Design Requirements – Reactor Physics

22.39 Elements of Reactor Design, Operations, and Safety

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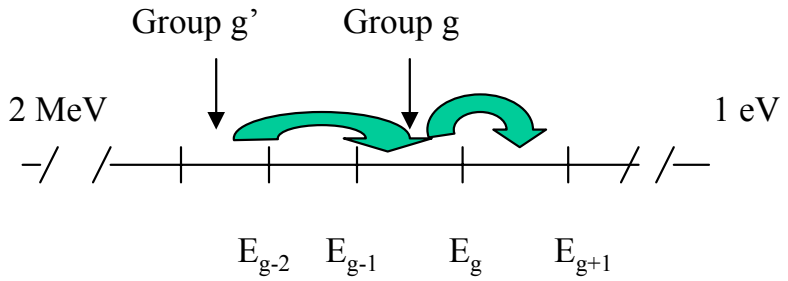
Principal Design Functions Involving Reactor Physics

- **Core criticality and power distribution**
  - **Are space and time dependent because of fuel burnup and isotope production over the core life**
  - **Depend on moderator-to-fuel ratio, core geometry, location and type of reactivity control, fuel element design**
- **Reactivity and control analysis (safety)**
  - **Must control excess reactivity in initial fuel loading**
  - **Describe short-term reactivity changes (and reactor kinetic behavior); reactivity coefficients.**
- **Depletion analysis (economic performance)**
  - **Monitor fuel composition and reactivity as a function of energy removal**

The Big Picture

The Multigroup Diffusion Equations:



$$\frac{1}{v_g}\frac{\partial \varphi_g}{\partial t} - \nabla \bullet D_g \nabla \varphi_g + \Sigma_{tg}\varphi_g(\underline{r},t) =$$

$$= \sum_{g'=1}^{G} \Sigma_{sg'g}\varphi_{g'} + \chi_g \sum_{g'=1}^{G} \nu_{g'}\Sigma_{fg'}\varphi_{g'} + S_g^{ext}$$

$$g = 1,2,...,G$$

Group constants:

$$\Sigma_{tg} \equiv \frac{\int_{E_{g-1}}^{E_g} dE \Sigma_t(E)\varphi(\underline{r},E,t)}{\int_{E_g}^{E_{g-1}} dE \varphi(\underline{r},E,t)}$$

Power Distribution

- **Problem: The group constants depend on the flux itself.**
- **Criticality: Set $\frac{1}{v_g}\frac{\partial \varphi_g}{\partial t}$ and S_g^{ext} equal to zero.**
- **Perform two multigroup calculations:**
  - **Treat spatial and time dependence very crudely and calculate the intragroup fluxes relying on models of neutron slowing down and thermalization, e.g., assuming that $\varphi(E) \propto \frac{1}{E}$ for energies between 1 eV and 10^5 eV, $\varphi(E) \propto \chi(E)$ in the high-energy range, and proportional to the Maxwellian distribution for thermal energies.**
  - **This *fine spectrum calculation* may involve as many as 1000 groups.**
  - **These intragroup fluxes are, then, used to calculate the group constants for a *coarse group calculation* with spatial dependence.**
- **LWRs: Usually three fast groups and one thermal.**
- **Fast Reactors: As many as 20 groups.**

Two-Group Criticality Calculation

$$-\nabla \bullet D_1 \nabla \varphi_1 + \Sigma_{R1} \varphi_1 = \frac{1}{k} [\nu_1 \Sigma_{f1} \varphi_1 + \nu_2 \Sigma_{f2} \varphi_2]$$

$$-\nabla \bullet D_2 \nabla \varphi_2 + \Sigma_{a2} \varphi_2 = \Sigma_{s12} \varphi_1$$

Assuming that

$$\varphi_1(\underline{r}) = \varphi_1 \psi(\underline{r}) \qquad \varphi_2(\underline{r}) = \varphi_2 \psi(\underline{r})$$

$$\nabla^2 \psi(\underline{r}) + B^2 \psi(\underline{r}) = 0, \quad \psi(\underline{r}_s) = 0$$

We get the *(effective) multiplication factor*

$$k = \frac{\nu_1 \Sigma_{f1}}{\Sigma_{R1} + D_1 B^2} + \frac{\Sigma_{s12}}{(\Sigma_{R1} + D_1 B^2)} \frac{\nu_2 \Sigma_{f2}}{(\Sigma_{a2} + D_2 B^2)}$$

