fs pulses, respectively.

In future work with BiEDF, the high gain and high Kerr nonlinearity per length of the BiEDF might be utilized to produce high repetition rate (100+ MHz) fiber lasers. Investigations into the effects of resonant (gain) dispersion [92], a potential limitation on the short pulse potential of these lasers, may also be pursued.
Chapter 6

Conclusions and Future Research

The work presented here sought to increase the achievable repetition rate and pulse energy (through amplification), and decrease the achievable pulse duration of mode-locked EDFLs. These improvements were pursued through careful design and the use of new devices and components (SBRs, BiEDF, highly-doped gain fiber, new pump lasers), with a focus on OAWG applications. The following were accomplished:

- Demonstration of carrier-envelope offset locking at a 200 $MHz$ repetition rate
- Demonstration of DFG for the locking of the MFL repetition rate using a methane-stabilized HeNe laser
- Improvement of EDFL fundamental repetition rates up to 234 $MHz$ (stretched-pulse with SBR and P-APM) and 200 $MHz$ (soliton regime with P-APM)
- Demonstration of repetition rate multiplication to 2 $GHz$ in manner compatible with carrier-envelope phase slip stabilization using an external FPI
- Progress towards an optimized high-power EDFAs for ultra-short pulse amplification using commercially available components
- Exploration of mode-locked erbium-doped bismuth oxide fiber lasers to create ultrashort pulses
Extending the MFL’s range of operating parameters makes them more competitive with respect to existing mode-locked laser applications. It also enables performance improvements in emerging applications that make use of stabilized frequency combs, such as frequency metrology, OAWG, and timing and frequency distribution systems. MFLs are currently among the best laser systems for stable frequency comb generation; further research will allow them to reach their full potential in these applications.

Even with this progress, there is more to be done. Many of the limitations of and methods for scaling up the repetition rate in stabilization-compatible ways remain unexplored. This includes the exploration of other novel high-gain waveguides and fibers, new high-power single-mode pumps, and different SA designs for cavity scaling, and pulse correlation techniques for harmonic mode-locking approaches. Concerning different SA designs, the achievement of a fast SA that is discrete in nature and can be integrated would be of great use. To this end, fast SA implementations based on KLM using chalcogenide glass, nonlinear mode coupling in multi-core bismuth oxide fibers, and general nonlinear transverse mode\(^1\) in step-index geometries, all implemented in ways compatible with integration, were investigated but not included in this text. While the nonlinear transverse mode effect was too small in step-index geometries, other geometries or systems enabled by PCFs remain an open question.

\(^1\)This research explored the waveguide-to-waveguide coupling of nonlinear modes, i.e. mode solutions that account for the effect of Kerr nonlinearity.
Bibliography


Appendix A

Thesis Conventions And Calculations

A.1 Notation

To describe electric fields, this thesis uses complex notation. Quantities that use complex notation are notated with a tilda; the corresponding real quantities do not have the tilda.

\[
E(t) = \frac{\tilde{E}(t) + \tilde{E}^*(t)}{2} \tag{A.1}
\]

\[
E(t) = \text{Re} \left[ \tilde{E}(t) \right] \tag{A.2}
\]

Unless otherwise noted, electric field quantities are normalized such that the time-averaged power is:

\[
\langle P(t) \rangle_t = \frac{1}{2} \left| \tilde{E}(t) \right|^2 \tag{A.3}
\]
A.2 Fourier Transform Pairs

A.2.1 Definitions and Identities:

Throughout this thesis, unitary linear (as opposed to angular) frequency Fourier transform pairs are used:

\[ x(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} dt\ x(t)\ e^{-i2\pi ft} \]  
\[ x(t) = \mathcal{F}^{-1}\{x(f)\} = \int_{-\infty}^{\infty} df\ x(f)\ e^{i2\pi ft} \]

From this choice of Fourier Transform pairs, the following relationships are true:

\[ \mathcal{F}\{A(t) \cdot B(t)\} = \mathcal{F}\{A(t)\} \otimes \mathcal{F}\{B(t)\} \]  
\[ \mathcal{F}\{A(t) \otimes B(t)\} = \mathcal{F}\{A(t)\} \cdot \mathcal{F}\{B(t)\} \]  
\[ \mathcal{F}^{-1}\{A(f) \cdot B(f)\} = \mathcal{F}^{-1}\{A(f)\} \otimes \mathcal{F}^{-1}\{B(f)\} \]  
\[ \mathcal{F}^{-1}\{A(f) \otimes B(f)\} = \mathcal{F}^{-1}\{A(f)\} \cdot \mathcal{F}^{-1}\{B(f)\} \]

A.2.2 Relevant Fourier Transform pairs

Using these definitions, Fourier Transform pairs for functions used in this thesis are given:

<table>
<thead>
<tr>
<th>Function</th>
<th>( x(t) )</th>
<th>( x(f) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Delta Function</td>
<td>( \delta(t \pm T_0) )</td>
<td>( e^{\pm i2\pi fT_0} )</td>
</tr>
<tr>
<td>Infinite Delta Function Train</td>
<td>( \sum_{n=-\infty}^{\infty} \delta(t - nT_0) )</td>
<td>( \sum_{n=-\infty}^{\infty} \delta(f - \frac{m}{T_0}) )</td>
</tr>
<tr>
<td>Gaussian</td>
<td>( A_0 e^{-\frac{t^2}{2\sigma^2}} )</td>
<td>( A_0 \tau \sqrt{\pi} e^{-\frac{(2\pi f)^2\tau^2}{\sigma^2}} )</td>
</tr>
<tr>
<td>Secant Hyperbolic</td>
<td>( A_0 sech \left( \frac{t}{\tau} \right) )</td>
<td>( A_0 \tau \sqrt{\frac{2}{\pi}} sech \left( \pi^2 f \tau \right) )</td>
</tr>
</tbody>
</table>
A.3 Useful Identities

\[ \int_{-\infty}^{\infty} dx \, e^{-(ax^2+bx+c)} = \sqrt{\frac{\pi}{a}} \exp \left[ \frac{b^2 - 4ac}{4a} \right] \] (A.10)

\[ \sum_{q=1}^{N} q \cos (qa) = \frac{1}{4} \left( \csc^2 \left( \frac{a}{2} \right) \right) \left( -N \cos ((N + 1)a) + (N + 1) \cos (Na) - 1 \right) \] (A.11)
Appendix B

Characteristics of Optical Fibers

The salient characteristics of optical fibers are reviewed here. Note that this section is intended as a summary, not as a complete treatment of the topic. The purpose is to be a practical reference for the reader to use when selecting and using optical fibers.

B.1 Dispersion

Dispersion describes the dependence of the propagation vector, $\beta(\omega)$, on frequency. This dependence originates from the material environment and waveguiding. Material dispersion arises from resonances in the material which, through the Kramers-Kronig relationship, give rise to a phase response. For most optical glasses, these resonances occur at ultraviolet frequencies and are typically modeled by the Sellmeier equations. Waveguide dispersion arises from modal dependence on optical frequency.

For any non-monochromatic signal, it is important to understand the origins of dispersion, and the ways to describe it. Two notations are commonly used for dispersion; this section outlines the derivations of and relationships between those notations.
B.1.1 Wavevector (β) Notation

In optical fibers and waveguides, the propagation constant is commonly notated as $\beta$.

If the center frequency of interest is $\omega_0$, it is natural to then Taylor expand $\beta(\omega)$ about $\omega_0$:

$$\beta(\omega) = \beta(\omega_0) + \frac{\partial \beta}{\partial \omega} |_{\omega_0} (\omega - \omega_0) + \frac{1}{2!} \frac{\partial^2 \beta}{\partial \omega^2} |_{\omega_0} (\omega - \omega_0)^2 + \ldots + \frac{1}{n!} \frac{\partial^n \beta}{\partial \omega^n} |_{\omega_0} (\omega - \omega_0)^n + \ldots \quad (B.1)$$

$$\beta(\omega) = \beta_0(\omega_0) + \frac{1}{v_g(\omega_0)} (\omega - \omega_0) + \frac{1}{2!} \beta_2(\omega_0) (\omega - \omega_0)^2 + \ldots + \frac{1}{n!} \beta_n(\omega_0) (\omega - \omega_0)^n + \ldots \quad (B.2)$$

where $\beta_n(\omega) = \frac{\partial^n \beta}{\partial \omega^n}$, $\beta_1(\omega) = \frac{\partial \beta}{\partial \omega} = \frac{1}{v_g(\omega)}$, and $v_g(\omega)$ is the group velocity. As such, each coefficient can be written as:

$$\beta_n(\omega_0) = \left[ \frac{\partial^{n-1}}{\partial \omega^{n-1}} \left( \frac{1}{v_g(\omega)} \right) \right]_{\omega = \omega_0} \quad (B.3)$$

The coefficient of the second term in the Taylor expansion quantifies the group velocity, the third term is the GVD, and so on.

B.1.2 Group Delay (D) Notation

This notation expresses dispersion in terms of the group delay, $T(\lambda)$, that results from propagation over the physical length $L$, $L$:

$$T(\lambda) \equiv \frac{L}{v_g(\lambda)} \quad (B.4)$$

The group delay dispersion (GDD) in this notation is defined as [93]:

1The free-space equivalent of β is $|k|$.

