STRUCTURAL ANALYSIS AND ASSESSMENT OF GUASTAVINO VAULTING

by

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B.S. Civil Engineering
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in Partial Fulfillment of the Requirements for the Degree of

Master of Science of Building Technology

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Abstract

This thesis studies the behavior and pathologies of the masonry tile structures built by the R. Guastavino Company in order to provide recommendations on their analysis and assessment. Structural analyses of two specific geometries – domes and barrel vaults – are carried out with equilibrium and elastic methods to determine how well each assesses the safety of Guastavino shells. Results show that stresses are relatively low in these structures, so they are unlikely to fail due to inadequate material capacity. The safety, then, is dependent on the stability of the structure rather than its material strength. Analysis of a Guastavino structure should demonstrate its stability, and graphical equilibrium analysis is well-suited to this task.

Case studies of three Guastavino projects – the Grace Universalist Church, the Saint Louis Art Museum, and the Army War College – provide examples of pathologies specific to masonry tile structures and demonstrate how they were successfully or unsuccessfully analyzed and rehabilitated in the past. Guastavino shells exhibit behavior similar to other masonry structures, but have an additional characteristic that sets them apart: soffit tiles can debond and fall as a result of cracking or water damage. Falling tiles pose a serious mortal danger but do not necessarily threaten a structure’s safety. Nonetheless, they elicit dramatic structural repairs and retrofits. The case studies present an opportunity to critically evaluate structural interventions with an understanding of masonry tile vault behavior. By bringing both sensitive and unsuccessful rehabilitations to light, hopefully Guastavino shells will be protected from unnecessary retrofits in the future.

Thesis Supervisor: John A. Ochsendorf
Title: Associate Professor of Building Technology
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Table of Contents

Chapter 1. Introduction ............................................................................................................. 9
  1.1 Motivation ......................................................................................................................... 9
  1.2 Problem Statement and Proposed Study ........................................................................... 9
  1.3 Scope of Research ............................................................................................................ 11
  1.4 Objectives of Research ................................................................................................. 12

Chapter 2. Literature Review ........................................................................................................ 13
  2.1 The Design Practices of the R. Guastavino Company ................................................ 13
    2.1.1 Rafael Guastavino Sr. and “Cohesive Construction” ........................................ 13
    2.1.2 Design Practices under Rafael Guastavino Sr. ..................................................... 17
    2.1.3 Rafael Guastavino Jr. and Graphical Analysis .................................................. 19
    2.1.4 Design Practices under Irving Berg ........................................................................ 20
  2.2 The Analysis of Masonry Vaults and Domes ................................................................ 21
    2.2.1 Graphical Analysis and Plastic Theory ..................................................................... 23
  2.3 The Analysis of Guastavino Vaulting ............................................................................. 26
  2.4 The Assessment and Restoration of Guastavino Vaulting .............................................. 29
  2.5 Chapter Summary ............................................................................................................ 31

Chapter 3. Structural Analysis Methods for a Guastavino Dome ........................................ 33
  3.1 Graphical Analysis ......................................................................................................... 34
  3.2 Membrane Analysis ........................................................................................................ 42
  3.3 Finite Element Analysis .................................................................................................. 44
  3.4 Discussion of Analysis Results ....................................................................................... 46
  3.5 Conclusions .................................................................................................................... 50

Chapter 4. Structural Analysis Methods for a Guastavino Barrel Vault ............................... 52
  4.1 Graphical Analysis ......................................................................................................... 53
  4.2 Finite Element Analysis .................................................................................................. 56
  4.3 Discussion of Analysis Results ....................................................................................... 58
  4.4 Conclusions .................................................................................................................... 62

Chapter 5. The Guastavino Dome of the Grace Universalist Church .................................... 63
  5.1 Geometry ....................................................................................................................... 65
  5.2 Structural History .......................................................................................................... 69
  5.3 Graphical Analysis ........................................................................................................ 70
  5.4 Discussion of Analysis Results ....................................................................................... 74
  5.5 Chapter Summary ............................................................................................................ 75

Chapter 6. The Guastavino Vault of the Saint Louis Art Museum ....................................... 76
  6.1 Geometry ....................................................................................................................... 77
  6.2 Structural History .......................................................................................................... 81
    6.2.1 Structural Problems: Settlement, Cracks, and Falling Tiles .................................. 81
    6.2.2 Retrofit of the Vault ............................................................................................... 84
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.3 Structural Analysis</td>
<td>84</td>
</tr>
<tr>
<td>6.3.1 Graphical Analysis: Investigation of the Vault’s Stability</td>
<td>85</td>
</tr>
<tr>
<td>6.3.2 Finite Element Analysis: Investigation of the Cause of Cracks</td>
<td>90</td>
</tr>
<tr>
<td>6.4 Discussion of Analysis Results and Retrofit Solutions</td>
<td>92</td>
</tr>
<tr>
<td>6.5 Chapter Summary</td>
<td>94</td>
</tr>
</tbody>
</table>

Chapter 7. Guastavino Vaulting at the Army War College | 95
| 7.1 Geometry | 95 |
| 7.2 Structural History | 100 |
| 7.2.1 Investigations and Repairs of the 1940s | 100 |
| 7.2.2 Investigation and Repairs of the 1970s and 1980s | 106 |
| 7.3 Discussion of Investigations and Repairs | 111 |
| 7.3.1 The 1940s | 112 |
| 7.3.2 The 1970s and 1980s | 113 |
| 7.4 Chapter Summary | 115 |

Chapter 8. Conclusions | 116
| 8.1 Summary | 116 |
| 8.2 Recommendations | 117 |
| 8.3 Future Research | 120 |

References | 122

Appendix A. Derivation of the Point of Zero Hoop Stress for a Dome | 126

Appendix B. A Step-by-Step Explanation of the Graphical Analysis of a Dome | 127

Appendix C. Graphical Analysis for a Guastavino Dome without Tensile Capacity | 133

Appendix D. Load Calculations for the Saint Louis Art Museum | 136

Appendix E. A Guide to the Structural Assessment of Guastavino Vaulting | 140
Chapter 1. Introduction

1.1 Motivation

In 1881, Rafael Guastavino Sr. immigrated to the United States with his son, Rafael Jr., and a building method never before seen in the United States. As an economical means to span long distances coupled with Rafael Sr.'s considerable salesmanship, his method of building thin structural shells out of tile and mortar gained popularity with prominent Beaux-Art architects around the turn of the century. The R. Guastavino Company continued to earn commissions for momentous projects through World War I, but the rising cost of labor caused their business to taper off after World War II (Collins 1968; Ochsendorf, forthcoming).

Though the company was liquidated in 1962, its legacy lives on in hundreds of projects throughout the United States. Like many historic structures, these feats of engineering are threatened with neglect, deterioration, and demolition. But while rising prominence of the name "Guastavino" amongst architecture aficionados can protect the company’s work against overzealous developers, engineers’ lack of understanding their structural behavior and inability to assess their safety puts Guastavino vaults at risk of destruction or unnecessary strengthening. Guastavino masonry tile structures often show signs of structural distress that are completely innocuous but make engineers fearful nonetheless, so there is a serious need for a means to assess the safety of these structures and to better understand their structural behavior.

1.2 Problem Statement and Proposed Study

There is no record of a Guastavino structure ever collapsing in service (Ochsendorf, forthcoming). Nonetheless, as many buildings with Guastavino vaults or domes are reaching their centennial, the need to assess the safety of these structures becomes more and more essential. In order to make such an evaluation, one must first define what makes a Guastavino shell safe or unsafe. In other words, what conditions should cause concern? Might the vault fail due to a lack of strength? Or will it collapse because of a lack of stability? Does a crack or fallen tile suggest impending collapse? A means of analyzing Guastavino vaults in order to assess their strength and stability is then needed. There are two dominant structural analysis methods available: limit analysis and elastic analysis (Heyman 1995, 3-5).

According to limit analysis, masonry structures are a collection of rigid blocks which are stable because gravitational forces induce compression into the assemblage (Heyman 1995, 12).
Limit analysis then investigates the safety of masonry structures based on equilibrium and geometry rather than the strength of the material (Heyman 1995, 14). Equilibrium methods, such as graphical analysis and membrane analysis, rely on equations of equilibrium to find the forces within a structure (Heyman 1995, 8). Graphical analysis holds the distinction of being one method used historically in the design of Guastavino shells and thus becomes a promising means of analyzing these structures today. Intuitive, quick to execute and easily adaptable to changes such as support movements and multiple load conditions, graphical analysis proposes forces within a Guastavino vault without taking the material properties into consideration. Unfortunately, equilibrium methods such as graphical analysis have been forgotten over time, though they may offer the best means to analyze a Guastavino vault.

Elastic analysis methods, such as finite element analysis, utilize constitutive relations, boundary conditions, and imposed deformations to find the forces within a Guastavino vault (Heyman 1995, 8). In order to analyze a structure using finite element analysis, material properties of the structure such as the modulus of elasticity are needed. Elastic solutions are highly sensitive to small movements of the supports, which are inevitable, and make the exact stress state of a structure unknowable (Heyman 1995, 9). Additionally, movements of the supports can induce cracks in brittle masonry shells, and cracks render the use of linear elastic finite element analysis inadequate for understanding the structure after cracking.

Besides a reliable analysis technique, another key element to the accurate assessment of Guastavino shell safety is an understanding of their behavior. Familiarity with the behavior of these structures and some common types of distress will aid an engineer in assessment by helping them to focus on the most critical aspects and avoid senseless investigations of non-threatening characteristics.

A unique challenge to the assessment of Guastavino shells is the propagation of inaccurate information on their behavior which can be traced in part to Rafael Guastavino Sr. Although thin tile vaults have a history predating the R. Guastavino Company by hundreds of years, Rafael Guastavino Sr. himself appears to be the first person to attempt to quantify their material properties and propose methods for their design (Huerta 2003, 98). Guastavino’s 1893 book, *Essay on the Theory and History of Cohesive Construction*, presents explanations of masonry tile vault behavior that attempt to differentiate it from other masonry structures, a misguided notion that engineers and architects still fall victim to today. The book is shaky in its
explanations and contradicts itself at times, but because the Guastavinos were masters of their craft, few people have questioned the validity of his claims (Huerta 2003).

This thesis will attempt to shed light on the behavior of Guastavino vaults through the study of pathologies and structural analysis. Instances of past structural problems will be considered for what they reveal about the behavior of masonry tile shells. Both elastic and equilibrium analysis methods will be considered for their ability to assess the safety of these structures. Finally, case studies will be used to investigate how well architects and engineers faced with making assessments of these structures in the past understood the behavior of Guastavino vaults, and will highlight instances of success and failure in their treatment of structural problems.

1.3 Scope of Research

This thesis aims to serve practicing architects and engineers as a guide in the assessment and restoration of Guastavino vaults and domes. The first step is to investigate the behavior of these structures by identifying common structural problems and considering the results of structural analyses. Then, general guidelines to making an accurate assessment of the safety of a masonry tile structure can be recommended.

One important consideration is that the recommendations presented herein be straightforward, practical, and reproducible. Of course, due to their complicated interactions with surrounding structural components and frequently cracked states, the complexity of these structures in real life is considerable. While sophisticated analyses methods may seem well suited for the assessment of these structures, this research will demonstrate that less complicated analyses are nonetheless useful and valuable tools. In the interest of protecting these structures against unnecessary retrofits and demolition, a clear-cut means to assess these structures may be more powerful than an expensive and arduous structural analysis program. Furthermore, if an analyst is well informed as to the behavior of these structures, there may be little need for complex or even simple analyses in most cases.

This thesis attempts to answer the following questions in regards to the overall goal: the assessment of Guastavino vaults.

- How does one evaluate the safety of a Guastavino vault or dome? Is the safety limited by the strength or stability of the structure?
1.4 Objectives of Research

The purpose of this thesis is to recommend methods suitable for the analysis of Guastavino structures, to provide insight into their structural behavior, and to highlight good and bad restoration decisions that have been made over the years in the rehabilitation of Guastavino vaulting. The potentials and limitations of three analysis methods - graphical analysis, membrane analysis, and finite element analysis - will be studied by their application to generic Guastavino forms and to existing structures. Additionally, past instances of Guastavino vault analysis will be considered in case studies to gauge the success or failure of previous interventions. The case studies will also serve as a means to understand the pathologies and behavior of Guastavino structures, and to demonstrate how their structural issues were dealt with in the past and to identify instances of good practice.
Chapter 2. Literature Review

This chapter provides background information on the design methodologies of the R. Guastavino Company and how the assessment of tile vaults has been approached in the past. The first part of the chapter investigates the Guastavinos’ own understanding of masonry tile vault behavior and the design methods used by their company. Next, this chapter reviews the literature pertaining to the field of unreinforced masonry analysis. Finally, sources specifically on the analysis of Guastavino shells and case studies of actual restorations are reviewed.

2.1 The Design Practices of the R. Guastavino Company

The Guastavinos’ collective career spanned for almost a century, from the Batlló Factory in Spain in 1868 to the Cathedral of St. Philip in 1961, so it is unsurprising that their design methodologies evolved over time (Ochsendorf, forthcoming). Rafael Guastavino Sr. was educated as a “master builder” in Spain and took classes in descriptive geometry, construction, and architecture (Ochsendorf, forthcoming). In 1892, he published *Essay on the Theory and History of Cohesive Construction*, eleven years after his arrival in the United States. This book reveals how vaults were designed during the early life of the company and provides insight into Rafael Sr.’s understanding of the structures he erected. Rafael Jr. did not have any formal education as an architect or engineer, but started working under his father at age fifteen and continued to study on his own (Ochsendorf, forthcoming).

2.1.1 Rafael Guastavino Sr. and “Cohesive Construction”

In his book, Rafael Guastavino Sr. imparts his theories on tile vault behavior, establishes mechanical properties of the vault material, and presents two design methodologies: one derived from equilibrium methods and one likely from elastic methods. Guastavino bases his theory of vault behavior around this idea of “cohesion,” that is, that tile vaults act as a solid mass as opposed to arches or domes composed of voussoirs, which rely on gravity for their stability. He supposes that this cohesion gives tile vaults resistance to shear and tensile forces that are much greater than for voussoir arches, and he conducts mechanical tests to find the values of these properties, the results of which are listed in Table 2.1. The transverse strength test (Figure 2.1) gives the bending strength for a three-course thick section of vault while the shear strength tests (Figure 2.2) give the strength between layers of tile bonded with both Portland cement and Plaster of Paris.
Compressive Strength, 5-day 2060 psi
Compressive Strength, 360-day 3290 psi
Tensile Strength 287 psi
Transverse (Bending) Strength 90 psi
Shear Strength, Portland Cement 123.7 psi
Shear Strength, Plaster of Paris 34 psi

Table 2.1 Test results obtained by Guastavino (1892, 58-59).

In his explanation of the behavior of tile arches, Guastavino uses three examples: a solid lintel, a solid arch-shaped lintel, and a shallow voussoir arch (Figure 2.3). He characterizes the solid lintel as having no thrust because it behaves like a tied arch, the solid arch-shaped lintel as having some thrust, and the voussoir arch as having full thrust. He argues that the solid arch-shape lintel can resist some thrust through its “cohesion,” or rather, that it can act as a beam in bending because of its tensile strength. Later in the book, Guastavino disputes a rumor that tile arches have no thrust, which was circulating amongst his colleagues in Spain at the time. As demonstrated by Huerta (2003, 125), the notion of thrust was well recognized in practice, since builders such as Guastavino always included buttresses or tension ties in their designs. Although he seemed confused on the issue of thrust, Guastavino always accounted for it in his designs. Nonetheless, his continued insistence on the importance of the “cohesion,” or tensile capacity, of
the material illustrates what Huerta (2003, 112) called “schizophrenia” in regards to Guastavino’s understanding of the thrust of vaults.

Guastavino used a similar analogy to describe the behavior of a dome, hollowing out the underside of a round, flat plate as illustrated in Figure 2.4. Just as Guastavino describes the solid lintel as having no thrust due to tensile capacity which acts as a natural tension tie, he describes the flat plate as having no thrust because of a natural tension ring. However, later in the book, Guastavino suggests including a steel ring at the base of a dome, but he does not say that it is to contain the thrust of the vault. In fact, he says that this ring can be very small since the dome is able to equilibrate all its own forces through tensile capacity. This notion is incorrect, as all masonry domes thrust outwards (Heyman 1995, 27-47). Although Guastavino does not understand dome behavior, his intuition prompted him to include the tension ring. Since Guastavino domes are relatively lightweight (versus other masonry domes), his use of small tension rings likely provides adequate resistance against thrust.
On the subject of concentrated loads on tile arches, Guastavino wrote that “if an arch is built with the condition that the curve of pressure is inside of the middle third, the arch is safe,” demonstrating some familiarity with graphical methods of analysis and the concept of a geometric factor of safety (Heyman 1995, 20). He draws thrust lines for an arch under a concentrated load at different locations, and these lines lie not within the arch but rather in a space above the arch (Figure 2.5), leading him to comment that this space must be “solid” or strengthened with elements that “take the place of solid materials.” This passage in the book demonstrates Guastavino’s understanding of vault behavior and explains how he knew to include stiffeners for his vaults, as evidenced by vaults at the Boston Public Library (Figure 2.6).
Huerta (2003) identifies the comte d’Espie and his 1754 book as the originating source of the theory that tile vaults have no thrust and were “monolithic” in nature, and although this theory was discounted by Ventura Rodríguez in the late 18th century, the idea still perpetuates today. Guastavino essentially reproduced these ideas with his cohesive construction theory, which includes the assertion that tile vaults have minimal thrust – the same idea he contradicts in other parts of the book, such as the use of thrust to calculate arch crown thickness. He probably included the table of elastic stresses because at the time elastic analysis was considered to be at the forefront of engineering research, maybe in hopes that this would lend some academic credence to his tile vaults. Huerta maintains that although Guastavino fell victim to the fallacious “monolithic vault” theory and predominant elastic ideas of his time, the structures he built were testament to his mastery of masonry tile construction.

2.1.2 Design Practices under Rafael Guastavino Sr.

Rafael Guastavino Sr. presents two different analysis methods in his book: equilibrium and elastic. He does not venture to relate the two, demonstrating his shaky understanding of vault behavior. For the design of masonry tile vaults, he gives an equation to calculate the required thickness at the crown of an arch based on an allowable compressive strength \( C \) reduced by a safety factor of 10.

\[
\text{Thickness at Crown} = \frac{(\text{load} \cdot \text{span})}{8 \cdot (\text{rise}) \cdot 12 \cdot C}
\]  

This formula was found using a simple static equilibrium calculation for half an arch to find the horizontal force at the crown (Figure 2.7). The reaction is then set equal to the thickness at the

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1 No. 175, Archives of the Trustees of the Boston Public Library.
crown times a unit width times the allowable compressive strength of 2060 psi reduced by a factor of 10. Guastavino uses the same formula for dome design, except divided by two. He slices it into sections (Figure 2.8) and rearranges these pieces into a barrel vault (Figure 2.9) with a length of half the dome circumference. This vault then has half of the surface area of a regular vault of the same length and span, and thus Guastavino halves the thickness equation. This approximation of a dome as an arch in order to analyze is an acceptable method of design, albeit a conservative one as noted by Heyman (1995, 46): “the use of two-dimensional slices [to analyze a dome] can lead to much simpler solutions; although these solutions are safe, they may oversimplify the problem as to give too conservative results.” Hodge (2006) found that this equilibrium equation was in fact used in Rafael Sr.’s vault and dome designs, as suggested by calculations for the Boston Public Library and Arion Club.

Figure 2.7 The source for Guastavino’s arch thickness calculation (Atamturktur and Boothby 2007, 28).
At the end of the book, with no introduction, is a table to calculate the bending stress in arches of different geometries. The table was generated by Massachusetts Institute of Technology Applied Mechanics Professor Gaetano Lanza using elastic analysis, although no details of this analysis are given (Huerta 2003, 109). Using the coefficient for the maximum stress at the crown of the vault, the allowable compressive strength of 2060 psi, and a safety factor of 10, a maximum uniform load on the vault can be calculated. Hodge (2006) found that the maximum internal forces predicted by this table were consistently less than those calculated with the equilibrium equation presented earlier. Rafael Sr. does not address this fact anywhere in his book, nor does he recommend one method over the other.

2.1.3 Rafael Guastavino Jr. and Graphical Analysis

Huerta (2003, 114) notes that membrane analysis was developed for domes by Rankine in 1858, which lead to Eddy’s development of graphical analysis for domes. Eddy’s method was published in an article by Dunn in 1904, where Rafael Guastavino Jr. is likely to have found it (Huerta 2003, 114-19). Another possibility is that he was taught graphical analysis by an engineer hired by the project architect to analyze the dome of St. Paul’s Chapel at Columbia University when the R. Guastavino Company was unable to do so (Ochsendorf, forthcoming).
The analysis of this dome from 1906 seems to be the first evidence of graphical analysis in the R. Guastavino Company’s body of work. ²

An article by Dunn (1908) discusses the behavior of several R. Guastavino Company domes, so it is likely that by the time Rafael Jr. ran the company (Rafael Sr. passed away in February 1908) he understood their behavior better than his father when he penned his book.³ Dunn cites a shallow dome at Yale University and the necessity of the steel tension ring at the base to support the thrust. Another example is a dome at Columbia with a greater angle of embrace, where Dunn comments about the need for the tension ring at the base to counteract thrust without reliance on abutments. Ochsendorf (forthcoming) provides further proof of Rafael Jr.’s understanding of dome behavior, citing an unpublished manuscript that quotes him saying that steel bars were placed within the dome thickness at the Cathedral of St. John the Divine to “increase the tensile strength,” an idea he had patented in 1910.

