BROADBANDING OF A KLYSTRON OUTPUT CAVITY

by

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ABSTRACT

An analysis of some two-cavity resonators to be used as output cavities of a klystron amplifier is presented. The emphasis in the analysis is put on the double-tuned, double-gap cavity system in which a single electron beam traverses both cavities. However, the behavior of a single-tuned cavity and of a double-tuned, single-gap cavity system is given for comparison purposes. Types of electromagnetic coupling of cavities considered were: (1) electric coupling of fringing fields through the short interconnecting drift region between the gridless gaps; (2) magnetic loop coupling between the cavities; (3) waveguide coupling of the two cavities; (4) coaxial line coupling of cavities and (5) electromagnetically decoupled cavities. In the analysis of the equivalent circuits of the double-tuned, coupled cavity systems it was assumed that the transit angle between the two cavities was small so that the electron beam remodulation at the first gap could be neglected. However, the effect of remodulation of the beam on the approximate results was considered for the first two types of coupling. The obtained results for these two types of coupling show that if the load is connected to the second cavity along the path of the beam, then the maximum of the power dissipated in the external load will be of the order of the maximum power dissipated in the external load of a single-tuned cavity system, provided the beam parameters and the interaction gap capacitance of the individual cavities are the same. However, the bandwidth of the composite cavity system will be about two to three times the bandwidth of a single-cavity system. The limitation in the bandwidth is due to the ripple in the power dissipation curve. The approximate analysis of the third and fourth types of coupling, using high-Q approximation and neglecting the electron beam remodulation at the first gap, shows that the maximum of the power delivered to the external load is about equal to the maximum of the power dissipated in a single-cavity system. However, the bandwidth is about twice the bandwidth of a single-tuned cavity. Furthermore, the transit angle between the individual cavities for the strong waveguide coupling must be as close to zero as possible. On the other hand, for optimum bandwidth, a strong coaxial line coupling requires the transit angle to be \( \pi \) radians.

The approximate analysis of decoupled cavities shows poorer performance than the other cavity systems analyzed.

Thesis Supervisor: Louis D. Smullin
Title: Associate Professor of Electrical Engineering
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<td>34</td>
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<td>( R_L )</td>
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1.1 Statement of the Problem

The requirement for high microwave power sources in recent years has led to an advanced development of microwave amplifiers. From these amplifiers, the klystron, the traveling wave tube, the magnetron traveling wave tube, the amplitron and the conventional UHF triode are capable of producing pulsed power of the order of one megawatt or greater and an average power of the order of one kilowatt or greater. Because of its high gain and of its power handling capabilities, the klystron amplifier is a very useful tube type. However, a klystron amplifier, which operates on the principle of interaction of a drifting electron beam with the RF fields of a high Q cavity, is inherently a low bandwidth device.

The first microwave amplifier in which two resonant cavities were used to velocity modulate and demodulate the beam was first described by the Varian brothers in 1939. This two-cavity amplification system they named a klystron. A later modification of the two cavity klystron led to the development of two other important microwave tubes: the reflex and the multicavity klystrons. Both of these tubes have found a large practical application. They are the most important examples of non-continuous interaction tubes; however, in recent years, the use of a large number of intermediate cavities in the multicavity klystron have increased the resemblance between this tube and the traveling wave tube. The reflex-klystron is a self-excited microwave oscillator, whereas a multicavity klystron can be used both as an oscillator and as a microwave amplifier. The gain-bandwidth characteristic of a multi-cavity klystron has been extensively investigated.
by a number of workers.\textsuperscript{5, 6, 7, 8} For an extensive bibliography on multi-
cavity klystrons, see reference 8; for a brief discussion of operation and
problems existing in multi-cavity klystrons, see, for example, reference 9.

Multicavity klystrons have been built with three and more
cavities.\textsuperscript{10, 11, 12, 13} The smaller ones can be focused either electrostatically
or through the magnetic field.\textsuperscript{10, 11, 12} Reasons for using more cavities than
the necessary number of two are summarized by Niestand: \textsuperscript{9}

"The principal advantage of using a multiresonator tube instead of several
single-stage amplifiers (two resonators) are: 1) single vacuum envelope-
greater reliability; 2) one electron beam - less d-c power input;
3) only one additional resonator to tune per stage instead of two; 4) no
coupled resonators present; 5) higher efficiency of multi-resonator tubes
and 6) gain of an n-stage tube is $2^n$ times as high as an equivalent number
of single-stage amplifiers."

Another important reason for using a multicavity klystron is the fact that
through stagger-tuning of the intermediate cavities a significant increase in
the bandwidth can be achieved. Unfortunately, the bandwidth of a multicavity
klystron is not independent of other important factors like efficiency, voltage
and power gains. Some of these factors important for the design of the
multicavity klystrons have been studied, for example, by Bryant, \textsuperscript{14} Curnow\textsuperscript{15}
and Muller. \textsuperscript{16} However, the effect of the intermediate structure and of the
beam parameters on the bandwidth of the multicavity klystron will not be
considered in this paper. In this analysis it will be assumed that the ampli-
tude of the first harmonic of the electron beam arriving at the first gap is
constant with frequency and that higher order harmonics of the electron
beam are negligibly small.

The broadband effect produced by the stagger-tuning of the inter-
medial cavities cannot be completely utilized unless the output cavity,
through which the amplified RF signal is taken out, is also broadband. Now,
a low Q cavity has a broadband characteristic, so that by heavy loading of
the output cavity the overall bandwidth of the multicavity klystron can be increased. Unfortunately, the power that can be extracted from the bunched electron beam is approximately inversely proportional to the effective load conductance of the output cavity. Therefore, the RF power extracted from the beam will decrease for heavier loading of the output cavity; i.e., the loaded $Q$ of the output cavity limits the bandwidth of the multicavity klystron. Dodds et al. have shown that the 3 db bandwidth of the output cavity of a conventional klystron amplifier, as determined by the loaded $Q$ for maximum efficiency at the band center, is given by

$$\Delta f/f = \frac{2}{A^2} \frac{R_{sh}}{Q} \eta^{4/5} k^{4/5} P^{1/5}$$

(1.1)

Where $\Delta f/f = 1/Q_L = 3$db fractional bandwidth, $A$ is the ratio of peak RF voltage to beam voltage, $R_{sh}/Q$ is the characteristic impedance of the cavity, $\eta$ is the efficiency at band center, $k$ is the perveance of the electron beam and $P$ is the RF power out. On the basis of Eq. (1.1), they indicate five fields for the improvement of bandwidth in the klystron amplifier. These are: 1) The factor $R_{sh}/A^2Q$, which they call the figure of merit for a klystron amplifier output circuit, can be increased by using conventional cavity resonators of modified geometry; 2) the 3 db bandwidth can be increased by the use of simple double-tuned circuits. The use of the double-tuned circuit will permit the bandwidth to exceed the value given by Eq. (1.1); 3) the bandwidth of the output cavity can be increased by overcoupling of the load to the output circuit. However, to keep the RF output power at its previous value, the beam voltage and current must be increased; 4) increase in perveance, for example, by the use of hollow beams and 5) the use of circuits with high $R_{sh}/Q$ in particular through the use of distributed circuit resonator.

On the basis of the above discussion, there is a need for a more
detailed analysis of some double-tuned cavity resonators which might be used to increase the overall bandwidth of the multicavity klystron amplifier. In the following section it will be shown that there is very little information available in the literature today that would enable the engineer, faced with the problem of designing a broadband output cavity, to choose the right type of the output cavity and to give him an idea of its approximate behavior. For this reason, in this paper different types of these double-tuned cavity resonators will be analyzed with the emphasis put on the improvement of the bandwidth of the output cavity of a klystron amplifier. All calculated results will be determined on the basis of maximum efficiency of the output cavity. That is, in all calculations the shunt conductance will be chosen in such a way that at some frequency the electrons in the beam will be on the verge of being turned back to the cathode. The performance of the various cavity systems will be expressed (normalized) in terms of the corresponding values of a single-tuned cavity system.

1.2 Historical Review of the Problem

The idea of using double-tuned microwave resonators to increase the bandwidth of a cavity system is not a new one. Double-tuned circuits have been used to increase the bandwidth of the conventional amplifiers for years, and the amount of literature treating the lumped-constant, double-tuned circuit is very large. For a complete, but uncomplicated, discussion of a double-tuned circuit, the reader is referred to Terman.\textsuperscript{18} For a more detailed treatment of the subject, the article by Aiken\textsuperscript{19} will be found to be very useful. At this point, it would be impossible to give a complete bibliography on the double-tuned circuit. However, in general, the treatment of the double-tuned circuit by all the workers is about the same and differs
either in the notation used or in the development for a particular field of application only. For example, Twiss\textsuperscript{20} gives an extensive analysis of the behavior of the double-tuned circuit for interstage coupling of amplifiers. For more references, the reader is referred to Smith.\textsuperscript{21}

One of the main advantages of a double-tuned circuit is the fact that its use leads to an increase in bandwidth without any significant loss in the efficiency of the system. Therefore, since the equivalent circuit of two weakly-coupled, electromagnetic cavity systems can be represented by coupled, lumped-constant, tuned circuits\textsuperscript{22}, it is not surprising to find the use of coupled microwave resonators for the improvement of bandwidth of a cavity system. At this point, it would seem that an intricate but a judicious connection of RLC elements would lead to an indefinite increase in bandwidth. That this is not so can be shown, for example, using the resistance integral\textsuperscript{23} and the principle of conservation of bandwidth in frequency transformation\textsuperscript{24} for a two-terminal network with a shunt capacitance $C$. For this type of network the upper bound on the bandwidth is $\pi/2$ times the bandwidth of a single-tuned circuit with the same shunt capacitance. Therefore, even with a complicated connection of cavities, the bandwidth of a single-gap cavity system cannot be increased by more than $\pi/2$ times the bandwidth of a singly-resonant cavity system with the same gap capacitance. However, in microwave practice, the use of a large number of coupled microwave cavities for the purpose of increasing the bandwidth of a single-gap system would pose many difficulties if more than two cavities were used. Fortunately, the use of only two coupled resonators increases the bandwidth of the single-gap system almost to its upper limit. For example, in the analysis of Appendix 2 it is shown that the maximum approximate bandwidth
of a double-tuned, single-gap system at 100 per cent efficiency is about square root of two times the bandwidth of a single-tuned cavity system (Fig. A2.2 at p = 0.811). This corresponds to about 82 per cent of its optimum value, or an increase in bandwidth of about 41 per cent.

There are fundamentally two types of two-coupled-cavity systems: 1) the double-tuned, single-gap system in which the electron beam traverses one cavity-gap only and 2) the double-tuned, double-gap system in which the electron beam traverses both cavities. The first system is more simple and, therefore, is easier to analyze and to build. Kreuchen et al have suggested the use of double-tuned, single-gap cavity systems in multicavity klystrons in 1956. However, no separate analysis of the behavior of a double-tuned, single-gap system, for example, as an output cavity, was presented. Beaver et al give a brief discussion for the improvement of bandwidth in a klystron amplifier through the use of double-tuned, single-gap cavity in the output circuit. One power dissipation curve for one particular loading and for the critical coupling is presented to illustrate the discussion. The results of this discussion are compared in a gross way with the performance of a special modification of the VA-87 klystron. However, no general discussion of the behavior of a double-tuned, single-gap output cavity is presented. (In Appendix 2 of this paper the analysis of this type of output structure is briefly analyzed.) The VA-87 tube mentioned above is not the only tube, which makes the use of a double-tuned, single-gap cavity. More tubes have been built using this type of cavity structure. Some of these tubes use a double-tuned, single-gap cavity for an output cavity only. Others use these cavities throughout. In the analysis of the performance of the above tubes it is very difficult to obtain the exact behavior of the
double-tuned, single-gap cavity as an output structure from the over-all performance of the tube; however, in all the tubes the use of the composite output cavity of this type shows an improvement in bandwidth.

At this point, it is important to note that in spite of the fact that there exists in the literature an extensive theoretical treatment of coupled cavities,\textsuperscript{22,29,30} only very little of the actual design data is currently available. The problem of designing two coupled microwave cavities, it seems therefore, must begin with the designing of individual cavities for which an ample amount of the design data is available. See, for example, references \textsuperscript{31,32,33,34}. The proper coupling between the individual cavities can then be obtained either experimentally, through the use of some data on couplings or using Bethe's\textsuperscript{35,36} diffraction method. For more data on the design of coupled cavities, see references \textsuperscript{37,38 and 39}. In the last reference, the coupling between two klystron cavities through the interconnecting drift tube is analyzed in greater detail. Only when sections of a rectangular waveguide are used as a double-tuned, single-gap output cavity, the problem of coupling the re-entrant cavity to the transmission cavity, etc., seems to be straightforward.\textsuperscript{27} At this point, it is worthwhile to mention the extensive theoretical treatment of gridless klystron cavities by Wang\textsuperscript{40} and Fujisawa.\textsuperscript{41} For a good analysis of a broadband single-tuned, single-gap cavity, the reader is referred to an article by Bryant.\textsuperscript{42}

As pointed out before, the double-tuned, double-gap cavity is more difficult to analyze, even though, it may not be much more difficult to build, than the double-tuned, single-gap cavity. The reason for the difficulty in the analysis of a double-tuned, double-gap cavity is due to the
fact that in a double-gap system, the first cavity remodulates the beam. Now, if the transit angle between the two gaps is large, the effect of the remodulation of the beam must be taken into account. This in turn makes calculations exceedingly complicated even with a number of simplifying assumptions. The behavior of a double-tuned, double-gap cavity system was first analyzed by Feenberg. In his analysis, besides the usual weak coupling and high-\( Q \) approximations, it was assumed that the RF voltage level induced in the single catcher by the beam was only a small fraction of the beam voltage and that the separation of the two cavities was small so that the remodulation of the beam at the first gap could be neglected. Because of the assumption of the very small gap voltages as compared to the beam voltage, it was possible to optimize the asymmetric loading of the cavities and the coupling parameter for maximum output power. Under these conditions, it was shown that it is possible to extract through one cavity twice the power of a single catcher at no cost in bandwidth. Similarly, it was shown that the bandwidth can be increased at some cost in this maximum obtainable power. The analysis by Feenberg differs from the analysis in this paper in that the electron beam is at no time subjected to stopping voltages, i.e., voltages of the order of the DC accelerating potential \( V_0 \). This explains the doubling of the output power. In this paper, because of the definition of the minimum allowable stopping conductance, the output power to the useful load will never be larger than the available power of a single-tuned cavity system. Double-tuned, double-gap output cavities have been built by Litton Industries and Eimac. The use of these composite cavities showed increased bandwidth. In reference 44, the two individual cavities were coupled
electromagnetically, whereas in reference 45 the cavities were decoupled electromagnetically from each other. At this point it is interesting to quote (without comment) a discussion between Mr. J. Dain (English Electric Valve Company) and Messrs. H. J. Curnow and L. E. W. Mathias (Services Electronics Research Laboratory) about the possibilities of a double-tuned, double-gap output cavity.

"On the atomic scale Nature works in discrete, quantized steps; on the macroscopic scale she prefers to work gradually and avoid sudden changes as much as possible. I suggest this should be borne in mind when designing the output system of a high-power klystron. The r.f. energy carried by the beam should not be extracted at one resonator gap spanned by a voltage which is about equal to the direct beam voltage. Instead, for example, it should be removed at two resonator gaps spanned by voltages of suitable phase and each of magnitude approximately one-half the beam voltage. Theoretically this system should have a bandwidth twice that of the single cavity. The logical conclusion of this process is to increase the number of cavities even further and so, perhaps, arrive at an output system which is a compromise between a single-cavity circuit and a travelling-wave circuit."

Messrs. H. J. Curnow and L. E. S. Mathias (in reply):

"The possibility of using two output cavities, coupled to each other and to the beam, was considered before adopting the scheme described. (Double-tuned, single-gap output cavity.) Such an arrangement would at first sight give the same bandwidth, with possibly higher efficiency, but at the expense of greater mechanical complexity. Moreover, the coupling of the cavities by the electron beam complicates the situation, and may even lead to oscillation.

In general, cross-breeds between klystrons and travelling-wave tubes are unlikely to be successful, as the high-signal bunches produced by one system are not suitable for injection into the other."

1.3 Outline of the Thesis

In Chap. 2 some double-tuned resonators, to be used as a composite output cavity system of a klystron amplifier, are considered. Lumped-constant equivalent circuits, the behavior of which will be analyzed in the consecutive chapters, are given.

In Chap. 3 the behavior of the equivalent circuit of a capacitively coupled, double-tuned, double-gap cavity is analyzed for the case when
the cavities are loaded symmetrically. The analysis of the equivalent circuit begins with the derivation of the approximate impedance functions from the locus of poles and zeros of the exact impedance functions in the complex frequency plane. With the approximate impedance functions determined, the behavior of the circuit is analyzed for different values of the coupling. It is assumed that the induced gap current at the second gap differs from the induced gap current at the first gap by a phase angle, equal to the transit angle $\psi$, only. In other words, the effect of the electron beam remodulation at the first gap is completely neglected. The power dissipated in individual cavities is then calculated and plotted. Finally, the bandwidth is determined from these power curves and also plotted. In the following chapter the effect of asymmetric loading on the behavior of the capacitively coupled, double-tuned, double-gap cavity is analyzed.

In Chap. 5 the effect of the electron beam remodulation at the first gap is considered. New power dissipation curves are derived using the same approximate impedance equations; however, the approximate expression for the induced gap current at the second gap is modified to include the first order effect of the electron beam remodulation at the first gap. The effect of remodulation of the beam at the first gap is calculated using small-signal, kinematic theory and the assumption of short and uniform gaps. Additionally, it is assumed that the amplitude of the first harmonic of the bunched electron beam arriving at the first gap is equal to the DC beam current and that the electrons arriving at the first gap have no velocity modulation. The amount of calculations are limited to a minimum; however, enough calculations are done to determine
the effect of the electron beam remodulation on the approximate results of Chaps. 3 and 4.

In Chap. 6 the equivalent circuit of a magnetically coupled, double-tuned, double-gap cavity system is analyzed. The results for this two-cavity resonator are expressed in terms of the results of the capacitively coupled, double-tuned, double-gap cavity system analyzed in Chaps. 3, 4 and 5.

In Chap. 7 the equivalent circuits of two waveguide-coupled and of two coaxial-line-coupled resonators of Figs. 2, 2a and 2.2b, respectively, are analyzed. In the following chapter the behavior of two uncoupled cavities is considered. In Chap. 9 the results of this paper are summarized.

In Appendix 1 the behavior of a single-tuned cavity resonator is presented for comparison purposes. In Appendix 2 the behavior of a magnetically coupled, double-tuned, single-gap cavity system is given for completeness. Finally, in Appendices 3 and 4, a study of the locus of the poles of the impedance functions and their exact expressions of some double-tuned circuits is presented for the convenience of the reader.
CHAPTER 2
THE EQUIVALENT CIRCUIT REPRESENTATION OF TWO
COUPLED CAVITIES

2.1 Introductory Remarks

The conventional, low-frequency circuit analysis presents the easiest way of analyzing the behavior of coupling to or between the microwave cavities. The number of lumped-constant equivalent circuits that can represent one particular type of coupling is large. Some of these equivalent circuits are simple, some are very complicated even for a small frequency range of operation. In general, the accuracy of the representation increases with the complexity of the equivalent circuit. However, by a judicious choice of an equivalent circuit, a simple network can be picked which gives a very good approximation to the behavior of the coupling for a relatively small frequency range.

The number of different ways of coupling microwave cavities and the amount of literature treating this subject, both from the circuit and the field point of view, is quite numerous (for references, see Sec. 1.2). The most common method of coupling is with loops and irises. Depending on the mechanism, its dimensions and its location in the system, the coupling is achieved primarily through the magnetic field or through the electric field. Since the resonant coupling system is more frequency sensitive than the magnetic or the electric coupling, it will not be considered. The coupling, together with the transit angle $\psi$ which will be defined later, determine the phase relationship of the gap voltages $V_1$ and $V_2$ of a particular cavity system. Figure 2.1a shows one of these cavity systems. The transit angle $\psi$, mentioned above, is the product of angular
frequency and the time it takes the beam to traverse the distance between the centers of two cavities as shown in Fig. 2.1a. If the fields in the adjoining cavities have the phase relationship shown in Fig. 2.1a, then they are said to resonate in the zero mode. In Fig. 2.1b, the cavities resonate in the $\pi$ mode.

Figure 2.1a shows two microwave resonant cavities. The possible location of a coupling aperture is indicated in this figure. However, in the cavity system of Fig. 2.1a, two other coupling methods can be used. In the first method, if the gaps are gridless, the interconnecting drift tube can be used as a capacitive coupling between the adjoining cavities (fringing fields). Here the drift region acts as a waveguide below cutoff; therefore, the amount of coupling will depend on the dimensions of the drift tube. The second method of coupling two adjoining cavities is shown in Fig. 2.2. In Fig. 2.2a, a rectangular waveguide which takes out the RF power from the cavities provides the coupling. In reference 47, the coupling waveguide was used in this way to couple the input and the output cavities of a two cavity klystron oscillator. It is interesting to note here, that in order for the dominant mode to propagate in the waveguide, the cavities must resonate in the zero mode. In Fig. 2.2b, the rectangular waveguide is replaced by a coaxial line. Note the change in the mode of oscillation of the cavity system of Fig. 2.2b required to couple out the power. Since a transmission line must always be connected to an output cavity of a klystron, the transmission line coupling is particularly useful when the cavity system of Fig. 2.2 is used as an output cavity of a klystron. Note that this type of coupling could sometimes be used between the two cavities of a double-tuned, double-gap intermediate cavity, where the individual cavities have to be loaded externally. However, at this point
it is worthwhile to mention, that the analysis of the intermediate cavity system differs from the analysis of an output cavity. In the analysis of the output cavity, the problem is attacked from the point of view of the maximum power extraction from the beam at the maximum bandwidth, whereas in the analysis of an intermediate cavity, the problem is approached from the point of view of the most efficient electron beam modulation. Here the term "efficient" may denote bandwidth, loading, gain per stage, etc. This paper will analyze the behavior of a double-tuned, double-gap output cavity only. The behavior of a double-tuned, double-gap cavity which could be used as a composite intermediate cavity will not be treated here. However, some of the expressions derived for the output cavity could very well be used for the analysis of a composite, intermediate cavity.

One may now proceed with the derivation of the approximate lumped-constant, equivalent circuit of the double-tuned, double-gap cavity. The term double-gap signifies that the electron beam passes through the two cavities as shown in Fig. 2.1. The term double-tuned will signify that one has two resonant cavities that are tunable to either the same or to two different frequencies as the case might require. From now on, a double-tuned, double-gap cavity will be designated as a DTDG cavity. In the derivation of the equivalent circuits, it will be assumed that this composite cavity as shown in Fig. 2.1 is made up of two identical cavities shown in Fig. Al.1. This implies that the effective inductance $L$, the effective capacitance $C$ and the effective conductance $G$, representing the internal losses in the cavity, are the same for both cavities unless stated to the contrary. The introduction of this restriction reduces the complexity of calculations considerably without too much loss in generality.
for practical application. The derivation of the equivalent circuits will be limited to coupling mechanisms with practical importance only. These will include electric coupling of gridless gaps through the interconnecting drift tube, magnetic loop coupling between the cavities and the coupling through the transmission line as shown in Fig. 2.2.

2.2 Capacitive Coupling of Gridless Gaps Through the Short Drift Tube

Consider now, for example, Fig. 2.1a. Let the voltage across the first gap along the electron path be $V_1$ with the polarity as indicated, and let the voltage across the second gap be $V_2$. Here $V_1$ and $V_2$ are complex quantities as will be all other variables designated by capital letters unless stated to the contrary. The interconnecting drift tube, if the gaps are gridless, acts as an electric energy storage space; this means, that the effect of the connecting region can be represented as an effective capacitance. Finally, denoting the induced gap current at the first interaction gap as $I_1$, and at the second gap as $I_2$, one has the equivalent circuit shown in Fig. 2.3. Here capital letters designating lumped-constant circuit elements are positive, real numbers. In Fig. 2.3, $C_c$ denotes the effective coupling capacitance. As one can see, two cavities with gridless gaps, coupled capacitively through a short drift tube, can be represented approximately as a "double-tuned" circuit utilizing lumped-constant elements. In Fig. 2.3, the induced gap currents $I_1$ and $I_2$ are clearly indicated. These induced gap currents $I_1$ and $I_2$ in Fig. 2.3 represent the excitation variables. However, since one is interested in the steady state response of the system, only the magnitude and the phase relationship between the two induced gap currents are of importance. The gap voltages $V_1$ and $V_2$ in Fig. 2.3 are the response variables so that their reference polarity can be fixed arbitrarily. With these facts in mind, one
can interchange, for example, the signs of the individual induced gap currents $I_1$ and $I_2$ and put the circuit of Fig. 2. 3b into the form given by Fig. 2. 4 without any loss of generality. The "double-tuned" circuit of Fig. 2. 4 is then an approximate, equivalent circuit of a capacitively coupled DTDG cavity.

2. 3 Magnetic Coupling Through a Loop

When the coupling of Fig. 2. 1 is a loop, the resonant cavities are coupled primarily through the magnetic field. For two magnetically coupled resonant cavities with the loop located as shown in Fig. 2. 1, one has an approximate equivalent circuit shown in Fig. 2. 5. (See, for example, references 36, 48 and 49.) There $M$ is the mutual inductance between the individual cavities. Its sign and its absolute value determine the behavior of the coupled cavities $^{38,49}$. The equivalent circuit of Fig. 2. 5 can be rewritten and put into the form of Fig. 2. 6 which reduces with one $T$ to $\pi$ transformation to a circuit shown in Fig. 2. 7. The magnetically coupled "double-tuned" circuit of Fig. 2. 7 is of the same form as the capacitively coupled "double-tuned" circuit of Fig. 2. 4. It is expected, therefore, that the results obtained in the analysis of the circuit of Fig. 2. 7 will resemble the results obtained in the analysis of the circuit of Fig. 2. 4. Both inductively coupled circuits, i.e., the circuit of Fig. 2. 7 with the positive value of $M$ as well as the circuit with the negative value of $M$, will be analyzed.

2. 4 Waveguide Coupling

Derivation of the equivalent circuit for the case when the output transmission line provides the coupling between the cavities as shown in Fig. 2. 2 requires special attention. At this point it will be assumed that the rectangular waveguide propagates the dominant mode only and the coaxial line propagates the TEM mode only. Both lines will be located
symmetrically between the cavities so that a symmetric equivalent circuit will result.

Consider now the rectangular waveguide coupling of Fig. 2.2a. In the region where the waveguide is connected to both cavities, the electric field \( E \), the magnetic field \( H \) and the wall current \( J \) are indicated. In Fig. 2.2a the two cavities and the connecting waveguide form regions capable of electromagnetic energy storage. These regions are interconnected so that the electromagnetic power can propagate from one region into another. Now, when the coupling between the cavities and the waveguide is primarily through the magnetic field, the magnetic flux lines of one region protrude into the adjoining region. An increased concentration of the magnetic flux lines in microwaves, however, corresponds to a magnetic energy storage in an inductance of the conventional lumped-constant circuit analysis. One would expect, therefore, that an inductance provides the coupling between the individual energy storage regions. At this point, however, it is probably easier to use the wall currents for the detailed derivation of the lumped-constant, equivalent circuit for the DTDG cavity system of Fig. 2.2a. The use of the wall current \( J \) in the derivation of the equivalent circuit, however, does not mean that the wall current \( J \) provides the coupling. As pointed out previously, the coupling between the regions was through the magnetic field and the wall current \( J \) was merely a result of the solution of the boundary condition for the magnetic field \( H \). The study of the flow of the wall current \( J \) is only helpful in determining the type of the connection of the inductive coupling in the equivalent circuit. From Fig. 2.2a, one can see that the current which flows through the load resistance \( R_L \) (not shown in Fig. 2.2a) flows also in series through both cavities. However, since the wall current in the connecting
waveguide which also flows through the load resistance \( R_L \) is not the same in magnitude as the current that flows through the equivalent inductance of each cavity (Fig. 2.2a), one has effective transformers at the junctions of the waveguide to each cavity. Hence, these effective transformers are connected in series. For this type of coupling, one therefore, has the equivalent circuit shown in Fig. 2.8a. In Fig. 2.8a, \( R_L \) stands for the load resistance, \( L_o \) stands for the additional magnetic energy storage in the junction and \( M \), representing the mutual inductance between the coils \( L \) and \( L_o \), stands for the previously described transformer action. In Fig. 2.8a, the direction of flow of the wall currents of Fig. 2.2a was used for the definition of the positive mutual inductance \( M \) between the coils \( L \) and \( L_o \). The circuit of Fig. 2.8b is identical with the circuit of Fig. 2.8a, but different in form. When the mutual inductance \( M \) is included into the circuit elements, the circuit of Fig. 2.8b reduces to the circuit of Fig. 2.8c. Now, when the circuit of Fig. 2.8c is transformed first by a \( \pi \) to \( T \) transformation and then by a \( T \) to \( \pi \) transformation, one obtains the equivalent circuit shown in Fig. 2.9. Fig. 2.9, therefore, shows an approximate equivalent circuit of a DTDG cavity coupled together by the connecting waveguide (Fig. 2.2a).

2.5 Coaxial Line Coupling

The method used for the derivation of the equivalent circuit for the waveguide coupling of Sec. 2.4, can also be used to derive the equivalent circuit of a coaxial line coupling shown in Fig. 2.2b. If this is done, one will obtain the equivalent circuit shown in Fig. 2.10a. Again, as in the case of the waveguide coupling, it was assumed that the magnetic field provided the coupling. Actually, in both types of transmission line coupling, the transmission line was connected to the cavities in the region of
strong magnetic fields and only weak electric field. Therefore, it seems to be logical, that the magnetic coupling should be used for the coupling.