2A physical length refers to the spatial length. It is called “physical” to differentiate it from an optical length, which is the product of the index of refraction and physical length.
Higher-order dispersion expressions are defined as derivatives of $D$ with respect to $\lambda$, evaluated at $\lambda_0$. Therefore, $n^{th}$-order dispersion ($n \geq 2$) is given by:

$$D_{n-2}(\lambda_0) = \left[ \frac{\partial^{n-2} D(\lambda)}{\partial \lambda^{n-2}} \right]_{\lambda=\lambda_0} = \frac{1}{L} \left[ \frac{\partial^{n-1} T(\lambda)}{\partial \lambda^{n-1}} \right]_{\lambda=\lambda_0} \quad (B.6)$$

which can be rewritten as:

$$D_{n-2}(\lambda_0) = \left[ \frac{\partial^{n-1} \left( \frac{1}{v_0(\lambda)} \right)}{\partial \lambda^{n-1}} \right]_{\lambda=\lambda_0} \quad (B.7)$$

### B.1.3 Relating $\beta$ and $D$

One notation can be converted to the other. Looking at Equation B.3 and Equation B.7, it is clear that the $\beta_n$ and $\omega$ are duals with $D_{n-2}$ and $\lambda$, respectively. As such, by deriving expressions for $D_{n-2}(\beta_0, \beta_1, ..., \beta_n)$, expressions for $\beta_n (D, ..., D_{n-2})$ are readily obtained by interchanging variables.

To begin, the following relationships between derivatives of $\omega$ with respect to $\lambda$ are summarized. The derivatives are given in three forms for convenience - in terms of lower order derivatives and $\lambda$, in terms of $\lambda$, and in terms of $\omega$:

$$\omega = \frac{2\pi c}{\lambda}$$

$$\frac{\partial \omega}{\partial \lambda} = -\frac{2\pi c}{\lambda^2} = -\frac{\omega^2}{2\pi c}$$

$$\frac{\partial^2 \omega}{\partial \lambda^2} = -\frac{2 \partial \omega}{\lambda \partial \lambda} = \frac{4\pi c}{\lambda^3} = \frac{\omega^3}{2\pi^2 c^2}$$

$$\frac{\partial^3 \omega}{\partial \lambda^3} = -\frac{3 \partial^2 \omega}{\lambda \partial \lambda^2} = \frac{6 \partial \omega}{\lambda^2 \partial \lambda} = -\frac{12\pi c}{\lambda^4} = -\frac{3\omega^4}{4\pi^3 c^3}$$

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\[
\frac{\partial^4 \omega}{\partial \lambda^4} = -4 \frac{\partial^3 \omega}{\partial \lambda^3} = 12 \frac{\partial^2 \omega}{\partial \lambda^2} = -24 \frac{\partial \omega}{\partial \lambda} = \frac{48 \pi c}{\lambda^5} = \frac{3 \omega^5}{2 \pi^4 c^4}
\]

\[
\frac{\partial^n \omega}{\partial \lambda^n} = \frac{n!}{(n-k)!} \frac{\partial^{n-k} \omega}{\partial \lambda^{n-k}} = \frac{n!}{(-\lambda)^n} \frac{2 \pi c}{\lambda} = n! \left( \frac{-\omega}{2 \pi c} \right)^n
\]

Using Equation B.7 and Equation B.3, the following general result can readily be written, valid for \( n \geq 2 \):

\[
D_{n-2}(\lambda_0) = \left[ \frac{\partial^{n-1} \left( \frac{1}{v_g(\lambda)} \right)}{\partial \lambda^{n-1} \beta_1(\omega)} \right]_{\lambda=\lambda_0, \omega=\omega_0} = \left[ \frac{\partial^{n-1} \beta_2(\omega)}{\partial \lambda^{n-1} \beta_1(\omega)} \right]_{\lambda=\lambda_0, \omega=\omega_0} \tag{B.8}
\]

where \( \lambda_0 \) corresponds to \( \omega_0 \).

Using Equation B.8 and the expressions for the \( \lambda \) derivatives of \( \omega \), the expressions to convert from wavevector notation to group delay notation for the first few orders of dispersion can be derived. The results for GVD, third-order dispersion (TOD), fourth-order dispersion (FOD), and fifth-order dispersion (5OD) are given below in three forms for convenience - in terms of the lower-order derivatives and \( \lambda \), in terms of \( \lambda \), and in terms of \( \omega \):

\[
D(\lambda_0) = \left[ \frac{\partial \omega}{\partial \lambda} \beta_2(\omega) \right]_{\lambda=\lambda_0, \omega=\omega_0}
\]

\[
= -\frac{\omega_0^2}{2 \pi c} \beta_2(\omega_0)
\]

\[
= -\frac{2 \pi c}{\lambda_0^2} \beta_2(\lambda_0)
\]

\[
\left[ \frac{\partial D(\lambda)}{\partial \lambda} \right]_{\lambda=\lambda_0} = \left[ \frac{\partial^2 \omega}{\partial \lambda^2} \beta_2(\omega) + \left( \frac{\partial \omega}{\partial \lambda} \right)^2 \beta_3(\omega) \right]_{\lambda=\lambda_0, \omega=\omega_0}
\]

\[
= \frac{\omega_0^3}{2 \pi^2 c^2} \left( \beta_2(\omega_0) + \frac{\omega_0}{2} \beta_3(\omega_0) \right)
\]
\[ = \frac{4\pi c}{\lambda_0^3} \left( \beta_2(\lambda_0) + \frac{\pi c}{\lambda_0} \beta_3(\lambda_0) \right) \]

\[
\left[ \frac{\partial^2 D(\lambda)}{\partial \lambda^2} \right]_{\lambda=\lambda_0} = \left[ \frac{\partial^3 \omega}{\partial \lambda^3} \beta_2(\omega) + 3 \frac{\partial^2 \omega}{\partial \lambda^2} \frac{\partial \omega}{\partial \lambda} \beta_3(\omega) + \left( \frac{\partial \omega}{\partial \lambda} \right)^3 \beta_4(\omega) \right]_{\lambda=\lambda_0, \omega=\omega_0}
\]

\[ = -\frac{3\omega_0^4}{4\pi^3 c^3} \left( \beta_2(\omega_0) + \omega_0 \beta_3(\omega_0) + \frac{\omega_0^2}{6} \beta_4(\omega_0) \right) \]

\[ = -\frac{12\pi c}{\lambda_0^4} \left( \beta_2(\lambda_0) + \frac{2\pi c}{\lambda_0} \beta_3(\lambda_0) + \frac{2\pi^2 c^2}{3\lambda_0^2} \beta_4(\lambda_0) \right) \]

\[
\left[ \frac{\partial^3 D(\lambda)}{\partial \lambda^3} \right]_{\lambda=\lambda_0} =
\]

\[
\left[ \frac{\partial^4 \omega}{\partial \lambda^4} \beta_2(\omega) + \left( 4 \frac{\partial^3 \omega}{\partial \lambda^3} \frac{\partial \omega}{\partial \lambda} + 3 \left( \frac{\partial^2 \omega}{\partial \lambda^2} \right)^2 \right) \beta_3(\omega) + 6 \frac{\partial^2 \omega}{\partial \lambda^2} \left( \frac{\partial \omega}{\partial \lambda} \right)^2 \beta_4(\omega) + \left( \frac{\partial \omega}{\partial \lambda} \right)^4 \beta_5(\omega) \right]_{\lambda=\lambda_0, \omega=\omega_0}
\]

\[ = \frac{3\omega_0^5}{2\pi^4 c^4} \left( \beta_2(\omega_0) + \frac{3\omega_0}{2} \beta_3(\omega_0) + \frac{\omega_0^2}{2} \beta_4(\omega_0) + \frac{\omega_0^3}{24} \beta_5(\omega_0) \right) \]

\[ = \frac{48\pi c}{\lambda_0^5} \left( \beta_2(\lambda_0) + \frac{3\pi c}{\lambda_0} \beta_3(\lambda_0) + \frac{2\pi^2 c^2}{\lambda_0^2} \beta_4(\lambda_0) + \frac{\pi^3 c^3}{3\lambda_0^3} \beta_5(\lambda_0) \right) \]

**B.2 Nonlinear Coefficient**

The nonlinear coefficient is defined as:

\[ \eta = \frac{2\pi n_2}{\lambda A_{eff}} = \frac{\omega n_2}{c A_{eff}} \]  

where \( n_2 \) is the Kerr coefficient of the material, \( A_{eff} \) is the effective area of the beam or mode, \( c \) is the speed of light, and \( \lambda \) and \( \omega \) are the wavelength and frequency of the
light, respectively.

B.3 Mode Field Diameter

Generally, the Petermann definition of mode field diameter is the standard definition of mode field diameter used by most, including optical fiber manufacturers. When the mode is gaussian, the Petermann definition is equivalent to the beam waist. The reader is referred to [94] for further information.

B.4 Numerical Aperture

B.4.1 Microscopy Definition

In microscopy, the numerical aperture is defined as:

\[ NA = n \sin\theta \]  

(B.10)

In this case, the NA describes the way in which a beam or mode diverges. For gaussian beams/modes, the information contained in the NA is also contained in the beam waist/mode-field diameter (MFD).