2.1.4 Design Practices under Irving Berg

Irving Berg was a consulting engineer who produced calculations for the R. Guastavino Company beginning sometime in the 1920s or 1930s, though his exact start date is unknown. Berg used graphical analysis to assess cracked Guastavino vaults and arches at the Army War College in 1944, demonstrating that equilibrium methods were still in use by the company in the mid-1940s.⁴ In a letter to company president Malcolm Blodgett, Berg mentions that “in recent years the allowable working compressive stress used for construction of this kind was 300 psi… [and] present day designing calls for a factor of safety of 10,” demonstrating that the company was assuming a higher compressive strength than in the past but maintained a large safety factor.⁵ Berg reports that based on his analysis, the vaults of the Army War College only have a safety factor between 5 and 8 but he is not concerned by this. The R. Guastavino Company seems to have known that their vaults depended on geometry for their stability and that the stresses were generally low.

⁵ Irving Berg to Malcolm Blodgett, 14 November 1944, Army War College Records, Guastavino Archive.
Three decades later, a former R. Guastavino Company Vice President was interviewed about the company and about analysis of their forms:

Guastavino's early methods of analysis were not known to Mr. Bartlett however he discussed force distribution and the need to increase the shell thickness and provide buttresses where necessary to direct the line of pressure. Subsequently, Mr. Ira [sic] Berg who began working with the company and studied for his Architectural degree and then Engineering degree whilst with the company, would analyze the structures. Mr. Bartlett indicated that Mr. Berg would prepare force diagrams which assumed the formation of cracks which turned the highly indeterminate shells into determinate arches. Graphical analysis of the arches then gave forces and directions of thrust.

It is likely that the R. Guastavino Company used graphical analysis for its design and analyses through its end.

2.2 The Analysis of Masonry Vaults and Domes

Before the advent of structural mechanics, builders used rules of proportion to design their structures (Heyman 1995, 4). Modern engineers design structures using the concepts of stress and strain from structural mechanics (based on elastic analysis), whose origins date back to Galileo in 1638 (Heyman 1995, 5). Robert Hooke published an anagram in 1675 which translates to "as hangs the flexible line, so but inverted will stand the rigid arch," and this theory is essentially the basis of equilibrium analysis (Heyman 1995, 7). Elastic analysis methods and equilibrium analysis methods did not necessarily develop at the same rate, but they have coexisted for several centuries. Within the field of unreinforced masonry design and analysis today, these two methods still coexist though not quite peacefully, and there is still much debate over which is the "correct" method for the analysis of masonry structures. Dunn (1908) indicates that this rivalry existed even a century ago when he addresses the fallacy of applying elastic theory to masonry domes and arches. He notes that engineers of that time used elastic theory to design metallic bridges, but that elastic theory is not applicable to masonry because of the nonlinear behavior of the material and its lack of tensile capacity.

Boothby (2001) considers both equilibrium and elastic analysis methods in his state of the art review of the field of unreinforced masonry analysis. The author considers both lower bound (equilibrium) and upper bound theorems to be intuitive and useful to understanding

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6 Barlett, interview by John Reilly on 29 October 1974 (JMC [1979?], section IV.2).
masonry structures, but says that these rigid-plastic analysis methods are limited because they do not consider deformations of masonry over time, are limited in their ability to analyze complex structures, are tedious in comparison to finite element analysis, are unable to predict movement at the springings, and unable to find the “overall effect” on a structure. Boothby then discusses the history of elastic analysis application to historic masonry structures and its refinement through the use of more representative constitutive laws and the development of discrete element methods. Overall, the author claims that finite element analyses that take the specific properties of mortar into account are the best suited for analysis of masonry vaults and arches.

Block et al. (2006) considered two masonry arches of different thickness analyzed with finite element analysis and thrust line analysis. The finite element results for the two models show little difference, but the thrust line analyses show that while one of the vaults is stable, the other would not stand as it is too thin to contain the thrust line (Figure 2.10). The results of this study make evident that linear elastic finite element analysis is unable to tell the analyst anything about the stability of the arch while the thrust line analysis demonstrates this very clearly, making it useful for the assessment of historic masonry structures. Furthermore, the authors note that while non-linear finite element analysis can be useful for crack propagation with the correct material properties, these properties are hard to predict. Such analyses can be time consuming, but limit analysis provides fast and reliable means to predict the collapse mechanism.

Figure 2.10 Finite element and thrust line analysis of two masonry vaults, where only thrust line analysis reveals that vault 'a' could not support itself in real life (Block et al. 2006, 1842).
2.2.1 Graphical Analysis and Plastic Theory

Graphical analysis investigates the equilibrium of a structure and can be applied to masonry structures in particular. A doctoral thesis by Maurer (1998) studies the development of graphic statics by Karl Culmann. The author gives some earlier graphical methods, such as the development of force polygons (Figure 2.11), descriptive geometry and projective geometry, as the precursors of Culmann’s graphical statics method, and remarks that engineers were hesitant to accept the method and mathematicians criticized it as “primitive” (Hashagen 2004).

![Figure 2.11 Images from Varignon's Nouvelle mécanique ou statique, 1725 (reprinted from Maurer, 1998).](image)

Dunn (1904) provides a graphical method for finding the forces within a metal dome (Figure 2.12). The metal dome differs from a masonry dome in that it has both inherent tensile and compressive capacity. Dunn explains that there is a transition in the hoop stresses (defined in the next chapter) from compression to tension at a certain point of the dome, and that the compression region has a tendency to fall inwards while the tension region collapses out. In a later article, Dunn (1908) suggests Eddy’s “New Constructions in Graphic Statics” to analyze masonry domes. For masonry, the point where the forces change from compression to tension is critical (Figure 2.12). Below this point, he notes, a dome without reinforcement will tend to act as a series of arches. Dunn ends the article with a discussion of stresses in domes, and points out that increasing thickness does not increase the strength of the dome, although increasing thickness is beneficial for resisting “casual loads, or unequal distribution of pressure due to elastic deformation, which the present state of knowledge does not enable us to deal with.”
Wolfe (1921, 250-53) introduced a method of graphically analyzing domes by drawing a segmental line following the centerline of the dome cross-section to find forces in the meridional direction and the necessary hoop forces to confine the thrust line there. He proposes two methods to analyze a dome – one for material with tensile capacity and the other for domes without, which neglects hoop forces below 51.8°. Wolfe’s method will be discussed in more detail in Chapter 3.

Traditional equilibrium analyses of masonry structures, such as those presented by Dunn and Wolfe, are still considered valid today since they satisfy the rules of a more modern development known as plastic theory. Heyman (1995) proposes that if a solution of compressive forces in equilibrium can be found to lie within the masonry, such as those found by Wolfe or Eddy, then by the Master Safe Theorem of Plasticity, the structure is safe. Plastic theory developed from tests conducted on steel structures in the 1930s, which found that the elastic stresses calculated by engineers did not correspond with the actual stresses in the structures. This difference in the actual stress state comes from initial imperfections. Two of the same structures with different initial imperfections (and consequently different initial stress states) will still collapse at the same load. Thus, trying to calculate the actual state of stress in a statically indeterminate structure – which is the approach of elastic analysis - is futile because only the structure knows the actual state of stress.

Heyman (1995) describes the pathologies of masonry arches and domes, which due to their brittle nature and little to no tensile capacity, must crack in response to support movements. For an arch, such cracks do not necessarily indicate that the structure is unstable, but only that it is now statically determinate (i.e. the path of the thrust line is known) like the arch in Figure 2.12 Graphical analysis of a metal dome (Dunn 1904, 404).
2.13. Collapse occurs when the thrust line no longer can fit within the thickness of the arch. The dome, similarly, carries loads through forces within its thickness, but in two directions rather than one like the arch. When the supports of a dome spread it tends to cause cracks along the meridian (Figure 2.14), at which point the dome can be analyzed like an arch as Poleni demonstrated with his analysis of the cracked dome of St. Peter’s.

![Figure 2.13 An arch on spreading supports can still contain a thrust line (Heyman 1995, 15).](image1)

![Figure 2.14 Exaggerated cracks in a dome on spreading supports (Heyman 1995, 36).](image2)

Lau (2006) developed a modified version of Wolfe’s graphical analysis method for domes which allows the thrust line to pass through any part of the material of a dome cross-section (i.e. the thrust line is not constrained to the centerline of the dome thickness) based on Heyman’s Master Safe Theorem. Her Modified Thrust Line Analysis provides a means of analyzing a wide variety of axisymmetric dome geometries because the hoop stresses can be varied to restrain the thrust line anywhere within the cross-sectional thickness, rather than being limited to equilibrating the meridional forces.

Equilibrium methods were used for the structural design of two recently completed Guastavino-inspired shells in England (Ramage et al. 2007). The two domes were analyzed during phases of construction and under uniform load using membrane theory. Additionally, the
completed domes were analyzed under an asymmetric live load using the modified thrust line analysis technique.

Block and Ochsendorf (2007) have developed a methodology called thrust network analysis that is capable of finding funicular solutions for three-dimensional shell structures. Using their methodology, finding a compression-only thrust network that fits within a three-dimensional vault shape demonstrates that the structure is safe based on limit analysis. This method holds great potential to establish three-dimensional equilibrium solutions for masonry tile shell structures, but its application to Guastavino vaulting is beyond the scope of this thesis.

2.3 The Analysis of Guastavino Vaulting

Santiago Huerta (2003) provides a comprehensive study of the evolution of the design and analysis of masonry tile vaults. There appear to be two schools of thought since the nineteenth century: those in favor of elastic analysis, and those for equilibrium methods. Rafael Guastavino Sr., who included a table of stresses in arches found with elastic analysis by Gaetano Lanza, was the first to apply elastic analysis to tile vaults, and he was not the last. A succession of Spanish engineers throughout the early 20th century tried to assert that tile vaults possessed the ability to carry bending stresses, sometimes comparing them to metallic shells. For example, Bayó insisted that the vaults did not work only in compression, but also tension, and recommended thickening arches at the location of bending stresses (Figure 2.15). Huerta points out that such an arch would not stand in reality. Today, the attempts to use elastic analysis for tile vaults continue with the use of linear elastic finite element software, which assume elastic properties, are sensitive to support conditions, and are unable to model cracks. However, regardless of what was taking place amongst the academics, actual builders of tile vaults - from nineteenth century French engineers to the Guastavinos - relied on equilibrium methods for their design. It is worth noting that in some instances, their use of equilibrium methods seems reluctant. For instance, Bergós spent decades trying to justify the application of elastic theory to masonry tile vaulting, but presented equilibrium methods in his books. In the end, Huerta asserts that masonry tile vaulting exhibits the same pathologies as other masonry vaults, and should be analyzed under the same assumptions - most importantly, no tensile resistance - within the framework of Limit Analysis using equilibrium methods.
Atamturktur (2006) analyzes two Guastavino domes under impact excitation with finite element analysis, experimental modal analysis, and thin elastic shell theory, and compares the resulting mode shapes and natural frequencies. The experimental modal analyses results are used to verify the boundary conditions and material properties assumptions used in the finite element analyses for the purposes of calibrating a model. The model can then be used for analysis under static loads, which gives detailed information on the reactions of the structure. Though experimental analysis of a cracked dome exhibited non-linear behavior, the model was still able to be calibrated and used for a static analysis. This ability to get accurate values for the reactions and thrust is a significant development in the analysis of Guastavino domes. The resulting tensile and compressive stresses from the finite element analysis of the uncracked dome are low in comparison to strength found by Rafael Guastavino Sr. (Table 2.1) However, the question of either dome’s stability or safety is never addressed. The author also conducted compression tests on a tile sample and a mortar sample to obtain the material properties. The author uses a weighted average of the tile and mortar results based on the ratio of the materials in a dome to obtain the values listed in Table 2.2 (Poisson’s ratio is linearly averaged).

<p>| | |</p>
<table>
<thead>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus (E)</td>
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</tr>
<tr>
<td>Poisson’s Ratio (ν)</td>
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</tr>
<tr>
<td>Density (ρ)</td>
<td>1800 kg/m³ (112 pcf)</td>
</tr>
</tbody>
</table>

Table 2.2 Results of materials tests (Atamturktur 2006, 119).

Atamturktur and Boothby (2007) present a study of the presence of thrust and finite element modeling of Guastavino domes. The authors note that Rafael Guastavino Sr. contradicted himself by claiming that his domes had no thrust but then accounted for thrust in his designs, as previously shown by the historical analyses of Huerta (2003). Using the calibrated
finite element models studied previously by Atamturktur (2006), finite element analysis is able to quantify the magnitude of the thrust for two existing buildings.

Saliklis, Kurtz and Furnback (2003) study the material properties of Guastavino tile and create linear elastic finite element models. Five tile samples were tested nondestructively to obtain the Young’s Modulus, compressive strength, and flexural strength. All tests revealed different results in orthogonal directions, suggesting that Guastavino tile possesses orthotropic behavior. Nonetheless, they used an average value of Young’s Modulus (16500 MPa) for the finite element model, assuming that the tiles were alternatively oriented in different directions. Multiple analyses were run to investigate the effects of varying the Young’s Modulus and results showed little difference. Twelve finite element models were created representing arches of different dimensions and meshed into separate tile and mortar components using shell elements. The resulting maximum stresses (the direction of this stress is unclear) is compared to the allowable compressive stress presented in Guastavino’s Essay, and the finite element models consistently had a safety factor of about 4 for compressive strength. It is never revealed in the study whether any tensile stresses resulted from the finite element analysis, nor were the finite element analysis results compared with any other analysis results for verification.

Kaup and Matteo (2008) use membrane theory, graphical analysis, and finite element analysis to analyze the Guastavino dome at the University of Virginia. The results are compared with Rafael Sr.’s formula for dome design (Equation 2.1), and the results of the membrane analysis show some correspondence to the thrust calculated with this equation. The hoop stresses resulting from the graphical analysis seem to be incorrect (they are absent from the force polygon for the analysis as well). The graphical analysis appears to be based on Wolfe’s method as the book is cited in the reference list. If this is the case, the graphical analysis should yield results close to that from membrane analysis since they are both equilibrium methods that constrain the meridional and hoop forces to the mid-surface layer of the shell (Lau 2006, 26). Also, the results of the finite element analysis differ somewhat from the other results, but no speculation or explanation is given for their deviation. Nonetheless, the paper is valuable for providing an instance of equilibrium methods applied to a Guastavino dome in practice and for comparing results across the different analysis methods. Finally, this paper is one of the first to realize the importance of calculating the force in the tension tie and comparing the expected value with the size of the tie specified by the R. Guastavino Company.
2.4 The Assessment and Restoration of Guastavino Vaulting

Prominent thin-shell engineer Anton Tedesko opposed heavy-handed restoration solutions and advocated for "leav[ing] the structure alone as much as possible" during his involvement with the rehabilitation of the Army War College. Silman’s (1999) treatment of Guastavino vaults in the Oyster Bar in the basement of Grand Central Terminal provides an excellent example of just that. After a fire in 1997, the decorative face tiles on the vaults of the Oyster Bar debonded from the structural tiles in numerous places. Many of these tiles fell from the vault as a result. The remaining decorative tiles were sounded with a rubber mallet and removed if they had debonded, thereby removing the threat of their falling. The next step was to establish the stability of the vaults without this layer of tile. Recognizing that the vaults had stood on their own before the addition of the decorative tiles, the author did not attempt a structural analysis of the vaults. Instead, he cites fire tests conducted by the R. Guastavino Company showing a test vault could safely stand under loading of 600 pounds per square foot after face tiles fell off due to thermal shock. Results of tests conducted by the Guastavinos are as valid today as they were in the past, and they should be considered during the assessment of existing vaults. The last stage of the Oyster Bar restoration was to simply replace the decorative tile layer.

The rehabilitation of the Queensboro Bridge, described by DiSanto (1999), serves as another example of a sensitive restoration. In his words, “the project was primarily conceived of as a structural stabilization but was approached from a preservation perspective.” This demanded a thorough inspection of the vaults, both underneath and above, and the sounding of all face tiles with a mallet. Inspections revealed significant water damage (efflorescence, tile delamination) and structural damage (cracks) due to water infiltration, thermal effects, and vibrations from bridge traffic. To address the water infiltration, the back sides of the vaults were coated in waterproofing and a drainage system was added to prevent water from collecting near the columns on the back side of the vault. It was recognized that major cracks in the vaults might reoccur if simply filled with grout, so flexible reinforcing bars were installed under the face tiles and the face tiles were then bonded only along the edges with flexible sealant. As a result of this restoration, not only was the damage to the vaults fixed, but preventative measures minimize future harm to the vaults. The use of the "soft joints" is particularly laudable as they allow the

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7 Anton Tedesco to Charles Shores, 18 August 1978, Guastavino Archive.
structure to continue to move as needed. However, the apparent function of the reinforcing bars seems to be to add tensile capacity to the vaults. Cracks in the vault are likely a result of foundation settlement or other changes to the geometry, and furthermore, may be serving as expansion joints that allow for safe temperature movements in the vault. If the added reinforcing bars hinder movement of the vault, they could induce new cracking in other parts of the structure in the future and cause damage to intact sections of the tile vaulting.

In his article on the seismic retrofit of the Hearst Memorial Mining Building, Robertson (1999) addresses the vulnerability of Guastavino structures to seismic activity. The author notes that the performance of Guastavino vaulting under seismic loads is essentially unknown save for minimal responses of the Hearst Memorial Mining Building and Grace Cathedral to the Loma Prieta earthquake of 1989. Instead he looks at the performance of other unreinforced masonry structures in regions with considerable seismic activity. He mentions the complete collapse of buildings in the Middle East with floor systems of masonry arches spanning between I-beams, something used often by the R. Guastavino Company except with thin tiles rather than bricks. He also cites the collapse of the Basilica of St. Francis of Assisi as indication that gothic vaults are vulnerable to seismic activity, but does not consider the fact that Guastavino vaults weigh much less than traditional masonry structures. The Hearst Memorial Mining Building is of particular concern because of its proximity to an active fault. In 1996, the building was scheduled for retrofit with base isolation bearings to reduce lateral loads during an earthquake, but because the building is so close to the fault, engineers were concerned about the vertical forces from an earthquake as well. Finite element models of the Guastavino vaults (presumably only the vaulted parts above the perimeter walkways were modeled) were analyzed first under vertical load only, revealing “flexural” behavior and high tensile stresses in the face tiles. The author interprets this to mean that the vault does not behave like “a properly designed arch” - regardless of the fact that it had survived the previous ninety years without dropping any tiles - rather than questioning the validity of the linear elastic model. Disbelieving the results of tile sounding tests, destructive tests are implemented, which revealed poor bonding of the tiles. The author notes that the results of this test are potentially unreliable because the vibrations from the coring could initiate failure and that this is the exact reason why this type of testing is not used on other masonry structures. For this reason, the testing method is particularly suspect since it likely caused or exacerbated the very debonding that it was intended to investigate. Impact echo, developed between 1983 and
1997 at the National Bureau of Standards and Cornell University (Sansalone and Streett 1997, 13-26), would have been a more appropriate testing method. Results of the analysis and testing led to a retrofit consisting of three components: epoxy-fiberglass ribs intended to reduce deformation in the vaults, wire mesh and spray-on foam as a “back up system” for the vaults if they would fail in the event of an earthquake, and pins to connect the face tiles to the wire mesh and keep tiles from falling on people should they come loose. While the first two items are at least hidden from view, the pins are visible on the face of each tile (Figure 2.16), not to mention that installing them could have potentially amplified any bonding deficiencies of the tiles. Owing to the fact that it is unknown exactly how these structures behave under seismic loads, it may have made more sense to use something reversible like netting over the face tiles to prevent falling tiles from harming anyone. The retrofit at the Hearst Memorial Mining Building illustrates the difficult task of preserving the integrity of Guastavino structures while ensuring public safety, but this retrofit seems especially drastic.

![Figure 2.16 Image of the Hearst Memorial Mining Building showing holes in the face tiles.](image)

### 2.5 Chapter Summary

This chapter introduced the design and analysis methods of the Guastavinos’ as well as their own understanding of masonry tile shell behavior. Two types of analysis methods used to analyze Guastavino structures and other masonry structures – equilibrium and elastic analysis.

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methods – have been used in the past, and the question of which method is better suited for their analysis is still debated today. Finally, case studies provide examples of both analysis methods and restoration techniques that have been used over the years by practicing architects and engineers faced with assessing and rehabilitating Guastavino vaults. The Oyster Bar and Queensboro Bridge repairs serve as models to be emulated, but the drastic investigative techniques and dramatic restorations at the Hearst Memorial Mining Building illustrate a need for more information on the behavior and analysis of these structures.
Chapter 3. Structural Analysis Methods for a Guastavino Dome

Today there are three principle structural analysis methods for Guastavino domes: graphical analysis, membrane analysis, and finite element analysis. This chapter applies these three methods to a theoretical Guastavino shell – a spherical dome of dimensions similar to existing structures. Looking at this geometrically simple structure with no cracks and loaded only by self-weight will allow for comparison of analysis results. The results will confirm the applicability of each method to thin, brittle masonry shells and will help to reveal the shortcomings of each method.

A generic dome is used to demonstrate the analysis methods. Its properties are given in Table 3. and it is illustrated in Figure 3.1. The density used for the analysis is from Atamturktur (2006, 112), as given in Table 2.2. The dome has a span to thickness ratio of about 200; for comparison, a typical eggshell has a span to thickness ratio of about 100 (Heyman 1977, 5). Thin shells are usually defined by the ratio of their radius to thickness, and a ratio over 20 is considered thin (Heyman 1977, 5). The radius to thickness ratio for the generic Guastavino dome is 120.

| Radius (R) | 40 feet |
| Thickness (t) | 4 inches |
| Angle of embrace (α) | 70° |
| Density (ρ) | 112 pcf |

Table 3.1 Properties of the generic Guastavino dome.

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9 This density is somewhat lower than that used by the R. Guastavino Company for the design of their structures. Company engineer Irving Berg used about 150 pcf in his analysis of the Army War College. Irving Berg, “War College Vault Panel ‘A’ Loading,” 1 November 1944, Guastavino Archive.
3.1 Graphical Analysis

Rafael Guastavino Jr. and Irving Berg employed graphical equilibrium methods to design and analyze R. Guastavino Company projects during the company’s life, so it makes good sense to utilize it for analysis of their domes today. Graphical analysis allows one to estimate the forces within the dome thickness in the directions of latitude and longitude, called hoop and meridional forces, respectively (Figure 3.2). Graphical analysis is an equilibrium method that does not consider material properties or rely on assumptions of elastic behavior to estimate the internal forces in the dome.

