Probing the Epoch of Reionization with Redshifted 21 cm
HI Emission

by
Judd D. Bowman

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2007

© Massachusetts Institute of Technology 2007. All rights reserved.

Author .............

Department of Physics
May 4, 2007

Certified by .............

Jacqueline N. Hewitt
Professor of Physics
Thesis Supervisor

Accepted by .............

Thomas J. Greytak
Associate Department Head for Education
Probing the Epoch of Reionization with Redshifted 21 cm HI Emission

by

Judd D. Bowman

Submitted to the Department of Physics
on May 4, 2007, in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Abstract

Emission and absorption features in the spectrum of the diffuse radio background below 200 MHz due to the 21 cm hyperfine transition line of neutral hydrogen gas in the high-redshift intergalactic medium offer a new and potentially valuable probe into the evolution of the Universe during the poorly constrained period when the first stars, galaxies, and quasars formed between approximately 370,000 to 700 million years after the Big Bang. We place an initial upper limit of 450 mK on the relative brightness temperature of the redshifted 21 cm contribution to the all-sky spectrum, assuming a rapid transition to a fully ionized intergalactic medium at a redshift of 8. This limit is approximately a factor of 20 greater than the expected contribution of 25 to 35 mK.

We analyze the ability of a new class of interferometric radio arrays to measure the statistical properties of expected angular and spectral fluctuations in the diffuse redshifted 21 cm emission. We calculate that thermal noise should not prevent the Mileura Widefield Array (MWA) from detecting the power spectrum of the fluctuations between redshifts 6 and 12, as long as the intergalactic medium is not fully ionized. Measurements of the redshifted 21 cm signal will be contaminated from intense Galactic synchrotron radiation and extragalactic continuum sources. We find that the instrumental response of the MWA does not prevent the spectral properties of the anticipated foregrounds from being used to separate the foregrounds from the redshifted 21 cm fluctuations in "dirty" sky maps. We also test whether observations with the MWA will be able to constrain the fundamental cosmological model if the hydrogen in the IGM remains fully neutral until redshift 8. The MWA cannot constrain the underlying cosmology, but a similar experiment with a 10-fold increase in collecting area could provide useful constraints on the slope of the inflationary power spectrum and the running of the spectral index. Observations acquired during field tests with prototype equipment at the MWA’s remote site in western Australia confirmed the predicted sensitivity of the antennas, sky-noise dominated system temperatures, and phase-coherent interferometric measurements. The radio spectrum was found to be remarkably free of strong terrestrial signals between 80 and 300 MHz.

Thesis Supervisor: Jacqueline N. Hewitt
Title: Professor of Physics
Acknowledgments

The work presented in this thesis, and my time at MIT over the last five years, has benefited from the contributions and support of many individuals. At MIT and Haystack, I would to thank my advisor, Jackie Hewitt, my thesis committee, Max Tegmark and Adam Burgasser, and Miguel Morales, Alan Rogers, Colin Lonsdale, Joe Salah, Divya Oberoi, Roger Cappallo, Shep Doleman, Justin Kasper, along with Omri Schwarz and Rich Crowley. At the Center for Astrophysics, I would like to thank Lincoln Greenhill, Randall Wayth, and Daniel Mitchell, along with Matt McQuinn and the members of the ITC working on reionization science. In Australia, I would especially like to thank Merv Lynch and David Herne at Curtin University and Frank Briggs, as well as all the collaborators on the MWA project, especially Rachel Webster and Stuart Wyithe at the University of Melbourne, and Lian and Patrick Walsh at Mileura Station. I would also like to thank Priya Natarajan and Chuck Keeton. The astrograds have been a constant source of support and entertainment, especially Jake Hartman and Miriam Krauss, and in earlier times, Matt Muterspaugh, Adam Bolton, John Fregeau, and Kristin Burgess. And my family, Cassie, Lizz, Dave, Ethan, Jill, Connie, and Warren.
4.3 Forecasted Parameter Constraints ........................................ 67
  4.3.1 Parameter Dependencies ........................................... 67
  4.3.2 Degeneracies in Amplitude ......................................... 71
  4.3.3 Residual Foreground Contamination ................................. 72
  4.3.4 Dark Energy Equation of State ..................................... 74

5 Initial Field Tests of Prototype MWA Antenna Tiles .......................... 77
  5.1 Instrument Design ...................................................... 79
      5.1.1 Wide-band Active Dipole Elements ............................... 79
      5.1.2 Antenna Tile and Delay-Line Beamformer ...................... 82
      5.1.3 Receiver and Digitizer System .................................. 87
      5.1.4 Array Layout ................................................... 88
      5.1.5 Site Infrastructure .............................................. 88
  5.2 Calibration and Performance ........................................... 89
      5.2.1 Antenna Power Response Pattern ................................. 89
      5.2.2 Gains and System Temperatures .................................. 89
      5.2.3 Baseline Determination ......................................... 90
      5.2.4 Antenna Delays .................................................. 91
      5.2.5 Cross Talk ...................................................... 91
      5.2.6 Noise and Integration Time ..................................... 92
  5.3 Site Characterization .................................................. 92
      5.3.1 The Western Australian Environment ............................. 92
      5.3.2 Radio Frequency Interference .................................. 94
  5.4 Observational Results ................................................ 96
      5.4.1 Fornax A Imaging ............................................... 96
      5.4.2 Solar and Ionosphere Measurements .............................. 97

6 Experiment to Detect the Global EOR Signature (EDGES) ......................... 110
  6.1 Method ......................................................................... 111
  6.2 System Design ........................................................... 112
      6.2.1 Hardware Configuration .......................................... 113
      6.2.2 Data Acquisition and Processing ............................... 114
  6.3 Initial Results ............................................................ 115
      6.3.1 Limits on Reionization History ................................ 117
  6.4 Absolute Sky Temperature and Spectral Index ............................. 122
      6.4.1 Calibration and Corrections ...................................... 126
      6.4.2 Model Parameters ................................................. 128
      6.4.3 Uncertainty in Model Parameters ................................. 129
      6.4.4 Variations in Spectral Index and Temperature ................... 130

7 Conclusion ........................................................................... 134
  7.1 Looking Ahead ............................................................. 136
List of Figures

1-1 Photograph of the first production antenna tile for the MWA ............... 18
2-1 Estimated division of observing time for the MWA .......................... 22
2-2 Four model antenna configurations and corresponding thermal uncertainty in measured power spectrum ........................................... 28
2-3 1-σ uncertainty in measured power spectrum at redshift $z = 8$ .......... 30
2-4 1-σ uncertainty in measured power spectra at redshifts $z = \{6, 8, 10, 12\}$ 31
2-5 Simulated measurement of redshifted 21 cm power spectrum at redshift $z = 8$ 34

3-1 Brightness temperature of the Galactic synchrotron foreground ........... 39
3-2 Realization of the MWA antenna layout and corresponding baseline distribution .......... 43
3-3 Profiles for the uniform, natural, and Gaussian weighting functions ...... 44
3-4 Model of extragalactic continuum sources .................................. 46
3-5 Residuals for sky model with no instrumental response and no thermal noise 49
3-6 Residuals for uniform weighting with no thermal noise ..................... 50
3-7 Residuals for natural weighting with no thermal noise ..................... 51
3-8 Residuals for Gaussian weighting with no thermal noise ................... 52
3-9 Thermal noise in dirty sky maps ............................................. 54
3-10 Magnitude of residuals after complex averaging over independent cells within a circular ring in the $uv$-plane. ............................................. 55
3-11 Distribution of residuals between $445 < |u| < 455\lambda$ at 156 MHz .......... 56

4-1 Comparison of uncertainties for MWA and MWA5000 ....................... 64
4-2 Marginalized elliptical error regions ........................................ 68
4-3 Dependency of redshifted 21 cm power spectrum on cosmological parameters 69
4-4 Unmarginalized error ellipses for nuisance parameters ..................... 73

5-1 Photographs of the first prototype antenna being assembled in western Australia ................................................................. 78
5-2 Photographs of individual dipole assembly and beamformer .................. 80
5-3 Predicted antenna tile beam patterns ............................................ 81
5-4 Diagram of relative antenna tile locations .................................... 82
5-5 Measured antenna tile power response profiles ................................ 83
5-6 Best-fit dipole power envelope .................................................. 84
5-7 Time series of sky maps at 200 MHz .......................................... 85
5-8 Effective receiver temperature and gain as functions of LST ................. 86
5-9 Thermal noise as a function of integration time ................................ 93
5-10 Measured spectrum between 80 and 300 MHz ................................ 101
5-11 Deep integrations for 11 spectral windows .................................. 102
List of Tables

1.1 Summary of Performance Properties of the MWA .................. 17
2.1 Fiducial Observation Parameters ........................................ 24
2.2 Redshift Dependent Parameters ....................................... 24
2.3 Data Cylinder Dimensions for Redshift $z = 8$ .................... 25
4.1 Fiducial Observation Parameters ........................................ 63
4.2 Redshift Dependent Parameters ....................................... 65
4.3 Model Power Spectrum Parametrization ............................... 66
4.4 Forecasted Uncertainties for the Model Parameters ............... 75
4.5 Covariance Matrix for MWA ............................................ 75
4.6 Covariance Matrix for MWA5000 ...................................... 76
5.1 Site Location ............................................................... 88
5.2 Antenna Tile Locations .................................................. 90
5.3 Baseline Determination Comparison ................................... 91
5.4 Weather in Western Australia ......................................... 94
6.1 Calibration Corrections and Uncertainties ............................ 128
6.2 Calibrated Observing Runs .............................................. 131
6.3 Calibrated Sky Measurements .......................................... 132
6.3 Calibrated Sky Measurements .......................................... 133
Chapter 1

Introduction

Over the last decade, astrophysical observations have succeeded in charting much of the history of the Universe at unprecedented levels. Ground- and space-based observatories have detected and characterized galaxies and quasars up to high redshifts \( z \approx 6 \) corresponding to times when the Universe was just 700 million years old (or about 5% of its current age of 13.7 billion years). At present, thousands of galaxies and quasars have been cataloged above redshift \( z > 4 \), and hundreds above \( z \gtrsim 6 \), a quantity that is due in large part to the remarkable achievements of the Sloan Digital Sky Survey and the Hubble Space Telescope. Similarly, gamma-ray bursts from objects at \( z \approx 6 \) have also begun to be detected in recent years with new satellite observatories such as the Swift Gamma Ray Burst Mission. Together, these observations have yielded important insights for understanding the evolution of the Universe, including the star formation history, properties of the intergalactic medium (IGM), and galaxy formation.

At the same time, precise measurements of the cosmic microwave background (CMB), and in particular of angular anisotropy in the intensity of the background, by the Wilkinson Microwave Anisotropy Probe (WMAP) have provided—from only 370,000 years after the Big Bang \( z \approx 1000 \)—the effective boundary conditions that govern the large-scale evolution of the Universe and ultimately the nature of the subsequent gravitational collapse of structures such as stars, galaxies, and quasars. Coupled with powerful cosmological probes at relatively low redshifts \( z \lesssim 1 \), including statistical analysis of the distribution of galaxies, measurements of the acceleration history of the Universe using Type-Ia supernovae, and constraints on the abundance of dark matter from observations of galaxy clusters, astrophysicists are closing in on a self-consistent and comprehensive model describing the evolution of the Universe from a dense, almost featureless plasma following the Big Bang to the complicated, structure-filled environment we see today.

Despite this amazing progress, a large gap remains in the history of the Universe for which there are currently no direct probes. The period between 370,000 and 700 million years after the Big Bang—referred to as the cosmological “Dark Ages”—is completely invisible to contemporary observations. Information from this period is not explicitly contained in the CMB because baryonic matter and radiation have already decoupled and CMB photons stream freely through the IGM. Yet, there has been insufficient time for significant numbers of stars, galaxies, and quasars to form and produce emission that could be detected with the telescopes available today. Not coincidentally (and almost by definition), the existing capabilities to detect high-redshift galaxies and quasars reach only to the very end of the Dark Ages.
Probing this epoch is at the forefront of modern astrophysics and cosmology. It is a daunting challenge to study the sources of the "first light", but several options do exist. According to theory, the first generation of stars could form as early as 100 to 200 million years after the Big Bang. These stars will be extremely massive, of order 100 solar masses, and therefore very short-lived. In principle, their deaths should produce prodigious numbers of gamma-ray bursts that may eventually be detected. Additionally, extremely over-dense regions in the Universe will collapse particularly early due to gravitational instability, and thus rare, but massive galaxies and quasars may exist, even during early times. The James Webb Space Telescope, scheduled for launch in 2013, and next-generation 30 m ground-based telescopes should be able to detect these objects out to very high redshifts ($z \lesssim 15$). Finally, there is a third major potential probe of the Dark Ages. The bulk of the baryonic matter in the Universe during this period is in the form of neutral hydrogen gas in the IGM. Rather than target observations at the galaxies and quasars that are the rare, early products of gravitational collapse, it should be possible to detect directly the presence of the ubiquitous hydrogen gas. The most promising method of achieving this detection—and the subject of this thesis—is to search for signatures of the (highly redshifted) 21 cm hyperfine transition line of neutral hydrogen in the radio spectrum.

1.1 The Epoch of Reionization

The transition period at the end of the Dark Ages is known as the epoch of reionization (EOR). During this epoch, radiation from the very first luminous sources—early stars, galaxies, and quasars—succeeded in ionizing the neutral hydrogen gas that had filled the Universe since the “recombination” event that occurred as the Universe cooled following the Big Bang. Reionization marks a significant shift in the evolution of the Universe. For the first time, gravitationally-collapsed objects exerted substantial feedback on their environments through electromagnetic radiation, initiating processes that have dominated the evolution of the visible baryonic Universe ever since. The epoch of reionization, therefore, can be considered a dividing line, of sorts, when the relatively simple evolution of the early Universe gave way to more complicated and more interconnected processes. This is the period that will be probed most thoroughly by the next generation of experiments.

Although the Dark Ages are known to end with reionization, constraints on when this transition occurred are limited. The best existing constraints come from two sources: CMB anisotropy measurements and absorption features in the spectra of high-redshift quasars. The amplitude of the temperature anisotropy in the CMB is affected by Thomson scattering due to electrons along the line of sight between the surface of last scattering and the detector. It is sensitive, therefore, to the ionization history of the IGM through the electron column density along the line of sight. Additionally, if there is sufficient optical depth to CMB photons due to free electrons in the IGM after reionization, some of the angular anisotropy in the unpolarized intensity can be converted to polarized anisotropy. This produces a peak in the polarization power spectrum at the angular scale size equivalent to the horizon at reionization with an amplitude proportional to the optical depth [Zaldarriaga et al., 1997]. Measurements by the WMAP satellite of these effects indicated that the redshift of reionization is $z_r \approx 11 \pm 4$ [Spergel et al., 2006], assuming an instantaneous transition.

Lyman-α absorption by neutral hydrogen is visible in the spectra of many high-redshift quasars, and thus offers the second probe of the ionization history of the IGM. Continuum emission from quasars is redshifted as it travels through the expanding Universe to the
observer. Neutral hydrogen along the line of sight creates absorption features in the continuum at wavelengths corresponding to the local rest-frame wavelength of the Lyman-α line. This effect can create an apparent “forest” or trough [Gunn and Peterson, 1965] in the spectrum that charts the density of neutral hydrogen as a function of redshift. Whereas CMB measurements place an integrated constraint on reionization, quasar absorption line studies are capable of constraining the detailed history. There is a significant limitation to this approach, however. The Lyman-α absorption saturates at very low fractions of neutral hydrogen (of order $x_{HI} \approx 10^{-4}$). Nevertheless, results from these studies have been quite successful and show that, while the universe is highly ionized below $z \gtrsim 6$ (with typical $x_{HI} \lesssim 10^{-5}$), a significant amount of neutral hydrogen is present above, although precisely how much remains unclear [Djorgovski et al., 2001, Becker et al., 2001, Fan et al., 2002, 2003, Wyithe and Loeb, 2004b, Fan et al., 2006].

The existing CMB and quasar absorption measurements are somewhat contradictory. Prior to these studies, the reionization epoch was assumed generally to be quite brief, with the transition from an IGM filled with fully neutral hydrogen to an IGM filled with highly ionized hydrogen occurring very rapidly. These results, however, open the possibility that the ionization history of the IGM (and the transition out of the Dark Ages) may be more complicated than previously believed [Haiman and Holder, 2003, Cen, 2003, Sokasian et al., 2003, Madau et al., 2004]. Resolving this conflict is one of the primary motivations driving the current interest in upcoming EOR observations. Direct observations of neutral hydrogen during this epoch would reveal the evolving properties of the IGM, and thus provide a definitive answer to how reionization unfolded and ended the Dark Ages.

### 1.2 The 21 cm Hyperfine Line of Neutral Hydrogen

Since hydrogen gas is the dominant component of the high-redshift IGM, it is potentially an extraordinary resource for studying the Dark Ages and the processes responsible for reionization, as well as other important topics (as we shall see below), including inflationary cosmology. Although the Lyman-α transition line used in quasar absorption studies below $z \approx 6$ has serious limitations when the neutral fraction of hydrogen gas is anything but minuscule, other transition lines in hydrogen (such as Lyman-β and Lyman-γ) are not as susceptible to this problem. The most promising hydrogen line for EOR observations is not an orbital transition, however, but rather the hyperfine transition of the electron’s spin state. The hyperfine transition of hydrogen has energy $E_{10} = 5.9 \times 10^{-6}$ eV, corresponding to a wavelength $\lambda_0 = 21$ cm and frequency $v_0 = 1420$ MHz. It is a forbidden transition with a mean lifetime over 10 million years. Thus, the optical depth to 21 cm photons in the high-redshift IGM is only of order $\tau_{21} \approx 10^{-3} x_{HI}$ [Furlanetto et al., 2006], and the line does not suffer the saturation limitations of the Lyman-α line.

The observed intensity, $I_\nu$ (typically given in units of J s$^{-1}$ m$^{-2}$ Hz$^{-1}$ str$^{-1}$ $\equiv 10^{26}$ Jy), of any emission or absorption by the hyperfine line of hydrogen in a localized region of the IGM can be expressed as an effective brightness temperature, $T_b$, that a black-body radiator would require to produce the equivalent intensity at the same frequency. At the low radio frequencies relevant to redshifted 21 cm observations ($\nu < 1420$ MHz), the Rayleigh-Jeans equation is a good approximation to the black-body spectrum, and yields

$$T_b \approx \frac{c^2 I_\nu}{2k_B\nu^2}, \quad (1.1)$$
where $c$ is the speed of light and $k_B$ is Boltzmann's constant. Just as the Lyman-$\alpha$ line is seen through absorption features in the spectra of quasars at higher frequencies, the presence of neutral hydrogen may also be detected as absorption due to the 21 cm line in the radio spectra of early quasars [Carilli et al., 2002]. Since quasars are extremely intense, the observed differential brightness temperature is well approximated in this scenario by

$$\delta T_b(z) \approx T_\star \left[ 1 - e^{-\tau_\star(z)} \right],$$

(1.2)

where $T_\star \gtrsim 10^4$ K is the observed brightness temperature of the quasar. In order to penetrate into the Dark Ages, however, this technique would require detecting very high-redshift quasars. It suffers an inherent limit to the earliest times that can be probed, therefore, since quasars need to have formed to act as background sources. Another disadvantage to this approach is that it would produce information along only a limited number of sight-lines, whereas, ideally, we would like to characterize the properties of the IGM in all directions.

Detecting the emission or absorption of the redshifted 21 cm line relative to the CMB would avoid this last disadvantage and provide an essentially unlimited number of sight-lines. For diffuse neutral hydrogen in the high-redshift IGM, the observed differential brightness temperature of the redshifted 21 cm line relative to the pervasive CMB is readily calculable from basic principles and is given by

$$\delta T_b(\vec{\theta}, z) = \frac{T_S - T_\gamma}{1 + z} \left[ 1 - e^{-\tau_\gamma} \right] \approx 9 \left( 1 + \delta \right) (1 + z)^{1/2} x_{HI} \left[ 1 - \frac{T_\gamma}{T_S} \right] \text{mK},$$

(1.3)

where $\delta$ is the local over-density, $T_\gamma = 2.73 (1 + z)$ K is the temperature of CMB at the redshift of interest, and $T_S$ is the spin temperature that describes the relative population of the ground and excited states of the hyperfine transition [Furlanetto et al., 2006]. The observed brightness temperature is very sensitive to the spin temperature. When the spin temperature is greater than the CMB temperature, the line is visible in emission. For $T_S > T_\gamma$, the magnitude of the emission saturates to a maximum (redshift-dependent) brightness temperature that is about 25 to 35 mK for a mean-density, fully neutral IGM between redshifts 6 and 15. At the other extreme, when the spin temperature is very small and $T_S \ll T_\gamma$, the line is visible in absorption against the CMB with a potentially very large (and negative) relative brightness temperature.

A significant amount of theoretical effort has been invested recently in predicting the typical differential brightness temperature of the redshifted 21 cm line as a function of redshift. At its core, this work reduces to modelling the evolution with redshift of the spin temperature of hydrogen in the IGM. A number of factors are involved in determining this quantity and must be treated in detail, including collisional coupling between the spin and kinetic temperatures of the gas, absorption of CMB photons, and heating by ultraviolet radiation from the first luminous sources. We will not go into detail describing these processes here, but instead direct the reader to Furlanetto et al. [2006] for a good introduction to the topic. The results of these efforts [Shaver et al., 1999, Gnedin and Shaver, 2004, Furlanetto, 2006] have yielded the prediction that, at sufficiently high redshifts ($z \gg 20$), the hyperfine line should be seen in absorption against the CMB with relative brightness temperatures of up to $|\delta T_b| \lesssim 100$ mK. This is because the IGM cools more rapidly than the CMB following recombination [Shapiro et al., 1994]. At lower redshifts, the radiation from the first generations of luminous sources has had sufficient time to elevate
the spin temperature of neutral hydrogen in the IGM above the CMB temperature and the redshifted 21 cm line should be detected in emission with relative brightness temperatures up to the expected maximum values (at least until the hydrogen becomes significantly ionized).

1.3 The Redshifted 21 cm Background

For the remainder of this thesis, we will be concerned entirely with detecting redshifted 21 cm signatures against the CMB. This signal from the Dark Ages appears as a faint, diffuse background in radio frequencies below 200 MHz (for redshifts above \(z = 6\)). From Equation 1.3, it is clear that measuring the brightness temperature of the redshifted 21 cm background could yield information about both the global and local properties of the IGM. Determining the average brightness temperature over a large solid angle as a function of redshift would eliminate any dependence on local density perturbations (\(\delta\)) and constrain the evolution of the product \(\bar{x}_{HI}(1 - T_r/T_s)\), where we use the bars over \(x_{HI}\) and \(T_s\) to denote spatial averages. During the reionization epoch, it is, in general, a good approximation to assume that \(T_s \gg T_r\) and, therefore, that the brightness temperature is proportional directly to \(\bar{x}_{HI}\). Global constraints on the brightness temperature of the redshifted 21 cm line during the EOR, therefore would directly constrain the neutral fraction of hydrogen in the IGM. This would yield significant improvements in estimates of the optical depth to CMB photons and, thus, would help to break existing degeneracies in CMB measurements between the optical depth and properties of the primordial matter density power spectrum [Tegmark et al., 2006]. They would also provide a basic foundation for understanding the astrophysics of reionization by setting bounds on the duration of the epoch, as well as identifying unique features in the ionization history (for example if reionization occurred in two phases or all at once).

On small scales, local perturbations in the density, ionization fraction, or spin temperature may produce significant deviations from the typical, global averages of the observed brightness temperature. Characterizing these fluctuations would be a powerful approach to exploiting the information in the redshifted 21 cm background. As primordial hydrogen cools following recombination and later reheats, density contrasts in the baryonic matter distribution should be revealed as fluctuations in the brightness temperature of the redshifted 21 cm line [Sunyaev and Zeldovich, 1972, Hogan and Rees, 1979, Scott and Rees, 1990, Iliev et al., 2002, 2003, Loeb and Zaldarriaga, 2004, Barkana and Loeb, 2005b]. At high redshifts prior to reionization, fluctuations in the redshifted 21 cm background are expected to follow closely the matter density fluctuations—at a time when baryon perturbations were still substantially in the linear regime—and should contain information regarding the fundamental cosmological model. Redshifted 21 cm observations may help constrain the geometry of the high redshift universe between recombination and reionization [Barkana and Loeb, 2005c]. In particular, at very small spatial scales, where neither CMB anisotropy measurements nor large-scale structure surveys are able to directly probe the matter power spectrum, redshifted 21 cm measurements may dramatically improve constraints on alternatives to the standard inflationary model. Loeb and Zaldarriaga [2004] calculate that the number of independent modes accessible through redshifted 21 cm measurements of the matter power spectrum is up to nine orders of magnitude greater than for CMB measurements. Thus, they may one day yield the definitive data set for precision cosmology.

During the reionization epoch (\(z < 15\)), a unique pattern will be imprinted in the
redshifted 21 cm signal that reflects the processes responsible for the ionizing photons and that evolves with redshift as reionization progresses. As the first luminous sources ionize their surroundings, voids are expected to appear in the fluctuating emission [Madau et al., 1997, Tozzi et al., 2000, Ciardi and Madau, 2003, Zaldarriaga et al., 2004, Furlanetto et al., 2004b]. In principle, these features may be studied through direct imaging or through the determination of the power spectrum (and higher-order statistics) of the spatial fluctuations. The fluctuations may be probed along a single line-of-sight (resulting in spectral features that are similar to the Lyman-α forest and that are dubbed the 21 cm forest), angularly in the plane of the sky (yielding maps of the fluctuations like those produced by WMAP for the CMB), or three-dimensionally in an observed volume of space. The last method gives rise to 21 cm tomography.

The properties of the three-dimensional power spectrum of the spatial fluctuations in the redshifted 21 cm brightness temperature are expected to be dominated by the characteristics of the reionized voids in the background emission due to the first luminous objects [Zaldarriaga et al., 2004, Furlanetto et al., 2004a]. Measurements of the power spectrum from this period would provide insight into many of the poorly understood processes responsible for reionization and structure formation, such as the radiative feedback mechanisms in star-forming regions, the physics of the first (Population III) stars, and the role of quasars. Tracing the power spectrum as a function of redshift during this epoch will chart the history of the formation of structures. Directly imaging voids in redshifted 21 cm brightness temperature from individual H II regions surrounding quasars during this epoch would probe quasar physics [Wyithe and Loeb, 2004a,c, Kohler et al., 2005] and could provide guides in searches for high-redshift galaxies [Wyithe et al., 2005].

Measurements of the redshifted 21 cm background during the EOR may also be useful for constraining cosmological models. Ali et al. [2005] and Barkana and Loeb [2005a] have shown that differences in the line-of-sight versus angular components of the observed redshifted 21 cm power spectrum can be used to separate primordial density perturbations from features caused by the radiative processes responsible for reionization, and Barkana [2006] has discussed the application of the Alcock-Paczynski (AP) test [Alcock and Paczynski, 1979] to redshifted 21 cm measurements. Additionally, Barkana and Loeb [2005b] consider the effects of the earliest galaxies on the redshifted 21 cm fluctuations and Naoz and Barkana [2005] discuss using redshifted 21 cm observations to study the thermal history of hydrogen gas by detecting a small-scale cutoff in the power spectrum due to thermal broadening of the hyperfine line.

1.3.1 Experimental Approaches

There are two broad approaches to using the redshifted 21 cm signal to probe the Dark Ages. The first approach is to constrain the global evolution of the average redshifted 21 cm differential brightness temperature with redshift, and the second is to characterize the local fluctuations in the background. Both types of observations would provide important information about the Dark Ages and the epoch of reionization. In principle, the easier approach to observing the redshifted 21 cm background is to chart the evolution of the global differential brightness temperature with redshift. Since the goal in this case is to average over a large solid angle at multiple frequencies, global signature experiments do not need necessarily to image the sky. Furthermore, since the redshifted 21 cm signal is visible in all directions, the signal will fill the primary beam of any antenna. This provides a significant simplification and means that there is no loss in sensitivity by increasing the field
of view (as would be typical if one were observing a point source). Thus, global signature experiments are able, in principle, to use very simple antennas, such as individual dipoles.

Several small experiments are underway that are designed to detect distinct features in the global redshifted 21 cm background [Shaver et al., 1999, Gnedin and Shaver, 2004, Furlanetto, 2006], such as a sharp step transition in the all-sky spectrum that would be present if reionization occurred very rapidly. Amazingly, these modest experiments could have been performed easily (for the most part) anytime in the past few decades, but were not conceived until significant attention was turned to understanding the reionization epoch. Two primary efforts in this category are the Compact Reionization Experiment (CORE) lead by Ron Ekers at the Australian Telescope National Facility, and the Experiment to Detect the Global EOR Signature (EDGES), lead by Alan Rogers at the MIT Haystack Observatory. We report initial results from EDGES in Chapter 6.

The second approach discussed above to observing the redshifted 21 cm background is to characterize the fluctuations in the signal. Unlike the global signature efforts, experiments using this approach are required to provide information about the sky on small angular scales. The ideal outcome of observations for this approach would be true maps of the redshifted 21 cm background. However, due to the extremely intense synchrotron radiation from our own galaxy (see Chapter 3), directly imaging the fluctuations and voids in the redshifted 21 cm background with the desired arc-minute or better resolution will require the sensitivity of the planned Square Kilometer Array [Furlanetto and Briggs, 2004], and thus will not be feasible until at least 2020. Statistical observations of the fluctuation power spectrum, on the other hand, should be obtainable with much smaller radio telescope arrays since statistical measurements allow a greater degree of information compression, thus increasing the effective signal to noise ratio in the measurements. As we discussed in the previous section, characterizing the power spectrum and its evolution would provide a wealth of information about structure formation and the fundamental astrophysics behind reionization. In large part because of this promise of opening the Dark Ages to scrutiny, as well as because of new enabling technologies, several radio-frequency experiments are underway that hope to detect the redshifted 21 cm background produced by neutral hydrogen above $z \gtrsim 6$ and constrain its statistical properties.

### 1.3.2 First Generation EOR Experiments

With the advent of high-performance, low-cost digital signal processing capabilities, a new approach to low-frequency radio astronomy instrumentation has become possible in recent years. At these frequencies, it is now feasible to directly sample radio-frequency waveforms from antennas and perform traditionally analog functions, such as filtering and mixing, in the digital domain. This minimizes analog devices in the signal path and leads to stable and well-calibrated systems. The low cost of such approaches opens up the possibility to deploy a very large number of small antennas, each equipped with direct-sampling digital receivers, and thereby to gain access to seamless, wide fields of view.

These technological advances make it feasible for the first time to construct radio arrays that are optimized [Morales and Hewitt, 2004, Morales, 2005] to detect and study fluctuations in the diffuse redshifted 21 cm emission from the EOR. The initial detection of the signal is anticipated to be a challenging experimental undertaking and a several approaches exploiting the new signal processing capabilities are being implemented. This first generation of radio observatories consists of three instruments that should be operational by the end of the decade. The instruments are the 21CMA, LOFAR, and MWA. In this thesis,
we characterize the expected performance of the MWA in detail. Thus we provide a brief
description of the 21CMA and LOFAR here, followed by a more thorough introduction
to the MWA below. Additional details about the MWA are also given in the subsequent
chapters.

The 21 Centimeter Array (21CMA) — previously called the Primeval Structure Tele-
scope (PAST) — is a project of the National Astronomical Observatories, Chinese Academy
of Sciences. The 21CMA is located in the radio-quiet Ulastai valley in northwestern China
and will consist of 10,000 single-polarization directional Yagi-Uda antennas arranged gen-
erally in east-west and north-south arms. The array will observe the north celestial pole
with a field of view of $\sim 10^\circ$. The 21CMA is expected to be the first of the three arrays to
begin acquiring data and features a relatively flexible design that will allow the experiment
to change based on initial experiences. [Peterson, 2004]

The Low Frequency Array\(^1\) (LOFAR) is being built by ASTRON in the Netherlands, and
is expected to consist initially of two sets of 7,700 dipole-based dual-polarization antennas
that are designed to cover 30 to 80 MHz and 110 to 240 MHz, respectively. The antennas
will be divided between a core region and an extended array. For EOR studies, the core
region will be most important and is expected to contain 3,200 of the antennas within a 2 km
diameter. The core antennas will be grouped in stations of 100 antennas to provide beam-
forming. The shortest baselines will be approximately 100 m. LOFAR will have the largest
collecting area (up to 79,000 m\(^2\), depending on frequency) of the three arrays, although
for successful observations of faint targets, including redshifted 21 cm measurements, the
unfavorable radio frequency interference (RFI) environment of the Netherlands may present
added complications for LOFAR. [Kassim et al., 2004]

1.3.3 The Mileura Widefield Array

The Mileura Widefield Array\(^2\) (MWA) is a low-frequency (80-300 MHz) radio array under
development at the SKA candidate site in western Australia. The project is a collaboration
between the Massachusetts Institute of Technology, the Harvard-Smithsonian Center for
Astrophysics, several Australian universities, the Australian Commonwealh Scientific and
Research Organization/Australian Telescope National Facility, and the Raman Research
Institute in India, with infrastructure support from the Western Australian government.
US funding is provided by the National Science Foundation and the Air Force Office of
Sponsored Research.