B.4.2 Optical Fiber Definition

As of the writing of this thesis, the following standard NA definition is used for step-index optical fiber\(^3\):

\[ NA = \sqrt{n_{core}^2 - n_{cladding}^2} \]  

(B.11)

This definition is derived using ray optics and Snell’s Law, \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \). When the core size approaches the wavelength, i.e. when wave effects becomes important and geometric optics is inadequate, this argument is no longer a valid way to evaluate the

\(^3\)This was verified with CorActive, Corning, Inc. (unless noted in their specification), Liekki Corporation, Nufern, and OFS.
mode divergence as it exits the fiber. For that purpose, the microscopy definition of numerical aperture, Equation B.10, should be used.

While this definition may be less accurate for mode-behavior purposes, it does provide valuable information in that it a) gives a relation between the core and cladding index, and b) can be conveniently used in the normalized frequency equation, \[ V = \frac{2\pi}{\lambda} a NA, \] for calculating mode cut-off wavelengths.
Appendix C

Pulses

This section reviews general concepts concerning temporal and spectral characteristics of pulses. Results for specific, commonly encountered pulse shapes are provided as well.

C.1 Non-interferometric Autocorrelation

For a non-interferometric autocorrelation, the resulting trace can be derived from the intensity profile of the pulse as follows:

\[ AC(t') = \int_{-\infty}^{+\infty} dt \ [E(t)]^2 \cdot [E(t - t')]^2 \]  

where \( t' \) is the temporal path difference between the paths in the autocorrelation interferometer, and \( E(t) \) is the electric field envelope (i.e. with the carrier frequency removed). When taking an autocorrelation measurement, the pulse duration can be estimated by dividing the FWHM of \( AC(t') \) by an appropriate factor. However, this does not make use of all of the autocorrelation data - it only uses those points just above and just below the half-maximum level. A better method that incorporates all of the data taken uses the \( AC(t') \) definition and an expected pulse shape and derives a functional form for the autocorrelation trace. This functional form can then be fit to the data. In the following sections, the functional forms for common pulse shapes are summarized.
C.2 Frequency and Wavelength Bandwidth Conversion

The expressions for converting a bandwidth from wavelength to frequency is done using:

\[ \Delta f = c \left( \frac{\Delta \lambda}{\lambda^2 - \left(\frac{\Delta \lambda}{2}\right)^2} \right) \]  \hspace{1cm} (C.2)

where \( \Delta f \) and \( \Delta \lambda \) are the bandwidths in frequency and wavelength, respectively, \( \bar{\lambda} \) is the wavelength at the center of the bandwidth, and \( c \) is the speed of light. The expression for conversion from frequency to wavelength is found when \( \lambda \leftrightarrow f \).

C.2.1 Gaussian Pulses

Using the following functional form for the electric field:

\[ E(t) = E_0 e^{-\frac{t^2}{\sigma^2}} \]  \hspace{1cm} (C.3)

The intensity profile is:

\[ I(t) = |E_0|^2 e^{-\frac{t^2}{\sigma^2}} \]  \hspace{1cm} (C.4)

for which the FWHM is:

\[ \tau_{I,FWHM} = 2\sqrt{\ln(2)}\tau \approx 1.665\tau \]  \hspace{1cm} (C.5)

The time-bandwidth product, given by the product of the intensity FWHM duration and the power spectrum FWHM, is given by:

\[ \tau_{I,FWHM}f_{FWHM} = \frac{2}{\pi} \ln(2) \approx 0.4413 \]  \hspace{1cm} (C.6)

And the autocorrelation function is then:
\[ AC(t') = |E_0|^4 \tau \sqrt{\frac{\pi}{2}} e^{-\frac{t'^2}{2\tau^2}} = |E_0|^4 \tau_{I,FWHM} \sqrt{\frac{\pi}{8 \ln(2)}} e^{-\frac{2 \ln(3) t'^2}{\tau_{I,FWHM}^2}} \]  

(C.7)

### C.2.2 Secant Hyperbolic

Using the following functional form for the electric field:

\[ E(t) \propto E_0 \text{sech} \left( \frac{t}{\tau} \right) \]  

(C.8)

The intensity profile is:

\[ I(t) = |E_0|^2 \text{sech}^2 \left( \frac{t}{\tau} \right) \]  

(C.9)

for which the FWHM is:

\[ \tau_{I,FWHM} = \ln(3 + \sqrt{8}) \tau \approx 1.7627 \tau \]  

(C.10)

The time-bandwidth product, given by the product of the intensity FWHM duration and the power spectrum FWHM, is given by:

\[ \tau_{I,FWHM} f_{FWHM} = \left( \frac{2 \ln(\sqrt{2} + 1)}{\pi} \right)^2 \approx 0.3148 \]  

(C.11)

The autocorrelation integral is then:

\[ AC(t') = 3A \left( T \coth(T) - 1 \right) \text{csch}^2(T) \]  

(C.12)

where:

\[ T = \frac{t' - t_{offset}}{\tau} = \frac{t' - t_{offset}}{\left( \frac{\tau_{I,FWHM}}{\ln(3 + \sqrt{8})} \right)} \]  

(C.13)

and \( A \) and \( t_{offset} \) are fitting parameters.
Appendix D

Mode-locked Lasers

This chapter provides miscellaneous analyses and derivations relating to mode-locked lasers. First, a set of back-of-the-envelope set of calculations to estimate the performance of a soliton laser design is outlined. This is followed by a summary of the Master Equation treatment that can serve as a basis for numerical simulations. Finally, an analysis of laser stability against CW and mode-locked Q-switching is summarized.

D.1 Soliton Fiber Laser Design

This section summarizes some basic considerations for soliton fiber laser design. This is meant to provide a “back-of-the-envelope” first-cut at predicting the performance characteristics of a soliton laser design. These calculations ignore spectral filtering, and the influence of the SA mechanism on pulse shaping and pulse stability, among other potential influences, which will generally cause the operating characteristics to change.

Three constraints can be considered to guide soliton laser design:

1. the Area Theorem

2. the roundtrip nonlinear phase shift

3. the steady-state intracavity power
D.1.1 The Area Theorem

From the nonlinear Schrödinger Equation, which describes propagation in a medium with GVD and SPM, it is known that the hyperbolic secant solution must adhere to the following requirement, known as the Area Theorem:

\[ E_p \tau_p = \frac{2|\beta_2|}{\eta} \]  

(D.1)

where \( \beta_2 \) is the GVD per unit length, given by Equation B.3, and \( \eta \) is the SPM per unit length, given by Equation B.9. In the context of soliton lasers and arising from the Master Equation description [95], the soliton-like pulse solution obeys a similar relationship, where the GVD per length becomes the net roundtrip GVD \( (\beta_2 \rightarrow D_2 = \frac{1}{2} \beta_2 L_{\text{roundtrip}}) \), and the SPM per length becomes the net roundtrip SPM \( (\eta \rightarrow \delta = \eta L_{\text{roundtrip}}) \). In both cases, Equation D.1 states that the pulse energy-duration product will be determined by the GVD and SPM of the medium.

D.1.2 The Roundtrip Nonlinear Phase Shift

The peak of a pulse circulating in the laser cavity will experience a nonlinear phase shift each roundtrip. If this nonlinear phase shift is too great, certain frequencies will be resonant with the cavity periodicity, and will build up [72]. This will effectively place a limit on how short the soliton duration can be. The requirement that the nonlinear phase shift be less than some maximum value can be written as:

\[ \phi_{NL}^{\text{max}} > \eta P_{\text{peak}} L_{\text{roundtrip}} \]  

(D.2)

where \( \phi_{NL}^{\text{max}} \) is the maximum nonlinear phase shift that can still lead to stable mode-locking, \( \eta \) is the SPM per unit length, \( P_{\text{peak}} \) is the pulse peak power, and \( L_{\text{roundtrip}} \) is the roundtrip cavity length. The maximum stable roundtrip nonlinear phase shift has been seen to be near \( \frac{3}{8} \pi \) [70]. Using the expression for the pulse energy of a hyperbolic secant pulse:

\[ E_p = 2P_{\text{peak}} \tau_p \]  

(D.3)
a limitation on the maximum pulse energy is found:

\[ E_p < \frac{2\phi_{NL}^{\text{max}} \tau_p}{\eta L_{\text{roundtrip}}} \]  \hspace{1cm} (D.4)

By incorporating Equation D.1, the maximum pulse energy can be rewritten solely in terms of cavity parameters:

\[ E_p < \frac{2\phi_{NL}^{\text{max}} |\beta_2|}{\eta L_{\text{roundtrip}} \eta E_p} \]  \hspace{1cm} (D.5)

\[ E_p < \frac{2}{\eta} \sqrt{\frac{|\beta_2| \phi_{NL}^{\text{max}}}{L_{\text{roundtrip}}}} = \frac{\lambda A_{\text{eff}}}{\pi n_2} \sqrt{\frac{|\beta_2| \phi_{NL}^{\text{max}}}{L_{\text{roundtrip}}}} \]  \hspace{1cm} (D.6)

**D.1.3 The Steady-State Intracavity Power**

The final consideration is the steady-state intracavity power. This incorporates all intracavity losses (including output coupling), and the amount of gain available (including doping, pump levels, level of saturation, etc.). With a calculated steady-state intracavity power, \( P_{\text{intracavity}} \), the maximum pulse energy that is sustainable can be written in terms of the laser repetition rate, \( F_R \). In the case of a FML laser, this will be:

\[ E_p < \frac{P_{\text{intracavity}}}{F_R} \]  \hspace{1cm} (D.7)

**D.1.4 Summary**

Either the nonlinear phase shift, through Equation D.6, or the sustainable steady-state intracavity power, through Equation D.7, will enforce a tighter constraint on the pulse energy. Through the Area Theorem in Equation D.1, this restriction translates into a minimum possible pulse duration.
D.2 Mode-locking and the Master Equation

To quantitatively address steady-state pulse solutions, a self-consistent description of the laser operation that simultaneously accounts for all influences (i.e. saturable gain, linear loss, GVD, SPM, filtering, self-amplitude modulation) is needed. Furthermore, to be applicable to erbium fiber/waveguide lasers, the treatment must allow highly saturated gain, and must be able to account for the use of a slow SA (to model an SBR), a fast SA (to model P-APM), or both simultaneously. An analytic solution of the Master Equation for fast SA mode-locking that meets these criteria has already been explored [96]. A Master Equation treatment of slow SA mode-locking has also been done [97], but this treatment assumes that the gain partially recovers on the timescale of the repetition rate, and is thus not applicable to erbium fiber/waveguide lasers. Therefore, solutions that are applicable to erbium fiber/waveguide lasers with slow SAs, or both slow and fast SAs, must be solved numerically.