Figure 3.2 Direction of meridional and hoop forces (Billington 1965, 3)

This section uses Wolfe’s method for the graphical analysis of a generic Guastavino dome, which is fundamentally similar to membrane analysis since it constrains the meridional
forces to the mid-surface of the dome thickness (Lau 2006, 26). The variation of the meridional and hoop forces is therefore the same as for membrane analysis: meridional forces are always compressive and increase in magnitude from crown to base, and hoop forces vary linearly from compression at the crown to tension at the base (Figure 3.3). The theoretical point at which the hoop forces change from compressive to tensile is the same for all spherical domes (without openings or lanterns at the top): 51.8°. See Appendix A for a derivation of this value. This point of zero hoop force is significant because domes constructed of materials with little or no tensile strength will potentially crack below this point (that is, in the region of tensile hoop forces) if not provided with buttressing or tensile reinforcing.

Figure 3.3 Variation of meridional (N_φ) and hoop (N_θ) stress resultants in a spherical dome (Billington 1965, 42).

Figure 3.4 Angle of embrace (Heyman 1995, 41).

For a spherical dome with an angle of embrace (Figure 3.4 illustrates angle of embrace), α, greater than 51.8°, hoop stresses are tensile in the region of the dome where 51.8° < φ < α (Heyman 1995, 33). Since Guastavino shells are constructed of brittle masonry tile and mortar, their material does not have considerable tensile capacity. However, the lower portion of many Guastavino domes contains reinforcing steel between the layers of tile, which can provide local tensile capacity for the development of tensile hoop forces and to contain the outward thrust of

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10 Wolfe's method is basically a membrane analysis carried out graphically instead of numerically.
the dome.\footnote{Rafael Guastavino Jr. received a patent in 1910 for a means of “strengthening” masonry structures with reinforcing bars or strips between layers of tile.} Graphical analysis conveniently provides a means of estimating the required force in a tension tie at the base of the dome necessary to contain this thrust, as demonstrated later in this section.

As explained in detail by Lau (2006, 26), Wolfe presents two methods for the analysis of domes: one for domes with tensile capacity below $\phi = 51.8^\circ$ and one for domes without (Wolfe 1921, 250-53). While the first method (for domes with tensile capacity) is applicable to Guastavino domes because they often have reinforcing steel or buttressing where $\phi > 51.8^\circ$, damage to the steel or buttressing of a dome could relieve it of tensile capacity, and then the second method would be more appropriate. Both methods are depicted in Wolfe’s figures, which are reproduced in Figure 3.5. The analysis method for a dome with tensile capacity where $\phi > 51.8^\circ$ corresponds to the left half of the dome in Wolfe’s Fig. 500 and Fig. 501. The analysis method for the dome without tensile capacity is given in the right half of Fig. 500 and Fig. 502. The methodology for each analysis case is explained in Appendices B and C.
Figure 3.5 Wolfe (1921, 252) presents two analysis methods for a dome.

The graphical analysis of a dome considers only one slice of the dome, called a "lune" (Figure 3.6). For this example, a lune 15° wide in plan (or 1/24th of the total dome) is used. This lune is then divided into ten segments (as shown in Figure 3.7). For a more refined analysis, more segments could be used to better represent the actual distribution of self-weight, which must be approximated as point loads for the graphical analysis. The next step is to calculate the weights of the ten segments based on the volume of the segment and the density of the dome material. The weights for the lune segments of the generic Guastavino dome are shown in Figure 3.7 (given in units of kips, where 1 kip = 1000 pounds-force). Since this analysis considers only self-weight of the dome, these values will be the only forces applied to the lune. Any additional
dead load (such as the weight of roofing material) or live load for the analysis of a real structure would be added to these segment weights.\textsuperscript{12}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3_6}
\caption{A dome showing a 15° lune.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3_7}
\caption{A lune divided into ten segments and labeled with their self weights.}
\end{figure}

The graphical analysis results presented in the next sections are for a generic Guastavino dome with tensile capacity in the form of steel tension ties, so it will follow Wolfe’s analysis method for a dome with tensile capacity where $\varphi > 51.8\degree$. A step-by-step explanation of this analysis is presented in Appendix B. For reference, an analysis of the same dome using Wolfe’s method for a dome without tensile capacity is presented in Appendix C. For further explanation of the graphical analysis of domes, readers are encouraged to consult Wolfe’s \textit{Graphical Analysis} (1921) and the thesis by Lau (2006).

\textsuperscript{12} Additional dead loads and live loads are neglected from this simple analysis, but they will be included in the analysis of the Guastavino dome of the Grace Universalist Church in Chapter 5.
The force polygon generated by the graphical analysis of the generic Guastavino dome is depicted three times in Figure 3.8, each copy in turn highlighting the load line, meridional forces, and hoop forces. The polygons are labeled with select magnitudes from each set of forces (given in units of kips).

Figure 3.8 Resulting force polygon for the generic Guastavino dome labeled with selected magnitudes for the (a) load line, (b) meridional forces and (c) hoop forces.

Figure 3.9 shows the thrust line assumed for the analysis, restrained approximately to the centerline of the dome. Because only ten segments of the dome were used to approximate the thrust line, the straight line segment approximation of the curve deviates somewhat from the centerline. This could be improved by taking more segments. An infinite number of segments would mean the solution could lie exactly on the centerline, and the solution would be exactly the same as that from membrane theory for a spherical shell.
Figure 3.9 Cross-section of the lune showing a possible thrust line.

Figure 3.9 shows the reaction at the base of the dome acting parallel to the last force in the thrust line. If the dome is not on supports perpendicular to this resultant (illustrated in (a) of Figure 3.10) and is only supported vertically (illustrated in (b) of Figure 3.10), something must be done to contain the thrust, which is equal to the horizontal component of the reaction in Figure 3.9. This horizontal component, that is, the magnitude of thrust, can be determined from the graphical analysis and is labeled in Figure 3.11, showing that the lune experiences a horizontal thrust of 3.8 kips. The thrust pushes out on the supports and tries to displace them laterally. If the supporting structure is rigid enough, thrust will have a minimal effect on the supports and they will not displace. If the supporting structure is not rigid, a tension tie at the base of the dome can contain the horizontal thrust and prevent it from displacing the supports. The steel tie allows the dome to theoretically be supported by vertical reactions only. Since the Guastavinos included a steel ring at the base of their domes, this ring can provide resistance to thrust and the required tie force to achieve this can be added graphically to the force polygon (Figure 3.11). Using this methodology, the minimum required capacity of the circular steel tension tie at the base of the dome is found to be 14.4 kips.
Figure 3.10 A dome supported (a) tangentially and (b) vertically (Billington 1965, 42).

Figure 3.11 Force polygon for the generic Guastavino dome with tie force added.

Table 3.2 and Table 3.3 list the results of the graphical analysis, where the forces (in units of kips) are given at angles $\phi$ measured with respect to the vertical (defined in Figure 3.) and a negative value denotes compression. The results are also given in terms of stress in units of pounds per square inch.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$P$</th>
<th>$\sigma$</th>
</tr>
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<tbody>
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<td>(*)</td>
<td>(k)</td>
<td>(psi)</td>
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<td>7</td>
<td>-0.96</td>
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Table 3.2 Meridional forces and stresses for the generic Guastavino dome found with graphical analysis.

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<th>φ</th>
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</table>

Table 3.3 Hoop forces and stresses for the generic Guastavino dome found with graphical analysis.

3.2 Membrane Analysis

Since the generic Guastavino dome has a relatively small thickness relative to its radius of curvature (1:120), it can be analyzed as a shell using membrane analysis. Membrane analysis assumes that a shell carries load through membrane action (forces parallel to the mid-surface of the shell) rather than through bending (Timoshenko and Woinowsky-Krieger 1959, 2). Wolfe's method of graphical analysis is based on many of the same assumptions, such as the meridional forces being constrained to the centerline of the dome by the hoop forces, the dome having constant stress throughout the thickness, and material properties not being relevant to the analysis (Lau 2006, 26). The following calculations are presented for comparison with the graphical analysis results.

The meridional and hoop stress resultants were found with membrane analysis for the generic Guastavino dome using closed-form equations for spherical domes subject to uniform load. The notation used is from plate and shell theory, where 'a' denotes the dome’s radius, 'q' is a uniformly distributed load on the surface of the dome and/or self-weight, and ‘φ' is the angle with respect to the vertical (as defined in Figure 3.). The quantity for the self-weight (q) comes from the density of the vault material (given in Table 3.) multiplied by the thickness of the dome.
The formulas to calculate meridional and hoop stress for a spherical dome by membrane theory are given by Equations 3.1 and 3.2, respectively.

\[ N_\phi^{'} = -aq \frac{1}{1 + \cos \phi} \]  

Equation 3.1

\[ N_\theta^{'} = aq \left( \frac{1}{1 + \cos \phi} - \cos \phi \right) \]  

Equation 3.2

Since the stress resultants are in terms of force per unit length (specifically, kips per square foot), \( N'_\phi \) and \( N'_\theta \) must be divided by the dome thickness to put the membrane analysis results in terms of stress (in units of pounds per square foot). Membrane analysis results for the generic Guastavino dome are given in Table 3.4 and Table 3.5.

<table>
<thead>
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<th>( N'_\phi ) (k/ft)</th>
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<td>-0.780</td>
<td>-16.2</td>
</tr>
<tr>
<td>35</td>
<td>-0.807</td>
<td>-16.8</td>
</tr>
<tr>
<td>42</td>
<td>-0.842</td>
<td>-17.5</td>
</tr>
<tr>
<td>49</td>
<td>-0.886</td>
<td>-18.5</td>
</tr>
<tr>
<td>56</td>
<td>-0.942</td>
<td>-19.6</td>
</tr>
<tr>
<td>63</td>
<td>-1.010</td>
<td>-21.0</td>
</tr>
</tbody>
</table>

Table 3.4 Meridional stresses for a 15° lune of the generic Guastavino dome found with membrane analysis.

<table>
<thead>
<tr>
<th>( \phi ) (°)</th>
<th>( N'_\theta ) (k/ft)</th>
<th>( \sigma ) (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-0.720</td>
<td>-15.0</td>
</tr>
<tr>
<td>14</td>
<td>-0.679</td>
<td>-14.2</td>
</tr>
<tr>
<td>21</td>
<td>-0.611</td>
<td>-12.7</td>
</tr>
<tr>
<td>28</td>
<td>-0.517</td>
<td>-10.8</td>
</tr>
<tr>
<td>35</td>
<td>-0.396</td>
<td>-8.2</td>
</tr>
<tr>
<td>42</td>
<td>-0.249</td>
<td>-5.2</td>
</tr>
<tr>
<td>49</td>
<td>-0.077</td>
<td>-1.6</td>
</tr>
<tr>
<td>56</td>
<td>0.121</td>
<td>2.5</td>
</tr>
<tr>
<td>63</td>
<td>0.343</td>
<td>7.1</td>
</tr>
</tbody>
</table>

Table 3.5 Hoop stresses for a 15° lune of the generic Guastavino dome found with membrane analysis.

Additionally, membrane analysis offers equations to calculate the thrust at the base of the dome and the force that would be required in a tension ring to contain that thrust. Billington (1965) presents the following equations for horizontal thrust (Equation 3.3) and the required tension ring force (Equation 3.4).
\[ H_\phi = N' \phi \cos \phi \]  
\[ T_\phi = N' \phi \ a \sin \phi \cos \phi \]

(3.3)
(3.4)

Assuming the tension tie is at the base of the generic dome (\( \phi = 70^\circ \)), these equations yield a thrust of 3.7 kips and a tension ring force of 14.1 kips.

### 3.3 Finite Element Analysis

The finite element program ADINA was used to analyze the generic Guastavino dome (ADINA R&D 2006). A dome of the same geometry as the previous two sections will be used, and in order to analyze it elastically, material properties must be provided as well. The properties used for the analysis given in Table 3.6 are from Atamturktur (2006, 112).

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity (E)</td>
<td>1.58 x 10^8 psf</td>
</tr>
<tr>
<td>Poisson’s ratio (v)</td>
<td>0.32</td>
</tr>
</tbody>
</table>

**Table 3.6 Material properties for the finite element analysis of the generic Guastavino dome.**

The full dome was modeled with 9-node shell elements and simply-supported at the base, meaning the base of the dome was restrained for translation in all directions but free to rotate. Shell elements have six degrees of freedom per node, use small displacement formulation, and can carry stresses both by membrane action and bending (ADINA R&D 2006, 100). The model is loaded only with self-weight.

In the post-procession phase, a spherical coordinate system is used to view the results in terms of meridional and hoop stress. Stress plots of the meridional and hoop stresses are shown in Figure 3.12 and Figure 3.13 (given in units of kips per square foot). The thrust of the dome can be estimated by looking at the horizontal reactions at the dome base. For one lune, the horizontal reaction given by ADINA is about 4.1 kips. Though there may be a way to estimate the force required by a tension ring to counteract this thrust with finite element analysis, it is not an obvious calculation and it is not considered here.
Figure 3.12 Meridional stresses in the generic Guastavino dome generated with ADINA.

Figure 3.13 Hoop stresses in the generic Guastavino dome generated with ADINA.

The finite element analysis predicts deflection of the generic Guastavino dome under self-weight, and Figure 3.14 shows a greatly magnified version of deflected shape. The deflection at the crown is 0.016 inches down.
3.4 Discussion of Analysis Results

The meridional and hoop stresses from each of the three analysis methods are given Table 3.7 and Table 3.8. The graphical analysis results for hoop stress have been interpolated to put the results in terms of different locations (φ) than the results presented in Table 3.3. The values for the finite element analysis results were found by averaging all calculated stresses calculated at 18 integration points in the two elements on either side of the desired location on the dome. Figure 3.15 and Figure 3.16 plot the results of the three analyses.

<table>
<thead>
<tr>
<th>φ</th>
<th>( \sigma_{\text{graphical}} ) (psi)</th>
<th>( \sigma_{\text{membrane}} ) (psi)</th>
<th>( \sigma_{\text{finite element}} ) (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-15.7</td>
<td>-15.3</td>
<td>-15.7</td>
</tr>
<tr>
<td>14</td>
<td>-15.8</td>
<td>-15.5</td>
<td>-15.8</td>
</tr>
<tr>
<td>21</td>
<td>-16.1</td>
<td>-15.8</td>
<td>-16.1</td>
</tr>
<tr>
<td>28</td>
<td>-16.6</td>
<td>-16.2</td>
<td>-16.6</td>
</tr>
<tr>
<td>35</td>
<td>-17.2</td>
<td>-16.8</td>
<td>-17.2</td>
</tr>
<tr>
<td>42</td>
<td>-17.9</td>
<td>-17.5</td>
<td>-17.9</td>
</tr>
<tr>
<td>49</td>
<td>-18.8</td>
<td>-18.5</td>
<td>-18.9</td>
</tr>
<tr>
<td>56</td>
<td>-20.0</td>
<td>-19.6</td>
<td>-20.0</td>
</tr>
<tr>
<td>63</td>
<td>-21.5</td>
<td>-21.0</td>
<td>-21.4</td>
</tr>
</tbody>
</table>

Table 3.7 Meridional stresses: Comparison of analysis results.

<table>
<thead>
<tr>
<th>φ</th>
<th>( \sigma_{\text{graphical}} ) (psi)</th>
<th>( \sigma_{\text{membrane}} ) (psi)</th>
<th>( \sigma_{\text{finite element}} ) (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-15.3</td>
<td>-15.0</td>
<td>-15.3</td>
</tr>
<tr>
<td>14</td>
<td>-14.4</td>
<td>-14.2</td>
<td>-14.4</td>
</tr>
<tr>
<td>21</td>
<td>-12.9</td>
<td>-12.7</td>
<td>-13.0</td>
</tr>
<tr>
<td>28</td>
<td>-10.9</td>
<td>-10.8</td>
<td>-10.9</td>
</tr>
<tr>
<td>35</td>
<td>-8.4</td>
<td>-8.2</td>
<td>-8.4</td>
</tr>
<tr>
<td>42</td>
<td>-5.2</td>
<td>-5.2</td>
<td>-5.3</td>
</tr>
<tr>
<td>49</td>
<td>-1.8</td>
<td>-1.6</td>
<td>-1.6</td>
</tr>
<tr>
<td>56</td>
<td>2.7</td>
<td>2.5</td>
<td>3.3</td>
</tr>
<tr>
<td>63</td>
<td>7.6</td>
<td>7.1</td>
<td>6.9</td>
</tr>
</tbody>
</table>

Table 3.8 Hoop stresses: Comparison of analysis results.
Figure 3.15 Plot of meridional stresses calculated by the three analysis methods.

Figure 3.16 Plot of hoop stresses calculated by the three analysis methods.
Results of the three methods show good consistency for both meridional and hoop stresses. The compressive meridional stresses given by membrane analysis are slightly greater than for the other methods, but vary in a similar manner. The hoop stresses given by finite element analysis show the biggest deviation from the other results near the base of the dome, where the trend line for the finite element analysis starts to fluctuate. A closer look at the stresses computed with finite element analysis shows the variation more clearly. Figure 3.17 plots the hoop stress for 20 elements, corresponding to a lune divided into 20 segments and labeled from top to bottom. The hoop stresses in the elements vary along a smooth curve from compression to tension, but there is a sudden change in the direction of the line near the base. This does not follow the same pattern as the results of the graphical and membrane analysis results shown in Figure 3.16. There are discrepancies in the stresses calculated with finite element analysis in the two layers at the base (Elements 19 and 20). In the rest of the lune, the stresses in each layer are relatively close, showing consistency with membrane theory in that the stresses in the shell are evenly distributed through the thickness (Timoshenko and Woinowsky-Krieger 1959, 3). The deflected shape from the finite element analysis (Figure 3.14) provides a clue as to the origin of the discrepancy. The overall deflected shape shows the dome flattening out except near the edges where the dome is restrained by the boundary conditions. The resulting dramatic change in curvature at the dome’s base seems to correspond with the change in direction of the hoop stress curve in Figure 3.17. Thus, the assumed boundary conditions and resulting deflected shape have a major impact on the elastic analysis results and could lead an analyst to believe there are large stress concentrations at the base of the dome which do not actually exist. Graphical and membrane analysis, on the other hand, are free from this effect.
The magnitude of the deflection given by the finite element analysis (0.016 inches) is very low compared to those that an actual structure might experience. A real Guastavino dome may experience deflections ten or even one hundred times greater due to support movement or construction defects, and a finite element analysis might find drastically different stresses within the dome for such deflections. The results of graphical analysis, on the other hand, would not show greatly different forces within the dome for a model including realistic deflections.

The results of the three analysis methods yield stresses within the dome between 22 psi in compression to 7.6 psi in tension. These values can be compared to test results found by Rafael Guastavino Sr. to evaluate the strength of the dome (Table 2.1). The results provided in his book give a five day compressive strength of about 2060 psi and an allowable tensile strength of 287 psi. This means that the generic Guastavino dome has a safety factor of almost 100 for compression and 40 for tension, indicating that these structures have ample capacity under self-weight. It should be noted that the dome will unlikely have to rely on tensile capacity of the material since steel reinforcing within the dome will provide that capacity.
In addition to finding stresses within the dome, all three analysis methods provide a value for the thrust at the base of the dome. The graphical analysis gave the thrust for one lune as 3.8 kips and membrane theory gave 3.7 kips. Finite element analysis shows a slightly higher thrust of 4.1 kips. These results show good correspondence for the graphical and membrane analysis results and an 8% greater results for the finite element analysis. The fact that each method provides reliable results for the thrust is significant because the thrust is important to know when assessing the stability of a dome. One way thrust can be contained is with a tension tie at the base of a dome, as was typical for Guastavino projects. Graphical and membrane analysis provide a means of estimating the minimum required force in the tie, both yielding results of about 14 kips. If an actual Guastavino dome has a steel ring but no other means to resist thrust (i.e. sufficient buttressing), the ability of this ring to carry the required tie force is paramount.

3.5 Conclusions

The consistency of results from the three analysis methods for the generic Guastavino dome demonstrates that any of these three methods can be used to estimate the stresses in an uncracked Guastavino dome with a regular spherical geometry. Since finite element analysis reports hoop stresses near the base of a dome that are inconsistent with the results of the other two methods, users of finite element analysis should be aware that assumed boundary conditions may produce stress concentrations which do not exist in reality. It also illustrates the importance of verifying finite element analyses with equilibrium calculations as a check on the results. However, all stresses were found to be significantly lower than the allowable stresses for Guastavino vaulting, indicating that the material strength of Guastavino domes should not typically be a cause for concern.

Of significance, therefore, is the ability of each analysis method to provide insight into the stability of the dome. The membrane and finite element analyses correspond to a graphical analysis of a dome with tensile capacity, that is, they are only capable of analyzing a dome with tensile capacity. Membrane or finite element analyses of domes without tensile capacity or a potentially compromised tensile capacity should be compared with results of a graphical analysis for Wolfe’s second method. All three methods provide a reliable means of assessing the thrust of a dome. This thrust needs to be accounted for in order for the dome to be stable, and one way this is typically done for Guastavino domes is with a tension ring. The minimum capacity of such a
ring can be found without difficulty using either graphical or membrane analysis, but not easily with finite element analysis.

Finally, it should be noted that while graphical and membrane analyses require information on the geometry and density of the dome material, finite element analysis needs additional material properties. Since the graphical and membrane analysis results do not depend on material properties, the dome is assessed based purely on equilibrium.
Chapter 4. Structural Analysis Methods for a Guastavino Barrel Vault

A structure type used frequently by the R. Guastavino Co. was that of a simple barrel vault. Examples can be seen at the Saint Louis Art Museum, Our Lady of Victory Chapel at the College of St. Catherine, and the Buffalo Central Terminal. Each of these vaults is a thin, three-dimensional shell structure, supported along its base except where there are window openings. When a cylindrical vault is supported all along its edges, it carries loads like an arch and not like a membrane (Billington 1965, 155). Therefore, in this analysis of a generic Guastavino barrel vault, the vault is analyzed like a two-dimensional arch and membrane action (of the arch as a three-dimensional spatial structure) is not considered.