The scientific objectives of the MWA project include characterization of redshifted 21 cm
H\(_i\) emission from the cosmological epoch of reionization [Furlanetto et al., 2006, Bowman
et al., 2007b, 2006, McQuinn et al., 2006, Wyithe et al., 2005, Babich and Loeb, 2005,
Morales and Hewitt, 2004, Zaldarriaga et al., 2004], investigation of the heliosphere through
scintillation and Faraday rotation effects in order to demonstrate the ability of such mea-
surements to improve the prediction of space weather [Salah et al., 2005], and a survey of the
sky for astronomical radio transient sources. Each of these science objectives requires a high
survey speed and large angular and spectral dynamic ranges. For reionization science, a
high dynamic range of order $10^5$ is needed to limit contamination in the measured redshifted
21 cm background from bright extragalactic sources in the field of view (see Chapter 3).
These requirements naturally lead to an array design based on a large number of small
antennas. Small antennas inherently provide large fields of view (since the angular size of

\(^1\)http://www.lofar.org
\(^2\)http://haystack.mit.edu/ast/arrays/mwa
Table 1.1. Summary of Performance Properties of the MWA

<table>
<thead>
<tr>
<th>$\nu$ [MHz]</th>
<th>$z$</th>
<th>$G_i$</th>
<th>$A_e$ [m$^2$]</th>
<th>$\delta\theta$ [arcmin]</th>
<th>$T_{sky}$ [K]</th>
<th>$T_R$ [K]</th>
<th>Sensitivity [K/Jy]</th>
<th>1-$\sigma$ Point Source Detection (1 s) [mJy]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>13.2</td>
<td>14.31</td>
<td>9600</td>
<td>6.86</td>
<td>824</td>
<td>158</td>
<td>3.48</td>
<td>53</td>
</tr>
<tr>
<td>125</td>
<td>10.4</td>
<td>16.09</td>
<td>9200</td>
<td>5.50</td>
<td>458</td>
<td>54</td>
<td>3.33</td>
<td>30</td>
</tr>
<tr>
<td>150</td>
<td>8.5</td>
<td>17.42</td>
<td>8800</td>
<td>4.58</td>
<td>200</td>
<td>34</td>
<td>3.19</td>
<td>16</td>
</tr>
<tr>
<td>175</td>
<td>7.1</td>
<td>18.82</td>
<td>9000</td>
<td>3.93</td>
<td>130</td>
<td>27</td>
<td>3.26</td>
<td>12</td>
</tr>
<tr>
<td>200</td>
<td>6.1</td>
<td>19.79</td>
<td>8600</td>
<td>3.44</td>
<td>96</td>
<td>26</td>
<td>3.12</td>
<td>10</td>
</tr>
<tr>
<td>225</td>
<td>5.3</td>
<td>20.59</td>
<td>7900</td>
<td>3.06</td>
<td>72</td>
<td>26</td>
<td>2.86</td>
<td>10</td>
</tr>
<tr>
<td>250</td>
<td>4.7</td>
<td>20.86</td>
<td>6800</td>
<td>2.75</td>
<td>55</td>
<td>29</td>
<td>2.46</td>
<td>10</td>
</tr>
<tr>
<td>275</td>
<td>4.2</td>
<td>19.79</td>
<td>4500</td>
<td>2.50</td>
<td>44</td>
<td>30</td>
<td>1.63</td>
<td>14</td>
</tr>
<tr>
<td>300</td>
<td>3.7</td>
<td>18.42</td>
<td>2800</td>
<td>2.29</td>
<td>36</td>
<td>32</td>
<td>1.01</td>
<td>22</td>
</tr>
</tbody>
</table>

Note. — Overview of the performance expectations for the MWA as a function of the frequency, $\nu$. For convenience, the second column lists the redshift of the 21 cm line that corresponds to each frequency. The third column lists the peak gain of the antenna tiles at the zenith relative to an ideal isotropic radiator. The fourth column gives the total effective collecting area for all 500 antenna tiles. The fifth column gives the approximate angular resolution according to $\delta\theta = \lambda/D$, were $\lambda$ is the wavelength and $D = 1500$ m is the diameter of the array. The sixth and seventh columns list the expected sky temperature due to Galactic synchrotron radiation and the design target for the effective receiver temperature, respectively. Column eight lists the sensitivity of the array given by $A_e/(2k_B)$, and column nine gives the 1-$\sigma$ detection threshold for point sources assuming a 1 s integration and 32 MHz bandwidth. Additional information about the designed performance of the MWA can be found in Chapters 2 and 5. Adapted from Oberoi [2006, MWA project document].
Figure 1-1 This photograph shows the first MWA antenna tile being tested on site in western Australia. The antenna consists of 16 crossed-dipoles in an approximately four-by-four meter grid. Five hundred antennas will be placed in a 1500 m diameter region by the end of 2008.
the primary beam is proportional to $\Theta_p \sim \lambda/d$, where $\lambda$ is the wavelength and $d$ is the characteristic size of the antenna). The effective collecting area of a small antenna is also correspondingly small, however, and thus a large number of antennas is needed to achieve high sensitivities. This turns out to be advantageous in the case of the MWA because a large number of antennas provides high angular dynamic range in interferometric observations. In general, for fixed collecting area, the field of view and angular dynamic range are each proportional to the inverse of the antenna size. As a result of these properties, the MWA design features a large number of small antennas with full-width half-maximum (FWHM) fields of view of $\sim 200\lambda^2$ deg$^2$, where $\lambda$ is in the range $1 \lesssim \lambda \lesssim 4$ m.

The antenna design for the MWA is different than that of most existing radio telescopes. Rather than constructing large collecting dishes for each antenna, the MWA design consists of a four-by-four phased-array of dual-polarized, wide-band active dipoles placed over a conductive ground screen, as shown in Figure 1-1. This unit forms an antenna “tile”. A significant advantage of constructing antennas tiles in this manner is that the primary beam of each antenna tile is steered electronically using a switched delay-line beamformer, and thus the array has no moving parts. The dipoles used in the antenna tiles are based on a vertical bowtie design, with symmetry above and below the midline in order to minimize gain along the horizon (where radio interference from terrestrial transmitters is likely to be strongest). The full array will consist of 500 antenna tiles spread over an area 1500 m in diameter, yielding a total collecting area of order 8000 m$^2$ per polarization (depending on frequency, see Table 1.1). This is approximately 1% of the $10^6$ m$^2$ planned for the SKA.

An additional advantage of the compact MWA antenna tile design is that the layout of the antenna tiles will be able to contain numerous short baselines ($\lesssim 10$ m) in order to preserve sensitivity to the diffuse (large angular scale) sky emission from the epoch of reionization.

Digital receiver units deployed across the array will sample directly the radio-frequency waveforms from each antenna tile and will transmit a selected 32 MHz of sky bandwidth to a central processing facility at high spectral resolution ($\sim 8$ kHz). At the central facility, the sampled waveforms will be cross-correlated for all 124,750 unique antenna pairs and 4 polarization combinations in a single-mode correlator. The resulting visibility measurements will be integrated for 0.5 s and sent to an on-site computer for additional processing. This correlator scheme will preserve the wide field of view and avoid multiple layers of beamforming and the consequent boundaries in sky coverage. The basic performance expectations of the MWA resulting from the overall design described in this section are summarized in Table 1.1.

The use of a large number of small antennas is a central element of the MWA design. Not only does it affect the physical properties of the array, but it also determines the data processing requirements. Unlike most radio facilities, the MWA will not be able to store its cross-correlated visibility measurements and process them offline. With 16 GB/s (1400 TB/day) of data flowing from the correlator, the MWA must implement real-time post-correlation processing in order to reduce the data rate to a manageable level that can be stored for later analysis. The primary significance of this aspect of the array is that instrumental (and ionospheric) calibration will be performed in real-time, with little or no ability to reassess the solutions. This is a major departure from the traditional practices of radio astronomy and much of the developmental effort for the MWA has been focused on understanding the calibration of the instrument. Calibration is typically not a robust process with existing facilities and is often done offline so that an operator can guide the process. The reason for this is that most radio telescopes produce only sparse measurements of the sky, and thus it is difficult for algorithms to converge on calibration solutions (in other
words, the goodness-of-fit surface in parameter space is not smooth, but has many local minima). The MWA should not be in this regime, however, because its large number of antennas will create a data-rich environment that should produce a comparatively smooth goodness-of-fit surface that traditional algorithms will able to solve for the best calibration model.

Deployment of the MWA is underway. Initial manufacturing prototype antenna tiles were delivered in March 2007 and the project schedule calls for the first 32 antenna tiles to be installed at the site by the end of 2007. The development sub-array formed by this initial deployment will grow to include all 500 antenna tiles by the end of 2008. Primary science operations will occur during 2009 and later.
Chapter 2

Expected Sensitivity of the Mileura Widefield Array

This chapter is adapted from the paper “The Sensitivity of First-Generation Epoch of Reionization Observatories and Their Potential for Differentiating Theoretical Power Spectra” by Bowman et al. [2006].

The first step toward using the MWA to characterize the properties of neutral hydrogen in the IGM during the reionization epoch at the end of the Dark Ages is to understand the fundamental sensitivity of the telescope. For an interferometer such as the MWA, thermal noise in the individual visibility measurements sets this basic limit. Since the MWA will not image the redshifted 21 cm background directly, but rather will attempt to measure the statistical properties of the fluctuations in the background, the thermal noise in the visibility measurements must be translated into uncertainty in the appropriate statistics.

The lowest-order statistical measurement containing significant information about the EOR is the power spectrum. Measuring the power spectrum of spatial fluctuations in the redshifted 21 cm backgrounds builds on the statistical techniques developed for analyzing the CMB anisotropy, and thus shares many similarities with CMB experiments. Unlike the CMB, however, the EOR signal is fully three-dimensional since the frequency of the redshifted 21 cm line maps to the line-of-sight distance. Recent efforts [Morales and Hewitt, 2004, Zaldarriaga et al., 2004, Bharadwaj and Ali, 2005] have shown that experiments with large bandwidths should be able to use the additional information contained in the spectral domain of the redshifted 21 cm signal to effectively increase their sensitivities in statistical measurements independently of their effective collecting areas. With its relatively limited collecting area, the MWA must exploit this advantage to detect the redshifted 21 cm power spectrum.

For this large-bandwidth regime, Morales and Hewitt [2004] and Morales [2005] have outlined a formalism for translating the thermal noise in the visibility measurements into uncertainty in the power spectrum measurements. In this chapter, we will utilize their formalism to calculate the thermal uncertainty in the three dimensional power spectrum derived from observations with the MWA. The calculations will include realistic observational parameters such as the antenna layout, field of view, and antenna temperature, and provide a fiducial mark for the capabilities of the first generation EOR observations. We begin in Sections 2.1 through 2.4 by detailing the observational parameters and techniques used for our calculation. Section 2.5 then presents the results, analyzing the sensitivity as a function of length scale, redshift, and global ionization fraction.
Figure 2-1 Estimated allocation of observing time for the MWA as a function of time of day and day of year. The EOR observing will occur during the night and primarily during the local summer, when the Galactic center is above the horizon during the day. This reduces the available time for EOR observations slightly due to the early sunrise and late sunset, but still provides approximately 1700 hr for observing multiple fields over one year.

2.1 The Fiducial Observation

To calculate accurately the sensitivity of an EOR measurement, we need to specify both the details of the instrument and the observing strategy. Many factors contribute to the instrumental response of an array. Among the most important to consider are the field of view, angular resolution, collecting area, antenna distribution, and bandwidth. These properties are specified in the reference array design for the MWA that is introduced in Chapter 1. The array consists of \( N = 500 \) antennas distributed within a \( D = 1500 \) m diameter circle. The density of antennas as a function of radius is taken to be approximately \( \rho(r) \sim r^{-2} \). The angular resolution is given by \( \lambda/D \) and the collecting area by \( N \, dA \), where \( dA \) is the collecting area of each antenna and scales like \( dA \sim 16(\lambda^2/4) \) for wavelengths below two meters. Although the bandwidth of the MWA is 32 MHz, we restrict the bandwidth of our reference observation to \( B = 8 \) MHz, which spans approximately \( 154 \lesssim \nu \lesssim 162 \) MHz for an observation centered on redshift \( z = 8 \). This avoids complications introduced by cosmic evolution, yet still provides measurements along the line-of-sight at the length scales of the strongest fluctuations in the 21 cm signal. It is also the minimum bandwidth suggested by Wyithe and Loeb [2004c] to ensure sensitivity to fluctuations during the final stage of reionization when ionized bubbles first overlap completely.

The observing schedule for the MWA is naturally divided by the presence of two astro-
nomic objects, the sun and the Galactic center. Due to large side lobes in the antenna tile beam (see Chapter 5), the reionization observations require the sun and Galactic center to both be below the horizon in order to minimize the thermal noise. This requirement fits naturally with the science objectives of the MWA, leaving the daylight hours for solar observations and many periods of the night free for pulsar and other transient event observations since these objects tend to be located in directions toward the Galactic center. Figure 2-1 illustrates how the science campaigns of the MWA will fill one year of observing.

In order to provide maximum duty time, the observing schedule will contain of order three EOR target fields at approximately $0^h$, $6^h$, and $12^h$ right ascension $(R.A.)$. The primary EOR target field will be opposite the Galactic center at $R.A. \approx 6^h$. The primary field will receive about 800 hours of integration per year and the secondary fields will each receive over 400 hours. The complete integration time available for reionization measurements will total approximately 1700 hours for one year. This observing plan is built on simple geometrical constraints. First, the nominal MWA observing window extends to $45^\circ$ from the zenith so a target can only be observed for up to about 6 hours at a time as the sky drifts through the observing window. Therefore, the observing schedule must contain a series of target fields spaced no more than $6^h$ apart in right ascension in order to maximize the duty time when the sun and Galactic center are below the horizon. To achieve the maximum integration time on a single field, the field must be located exactly opposite (in right ascension) the Galactic center. Since the Galactic center has $R.A. = 17.75^h$, this places the primary target EOR field at about $R.A. \approx 6^h$. Combining this with the previous requirements, two secondary target fields should be located near $R.A. \approx 0^h$ and $12^h$, for a total of three target fields. Figure 2-1 includes a depiction of the observing time per EOR target field for this plan. Conservatively, rejecting half the observations for non-ideal conditions yields 400, 200, and 200 hours of integration per year during the most favorable circumstances for the three fields. For our fiducial observation, we will consider a single target field that has been observed for 360 hours under the most favorable circumstances. This represents a conservative estimate of the total integration on the primary field in the first year of observing, or an ambitious estimate for the integration time that would be available on the secondary fields.

The full parameter set for our fiducial observation is summarized in Tables 2.1 and 2.2.

2.2 The Data Cylinder

Since neutral hydrogen is optically thin to the 21 cm line, the visibility measurements of the MWA inherently sample the emission from a three dimensional volume of space at high redshift. To good approximation, these measurements form a three dimensional data cylinder in visibility space $(u, v, f)$ due to the overall circular shape of the baseline distribution (see Chapter 3).

By applying Fourier transforms along one or more of the coordinates of the data cylinder, the measurements also may be represented as cylinders in several additional useful coordinate spaces. These include real space (with units of comoving Mpc) and its Fourier transform pair cosmological Fourier space $(k = k_1, k_2, k_3)$, image space $(\theta_x, \theta_y, f)$, and instrumental Fourier space $(u, v, \eta)$. Each coordinate space possesses advantages for different stages of the analysis. For example, the astrophysical foreground removal that we discuss in the next chapter is most conveniently accomplished in the image space, while the power spectrum of the redshifted 21 cm signal has symmetries that are most easily exploited in
### Table 2.1. Fiducial Observation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Array configuration, $\rho(r)$</td>
<td>$\sim r^{-2}$</td>
</tr>
<tr>
<td>Array diameter, $D$</td>
<td>1500 m</td>
</tr>
<tr>
<td>Bandwidth, $B$</td>
<td>8 MHz</td>
</tr>
<tr>
<td>Frequency resolution</td>
<td>8 kHz</td>
</tr>
<tr>
<td>Integration time, $t$</td>
<td>360 hours</td>
</tr>
<tr>
<td>Number of antennas, $N$</td>
<td>500</td>
</tr>
</tbody>
</table>

Note. — Observation parameters used in the sensitivity analysis.

### Table 2.2. Redshift Dependent Parameters

<table>
<thead>
<tr>
<th></th>
<th>$z = 6$</th>
<th>$z = 8$</th>
<th>$z = 10$</th>
<th>$z = 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular resolution ($^\circ$)</td>
<td>0.06</td>
<td>0.07</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>Antenna collecting area, $dA$ (m$^2$)</td>
<td>9</td>
<td>14</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>Antenna response scale, $\Theta_P$ ($^\circ$)</td>
<td>19</td>
<td>31</td>
<td>38</td>
<td>43</td>
</tr>
<tr>
<td>Frequency (MHz)</td>
<td>203</td>
<td>158</td>
<td>129</td>
<td>109</td>
</tr>
<tr>
<td>System temperature, $T_{sys}$ (K)</td>
<td>250</td>
<td>440</td>
<td>690</td>
<td>1000</td>
</tr>
</tbody>
</table>

Note. — Characteristics of the fiducial observation that depend on frequency, and thus on redshift. Note that the antenna collecting area is capped above $z = 10$ due to self-shadowing by the antennas at low frequencies. The system temperature is dominated by sky noise and depends significantly on frequency. We use a more conservative system temperature for these calculations than what would be derived from Table 1.1 in order to account for ground loss and other instrumental complications (see Chapter 5).
Table 2.3. Data Cylinder Dimensions for Redshift $z = 8$

<table>
<thead>
<tr>
<th>Frame</th>
<th>Diameter</th>
<th>Depth (Line-of-sight)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cosmological Fourier ($k$)</td>
<td>0.55 Mpc$^{-1}$</td>
<td>48 Mpc$^{-1}$</td>
</tr>
<tr>
<td>Image ($\theta_x, \theta_y, f$)</td>
<td>62°</td>
<td>8 MHz</td>
</tr>
<tr>
<td>Instrumental Fourier ($u, v, \eta$)</td>
<td>790 λ</td>
<td>1.2 $\cdot 10^{-4}$ Hz$^{-1}$</td>
</tr>
<tr>
<td>Real</td>
<td>9400 Mpc</td>
<td>130 Mpc</td>
</tr>
<tr>
<td>Visibility ($u, v, f$)</td>
<td>790 λ</td>
<td>8 MHz</td>
</tr>
</tbody>
</table>

Note. — Dimensions of the data cylinder at redshift $z = 8$ for the example array configuration in different frames. The diameter of the cylinder is in the plane of the first two coordinates and the depth is along the third coordinate.

the cosmological Fourier space. Since space is isotropic (rotationally invariant) the redshifted 21 cm signal is approximately spherically symmetric in the cosmological Fourier space (although there are distortions due to velocity field effects in distances derived from redshift). All of the measurements from a spherical shell are thus drawn from the same statistical ensemble and can be averaged together to maximize the signal to noise [Morales and Hewitt, 2004, Morales, 2005]. The spherical symmetry is employed in our sensitivity calculation below.

At redshift $z = 8$, the data cylinder in cosmological Fourier space has a diameter of 0.55 Mpc$^{-1}$ in the $k_1k_2$-plane and spans 48 Mpc$^{-1}$ in the line-of-sight direction. The data cylinder is very elongated in cosmological Fourier space, with the line-of-sight axis responsible for the high spatial frequency (small-scale) contribution. Table 2.3 lists the dimensions of the data cylinder in additional frames. We will see in the following sections that the dimensions of the data cylinder play an important role in the sensitivity of the planned statistical measurements.

2.3 The Instrumental Response

The basis of the statistical measurement of the redshifted 21 cm power spectrum using low-frequency, wide-field radio observations has been developed in the literature by Morales and Hewitt [2004], Zaldarriaga et al. [2004], Morales [2005], and Bowman et al. [2006]. These efforts are built on the similar approach employed for interferometric measurements of CMB anisotropies [White et al., 1999, Hobson and Maisinger, 2002, Myers et al., 2003]. Here, we review the relevant properties of the measurement; we direct the reader to the previous works for additional details.

In order to understand the sensitivity of the experiment, the redshifted 21 cm power spectrum must be mapped to an instrumental response. Following the development of Morales and Hewitt [2004], this response is given by the convolution of the power spectrum,
The window function is given by the transformation into the Fourier domain of the array’s angular and spectral power response. We approximate it with a function that is independent of frequency within the band and that depends on $\Omega$ as

$$W(\Omega) = \cos^2 \left( \frac{\pi}{2} \theta / \Omega_P \right), \quad \theta < \Omega_P$$

(2.2)

where $\Omega_P$ is the angular scale size of the primary beam of an antenna tile and $W(\Omega)$ is zero outside the defined region. In general, the cosmological Fourier domain window function, $W(k)$, is a very sharply peaked function since the field of view and bandwidth of the MWA are both large. At redshift $z = 8$, the width of $W(k)$ is approximately $10^{-3}$ Mpc$^{-1}$ in the $k_1k_2$-plane and $10^{-2}$ Mpc$^{-1}$ along the $k_3$-direction. Equivalently, the visibility measurements are localized in the $uv$-plane.

Inherent in every visibility measurement is thermal noise. The contribution of this noise to the measured power spectrum can be estimated by dividing the Fourier space into a large number of independent cells, where the size of each cell is given by the characteristic size of the window function, $W(k)$. The uncertainty in the measured power due to thermal noise in an independent cell can then be approximated by considering the density of visibility measurements in the $uv$-plane using [Morales, 2005, His Equation 11]

$$[C_{ij}^N(\mu)]_{\text{rms}} = 2 \left( \frac{2k_B T_{\text{sys}}}{\epsilon dA \, d\eta} \right)^2 \frac{1}{B \bar{n}(\mu) \, t} \delta_{ij},$$

(2.3)

where $dA$ is the physical antenna area, $d\eta$ is the inverse of the total bandwidth, $k_B$ is Boltzmann’s constant, $T_{\text{sys}}$ is the total system temperature, $\epsilon$ is the efficiency, $B$ is the total bandwidth, $\bar{n}$ is the time average number of baselines in an observing cell, $\mu \equiv \sqrt{u^2 + v^2}$, and $t$ is the total observation time. This equation is similar to the corresponding relationship for interferometric measurements of CMB anisotropy in White et al. [1999, Their Eqn. 16]. The noise and its uncertainty is taken to be diagonal in the data cylinder since the cell size has been chosen to correspond to the localization scale of the independent visibility measurements.

The thermal noise is approximately independent of frequency within the observing band and has a cylindrical symmetry in the Fourier domain. For our purposes, the measured value of the power spectrum, however, is an average over spherical shells in Fourier space. Thus, to calculate the uncertainty due to thermal noise in the measurement, we must average the uncertainty contributions from all of the independent cells in the data cylinder over spherical shells. The result is that the uncertainty at a given length scale has a somewhat complicated dependence on thermal noise, and therefore, on the antenna distribution of the array.

The top panel of Figure 2-2 shows four candidate antenna profiles for the MWA. All of the distributions contain 500 antennas and respect the minimum physically possible antenna spacing (which is responsible for the density plateaus as $r \rightarrow 0$). The four distributions are labelled by the antenna density profiles at large radii: $\rho(r) \sim r^0$, $r^{-1}$, $r^{-2}$, and $r^{-3}$. The second panel shows the density of visibility measurements in the $uv$-plane for each of the antenna distributions. The density is related to the average number of measurements
per cell, \( \bar{n}(\mu) \), in Equation 4.2 by the cell size (which is of order \( 2\lambda^2 \)). The third panel of Figure 2-2 gives the uncertainty due to thermal noise per spherical logarithmic shell in cosmological Fourier space with five bins per decade. Two limiting regimes are evident in this plot. For small shells in Fourier space, the noise depends strongly on the density profile since only visibility measurements from baselines smaller than a shell's radius can contribute to that shell and since large scales are only probed by angular measurements with the MWA. Increasing the steepness of the antenna profile condenses the visibility measurements toward the origin of the uv-plane (\( k_1 k_2 \)-plane), reducing the noise in small shells. In the other limiting case, shells in Fourier space that have extended beyond the radius of the largest baseline in the uv-plane (\( k_1 k_2 \)-plane), about \( u \gtrsim 790 \lambda (k \gtrsim 0.5 \text{ Mpc}^{-1}) \), include information from every antenna, and the density profile has little effect on the noise.

Second order effects such as the distinction between coherent integration of visibility measurements within a cell and the incoherent averaging of independent cells leads to the small differences seen at large \( k \). Between these limiting cases, the interaction of the cylindrical symmetry of the thermal noise with the spherical symmetry and logarithmic widths of the shells causes a more complicated behavior. At length scales comparable to the diameter of the data cylinder, where visibility measurements are sparse over much of a shell, the uncertainty even has a local maximum for the centrally condensed arrays.

It is clear from Figure 2-2 that the sensitivity of the statistical measurement is tightly linked to the array antenna distribution. The difference in uncertainty between the uniform distribution and the steeper power-law distributions is as much as two orders of magnitude, depending on the length scale of interest. As indicated in Table 2.1, we will use the \( \rho(r) \sim r^{-2} \) antenna distribution as our reference in the remainder of the thesis.

### 2.4 The Model Power Spectrum

For our calculations we use a simple model of the redshifted 21 cm power spectrum that has been used commonly in the literature. This model assumes that the hydrogen in the IGM is fully neutral, follows the dark matter distribution, and has a spin temperature much larger than the CMB temperature [Madau et al., 1997, Tozzi et al., 2000, Zaldarriaga et al., 2004]. This is a reasonable model for the fluctuations during the early stages of the reionization epoch after the spin temperature has been heated the processes described in Chapter 1, but before the hydrogen in the IGM has been significantly ionized by the first luminous objects, though over what redshift range this may be observed is uncertain. This model power spectrum is computed using CMBFAST [Seljak and Zaldarriaga, 1996] and does not include velocity distortions, but Barkana and Loeb [2005a] have shown that including peculiar velocities increases the signal amplitude in spherically averaged shells by about a factor of two. Distortions due to geometrical effects (such as a scaling between the line-of-sight and perpendicular axes) and peculiar velocities could allow separation of cosmological and astrophysical effects and provide sensitive probes of the underlying cosmology, but are not included in our simple model [Alcock and Paczynski, 1979, Kaiser, 1987, Barkana and Loeb, 2005a].

Given the limited information currently available, it may be possible that the redshift range accessible to the first generation experiments spans periods before, during, or after reionization. Current theoretical predictions [Zaldarriaga et al., 2004, Furlanetto et al., 2004a,b] of the redshifted 21 cm power spectrum during reionization are generally similar in shape to the simple model stated above, although their amplitudes vary (both above
Figure 2-2 Four model antenna configurations (top), the corresponding densities of visibility measurements (middle), and the 1-σ uncertainties in the measured power spectrum due to thermal noise (bottom) in an observation targeted at redshift $z = 8$. The antenna configurations are characterized by power-law density profiles, $\rho(r) \sim r^{-3}$ (solid), $r^{-2}$ (dash), $r^{-1}$ (dot), and $r^{-0}$ (dash-dot). The abscissas of the three panels are aligned so that the $r$, $u$, and $k$ coordinates correspond. The vertical gray bars represent the 4 m width of an antenna (far left), the 750 m radius of the array (middle) and the 1500 m maximum baseline in the $uv$-plane (right). The uncertainties in the bottom panel extend beyond the bounds of the maximum baseline due to the elongated $k_3$ axis, as described in Section 2.2.
and below the fully neutral model) depending on global ionization fraction. In addition, power shifts from large scales to smaller scales as reionization progresses. In particular, it seems unlikely that the universe is fully ionized by redshift $z = 8$, the target of our fiducial observation, given the quasar absorption and WMAP findings and, therefore, that the simple model power spectrum represents a plausible approximation. However, even if this were not true, an observed null result would be interesting and help to constrain the history of reionization.

Figure 2-3 shows the results of performing the above calculations for the fiducial experiment using our simple model of the redshifted 21 cm power spectrum at redshift $z = 8$ with a standard cosmology ($\Omega_M = 0.3, \Omega_A = 0.7, h = 0.7$). The model power spectrum shown in the figure has been convolved with the instrumental window function to produce the solid black line, which is plotted in the instrumental units Jy$^2$ Hz$^2$ (see Morales and Hewitt [2004] for a discussion of units).

Just as there is an inherent uncertainty due to thermal noise, there is also an inherent cosmic sample variance in the observed power spectrum. For a spherical shell in Fourier space, the sample variance can be estimated by considering the number of independent samples in the shell and assuming Gaussian statistics. This uncertainty is plotted as the dark gray region around the model power spectrum in Figure 2-3, while the combined uncertainty due to sample variance and thermal noise is added in quadrature, and the light gray region shows the full 1-\sigma uncertainty.

2.5 Predictions

2.5.1 The Measured Power Spectrum

Several effects of the instrumental response are contained in the measured power spectrum shown in Figure 2-3. Since the shape of the power spectrum is smeared by convolution with the instrumental window function, both the relative amplitude of the peak and the distinction of the baryon bump at $k \approx 0.04$ are slightly reduced. In addition, the uncertainty due to thermal noise and cosmic sample variance increases rapidly as $k \to 0$, creating unfavorable sensitivity on large length scales. The field of view is a significant factor determining the uncertainty at low $k$ since the number of measurements at these scales is proportional to $\Theta^2$. Since the line-of-sight depth of the data cylinder in real space is less than these length scales, the advantages of a three-dimensional data set and large bandwidth do not apply at low $k$. Only spherical shells with radius $k \gtrsim 0.02$ contain contributions from measurements with $k_3 \neq 0$.

The antenna distribution of the array, as we saw above, also affects the uncertainty of the measurement. Although more condensed arrays produced less uncertainty at large length scales (see Figure 2-2), other experimental considerations oppose the contraction of the array. Applications that rely on synthesis imaging, such as removing astrophysical foregrounds in the EOR observations (see Chapter 3), are adversely affected by condensing the antenna distribution and could potentially negate any benefits from such a change.

The sensitivity decreases again for large $k$ since the power spectrum falls off more rapidly than the thermal noise. For the reference design of the MWA, the best range for constraining the measured power spectrum at redshift $z = 8$ is approximately $10^{-2} < k < 10^{-1}$ Mpc$^{-1}$.
Figure 2-3 Combined 1-σ uncertainty in the measured power spectrum due to sample variance (dark gray) and combined sample and thermal variance (light gray) for logarithmic shells of width equivalent to five spectral points per decade. The instrumental response to the model redshifted 21 cm power spectrum based on a fully neutral IGM at redshift $z = 8$ is shown in black. In the upper panel the signal is plotted with linear scaling in $P$ (so that the measurement errors are Gaussian), while the lower panel uses the theoretical convention by changing the ordinate to $(k^3 P / 2\pi^2)^{1/2}$ and plotting on log-log axes.
Figure 2-4 Same as Figure 2-5, but computed for observations at four redshifts. From left to right, the redshifts are $z = 6, 8, 10,$ and $12$. Note that the vertical scales are different for each of the upper plots. The small peaks in uncertainty at $k \approx 0.5$ for $z = 10$ and 12 correspond to the increase in uncertainty due to thermal noise at length scales sampled by the largest baselines (see Figure 2-2).
2.5.2 Redshift Range

We can estimate the sensitivity of the array at additional redshifts by modifying the frequency-dependent parameters of the fiducial observation. Four primary characteristics of the array are frequency dependent: the field of view (and thus the instrumental window function, $W$), the collecting area, the angular resolution, and the system temperature, $T_{\text{sys}}$. Table 2.2 lists the values of these parameters at four frequencies corresponding to redshifts of $z = 6, 8, 10, \text{ and } 12$. The changes in these parameters require both the measurement uncertainty and the instrumental response to the redshifted 21 cm emission to be calculated for each redshift since the independent cell size, data cylinder dimensions, and characteristic thermal noise are directly affected. Figure 2-4 displays the results of such calculations for observations at the redshifts listed in Table 2.2. Again, the reference signal was a fully neutral IGM and the measurement was averaged over spherical logarithmic bins of width corresponding to five points per decade. The panels for redshift $z = 6$ illustrate the measurement with the greatest sensitivity. Two factors contribute to this performance: the amplitude of the underlying matter density power spectrum increases as $z \rightarrow 0$, and the system temperature of the instrument decreases. On the other hand, the field of view and collecting area are reduced considerably, limiting the improvement in sensitivity for lower redshifts. The net result is that, in instrumental units, the observed amplitude of the power spectrum increases by a factor of $\sim 8$ between redshifts $z = 12$ and 6, while the system temperature, dominated by Galactic foreground emission, decreases by a factor of $\sim 3.5$. The sensitivity is sufficiently great at redshift $z = 6$ that the dominant source of uncertainty at large scales is sample variance. Although it is already ruled out that the IGM remains fully neutral until redshift $z = 6$, even significantly weaker signals from a partially ionized IGM should be detectable. There is a clear degradation of the measurement sensitivity in Figure 2-4 as the redshift increases until, by redshift $z = 12$, the observation is infeasible without longer integrations or additional collecting area.