The Master Equation treatment of SA mode-locking with highly-saturated gain is reviewed below. These results are adapted from the references [96,98]. For clarity, the assumptions required for the validity of the Master Equation treatment are given:

1. All effects per pass are small and, therefore, are additive.

2. The gain medium is homogeneously broadened. This allows Taylor expansion of the gain in the time domain and facilitates analytic solutions.

3. In steady-state operation, the gain medium is highly saturated such that it does not recover on the timescale of the roundtrip time.

With these assumptions, the Master Equation can be written as:

\[
\begin{align*}
\begin{bmatrix}
-l_0 + g + jD^2_2 \frac{\partial^2}{\partial t^2} - j\delta |A(t, T)|^2 + \frac{1}{\Omega_f^2} \frac{\partial^2}{\partial t^2} - q_1(t) - q_2(t) \\
\text{linear loss} & \\
\text{gain} & \\
\text{GVD} & \\
\text{SPM} & \\
\text{SPM} & \\
\text{filtering} & \\
\text{1st SA} & \\
\text{2nd SA} & \\
\end{bmatrix} A(t, T) &= T_R \frac{\partial A(t, T)}{\partial T}
\end{align*}
\]

(D.8)
where \( t \) refers to timescales on the order of the pulse duration, and \( T \) refers to timescales on the order of the roundtrip time, \( T_R \). The net GVD is given by \( D_2 = \frac{1}{2} \beta_2 L_{cavity} \), spectral filtering is represented by \( D_f = \frac{1}{\Omega_f} \), and \( \delta \) is the SPM coefficient. Two saturable absorbers have been included, which may represent two absorbers with two different time constants; additional SA effects can be included by adding more SA terms. The sum of these effects are set equal to \( T_R \frac{\partial A(t,T)}{\partial T} \) because the described changes are taken to happen over many roundtrips.

Generally, the equation of motion for the gain is given by:

\[
T_R \frac{\partial g}{\partial t} = -T_R \frac{g - g_0}{\tau_L} - \frac{gT_R P}{E_L}
\]

The first term describes the recovery term back to the small-signal value. The second term describes the tendency for incident power to cause a deviation from the small-signal value.

Under various assumptions, the solution to the differential equation can be approximated. If the gain recovery time is much less than the pulse duration, which is much less than the roundtrip time \( (T_R \gg \tau_p \gg \tau_L) \), the gain follows the instantaneous power. Thus, Equation D.9 simplifies to:

\[
g = \frac{g_0}{1 + \frac{P(t)}{P_L}}
\]

where \( P_L = \frac{E_L}{\tau_L} \). If the gain recovery time is much greater than the pulse duration, but much less than the roundtrip time \( (T_R \gg \tau_L \gg \tau_p) \), the gain will respond to the pulse energy. Equation D.9 simplifies to [98]:

\[
g = g_0 \frac{1 - exp \left[ -\frac{E_p}{E_L} \right]}{\frac{E_p}{E_L}}
\]

If the gain recovery time is much greater than the roundtrip time and the pulse duration
\( \tau_L \gg T_R, \tau_p \), Equation D.9 simplifies to:

\[
g = \frac{g_0}{1 + \frac{P_{av}}{P_L}} \quad (D.12)
\]

Saturable absorbers obey an equation of motion identical to the form of Equation D.9, and may be simplified under analogous assumptions. Changing \( g \to q \) and \( L \to A \) will result in equations of motions for the saturable absorber.

## D.3 Stability Against CW Q-Switching

To analyze stability against relaxation oscillations, and hence stability against Q-switching, perturbations on a time scale much longer than the cavity roundtrip time must be considered. Should these perturbation grow with time (i.e. have positive net gain), the laser is concluded to be unstable against Q-switching. The content of this section is adapted from the references [38, 98].

To obtain these results, the following is done: The laser is assumed to be in CW steady-state operation. A perturbation is then added to each of the intracavity power, gain, and saturable absorber(s), and the equations of motion for each parameter are linearized in the neighborhood of the CW steady-state parameters. The perturbation is taken to be on a timescale longer than \( T_R \). This results in equations of motion for the perturbations, which results in conditions required for the perturbation in the power to grow with time that will indicate (in)stability against relaxation oscillations that cause Q-switching.

Looking back to Equation D.8, the GVD, SPM, and filtering terms are negligible because of the quasi-monochromatic spectrum. This leads to the equation of motion for the field:

\[
T_R \frac{\partial A(t)}{\partial t} = \left[ -l_0 + g - \sum_{x=1}^{X} q_x \right] A(t) \quad (D.13)
\]

where the long and short timescales, \( T \) and \( t \) have been dropped because separate
timescales for the dynamics being considered is not yet defined. In this form, $X$ saturable absorbers, each with a different saturable loss, $q_{ox}$, and saturation time, $\tau_{Ax}$, are in the cavity. Equation D.13 can be cast in terms of power as:

$$T_R \frac{\partial P(t)}{\partial t} = 2 \left[ -l_0 + g - \sum_{x=1}^{X} q_x \right] P(t) \quad (D.14)$$

Now the parameters are perturbed, so $P \rightarrow P_{cw} + \Delta P$ and $g \rightarrow g_{cw} + \Delta g$, where the perturbations are assumed to be on a timescale longer than the cavity roundtrip time. The resulting equations of motion for the power and gain perturbations are then:

$$T_R \frac{\partial \Delta P}{\partial t} = \Delta P \left[ -2P_{cw} \left( \sum_{x=1}^{X} \frac{\partial q_x}{\partial P} \right) |_{cw} \right] + \Delta g \left[ 2P_{cw} \right] \quad (D.15)$$

$$T_R \frac{\partial \Delta g}{\partial t} = \Delta P \left[ -\frac{g_{cw} T_R}{E_L} \right] + \Delta g \left[ \frac{T_R}{\tau_L} - \frac{P_{cw} T_R}{E_L} \right] \quad (D.16)$$

This constitutes a matrix equation. The coefficient matrix will have eigenvalues with negative real parts (i.e. the perturbation will decay and the laser will be stable against these perturbations) if the trace of the coefficient matrix is negative and the determinant is positive. This results in, respectively:

$$-2P_{cw} \left( \sum_{x=1}^{X} \frac{\partial q_x}{\partial P} \right) |_{cw} < \frac{T_R}{\tau_L} \left( 1 + \frac{P_{cw}}{P_L} \right) \quad (D.17)$$

$$\left( \sum_{x=1}^{X} \frac{\partial q_x}{\partial P} \right) |_{cw} > \left. \frac{\partial g}{\partial P} \right|_{cw} \quad (D.18)$$

For a laser that will lase on its own, Equation D.18 is automatically satisfied, because the small signal gain must be greater than the cavity losses. Assuming the SA recovery times are much faster than the roundtrip time ($\tau_{Ax} \ll T_R$ for all $x$), and hence faster than the perturbation, the absorbers respond to the instantaneous power according to Equation D.10, and hence Equation D.17 can be rewritten as:
where $\chi_x = \frac{P_{Ax}}{P_{L}}$ is the "stiffness" of absorber $x$ against CW saturation. The laser approaches instability against CW Q-switching when:

1. the recovery time of the gain medium, $\tau_L$, increases
2. the roundtrip time, $T_R$, decreases
3. the saturable loss, $q_{0x}$, increases
4. the stiffness, $\chi_x$, decreases, corresponding to any of the following
   - a decrease in the saturation power of a SA, $P_{Ax}$
   - a decrease in the saturation energy of the SA, $E_{Ax}$
   - an increase in the recovery time, $\tau_{Ax}$

### D.4 Stability Against Q-Switched Mode-locking

To consider stability against Q-switched mode-locking, a similar approach to Section D.3 can be taken. In this case, instead of perturbing the laser from CW steady-state, it is perturbed from a mode-locked state. As with the Master Equation in Section D.2, two time scale are designed, $t$ for changes that occur over times on the order of the pulse duration, and $T$ for changes that occur over times on the order of a roundtrip time, $T_R$. The steady-state power is then written in a pulse form:

$$P(t,T) = E_p(T) \sum_{n=-\infty}^{\infty} f_n(t - nT_R)$$

where $f_n(t - nT_R)$ represents the pulse shape, whose integral is normalized over a roundtrip time, and $E_p(T)$ is the pulse energy.