For criticality: $k = 1$

More on the Multiplication Factor

$$k = \left[\frac{\nu_1 \Sigma_{f1}}{\Sigma_{R1} + D_1 B^2}\right] + \left[\frac{\Sigma_{s12}}{(\Sigma_{R1} + D_1 B^2)} \frac{\nu_2 \Sigma_{f2}}{(\Sigma_{a2} + D_2 B^2)}\right]$$

k_1: due to fissions in the fast group

k_2: due to fissions in the thermal group

$$k_2 = \frac{\left(\frac{\Sigma_{s12}}{\Sigma_{R1}}\right)}{\left(1 + \frac{D_1}{\Sigma_{R1}} B^2\right)} \frac{\nu_2 \left(\frac{\Sigma_{f2}}{\Sigma_{a2}}\right)}{\left(1 + \frac{D_2}{\Sigma_{a2}} B^2\right)} =$$

$$= \frac{p}{\left(1 + L_1^2 B^2\right)\left(1 + L_2^2 B^2\right)} = \eta_2 f_2 p P_{NL1} P_{NL2}$$

The Six-Factor Formula

$$k = \eta_2 f_2 p \varepsilon P_{NL1} P_{NL2} = (\eta_{th} f_{th} p \varepsilon) P_{NL1} P_{NL2} = k_\infty P_{NL1} P_{NL2}$$

$\eta_2 \equiv \nu_2 \left(\frac{\Sigma_{f2}}{\Sigma_{a2}^F} \right)$ **Average number of neutrons produced per thermal neutron absorbed in the fuel**

$f \equiv \left(\frac{\Sigma_{a2}^F}{\Sigma_{a2}} \right)$ **Thermal utilization: conditional probability of absorption of a thermal neutron in the fuel**

$L_1^2 \equiv \frac{D_1}{\Sigma_{R1}}$ **Diffusion area for fast neutrons**

$P_{NL1} \equiv \frac{1}{(1 + L_1^2 B^2)}$ **Nonleakage probability for fast neutrons**

$p \equiv \left(\frac{\Sigma_{s12}}{\Sigma_{R1}} \right)$ **Resonance escape probability**

$\varepsilon \equiv \left(1 + \frac{k_1}{k_2} \right)$ **Fast fission factor**

Comments

η_{th} ^{233}U: 2.29, ^{235}U: 2.07, ^{239}Pu: 2.15, natural uranium: 1.34, enriched uranium: 1.79. Increases initially as Pu is produced from ^{238}U, decreases later as fission products are produced

f_{th} About 0.9 for natural uranium. Larger as absorptions in nonfuel material decrease.

p About 0.70 for homogeneous mixtures, 0.9 for heterogeneous mixtures, increases as the ratio of moderating atoms to fuel atoms becomes large.

ε About 1.05 for natural uranium.

L_1 Water: 0.052 m, heavy water: 0.114 m, graphite: 0.192 m

L_2 Water: 0.027 m, heavy water: 1.0 m, graphite: 0.54 m

B^2 Typically less than 10 m^{-2}, therefore $P_{NL1} > 0.97$ and $P_{NL2} > 0.99$ for H_2O

Time Dependence

- **To study the time-dependence of the flux we have to solve the multigroup equations in slide 3 augmented to include the equations for delayed neutrons.**

- **There are two time scales:**
  - **Short-term changes (seconds) due to temperature effects and external deliberate changes**
  - **Long-term changes (hours or more) due to fuel depletion and fission-product buildup.**