A generic Guastavino barrel vault was used to compare two methods of analysis, namely graphical analysis and finite element analysis. In order to define appropriate dimensions for an arch as daringly thin as a Guastavino barrel vault, it is necessary to consider the limiting thickness, which is determined by the arch thickness (t), radius (R), and angle of embrace (α). A full barrel vault may have an angle of embrace of 90°, but the presence of diaphragm walls effectively reduces this dimension. This new value will be denoted α'. Based on the diaphragm walls of the Boston Public Library (Figure 2.6) and the Saint Louis Art Museum (Figure 6.3), a reasonable estimate is that the diaphragm walls restrain the vault between α < φ < 45° and thus α' = 45°. For this geometry, the minimum value for t/R is 0.0113, so for a radius of 20 feet, the

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13 Guastavino Fireproof Construction Company/George Collins Archive, Drawings and Archives Department, Avery Architectural and Fine Arts Library, Columbia University (hereafter cited as Guastavino Archive).
minimum thickness is 2.7 inches (Ochsendorf 2006, 30). Thus, a generic Guastavino vault with a radius of 20 feet must have a minimum thickness of 3 inches to be of a geometry that could exist in reality. If the generic vault is assumed to be three courses of tile thick, each course with a nominal thickness of 1 inch, then with mortar, the thickness of the vault is between 3 and 4 inches thick. The generic vault was generated with the properties listed in Table 4.1 and illustrated in Figure 4.2. The density used is from Atamturktur (2006,112), as given in Table 2.2.

<table>
<thead>
<tr>
<th>Radius (R)</th>
<th>20 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of embrace (α)</td>
<td>90°</td>
</tr>
<tr>
<td>Thickness (t)</td>
<td>4 inches (approximately three tiles thick)</td>
</tr>
<tr>
<td>Width</td>
<td>1 foot (unit width)</td>
</tr>
<tr>
<td>Density (ρ)</td>
<td>112 pcf</td>
</tr>
</tbody>
</table>

Table 4.1 Properties of the generic Guastavino barrel vault.

Figure 4.2 Cross-section of vault used for analyses showing the diaphragm wall.

4.1 Graphical Analysis

This section presents a summary of the graphical analysis of a generic Guastavino barrel vault. For further explanation of the graphical analysis of vaults, readers are encouraged to consult Zalewski and Allen’s *Shaping Structures* (1998).

The first step in the analysis of a vault is to establish the load case or cases, which will be limited to self-weight of the vault for the analysis presented here. The weight of the diaphragm

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14 The minimum value for t/R of 0.0113 is actually for α' = 50°, which was used because there is no available t/R value for α' = 45° (Ochsendorf 2006). Using the t/R value for α' = 50° results in a conservative estimate for the minimum thickness.
walls, which are necessary for the stability of the vault (as will be demonstrated later), is neglected for this simple analysis. In actual structures, sometimes the vault supports the roof and thus the weight of the roof and any applicable live load would need to be estimated and included in the analysis as well (see Chapter 6 for an example in the analysis of the Saint Louis Art Museum). Since the generic vault is symmetric, only one half need be analyzed. The half-vault was divided into fifteen equal sections, and the weight of each section was calculated to be 78 pounds. These weights are concentrated into point loads acting at the centroid of each section, as depicted in Figure 4.4.

Next the force polygon is constructed, starting with the loads on the vault. These loads are represented by vertical lines (to represent the direction of the force) and scaled to the magnitude of the force, using a scale such as 1 foot = 100 pounds. The fifteen lines are connected end to end to become the load line, represented by the vertical line on the left in Figure 4.3. The horizontal line across the top of the force polygon represents the thrust horizontal reaction at the crown of the vault (and, by equilibrium, the horizontal thrust at the spring line). The length of this line is determined by trial and error, so that the resulting thrust line (Figure 4.4) lies within the cross-section of the vault or the diaphragm wall. For the generic Guastavino vault, a horizontal thrust of 700 pounds resulted in a satisfactory thrust line. The lengths of the fifteen diagonal segments establish the magnitude of the corresponding segments of the thrust line. The force polygon for the generic vault is depicted in Figure 4.3 and the corresponding thrust line is shown in Figure 4.4.
Figure 4.3 A possible force polygon for the generic Guastavino barrel vault.

Figure 4.4 A possible thrust line for the generic Guastavino barrel vault.
The thrust line in Figure 4.4 illustrates how the vault carries loads. Any loads the vault supports are transferred through a thrust line to the supporting structure below. The thrust line must have material to pass through in order for the vault to be stable. Figure 4.4 shows the thrust line leaving the vault thickness at around $\varphi = 45^\circ$, where a buttressing element such as a diaphragm wall would be necessary to carry the thrust down to the supporting walls. The force polygon in Figure 4.3 gives the magnitude of force (in units of kips) in each segment of the thrust line. Table 4.2 gives the magnitudes of the thrust line and compressive stresses (in units of pounds per square inch) for the region where the thrust line passes through the vault thickness (negative values indicate compression).

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>$P$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(°)</td>
<td>(kips)</td>
<td>(psi)</td>
</tr>
<tr>
<td>0</td>
<td>-0.70</td>
<td>-14.6</td>
</tr>
<tr>
<td>6</td>
<td>-0.70</td>
<td>-14.7</td>
</tr>
<tr>
<td>12</td>
<td>-0.72</td>
<td>-14.9</td>
</tr>
<tr>
<td>18</td>
<td>-0.74</td>
<td>-15.4</td>
</tr>
<tr>
<td>24</td>
<td>-0.77</td>
<td>-16.0</td>
</tr>
<tr>
<td>30</td>
<td>-0.80</td>
<td>-16.7</td>
</tr>
<tr>
<td>36</td>
<td>-0.84</td>
<td>-17.6</td>
</tr>
<tr>
<td>42</td>
<td>-0.89</td>
<td>-18.5</td>
</tr>
<tr>
<td>48</td>
<td>-0.94</td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>-0.99</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>-1.05</td>
<td></td>
</tr>
<tr>
<td>66</td>
<td>-1.11</td>
<td></td>
</tr>
<tr>
<td>72</td>
<td>-1.17</td>
<td></td>
</tr>
<tr>
<td>78</td>
<td>-1.24</td>
<td></td>
</tr>
<tr>
<td>84</td>
<td>-1.30</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>-1.37</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2 Magnitude of the thrust line and resulting stress in the arch.

4.2 Finite Element Analysis

A finite element analysis of the generic Guastavino barrel vault is conducted with ADINA using a 2-dimensional model of the vault and its diaphragm walls (Figure 4.5). The model is fixed along the base for translation and rotation in all directions. Nine node shell elements are used to mesh both the vault and the walls, with a unit width assigned to the vault and a 3 inch width to the walls. The entire model is loaded with self-weight only, and the walls are assumed to have the same density and material properties as the vault. The geometric and material properties for the vault are listed in Table 4.1 and Table 4.3, and the Guastavino tile is
modeled as an isotropic, linear elastic material. The properties used for the analysis are from Atamturktur (2006, 112).

<table>
<thead>
<tr>
<th>Modulus of elasticity (E)</th>
<th>$1.58 \times 10^8$ psf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson’s ratio (v)</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Table 4.3 Material properties for the finite element analysis of the generic Guastavino vault.

![Figure 4.5 Finite element model of the generic Guastavino vault.](image)

Output is given in terms of stresses at 18 integration points within each element, and the stresses in each element on either side of a desired location ($\varphi$) are averaged to approximate the stress at that point. A cylindrical coordinate system was created in the post-processing stage to put the analysis results in terms of normal stress, that is, stresses parallel to the mid-surface layer of the vault in the direction of $\varphi$ (Flügge 1960). Table 4.4 gives results in the vault in terms of pounds per square inch (negative values indicate compression). Figure 4.6 shows a normal stress plot for the vault in terms of kips per square foot.

<table>
<thead>
<tr>
<th>$\varphi$ (°)</th>
<th>$\sigma$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-14.2</td>
</tr>
<tr>
<td>6</td>
<td>-14.3</td>
</tr>
<tr>
<td>12</td>
<td>-14.6</td>
</tr>
<tr>
<td>18</td>
<td>-15.0</td>
</tr>
<tr>
<td>24</td>
<td>-15.6</td>
</tr>
<tr>
<td>30</td>
<td>-16.4</td>
</tr>
<tr>
<td>36</td>
<td>-17.3</td>
</tr>
<tr>
<td>42</td>
<td>-18.2</td>
</tr>
</tbody>
</table>

Table 4.4 Finite element analysis results for stresses in the generic Guastavino vault.
Figure 4.6 Finite element analysis results for the generic Guastavino arch in terms of normal stress.

The horizontal reactions given by finite element analysis at the base of the vault, which resist the outward movement of the supports, are 680 pounds. The displacement at the crown given by the analysis is 0.022 inches downwards, and the deflected shape is shown in Figure 4.7.

Figure 4.7 The deflected shape of the barrel vault (magnified) shown with the undeflected shape.

4.3 Discussion of Analysis Results

In this section, the results of the graphical and finite element analysis are evaluated for what they each reveal about the safety of the barrel vault. Results are compared in Table 4.5 and plotted in Figure 4.8 for the unstiffened portion of the vault ($0^\circ < \varphi < 45^\circ$).

<table>
<thead>
<tr>
<th>$\varphi$ ($^\circ$)</th>
<th>(\sigma_{\text{graphical}}) (psi)</th>
<th>(\sigma_{\text{finite element}}) (psi)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-14.6</td>
<td>-14.2</td>
<td>2.8</td>
</tr>
<tr>
<td>6</td>
<td>-14.7</td>
<td>-14.3</td>
<td>2.7</td>
</tr>
<tr>
<td>12</td>
<td>-14.9</td>
<td>-14.6</td>
<td>2.6</td>
</tr>
<tr>
<td>18</td>
<td>-15.4</td>
<td>-15.0</td>
<td>2.4</td>
</tr>
<tr>
<td>24</td>
<td>-16.0</td>
<td>-15.6</td>
<td>2.2</td>
</tr>
</tbody>
</table>
Table 4.5 Analysis results for the generic Guastavino barrel vault.

<table>
<thead>
<tr>
<th>Location</th>
<th>Graphical Stress (psi)</th>
<th>Finite Element Stress (psi)</th>
<th>Safety Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>-16.7</td>
<td>-16.4</td>
<td>2.0</td>
</tr>
<tr>
<td>36</td>
<td>-17.6</td>
<td>-17.3</td>
<td>1.8</td>
</tr>
<tr>
<td>42</td>
<td>-18.5</td>
<td>-18.2</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Figure 4.8 Hoop stresses generated with graphical and finite element analysis.

Both methods give similar results for the stresses in the unstiffened vault portion. The stresses are all compressive and vary between about 14 and 19 psi, which can be compared to the compressive strength determined by Rafael Guastavino Sr. and given in Table 2.1. This compressive strength of 2060 psi yields a safety factor of over 100 for the vault, and thus the strength of a Guastavino barrel vault under self-weight is unlikely to be a limiting factor as far as safety is concerned.

While it would be difficult to compare the two analysis results in the region of the diaphragm walls, superimposing the thrust line from the graphical analysis onto the stress plot of the finite element analysis yields interesting results (Figure 4.9). The diagonal bands in the finite element stress plot reflect the orientation of the thrust line from the graphical analysis. However, while the thrust line provides a clear illustration of the load path through the vault, the results of the finite element analysis are difficult to interpret. The thrust line clearly demonstrates the necessity of the diaphragm walls for the vault’s stability, but an analyst presented only with the finite element analysis results would not necessarily be able to make such a determination.
Since the material strength of a Guastavino barrel vault is unlikely to jeopardize its safety, the most important data from a vault analysis pertains to its stability. The thrust of a vault is important to know since it must always be counteracted to ensure stability, and both analyses determine a reliable value for this thrust. The thrust from the graphical and finite element analyses are 700 pounds and 680 pounds, respectively – within 3% of each other.

Although the results of the finite element analysis show good correspondence with the graphical analysis, there are suspicious characteristics. Figure 4.6 shows stress concentrations in the vault at $\varphi = 45^\circ$, that is, at the top of the diaphragm walls), so it would be wise to investigate the finite element results further. Figure 4.10 plots the stresses at three layers within the vault for the portion of the vault not restrained by the diaphragm walls. The stresses at the mid-surface layer correspond to the stresses in Figure 4.8, but the stresses at the outer and inner layer of the vault vary in a manner reminiscent of the deflected shape (Figure 4.7). This phenomenon of stresses being influenced by the deflected shape predicted by finite element analysis was observed with the generic Guastavino dome in Chapter 3, where the deflected shape showed a high rate of curvature near the base and corresponding to a change in pattern of the stresses. For the dome, the problem was limited to one position on the model - that is, near the base - and thus had a minimum effect on the results. For the barrel vault, the deflected shape predicted by finite element analysis for a thin elastic arch induces widely varying stresses through the thickness that affect the entire unstiffened portion of the vault. For the vault analyzed here, the stresses
predicted at the inner and outer are within the range of allowable compressive and tensile stress for the material, but for different geometries this may not be the case. For example, Robertson (1999) cites these stress variations at the intrados face resulting from a finite element analysis of the vaults at the Hearst Memorial Mining Building as one reason to strengthen the vaults. Furthermore, it is hard to imagine a brittle masonry tile shell structure deflecting in an elastic manner as depicted in Figure 4.7.

![Graph showing stress variation over the thickness of the vault](image)

*Figure 4.10 Variation of stresses over the thickness of the vault given by finite element analysis.*

A final comment on the deflections predicted by the finite element analysis is that they are very small in comparison to actual deflections of a structure due to support movements or construction defects. For example, the vaults of the Army War College (the topic of Chapter 7) deflected 1½ inches at the crown due to support movements.\(^\text{15}\) Inducing a finite element model with this magnitude of deflection might result in considerably different stresses than the undeflected model since the finite element analysis is so sensitive to small deformations. Attempting a graphical analysis of a vault with the same deflection would likely not show a great difference in results.

\(^{15}\) Malcolm Blodgett to Rafael Guastavino, 16 October 1944. Guastavino Archive.
4.4 Conclusions

Graphical and finite element analyses give similar results for the stresses within a Guastavino barrel vault and comparable results for the thrust as well, thus it seems reasonable that either method could be used for analysis of these structures. The stresses in the vault are so small relative to the strength of the vault material that it hardly seems useful to calculate these stresses, other than to prove that the vault is safe with regards to strength beyond a shadow of a doubt. The horizontal thrust, on the other hand, is important to know because the supporting structure is responsible for supporting that force. Although it does provide the magnitude of horizontal thrust, finite element analysis is limited in its ability to illustrate the stability of a Guastavino barrel vault since it does not have an equivalent to the thrust line of the graphical method. The graphical analysis clearly demonstrates the necessity of the diaphragm walls while the finite element analysis does not.

Finite element analysis requires material properties and the assumption of linear elastic behavior, assumes elastic deformation (and, as a result, stress concentrations at the faces of the vault) and presumes that there is only one solution for a given problem, so its use in the analysis of Guastavino structures is questionable. If a finite element model is used to analyze an arch or barrel vault, an equilibrium method such as graphical analysis should be used to check the results. Graphical analysis recognizes that the vault is an indeterminate form with multiple possible thrust lines and is better suited to assess its stability since it clearly demonstrates the necessity of the diaphragm walls. For the analysis of Guastavino barrel vaults, graphical analysis is the better choice.
Chapter 5. The Guastavino Dome of the Grace Universalist Church

The Grace Universalist Church (Figure 5.1) in Lowell, Massachusetts was built in 1895 and designed by architect Chester Chase. The church is today known as Saint George Greek Orthodox Church, but will be referred to here by its name at the time it was constructed by the R. Guastavino Company. The auditorium of the church is circular in plan (Figure 5.2) and covered with a domed ceiling. This dome, designed and built by the Guastavino Fireproof Construction Company (the name of the Guastavinos’ company until 1897), was constructed with little formwork within two months’ time (Ochsendorf, forthcoming). However, the Guastavinos were almost relieved of this job before their work even began.

Figure 5.1 Grace Universalist Church ca. 1935.16

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16 Grace Universalist Church, service bulletin, 7 May 1935, bMS 244/2 (10), Unitarian Universalist Congregational Records, Manuscripts and Archives Department, Andover-Harvard Theological Library, Harvard University (hereafter cited as Unitarian Universalist Records).
Meeting minutes of the church building committee document a change in heart of the architect with regards to the Guastavino dome during the planning phase of the project. At a meeting in late 1893, Chase encouraged the men to visit the recently completed Central Congregational Church in Providence, Rhode Island, to see an example of a Guastavino dome. There is no mention of the committee visiting that particular building, but they had visited the Boston Public Library for the purpose of seeing the Guastavinos’ work. Afterwards, they instructed Chase to start on his drawings right away. From this information, one would assume that Chase was advocating for the Guastavino dome and that the committee was impressed enough by the library to let him proceed. However, in November 1894, Chase suggests “monolithic construction, cement instead of brick,” prompting the Aberthaw Construction Company to submit a bid for a concrete dome costing $5100. About a year later, Chase again suggests an alternative to the tile dome, this time a copper roof. The committee decides to hold off on considering his suggestion until they can speak with “Mr. Guastavino” himself –

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17 First Floor Plan, n.d., bMS 581/5, Unitarian Universalist Records.
18 Building Committee, meeting minutes, 1893-96, bMS 581/19, Unitarian Universalist Records.
presumably Rafael Sr.\textsuperscript{19} When Guastavino came to a meeting in April of 1895 to explain his system, it was enough to persuade the committee to continue with his work and accept his proposal to use a corrugated zinc tile on the outside of the dome. The Guastavinos were paid $4500 for their work on this project in the end.

Figure 5.3 An early picture of the domed ceiling showing the exposed tile (Guastavino Archive, reprinted from Ochsendorf, forthcoming).

5.1 Geometry

The dome of Grace Universalist is noteworthy for its thinness and rapid completion (Ochsendorf, forthcoming). As an early project of the Guastavinos, it is unsurprising that only one drawing exists today for the church (Figure 5.4). The sectional drawing shows a dome with a 35 foot radius and four semi-circular openings spaced evenly around the dome. The angles circled in Figure 5.4 represent a tie rod within the thickness of the dome right above the top of the openings. Wight (1901, 186) mentions this tie specifically in his writings on Rafael Sr.: “…

\textsuperscript{19} While Rafael Jr. eventually superintended the project, his father would have likely still attended to the business affairs of the company.
the penetrations of twenty feet on the four sides required extraordinary precautions, accurate calculations and great care in execution. It was thought wise, as will be seen in the drawings, to insert steel angles in the intersections.” While surviving drawings of other Guastavino projects frequently depict each layer of tile or call out the number of courses, the drawing for Grace Universalist Church has only dimensions of four inch thickness at the crown and six inches above the openings. Rafael Jr.’s writings indicate that this corresponds to a three courses thickness at the crown (Ochsendorf, forthcoming).

Figure 5.4 Drawing of the dome (Guastavino Archive, reprinted from Ochsendorf, forthcoming).

As the building stands today, there are tie rods at the base of four arched openings (see Figure 5.3 and Figure 5.5) that are not shown in the sectional drawing. Since there are few details on the construction of the dome, it is unknown whether or not these ties run continuously through the dome or not.
Only the top portion of the dome is visible from the outside (Figure 5.6). A cylindrical wall is built up around the base of the dome, and a metal roof is draped across the top of this wall and over the dome. It is possible that between the cylindrical wall and the bottom portion of the dome there are buttressing walls (Figure 5.7) or ribs (Figure 5.8). However, a picture taken of the dome while it was under construction (Figure 5.9) appears to show the dome free-standing without walls, so they are probably not a part of this particular structure.
Figure 5.7 Example of a Guastavino dome with buttressing walls at East Boston High School, Boston, MA, 1899 (Guastavino Archive, reprinted from Huerta 1999, plate 6).

Figure 5.8 Example of a Guastavino dome supported by ribs or flying buttresses at the National Museum, Washington DC, 1906 (Guastavino Archive, reprinted from Huerta 1999, plate 27).
5.2 **Structural History**

There is no known record of damage to the dome since its construction, which is rather remarkable given its age and relatively unprotective roof. Since the roof on the dome is non-structural and only serves to protect the dome from the elements, the top portion of the dome is subject to the same forces as the roof (for example, snow loads and hurricane winds). But while there are records of damage to the roof over the years, and several mentions of water damage inside the church, there are no mentions of cracks in the dome or falling tiles.

The dome of the Grace Universalist Church has survived severe loading during its lifetime, including three hurricanes: The Great New England Hurricane in 1938, and Hurricanes Carol and Edna, both in 1954. The storm of 1938 blew tiles off the roof, but there is no record of damage to the dome for this hurricane. Similarly, after the 1954 storms, it was noted that the roof was damaged but there is again no mention of the dome. Based on the information available, it appears that the dome weathered the hurricanes very well.

Looking at the history of repairs to the roof can provide some clues on the state of the Guastavino dome over the last 113 years. Presumably, the original roof of the building was constructed of corrugated zinc tile. After the hurricane of 1938, meeting minutes refer to the roofing material as ‘slates’, indicating that the original zinc roof was replaced with slate tiles.