The saturable absorbers are then assumed to have a recovery time much greater than the pulse duration, but much less than the roundtrip time ($T_R \gg \tau_{Ax} \gg \tau_p$). As
a result, the recovery term of the absorber can be ignored and the short timescale, \( t \),
can be integrated out of the SA equation of motion, resulting in the total saturable
absorber loss for a given pulse:

\[
q_x(nT_R, t) = q_o x \exp \left( - \frac{E_p(T)}{E_A} \int_{-\frac{T_R}{2}}^{t} dt' f_n(t') \right)
\]

(D.21)

\[
q_{px}(nT_R) = q_ox \int_{-\frac{T_R}{2}}^{\frac{T_R}{2}} dt f_n(t) q(mT_R, t) = q_ox \frac{1 - e^{x \left( - \frac{E_p(T)}{E_A} \right)}}{E_p(T) E_A}
\]

(D.22)

By averaging over one roundtrip time, Equation D.14 becomes:

\[
T_R \frac{\partial E_p}{\partial T} = 2 \left[ -l_0 + g - \sum_{x=1}^{X} q_{px} \right] E_p
\]

(D.23)

and, assuming the gain saturation with one pulse is negligible, Equation D.9 becomes:

\[
T_R \frac{\partial g}{\partial T} = -T_R \left( g - g_0 \right) - \frac{gE_p}{E_L}
\]

(D.24)

These equations take the same form as Equation D.14 and Equation D.9, where
\( P_{cw} \rightarrow \frac{E_p(T)}{T_R} \) and \( q_x \rightarrow q_{px} \):

\[
T_R \frac{\partial E_p}{\partial t} = \Delta E_p \left( - \sum_{x=1}^{X} \frac{\partial q_{px}}{\partial P} \right)_{cw-MLing} + \Delta g \left[ 2E_p \right]
\]

(D.25)

\[
T_R \frac{\partial g}{\partial t} = \Delta E_p \left( - \frac{g_{cw-MLing} T_R}{E_L} \right) + \Delta g \left[ - \frac{T_R}{\tau_L} - \frac{E_p}{E_L} \right]
\]

(D.26)

thus, the same form of conditions on stability against perturbations arises. As before,
one condition is met when the laser is above threshold. The other condition is:

\[
-2E_p \left( \sum_{x=1}^{X} \frac{\partial q_{px}}{\partial P} \right)_{cw-MLing} \left( 1 + \frac{P_{avg}}{P_L} \right)
\]

(D.27)

Given the form of the saturable absorber response, given in Equation D.22, the
stability against Q-switched mode-locking condition can be rewritten as:

\[ 2 \frac{\tau_L}{T_R} \left( \sum_{x=1}^{X} q_{0x} \frac{1 - \exp \left( -\frac{P_{avg}}{\chi_{px} P_L} \right)}{\frac{P_{avg}}{\chi_{px} P_L} \left( 1 + \frac{P_{avg}}{\chi_{px} P_L} \right)} \right) < 1 + \frac{P_{avg}}{P_L} \]  

(D.28)

where \( \chi_{px} = \frac{P_{Ax} \tau_{Ax}}{P_L T_R} = \chi_x \frac{\tau_{Ax}}{T_R} \) is the “stiffness” of absorber \( x \) against CW mode-locked saturation. Again, the laser approaches instability against Q-switched mode-locking when:

1. the recovery time of the gain medium, \( \tau_L \), increases
2. the roundtrip time, \( T_R \), decreases
3. the saturable loss, \( q_{0x} \), increases
4. the stiffness, \( \chi_{px} \), decreases, corresponding to any of the following

(a) a decrease in the saturation power of a SA, \( P_{Ax} \)
(b) a decrease in the saturation energy of the SA, \( E_{Ax} \)
(c) an increase in the recovery time, \( \tau_{Ax} \)
Appendix E

Harmonically Mode-locked Laser
Pulse Train Derivations

E.1 Derivation of an Harmonically Mode-locked Laser
Pulse Train Description

In this section, the full derivation for the expressions for the harmonically mode-locked
output in the time and frequency domains are presented.

The Uncorrelated Harmonic Pulse Train and Frequency Comb

The following is assumed:

- There are $N$ intracavity pulses.
- $v(t)$ is the same for all pulses, and has the same properties as in Section 2.1.1. It is
  assumed that $v(t) = \frac{E_0}{\sqrt{N}} \delta(t)$ in order to simplify the analysis. The constant phase
  of the carrier, $\phi_k$, can account for the phase that a complex $E_0$ would represent,
  thus $E_0$ is taken to be real without a loss of generality.

In the time domain, the portion of the output pulse train electric field generated by
intracavity pulse $k$ is given by:
\[ E_k(t) = \frac{E_0}{2\sqrt{N}} \left( e^{i(2\pi f_0 t + \phi_k)} + c.c. \right) \sum_{n=-\infty}^{\infty} \delta \left( t - nT_R - \frac{k-1}{N}T_R \right) \]  

(E.1)

where \( n \) is a number that labels each pulse consecutively in time, \( T_R = \frac{1}{f_R} \) is the cavity round-trip time and equals the inverse fundamental repetition rate, \( f_0 \) corresponds to the carrier frequency, \( k \) is a natural number ranging from 1 to \( N \) that labels each fundamental pulse train that is a subset of the full harmonic pulse train, and \( \phi_k \) is the absolute carrier phase of the fundamental subset pulse train, \( k \). The electric field amplitude is normalized such that \( \frac{E_k^2}{N} \) is the pulse energy, and is divided by the harmonic number, \( N \), in order to keep the average power independent of \( N \). The amplitude spectrum of this electric field is:

\[ E_k(f) = \frac{E_0}{2\sqrt{N}} \int_{-\infty}^{\infty} dt e^{-i2\pi ft} \left[ e^{i(2\pi f_0 t + \phi_k)} + c.c. \right] \sum_{n=-\infty}^{\infty} \delta \left( t - nT_R - \frac{k-1}{N}T_R \right) \]  

(E.2)

\[ E_k(f) = \frac{E_0}{2\sqrt{N}} \int_{-\infty}^{\infty} dt \left( e^{-i(2\pi(f-f_0)t - \phi_k)} + e^{-i(2\pi(f+f_0)t + \phi_k)} \right) \sum_{n=-\infty}^{\infty} \delta \left( t - nT_R - \frac{k-1}{N}T_R \right) \]  

(E.3)

letting \( f_\pm = f \pm f_0 \) and \( t = t - \frac{k-1}{N}T_R \):

\[ E_k(f_\pm) = \frac{E_0}{2\sqrt{N}} \int_{-\infty}^{\infty} d\tilde{t} \left[ \left( e^{-i2\pi\tilde{t}} e^{-i(2\pi T_R(\frac{k-1}{N}) - \phi_k)} + e^{-i2\pi\tilde{t}} e^{-i(2\pi T_R(\frac{k-1}{N}) + \phi_k)} \right) \right. \]  

\[ \left. \times \left( \sum_{n=-\infty}^{\infty} \delta \left( \tilde{t} - nT_R \right) \right) \right] \]  

(E.4)

using the identities in Appendix A.2.2 gives:

\[ E_k(f) = \frac{E_0}{2\sqrt{N}} \sum_{m=-\infty}^{\infty} \left[ e^{-i(2\pi m f_R(\frac{k-1}{N}) - \phi_k)} \right] \delta \left( f - m f_R \right) \]  

(E.5)
where $m$ is an integer that labels consecutively the modes in the frequency domain. Written in terms of $f$ and $f_0$:

$$E_k(f) = \frac{E_0}{2\sqrt{N}} \sum_{m=-\infty}^{\infty} \left[ e^{-i\left(2\pi \frac{f-f_0}{F_R} \frac{k}{N} - \phi_k\right)} \delta(f - f_0 - mF_R) + e^{-i\left(2\pi \frac{f+f_0}{F_R} \frac{k}{N} + \phi_k\right)} \delta(f + f_0 - mF_R) \right]$$

The full electric field of the output from a harmonically mode-locked laser is the superposition of the individual pulse trains from each intracavity pulse, thus it can be written:

$$E(t) = \sum_{k=1}^{N} \left[ \frac{E_0}{2\sqrt{N}} \left( e^{i(2\pi f_0 t + \phi_k)} + c.c. \right) \sum_{n=-\infty}^{\infty} \delta\left(t - nT_R - \frac{k-1}{N} T_R\right) \right]$$

$$E(f) = \sum_{k=1}^{N} \frac{E_0}{2\sqrt{N}} \sum_{m=-\infty}^{\infty} \left[ e^{-i\left(2\pi \frac{f-f_0}{F_R} \frac{k}{N} - \phi_k\right)} \delta(f - f_0 - mF_R) + e^{-i\left(2\pi \frac{f+f_0}{F_R} \frac{k}{N} + \phi_k\right)} \delta(f + f_0 - mF_R) \right]$$

The time-average power in the time domain is:

$$\langle P(t) \rangle_t = \frac{1}{2} |\bar{E}(t)|^2 = \frac{1}{2} \sum_{k=1}^{N} \left[ \frac{E_0}{\sqrt{N}} e^{i(2\pi f_0 t + \phi_k)} \sum_{n=-\infty}^{\infty} \delta\left(t - nT_R - \frac{k-1}{N} T_R\right) \right]$$

$$\times \sum_{k=1}^{N} \left[ \frac{E_0}{\sqrt{N}} e^{-i(\phi_k - \phi_k)} \sum_{\bar{n}=-\infty}^{\infty} \delta\left(t - \bar{n}T_R - \frac{k-1}{N} T_R\right) \right]$$

$$\langle P(t) \rangle_t = \frac{E_0^2}{2N} \sum_{k=1}^{N} \sum_{\bar{k}=1}^{N} e^{i(\phi_k - \phi_{\bar{k}})} \sum_{n=-\infty}^{\infty} \delta\left(t - nT_R - \frac{k-1}{N} T_R\right)$$
\[ \times \sum_{\bar{n} = -\infty}^{\infty} \delta \left( t - \bar{n}T_R - \frac{k - 1}{N} T_R \right) \]  

(E.10)

\[ \langle P(t) \rangle_t = \frac{E_0^2}{2N} \sum_{k=1}^{N} \sum_{n=-\infty}^{\infty} \delta \left( t - nT_R - \frac{k - 1}{N} T_R \right) \]  

(E.11)

As defined, the product of the summations over \( n \) and \( \bar{n} \), and \( k \) and \( \bar{k} \) is only non-zero when \( n = \bar{n} \) and \( k = \bar{k} \), and hence collapses the summations in \( n \) and \( \bar{n} \) to one summation in \( n \), and \( k \) and \( \bar{k} \) to one summation in \( k \). Also, because \( k = \bar{k} \) is true everywhere the power is non-zero, the absolute carrier phase drops out, as expected.