Delayed Neutrons

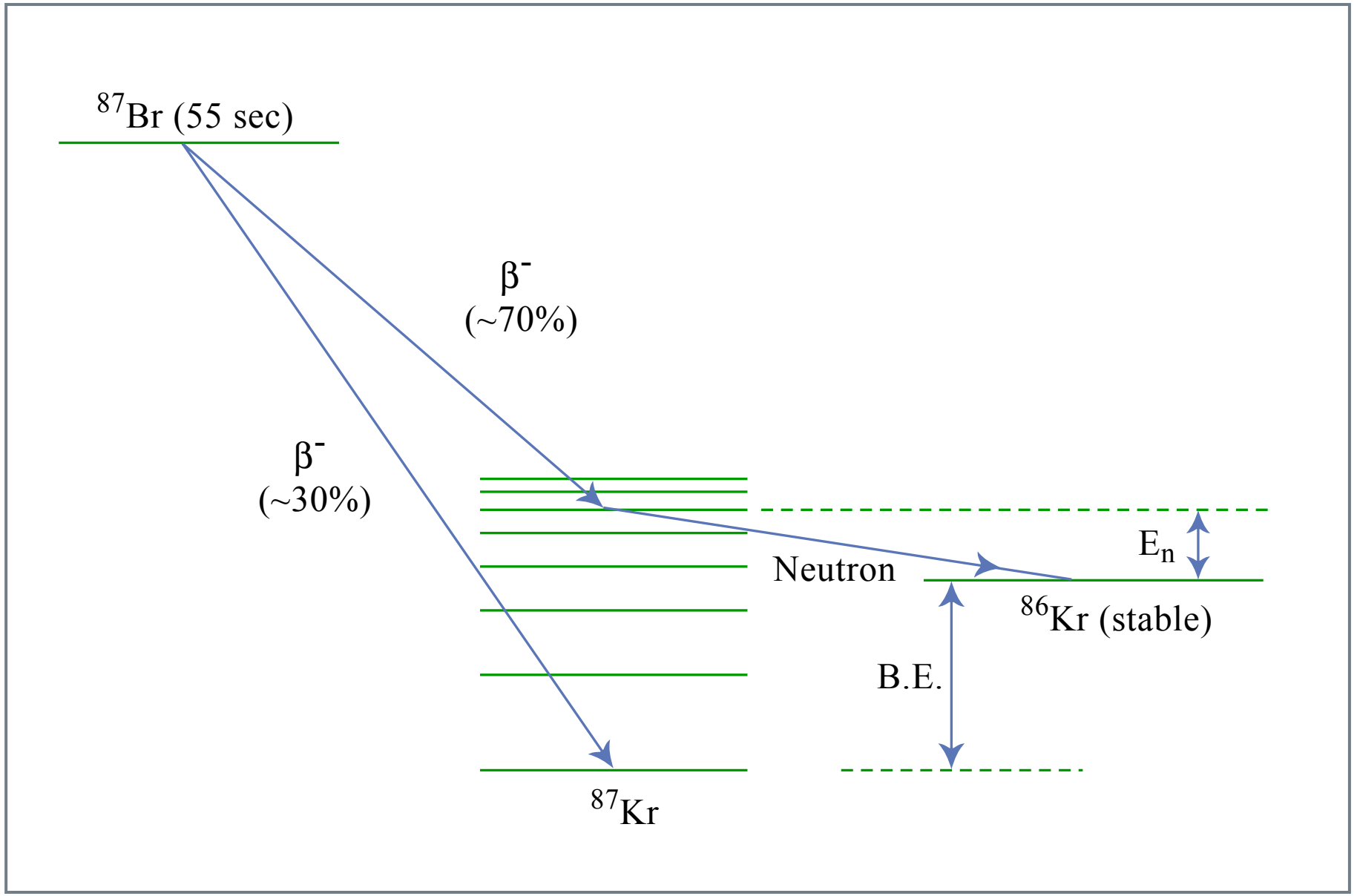


Figure by MIT OCW.

Delayed Neutron Precursors

Decay Constants and Yields of Delayed-Neutron Precursors
in Thermal Fission of Uranium-235

$t_{1/2}$ (s)	λ_i (s^{-1})	β_i	β_i/λ_i (s)
55.7	0.0124	0.000215	0.0173
22.7	0.0305	0.00142	0.0466
6.22	0.111	0.00127	0.0114
2.30	0.301	0.00257	0.0085
0.61	1.1	0.00075	0.0007
0.23	3.0	0.00027	0.0001
Total		**0.0065**	**0.084**

Precursor Data

Approximate Half-Life (seconds)	Number of Fission Neutrons Delayed / Fission			Energy (MeV)
	U-233	U-235	Pu-239	
55	5.7×10^{-4}	5.2×10^{-4}	2.1×10^{-4}	0.25
23	19.7	34.6	18.2	0.46
6.2	16.6	31.0	12.9	0.41
2.3	18.4	62.4	19.9	0.45
0.61	3.4	18.2	5.2	0.41
0.23	2.2	6.6	2.7	–
Total Delayed	0.0066	0.0158	0.0061	
Total Fission Neutrons	2.49	2.42	2.93	
Fraction Delayed	0.0026	0.0065	0.0020	

Point Kinetics

- **Recall (slide 5):** $\begin{pmatrix} \varphi_1(\underline{r}) \\ \varphi_2(\underline{r}) \end{pmatrix} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \psi(\underline{r})$ $\quad \nabla^2\psi(\underline{r}) + B^2\psi(\underline{r}) = 0, \quad \psi(\underline{r}_s) = 0$