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20 Board of Assessors, minutes, 13 October 1938, bMS 581/12, Unitarian Universalist Records.
21 First-Grace Universalist Parish, annual report, 1954, bMS 581/1, Unitarian Universalist Records.
22 Building Committee, minutes, n.d. [ca. October 1893], bMS 581/19, Unitarian Universalist Records.
some time prior. These same minutes say that the areas where slates were blown off by the hurricane are to be covered in asbestos tile.\textsuperscript{23} It is possible this was a temporary solution, as later records never mention asbestos tile. Records indicate that significant roof repairs were made in 1947 by the Douglas Roofing Company, although the exact nature of the repairs is unknown.\textsuperscript{24} In 1954, damage was done to the “copper and tin” roof by the two hurricanes, and inspections revealed that the roof was “badly eroded above the copper covered part of the dome... [and] the eroded part of the dome should be replaced,” although this seems like a misprint and that really it was the roof that was eroded and not the structural tile dome.\textsuperscript{25} The following year, a new tin roof was installed on the church by the Douglas Roofing Company, who was then hired to make semi-annual inspections of the dome.\textsuperscript{26} More recent roof work has taken place as well – in 2005, the roof was stripped down to the sheathing and replaced.\textsuperscript{27}

5.3 Graphical Analysis

Using Wolfe’s graphical analysis method introduced in Chapter 3, the dome can be analyzed in order to estimate its internal stresses and investigate its stability. Graphical analysis can determine the stresses in the dome, which are expected to be significantly lower than the capacity of the material. If a size for the tie circled in Figure 5.4 were provided, it could be checked for adequate capacity after determining the minimum required force in the tie with graphical analysis. Since that information is unavailable, the minimum required size can be determined by finding that force and assuming an allowable tensile stress for the tie material.

The first step of the analysis is to establish an appropriate model of the dome. Figure 5.4 shows that the dome forms almost a full hemisphere, with an angle of embrace ($\alpha$) of about 80°. There are four openings at the base of the dome, located in the region of $80^\circ < \varphi < 58^\circ$. Barrel vaults or arches abut the dome on four sides, corresponding to the location of the four openings, and these vaults and arches provide buttressing to the lower portion of the dome. For this reason, the bottom portion ($80^\circ < \varphi < 60^\circ$) will not be considered for analysis. The dome is approximated as having a constant thickness of four inches because the dome appears to be of near-constant

\begin{itemize}
  \item \textsuperscript{23} Board of Assessors, minutes, 13 October 1938, bMS 581/12, Unitarian Universalist Records.
  \item \textsuperscript{24} First-Grace Universalist Parish, annual report, 1947, bMS 581/1; Trustees, minutes, 20 October 1947, bMS 581/12, Unitarian Universalist Records.
  \item \textsuperscript{25} First-Grace Universalist Parish, annual report, 1954, bMS 581/1; Trustees, minutes, 1 November 1954, bMS 581/12, Unitarian Universalist Records.
  \item \textsuperscript{26} Trustees, minutes, 7 February – 7 March 1955, bMS 581/12, Unitarian Universalist Records.
  \item \textsuperscript{27} Building Permit, 16 November 2005, Office of Inspectional Services, Lowell City Hall, Lowell, MA.
\end{itemize}
thickness until fairly close to $\varphi = 58^\circ$ (Figure 5.4). The geometry of the dome used for analysis is presented in Table 5.1.

<table>
<thead>
<tr>
<th>Radius (R)</th>
<th>35 feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of embrace ($\alpha$)</td>
<td>60°</td>
</tr>
<tr>
<td>Thickness (t)</td>
<td>4 inches</td>
</tr>
</tbody>
</table>

Table 5.1 Geometric properties of the Grace Universalist Church dome for analysis.

A simple load case with only uniform dead load and live load is considered here, although a more critical load case might be an asymmetric load with live load on only one half of the vault. The weight of the dome material and the roof cladding contribute to the dead load. The thickness of the dome and a density of 150 pcf yield a dead load of 50 psf for the structural dome. Since there is no data on the roofing material but it seems to be constructed of metallic tiles that are probably fairly thin, the dead load of the roofing material is estimated to be 5 psf. Since the roofing material of the church is assumed to be laid right on of the dome, the dome will be subject to live load from wind and snow on the roof. The live load used for analysis is 30 psf, approximated from the 2003 International Building Code (ICC 2002, 279-82). A summary of the load case used for the graphical analysis is given in Table 5.2.

| Dead Load of Dome | 50 psf |
| Dead Load of Roofing | 5 psf |
| Live Load | 30 psf |

Table 5.2 Load case for the graphical analysis of the Grace Universalist Church.

As introduced in Chapter 3, a dome has tensile hoop stresses in the region of $\alpha < \varphi < 51.8^\circ$ (or no hoop stresses if the dome cannot provide tensile capacity). For the geometry considered here, a steel tie or buttressing would have to provide the tensile hoop forces (which serve to constrain the thrust line to the mid-surface of the dome thickness) in the region of the dome of $51.8^\circ < \varphi < 60^\circ$. Since the tie in Figure 5.4 is located at approximately $\varphi = 56^\circ$, it is assumed to provide this tensile force. Thus, this tie will provide both the tensile hoop stress and will contain the outward thrust of the dome.

The results of the graphical analysis are presented in Figure 5.10, Figure 5.11, Table 5.3, and Table 5.4. Figure 5.10 shows a cross-section of the dome with the thrust line approximately at mid-surface and the loads on the 12 segments of the lune. Figure 5.11 shows the force polygon.

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28 Such a load case was used in the analysis of the Pines Calyx domes (Ramage et al. 2007).
29 The density of 150 pcf comes from Irving Berg’s analysis of the Army War College, where he assumes that a vault of 2 courses thickness (thus, approximately 2 inches thick) has a weight of 25 psf. Irving Berg, “War College Vault Panel ‘A’ Loading,” 1 November 1944, Guastavino Archive.
and selected lines labeled with their magnitude. Table 5.3 and Table 5.4 present the meridional and hoop stresses, respectively (negative values indicate compression). From membrane theory, the point of zero hoop stress is $\varphi = 51.8^\circ$, so in this analysis the hoop force at $\varphi = 52.5^\circ$ is approximately zero.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.10.png}
\caption{A possible thrust line for the dome of Grace Universalist Church.}
\end{figure}
Figure 5.11 Force polygon for the dome of Grace Universalist Church showing selected values.

<table>
<thead>
<tr>
<th>$\theta$ (°)</th>
<th>$P$ (kips)</th>
<th>$\sigma$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-1.2</td>
<td>-31.0</td>
</tr>
<tr>
<td>10</td>
<td>-2.4</td>
<td>-31.2</td>
</tr>
<tr>
<td>15</td>
<td>-3.6</td>
<td>-31.5</td>
</tr>
<tr>
<td>20</td>
<td>-4.8</td>
<td>-31.9</td>
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<tr>
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<td>-6.0</td>
<td>-32.5</td>
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<td>-34.1</td>
</tr>
<tr>
<td>40</td>
<td>-9.9</td>
<td>-35.1</td>
</tr>
<tr>
<td>45</td>
<td>-11.3</td>
<td>-36.3</td>
</tr>
<tr>
<td>50</td>
<td>-12.7</td>
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<tr>
<td>60</td>
<td>-15.7</td>
<td>-41.3</td>
</tr>
</tbody>
</table>

Table 5.3 Meridional forces and stresses in the dome.

<table>
<thead>
<tr>
<th>$\theta$ (°)</th>
<th>$P$ (kips)</th>
<th>$\sigma$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>-4.5</td>
<td>-31.0</td>
</tr>
<tr>
<td>7.5</td>
<td>-4.5</td>
<td>-30.4</td>
</tr>
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<td>12.5</td>
<td>-4.3</td>
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<td>-4.0</td>
<td>-27.4</td>
</tr>
<tr>
<td>22.5</td>
<td>-3.7</td>
<td>-25.1</td>
</tr>
<tr>
<td>27.5</td>
<td>-3.3</td>
<td>-22.2</td>
</tr>
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<tr>
<td>47.5</td>
<td>-0.7</td>
<td>-4.9</td>
</tr>
<tr>
<td>52.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>57.5</td>
<td>1.2</td>
<td>7.9</td>
</tr>
</tbody>
</table>

Table 5.4 Hoop forces and stresses in the dome.

The outward horizontal thrust at the dome base is 7.9 kips, obtained from the horizontal component of the forces in the tie. Since the tie rod is assumed to both contain the meridional force near the bottom of the dome to the mid-surface layer as well as contain the thrust, the force in the tie rod will be the sum of the 1.2 kip hoop force at \( \varphi = 57.5^\circ \) and the 30.1 kip tie force due to the thrust (labeled in Figure 5.11), for a total of 31.3 kips.

5.4 Discussion of Analysis Results

The maximum compressive stress in the dome from the analysis is 41 psi. From Table 2.1, the compressive strength of the dome material is about 2060 psi, which yields a safety factor of about 50 for the vault material. Thus, the actual stresses in the dome are much lower than the capacity and the material still has considerable reserve strength under this load case.

Graphical analysis determined the force required in the steel tie to contain the thrust and provide the tensile hoop force. The 31.3 kip force calculated from this analysis can be used to establish the minimum steel tie size. Assuming an allowable strength of steel of 18 ksi (Kaup and Matteo, forthcoming), the minimum cross-sectional area for a tie would be 1.7 square inches. This area corresponds to an L3x3x\( \frac{3}{8} \) steel angle with an area of 2.11 square inches (AISC 1989, 1-49). From Figure 5.4, the thickness of the dome in the region of the tie is about 6”, where an L3x3x\( \frac{3}{8} \) steel angle would fit without difficulty. However, the actual size of the angle used by the R. Guastavino Company is not known, so assessing the safety of the tension tie is beyond the scope of this thesis.

The results of the graphical analysis and knowledge of the dome’s structural history should be considered when evaluating the strength and stability of the dome. The low stresses

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30 This specific angle size is from a modern steel table and may not have been available during the construction of the Grace Universalist Church.
indicate that the strength of the dome material is not threatened. Given that the dome has been standing for over 100 years and that the calculated minimum size for the tensile steel is reasonable, the stability of the dome is assured. However, this stability would be undermined if the steel ring were damaged. The history of the church documents several instances of roof leaks, and water infiltrating the dome in the region of the tie could cause it to rust and lose cross-sectional area (and consequently, its ability to carry the 31.3 kip tie force). Thus, if there were evidence of water damage to the dome near \( \varphi = 56^\circ \), an inspector should investigate the integrity of the steel tie by looking for evidence of rusting. In the particular case of Grace Universalist, it is possible the dome would remain stable even with a compromised tie because of the buttressing arches around the bottom portion of the dome, but this may not be the case with all Guastavino domes.

5.5 Chapter Summary

The Grace Universalist Church is a testament to the strength and durability of Guastavino shells. Having been subjected to hurricane force winds and a leaking roof on several occasions, the dome has never suffered any visible cracks or falling tiles. Graphical analysis established that despite its thinness, the dome has relatively low stresses under general load conditions. Also, the required cross-sectional area of the steel tie was found to be of a reasonable size. The results of the graphical analysis coupled with knowledge of the structural history of the dome are useful for evaluating the stability of the dome as well as possible threats to its integrity.
Chapter 6. The Guastavino Vault of the Saint Louis Art Museum

Architect Cass Gilbert designed the Palace of Fine Arts for the 1904 World’s Fair in St. Louis, Missouri. As the only permanent building erected for use both during and after the fair, Gilbert designed a monumental structure inspired by the Baths of Caracalla, with a Guastavino barrel vaulted ceiling in the large central room known as the Sculpture Hall (Overby 1987, 8). After the fair, it became an art museum for St. Louis and was called the City Art Museum until 1971 (Overby 1987, 26). Today, the building is known as the Saint Louis Art Museum.

Figure 6.1 The Sculpture Hall of the Saint Louis Art Museum in 1977 (Photograph by C. Robinson, reprinted from Overby 1987, 29).

The Guastavino vault of the Saint Louis Art Museum provides an interesting case study because of its history of damage and rehabilitations. Cracks appeared in the vault early on in its life, eventually causing a tile to fall. In response to the serious mortal threat posed by a sizable ceramic tile falling from over 50 feet overhead, museum officials ordered the lower surface of
the vault to be covered with reinforced concrete, forever obscuring the tile from view. Given the architectural significance of the Guastavino vaulting to the original appearance of Gilbert’s design, this heavy-handed intervention permanently altered a significant work of American architecture. The following sections will investigate the structural behavior of the Guastavino barrel vault of the museum to assess whether the concrete was necessary and if it improved the safety of the vault.

6.1 Geometry

The roof system of the Saint Louis Art Museum consists of a Guastavino barrel vault below a timber roof (Figure 6.2). The semi-circular vault has the dimensions listed in Table 6.1, which are taken from drawings but have not been verified in the field.\(^{31}\) The roof is supported by vertical and diagonal members that bear directly on the vault in some places, making its geometry a great deal more complex than the simple vault presented in Chapter 4. At these locations, the vault is thickened with additional layers of tile (Figure 6.8). The nominal thickness of one course of tile is 1 inch.

<table>
<thead>
<tr>
<th>Radius (R)</th>
<th>27 feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of embrace ((a))</td>
<td>90°</td>
</tr>
<tr>
<td>Thickness below roof rafters</td>
<td>5 courses of tile</td>
</tr>
<tr>
<td>Thickness elsewhere</td>
<td>3 courses of tile</td>
</tr>
</tbody>
</table>

Table 6.1 Dimensions of the barrel vault of the Saint Louis Art Museum.

\(^{31}\) The author has inspected the vault in person for damage, but did not attempt to confirm any dimension.
Figure 6.2 Cross-section of one-half of the vault (locations of roof supports are approximate).

The vault is braced by diaphragm walls between the vault and the clerestory walls, spaced at about 4½ feet (Figure 6.3). This spacing is equivalent to half of the roof rafter spacing. The vault has three window openings on each side (see Figure 6.1 - note that only 2 windows are visible per side in this image) and additional buttressing between windows. This buttressing consists of “through walls,” which are flanked on either side by a shorter wall and a buttress (see Figure 6.4 and Figure 6.5).

Figure 6.3 Three diaphragm walls buttressing the vault (Photograph by D. Esarey).
Figure 6.4 Drawing of space above the vault showing the through wall (labeled “thru wall”) with a shorter wall and buttress on either side.  

Figure 6.5 A through wall and a flying buttress.

The timber roof is supported by rafters which span from the crown of the vault to the clerestory walls (Figure 6.6). The exterior of the roof is covered in a layer of clay tiles (Figure 6.7). Roof rafters are spaced at approximately 9 feet so that each rafter lines up every other

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diaphragm wall. Vertical and diagonal members support the rafters and bear on both the vault and the diaphragm walls (Figure 6.6 and Figure 6.8).

Figure 6.6 A roof rafter and a diagonal element bearing on a diaphragm wall (Photograph by D. Esarey).

Figure 6.7 Roof tiles.
6.2 Structural History

The history of the vault of the Saint Louis Art Museum includes structural problems appearing soon after the building’s completion and a dramatic retrofit in the 1930’s, so its structural safety is worth examining in detail.

6.2.1 Structural Problems: Settlement, Cracks, and Falling Tiles

Foundation problems were observed early in the life of the art museum and were likely the cause of cracks in the vault. Differential settlement of the foundation was documented in the building as early as 1910 (Overby 1987, 14). This settlement and other problems, such as leaks, were addressed by Gilbert in his 1916 design plans for the future of the museum (Overby 1987, 16). In the late 1930’s, measurements were taken at eight points of the Sculpture Hall between January 1937 and May 1938 to check for settlement in the building. Over that time period no movement was recorded, but the measurements do show small differences in elevation over the Sculpture Hall floor that are probably the result of earlier differential settlement. Since these measurements were taken over 30 years after the museum was erected, the soil may have been consolidated by that time and the foundation was likely stable by the “generation rule” (Heyman 1995, 25).

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33 Pitzman’s Co. of Surveyors & Engineers, “Statuary Hall: City of St. Louis Art Museum; Elevations Taken for La Beaume & Klein, Architects,” 23 January 1937 – 2 May 1938, Guastavino Archive.
In addition to cracks, the vault suffered at least one fallen tile by the 1930's, which prompted the museum to investigate the safety of the structure (Overby 1987, 22). The R. Guastavino Company was consulted to inspect the ceiling, make repairs, and replace lost tiles in 1937. Blueprint copies of R. Guastavino Company drawings are marked-up with inspection notes and repair suggestions, although it is not clear if these notes were necessarily made by someone at the company. These mark-ups map the location of cracks in the vault, walls and floor, locations of loose pointing in the vault, and displacements of the walls (both the East and West clerestory walls were reported as leaning out by 1¼ inches). This outward lean was also noted on a 1937 survey by another company, which records displacements of the west wall between 0 inches and 2¼ inches (the average of the five values is 1¼ inches). There is also a copy of a marked-up photograph of Sculpture Hall with the location of cracks in the vault, walls and floor (Figure 6.9). A note at the bottom of the image reads “Shows West side of room where major portion of cracks occur,” and marks where a tile fell from the vault (circled in Figure 6.9).

Current inspection of the vault provides clues as to which suggested repairs were completed and which were disregarded. One proposal was to tie the roof rafters together with 2 inch by 8 inch wooden planks in the space above the vault, and the light-colored boards in Figure 6.10 are likely the result since they appear to be a different kind of wood than that used for the roof construction. This may have been an attempt to reduce the horizontal thrust of the roof and prevent further leaning of the clerestory walls. The R. Guastavino Company also proposed a steel tie across the base of the arch inside the Sculpture Hall, but there is no evidence that this tie rod was ever installed.

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34 Louis Le Beaume to F.M. Summerville, 15 January 1937, Guastavino Archive.
35 “Art Building at St. Louis Exposition: St. Louis, Mo.,” 21 January 1937, Guastavino Archive.
36 Pitzman’s Co. of Surveyors & Engineers, “Upper West Wall of Art Museum,” 15 April 1937, Guastavino Archive.
37 “Art Building at St. Louis Exposition: St. Louis, Mo.,” 21 January 1937, Guastavino Archive.
Figure 6.9 Location of cracks in the vault (Guastavino Archive).
6.2.2 Retrofit of the Vault

According to Overby (1987, 22), the vault was covered in a layer of “gunite” (sprayed on concrete) sometime during 1937 or 1938 (compare Figure 6.9, where the R. Guastavino Company’s signature herringbone pattern of finishing tile is exposed, to the covered vault in Figure 6.1). A 1¼ inch thick layer of gunite with wire mesh was proposed by Mr. Baxter Brown, President of the Board of Public Service, and Mr. Becker, City Engineer, as reported in minutes for the Administrative Board of Control from February 4, 1937.\(^3\) In July of 1937, the museum was billed for 200 face tiles and almost 400 backing tiles by the R. Guastavino Company, so loose and fallen face tiles of the vault may have been replaced before the gunite was applied.\(^4\) It is unclear what the backing tiles may have been used for.

6.3 Structural Analysis

After considering the cracks, settlement problems, and outward rotation of the clerestory walls, the structural stability of the vault comes into question. Was the vault safe before the retrofit? Did the 1930’s addition of gunite improve or detract from the vault’s safety? Additionally, what caused the main cracks in the vault and what do they mean for the structure?

As demonstrated in Chapter 4, graphical equilibrium analysis provides a fast and straightforward

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\(^4\) R. Guastavino Co. to Louis La Beaume, 13 July 1937, Guastavino Archive.
means of answering questions about vault stability and will be used in the following section. The cause of cracks in the vault will be investigated with a three-dimensional finite element model.

6.3.1 Graphical Analysis: Investigation of the Vault’s Stability

Graphical analysis is applied to a 4½ foot wide section of the vault (corresponding to the spacing of the diaphragm walls) at a location with an angle of embrace (α) of 90° (Figure 6.11). Only one half of this section need be analyzed because of symmetry.

![Figure 6.11 Section of vault for thrust line analysis (roof not shown).](image)

Two load cases are considered for the graphical analysis, which are combinations of dead load of the vault, dead load of the gunite, dead load of the diaphragm walls, dead load of the roof, and live load on the roof. The determination of each dead load is straightforward except for the roof, which is only partially supported by the vault (see Figure 6.8). To calculate the dead and live loads for the analysis, one must consider the loads on a 4½ foot wide section of vault loaded with the weight of a 9 foot wide section of the roof (based on the spacing of the diaphragm walls and roof supports, respectively). The density of the vault is estimated to be 150 pounds per cubic foot, and the weight of the vault is calculated by multiplying this density by the thickness.\(^{41}\) The same density was assumed for the gunite, and its weight was calculated based on the 1¼ inch thickness. The 15 psf live load estimate is from the 2003 International Building code (ICC 2002, 279-82). Details of the load calculations for the roof can be found in Appendix D and are summarized in Table 6.2. The load from the diaphragm wall is not included in this table as its weight is broken into point loads based on the area of wall over a vault segment.

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Table 6.2 Summary of Loads on the vault (excluding diaphragm walls).

For purposes of the analyses one must estimate the proportion of roof weight supported by the vault and how to distribute that weight. Here, it is assumed that three-quarters of one-half of the roof weight is carried by the half-vault. This load is distributed uniformly over the first eleven of fifteen sections that the half-vault is divided into (illustrated in Figure 6.12). The remaining one-quarter of the weight is then supported by the clerestory wall.

![Diagram](image)

Figure 6.12 Section of vault showing loads used for graphical analysis.

Two cases of the vault are analyzed: before and after the application of the gunite. Figure 6.13 and Figure 6.14 show the loads used for the graphical analysis and the resulting thrust lines through the cross-sections. Note that the vault in Figure 6.14 represents the vault with the gunite, so it has a slightly thicker cross section to reflect the extra material that the thrust line can safely pass through and also has higher segment loads. Given the unknown size and location of wire
steel mesh, it is assumed that the added concrete has no appreciable tensile capacity and simply acts as an unreinforced concrete shell.

Figure 6.13 A possible thrust line for the vault before the addition of gunite.
Figure 6.14 A possible thrust line for the vault after the addition of gunite.

Table 6.3 Results of graphical analysis for the vault before and after the application of gunite.