2.5.3 Sensitivity to Reionization Models

Based on these estimates of the uncertainty in measurements of the redshifted 21 cm fluctuation power spectrum with the MWA and assuming that other experimental challenges, such as astrophysical foreground mitigation, can be overcome, it is worth considering whether the MWA may be able to go beyond simple detections of high redshift neutral hydrogen and distinguish between different reionization scenarios. Several theoretical reionization models have been discussed in the literature [Santos et al., 2003, Zaldarriaga et al., 2004, Furlanetto et al., 2004b,a, Santos et al., 2005]. Figure 2-5 shows the results of a simulation based on the example array configuration for redshift $z = 8$ and includes, for comparison, the models presented by Furlanetto et al. [2004b,a] for several power spectra with different global ionization fractions. The solid black line and shaded gray regions are the same as in Figure 2-4, Column 2. It is evident from the figure that the large changes in peak amplitude of the power spectra for these models would be suitable for constraining the ionization fraction. There are many other factors that contribute to the amplitude and shape of the redshifted 21 cm fluctuation power spectrum that may also be constrained by measurements. We offer this example as an illustration of the potential of the upcoming measurements, not as an analysis of the expected scientific return. Additional efforts are needed (and are ongoing) in both the development of a useful parametrization of the astrophysical content in the power spectrum and a better estimates of the complete uncertainty in the measurements.
Figure 2-5 Same as Figure 2-4, Column 2 for redshift $z = 8$, but the data points show a simulated realization of the measured power spectrum. The error bars are the $1 - \sigma$ uncertainties calculated from the thermal noise. The dashed lines are different values of the ionization fraction in the Furlanetto et al. [2004b,a] models and are, from top to bottom, $x_i = 0.51, 0.0$ (solid), 0.43, 0.38, 0.25, and 0.13. Note that the amplitude of the redshifted 21 cm power spectrum for the fully neutral IGM is within those of the other ionization fractions. In general, the amplitudes of the model power spectra drop rapidly between $x_i = 0.0$ and 0.13, and then slowly increase with ionization fraction as large bubbles increase the contrast.
Chapter 3

Astrophysical Foreground Contamination

In the last chapter, we estimated the fundamental uncertainty in the measurements of the redshifted 21 cm power spectrum due to thermal noise and cosmic sample variance. These are not the only sources of uncertainty in the measurements. Radio observations in the meter bands are notoriously difficult and there are many other observational difficulties which must be overcome by the MWA and other first-generation experiments that will reduce the sensitivity below the baseline values.

The redshifted 21 cm radiation targeted by EOR experiments falls in the frequency range commonly used for television, FM radio, and satellite transmissions. Both the MWA and 21CMA have chosen very remote locations to avoid these radio communications that are now ubiquitous in many parts of the world, and all three of the first-generation experiments are developing advanced radio frequency interference (RFI) mitigation techniques, but it is impossible to completely eliminate these interferers (see Chapter 5) and some fraction of the observed frequency band will necessarily be excised from the power spectrum analysis. This excision will complicate the line-of-sight axis of the instrumental window function discussed in Chapter 2 and increase the correlation between different length scales in the measured power spectrum.

Turbulence in the Earth’s ionosphere refracts low frequency radio waves through a process that acts much like atmospheric turbulence at optical wavelengths and causes distortions in the apparent location and magnitude of signals originating from above the ionosphere. These distortions must be corrected in low-frequency radio observations to make accurate maps of the true sky brightness using techniques similar to wide-field adaptive optics. To facilitate this calibration process, the longest baselines of the MWA have been restricted to $\sim 1500$ m so that the instrumental calibration process does not require the “global self-calibration” techniques that have been proposed to handle wide-field imaging on arbitrarily long baselines.

After RFI and ionospheric distortions have been removed from the observations, there are still astrophysical foreground contaminants which are five orders of magnitude brighter than the $\sim 10$ mK redshifted 21 cm fluctuations. These foregrounds will severely complicate planned experiments and the interpretation of their results. At the initial target redshifts of $z \approx 8$, the wavelength of the 21 cm line is $\lambda = 0.21(1 + z) \text{ m} \approx 2$ m, or equivalently, $\nu \approx 150$ MHz. Galactic synchrotron radiation dominates the sky at these long radio wavelengths, accounting for $\gtrsim 70\%$ of the (200 to 10,000 K) total brightness temperature [Shaver et al.,
Extragalactic continuum point sources are also especially strong and numerous, comprising the bulk of the remaining ~ 30% of the sky brightness temperature. Galactic radio recombination lines (RRLs) and free-free emission from electrons in both the Galaxy and the IGM will additionally complicate the planned measurements. Another concern is the instrumental effects related to Galactic emission and imperfect instrumental calibration. Not only is the Galactic synchrotron radiation responsible for the large system temperatures in low-frequency radio receivers, but since it is Faraday rotated by the interstellar medium, it presents a bright (~ 1 K), highly structured polarized emission pattern across the sky. This places very tight constraints on the required precision of the instrumental polarization calibration.

Although all of these experimental effects will significantly complicate the analysis of the redshifted 21 cm power spectrum in actual observations, and all of them have the potential to adversely affect the sensitivity of the MWA, the effects of the astrophysical foregrounds are the source of the most concern. Initial analyses indicated that these foregrounds were an insurmountable obstacle [Di Matteo et al., 2002, Oh and Mack, 2003] because their angular variance dominates the expected fluctuations in the redshifted 21 cm background, but subsequent studies have suggested that the multi-frequency observations of the MWA and the application of appropriate statistical techniques should provide methods to separate the foregrounds from the redshifted 21 cm signal by exploiting the large coherence of the foregrounds with frequency [Di Matteo et al., 2004, Zaldarriaga et al., 2004, Morales and Hewitt, 2004, Furlanetto and Briggs, 2004, Gnedin and Shaver, 2004, Santos et al., 2005, Wang et al., 2006, McQuinn et al., 2006]. However, even these studies have made simplifying assumptions. Most notably, they have neglected any complications due to the instrumental response function. Thus, the ability of these techniques to isolate foregrounds has yet to be demonstrated in realistic treatments of the planned measurements, and the effects of their residuals on the constraints placed by redshifted 21 cm experiments on reionization and cosmological model parameters are not understood. In this chapter, we consider in detail the astrophysical foreground contamination that will be present in measurements of redshifted 21 cm emission from the EOR by the MWA and include the effects of the non-ideal instrumental response function in our analysis.

3.1 The Mitigation Strategy

The design of the MWA was chosen carefully to help mitigate some of the problems associated with astrophysical foregrounds. Since the equivalent brightness temperature of the Galactic synchrotron background is between about 200 and 10,000 K at the frequencies of interest, it dominates the system temperature of the receiver system. This inescapable source of thermal uncertainty in the measurements allows the electronics used in the receivers to be relatively inexpensive (since they do not have to perform to such exacting standards as those at higher frequencies). Although the array design takes advantage of the fact that the sky noise dominates the receiver noise, it is still preferable to have as little noise as possible in the measurements. For this reason, the angular size of the primary antenna beam (field of view) is matched to fit within the relatively cold regions regions that exist in the Galactic synchrotron emission at high Galactic latitudes. Figure 3-1 illustrates the typical brightness temperature of the synchrotron foreground at 150 MHz, as well as the primary field of view for an MWA antenna tile that is targeting a cold part of the sky.

The MWA has also been designed to aid in the mitigation of the second most intense
category of foreground contaminants, extremely intense extragalactic continuum sources (and a few Galactic sources, such as the Crab Nebula). It is common practice in radio astronomy to “peel” away bright sources in the field of view to improve the overall dynamic range in derived maps of the sky brightness and, thus, to increase the sensitivity to fainter signals. This process must be applied to the bright point sources in measurements by the MWA to a high degree of precision (better than 1 part in $10^5$) in order to reveal the fluctuations in the redshifted 21 cm background. For typical interferometers, the sparse coverage of visibility measurements in the $uv$-plane means that a substantial amount of power from these point sources is spread across the derived map of the sky brightness due to the side lobes of the point spread function of the synthesized beam. This poses a considerable deconvolution problem and it makes it difficult to subtract them from the measurements since the locations and intensities of the point sources must be determined very accurately before their contributions to the individual visibility measurements can be removed to high precision. The large number of antennas used for the MWA reduces the side lobes of the synthesized beam and provides high angular dynamic range in uncorrected maps of the sky (see the introduction to the MWA in Chapter 1). This is advantageous for isolating and subtracting the power from extremely intense extragalactic continuum sources and alleviates the severity of the bright source contamination.

3.1.1 Spectral Coherence

Targeting a cold region of the sky and subtracting the bright, easily identifiable point sources in the field of view are necessary first steps toward mitigating the astrophysical foreground contamination, but they will not be sufficient. Contributions from a sea of “confused” extragalactic continuum sources that cannot be identified individually will remain in the measurements, as will the Galactic synchrotron radiation (albeit at the $\sim 200$ K levels of the cold regions and not the worst-case $\gtrsim 1000$ K levels in the Galactic plane) and any free-free emissions. These foregrounds are still substantially more intense than the redshifted 21 cm background. Although the Galactic synchrotron continues to dominate the sky brightness, the integrated flux from the confused source population is also some two orders of magnitude greater the fluctuations in the redshifted 21 cm background (see Section 3.2.2), as are the expected free-free emissions.

These remaining contaminants share common spectral properties. Along any one line of sight, each of the foregrounds has a slowly varying power-law-like spectrum. This results in a long spectral coherence scale and is in contrast to the redshifted 21 cm signal from the reionization epoch, which fluctuates relatively rapidly in all three spatial dimensions, and thus has a short coherence scale, both in frequency and angle. In general, the spatial coherence length of the reionization signal is of order 10 Mpc, which translates to sub-degree scale fluctuations on the plane of the sky and sub-MHz fluctuations in frequency. The spectral fluctuations of the redshifted 21 cm background, therefore, are much more rapid than those of the foregrounds.

The proposed strategies for isolating the remaining foreground contamination all exploit this difference in the spectral properties between the foreground sources and the signal. Several techniques have been considered. In the image domain, subtracting a smooth spectral component from each pixel in a data cube should be able to effectively remove the foreground contribution [Furlanetto and Briggs, 2004, Santos et al., 2005, Wang et al., 2006, McQuinn et al., 2006] and only sacrifice a few of the largest modes of the redshifted 21 cm fluctuations. This is similar to the technique that is employed to remove the monopole
and dipole contributions in CMB anisotropy measurements. In the image domain, another (equivalent) foreground subtraction process may also be applied to redshifted 21 cm observations that mirrors the approach used to subtract Galactic emission from CMB anisotropy measurements [Gnedin and Shaver, 2004, Di Matteo et al., 2004] by differencing neighbor sky maps along the spectral axis. In the Fourier domain, it may be possible to isolate the foreground contribution by matching templates of the integrated properties of the different components to the derived power spectrum [Zaldarriaga et al., 2004, Morales and Hewitt, 2004, Morales et al., 2006].

Both approaches have inherent advantages and disadvantages. Subtracting foregrounds in the image domain is natural because the intensity contributions from individual sources are localized (unless the point spread function of the instrument significantly distributes their power). The attraction to isolating foreground contamination in the Fourier domain is that it is the fundamental domain of both the power spectrum and the experiment. In the Fourier domain, the thermal noise in the visibility measurements of an interferometer is predominantly white (uncorrelated). However, the foreground contributions are not localized in the Fourier domain. The received power from individual extragalactic sources is distributed between all baselines. Although the form of the mapping is simple and easily modelled, it may not be computationally feasible to solve self-consistently for a model based on individual source contributions. Various simplifications or assumptions must be employed to reduce the computational burden. If the integrated contribution from all the sources can be predicted well enough, however, a template could be constructed and fit to the measured power spectrum. This would be most practical if a model for the redshifted 21 cm emission were simultaneously fit to the measured power spectrum.

To date, the first strategy of subtracting foregrounds from the image domain representation of the measurements has been explored more thoroughly. For this work, we will also use the approach of subtracting foregrounds from the image domain. Regardless of the final strategy chosen for interpreting observations of the MWA, it is instructive to begin with a simple approach to establish a baseline for future comparison. Furthermore, it is likely that the final strategy will employ aspects of both methods [Morales et al., 2006, Wang et al., 2006].

3.1.2 Method

The fundamental assumption in image domain foreground mitigation techniques is that, after targeting a cold part of the sky and peeling away the bright point sources, the foreground contribution remaining in redshifted 21 cm observations can be subtracted by fitting a simple model along the spectral component of each pixel in a derived map of the sky brightness. Furlanetto and Briggs [2004] showed that, neglecting any complication caused by the instrument, a low-order polynomial is sufficient to match the spectral contribution in a single pixel from the sum of the faint continuum sources along that line of sight. The residuals after subtracting a model fit in this style are at the $\sim 1$ mK level, below the expected redshifted 21 cm contribution. Wang et al. [2006] added support to this approach by including thermal noise in the simulated spectra and successfully recovering the redshifted 21 cm signals by averaging over a large number of independent site lines after removing the foreground contribution. Our goal is to extend these initial efforts and include the effects of the most dominant aspects of the instrumental response.

We start by assuming a model of the true sky brightness, $I(\vec{\theta}, \nu)$, that is defined over the solid angle of the antenna tile beam and the frequency range of the measurement. For
Figure 3-1 Brightness temperature of the Galactic synchrotron foreground at $\nu = 150$ MHz. The all-sky measurements of Haslam et al. [1982] were scaled from 408 MHz using a constant spectral index of $\beta = 2.6$ to produce this map. The horizontal lines indicate the maximum and minimum declinations visible to the MWA from Western Australia (dotted) assuming a limiting zenith angle of 45°. The declination of the zenith is also shown (dashed). The primary beam of the MWA antenna tiles is shown centered at a possible target field. The center of the field of view (marked by the plus) is at 60 R.A by -30 Dec., and the concentric circles indicate the 90% (dashed), 50% (solid), and 10% (dash-dot) power response relative to the peak response. The MWA will have access to cold regions ($\sim 250$ K) of the sky both north and south of the Galactic plane.
notational simplicity, we drop the explicit dependence on $\nu$ in the following discussion. The result of observing the true sky with the MWA is a vector, $m$, of complex visibility measurements. This process can be expressed as

$$ m(v) = A(v, \bar{\theta}) I(\bar{\theta}) + n(v), \quad (3.1) $$

where $v$ indexes the individual measurements, $A$ is the instrumental response matrix that maps the true sky to the resulting measurements, and $n$ is a vector containing the system noise added to each measurement. In observations with the MWA, the visibility data must be compressed so that long integrations can be archived for later analysis. This compression most likely will be accomplished by mapping the individual visibility measurements onto a cartesian grid of cells and storing only an average value at each cell. We can express the regularly sampled measurements by applying a gridding convolution operator, $C$, to the true measurement expression, according to

$$ m(u) = C(u, v) m(v) \quad (3.2) $$

where $u$ indexes the individual pixels in the gridded representation of the measurements. In general, mapping the visibility measurements to a regular cartesian grid can introduce significant aliasing into the derived sky brightness map. The form of the gridding convolution function plays an important role in reducing the effects of this aliasing, and can range from a very simple function, such as a rectangular pillbox, to more complicated functions, such as truncated gaussians or sinc functions [Greisen, 1976, Clark, 1976, Schwab, 1978, Greisen, 1979, Schwab, 1980, Briggs et al., 1999, Thompson et al., 2001]. In all practical implementations, the gridding convolution function has very limited support (it is very localized) in order to reduce the computational burden associated with its application. The result of the MWA data processing pipeline will be gridded maps in the Fourier domain that are given by

$$ m(u) = C(u, v) A(v, \bar{\theta}) I(\bar{\theta}) + C(u, v) n(v), \quad (3.3) $$

For our calculations, we approximate this expression by first gridding the ideal sky model and then applying the Fourier transformation operator, $F$. The measurement equation is then expressed as

$$ m(u) = A(u, u') I(u') + n(u). \quad (3.4) $$

In this form, $I(u) = F(u, \theta) C(\theta, \bar{\theta}) I(\bar{\theta})$ is a vector containing the Fourier domain representation of the ideal sky model sampled on a cartesian grid, $A(u, u')$ is the instrumental response function similarly recast to a sampled grid in the Fourier domain, and $n(u) = C(u, v) n(v)$. Relating this expression to the previous efforts of Furlanetto and Briggs [2004] and Wang et al. [2006], we find that, in essence, they have simplified their analyses by assuming complete coverage of the visibility measurements in the $uv$-plane, equivalent to using $A = I$, where $I$ is the identity matrix, and by setting $n(u) = 0$ or assuming $n(u)$ to be constant, uncorrelated noise, even though this is only true for $n(v)$.

We improve on these efforts by treating $A$ and $n$ more realistically. We expand $A$ into two primary constituents such that

$$ A(u, u') = A_B(u, u') A_P(u, u'), \quad (3.5) $$

where $A_B$ accounts for the synthesized array beam due to the coverage of the visibility measurements in the $uv$-plane and $A_P$ accounts for the response of the primary beam of
the antenna tiles. For $A_P$, we use the same window function as in Equation 2.2, such that
\[ A_P(u, u') = F(u, \theta) \quad W(\theta, \theta') \quad F'(\theta', u'). \]
For $A_B$, we compute the typical coverage of the visibility measurements by creating a realization of the antenna distribution of the MWA. We then use the observing rules described in Chapter 2 and include earth-rotation synthesis assuming 2 s accumulations to build a table of the projected baselines toward the center of a plausible target field at $R.A. = 60^\circ$ and $Dec. = -30^\circ$. The result of this process is shown in Figure 3-2.

For the noise contribution to the simulated observations, we take $n(u)$ to be a vector of Gaussian random variables with standard deviation [Morales, 2005]

\[ \sigma_n = \frac{2k_B T_{sys}}{dA \sqrt{d\nu \tau}}, \]

where $T_{sys}$ is the system temperature, $dA$ is the effective collecting area of individual antenna tiles, $d\nu$ is the spectral resolution of a single channel, and $\tau$ is the accumulation duration for each visibility measurement. We approximate $n(u)$ by treating $C(u, v)$ as block diagonal, such that the contribution of any single visibility measurement is applied to only one grid cell. In this limit, the thermal noise is uncorrelated in the Fourier domain maps and given by, $n(u) \approx \sigma_n/\sqrt{N(u)}$, where $N(u)$ is the number of visibility measurements contributing to each grid cell. We set $n(u)$ to zero wherever $N(u)$ is zero, thus the noise will be correlated in the derived maps of the sky brightness. This approximation is more realistic than that of Wang et al. [2006] since our thermal noise contribution is not white in $n(u)$ and since it is correlated in derived maps in the image domain.

The final component of modelling the foreground subtraction is to actually fit and subtract the foreground model from the sky brightness map derived from the observations. There are several reasonable methods that can be used to derive the sky brightness from the archived measurements. The most ideal map to use would be a minimally biased estimate of the true sky. This would require inverting the instrumental response matrix, $A$, and the noise covariance matrix [Tegmark et al., 1997]. In practice, the extent to which it will be feasible to invert $A$, is not yet clear. At any rate, our goal is to set a worst-case limit due to the effects of the instrumental response of the MWA on subtracting foreground contamination, and a minimally biased estimate of the sky should remove many of these effects. Thus, we turn instead to what is commonly called the “dirty” sky map in radio astronomy. This is the map that results by simply transforming the measurements stored in the Fourier map to the image domain without applying any correction (deconvolution) for the instrumental response. The dirty sky map is given by

\[ I_D(\theta) = F(\theta, u) \quad U(u, u') \quad m(u'), \]

where $U$ is a weighting function for each pixel in the Fourier map [Sault, 1984, Briggs et al., 1999, Thompson et al., 2001]. The weighting function is typically chosen to give the least uncertainty in the derived sky map due to thermal noise and, thus, is equal to the inverse of the variance due to the (Gaussian) thermal noise in each pixel in the Fourier domain. This is called natural weighting. Although it produces the least uncertainty, this method typically emphasizes the information contained in short baselines because short baselines are more numerous in radio interferometers than long baselines. Thus, the effective resolution of the derived sky brightness map is lower than the $\lambda/D$ expectation. Another common method, called uniform weighting, alleviates this under-resolution at the expense of introducing more uncertainty into the derived sky map. Uniform weighting, like its name
suggests, is constant for each pixel in the Fourier domain. This allows the information in the long baselines to be emphasized and restores the effective resolution of the sky map to the expected level. Figure 3-3 illustrates the difference in the weighting functions. We will test both of these weighting methods, along with an intermediate weighting, to see if the foreground subtraction technique is robust to any effects they might introduce.

Once a dirty sky map has been generated, a low-order polynomial is fit to each pixel in the map and subtracted. The residual contamination, \( r(\theta) \), after this subtraction is given by

\[
r(\theta) = I_D(\theta) - d(\theta)
\]

where \( d(\theta) \) is the model generated from the polynomial fits. Since the natural regime of the power spectrum is the Fourier domain, we transform this residual map back to the Fourier domain and undo the weighting using

\[
r'(u) = U^{-1}(u, u') F(u', \theta) r(\theta)
\]

\[
= U^{-1}(u, u') F(u', \theta) [I_D(\theta) - d(\theta)]
\]

\[
= m(u) - U^{-1}(u, u') F(u', \theta) d(\theta)
\]

(3.9)

where \( d'(u) = U^{-1}(u, u') F(u', \theta) d(\theta) \). The residual, \( r' \), in the Fourier domain map is the quantity we are interested in determining, and that we hope will be less than the amplitude of the expected redshifted 21 cm fluctuations.

### 3.2 The Sky Model

We are nearly ready to analyze the effects of the instrumental response in the foreground subtraction process. The final step is to specify the details of the sky model for the astrophysical foregrounds. We begin by constructing a model of the astrophysical foregrounds that consists of two components: discrete extragalactic continuum sources and the Galactic synchrotron emission. Together, these are expected to account for approximately 98% of the total intensity in the radio spectrum below 200 MHz [Shaver et al., 1999, Bridle, 1967]. We exclude free-free emissions as a separate component in our analysis since they have power-law spectra as well [Kogut et al., 1996] and, as we will see below, they are easily subsumed by the uncertainty in the discrete continuum source contribution, although the free-free emissions have been shown to correlate with dust clouds at high Galactic latitudes, and thus have different angular structure than the discrete source population. We have also chosen to ignore RRLs in this analysis. The frequencies at which they occur are known (or easily calculable) and can be excised from observations if they are determined to be a significant contribution. The spectral lines have never been observed at high Galactic latitudes. They are expected to be narrower than the 32 kHz frequency channel size of the MWA and occur approximately every few MHz, and therefore in only a few percent of the spectral channels (Erickson et al. [1995], and Shaver [1975] and Walmsley and Watson [1982], are good starting points for additional details about the observed properties and theoretical treatment of RRLs, respectively). Thus, the cost of excising the RRLs is a minor complication to the window function, but one that is expected to be no more severe than that caused by excising radio-frequency interference.
Figure 3-2 Realization of the MWA antenna layout and corresponding distribution of baselines. The 500 antennas are coplanar and distributed over a 1500 m diameter region with a density that falls off as \( \sim r^{-2} \) (left panel), as described in Chapter 2. The baselines in the \( uv \)-plane formed by the pairwise combination of all the antennas for an instantaneous observation of a target at the zenith are shown in the middle panel. The right panel illustrates the relative density of visibility measurements after earth-rotation synthesis has been achieved by tracking a target field at \( R.A. = -60^\circ \) and \( Dec. = -30^\circ \) according to the observing rules outline in Chapter 2.
Figure 3-3 Illustration of three different weighting functions used for forming the dirty sky maps in Figures 3-6 through 3-8. Each profile is for a north-south cross-section through the center of the baseline distribution after earth-rotation synthesis on the target field, and are for natural weighting (solid line), uniform weighting (dotted line), and Gaussian weighting (dashed line). The profiles are normalized in this plot such that the maximum weight is one. The natural weighting profile favors the short baselines, while the uniform weight accentuates the long baselines. The Gaussian profile was chosen to be intermediate between the other two, and is characterized by an e-folding half width of 515 m.
3.2.1 Galactic Synchrotron Radiation

We will rely on empirical findings as much as possible to build our sky model. For the synchrotron foreground, there are a number of observations on which to base the model [Bridle, 1967, Landecker and Wielebinski, 1970, Haslam et al., 1982, Reich and Reich, 1988, Alvarez et al., 1997, Roger et al., 1999]. The spectrum of the Galactic synchrotron emission has been found to be a nearly featureless power-law with modest variations in the spectral index as a function of direction and frequency. At the frequencies of interest, the spectral index is given by $T \sim \nu^{-\beta}$, and has a typical value of $\beta \approx 2.5$. In general, it is steeper at high Galactic latitudes than toward the Galactic plane. The spectral index also steepens as a function of frequency to a maximum of $\beta \approx 2.8$ by $\nu \approx 1$ GHz, but is generally constant below about 200 MHz (see Chapter 6 and Shaver et al. [1999] for more details). The variation of the spectral index across the sky has a standard deviation of order $\sigma_{\beta} \approx 0.1$ on degree scales [Bridle, 1967, Reich and Reich, 1988, Platania et al., 1998, Roger et al., 1999].

To construct a model of the Galactic synchrotron radiation, we begin by extracting a region of the 408 MHz all-sky map of Haslam et al. [1982] centered on our target field at R.A. = 60° and Dec. = −30°. This region is comfortably within the range of declinations visible to the MWA and it has relatively low brightness temperature. In order to produce a reference map for the synchrotron emission at the frequencies of interest for the MWA, the temperatures in the Haslam et al. [1982] map are scaled from their 408 MHz values to 150 MHz using a constant spectral index of $\beta = 2.6$. The results of this scaling are shown in Figure 3-1. The reference map is then used to populate the spectra associated with every pixel in the sky model by randomly selecting a spectral index for each pixel from a Gaussian distribution with $\bar{\beta} = 2.6$ and $\sigma_{\beta} = 0.1$.

3.2.2 Extragalactic Continuum Sources

Normal galaxies, radio galaxies, and active galactic nuclei, form the majority of the extragalactic continuum sources [Santos et al., 2005]. Their contribution to the brightness temperature has been found to be of order 30 to 70 K at 178 MHz by Bridle [1967] by fitting an isotropic background component to measurements of spatial variations in the spectral index of the diffuse emission between 18 and 404 MHz. In the same analysis, it was found that the integrated brightness temperature due to extragalactic sources has a characteristic spectral index of $\beta = 2.7$, although with $\sim 10\%$ uncertainty, and therefore is somewhat steeper than the Galactic synchrotron spectral index. Individual sources have been observed to have significantly more variation in their spectral indices, however, spanning $-2 < \beta < 3$. The inverted cases ($\beta \approx -2$) are due to sources with strong synchrotron self absorption, such that the spectrum has already flattened at the frequencies of interest (as opposed to flattening below $\sim 10$ MHz, as would be more typical). In general, the spectral index of discrete extragalactic sources is related to the intensity of the source. It is steeper for brighter sources (with maximum $\beta \approx 3$) than for fainter sources, where $\beta \approx 2.5$ typical for sources with fluxes less than $S < 0.1$ Jy [Kellermann et al., 1969, Zhang et al., 2003, Cohen et al., 2004].

A number of surveys of radio sources have been performed at frequencies relevant to the MWA (see Cohen et al. [2004, their Fig. 1]). They include a survey at 74 MHz by Cohen et al. [2004], the 7C [McGilchrist et al., 1990] and 6C surveys [Hales et al., 1988] at 151 MHz, the 3CR survey [Laing et al., 1983] of bright ($> 10$ Jy) sources at 178 MHz,
WENSS [Rengelink et al., 1997] at 327 MHz, and the 5C5 survey [Pearson, 1975] at 408 MHz, as well as the FIRST survey [White et al., 1997] and NVSS [Condon et al., 1998] at 1.4 GHz. Analysis of the catalogs of radio sources derived from these and other surveys have yielded a solid understanding of the statistical properties of radio sources [Perley and Erickson, 1984, Windhorst et al., 1985, Wieringa, 1991, Fomalont et al., 1991, Windhorst et al., 1993].

The distribution of radio sources is found to obey Poisson statistics with only very weak angular clustering [Blake and Wall, 2002]. The brightest sources in the catalogs are relatively nearby objects and, thus, have differential source counts given by $N(S) \sim S^{-2.5}$, as would be expected for a population of objects distributed evenly in a Euclidean space. Weaker sources are generally more distant and the observed differential counts fall off more gradually, as $N(S) \sim S^{-1.8}$, due to evolution of the population and redshift effects. At extremely faint flux levels (in the $\mu$Jy regime), a new population of local starburst galaxies becomes visible and the differential counts again become steeper, with $N(S) \sim S^{-2.2}$. Differential source counts cannot asymptote to this power-law as $S \to 0$ since it is steeper than $N(S) \sim S^{-2}$ and an infinite radio flux would result. The differential counts must eventually flatten again with decreasing flux.

For the foreground subtraction strategy planned for the MWA, the brightest sources are subtracted individually. We therefore only need to include faint sources in our sky model up to a maximum flux threshold, $S_{\text{max}}$. In this scenario, it can be shown that most of the flux due to the faint sources is due to the brightest remaining sources near the cutoff limit. Thus, for a simple model of the differential source counts, it is adequate to use a single power-law dependence that is appropriate near $S_{\text{max}}$. In our case, we use the expression for
differential source counts given by Subrahmanyan and Ekers [2002],

\[ N(S) = 100 \, S_{\mu Jy}^{-2.2} \, \nu_{\text{GHz}}^{0.8} \, \text{arcmin}^{-2} \, \mu\text{Jy}^{-1} \]  

(3.10)

where \( S_{\mu Jy} \) is the flux measured in \( \mu Jy \) and \( \nu_{\text{GHz}} \) is the frequency measured in GHz. Using this expression, we can determine the value of \( S_{\text{max}} \) appropriate for the MWA. If thermal noise is not a limiting factor, then the flux at which at least one source can be expected on average per pixel in a map of the sky will roughly define the distinction between the faint sources responsible for confusion noise (with \( S < S_{\text{max}} \)) and the relatively bright sources (with \( S > S_{\text{max}} \)) that can be identified individually in a map and subtracted from the interferometric data to high precision. From Subrahmanyan and Ekers [2002, their Equations 2 and 3], we can use Equation 3.10 to write

\[ S_{\text{max}} = 40 \, \Theta_B^{1.7} \, \nu_{\text{GHz}}^{-0.7} \, \mu\text{Jy} \]  

(3.11)

where \( \Theta_B \) is the FWHM radius of the synthesized array beam in arcmin. For the MWA, we estimate \( \Theta_B \approx \lambda/D \), where \( D = 1500 \, \text{m} \) and \( 1 \lesssim \lambda \lesssim 4 \, \text{m} \). This yields a typical beam size of \( \theta \approx 5' \) at 150 MHz, with \( S_{\text{max}} \approx 2 \, \mu\text{Jy} \). Furthermore, if we consider a map that has been “cleaned” of the bright sources to a flux level, \( S_{\text{clean}} \); the residual flux per pixel, \( \Delta S \), due to sources fainter than \( S_{\text{clean}} \) (measured also in \( \mu\text{Jy} \)) is

\[ \Delta S = 8 \, \Theta_B \, \nu_{\text{GHz}}^{-0.4} \, S_{\text{clean}}^{0.4} \, \mu\text{Jy} \]  

(3.12)

and the differential surface brightness temperature due to these sources is given by

\[ \Delta T = 3 \, \Theta_B^{-1} \, \nu_{\text{GHz}}^{2.4} \, S_{\text{clean}}^{0.4} \, \text{mK} \]  

(3.13)

In practice, a flux cutoff level that yields a density of sources equivalent to one source per pixel underestimates \( S_{\text{max}} \), so it is common to assume a higher threshold by a factor of about 5 to 10. In this case, we set \( S_{\text{clean}} \approx 5S_{\text{max}} \approx 10 \, \mu\text{Jy} \), and calculate for the MWA at 150 MHz, \( \Delta S \approx 9 \, \text{mJy} \) and \( \Delta T \approx 7 \, \text{K} \).