Following the same procedure as in Sec. 2.4, one finally obtains the approximate equivalent circuit (Fig. 2.11) for the coaxial line coupling shown in Fig. 2.2b.

2.6 Summary

This concludes the derivation of the equivalent circuits, the behavior of which will be studied in the following chapters. Coupling methods which were considered in this Chapter were the following. Capacitive coupling through the short interconnecting drift tube as shown in Fig. 2.1a. The equivalent circuit for this type of coupling is shown in Fig. 2.4. The second coupling considered was a magnetic loop coupling as indicated in Fig. 2.1a. Its equivalent circuit is given in Fig. 2.7. The next coupling to be analyzed, was magnetic coupling through the connecting waveguide as shown in Fig. 2.2a. The equivalent circuit for this type of coupling is given by Fig. 2.9. The last coupling which was studied was magnetic coupling through the connecting coaxial line as shown in Fig. 2.2b. This coupling has an equivalent circuit of Fig. 2.11. One may now proceed with the analysis of individual coupling mechanisms whose equivalent circuits were derived in this chapter.
TO LOAD

FIG. 2.2 a

ELECTRIC FIELD $E$

MAGNETIC FIELD $H$

WALL CURRENT $J$

FIG. 2.2 b
FIG. 2.3a

FIG. 2.3b

FIG. 2.4
FIG. 2.8a

FIG. 2.8b

FIG. 2.8c

FIG. 2.9
CHAPTER 3

THE BEHAVIOR OF TWO CAPACITIVELY COUPLED, SYMMETRICALLY LOADED RESONANT CAVITIES

3.1 Description of the Method of Analysis

In Chap. 2, the equivalent circuits of two resonant cavities were derived for different couplings. Before proceeding with the actual analysis of these equivalent circuits, one should briefly discuss the method of investigation that will be used.

As one can see, the equivalent circuits are composed of linear, passive and bilateral elements. This means that the response of the system can be calculated from a set of two linear equations

\[ V_1 = Z_{11} I_1 + Z_{12} I_2 \]  \hspace{1cm} (3.1a)

and

\[ V_2 = Z_{12} I_1 + Z_{22} I_2 \]  \hspace{1cm} (3.1b)

where the reciprocity of the network is taken into account by letting

\[ Z_{21} = Z_{12} \]  \hspace{1cm} (3.2)

Here \( Z_{11} \) and \( Z_{22} \) are the driving point impedances, and \( Z_{12} \) is the transfer impedance of the coupled network. Knowing \( Z_{11}, Z_{12}, \) and \( Z_{22} \) and the gap currents \( I_1 \) and \( I_2 \), the response voltages \( V_1 \) and \( V_2 \) can be determined. But if \( V_1 \) and \( V_2 \) are known, then the complete behavior of the system is known. Therefore, \( Z_{11}, Z_{12} \) and \( Z_{22} \) will be derived for each equivalent circuit of Chap. 2. In the determination of the impedance functions, high-Q approximations will be used throughout this analysis. A number of other approximations, which will reduce the complexity of the analysis without great loss in the accuracy, will also be used.
Now, consider the equivalent circuit obtained for the capacitive coupling through the interconnecting drift tube as shown in Fig. 2.4. Without any loss of generality this circuit may be rewritten and put in the form shown in Fig. 3.1. In this circuit, the conductances \( G \), originally representing internal losses of the cavities only, were replaced by \( G_1 \) and \( G_2 \). Here, \( G_1 \) now not only represents internal losses in the first cavity, but also takes into account the transformed external loading conductance if such is present. \( G_2 \) plays a similar role in the second cavity.

For the circuit of Fig. 3.1 one has

\[
Z_{11} = \frac{Ks}{C\Delta} \left( s^2 + \frac{G_2}{C} s + \frac{1}{LC} \right) \quad (3.3a)
\]

\[
Z_{22} = \frac{Ks}{C\Delta} \left( s^2 + \frac{G_1}{C} s + \frac{1}{LC} \right) \quad (3.3b)
\]

and

\[
Z_{12} = \frac{s^3 KC_c}{C^2 \Delta} \quad (3.3c)
\]

where

\[
\Delta = s^4 + \frac{G_1 + G_2}{C} Ks^3 + \left( \frac{2}{CL} + \frac{G_1 G_2}{C^2} \right) Ks^2 + \frac{G_1 + G_2}{C^2 L} Ks + \frac{K}{C^2 L^2}
\]

\[
K = \frac{C^2}{C^2 - C_c^2} = \frac{1}{1 - C_c^2/C^2} \quad (3.4a)
\]

and \( s \) is the complex frequency given by \( s = \sigma^* + j\omega \).

From these equations the behavior of the circuit can now be determined for all values of frequency. However, approximations made previously in the derivation of the equivalent circuits make exact but lengthy calculations at this point unwarranted. It is advisable, therefore, to express the driving point and transfer impedances in some approximate form. The best way to attack this problem is by studying the loci of poles and zeroes of the driving point impedances and the transfer impedance in the complex plane.
In general, it is not possible to find the roots of Eq. (3.4a) i.e., the coordinates of the poles of $Z_{11}$, $Z_{12}$ and $Z_{22}$, in closed form. Exact analytic solutions for any $K$ exist only for the case when $G_1 = G_2$, i.e., symmetric loading. This solution can be obtained by inspection of the circuit of Fig. 3.1, remembering that the roots of $Z_{11}$, $Z_{12}$ and $Z_{22}$ are the open-circuit resonant frequencies of the circuit. When $G_1 \neq G_2$, solutions in closed form exist only for $K = 1$, i.e., for zero coupling. Solutions for any other choice of $G_1$, $G_2$ and $K$ can be found with any degree of accuracy through numerical calculations.

3.2 Derivation of the Impedance Functions for the Symmetric Loading

At this point, it seems advisable to study first the behavior of the circuit of Fig. 3.1 when it is symmetrically loaded. Later, the effect of asymmetrical loading on the performance of the circuit can be analyzed.

As previously stated, the roots of Eq. (3.4a) for the case when $G_1 = G_2$ can be easily expressed in closed form. They are:

$$s_{1,4} = -\alpha_1 \pm j \sqrt{\omega_1 - \alpha_1^2}$$  \hspace{1cm} (3.5a)

$$s_{2,4} = -\alpha_2 \pm j \sqrt{\omega_2 - \alpha_2^2}$$  \hspace{1cm} (3.5b)

where

$$\alpha_1 = \frac{G}{2(LC - C_c)}$$  \hspace{1cm} (3.6a)

$$\alpha_2 = \frac{G}{2(LC + C_c)}$$  \hspace{1cm} (3.6b)

$$\omega_1^2 = \frac{1}{LC - C_c}$$  \hspace{1cm} (3.6c)

and

$$\omega_2^2 = \frac{1}{LC + C_c}$$  \hspace{1cm} (3.6d)
The locus of poles $s_{1,4}$ and $s_{2,3}$, which are the complex natural frequencies of the circuit, is shown in Fig. 3.2 for increasing values of $K$. A cross denotes a pole and a circle denotes a zero in the complex plane. As one can see, the poles lie on the circle of radius $1/LG$ through the origin. Therefore, knowing $a_1$, $a_2$ and the radius of the circle, the location of the poles can be quickly found.

When $Q = \sqrt{C/L}/G > 10$ and the coupling is weak, i.e., $K \approx 1$, the ratio of $a_1$ to $\omega_1$ and of $a_2$ to $\omega_2$ is of the order of $1:20$ and smaller. This means that the arc of the circle connecting the two poles is almost a straight line parallel to the $j\omega$-axis. In Figure A4.1, the exact and the approximate loci of poles for different couplings were drawn for a circuit with a $Q = 20$. Introducing the approximation

$$a_1 \approx a_2 \approx \frac{G}{2C} = \alpha,$$  \hspace{1cm} (3.7)

one has the new approximate location of poles as shown in Fig. 3.3 for the upper half-plane only. Therefore, with large-$Q$ and weak coupling approximations the new coordinates of the poles are

$$s_{1,4} \approx -\alpha \pm j(\omega_0 + p\alpha)$$ \hspace{1cm} (3.8a)

and

$$s_{2,3} \approx -\alpha \pm j(\omega_0 - p\alpha).$$ \hspace{1cm} (3.8b)

Here $p$ is defined in Fig. 3.3 and is approximately equal to

$$p \approx \frac{\omega_1 - \omega_2}{2\alpha} \approx \frac{C}{2C} \frac{\omega_0}{\omega_0} = \frac{C}{\alpha} Q_o,$$ \hspace{1cm} (3.9a)

where

$$\omega_0^2 = \frac{1}{LC} = \omega_1^2 + \alpha^2 \approx \omega_0^2$$ \hspace{1cm} (3.9b)

and

$$Q_o = \frac{\omega_0}{2\alpha}.$$ \hspace{1cm} (3.9c)

With the coordinates of Eq. (3.8), $Z_{11}$, $Z_{22}$ and $Z_{12}$ become
\[ Z_{11} = Z_{22} = \frac{-30}{(j\omega_o/C)(2j\omega_o)} \left[ \alpha + j(\omega - \omega_o) \right] \]

\[
= \frac{1}{G} \frac{(1 + jx)}{(1 + p^2) + j2x - x^2}
\]

and

\[ Z_{12} = \frac{jp/G}{(1 + p^2) + j2x - x^2} \]

where \( x \) is the normalized relative frequency variable defined as

\[ x = \frac{2(\omega - \omega_o)}{2\alpha} \]

and \( p \) is a coupling parameter defined in Eq. (3.9a). Note, that because of the high-Q and the weak coupling approximation introduced in this section, expressions for \( Z_{11} \) and \( Z_{12} \) contain an error which increases in amplitude as \( x \) and \( p \) are increased or \( Q \) is decreased. The exact expressions for \( Z_{11} \) and \( Z_{12} \) of a symmetrically loaded circuit of Fig. 3.1 \((G_1 = G_2 = G)\) are derived in Sec. A3.1. The analysis of Appendix 3 shows that even though the exact and the approximate power dissipation curves differ somewhat in general appearance, from the maximum power dissipation point of view and from the point of view of bandwidth, expressions for \( Z_{11} \) and \( Z_{12} \) of Eq. (3.10) can be used freely if \( p \) is of the order of unity or less and if \( Q_0 > 10 \). See, for example, Fig. A3.1, where the approximate power dissipation curve (heavy curve) was plotted using \( Z_{11} \) and \( Z_{12} \) of Eq. (3.10), together with an exact power dissipation curve (light, continuous curve). Observe the appreciable error in the power dissipation curve for small values of \( P_{dn} \). Fortunately, the region where the power dissipation curve is so low is of very little interest in the analysis of an output cavity.
3.3 Approximation of the Gap Currents

At this point, a few words must be said about the gap currents $I_1$ and $I_2$, and the magnitude of the transit angle $\psi$. In the following discussion, the induced gap currents $I_1$ and $I_2$ will represent the effect of the electron beam as seen at the gap terminals. Both currents are assumed to be sinusoidal and of fundamental frequency. Their amplitudes depend on the bunching of the beam and on the beam-to-circuit coupling. Furthermore, it will be assumed that their amplitudes are constant for an arbitrary frequency range and that the electrons reaching the first gap have no velocity modulation. One may now define the gap currents $I_1$ and $I_2$ as

$$I_1 = I_1$$

and

$$I_2 = I_1 e^{-j\psi}.$$  

In Eq. (3.12), $\psi$ is the transit angle between the symmetry planes of the two cavities as shown in Fig. 2.1a.

From Eq. (3.12) it can be seen that the gap current $I_1$ differs from the gap current $I_2$ only in phase by the transit angle $\psi$. This implies that the effect of the current modulation produced by the gap voltage $V_1$ at the first gap was neglected. However, the first order effect of the electron beam current remodulation at the first gap can be taken into account by using small-signal, kinematic theory and the assumption of short and uniform gaps. With these assumptions one can obtain approximately, for example, from Chap. 2 of reference 8, the beam current at the second gap taking into account the beam remodulation at the first gap

$$I_{b2} \approx I_{b1} e^{-j\psi} \left(1 - jM_1 \frac{V_1 I_0 \psi}{2V_0 I_{b1}} \right).$$  

Here $I_{b1}$ and $I_{b2}$ are the amplitudes of the first harmonic of the bunched
electron beam current at the respective gaps. $M_1$ is the beam coupling coefficient of the first gap. $I_0$ is the DC beam current, $V_o$ is the DC potential of the electron beam and $V_1$ is the gap voltage across the first gap. The minus sign was introduced into Eq. (3.13a) in order to take the reference polarity at the first gap of Fig. 3.1 into account. Equation (3.13a) can be put into a different form using the fact that the beam current and the gap current at a particular gap are proportional to each other. The proportionality factor in this case is the beam coupling coefficient which is very near equal to unity so that one can write approximately for the gap current at the second gap,$$
abla_1 I_2 = I_1 e^{-j\psi} (1 - \frac{\psi V_1 I_o}{2V_o I_1}) \quad (3.13b)$$

Here $I_1$ and $I_2$ now represent the respective gap currents. $I_2$ of Eq. (3.13b) reduces to $I_2$ of Eq. (3.12b) if the term $\frac{\psi V_1 I_o}{2V_o I_1}$ in Eq. (3.13b) is neglected. Therefore, the above-mentioned term represents the effect of the beam current remodulation at the first gap. If $I_2$ of Eq. (3.13b) were used to find the power dissipated in the loads of the circuit of Fig. 3.1, the amount of algebra involved would be prohibitive. Therefore, in order to keep the amount of calculations to a minimum, $I_2$ defined by Eq. (3.12b) will be used throughout the analysis. That is, the general behavior of a DTDG cavity will be analyzed neglecting the effect of the current remodulation at the first gap.

Before proceeding with the actual determination of the power dissipated in the individual loads of Fig. 3.1, one should briefly discuss the magnitude of the error in $I_2$ of Eq. (3.12b). From Eq. (3.13b) one can see that the error in $I_2$ given by Eq. (3.12b) will be small if either $V_1/V_o$ is small or $\psi/2$ is small so that $\left|\frac{V_1 I_o \psi}{2V_o I_1}\right| \ll 1$, where the vertical bars denote that the absolute value of the inclosed expression is taken. However, in a
klystron output cavity, the voltages produced across the two gaps of the
double-gap output cavity will be of the order of the DC electron potential $V_o$. As a matter of fact, to get the maximum efficiency out of the beam, an electron stopping conductance $G_{stop}$ will be defined later on, which will require that the vectorial addition of the gap voltages be less than or equal to the DC accelerating voltage $V_o$ of the electron beam. So that, if an output cavity system has two gaps, the amplitude of the voltage across the gaps may be of the order of $V_o/2$. Since the vectorial sum of $V_1$ and $V_2$ will be fixed by the stopping condition in Eq. (3.22), from the point of view of the approximation used in Eq. (3.12b) it is therefore desirable to impose another restriction on the magnitudes of the gap voltages. This restriction is

$$|V_1| < |V_2|.$$  \hspace{1cm} (3.14)

In the case of asymmetric loading of the output cavity, the load must be coupled into the cavities in such a way that the voltages produced across the first gap are smaller than those produced across the second gap so that the restriction of Eq. (3.14) will be satisfied. From the above consideration it can be assumed that the magnitude of the voltage across the first gap of the output cavity will never be much larger than $V_o/2$. Assuming that $|V_1|_{\text{max}} \approx V_o/2$ one obtains, for example for $\psi = \pi/2$, for $I_2$ from Eq. (3.13b)

$$|I_2|_{\text{max}} < 1.4 |I_1|$$ \hspace{1cm} (3.15a)

and

$$|I_2|_{\text{min}} > 0.6 |I_1|.$$ \hspace{1cm} (3.15b)

As one can see, when $\psi = \pi/2$ and when $|V_1|_{\text{max}} \approx V_o/2$, the maximum error in the amplitude of the gap current $I_2$ will be quite appreciable. Therefore, it is to be expected that the results obtained using the approximate definition of $I_2$ from Eq. (3.12b) will show the greatest error when the
gap voltage across the first gap is the greatest. Fortunately, \( V_1 \) adds vectorially in Eq. (3.13b), so that the error in the amplitude of \( I_2 \) will also depend to a large extent on the phase of the gap voltage \( V_1 \). Note that the use of Eq. (3.12b) not only introduces an error in the amplitude of \( I_2 \) but also an error in the phase of \( I_2 \).

3.4 Power Delivered to the Loads

One may now determine the power delivered to the loads of the circuit of Fig. 3.1. However, before doing this, one can rewrite Eq. (3.1) taking into account the minus sign in front of \( I_2 \) of Fig. 3.1. With this change of sign, Eq. (3.1) becomes

\[
V_1 = Z_{11} I_1 - Z_{12} I_2 \quad (3.16a)
\]

and

\[
V_2 = Z_{12} I_1 - Z_{22} I_2 \quad . \quad (3.16b)
\]

If the circuit of Fig. 3.1 is loaded symmetrically, i.e., \( G_1 = G_2 = G \), one has additionally

\[
Z_{11} = Z_{22} \quad . \quad (3.17)
\]

The power dissipated in the two-terminal-pair network is given by

\[
P_d = \frac{1}{2} \text{Re} V_1 I_1^* + \frac{1}{2} \text{Re} V_2 I_2^* \quad . \quad (3.18a)
\]

In Eq. (3.18a), the star denotes the complex conjugate of the particular gap current and Re stands for the fact that only the real part of the complex quantity is taken. In the case when the load conductances are connected as shown in Fig. 3.1, the power delivered to the circuit is also given by

\[
P_d = \frac{G_1}{2} \left| V_1 \right|^2 + \frac{G_2}{2} \left| V_2 \right|^2 \quad . \quad (3.18b)
\]

In Eq. (3.18b), \( \left| V_1 \right| \) denotes the absolute value of the gap voltage \( V_1 \).

Later on in this chapter it will be shown that the absolute values of the gap voltages, and therefore the values of the power dissipated in each
load, in general, are not equal at the same frequency even for the case when
the circuit of Fig. 3.1 is loaded symmetrically, i.e., \( G_1 = G_2 = G \). However,
in the derivation of the equivalent circuit of Fig. 3.1, it has been assumed
that each individual cavity was loaded separately. Therefore, one must now
evaluate the power dissipated in each load of the circuit of Fig. 3.1 also
separately, since the expression for the total dissipated power given either
by Eq. (3.18a) or Eq. (3.18b) alone has little meaning in terms of maximum
power out and bandwidth. For this reason, the form of the expression for
the total dissipated power given by Eq. (3.18b) is advantageous, because
it gives the total dissipated power in the circuit of Fig. 3.1 in terms of the
sum of powers dissipated in the individual loads. From Eq. (3.18b), one
obtains, by using Eqs. (3.16), (3.17), (3.10) and (3.12) for the dissipated
power in the first load of the symmetrically loaded network \([G_1 = G_2 = G]\),
\[
    P_{d1} = \frac{G|I_1|^2}{2} \left| Z_{11} - Z_{12} e^{-j\psi} \right|^2 = \frac{|I_1|^2}{2GA_1} \left[ 1 + p^2 + x^2 - 2p \sin\psi \cos\psi \right], \tag{3.19a}
\]
and similarly for the power dissipated in the second load,
\[
    P_{d2} = \frac{|I_1|^2}{2GA_1} \left[ 1 + p^2 + x^2 + 2p \sin\psi - x \cos\psi \right], \tag{3.19b}
\]
where
\[
    \Delta_1 = (1 + p^2)^2 + 2x^2 (1 - p^2) + x^4 \tag{3.19c}
\]
and the total power dissipated in the whole network is then given by the sum
of Eqs. (3.19a) and (3.19b). In general, \( p \neq 0 \), so that Eq. (3.19) is an
asymmetric function of \( x \) if \( \cos\psi \neq 0 \). If \( \psi \) in Eq. (3.19) is allowed to assume
any arbitrary value, the analysis of the power expression becomes very
lengthy. However, the range of values of \( \psi \) which are of interest is fortunately
small. First of all, \( \psi \) can never be zero nor can it be very small, because
\( \psi = 0 \) corresponds to two overlapping gaps. On the other hand, when \( \psi \) is
small, the two gaps are close together and the coupling between the driftless
gaps is quite strong. However, in the beginning of this chapter it has
been assumed that the capacitive coupling through the connecting drift tube
was weak. Therefore, from the above reasoning one can assume that the
values of \( \psi \) approximately less than \( \pi/2 \) are of no interest for this type of
coupling of DTDG cavities. One may now investigate the limitation imposed
on the large values of \( \psi \). In Sec. 3.3 the gap current \( I_2 \) was defined by
Eq. (3.12b). In the definition of Eq. (3.12b) it has been assumed that
the term \( V_1 \psi/2V_0 \) remains small. However, as it was pointed out before,
\( V_1 \) may be of the order of \( V_0/2 \) so that even with \( \psi = \pi/2 \) the term \( V_1 \psi/2V_0 \)
becomes appreciable. Therefore, let \( \psi \) be as near \( \pi/2 \) as possible. If
\( \psi = \pi/2 \), Eq. (3.19) can be greatly simplified. It becomes
\[
\begin{align*}
P_{d1} &= \frac{|I_1|^2}{2G\Delta_1} \left[ (1-p)^2 + x^2 \right] \\
\text{(3.20a)}
\end{align*}
\]
and
\[
\begin{align*}
P_{d2} &= \frac{|I_1|^2}{2G\Delta_1} \left[ (1+p)^2 + x^2 \right], \\
\text{(3.20b)}
\end{align*}
\]
where \( \Delta_1 \) is given by Eq. (3.19c). Equation (3.20) represents the power
dissipated in the individual loads of the symmetrically loaded network
of Fig. 3.1 \([G_1 = G_2 = G]\) as a function of the normalized relative frequency
variable \( x \) with the condition that \( \psi = \pi/2 \). From Eq. (3.20) the half-power
point \( x_o \) can now be determined as a function of the parameter \( p \). However,
because of the definition of \( x \) [Eqs. (3.11) and (3.7)], the bandwidth depends
on the value of the conductance \( G \).
3.5 Definition of Electron Stopping Conductance

From Eq. (3.20) one can see that as $G$ is made smaller, the power delivered to the symmetric loads increases. But not only the dissipated power is increasing with decreasing $G$; the RF voltages $V_1$ and $V_2$ are also inversely proportional to $G$. This means that, as $G$ is made smaller, a voltage will soon be reached at which the electrons in the beam accelerated to a potential $V_o$ will be stopped and returned to the cathode. The value of $G$ for which the electrons will be stopped for any value of frequency shall be called the electron stopping conductance $G_{stop}$. Therefore, the stopping conductance is defined as the smallest value that $G$ can assume when

$$\left|V_1\right| + \left|V_2\right| \leq V_o.$$

(3.21)

Note that, with a larger value of $G$ than the one given by Eq. (3.21), the bandwidth can be increased, but the maximum of $P_{d1}$ and $P_{d2}$ will go down. Equation (3.21) in itself is sufficient to guarantee that no electrons will be returned to the cathode. However, Eq. (3.21) has a margin of safety, so that the available power in the electron beam is not completely extracted when Eq. (3.21) is used to find the value of $G_{stop}$, because the phase relationship of the gap voltages $V_1$ and $V_2$ with respect to the moving electron was not taken into account. When this phase relationship is utilized, one obtains a second, marginal stopping condition, which is

$$\left|V_1 - V_2 e^{j\psi}\right|_{\text{max}} = V_o.$$

(3.22)

The subscript max in both equations indicates maximum value of the expression for any frequency. Equation (3.22) will be used throughout the analysis for the calculation of the electron stopping conductance. Note that, in Eq. (3.22), the transit angle $\psi$ and the direction of the motion of
electrons through the gaps were taken into account. In this case, for example, the electrons may drop through the potential \( V_1 \) at gap one, but then they must rise through the potential \( V_2 \) at gap two (Fig. 3.1). This explains the minus sign in front of \( V_2 \) in Eq. (3.22).

In Eq. (3.22) one has the absolute value of the difference of two vectors. However, the amplitude of the voltage across the first gap alone can never be larger than \( V_0 \). This is because, if an electron is slowed down at the first gap by a potential greater than \( V_0 \) and then it is speeded up again at the second gap, Eq. (3.22) may apply; however, the electron will never reach the second gap since it was returned to the cathode by the excessive voltage at the first gap. Therefore, when using Eq. (3.22) additional requirement on the gap voltage should be for any frequency

\[
| V_1 | \leq V_0 \quad . \tag{3.23}
\]

One may now determine \( G_{\text{stop}} \) for the circuit of Fig. 3.1.

From Eq. (3.22) one has, using Eqs. (3.16), (3.17), (3.10) and (3.12) for \( \psi = \pi/2 \),

\[
| V_1 - V_2 e^{j\psi} |_{\text{max}} = \frac{2 |I_1|}{G} \left[ \frac{1 + \frac{x^2}{2}}{\left(1 + \frac{x^2}{2}\right)^2 + 2(1-p^2) x^2 + 4} \right]^{1/2} = V_0 \quad . \tag{3.24}
\]

The location of the maximum of the expression in the brackets and its value may now be found. The maximum is at \( x = 0 \), if \( p^2 < (\sqrt{5} - 2) \). For these values of \( p \), \( G_{\text{stop}} \) is given by

\[
G_{\text{stop}} = \frac{2 |I_1|}{V_0 (1 + p^2)} . \tag{3.25}
\]

The maximum of Eq. (3.24) is at \( x^2 = p \sqrt{4 + p^2} - 1 \), if \( p^2 > (\sqrt{5} - 2) \). For these values of \( p \), \( G_{\text{stop}} \) is
3.6 Normalized Power Expressions

Now that $G_{\text{stop}}$ has been determined, one can introduce these values of $G_{\text{stop}}$ into Eq. (3.20). The new dissipated power expressions then become after normalizing to the maximum value of the power dissipated in one single-cavity system [Eq. (A1.12)]

\[
P_{d1} = \Delta_2 (1+p^2) \left[ (1-p)^2 + x^2 \right] \tag{3.27a}
\]

\[
P_{d2} = \Delta_2 (1+p^2) \left[ (1+p)^2 + x^2 \right] \tag{3.27b}
\]

\[
P_{dn} = 2 \Delta_2 (1+p^2) \left( 1+p^2 + x^2 \right) \tag{3.27c}
\]

if $p^2 < (\sqrt{5} - 2)$, where

\[
\Delta_2 = \frac{1}{2} \left[ (1+p^2)^2 + 2x^2 (1-p^2) + x^4 \right], \tag{3.28a}
\]

and

\[
P_{d1} = \Delta_2 \left[ \frac{2p \sqrt{4+p^2} - 2p^2}{2p \sqrt{4+p^2} - 2p^2} \right] \frac{1}{2} \left[ (1-p)^2 + x^2 \right] \tag{3.28b}
\]

\[
P_{d2} = \Delta_2 \left[ \frac{2p \sqrt{4+p^2} - 2p^2}{2p \sqrt{4+p^2} - 2p^2} \right] \frac{1}{2} \left[ (1+p)^2 + x^2 \right] \tag{3.28c}
\]

\[
P_{dn} = 2 \Delta_2 \left[ \frac{2p \sqrt{4+p^2} - 2p^2}{2p \sqrt{4+p^2} - 2p^2} \right] \frac{1}{2} \left[ 1+p^2 + x^2 \right] \tag{3.28d}
\]

if $p^2 \gg (\sqrt{5} - 2)$, where $\Delta_2$ is given by Eq. (3.28a).

In Fig. 3.4 the power dissipated in the first load of the symmetrically loaded $[G_1 = G_2 = G]$, capacitively coupled circuit of Fig. 3.1 as given by
Eqs. (3.27a) and (3.28b) was plotted in light curves as a function of $x$ and $p$. The normalized relative frequency variable $x$ and the coupling parameter $p$ were defined by Eqs. (3.11) and (3.9a), respectively. The power dissipated in the second load as given by Eqs. (3.27b) and (3.28c) was plotted in heavier curves as a function of $x$ and $p$ in the same figure. All the power dissipation curves in Fig. 3.4 are normalized to the maximum power dissipated in one single-cavity system given by Eq. (A1.12). Figure 3.4 shows that the power dissipated in the individual loads of the symmetrically loaded $[G_1 = G_2 = G]$, capacitively coupled circuit of Fig. 3.1, in general, is not the same for the same $x$ and $p$. For example, at $x = 0$ and $p = 1$, the power dissipated in the second load is 0.786 times the power dissipated in one single-cavity system whereas no power is dissipated in the first conductance at all. Or again, at $x = 0$ and $p = 0.5773$, $P_{d2_n} = 0.922$ but $P_{d1_n} = 0.066$. Only for $p = 0$ is $P_{d1_n} = P_{d2_n}$ for all values of $x$. Furthermore, when $p = 0$, the power dissipated in one single load of the symmetrically loaded $[G_1 = G_2 = G]$ circuit of Fig. 3.1 has the same characteristic shape as the power dissipated in one single-cavity system [Eq. (A1.11)], but its maximum amplitude in only one-half of the maximum amplitude of one single-cavity system [Eq. (A1.12)]. However, later on, it will be shown that its bandwidth is twice the bandwidth of one single-cavity system [Eq. (A1.12)]. Note that, when $p = 0$, the maximum power dissipated in both cavities combined is equal to the maximum power dissipated in one single-cavity system [Eq. (A1.12)].