<table>
<thead>
<tr>
<th></th>
<th>Vault before Gunite</th>
<th>Vault after Gunite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Thrust</td>
<td>16,200 lbs</td>
<td>18,000 lbs</td>
</tr>
<tr>
<td>Rankine Safety Factor</td>
<td>2.5</td>
<td>2.4</td>
</tr>
</tbody>
</table>

The 1¼ inches of gunite added about 3000 pounds of dead load to one half of the vault section shown in Figure 6.11. This additional weight results in an increase in thrust of 1800 pounds (11%), or 400 pounds per linear foot across the width of the vault. The safety factor of the two cases is calculated using the Rankine Safety Factor to investigate the stability of the supporting brick wall. Figure 6.15 provides an explanation of the Rankine Safety Factor. Comparing the safety factors shows that the addition of gunite lowered the factor of safety slightly, but both cases have a considerable safety factor by this method. It should be noted that this safety factor is based on an estimate of the wall thickness. If the wall is less massive than estimated, the safety factors will decrease. The gunite could potentially have a detrimental effect on the stability of a vault or dome with less massive walls. This is especially true considering the manner in which Guastavino vaults were designed, where a vault such as this one would have been designed to the spring line and then given to the project engineer along with the magnitude
of the thrust (JMC [1979?], Section IV.2). If building walls were designed for only the original value of thrust, the extra thrust due to gunite could be problematic for the building's stability.

![Thrust Line Centerline of Abutment](image)

**Figure 6.15 Definition of Rankine Safety Factor from Rankine's Manual of Applied Mechanics, 1858 (reprinted from Lancaster 2005, 159).**

Looking at the analysis with the gunite can give us an idea of the stresses in the present day vault. Table 6.4 gives the magnitude of the thrust line at each location in the vault, and the corresponding stress is calculated where the force is contained within the vault thickness (as opposed to the diaphragm walls). From Rafael Guastavino Sr.'s test results given in Table 2.1, the compressive strength for Guastavino vaulting is about 2060 psi, yielding a factor of safety for the vault of about 30 in its most highly stressed region. For comparison, the generic vault in Chapter 4 had a safety factor of over 100 under self-weight only. The Saint Louis Art Museum demonstrates that with addition dead load and live load, the stresses in the vault are higher but still pose no threat to the strength of the vault.

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<thead>
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<th>σ</th>
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<tbody>
<tr>
<td>(°)</td>
<td>(k)</td>
<td>(psi)</td>
</tr>
<tr>
<td>0</td>
<td>-18.0</td>
<td>-46.0</td>
</tr>
<tr>
<td>6</td>
<td>-18.1</td>
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<tr>
<td>66</td>
<td>-30.1</td>
<td>-</td>
</tr>
<tr>
<td>72</td>
<td>-31.0</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 6.4 Magnitude of thrust line and stresses within the vault (negative values indicate compression).

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<table>
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<tbody>
<tr>
<td>78</td>
<td>-31.9</td>
<td>-</td>
</tr>
<tr>
<td>84</td>
<td>-32.8</td>
<td>-</td>
</tr>
</tbody>
</table>

6.3.2 Finite Element Analysis: Investigation of the Cause of Cracks

From the structural history of the Saint Louis Art Museum, it is known that the cracks in the vault formed early on and that foundation settlement was documented around the same time. From this information, it seems likely that differential settlement of the foundation was the likely cause of the cracks in the vault. As shown in Chapter 4, finite element analysis results for a barrel vault are suspect because of the high variability of stresses over the thickness of the vault and the related deflected shape of the model under distributed loads. Also, a linear elastic finite element model is less accurate once a vault has cracked. Yet the potential remains for a finite element model to give a qualitative understanding for the cause and location of cracks, and a three dimensional model of the vault will be considered here to investigate the cause of cracks documented in the 1930s.

Figure 6.16 shows the geometry of the vault used in the finite element analysis and the corresponding vertical displacements (in feet) applied at five of the supporting edges. Using the survey information mentioned previously, the elevation of the edges labeled “fixed” was used to calculate the relative displacements of the remaining five edges.42 The first three edges are fixed for all six degrees of freedom, and the remaining five fixed for all translations and rotations except for vertical translation. The vault was modeled using the program ADINA and 9-node shell elements with elastic-isotropic material properties. Other than the five prescribed displacements, no other load conditions are applied to the vault. The diaphragm walls and window openings that serve to buttress the vault are not included in the model, and would likely have a minimal effect on the results of a model without gravity loads included.

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42 Pitzman’s Co. of Surveyors & Engineers, “Statuary Hall: City of St. Louis Art Museum; Elevations Taken for La Beaume & Klein, Architects,” 23 January 1937 – 2 May 1938, Guastavino Archive.
Figure 6.16 Model used for the finite element analysis showing displacements at five supporting edges.

Figure 6.17 and Figure 6.18 show the resulting stress plots for the East and West sides of the vault (the range of stresses illustrated have been limited to tensile stresses between 0 and 25 ksf to highlight the stresses in the regions between the windows). Since the condition surveys from the 1930s show the biggest cracks in the regions labeled A to D in Figure 6.16, only the stresses in those regions will be considered in the next section. The highly stressed regions in the four corners will be disregarded as their small dimensions may be influencing the stresses.
6.4 Discussion of Analysis Results and Retrofit Solutions

As demonstrated by the comparison of stresses in the present day vault to the allowable stress found by Rafael Guastavino Sr., the determination of whether or not the vault is safe is a matter of its stability, not its strength. Graphical analysis of the Saint Louis Art Museum vault provides an effective means of checking this stability and serves as a useful tool to check how modifications affect that stability, such as the addition of gunite. The first vault analysis (Figure 6.13) demonstrates that the thrust line was contained within the arch thickness, so by the master safe theorem of plasticity, the vault is safe. The gunite was unnecessary to improve the safety of the vault other than by physically preventing falling tiles, as graphical analysis reveals that the gunite increased the thrust on the walls and lowers the Rankine Safety Factor. One might be inclined to think that because no cracks have been documented in the vault since the gunite retrofit that the gunite helped to stabilize the vault, but this is probably not the case. More than likely, no cracks have formed in the vault since the addition of gunite because the foundation settlement had stopped some time prior. The graphical equilibrium analysis shows that the gunite did not improve the stability of the vault.

The equilibrium analyses presented here only consider a two-dimensional cross-section of the vault that did not account for cracks that run circumferentially. The cracks in Figure 6.9 essentially divide the vault into three barrel vaults of shorter length. Since the behavior of a vault can be approximated as a series of side-by-side arches, these cracks do not affect the ability of the vault to span the width of the Sculpture Hall (Heyman 1995, 50). This characteristic of vault behavior proves that circumferential cracks do not threaten the vault’s safety; however, the cause
of cracks could eventually undermine the vault if the problem continues to progress. The cause should be identified and studied to see if it might cause problems in the future. The finite element analysis appears to verify that differential settlement caused the cracks by showing stress concentrations in regions A through D of Figure 6.16. Figure 6.9 shows that the longest crack (and the one that resulted in a fallen tile) is located in region A (the North-West region) of the vault, and this corresponds to the zone of highest tensile stress in Figure 6.18. Thus, finite element analysis of the Saint Louis Art Museum suggests that the cause of cracks was probably differential settlement. Monitoring the foundation to show that settlement has ceased, as was done in the 1930s, will ensure that differential settlement will not affect the vault further.

The Saint Louis Art Museum covered a great architectural feature – the signature herringbone tile pattern of the R. Guastavino Company - when they covered the vault. Though the gunite does serve the important role of protecting museum staff and visitors from falling tiles, there are ways to accomplish this that are less heavy-handed and (more importantly) reversible. A common but perhaps unsightly means of protection from falling tiles is the installation of nets below Guastavino vaulting to catch any falling tiles (Figure 6.19). Another approach would have been to regularly inspect the vault for loose tiles, and repair those that show evidence of debonding. This would ensure that there were no loose tiles to put staff and visitors at risk, while at the same time alerting inspectors to any potential problems with the vault (such as leaks in the roof, which could subject the vault to water and loosen tiles).

Figure 6.19 Barely visible net under the dome of St. Paul's Chapel, Columbia University, 2007.
If repairs were being made to the Saint Louis Art Museum today, a less drastic approach could be taken to repair cracks and fix falling tiles. For example, the rehabilitation of the Bridgemarket space under the Queensboro Bridge used flexible material to fill in cracks that were likely to reopen and devised a safe way to replace tiles along the cracks so they would not be in danger of falling as a result of joint movement (DiSanto 1999). Engineers involved with the assessment and possible repair of Guastavino vaults have the great responsibility of guaranteeing that the structures are safe. Nevertheless, they should strive for solutions that are respectful of the behavior of masonry tile shells as well as the legacy of the R. Guastavino Company.

6.5 Chapter Summary

The barrel vault of the Saint Louis Art Museum was presented to illustrate the behavior of Guastavino masonry tile structures and to demonstrate the applicability of analysis methods to a real structure. Graphical equilibrium analysis was used to analyze the vault before and after the gunite retrofit. Comparison of the two cases shows that the addition of gunite increased the thrust of the vault, highlighting a very negative side effect and bringing into question the suitability of the retrofit. The stresses in the vault were compared to the compressive strength of the vault material and still found to be relatively low. Finite element analysis was used to investigate the likelihood of differential settlement of the foundation causing the major cracks in the vault and showed some confirmation of this. From the first graphical analysis, it was shown that the vault was stable regardless of the circumferential cracks. But cracks can be a threat to the safety of the museum-goers because they can loosen tiles to the point that they fall from the vault. In this regard, the museum officials were correct to take action, but the gunite retrofit was an ill-advised solution from both a structural and an aesthetic standpoint.
Chapter 7. Guastavino Vaulting at the Army War College

For its extensive use of tile vaulting as both a structural and decorative system, the Army War College is one of the most significant projects the R. Guastavino Company completed with architects McKim, Mead and White. Its structural problems, investigations and rehabilitations span several decades and involved numerous persons and organizations, including the R. Guastavino Company and thin-shell expert Anton Tedesco.

In early 1905, Captain John Stephen Sewell, U.S. Army Corps of Engineers, began submitting requests to Brigadier General A. Mackenzie, Chief of Engineers, requesting the hire of the R. Guastavino Company for construction of tile vaults in the Army War College Building. Reasons he specified were their low cost relative to steel construction, resistance to fire, and the impressive architectural space that results from the construction without additional cost for aesthetics. After a formal call for proposals for “timbrel arch construction,” the R. Guastavino Company was hired for the project. The company made estimates for (and presumably built) the paneled, coffered dome of the central lobby area, the double dome of the roof, the second and third floor balconies, the vaulted ceilings of the Map Room and Library, the room over the Lecture Hall, four arches at the corners of the Entrance Hall, a rough roof, and two staircases.

7.1 Geometry

The Army War College building consists of a central lobby with two wings in the east and west direction and a half-circular space to the north. The central lobby, also called the

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Entrance Hall or Rotunda, has a coffered dome ceiling divided by ribs (Figure 7.2). The coffered dome is supported on piers 35 feet apart and varies in thickness between 2 and 6 courses of tile.\textsuperscript{46} There are impressive flat vaults in the balconies (called “galleries” by the Guastavinos) at the second and third level of the lobby, the third level of which is shown in Figure 7.2.

![Image of the coffered Rotunda dome (Guastavino Archive).](image)

Above the coffered interior dome of the rotunda is a spherical double dome by Guastavino (visible in Figure 7.1), supported by an octagonal wall that bears on the coffered dome (Figure 7.3). The outer dome is four courses of tile thick, has a radius of 27 feet and 8 inches, an angle of embrace of about 30°, and a 4 inch steel band all around the base.\textsuperscript{47} The inner dome, with a radius of 20 feet and 7 inches, is constructed of either two courses of Guastavino tile or lath and plaster, though its exact construction is not clear from available drawings.\textsuperscript{48}

\textsuperscript{46} R. Guastavino Co., “Transverse Section: War College,” 9 October 1905, Guastavino Archive.
\textsuperscript{47} R. Guastavino Co., “Transverse Section: War College,” 9 October 1905, Guastavino Archive.
\textsuperscript{48} R. Guastavino Co., “Preliminary Details of Dome,” 16 November 1904, Guastavino Archive.
The East Wing (originally the Map Room and later the Eisenhower Theater) and West Wing (also called the Library Wing) have Guastavino vaulted ceilings, which are barrel vaults with cross vaults inserted (Figure 7.4). The cross vault openings are supported at the walls with tile arches (referred to here as “longitudinal rib arches”), which open to small vaulted alcoves. The third floor gallery encircles the wing just below the level of these arches. The vaults are protected from the elements by a ridged roof (Figure 7.5). The roof is supported by another set of tile arches (referred to here as “roof support arches”) that bear alternately on the piers (shown in Figure 7.7 and on the right in Figure 7.5) and the longitudinal rib arches of the cross vaults (shown in Figure 7.8 and on the left in Figure 7.5). Above the vaulting, a tie restrains the outward thrust of the roof support arches (Figure 7.6).

Figure 7.4 Guastavino vaults in the West Wing (Guastavino Archive).

Figure 7.5 Section through the East and West Wings.\textsuperscript{50}

\textsuperscript{50} R. Guastavino Co., “Ceiling of Library & Map Room,” 12 July 1905, Guastavino Archive.
Figure 7.6 Space between vaults of the East Wing and the roof, showing the roof support arches and ties (McGaughan 1980, 10).

Figure 7.7 A roof support arch bearing on a pier (McGaughan 1980, 10).
7.2 Structural History

The Guastavino vaults of the Army War College have suffered structural damage in the form of cracks, water damage, and fallen tiles, prompting two main periods of investigation and rehabilitation. The first was in the 1940s, when the R. Guastavino Company investigated the vaults they had built 35 years earlier. The second was in the late 1970s, involving multiple firms and costing millions of dollars.

7.2.1 Investigations and Repairs of the 1940s

In 1943, Captain Clifford Colbert, Post Engineer, wrote to the R. Guastavino Company requesting a representative to inspect the “loosening of tile and cracks in the arches” after tiles began falling in one or both of the wings over the third floor gallery.\footnote{Clifford J. Colbert to Malcolm Blodgett, 19 August 1943, Guastavino Archive.} Malcolm Blodgett, President and Treasurer of the company, went down for a preliminary inspection and found cracks in the wing vaults “running from the crown of the rib through the two course vault over the gallery” (Figure 7.9).\footnote{Malcolm Blodgett to Rafael Guastavino, 16 October 1944, Guastavino Archive.} He also mentioned that soffit tiles adjacent to the cracks were loose.\footnote{The tiles of the first layer on the underside of the vault are referred to as soffit tiles.} Blodgett observed that the high vaults are in good condition with no cracks.\footnote{Malcolm Blodgett, “Memorandum – Washington D.C.,” 23 September 1943, Guastavino Archive.} Because the country was involved in World War II at the time, Colbert was concerned with the cost of inspection and repair, especially the costly scaffolding necessary to inspect the higher ceiling...
vaults, and thus decided to forgo inspecting the high vaults.\textsuperscript{55} The R. Guastavino Company signed a contract to inspect the vaults above the gallery for $50.\textsuperscript{56} Blodgett inspected the cracks and recommended sounding the tiles, removing any debonded ones, and then replacing removed tiles once the war was over.\textsuperscript{57}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image1.jpg}
\caption{Location of cracks found in 1943 (author's interpretation).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image2.jpg}
\caption{Close-up of the crack in the crown of a longitudinal rib arch, ca. 1978 (JMC [1979?], figure III.4-7).}
\end{figure}

\begin{flushright}
\footnotesize
\textsuperscript{56} War Department, "Purchase Order," 18 October 1943, Guastavino Archive.
\textsuperscript{57} Malcolm Blodgett to Frank J. Sullivan, 7 August 1944, Guastavino Archive.
\end{flushright}
A new post engineer started in 1944 and was alarmed by the vault cracks and very concerned about the vault's stability. This time, the R. Guastavino Company was hired for $350 to inspect the longitudinal rib arches and provide stress analyses. Blodgett returned to the site for a more thorough inspection of the vaults. In his letter to Rafael Guastavino Jr., Blodgett notes that there are no cracks in the coffered dome but several in the wing vaults, including vertical cracks in *all* the longitudinal rib arches and that in most places these cracks extend across the adjacent vault as described earlier (Figure 7.9). Blodgett exhibits good instinct as far as masonry arch behavior is concerned by proposing that these cracks in the ribs are not due to overloading. He describes other cracks in the high vaults, about a foot away from the longitudinal rib arches, between 8 and 10 feet long. The description corresponds to what are known as Sabouret cracks for masonry groin vaults whose supports have moved apart. Also in the letter, Blodgett observes that the tie rods above the wing vaults originally rested on the vaults but are now located about 1½ inches above the vaults, indicating that the crown of the vault has dropped, probably as a result of support movement.

Irving Berg analyzed the wing vaults with graphical equilibrium analysis and determined that the greatest compressive stress in the vault rib was 388 psi (Figure 7.11). Berg insisted that the vaults were safe and that the cracks were likely due to differential settlement of the supports and not caused by a lack of load capacity.

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58 Malcolm Blodgett to Rafael Guastavino, 16 October 1944, Guastavino Archive.
59 War Department, “Purchase Order,” 5 December 1944, Guastavino Archives.
60 Malcolm Blodgett to Rafael Guastavino, 16 October 1944, Guastavino Archive.
61 Sabouret cracks will be explained in more detail later in this chapter.
63 Irving Berg to Malcolm Blodgett, 14 November 1944, Guastavino Archive.
The R. Guastavino Company produced drawings for proposed repairs based on the inspections and analysis. One detail on a drawing shows reinforcing of the longitudinal rib arches with an additional layer of tile. Another detail shows reinforcement over the cracks in the wing vaults with reinforcing steel embedded within new tile layers. As will be discussed in more detail in a later section, reinforcing of Guastavino vaulting with new layers of tile and reinforcing steel is not necessarily a good idea. It is unknown if any of the repair suggestions were completed, but preparations for repairs were carried out the following year as temporary shoring was erected under the 16 longitudinal rib arches in both wings (Figure 7.12).

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66 Frank J. Sullivan to Malcolm Blodgett, 31 August 1945, Guastavino Archive.
In 1946, the R. Guastavino Company was requested for a more detailed inspection and repair of the vaults. In a letter, the post engineer outlines the prospective work, including:

1 - Investigation and study of the arches and vaults at the site.
2 - Preparation of complete contract drawings and specifications covering necessary repairs.
3 - Estimate of cost of repairs, including complete breakdown of materials and labor.
4 - Construction, at a site in Washington, D.C. [sic] to be selected by the Government, of three (3) full scale arches for testing purposes. The Government will conduct the tests and assume all costs of same. 68

Another letter states that the reinforcing work on the longitudinal rib arches will be completed by others but requests that personnel of the R. Guastavino Company do the vault inspection for the wing vaults and Rotunda. 69 In March of 1946, Blodgett confirms that the Rotunda, East and West Wing vaults and gallery vaults were all inspected from the ground - that is, without scaffolding - and that cracks were mapped (Figure 7.13). 70

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70 Malcolm Blodgett to P.C. Dorr, 7 May 1946, Guastavino Archive.
A specification document from this time period says that "cracked areas shall be reinforced by the addition of one layer of rough tile cemented in place on the upper surface of the arch..." and "cracked tile shall be carefully removed, including adjacent mortar... [and] new finish soffit tile... shall be carefully cemented in place on a full mortar bed." In July of 1946,

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72 R. Guastavino Co., 29 August 1946, Guastavino Archives.
five hundred 6 inch by 12 inch “regular buff ceramic unglazed corrugated tile” were sent to the Army War College.\footnote{Factory order card, 31 July 1946, Guastavino Archive.} These tiles were likely used to replace soffit tiles where indicated in Figure 7.13. An estimate sheet from April 1946 calls for about 950 rough tiles to reinforce cracks on the top side of the vaults, but there are no order cards for these tiles.\footnote{R. Guastavino Co., 25 April 1946, Guastavino Archive.}

Some recommended repairs appear to have been carried out and some not. Reinforcing of the cracks parallel to the longitudinal rib arches (the earlier mentioned Sabouret cracks) was supposedly completed in 1946 (JMC [1979?], section III.4.5), but it is unknown whether the steel reinforcing bars recommended in the drawings were included.\footnote{John J. Reilly to J. Van Fossen, 27 March 1980, Guastavino Archive.} The steel bars were probably not added because they are not stipulated in the specifications.\footnote{U.S. Engineer Office, “Specifications for Reinforcing and Replacing Soffit Tile in Ceilings: Part IV; Technical Provisions,” 24 June 1946, Guastavino Archive.} Another recommendation that called for reinforcing over the backs of the vaults diagonally from pier to pier was not completed (JMC [1979?], Section III.4.5). Reinforcing of the longitudinal rib arches with heavy concrete ribs was completed by another company sometime after 1946, as evidenced by photos from a Historic American Building Survey in 1974 (Figure 7.15).

\begin{figure}[h]
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\includegraphics[width=\textwidth]{figure715.jpg}
\end{figure}

7.2.2 Investigation and Repairs of the 1970s and 1980s

Several investigations were made of the Guastavino vaulting at the Army War College in the 1970s and 1980s, involving various people and organizations who did not always agree with...
the opinions and recommendations of one another. Although there were repairs to other parts of the building during this time period, the materials presented in this section focus on the Rotunda dome and the wing vaults.

*Universal Restoration Incorporated Report*

In 1974, emergency investigation and restoration was ordered on the Guastavino vaulting of the Army War College after two tiles fell from the Rotunda dome, and Universal Restoration Incorporated was hired as the principal contractor for the work under an open-ended contract. They amended their contract seven times to repair defects as they found them and, as a result, went nearly $1.9 million over budget. This was highlighted by an article title in the Washington Post reading “Hill Hits Ceiling as Army Repair Bill Soars 3,000%.” According to Universal Restoration Incorporated, their work was to focus on the compression ring and surrounding decorative tile of the Rotunda dome, but once they started inspection on loose tiles near the compression ring, they found more than the expected amount of loose tiles as well as large cracks (BCT [1976?]). This set them off on a series of investigations that included not only sounding of tiles, but investigations into settlement and vibration effects as well as structural analysis.