The power spectrum is:

\[ P(f) = |E(f)|^2 = \sum_{k=1}^{N} \frac{E_0}{2\sqrt{N}} \left\{ \sum_{m=-\infty}^{\infty} \left[ e^{-i\left(2\pi f f_R \frac{k-1}{N} \phi_k\right)} \delta(f - f_0 - mF_R) \right] \right\} \]

\[ \times \sum_{k=1}^{N} \frac{E_0}{2\sqrt{N}} \left\{ \sum_{\bar{m}=-\infty}^{\infty} \left[ e^{-i\left(2\pi f f_R \frac{k-1}{N} \phi_k\right)} \delta(f - f_0 - \bar{m}F_R) \right] \right\} \]

(E.12)

As stated in Section 2.1.1, the spectral envelope prevents the combs centered at the positive and negative component of the carrier frequency from spectrally coinciding, resulting in only two cross-terms that are non-zero:

\[ P(f) = \frac{E_0^2}{4N} \sum_{k=1}^{N} \sum_{\bar{k}=1}^{N} \left\{ \sum_{m=-\infty}^{\infty} \sum_{\bar{m}=-\infty}^{\infty} \left[ e^{-i\left(2\pi f f_R \frac{k-1}{N} \phi_k\right)} \delta(f - f_0 - mF_R) \delta(f - f_0 - \bar{m}F_R) \right] \right\} \]

(E.13)
\[ P(f) = \frac{E_0^2}{4N} \sum_{k=1}^{N} \sum_{k=1}^{N} \sum_{m=-\infty}^{\infty} \left\{ e^{i\left(2\pi \frac{f - f_0}{F_R} \frac{k - k}{N} \left(\phi_k - \phi_k\right)\right)} \delta (f - f_0 - mF_R) + e^{i\left(2\pi \frac{f + f_0}{F_R} \frac{k - k}{N} \left(\phi_k - \phi_k\right)\right)} \delta (f + f_0 - mF_R) \right\} \tag{E.14} \]

where the summation in \( m \) and \( \bar{m} \) collapses to a single summation, as before. The following notation is introduced:

\[ A_{\pm \bar{k}k}(f) = e^{i\left(2\pi \frac{f \pm f_0}{F_R} \frac{k - k}{N} \left(\phi_k - \phi_k\right)\right)} \delta (f \pm f_0 - mF_R) \tag{E.15} \]

allowing the power to be written as:

\[ P(f) = \frac{E_0^2}{4N} \sum_{k=1}^{N} \sum_{k=1}^{N} \sum_{m=-\infty}^{\infty} (A_{-\bar{k}k}(f) + A_{+\bar{k}k}(f)) \tag{E.16} \]

Each summation of \( A_{\pm \bar{k}k}(f) \) over \( k \) and \( \bar{k} \) can be separated according to when \( k = \bar{k} \) and \( k \neq \bar{k} \), and written as:

\[ P(f) = \frac{E_0^2}{4N} \left\{ 2N + \sum_{k=1}^{N-1} \sum_{k+1}^{N} \sum_{m=-\infty}^{\infty} [(A_{-\bar{k}k}(f) + A_{+\bar{k}k}(f)) + (A_{-\bar{k}k}(f) + A_{+\bar{k}k}(f))] \right\} \tag{E.17} \]

\[ P(f) = \frac{E_0^2}{4N} \left\{ 2N + \sum_{k=1}^{N-1} \sum_{k+1}^{N} \sum_{m=-\infty}^{\infty} [(A_{-\bar{k}k}(f) + A_{+\bar{k}k}(f)) + (A_{+\bar{k}k}(f) + A_{+\bar{k}k}(f))] \right\} \tag{E.18} \]

Observe that:

\[ A_{\pm \bar{k}k}(f) = A^*_{\pm \bar{k}k}(f) \tag{E.19} \]

and:

\[ A_{\pm \bar{k}k}(f) + A^*_{\pm \bar{k}k}(f) = 2 \cos \left[ 2\pi \frac{f \pm f_0}{F_R} \frac{\bar{k} - k}{N} \pm (\phi_k - \phi_k) \right] \delta (f \pm f_0 - mF_R) \tag{E.20} \]
This allows the expression to be further simplified:

\[
P(f) = \frac{E_0^2}{4} \left\{ 2 + \frac{1}{N} \sum_{k=1}^{N-1} \sum_{\bar{k}=k+1}^{N} \sum_{m=-\infty}^{\infty} \left[ \left( A_{-\bar{k}}(f) + A_{-\bar{k}}^*(f) \right) + \left( A_{+\bar{k}}(f) + A_{+\bar{k}}^*(f) \right) \right] \right\}
\]

(E.21)

\[
P(f) = \frac{|E_0|^2}{2} \left\{ 1 + \frac{1}{N} \sum_{k=1}^{N-1} \sum_{\bar{k}=k+1}^{N} \sum_{m=-\infty}^{\infty} \left[ \cos \left( 2\pi \frac{f-f_0}{F_R} \left( \frac{\bar{k}-k}{N} \right) \right) - (\phi_{\bar{k}} - \phi_k) \right] \delta (f - f_0 - mF_R)
\]

+ \cos \left( 2\pi \frac{f+f_0}{F_R} \left( \frac{\bar{k}-k}{N} \right) + (\phi_{\bar{k}} - \phi_k) \right) \delta (f + f_0 - mF_R) \right\}
\]

(E.22)

The frequency of mode \( m \) is \( f_m \), and thus the power can be written as:

\[
P(f_m) = \frac{|E_0|^2}{2} \left\{ 1 + \frac{1}{N} \sum_{k=1}^{N-1} \sum_{\bar{k}=k+1}^{N} \left[ \cos \left( 2\pi m \left( \frac{\bar{k}-k}{N} \right) \right) - (\phi_{\bar{k}} - \phi_k) \right]
\]

+ \cos \left( 2\pi m \left( \frac{\bar{k}-k}{N} \right) + (\phi_{\bar{k}} - \phi_k) \right) \right\}
\]

(E.23)

**The Correlated Harmonic Pulse Train and Frequency Comb**  In the special case that all of the \( k \) subset pulse trains have the same absolute carrier phase, i.e. \( \phi_k \) are equal for all \( k \), the power at mode \( f_m \) becomes:

\[
P(f_m) = \frac{|E_0|^2}{2} \left\{ 1 + \frac{1}{N} \sum_{k=1}^{N-1} \sum_{\bar{k}=k+1}^{N} 2 \cos \left( 2\pi m \left( \frac{\bar{k}-k}{N} \right) \right) \right\}
\]

(E.24)

In the \( k \) and \( \bar{k} \) summations, one will observe that there are \( N - 1 \) equal terms in the summations when \( k - \bar{k} = 1 \), \( N - 2 \) equal terms in the summations when \( k - \bar{k} = 2 \), etc. Letting \( q = k - \bar{k} \), there are \( N - q \) equal terms in the summations, and thus the expression can be rewritten as:

\[
P(f_m) = \frac{|E_0|^2}{2} \left\{ 1 + \sum_{q=1}^{N-1} \frac{2(N-q)}{N} \cos \left( 2\pi m \left( \frac{q}{N} \right) \right) \right\}
\]

(E.25)
\[ P(f_m) = \frac{|E_0|^2}{2} \left\{ 1 + 2 \sum_{q=1}^{N-1} \cos \left( 2\pi q \left( \frac{m}{N} \right) \right) - \frac{2}{N} \sum_{q=1}^{N-1} q \cos \left( 2\pi q \left( \frac{m}{N} \right) \right) \right\} \]  \hspace{1cm} (E.26)

\[ P(f_m) = \frac{|E_0|^2}{2} \left\{ 1 + 2 \sum_{q=1}^{N} \cos \left( 2\pi q \left( \frac{m}{N} \right) \right) - \frac{2}{N} \sum_{q=1}^{N} q \cos \left( 2\pi q \left( \frac{m}{N} \right) \right) \right\} \]  \hspace{1cm} (E.27)