- **Local perturbations, e.g., by moving the control rods, leads to a readjustment of $\psi(\underline{r})$ that is usually slight and happens in a few milliseconds. Then, the readjusted shape rises or falls "as a whole" (harmonics) depending on whether the perturbation increased or decreased k.**
- **Point kinetics allows us to investigate the level (or average) flux assuming that the shape does not change appreciably.**
- **We average over all energy groups and write the neutron density as**

$$n(\underline{r}, t) = n(t)\psi(\underline{r})$$

- **n(t) is the total neutron density or the total power**

$$\text{Power}(t) = \sum_{g'=1}^{G} \int_V dV w_{fg'} \Sigma_{fg'} \varphi_{g'}(\underline{r}, t) \propto n(t)$$

- **Using this equation in the space- and time-dependent equations and including delayed neutrons leads to**

Point Kinetics Equations

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{1}^{6} \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \qquad i = 1,2,\ldots, 6$$

$$\rho(t) \equiv \frac{k(t) - 1}{k(t)}$$ **Reactivity**

$$\Lambda \equiv \frac{\ell}{k(t)} \cong \ell \equiv \frac{1}{v\Sigma_a(1 + L^2B^2)}$$

Mean generation time between birth of a neutron and absorption inducing fission

ℓ **Prompt neutron lifetime between birth of a neutron and absorption;**

10^{-3} to 10^{-4} for thermal reactors; 10^{-7} for fast reactors

Importance of Delayed Neutrons

$$n(t) \propto \exp\left(\frac{k-1}{\langle \ell \rangle} t\right) \equiv \exp\left(\frac{t}{T}\right)$$

$$T \equiv \frac{\langle \ell \rangle}{k-1}$$ **Reactor Period**

$$\langle \ell \rangle = (1-\beta)\ell + \sum_1^6 \beta_i \left[\frac{1}{\lambda_i} + \ell\right]$$

Prompt neutron lifetime with delayed neutrons

$$T \equiv \frac{\ell}{k-1}$$

Reactor period without delayed neutrons

The reactor period is less than 10^{-3} s for thermal reactors, if delayed neutrons are neglected. It is about 0.1 s with delayed neutrons, thus making reactor control possible.

Reactivity Feedback

- The reactivity depends on the neutron density (or power level) itself.
- This is due to the fact that k depends on macroscopic cross sections, which themselves involve the atomic number densities of the materials:

$$\Sigma(\underline{r},t) = N(\underline{r},t) \bullet \sigma(\underline{r},t)$$

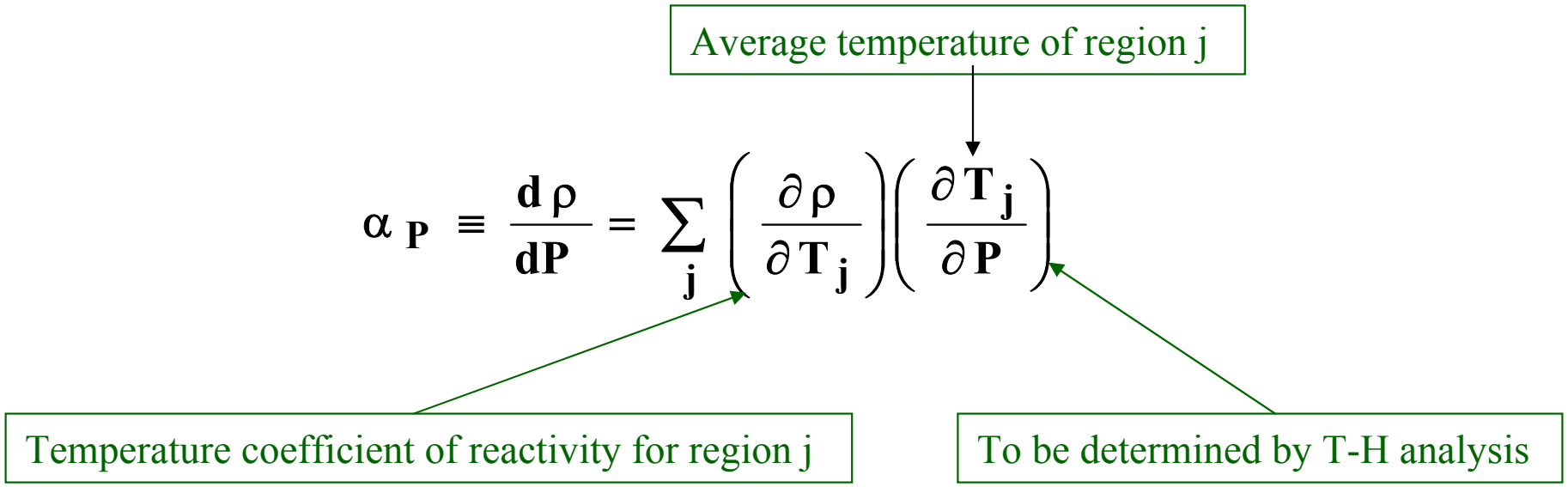
- The atomic density depends on the power level because:
  - Material densities depend on temperature, which, in turn, depends on the power distribution
  - The buildup of poisons and burnup of fuel (long-term effect).
- We write