In Fig. 3.4, the values of $p$ were chosen in such a way that they cover the range from $p = 0$ to a value of $p$ slightly larger than the values of $p$ which gives the 3-db ripple at $x = 0$. Needless to say, the curves in Fig. 3.4 are
symmetric about the \( x=0 \) coordinate. In the figure some values of \( p \) are of significance. For example, when \( p < 0.4859 \), the electrons are stopped always at the same value of the normalized frequency \( x = 0 \). Also, when \( p < 0.4859 \), the power dissipated in the first and the second loads, if combined, will give a single-humped power curve with the maximum value equal to unity at \( x = 0 \) (Fig. 3.5). When \( p = 0.5773 \) the power dissipated in the first and second loads combined will still give the single-hump shape (Fig. 3.5). For all other values of \( p \), i.e., \( p > 0.5773 \), the total dissipated power curve of Fig. 3.5 will exhibit a double-hump characteristic. Note that, from Eq. (3.28c), the power dissipated in the second load has a maximally flat curve (still with one hump) for \( p = 0.7711 \). The value of \( p = 1.414 \) was chosen only because \( p^2 = 2 \), and finally, the value of \( p = 1.638 \) is that value for which the total dissipated power given by Eq. (3.28d) gives the 3-db ripple at \( x = 0 \).

In conclusion, the total power dissipated in the symmetrically loaded \((G_1 = G_2 = G)\), capacitively coupled network of Fig. 3.1, as given by Eqs. (3.27c) and (3.28d), was plotted in Fig. 3.5 where the total dissipated power is again given in terms of the maximum dissipated power in one cavity system [Eq. (A1.12)]. In Fig. 3.5, the curves for \( P_{dn} \) have horizontal slopes at \( x = 0 \) and

\[
x^2 = 2p \sqrt{1+p^2} - (1+p^2).
\]

When \( p^2 \leq 1/3 \), there is only one maximum at \( x = 0 \). When \( p^2 > 1/3 \) there are two symmetric maxima given by Eq. (3.29) and one minimum at \( x = 0 \). Therefore, from Eq. (3.28) one can see that when \( p^2 \leq (\sqrt{5} - 2) \) there is one maximum at \( x = 0 \) and its value is \( P_{dn} \mid_{\text{max}} = 1 \). That is, there is no loss
in the total maximum dissipated power as compared to a single-cavity system (Appendix 1). When \( (\sqrt{5} - 2) \leq p^2 \leq 1/3 \), the maximum occurs again at \( x = 0 \) but its value now is

\[
P_{d_n} \max = \frac{[2p \sqrt{4+p^2} - 2p^2]^{1/2}}{(1+p^2)}.
\]

(3.30)

When \( p^2 > 1/3 \), the maxima are at \( x \) given by Eq. (3.29), and their values are

\[
P_{d_n} \max = \frac{[\sqrt{4+p^2} - p]^{1/2}}{\sqrt{8p}} [\sqrt{1+p^2} - p]
\]

(3.31)

The minimum is at \( x = 0 \), and its value is given by Eq. (3.30).

3.7 Absolute Amplitude of the Gap Voltages

In order to see whether Eq. (3.23) is satisfied, one should calculate the absolute amplitude of the complex voltage \( V_1 \) across the first gap of the symmetrically loaded (\( G_1 = G_2 = G \)), capacitively coupled DTDG cavity with the approximate equivalent circuit given in Fig. 3.1. From Eqs. (3.18b), (3.20a), (3.25) and (3.26) one has for \( |V_1| \) with \( \psi = \pi/2 \),

\[
|V_1| = \left[ \frac{2p_{d1}}{G_{stop}} \right]^{1/2} = V_0 \Delta_2 (1+p^2) [(1-p^2) + x^2]^{1/2}
\]

(3.32a)

if \( p^2 \leq \sqrt{5} - 2 \), but when \( p^2 > \sqrt{5} - 2 \), \( |V_1| \) becomes

\[
|V_1| = V_0 \Delta_2 \left[2p \sqrt{4+p^2} - 2p^2 \right]^{1/2} \left[(1-p^2) + x^2 \right]^{1/2}
\]

(3.32b)

where \( \Delta_2 \) is given by Eq. (3.28a). Even though the general behavior of \( |V_1| \) and \( |V_2| \) as a function of \( x \) and \( p \) can be obtained from Fig. 3.4 since \( |V_1| \) is proportional to the square root of \( P_{d1} \) and \( |V_2| \) is proportional to the square root of \( P_{d2} \), \( |V_1| \) and \( |V_2| \) were plotted in Figs. 3.6 and 3.7, respectively. From Fig. 3.6 one can see that Eq. (3.23) is satisfied for all values of \( p \) and \( x \). Furthermore, in view of the approximation of \( I_2 \) of
Eq. (3.13b) by $I_2$ of Eq. (3.12b), it is to be expected that, for the values of $p$ and $x$ for which $|V_1|$ is the largest, the approximate results will also exhibit the largest error. However, the effect of electron remodulation at the first gap will be considered in greater detail in Chap. 5.

Again, as it was expected, $|V_1| \neq |V_2|$. From Eqs. (3.19a) and (3.19b) one can see that $P_{d_1}$ is different from $P_{d_2}$ for all values of the transit angle $\psi$ and for all values of the coupling parameter $p$, if $\sin \psi \neq 0$ and $p \neq 0$. But, since $P_{d_1}$ is different from $P_{d_2}$, $|V_1|$ is also different from $|V_2|$ if $\sin \psi \neq 0$ and $p \neq 0$. Therefore, the choice of $\psi$ determines the relationship between $|V_1|$ and $|V_2|$. In general, $|V_1|$ is not equal to $|V_2|$ even though the circuit itself is symmetric. For example, if one had chosen $\psi = 3\pi/2$, the individual power expressions of Eq. (3.19) would have to be interchanged as compared to the case when $\psi = \pi/2$. Similarly, the absolute gap voltage plots of Figs. 3.6 and 3.7 would have to be interchanged also. That is, if $\psi = 3\pi/2$, Fig. 3.6 would now give the absolute magnitude of the voltage across the second gap and Fig. 3.7 would give the absolute magnitude of the voltage across the first gap. Even though Eq. (3.23) is still satisfied if $\psi = 3\pi/2$ the approximation of Eq. (3.13b) by Eq. (3.12b) is, however, no more permissible.

3.8 Definition and Calculation of Bandwidth

For a symmetric power curve, the 3 db bandwidth (BW) is given with the help of Eq. (3.11)

$$BW = 2(\omega_{1/2} - \omega_0) = 2\alpha x_0.$$  \hspace{1cm} (3.33)

In Eq. (3.33), $\omega_{1/2}$ denotes the value of the radian frequency $\omega$ for which the power dissipation curve is down to one-half of its maximum value. Similarly, $x_0$ denotes the value of the normalized frequency variable $x$, for which the power dissipation curve is down to one-half of its maximum value. From now on, $x_0$ will be called the half-power point. Therefore,
for any $x_o$, the bandwidth becomes using Eqs. (3.25) and (3.26) and after normalizing to the bandwidth of one single-cavity system given by Eq. (A1.13),

$$BW_n = \frac{CV_o}{|I_1|} \frac{2x_o G_{\text{stop}}}{2C} = \frac{2x_o}{1 + p^2}$$  \hspace{1cm} (3.34)

if $p^2 \ll \sqrt{5} - 2$. But when $p^2 \gg \sqrt{5} - 2$, the normalized bandwidth is given by

$$BW_n = \frac{2x_o}{[2p \sqrt{4 + p^2} - 2p^2]^{1/2}}.$$  \hspace{1cm} (3.35)

The bandwidth given by Eqs. (3.34) and (3.35) was calculated for the power dissipation curve $P_{d2_n}$ of Fig. 3.4 and is given in Fig. 3.8 by the heavier, continuous curve. The continuous, light curve in Fig. 3.8, on the other hand, represents the efficiency $\eta_2$ in per cent. The efficiency, in this case, was defined as the amount of power dissipated in one particular load as compared to the maximum power that can be dissipated in one single-cavity system [Eq. (A1.12)]. Note that the total power dissipated in the symmetrically loaded DTDG cavity, i.e., the power dissipated in the first and the second cavities combined, is sometimes less than the maximum power dissipated in one single-cavity system (Fig. 3.5). This means that if $P_{d1_n} + P_{d2_n}$ is less than unity for some particular choice of $x$ and $p$, then $\eta_1 + \eta_2$ will add up to less than 100 per cent for the same values of $x$ and $p$ because of the above definition of $\eta$. It is worthwhile to point out once again that, under the above assumptions, and in particular for a choice of $\psi = \pi/2$, a DTDG cavity loaded symmetrically will dissipate more power in the second cavity than in the first cavity. For example, from Fig. 3.8 it follows that, at $p \approx 0.63$, the maximum power that can be dissipated in the second cavity of the symmetrically loaded DTDG cavity will be of the order of 92 per cent of the total power that can be dissipated in one single-cavity system [Eq. (A1.12)]. Therefore, if one, for example, connects a useful
load to the second cavity and connects a dummy load of the same value to the first cavity, and if one lets $\psi=\pi/2$ and $p = 0.63$, then the maximum useful power that can be extracted from the beam will be about 0.92 times the useful power that can be extracted from the beam using a single-cavity system. However, the bandwidth of the DTDG cavity system will be about 1.85 times the bandwidth of a single-cavity system. Note that in the above discussion no mention is made of the internal losses of the cavities. In a more accurate analysis, these losses must be taken into account.

In Fig. 3.8, the bandwidth and the efficiency $\eta_2$ was calculated and plotted for the power dissipation curves $P_{d2}$ of Fig. 3.4 only. It is clear that the same thing could have been done for $P_{d1}$, however, from Fig. 3.4 one can see that hardly ever will the useful load be connected to the first cavity and the dummy load be connected to the second cavity for the choice of the transit angle $\psi = \pi/2$, because of the very low power dissipation in the first cavity. For this reason, neither the bandwidth nor the efficiency $\eta_1$ were plotted for the power dissipation curves $P_{d1}$ of Fig. 3.4. However, for the sake of comparison only, the bandwidth and the efficiency of the total power dissipated in the symmetrically loaded ($G_1=G_2=G$) circuit of Fig. 3.1 were plotted in Fig. 3.8 in dashed curves. At this point it is worthwhile to mention that the power dissipation curves of both the individual and the combined cavities exhibit a double-hump characteristic for a sufficiently large value of the coupling parameter $p$. This means that, even though the half-power point $x_o$ of each power dissipation curve is increasing with increasing $p$, a point will soon be reached for which the power dissipated at $x=0$ of either of Fig. 3.4 or Fig. 3.5 will be one-half the maximum power dissipated for that particular
choice of p. The change in the ripple, defined here as r and representing
the ratio of the maximum of the power dissipation curve to its minimum at
x = 0, was calculated and plotted on the bandwidth curves of Fig. 3.8
for P_{d2_n} and P_{d_n}. The values of r along these curves are given in decibels.
Hence, when the power dissipation curve at x=0 is equal, for example, to
one-half of its maximum value for a particular choice of p, the value of the
ripple is marked as 3 db on the bandwidth curve for that particular value
of p. For the values of p less than that value of p for which r=0 db, the
power dissipation curves exhibit no ripple. The value of p for which r = 3db
for the total power dissipation curves P_{d_n} of Fig. 3.5 can be obtained from
Eqs. (3.30) and (3.31). One must have

\[
2 \left[ \frac{2p \sqrt{4 + p^2} - 2p^2}{(1 + p^2)} \right]^{1/2} = \frac{\sqrt{4 + p^2} - p}{2 \sqrt{2p} \left[ \sqrt{1 + p^2} - p \right]} \quad (3.36)
\]

From which it follows that p = 1.638. From Fig. 3.8 the maximum, approxi-
mate, theoretical 3 db bandwidth, for the case when the useful load is
connected to the second cavity, i.e., of P_{d2_n} of Fig. 3.4, is then about
three times the bandwidth of the single-cavity system. However, at this
value of bandwidth, the power that can be extracted from the beam is only
about 70 per cent of the power that can be extracted using one single-cavity
system.

3.9 Conclusion

This concludes the analysis of a capacitively coupled, symmetrically
loaded DTDG cavity. In this chapter it has been shown that a cavity of this
type can indeed have a larger bandwidth as compared to a single-cavity
system analyzed in Appendix 1 or to a double-tuned, single-gap cavity
system of Appendix 2.
The assumptions used in this analysis were the following. In Chap. 2 an equivalent circuit was derived which approximately represented the behavior of a capacitively coupled, symmetrically loaded DTDG cavity. The impedance functions, which were derived for the equivalent lumped-constant circuit, were valid only for high-Q and weak coupling. That is \( Q > 10 \) and \( Q_o C_c/C \lesssim 1 \). However, the assumption that will probably give the largest error was the assumption of the gap currents \( I_1 \) and \( I_2 \) as given by Eq. (3.12). That is, the effect of current remodulation at the first gap was completely neglected.

The individual power dissipation curves of the symmetric loads were calculated and plotted in Fig. 3.4 for different values of the coupling parameter \( p \). The power curves if Fig. 3.4 are normalized to the maximum power dissipated in one single-cavity system of Appendix 1, and are symmetric about the \( x = 0 \) point. The bandwidth was calculated for \( P_{d2n} \) curves of Fig. 3.4 only. The reason for this was the fact that the power dissipated in the second load, because of the choice of \( \psi = \pi/2 \), was much larger than the power dissipated in the first load. Therefore, it is logical that the useful load would always be connected to the second cavity for this particular choice of \( \psi \). The bandwidth curve of \( P_{d2} \) is given in Fig. 3.8 by the continuous, heavy curve. The continuous, light curve in Fig. 3.8 denotes the efficiency \( \eta_2 \) defined as the maximum power dissipated in the second cavity as compared to the maximum power dissipated in the one single-cavity system of Appendix 1. The little cross-lines on the bandwidth curve denote the ripple of \( P_{d2n} \) curves in decibels. The comparison of the bandwidth curve with the efficiency curve in Fig. 3.8 indicates clearly the advantage of a capacitively coupled, symmetrically loaded DTDG cavity.
FIG. 3.1

FIG. 3.2
FIG. 3.3
FIG. 3.5
FIG. 3.6

FIG. 3.7
CHAPTER 4

THE EFFECT OF ASYMMETRIC LOADING ON THE PERFORMANCE OF TWO CAPACITIVELY COUPLED RESONANT CAVITIES.

4.1 Impedance Functions for Undercoupled Cavities

When $G_1 \neq G_2$ in the circuit of Fig. 3.1, the locus of poles of the impedance functions can be found through calculation only. However, the case when the cavities are loaded asymmetrically is of greater practical value than a symmetrically loaded double-tuned double-gap (DTDG) cavity. Therefore, in order to derive the approximate expressions for the impedance functions, the locus of the poles of $Z_{11}$, $Z_{22}$, and $Z_{12}$ was studied in Appendix 4 for a few different values of $G_1$, $G_2$ and the $Q$ of the circuit. In order to simplify the analysis, it was assumed that

$$G_1 = G(1 + g)$$   \hspace{1cm} (4.1a)

and

$$G_2 = G(1 - g).$$   \hspace{1cm} (4.1b)

Figure A4.1 shows the locus of poles of the impedance functions for $g = 0$, $g = 0.5$ and $g = 1$ for a circuit with $Q = 20$. The numbers along the curves represent the values of the coupling coefficient $c$ defined as

$$\frac{C_c}{C} = c.$$   \hspace{1cm} (4.2)

The effect of the $Q$ of the circuit is shown in Figure A4.2.

As one can see from Figure A4.2, if the $Q$ of the circuit is higher than ten, the locus of poles of the impedance functions of an asymmetrically loaded DTDG cavity can be split up approximately into two distinct parts.

The separations of the complete locus of the impedance functions is shown in Figures 4.1 and 4.2. In Figure 4.1, the capacitive coupling is such that the poles of the impedance functions move approximately parallel to the real
frequency coordinate axis $\sigma'$. Resonant cavities with capacitive coupling such that the poles of the impedance functions are as shown in Figure 4.1 will be called undercoupled. Similarly, when the locus of the poles is as shown in Figure 4.2, the cavities will be said to be overcoupled.

Consider first the case when the cavities are undercoupled. From Figure 4.1 one has, with the help of Eq. (4.1),

$$a_1 = \frac{G_1}{2C} = a (1 + g) \quad (4.3a)$$

and

$$a_2 = a (1 - g), \quad (4.3b)$$

where

$$a = \frac{G}{2C}. \quad (4.4)$$

From Eq. (4.1) it follows that, when $g > 0$, the first cavity is loaded more heavily than the second and vice versa for $g < 0$. In Figure 4.1, $p$ is some coupling parameter, which depends nonlinearly on the coupling and the loading coefficients $c$ and $g$, respectively (Figure A4.1). However, the value of the parameter $p$, which will be determined later, is limited. From Figure 4.1, one can see that $p$ cannot be less than zero and that it cannot be greater than the value of $g$. Since the value of $g$ is also limited by Eq. (4.1), one can write for the coupling parameter $p$,

$$0 \leq p \leq \left| g \right| \leq 1. \quad (4.5)$$

In Figure 4.1, $P_1$ and $P_2$ denote the location of the poles of the impedance functions for some particular values of $c$ and $g$. In the same figure, $Z_1$ denotes the location of the zero of $Z_{11}$ for the same value of $g$, and $Z_2$ stands for the zero of $Z_{22}$. When the high $Q$ approximation is used for the zeropole distribution of Figure 4.1, one obtains from Eq. (3.3) the approximate expressions for the impedance functions for the case when the cavities
are undercoupled

\[ Z_{11} = \frac{(1 - g + jx) j/G}{1 - p^2 - x^2 + 2jx} \]  
\[ Z_{22} = \frac{(1 + g + jx) j/G}{1 - p^2 - x^2 + 2jx} \]  
(4.6a)

and

\[ Z_{12} = \frac{\left(\frac{(g^2 - p^2)^{1/2}}{1 - p^2 - x^2 + 2jx}\right) j/G}{1 - p^2 - x^2 + 2jx} \]  
(4.6b)

where \( x = \frac{\omega - \omega_0}{g} \)  
(4.7a)

and

\( (g^2 - p^2)^{1/2} = \frac{\omega_0 C_c}{G} = \frac{\omega_0 C}{G} \frac{C_c}{C} = Q_0 c \).  
(4.7b)

Note that, due to the definition of the coupling parameter \( p \) in Eq. (4.7b), \( p = 0 \) does not correspond to zero coupling as it did in the case of symmetric loading. Only when \( p = g \), one has no coupling between the cavities, i.e., \( Z_{12} = 0 \).

4.2 Normalized Power Expressions for the Undercoupled Cavities

With the impedance functions \( Z_{11}, Z_{22} \) and \( Z_{12} \) derived for the undercoupled cavities and given by Eq. (4.6), one can calculate the power dissipated in each individual load of the circuit of Figure 3.1 by using Eq. (3.18b).

Again, as in the case of the symmetric loading, the approximate expression for the induced gap currents \( I_1 \) and \( I_2 \), given by Eq. (3.12), will be used. Similarly, \( \Psi' \) will be taken to be equal to \( \pi/2 \). With these assumptions in mind, and using the expressions for the impedance functions given by Eq. (4.6), one has for the dissipated power in the individual loads of the circuit of Figure 3.1.
P_{d1} = \frac{G_1}{2} \left| V_1 \right|^2 = \frac{G(1 + g)}{2} \left| Z_{11} + j Z_{12} \right|^2 \left| I_1 \right|^2 \tag{4.8a}

= \frac{\left| I_1 \right|^2 (1 + g)}{2G} \left[ \frac{1 - g - (g^2 - p^2)^{1/2}}{(1 - p^2)^2} + 2 \frac{(1 + p^2) x^2 + x^4}{(1 + p^2)^2 + 2 (1 + p^2) x^2 + x^4} \right]^2 + x^2

and similarly

P_{d2} = \frac{\left| I_1 \right|^2 (1 + g)}{2G} \left[ \frac{1 + g + (g^2 - p^2)^{1/2}}{(1 - p^2)^2} + 2 \frac{(1 + p^2) x^2 + x^4}{(1 + p^2)^2 + 2 (1 + p^2) x^2 + x^4} \right]^2 + x^2 \tag{4.8b}

The total dissipated power in the circuit of Figure 3 is then given by the sum of Eqs. (4.8a) and (4.8b)

P_d = \frac{\left| I_1 \right|^2}{G} \frac{1 - p^2 + x^2}{(1 - p^2)^2 + 2 (1 + p^2) x^2 + x^4} \tag{4.8c}

The stopping condition for undercoupled cavities becomes, using Eq. (3.22) with \( \psi = \pi/2 \),

\[ V_o = \left| I_1 \right| \left| Z_{11} + Z_{22} \right| \text{max} = \frac{2 \left| I_1 \right| \left[ \frac{1 + x^2}{(1 - p^2)^2 + 2 (1 + p^2) x^2 + x^4} \right]^{1/2} \text{max}}{G} \ \tag{4.9} \]

Remembering that the value of \( p \) is limited by Eq. (4.5), one finds that the maximum of the expression in the brackets of Eq. (4.9) occurs at \( x = 0 \).

With this maximum value, one has

\[ G_{\text{stop}} = \frac{2 \left| I_1 \right|}{V_o (1 - p^2)} \tag{4.10} \]

With this value of \( G_{\text{stop}} \), the power expressions given by Eq. (4.8) become, after normalizing to the maximum power dissipated in one single-cavity system [Eq. (A1.12)],
and the total power dissipated in both loads is then given by the sum of Eqs. (4.11a) and (4.11b). If one now remembers that $g$ can assume any value such that

$$-1 \leq g \leq 1$$ (4.12)

and if the numerator of Eq. (4.11a) is compared with the numerator of Eq. (4.11b), one will notice that $P_{d2_{n}} \geq P_{d1_{n}}$ for all values of $p$ and identical asymmetric loading of the cavities. Note, that this was also the case for the symmetrically loaded network of Figure 3.1 ($G_{1} = G_{2} = G$). The reason for this fact, as it was explained in Chap. 3, was the choice of the transit angle $\Psi = \pi/2$.

Consider the power dissipated in the second load only, i.e., consider $P_{d2_{n}}$ of Eq. (4.11b) only. It can be shown (Appendix 5), that

$$P_{d2_{n}} \left|^{\text{max}}_{x=0} \right. = P_{d2_{n}} \left|^{\text{max}}_{x=0} \right. (4.13)$$

for all values of $g$ and $p$. So that one has for the maximum value of $P_{d2_{n}}$

$$P_{d2_{n}} \left|^{\text{max}} = \frac{(1 - g)}{2} \left[ 1 + g + \left( \frac{g^2 - p^2}{1 - p^2} \right)^{1/2} \right]^2 \right. (4.14)$$

This means that the power dissipated in the second load exhibits a single-hump characteristic with a maximum occurring at $x = 0$ and the magnitude given by Eq. (4.14).

4.3 Results for Complete Asymmetric Loading of Undercoupled Cavities with the Useful Load in the Second Cavity
The curve of the dissipated power given by Eq. (4.15), therefore, has a simple, single-hump shape. For this reason, no plot of Eq. (4.15) as a function of \( x \) and \( p \) is given here.

The half-power point is at

\[
x_o = \left[ \sqrt{1 - 2p^2 + 5p^4} - 2p^2 \right]^{1/2}.
\]

(4.17)

The bandwidth (BW) using Eq. (4.7a) becomes, after normalizing to the bandwidth of a single-cavity system [Eq. (A1.13)],

\[
BW_n = \frac{2}{(1 - p^2)} \left[ \sqrt{1 - 2p^2 + 5p^4} - 2p^2 \right]^{1/2}.
\]

(4.18)

Because of Eq. (4.5) and since in this case \( g = -1 \), \( p \) can assume any value from zero to unity. The bandwidth was calculated for different values of \( p \) and is given in Figure 4.3. From Figure 4.3 one can see that the theoretical
bandwidth is never larger than two. However, for small values of \( p \), the
bandwidth is very near equal to two times the bandwidth of a single-cavity
system. Note, that the bandwidth given by Fig. 4.3 was calculated for the
power dissipated in the second load of Fig. 3.1 with \( g = -1 \), i.e., \( G_1 = 0 \) and
\( G_2 = 2G \). The maximum value of the power dissipation curve was unity
[Eq. (4.16)] for all values of \( p \).

As a last step, one must investigate whether Eq. (3.23) is satisfied.

With the help of Eq. (4.8a) one has

\[
\left| V_1 \right|_{\text{max}} = \frac{|I_1|}{G_{\text{stop}}} \left[ \frac{(1 - g - (g^2 - p^2)^{1/2})^2 + x^2}{(1 - p^2)^2 + 2(1 + p^2)x^2 + x^4} \right]_{\text{max}}^{1/2},
\]

(4.19)

The expression in the numerator of Eq. (4.19), which is independent of \( x \), is
the greatest if \( g < 0 \) and \( |g| = p \). That is, one can write for Eq. (4.19),

\[
\left| V_1 \right|_{\text{max}} \leq \frac{|I_1|}{G_{\text{stop}}} \left[ \frac{(1 + p)^2 + x^2}{(1 - p^2)^2 + 2(1 + p^2)x^2 + x^4} \right]_{\text{max}}^{1/2},
\]

(4.20)

where, in Eq. (4.19), \( g \) was replaced by \(-p\). Using the procedure described
in Appendix 5, it can be shown that the maximum of the expression in the
brackets of Eq. (4.20) occurs only at \( x = 0 \). Hence one can write

\[
\left| V_1 \right|_{\text{max}} \leq \frac{(1 - p^2)}{2} \frac{V_o}{(1 - p^2)} \leq V_o,
\]

(4.21)

where Eq. (4.21) is true for all values of \( p \). This means that Eq. (3.23) is
indeed satisfied. Furthermore, when \( p \approx 0 \), \( BW_n \approx 2 \) and \( \left| V_1 \right|_{\text{max}} \approx V_o / 2 \).

At this point, it is interesting to note that if \( V_2 \) is investigated for the maxi-
mum value it can have, one will find, using Eq. (4.14), that at \( g = 1 \), \( p = 0 \)
and \( x = 0 \), \( \left| V_2 \right| = 3V_o / 2 \). \( \left| V_1 \right| \) for the same values of \( g \), \( p \) and \( x \) is equal to
\( V_o / 2 \). Despite the fact that \( \left| V_2 \right| = 3V_o / 2 \), no electrons will be returned to
the cathode because Eqs. (3.22) and (3.23) are satisfied. It can actually
be shown that when $|V_2| > V_o$, the voltage across the first gap is such that the electrons at the first gap are accelerated beyond the DC beam potential $V_o$, so that at no time does an electron fall through a combined potential [Eq. (3.22)] greater than $V_o$.

4.4 Impedance Functions for Overcoupled Cavities

When the capacitive coupling is such that the individual tank circuits of Figure 3.1 are overcoupled, i.e., the poles of the impedance functions are shown in Figure 4.2, one has for the impedance functions,

$$Z_{11} = \frac{(1 - g + jx)\, 1/G}{1 + p^2 - x^2 + 2\, jx}$$  \hspace{1cm} (4.22a)

$$Z_{22} = \frac{(1 + g + jx)1/G}{1 + p^2 - x^2 + 2\, jx}$$  \hspace{1cm} (4.22b)

and

$$Z_{12} = \left[\frac{(g^2 + p^2)^{1/2}}{1 + p^2 - x^2 + 2\, jx}\right] j/G,$$  \hspace{1cm} (4.22c)

where $(g^2 + p^2)^{1/2} = \frac{\omega C}{G} = cQ_o$  \hspace{1cm} (4.23)

and where $x$, $c$, $g$ and $a$ are given by Eqs. (4.7a), (4.2), (4.1) and (4.4), respectively. From Figure 4.2 one can see that, for this type of capacitive coupling, $g$ is still given by

$$|g| \leq 1$$  \hspace{1cm} (4.24)

but $p$ can assume any value from zero up. However, because of the fact that the impedance functions in Eq. (4.22) were derived using high $Q$ (i.e., $Q > 10$) and weak coupling (i.e., $p \approx 1$ or less) approximations, the value of $p$ cannot increase indefinitely. Fortunately, the most interesting range is when $p$ is small, i.e., $p \approx 1$ or less. Furthermore, from Eq. (4.22c) one can see that $Z_{12}$ is never zero when the overcoupled circuit of Figure 3.1 is loaded asymmetrically, i.e., when $G_1 \neq G_2$ or $g \neq 0$. 
4.5 Normalized Power Expressions for Overcoupled Cavities

With the impedance functions given by Eq. (4.22) and the approximate definition of the gap currents given by Eq. (3.12) one has for individual power expressions using Eqs. (3.18b) and (3.16) with \( \Psi = \pi/2 \),

\[
P_{d1} = \frac{G_1}{2} \left| V_1 \right|^2 = \frac{G(1 + g)}{2} \left| \frac{I_1}{2} \right|^2 \left| Z_{11} + jZ_{12} \right|^2
\]

\[
= \frac{(1 + g) \left| I_1 \right|^2}{2G} \left[ \frac{1 - g - (g^2 + p^2)^{1/2}}{1 + p^2} + \frac{1}{2} \right] + x^2
\]

\[
\frac{(1 + p^2)^2 + 2(1 - p^2)x^2 + x^4}{(1 + p^2)^2 + 2(1 - p^2)x^2 + x^4}
\]

(4.25a)

\[
P_{d2} = \frac{(1 - g) \left| I_1 \right|^2}{2G} \left[ \frac{1 + g + (g^2 + p^2)^{1/2}}{1 + p^2} + \frac{1}{2} \right] + x^2
\]

\[
\frac{(1 + p^2)^2 + 2(1 - p^2)x^2 + x^4}{(1 + p^2)^2 + 2(1 - p^2)x^2 + x^4}
\]

(4.25b)

and

\[
P_d = \frac{G \left| I_1 \right|^2}{2} \left[ \frac{1 + p^2 + x^2}{1 + p^2 + 2(1 - p^2)x^2 + x^4} \right]_{\text{max}}
\]

(4.25c)

The stopping conductance for this case becomes

\[
G_{stop} = \frac{2 \left| I_1 \right|^2}{V_o} \left[ \frac{1 + x^2}{1 + p^2 + 2(1 - p^2)x^2 + x^4} \right]_{\text{max}}^{1/2}
\]

(4.26)

Eq. (4.26) is identical with Eq. (3.24), so that the value of \( G_{stop} \), depending on the value of \( p \), is given either by Eq. (3.25) or by Eq. (3.26).