Because \( m \) is an integer, some of the sinusoidal factors go to unity and cancel. The constant-coefficient sinusoid summation represents the constant-interval sampling of an integer number of cosine periods, and thus their sum is zero when \( m \neq rN \), and \( N \) when \( m = rN \), where \( r \) is an integer. The latter sinusoidal summation can be simplified using the following identity:

\[ \sum_{q=1}^{N} \cos(qa) = \frac{1}{4} \left( \csc^2 \left( \frac{a}{2} \right) \left( -N \cos \left( \left( N + 1 \right) a \right) + (N + 1) \cos \left( Na \right) - 1 \right) \right) \]  \hspace{1cm} (E.28)

where, in this case, \( a = 2\pi \frac{m}{N} \). Using the values for \( a \), this simplifies:

\[ \sum_{q=1}^{N} \cos(qa) = \frac{1}{4} \left( \csc^2 \left( \frac{a}{2} \right) \left( -N \cos \left( \left( N + 1 \right) a \right) + (N + 1) - 1 \right) \right) \]  \hspace{1cm} (E.29)

\[ \sum_{q=1}^{N} \cos(qa) = \frac{1}{4} \left( \frac{N \left( 1 - \cos \left( 2\pi m + 2\pi \frac{m}{N} \right) \right)}{\sin^2 \left( \frac{\pi m}{N} \right)} \right) \]  \hspace{1cm} (E.30)

\[ \sum_{q=1}^{N} \cos(qa) = \frac{1}{4} \left( \frac{2N \left( \sin^2 \left( \pi m + \frac{\pi m}{N} \right) \right)}{\sin^2 \left( \frac{\pi m}{N} \right)} \right) \]  \hspace{1cm} (E.31)
\[
\sum_{q=1}^{N} q \cos(qa) = \frac{N}{2} \left( \frac{(-1)^{2m} \sin^2 \left( \frac{\pi m}{N} \right)}{\sin^2 \left( \frac{\pi m}{N} \right)} \right) = \frac{N}{2} \quad (E.32)
\]

which gives the final power spectrum:

\[
P(f_m) = |E_0|^2 \left[ \begin{array}{c}
N \text{ when } (m = rN) \\
0 \text{ otherwise }
\end{array} \right] \quad (E.33)
\]
Appendix F

Experiments with High Repetition Rate Fundamentally Mode-locked Fiber Lasers

This appendix provides supplemental information for the high repetition rate fundamentally mode-locked fiber laser experiments.

F.1 In-Situ SBR Behavior Study

F.1.1 Component Loss Measurements

The transmission losses of many of the components used in the experiments outlined in Section 3.1.1 were measured. Unless otherwise noted, these measurements were taken at normal incidence. An ASE source\(^1\) was collimated\(^2\) to approximately a 0.7 mm radius beam. The power of the beam was directly detected with an optical power meter\(^3\), both before and after placing the device under test (DUT) in the beam path. The ratio of the powers was taken, the results of which are given in Table F.1. The distance between

\(^1\)Oprel EDFA, Model: OFP14D-12241S, Serial: 102-00028, with no input signal
\(^2\)Thorlabs, Inc., Model: F240APC-1550
\(^3\)Hewlett-Packard , Base Model: HP8152A, Serial: 2846G01764, Head Model: HP81525A, Serial: 3815 G 03573
Figure F.1: Setup used to measure the component losses.

<table>
<thead>
<tr>
<th></th>
<th>HWP at gain input</th>
<th>QWP at gain input</th>
<th>QWP before SBR</th>
<th>HWP at gain output</th>
<th>AR Coated Silicon Slab</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission</td>
<td>96.5%</td>
<td>98.83%</td>
<td>98%</td>
<td>98%</td>
<td>95.4%</td>
</tr>
<tr>
<td>Reflection</td>
<td>0.27%</td>
<td>0.52%</td>
<td>n/a</td>
<td>0.07%</td>
<td>1.5%</td>
</tr>
</tbody>
</table>

Table F.1: Losses of the various components used in this set of experiments.

the source and the DUT was approximately 40 cm.

For the polarization-dependent devices, a linear polarizer was placed after the collied ASE beam. The polarizer was then rotated to vary the input polarization orientation, and the transmission was measured as a function of the polarization angle. The results for the polarizing isolator and one of the dichroics are shown in Figure F.2. As expected of a polarizing component, the isolator shows a sinusoidal dependence. The minimum and maximum transmission were 0.00314% and 94.2%, respectively. Only one dichroic was measured, as the two used in the laser were fabricated together and will therefore be extremely similar. The reflectivity at 1550 nm shows slight polarization variation, as expected, ranging from 98.690% (TM polarized) to 99.064% (TE polarized).

For the PBS, linearly polarized light was again used, and it’s polarization state rotated. The output power from both output ports was measured. The sum of these two powers represents the power that will remain in the laser cavity, and is not lost to scattering and surface reflections. The sum of those powers, divided by the input power, is shown in Figure F.3a. The transmission associated with laser cavity loss induced by
the PBS varied from 97.433% to 98.951%.

Shown in Figure F.3b is the measured back-reflected power from the PBS. The sinusoidal variation is due to the fact that the reflected power is the sum of the reflections from three surfaces: at the input port, and each of the two output ports. The coatings for each of these ports must have been deposited during different coat runs, and thus will be have slightly different reflectivities. More reflection will be seen when more power is directed to the output port with the less ideal AR coating; less reflected power will be see when more power is directed to the output port with the better AR coating. The values are small, ranging from 0.434% to 0.467%, and are consistent with three standard AR coatings, which typically exhibit reflections ranging from 0.1% to 0.2%.

**F.1.2 Complete Characterization Data Sets**

This section presents more complete data sets for the laser output when using each SA. Data for the maximum spectral bandwidth and minimum spectral bandwidth are shown.
Figure F.3: Plots of the measured PBS losses. Shown in a) is the total transmission into the two output powers, and in b) is the measured back-reflection from the PBS.

F.1.2.1 Pump Configuration

Two pump diodes\(^4\) were used to pump the laser. Figure shows the measured output pump power as a function of diode current:

Figure F.4: Measured pump power as a function of pump current for Pump 1 (left) and Pump 2 (right).

The pump powers given below are calculated from the pump currents and this data, and so represent the total pump power exiting the pump collimator.

\(^{4}\)Bookham, Inc., Model: LC96UF74-20R, Serials: SO224692.001 0746 (“Pump 1” or “Slot 3”) and SO224693.001 0746 (“Pump 2” or “Slot 2”)
F.1.2.2 BATOP, SAM-1550-09-25.4s, Maximum Bandwidth

<table>
<thead>
<tr>
<th>Output</th>
<th>Pre-gain Tap</th>
<th>Post-gain Tap</th>
<th>Residual Pump</th>
<th>Pump Current 1</th>
<th>Pump Current 2</th>
<th>Pump Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.4 mW</td>
<td>31.9 μW</td>
<td>197.5 μW</td>
<td>45 mW</td>
<td>1050 mA</td>
<td>275 mA</td>
<td>831 mW</td>
</tr>
</tbody>
</table>

Table F.2: Measured power values for SAM-1550-09-25.4s with maximum bandwidth

Figure F.5: Optical power spectrum of output, pre-gain tap, and post-gain tap (left), and chirp compensation data (right).

Figure F.6: Autocorrelation of the direct laser output indicating a 418 fs pulse (left) and the shortest autocorrelation trace obtained indicating a 141 fs pulse (right).
Figure F.7: The RF spectrum of the detected fundamental beat frequency (left) and the beat spectrum spanning from the fundamental to the second harmonic (right).

Figure F.8: The RF spectrum of detected beat (left) and a snapshot of the detected pulse train on an oscilloscope (right).

F.1.2.3 BATOP, SAM-1550-09-25.4s, Minimum Bandwidth

<table>
<thead>
<tr>
<th>Output</th>
<th>Pre-gain Tap</th>
<th>Post-gain Tap</th>
<th>Residual Pump</th>
<th>Pump Current 1</th>
<th>Pump Current 2</th>
<th>Pump Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.2 mW</td>
<td>17.45 μW</td>
<td>123.5 μW</td>
<td>32 mW</td>
<td>900 mA</td>
<td>0 mA</td>
<td>568 mW</td>
</tr>
</tbody>
</table>

Table F.3: Measured power values for SAM-1550-09-25.4s with minimum bandwidth
Figure F.9: Optical power spectrum of output, pre-gain tap, and post-gain tap (left), and chirp compensation data (right).

Figure F.10: Autocorrelation of the direct laser output indicating a 439 fs pulse (left) and the shortest autocorrelation trace obtained indicating a 322 fs pulse (right).
Figure F.11: The RF spectrum of the detected fundamental beat frequency (left) and the beat spectrum spanning from the fundamental to the second harmonic (right).

Figure F.12: The RF spectrum of detected beat (left) and a snapshot of the detected pulse train on an oscilloscope (right).

F.1.2.4 BATOP, SAM-1550-20-25.4s, Maximum Bandwidth

<table>
<thead>
<tr>
<th>Output</th>
<th>Pre-gain Tap</th>
<th>Post-gain Tap</th>
<th>Residual Pump</th>
<th>Pump Current 1</th>
<th>Pump Current 2</th>
<th>Pump Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>57.1 mW</td>
<td>53.1 µW</td>
<td>333 µW</td>
<td>76.5 mW</td>
<td>1050 mA</td>
<td>1050 mA</td>
<td>1327 mW</td>
</tr>
</tbody>
</table>

Table F.4: Measured power values for SAM-1550-20-25.4s with maximum bandwidth
Figure F.13: Optical power spectrum of output, pre-gain tap, and post-gain tap (left), and chirp compensation data (right).