These estimates are useful to understand the relative contributions of the faint sources to the foreground contamination. Rather than generate a sky model based on these numbers, however, we assemble a simulated catalog of radio sources. Since the integrated flux due to faint radio sources converges around 10 \( \mu\text{Jy} \), and since we want to limit the number of sources in our simulated catalog, we only model sources between 10 \( \mu\text{Jy} < S < 100 \, \mu\text{Jy} \). The number of sources per pixel in the target field is set by sampling the simulated catalog. In order to build an accurate catalog, the angular correlation of radio sources must be taken into account. Blake and Wall [2002] analyzed the 1.4 GHz NRAO VLA Sky Survey (NVSS) and found evidence for weak clustering of the continuum point sources. The angular correlation function that they derived is

\[ w(\theta) = 10^{-3} \, \theta^{-0.8}, \]  

(3.14)

for \( \theta \) measured in degrees. The magnitude of this clustering is much lower than for the galaxy correlation functions determined optically, and is the result of projection effects. Deep radio catalogs contain sources spanning a large range of redshifts since their continuum spectra do not allow for grouping by photometric redshift measurements. This causes the angular correlation function of radio continuum sources to dilute since unrelated volumes of the
Universe are superimposed in the observations.

We follow the method of González-Nuevo et al. [2005] to make a realization of the source counts in pixels that includes both the Poisson variance and the proper angular clustering. For each source in our catalog, we randomly assign a flux such that Equation 3.10 is satisfied. We also randomly assign a spectral index for each source from a distribution of spectral indices that is the sum of two Gaussian distributions. One distribution has $\bar{\beta} = 2.7$ and $\sigma_\beta = 0.1$, and the second distribution has $\bar{\beta} = 0$ and $\sigma_\beta = 0.6$ in order to account for the large variation of spectral indices in extragalactic continuum sources. Only 10% of the sources have their spectral indices drawn from the second distribution.

The large scale structure introduced by the angular clustering adds an additional variance to the distribution of point sources and the resulting brightness temperature in the sky map. Di Matteo et al. [2002] demonstrated that this clustering variance dominates the Poisson variance for measurements similar to those planned for the MWA. The combined variance, $\sigma_2^2$, in our sky map is given by

$$\sigma_2^2 = \sigma_P^2 \left(1 + \langle y \rangle \sigma_P^2 \right), \tag{3.15}$$

where $\sigma_P^2$ is the variance in the pixels due to Poisson statistics alone and $\langle y \rangle$ is a factor that accounts for the extra variance due to the angular correlation after taken into account the pixel shape and is given by Blake and Wall [2002, their Equation 2] for square pixels. For the angular correlation function in Equation 3.14, $\langle y \rangle \approx 2.2 \times 10^{-3} \Theta_B^{-0.8}$, where $\Theta_B$ in this case is the dimension of the square pixel in degrees. Thus, for the MWA, $\langle y \rangle \approx 0.02$. Combining this with $\sigma_P^2 = \langle N_S \rangle \approx 100$ for the typical number of sources per pixel in our sky model catalog, we find that $\sigma_2^2 \approx 3\sigma_P^2 \approx 300$.

Figure 3-4 displays our faint point source model. The left panel shows the number of faint continuum sources between $10 \mu$Jy and $10$ mJy per pixel in the sky model and the right panel shows the differential brightness temperature at 158 MHz due to summing the contributions of all the sources in each pixel.

### 3.3 Results

We begin by analyzing an ideal case as a reference model. Before we consider the effects of the instrumental response, we first review the ability of the polynomial fit technique to remove the foreground contributions in the image domain representation of our sky model itself. For all the cases discussed in this section, a 3\textsuperscript{rd}-order polynomial has been used for the fits. The sky model is shown in the top row of Figure 3-5 and the results of performing the foreground subtraction on the sky model are shown in the second row. The residuals are of order $\sim 1$ mK and represent the best-case scenario that can be achieved by the 3\textsuperscript{rd}-order polynomial fit method. In general, the amplitude of the residuals could be made arbitrarily low by increasing the order of the polynomial used in the fits, but only at the expense of extracting additional power from the more rapidly varying spectral fluctuations. Since, in actual observations, the polynomial fit will remove power from the foreground contributions as well as the redshifted 21 cm fluctuations, it is desirable to use the lowest order polynomial feasible because this will limit the effects on the redshifted 21 cm power spectrum to only the longest length scales in the spectral domain. Our reference case is equivalent to the studies performed by Furlanetto and Briggs [2004], and agrees well with their results.

Next, we include the instrumental response in the analysis, but no thermal noise. The definition of the MWA fiducial observation is the same as that used in Chapter 2. However,
Figure 3-5 Polynomial foreground subtraction technique applied to the sky model without including the instrumental response or thermal noise. The first row shows two cuts through the sky model. The left panel is an image plane for a spectral channel at $\nu = 157$ MHz, thus the pixels in the plot are indexed by $I(\theta_x, \theta_y)$. The right panel is a slice through frequency and one angular dimension, thus the pixels in the plot are indexed through $I(\nu, \theta_y)$. The smooth spectral properties of the foreground model are easily visible in the righthand plot. The middle row displays the residuals, $r$, in the image domain after a 3rd-order polynomial has been fit and subtracted from each line of sight in the data cube. Again, the left panel is an image plane and the right panel is slice through frequency and one angular dimension. The bottom row displays the unweighted residuals, $r'$, in the Fourier domain. In the middle and bottom rows, the residuals are due entirely to errors in the model and represent the best-case fits to the sky model with the 3rd-order polynomial.
Figure 3-6 Same as Figure 3-5, but for the polynomial subtraction technique applied to the dirty sky map generated with a uniform weighting of the visibility map. No thermal noise was included. In this case, the residuals, \( r \), in the image domain appear much larger than for the best-case fits to the sky model alone. However, after transforming back to the Fourier domain and unweighting, it becomes evident that the polynomial fit and subtraction removed the foreground contribution well for baselines shorter than \( \sim 500\lambda \).
Figure 3-7 Same as Figure 3-5, but for the polynomial subtraction technique applied to the dirty sky map generated with a natural weighting of the visibility map. No thermal noise was included. As with the uniform weighting, the residuals, $r$, in the image domain appear much larger than for the best-case fits to the sky model alone. In this case, however, even after transforming back to the Fourier domain and unweighting, the residuals remain much larger than the best-case scenario.
Figure 3-8 Same as Figure 3-5, but for the polynomial subtraction technique applied to the dirty sky map generated with a Gaussian weighting of the visibility map. No thermal noise was included. For this case, the residuals, $r$, in the image domain are the lowest for the three dirty sky maps tested, but are still two orders or magnitude greater than the best-case subtraction for the sky model with no instrumental response. Transforming back to the Fourier domain and unweighting yields residuals that are intermediate in amplitude compared to the other two dirty sky map cases.
in this case, there is an additional degree of freedom. The weighting function, $U$, that is applied to the simulated gridded measurements before transforming from the Fourier to the image domain (see Equation 3.7) must be specified before the calculation can be performed. We use three variations of the weighting function in order to study the robustness of the image domain foreground subtraction technique to this extra degree of freedom. As discussed in the last section, we model uniform weighting, natural weighting, and Gaussian weighting (see Figure 3-3). The results of the foreground subtraction technique performed on the simulated dirty sky maps for each of these weightings are presented in Figures 3-6 through 3-8, respectively.

The first case is that of uniform weighting and is shown in Figure 3-6. Here, the top row of the figure shows the dirty sky map, $I_D(\theta)$, produced by this weighting, the second row shows the results of the foreground subtraction in the image domain, $r(\theta)$, and the bottom row shows the results of the foreground subtraction after transforming back to the Fourier domain and removing the weighting, $r'(u)$. As discussed in Section 3.1.2, this weighting gives the best effective angular resolution, but would increase the uncertainty due to thermal noise in the derived sky map (if we had included thermal noise). At first glance, the results of the foreground subtraction in this case do not look especially promising since there are of order $\sim 1$ K fluctuations in the residual map, $r(\theta)$. However, after the residual map is transformed back to the Fourier domain, it becomes evident that the polynomial fit has actually done an excellent job of subtracting the foreground contamination from baselines within a radius of $\sim 500\lambda$, and only a poor job for baselines beyond this radius. The transition from the region where the foreground subtraction was successful to the region where it failed is rapid and coincides with the radius where visibility measurements become sufficiently sparse that there is no longer complete coverage.

Similar results are found for both the natural and Gaussian weightings, shown in Figures 3-7 and 3-8, although both of these cases fail to provide the level of foreground mitigation in the short baselines that the uniform weighting provided. The natural weighting is found to be the worst of the three cases. It is clear even in the second row of Figure 3-7 that the residual fluctuations are on much longer scales and have larger amplitudes than for the uniform weighting case, and this is confirmed explicitly in the plots of $r'(u)$ shown in the third row of the figure. The best explanation for the poor subtraction obtained with the natural weighting can be found in the profiles of the weighting functions shown in Figure 3-3. The hash-like fluctuations that are visible in the profile for the natural weighting are responsible for introducing ripples and fluctuations into the dirty sky map generated with this weighting. So, although natural weighting would result in the least thermal uncertainty in the dirty sky map, it does so at the expense of coupling the variations in the density of visibility measurements in the $uv$-plane to the dirty sky map. This is exactly what we would like to avoid in the sky map used for the foreground subtraction.

We have yet to explore the effects of introducing thermal noise into the simulated measurements, but in general, it would seem prudent to have as little thermal noise as possible in the estimate of the sky used for the foreground subtraction so as not to confuse the polynomial fits along each pixel. For this reason, we explored the Gaussian weighting function that retains the emphasis on the short baselines that exists in the natural weighting case, but without introducing the fine-scale variations in the density of the visibility measurements into the weighting function. The width of the Gaussian was arbitrarily set to 515 m in the $uv$-plane. The results of the foreground subtraction for this case are shown in Figure 3-8. Although this weighting function appears to provide the best results in $r(\theta)$, it is only intermediate between the uniform and natural weighting cases in $r'(u)$. Again, this
3.3.1 Thermal Uncertainty

In the preceding section, we found that, in the absence of thermal noise, the simple polynomial fit model was sufficient to remove the foreground contribution in simulated dirty sky maps for the MWA as long as uniform weighting of the gridded visibility measurements was used when producing the dirty sky map. In this section we explore the effects of including thermal noise. We simulated the thermal noise contribution using Equation 3.6. For the MWA, the thermal noise is a factor of $\sim 10^3$ larger than the best-case residuals after the foreground model is fit and removed from the dirty sky maps. Figure 3-9 illustrates the simulated thermal noise contribution in the gridded visibility measurements and in the dirty sky maps produced by the three fiducial weighting functions.

The analysis of the foreground subtraction technique proceeds as it did above for the cases without a thermal noise contribution. Again, three weighting functions were tested.
Figure 3-10 Magnitude of residuals, \( r'(u) \), after complex averaging over \( N_{\text{cells}} \) independent cells within a circular ring in the \( uv \)-plane. If the residuals are uncorrelated, the complex average will trend to zero like \( N_{\text{cells}}^{-1/2} \) as more cells are included. Circular rings are used since the residuals and thermal noise in the \( uv \)-plane are both expected to have circular symmetry. The panels show the trends (black lines) for 45 rings between \( 0 < |u| < 450\lambda \) in a single plane at \( \nu = 156 \) MHz. The left panel is for the residuals found without any thermal noise in the analysis. The middle panel is for the residuals found when thermal noise is included, and the right panel is for the thermal noise itself. Uniform weighting was used to produce the dirty maps used to find the residuals considered in the left and middle panels. The gray lines are guides for the eye proportional to \( N_{\text{cells}}^{-1/2} \), and are the same between the three panels. The residuals for the case including thermal noise (middle panel) and the thermal noise itself (right panel) follow very similar trends, indicating that the residuals after foreground subtraction in the presence of thermal noise are noise-like themselves.
Figure 3-11 Distribution in a circular region in the $uv$-plane defined by $445 < |u| < 455\lambda$ at 156 MHz of the real component of the residuals, $r'(u)$, after foreground subtraction. The three panels are for three variations of the analysis. The left panel shows the distribution of the residuals when thermal noise is not included in the analysis (in this case, only the sky model is processed with the instrumental response, as in Figures 3-5 through 3-8). The center panel shows the distribution of residuals when thermal noise is included, and the right panel displays the distribution of the thermal noise itself (equivalent to setting the sky model $I(\theta) = 0$). Both subtraction cases (left and center panels) used uniform weighting, although the weighting was found to make little difference in the thermal noise dominated residuals (in contrast to the noise-free cases). As in Figure 3-10, the typical magnitude of the residuals due to the model fit alone are a factor of $\sim 10^4$ lower than the residuals including the expected level of thermal noise in the MWA measurements. The units of the left panel reflect this difference. Again, we see that the thermal noise dominated residuals are very similar to the thermal noise itself, reaffirming that they will average down over many cells in the measured three-dimensional power spectrum.
In this scenario, however, we find that the residuals, \( r'(u) \), after subtracting the derived foreground model are similar between the weighting cases, and that the residuals are consistent with the simulated thermal noise (although not identical to input noise contribution) and show no obvious evidence of the foreground contribution. Since the MWA must use the anticipated symmetry of the redshifted 21 cm signal in the Fourier domain to average over many independent cells in order to achieve the necessary signal-to-noise ratio to constrain the power spectrum of the fluctuations, the remaining task, is to demonstrate that the residuals after foreground subtraction in the presence of thermal noise are themselves noise-like in that they will average down over many measured cells in the \( uv \)-plane. Figures 3-11 and 3-11 illustrate that this requirement appears to be satisfied. This is not an exhaustive analysis, but is a necessary first step and suggests that the contribution of the instrumental response in the presence of thermal noise does not cause the foreground subtraction technique to behave differently than for the ideal sky results of Wang et al. [2006] and McQuinn et al. [2006].

3.4 Discussion

The results presented in the preceding sections indicate that fitting and subtracting foreground components from dirty sky maps produced by the MWA may be sufficient to mitigate the foreground contamination that remains in the measurements after the target field has been chosen carefully and the bright extragalactic continuum sources have been identified and subtracted individually. This is particularly encouraging because the dirty sky map is the estimate of the sky that is the most sensitive to instrumental effects. Thus, we have established, in some sense, a worst-case scenario that appears manageable. But the results of the simulations also provide a note of caution. Care must be taken when forming the dirty sky maps to minimize the coupling between the distribution of the visibility measurements and the weighting function. In our analysis, this coupling is the primary influence on the ability of the foregrounds to be subtracted from dirty maps. It is not a coincidence that, even for the uniform weighting case, the residuals in the Fourier domain after subtraction increased rapidly for baselines longer than \( \sim 500\lambda \). This is the length scale at which significant variations and gaps in the \( uv \) coverage begin to occur. There is no way to mitigate these holes—as was possible for the variations induced by the natural weighting function by switching to the Gaussian weighting function—because there simply are not enough measurements at these length scales to fill the \( uv \)-plane. The only options are to either disregard the measurements that do exist and only retain the data within a radius of \( \sim 500\lambda \), or effectively interpolate in the missing regions.

That fact that this is even a possibility is a fundamentally new phenomenon only possible with the MWA (and other new radio telescopes with many antennas). In the paradigm of the VLA, the \( uv \)-plane is almost always sparsely sampled, and there are essentially no regions where the coverage can be said to be “complete”, as is the case for the MWA within \( \sim 500\lambda \). The reason that this is generally not a problem for telescopes like the VLA is that, in most cases, they are not attempting to make observations at or below the confusion limit imposed by faint extragalactic continuum sources. So, although the VLA has sparse coverage in the \( uv \)-plane, the sky that it is observing is also essentially sparse and algorithms like CLEAN are able to reconstruct a reasonable model of the point sources in the field of view. The fact that the MWA breaks both of these paradigms at the same time is extremely fortunate.
3.4.1 An Analytic Approximation

It is useful to construct an analytic approximation for the simulations discussed above in order to foster a more intuitive understanding of the processes responsible for the results. Condon [1974], Perley and Erickson [1984], and Artyukh [2003] provide a foundation for such a model, and Sault and Wayth [2006, 2007, MWA project documents] have adapted the results of these efforts for analyzing confusion noise in measurements of the MWA.

As we outlined in Section 3.1.2, a dirty sky map produced by an interferometer is, to a good approximation, the result of multiplying the true sky by the primary antenna tile beam and then convolving by the synthesized array beam. This statement is equivalent to the expressions given in Equations 3.1 through 3.7, as well as the instrumental response applied in Chapter 2 (Equations 2.1 through 2.3). If the sky consists only of the faint extragalactic continuum sources with Poisson statistics, then the variance in the intensity of the dirty sky map, $\sigma_D^2$, will be related to the variance in the true sky by

$$\sigma_D^2(\theta_x, \theta_y) = \sigma_T^2 \int \int B^2(\theta_x - \theta'_x, \theta_y - \theta'_y) P^2(\theta_x, \theta_y) \, d\theta_x \, d\theta_y$$

(3.16)

where $\sigma_T^2$ is the variance in the intensity of the true sky, $B$ describes the response of the synthesized array beam as a function of angle, and $P$ describes the primary antenna tile beam. This equation can be simplified by assigning simple models for the response profiles of the synthesized array beam and the primary antenna tile beam. We take the beams to be described by tophat functions, such that the response is defined to be one within a region of diameter $\Theta_B$ (for the synthesized array beam; $\Theta_P$, for the primary antenna tile beam). Outside this region, the response is taken to be $B_{\text{rms}} \ll 1$ for the synthesized beam, and zero for the primary antenna tile beam. Thus, we include an allowance for the side lobes of the synthesized beam (with the $B_{\text{rms}}$ contribution), but not for the primary beam. With these simplifications, the integrals in Equation 3.16 can be performed and the expression reduces to

$$\sigma_D^2 \approx \sigma_T^2 \left(1 + B_{\text{rms}}^2 \frac{\Omega_P}{\Omega_B} \right)$$

(3.17)

where $\sigma_T^2$ is the variance due to the faint sources (between $S_{\text{min}}$ and $S_{\text{max}}$) given in Equation 3.15 and accounts for the extra variance due to the weak angular clustering, $\Omega_B \approx \Theta_B^2$ is the solid angle of the synthesized array beam, and $\Omega_P \approx \Theta_P^2$ is the solid angle of the primary antenna tile beam.

Inspecting this simplified form of Equation 3.16, we see that when the synthesized array beam has no side lobes, the variance in the dirty sky map is equal to that of the true sky map (after gridding). This is equivalent to complete coverage of the $uv$-plane with visibility measurements, and corresponds to the cases considered by Furlanetto and Briggs [2004] and Wang et al. [2006]. But when side lobes are included in the model of the synthesized array beam, the variance in the dirty map is increased above that of the true sky map. For sparse coverage in the $uv$-plane, $B_{\text{rms}}^2$ will be given approximately by the inverse of the number of independent measurements in the $uv$-plane. In the limiting case of a single, very short integration, (such as a "snapshot" observation), the number of independent measurements is equal to the number of baselines. For the MWA, with its $\sim 125,000$ instantaneous baselines, $B_{\text{rms}}^2 \approx 10^{-5}$ in the worst-case, and is even better (lower) for long integrations when earth-rotation synthesis increases the number of independent measurements. Using $\Theta_B \approx 5'$ and $\Theta_P \approx 30^\circ$, we can estimate

$$B_{\text{rms}}^2 \frac{\Omega_P}{\Omega_B} \approx 1.$$  

(3.18)
Thus, the variance due to the side lobes in the synthesized array beam is comparable in magnitude to the inherent variance in the intensity of the true sky. This situation is brought on almost entirely by the large field of view of the MWA, a feature which has, up to this point, been considered an advantage because it improves the sensitivity of the telescope to the power spectrum of fluctuations in the redshifted 21 cm signal. The large field of view acts to oppose the advantages of the dense coverage of the $uv$-plane in this instance by introducing more variance into the dirty sky map. In this regime, in order to produce an estimate of the sky without the effects of the side lobes of the synthesized array beam, the full inversion of $A$ is required since it cannot be approximated as a sparse matrix.

3.4.2 Translating Spatial Variations into Spectral Variations

Just because there is additional side lobe-induced noise in the dirty sky maps does not necessarily mean that the foreground subtraction will be adversely affected. If the side lobe patterns were the same for each image plane in the data cube, then the spectrum along each pixel would simply receive a faint mirror contribution from the spectra along all the other pixels. This would be essentially equivalent to extending our simulated radio catalog below $\mu$Jy levels, or adding more faint sources to each pixel. It is the variation with frequency of the side lobe-induced noise that causes trouble in the foreground subtraction technique studied in this chapter.

We can estimate the importance for foreground subtraction of gaps in the $uv$ coverage at a given length scale. In general, the synthesized array beams at each frequency measured by the MWA will be identical, but proportional in overall size to the inverse of the frequency, since the distribution of visibility measurements is identical, but scales with the wavelength in the $uv$-plane. The coherence length of fluctuations in visibility coverage in the $uv$-plane is mapped to the spectral domain by this frequency-dependent scaling. The relationship between the coherence length in the $uv$-plane, $l_u$, and the coherence length in the frequency domain, $l_\nu$, is given by

$$\frac{l_u}{|u|} = \frac{l_\nu}{\nu_0}$$

where $|u|$ is the radius of interest in the $uv$-plane at frequency $\nu_0$. From this expression, it is easy to see that smooth features in the coverage of the visibility measurements near the origin of the $uv$-plane do not introduce significant fluctuations into the spectral domain because $l_\nu \to \infty$ as $|u| \to 0$, whereas variations in the coverage or weighting at large radii in the $uv$-plane will have much shorter coherence lengths in the spectral domain because $l_\nu \to 0$ as $|u| \to \infty$. In order to produce fluctuations in the spectral domain that have sufficiently long periods that they can be fit by a low-order polynomial across the observed bandwidth requires $l_\nu \gtrsim B$, where $B$ is the bandwidth. For the MWA, with $\nu_0 \approx 150$ MHz and $B = 8$ to 32 MHz, this condition requires that

$$l_u \gtrsim |u|\frac{B}{\nu_0} \approx \frac{|u|}{10}$$

in order to prevent fluctuations in the spatial coverage from impacting the polynomial fits used in the foreground subtraction. Referring to Figures 3-6 through 3-8, this condition is clearly not met above $|u| \approx 500 \lambda$, where the foreground subtraction fails in all cases.

Another useful metric is to determine the radius in the $uv$-plane where the condition expressed in Equation 3.20 is always satisfied, regardless of the density of the visibility
measurements. This is the radius at which \( l_u \) becomes equal to the typical extent of a single visibility measurement. The localization of a single visibility is determined by the field of view, and thus, the Fourier domain representation of the window function, \( W(\vec{u}) \), used in Chapter 2. This characteristic length (in wavelengths) is approximately \( l_u \approx \Theta^{-1} \).

Combining this information with the condition derived above, we find for the MWA

\[
|u| \lesssim \Theta^{-1} \frac{\nu}{B} \approx 20, \tag{3.21}
\]

where we note that \( |u| \) is measured in wavelengths. Thus, at \( \nu = 150 \) MHz, only baselines less than approximately 40 m are guaranteed not to produce disruptive fluctuations in the spectral domain (this says nothing about the properties of the actual sky preventing a good polynomial fit, however). This distance scale encompasses the core of the MWA, where the antennas will be packed at maximum density, and should, therefore, account for approximately 80 of the antennas, or no fewer than about 6400 baselines. This is a small fraction of the 125,000 baselines, however, so it is not a significant guarantee on its own that the MWA will be able to successfully apply the polynomial fit method to foreground subtraction in the dirty sky maps.

### 3.4.3 The Role of Long Baselines

Despite the increased susceptibility of long baselines to introducing undesirable fluctuations into the spectral domain, and the inability of the foreground subtraction technique to remove the contamination from long baselines, it is important to point out that they still serve an important purpose. We have considered only one aspect of the full analysis pipeline for EOR observations in this chapter, that of faint source subtraction. Long baselines are still expected to be necessary in order to peel away the bright sources in the field of view since the peeling process requires a precise knowledge of the position of each bright source, which is greatly improved with long baselines. Given this division of labor between baseline lengths, the appropriate balance between the relative number of antennas contributing to long baselines versus short baselines becomes an interesting question. We saw in Chapter 2 that the ideal baseline distribution for measurements of the redshifted 21 cm power spectrum is extremely condensed. Now, we have shown that the subtraction of faint continuum sources in dirty sky maps is most successful on these short baselines, as well. In principle, it would seem that only relatively few long baselines should be included in the design since they primarily contribute to subtraction of bright sources, for which sensitivity will be much less critical. With additional study, perhaps this will prove to be another of the design choices with (sometimes unexpected) dual advantages that can be exploited by the MWA.
Chapter 4

Forecasted Constraints on Fundamental Cosmological Parameters

This chapter is adapted from the paper “Constraints on Fundamental Cosmological Parameters with Upcoming Redshifted 21 cm Observations” by Bowman et al. [2007b].

In this chapter, we employ the calculations of the sensitivity of the MWA from Chapter 2 to forecast the ability of the instrument to place constraints on cosmological models using measurements of the redshifted 21 cm power spectrum. In order to treat the best-case scenario and to simplify the analysis, we have chosen to assume that reionization has not yet begun by the target redshift range of $8 < z < 10$. This assumption represents the most optimistic scenario generally consistent with the existing evidence about when reionization began from quasar absorption line measurements [Djorgovski et al., 2001, Becker et al., 2001, Fan et al., 2003, Wyithe and Loeb, 2004b] and from Thomson scattering measurements by the WMAP satellite [Spergel et al., 2006]. These measurements suggest reionization occurred at redshifts $z \gtrsim 6$ and $z \approx 11$, respectively.

We begin in Section 4.1 by briefly reviewing the results of Chapter 2. In Section 4.2, the method to forecast constraints on cosmological parameters is described in terms of statistical errors using a Fisher matrix treatment of the full three-dimensional power spectrum. The calculation marginalizes over several cosmological parameters and two anticipated contributions related to the astrophysical foreground contaminants. The results are discussed in Section 4.3 and encapsulated in Figure 4-2. A similar study has been performed by McQuinn et al. [2006]. While we analyze the characteristics relevant to constraining cosmological models with the initial generation of experiments and in the presence of two nuisance contributions, their analysis explores the benefit of combining redshifted 21 cm measurements with information from other experiments, such as WMAP and Planck. Together, the two efforts provide a thorough overview of the potential of future redshifted 21 cm power spectrum measurements to contribute to cosmology.
4.1 The Measurement

The fluctuation power spectrum of redshifted 21 cm emission in the sampled volume of space is mapped to an instrumental response by the convolution of the power spectrum, \( P_{HI}(k) \), with the instrumental window function, \( W(k) \), according to

\[
C^{HI}(k) = \langle |\Delta I(k)|^2 \rangle = \int P_{HI}(k) |W(k-k')|^2 d^3k',
\]

where the window function is given by the Fourier transform of the instruments angular and frequency response, and is very sharply peaked for the cases we will consider (see Section 4.1.2).

There is an inherent uncertainty in the measurement of the redshifted 21 cm power spectrum from cosmic sample variance. The uncertainty can be estimated by dividing the observed three-dimensional Fourier space into a large number of independent cells, where the volume of each cell is approximately the size of the window function, \( W(k) \). For the regime of interest in our calculations, the variance in the measured power spectrum is then

\[
C_{ij}^{V}(k) \approx C^{HI}(k) \delta_{ij},
\]

where the indices \( i \) and \( j \) run over all the independent cells in the sampled volume.

4.1.1 Contributions from Astrophysical Foregrounds

The redshifted 21 cm emission is not the only source of power in low-frequency radio observations. Astrophysical foregrounds are several orders of magnitude stronger and dominate the measured signal. The foregrounds include free-free and synchrotron emission from the Galaxy, extragalactic point sources, and free-free emission from electrons in the IGM.

The bright foregrounds manifest themselves in several ways. First, the Galactic emission dominates the thermal noise in the measurements, especially at lower frequencies. Although much of this large-scale emission is expected to be resolved-out by interferometric observations, its effect on antenna temperature remains.

The contribution to the measured power spectrum due to thermal noise in the individual visibility measurements is substantial, but white \( \langle C^N(k) \rangle = \text{constant} \), and therefore should be readily removed. In our calculations, we will assume that it has been removed (but allow for an imperfect subtraction by including an offset parameter in the model). After this subtraction, the thermal uncertainty per independent cell in the three-dimensional measured power spectrum can be approximated, in instrumental coordinates, using [Morales, 2005, His Eqn. 11]

\[
[C_{ij}^N(u)]_{\text{rms}} = 2 \left( \frac{2k_B T_{sys}}{\epsilon dA d\eta} \right)^2 \frac{1}{B \bar{n}(u) t} \delta_{ij},
\]

where \( dA \) is the physical antenna area, \( d\eta \) is the inverse of the total bandwidth, \( k_B \) is Boltzmann’s constant, \( T_{sys} \) is the total system temperature, \( \epsilon \) is the efficiency, \( B \) is the total bandwidth, \( \bar{n} \) is the time average number of baselines in an observing cell, and \( t \) is the total observation time. Although the measured power spectrum in this case is three-dimensional, the parameters in Equation 4.2 are approximately independent of frequency within the observing band. The thermal uncertainty per independent cell, therefore, is taken also to be independent of \( \eta \) (and \( k_B \)). Additionally, for the MWA, the time-averaged visibility distributions are expected to be dense and to have nearly circular symmetry, thus \( \bar{n}(u) \) depends only on \( \sqrt{u^2 + \delta^2} \).
Table 4.1. Fiducial Observation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MWA</th>
<th>(MWA5000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Array configuration, $\rho(r)$ (m$^{-2}$)</td>
<td>$\sim r^{-2}$</td>
<td></td>
</tr>
<tr>
<td>Array diameter, $D$ (m)</td>
<td>1500 (3000)</td>
<td></td>
</tr>
<tr>
<td>Bandwidth, $B$ (MHz)</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>Frequency resolution (kHz)</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Number of antennas, $N$</td>
<td>500 (5000)</td>
<td></td>
</tr>
</tbody>
</table>

Note. — Array parameters for the MWA and MWA5000 reference experiments.

The second important effect of astrophysical foregrounds is that they produce their own signatures in the measured power spectrum using a technique like that studied in Chapter 3. The dominate foreground signature is expected to be due to extragalactic point sources. We will assume that the foregrounds have been cleaned from the measured signal. Even the best subtraction left residual traces of the contaminant, however, and a perfect model may produce an imperfect subtraction simply due to thermal uncertainty in the measurement. Such a subtraction error would result in a power spectrum signature following the shapes of the model components [Morales et al., 2005]. For the low-order polynomial model used in Chapter 3, the principle residual signature due to this kind of subtraction error can be quantified as a $k_3$-dependent factor, $C_F(k_3) \propto k_3^{-2}$, which is the Fourier transform of the linear contribution to the polynomial. Since extragalactic point sources are expected to be the most significant astrophysical foreground contaminant we include this contribution in our analysis as a nuisance term.

4.1.2 Reference Experiments

For the analysis in this chapter, we define the fiducial observation with the MWA to be of a single target field with 1000 hours integration during the most favorable circumstances. Additionally, we set the frequency coverage to $125 < f < 157$ MHz, which spans $10 > z > 8$, and treat the bandwidth as four consecutive 8 MHz regions, each of which spans approximately $\Delta z = 0.5$. In principle, the observed 21 cm power spectrum is varying continuously with redshift due to cosmology-dependent effects. By dividing our measured volume of space into thin regions in redshift we may ignore cosmic evolution within each region. We also include a second reference design that is based on an expanded MWA configuration, similar to the one considered by Wyithe et al. [2005]. Dubbed the MWA5000, the expanded array contains 5000 antennas distributed over a 3000 m diameter region. The density of antennas remains $\sim r^{-2}$ and capped at one per 18 m$^2$. The antenna response, instrument bandwidth, and frequency resolution are also unchanged. Tables 4.1 and 4.2 summarize the properties of the reference experiments, and the same as Tables 2.1 and 2.2, but now with the MWA5000 parameters included. In both experiments all antenna elements are correlated to preserve the large field of view.