With these values of the stopping conductance, the power expressions of Eq. (4.25) finally become, after normalizing to the maximum power dissipated in one single-cavity system,

\[
P_{d1} = \frac{(1 + g)(1 + p^2) \left[ 1 - g - (g^2 + p^2)^{1/2}\right]^2 + x^2}{(1 + p^2)^2 + 2(1 - p^2)x^2 + x^4}
\]

(4.27a)
When \( p^2 \leq (\sqrt{5} - 2) \),

\[
P_{d2_n} = \frac{(1-g)(1+p^2)}{2} \left[ 1 + g + (g^2 + p^2)^{1/2} \right]^2 + x^2
\]

(4.27b)

and

\[
P_{d_n} = \frac{(1+p^2)}{(1+p^2)^2 + 2(1-p^2)x^2 + x^4}
\]

(4.27c)

if

\[
p^2 \leq (\sqrt{5} - 2)
\]

(4.27d)

When

\[
p^2 \geq (\sqrt{5} - 2)
\]

(4.28a)

\[
P_{d1_n} = (1+g) \left[ \frac{\sqrt{4+p^2} - p^2}{2} \right]^{1/2} \left[ 1 - g - (g^2 + p^2)^{1/2} \right]^2 + x^2
\]

(4.28b)

\[
P_{d2_n} = (1-g) \left[ \frac{\sqrt{4+p^2} - p^2}{2} \right]^{1/2} \left[ 1 + g + (g^2 + p^2)^{1/2} \right]^2 + x^2
\]

(4.28c)

and

\[
P_{d_n} = 2 \left[ \frac{\sqrt{4+p^2} - p^2}{2} \right]^{1/2} \frac{1+p^2 + x^2}{(1+p^2)^2 + 2(1-p^2)x^2 + x^4}
\]

(4.28d)

When Eqs. (4.27) and (4.28) are compared with Eqs. (3.27) and (3.28), one will notice that the power dissipated in individual loads is different if \( g \neq 0 \). But the total power dissipated in both loads combined is the same regardless of whether or not the loading is symmetric.

4.6 Useful Load in the Second Cavity

Consider first the power dissipated in the second load of the circuit of Figure 3.1, i.e., consider the expressions given by Eqs. (4.27b) and (4.28c). As pointed out previously, when \( g = 0 \), Eqs. (4.27b) and (4.28c) reduce to Eqs. (3.27b) and (3.28c), respectively. Note that Eqs. (3.27b) and (3.28c) were analyzed extensively in Chap. 3. Figure 3.4, for example, shows graphically the dependence of these equations on \( p \) and \( x \) (heavy curves). Figure 3.8 shows the bandwidth and the efficiency (continuous curves) for the power dissipation curves of Figure 3.4. When \( g = 1 \), i.e., no load in
the second cavity, the power dissipated in the second cavity is zero. When
$g = -1$, i.e., the first cavity contains no load, the power dissipated in the
second load can be obtained from Eqs. (4.27b) and (4.28c) by letting $g = -1$.
For this value of $g$, one will obtain expressions identical with the expres-
sions given by Eqs. (4.27c) and (4.28d), which give the total dissipated
power in both loads combined. This result is not surprising, because the
total dissipated power expressions given by Eqs. (4.27c) and (4.28d) do not
depend on $g$. Therefore, when the first cavity contains no load, i.e.,
$g = -1$, the expressions for the power dissipated in the second load must be
identical with the expressions for the total dissipated power as given by
Eqs. (4.27c) and (4.28d). When $g = +1$, i.e., the load is in the first cavity
only, $P_{d2_n}$ is zero for all values of $p$. When $g = 0$, i.e., the circuit is
loaded symmetrically, $P_{d2_n}$ is given by Eqs. (3.27b) and (3.28c) or in
Figure 3.4 as a function of $x$ and the parameter $p$ (heavy curves). The
continuous curves in Figure 3.8 show the corresponding bandwidth and effi-
ciency. When $g = -1$, i.e., the load is in the second cavity only, $P_{d2_n}$ is
given by Eqs. (3.27c) and (3.28d) or (4.27c) and (4.28d) or in Figure 3.5 as
a function of $x$ and the parameter $p$. Figure 3.8 gives bandwidth and effi-
ciency for this type of loading (dashed curves). When the loading is differ-
ent from the cases discussed above, the power dissipation curves given by
Eqs. (4.27b) and (4.28c) are also different from the curves of Figures 3.4
and 3.5. It would be a difficult and hardly a worthwhile job to calculate and
plot more curves of $P_{d2_n}$ for different values of $g$. However, the general
behavior of the power dissipation curves as a function of $g$ is known because
of the extensive analysis available for the three cases of loading: $g = -1$,
$g = 0$, and $g = +1$. When a power dissipation curve must be found for a dif-
ferent value of $g$, it is worthwhile to remember that the power dissipation
curve changes as a function of $g$ in two different ways. First of all, the over-all expression is multiplied by a factor which depends on $g$ and which will tend to change the amplitude of the over-all power dissipation curve. Second, the term in the numerator, which is independent of $x$, will change for varying $g$. This change will produce a variation in the power dissipation curve, which will be particularly noticeable at $x \approx 0$. Note that the shape of the double hump will be slightly affected. Therefore, in order to obtain a power dissipation curve $P_{d2}^{2n}$ for a value of $g$ different from 0, 1 or -1, first modify one of the available curves, which is closest to the required values of $g$ and $p$, at $x = 0$ (exactly) and near the point $x = 0$ (approximately). Note that for $x > 1$ the change will hardly be noticeable.

As a second step, reduce or increase the magnitude of the newly obtained curve by the $g$-dependent multiplying factor.

As a last step, one should investigate whether Eq. (3.23) is satisfied. With the impedance function of Eq. (4.22), one has for Eq. (3.23) using Eq. (4.25a),

$$|V_1| = \left[ \frac{2P_{d1}}{G(1+g)} \right]^{1/2} = \frac{|I_1|}{G} \left[ \frac{1-g-(g^2+p^2)^{1/2}}{(1+p^2)^2 + 2(1-p^2)x^2 + x^4} \right]^{1/2}.$$  \hspace{1cm} (4.29)

Comparing Eq. (4.29) with Eq. (4.19), one can write for $|V_1|$:

$$|V_1|_{\text{max}} \leq \frac{|I_1|}{G_{\text{stop}}} \left[ \frac{(1+p)^2 + x^2}{(1+p^2)^2 + 2(1-p^2)x^2 + x^4} \right]^{1/2} \leq \frac{|I_1| V_0}{G_{\text{stop}}} \left. \frac{P_{d2}^{2n}}{g=0} \right|_{\text{max}}^{1/2}.$$  \hspace{1cm} (4.30)

From Figure 3.4 one can see that $P_{d2}^{2n}$, for $g = 0$ (symmetric loading), is never larger than unity, so that Eq. (4.30) becomes
When \( p^2 \leq (\sqrt{5} - 2) \),
\[
|V_{1\text{max}}| \leq \left[ \frac{V_o}{2 (1 + p^2)} \right]^{1/2} < V_o .
\] (4.32a)

When \( p^2 \geq (\sqrt{5} - 2) \),
\[
|V_{1\text{max}}| \leq V_o \left[ \frac{p^4 + p^2 - p^2}{2} \right]^{1/4} \leq V_o .
\] (4.32b)

Therefore, Eq. (3.23) is satisfied.

4.7 Useful Load in the First Cavity

If one considers the power dissipated in the first cavity, one will obtain the following results. When \( g = -1 \), i.e., when there is no load in the first cavity, the power dissipated in the first cavity is zero for all values of the coupling. When \( g = 0 \), i.e., when the cavities are loaded symmetrically, then \( P_{d1} \) is given by Eqs. (3.27a) and (3.28b) or by Figure 3.4 (light curves). When \( g = 1 \), the power dissipated in the first cavity is given either by Eqs. (3.27c) and (3.28d) or by Eqs. (4.27c) and (4.28d) or by Figure 3.5. That is, the maximum power dissipated in the first cavity is equal to the maximum power dissipated in the second cavity if the cavities are loaded identically.

4.8 Summary

When the two capacitively coupled resonant cavities, the approximate equivalent circuit of which is shown in Figure 3.1, are loaded asymmetrically \( [G_1 \neq G_2] \), one can differentiate two types of couplings: the undercoupled and the overcoupled cases. On the basis of the analysis of Appendix 4, it was possible to split up the locus of poles of the impedance functions of
the asymmetrically loaded network of Figure 3.1 into two approximate, distinct cases as shown in Figures 4.1 and 4.2. When the coupling was such that the poles and zeroes were located as shown in Figure 4.1, the cavities were said to be undercoupled. Figure 4.2, on the other hand, shows the locus of poles and zeroes for the overcoupled cavities. On the basis of Figure A4.2, it was assumed that this separation into two distinct cases was valid for circuits with \( Q > 10 \).

The power expressions for the undercoupled cavities are given by Eq. (4.11). From Eq. (4.11) it was shown that the power dissipated in the second cavity is at least as large as the power dissipated in the first cavity for identical asymmetric loading. For this reason, only the analysis of the power dissipated in the second cavity was performed. It was found that the power dissipation curve \( P_{d2n} \) had a single-hump characteristic and that its maximum value was given by Eq. (4.14) for all values of \( g \) and \( p \), where the relationship between \( g \) and \( p \) was defined by Eq. (4.5). Further analysis of \( P_{d2n} \) was limited to one particular value of \( g \): \( g = -1 \), i.e., no load in the first cavity. For this value of \( g \), the maximum value of \( P_{d2n} \) curve was found to be unity for all values of \( p \). Figure 4.3 shows the normalized bandwidth as a function of \( p \). From Figure 4.3, one can see that the bandwidth of a DTDG cavity for this value of the loading coefficient \( g \) was about two times the bandwidth of a single-cavity system if \( p \approx 0 \). [Note that \( p = 0 \) does not mean that the cavities are decoupled \( (Z_{12} = 0) \), \( Z_{12} = 0 \) only if \( p = g \).] The analysis of the gap voltage across the first gap has shown that Eq. (3.23) is satisfied for all values of \( g \) and \( p \). Even though it was found that \( V_2 \leq \frac{3V_0}{2} \), no electrons will be returned to the cathode since Eqs. (3.22) and (3.23) are satisfied. Equation (3.22), which defines the stopping conductance \( G_{stop} \), shows that at no time is one single electron
slowed down by a potential greater than $V_o$ by traversing both gaps. Because of the choice of $\psi = \pi/2$, $|V_2|_{\text{max}} > |V_1|_{\text{max}}$; however, if $\psi$ were $3\pi/2$ the condition $|V_2|_{\text{max}} > |V_1|_{\text{max}}$ would have to be reversed. The effect of the choice of $\psi = \pi/2$ was discussed more carefully in Chap. 3.

When the individual tank circuits of Figure 3.1 were overcoupled, the normalized power expressions were found to be as given by Eqs. (4.27) and (4.28). It was interesting to note, that within the high Q and weak coupling approximations, from the point of view of the total power dissipated in both loads of Figure 3.1 combined, it did not matter how the cavities were loaded. Furthermore, it was shown that when $g = -1$, i.e., the load connected to the second cavity only, the power dissipated in the first cavity was zero and the power dissipated in the second cavity was equal to the total power dissipated in both cavities combined. Therefore, for this particular value of $g$, the normalized power dissipated in the second cavity is given either by Eqs. (3.27c) and (3.28d) or by Eqs. (4.27c) and (4.28d) or by the curves of Figure 3.5. The dashed curves of Figure 3.8 represent the corresponding bandwidth (heavy curve) and efficiency (light curve). When $g = 0$, i.e., when the cavities are loaded symmetrically, the dissipated power is given by Eqs. (3.27) and (3.28) or in Figure 3.4. The corresponding bandwidth and efficiency are again given by Figure 3.8, in which the continuous, heavy curve represents the bandwidth and the continuous, light curve represents efficiency. When $g = 1$, i.e., when there is no load in the second cavity, the power dissipated in the second cavity is now equal to zero and the power dissipated in the first cavity, its bandwidth and its efficiency are again given by the same expressions as for the case of power dissipated in the second cavity with $g = -1$. Even though the number of different loadings which were discussed was small ($g = -1$, $g = 0$ and $g = 1$),
the power dissipation curves for any other value of \( g \) can be obtained very easily. This can be done either through the use of the derived equations or through a simple approximation method from the available curves of Figures 3.4 or 3.5. This method was briefly discussed in Sec. 4.6. Note that in the analysis of this chapter the remodulation of the electron beam at the first gap was neglected.
FIG. 4.1

FIG. 4.2

FIG. 4.3
CHAPTER 5

THE EFFECT OF ELECTRON BEAM REMODULATION AT THE FIRST GAP

5.1 General Gap Voltage Equations

In the derivation of the results in Chaps. 3 and 4, it has been assumed that the induced gap current at the second gap differed from the induced gap current at the first gap by a phase angle $\psi$ only. See Eq. (3.12b), where $\psi$ was the transit angle between the cavities as shown in Fig. 2.1a.

In other words, the effect of the beam current remodulation at the first gap was completely neglected. However, Eq. (3.13b) gives an expression for the gap current at the second gap taking into account the first-order effect of the beam current remodulation at the first gap. Equation (3.13b) was obtained from Eq. (3.13a) with the assumption that the beam coupling coefficient at both gaps were nearly equal to unity. Equation (3.13a), in turn, was obtained by using small-signal, kinematic theory and the assumption of short and uniform gaps. Additionally it was assumed in the derivation of Eq. (3.13a) that the electrons arriving at the first gap had no velocity modulation.

Note that for a short transit angle $\psi$ ($\psi \approx \frac{\pi}{2}$), one did not have to take space charge into account. For convenience, Eq. (3.13b) is given here once again

$$I_2 = I_1 e^{-j\psi} (1 - j \frac{V_1 I_o \psi}{2N_0 I_1}) . \quad (5.1)$$

In Eq. (5.1), $I_1$ and $I_2$ are the induced gap currents at the respective gaps.

In the following analysis it will be assumed that the amplitude of the induced gap current at the first gap is approximately equal to the DC beam current, i.e.,

$$|I_1| \approx I_0 . \quad (5.2)$$
When Eq. (5.1) is introduced into Eq. (3.16) one obtains for the gap voltages

\[
V_1 = \frac{I_1}{\Delta} \left( Z_{11} - Z_{12} e^{-j\psi} \right) \tag{5.3a}
\]

\[
V_2 = \frac{I_1}{\Delta} \left[ Z_{12} - Z_{22} e^{-j\psi} + j \frac{I_0 \psi}{2V_o} e^{-j\psi} \left( Z_{11} Z_{22} - Z_{12}^2 \right) \right], \tag{5.3b}
\]

where

\[
\Delta = 1 - jZ_{12} \frac{I_0 \psi}{2V_o} e^{-j\psi}. \tag{5.4}
\]

From Eq. (5.3) one can see that if the cavities are uncoupled, \( Z_{12} = 0 \) and the voltage across the second gap is given by

\[
V_2 = -I_1 Z_{22} e^{-j\psi} \left( 1 - jZ_{11} \frac{I_0 \psi}{2V_o} \right). \tag{5.5}
\]

This expression for \( V_2 \) could have been obtained from Eq. (5.1) directly, remembering that when \( Z_{12} = 0 \), \( V_1 = Z_{11} I_1 \) and \( V_2 = -Z_{22} I_2 \). Equation (5.4) represents the feedback effect through the circuit coupling. Here, \( \Delta \) depends on the parameters of the beam, the transfer impedance \( Z_{12} \) and the transit angle \( \psi \). For a particular set of these variables, it is possible that \( \Delta \) will become zero. When \( \Delta = 0 \), a double-tuned, double-gap (DTDG) cavity will behave as a two-cavity klystron oscillator. The term \( Z_{11} Z_{22} - Z_{12}^2 \) of Eq. (5.3b) is never zero. If this term could become zero, Eq. (3.1b) would indicate that the cold circuit contained some gain. However, the impedance functions \( Z_{11}, Z_{22} \) and \( Z_{12} \) were derived for a passive and at least slightly lossy cavity system. Therefore, \( Z_{11} Z_{22} - Z_{12}^2 \) can never become zero.

### 5.2 General Power Expressions

The power dissipated in the asymmetrically loaded circuit of Fig. 3.1 is given by Eq. (3.18b)
When the new expressions for $V_1$ and $V_2$ of Eq. (5.3) are introduced into Eq. (5.6), the dissipated power in individual loads of the circuit of Fig. 3.1 becomes

$$P_{d1} = \frac{G_1}{2} \left| \frac{I_1}{\Delta_1} \right|^2 \left| Z_{11} - Z_{12} e^{-j\psi} \right|^2$$  \hspace{1cm} (5.7a)

and

$$P_{d2} = \frac{G_2}{2} \left| \frac{I_1}{\Delta_1} \right|^2 \left| Z_{12} - Z_{22} e^{-j\psi} + j \frac{I_{o1}}{2V_o} e^{-j\psi} (Z_{11} Z_{22} - Z_{12}^2) \right|^2,$$  \hspace{1cm} (5.7b)

where $\Delta$ is given by Eq. (5.4). For the circuit of Fig. 3.1, Eq. (5.7) becomes, with the approximate impedance functions for the overcoupled case

$$P_{d1} = \frac{(1 + g)}{2G} \left| \frac{I_1}{\Delta_1} \right|^2 \left\{ \left[ 1 - g - (g^2 + p^2)^{1/2} \sin \psi \right]^2 + \left[ x - (g^2 + p^2)^{1/2} \cos \psi \right]^2 \right\}$$  \hspace{1cm} (5.8a)

$$P_{d2} = \frac{(1 - g)}{2G} \left| \frac{I_1}{\Delta_1} \right|^2 \left\{ \left[ 1 + g + (g^2 + p^2)^{1/2} \sin \psi \right]^2 + \left[ x + (g^2 + p^2)^{1/2} \cos \psi \right]^2 \right\} \frac{I_{o1}}{2G V_o} + \frac{2(g^2 + p^2)^{1/2}}{2G V_o} \cos \psi - 2x \right\}.$$  \hspace{1cm} (5.8b)

Here

$$\Delta_1 = \left( 1 + p^2 \right)^2 + 2(1-p^2) x^2 + x^4 + \frac{I_{o1}}{2G V_o} (g^2 + p^2)^{1/2} \left[ \frac{I_{o1}}{2G V_o} (g^2 + p^2)^{1/2} + \frac{2(1 + p^2 - x^2) \cos \psi - 4x \sin \psi}{2G V_o} \right].$$  \hspace{1cm} (5.9)

When the effect of the beam current remodulation at the first gap is eliminated from Eq. (5.8), it reduces to Eqs. (4.25a) and (4.25b). As one can see, if Eq. (5.8) were used to find the power dissipated in the individual loads of the
asymmetrically loaded circuit of Fig. 3.1 (for example, as a function of the coupling parameter $p$ and the loading parameter $g$), the algebra would soon become impossible. In order not to complicate the analysis by a very large amount of algebra, in Chaps. 3 and 4 the behavior of a symmetrically and of an asymmetrically loaded DTDG cavity was analyzed, neglecting the beam current remodulation that normally takes place at the first gap. In particular, it was assumed that the beam current at the second gap differed from the beam current at the first gap by a phase angle $\psi$ only. One can now study the effect of the beam current remodulation at the first gap on the approximate results obtained in Chaps. 3 and 4. Probably the simplest way of doing this is by evaluating $P_{d1}$ and $P_{d2}$ of Eq. (5.8) for a few particular values of $p, g$ and $\psi$ and by comparing the obtained results with the approximate results of Chaps. 3 and 4. However, before doing this, one ought to derive a general expression for the stopping condition of Eq. (3.22).

5.3 **General Stopping Condition**

When the effect of the beam current remodulation at the first gap is included in the analysis, the stopping condition [Eq. (3.22)] becomes, using Eq. (5.3),

$$
\left| V_1 - V_2 e^{j\psi} \right|_{\text{max}} = \left| \frac{I_1}{\Delta} \right| \left[ Z_{11} + Z_{22} - 2Z_{12} \cos \psi - \frac{j I_0 \psi}{2V_o} (Z_{11} Z_{22} - Z_{12}^2) \right]_{\text{max}} = V_o.
$$

(5.10)

With Eq. (4.22), Eq. (5.10) becomes for the overcoupled case

$$
V_o = \frac{2 |I_1|}{G (\Delta_1)^{1/2}} \left\{ 1 + x^2 + (g^2 + p^2) \cos^2 \psi - 2x (g^2 + p^2)^{1/2} \cos \psi + \frac{I_0 \psi}{2GV_o} \left[ \frac{1}{4} - \frac{I_0 \psi}{2GV_o} + (g^2 + p^2)^{1/2} \cos \psi - x \right] \right\}^{1/2},
$$

(5.11)
where $\Delta_1$ is given by Eq. (5.9). However, as it was pointed out in Chap. 3, the use of Eq. (3.22) for the definition of the electron stopping conductance $G_{\text{stop}}$ imposes an additional requirement on the gap voltage across the first gap. This additional requirement was given by Eq. (3.23). That is, together with Eq. (5.11) one must have additionally

$$V_o \geq \left[ \frac{2P_d}{G(1+g)} \right]^{1/2}.$$  \hspace{1cm} (5.12)

The value of the electron stopping conductance can now be found from Eq. (5.11) (for example, through graphical interpolation), provided $g$, $p$ and $\psi$ are known.

5.4 Symmetric Loading with Zero Coupling and $\psi = \pi/2$

As it was pointed out previously, the general behavior of the capacitively coupled DTDG cavity is known from the approximate analysis of Chaps. 3 and 4. For example, in Chap. 3 the power dissipation curves for $p = 0$, and $\psi = \pi/2$ were calculated and plotted in Fig. 3.4. In order to find the effect of the beam current remodulation at the first gap, one can calculate the power dissipation curves from Eq. (5.8), using the above values of $p$, $g$ and $\psi$. With $p = 0$, $g = 0$ and $\psi = \pi/2$, Eq. (5.8) becomes

$$P_{d1} = \frac{|I_1|^2}{2G(1 + x^2)}.$$ \hspace{1cm} (5.13a)

and

$$P_{d2} = \frac{|I_1|^2}{2G(1 + x^2)^2} \left[ 1 + \frac{I_o}{4GV_o} \left( \frac{\pi}{2} - 2x \frac{I_o}{4GV_o} + x^2 \right) \right].$$ \hspace{1cm} (5.13b)

When the above values of $p$, $g$ and $\psi$ are introduced into Eq. (5.11), one obtains

$$V_o = \frac{2|I_1|}{G} \left[ 1 + \frac{1}{4} \left( \frac{I_o}{4GV_o} \right)^2 - \frac{I_o}{4GV_o} + x^2 \frac{I_o}{4GV_o} \right]^{1/2} \left[ \frac{\pi}{2} - 2x \frac{I_o}{4GV_o} + x^2 \right]^{1/2}. \hspace{1cm} (5.14)$$
However, instead of finding the maximum of the expression in Eq. (5.14) through differentiation, it is easier to do the following. Replace the G's of the expression in the brackets of Eq. (5.14) by the value of the stopping conductance found for the case of no current modulation at the first gap \[ \text{Eq. (3.25) with } p = 0 \]. Then calculate the new \( G_{\text{stop}} \) by finding the maximum of the expression in Eq. (5.14) graphically. (Note that only a few points are required to find the maximum.)

When \( p = 0, \ g = 0 \) and \( \psi = \pi/2 \), \( G_{\text{stop}} \) for the case of no current modulation at the first gap was \( 2 \left| I_1 \right| / V_0 \). With this value of \( G_{\text{stop}} \) and Eq. (5.2), one obtains from Eq. (5.14)

\[
G_{\text{stop}} = \frac{2 \left| I_1 \right|}{V_0} \left[ \frac{1.039 - 0.394x + x^2}{(1 + x^2)^2} \right]^{1/2} \max = 2.070 \frac{\left| I_1 \right|}{V_0}. \tag{5.15}
\]

Repeating the above procedure with the \( G_{\text{stop}} \) given by Eq. (5.15) instead of the previous value, one obtains \( G_{\text{stop}} = 2.066 \frac{\left| I_1 \right|}{V_0} \). The recalculated value of \( G_{\text{stop}} \) differs very little from \( G_{\text{stop}} \) given by Eq. (5.15). When the last value of \( G_{\text{stop}} \) is introduced into Eq. (5.13), the expressions for the dissipated power become after normalizing to the maximum power dissipated in a single-cavity system \[ \text{Eq. (Al.12)}, \]

\[
P_{d1} = \frac{.484}{1 + x^2} \tag{5.16a}
\]

and

\[
P_{d2} = \frac{.554 - .368x + .484x^2}{(1 + x^2)^2}. \tag{5.16b}
\]

The plot of Eqs. (5.16a) and (5.16b) is shown in Fig. 5.1a in light curves. The heavier curve in Fig. 5.1a represents the power dissipated in the first and in the second cavity, neglecting the electron beam remodulation at the first gap. From Fig. 5.1a one can see that the power dissipated in the first
cavity is now slightly smaller because the value of the stopping conductance has increased. However, $P_{d1n}$ is still symmetric because the cavities are decoupled. When the beam remodulation is considered in the analysis, the maximum power dissipated in the second cavity is about 0.6 times the power dissipated in one single-cavity system but its bandwidth is now only 1.75 times the bandwidth of the single-cavity system of Appendix 1. In the approximate analysis, it was found that the maximum of $P_{d2n}$ was 0.5 and $BW_n = 2$.

5.5 Symmetrical Loading with the Coupling Parameter $p = 0.486, 1, 1.638$ and $\psi = \pi/2$

When $p \neq 0$, the same procedure can be applied to find the power dissipation curves as was done in Sec. 5.4 for $p = 0$. However, now the algebra is slightly more complicated. Consider first the coupling for $p = 0.486$. From Fig. 3.5 one can see that this coupling is such that the maximum power dissipated in both cavities combined is still equal to the maximum power dissipated in the single-cavity system of Appendix 1. With $p = 0.486$, $g = 0$ and $\psi = \pi/2$, one obtains from Eq. (5.11) using the procedure described in Sec. 5.4, $G_{\text{stop}} = 1.667 \left| I_1 \right| / V_0$. With these values of $p$, $g$, $\psi$ and $G_{\text{stop}}$, Eq. (5.8) becomes after normalizing to the maximum power dissipated in one single-cavity system [Eq. (A1.12)]

$$P_{d1n} = \frac{0.60 \left( 0.264 + x^2 \right)}{1.580 - 0.916x + 1.528x^2 + x^4} \quad (5.17a)$$

and

$$P_{d2n} = \frac{0.60 \left( 2.430 - 0.942x + x^2 \right)}{1.580 - 0.916x + 1.528x^2 + x^4}. \quad (5.17b)$$

The plot of Eqs. (5.17a) and (5.17b) is given in Fig. 5.1b in light curves. Heavier curves represent the power dissipated in the first and second cavities, neglecting the electron beam remodulation at the first gap. Again,
the maximum of the power dissipated in the second cavity is larger by about 5.2 per cent than the approximate value found in Chap. 3. The new normalized bandwidth is now 1.77 times the bandwidth of a single-cavity system.

The approximate value was 1.87, a difference of only 5.4 per cent.

When \( p = 1, \ g = 0 \) and \( \psi = \pi/2 \), the stopping conductance is

\[
G_{\text{stop}} = 1.419 \left| \frac{I_1}{V_0} \right|.
\]

With these values of \( p, g, \psi \) and \( G_{\text{stop}} \), the normalized power expressions become

\[
P_{d1_n} = \frac{0.705x^2}{4.306 - 2.214x + x^4} \tag{5.18a}
\]

and

\[
P_{d2_n} = \frac{0.705 (4.306 - 1.107x + x^2)}{4.306 - 2.214x + x^4} \tag{5.18b}
\]

The plot of Eqs. (5.18a) and (5.18b) is shown in Fig. 5.1c in light curves. Heavier curves in Fig. 5.1c give the approximate values of \( P_{d1_n} \) and \( P_{d2_n} \) of Chap. 3. As one can see from Fig. 5.1c, when the electron beam remodulation at the first gap is taken into account, the power dissipation curves differ quite strongly from the approximate results in which the remodulation of the beam was neglected. Both curves, i.e., \( P_{d1_n} \) and \( P_{d2_n} \), exhibit a pronounced nonsymmetric characteristic. The maximum of the power dissipated in the second cavity is now about 17.2 per cent larger than the approximate value obtained in Chap. 3. The bandwidth is 2.12 times the bandwidth of the single-cavity system of Appendix 1, or a 9.1 per cent decrease in bandwidth as compared to the approximate value of Chap. 3. Note that, by using a different value of \( \psi \), the shape of the power dissipation curve \( P_{d2_n} \) of Fig. 5.1c can be made more symmetric. By doing this, one might also increase the bandwidth.