The report labels two of the large cracks found as “compression ring cracks,” likely because they were found at the compression ring (BCT [1976?], 5-16). These two cracks run in the east-west direction, are between 3 and 5 feet long, and extend through the full depth of the compression ring, which is 11 layers of tile thick. The authors characterize these cracks as showing evidence of shearing, which is unlikely. Another large crack in the dome runs horizontally part-way down the dome. The authors report evidence of water damage to the Rotunda dome and postulate that this contributed to loose tiles, specifically those of the soffit layer because they were constructed with Plaster of Paris. After sounding the Rotunda dome and galleries, the firm came up with the loose tile statistics listed in Table 7.1.

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80 Ibid.
81 Ibid.
82 Although the organization listed on the cover of the report is Building Conservation Technology Inc., it seems that the report comes from Universal Restoration Incorporated (Link 1995).
<table>
<thead>
<tr>
<th></th>
<th>Rotunda Dome</th>
<th>Galleries</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of tiles removed</td>
<td>178</td>
<td>3</td>
</tr>
<tr>
<td>No. of loose tiles needing removal</td>
<td>124</td>
<td>74</td>
</tr>
<tr>
<td>No. of suspect tiles</td>
<td>2032</td>
<td>3666</td>
</tr>
</tbody>
</table>

Table 7.1 Results of sounding tests by Universal Restoration Incorporated (BCT [1976?], 13-14).

James Madison Cutts Report

In their report from 1979, the engineering firm of James Madison Cutts (JMC), particularly engineer John Joseph Reilly, investigate the behavior of the Guastavino vaults and explain the cracks in the Rotunda dome and wing vaults using finite element analysis, though they acknowledge the inability of finite element programs to analyze cracked shells (JMC [1979?]). NASTRAN was used to model the Rotunda dome and the east and west wing vaults.

The JMC report ([1979?], section III.3) mentions the same three East-West cracks in the Rotunda dome as the Universal Restoration Incorporated report and an additional “shear crack” along a groin. The report notes that the Rotunda dome appears symmetric about North-South and East-West axes, but the finite element analysis of the dome revealed a lack of thrust support on the South side, resulting in tensile stresses in the East-West direction that corresponding to the three main cracks observed. The shear crack, which is probably not actually the result of shear stresses, is not explained by the model or otherwise. Additionally, the NASTRAN model shows highly stressed regions in the Rotunda dome where beams supporting the upper double dome bear on the vaulting. There are many layers of built up tile under the beam, probably similar to those under the roof rafters at the Saint Louis Art Museum (Figure 7.16). During inspection, no cracks or loose tiles were found on the Rotunda dome under these supports, conflicting with the results of the NASTRAN analysis and demonstrating an instance where finite element analysis is unable to predict the real behavior of Guastavino shells. The fact that the structure did not show any signs of undue stress at these locations demonstrates that masonry tile shells can carry concentrated loads sufficiently with membrane action due to very high stiffness within the surface of the shell.
In addition to the Rotunda dome, the west wing vaults were analyzed with NASTRAN in the JMC report ([1979?], section III.4). The vaults were modeled in conjunction with the roof structure, and subjected to dead and live load, temperature load, and lateral support movement in several combinations. The resulting stress plots show areas of tension that the authors compare to cracks in the shell. The first set of cracks mentioned are the ones documented in the 1940s – the cracks in the longitudinal rib arches that extend through the side arches – said to be the result of thrust from the roof support arches and tensile stresses caused by a “divergence of load,” that is, the change in direction of the load path. In fact, the thrust of the roof support arches likely influenced the formation of the cracks, but a change in the direction of the load path will not induce tensile stresses. Another crack cited is a longitudinal crack along the crown of the vault, which is attributed to the inability of a parabolic load path to fit within a semi-circular vault, thus inducing bending. Similarly, there are cracks at the quarter points on the top sides of the shell, explained in the same manner. These explanations indicate some familiarity with funicular forms, but fall short of recognizing the capability of shell structures to carry forces in three-dimensions. Furthermore, the reported vault cracks exactly correspond to the description of crack formation in a groin vault on spreading supports, as will be explained later in this chapter. Finally, high bending stresses in the finite element model of the vault are found around piers and stiffeners that are disputed; the authors explain them as an artifact of the finite element analysis and not representative of stresses in the actual structure.
The report makes recommendations for the repair of the wing vaults to contain the lateral thrust from the roof support arches and "strengthen" the cracked areas of the vault (JMC [1979?], section III.4.5). The first recommendation is that lateral thrust be controlled with a complicated system of tie rods - a completely unnecessary measure if the vaults had cracked and settled long ago and there is no continued movement. Similarly, tie rods are recommended at the bases of the longitudinal rib arches running the length of the wings to tie back the exterior walls that had leaned out slightly, which are also superfluous if the leaning of the walls has ceased. Other recommendations include completion of some repairs proposed by the Guastavinos in the 1940s that were never carried out, such as installing extra layers of tile with embedded reinforcing bars over cracks and reinforcing on the backs of the vaults diagonally from pier to pier with extra tile. These are poor recommendations since the cracks in the vault allow the structure some movement, such as that due to temperature fluctuations, and reinforcing those cracks might cause new ones to open in undamaged sections of the vault.

Response to the James Madison Cutts Report

The engineering report by James Madison Cutts was reviewed by Anton Tedesko, a leading designer of thin concrete shells in the United States (Billington 1982).83 Tedesko raises concern about relying on the results of the NASTRAN analysis to understand how the Guastavino structures behave. One reason is that since the finite element analysis cannot model the cracks, the analyses represent how the Rotunda dome and wing vaults behaved upon completion in 1906 and "does not consider the stress adjustments in the structure within past decades." The JMC report ([1979?], section III.4.4) does in fact acknowledge this by recognizing that their analysis was only valid to the point of cracking. Another point of Tedesko's is that the JMC report says bending is induced in the wing vaults because a parabolic pressure lines deviates from their spherical shape, but this assumption is for arches and not 3-dimensional shell structures which carry forces through membrane action. Tedesko also opposes some of the repair suggestions, such as the proposed ribs diagonally from pier to pier, because they will only add weight as the structure has already established load paths. Similarly, he comments that cracked alcove vaults do not necessarily pose a risk and that "they will attract smaller forces." Tedesko is hesitant to consent to the installation of tension ties because they cannot prevent the movement

83 Anton Tedesko to Charles Shores, 18 August 1978, Guastavino Archive.
that leads to falling tiles and he notes that the high prestressing force (60 kips) proposed for the
ties could in fact damage the building. Tedesko rounds out his comments by recommending that
the vaults be left alone as much as possible, citing that Europeans do not frequently retrofit their
buildings with tension ties and the like. As a leading designer and builder of thin shell structures,
Tedesko’s expert opinion is important and he offers lucid advice against the confused reports of
the engineers.

Investigation of the East Wing Vaults

McGaughan & Johnson and Weidlinger Associates produced a report in 1980 focusing on
recommendations for the east wing of the Army War College. They report that soffit tiles are
missing from the vaults in a number of places, some having fallen and others removed to replace
tiles in the Rotunda or in the west wing. The authors speculate that the vaults were not
constructed properly and that this lack of cohesion was a result of poor craftsmanship and delays
in tile delivery, a claim Reilly rebukes, speculating that the poor adhesion of the cement was
likely due to its quality and not delivery delays. 84 The report’s recommendations for repair are to
remove all face tiles from the vault and replace them with new tiles, and estimates that this will
require about four thousand special order tiles. To remove all face tiles, including ones that are
not at risk of falling, is an especially drastic and inappropriate rehabilitation decision. In a later
correspondence, Johnson states “everyone we have questioned regarding the recent restoration
work in the [west wing] and Rotunda [has said] that all of the soffit tile were removed and
replaced with epoxy mortar” (emphasis added). 85 It is likely that this precedent lead them to
recommend the same treatment for the east wing. In her thesis on Guastavino history and
restoration, Link (1995) mentions that the east wing was not open to visitors in late 1993, which
might indicate that the restoration of the east wing was never carried out.

7.3 Discussion of Investigations and Repairs

Damage to the Guastavino vaulting at the Army War College has been characterized as
extreme. While the masonry tile shells there seem to have suffered their share of debonded tiles,
loose tiles and leaks appear to be the building’s primary problems. The response to cracks in the

84 John J. Reilly to J. Van Fossen, 27 March 1980, Guastavino Archive.
85 Hugh B. Johnson to John Van Fossen, 5 June 1980, Guastavino Archive.
Guastavino structures was frequently an overreaction, sometimes resulting from not recognizing documented patterns of cracks in vaults and arches.

7.3.1 The 1940s

The cracking patterns documented in the 1940s illustrate fairly typical behavior as far as unreinforced masonry is concerned. Minor support movements can cause cracks, and in many cases a cracked vault is perfectly safe. However, Guastavino vaulting has one very particular characteristic: where there are cracks, adjacent tiles may loosen and fall. Falling tiles are of great concern because of the danger they pose to building occupants, but they are not necessarily a sign that the vault is in danger of collapsing.

Cracks in the longitudinal rib arches (Figure 7.10) were first noticed in 1943 because tiles were falling adjacent to the cracks – the post engineer even mentions that no one noticed the cracks until the tiles started to fall. This intrados crack at the crown is typical of arches since a hinge often forms there as the supports move apart slightly, corresponding to a state of minimum thrust in the arch (Heyman 1995, 15-20). For a masonry arch, a three-hinge arch is stable, so if the arch had settled into this form and the supports were no longer displacing, the crack was not a threat to the stability of the building. Blodgett mentioned that the original post engineer “did not appear to be disturbed about the appearance of any cracks.” Unfortunately for the Army War College, the next post engineer was seriously concerned with these cracks, and as a result the longitudinal rib arches were retrofitted with reinforced concrete arches sometime after 1946 (Figure 7.15). The addition of concrete increased the strength of the arches, but this additional strength is only necessary for the unlikely event that the arches become overloaded. However, this reinforcing did not improve the stability of the arches and did not address the cause of cracking. Also, the formwork placed under the arches (Figure 7.12) could have caused damage if it was exerting considerable force because arches work in compression and a force on the underside introduces tension into the arch. Pushing upwards on compression structures such as Guastavino vaulting is a practice that should be avoided.

Another set of cracks discovered in the 1940s were the cracks in the high vaults of the east and west wings that ran parallel to the length of the vaults. This type of crack is typical in quadripartite vaults and is known as a Sabouret crack. These cracks indicate that the vault has

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86 Clifford J. Colbert to Malcolm Blodgett, 19 August 1943, Guastavino Archive.
87 Malcolm Blodgett to Rafael Guastavino, 23 September 1943, Guastavino Archive.
separated from the wall, and that the vaulting action is now taking place in the direction parallel
to the crack (Heyman 1995, 66-71). Sabouret’s cracks are discussed in more detail later in this
chapter.

7.3.2 The 1970s and 1980s

The repairs in the 1970s and 1980s are characterized by excessive costs and drastic
measures. Expensive finite element models were used to investigate the behavior of the
Guastavino structures. Tiles showing no evidence of debonding were removed and regrount, a
very drastic measure that could have potentially disturbed the integrity of the structures.

Universal Restoration Incorporated’s Washington DC office, who carried out the initial
investigation for the building, was closed soon after their costly examination was revealed during
a hearing of the House Government Operations Subcommittee, so while not confirmed, it is
possible that their closing was a result of overspending on this job.\(^{88}\) Their approach was overly
cautious; the vibrations analysis was extraneous and their concern with over 5000 “suspect tiles”
rather than just the obviously debonded ones indicates that they did not understand the behavior
of Guastavino shells (BCT [1976?]). In fact, they say that loose and fallen tiles lead to a
“redistribution of cohesive forces,” indicating that they took Rafael Guastavino Sr.’s explanation
of masonry tile structure behavior as fact and overreacted as a result when they thought the all-
important cohesive forces were failing. Certainly suspect tiles – a term they never define –
should be monitored periodically to see if they are becoming loose, but it is unnecessary and
irresponsible to disturb adequately bonded tiles of a masonry tile vault or dome.

The 1979 James Madison Cutts report uses finite element analysis to explain cracks in the
vault, which seems to be the best use for linear-elastic finite element analysis of Guastavino
shells. The report recognizes that this analysis models an uncracked condition, and Tedesko
reiterates this point by saying that the model represents the vault only as it stood immediately
after completion. Their models serve the purpose of verifying the boundary conditions they
assumed caused cracks in the first place, and they wisely took steps to monitor either cracks or
support movements to identify changing boundary conditions that might cause instability in the
structure.

\(^{88}\) Joseph D. Whitaker, “Hill Hits Ceiling as Army Repair Bill Soars 3,000%,” The Washington Post, 11 December
1975.
In the discussion of cracks in the wing vaults, the JMC report suggests that the longitudinal cracks at the crown and quarter points of the vault are the result of bending induced by a parabolic load path being forced into a circular shape. In actuality, these cracks match the description of a quadripartite vault on spreading supports as described by Heyman (1983). If the supports of a cylindrical vault give way, the first crack will form at the crown of the vault and will be visible from underneath. But a brittle structure like a masonry vault cannot form just one crack, it must form three to create a kinematically admissible mechanism and thus becomes a three-hinge arch. In order to accommodate this new geometry, the crown of the vault will descend relative to its initial position, and this deflection was observed by Blodget. This hinging and dropping essentially lengthens the arch, as illustrated on the left (South) side of (d) in Figure 7.17. The masonry next to this large crack is now under a lot of strain, which can be greatly relieved if another crack opens to accommodate this extra distance as well. This new crack is illustrated on the right side of (d) and referred to as a Sabouret crack. The resulting sliver between the wall crack and Sabouret crack is separated from the rest of the vault and essentially behaves as an arch. Heyman points out that while the crack at the top (the hinge) can transfer loads perpendicular to its own direction, the nature of the Sabouret crack is that it is a total separation of the vault and thus no loads can transfer across it, that is, perpendicular to the crack.

In reference to the hinge crack, Sabouret crack, and wall crack, Heyman (1983, 132) writes:

*All three of these types of cracks may have existed for many years, or for centuries, in the vaults of a particular church. They do not, in themselves, indicate that the vaults are in a dangerous state; rather, they are all related, and arise from a simple pattern of movement that has occurred in the past and is not necessarily continuing.*

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89 Malcolm Blodgett to Rafael Guastavino, 16 October 1944, Guastavino Archive.
7.4 Chapter Summary

The repairs to the Guastavino shells and arches of the Army War College could be considered even more extreme than the gunite retrofit of the barrel vault of the Saint Louis Art Museum given the money spent on their inspection, investigation, analysis, and repairs. Some of the cracking patterns—such as the intrados cracks at the crown of the longitudinal rib arches and the Sabouret cracks in the wing vaults—are typical for masonry, but went unrecognized by several different investigations. Some investigators fell victim to the misguided notions of Rafael Guastavino Sr. The repair suggestions—even those suggested by the R. Guastavino Company—were wrongly focused on “fixing” cracks and “strengthening” the structures with repairs that could do more harm than good. Cracks do not necessarily endanger the safety of the vault, but may become a threat to persons passing beneath if debonded tiles adjacent to the cracks fall. Therefore, the inspections should have focused on identifying loose tiles in the vicinity of cracks and water damage and monitoring supports to ensure that there is not further movement.
Chapter 8. Conclusions

This thesis has provided insight into the behavior of Guastavino vaults and domes in order to help practicing architects and engineers faced with evaluation and rehabilitation of these important historical structures. By presenting structural analysis methods, typical pathologies, and past restoration techniques, it is possible to make recommendations for the safety assessment and rehabilitation of Guastavino shells to maintain them for generations to come.

8.1 Summary

Two generic Guastavino geometries – a dome and a barrel vault – were analyzed with different methods to establish their safety. Graphical analysis, membrane analysis and finite element analysis were used to analyze the dome and the three methods gave consistent results for the stresses in the dome and its thrust, but only graphical and membrane analysis provided a straightforward means to find the force in the tension tie. Graphical and finite element analyses were used to analyze the barrel vault, again yielding similar results for the stresses in the vault and the thrust. For both geometries, the calculated stresses were significantly lower than the strength of material, indicating that the strength of masonry tile structures is unlikely to limit their safety as opposed to stability. Furthermore, this brings into question the point of calculating stresses in the first place. In order for an analysis method to be useful for assessing the safety of a Guastavino structure, it should reveal something about its stability. That stability is based on the horizontal thrust, the load path, and in the case of domes, the tension tie force, and graphical equilibrium analysis is an effective tool to determine all these attributes.

Grace Universalist Church was a case study demonstrating the safety of an actual Guastavino dome. Its assessment is different from that of the generic structures because the structural history of the dome and more complex geometry must be considered as well. As an early project of the Guastavinos that has survived extreme weather with no real signs of distress, its longevity should in itself reassure one of its safety. Regardless, a graphical equilibrium analysis was carried out to verify this. The stresses within the dome thickness are found to be relatively small, so the strength of the material is not an issue for the dome. By calculating the size of the tension tie and establishing that it is reasonable, one should be confident of the dome’s stability. The last stage of assuring the safety of the dome is to consider the frequent incidents of leakage – as long as there are no signs of water damage in the vicinity of the tension tie, the dome can be considered sound.
The Guastavino barrel vault of the Saint Louis Art Museum was analyzed at three different stages of life – before cracking, after cracking, and post-retrofit. The three-dimensional finite element model of the barrel vault allowed investigation of the effect of differential foundation settlement on the vault and provided some evidence that this settlement was likely the cause of the cracks. While typical differential settlement can cause cracking, it is usually no threat to the long-term stability of a Guastavino structure. Once a shell is cracked, a three-dimensional linear elastic finite element model is of limited use. For the cracked vault, graphical analysis is useful to demonstrate that the stresses in the vault are low and also to establish the vault’s stability by finding a thrust line that fits within the material. The Rankine Safety Factor was used to quantify the safety of the vault and shows that the clerestory wall is integral to its stability. Employing graphical analysis again to analyze the vault after the gunite retrofit allows one to weigh its intended use (to improve the safety of the vault) versus its real outcomes (no real improvement to safety since it increases thrust, lowers the safety factor, and detracts from the architecture of the museum). The heavy-handed, irreversible gunite retrofit was inappropriate for the vault from both a preservation and structural standpoint. Additionally, the Saint Louis Art Museum chapter introduces one of the most critical pathologies of masonry tile structures: while cracks do not usually detract from the overall safety of the vault, they can instigate dangerous falling tiles.

A study of past investigations and repairs of the Guastavino vaulting at the Army War College provided insight into the behavior of masonry tile structures and review of assessment techniques, but also serves as a cautionary tale against uneducated assessments. The over-reaction to cracks in the arches and shells, the blind faith in Rafael Guastavino Sr.’s description of masonry tile shell behavior, and the reliance on linear elastic finite element analysis all contributed to expensive and inelegant repair solutions. The thick, concrete arches (Figure 7.15) detract enormously from the graceful curves of the tiled vaults, and is yet another testament to the need for educated engineers who can provide sensitive and appropriate rehabilitations.

8.2 Recommendations

Even with their extreme thinness, stresses within Guastavino shells under typical load cases are still well below the compressive strength determined by Rafael Guastavino Sr. in 1893. Thus, for most Guastavino structures, the important parts of a safety assessment are investigation into a structure’s stability and identification of threats to that stability. One can accomplish this
by understanding the behavior of masonry tile structures, employing structural analyses when useful and necessary, knowing the structural history of the building, and monitoring suspicious cracks or support movements. A short guide to the assessment of Guastavino masonry tile structures is given in Appendix E, and recommendations based on the material in this thesis are listed in the following paragraphs.

Based on the analyses presented in this thesis, graphical equilibrium analysis seems the best suited for investigating the stability of a structure. In addition to determining stresses in a dome or vault, graphical analysis can determine the magnitude of thrust, an important quantity for either structure type as far as stability is concerned. Additionally, graphical analysis can determine the required tension tie force to contain the thrust of a dome. For vaults, graphical analysis illustrates the load path through a structure, which is essential for understanding the behavior of a vault.

Finite element analysis, when implemented correctly, will give stresses in a dome or vault similar to the results of a graphical equilibrium analysis and predict a comparable value of thrust. However, finite element analyses of both dome and vault exhibited some inconsistencies with the results of the graphical analysis. The assumed boundary conditions for the dome and resulting deflected shape influenced the hoop stresses near the base of the dome, resulting in discrepancies with the results of the graphical and membrane analyses. The elastic deflected shape of the vault finite element model caused large stress variations through the thickness of the vault that for certain geometries could predict high tensile and compressive stresses at the faces. In both cases, an analyst should be aware of these idiosyncrasies and be critical of finite element analyses that show high stresses in areas of an actual structure that are perfectly intact.

Furthermore, deflections predicted by the finite element analyses are very small compared to deflections experienced by actual structures due to support movements and construction defects, and a finite element model subjected to real deflections might find drastically different stresses in the structure. These issues present a compelling reason to avoid the use of finite element software for the analysis of Guastavino shells. However, if an analyst must conduct a finite element analysis of a shell (perhaps for the execution of a nonlinear finite element analysis), it is essential that he or she make certain that their model gives results close to that of graphical or membrane analysis.
One aspect of finite element analysis that does help in determining the stability of a structure is its ability to investigate support movements or changes in boundary conditions that have caused cracks in Guastavino structures. If an analyst can show evidence that cracks in a vault are the result of foundation settlement or support movements, he or she will be able to monitor the underlying cause of the cracks to ensure that settlement or movements have ceased, or else devise a plan for stabilizing the structure against such movements.

Huerta (2003, 126) has stated definitively that Guastavino vaulting behaves the same way as other masonry structures and does not have special "cohesive" behavior that gives a structure tensile capacity as asserted by Rafael Guastavino Sr. The case studies presented here serve to support Huerta's claim, but highlight a crucial difference specific to Guastavino vaulting: the threat of falling tiles. An innocuous intrados crack at the crown of a voussoir masonry arch poses no serious threat, but that same crack in a Guastavino arch may loosen nearby tiles that might fall if not properly dealt with. Along those same lines, soffit tiles in regions of water damage may be suspect as well, and such areas might be identifiable by efflorescence on the underside of a vault. As falling tiles are a very serious matter, regular sounding of soffit tiles is essential to ensuring the safety of persons passing beneath Guastavino vaulting, and inspections should focus considerable attention in regions of cracking or water damage. All structures, such as timber covered bridges and building facades, require regular inspection and maintenance, and Guastavino masonry tile structures are no exception.