Figure 4-1 illustrates the relative sensitivities of the two reference experiments by plot-
Figure 4-1 Uncertainties (stepped lines) in spherically averaged power spectrum measurements due to thermal and cosmic sample variance for a single $\Delta z \simeq 0.5$ (8 MHz) region at redshift $z = 8$ observed by the MWA and MWA5000 reference experiments relative to the fiducial, redshifted 21 cm power spectrum (dashed line). The thermal uncertainty (thin gray lines) is overshadowed by uncertainty due to cosmic sample variance at large scales (small $k$). The combined uncertainty is shown as black lines. Due to the limited redshift range over which cosmic evolution can be neglected ($\Delta z \simeq 0.5$), the measurements in the light-gray region to the left of the vertical line are constrained only by angular fluctuations and will thus be most affected by astrophysical foreground contamination. The predicted constraints on the cosmological model parameters in Figure 4-2 and Tables 4.5 and 4.6 were derived using the full three-dimensional power spectrum, and thus, this plot of the uncertainty in the spherically averaged measurement is best used as a guide to illustrate which scale sizes will be most important in the constraints.
Table 4.2. Redshift Dependent Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$z = 8$</th>
<th>$z = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular resolution ($^\circ$)</td>
<td>0.073 (0.036)</td>
<td>0.089 (0.044)</td>
</tr>
<tr>
<td>Antenna collecting area, dA (m$^2$)</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>Antenna response scale, $\Theta_P$ ($^\circ$)</td>
<td>31</td>
<td>38</td>
</tr>
<tr>
<td>Frequency (MHz)</td>
<td>158</td>
<td>129</td>
</tr>
<tr>
<td>System temperature, $T_{sys}$ (K)</td>
<td>440</td>
<td>690</td>
</tr>
</tbody>
</table>

Note. — Characteristics of the fiducial observation that depend on frequency, and thus on redshift, are listed along with their values at the $z = 8$ and $z = 10$ edges of the observation redshift range. In the calculations, the values of the fiducial parameters at intermediate redshifts were linearly interpolated from the end points (except for $\Theta$, see text). For the angular resolution, the MWA and MWA5000 have different properties due to the larger size of the MWA5000 (whose values are listed parenthetically).

ting their uncertainties in spherically averaged bins due to thermal noise and cosmic sample variance along with a fiducial, redshifted 21 cm power spectrum. The two reference experiments share a common level of uncertainty at large spatial scales (small $k$) due to cosmic sample variance since their fields of view are identical. At smaller spatial scales ($k \gtrsim 0.1$), however, the larger collecting area of the MWA5000 reduces its thermal uncertainty in a given integration time compared to that of the MWA. The MWA5000 is nearly an order of magnitude more sensitive than the MWA at these scales.

4.2 Forecasting Method

The Fisher information matrix provides a convenient method for translating uncertainties in power spectrum measurements to constraints on cosmological parameters [Tegmark et al., 1997]. The minimum errors are calculated using the Fisher matrix:

$$ F_{ab} = \sum_i \frac{1}{\sigma^2_{(k_i)}} \frac{\partial P(k_i)}{\partial p_a} \frac{\partial P(k_i)}{\partial p_b}, $$

(4.3)

where $\sigma^2 = \sigma_N^2 + \sigma_F^2$ is the combined uncertainty per independent cell due to thermal noise and cosmic sample variance, $P$ is the measured power in a cell given by Equation 4.4, and $p$ is the set of model parameters. Taking the square roots of the diagonal elements of the inverse of Fisher matrix gives the errors.

The model we use for the observed power spectrum represents the measurement after substantial data reduction has occurred, including instrumental calibration and astrophysical foreground subtraction. At this stage, the observed power spectrum is parameterized
Table 4.3. Model Power Spectrum Parametrization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Fiducial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalization (at $k = 0.05 \text{ Mpc}^{-1}$)</td>
<td>$A$</td>
<td>1.00</td>
</tr>
<tr>
<td>Matter density</td>
<td>$\Omega_M$</td>
<td>0.30</td>
</tr>
<tr>
<td>Baryon density</td>
<td>$\Omega_b$</td>
<td>0.05</td>
</tr>
<tr>
<td>Hubble constant</td>
<td>$h$</td>
<td>0.70</td>
</tr>
<tr>
<td>Scalar spectral index (at $k = 0.05 \text{ Mpc}^{-1}$)</td>
<td>$n_s$</td>
<td>1.00</td>
</tr>
<tr>
<td>Running index slope (at $k = 0.05 \text{ Mpc}^{-1}$)</td>
<td>$\alpha_s \equiv \frac{dn_s}{d\ln k}$</td>
<td>0.00</td>
</tr>
<tr>
<td>Residual foreground amplitude</td>
<td>$F$</td>
<td>0.00</td>
</tr>
<tr>
<td>Residual thermal offset</td>
<td>$N$</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note. — Eight parameters used to describe the measured power spectrum, and their values for the fiducial model. The first six rows give the basic parameters used to describe the matter power spectrum and the last two rows give the parameters to quantify the residual astrophysical foreground and residual thermal offset terms. In addition, we constrain $\Omega_K = 0$ and $\Omega_A = 1 - \Omega_M$.

Based on three contributions:

$$ P(k) \equiv P_{HI}(k) + P_F(k_3) + P_N, \quad (4.4) $$

where the first term is the redshifted 21 cm contribution, the second is the primary residual astrophysical (extragalactic point source) foreground contribution discussed in Section 2.1, and the third is the residual white thermal noise contribution (also discussed in Section 2.1).

For our assumption that reionization has heated the neutral hydrogen in the IGM but not yet produced significant features in the emission patterns of the gas by the target redshifts, the redshifted 21 cm contribution to the model power spectrum follows the matter power spectrum and can be expressed at a given redshift as

$$ P_{HI}(k) \equiv C_{Jy}^2 P_{\delta\delta}(k \mid A, \Omega_M, \Omega_b, h, n_s, \alpha_s), \quad (4.5) $$

where $C_{Jy}$ is the strength of the 21 cm emission from mean-density neutral hydrogen gas in the IGM and $P_{\delta\delta}(k)$ is the matter power spectrum. At the target redshifts, fluctuations in the matter density are expected generally to be in the linear regime over the range of spatial scales sampled. For redshift $z = 8$, the perturbations at the smallest scales constrained by the measurements ($k \simeq 1 \text{ Mpc}^{-1}$) will be approximately 40%; and at the largest scales ($k \simeq 0.01 \text{ Mpc}^{-1}$), the perturbations will be less than 1%. In contrast to measurements of the matter power spectrum through large scale structure surveys of galaxies, redshifted 21 cm measurements probe directly the baryon density perturbations in the IGM, which follow closely the dark matter density perturbations. The effect of small deviations from the purely linear regime can be modeled easily in this case and we omit, therefore, this complication in our analysis. We use six parameters, $p = \{A, \Omega_M, \Omega_b, h, n_s, \alpha_s\}$, to describe
the matter power spectrum. These parameters are summarized in Table 4.3 along with their fiducial values. Additionally, we constrain $\Omega_K = 0$ and $\Omega_\Lambda = 1 - \Omega_M$. To compute the matter power spectrum, we use CMBFAST [Seljak and Zaldarriaga, 1996] and do not include velocity distortions or deviations from linear dynamics in the model.

As described in Section 2.1, the primary residual foreground contribution is a function of $k_3$ and is expressed as

$$P_F(k_3) = F \times c_{\text{norm}} \left( \frac{k_3}{k_{\text{norm}}} \right)^{-2},$$

where $F$ is an amplitude scale factor of order unity and $c_{\text{norm}}$ is a normalization constant. The thermal offset contribution does not vary with $k$ and is defined as

$$P_N = N \times c_{\text{norm}},$$

where $N$ is also an amplitude scale factor of order unity. For the Fisher matrix analysis, both $P_F$ and $P_N$ are normalized by setting $k_{\text{norm}} = 0.01$ Mpc$^{-1}$ and evaluating $c_{\text{norm}} = P_{HI}(k_{\text{norm}})$ at redshift $z = 8$. Thus, the magnitudes of $F$ and $N$ give the amplitudes of the residual contributions approximately relative to the peak value of the redshifted 21 cm power spectrum. Adding $F$ and $N$ to the set of six cosmological parameters yields the eight parameters, $p = \{A, \Omega_M, \Omega_b, h, n_s, \alpha_s, F, N\}$, which constitute the complete set of free variables of our base model.

4.3 Forecasted Parameter Constraints

The results of performing the Fisher matrix calculations for the two reference experiments described in Section 4.1.2 are summarized in Table 4.4, which lists the forecasted 1-$\sigma$ uncertainties on the model parameters. These values indicate that observations with the MWA would not constrain the cosmological parameters significantly. The only exception to this finding is for $\alpha_s$, which could be constrained to $0 \pm 0.04$ under the favorable assumptions used in the model. The second reference experiment, MWA5000, does constrain reasonably all the model parameters. Table 4.4 shows that this reference experiment provides constraints at levels approximately equivalent to those from the first-year WMAP results [Spergel et al., 2003, Their Table 10].

The elements of the covariance matrices for each reference experiment are given in Tables 4.5 and 4.6, and additional information about the forecasted constraints for the MWA5000 is provided in Figure 4-2, which shows marginalized error ellipses for two-parameter combinations of the model parameters. The properties of these results are discussed in detail in the remainder of this section.

4.3.1 Parameter Dependencies

To facilitate a conceptual understanding of the origin of the constraints on the model parameters, it is instructive to individually vary the cosmological parameters and compare the resulting redshifted 21 cm signals. On all but the largest length scales, the experimentally observed power spectrum is well approximated by

$$P_{\text{HI}}(u, v, \eta) \approx W^2 C_{\text{HI}}^2 \left[ P_{\delta\delta}(k) \frac{d^3k}{dudvd\eta} \right],$$

(4.8)
Figure 4-2 Marginalized elliptical error regions for pairs of model parameters for the MWA5000 reference experiment. The contours are for 68% (solid) and 95% (dashed) likelihood. The first two columns are set apart to emphasize the distinction between the nuisance components in the model due to astrophysical foregrounds and the cosmological terms. The bounds of the plotted regions for the cosmological parameters are set at twice the uncertainties (± 2-σ) reported in the WMAP first-year results for the respective parameters. Thus, if the dotted ellipse is visible in a particular plot, the constraints on the parameters would be an improvement over WMAP. It is evident that the MWA5000 would do a relatively good job of constraining $\Omega_M$, $n_s$, and $\alpha_s$, and a comparatively poor job of constraining $\Omega_b$ and $h$. The inability of the MWA5000 to constrain the scalar amplitude, $A$, of the power spectrum is due to degeneracies with other parameters, in particular $\Omega_b$ and $h$, but also $n_s$. 
Figure 4-3 Redshifted 21 cm power spectrum for the fiducial cosmological model parameters, shown with four example variations of the parameter values. The top and bottom panels show the same elements plotted in different units. The thick solid line is for the fiducial model with standard cosmological parameters ($\Omega_0 = 0.05$, $\Omega_M = 0.30$, $h = 0.70$, $n_s = 1.00$, $\alpha_s = 0$). The other curves are produced by adjusting one cosmological parameter from the fiducial: $\Omega_0 + 10\%$ (dotted), $h + 10\%$ (dash), $n_s + 10\%$ (dash dot), and $\alpha_s = 0.1$ (thin solid). The dark shaded region indicates the 1-$\sigma$ uncertainty due to cosmic variance within the observed field and the light shaded region (at large $k$) is the thermal uncertainty for the MWA5000 reference experiment. As in Figure 4-1, the measurements to the left of the vertical line are constrained by angular fluctuations only and will be most sensitive to astrophysical foreground contamination.
where $W$ is the integrated value of the observational window function and represents the signal strength for a given instrument, and $d^3k/dudv\eta$ is the Jacobian for converting the units of the matter power spectrum to the observed units. The relationships between $u, v, \eta$ and $k$ are [Morales and Hewitt, 2004]:

$$\begin{align*}
[k_1, k_2] = \frac{2\pi}{D_M(z)} [u, v] \\
k_3 \approx \frac{2\pi \nu_{21} H_0 E(z)}{c (1+z)^2} \eta,
\end{align*}$$

(4.9)

(4.10)

where

$$E(z) \equiv \sqrt{\Omega_M (1+z)^3 + \Omega_k (1+z)^2 + \Omega_{\Lambda} (1+z)^{3(1+w)}},$$

(4.11)

and $D_M(z)$ and $\nu_{21} = 1420.4$ MHz are the transverse comoving distance and the rest-frame frequency of the hyperfine line, respectively.

In Equation 4.8, there are three separate contributions which may be affected by changing the cosmological parameters: the matter power spectrum itself, the neutral hydrogen emission factor, and the coordinate mapping between $k$ and $u, v, \eta$. The mapping between $k$ and $u, v, \eta$ produces two effects when measuring the power spectrum. First, the region in $u, v$ and $\eta$ occupied by the observed power spectrum depends on the cosmological model. This effect is analogous to the observed CMB power spectrum shifting in $\ell$ for different cosmological models, but since the redshifted 21 signal is three-dimensional, there may be a difference in scaling between the sky-plane and line-of-sight dimensions. This additional degree of freedom gives rise to the AP test [Alcock and Paczynski, 1979] (see also Seo and Eisenstein [2003] for an applicable discussion of constraining dark energy using baryon acoustic oscillations in power spectra from three-dimensional galaxy redshift surveys).

The second effect of coordinate mapping in the observed power spectrum is the presence of a significant cosmology-dependent amplitude factor in the measured power spectrum due to the Jacobian contribution in Equation 4.8. Differentiating and combining Equations 4.9 and 4.10 yields the Jacobian:

$$\frac{d^3k}{dudv\eta} = \frac{(2\pi)^3 \nu_{21} H_0 E(z)}{c (1+z)^2 D_M^2(z)}.$$  

(4.12)

The effect of the Jacobian amplitude factor is further compounded by the hydrogen emission factor, $C_{J_H}$. This term is unique to redshifted 21 cm experiments and provides a significant boost to the cosmology-dependent amplitude factor. Assuming the spin temperature of the neutral hydrogen in the IGM is significantly warmer than the CMB, the hydrogen emission factor goes as [Morales and Hewitt, 2004, Their Equation 13 and Their Appendix A]

$$C_{J_H} \propto \frac{\Omega_b h}{E(z)} \bar{x}_{HI},$$

(4.13)

where $\bar{x}_{HI}$ is the mean neutral fraction (taken to be unity in this paper). Using Equations 4.12 and 4.13 and recalling that $D_M(z) \propto h^{-1}$, the amplitude of the observed power spectrum is now seen to be proportional to

$$P_{HI} \propto \frac{\Omega_b^2 h^5}{(1+z)^2 E(z)} P_{\delta \delta}.$$  

(4.14)
Equation 4.14 indicates that the amplitude of the redshifted 21 cm fluctuations does contain useful information about the cosmological model (with the caveat that the HII spin temperature must be known). This is in contrast to measurements of the matter power spectrum by galaxy clustering surveys, where the connection between the observed amplitude of the power spectrum and the underlying physical amplitude remains weakly understood.

Figure 4-3 shows the spherically-binned redshifted 21 cm power spectrum signal for the fiducial model (thick line, \( \Omega_b = 0.05, \Omega_M = 0.30, h = 0.70, n_s = 1.00, \alpha_s = 0 \)) along with the expected signals obtained by varying individual parameters. We can see that varying \( \Omega_b \) or \( h \) primarily results primarily in a simple scaling of the observed power spectrum, as expected from Equation 4.14. Thus these parameters are highly degenerate with each other and with the amplitude factor A, as shown in Figure 4-2 and Table 4.6.

The primordial power spectrum slope \( n_s \) and running of the spectral index \( \alpha_s \) do not enter into the overall amplitude factor in Equation 4.14 for the observed power spectrum, but instead affect the shape of the underlying matter power spectrum. Increasing \( n_s \) lowers the power at small \( k \) and increases the the power at large \( k \), whereas increasing \( \alpha_s \) boosts the power spectrum at both extremes. Because these parameters do not strongly affect the amplitude normalization, they are relatively independent of \( A, \Omega_b, \) and \( h \) as shown in Figure 4-2 and Table 4.6. They are also independent of our foreground parameter, \( F \), because the shape they introduce is different than the \( k_3^{-2} \) shape of the residual foreground contamination. Thus \( n_s \) and \( \alpha_s \) are the best constrained cosmological parameters for the planned redshifted 21 cm observations.

The last class of observational effects due to the cosmological model is from the coordinate mapping. Most significantly, for an increase in \( h \), a radially inward shift of scales (see Equations 4.9 and 4.10, with \( D_M(z) \propto h^{-1} \)) counteracts the amplitude increase from the \( C_{Jy} \) and Jacobian factors over much of the observed range in \( k \)-space. Increasing \( \Omega_M \) also shifts scales inward (through \( E(z) \) and \( D_M(z) \)), but does so differently for the line-of-sight direction and the transverse directions. This AP effect cannot be represented easily in Figure 4-3, however, we have included it in all the constraints. The significance of the effect is limited since we take \( \Omega_A = 1 - \Omega_M \). A 10% increase in \( \Omega_M \) yields an approximately 4% reduction in scale, but only about 1% distortion between the directions. In general, coordinate based effects may be better constrained with observations targeting the quasar bubbles during reionization since sharper spatial features in the power spectrum may emerge during reionization due to characteristic sizes of reionized regions and Stromgren spheres around quasars.

### 4.3.2 Degeneracies in Amplitude

The uncertainty in \( A \) is considerable. Table 4.4 shows that the MWA5000 only constrains \( A \) to within 45%. The origin of this uncertainty is explained by the amplitude factor discussed above and is clearly evident in Table 4.6, which displays the large degeneracies (as normalized covariance factors with magnitudes close to unity) between \( A, \Omega_b, h, \) and more unexpectedly, \( n_s \). The degeneracy with \( n_s \) is due to the range of scales to which the experiments are sensitive, which is primarily after the pivot point for the primordial power spectrum spectral index \( (k = 0.05 \text{ Mpc}^{-1}) \). Therefore, changes in \( n_s \) tend to appear less similar to tilting to the power spectrum and more like changing the amplitude, although \( n_s \) remains well constrained. Additionally, the covariance between \( A \) and \( \Omega_b \) confirms that the baryon acoustic oscillations in the power spectrum are not contributing much information about the baryon density relative to the amplitude changes from the \( C_{Jy} \) and Jacobian
factors.

4.3.3 Residual Foreground Contamination

Mitigating astrophysical foreground contamination is an important issue for redshifted 21 cm experiments, whether attempting to constrain cosmological models or determine reionization processes and scenarios. To this end, the relationships between \( P_{HI} \), \( P_F \), and \( P_N \), as described by the covariance between our model parameters \( A \), \( F \), and \( N \) are very useful to study in some detail.

The constraints on the amplitudes of the residual astrophysical foreground and residual thermal offset terms are especially encouraging. Neither contribution is significantly degenerate with changes to the redshifted 21 cm power spectrum. In particular, the long, diminishing tail of the redshifted 21 cm power spectrum provides many statistical samples which contribute little to knowledge of the cosmological parameters, yet provide good references to prevent degeneracy with changes in the magnitude of the residual thermal offset. Thus, \( P_N \) is constrained to better than 2% of the peak value of \( P_{HI} \) for the MWA and to better than 0.01% for the MWA5000.

As discussed in Section 4.1.1, the residual foreground contamination is expected to be a power-law along the line-of-sight direction and to have little structure in the transverse directions. This is distinct from the redshifted 21 cm power spectrum with its generally spherical symmetry, although geometrical effects due to the shape of the observed volume of space work to negate this distinction. The observed region has dimensions of \( \sim 9000 \) by \( \sim 9000 \) Mpc in the sky-plane, but only \( \sim 100 \) Mpc along the line-of-sight [Bowman et al., 2006]. Since all three dimensions are sampled with an approximately equal number of divisions, this flattened shape is inverted when transforming into \( k \)-space, leading to a highly elongated data cube in the \( k_3 \)-direction. For the MWA, the maximum \( k \) in the sky-plane \((k_1,k_2)\) is approximately 0.05 Mpc\(^{-1}\), while along \( k_3 \) it is over 10 Mpc\(^{-1}\). Therefore, much of the information in the measurement about the redshifted 21 cm power spectrum is actually coming from the line-of-sight direction. Furthermore, the lack of depth in the observed volume prevents the experiments from probing large scales in the line-of-sight direction and, thus, the peak of the power spectrum (at \( k \approx 0.01 \)) is not well constrained. This means that not only is most of the information coming from the line-of-sight direction, it is coming from the power law-like tail \((\sim k^{-3})\) of the power spectrum. These effects combine to make the redshifted 21 cm contribution and residual foreground contamination less distinguishable in the measured power spectrum.

Fortunately, the analysis indicates that the residual foreground component is well separated from the redshifted 21 cm power spectrum and does not affect substantially the ability to constrain the cosmological parameters. From Table 4.6 and Figure 4-2, we see that the largest degeneracy is between \( F \) and \( \Omega_M \). Removing \( P_F \) from the model power spectrum, and thus \( F \) from the parameter set such that \( p = \{A, \Omega_M, \Omega_b, h, n_s, \alpha_s, N\} \), would reduce the forecasted uncertainty on \( \Omega_M \) by less than 30% in an observation with with MWA5000. The contributions from the residual thermal offset, \( P_N \), are even less.

In our analysis so far we have treated the full three-dimensional measured power spectrum. This has maximized the effects of the symmetry differences between the 21 cm power spectrum and primary residual foreground power spectrum. We might have instead considered a reduced one-dimensional power spectrum produced by averaging over spherical bins in \( k \)-space. This approach is used commonly to reduce data in which the expected signal has approximately spherical symmetry.
Figure 4-4 Unmarginalized error ellipses for the amplitudes of the redshifted 21 cm power spectrum and primary residual foreground contamination, $A$ and $F$, forecasted for the MWA. The thick curves are for a Fisher matrix calculation utilizing the full three-dimensional measured power spectrum, while the thin curves are for the same calculation using spherical bins in $k$-space. The contours are for 68% and 95% likelihood. The behavior is similar for the MWA5000, although the errors are smaller.
In Figure 4-4, we compare the full three-dimensional treatment with the spherically-binned approach by plotting unmarginalized error ellipses in the A-F plane of parameter space for both cases. It is clear that the binned method produces a significantly larger degeneracy between the magnitude of the residual foreground contribution and the amplitude of the 21 cm neutral hydrogen power spectrum than the full three-dimensional treatment.

4.3.4 Dark Energy Equation of State

Determining the nature of dark energy has become an important goal in astrophysics and the applicability of future large-scale structure surveys to this topic has been considered recently with generally favorable results [Seo and Eisenstein, 2003, Linder, 2003, Hu and Haiman, 2003, Wang et al., 2004, Abdalla and Rawlings, 2005]. With their similarities to large-scale structure surveys, would measurements of the redshifted 21 cm neutral hydrogen power spectrum also improve knowledge of dark energy?

Including the dark energy equation of state parameter, w, as a free variable in the Fisher matrix analysis, so that \( p = \{ A, \Omega_M, \Omega_b, h, n_s, \alpha_s, w, F, N \} \), yields little information about the nature of dark energy. Neither the MWA nor the MWA5000 are able to provide meaningful constraints on w without priors from other experiments. Furthermore, the constraints on \( \Omega_M \), \( \Omega_b \), and h are significantly weakened (by up to an order of magnitude), although information is retained about the primordial power spectrum.

Although the MWA5000 reference experiment would be able to detect the baryon acoustic oscillations (see Figure 4-3), the experiment targets emission from high redshifts prior to the epoch of dark energy domination. The value of the experiment would be relevant only as an in intermediate measurement between the CMB and low redshift large-scale structure surveys [Barkana, 2006], but could be important if dark energy had unusual behavior at high redshifts.
Table 4.4. Forecasted Uncertainties for the Model Parameters

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>F</th>
<th>D</th>
<th>(\Omega_M)</th>
<th>(\Omega_b)</th>
<th>h</th>
<th>(n_s)</th>
<th>(\alpha_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MWA</td>
<td>6.69</td>
<td>0.09</td>
<td>2e-3</td>
<td>0.21</td>
<td>0.07</td>
<td>0.88</td>
<td>0.2</td>
<td>0.04</td>
</tr>
<tr>
<td>MWA5000</td>
<td>0.45</td>
<td>2e-3</td>
<td>1e-4</td>
<td>0.01</td>
<td>6e-3</td>
<td>0.05</td>
<td>0.02</td>
<td>4e-3</td>
</tr>
<tr>
<td>WMAP</td>
<td>0.09</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.04</td>
<td>—</td>
<td>2e-3</td>
</tr>
<tr>
<td>SDSS</td>
<td>0.10</td>
<td>—</td>
<td>—</td>
<td>0.05</td>
<td>2e-3</td>
<td>0.04</td>
<td>0.03</td>
<td>—</td>
</tr>
</tbody>
</table>

Note. — Forecasted 1-\(\sigma\) uncertainties for the model parameters. The first row gives the fiducial values for reference. The MWA reference experiment does not significantly constrain cosmological parameters, while the MWA5000 does a reasonably good job, particularly for \(\Omega_M\) and the primordial power spectrum spectral index descriptors, \(n_s\) and \(\alpha_s\). For comparison, the last two rows display approximate constraints for similar models from WMAP [Spergel et al., 2003, Their Table 10] and SDSS [Tegmark et al., 2004, Their Table 3].

Table 4.5. Covariance Matrix for MWA

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>F</th>
<th>D</th>
<th>(\Omega_M)</th>
<th>(\Omega_b)</th>
<th>h</th>
<th>(n_s)</th>
<th>(\alpha_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>-0.09</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>-0.09</td>
<td>0.01</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Omega_M)</td>
<td>-0.04</td>
<td>-0.72</td>
<td>-0.04</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Omega_b)</td>
<td>-0.87</td>
<td>-0.24</td>
<td>0.04</td>
<td>0.49</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>-0.97</td>
<td>0.24</td>
<td>0.11</td>
<td>-0.20</td>
<td>0.72</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n_s)</td>
<td>0.95</td>
<td>0.04</td>
<td>-0.15</td>
<td>-0.12</td>
<td>-0.82</td>
<td>-0.92</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>(\alpha_s)</td>
<td>-0.41</td>
<td>-0.37</td>
<td>-0.15</td>
<td>0.63</td>
<td>0.69</td>
<td>0.23</td>
<td>-0.32</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note. — Elements of the covariance matrix for the MWA calculated by taking the inverse of the Fisher matrix, \(F\). Each element has been normalized according to \(c_{ab}/\sqrt{c_{aa}c_{bb}}\), where the diagonal elements are the squares of the 1-\(\sigma\) uncertainties given in Table 4.4.
### Table 4.6. Covariance Matrix for MWA5000

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>F</th>
<th>D</th>
<th>Ωₘ</th>
<th>Ωₜ</th>
<th>h</th>
<th>nₛ</th>
<th>αₛ</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0.20</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>-0.07</td>
<td>0.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ωₘ</td>
<td>-0.40</td>
<td>-0.60</td>
<td>-0.02</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ωₜ</td>
<td>-0.93</td>
<td>-0.35</td>
<td>0.05</td>
<td>0.66</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>-0.97</td>
<td>-0.08</td>
<td>0.08</td>
<td>0.19</td>
<td>0.81</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>nₛ</td>
<td>0.93</td>
<td>0.27</td>
<td>-0.07</td>
<td>-0.46</td>
<td>-0.83</td>
<td>-0.92</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>αₛ</td>
<td>-0.48</td>
<td>-0.28</td>
<td>-0.20</td>
<td>0.53</td>
<td>0.50</td>
<td>0.42</td>
<td>-0.62</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note. — Same as Table 4.5, but for the MWA5000.
Chapter 5

Initial Field Tests of Prototype MWA Antenna Tiles

This chapter is adapted from the paper "Field Deployment of Prototype Antenna Tiles for the Mileura Widefield Array Low Frequency Demonstrator" by Bowman et al. [2007a].

The design of the MWA is a departure from traditional approaches to radio astronomy instrumentation. The wide field of view and large bandwidth coverage of the instrument, combined with the phased-array design of the antenna tiles and low-cost digital receiver system, create exciting new opportunities for scientific exploration in a part of the electromagnetic spectrum that has been largely neglected for decades. But for the array to fully succeed in achieving its science objectives requires an understanding of the operational properties of the instrument at unprecedented levels. The field testing campaign reported on here was conducted to begin addressing this requirement at an early stage in the development of the array.

As part of the development effort for the MWA, prototypes of the antenna tiles were constructed. In order to demonstrate the performance of the prototype hardware, as well as to characterize the planned site for RFI and a variety of environmental factors, three antenna tiles were deployed in western Australia, along with a simple data capture and software correlation system, and necessary infrastructure. Four expeditions to the site were conducted from March through September, 2005. During this 6-month period, the equipment was operated for a total of 8 weeks and several terabytes of data were gathered. This program was supported by MIT, the University of Melbourne, the Australian National University, and Curtin University of Technology.

In this chapter, we describe the prototype system and the results that we derived from it. In Sections 5.1 and 5.2 we describe the specific equipment and the physical deployment at the Mileura Station site, along with the experimental results relevant to characterization of the performance of the system. Section 5.3 addresses site characterization, with particular emphasis on the RFI environment as seen by our systems. In Section 5.4, a variety of results are presented showing the characterization of astronomical sources with the prototype equipment.
Figure 5-1 Photographs of the first MWA prototype antenna tile being tested on site at Mileura Station, Western Australia. The antenna consists of 16 crossed-dipoles in a four-by-four meter grid and is elevated approximately 0.8 m above the ground. In the top panel, the antenna tile is being assembled near the caravan used for housing the computers and receiver electronics. The middle panel is an aerial view after the antenna tile was moved to its final location approximately 150 m northwest of the caravan. The bottom panel shows the completed antenna tile in position. The solar panel is visible leaning against the tile and the beamformer box and heat shield are under the tile. Two additional antenna tiles were subsequently installed at the site.
5.1 Instrument Design

The antenna tiles used for the prototype deployment (shown in Figure 5-1) were early designs of the production style antenna in the Figure 1-1. The dipole design is identical between the two versions of the antenna tiles, as is the spacing of the dipoles in the four-by-four grid. The choice of spacing is 1.07 meters, or $\lambda/2$ at 140 MHz, and is aimed at optimizing performance for EOR science, but leads to significant grating lobes in the antenna tile beam at frequencies above ~ 200 MHz. An active balun is located at the junction of the dipole arms, and the amplified signal is fed to the beamformer unit via coaxial cable. For the prototype units, structural support for these dipole elements was provided by a Lexan tube of 6.25 cm diameter, where as for the production antenna tiles, the dipoles are supported at the bottom of the bowtie elements.

Because of imperfect impedance matching between the dipole and the balun, much of the antenna power received from the sky is reflected back to the sky, particularly at the low end of the frequency range. The sky brightness temperature is so much higher than the first-stage amplifier noise temperature, however, that the overall system temperature is dominated by sky noise, despite the loss of the reflected power (see Section 5.2.2 for additional discussion of the system temperature).

The dipoles need to be positioned over a ground plane, and due to the Lexan tube support structure of the prototypes, it was necessary to construct the ground planes as raised platforms, with a tube clamping arrangement below the conductive screen. This arrangement is visible in Figure 5-1. Ground planes for each of the three antenna tiles were custom-built, with successive refinements for each ground plane based on experience from the previous antenna tile. For the full array, we expect to use conductive mesh at ground level.

The beamformer units are based on switchable delay lines implemented as coplanar waveguides via traces on printed circuit boards. Switch settings are controlled by a computer workstation communicating with a microcontroller over an RS-232 serial line. Certain beamformer functions, such as power combination, were implemented in the prototype units with commercially built, connectorized modules, and the general degree of component integration was lower than that to be implemented in the full array. However, the prototype beamformer is expected to match the production version closely in performance.

5.1.1 Wide-band Active Dipole Elements

Each dipole element, shown in Figure 5-2, for the phased-array antenna tile is an “active antenna,” with a low-noise amplifier and a balun integrated into the antenna structure. The combination provides low-noise amplification at the point where the received signal is weakest, and converts it from a balanced to an unbalanced signal, as is necessary before transmitting the signal over coaxial cable to the downstream electronics.

The open-frame, vertical bowtie construction of the antenna elements was chosen with several objectives in mind: wide frequency range, broad angular coverage, low sensitivity at the horizon (to minimize reception of RFI from terrestrial sources), and low manufacturing cost. A simple, horizontal, wire dipole over a ground plane satisfies the latter two, but has difficulties with the first two—especially with a two octave frequency range. The impedance of a dipole has a much stronger frequency dependence than does the input impedance of a typical amplifier. As a result, there is a large impedance mismatch between dipole and amplifier except over a narrow frequency range, and most of the power received from the
Figure 5-2 Individual bowtie dipole assembly (top) with close-up of the low-noise amplifier and balun (inset), and interior of a beamformer (bottom). The signals from sixteen dipole assemblies are combined by the beamformer to create the output for a single, phased-array antenna tile.
Figure 5-3 Predicted beam patterns at two frequencies for antenna tiles phased to point at the zenith. The left panel shows the beam pattern at 110 MHz and the right panel at 200 MHz. The gray scale is linear in dB and, from black to white, spans -50 to 0 dB. The contour lines are at -30 dB (solid), -20 dB (dash), -10 dB (dash-dot), and -3 dB (dot). The plots are polar projections spanning the full sky. The predicted beam patterns were derived using a simple model based on canonical dipoles placed one-quarter wavelength above an infinite ground plane. No coupling between the dipole elements was included.
The caravan control center was located approximately in the center of the array.

Extending the dipole arms vertically into bowties significantly flattens the frequency dependence of the antenna impedance, thereby improving the match with the amplifier over most of the 80 to 300 MHz range. The conversion to bowties also widens the beamwidth. Constructing the bowtie in outline form, with spars around the outer edges and a single horizontal crosspiece, gives performance similar to a solid-panel bowtie, but with the advantages of reduced wind loading and lighter weight, both of which allow a simplified mechanical support structure and reduce the manufacturing cost.