Consider one more coupling for the symmetric loading (\( g = 0 \)).
Let $p = 1.638$. This value of the coupling parameter $p$ corresponds to that total power dissipation curve of Fig. 3.5 for which the ripple at $x = 0$ is 3 db down from its maximum value. With $p = 1.638$, the value for the stopping conductance becomes from Eq. (5.11), $G_{\text{stop}} = 1.345 \left| I_1 \right| / V_o$. With these values of $g$, $p$, $G_{\text{stop}}$ and $\psi = \pi/2$, the power expressions of Eq. (5.8) become after normalizing to the maximum power dissipated in one single-cavity system [Eq. (Al.12)],

$$P_{d1} = \frac{0.743 \left(0.407 + x^2 \right)}{14.475 - 3.826x - 3.366x^2 + x^4}$$

and

$$P_{d2} = \frac{0.743 \left(7.300 - 1.168x + x^2 \right)}{14.475 - 3.826x - 3.366x^2 + x^4}$$

Figure 5.1d shows the plot of Eqs. (5.19a) and (5.19b) in light curves. The results for $p = 1.638$ are similar to those of $p = 1$. However, in this case the bandwidth is much smaller than the bandwidth obtained by using the assumption of no electron beam remodulation at the first gap. It is possible that a different choice of $\psi$ would give a better characteristic.

**5.6 Power Expressions for Different Values of $\psi$ with $p = 0.4$ and $g = \pm 0.9165$**

Consider the more important case when one of the cavities contains almost all the load. At first, let the coupling be weak ($p = 0.4$). Now let either the first cavity or the second cavity have almost all the load, i.e., let $g \approx 1$ and then let $g \approx -1$. The value of $g = 0.9165$ was chosen because $p^2 + g^2 = 1$. Therefore, $G_1 \approx 23 G_2$ or vice versa. With the above values of $p$ and $g$, Eqs. (5.11) and (5.8) become

$$G = \frac{2 \left| I_1 \right|}{V_o \left(2G \Delta \right)^{1/2}} \left[ 1 + \cos^2 \psi + x^2 - 2x \cos \psi + \frac{I_0 \psi}{2GV} \left( \frac{1}{4} \frac{I_0 \psi}{2GV} + \cos \psi - x \right) \right]^{1/2}$$

(5.20)
\[ P_{d1_n} = \frac{1.917 |I_1|}{GV_o \Delta_2} \left[ 1.007 - .167 \sin \psi - 2x \cos \psi + x^2 \right], \quad (5.21a) \]

\[ P_{d2_n} = \frac{0.084 |I_1|}{GV_o \Delta_2} \left[ 4.673 + 3.833 \sin \psi - 2x \cos \psi + x^2 + \frac{I_o \psi}{2GV_o} \left( \frac{I_o \psi}{2GV_o} + 2 \cos \psi - 2x \right) \right], \quad (5.21b) \]

\[ P_{d1_n} = \frac{0.084 |I_1|}{GV_o \Delta_2} \left[ 4.673 - 3.833 \sin \psi - 2x \cos \psi + x^2 \right], \quad (5.22a) \]

and

\[ P_{d2_n} = \frac{1.917 |I_1|}{GV_o \Delta_2} \left[ 1.007 + .167 \sin \psi - 2x \cos \psi + x^2 + \frac{I_o \psi}{2GV_o} \left( \frac{I_o \psi}{2GV_o} + 2 \cos \psi - 2x \right) \right], \quad (5.22b) \]

where

\[ \Delta_2 = 1.346 + 1.68x^2 + x^4 + \frac{I_o \psi}{2GV_o} \left[ \frac{I_o \psi}{2GV_o} + 2 \left( 1.16 - x^2 \right) \cos \psi - 4x \sin \psi \right]. \quad (5.23) \]

In Eq. (5.21) \( g = .9165 \), i.e., the first cavity has almost all the load. In Eq. (5.22) \( g = -.9165 \). Note that the stopping conductance is independent of which cavity is loaded more heavily.

Using the procedure of finding the stopping conductance described in Sec. 5.4, \( G_{stop} \) was found for four different values of \( \psi \). When \( \psi = 0.7 \pi/2 \), the stopping conductance is 1.817 \( |I_1|/V_o \). For \( \psi = 0.8 \pi/2, 0.9 \pi/2 \) and \( \pi/2 \), the stopping conductance was found to be 1.688, 1.766 and 1.922 times \( |I_1|/V_o \), respectively. The solid curves in Fig. 5.2 give the plot of Eqs. (5.21) and (5.22) for the chosen values of the transit angle \( \psi \) and their cor-
responding values of the stopping conductance. The dashed curves in Fig. 5.2 give the approximate power dissipation curves, neglecting the electron beam remodulation at the first gap with \( \psi = \pi/2 \) only. The effect of the transit angle \( \psi \) on the power dissipation curves is clearly indicated in Fig. 5.2. From Fig. 5.2a one can see that, if the load is almost entirely in the second cavity, the maximum power dissipated in the first cavity is less than 0.04 times the maximum of the power dissipated in the single-cavity system of Appendix 1. However, the power dissipation curve of the second cavity has almost a constant maximum value independent of the value of \( \psi \). Therefore, for the above values of \( p \) and \( g \), the maximum power dissipated in the second cavity is about equal to the maximum value of the power dissipation curve of one single-cavity system. From Fig. 5.2a, the maximum obtainable bandwidth occurs at \( \psi \approx 0.9 \pi/2 \) and its value is 2.2 times the bandwidth of one single-cavity system. Note that the approximate analysis (dashed curve of Fig. 5.2a) indicates that \( BW_n = 2.11 \) at \( \psi = \pi/2 \).

When the entire load is in the first cavity, the power dissipated in the first and second cavities is given by Fig. 5.2b. The approximate power dissipation curves of the two cavities are shown in dashed curves for \( \psi = \pi/2 \) only. Note that, in spite of the fact that there is almost no load in the second cavity, the power dissipated there is quite large, indicating existence of large voltages across the second gap. Observe also the strong effect of the transit angle \( \psi \) on the power dissipated in the first cavity, which indicates that, in spite of the fact that \( p \) is only 0.4, the effect of the coupling is quite pronounced. The reason for this, as it will be shown subsequently, is the very high voltages produced across the second gap.

Figure 5.2a can be used to find the amplitude of the voltages across
the first and second gaps for the case when the load is almost entirely in the second cavity. For this value of the loading, the maximum value of $|V_1|$ occurs at $\psi = \pi/2$ for all known values of $\psi$. This maximum value is

$$|V_1|_{\text{max}} = \left[ \frac{2 P_{d1}}{G(1+g)} \right]^{1/2} \approx 0.506 V_o . \quad (5.24)$$

The maximum value of $|V_2|$ occurs at $\psi = 0.8 \pi/2$. This value of $|V_2|$ is

$$|V_2|_{\text{max}} = \left[ \frac{2 P_{d2}}{G(1-g)} \right]^{1/2} \approx 0.556 V_o . \quad (5.25)$$

Figure 5.2b can be used in a similar way to find the maximum of the gap voltages for the case when the load is almost entirely in the first cavity. In this case, the maximum of $|V_1|$ occurs at $\psi = \pi/2$ and its value is $0.508 V_o$. The maximum of $|V_2|$ occurs at $\psi = 0.9 \pi/2$ and its value is $1.46 V_o$. Comparison of these two values shown that the amplitude of the voltages across the second gap is indeed very large. Therefore, in spite of the fact that the coupling is small ($p = 0.4$), the behavior of the second cavity is quite pronounced on the performance of the first cavity. This explains why the transit angle $\psi$ is so effective on the power dissipation curve of the first cavity and why these curves are so much different from the approximate curve of the power dissipated in the first cavity (shown by the dashed curve of Fig. 5.2b). Therefore, from the point of view of bandwidth and efficiency, i.e., from the maximum dissipated power point of view, the useful load should be connected to the second cavity. Then, if $g \approx 0.9165$ and the coupling is such that $p \approx 0.4$, the power dissipation curves will be shown in Fig. 5.2a. The bandwidth of the power curves of Fig. 5.2a is between 2.13 and 2.2 times the bandwidth of the single-cavity system of Appendix 1. The maximum normalized bandwidth occurs at $\psi = 0.9 \pi/2$ and its value is 2.2. Note that
when $\psi = 0.8 \pi/2$, $P_{d1n}$ of Fig. 5.2b has a normalized bandwidth of 2.78; however, its normalized maximum value is only 0.614. It will be shown subsequently that the same order of bandwidth can be obtained at much larger efficiency with heavier coupling.

5.7 Power Expressions for Different Values of $\psi$ with $p = 1$ and $g = \pm 1$

Consider one more value of the coupling. Let $p = 1$. However, in this case, assume that $g$ is equal to plus or minus unity, i.e., assume that either the second cavity or the first cavity has no dissipation whatsoever.

The value of the stopping conductance can now be calculated from Eq. (5.11) for the above values of $g$ and $p$. With $\psi = 0.7 \pi/2$, $0.8 \pi/2$, $0.9 \pi/2$, $\pi/2$, $1.1 \pi/2$ and $1.2 \pi/2$, the stopping conductance was found to be equal to 1.524, 1.360, 1.365, 1.557, 1.768 and 1.985 times $|I_1| V_o$, respectively. The individual power expressions for the above values of $g$, $p$, $\psi$ and their corresponding values of $G_{stop}$ become, after normalizing to the maximum of the power dissipated in the single-cavity system Eq. [(A1.1)],

$$P_{d1n} = \frac{2|I_1|}{GV_o \Delta_3} \left(2 - 2 \sqrt{2} x \cos \psi + x^2\right) = P_{d_n}$$

(5.26a)

and

$$P_{d2n} = 0,$$

(5.26b)

when $g = 1$. However, when $g = -1$, the normalized power expressions are

$$P_{d1n} = 0$$

(5.27a)

$$P_{d2n} = \frac{2|I_1|}{GV_o \Delta_3} \left[2 - 2 \sqrt{2} x \cos \psi + x^2 + \frac{I_0 \psi}{2GV_o} \left(\frac{I_0 \psi}{2GV_o} + 2 \sqrt{2} \cos \psi - 2x\right)\right] = P_{d_n}$$

(5.27b)

where
\[ \Delta_3 = 4 + x^4 + \frac{I_0 \psi}{2GV_o} \sqrt{2} \left[ 1 - \frac{I_0 \psi}{2GV_o} \sqrt{2 + 2(2-x^2) \cos \psi - 4x \sin \psi} \right] . \tag{5.28} \]

The plot of Eq. (5.27) for the above values of \( \psi \) and their corresponding values of the stopping conductance is shown in Fig. 5.3a in light curves. The heavier curve in Fig. 5.3a represents the approximate value of \( P_{d_{2n}} \) calculated, neglecting the electron beam remodulation at the first gap for \( \psi = \pi/2 \). Figure (5.3b) shows the plot of Eq. (5.26), where the heavier curve represents \( P_{d_{1n}} \) for \( \psi = \pi/2 \) as calculated approximately using the procedure of Chap. 4.

The results of Fig. 5.3 resemble the results obtained for the weaker coupling, i.e., for \( p = 0.4 \) of Fig. 5.2. Therefore, the analysis of Fig. 5.2 can be applied to Fig. 5.3 also. When \( \psi = 0.8 \pi/2 \), the normalized bandwidth of \( P_{d_{2n}} \) is 2.67. When \( \psi = 0.9 \pi/2 \), the normalized bandwidth of \( P_{d_{2n}} \) is 2.59. Note that, from the approximate analysis with \( \psi = \pi/2 \), the normalized bandwidth was given by Fig. 3.8 as 2.55. In Fig. 5.2a the maximum value of \( |V_1| \) is of the order of \( V_o/2 \) and the maximum value of \( |V_2| \) is of the order of 0.6\( V_o \). However, the results of Fig. 5.2b must be used with caution. The reason for this is the fact that all the results in this chapter were derived using the small-signal analysis, but the amplitude of the voltages produced across the second gap for the case when the load is in the first cavity exceeds the DC accelerating voltage by almost 50 per cent. Note that the expression for the definition of the stopping conductance given by Eq. (3.22) is always satisfied.

5. Summary

The results in this chapter were derived using the approximate expressions for the impedance functions of Chaps. 3 and 4. However, instead
of using the approximate definition of the gap current at the second gap given by Eq. (3.12b), a more accurate definition of $I_2$ given by Eq. (3.13b) or by Eq. (5.1) was used throughout this analysis. Note that $I_2$ of Eq. (5.1) was derived using the small-signal, kinematic theory and the assumption of short and uniform gaps. Furthermore, it was assumed that the coupling coefficients between the electron beam and either of the two gaps was equal to unity. In the analysis of this chapter, it was additionally assumed that the amplitude of the first harmonic of the bunched electron beam arriving at the first gap was equal to the DC beam current $I_0$. Therefore, since in general $|I_1| < I_0$, the effect of the electron beam remodulation at the first gap was slightly reduced [Eq. (5.1)].

The results of this chapter indicate that the approximate power dissipation curves for the second cavity are in fairly good agreement with the results obtained in this chapter if the coupling parameter $p$ is less than unity. However, for larger values of $p$, the power dissipation curves of the second cavity are still in good agreement, both from the point of view of maximum dissipated power as well as bandwidth, if the value of the transit angle is slightly reduced. For example, in Fig. 5.3a the approximate curve for $P_{d2n}$ with $\psi = \pi/2$ shown by the heavier curve is in fairly good agreement with the exact curve for $P_{d2n}$ with $\psi \approx 0.85 \pi/2$. However, the approximate and the exact power dissipation curves of the first cavity show a much larger discrepancy even for small values of the coupling parameter $p$. The primary reason is the fact that, if the load is almost entirely in the first cavity or even when the cavities are loaded symmetrically, the voltages produced across the second gap are much larger than voltages produced across the first gap. Therefore, the behavior of the second cavity is now much more effective on
the behavior of the first cavity.

On the basis of the analysis of this chapter, the external load must always be connected to the second cavity. When this requirement is fulfilled, the power dissipation curve of the external load will have a larger bandwidth at higher efficiency, i.e., the maximum RF power that will be extracted from the beam will be greater. Fortunately, as was pointed out before, the approximate results of Chaps. 3 and 4 derived for $\psi = \pi/2$ will then be in better agreement with the results obtained considering the electron beam remodulation at the first gap. Therefore, if a capacitively coupled DTDG cavity, the equivalent circuit of which is shown in Fig. 3.1, is built with the external load connected to the second cavity and if the transit angle $\psi$ is kept between $0.8 \pi/2$ and $0.9 \pi/2$, then the behavior of this composite cavity is determined almost directly by the results obtained in Chaps. 3 and 4. Note that the approximate behavior of the capacitively coupled circuit of Fig. 3.1 is identical with the behavior of the magnetically coupled circuit of Fig. 6.1 if the mutual inductance $M$ of Fig. 2.7 or of Fig. 2.5 is negative. (A small error between the two cases exists only in the approximate definition of the impedance functions.) When the mutual inductance $M$ is positive, the results require some modification. Note that a similar procedure can be applied for the analysis of a DTDG cavity with a larger value of the transit angle $\psi$ ($\pi/2 < \psi < 2\pi$).
FIG. 5.1
CHAPTER 6

BEHAVIOR OF TWO MAGNETICALLY COUPLED, D.T.D.G. CAVITIES

6.1 Derivation of the Impedance Functions for the Symmetric Loading

In the preceding three chapters the behavior of two capacitively coupled doubled-tuned, double gap (DTDG) resonant cavities was analyzed. In this chapter, similar analysis will be performed on two magnetically coupled resonant cavities of Fig. 2.1.

Figure 2.7 shows an equivalent circuit for two magnetically coupled cavities. Taking into account the possibility of asymmetric loading of the cavities and letting

\[ \frac{L^2 - M^2}{L} = L_o, \]  \hspace{1cm} (6.1)

the original circuit of Fig. 2.7 can be slightly changed. The best way to do this is by replacing the original \( L \) in the circuit by

\[ L = \frac{L_o}{1 - k^2}. \]  \hspace{1cm} (6.2a)

Here

\[ M = k \sqrt{LL} = kL. \]  \hspace{1cm} (6.2b)

Finally, the subscript \( o \) can be dropped from the new \( L_o \). The new circuit for magnetically coupled cavities is shown in Fig. 6.1.

As in the case of capacitively coupled cavities of Chap. 3, one has for the circuit of Fig. 6.1

\[ Z_{11} = \frac{s/C}{\Delta} (s^2 + \frac{G_2}{C} s + \frac{1}{CL}) \]  \hspace{1cm} (6.3a)

\[ Z_{22} = \frac{s/C}{\Delta} (s^2 + \frac{G_1}{C} s + \frac{1}{CL}) \]  \hspace{1cm} (6.3b)
and \( Z_{12} = \frac{s/L}{C^2} \) \( \Delta \) (6.3c)

Here \( \Delta \) is given by

\[
\Delta = s^4 + \left( \frac{G_1 + G_2}{C} \right) s^3 + \left( \frac{2}{CL} + \frac{G_1 G_2}{C^2} \right) s^2 + \left( \frac{G_1 + G_2}{C^2 L} \right)s + \frac{K}{C^2 L^2} ,
\]

where

\[ K = (1 - k^2) \] (6.5a)

and

\[ k = \frac{L}{L_C} . \] (6.5b)

Again, as in the case of capacitive coupling, the roots of Eq. (6.4a) can be written in closed form, only for particular values of \( G_1, G_2 \) and \( K \). Exact analytic solutions for any \( K \) exist only when \( G_1 = G_2 \). When \( G_1 \neq G_2 \), solutions exist in closed form if \( K = 1 \) (\( k = 0 \)) only.

Let \( G_1 = G_2 = G \), so that the circuit of Fig. 6.1 is symmetrically loaded. Here \( G \) may or may not contain external loading. By inspection of the open-circuit resonant frequencies of the symmetric circuit of Fig. 6.1, i.e., \( G_1 = G_2 = G \), one will obtain the roots of the poles of \( Z_{11} \) and \( Z_{12} \) as given by Eq. (3.5). However, in this case one has

\[ a_1 = \frac{G}{2C} = \alpha \] (6.6a)

\[ a_2 = \frac{G}{2C} = \alpha \] (6.6b)

\[ \omega_1^2 = \frac{1 - k}{CL} \] (6.6c)

and

\[ \omega_2^2 = \frac{1 + k}{CL} . \] (6.6d)

Two out of four poles of \( Z_{11} \) and \( Z_{12} \) are shown in Fig. 6.2. One can
see that, as $K$ is varied, these poles move parallel to the $j\omega$-axis, so that their real part remains constant. (The poles of the capacitively coupled circuit with $G_1 = G_2 = G$ lay on the $G_1 = G_2$ circle of Fig. 3.2.)

Note that for the weak coupling and the high-$Q$ approximations of the impedance functions of a capacitively coupled DTDG cavity of Chap. 3, one of the assumptions was that the poles lay on the line parallel to the $j\omega$-axis. With the help of Eq. (6.6) and the use of weak coupling and high-$Q$ approximations, the roots of Eq. (3.5) can be put into the following form:

\begin{equation}
 s_{1,4} \approx - \alpha \pm j(\omega_0 - p\alpha) \quad (6.7a)
\end{equation}

and

\begin{equation}
 s_{2,3} \approx - \alpha \pm j(\omega_0 + p\alpha) \quad (6.7b)
\end{equation}

where, in Eq. (6.7), one has defined

\begin{equation}
 \omega_0 \frac{2}{\alpha} = \frac{1}{LC} \quad (6.8a)
\end{equation}

and

\begin{equation}
 p = \frac{\omega_0}{2\alpha} \frac{L}{L_c} = Q_0 k \quad (6.8b)
\end{equation}

With the coordinates of poles of $Z_{11}$, $Z_{12}$ given by Eq. (6.7), one has approximately for the impedance functions of Eq. (6.3),

\begin{equation}
 Z_{11} = \frac{\frac{1}{G}(1 + jx)}{1 + \frac{p}{G} - x^2 + 2jx} \quad (6.9a)
\end{equation}

and

\begin{equation}
 Z_{12} = \frac{\frac{-j}{G} + \frac{p}{G}}{1 + \frac{p}{G} - x^2 + 2jx} \quad (6.9b)
\end{equation}

where $p$ and $x$ in Eq. (6.9) are as defined by Eqs. (6.8b) and (3.11),
respectively. The minus sign in Eq. (6.9b) corresponds to the positive value of the mutual inductance \( M \) as shown, for example, in Fig. 2.7. The positive sign, on the other hand, corresponds to the negative value of \( M \).

6.2 Power Relations for Symmetric Loading

Except for the two different signs of \( Z_{12} \) of Eq. (6.9b), the expressions for \( Z_{11} \) and \( Z_{12} \) of the magnetically coupled, symmetrically loaded DTDG cavity are identical with the corresponding expressions for capacitive coupling [Eq. (3.10)]. The two different signs of \( Z_{12} \) can be associated with the single power term \( p \) of Eq. (6.9b). From Eq. (3.24) one can see that the stopping conductance does not depend on the sign of \( Z_{12} \) since \( p \) in Eq. (3.24) appears to the even power only. However, the individual power expressions given by Eqs. (3.27) and (3.28) contain a single power term in \( p \). Therefore, when \( p \) is positive, i.e., \( M \) is negative, the individual power expressions for magnetic coupling are still given by Eqs. (3.27) and (3.28). On the other hand, when \( p \) is negative, i.e., \( M \) is positive, Eqs. (3.27) and (3.28) still apply for magnetic coupling, but the subscripts \( P_{d1n} \) and \( P_{d2n} \) in Eqs. (3.27) and (3.28) must now be interchanged. Equation (3.33) shows that the absolute magnitudes of the gap voltages should be handled in a similar way.

6.3 Results for Asymmetrically Loaded, Undercoupled Cavities

In Appendix 4, an analysis of the locus of poles of the impedance functions of the capacitively and magnetically coupled circuits of Figs. 3.1 and 6.1 respectively, was performed for a few particular values of the loading parameter \( g \) and the \( Q \) of the circuit. On the basis of this
analysis, the locus of poles of $Z_{11}$, $Z_{22}$ and $Z_{12}$ for the magnetically coupled circuit of Fig. 6.1 with some arbitrary value of $g$ and for increasing values of $K$ is shown approximately in Fig. 6.3. From Fig. 6.3 or again on the basis of the analysis of Appendix 4, if the $Q$ of the circuit is large ($Q > 10$), then the locus of poles of the impedance functions can be broken up into two distinct, approximate loci as shown in Figs. 4.1 and 4.2. As in the case of capacitive coupling, one can define two types of couplings: the undercoupled and the overcoupled cases.

Consider first the case when the magnetically coupled cavities are undercoupled, i.e., the locus of poles of the impedance functions is given by Fig. 4.1. For this case $Z_{11}$, $Z_{22}$ and $Z_{12}$ are again given by Eq. (4.6). However, in this case $Z_{12}$ has two possible signs associated with the two possible signs of the mutual inductance $M$ (Fig. 2.7), and the numerator of $Z_{12}$ is given by

$$\frac{2}{\omega} \frac{2}{\omega} \sqrt{\frac{1}{L_c \omega G}} = \frac{L}{L_c} \frac{\omega}{\omega} = k Q_o. \quad (6.10)$$

When $M$ of Fig. 2.7 is negative, the impedance functions of the magnetically undercoupled circuit of Fig. 6.1 are given identically by Eq. (4.6). Only in this case $p$ is given by Eq. (6.10). Therefore, when $M$ of Fig. 2.7 is negative and when the magnetically coupled circuit of Fig. 6.1 is undercoupled, then the behavior of this type of circuit is given by the results of Secs. 4.2 and 4.3. However, when $M$ of Fig. 2.7 is positive, $Z_{12}$ of Eq. (4.6c) will have a negative sign associated with it. Inspection of Eq. (4.9) indicates that the value of $G_{\text{stop}}$ remains unaffected by the change of the sign of $Z_{12}$. However, the normalized power expressions of Eq. (4.11) are slightly changed if $Z_{12}$ is negative. Analysis of
Eq. (4.11) shows that if \( Z_{12} \) is negative, i.e., the sign in front of the expressions \( (g^2 - p^2)^{1/2} \) is now changed, then the power dissipated in the first cavity is given by Eq. (4.11b) where \( g \) is now defined as

\[
G_1 = G (1 - g) \quad \text{(6.11a)}
\]

and

\[
G_2 = G (1 + g) \quad \text{(6.11b)}
\]

Similarly, when \( Z_{12} \) is negative, the power dissipated in the second cavity is given by Eq. (4.11a), where \( g \) is now defined by Eq. (6.11). The remaining results of Secs. 4.2 and 4.3 can be treated in a similar way.

However, it must be noted here that if \( Z_{12} \) is negative, \( |V_2|^\text{max} \) is given by Eq. (4.19) where \( g \) is defined by Eq. (6.11). Therefore \( |V_2|^\text{max} \) is now

\[
|V_2|^\text{max} \leq V_0, \quad \text{(6.12)}
\]

and \( |V_1|^\text{max} \) for negative \( Z_{12} \) is given by

\[
|V_1|^\text{max} = \frac{|I_1|}{G_\text{stop}} \left[ \left[ \frac{1 + g + (g^2 - p^2)^{1/2}}{(1-p^2)^2 + 2(1+p^2)x^2 + x^4} \right] \right]^{1/2} \max.
\]

Using the procedure described in Appendix 5, one can show that the maximum of the expression in the brackets of Eq. (6.13) occurs at \( x = 0 \) only, so that \( |V_1|^\text{max} \) becomes, with \( G_\text{stop} \) given by Eq. (4.10),

\[
|V_1|^\text{max} \leq \frac{V_0}{2} \left[ 1 + g + (g^2 - p^2)^{1/2} \right] \leq \frac{3V_0}{2}. \quad \text{(6.14)}
\]

This means that if \( Z_{12} \) is negative, the absolute amplitude of the voltages across the first gap may exceed the value of the DC accelerating potential \( V_0 \). In that case, electrons will be returned to the cathode at the first gap. For this reason, when the magnetically coupled circuit of Fig. 6.1 is undercoupled and when the magnetic coupling is such
that $Z_{12}$ is negative, then the values of $g$ and $p$ that can be used must be determined from Eq. (6.14) such that

$$|V_1| \leq V_o.$$  \hspace{1cm} (6.15)

For example, from Eq. (6.14) one can see that if $g \approx -1$, then $|V_1|_{\text{max}}$ is of the order of $V_o/2$ only. But in Eq. (6.11) $g=-1$ means that the first cavity is loaded more heavily. This means that if $Z_{12}$ is negative and the first cavity is loaded more heavily, $|V_1|_{\text{max}}$ will be smaller than $V_o$. Note that if $Z_{12}$ is negative a choice of $\psi = 3\pi/2$ will make the results of Secs. 4.2 and 4.3 directly applicable. However, from the point of view of the approximation of Eq. (3.13b) by Eq. (3.12b), the transit angle $\psi = 3\pi/2$ cannot be used without taking into account the electron beam remodulation at first gap.

6.4 Results for Asymmetrically Loaded, Overcoupled Cavities

When the magnetically coupled circuit of Fig. 6.1 is overcoupled so that the locus of poles is as shown in Fig. 4.2, the impedance functions $Z_{11}$, $Z_{22}$ and $Z_{12}$ are given by Eq. (4.22). However, here again, $-Z_{12}$ corresponds to the positive value of the mutual inductance $M$ of Fig. 2.7. The positive value of $Z_{12}$ corresponds to the negative value of $M$. Additionally, for magnetic coupling Eq. (4.23) is now given by

$$\left( g^2 + p^2 \right)^{1/2} = \frac{\omega_0}{Z_o} \frac{L}{L_c} = Q_o k$$  \hspace{1cm} (6.16)

Following the procedure used in the analysis of the magnetically undercoupled DTDG cavity of Sec. 6.3, one obtains the following results for magnetically overcoupled DTDG cavity. When the mutual inductance $M$ of Fig. 2.7 is negative, $Z_{12}$ is positive and except for the definition of $p$ given in Eq. (6.16) all the results of Secs. 4.5, 6, 7 and 8 can be
applied to the magnetically overcoupled DTDG cavity directly. When $M$ is positive, $Z_{12}$ is negative and the results of Secs. 4, 5, 6, 7 and 8 may be used after some modification. $P_{d1n}$ is now given by Eqs. (3.27b) and (3.28c) where $g$ is as defined by Eq. (6.11). Similarly, $P_{d2n}$ is given by Eqs. (3.27a) and (3.28b) with $g$ defined by Eq. (6.11). The remaining results can be used after similar modification. As a final remark, it must be pointed out that if the second cavity is loaded more heavily, then the absolute magnitude of the voltage across the first gap may be in excess of the DC accelerating potential $V_0$. Therefore, the loading of the second cavity for negative $Z_{12}$ must be done with caution. $|V_1|_{\text{max}}$ is now given by

$$|V_1|_{\text{max}} = \frac{I_1}{G_{\text{stop}}} \left[ \frac{1}{(1 + p)^2} + \frac{1/2}{2(1 - p)^2} \right] \frac{V_2}{\text{max}}, \quad (6.17)$$

where in Eq. (6.17) $g$ is defined by Eq. (6.11) and $G_{\text{stop}}$ is given by Eqs. (3.25c) and (3.26c).