Finally, there are no known instances of Guastavino vaults ever collapsing, and while this does not mean that they are invincible, it does speak to their sturdiness (Ochsendorf, forthcoming). They are robust and do not require the heavy-handed repairs often given to them. This point is best illustrated by the graphical analysis of the Saint Louis Art Museum, which demonstrated that the vault was stable before the addition of gunite and that this retrofit was superfluous. The museum forever lost a great architectural treasure when it covered its Guastavino tile vault and gained little structural benefit. Guastavino vaults are worth saving, and sensitive retrofits will ensure that these spaces endure for people to enjoy for decades to come.

In closing, it should be emphasized that any potential rehabilitation solutions to Guastavino shells may do more harm than good. All recommendations should be carefully considered. In a correspondence regarding the investigation and suggested repairs of the Army War College, Anton Tedesko perhaps put it most eloquently:
I am greatly impressed by what was done 75 years ago in the construction of this building, by the judgement [sic] used, and by the skill of the masons of that time... It is from my observation, my studies and my experience with other structures that as a basic philosophy I would propose to leave the structure alone as much as possible. Remedies which might be proposed with good intentions could turn out to be more harmful than helpful. They may lead to an increase of loads in some parts of the structure and to secondary moments where forces seem fairly in balance now. I am reminded of monumental buildings in Europe, much older than the War College Building, where shells have deflected considerably, where columns have shown several inches of translation and where authorities so far have not resorted to the use of tension ties or of other stabilizing devices. Localized heavy cracks and holes in similar war-damaged European buildings did not appreciably affect the carrying capacity of these structures.  

8.3 Future Research

The research presented in this thesis could be expanded upon in several ways. The equilibrium analyses were limited to lower bound solutions, so one proposal for future research would be to find upper bound solutions, that is, the conditions leading to the collapse of a structure. Also, the load cases considered in this thesis were rather limited, and investigating others, such as asymmetrical live loads, would be of value. The effect of seismic activity on Guastavino shells was not considered in this thesis, and would be of use to those structures located in seismic regions, such as the Saint Louis Art Museum.

The findings of this thesis show that an equilibrium method such as graphical analysis is overall the best-suited for assessment of Guastavino shells, but the graphical analyses presented in this thesis were limited to the three-dimensional analysis of a simple spherical dome and two-dimensional analysis of a three-dimensional barrel vault. A method of three-dimensional equilibrium analysis, such as thrust network analysis (Block and Ochsendorf 2007), would perhaps give a better understanding of the behavior of a vault like that at the Saint Louis Art Museum. Also, the basic geometries considered here – the dome and the barrel vault – are only two of the wide variety of forms created by the R. Guastavino Company, and a three-dimensional equilibrium analysis method could explore the behavior of more complex shapes.

Finally, countless studies could be conducted on specifics of Guastavino structures and the very company that built them. The exact location and size of tension ties within Guastavino

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90 Anton Tedesko to Charles E. Shores, 13 August 1978, Guastavino Archive.
domes could be explored, or the methods used to determine the spacing of diaphragm walls and buttresses. The case studies of the Saint Louis Art Museum and Army War College present examples of structural repairs proposed by the R. Guastavino Company, and similar studies could be carried out on other structures to further examine how they dealt with repairs of their own work. Given that the company was active for almost a century and that structural engineering was evolving as a field by leaps and bounds during this same time, establishing absolutes for the practices of the company or their structures might be impossible. But it is that same aspect that contributes to the lure of the R. Guastavino Company and that makes the Guastavinos and their masonry tile structures such a fascinating topic of study.
References


AISC. See American Institute of Steel Construction.


BCT. See Building Conservation Technology Inc.


Guastavino Fireproof Construction Company/George Collins Archive, Drawings and Archives Department, Avery Architectural and Fine Arts Library, Columbia University.


JMC. See James Madison Cutts, Consulting Engineers.


Unitarian Universalist Congregational Records, Manuscripts and Archives Department, Andover-Harvard Theological Library, Harvard University.


Appendix A. Derivation of the Point of Zero Hoop Stress for a Dome

The derivation of the point of zero hoop stress for a dome is as follows. It starts by setting the closed-form equation (Billington 1965, 41) for hoop stress to zero and simplifying the equation.

\[ N'_{\theta} = aq \left( \frac{1}{1 + \cos \varphi} - \cos \varphi \right) \]

\[ 0 = aq \left( \frac{1}{1 + \cos \varphi} - \cos \varphi \right) \]

\[ \frac{1}{1 + \cos \varphi} = \cos \varphi \]

\[ 1 = \cos \varphi + \cos^2 \varphi \]

Plugging in \( \varphi = 51.8^\circ \) yields approximately a value of 1 on the right hand side of the equation.

\[ 1 \approx \cos 51.8^\circ + \cos^2 51.8^\circ \]
Appendix B. A Step-by-Step Explanation of the Graphical Analysis of a Dome

Graphical analysis of the generic Guastavino dome presented in Chapter 3 using Wolfe’s method for a dome with tensile capacity is explained here in greater detail. The figures referred to are given on the pages following the explanation. Each figure depicts a cross-section of the lune on the left side of the figure and the force polygon on the right side. Each line of the force polygon represents an internal or external force acting on or within the lune. The orientation of each line in the force polygon is the same as that of the force on or within the lune. The length of each line in the force polygon corresponds to the magnitude of each force.

- Figure (a). Apply the ten lune segment weights as point loads to the lune cross-section. Draw ten vertical line segments to a scale representing each load, and connect them end to end. This line is called the “load line,” and it represents the start of the force polygon. Draw a horizontal line at the top of the load line, extending to the right. This line will be used to represent the horizontal forces in the system, and its length will be determined later. This horizontal line corresponds to the first segment of the “thrust line” at the crown of the lune cross-section. The thrust line represents one set of internal forces within the lune – the meridional forces.

- Figure (b). The next step is to approximate the thrust line within the dome using the assumption from membrane theory that the forces are restrained to the mid-surface layer of the dome thickness. Draw a line connecting the centers of the top two segments of the lune. Maintaining the orientation of this line, copy it to the force polygon so that it starts between the load line segments of the top two segments. End the line wherever it intersects with the horizontal line. The length of this line in the force polygon approximates the meridional force in that region of the lune. Drawing this first meridional force also determines the magnitude of the horizontal force acting at the crown of the lune.

- Figure (c). Repeat the last step, drawing the next segment of the thrust line to determine the next meridional force.

- Figure (d). Repeat eight more times. The inclusion of tensile capacity allows the thrust line to “turn downwards” and follow the non-funicular form of the hemisphere.\(^91\) The last

\(^91\) A funicular form is a pure tension or pure compression solution for a specific set of loads, which can be conceived of as the shape that a hanging chain would take under that set of loads (Heyman 1995, 7).
segment of the thrust line is parallel to the reaction at the base of the lune, so this reaction is of the same magnitude as the last segment of the force polygon. The length of the horizontal line drawn in Figure (c) is determined by its furthest right intersection with a meridional force on the force polygon.

- Figure (e). A plan-view of the lune has been added to the remaining figures above the lune cross-section. Two forces are drawn perpendicular to the edge of the lune pointing towards the center of the first lune segment. Since these forces are pushing on the lune, they are compressive forces. Maintaining the orientation of these lines, copy them to the force polygon as shown. These segments will represent the other set of internal forces for the lune – the hoop forces.

- Figure (f). Repeat the last step to find the next hoop force.

- Figure (g). Repeat five more times to find the magnitudes of the compressive hoop forces. This brings one to the right end of the horizontal line. The same process will be repeated to find the tensile hoop forces, now working from right to left below the horizontal line.

- Note that one lune segment does not have a hoop force. This is because the two points that would be used to determine that hoop force on the force polygon are very close to each other, thus the hoop force lines would be so short to be practically zero.

- Figure (h). Using the same methodology as for the hoop forces, draw two forces at the base of the lune plan view perpendicular to the edges of the lune. Maintaining the orientation of these lines, “close” the force polygon by connecting one line to the end of the last tensile hoop force line and the other to the left end of the horizontal line. These force represent the required force in the tension tie to carry the thrust of the dome.
Appendix C. Graphical Analysis for a Guastavino Dome without Tensile Capacity

Guastavino domes were built with steel reinforcing or buttressing that provides support for a dome in the region of tensile hoop forces (\( \varphi > 51.8^\circ \)). If steel tensile reinforcing within the dome material has rusted or if the buttressing is insufficient, it is possible to analyze the dome using the second graphical method for a dome without tensile capacity introduced by Wolfe (1921, 250-53) to evaluate its stability. The methodology is essentially the same as that described in Appendix B, except that there are no tensile hoop forces to restrain the thrust line to the centerline of the dome thickness for \( \varphi > 51.8^\circ \). In Figure 3.5, the right half of Wolfe’s Fig. 500 shows that the thrust line for the dome without tensile capacity does not have the same spherical shape below line \( Y_1-Y_1 \) as the thrust line on the left half of Fig. 500.

The generic dome from Chapter 3 is analyzed with this alternative method. The analysis begins by constructing the load line and drawing the horizontal line in the exactly the same way as was done in Figure (a) of Appendix B. Next, the thrust line is drawn within the dome thickness in the region of \( 0^\circ < \varphi < 51.8^\circ \) following the methodology illustrated in Figures (b) through (d) of Appendix B, and these lines are transferred to the force polygon as before. The thrust line and force polygon at this stage is given in Figure C.1.

![Figure C.1 Thrust line (left) and force polygon for dome in the region of \( 0^\circ < \varphi < 51.8^\circ \).](image)

Constructing the rest of the thrust line deviates from the method in Appendix B at this point. To complete the thrust line, line segments are drawn first on the force polygon, starting at

\[ \text{Scale: } 1 \text{kip} \]
the point labeled ‘o’ in Figure C.1 and ending between the load line segments. While maintaining the same orientation, these line segments are copied to the thrust line. The hoop forces are then determined in the region of $0^\circ < \varphi < 51.8^\circ$ in the same manner as described by Figures (e) through (g) of Appendix B. There are no hoop forces in the part of the dome where $\varphi > 51.8^\circ$.

The force polygon and thrust line generated by the analysis are given in Figure C.2 and Figure C.3, respectively. Comparison of Figure 3.11 to Figure C.2 (b) shows an increase of 24% in the thrust of the dome when tensile capacity is no longer present. Additionally, Figure C.4 compares the thrust line at the base of the dome given in Figure C.3 to that from the generic Guastavino dome with tensile capacity (Figure 3.9) and shows that the analysis presented here results in a thrust line exiting the dome thickness. This means that either tensile stresses would be induced in the dome material, or for materials without tensile capacity, that the dome would develop cracks.

Figure C.2 Force polygon for the generic Guastavino dome without tensile capacity labeled (a) with select magnitudes and (b) with the magnitude of thrust.
Figure C.3 The thrust line generated by Wolfe’s method of graphical analysis for a dome without tensile capacity.

Figure C.4 Comparison of thrust lines at the base of the dome for a dome without tensile capacity and with tensile capacity (in grey).
Appendix D. Load Calculations for the Saint Louis Art Museum

Saint Louis Art Museum - Thrust Line Analysis without Gunite

\[ P_{tile} = 150 \text{ pcf} \]
\[ P_{timmer} = 60 \text{ pcf} \]
\[ P_{masonry} = 150 \text{ pcf} \]

**DEAD LOAD (vault)**

dead load calculation is based on the weight of the vault at a typical cross section for the entire vault

\[
\begin{align*}
\text{radius} &= 27 \text{ ft} \\
\text{1/2 circumference} &= 84.82 \text{ ft} \\
\text{width} &= 4.5 \text{ ft} \\
\text{thickness} &= 0.3333333 \text{ ft} \quad \text{estimate: 3 tiles thick} \\
DL_{V1} &= 19.09 \text{ kips} \\
1/2 DL_{V1} &= 9.543 \text{ kips} \quad \text{thrust line analysis is for one half of vault} \\
\text{width} &= 2 \text{ ft} \quad \text{width of extra tile layers under roof rafters} \\
\text{thickness} &= 0.1666667 \text{ ft} \quad \text{estimate: 2 tiles thick} \\
DL_{V2} &= 4.24 \text{ kips} \\
1/2 DL_{V2} &= 2.121 \text{ kips} \\
1/2 DL_{V} &= 11.663 \text{ kips} \\
DL_{V \text{ per section}} &= 0.778 \text{ kips} \quad \text{dead load of vault for each of 15 sections}
\end{align*}
\]

**DEAD LOAD (diaphragm walls)**

\[
\begin{align*}
\text{area}_{10} &= 1.28 \text{ ft}^2 \quad \text{area of diaphragm wall above point on vault} \\
\text{area}_{11} &= 4.24 \text{ ft}^2 \\
\text{area}_{12} &= 6.01 \text{ ft}^2 \\
\text{area}_{13} &= 6.34 \text{ ft}^2 \\
\text{area}_{14} &= 5.07 \text{ ft}^2 \\
\text{area}_{15} &= 2.12 \text{ ft}^2 \\
\text{thickness} &= 0.167 \text{ ft} \quad \text{diaphragm wall constructed of two 1" thick tiles, 1" apart} \\
DL_{Dw10} &= 0.0320 \text{ kips} \quad \text{additional DL on vault section 10} \\
DL_{Dw11} &= 0.106 \text{ kips} \quad \text{additional DL on vault section 11} \\
DL_{Dw12} &= 0.150 \text{ kips} \quad \text{additional DL on vault section 12} \\
DL_{Dw13} &= 0.158 \text{ kips} \quad \text{additional DL on vault section 13} \\
DL_{Dw14} &= 0.127 \text{ kips} \quad \text{additional DL on vault section 14} \\
DL_{Dw15} &= 0.0529 \text{ kips} \quad \text{additional DL on vault section 15}
\end{align*}
\]

**DEAD LOAD (roof)**

live load is based on assumptions of roof dimensions and that 3/4 of weight bears on the vault (1/4 transferred to walls)
live load is for one half of the roof (since thrust line analysis is for one half of vault)

\[
\begin{align*}
\text{length} &= 32.8 \text{ ft} \\
\text{area} &= 1 \text{ ft}^2 \quad 1' \times 1' \text{ roof rafters} \\
DL_{R1} &= 1.966 \text{ kips} \\
\text{length} &= 9 \text{ ft} \quad \text{rafters are 9' apart} \\
\text{area} &= 0.25 \text{ ft}^2 \quad 1' \times 3' \text{ purlins} \\
\text{number} &= 6.5 \\
DL_{R2} &= 0.8775 \text{ kips}
\end{align*}
\]
rafters are 9’ apart  
assume roof is built of 2" thick timbers

weight of timber component of roof

weight of roofing tiles

3/4 DLr = 9.883 kips 
assume 3/4 roof dead load transfers to vault

3/4 DLr per section = 0.898 
deep load of roof for each of 11 sections

1/4 DLr = 3.294 kips 
assume 1/4 roof dead load transfers to clerestory wall

LIVE LOAD (on roof)

loading = 0.015 ksf

length = 32.8 ft

width = 9 ft

LL = 4.428 kips

3/4 LL = 3.321 kips 
assume 3/4 roof dead load transfers to vault

3/4 LL per section = 0.302 
deap load of roof for each of 11 sections

1/4 LL = 1.107 kips 
assume 1/4 roof dead load transfers to clerestory wall

DEAD LOAD (clerestory wall)

length = 4.5 ft  
length between diaphragm walls

thickness = 3.8333333 ft  
1’-10” thick wall

height = 23 ft  
height above wing roof

DLcw = 59.51 kips

DLcw + DLr = 62.807 kips

THRUST LINE ANALYSIS

half-vault section for analysis is divided into: 15 sections
DLv divided equally between all sections
DLr + LL divided equally amongst top: 11 sections
DLr + LL applied as a distributed load

LOAD LINE

labeled from top (phi = 3) to bottom (phi = 87)

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<th>DLtw</th>
<th>DLr</th>
<th>LL</th>
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</table>
Saint Louis Art Museum - Thrust Line Analysis with Gunite

\[ \rho_{\text{tile}} = 150 \text{pcf} \quad \rho_{\text{timber}} = 60 \text{pcf} \quad \rho_{\text{masonry}} = 150 \text{pcf} \quad \rho_{\text{gunite}} = 150 \text{pcf} \]

**DEAD LOAD (vault)**

Dead load calculation is based on the weight of the vault at a typical cross section for the entire vault

- radius = 27 ft
- width = 4.5 ft
- thickness = 0.333333 ft
- \( DLv_1 = 19.09 \text{ kips} \)
- 1/2 \( DLv_1 = 9.543 \text{ kips} \)

- thickness = 0.1666667 ft
- \( DLv_2 = 4.24 \text{ kips} \)
- 1/2 \( DLv_2 = 2.121 \text{ kips} \)

- **1/2 \( DLv = 14.645 \text{ kips} \)**

**DEAD LOAD (diaphragm walls)**

- area\(_{10} = 1.28 \text{ ft}^2 \)
- area\(_{11} = 4.24 \text{ ft}^2 \)
- area\(_{12} = 6.01 \text{ ft}^2 \)
- area\(_{13} = 6.34 \text{ ft}^2 \)
- area\(_{14} = 5.07 \text{ ft}^2 \)
- area\(_{15} = 2.12 \text{ ft}^2 \)

- thickness = 0.167 ft

Diaphragm wall constructed of two 1" thick tiles, 1" apart

- \( DLW_{10} = 0.0320 \text{ kips} \)
- \( DLW_{11} = 0.106 \text{ kips} \)
- \( DLW_{12} = 0.150 \text{ kips} \)
- \( DLW_{13} = 0.158 \text{ kips} \)
- \( DLW_{14} = 0.127 \text{ kips} \)
- \( DLW_{15} = 0.0529 \text{ kips} \)

**DEAD LOAD (roof)**

Live load is based on assumptions of roof dimensions and that 3/4 of weight bears on the vault (1/4 transferred to walls)

- length = 32.8 ft
- area = 1 ft\(^2\)
- \( DLr1 = 1.968 \text{ kips} \)

1/2 \( DLv = 14.645 \text{ kips} \)

Dead load of vault for each of 15 sections
length = 9 ft
area = 0.25 ft²
number = 6.5
DL₉₂ = 0.8775 kips

length = 32.8 ft
width = 9 ft
thickness = 0.167 ft
DL₉₃ = 2.852 kips

length = 32.8 ft
width = 9 ft
thickness = 0.167 ft
DL₉₄ = 7.380 kips

3/4 DL₉ = 9.833 kips
3/4 DL₉ per section = 0.898

1/4 DL₉ = 3.294 kips

LIVE LOAD (on roof)
loading = 0.015 ksf
length = 32.8 ft
width = 9 ft
LL = 4.428 kips

3/4 LL = 3.321 kips
3/4 LL per section = 0.302
1/4 LL = 1.107 kips

DEAD LOAD (clerestory wall)
length = 4.5 ft
thickness = 3.8333 ft
height = 23 ft
DLcw = 59.51 kips

DLcw + DLr = 62.807 kips

THRUXT LINE ANALYSIS
half-vault section for analysis is divided into: 15 sections
DLv divided equally between all sections
DLr + LL divided equally amongst top: 11 sections
DLr + LL applied as a distributed load

LOAD LINE
labeled from top (phi = 3) to bottom (phi = 87)
Appendix E. A Guide to the Structural Assessment of Guastavino Vaulting

The following guidelines are meant to serve as a starting point for the assessment of Guastavino vaulting. The assessment of a Guastavino masonry tile structure should commence with a visual inspection of the vaulting for cracks and water damage. Based on the findings, the inspection may continue as follows.

For cracks in the vaulting:

- Soffit tiles in the regions of cracking should be sounded with a rubber mallet to check for tiles that have become debonded and might fall from the vaulting. Loose tiles should be regouted.
- The source of cracks should be investigated to determine that their cause has ceased to progress. This can be accomplished by:
  - Knowing the pathologies of masonry vaulting and recognizing if cracking is of a common type;
  - Creating a three-dimensional finite element model and investigating different possible causes of the cracks (such as foundation settlement and temperature fluctuations). This step may be overly complicated and expensive for some applications.
- If the cause of cracks cannot be determined absolutely, cracks should be instrumented and monitored to ensure they are not opening wider (other than for seasonal variations). If the size of a crack continues to increase, more investigation should be done to identify the problem and intervention may need to be taken to assure the stability of the structure.

For water damage:

- Soffit tiles in the regions of water damage should be sounded with a rubber mallet to check for tiles that have become debonded and might fall from the vaulting. Loose tiles should be regouted.
- If water damage is in the region of tensile reinforcing (such as near a tension tie at the base of a dome), any tensile capacity needed from that reinforcing and the corresponding required cross-sectional area should be determined along with the potential for section-loss due to rusting. If the integrity of the tensile tie is in question, steps may need to be taken to assure the stability of the structure.

After assessing any damage to the structure, its stability should be investigated. This is most easily accomplished with a graphical equilibrium analysis to find a thrust line that can be contained within the material. The results of the graphical analysis might help recognize threats to that stability.

For stability:

- The thrust line should be used to evaluate the elements of a structure that are critical to its stability. These may include stiffening elements such as diaphragm walls, parts of the surrounding structure that direct the line of thrust downwards such as clerestory walls, and tension ties.
- The magnitude of thrust for a structure should be determined.
- For a dome, the required minimum force for the tension tie should be evaluated.

Once an analyst has determined the requirements for stability of a masonry tile structure, he or she should make sure that any renovations or changes being made to the building do not affect elements necessary for the stability and to contain the thrust in order to ensure the Guastavino vaulting is safe.