Each pair of diametrically opposite dipole arms feeds a low-noise amplifier housed in the Lexan tube. The amplifier employs two Agilent ATF-54143 HEMT amplifiers in a balanced configuration with a balun on the output. The amplifier noise temperature is 15 to 20 K (compared with approximately 100 to 5000 K for the sky temperature). The balanced design and intrinsically low distortion characteristics of the HEMT ensure an acceptably low level for any output intermodulation products arising from RFI signals received by the antenna.

5.1.2 Antenna Tile and Delay-Line Beamformer

For each of the 16 signals of the same polarization from the dipole elements of an antenna tile, the beamformer filters the signal to attenuate out-of-band interference, and then applies a delay between 0 and 10.4 ns, as appropriate to steer the phased-array beam in the desired direction. The delay maximum of 10.4 ns is sufficient to phase the beam to the horizon in the principle planes of the tile, but corner dipoles may be under-delayed in the case of off-axis incident waves with zenith angles greater than 45°. At a zenith angle of 60° and
Figure 5-5 Measured power response profiles in the E-plane of an antenna tile. The left panels show the measured profiles at 110 MHz and the right panels at 200 MHz. From top to bottom the panels show different antenna tile pointing angles, and are 0, 15, 30, and 45° relative to the zenith in the E-plane. In both columns, the dotted lines show the predicted profiles assuming ideal dipoles and no mutual coupling. The grating lobes of the antenna tiles are especially evident in the measured data for the 45° pointing angle at 200 MHz.
Figure 5-6 Best-fit dipole power envelope in the north-south polarization at 120 MHz from the baseline formed by antenna tiles 1 and 3. Seven point sources were isolated interferometrically and tracked across the sky to estimate the profile. The sources were PKS2152-69 (circle), PKS2356-61 (star), Pic A (dot), 3C161 (plus), 3C353 (cross), Her A (square), and Tau A (diamond). The amplitudes of the resulting visibility measurements were normalized and fit to a $\cos^\beta(\theta)$ profile, where $\theta$ is the zenith angle and the best-fit (solid line) was found for $\beta = 2.6 \pm 0.4$. Negative zenith angles indicate measurements where the source was in the eastern half of the sky, and positive zenith angles indicate the western half. The dotted line shows the equivalently normalized ideal dipole ($\beta = 2$) envelope.
Figure 5-7 Time series of sky maps at 200 MHz produced by scanning repeatedly an antenna tile beam through a raster of pointing directions. The Galaxy center is shown rising and transiting in the top row, and the sun is rising in the bottom row. The maps are polar projections from 0 to 60 deg zenith angle with north at the top, and the shading indicates the sky temperature on a log scale spanning approximately 250 (black) to 1000 K (white). The effect of the diffraction grating sidelobes in the antenna tile power response function are evident in the observations of the sun, producing mirror images at each corner of the map. The actual image of the sun is toward the top-left (northeast) corner of the maps, although the mirror image in the bottom-left (southeast) is nearly as strong. No corrections were applied to the data used to generate these maps; in particular, there is no compensation for the reduced gain at large zenith angle due to the dipole element power envelope.
Figure 5-8 Effective receiver temperature (top) and gain (second) for 37 frequencies spanning 80 MHz to 228 MHz for the east-west polarization of antenna tile 1. Error bars are at 95% confidence. The instrumental performance was determined by fitting Galactic drift scan profiles to observed antenna power measurements. The third panel indicates the quality of the fit at each frequency by plotting the $\chi^2$ value. The bottom panel shows example best fits at 100, 120, 140, 160, 180, and 200 MHz (from top to bottom), where the solid line is the model and the dots are the data. Due to an equipment malfunction affecting the impedance matches within the beamformer at the time of the observations, these results represent a lower limit on the system performance.
azimuth of 45°, the corner dipole opposite the reference has a serious delay error of 2.7 ns (0.81 turn at 300 MHz), while the next worst delay errors are only 0.5 ns (0.15 turn at 300 MHz).

The delay for each dipole is generated from five delay lines that may be independently switched in or out of the signal path via paired GaAs switches. The delays differ by powers of two between sections, so the minimum delay step is 0.34 ns, or 0.1 turn at 300 MHz. The coplanar-waveguide construction of the delay lines affords excellent delay repeatability between units and extremely low temperature sensitivity. Following the delay line sections, the signal is combined with the other 15 signals of the same polarization, further amplified, and then sent on to the receiver via coaxial cable.

Each beamformer chassis, shown in Figure 5-2, contains the delay lines and associated electronics for the two polarizations from a single tile, voltage regulators to step down the DC voltage from an external power source, and a microcontroller-based digital interface to convert RS-232 serial data commands into parallel data. These signals control the delay line switches and also a second set of switches, one per dipole signal, which allow individual dipoles to be switched in or out of the signal path for test purposes or in the event of a dipole failure. During astronomical observations, the control electronics are deactivated to reduce interference except for the brief instants when the delays are reset to change the pointing of the antenna tile beam.

The mutual coupling between antenna elements is less than -20 dB, except between 90 and 140 MHz, where it peaks at about -12 dB at 110 MHz. With this low mutual coupling between antenna elements, the antenna tile power response pattern is modelled as the single-element dipole pattern multiplied by the array factor, which is the pattern the antenna tile would have if all the elements were replaced by ideal, isotropic antennas. The half-power beamwidth of a single dipole element mounted above a ground plane is > 80 deg over most of the frequency range, with peak response at the zenith except near 300 MHz. The beamwidth of a full antenna tile varies approximately inversely with frequency, and has zenith values of 15 to 45 deg. The beamwidth increases as the beam is steered down toward the horizon due to foreshortening. Figure 5-3 shows two example antenna tile power response patterns calculated assuming ideal dipole elements with no coupling. The small actual coupling does cause slight deviations from these patterns, as discussed in Section 5.2.1.

5.1.3 Receiver and Digitizer System

For the prototype field deployment, the receiver functions were handled by a simple, dual-frequency conversion system, wherein a 4 MHz band was selected and mixed to 28 MHz center frequency. The signal was digitally sampled at 64 MHz using the “Stromlo Streamer” [Briggs et al., 2006], but only one in four samples was kept, producing an effective sampling rate of 16 MHz.

The system used a computer workstation equipped with a terabyte RAID array to record four input channels at the effective sampling rate with 8 bit precision, giving a data rate of 64 MB/s that was recorded to disk. Normally, only the low order bits were toggled by the received signals. The dynamic range of the 8 bits allowed operation of the system over the full frequency range without further gain adjustment. There was ample head-room for RFI signals, and in those few instances where higher order bits were needed, the IF conversion system entered a non-linear regime due to the high signal power concentrated in the 4 MHz passband at the output of the IF system. The data were processed in
software by playing the recordings back into an FX correlator code which computed both auto- and cross-correlation power spectra. There were two principal modes of operation: a high spectral resolution mode, which implemented a polyphase filter bank to obtain 1 kHz spectral channels with a high level of channel-to-channel rejection, and a 16 kHz resolution mode, which used a straight FFT for spectral dispersion in order to cut the correlation time to a little longer than recording time. This latter mode produced 512 channels and was used in the analysis in Section 5.4.2.

5.1.4 Array Layout

The location of the prototype deployment array is given in Table 5.1. At the site, the three antenna tiles were deployed in a triangular configuration roughly 300 m across, as illustrated in Figure 5-4. This provided interferometer baselines of sufficient length to resolve out the diffuse galactic emission and to effectively isolate signals from bright individual point sources through phase-stable interferometry. The configuration also allowed the potential of the array to yield scientifically interesting astronomical information.

5.1.5 Site Infrastructure

The prototype field deployment campaigns had modest infrastructure requirements. A small caravan (trailer) was purchased and placed on-site to act as the control center and house the receivers and computer equipment. Battery-backed solar power units were installed to power each of the three antenna tiles and a portable diesel generator was used at the caravan to provide power for the electronics and daytime air conditioning. Coaxial cables of 200 m length were used to transport signals from the antenna tile beamformers to the caravan, and the cables were simply laid on the ground. Between campaigns, the beamformers and solar power units were removed and placed in storage.
5.2 Calibration and Performance

5.2.1 Antenna Power Response Pattern

The actual antenna power response pattern for the prototype antenna tiles was constrained using two methods. Prior to installing the antenna tiles in the field, the response function was determined at two frequencies by systematically scanning a transmitter above an antenna tile using facilities at the MIT Haystack Observatory. Figure 5-5 shows resulting beam profiles for line scans in the E-plane of the antenna tile at four different antenna tile pointing angles. The structures in these profiles agree well with the patterns produced by assuming that the antenna tiles consist of ideal dipole elements above an infinite ground plane with no mutual coupling, which are shown as the dotted lines in Figure 5-5. There are a few notable deviations. Most prominently, the primary beam appears larger than predicted at 110 MHz, as can be seen in the upper-left two panels in the Figure. Additionally, the nulls in the beam patterns are not as deep as would be expected from the ideal calculations, and at the larger pointing angles, there are some significant deviations from the expected patterns. Some of these discrepancies may be the result of the test conditions. The distance (~15 m) between the transmitter and antenna tile was not sufficient to reach fully the far-field limit for the radiation patterns, and the alignment and pointing of the transmitter at the antenna tile was performed manually with an estimated margin of error of ~5° in each, allowing ambiguities in the effective zenith angle and in gain due to the dipole pattern of the transmitter.

In the field, additional measurements were achieved by interferometrically isolating point sources and tracking them as they drifted across the sky. This technique effectively separated the dipole element power response pattern from that of the full antenna tile pattern and resulted in an estimate of the effective dipole power envelope as a function of zenith angle. The amplitudes of the interferometrically isolated point sources were fit to a \( \cos^\beta(\theta) \) profile, where \( \theta \) is the zenith angle and \( \beta \) is a constant. The best-fit at 120 MHz is shown in Figure 5-6 and was found to be given by \( \beta = 2.6 \pm 0.4 \), whereas an ideal dipole has \( \beta = 2 \) and is excluded by the measurements at the 85% confidence level. This indicates that the effective power pattern for the antenna tile dipole elements has a somewhat narrower shape than the ideal case.

5.2.2 Gains and System Temperatures

The gain and noise temperature of an antenna tile system was measured by setting its beamformer delays to point the antenna tile towards zenith and then recording the signal over a long period of time as the Earth turns and the beam transits the Galactic plane. In this technique, the measured power is compared to the predicted power by convolving a map of the low frequency Galactic synchrotron emission with the computed antenna power response function. The data are fit by [Rogers et al., 2004, Their Equation 1],

\[
P(t) = g \left[ T_{\text{sky}}(t, \theta) \ast W(\theta) + T_{\text{receiver}} \right],
\]

where \( P \) is the measured power in arbitrary units as a function of local sidereal time (LST), \( g \) is the receiver gain (assumed to be constant with time), \( T_{\text{sky}} \) is the sky model as function of direction in the sky \( \theta \), \( W \) is the antenna power response pattern, \( T_{\text{receiver}} \) includes the remaining system noise contributions (including amplifier noise and ground spillover) referred to the antenna output, and \( \ast \) is the convolution operator.
Table 5.2. Antenna Tile Locations

<table>
<thead>
<tr>
<th>Tile</th>
<th>Equatorial $(x, y, z)$ [m]</th>
<th>Local $(x_{topo}, y_{topo}, z_{topo})$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tile 1</td>
<td>(0, 0, 0)</td>
<td>(0, 0, 0)</td>
</tr>
<tr>
<td>Tile 2</td>
<td>(-1.08, 145.64, -2.56)</td>
<td>(0.175, 145.64, -2.77)</td>
</tr>
<tr>
<td>Tile 3</td>
<td>(-76.43, 278.45, -149.28)</td>
<td>(-1.993, 278.45, -167.70)</td>
</tr>
</tbody>
</table>

Note. — Antenna tile locations determined by observations of unresolved continuum sources at 120 and 275 MHz. The left set of locations is in conventional coordinates for interferometer baselines, described in Section 5.2.3, and the lower set is in a topocentric coordinate system, where $x_{topo}$ is aligned with up, $y_{topo}$ is aligned with east, and $z_{topo}$ completes a right-handed system by pointing into the north.

We used the 408 MHz sky map made by Haslam et al. [1982] and assumed a spectral index of $\alpha = 2.6$ to extrapolate to the observed frequencies. The power response pattern for the prototype antenna tiles was taken to be that of ideal dipole elements with no mutual coupling above an infinite ground plane, as described at the end of Section 5.1.2. Although this simplified model was shown in Section 5.2.1 to produce a larger dipole power envelope than that of the actual antennas tiles, it deviates by only approximately 10% at the edge of the primary antenna tile beam when phased to the zenith.

Figure 5-7 gives examples of the actual sky as seen by a prototype antenna tile, and Figure 5-8 shows the results of the system gain and noise temperature calibration between 80 and 230 MHz. In general, the effective receiver temperature as measured at the output of the antenna was found to be below about 200 K except at the lowest frequencies measured, although, the results are considered a lower limit on the performance of the system due to an equipment problem at the time of the observations which increased impedance mismatches within the beamformer of the antenna tile used for the observations. The predicted antenna temperature as a function of LST and examples of the best least-squares fits of the data at six frequencies are also given.

5.2.3 Baseline Determination

Baselines of the final array configuration were determined using observations of unresolved astronomical continuum sources. The antenna coordinates were defined in the conventional right-handed, cartesian coordinate system with the $x$- and $y$-axes lying in a plane parallel to Earth’s equator and the $z$-axis parallel to Earth’s rotation axis pointing towards the north pole. In terms of hour angle (HA) and declination (Dec), $x$, $y$, and $z$ are measured towards (HA = 0h, Dec = 0°), (HA = -6h, Dec = 0°) and (Dec = 90°) respectively. In this coordinate system, a large HA coverage is useful to constrain the $x$ and $y$ components of the baseline, and the $z$ component is best constrained by observing sources spanning a large range in declination.

An observing session optimized for baseline calibration was conducted on the night of
Table 5.3. Baseline Determination Comparison

<table>
<thead>
<tr>
<th>Celestial [m]</th>
<th>Tape [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tile 1 - Tile 2</td>
<td>145.67</td>
</tr>
<tr>
<td>Tile 1 - Tile 3</td>
<td>325.06</td>
</tr>
<tr>
<td>Tile 2 - Tile 3</td>
<td>211.76</td>
</tr>
</tbody>
</table>

Note. — Comparison of the baselines determined using astronomical observations and those measured by tape.

September 15, 2005 which continued through mid-morning the next day. The observations were carried out at 120 and 275 MHz using the north-south aligned polarization on all three tiles and observed seven bright, unresolved sources: Tau-A, Her-A, 3C252, 3C161, Pic-A, PKS 2152-69 and PKS 2356-61. The relative antenna tile locations are given in Table 5.2, and Table 5.3 shows a comparison between the baseline vectors determined by astronomical observations and those obtained manually by using a tape measure.

The resulting antenna tile positions are sufficiently accurate to correct the interferometer phases for changing geometrical delays as a function of hour angle and declination. The least square fitting uncertainties in x and y were determined to be about 2 cm and those in z to be about 5 cm. The dominant sources of error in the antenna tile coordinates due to pseudo-random effects were cross talk in the receiver chains (see Section 5.2.5) and ionospheric variations (see Section 5.4.2).

5.2.4 Antenna Delays

In addition to determination of the antenna coordinates, calibration of interferometric data requires the determination of antenna based delays (or phase offsets) due to differences in the electrical path lengths to different antennas. These offsets were determined using observations of Pic-A conducted on September 18, 2005. An analysis using all three baselines sensing the east-west aligned polarization led to the conclusion that the electrical paths to antenna tiles 1 and 2 differ by 1.5 m and that those to antenna tiles 1 and 3 differ by 0.6 m. The major contributor to these difference is differing lengths of the cables between the antennas and the inputs to the digitizers. The measured instrumental phase offsets were found to be well behaved, varying with frequency $f$ as $\Delta \phi = 2\pi \Delta L f/c$ where $\Delta L$ is the electrical path difference and c is the speed of light.

5.2.5 Cross Talk

When the array was pointed at a bright radio source, the interferometers showed a response with two components: one corresponding to the natural fringe rate due to the Earth's rotation, and a second that was invariant with time. Due to the simplicity of the receiving system, it was deduced that the invariant component is dominated by an instrumental component, caused by cross talk (faint electrical coupling) between the signal paths in...
the receiving system. In more sophisticated receivers, this effect can be avoided by phase
switching the signal close to the antenna, followed by synchronous demodulation in the
digital section, and the full array will employ this technique.

For the purposes of this prototyping exercise, it was sufficient to approximate the in-
variant term by a long average of the interferometer response at “zero fringe rate” (i.e.,
computed without compensation for Earth rotation) and to remove the cross talk compo-
nent by subtraction of the average from the visibility data prior to application of the fringe
de-rotation for the celestial sources.

5.2.6 Noise and Integration Time

In order to assess the noise characteristics of the system, a deep integration was acquired in
an empty part of the spectrum with one antenna tile. The observation was conducted over
three days beginning June 23, 2005, and resulted in 10 h of total integration in a 4 MHz
band centered at 187 MHz with 1 kHz spectral resolution. The integration was processed
to remove the bandpass and corrected for amplitude variations due to variations in Galactic
noise with LST. From these cleaned data, 1000 contiguous channels spanning 188 to 189
MHz were selected and analyzed as a function of the cumulative integration time. For pure
thermal noise, the variation of the individual channel powers about the mean should have
a Gaussian distribution and their RMS should be proportional to the inverse square root of
the integration time ($t^{-1/2}$). Figure 5-9 shows that the observed trend is consistent with
thermal noise and that the distribution of channel powers at the end of the integration is
approximately Gaussian.

5.3 Site Characterization

Western Australia is a new venue for radio astronomy and the prototype field deployment
system was one of the first instruments to be deployed in the region. The effort, therefore,
was able to provide valuable experiences regarding working conditions in the remote area,
the ability of the equipment to withstand the harsh environment, and the suitability of the
site for low frequency radio astronomy. In this Section, we report significant findings on
these topics.

5.3.1 The Western Australian Environment

Mileura Station is an active sheep and cattle station (ranch) in a remote area approximately
620 km north of Perth, Western Australia. The nearest towns are Meekathara (~100 km
east) and Cue (~150 km southeast), and the nearest small city is Geraldton (~350 km
southwest). During dry seasons, Mileura can be reached from Perth by car or truck in
approximately 10 hours over paved and dirt roads. In wet weather, the dirt roads may
become virtually impassable for short periods of time, cutting off access to the site. The
ranch consists of about 620,000 acres (~2500 km$^2$) of natural grazing land for sheep and
cattle, but kangaroos, emus, feral goats, lizards, snakes, birds, and many insects are also
abundant. The climate of the region is arid (see Table 5.4).

Prominent concerns prior to the field deployment effort regarded the logistics of working
at the site and whether the equipment would be able to endure the harsh environment,
including the climate and the wildlife. Over the course of the six-month deployment, the
equipment was operated successfully in a variety of conditions including extreme heat, severe
Figure 5-9 Analysis of the 10 h integration centered at 187 MHz. The top panel shows the RMS channel power for the 1000 frequency channels spanning 188 to 189 MHz as a function of integration time and clearly follows a $t^{-1/2}$ trend (shown by the dashed line). The lower panel gives the distribution of channel powers at the end of the integration, which is close to an ideal Gaussian distribution (shown by the solid line).
Table 5.4. Weather in Western Australia

<table>
<thead>
<tr>
<th>Month</th>
<th>Mean High Temp [°C]</th>
<th>Mean Low Temp [°C]</th>
<th>Mean Rainfall [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>37.8</td>
<td>22.8</td>
<td>25.4</td>
</tr>
<tr>
<td>Feb</td>
<td>36.8</td>
<td>22.4</td>
<td>29.8</td>
</tr>
<tr>
<td>Mar</td>
<td>34.0</td>
<td>20.1</td>
<td>22.9</td>
</tr>
<tr>
<td>Apr</td>
<td>29.0</td>
<td>15.7</td>
<td>19.6</td>
</tr>
<tr>
<td>May</td>
<td>23.2</td>
<td>11.1</td>
<td>25.5</td>
</tr>
<tr>
<td>Jun</td>
<td>19.1</td>
<td>8.2</td>
<td>29.1</td>
</tr>
<tr>
<td>Jul</td>
<td>18.4</td>
<td>6.9</td>
<td>25.0</td>
</tr>
<tr>
<td>Aug</td>
<td>20.4</td>
<td>7.8</td>
<td>17.8</td>
</tr>
<tr>
<td>Sep</td>
<td>24.6</td>
<td>10.1</td>
<td>6.9</td>
</tr>
<tr>
<td>Oct</td>
<td>28.3</td>
<td>13.1</td>
<td>6.7</td>
</tr>
<tr>
<td>Nov</td>
<td>32.8</td>
<td>17.2</td>
<td>8.4</td>
</tr>
<tr>
<td>Dec</td>
<td>36.2</td>
<td>20.7</td>
<td>14.0</td>
</tr>
</tbody>
</table>

Note. — Climate averages at Cue, Western Australia. From the Australian Government Bureau of Meteorology.

winds, cool nights, and light rain. Additionally, although numerous kangaroos and emus were seen in the vicinity of the array, no damage or evidence of any interaction with the equipment was observed.

Several severe storms occurred while the array was not in use. One of these storms produced flooding at the site. Inspection of the antenna tiles after the flooding showed clear signs that running water passed under the elevated ground planes, eroding material around the ground plane supports. A second storm produced large hail (up to ~3 cm in diameter) at the site, but no significant damage was detected to the antenna tiles, including the dipole elements and ground planes.

5.3.2 Radio Frequency Interference

Characterizing the RFI at the Mileura Station site was an important objective of the prototype field deployment effort and a significant amount of observing time was dedicated to this task. All RFI studies were processed with the high spectral dynamic range, polyphase filter bank mode to obtain 1 kHz resolution. Figure 5-10 gives an overview of the strongest signals in the 80 to 300 MHz band. This full spectrum scan results from the stitching together of a sequence of 4 MHz bands. Each individual band was observed for 15 seconds by a single antenna tile phased to point at the zenith and was bandpass corrected. With 1 kHz channels, less than 1% of the spectrum was occupied by interferers above 5-σ in this observation. The strongest identified features, most notably at 137, 180, and 240 through 270 MHz, originate from satellites. The temporary receiving system used in these prototyping exercises emits signals at multiples of the 64 MHz sampling clock that could be detected in the deep spectra. The full array will use a much higher sampling rate, and the critical
components will be contained in RFI tight enclosures.

Long Integrations

Figure 5-11 shows spectra for long integrations in 11 selected bands. The observations vary between 30 min and 10 h and each was acquired with a single antenna tile phased to point at the zenith. The sensitivity levels probed with these deep integrations, where the time-bandwidth product is $\sim 10^6-10^7$, achieve RMS channel powers more than 30 dB below the galactic background at 1 kHz resolution. These observations illustrate that the spectrum is remarkably free of RFI.

The RFI in a particular observation can be characterized by considering the histogram of individual 1 kHz by 1.3 sec samples comprising the full integration. Absent of RFI and systematic processing errors, the distribution of samples should approach a Gaussian distribution. Figure 5-12 shows two such histograms for the 42 min and 10 h observations centered at 107 and 187 MHz, respectively (panels 2 and 8 in Figure 5-11). The significant presence of RFI due to FM radio transmissions in the observation centered 107 MHz is quite clear as a deviation above a pure Gaussian distribution at high $\sigma$, while the very clean spectrum around 187 MHz reproduces a Gaussian distribution more accurately. In both cases, there appears to be a slight systematic deviation (shown by the bottom plot in Figure 5-12) from the best-fit Gaussian at all levels. The deviation is most likely an artifact of the bandpass correction applied to both observations.

The 10 h integration centered at 187 MHz (also discussed in Section 5.2.6) is a particularly clean observation and has significant implications for the measurements of redshifted 21 cm HI emission that will be targeted by EOR experiments with the full MWA–LFD. The planned EOR observations will require very deep integrations, of order 100 to 1000 h, to achieve the required sensitivity levels. The duration of the observation centered at 187 MHz, therefore, represents a substantial step towards characterizing the properties of the RFI at levels approaching those of interest, and the lack of detectable interferers (even if over only a small portion of the total spectrum) bolsters confidence that the full MWA–LFD will be able to achieve the very deep integrations needed to detect evidence of fluctuations in the redshifted 21 cm HI emission from the EOR.

Reflections from Meteor Trails and Aircraft

The strong interferers at the frequencies of greatest interest for EOR experiments (below 200 MHz) tend to be highly time variable. Figure 5-13 presents a dynamic spectrum or “waterfall” plot in order to illustrate the time variability of signals in the portion of the FM band between 105 and 109 MHz. Two consistent interferers with variable amplitudes are observed at 106.3 MHz and 107.9 MHz, and are typically detected at $\sim 15-\sigma$ in 1 kHz channels with 1.3 sec integrations. Figure 5-13 also contains two events at approximately 7 min and 20 min elapsed time when the power in individual frequency channels, including many outside the consistent carrier channels, deviates by up to 20 dB, or well above 100-\sigma. Much of the power in the interferers in the FM band is from short-duration bursts like these. The two interferers visible in the top panel in Figure 5-11 are due entirely to short-duration events, and the peak at 112.4 MHz in the third panel is significantly dominated by another two such events, one of which increases the power in the peak channel by 25 dB.

These events are believed to be due to radiation from distant, over-the-horizon transmitters scattering off the plasma in meteor trails toward the array [Yellaiah et al., 2001,
Mallama and Espenak, 1999]. Meteoroids enter the Earth’s atmosphere at speeds of order \(10 \text{ km s}^{-1}\) and ablate significantly around 100 km altitude. The ablated atoms interact with molecules in the atmosphere producing meteor trails, expanding clouds of plasma. Radio reflections can be produced from radiation scattering off the plasma immediately surrounding the moving meteoroid or off the trail. Scattering events from the vicinity of the moving meteoroid are very brief, \(\sim 0.1 \text{ sec}\), while scattering from meteor trails last generally around one second (although some may persist for minutes).

Figure 5-14 shows a high time resolution measurement of a meteor trail event in which carrier waves from two distant transmitters are briefly visible and seen to modulate with their audio signals. All three antenna tiles recorded this event with sufficient signal to noise that interferometric phases were available. Solving for the centroid of the reflected radiation (Figure 5-14, bottom panel) reveals two distinct distributions separated by approximately 0.1 deg. At an altitude of 100 km, this angular separation corresponds to an offset of about 175 m. The two distributions are divided in time, as well, with one coming from the first 300 ms of the event and the other from the final 200 ms. The transition between the two features lasts approximately 50 ms. This is consistent with observing the transition from “head” scattering due to plasma immediately surrounding the meteoroid as it enters the atmosphere to trail scattering due to the lingering partially ionized cloud that forms in its wake [Close et al., 2002].

The metallic surfaces on aircraft are also capable of reflecting distant transmissions toward the array. Unlike meteoroids entering the atmosphere, aircraft are typically moving at speeds of order \(0.2 \text{ km s}^{-1}\) at altitudes around 30 km; and the rounded surfaces on aircraft may favorably reflect radiation for up to a few minutes as the aircraft moves across the sky.

The sporadic nature of the FM signals detected with the prototype system appears largely to be due to scattering from meteor trails and reflections from the metallic surfaces on aircraft. Overall, ten unique transient events were recorded in the FM band during 2 h of discontinuous observing. There were seven events lasting less than 3 sec, one event lasting 40 sec, and two events lasting about 65 sec. The lengths of these events suggest that the seven short-lived reflections are due to meteor trail reflections and the three longer events are most likely due to aircraft reflections. The total temporal occupancy was about 2.5% and the spectral occupancy during the events increased by as much as a factor of 100 from \(< 0.1\%\) to \(\sim 1\%\) over a 4 MHz band.

Since the integrated spectrum in the FM band is dominated by meteor reflections that have very low temporal occupancy, the implication is that whatever RFI exists in other bands (even well below the sensitivity threshold achieved in extended integrations by the prototype system) may be similarly dominated. It may be possible to eliminate most of the weak RFI in other bands by full-band excision of time slots when the FM band is observed to contain strong reflections from over-the-horizon interferers. Using such a scheme, the MWA–LFD should be able to go much deeper than the -30 dB achieved with the prototype system, without picking up any significant interference.

5.4 Observational Results

5.4.1 Fornax A Imaging

As a final test of the stability of the prototype system and of our consequent ability to achieve consistent amplitude and phase calibration over both time and frequency, it is
useful to attempt imaging observations of bright, discrete radio sources. We have done this for the well-known source For-A (R.A. 03h 22m 40s, Dec. -36° 14' 20''), which has a prominent double morphology of ~50 arcmin angular extent, sufficient to be resolved by the prototype array. The total flux density of For-A is ~475 Jy at 150 MHz, rising to ~900 Jy at 80 MHz [Ekers et al., 1983].

Data were acquired on September 18, 2005, using 4 MHz bands centered at 80, 100, 120, 140 and 160 MHz. The observations cycled between these and other frequencies every ~7 minutes over a period of ~10.5 hours, with an on-source integration duty cycle of roughly 3.3% at each frequency. The data were correlated on-site using the 512-channel mode and the resulting visibility measurements were stored for later analysis.

The For-A dataset included regular observations of Pic-A, which is not resolved by the prototype system interferometers. The observations provided the gain and phase calibration required in addition to that needed for the baseline determination (see Section 5.2.3) used to correct for geometric delay and Earth rotation. We also calculated the cross-talk contribution from the data itself and subtracted it before exporting the data to FITS files for subsequent analysis and imaging.

Team members experimented with creating images in several imaging packages. This included the use of self-calibration to correct phases. All single-frequency images showed clear double structure, but due to the sparse visibility coverage in the uv-plane from only 3 baselines, strong residual sidelobe structures were widespread.

In order to improve the image quality, we obtained more complete uv-coverage by implementing “multifrequency synthesis” (Sault & Conway 1999). The Miriad package turned out to be the most convenient environment in which to perform this task. The multifrequency synthesis algorithm in Miriad generates both an intensity and a spectral index map of the source, in effect scaling the visibilities to a compromise flux scale. The formal dynamic range of the resulting image in Figure 5-15 is ~100:1 (peak/rms), and the observed structure is in excellent agreement with high fidelity images of Fornax A at higher frequencies [Jones and McAdam, 1992, Fomalont et al., 1989].

This successful mapping exercise indicates that the calibration of the prototype antenna tiles and receivers is reasonably well understood. There are various effects which will limit the fidelity of the image shown in Figure 5-15, including unmodelled spectral gradients in the For-A structure, unmodelled time-variable instrumental response to any linearly polarized emission in For-A, and the presence of other sources within the antenna tile field of view. Of these, the biggest source of uncertainty is most certainly the cumulative effect of other sources entering both the main beam of the antenna tile and the strong sidelobes, which, in some cases, are only ~ 10 dB below the peak response (see Figure 5-5). There has been no attempt to perform an all-sky self-calibration or source subtraction in order to achieve a more global solution.

5.4.2 Solar and Ionosphere Measurements

During September 12 through 21, 2005, the Sun was observed in an 8 MHz band centered at 100 MHz for several hours per day. The data collection mode consisted of an alternating pattern of observing for 64 contiguous seconds and then calculating the auto- and cross-correlations for all three antenna tiles for a similar amount of time. This resulted in an approximately 50% duty cycle. To maintain a high time resolution in the processed data while minimizing the downtime for computation, the 512-channel correlator mode was used to produce 16 kHz-wide frequency channels. The resulting spectra were saved with 50 ms
temporal resolution.

A single, large cluster of sunspots, NOAA active region 0808, was present on the Sun during the entire prototype field deployment (March through September, 2005) and was the source of all flares occurring during the deployment (flare information courtesy of the NOAA Space Environment Center\(^1\)). During periods when no sunspots were visible on the solar surface, such as on September 20, 2005, the measured fluxes at 100 MHz from the Sun were \(\sim 10^4\) Jy.

On September 16, 2005, an M4.4-class flare commenced at 01:41 UTC from active region 0808. The active region was located at S11W26 on the solar disk. The x-ray flux peaked at 01:49 UTC and decayed to 50% peak intensity by 01:56 UTC. Observations with the prototype equipment began at 02:00 UTC and covered the subsequent hour as the flare continued to decay. In contrast to observations of the quiet Sun, the measured background power at all frequencies in the 8 MHz band during this period was elevated by a factor of 40, with typical values of \(\sim 4 \times 10^5\) Jy.

Dozens of short duration radio bursts were observed during this period. The bursts had durations of 0.5 sec or less in any one frequency channel, descended rapidly across the observing band to low frequencies within seconds, and had peak power levels that could exceed \(5 \times 10^6\) Jy. These are signatures of Type-III solar radio bursts, which are generated by beams of electrons travelling along magnetic field lines at speeds above 20,000 km s\(^{-1}\) [Benz, 2002, Cairns and Kaiser, 2002].