**Summary**

In this chapter it was shown that within the high $Q$ and weak coupling approximations, the behavior of the magnetically coupled circuit of Fig. 6.1 can be described in terms of the results of Chaps. 3, 4 and 5 if the mutual inductance $M$ of Fig. 2.7 is negative. (Chaps. 3 and 4 describe the behavior of symmetrically and asymmetrically loaded circuit). The coupling parameter $p$ is now given by Eqs. (6.8b), (6.10) and (6.16). However, when the mutual inductance $M$ of Fig. 2.7 was positive, $Z_{12}$ was negative so that the results of Chaps. 3 and 4 could not be used directly. It was found, that when the circuit of Fig. 6.1 was loaded symmetrically, i.e., $G_1 = G_2 = G$, all one had to do to take
into account the negative value of $Z_{12}$ is to reverse the subscripts one and two on all of the results. When the circuit of Fig. 6.1 was loaded asymmetrically and $Z_{12}$ was negative, it was found that the subscripts one and two had to be reversed again, however, additionally a new loading parameter $g$ had to be defined by Eq. (6.11).

In this chapter, it was found that if $Z_{12}$ is negative and if the second cavity is loaded more heavily than the first one, the absolute amplitude of the voltages produced across the first gap may exceed the value of the DC accelerating potential $V_0$. Therefore, the loading of the second cavity must be done with caution, so that Eq. (3.23) is satisfied.
\[ L_c = \frac{L}{k} \]

**FIG. 6.1**

**FIG. 6.2**

**FIG. 6.3**
CHAPTER 7

THE BEHAVIOR OF TWO RESONANT CAVITIES COUPLED MAGNETICALLY THROUGH THE CONNECTING TRANSMISSION LINE

7.1 Magnetic Waveguide Coupling

In the preceding chapters the behavior of two resonant cavities, coupled capacitively through the interconnecting drift region and magnetically through a loop, was analyzed. In this chapter the behavior of a double-tuned, double gap (DTDG) cavity will be analyzed when the cavities are coupled magnetically either through a connecting waveguide or through a connecting coaxial line, as shown in Figure 2.2.

In Chap. 2, an equivalent circuit (Figure 2.9) was derived for the case when the two cavities were coupled magnetically through the waveguide.

Letting $M = k \sqrt{L L_o}$ \hspace{2cm} (7.1a)

and $K = L/L_o$ \hspace{2cm} (7.1b)

one has the new equivalent circuit shown in Figure 7.1. At this point, it is not obvious whether the weak coupling or the strong coupling, i.e., $k \ll 1$ or $k \ll 1$, will give the better results. When the useful load was included into the G's of the tank circuits as shown in Figure 3.1 or Figure 6.1, the weak coupling provided only a small detuning in the resonant frequencies of the composite resonant system. It was found that this resonant frequency detuning in terms of the coupling parameter $p$ then led to an increase in bandwidth without any significant change in the maximum power dissipated in the individual tank circuits. However, the magnitude of the current that flows through the coupling mechanism is negligibly small only if the coupling is very weak. When the coupling is such that $C/C_c$ or $L_c/L$ is of the order of the $Q$ of the circuit, the current through the coupling element
is comparable in amplitude with the induced gap currents \( I_1 \) and \( I_2 \).

In Figure 7.1, the load resistance is introduced in series with the coupling inductance. The presence of the load resistance in the coupling circuit, however, means that the impedance functions will now have an additional pole on the real axis of the complex frequency plane, as compared to the impedance functions of the networks of Figures 3.1 and 6.1. Similarly, \( Z_{11} \) will have a zero on the real axis. Therefore, the form of the impedance functions of the circuit of Figure 7.1 is complicated despite the fact that the circuit is symmetric. The \( k \) in the circuit of Figure 7.1 cannot be very small, for then the coupling inductance will be large. However, on the basis of the previous discussion, a very large coupling inductance means that the current through the coupling arm, and therefore through the load resistance, will be small. One is interested, therefore, in large values of \( k \), i.e., in values of \( k \) comparable to unity. For simplicity, in the following discussion \( k \) will be set approximately equal to unity, so that the coupling inductance of Figure 7.1 can be assumed to be equal to zero. The choice of \( k \approx 1 \) simplifies the analysis of the problem considerably. When \( k \neq 1 \), the results obtained for the case when \( k = 1 \) can still be applied approximately, as long as the impedance of the coupling inductance is small compared to the load resistance.

Let \( k \) be approximately equal to unity. Then an effective load conductance can be defined as

\[
G_L = \frac{k^2}{R_L K} \approx \frac{1}{R_L K} = pG, \tag{7.2}
\]

where in Eq. (7.2) \( G \) stands for the internal losses in the cavity. The new approximate equivalent circuit for two magnetically coupled cavities, where
the connecting waveguide provides the coupling, is shown in Figure 7.2.

For the circuit of Figure 7.2, the impedance functions can be written by inspection. They are:

\[
Z_{11} = \frac{s \left[ s^2 + 2 a (1 + p)s + \omega_o^2 \right]}{G(s^2 + 2as + \omega_o^2) \left[ s^2 + 2a(1 + 2p)s + \omega_o^2 \right]} \quad (7.3a)
\]

and

\[
Z_{12} = \frac{pG}{s^2/C^2} \frac{s^2/C^2}{(s^2 + 2as + \omega_o^2) \left[ s^2 + 2a(1 + 2p)s + \omega_o^2 \right]} \quad (7.3b)
\]

In Eq. (7.3) \( p \) is defined by Eq. (7.2),

\[
a = \frac{G}{2C} \quad (7.4a)
\]

and

\[
\omega_o^2 = \frac{1}{LC} \quad (7.4b)
\]

For a high \( Q \) approximation, the poles of the impedance functions of Eq. (7.3) and one zero of \( Z_{11} \) are shown in Figure 7.3 for the upper half-plane only. If now the normalized relative frequency variable \( x \) is defined as

\[
x = \frac{\omega - \omega_o}{a} \quad (7.5)
\]

one has approximately, for the impedance functions of Eq. (7.3),

\[
Z_{11} = \frac{(1 + p + jx) 1/G}{1 + 2p - x^2 + 2j (1 + p) x} \quad (7.6a)
\]

and

\[
Z_{12} = \frac{p/G}{1 + 2p - x^2 + 2j (1 + p) x} \quad (7.6b)
\]

With \( Z_{11} \) and \( Z_{12} \) given by Eq. (7.6), the dissipated power in the load conductance can be determined. It is

\[
P_{dL} = \frac{G_L}{2} \left| V_2 - V_1 \right|^2 = \frac{G_L}{2} \left| Z_{11} - Z_{12} \right|^2 \left| I_1 + I_2 \right|^2
\]

\[
= \frac{2p^2}{G_L} \left| I_1 \right|^2 (\cos \psi/2)^2 \frac{1 + x^2}{(1 + 2p + x^2)^2 + 4p^2 x^2} \quad (7.7)
\]
The maximum power will be dissipated in the load conductance \( G_L \) when \( \psi = 0 \) or \( \psi = 2\pi \). In view of the approximation of the induced gap current at the second gap given by Eq. (3.12b), it is better to let \( \psi \) be as near to zero as possible. Therefore, assuming that \( \psi \) in Eq. (7.7) is as close to zero as possible so that \( \cos^2 \frac{\psi}{2} \) is very near equal to unity, Eq. (7.7) becomes approximately

\[
P_{dL} = \frac{2p^2 |I_1|^2}{G_L} \frac{1 + x^2}{(1 + 2p)^2 + 2(1 + 2p + 2p^2)x^2 + x^4},
\]

whereas the power lost in both cavities combined becomes, with \( \psi \approx 0 \),

\[
P_{dC} = \frac{G}{2} \left[ |V_1|^2 + |V_2|^2 \right] = \frac{G |I_1|^2}{G_L} \frac{|Z_{11} - Z_{12}|^2}{1 + x^2} \frac{1}{(1 + 2p)^2 + 2(1 + 2p + 2p^2)x^2 + x^4}
= \frac{P_{dL}}{2p}.
\]

From Eq. (7.9) it can be seen that, by letting \( p \) become large, almost all of the power will be dissipated in the useful load. Therefore, the power dissipated in the useful load as compared to the power dissipated in the internal loads of both cavities combined is directly proportional to \( k \) [Eq. (7.2)]. This result is in agreement with the previous discussion of the coupling of the circuit of Figure 7.1.

When \( \psi \approx 0 \), the stopping conductance is given by Eq. (3.22) as

\[
\left| V_1 - V_2 e^{j\psi} \right|_{\text{max}} = 2 \left| I_1 \right| \left| Z_{11} - Z_{12} \right|_{\text{max}}
= \frac{2p |I_1|}{G_L} \frac{1 + x^2}{(1 + 2p)^2 + 2(1 + 2p + 2p^2)x^2 + x^4}^{1/2} = V_o.
\]

The maximum of the expression in the brackets of Eq. (7.10) occurs at \( x = 0 \). Therefore, the stopping conductance is given by

\[
(G_L)_{\text{stop}} = \frac{2p |I_1|}{V_o (1 + 2p)}.
\]
As one can see from Eq. (7.11), if $|I_1|$, $V_o$ and $G$ of the circuit of Figure 7.2 are given, the value of $G_L$ for which the electrons will be stopped is uniquely determined by Eqs. (7.11) and (7.2). Rewriting Eq. (7.11) so that $(G_L)_{\text{stop}}$ is expressed in terms of $|I_1|$, $V_o$ and $G$ only, one has

$$(G_L)_{\text{stop}} = \frac{|I_1|}{V_o} - \frac{G}{2} \quad . \quad (7.12)$$

In Appendix 1 it was found that the stopping conductance for a single-cavity system was equal to $|I_1| / V_o$ [Eq. (A1.10)]. Therefore, if the internal cavity losses represented by $G$ are small, the load conductance $G_L$ that can be connected to the system may be of the order of the stopping conductance for one single-cavity system [Eq. (A1.10)]. With the stopping conductance given by Eq. (7.11), $P_{dL}$ of Eq. (7.8) becomes, after normalizing to the maximum power dissipated in one single-cavity system [Eq. (A1.12)],

$$P_{dL} = \frac{2p(1 + x^2)(1 + 2p)}{(1 + 2p)^2 + 2(1 + 2p + 2p^2)x^2 + x^4} \quad . \quad (7.13)$$

The maximum power dissipated is then

$$P_{dL}\big|_{\text{max}} = \frac{2p}{1 + 2p} \quad . \quad (7.14)$$

This means that, for large values of $p$, the maximum power dissipated in the load conductance $G_L$ is approaching the maximum value of the power dissipated in one single-cavity system. Already for $p = 2$, $P_{dL}\big|_{\text{max}} = 0.8$.

The normalized bandwidth ($BW_n$) of the power curves of Eq. (7.13) is

$$BW_n = 2\alpha x_o \frac{CV_o}{|I_1|} = \frac{V_o}{|I_1|} G_{\text{stop}} (1 + 2p) = 2 \quad , \quad (7.15)$$

where in Eq. (7.15) $x_o$ is the half-power point, which in this case is equal to $(1 + 2p)$. Therefore, the bandwidth of this type of DTDG cavity is approximately twice the bandwidth of a single-tuned cavity system [Eq. (A1.13)]. Finally, it can be shown using Eqs. (7.7), (7.11) and (7.14) that Eq. (3.23) is satisfied.
7.2 Results for Magnetic Waveguide Coupling

In Sec. 7.1 it was shown that when a waveguide is used to couple two microwave cavities as shown in Figure 2.2a, the maximum power dissipated in the useful load is about the same as the maximum power dissipated in the load of a single-cavity system analyzed in Appendix 1. Furthermore, it was found that the bandwidth of the power dissipation curve is approximately twice the bandwidth of a single-cavity system. From the point of view of the maximum dissipated power, the \( k \) of the circuit of Figure 7.1, which is directly proportional to the mutual inductance between the cavities and the waveguide, must be of the order of unity. This means that the coupling of the waveguide to the cavities must be strong. Again, from the maximum dissipated power point of view, the transit angle between the two cavities was prescribed. It was found that if magnetic irises were used, the transit angle \( \psi \) must be as nearly zero as possible. Note that the results of Sec. 7.1 were derived using the high-Q approximation. Furthermore, in the above analysis the remodulation of the electron beam at the first gap was neglected.

7.3 Magnetic Coaxial Line Coupling

In Chap. 2, the equivalent circuit for this type of coupling was derived. Letting, in Figure 2.11,

\[
M = k \sqrt{LL_0} \quad (6.16)
\]

and

\[
K = \frac{L}{L_0} \quad (6.17)
\]

one has the simplified circuit of Figure 7.4. From this circuit it can be seen that when \( k \) is very small (weak coupling), one has two tank circuits which are coupled through a very small load resistance in parallel with a very small inductance. Even if the individual tank circuits were slightly detuned to provide a wide band characteristic, the amount of power dissipated in the
useful load would be very small so that a low efficiency DTDG cavity would result. One is not interested in this type of cavity.

When \( k \approx 1 \), one has the circuit shown in Figure 7.5. This circuit represents the equivalent circuit of a single cavity with the composite current equal to the difference of the induced gap currents \( I_1 \) and \( I_2 \). The analysis of this circuit is very simple if the conductance \( 2G \) representing the internal losses in the cavities is neglected and if the approximate definition of \( I_2 \) given by Eq. (3.12b) is used in the analysis.

Defining a new \( \alpha \),

\[
\alpha = \frac{G_L}{4C}
\]  
(7.18)

\[
x = \frac{\omega - \omega_0}{\alpha}
\]  
(7.19)

and

\[
\omega_0^2 = \frac{1}{LC},
\]  
(7.20)

one has for the driving point impedance,

\[
Z_{11} = \frac{1/G_L}{1+jx}.
\]  
(7.21)

The power dissipated in the load conductance \( G_L \) is given by

\[
P_{dL} = \frac{G_L}{2} \left| \frac{V_1}{Z_{11}} \right|^2 = \frac{G_L}{2} \left| Z_{11} \right|^2 \left| I_1 - I_2 e^{-j\psi} \right|^2
\]

\[
= \left| I_1 \right|^2 \frac{4 \sin^2 \frac{\psi}{2}}{2G_L (1 + x^2)}.
\]  
(7.22)

The stopping conductance is determined from Eq. (3.22)

\[
\left| \frac{V_1 - V_2 e^{j\psi}}{\max} \right| = \left| V_1 \right| \max \left| 1 - e^{j\psi} \right| = \left| I_1 \right| \left| Z_{11} \right| \left| 1 - e^{-j\psi} \right| \left| 1 - e^{j\psi} \right| = V_0.
\]  
(7.23a)
The stopping conductance is then

\[
(G_L)_{\text{stop}} = \frac{4|I_1|}{V_o} \left(\frac{(\sin \psi/2)^2}{1 + x^2} \right)^{1/2} \frac{1}{\max} = \frac{4|I_1|}{V_o} \left(\frac{(\sin \psi/2)^2}{1 + x^2} \right)^{1/2}. \tag{7.23b}
\]

With Eq. (7.23b) the power dissipated in the load conductance given by Eq. (7.22) becomes, after normalizing to the maximum power dissipated in one single-cavity system [Eq. (Al.12)],

\[
P_{dL_n} = \frac{1}{1 + x^2} \tag{7.24}
\]

and the bandwidth normalized to the bandwidth of one single-cavity system Eq. [(Al.13)] is

\[
B_{\text{W}_n} = \frac{G V_o}{|I_1|} = 2a x_o = 2 \sin^2 \psi/2. \tag{7.25}
\]

7.4 Results for Magnetic Coaxial Line Coupling

In Sec. 7.3 it was shown that when a coaxial line is used to couple two microwave cavities as shown in Figure 2.2b, the maximum power dissipated in the load is approximately equal to the maximum power dissipated in one single-cavity system. Note that, in the derivation of the power expressions, it was assumed that the conductance 2G representing the internal losses in the cavities was negligibly small as compared to the useful load conductance G_L. It is interesting to note that the power dissipated in the conductance G_L was independent of the transit angle \(\psi\). However, the bandwidth of the single-hump dissipation curve depends on \(\psi\). When \(\psi = \pi\), the bandwidth is two times the bandwidth of a single-cavity system of Appendix 1. Again, as in the case of the waveguide coupling, the coupling of the coaxial line and the cavities must be strong, i.e., \(k\) should be as near unity as possible. In the derivation of results of Sec. 7.3, high-Q approximation was used throughout, together with the approximate definition of the induced gap current at the second gap given by Eq. (3.12b).
FIG. 7.4

FIG. 7.5
8.1 Derivation of the Impedance Functions

In the previous chapters the behavior of two electromagnetically
coupled microwave cavities was analyzed. In this chapter a double-tuned,
double-gap (DTDG) cavity will be studied which has no electromagnetic
coupling between the individual cavities. However, it must be pointed out
that if such a DTDG cavity could be built, it would have limited application.
This is because complete electromagnetic decoupling over a fairly large
frequency range can be achieved only if the RF power from each individual
cavity is fed into two separate loads. Even if this requirement of delivering,
say, equal power at a fixed phase angle difference to different loads were
fulfilled, some weak electromagnetic coupling between the cavities would
probably exist because of the coupling of the fringing fields through the con-
necting drift tube, unless the tube is long. Note, that in Chaps. 3 and 4
the behavior of a DTDG cavity with exactly this type of coupling was ana-
alyzed. In the analysis of Chaps. 3 and 4 it was found that the value of the
coupling capacitance $C_c$, necessary to produce noticeable detuning between
the resonant frequencies of the individual cavities, was very small as com-
pared to the effective gap capacitance $C$ of each individual cavity. Neverthe-
less, assuming that there is no coupling between the cavities, one has for
this type of cavity the equivalent circuit as shown in Fig. 8.1. For the sim-
plicity of the analysis, let

$$\frac{G_1}{C_1} = \frac{G_2}{C_2} = 2\alpha$$

With this assumption, the poles of the impedance functions are given in
Fig. 8.2. The impedance functions themselves are then
With zero coupling, \( Z_{12} = 0 \). In Eq. (8.2), \( \omega_1^2 = 1/C_1L_1 \) and \( \omega_2^2 = 1/C_2L_2 \). With the high-Q and weak coupling approximations, one has for the frequency range of interest

\[
Z_{11} = \frac{1}{G_1} \frac{1}{1 + j(x - p)} \tag{8.3a}
\]

and

\[
Z_{22} = \frac{1}{G_2} \frac{1}{1 + j(x + p)} \tag{8.3b}
\]

where \( x \) is defined in Eq. (3.11), \( \omega_1 \approx \omega_o + p\alpha \) and \( \omega_2 \approx \omega_o - p\alpha \).

### 8.2 Analysis of the Power Expressions

With the impedance functions given by Eq. (8.3) and with the approximate definition of the induced gap currents given by Eq. (3.12), the power dissipated in individual cavities becomes

\[
P_{d1} = \frac{G_1}{2} \left| V_1 \right|^2 = \frac{|I_1|^2}{2G_1} \frac{1}{1 + (x - p)^2} \tag{8.4a}
\]

and similarly

\[
P_{d2} = \frac{|I_1|^2}{2G_2} \frac{1}{1 + (x + p)^2} \tag{8.4b}
\]

This means, that the power dissipation curves exhibit a single-hump characteristic and except for the normalized relative frequency displacement of \( \pm p \) are similar to the power dissipation curve of a single-cavity system [Eq. (A1.8)]. However, since the electron beam remodulation at the first gap was neglected, the magnitude of the induced gap current at the second gap is equal to the magnitude of the induced gap current \( I_1 \). This means,
that unless $G_1$ or $G_2$ is less than value of the stopping conductance for one single-cavity system [Eq. (A1.10)], the maximum power dissipated in each individual cavity of a decoupled DTDG cavity is less than the maximum power dissipated in one single-cavity system [Eq. (A1.12)]. However, it is unreasonable to expect that either $G_1$ or $G_2$ could be made smaller than the value of the stopping conductance for a single-cavity system [Eq. (A1.10)]. This can be shown to be true by calculating the stopping conductance. With Eqs. (4.1), (8.3), Eq. (3.22) becomes

$$\left| V_1 - V_2 e^{j\phi} \right|_{\text{max}} = \frac{2|I_1|}{G(1-g^2)} \left[ \frac{1 + (x + pg)^2}{(1 + p^2)^2 + 2(1 - p^2)x^2 + x^4} \right]^{1/2} = V_o .$$

(8.5)

From Eq. (8.5) it follows that

$$G_{\text{stop}} \geq \frac{2|I_1|}{V_o} \left[ \frac{1 + x^2}{(1 + p^2)^2 + 2(1 - p^2)x^2 + x^4} \right]_{\text{max}}^{1/2} .$$

(8.6)

Comparing Eq. (8.6) with Eq. (3.24) and analyzing the dependence of Eqs. (3.25) and (3.26) on $p$, one can see, that indeed, neither $G_1$ nor $G_2$ can be made smaller $|I_1|/V_o$. Therefore, the maximum of the power dissipated in each individual cavity of a decoupled DTDG cavity is always less than or equal to the maximum power dissipated in a single-cavity system. Similarly, the bandwidth of each individual cavity at a given value of the maximum dissipated power is less than or equal to the bandwidth of a single-cavity system. Therefore, the analysis, in which the electron beam remodulation at the first gap was neglected, shows that nothing is to be gained from the use of this type of composite cavity in which one single useful load must be connected to one of the electromagnetically decoupled cavities.

8.3 Results for Symmetric Loading

If the microwave power at large bandwidth must be delivered to two useful but separate loads, sometimes instead of using, for example, a
magic T, the loads may be connected to two separate cavities of a DTDG cavity. If the individual cavities additionally are coupled together, then the problem can be solved using the results obtained in the previous chapters with an arbitrary value of $\psi$. However, assuming again that there is no electromagnetic coupling either between the cavities or between the loads, the individual dissipated power expressions are again given by Eq. (8.4). At this point, it will be assumed that the maximum value of the power that will be dissipated in the separate loads should be the same even though the maximum value may occur at a different frequency. Therefore, introducing the condition of equal maximum power, one must have from Eq. (8.4), $G_1 = G_2 = G$. So that the power expressions of Eq. (8.6) now become:

\[
P_{d1} = \frac{|I_1|^2}{2G} \frac{1}{1 + (x-p)^2}
\]

\[
P_{d2} = \frac{|I_1|^2}{2G} \frac{1}{1 + (x+p)^2}
\]

and the total power dissipated in both loads combined is given by:

\[
P_d = \frac{|I_1|^2}{G} \frac{1 + p^2 + x^2}{(1+p^2)^2 + 2(1-p^2)x^2 + x^4}
\]

The stopping conductance for the symmetric loading is then given by Eq. (8.5) with $g = 0$. That is,

\[
G_{\text{stop}} = \frac{2|I_1|}{V_0} \left[ \frac{1 + x^2}{(1+p^2)^2 + 2(1-p^2)x^2 + x^4} \right]_{\text{max}}^{1/2}
\]

From Eq. (8.7) one can see that $P_{d1}$ is maximum when $x = p$. At this value of $x$, $P_{d1}$ is equal to $P_{d1}\text{max} = |I_1|^2/2G$. On the other hand, $P_{d2}$ for $x = p$ is only one fifth of this value. Comparison of Eq. (8.7c) with Eqs. (3.20) and (4.25c) shows that the total power dissipated in both cavities of
a symmetrically or an asymmetrically loaded DTDG cavity is the same regard-
less if the cavities are coupled electromagnetically or not. Note that
the expression for stopping conductance given by Eq. (8.8) is identical with
Eq. (3.24).

As a final step, one must determine if Eq. (3.23) is satisfied. With
\( Z_{11} \) given by Eq. (8.3a) and \( G_1 = G_2 = G \), one has
\[
|V_1| = \left| Z_{11} I_1 \right| = \left| \frac{I_1}{G_{\text{stop}}} \right| \left[ 1 + (x - p)^2 \right]^{-1/2}.
\] (8.9)

As it was pointed out before, \( G_{\text{stop}} \) is always larger than \( |I_1|/V_o \), so that
Eq. (8.9) can be written
\[
|V_1| \leq V_o \left[ 1 + (x - p)^2 \right]^{-1/2} \leq V_o.
\] (8.10)

8.4 Summary

In this chapter it was found that the behavior of one single cavity of a
DTDG, electromagnetically decoupled-cavity system in terms of maximum
dissipated power and bandwidth is worse than the behavior of a single-cavity
system analyzed in Appendix I. It was found that if the power is divided be-
tween two separate, useful loads, then the total power dissipated in both
loads combined is still the same regardless of the fact if the cavities are
coupled electromagnetically or not. However, the power dissipated in each
individual load as given by Eq. (8.7) does not have the same frequency de-
pendence if \( p \neq 0 \). For example, at \( x = p = 1 \), it was found that the value
of \( P_{dl} \) was five times the value of \( P_{d2} \).

The above stated conclusions were based on the results derived in this
chapter in which the electron beam remodulation at the first gap was neg-
lected entirely. Note that up to now no requirement was imposed on the
transit angle \( \psi \). However, in view of the approximation of Eq. (3.13b) by
Eq. (3.12b), the results of this chapter will be in better agreement with the
actual behavior of this DTDG cavity provided that the transit angle $\psi$ is kept
small, i.e., $\psi \leq \pi/2$. Now, in order that no electromagnetic coupling
will exist between the cavities, the use of grids seems to be necessary. As
a final remark, it must be pointed out that the value of $\psi$ might be pre-
scribed to a certain extent by the phase relationship of the electromagnetic
waves arriving at the two separate loads.

In conclusion, one can say that the use of electromagnetically de-
coupled cavities will not only present difficulties in the construction of such
a system for the reasons discussed at the beginning of this chapter, but the
behavior of the resulting system will be worse than the behavior of an
electromagnetically coupled DTDG cavity system.
FIG. 8.1

FIG. 8.2
CHAPTER 9

SUMMARY

9.1 Method of Analysis

In this paper the behavior of different types of output cavities of a klystron amplifier was analyzed. The analysis of the problem consisted first of devising some suitable microwave cavity systems that could be used as a broadband output cavity of a klystron amplifier. Some of these cavity systems and their corresponding lumped-constant equivalent circuits are shown in Chap. 2. In the second part of the analysis the behavior of these equivalent circuits was investigated.

In order to simplify the analysis of the equivalent circuits, impedance functions were derived by approximating the location of poles and zeros of the exact impedance functions in the complex frequency plane. The results of Appendixes 3 and 4 show that as long as the Q of the circuit is higher than ten, only the general shape of the power dissipation curve will be affected if the approximate expressions for the impedance functions were used. However, from the bandwidth and from the maximum dissipated power point of view, the results exhibit a negligible error. Another very important approximation used in the analysis was the assumption that the beam current arriving at the second gap of a double-gap cavity system differed from the beam current at the first gap by a phase angle \( \psi \) only. However, in Chap. 5, the effect of electron beam remodulation at the first gap on the approximate results of Chaps. 3 and 4 was considered.

In Appendix 1, a single-tuned cavity system, the simplest known
output cavity, was analyzed. The results of this analysis in terms of
the maximum dissipated power in the load and bandwidth, as given by
Eqs. (A1.12) and (A1.13), respectively, were derived using the maximum
efficiency condition. That is, the load conductance $G$ of Fig. A1.2
was chosen as that conductance for which the electron beam was stopped
at some frequency. This conductance was then called the stopping con-
ductance. The results for all other cavity systems are given (normalized)
in terms of the maximum dissipated power and bandwidth of this single-
cavity system.

9.2 DTSG Cavity System as an Output Cavity

In Appendix 2 a double-tuned, single-gap cavity system was an-
alyzed as an output cavity. The equivalent circuit of a magnetically
coupled cavity of this type is shown in Fig. A2.1. When the cavities
are undercoupled and Eq. (A2.5) is satisfied, the individual power ex-
pressions are given by Eqs. (A2.7) and (A2.8). However, when
Eq. (A2.5) is not satisfied, the individual power expressions are given
by Eqs. (A2.9) and (A2.10). When the cavities are overcoupled, the
results are more complicated, as shown in Sec. A2.2. For the most in-
teresting case, i.e., for the case when the load is almost entirely in the
first cavity, the power dissipation curve goes to zero at resonance; ob-
viously a very undesirable effect. However, when the load is connected to
the external cavity, so that approximately all the load is in the second
cavity, the maximum value of the normalized power dissipation curve and
its normalized bandwidth are plotted in Fig. A2.2 for both the undercoupled
and the overcoupled cases. It is important to note that the power dissipa-
tion curve has a single-hump characteristic for all values of the coupling
to the left of the 0-db ripple point. To the right of this point, the power
dissipation curve has a double-hump characteristic. Therefore, the maxi-
mum possible bandwidth at the highest efficiency for this type of an output
cavity is slightly less than two times the bandwidth of a single-cavity
system.

9.3 Capacitively Coupled DTDG Cavity System as an Output Cavity

The equivalent circuit of a capacitively coupled, double-tuned,
double-gap cavity, in which the electric coupling of individual cavities is
achieved through the coupling of fringing fields in the short interconnecting
drift tube, is shown in Fig. 3.1. When the cavities are loaded symmetri-
cally, i.e., when \( G_1 = G_2 = G \) of the circuit of Fig. 3.1, the individual
power curves are given by Fig. 3.4 for \( x > 0 \). The symmetric left-hand
side of the power curves is not shown. The normalized bandwidth and the
efficiency of the power dissipation curves of the second cavity are shown in
Fig. 3.8 by continuous curves as a function of \( p \). The total power dissipa-
tion curves of both cavities combined are shown in Fig. 3.5. Their normal-
ized bandwidth and efficiency are given in Fig. 3.8 by the dashed curves.