Figure F.14: Autocorrelation of the direct laser output indicating a 625 fs pulse (left) and the shortest autocorrelation trace obtained indicating a 93 fs pulse (right).
Figure F.15: The RF spectrum of the detected fundamental beat frequency (left) and the beat spectrum spanning from the fundamental to the second harmonic (right).

Figure F.16: The RF spectrum of detected beat (left) and a snapshot of the detected pulse train on an oscilloscope (right).

F.1.2.5 BATOP, SAM-1550-20-25.4s, Minimum Bandwidth

<table>
<thead>
<tr>
<th>Output</th>
<th>Pre-gain Tap</th>
<th>Post-gain Tap</th>
<th>Residual Pump</th>
<th>Pump Current 1</th>
<th>Pump Current 2</th>
<th>Pump Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.0 mW</td>
<td>30.5 $\mu W$</td>
<td>212 $\mu W$</td>
<td>57 mW</td>
<td>1050 mA</td>
<td>490 mA</td>
<td>969 mW</td>
</tr>
</tbody>
</table>

Table F.5: Measured power values for SAM-1550-20-25.4s with minimum bandwidth
Figure F.17: Optical power spectrum of output, pre-gain tap, and post-gain tap (left), and chirp compensation data (right).

Figure F.18: Autocorrelation of the direct laser output indicating a 425 fs pulse (left) and the shortest autocorrelation trace obtained indicating a 147 fs pulse (right).
Figure F.19: The RF spectrum of the detected fundamental beat frequency (left) and the beat spectrum spanning from the fundamental to the second harmonic (right).

Figure F.20: The RF spectrum of detected beat (left) and a snapshot of the detected pulse train on an oscilloscope (right).

F.1.2.6 BATOP, SAM-1550-23-25.4s, Maximum Bandwidth

<table>
<thead>
<tr>
<th>Output</th>
<th>Pre-gain Tap</th>
<th>Post-gain Tap</th>
<th>Residual Pump</th>
<th>Pump Current 1</th>
<th>Pump Current 2</th>
<th>Pump Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>57.6 mW</td>
<td>54.2 μW</td>
<td>344 μW</td>
<td>82 mW</td>
<td>1050 mA</td>
<td>1050 mA</td>
<td>1327 mW</td>
</tr>
</tbody>
</table>

Table F.6: Measured power values for SAM-1550-23-25.4s with maximum bandwidth
Figure F.21: Optical power spectrum of output, pre-gain tap, and post-gain tap (left), and chirp compensation data (right).

Figure F.22: Autocorrelation of the direct laser output indicating a 574 fs pulse (left) and the shortest autocorrelation trace obtained indicating a 100 fs pulse (right).
Figure F.23: The RF spectrum of the detected fundamental beat frequency (left) and the beat spectrum spanning from the fundamental to the second harmonic (right).

Figure F.24: The RF spectrum of detected beat (left) and a snapshot of the detected pulse train on an oscilloscope (right).

F.1.2.7  BATOP, SAM-1550-23-25.4s, Minimum Bandwidth

<table>
<thead>
<tr>
<th>Output</th>
<th>Pre-gain Tap</th>
<th>Post-gain Tap</th>
<th>Residual Pump</th>
<th>Pump Current 1</th>
<th>Pump Current 2</th>
<th>Pump Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.3 mW</td>
<td>15.9 μW</td>
<td>120.3 μW</td>
<td>36 mW</td>
<td>950 mA</td>
<td>0 mA</td>
<td>600 mW</td>
</tr>
</tbody>
</table>

Table F.7: Measured power values for SAM-1550-23-25.4s with minimum bandwidth
Figure F.25: Optical power spectrum of output, pre-gain tap, and post-gain tap (left), and chirp compensation data (right).

Figure F.26: Autocorrelation of the direct laser output indicating a 440 fs pulse (left) and the shortest autocorrelation trace obtained indicating a 283 fs pulse (right).
Figure F.27: The RF spectrum of the detected fundamental beat frequency (left) and the beat spectrum spanning from the fundamental to the second harmonic (right).

Figure F.28: The RF spectrum of detected beat (left) and a snapshot of the detected pulse train on an oscilloscope (right).

F.1.2.8 BATOP, SAM-1550-35-25.4s, Maximum Bandwidth

<table>
<thead>
<tr>
<th>Output</th>
<th>Pre-gain Tap</th>
<th>Post-gain Tap</th>
<th>Residual Pump</th>
<th>Pump Current 1</th>
<th>Pump Current 2</th>
<th>Pump Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>72.8 mW</td>
<td>47.0 μW</td>
<td>365 μW</td>
<td>74 mW</td>
<td>1050 mA</td>
<td>1050 mA</td>
<td>1327 mW</td>
</tr>
</tbody>
</table>

Table F.8: Measured power values for SAM-1550-35-25.4s with maximum bandwidth
Figure F.29: Optical power spectrum of output, pre-gain tap, and post-gain tap (left), and chirp compensation data (right).

Figure F.30: Autocorrelation of the direct laser output indicating a 860 fs pulse (left) and the shortest autocorrelation trace obtained indicating a 326 fs pulse (right).
Figure F.31: The RF spectrum of the detected fundamental beat frequency (left) and the beat spectrum spanning from the fundamental to the second harmonic (right).

Figure F.32: The RF spectrum of detected beat (left) and a snapshot of the detected pulse train on an oscilloscope (right).

F.1.2.9 BATOP, SAM-1550-35-25.4s, Minimum Bandwidth

<table>
<thead>
<tr>
<th>Output</th>
<th>Pre-gain Tap</th>
<th>Post-gain Tap</th>
<th>Residual Pump</th>
<th>Pump Current 1</th>
<th>Pump Current 2</th>
<th>Pump Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.6 mW</td>
<td>38.1 μW</td>
<td>288 μW</td>
<td>63.5 mW</td>
<td>1050 mA</td>
<td>750 mA</td>
<td>1135 mW</td>
</tr>
</tbody>
</table>

Table F.9: Measured power values for SAM-1550-35-25.4s with minimum bandwidth
Figure F.33: Optical power spectrum of output, pre-gain tap, and post-gain tap (left), and chirp compensation data (right).

Figure F.34: Autocorrelation of the direct laser output indicating a 920 fs pulse (left) and the shortest autocorrelation trace obtained indicating a 404 fs pulse (right).
Figure F.35: The RF spectrum of the detected fundamental beat frequency (left) and the beat spectrum spanning from the fundamental to the second harmonic (right).

Figure F.36: The RF spectrum of detected beat (left) and a snapshot of the detected pulse train on an oscilloscope (right).
Appendix G

Laser Performance Specifications

This section presents supplemental performance information for the lasers used in various experiments described in this thesis.

G.1 Stretched-Pulse Laser

Below is a full characterization data set for the stretched-pulse laser used as a source in these measurements:

![Optical power spectrum](image1)

![Chirp-compensated output pulse](image2)

Figure G.1: Optical power spectrum (left) and the chirp-compensated output pulse indicating a pulse intensity FWHM of 88 fs (right).
Figure G.2: Detected RF spectrum showing the fundamental repetition rate beat (left), and the RF beat spectrum spanning the fundamental and first harmonic beats (right).

Figure G.3: The RF beat spectrum (left) and an oscilloscope trace of the detected pulse train (right).

**G.2 Commercial Laser**

Figure G.4 shows the measured optical power spectrum and autocorrelation for the commercial laser. The system built in to the laser that mode-locks it sometimes locks on to other states that may include CW spikes or other structure; this state is the good state in which the laser was intended to be used. This 200 MHz soliton laser
was purchased from Menlo Systems GmbH. The laser output power is approximately 114 mW. Other performance data measured by the company is included in the laser manual.

Figure G.4: Output optical power spectrum (left) and pulse autocorrelation (right). Secant hyperbolic fit indicates an intensity FWHM of 90 fs.
Appendix H

Miscellaneous

H.1 Photodetector Response and Mixing

This section summarizes the square-law nature of photodiode detectors.

A typical photodiode detector uses photons to promote electrons to the conduction band such that they can flow through the supporting detector circuits. The result is a square-law detector, that is, a detector that generates a current proportional to the input intensity. The responsivity of the detector, \( \eta \), expresses this relationship:

\[
I = \eta P_{\text{optical}} \quad \text{(H.1)}
\]

The electrical power, then, can easily be written in terms of the optical power:

\[
P_{\text{electrical}} = (\eta P_{\text{optical}})^2 Z \quad \text{(H.2)}
\]

where \( Z \) is the impedance of the detector circuit. Because the electrical power is proportional to the square of the optical power, the detector acts as a mixer of the incident optical frequencies.
H.2 Shot Noise-Limited Beat Detection

For detection of the optical beat frequency on a detector, the SNR is of critical importance. If shot noise is limiting the SNR, then the SNR can be written as:

\[
SNR \propto \frac{\text{Beat Power}}{(\text{Shot}) \text{ Noise Power}} = \frac{\sqrt{P_{DFG}P_{HeNe}}}{P_{DFG} + P_{HeNe}}
\]  

(H.3)

where the shot noise is proportional to the total intensity. As a result, provided there is sufficient power that shot noise is the limiting noise source, an optimal beat signal will be detected when the powers, \( P_{DFG} \) and \( P_{HeNe} \), are equal.