In Type-III solar radio bursts, the motion of the electrons along the field lines excites fluctuations in the corona at the local plasma frequency, and these fluctuations are converted into radio emission at the plasma frequency or at a harmonic. As the electrons escape along an open field line into interplanetary space, the density of the local plasma decreases, resulting in a rapid drop in the emitting frequency. Figure 5-16 (right panel) shows the measured power as a function of frequency and time for a clear Type-III burst observed by the prototype system.

A more detailed study of the properties of the individual radio bursts will be the subject of a future paper. In the remainder of this section we examine the variation of the location of the emission centroid.

**Emission Centroid**

Since both the bursts and the background solar emission were very intense during the flare on September 16, 2005, each individual frequency and time interval had sufficient signal to noise to solve for the unique location of the centroid of emission using the phases of the cross-correlated powers. In general, the combination of the three baselines produced locations repeatedly to within 10 arcsec.

Two forms of variation were seen. On a timescale of several hundred seconds, the location of the centroid for the Sun wandered by about 7 arcmin, much larger than the error in the solutions for the emission location. Intense radio bursts were associated with sudden deflections in the source location, as can be seen in Figure 5-16 (left panel). We attribute the long-term motion to small changes in electron column density between each antenna tile and the Sun due to density gradients in the ionosphere; whereas the short term change in the emission centroid is believed to be due to the burst region itself. If the burst region is slightly offset from the location of the background emission and momentarily

\(^1\)http://sec.noaa.gov/
dominates the power along the baselines of the array, the location of the centroid will move
during the bursts.

Figure 5-17 is a plot of the location of the emission centroid as a function of wavelength
for one snapshot interval without radio bursts. The absolute location is unknown due to
uncertainties in the lengths of the cables between the antenna tiles and other experimental
effects, but there is a clear gradient in any given interval between displacement and wave-
length. A simple model for the angular offset, $\alpha$, that one would expect with wavelength
due to a difference in electron column density in the ionosphere between the antenna tiles
is given by:

$$\sin(\alpha) \sim \frac{\Delta_{TEC}}{\nu^2 d},$$

where $\Delta_{TEC}$ is the difference in the total electron content (TEC), $\nu$ is frequency, and $d$
is the length of the baseline. Although the observed frequency band was too narrow to
clearly see the $\sim \lambda^2$ dependency expected from Equation 5.2, the properties of the centroid
displacements were consistent overall with the effects of ionospheric gradients, as opposed to
poor calibration of the baselines, since they exhibited quasi-random behavior as a function
of time. For the observation shown in Figure 5-17, we identify a best-fit value for the $\Delta_{TEC}$
of $6.4 \times 10^{13}$ electrons m$^{-2}$. This is several thousand times smaller than the typical total
electron content for the daytime ionosphere.

We quantify the ionospheric variation associated with the long-term (several hundred
second) variation in the location of the emission centroid in Figure 5-18, which shows
a histogram of the relative frequency of occurrence of different values of the differential
electron column density between the antenna tiles. For the one hour period analyzed, 90%
of the time the difference in TEC was less than 0.005 TECU, where 1 TECU $= 10^{16}$ electrons
m$^{-2}$ This distribution should represent a relatively extreme case since the ionosphere was
active during this period, as a coronal mass ejection associated with a flare several days
earlier had arrived recently at Earth and triggered a geomagnetic storm.

Implications for Ionospheric Calibration

As demonstrated by Figure 5-17, the apparent positions of astronomical sources on the sky
shift due to variations in electron column density over the array. These positional errors
will contribute to the noise levels in MWA–LFD images and uncertainties in EOR statistical
measurements, thus their calibration and removal over the wide field of view will be a critical
component of the final image processing path for the full MWA–LFD. With the antenna
tile characteristics measured during the prototype field deployment, quantitative estimates
of the efficacy of the planned ionospheric calibration can be made.

Simulations of the ionospheric calibration process for the full MWA–LFD have been
performed and will be described in detail in a future paper. The simulated technique
is based on an algorithm that is used for Very Large Array data taken at 74 MHz [Cotton
et al., 2004, Lane et al., 2004, Cohen et al., 2004], where the longer baselines and lower
frequency make ionospheric calibration much more challenging than for the MWA–LFD
case. For a model ionosphere whose electron column density fluctuations are comparable
to those determined in Section 5.4.2 (as shown in Figures 5-17 and 5-18), a 9th-order two-
dimensional polynomial fit to the measured offset in the locations of 160 predetermined
calibrator sources results in RMS residuals in position of 4 to 6 arcsec (2 to 3% of the 200
MHz array beam) over an 8° by 8° field of view. The residual noise level was $F \cong 10 \mu$Jy,
equivalent to the thermal noise level that would be reached by the MWA–LFD in $\sim 1000$ h of
integration. Although the field of view of the actual array will be greater than 20° by 20° for frequencies below 200 MHz—and the model fit would be according less accurate given the same number of calibrator sources and order of polynomial—with refinement it is projected that ionospheric calibration will not limit MWA–LFD image fidelity or significantly interfere with EOR statistical measurements.
Figure 5-10 Measured RFI spectrum at Mileura Station from 80 to 300 MHz. The full spectrum was generated from 55 separate 15-second integrations, each covering 4 MHz. The resulting spectra, with 1 kHz resolution, were bandpass-corrected and stitched together into one long spectrum. At the lower frequencies, a low-amplitude ripple of 600 kHz periodicity, caused by cable reflections, was removed by median filtering. At all frequencies, the 0 dB level corresponds approximately to the Galactic background noise in a cold part of the sky. Although the physical resolution on the plot corresponds to several hundred channels, even a single 1 kHz channel with RFI will be visible. There are many long sections of spectrum completely free of RFI at this sensitivity.
Figure 5-11 Deep integrations with a single antenna tile for 11 spectral windows of 4 MHz each in order to characterize the low-level RFI environment. Integration times are given in the top left corner of each plot and range from 30 min to 10 h. The scales are in dB relative to the system noise, which is dominated by the sky background. As discussed in Section 5.3, much of the visible RFI in these observations is due to intermittent sources.
Figure 5-12 Normalized probability of individual channel power measurements deviating from the mean value for two deep integrations (top) and the residual variation from purely Gaussian noise (bottom). The solid lines are for the 42 min integration centered at 107 MHz, and the dashed lines are for the 10 h integration centered at 187 MHz. The dotted lines give a reference Gaussian distribution. Each integration consists of 4096 channels spanning 4 MHz and is divided into 1.3 sec time steps. Thus, the two observations have $4096 \times 1922 = 7,872,512$ and $4096 \times 30874 = 126,459,904$ individual samples, respectively. The high-$\sigma$ tail in the 107 MHz data is due to the presence of RFI.
Figure 5-13 Waterfall plot of RFI in the 4 MHz band spanning 105 to 109 MHz. The gray scale is such that white corresponds to a 3-σ variation in measured power. Two strong interferers are visible at 106.3 and 107.9 MHz. Large, short-duration bursts in power at approximately 7 min and 20 min elapsed time are taken as evidence of distant transmissions scattering off ionized gas in meteor trails.
Figure 5-14 Expanded view of signals from two distant transmitters scattering off ionized gas in a meteor trail. The top panel illustrates the high time resolution modulation of the carrier signals. The second panel shows the interferometric phase at 98.9 MHz during the length of the reflection event along the baseline between antenna tiles 1 and 2, and the bottom panel uses the interferometric phase information from all three baselines to plot the relative location of the centroid of the reflected emission. The relative position of the centroid undergoes a clear shift in its position approximately half way through the event. This observation was acquired with all three antenna tiles using the north-south polarization on September 20, 2005 at 21:22:41 UTC.
Figure 5-15 Multifrequency synthesis image of For-A (left) and the distribution of visibility measurements in the uv-plane (right).
Figure 5-16 Grayscale plots of the east-west location (left) and auto-correlation power (right) as a function of time and frequency, illustrating an example of a Type-III solar radio burst. The deflection of the burst is about 20 arcsec, and the burst is about 20 times more powerful than the background.
Figure 5-17 Deflection of the centroid of emission as a function of wavelength for a 50 ms interval without radio bursts. The dashed line is the best-fit of a model for the deflection expected due to a difference in electron column density in the ionosphere between the antenna tiles of $6.4 \times 10^{13}$ electrons m$^{-2}$. 
Figure 5-18 Histogram of the relative frequency of occurrence of differences in electron column density between the antennas. The black line is the difference in column density in the east-west direction, and the gray line is the difference in the north-south direction.
Chapter 6

Experiment to Detect the Global EOR Signature (EDGES)

When completed, the MWA will probe the reionization epoch by characterizing the properties of fluctuations in the expected redshifted 21 cm background. These fluctuations are predicted to be due to local perturbations in the density, spin temperature, and ionization fraction of hydrogen gas in the IGM at the time of reionization. By studying these fluctuations, it is believed that information will be revealed regarding both the history of how reionization unfolded on local scales (those of individual quasars groups of galaxies), as well as how the global (mean) spin temperature and ionization fraction of hydrogen in the IGM evolved with time.

As discussed in Chapter 1, there is a second, complementary, experimental setup that can be used to probe the time-evolution of just the global properties of the hydrogen in the early universe. The mean observed brightness temperature over a large solid angle of the sky for a given frequency, $\nu < 1420$ MHz, will be affected by the neutral hydrogen in the IGM at a redshift corresponding to $1 + z = 1420/\nu_{\text{MHz}}$. Depending on the spin temperature and ionization state of the neutral hydrogen, the observed brightness temperature may be reduced (due to absorption by HI of the CMB), enhanced due to emission by HI, or unchanged relative to the background level. The result is a pseudo-continuum that, for times earlier than redshift $z \gtrsim 6$, should be visible as a contribution to the radio spectrum below 200 MHz. Determining the precise frequency-dependence of this contribution would directly constrain $\bar{x}_{HI}(1 - T_e/T_S)$, and thereby the ionization history of hydrogen gas in the IGM and derived quantities such as the optical depth to CMB photons, $\tau$. Furlanetto [2006] illustrates possible examples of this contribution for several fiducial stellar population histories.

In principle, this measurement is much less complicated to perform than that planned for the MWA. Since the desired signal is the mean brightness temperature due to redshifted 21 cm emission (or absorption) over the entire sky, there is no need for high angular resolution or imaging, and a single dipole antenna tuned to the appropriate frequencies could reach the required sensitivity ($\sim 10$ mK) with only one hour of integration time assuming reasonable spectral resolution. There is a fundamental complication with such an experiment, however, arising from the global nature of the signal. Since the expected redshifted 21 cm emission fills the entire sky, there is no ability to perform comparison switching between the target field and a blank field. The problem this causes is two-fold. First, it makes it difficult to separate the contributions to the measured spectrum due to the redshifted HI signal
from any other all-sky emission, including the Galactic synchrotron and free-free radiation, the integrated effect of extragalactic continuum sources, and the CMB. Second, for similar reasons, it is difficult to avoid confusing any systematic effects in the measured spectrum due to the instrument or environment with received signal from the sky. The severity of these problems is exacerbated in single-antenna measurements by the intensity of the Galactic synchrotron emission. Unlike the interferometric observations of the MWA, a single antenna is sensitive to the large-scale emission from the Galaxy, providing a 200 to 10,000 K foreground in the measured spectrum. Determining the \(\sim 10 \text{ mK}\) redshifted 21 cm contribution to the spectrum requires identifying deviations in this power-law-like foreground spectrum at better than 1 part in 10,000.

Despite these complications, constraints on the global properties of hydrogen in the IGM would be a valuable independent probe of the reionization history of the universe. For this reason, several efforts are underway to make precise measurements of the radio spectrum below 200 MHz. In this chapter, we report on the initial results of one ongoing effort, the Experiment to Detect the Global EOR Signature (EDGES). This experiment is supported by Haystack Observatory and lead by Alan Rogers.

In order to be successful, any experiment attempting to constrain the low-frequency radio spectrum at the level required for reionization science must tackle the two challenges identified above: separate the redshifted 21 signal from the Galactic synchrotron emission, and eliminate any instrumental systematic errors in the measured spectrum to better than 1 part in 10,000. Both of these are very difficult tasks. In Section 6.1, we describe the specific approach used for EDGES to address the issue of separating the redshifted 21 signal from the Galactic emission. We then give an overview of the EDGES system in Section 6.2, followed by the results of the first observing campaign with the system in Section 6.3.

6.1 Method

As with the planned measurements of the fluctuation power spectrum with the MWA, a global reionization experiment must take advantage of existing information regarding the differences between the spectra of the Galactic and extragalactic foregrounds and the expected redshifted 21 cm contribution to isolate the reionization signal. In order to describe the approach used for EDGES to attempt to constrain the ionization history of the IGM, it is useful to begin by reviewing briefly these properties of both the foregrounds and the signal.

As we saw in Chapter 3, the Galactic synchrotron emission is the dominate component of the foregrounds, accounting for all but approximately 30 to 70 K (\(\sim 70\%\)) of the foregrounds at 178 MHz [Bridle, 1967, Shaver et al., 1999]. Its spectrum is roughly a power-law in temperature units given by \(T_{\text{gal}}(\nu) \sim \nu^{-\gamma}\), where \(\gamma \approx 2.5\) is the spectral index. The spectral index is generally constant over the frequencies of interest, although it is known to roll over (flatten) with decreasing frequency due to self-absorption. The amplitude of the synchrotron emission and the exact value of the spectral index depend on Galactic coordinate (see Chapter 3). The amplitude varies over an order of magnitude, between about 200 < \(T_{\text{gal}}\) < 10,000 K at 150 MHz (peaking toward the Galactic center), while the spectral index has small variations of order \(\sigma_{\gamma} \approx 0.1\) dependent largely on Galactic latitude, with the steepest regions occurring at high Galactic latitudes.

The free-free emission in the Galaxy and the extragalactic continuum sources also have power-law spectra with similar spectral indices. The extragalactic continuum is generally
isotropic on large scales and accounts for the majority of the remaining power in the low-frequency radio spectrum, with free-free emission making up only about 1% of the total power.

The global mean redshifted 21 cm emission, on the other hand, is expected to be negligible for $\nu \geq 200$ MHz and increase with decreasing frequency to a maximum contribution of order $\Delta T_{\text{eor}} \approx 25$ mK at frequencies corresponding to early in the reionization epoch, with the transition occurring roughly between 100 and 200 MHz. Below 100 MHz (at sufficiently high redshift), the redshifted 21 cm contribution will eventually turn over and decrease with decreasing frequency. It may even dip into absorption (again of order $|\Delta T_{\text{eor}}| \lesssim 100$ mK) before returning to zero permanently at very low frequencies ($\nu \lesssim 20$ MHz). For the large solid angles of a single antenna beam, the mean redshifted 21 cm signal should vary little from one location to another on the sky.

The approach employed with EDGES to overcome the difficulty in separating these contributions in the measured spectrum is to limit the scope of the experiment to test for fast reionization only. In the extreme case that the transition from a fully neutral IGM to a fully ionized IGM was virtually instantaneous ($\dot{\chi}_{\text{HI}}(z) \rightarrow \infty$), the contribution to the global spectrum at the frequencies corresponding to the reionization epoch would approach a step function. A sharp feature resembling a step function superimposed on the smooth power-law-like foreground spectrum should be relatively easy to identify, and certainly more separable than a prolonged, smooth transition that spanned a large range of redshifts and many tens of MHz. In principle, a simple low-order polynomial fit to the measured spectrum would reveal such a discontinuous feature and determine the redshift of reionization. This method parallels the approach planned for the MWA to separate the foregrounds from redshifted 21 cm fluctuations based on their characteristic coherence lengths in the spectral domain since it exploits the spectral smoothness of the foregrounds. An advantage of this approach for global reionization experiments is that, given sufficient sensitivity, even a null result would constrain $\dot{\chi}_{\text{HI}}(z)$, and thereby distinguish between slow and fast reionization scenarios.

6.2 System Design

By focusing (at least initially) on confirming or ruling out a fast reionization scenario, the design of the EDGES system is able to be made relatively simple. The primary need is to reduce any instrumental or systematic contributions to the measured power spectrum that vary rapidly with frequency, since these could be confused with a sharp feature in the spectrum due to a fast transition to a neutral IGM. Such contributions could be due to reflections of receiver or sky noise from nearby objects, undesirable resonances within the electronics or RFI enclosures, or spurious signals introduced by the digital sampling system.

In this section, we provide an overview of the system, highlighting aspects that are relevant to reducing the effects of systematic errors. Details of the design can be found in the EDGES memorandum series\(^1\). When appropriate, relevant memoranda are cited below. Ultimately, eliminating or counteracting sources of systematic errors in radio frequency electronics is a very iterative process of trial and error tailored to the specific instrument under consideration. Much of this critical effort for EDGES was performed by Alan Rogers and is documented in the project memoranda.

\(^1\)http://www.haystack.mit.edu/ast/arrays/Edges/
6.2.1 Hardware Configuration

The EDGES system consists of three primary modules: an antenna, an amplifier and comparison switching module, and an analog-to-digital conversion unit. The antenna, shown in Figure 6-1, is a “fat” dipole based design derived [Rogers, 2006a] from the four-point antenna of Suh et al. [2003, 2004]. The design was chosen for its simplicity and its relatively broad frequency response that spans approximately an octave. The response of the antenna was tuned to 100 to 200 MHz by careful selection of the dipole dimensions. In order to eliminate reflections from the ground and reduce gain toward the horizon, the antenna is placed over a conducting mesh that rests directly on the ground. The mesh is constructed from thin, perforated metal sheets to reduce weight and is shaped to match an octagonal support structure below the ground screen. The diameter of the ground screen is approximately 2 m.

Although the antenna is constructed with perpendicular dipoles capable of receiving dual linear polarizations, only one polarization of the crossed-dipole is sampled by the receiver system. A dipole antenna is naturally a balanced electrical system. To convert from the balanced antenna leads to the unbalanced receiver system, a short coaxial cable enclosed in a clamp-on split ferrite core with an impedance of 572 $\Omega$ at 100 MHz (Digi-Key part no. 240-2245-ND) is used as a balun and is connected directly to the terminals of the antenna with the central conductor fastened to one element and the braided shielding to the other.

The amplifier module consists of two stages that are contained in separate aluminum enclosures to reduce coupling between the low-noise amplifiers. Each stage provides 33 dB of gain for a total of 66 dB. Bandpass filtering of the signal is also performed in the second stage, and the resulting half-power bandwidth spans approximately 50 to 330 MHz. The amplifier chain can be connected through a voltage controlled three-position switch to the antenna, an ambient load, or an ambient load plus a calibration noise source [Rogers, 2006c]. Switching between the ambient load and the antenna provides a comparison to subtract spurious instrumental signals in the measured sky spectrum.

Impedance mismatch between the antenna and the amplifiers causes reflections of the sky noise within the electrical path of the instrument that produce an undesirable sinusoidal ripple in the measured spectrum due to the frequency-dependence of the phase of the reflections at the input to the amplifier. To reduce the effects of these reflections in EOR measurements, the input to the amplifier chain is connected directly to the balun on the antenna (with no intermediate transmission cable), as shown in Figure 6-1. While absolute calibration is limited in this configuration by the effect of the unknown phases of the reflections on the measured spectrum, the compact size of the antenna and the small signal path delays result in a very smooth spectral response.

The amplifier module is connected to the analog-to-digital conversion module by three 50 foot LMR-240 super-flex cables. The cables provide power, switching control, and signal transmission, respectively. Common-mode current on these cables (i.e. current that is on the outer surface of the shielding in the coaxial cable, or current that is unidirectional on both the central conductor and inner surface of the shielding) is also capable of producing reflections and additional sinusoidal ripples in the measured spectrum. The ferrite core balun used in EDGES allows common-mode current of approximately 10% of the differential mode. Although most of this current is transferred to the ground screen by direct contact between the amplifier module casing and the ground screen, some current persists and leaks through the casing of the amplifier module and onto the shielding of the three cables connecting the amplification module to the analog-to-digital conversion module. Additional
clamp-on ferrite cores with 471 Ω impedance at 100 MHz (Digi-Key part no. ND-240-2246) are placed every meter on the cables to reduce this current to less than 0.005% [Rogers, 2006f].

Finally, the analog-to-digital conversion is accomplished with an Acqiris AC240² commercial 8-bit digitizer with maximum dynamic range of 48 dB. The AC240 uses an embedded field programmable gate array (FPGA) to perform onboard Fourier transform and integration in realtime. The spectrometer is clocked at 1 GS/s and the Fourier transform processes 16,384 channels, giving a bandwidth of 500 MHz and a raw spectral resolution of about 30 kHz. An onboard digital filterbank is used to improve the isolation between neighboring frequency channels at the expense of reducing the effective spectral resolution to 122 kHz. The unit is contained on a CompactPCI card connected to a host computer running a Linux operating system and is capable of transferring data at more than 100 MB/s (although only a fraction of the available bandwidth was used for EDGES). The digitizer and host computer, along with a power transformer and interface circuitry for controlling the amplifier module with the serial port of the computer, are enclosed in an aluminum box to prevent interference.

6.2.2 Data Acquisition and Processing

To acquire a spectrum, software on the host computer cycles the amplifier module between the three switch positions and triggers the AC240 digitizer to acquire, Fourier transform, and accumulate data for a predefined duration at each of the switch positions. Typically, the integration durations per switch position are \( \tau_{(0,1,2)} = \{10, 5, 10\} \) seconds for the ambient load, ambient load plus calibration noise source, and antenna, respectively. Thus, the loop repeats approximately every 25 seconds for the duration of the observation. The three spectra from each cycle are recorded to a disk along with the end time of the cycle and ancillary configuration data.

If the frequency-dependence of the impedance match between the antenna and the receiver is neglected, the measured spectra from the three switch positions can be combined to produce a partially calibrated estimate of the sky temperature following the prescription of Rogers [2006b,d,e,h]. The measured spectra are given by:

\[
\begin{align*}
    p_0 &= g(T_L + T_R)(1 + n_0) \\
    p_1 &= g(T_L + T_R + T_{cal})(1 + n_1) \\
    p_2 &= g(T_A + T_R)(1 + n_2)
\end{align*}
\]  

(6.1)

where the explicit frequency dependence of each term has been dropped and \( p_0 \) is the spectrum for the ambient load, \( p_1 \) is the spectrum for the ambient load plus calibration noise, and \( p_2 \) is the spectrum for the antenna. In this terminology, \( g \) is the gain, \( T_L \) is the ambient load temperature, \( T_R \) is the receiver noise temperature, \( T_{cal} \) is the calibration noise temperature, and \( T_A \) is the antenna temperature. Uncertainty in the measurements is explicitly included in the Gaussian random variables \( n_0, n_1, \) and \( n_2, \) the magnitudes of which are given by \((b\tau_i)^{-1/2}\), where \( b = 122 \times 10^3 \) Hz is the resolution bandwidth and \( \tau_i \) is the integration time in seconds (for each switch position, \( i \)). Temporarily setting the noise terms to zero, \( n_i \to 0 \), and solving for the antenna temperature yields

\[
T_A = T_{cal} \frac{p_2 - p_0}{p_1 - p_0} + T_L.
\]  

(6.2)

Adding the noise terms back in and solving in the limit that \( T_{\text{cal}} \gg (T_L \approx T_A) > T_R \) results in an estimate of the thermal uncertainty per frequency channel of approximately

\[
\Delta T_{A,\text{rms}} \approx \sqrt{n_0^2(T_L + T_R)^2 + n_1^2(T_L)^2 + n_2^2(T_A + T_R)^2}.
\] (6.3)

For optimal efficiency, the three terms contributing to \( \Delta T_{A,\text{rms}} \) should be comparable in magnitude. Substituting \( T_L = 300 \text{ K}, T_A = 250 \text{ K} \) and \( T_R = 20 \text{ K} \), we find that the terms are comparable as long as approximately equal time is spent in each switch position, and that a 1 h integration in each switch position (3 h total) will result in a thermal uncertainty in the estimate of the antenna temperature of approximately \( \Delta T_{A,\text{rms}} \approx 25 \text{ mK} \) within each 122 kHz frequency channel.

### 6.3 Initial Results

The EDGES system was taken to Mileura Station in Western Australia and deployed near the homestead from 29 November through 8 December 2006 [Rogers and Bowman, 2006]. The expedition was conducted in order to test and refine the configuration of the instrument in a radio-quiet site and to attempt the first measurements capable of constraining the reionization history of the universe based on redshifted 21 cm emission. Mileura Station
Figure 6-2 Integrated spectrum used for upper limit analysis of reionization signal. The sky temperature, $T_{\text{sky}}$, is an estimate based on modeled values for cable losses and no correction for antenna reflections. The spectrum represents the best 10% of the data from observations over two nights. It is selected by discarding individual observation cycles (see Section 6.2.2) containing periods of particularly intense radio frequency interference. A total of approximately 1.5 h of integration is included (3.75 h including the ambient load and calibrator noise source measurements in each cycle). The black curve shows the spectrum after de-weighting the interferers (shown in gray) present in the retained observations.
was chosen as the first deployment site due to previous experience at the location and the beneficial orientation of the Galaxy during the prime nighttime observing hours for the target deployment dates. In general, the choice of deployment site (and date) for the system is important [Bowman, 2006b] in order to minimize both RFI and exposure to periods of high Galactic noise. At Mileura Station, the system was deployed approximately 100 m from the nearest buildings in a clearing with no nearby objects and no obstructions above \( \sim 5^\circ \) on the horizon. The antenna was aligned in an approximately north-south/east-west configuration before acquiring data. The precise alignment was found later using GPS to be \( 18^\circ \pm 7 \) clockwise of a north-south/east-west configuration. The north-south polarization was sampled for all EOR measurements.

The system was operated on 8 consecutive nights during the deployment, with 5 of the nights dedicated to EOR observing. In total, over 30 h of relevant observations were obtained, but strong, intermittent interference from satellites complicated the measurements and only approximately 8 h of high-quality observations were used as the primary data set. From that data, a stringent filter was applied to select the best 1.5 h of sky-time when transient satellite signals were weakest. The integrated spectrum generated from these measurements is shown in Figure 6-2. Frequency channels containing RFI were identified in the integrated spectrum using a sliding local second-order polynomial fit and iteratively removing channels with large errors until the fit converged. The affected channels were weighted to zero in subsequent analysis steps. To look for small deviations from the smooth Galactic synchrotron foreground over a large range of the spectrum, a seventh-order polynomial was fit to the measured spectrum between 130 and 190 MHz and subtracted. The residual deviations are shown in Figure 6-3.

The level of systematic contributions to the measured spectrum was found to be approximately \( \Delta T_{\text{rms}} \approx 75 \) mK. This is a factor of \( \sim 3 \) larger than the expected redshifted 21 cm contribution. Although it is not obvious in Figure 6-3 that the variations in the residuals are due to instrumental contributions and not thermal noise, the analysis of the dependence of \( \Delta T_{\text{rms}} \) on integration duration shown in Figure 6-4 clearly illustrates that the \( \text{rms} \) of the residuals follows a thermal profile initially and then saturates to a constant value. After smoothing to 2.5 MHz resolution, the instrumentally dominated 75 mK threshold is reached in approximately 20 minutes (1200 s) of integration on the sky (50 minutes of total integration in all three switch positions). Reordering the individual 25-second observation cycles used in the full integration does not change the behavior in Figure 6-4.

### 6.3.1 Limits on Reionization History

Although the sensitivity level of the initial observations with the EDGES system was limited by instrumental effects in the measured spectrum above the expected contribution due to redshifted 21 cm emission, weak constraints can still be placed on the reionization history. In addition, it is also possible to make a quantitative assessment of how much improvement must be made to the EDGES system before significant constraints are possible. To begin, we introduce a model for the sky spectrum such that

\[
T_{\text{sky}}(\nu) = T_{\text{gal}}(\nu) + T_{\text{cmb}} + \Delta T_{\text{eor}}(\nu)
\]

where \( T_{\text{cmb}} = 2.725 \) K is the CMB contribution, \( \Delta T_{\text{eor}} \) is a constant and is the maximum amplitude of the redshifted 21 cm contribution, and \( t(\nu) \) is the specific form for the frequency-dependence of the emission during the transition from the fully neutral to fully
Figure 6-3 Residuals after subtraction of seventh-order polynomial fit to measured spectrum shown in Figure 6-2. The gray line is the raw spectrum with 122 kHz resolution. The black line is after smoothing to 2.5 MHz resolution to reduce thermal noise to below the systematic noise. The rms of the smoothed fluctuations is approximately 75 mK (see Figure 6-4). All of the identifiable features in the spectrum are instrumental artifacts. The large variations between 163 and 170 MHz are due to the 166 MHz PCI-bus clock of the AC240 and computer. The gap between 136 and 138 MHz is due to RFI excision over a region spanning more than 2.5 MHz. Longer integrations (up to approximately 3 h of sky time), using observation cycles with more intense interference, continued to decrease the thermal noise, but the spurious signals and systematic effects remained unchanged.
Figure 6-4 Characteristic amplitude of the residuals to the polynomial fit as a function of integration time on the sky. The \textit{rms} follows a thermal \((br)^{-1/2}\) dependency until saturating at a constant 75 mK noise level due to the instrumental errors introduced into the measured spectrum. The dotted lines are guides for the eye showing a \((br)^{-1/2}\) profile and a constant 75 mK contribution.
Figure 6-5 Example of redshifted 21 cm contribution (solid) to $T_{\text{sky}}$ based on the model described in Section 6.3.1 with $\Delta T_{\text{cor}} = 25 \text{ mK}$, $z_0 = 8$, and $\Delta z = 0.6$. The residuals (dashed) are also shown for a seventh-order polynomial fit to the simulated combined spectrum between 130 and 190 MHz.
Figure 6-6 Constraints placed by EDGES on the redshifted 21 cm contribution to the sky spectrum. The dark-gray region at the top-left is the portion of the parameter space ruled out by the initial EDGES results with $\Delta T_{rms} = 75$ mK (solid line). The dashed line labelled $\Delta T_{rms} = 7.5$ mK and the dotted line labelled $\Delta T_{rms} = 3$ mK indicated the constraints that could be placed on reionization if the experimental systematics were lowered to the respective values. The light-gray region along the bottom is the general range of parameters believed to be viable. The redshifted 21 cm contribution to the spectrum is modelled according to the description in Section 6.3.1 with $z_0 = 8$. 

120
ionized IGM. For simplicity we take \( t(\nu) \) to be given by

\[
\begin{align*}
t(\nu) = \begin{cases} 
1, & \nu < (\nu_0 - \Delta \nu/2) \\
\frac{1}{2} + \frac{1}{2} \cos \left[ \pi (\Delta \nu/2 + \nu - \nu_0)/\Delta \nu \right], & (\nu_0 - \Delta \nu/2) \leq \nu \leq (\nu_0 + \Delta \nu/2) \\
0, & (\nu_0 + \Delta \nu/2) < \nu 
\end{cases}
\end{align*}
\]

(6.5)

where \( \nu_0 \) is the frequency corresponding to the redshift, \( z_0 \), when \( \dot{\tau}_{HI}(z_0) = 0.5 \), and \( \Delta \nu \) is the frequency range corresponding to the total duration, \( \Delta z \), of the reionization epoch. The frequency range and redshift duration are related by

\[
\Delta z = 1420 \left[ (\nu_0 - \Delta \nu/2)^{-1} - (\nu_0 + \Delta \nu/2)^{-1} \right].
\]

(6.6)

where \( \nu_0 \) and \( \Delta \nu \) are measured in MHz and we note that \( \dot{\tau}_{HI} \approx \Delta z^{-1} \). For convenience, this model is defined in frequency coordinates, thus the simple half-period sinusoidal form for the spectral contribution is distorted in redshift-space. However, the exact shape of the transition has little influence on the outcome of the constraints as long as it is reasonably smooth. Figure 6-5 illustrates the modelled redshifted 21 cm spectrum. The free parameters in the model are \( \Delta T_{\text{cor}}, \nu_0, \) and \( \Delta \nu \), or equivalently, \( \Delta T_{\text{cor}}, z_0, \) and \( \Delta z \).

For the EDGES best-response frequency range, a center redshift around \( z_0 = 8 \) allows the largest range of \( \Delta z \) to be explored. By simulating the combined sky spectrum, \( T_{\text{sky}} \), for a range of the two remaining free parameters, we can determine the \( \text{rms} \) of the residuals that would remain following the polynomial fit used in the EDGES data analysis. Comparing the residuals in the models to the 75 mK \( \text{rms} \) of the initial measurements gives the region of parameter space ruled out so far. Figure 6-6 illustrates the results of this process.