When the cavities are asymmetrically loaded and they are under-
coupled, the normalized power expressions are given by Eq. (4.11).
However, for asymmetrically loaded, overcoupled cavities the normalized
power expressions are given by Eqs. (4.27) and (4.28). It is interesting to
note that the total power dissipated in both cavities combined is the same
regardless of whether or not the cavities are loaded symmetrically and is
given by Fig. 3.5. Therefore, if either the first cavity or the second
cavity has all the load, the normalized power curves are given by Fig. 3.5
and their normalized bandwidth and efficiency are given in Fig. 3.8 by the
dashed curves.

The approximate analysis shows, therefore, that when a capacitively coupled, DTDG cavity is loaded in such a way that all the load is in one single cavity only, then the normalized bandwidth of this type of cavity is always larger than two. Furthermore, the maximum of the normalized power expression is never smaller than 0.94.

In Chap. 5, the effect of the electron beam remodulation at the first gap was taken into account using small-signal, kinematic theory and the assumption of short and uniform gaps. It was found that from the maximum dissipated power point of view and at the same time from the maximum bandwidth point of view, the second cavity must be loaded more heavily. Fortunately, the obtained results using the effect of the electron beam remodulation at the first gap and the approximate results of Chaps. 3 and 4 are in better agreement when the external load is connected to the second cavity. However, as one can see from Figs. 5.1, 5.2 and 5.3, if the transit angle $\psi$ is made somewhere between $0.8 \pi/2$ and $0.9 \pi/2$, the results of Chap. 5 are in fairly good agreement with the approximate results of Chaps. 3 and 4 calculated for $\psi = \pi/2$. Therefore, except for the condition of loading the second cavity more heavily and except for the required reduction of the transit angle $\psi$ from the value used in the approximate analysis, the results obtained neglecting the electron beam remodulation at the first gap give fairly accurate results. The results obtained in Chap. 5 for the case when the first cavity is loaded more heavily must be used with caution since the amplitude of the voltages across the second gap at times exceeds the DC accelerating voltage by almost 50 per cent.

9.4 Magnetically Coupled DTDG Cavity System as an Output Cavity

The equivalent circuit of a magnetically coupled, double-tuned,
double-gap cavity is shown in Fig. 6.1. Except for a different definition of
the coupling parameter \( p \), the approximate impedance functions of a mag-
netically coupled DTDG cavity are identical with the approximate impedance
functions of a capacitively coupled DTDG cavity, the behavior of which was
summarized in Sec. 9.3. Note that there is a small difference in the locus
of poles of the exact impedance functions of the circuits of Figs. 3.1 and 6.1
which does not appear in the approximate impedance expressions. However,
depending on the sign of the mutual inductance \( M \) of Figs. 2.5 or 2.7, the
transfer impedance can now be either positive or negative. When \( M \) is nega-
tive, \( Z_{12} \) is positive and the summary of the results of Sec. 9.3 applies
directly to the magnetically coupled DTDG cavity system. On the other
hand, when \( M \) is positive, the approximate behavior of a magnetically
coupled, DTDG cavity can be obtained by modifying the results of Chaps.
3 and 4 as described in Chap. 6. However, when the electron beam remodu-
lation at the first gap is taken into account, the results for a magnetically
coupled DTDG cavity system with a positive \( M \) can be obtained from the re-
results of Chap. 5 either by changing the sign of \( Z_{12} \) directly or, for example,
by changing the sign in front of the term \( (g^2 + p^2)^{1/2} \) in Eqs. (5.8), (5.9)
and (5.11).

9.5 Transmission Line Coupled DTDG Cavity as an Output Cavity

The equivalent circuit for the waveguide coupling of Fig. 2.2a, when
the coupling is strong, is shown in Fig. 7.2. By using the high \( Q \) approxi-
mation and neglecting the remodulation of the electron beam at the first gap,
it was found that the maximum of the power dissipated in the external load
occurs for \( \psi = 0 \). For a transit angle \( \psi \) very near equal to zero, this maxi-
mum value of the dissipated power is approximately equal to the maximum
of the power dissipated in a single-tuned cavity system. However, the bandwidth of this type of DTDG cavity system is approximately twice the bandwidth of a single-cavity system. Similar results were obtained for a strong coaxial line coupling of resonant cavities shown in Fig. 2.2b. The equivalent circuit for the above coupling is shown in Fig. 7.5. Note that for optimum bandwidth $\psi$ now has to be as near to $\pi$ as possible. The effect of electron beam remodulation on the approximate results for the waveguide and coaxial line couplings was not determined.

9.6 Two Uncoupled Resonant Cavities as an Output Cavity

The last composite cavity system to be analyzed was the output cavity consisting of two uncoupled cavities. The equivalent circuit of such an output cavity system is shown in Fig. 8.1. Without mentioning the construction difficulty and the usefulness of this type of an output cavity, two problems that were extensively discussed in Chap. 8, it was found that the bandwidth and the maximum dissipated power of each individual cavity was less than that of the single-tuned cavity system of Appendix 1. Additionally, it was found that the total power dissipated in both cavities combined was still the same, whether the cavities were coupled electromagnetically or not. However, the power dissipated in each cavity [Eq. (8.11)] did not have the same frequency dependence. For example, it was found that, at $x = p = 1$, the value of $P_{d1}$ is five times the value of $P_{d2}$ for symmetric loading of cavities. Therefore, the behavior of this composite output cavity is worse than either that of a capacitively coupled or a magnetically coupled DTDG cavity system. The effect of the electron beam remodulation on the approximate results of Chapter 8 was not considered.
APPENDIX 1

ANALYSIS OF A SINGLE-CAVITY SYSTEM

Al. 1  **Driving Point Impedance for a High-Q Circuit**

Figure A1.2 shows an equivalent circuit of the single cavity shown in Fig. A1.1. As one can see, two of these cavities coupled together comprise the composite double-tuned, double-gap cavity of Figs. 2.1 and 2.2. In the gap region, the field configuration is that of a distorted $\text{TM}_{010}$ cylindrical mode, whereas in the remaining cavity region the field configuration looks somewhat like a deformed TEM mode.

For the circuit of Fig. A1.2, the driving point impedance $Z_{11}$ is

$$Z_{11} = \frac{s/C}{s^2 + 2\alpha s + \omega_0^2} \quad \text{(A1.1)}$$

where

$$\alpha = \frac{G}{2C} \quad \text{(A1.2)}$$

and

$$\omega_0^2 = \frac{1}{LC} \quad \text{(A1.3)}$$

The coordinates of poles of $Z_{11}$ are shown in Fig. A1.3. They are

$$s_{1,2} = -\alpha \pm j\omega_1 \quad \text{(A1.4)}$$

where $s_1$ and $s_2$ are the complex natural frequencies. For a high-$Q$
circuit, i.e., for \( Q = \omega_0/2a > 10 \), the driving point impedance \( Z_{11} \) becomes approximately

\[
Z_{11}(j\omega) = \frac{j\omega_0/C}{[a + j(\omega - \omega_0)]} 2j\omega_0
\]

Letting \( x = \frac{\omega - \omega_0}{a} = 2Q \left[ \frac{\omega}{\omega_0} - 1 \right] \),

the approximate driving point impedance of Eq. (Al. 5) can be rewritten

\[
Z_{11}(x) = \frac{1/G}{1 + jx}
\]

Equation (Al. 7) is a good approximation to the actual \( Z_{11} \) if \( Q > 10 \) and if \( x \) is not much larger than unity. The exact expression for \( Z_{11} \) is somewhat complicated and will be derived in Sec. Al. 3. The form of this exact expression for \( Z_{11} \) will be such that the approximate results obtained, using for \( Z_{11} \) the expression given by Eq. (Al. 7), could be easily corrected for low-\( Q \) circuits and large \( x \) variations.

Al. 2 Electron Stopping Conductance, Power and Bandwidth Expressions

The dissipated power in the load conductance \( G \) as a function of the normalized relative frequency variable \( x \) is

\[
P_d(x) = \frac{G}{2} \left| V_1 \right|^2 = \frac{\left| I_1 \right|^2}{2G(1 + x^2)}
\]

Imposing the restriction that no electrons in the beam traversing the gap region should be returned to the cathode, one must have \( \left| V_1 \right| \leq V_0 \). Here \( V_0 \) is the DC accelerating voltage of the electron beam and is a real number. Since \( \left| V_1 \right| \) is maximum for \( x = 0 \), one can define the electron stopping conductance \( G_{\text{stop}} \) as

\[
\left| V_1 \right|_{\text{max}} = \left| I_1 \right| \left| Z_{11} \right|_{\text{max}} = \left| I_1 \right| \frac{1}{G_{\text{stop}}(1 + x^2)} \bigg|_{x = 0} = V_0
\]
Therefore, \[ G_{\text{stop}} = \frac{|I_1|}{V_o}. \] (A1.10)

With Eq. (A1.10), Eq. (A1.8) becomes
\[ P_d(x) = \frac{|I_1|}{2} \frac{V_o}{1 + x^2}. \] (A1.11)

The plot of \( P_d(x) \) as a function of \( x \) is shown in Fig. A1.4 (heavy curve). The maximum of \( P_d(x) \) occurs at \( x = 0 \) and its value is
\[ P_d \bigg|_{\text{max}} = \frac{|I_1|}{2} \frac{V_o}{1 + x^2}. \] (A1.12)

The bandwidth of the power dissipation curve is then
\[ \text{BW} = \frac{|I_1|}{CV_o}. \] (A1.13)

Note that the above results [Eqs. (A1.11), (A1.12) and (A1.13)] are given in terms of the electron stopping conductance defined in Eq. (A1.10).

A3.1 Exact Form of the Driving Point Impedance

The driving point impedance \( Z_{11} \) can be put into a different form without the use of an approximation. This can be done in the following way. Equation (A1.1) can be rewritten as
\[ Z_{11} = \frac{1}{2aC \left[ 1 + s^2 + \frac{\omega_0^2}{2a s} \right]} . \] (A1.14)

With \( s = j \omega \), \( Q = \omega_o/2a \) and Eq. (A1.6), Eq. (A1.14) becomes
\[ Z_{11} = \frac{1/G}{1 + jx \frac{1 + x/4Q}{1 + x/2Q}} . \] (A1.15)

Defining a new normalized relative frequency variable \( \bar{x} \) as
\[ \bar{x} = x \frac{1 + x/4Q}{1 + x/2Q} , \] (A1.16)
the exact driving point impedance \( Z_{11} \) becomes
Note that the same expression could have been obtained directly from Eq. (A1.14) by defining $\bar{x}$ as

$$
\bar{x} = \frac{\omega^2 - \omega_o^2}{2Q} = Q \left[ \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right].
$$

Equation (A1.17) is identical in form with Eq. (A1.7); however, in Eq. (A1.17) $Z_{11}$ is given in terms of $\bar{x}$ which, in turn, is determined by Eq. (A1.16) or Eq. (A1.18). In Fig. A1.5, $\bar{x}$ is plotted as a function of $x$ and $Q$ of the circuit. The error in the approximate definition of $x$ of Eq. (A1.6) is clearly seen from this figure. The identical form of Eqs. (A1.7) and (A1.17) allows for easy correction of the approximate power dissipation curve of Fig. A1.4 (heavier curve) with the use of Eq. (A1.16) or Fig. A1.5. In Fig. A1.4, the exact power dissipation curve was plotted for a $Q = 10$ (light curve). There is no noticeable change in the maximum dissipated power and in the bandwidth of the exact and the approximate power dissipation curves of Fig. A1.4. Therefore, as long as $Q > 10$, the approximate expression for the driving point impedance can be used without restriction. For the analysis of a broadband, single-gap output cavity see reference 42.
FIG. A1.1

GAP REGION

BEAM CURRENT

FIG. A1.2

ELECTRIC FIELD E — — — — — — — —
MAGNETIC FIELD H ◯ ◯
WALL CURRENT J — — — — — — — —

FIG. A1.3
APPENDIX 2

ANALYSIS OF A DOUBLE-TUNED, SINGLE-GAP CAVITY

A2.1 Asymmetrically Loaded, Magnetically Undercoupled Cavities

In Chapter 2, a coupled, two-cavity system in which a single electron beam was traversing both cavities was defined as a double-tuned, double-gap cavity system. It was designated as a DTDG cavity. Similarly, a coupled, two-cavity system in which the electron beam passes through only one single cavity will be defined as a double-tuned, single-gap cavity system and will be designated as a DTSG cavity.

Since in a DTSG cavity the electron beam passes through one single cavity only, the capacitive coupling through the short drift tube between the gridless gaps such as exists in a DTDG cavity, cannot be used to couple two cavities of a DTSG cavity system. Similarly, the transmission line couplings as shown in Fig. 2.2 cannot be used in their present form, because the beam now loads one single cavity only. Therefore, for example, in Fig. 7.1 the two G's are no more equal. The analysis of a DTSG cavity will be limited to the case when the cavities are coupled magnetically. The equivalent circuit for a magnetically coupled DTSG cavity is given in Fig. A2.1. The circuit of Fig. A2.1 differs from the circuit of Fig. 6.1 only in that the induced gap current $I_2$ in Fig. A2.1 is now equal to zero. However, since the equivalent circuit itself is the same for the magnetically coupled DTDG (Fig. 6.1) and DTSG (Fig. A2.1) cavities, their impedance functions are also the same. Therefore, when the loading of cavities is non-symmetric and the magnetic coupling is such that the cavities are undercoupled, the impedance functions given by Eq. (4.6) apply also for magnetically coupled DTSG cavity system. Note that in this case, where the beam traverses one cavity only, external, symmetric...
loading of the cavities will not make $G_1$ equal to $G_2$ in the circuit of Fig. A2.1, because the beam itself loads one cavity only. However, in this case where the coupling is magnetic, $Z_{12}$ of Eq. (4.6c) can have either a positive or a negative value which depends on the sign of the mutual inductance $M$ of Fig. 2.7. Also, $p$ is now defined by Eq. (6.10) and not as given by Eq. (4.7b).

With the impedance functions of a magnetically undercoupled DTSG cavity known from Eq. (4.6), the power dissipated in the individual cavities can now be determined. One has for the power dissipated in the first cavity

$$P_{d1} = \frac{G_1}{2} \left| V_1 \right|^2 = \frac{G}{2} \left( \frac{1 + g}{2} \right) \left| I_1 \right|^2 \left| Z_{11} \right|^2$$

$$= \frac{(1 + g) \left| I_1 \right|^2}{2G} \frac{(1 - g)^2 + x^2}{(1 - p^2)^2 + 2(1 + p^2) x^2 + x^4}$$

(A2.1a)

and

$$P_{d2} = \frac{G_2}{2} \left| V_2 \right|^2 = \frac{G(1 - g)}{2} \left| I_1 \right|^2 \left| Z_{12} \right|^2$$

$$= \frac{(1 - g) \left| I_1 \right|^2}{2G} \frac{g^2 - p^2}{(1 - p^2)^2 + 2(1 + p^2) x^2 + x^4}$$

(A2.1b)

where in Eq. (A2.1), $g$ is defined by Eq. (4.1). Note that $g$ and $p$ must also satisfy Eq. (4.5). It is interesting to note that the results of Eq. (A2.1) are not affected by the sign of $Z_{12}$.

Since the electron beam passes through one cavity only, in the stopping condition [Eq. (3.22)] $V_2$ must be set equal to zero. One must, therefore have

$$\left| V_1 \right|_{\text{max}} = V_o$$

(A2.2)
From Eq. (A2.1a) one can see that $P_{d1}$ and $|V_1|$ have the maximum occurring at the same value of $x$. So that the maximum value of $P_{d1}$ of Eq. (A2.1a) may be written after normalizing to the maximum power dissipated in one single cavity system

$$P_{d1\text{ max}} = (1 + g) \left[ \frac{(1 - g)^2 + x^2}{(1 - p^2)^2 + 2(1 + p^2)x^2 + x^4} \right]^{1/2}.$$

(A2.3)

The stopping conductance is, therefore,

$$G_{\text{stop}} = \frac{I_1}{V_0} \left[ \frac{(1 - g)^2 + x^2}{(1 - p^2)^2 + 2(1 + p^2)x^2 + x^4} \right]^{1/2}.$$

(A2.4)

Using the procedure described in Appendix 5, it can be shown that the maximum of the expression in the brackets of Eq. (A2.4) occurs at $x = 0$ if the following condition is satisfied

$$g \leq 1 - \left[ \frac{(1 - p^2)^2}{2(1 + p^2)} \right]^{1/2}.$$

(A2.5)

Therefore, when the maximum occurs at $x = 0$, $G_{\text{stop}}$ is given by

$$G_{\text{stop}} = \frac{I_1}{V_0} \frac{(1 - g)}{(1 - p^2)}.$$

(A2.6a)

When the condition of Eq. (A2.5) is not satisfied, the maximum of the expression in the brackets of Eq. (A2.4) does not occur at $x = 0$ and $G_{\text{stop}}$ is then given by

$$G_{\text{stop}} = \frac{I_1}{\sqrt{2} V_0} \left[ (1 + p^2) - (1 - g)^2 + \sqrt{(1 + p^2) - (1 - g)^2 - 4p^2} \right]^{1/2}.$$

(A2.6b)

On the basis of the above discussion, if $g$ is such that Eq. (A2.5) is satisfied, then the power expressions become after normalizing to
the maximum power dissipated in one single-cavity system [Eq. (A1.12)]

\[ P_{d1n} = \frac{(1 + g) (1 - p^2)}{(1 - g)} \left( \frac{(1 - g)^2 + x^2}{(1 - p^2)^2 + 2(1 + p^2) x^2 + x^4} \right) \]  

(A2.7a)

and

\[ P_{d2n} = \frac{(1 - p^2)}{(1 - p^2)^2 + 2(1 + p^2) x^2 + x^4} \left( \frac{2 - p^2}{1 - p^2} \right) \]  

(A2.7b)

Their respective maxima will occur at \( x = 0 \) and are equal to

\[ P_{d1n} \bigg|_{\text{max}} = \frac{1 - g^2}{1 - p^2} \]  

(A2.8a)

and

\[ P_{d2n} \bigg|_{\text{max}} = \frac{2 - p^2}{1 - p^2} \]  

(A2.8b)

However, when \( g \) is such that Eq. (A2.5) is not satisfied, then the

normalized power expression become

\[ P_{d1n} = \frac{(1 + g) \left| I_1 \right|}{V_o G_{\text{stop}}} \left( \frac{(1 - g^2 + x^2)}{(1 - p^2)^2 + 2(1 + p^2) x^2 + x^4} \right) \]  

(A2.9a)

and

\[ P_{d2n} = \frac{(1 - g) \left| I_1 \right|}{V_o G_{\text{stop}}} \left( \frac{2 - p^2}{(1 - p^2)^2 + 2(1 + p^2) x^2 + x^4} \right) \]  

(A2.9b)

Their respective maxima are

\[ P_{d1n} \bigg|_{\text{max}} = \frac{(1 + g) V_o}{\left| I_1 \right|} G_{\text{stop}} \]  

(A2.10a)

and

\[ P_{d2n} \bigg|_{\text{max}} = \frac{(1 - g) \left| I_1 \right|}{V_o G_{\text{stop}}} \left( \frac{2 - p^2}{(1 - p^2)^2} \right) \]  

(A2.10b)

Where in Eqs. (A2.9) and (A2.10), \( G_{\text{stop}} \) is determined by Eq. (A2.6b).

Note that the maximum of \( P_{d1n} \) given by Eq. (A2.10a) does not occur at
A2.2 Results for Complete Asymmetric Loading of Magnetically Undercoupled Cavities

A general analysis of the power expressions as given by Eqs. (A2.7) and (A2.9) involves an excessive amount of algebra. In order to keep the amount of algebra in the analysis of a DTSG cavity to a minimum, consider only the two most important cases of asymmetric loading. That is, consider the case when only the first cavity is loaded \((g = 1)\) and the case when the load is in the second cavity only \((g = -1)\).

When \(g = 1\), Eq. (A2.5) is not satisfied unless \(p = 1\) also. Therefore, \(G_{\text{stop}}\) is given by Eq. (A2.6b). It is

\[
G_{\text{stop}} = \frac{|I_1|}{2V_0} .
\]  

(A2.11)

The normalized power expressions are given by Eq. (A2.9). They become with \(g = 1\)

\[
P_{d1_n} = \frac{4x^2}{(1 - p)^2 + 2(1 + p^2)x^2 + x^4}
\]  

(A2.12a)

and

\[
P_{d2_n} = 0 .
\]  

(A2.12b)

The maximum of Eq. (A2.12a) is given by Eq. (A2.10a) and is equal to unity. From Eq. (A2.12) one can see that when the first cavity, which is traversed by the electron beam, contains all the load, the power dissipated in the external cavity is zero for all values of coupling. However, the power dissipated in the first cavity is zero at \(x = 0\) unless \(p = 1\). For values of \(x\) different from zero, the power dissipated in the first cavity is non-zero. As a matter of fact, for some value of the
frequency, the power dissipated in the first cavity is equal to the power
dissipated in one single-cavity system of Appendix 1. When \( p = 1 \),
\( P_{d_1} \) becomes from Eq. (A2.12a)
\[
\left. P_{d_1} \right|_{p=1} = \frac{4}{4 + x^2} \quad \text{(A2.13)}
\]
For these values of \( g \) and \( p \), the bandwidth of a DTSG cavity is equal to
the bandwidth of a single-cavity system of Appendix 1. Note that when
\( p = g \) the two cavities are decoupled.

When \( g = -1 \), Eq. (A2.5) is satisfied for all values of \( p \), so that
\( G_{\text{stop}} \) is now given by Eq. (A2.6a) and its value is
\[
G_{\text{stop}} = \frac{2 |I_1|}{V_0 (1 - p^2)} \quad \text{(A2.14)}
\]
The normalized power expressions are given by Eq. (A2.7a). They
become with \( g = -1 \)
\[
P_{d_1} = 0 \quad \text{(A2.15a)}
\]
and
\[
P_{d_2} = \frac{(1 - p^2)^2}{(1 - p^2)^2 + 2(1 + p^2) x^2 + x^4} \quad \text{(A2.15b)}
\]
The maximum of Eq. (A2.15b) is at \( x = 0 \) and its value is equal to unity.
The half power point of the expression given by Eq. (A2.15b) is at
\[
2 + 4x = 2(1 + p^2) - (1 + p^2) \quad \text{(A2.16)}
\]
The normalized bandwidth is then given by
\[
\text{BW}_n = \frac{2}{(1 - p^2)} \left[ \sqrt{2(1 + p^4)} - (1 + p^2) \right]^{1/2} \quad \text{(A2.17)}
\]
The normalized bandwidth given by Eq. (A2.17) is maximum when \( p = 0 \).
and its value is

\[ \text{BW}_n |_{\text{max}} = 2 \left[ \sqrt{2-1} \right]^\frac{1}{2} = 1.29. \quad (A2.18) \]

The plot of Eq. (A2.17) is shown in Fig. A2.2.

In conclusion one can say that when a magnetically undercoupled DTSG cavity is loaded completely asymmetrically, the most interesting case is when the external cavity has all the load, i.e., when \( g = -1 \). The single-hump power dissipation curve is then given by Eq. (A2.15b) for all values of \( p \ll 1 \). The maximum of the power dissipation curve occurs at \( x = 0 \) for all values of \( p \ll 1 \), and is equal to the maximum of the power dissipated in the single-tuned cavity system of Appendix 1.

The normalized bandwidth of this type of DTSG cavity is given by Eq. (A2.17), which is maximum when \( p = 0 \). From Eq. (A2.18) one can see that when the external cavity has all the load, the magnetically undercoupled DTSG cavity can have about 1.29 times the bandwidth of a single-cavity system of Appendix 1.

A2.3 Asymmetrically Loaded, Magnetically Overcoupled Cavities

When the loading of cavities is nonsymmetric and the magnetic coupling is such that the cavities are overcoupled, i.e., the locus of poles of the impedance functions is approximately as shown in Fig. 4.2, the impedance functions given by Eq. (4.22) apply also to a magnetically overcoupled DTSG cavity system. However, in this case \( p \) is defined by Eq. (6.16) and \( Z_{12} \) can be either positive or negative. 38, 49

With the impedance functions of Eq. (4.22), the power expressions for the magnetically overcoupled DTSG cavity system can now be determined with the help of Eq. (A2.1). They are
\[ P_{d1} = \frac{(1 + g) |I_1|^2}{2G} \left[ \frac{(1 - g)^2 + x^2}{(1 + p^2)^2 + 2(1 - p^2) x^2 + 4} \right] \]  \hspace{1cm} (A.19a) \\

and

\[ P_{d2} = \frac{(1 - g) |I_1|^2}{2G} \left[ \frac{g^2 + p^2}{(1 + p^2)^2 + 2(1 - p^2) x^2 + 4} \right] \]  \hspace{1cm} (A2.19b)

Note that again g is defined by Eq. (4.1), and Eq. (A2.19) does not depend on the sign of \( Z_{12} \). However, in this case, \( p \) can assume any value, that is, \( p \) does not have to satisfy Eq. (4.5) any more. Therefore, for large values of \( p \), \( P_{d2} \) will exhibit a double-hump characteristic.

The coupling for which this happens, in this case when \( p = 1 \), is often referred to as the critical coupling.

The stopping conductance in this case is given by

\[ G_{stop} = \frac{|I_1|}{V_o} \left[ \left( \frac{(1 - g)^2 + x^2}{(1 + p^2)^2 + 2(1 - p^2) x^2 + 4} \right) \right]^{1/2} \max. \]  \hspace{1cm} (A2.20)

Following the procedure of Sect. 4.1, it can shown that the maximum of the expression in the brackets occurs at \( x = 0 \) if \( p < 1 \) and if the following condition is satisfied

\[ g \leq 1 - \left[ \frac{(1 + p^2)^2}{2(1 - p^2)^2} \right]^{1/2} \]  \hspace{1cm} (A2.21)

When the maximum occurs at \( x = 0 \), \( G_{stop} \) is given by

\[ G_{stop} = \frac{|I_1|}{V_o} \frac{(1 - g)}{(1 + p^2)} \]  \hspace{1cm} (A2.22)

When the condition of Eq. (A2.21) is not satisfied or when \( p \gg 1 \), the maximum does not occur at \( x = 0 \) and \( G_{stop} \) is then given by Eq. (A2.6b) in which \( p^2 \) should be replaced by \(-p^2\). Therefore, when \( p < 1 \) and \( g \) is
such that Eq. (A2.21) is satisfied, the complete normalized power expressions and their respective maxima at $x = 0$ are given by Eqs. (A2.7) and (A2.8) in which $p^2$ must be replaced by $-p^2$. When $p < 1$, but $g$ is such that Eq. (A2.21) is not satisfied, then the normalized power expressions and their respective maxima are given by Eqs. (A2.9) and (A2.10). However, for the overcoupled case, $p^2$ in these equations must again be replaced by $-p^2$. When $p \geq 1$, the normalized power expressions and the maximum of $P_{d1n}$ are still given by Eqs. (A2.9) and (A2.10a) in which $p^2$ must be replaced by $-p^2$; however, when $p \geq 1$ the maximum of $P_{d2n}$ is now given by

$$P_{d2n}\max = \frac{(1 - g)|I_1|}{V_0 G_{\text{stop}}} \frac{g^2 + p^2}{4p^2}.$$  \hspace{1cm} (A2.23)

In Eq. (A2.23), $G_{\text{stop}}$ is given by Eq. (A2.6b) with $p^2$ replaced by $-p^2$.

A2.4 Results for Complete Asymmetric Loading of Magnetically Overcoupled Cavities

A general analysis of the power expressions of a magnetically overcoupled DTSG cavity system is difficult because of the amount of algebra involved. For this reason, consider only the most important cases of asymmetric loading, that is, consider only two values of $g$: $g = 1$ and $g = -1$.

When $g = 1$, the first cavity, which is traversed by the electron beam, contains all the load. When $g = 1$, Eq. (A2.21) is not satisfied so that $G_{\text{stop}}$ is now given by Eq. (A2.6b) with $p^2$ replaced by $-p^2$. This value of $G_{\text{stop}}$ is
The normalized power expressions are now given by Eq. (A2.12) with $p^2$ replaced by $-p^2$. Therefore, when the first cavity has all the load, the power dissipated in the second cavity is zero for all values of the coupling. The power dissipation curve of the first cavity exhibits a double-hump characteristic with the power curve going through zero at $x = 0$. The maximum power dissipated in the first cavity occurs at $x^2 = 1 + p^2$ and its value is the same as the power dissipated in one single-cavity system of Appendix 1.

When $g = -1$, the external (second) cavity has all the load. From Eq. (A2.19a) one can see that the power dissipated in the first cavity is zero for all values of the coupling. However, the power dissipation curve $P_{d2n}$ depends strongly on the coupling. One can differentiate three different power expressions corresponding to three different ranges of the coupling parameter $p$. Consider first the values of $p$ such that

$$0 \leq p \leq (4 \sqrt{2} - 5) \left( \frac{1}{2} \right) = .811$$

(A2.25)

When $p$ is such as shown by Eq. (A2.25), Eq. (A2.21) is satisfied, $G_{stop}$ is given by Eq. (A2.22) and $P_{d2n}$ can be obtained from Eq. (A2.19b). It is

$$P_{d2n} = \frac{(1 + p^2)^2}{(1 + p^2)^2 + 2(1 - p^2) x^2 + x^4}$$

(A2.26)

Eq. (A2.26) is identical with Eq. (A2.15b) if $p^2$ is replaced by $-p^2$. The maximum of $P_{d2n}$ of Eq. (A2.26) occurs at $x = 0$ and its value is unity.