The initial results constrain only a small portion of parameter space that is well outside the expected region for both the amplitude and duration of reionization. The best constraint in the case of a nearly instantaneous reionization is that the redshifted 21 cm contribution to the spectrum is not greater than about \( \Delta T_{\text{cor}} \lesssim 450 \) mK before the transition. Decreasing the systematic contributions in the measured spectrum by more than an order of magnitude to \( \Delta T_{\text{rms}} < 7.5 \) mK would allow meaningful constraints. However, an improvement of a factor of 25 or better to \( \Delta T_{\text{rms}} < 3 \) mK is required to rule out a significant portion of the viable parameter range. Below 3 mK the \( \text{rms} \) is dominated by errors in the polynomial fit due to the Galactic synchrotron spectrum contribution.

### 6.4 Absolute Sky Temperature and Spectral Index

In the 1960s and 1970s there was considerable interest in constraining the radio spectrum between 10 and 1400 MHz in order to investigate the physical structure of the Galaxy [Turtle et al., 1962, Andrew, 1966, Purton, 1966, Bridle, 1967, Webster, 1974, Sironi, 1974, 1976]. At the time, it was recognized that, along with extragalactic sources, the diffuse Galactic radio emission had structure that was composed of three components, originating from the disk, the spiral arms, and a radio halo, respectively [Sironi, 1976]. Several experiments were performed using sets of identically scaled horns, antennas, or dipole arrays, in addition to receivers on satellites, to measure accurately the spectrum of the Galactic non-thermal radiation. The individual components were identified and separated by utilizing analysis tools such as temperature-temperature ("T-T") plots so that their spectral properties could be investigated, both as functions of frequency and look-direction. These early measurements resulted in several key findings, including that the total diffuse spectrum flattens below 10
Figure 6-7 Antenna reflection coefficient as a function of frequency. A 15th-order polynomial is fit (solid) to the raw measurements (gray).
Figure 6-8 Example calibrated sky spectrum (gray) after subtraction of $T_{cmb} = 2.725$ compared to model fit (solid line). One 25 s integration cycle from observations in the north-south polarization with a 100 foot transmission cable at approximately 8 h LST on 7 Dec 2006 was used for this plot.
Figure 6-9 Derived spectral index between 100 and 200 MHz (top panel) and $T_{gal}$ at 150 MHz (bottom panel) as functions of LST. Four data sets were used in the plots. The data points indicated with circles and crosses are for observations on 7 Dec 2006 using a 100 foot transmission cable and the north-south and east-west polarizations, respectively, while the dots are from 30 Nov 2006 and the boxes are from 1 Dec 2006, both of which used 50 foot cables and the north-south polarization of the antenna. For the 1 Dec 2006 observations, the ground screen was extended to test the contribution of ground loss.
MHz [Sironi, 1976], the halo contribution is extremely faint, the spectral index of the disk contribution is dependent on look-direction, and the typical spectral index of the disk contribution steepens rapidly with increasing frequency between 200 and 400 MHz from about $\beta = 2.4$ to $\beta = 2.8$ [Bridle, 1967]. More recent investigations have confirmed these early results [Roger et al., 1999, Platania et al., 1998] and demonstrated that the total spectral index saturates to about $\beta = 2.9$ above 1 GHz. There has also been renewed interest in absolute sky temperature measurements around 1 GHz in order to investigate predicted deviations in the CMB from a purely black-body (Planckian) frequency distribution.

With the current interest in the 100 to 200 MHz frequency range for EOR science, it is useful to gather the collected knowledge of earlier experiments and confirm the results with modern approaches. Although, the design of EDGES is optimized for constraining the smoothness of the low-frequency radio spectrum, a small change to the standard configuration enables the additional calibration needed to constrain the absolute sky temperature and the spectral index of the diffuse emission. In this section, we describe the modifications made to the EDGES system to allow absolute calibration and report the results of the absolute temperature and spectral index measurements between 100 and 200 MHz. Because the EDGES antenna is a single dipole with a large field of view, it is incapable of performing the difference measurements employed in the pioneering efforts to isolate the Galactic and extragalactic contributions to the spectrum, and therefore, constrains only the total spectrum.

6.4.1 Calibration and Corrections

To calibrate the sky temperature measurements made with EDGES, the location of the amplification module is moved so that it is connected to the antenna via a long (either 50 or 100 foot) LMR-400 transmission cable (without any ferrite cores for common mode suppression). This configuration provides the ability to easily identify the contribution in the measured spectrum from reflections due to the impedance mismatch of the antenna and receiver system and, thus, to calibrate the antenna reflection coefficient. However, it also requires that the cable transmission coefficient is known since the ambient load and calibrator noise sources in the amplifier module are separated from the antenna by the transmission cable. The losses due to the ground, balun, antenna, and horizon must also be included in the analysis. The antenna reflection coefficient and cable transmission coefficients are dependent on frequency, while the last set of losses is reasonably independent of frequency. Below, we discuss briefly the methods used for acquiring or estimating these quantities and then present the resulting determination of the spectral index and absolute temperature of the non-CMB contribution to the spectrum.

The calibration of the loss due to the transmission cable between the antenna and the receiver system is determined in much the same way that the precise temperature of the internal calibration noise source is measured. A precision calibrated noise source is connected to either the amplifier module input directly, or through the transmission cable. When connected directly, the precision noise source provides a reference for calibrating the effect of the cable, as well as a temperature standard for determining the absolute temperature of the internal noise source. To complete the calibration of the internal noise source, a liquid nitrogen cooled load is applied directly to the amplifier input in the laboratory. To complete the transmission cable calibration, the transmission cable is inserted between the precision noise source and the input to the amplifier module. Measurements of the difference between the spectra give the loss due to the cable. For the LMR-400 cable, the transmission
coefficient, $\Upsilon$, was determined to be

$$\Upsilon(\nu) = \left(\frac{L}{100}\right) 10^a(b\nu^{1/2} + c\nu)$$  \hspace{1cm} (6.7)

where $a = -0.115$, $b = 0.12229$, and $c = 2.6 \times 10^{-4}$, and $L$ is the length of the cable measured in feet and $\nu$ is the frequency in MHz. For a 100 foot cable, this corresponds to $\Upsilon = 0.7184$ and 0.6239 at $\nu = 100$ and 200 MHz, respectively. Similarly, the calibration noise source was found to have frequency-dependent temperature, $T_{cal}$, according to

$$T_{cal}(\nu) = 495 + 30 \left[ \left(\frac{\nu}{150}\right)^{1.8150} - 1 \right].$$  \hspace{1cm} (6.8)

where $\nu_{150} = 150$ MHz. This information was also applied to the approximate calibration used in the EOR measurements.

To determine the antenna reflection coefficient, a strong noise source is injected at the input of the amplification module using a resistive power splitter so that the transmission cable leading to the antenna also remains connected. This configuration produces correlated reflections due to the impedance mismatch of the antenna [Rogers, 2006g] that are evident in the measured spectrum as a sinusoidal contribution to the spectrum of the noise source. Comparing the ratio of the sinusoidal contribution to the overall amplitude yields the reflection coefficient, $\Gamma$, according to

$$\Gamma(\nu) = \frac{2}{\Upsilon} \left( 1 - \sqrt{1 - \frac{R(\nu)^2}{R(\nu)}} \right),$$  \hspace{1cm} (6.9)

where

$$R(\nu) = \frac{T_S(\nu)}{T_N(\nu) - T_L}$$  \hspace{1cm} (6.10)

and $T_S$ is the amplitude of the sinusoidal ripple, $T_N$ is the measured noise temperature after the contribution of the reflection has been removed, and $T_L$ is the standard ambient load. In practice, $T_S$ and $T_N$ are found simultaneously by solving for the offset and amplitude of a sliding sine wave over a small range of frequencies corresponding to approximately one period of the ripple. The period is given by the inverse of the time delay, $\tau_d$, of the transmission cable. For the LMR-400 cable,

$$\tau_d(\nu) = 0.3048 \left[ 1 - \left(\frac{\nu - 200}{500}\right) \right] \frac{2L}{0.83 c},$$  \hspace{1cm} (6.11)

where $\nu$ is measured in MHz, $c$ is the speed of light, the factor of 0.3048 comes from the conversion from feet to meters, and the factor of 0.83 is the relative propagation speed compared to free space of a wave in the cable. Thus, the characteristic period in the measured spectrum of the sinusoidal ripple is $\tau_d^{-1} \approx 4$ MHz for a 100 foot cable. Figure 6-7 shows the antenna reflection coefficient for EDGES as a function of frequency. The best match (lowest reflection) is between 130 and 200 MHz. The reflection coefficient was also measured using a network analyzer at Haystack Observatory [Bowman, 2006a] with similar results.

Lastly, the balun, ground-screen, antenna, and horizon losses are determined using either laboratory measurements or numerical simulations. For the simple balun used with EDGES, the loss can be measured in the laboratory by connecting two identical balun assemblies
Table 6.1. Calibration Corrections and Uncertainties

<table>
<thead>
<tr>
<th>Source</th>
<th>Correction</th>
<th>Uncertainty</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
<td>see Fig. 6-7</td>
<td>&lt; 1.0</td>
<td>measured in field</td>
</tr>
<tr>
<td>$T$</td>
<td>see Eqn. 6.7</td>
<td>&lt; 1.0</td>
<td>measured in laboratory</td>
</tr>
<tr>
<td>$T_{cal}$</td>
<td>see Eqn. 6.8</td>
<td>&lt; 1.0</td>
<td>checked with precision source</td>
</tr>
<tr>
<td>horizon loss</td>
<td>0.05 dB</td>
<td>&lt; 0.5</td>
<td>model</td>
</tr>
<tr>
<td>ground loss</td>
<td>0.05 dB</td>
<td>&lt; 0.5</td>
<td>model</td>
</tr>
<tr>
<td>antenna loss</td>
<td>0.10 dB</td>
<td>&lt; 1.0</td>
<td>model</td>
</tr>
<tr>
<td>balun loss</td>
<td>0.10 dB</td>
<td>&lt; 1.0</td>
<td>measured in laboratory</td>
</tr>
<tr>
<td>statistical, $T_{150}$</td>
<td>...</td>
<td>&lt; 0.3</td>
<td>$\chi^2$</td>
</tr>
<tr>
<td>statistical, $\beta$</td>
<td>...</td>
<td>&lt; 0.1</td>
<td>$\chi^2$</td>
</tr>
</tbody>
</table>

Note. — Summary of corrections and their uncertainties applied to calibrated sky temperature measurements with EDGES. For comparison, the statistical uncertainties on the model parameters, $\beta$ and $T_{150}$, are also listed. The full uncertainty on each measurement of the model parameters is a combination of all the terms in the Table. Adapted from Rogers and Bowman [2006].

back-to-back and measuring the transmission. The loss due to applying the 572Ω ferrite core balun to a coaxial cable is found using this method to be about 0.1 dB ($\sim 2\%$) at the frequencies of interest. The losses due to the finite size of the ground screen and due to blockage of the sky by objects along the horizon are more difficult to estimate. Numerical simulations were performed to estimate these contributions and they were found to be each of order 0.05 dB ($\sim 1\%$). To further constrain the estimated loss due to the ground screen, measurements were made in the field after extending the ground screen to a diameter of about 3.5 m using aluminum foil. These measurements are discussed in the analysis below. The antenna loss was also modelled and it was found to be less than 0.1 dB.

The contribution of all the calibration corrections determines the total systematic uncertainty in the derived properties of the radio spectrum. Table 6.1 summarizes the corrections discussed in this section and the uncertainty that each contributes to the calibrated sky temperature measurements. The total systematic uncertainty on the temperature measurement in any given spectral channel is found to be less than $\Delta = 2.5\%$ by adding the individual uncertainties in quadrature.

6.4.2 Model Parameters

Combining the corrections above to calibrate observations performed with EDGES in the appropriate configuration while at Mileura Station yields accurate determinations of the absolute sky temperature. To study the foreground contribution to the spectrum (which we continue to refer to as $T_{gal}$ to distinguish it from the CMB contribution), we employ the
model
\[ T_{\text{gal}}(\nu) = T_{150} \left( \frac{\nu}{\nu_{150}} \right)^{-\beta} \]  
(6.12)

where \( T_{150} \) is the temperature at 150 MHz, and \( \beta \) is the spectral index introduced in Section 6.1. For a model of the complete sky temperature, we substitute \( T_{\text{gal}} \) back into Equation 6.4 and neglect the redshifted 21 cm contribution, such that
\[ T_{\text{sky}}(\nu) = T_{150} \left( \frac{\nu}{\nu_{150}} \right)^{-\beta} + T_{\text{cmb}}. \]  
(6.13)

Taking \( T_{\text{cmb}} = 2.725 \), we solve for \( \beta \) and \( T_{150} \). Figure 6-8 illustrates an example fit of the model to a typical observation made on the night of 7 Dec 2006 with the approximately north-south polarization of the antenna. It is clear in Figure 6-8 that the model is a good fit to the measurements, yielding in this case, \( \beta = 2.470 \) and \( T_{150} = 283.20 \).

6.4.3 Uncertainty in Model Parameters

The inherent systematic uncertainties in the model parameters are derived from the combined uncertainties due to the calibration and correction terms discussed above. For \( T_{150} \), the uncertainty is essentially equivalent to the systematic uncertainty, but for \( \beta \) the dependence is more complicated.

Two scenarios can be considered to determine a reasonable estimate of the uncertainty in \( \beta \). First, if errors in the calibration and correction terms conspire to scale all the temperature measurements within a spectrum by a scalar quantity, and second, if the errors produce a tilt of the spectrum. The first scenario would generally be the result of incorrect estimates for the loss terms, while the second would be the result of incorrect calibration of the frequency-dependence of the antenna reflection, cable transmission, or calibrator temperature. Considering only the measured sky temperatures at two frequencies \( \nu = 100 \) and \( 200 \) MHz, we can solve for the derived spectral index according to
\[ \beta = \frac{\log \left( \frac{T_{100} - T_{\text{cmb}}}{T_{200} - T_{\text{cmb}}} \right)}{\log \left( \frac{\nu_{100}}{\nu_{200}} \right)}. \]  
(6.14)

Consequently, if \( T_{100} \) and \( T_{200} \) are both scaled by an incremental amount, \( \Delta \), we have
\[ \beta_{\Delta} = -\frac{\log \left[ \frac{(1+\Delta)T_{100} - T_{\text{cmb}}}{(1+\Delta)T_{200} - T_{\text{cmb}}} \right]}{\log \left( \frac{\nu_{100}}{\nu_{200}} \right)}. \]  
(6.15)

Solving for \( \Delta_{\beta} = |1 - \beta_{\Delta}/\beta| \), we find that for \( \Delta = 2.5\% \), \( \Delta_{\beta} \approx 1\% \). Similarly, if \( T_{100} \) is increased by an incremental fraction \( \Delta \), while \( T_{200} \) is decreased, and we ignore the contribution of \( T_{\text{cmb}} \), we find that
\[ \beta_{\Delta} \approx -\frac{\log \left[ (1 + \Delta)^2 \left( \frac{T_{100}}{T_{200}} \right) \right]}{\log \left( \frac{\nu_{100}}{\nu_{200}} \right)} = \beta - \frac{2 \log (1 + \Delta)}{\log \left( \frac{\nu_{100}}{\nu_{200}} \right)}. \]  
(6.16)

and that \( \Delta_{\beta} \approx 3\% \) in this scenario.
To finish the uncertainty estimates, we must include the statistical uncertainty in the model-fits. The statistical uncertainty is generally dependent on the variance of the temperature measurements between individual spectral channels and the total number of channels in the sample. It can be assessed using the $\chi^2$ metric to determine the 95% confidence region for the parameter fits and is found to be $< 0.1\%$ for $\beta$ and $< 0.3\%$ for $T_{150}$, for all observation cycles. Combining the results, we conclude that the total uncertainty in $\beta$ is less than 4% (or $\Delta \beta \lesssim 0.1$), and the total uncertainty in $T_{150}$ is less than 3% (or about 8 K). Thus, the measurements provide reasonably sensitive tests of the shape of the diffuse radio background averaged over a large solid angle.

6.4.4 Variations in Spectral Index and Temperature

The spectral index and intensity of the non-thermal contributions to the low-frequency spectrum have been shown to vary across the sky. Drift scan observations with EDGES can measure these variations in $T_{\text{gal}}$ (albeit convolved with the large antenna beam) as a function of local apparent sidereal time (LST). In addition, the intensity of the measured spectrum should vary with the polarization direction of the dipole due to differences in the shape of the antenna beam. Sampling both the north-south and east-west polarizations allows additional tests for consistency with expectations from prior measurements.

Four configurations of the system were used during the calibrated observing runs at Mileura Station that provide relevant measurements. Table 6.2 lists these modes. The observations for each night span about 0 h to 8 h LST. The same algorithm used to excise individual observation cycles during the EOR observations with large RFI transient signals is applied to these data. The results of fitting the model for $T_{\text{sky}}$ to the high quality, RFI-free data from these runs are shown in Figure 6-9 and listed in Table 6.3.

The spectral index is typically of order $\beta = 2.45$, and is consistent with the measurements of the 1960s and 1970s for this frequency range. The temperature of the non-CMB contribution at 150 MHz varies with LST, as expected, between approximately 240 K and 300 K over the observed range of LSTs. The temperature measurements are consistent to within less than 1% for the north-south polarization measurements with overlapping LSTs, and the temperatures calculated for the east-polarization are about 2% lower than the equivalent north-south measurements, which is anticipated from extrapolation of all-sky measurements at higher frequencies. The large field of view of the EDGES dipole antenna beam convolved with the sky produces very smooth variations with LST and eliminates much of the structure in both $\beta$ and $T_{150}$ that would be observed with more localized observations.

The most notable irregularity in the measurements is the difference between the derived spectral index estimates made with and without the extended ground screen on 30 Nov 2006 and 1 Dec 2006. This discrepancy indicates that the ground screen is having a significant effect on the either the shape of the antenna beam or the ground loss. It raises concern about the estimate of the ground loss due to the finite size of the ground screen. More measurements are needed to resolve this issue.
Table 6.2. Calibrated Observing Runs

<table>
<thead>
<tr>
<th>Date</th>
<th>LST [h]</th>
<th>Polarization</th>
<th>Cable Length [feet]</th>
<th>Ground Screen</th>
<th>Cycle [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 Nov 2006</td>
<td>0 to 8</td>
<td>NS</td>
<td>50</td>
<td>normal</td>
<td>210</td>
</tr>
<tr>
<td>01 Dec 2006</td>
<td>0 to 8</td>
<td>NS</td>
<td>50</td>
<td>extended</td>
<td>210</td>
</tr>
<tr>
<td>07 Dec 2006</td>
<td>22 to 24</td>
<td>NS</td>
<td>100</td>
<td>normal</td>
<td>25</td>
</tr>
<tr>
<td>07 Dec 2006</td>
<td>0 to 8</td>
<td>EW</td>
<td>100</td>
<td>normal</td>
<td>25</td>
</tr>
</tbody>
</table>

Note. — List of the four calibrated observing runs, indicating the configuration of the system during each run and the approximate LST of the start and stop times. The polarizations are either approximately north-south (NS) or east-west (EW), as described at the beginning of the chapter. The cycle times in the last column are the total amount of time spent in one complete cycle of the three-position switch. For the 210 s cycles, the division of time between the antenna, the ambient load, and the calibrator load is [100, 100, 10] s, respectively, while for the 25 s cycles, it is [10, 10, 5] s, respectively.
<table>
<thead>
<tr>
<th>LST h  m</th>
<th>NS-50 $T_{150}$ [K]</th>
<th>$\beta$</th>
<th>$N_{bin}$</th>
<th>NS-50, Extended $T_{150}$ [K]</th>
<th>$\beta$</th>
<th>$N_{bin}$</th>
<th>NS-100 $T_{150}$ [K]</th>
<th>$\beta$</th>
<th>$N_{bin}$</th>
<th>EW-100 $T_{150}$ [K]</th>
<th>$\beta$</th>
<th>$N_{bin}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 45</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>315.13</td>
<td>2.399</td>
<td>2</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>23 00</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>305.70</td>
<td>2.403</td>
<td>12</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>23 15</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>295.31</td>
<td>2.406</td>
<td>8</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>23 30</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>286.11</td>
<td>2.407</td>
<td>4</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>23 45</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>00 00</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>00 15</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>262.00</td>
<td>2.517</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>00 30</td>
<td>260.81</td>
<td>2.389</td>
<td>2</td>
<td>258.33</td>
<td>2.520</td>
<td>2</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>00 45</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>249.56</td>
<td>2.431</td>
<td>2</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>01 00</td>
<td>252.03</td>
<td>2.387</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>248.18</td>
<td>2.435</td>
<td>8</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>01 15</td>
<td>249.82</td>
<td>2.391</td>
<td>1</td>
<td>250.37</td>
<td>2.517</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>244.94</td>
<td>2.449</td>
<td>9</td>
</tr>
<tr>
<td>01 30</td>
<td>249.04</td>
<td>2.393</td>
<td>1</td>
<td>248.39</td>
<td>2.516</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>243.45</td>
<td>2.452</td>
<td>30</td>
</tr>
<tr>
<td>01 45</td>
<td>247.34</td>
<td>2.396</td>
<td>1</td>
<td>246.57</td>
<td>2.511</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>241.91</td>
<td>2.459</td>
<td>18</td>
</tr>
<tr>
<td>02 00</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>240.89</td>
<td>2.464</td>
<td>18</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>02 15</td>
<td>245.95</td>
<td>2.405</td>
<td>2</td>
<td>244.73</td>
<td>2.513</td>
<td>2</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>240.37</td>
<td>2.468</td>
<td>13</td>
</tr>
<tr>
<td>02 30</td>
<td>245.94</td>
<td>2.408</td>
<td>1</td>
<td>244.65</td>
<td>2.511</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>239.97</td>
<td>2.482</td>
<td>6</td>
</tr>
<tr>
<td>02 45</td>
<td>246.05</td>
<td>2.415</td>
<td>3</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>239.97</td>
<td>2.488</td>
<td>22</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>03 00</td>
<td>246.71</td>
<td>2.419</td>
<td>2</td>
<td>246.06</td>
<td>2.499</td>
<td>2</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>240.41</td>
<td>2.493</td>
<td>26</td>
</tr>
<tr>
<td>03 15</td>
<td>247.12</td>
<td>2.427</td>
<td>2</td>
<td>247.20</td>
<td>2.495</td>
<td>3</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>241.21</td>
<td>2.499</td>
<td>3</td>
</tr>
<tr>
<td>03 30</td>
<td>249.92</td>
<td>2.423</td>
<td>3</td>
<td>249.40</td>
<td>2.492</td>
<td>2</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>242.51</td>
<td>2.506</td>
<td>24</td>
</tr>
<tr>
<td>03 45</td>
<td>252.49</td>
<td>2.425</td>
<td>2</td>
<td>251.24</td>
<td>2.492</td>
<td>2</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>244.21</td>
<td>2.507</td>
<td>22</td>
</tr>
<tr>
<td>04 00</td>
<td>256.22</td>
<td>2.424</td>
<td>3</td>
<td>253.50</td>
<td>2.492</td>
<td>4</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>246.21</td>
<td>2.510</td>
<td>21</td>
</tr>
<tr>
<td>04 15</td>
<td>259.59</td>
<td>2.423</td>
<td>2</td>
<td>256.22</td>
<td>2.493</td>
<td>3</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>248.15</td>
<td>2.511</td>
<td>9</td>
</tr>
</tbody>
</table>
Note. — Calibrated measurements of the model parameters described in the text for the non-CMB contribution to the diffuse radio spectrum as a function of LST. Individual observation cycles are grouped and averaged over 15 minute periods. Data from four instrumental configurations are listed. The title of the column group gives the polarization sampled (NS or EW) and the length of the transmission cable in feet (50 or 100). The ground screen was extended using aluminum foil for the group labelled NS-50, Extended. The columns labelled $N_{\text{bin}}$ give the number of individual cycles in each time bin. The duration of individual cycles is not constant between the different configurations. In particular, the observations used for the first two sets of columns had 110 s cycles, while the those used for the last two had 25 s cycles.
Chapter 7

Conclusion

We began in Chapter 1 by introducing the tremendous opportunities of radio observations of the redshifted 21 cm background to constrain the evolution of the high-redshift IGM and the radiative processes associated with the first luminous objects in the Universe. Measurement of the redshifted 21 cm line during the epoch of reionization at the end of the Dark Ages is the only foreseeable technique capable of providing direct probes of the properties of neutral hydrogen in the IGM from this time, and such measurements would complement and extend future observations of individual high-redshift objects by the James Webb Space Telescope and large ground-based facilities. Although the idea of using low-frequency radio experiments to study the early Universe has been around for some time, experimenters have just begun in earnest to bring the tools of radio astronomy to bare on this objective. Over the last several years, there has been a concerted effort by many in the field to gather the collective insight learned from previous projects (including all the way back to the pioneering days of Karl Jansky) and leverage new options enabled by advances in computational and signal processing technology. The proliferation of experiments that are now under development—including the MWA, 21CMA, and LOFAR, as well as EDGES and CORE—is a direct result of these initiatives. At the same time, members of the theoretical community, spurred in part by seemingly conflicting results about the history of reionization from quasar absorption studies and WMAP findings, have made tremendous advances toward predicting the redshifted 21 cm signal and enumerating the fundamental astrophysical processes that drive it.

The work performed for this thesis bridges these two ongoing efforts and serves to keep the experimental and theoretical developments synchronized so that they progress more effectively in tandem. For this purpose, we have taken the ideas and strategies devised for the new experiments and matched their inherent uncertainties to the expected statistical properties of the redshifted 21 cm signal, we have analyzed the connection between the MWA and the sky in increasing layers of sophistication, and we have tested that the basic elements of the new instrumental approaches perform as they are intended. And, we have begun to make the first measurements that may lead to new constraints of the time and duration of the reionization epoch. In undertaking this process, we have helped to transfer the rigorous analysis framework and mathematical tools developed for CMB anisotropy measurements and large-scale structure surveys to the new breed of low-frequency radio experiments, and, in doing so, to establish a firm foundation for a new era in radio astronomy.

With a wide field of view and large bandwidth coverage, the MWA departs from traditional approaches to radio astronomy. Featuring a phased-array design of the antenna
tiles and low-cost digital receiver system, it enables innovative and promising opportunities for scientific exploration in a largely-neglected slice of the electromagnetic spectrum. But only a thorough understanding of the operational properties of the instrument will allow the array to fully achieve its science objectives. Field testing (reported on in Chapter 5) began to address this requirement at an early stage in the development of the array and demonstrated the fundamental performance of the MWA prototype antenna tiles and receiver system in the western Australia environment. The findings also indicated that the RFI environment of the site is excellent and should not pose a significant hurdle for the planned observations, including those targeting the epoch of reionization.

Using the power spectrum of fluctuations in the redshifted 21 cm background as a characteristic statistical test of the properties of neutral hydrogen in the IGM during reionization, in Chapter 2 we illustrated explicitly the fundamental uncertainties for the MWA due to thermal noise and cosmic sample variance. The unique combination of angular and line-of-sight information in redshifted 21 cm measurements was found to provide a sufficient signal-to-noise ratio that the MWA could be expected reasonably to produce significant measurements of the fluctuation power at several relevant spatial scales if reionization occurred below redshift \( z \lesssim 12 \). Moving beyond this basic performance threshold, we sought in Chapter 3 to understand the additional limitations imposed on interpreting redshifted 21 cm measurements by the need to separate the desired signal from astrophysical foreground contaminants. Although astrophysical foregrounds were recognized early on as a potential problem for redshifted 21 cm experiments, and a correspondingly significant amount of attention has been given to identifying techniques to mitigate their effects, we established for the first time a bound on the detrimental consequences of including the effects of the instrumental response of a real array on the mitigation process. These results were encouraging and indicated that, for a large fraction of the visibility measurements of the MWA, the instrumental response has only a negligible effect on the foreground mitigation.

This work has strengthened the expectation that the first generation of redshifted 21 cm experiments has the potential to characterize the processes and history of reionization at redshifts \( 6 < z \lesssim 12 \). In Chapter 4, however, we also considered a scenario in which reionization is not the dominant contribution to the measured power spectrum during this period. Our findings demonstrated that the initial experiments would not contribute substantially to knowledge of the underlying cosmology under these circumstances. While they may provide some use to cosmological studies, the primary science return of the first generation of redshifted 21 cm experiments will be regarding the astrophysics of reionization. Under optimistic assumptions, a second-generation of experiments, with an order of magnitude increase in collecting area above the MWA, would be better suited to constrain \( \Omega_M \) and the primordial power spectrum through \( n_s \) and \( \alpha_s \), although it is difficult to envision a scenario where redshifted 21 cm observations alone could provide an unambiguous constraint on cosmological information since the signal could easily be confused if the universe had a small amount of reionization structure on small scales or spin temperature fluctuations in the IGM.

In principle, if the experimental challenges can be overcome, useful measurements of the redshifted 21 cm can be carried out with relatively small arrays. These measurements of the reionization history would be fundamental to understanding the evolution of the Universe and, with properly designed experiments, they may be obtained in the very near future. The statistical properties of fluctuations in the redshifted 21 cm background could be constrained by the first generation of low-frequency arrays such as the MWA. In the future, with the increased collecting area of larger arrays such as the SKA, mapping of individual features
in the redshifted 21 cm signal may even be possible. The global evolution of the mean spin temperature and mean ionization fraction could be constrained even more immediately by very compact instruments employing individual dipole antennas. In Chapter 6, we reported very preliminary results based on this approach from the first observing campaign with the EDGES system. These observations were limited by systematic effects that were an order of magnitude larger than the anticipated signal, and thus, ruled out only an already unlikely range of parameter space for the differential amplitude of the redshifted 21 cm brightness temperature and for the duration of reionization. Nevertheless, this experiment has effectively drawn the first line in the sand (metaphorically, and somewhat literally) and marks the beginning of a renewed period of low-frequency exploration in radio astronomy.

7.1 Looking Ahead

The time is rapidly approaching when the focus of developing radio techniques to characterize the redshifted 21 cm signal from the early Universe will transition from studying simulations and models to the practical considerations of testing, analyzing, and refining the first implementation of the MWA and the other new arrays (and indeed this transition has already occurred for EDGES, as well as some aspects of the MWA). Experimental and theoretical researchers have accomplished much in the last five years, but now actual measurements are needed to drive the next level of progress. The MWA project expects to complete a 32 antenna tile development sub-array by the end of 2007, so that testing can proceed throughout 2008 while the remainder of the 500 antenna tiles are installed. This development sub-array will be the immediate focus of attention in our ongoing efforts to connect experiment and theory, and it should provide valuable information regarding the polarization properties of the antenna tiles, the limitations of instrumental and ionospheric calibration, and the ability of the MWA to reconstruct an accurate estimate of the sky.

By far, the most significant under-addressed aspects of the measurements planned for the MWA involve polarization, both of the properties of the diffuse emission from the sky and of the response of the antenna tiles. In particular, the polarization response of the MWA has been difficult to estimate and to account for in simulations due to the unique design of the antenna tiles. Since there is expected to be significant polarized signal due to Faraday rotation of the Galactic synchrotron foreground by the interstellar medium, the ability to calibrate the polarization response of the array is critical to the foreground mitigation necessary for redshifted 21 cm studies. This deficiency is beginning to be addressed with the MIT Array Performance Simulator (MAPS) by researchers developing the calibration pipeline for the MWA. MAPS is a new tool for radio astronomy that is capable of producing full-stokes simulations of the instrumental response of an interferometer to the sky. For redshifted 21 cm science, we must utilize this resource, along with measurements of the polarization mixing in the actual system, to study the expected polarized foregrounds, the techniques necessary for mitigating them, and the residual contamination that will remain in the measurements.

At the same time, we must address additional approaches to capturing the astrophysical information in the upcoming experiments. The power spectrum of fluctuations in the redshifted 21 cm background is not the only statistical metric for probing the reionization epoch with the planned measurements, and may not even be the most optimal one since fluctuations of the surface brightness temperature of the 21 cm line are not expected to be Gaussian at the end of reionization. Cross-correlating the radio observations with maps of
the CMB anisotropy or catalogs of high-redshift galaxies from the same comoving volume of the Universe, or calculating the distribution of pixel intensity values in redshifted 21 cm maps are alternative tests that may have the potential to return significant cosmological and astrophysical information. We have used the power spectrum as a reference test for our discussion of sensitivity and the effects of foreground mitigation, but these other techniques will have different dependencies on the instrumental properties and unique susceptibilities to uncertainties and biases that need to be understood.

Taken together, these are the topics that must be resolved before meaningful results from redshifted 21 cm measurements can be produced and before any results will be accepted with confidence by the astrophysical community. The timeline for performing this work has been established by the construction schedule for the MWA and other first-generation experiments. For the next several years (through 2010), the goal is to keep experiment and theory in stride, so that when the first observations of the MWA are complete, the measurements and the planned tests are optimized for each other. If all goes well, this period will prove to be even more exciting the last five years and will culminate with the opening of an entirely new window on the early Universe.
Bibliography


J. D. Bowman. Comparison of galactic noise at potential edges sites. Memorandum EDGES–020, Massachusetts Institute of Technology, Haystack Observatory, 2006b.