The half power point is now at
\[ x^2_o = \sqrt{2(1+p^4)} - (1-p^2) \quad , \quad (A2.27) \]
and the normalized bandwidth is given by
\[ BW_n = \frac{2}{1+p^2} \left[ \sqrt{2 + 2p^4} - (1-p^2) \right]^{1/2} . \quad (A2.28) \]
The plot of the normalized bandwidth for the range of \( p \) given by Eq. (A2.25) is shown in Fig. A2.2.

When \( g = -1 \) and \( p \) is such that
\[ 4\sqrt{2} - 5 \leq p^2 \leq 1 \quad , \quad (A2.29) \]
Eq. (A2.21) is not satisfied and \( G_{\text{stop}} \) is now given by Eq. (A2.6b) with \( p^2 \) replaced by \(-p^2\). This value of \( G_{\text{stop}} \) is
\[ G_{\text{stop}} = \frac{1}{\sqrt{2} V_o} \left[ \sqrt{9 + 10p^2 + p^4} - (3 + p^2) \right]^{-1/2} . \quad (A2.30) \]
With this value of \( G_{\text{stop}} \), \( P_{d2_n} \) of Eq. (A2.19b) becomes
\[ P_{d2_n} = \frac{2 \sqrt{2} (1+p^2)}{(1+p^2)^2 + 2(1-p^2) x^2 + x^4} \left[ \sqrt{2 + 2p^4} - (1-p^2) \right]^{1/2} . \quad (A2.31) \]
The maximum of the above expression occurs at \( x = 0 \). The plot of this maximum is shown in Fig. A2.2 as a function of \( p \). The half power point of \( P_{d2_n} \) of Eq. (A2.31) occurs again at \( x_o \) given by Eq. (A2.27) and the normalized bandwidth is now given by
\[ BW_n = \left[ \frac{\sqrt{2(1+p^4)} - (1-p^2)}{2 \sqrt{9 + 10p^2 + p^4}} \right]^{1/2} . \quad (A2.32) \]
The plot of the normalized bandwidth for the range of \( p \) given by Eq. (A2.29) is again shown in Fig. A2.2.

When \( g = -1 \) and \( p > 1 \), \( G_{\text{stop}} \) and \( P_{d2_n} \) are still given by
Eqs. (A2.30) and (A2.31), respectively. However, for these values of \( p \), the power dissipation curve \( P_{d2} \) exhibits a double-hump characteristic with the maximum of \( P_{d2} \) occurring now at \( x^2 = p^2 - 1 \). This maximum value is equal to

\[
P_{d2,n} \bigg|_{\text{max}} = \frac{(1 + p^2) \left[ \sqrt{9 + 10p^2 + p^4} - (3 + p^2) \right]}{\sqrt{2} \, p^2} \bigg) \bigg]^{1/2}.
\]

(A2.33)

The plot of Eq. (A2.33) for the range of \( p \gg 1 \) is shown in Fig. A2.2.

Because of the double-hump characteristic of the power dissipation curve, the expression for bandwidth is now more complicated. The half power point is at

\[
x_o^2 = -1 + p^2 + 2p.
\]

(A2.34)

The plus sign in Eq. (A2.34) can be used to determine the bandwidth, whereas the minus sign can be used to find the value of \( p \) for which the ripple in the center of the power dissipation curve is 3 db down from its maximum value. Therefore, the power dissipation curve at \( x = 0 \) is one half of its maximum value when

\[
p = 1 + \sqrt{2}.
\]

(A2.35)

The normalized bandwidth as a functions of \( p \) is then

\[
B_{W_n} = \left[ \frac{p^2 + 2p - 1}{2 \sqrt{9 + 10p^2 + p^4 - 2(3 + p^2)}} \right]^{1/2}.
\]

(A2.36)

The plot of Eq. (A2.36) for \( p \gg 1 \) is shown in Fig. A2.2.

A2.5 Conclusion

In conclusion one can say, that if the first cavity of a magnetically coupled DTSG cavity, the equivalent circuit of which is shown in Fig.
A2.1, has all the load, the power dissipated in the second cavity is zero for all values of the coupling. The power dissipated in the first cavity, however, exhibits a double-hump characteristic and its value is equal to zero at \( x = 0 \). Therefore, there is nothing to be gained by coupling the useful load to the first cavity. On the other hand, when the second or the external cavity of DTSG cavity system has all the load, the power dissipation curve exhibits a single-hump characteristic up to a value of \( p = 1 \) for overcoupled cavities. For larger values of the coupling parameter \( p \), the power dissipation curve has a double-hump characteristic. The value of the coupling for which this happens \((p = 1)\) is often called the critical coupling. Note that when the cavities are undercoupled, \( p \) is defined by Eq. (6.10). For overcoupled cavities \( p \) is defined by Eq. (6.16). In Fig. A2.2 the value of the maximum power dissipated in the external cavity and its bandwidth were plotted as a function of the coupling parameter \( p \). The results of this type of DTSG cavity are evident from this figure. However, it must be pointed out that the results of this appendix apply only for high Q circuits, i.e., \( Q > 10 \). To obtain results with a higher degree of accuracy, the exact impedance expressions derived in Appendix 3 must be used.
FIG. A2.1

\[ L_C = \frac{L}{k} \]

FIG. A2.2

\[ P_d 2n \quad \text{and} \quad B.W. n \]

0 dB RIPLE

\[ r = 3 \text{ dB} \]

UNDERCOUPLED

OVERCOUPLED
A3.1 Exact Form of the Impedance Functions for a Symmetrically Loaded, Capacitively Coupled Circuit

In Sec. 3.2 the location of the poles of \( Z_{11} \) and \( Z_{12} \) for a symmetrically loaded, capacitively coupled circuit was approximated as shown in Fig. 3.3. The only other assumption used was that their conjugate poles lay approximately parallel to the \( j\omega \)-axis a constant distance \( 2\omega_o \) away from the real frequency variable \( \omega \). However, starting with Eqs. (3.3) and (3.4), and letting \( G_1 = G_2 = G \) for the symmetric loading, one has, for example, for \( Z_{11} \)

\[
Z_{11}(j\omega) = \frac{k \alpha}{2C \alpha_1 \alpha_2} \frac{(1 + j \frac{2}{2\alpha_1\omega})}{(1 + j \frac{2}{2\alpha_2\omega})} \left( 1 + j \frac{\omega - \omega_1}{2\alpha_1\omega} \right) \left( 1 + j \frac{\omega - \omega_2}{2\alpha_2\omega} \right),
\]

where \( \alpha_1, \alpha_2, \omega_1 \) and \( \omega_2 \) are given by Eq. (3.6), and where \( K, \omega_o, \alpha \) and \( Q_o \) are given by Eqs. (3.4b), (3.9b), (3.7) and (3.9c) respectively.

Additionally, in Eq. (A3.1), \( j\omega \) stands for \( s \) of Eqs. (3.3) and (3.4). When a high Q approximation is applied to Eq. (A3.1) directly, one will obtain the approximate form of \( Z_{11} \) given by Eq. (3.10a). However, using Eq. (3.11), Eq. (A3.1) becomes

\[
Z_{11}(x) = \left[ \frac{1 + j \frac{x}{4Q_o}}{(1 + j \frac{x}{2Q_o})} \right]^{\frac{1}{G}} \left[ \frac{C_c}{C} Q_o \left( 1 + \frac{x}{2Q_o} \right) \right]^{\frac{1}{2}}.
\]
With the new normalized relative frequency variable $x$ as given by Eq. (A1.16) or Eq. (A1.18) and the new coupling parameter $p$ defined as

$$\overline{p} = \frac{C_c}{C} Q_0 (1 + \frac{x}{2Q_0}) = \frac{C_c}{C} Q_0 \frac{\omega}{\omega_o}, \quad (A3.3)$$

Eq. (A3.2) finally becomes

$$Z(x) = \left(1 + \frac{j}{G} \frac{1}{2G} \right). \quad (A3.4a)$$

Similarly for $Z_{12}$ one has

$$Z_{12}(\overline{x}) = \left(1 + \frac{j\overline{p}}{G} \right). \quad (A3.4b)$$

Note that Eqs. (A3.4) and (3.10) have identical form. However, (Eq. (3.4) is exact whereas Eq. (3.10) is valid only for high $Q$ circuits and weak coupling, even then only approximately. The approximate results obtained in Chap. 3 can now be recalculated using the exact expressions for the impedance functions given by Eq. (A3.4), for example, for one particular choice of the coupling. Note that since the exact and the approximate expressions for the impedance functions [Eq. (A3.4) and Eq. (3.10), respectively] have the same form, the approximate results of Chapter 3 can be quickly corrected for lower $Q$ circuits using the new definitions of $\overline{x}$ and $\overline{p}$.

A comparison of Eq. (A3.4) together with the definition of $\overline{x}$ and $\overline{p}$ with Eq. (3.10) shows that by using the approximate definition of the impedance functions of Eq. (3.10) a large amount of calculations was avoided. The obtained results, even though only approximately true, have a simple form and the general behavior of a symmetrically loaded, capacitively coupled DTDG cavity is clearly seen from these approximate
results. The use of the exact expressions for $Z_{11}$ and $Z_{12}$ introduces only a second order correction, for example, into the shape of the power dissipation curve of Fig. 3.4. See, for example, Fig. A3.1 in which the power dissipated in the symmetrically loaded, capacitively coupled DTDG cavity was calculated using the exact impedance expressions [Eq. (A3.4)] (continuous light curve) and approximate expressions [Eq. (3.10)] (heavy curve) for the case when $p = C_c Q_o / C = 1$ and $Q_o = 10$. Note that from the maximum dissipated power point of view and from the point of view of bandwidth there is no marked difference between the two curves. The difference between the exact and the approximate power dissipation curves will be even less if $p < 1$ and if $Q_o > 10$. Therefore, the approximate expressions for the impedance functions can be used freely for the circuits with the coupling parameter $p < 1$ and $Q_o > 10$. Note that a higher order approximation for the impedance functions $Z_{11}$ and $Z_{12}$ for the symmetrically loaded circuit could have been obtained if one had assumed that the poles of $Z_{11}$ and $Z_{12}$ lie on the tangent to the circle $G_1 = G_2$ at the point $K = 1$ of Fig. 3.2. The slope of the tangent at this point is approximately minus $Q_o$. However, the impedance functions using this approximation show no practical improvement as compared to $Z_{11}$ and $Z_{12}$ of Eq. (3.10) or Eq. (A3.4).

A3.2 Higher Order Approximation for the Impedance Functions of an Asymmetrically Loaded, Capacitively Coupled Circuit

The case when the loads are unevenly distributed between the cavities is of greater practical value. However, for this case, when in the circuit of Fig. 3.1 $G_1 \neq G_2$, the locus of poles of $Z_{11}$, $Z_{22}$ and $Z_{12}$ for varying coupling has a somewhat complicated form as shown in Fig. A4.1
for $Q = 20$. Assume at first that the poles of $Z_{11}$, $Z_{22}$ and $Z_{12}$ for the overcoupled circuit lie on the $G_1 = G_2$ circle of Fig. 3.2. Note that in the approximation of Chap. 4, the poles of $Z_{11}$, $Z_{22}$ and $Z_{12}$ for the overcoupled circuit were assumed to lie as shown in Fig. 4.2.

One may now define a new set of $\alpha$'s and $\omega$'s

$$\alpha_1 = \frac{\alpha}{1 - m \frac{C_c}{C}} \quad \text{(A3.5a)}$$

$$\alpha_2 = \frac{\alpha}{1 + m \frac{C_c}{C}} \quad \text{(A3.5b)}$$

$$\omega_1^2 = \frac{\omega_o^2}{1 - m \frac{C_c}{C}} \quad \text{(A3.5c)}$$

and

$$\omega_2^2 = \frac{\omega_o^2}{1 + m \frac{C_c}{C}} \quad \text{(A3.5d)}$$

In Eq. (A3.5), $m$ is some, as yet unknown, factor introduced to take into account the effect of asymmetric loading on the locus of poles of $Z_{11}$, $Z_{22}$ and $Z_{12}$ (Fig. A4.1). When Eq. (A3.5) is introduced into Eq. (A3.1) one will obtain for $Z_{11}$

$$Z_{11} = \left[ \frac{1 - g + jx \frac{x}{4Q_o}}{(1 + \frac{x}{2Q_o})} \right]^{1/G} \left[ \frac{x}{4Q_o} \right]^{2} + \left[ \frac{m \frac{C}{C} Q_o (1 + \frac{x}{2Q_o})}{x \frac{x}{2Q_o}} \right]^{2}. \quad \text{(A3.6)}$$

In Eq. (A3.6) $g$ is given by Eq. (4.1). Note that the term $C_c/C$ of $K$ of Eq. (3.4b) had also to be multiplied by $m$. Defining a new $\bar{p}$ as

$$\bar{p} = m \frac{C_c}{C} Q_o \left( 1 + \frac{x}{2Q_o} \right), \quad \text{(A3.7)}$$
Eq. (A3.6) becomes with \( x \) given by Eq. (A1.16)

\[
Z_{11} = \frac{(1 - g + jx)^{1/2}}{(1 + jx)^2 + P^2}. \tag{A3.8a}
\]

Similarly

\[
Z_{12} = \frac{jP/mG_{12}G_{2}}{(1 + jx)^2 + P^2}. \tag{A3.8b}
\]

\( Z_{22} \) is given by Eq. (A3.8a) with a plus sign in front of \( g \). The value of \( m \) can now be determined by evaluating \( Z_{11} \) of Eq. (A3.8) and \( Z_{11} \) of Eq. (A3.1) at, say, \( \omega = \omega_o \). One obtains

\[
m^2 = 1 - \left( \frac{C g}{Q_o} \right)^2. \tag{A3.9}
\]

With this value of \( m \), \( \bar{p} \) of Eq. (A3.7) becomes

\[
\bar{p} = \left[ \left( \frac{C c Q_o}{C} \right)^2 - g^2 \right]^{1/2} (1 + \frac{x}{2Q_o}) = \left[ \frac{1}{2} \right]^{1/2} \frac{\omega}{\omega_o}. \tag{A3.10}
\]

Note that from Eq. (A3.10) it follows that \( \bar{p} \) and therefore \( Z_{12} \) is defined only if \( Q_o C_c / C \geq g \) (overcoupled cavities).

A3.3 Exact Form of the Impedance Functions of an Asymmetrically Loaded, Capacitively Coupled Circuit

When in the circuit of Fig. 3.1 \( G_1 \neq G_2 \), the impedance function \( Z_{11} \) of Eq. (A3.1) can be put into the form of Eq. (3.10) without the use of an approximation. For this case \( Z_{11} \) is still given by Eq. (A3.8a) with \( x \) defined in Eq. (A1.16); however, now \( \bar{p} \) is given by

\[
\bar{p}^2 = \left( \frac{C c Q_o}{C \omega_o} \right)^2 - g^2. \tag{A3.11}
\]

Note the difference between Eqs. (A3.11) and (A3.10). \( Z_{12} \) is now given by

\[
Z_{12} = \frac{j \left( \bar{p}^2 + g^2 \right)^{1/2}}{G \left[ (1 + jx)^2 + \bar{p}^2 \right]} \tag{A3.12}
\]
where $p$ is defined by Eq. (A3.11). Note that in this case (exact expression), $p$ in Eq. (A3.12) appears to the even power only, so that the impedance functions derived in this section apply for all values of coupling. That is, these equations are valid for undercoupled and the overcoupled circuits.

In Chap. 4, the power expressions for the asymmetrically loaded circuit of Fig. 3.1 were derived using the approximate expressions for the impedance functions. With the exact expressions for $Z_{11}$, $Z_{22}$ and $Z_{12}$ determined in this section, one may calculate the power dissipation curve exactly. This has been done for one particular value of $p$ and $g$. $g$ was taken to be equal to unity. This value of $g$ corresponds to a complete asymmetric loading of cavities, i.e., in Fig. 3.1 $G_1 = 2G$ and $G_2 = 0$. The case $g = 1$ was chosen because, as it can be seen from Fig. A4.1, it gives the maximum error in the locus of poles of the approximate impedance functions of Eqs. (4.6) and (4.22). With $g = 1$, the value of the coupling parameter $p$ was chosen such that at $\omega = \omega_0$, $p = 1$ for a $Q_0 = 10$. That is, $(Q_0 C_C / C)^2 = 2$. The new, exact power dissipation curve is given in Fig. A3.1 dashed. From the bandwidth and the maximum dissipated power point of view, there is again no significant difference between the exact and the approximate curves. Note the approximate power curve for the above values of loading and coupling is given in Fig. A3.1 by the heavy curve. Therefore, if $p$ at $\omega = \omega_0$ is less than unity and if $Q_0 > 10$, the approximate results of Chap. 4 can be used freely in the calculation of the maximum dissipated power and in the determination of bandwidth.

A3.4 Exact Forms of the Impedance Functions of a Symmetrically and an Asymmetrically Loaded, Magnetically Coupled Circuit.

As in the case of a capacitively coupled circuit of Fig. 3.1, one
can rewrite the impedance functions of an asymmetrically loaded, magnetically coupled circuit of Fig. (6.1) given by Eq. (6.3). Without the use of an approximation, $Z_{11}$ and $Z_{22}$ can be put into the same expression as it was done in Sec. A2.3 for the capacitively coupled circuit. However, now $Z_{12}$ can have a positive or a negative sign corresponding to a negative or a positive mutual inductance $M$ of Figs. 2.5 or 2.7, and $p$ is now defined by

$$p^2 = \left( \frac{L_{\omega_o}}{L_c} \right)^2 - g^2 .$$

(A3.13)

Note that in Eq. (A3.13) one has the term $\omega_o/\omega$, but in Eq. (3.11) one had the term $\omega/\omega_o$. In Fig. A3.1 the power dissipation curve, which was calculated for the inductive coupling with $(LQ_o/L_c)^2 = 2$, $Q_o = 10$ and $g = 1$, is indicated by dots for the purpose of comparison with other power curves. Note that the approximate expressions for the capacitive and the inductive coupling give identical power dissipation curves. The exact impedance functions of a magnetically coupled, symmetrically loaded DTDG cavity can be obtained from the above results by letting $g = 0$. 
A STUDY OF THE LOCUS OF POLES OF THE IMPEDANCE FUNCTIONS

A4.1 The exact Locus of Poles of the Impedance Functions for Capacitive Coupling

The locus of poles of $Z_{11}$, $Z_{22}$ and $Z_{12}$ of the capacitively coupled circuit of Fig. 3.1, in general, has to be found through numerical computation. Only when the circuit of Fig. 3.1 is loaded symmetrically, that is, when $G_1 = G_2 = G$, or when the coupling capacitance $C_c$ is equal to zero, can the roots be expressed in closed analytic form. The locus of poles of $Z_{11}$, $Z_{22}$ and $Z_{12}$ for this particular loading and coupling is shown in Fig. 3.2. However, in the analysis of the circuit of Fig. 3.1 one is interested more in the circuit in which the coupling capacitance $C_c$ is not equal to zero and in which the loading conductances are not equal to each other. In order to derive a simplified, approximate representation for the impedance functions of an asymmetrically loaded network, the locus of poles of $Z_{11}$, $Z_{22}$ and $Z_{12}$ was calculated and plotted for a set of particular values of the circuit parameters $C$, $L$, $G_1$, $G_2$ and $C_c$. In order to simplify the calculations, it was assumed that

$$G_1 = G (1 + g) \quad (A4.1a)$$

and

$$G_2 = G (1 - g) \quad (A4.1b)$$

Additionally, without any loss of generality, it was assumed that

$$\omega_0^2 = \frac{1}{LC} = 1 \quad (A4.2)$$

Now using the expression for the $Q$ of the circuit

$$Q_o = \frac{\omega_0}{2 \alpha} = \frac{1}{G} \sqrt{\frac{C}{L}} \quad (A4.3)$$
one has for $\Delta$ of Eq. (3.4a)

$$\Delta = s^4 + \frac{2}{Q_0} K s^3 + \left[2 + \left(1 - g^2 \frac{1}{Q_0^2}\right) K s^2 + \frac{2}{Q_0} K s + K\right]. \quad (A4.4)$$

Equation (A4.4) is a function of $K$, $g$ and $Q_0$. In Eq. (A4.4), $K$ is as defined by Eq. (3.4b). However, Eq. (3.4b) can be put into the following form:

$$K = \frac{1}{C^2} \frac{1}{1 - c^2}, \quad (A4.5)$$

where one had defined

$$c = \frac{C_c}{C}. \quad (A4.6)$$

Now, the roots of Eq. (A4.4) with $\Delta = 0$ correspond to the poles of the impedance functions $Z_{11}$, $Z_{22}$ and $Z_{12}$ of the circuit of Fig. 3.1. Note, that Eq. (A4.4) is still quite general despite of the normalization of the resonant frequency in Eq. (A4.2). The locus of roots of Eq. (A4.4) with $\Delta = 0$ was studied for three different values of $Q_o$: $Q_o = 10$, $20$ and $40$. The emphasis in the analysis was put on the circuit with $Q_o = 20$. With $Q_o = 20$ and $K$ given by Eq. (A4.5), Eq. (A4.4) becomes for $\Delta = 0$.

$$s^4 \left(1 - c^2\right) + \frac{1}{s^3} + \left(2 + 0.025 - 0.0025g^2\right) s^2 + 0.1s + 1 = 0. \quad (A4.7)$$

In Eq. (A4.7), $g$ was allowed to assume the following values: $g = 0$, $0.1$, $0.3$, $0.5$, $0.7$ and $1$. The value of $g = 0$ corresponds to the symmetrically loaded circuit of Fig. 3.1, that is, $G_1 = G_2 = G$, and with $Q_o = 20$. The value of $g = 1$, on the other hand, corresponds to complete asymmetrical loading of the circuit of Fig. 3.1 with $Q_o = 20$. With the help of Eq. (A4.1), one can see that the complete asymmetrical loading corresponds to the
circuit of Fig. 3.1 with $G_1 = 2G$ and $G_2 = 0$. The roots of Eq. (A4.7) were calculated for all of the above values of $g$ and a number of values of the coupling coefficient $c$. The locus of roots of the Eq. (A4.7) as a function of $c$ was found for each value of $g$ by plotting the roots of Eq. (A4.7) in the complex frequency plane. From the set of these locus plots three curves were chosen and are reproduced in Fig. A4.1. Note that Fig. A4.1 gives the locus of two poles of $Z_{11}$, $Z_{22}$ and $Z_{12}$ as a function of $c$. Their conjugate poles were not included into the Fig. A4.1. The values of $g$ that have been selected are: $g = 0$, $g = 0.5$ and $g = 1$. The reason for this selection is the following. When the locus of roots of Eq. (A4.7) were plotted out for six values of $g$, it was found that all curves were within the two limiting values of $g$: $g = 0$ and $g = 1$. None of the curves crossed and they all had the general shape of contour lines. Therefore, it was thought sufficient to give the curves of the two limiting values of $g$ and one value of $g$ in between. Another reason for not including more curves corresponding to other values of $g$ was the limitation in space between the $g = 0$ and $g = 1$ curves. From Fig. A4.1 the effect of asymmetric loading on the locus curve is clearly visible. Furthermore, Fig. A4.1 shows the effect of loading on a particular pole location corresponding to a particular value of $c$.

Figure A4.2 shows the effect of the $Q$ of the circuit on the locus of poles of the impedance functions of the circuit of Fig. 3.1. In Fig. 3.2, the locus of poles was plotted for three values of $Q_0$: $Q_0 = 10$, 20 and 40 and in each case for only two extreme values of $g$: $g = 0$ and $g = 1$. The effect of the $Q$ of the circuit is clearly seen from these plots.

A4.2 The Approximate Locus of Poles of the Impedance Functions for Capacitive Coupling

In Chap. 3, approximate expressions for the impedance functions
were derived for a capacitively coupled and symmetrically loaded circuit 
\((G_1 = G_2 = G)\) of Fig. 3.1. In this approximation it was assumed that the 
poles of the impedance functions lay on the line parallel to the \(j\omega\)-axis 
instead of on the \(G_1 = G_2\) circle of Fig. 3.2. See, for example, Fig. A4.1 
in which the poles for different values of \(c\) were plotted on the vertical line 
parallel to the \(j\omega\)-axis as given by the approximation of the impedance 
functions of Chapter 3. The exact and the approximate location of the 
poles of \(Z_{11}\) and \(Z_{12}\), corresponding to the same value of \(c\), were 
connected by a line in order to simplify the comparison. At this point, 
it is worth mentioning that in the derivation of the approximate expressions 
of the impedance functions of Chapter 3, one more approximation was 
used. In Chapter 3 it was additionally assumed that the conjugate poles 
of \(Z_{11}\) and \(Z_{12}\) lay vertically below and a constant distance \(2\omega_0\) away 
from the real frequency variable \(\omega\), where \(\omega \approx \omega_0\). In other words, one 
used, for example, the approximation for the fourth pole 
\[(s - s_4) \approx 2j\omega_0\]. \hspace{1cm} (A4.8)

Figure A4.2 shows clearly the validity of the approximation as a function 
of \(Q\) of the circuit.

In Chapter 4, using the locus plots of Fig. A4.1, approximate 
expressions for the impedance functions were derived for the 
asymmetrically loaded circuit of Fig. 3.1. In order to compare the 
approximate location of poles with their exact location, the approximate 
locus of poles was included in Fig. A4.1 for \(g = 1\). Poles corresponding 
to the same value of \(c\) were connected by a short line to simplify the 
comparison.
The approximate location of poles corresponding to a specific value of $c$ is denoted in Fig. A4.1 by a little square mark. Note, that the value of $g = 1$ was chosen, because it gives the maximum discrepancy in the location of the poles and, therefore, serves as an upper limit for the approximation. The validity of the approximation in the locus of poles can be seen from Fig. A4.1. The validity of the approximation for different $Q$ can be seen from Fig. A4.2. Again, as in the case of the symmetric loading, the additional approximation used to obtain the impedance expressions was the assumption for the conjugate poles given, for example, for the fourth pole by Eq. (A4.8).

A4.3 Locus of Poles of the Impedance Functions for the Inductive Coupling

The locus of poles of the impedance functions of the inductively coupled network of Fig. 6.1 was analyzed in a similar way. It was found that when the circuit is loaded symmetrically, the poles lie on the line parallel to the $j\omega$-axis as shown in Fig. 6.2. Furthermore, it was found that when the circuit is loaded asymmetrically, i.e., $G_1 \neq G_2$, the poles move as shown approximately in Fig. 6.3. Except for the fact that the poles now approach a line parallel to the $j\omega$-axis with increasing $K$ and for any value of $G_1$ and $G_2$, there is no basic difference between the capacitive and the inductive couplings. For this reason, no results in forms of curves were included here for the inductively coupled network of Fig. 6.2.
FIG. A4.1
APPENDIX 5

PROOF OF EQ. (4.13)

When Eq. (4.11b) is put into the form

\[ P_{d2} = \frac{(1-g)(1-p^2)}{2} \left[ \frac{(1 + g) + (g^2 - p^2)^{1/2}}{(1 - p^2)^2 + 2(1 + p^2)x^2 + x^4} \right]^2 + \frac{x^2}{4} \]  

(A5.1)

one can define

\[ A = \left[ (1 + g) + (g^2 - p^2)^{1/2} \right]^2 \geq 0 \]  

(A5.2a)

\[ B = (1 - p^2)^2 \geq 0 \]  

(A5.2b)

\[ C = 2(1 + p^2) > 0 \]  

(A5.2c)

and

\[ D = \frac{(1 - g)(1 - p^2)}{2} \geq 0 \]  

(A5.2d)

Then Eq. (A5.1) becomes

\[ P_{d2} = D \frac{A + x^2}{B + C x^2 + x^4} \]  

(A5.3)

It can be shown that for the maximum of Eq. (A5.3) to occur anywhere else but at \( x = 0 \), one must have

\[ A^2 + B \geq AC \]  

(A5.4a)

and

\[ B \geq AC \]  

(A5.4b)

Since \( A \) is positive for all values of \( g \) and \( p \), Eq. (A5.4b) is a more stringent requirement. Now, it can be shown that the requirement of Eq. (A5.4b) is never satisfied, i.e., the maximum of Eq. (A5.1) occurs at \( x = 0 \) only. For when \( g > 0 \), one has to satisfy the following requirement given by Eq. (A5.4b):
\[ 0 > (1 + 2g)^2 + p^2 (2 + 4g + 4g^2 - 3p^2) + \\
+ 4 (1 + g) (1 + p^2) (g^2 - p^2)^{1/2} . \] \hspace{1cm} (A5.5)

As one can see, Eq. (A5.5) cannot be satisfied by any value of \( p \) or \( g \) within the limit given by Eq. (4.5). When \( g < 0 \), Eq. (A5.5) becomes

\[ 0 > (1 - 2g)^2 + p^2 (2 - 4g + 4g^2 - 3p^2) + 4(1 - g) (1 + p^2) (g^2 - p^2)^{1/2} . \hspace{1cm} (A5.6a) \]

Now, with the help of Eq. (4.5), one must have

\[ 0 \gg (1 - 2g)^2 \] \hspace{1cm} (A5.6b)

Therefore, if \( g < (2 - \sqrt{2}) \) one must have

\[ 0 \gg > (1 - 2g)^2 , \hspace{1cm} (A5.6c) \]

which is never true. Or, if \( g > (2 - \sqrt{2}) \) one must have

\[ 0 \gg > (1 - 2g)^2 + 2g^2 (2-4g + g^2) = (1 - g)^4 , \hspace{1cm} (A5.6d) \]

which also impossible. Therefore, Eq. (A5.1) does indeed have only one maximum at \( x = 0 \) for any value of \( g \) and \( p \) given by Eq. (4.5).
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