$$\rho(t) = \delta\rho_{ext}(t) + \delta\rho_f[P]$$

$\delta\rho_{ext}(t)$ External reactivity from some reference power level P_0 for which ρ is zero.

$\delta\rho_f[P]$ Change in reactivity due to inherent feedback mechanisms.

Power Coefficient of Reactivity



Safety requirement: $\alpha_P < 0$

$$\Delta\rho_{PD} \equiv \int_0^{FP} dP\, \alpha_P$$

Power Defect: Total reactivity change from hot zero-power state to hot full-power state; compensated by control-rod insertion and soluble boron. It's less than about 0.05.

Isothermal Temperature Coefficients of Reactivity

- **Assume a uniform reactor with the temperature uniform throughout the reactor.**

$$k = (\eta_{th} f_{th} p \varepsilon) P_{NL1} P_{NL2} = k_\infty P_{NL1} P_{NL2}$$

$$\frac{d\rho}{dt} = \frac{1}{k^2}\frac{dk}{dt} \approx \frac{1}{k}\frac{dk}{dt} = \frac{1}{k_\infty}\frac{dk_\infty}{dt} + \frac{1}{P_{NL1}}\frac{dP_{NL1}}{dt} + \frac{1}{P_{NL2}}\frac{dP_{NL2}}{dt}$$

$P_{NL} \equiv \dfrac{1}{\left(1 + L^2 B^2\right)}$ Decreases with temperature because of thermal expansion; for large thermal reactors, a negligible effect.

η_{th} Usually negative effect; small for ^{235}U, larger (but still small) for ^{239}Pu.

ε Negligible effect

p Decreases due to Doppler broadening of the absorption resonances; not too important for homogeneous reactors, very important for heterogeneous reactors. Main contributor to the **prompt (fuel) temperature coefficient**.

$f \equiv \left(\dfrac{\Sigma_{a2}^F}{\Sigma_{a2}}\right)$ Cross sections depend on energy spectrum, hence on moderator temperature. Main contributor to the **delayed (moderator) temperature coefficient**. Could be positive.

Temperature Coefficients

	BWR	PWR	HTGR	LMFBR
Fuel-Temperature Coefficient				
Doppler (pcm/^{o}K)	-4 to -1	-4 to -1	-7	-0.6 to -2.5
Isothermal Temperature Coefficient				
Coolant Void (pcm / %void)	-200 to -100	0	0	-12 to +20
Moderator (pcm/^{o}K)	-50 to -8	-50 to -8	+1.0	
Expansion (pcm/^{o}K)	~0	~0	~0	- .92
Temperature Defect (%Δk/k)	2.0-3.0	2.0-3.0	0.7	0.5
Power Defect (%Δk/k)	1.5-2.5	1.5-2.5	4.0	0.8
Xe Worth (%Δk/k)	2.6	2.6	3.3	0.0
Sm Worth (%Δk/k)	0.7	0.7	0.5	0.0

Figure by MIT OCW.

General Design Criteria (10 CFR 50 Appendix A)

- **The principal design criteria establish the necessary design, fabrication, construction, testing, and performance requirements for structures, systems, and components important to safety; that is, structures, systems, and components that provide reasonable assurance that the facility can be operated without undue risk to the health and safety of the public.**

- **The General Design Criteria are also considered to be generally applicable to other types of nuclear power units and are intended to provide guidance in establishing the principal design criteria for such other units.**

General Design Criteria 27 and 28

- ***Criterion 27--Combined reactivity control systems capability.*** **The reactivity control systems shall be designed to have a combined capability, in conjunction with poison addition by the emergency core cooling system, of reliably controlling reactivity changes to assure that under postulated accident conditions and with appropriate margin for stuck rods the capability to cool the core is maintained.**

- ***Criterion 28--Reactivity limits.*** **The reactivity control systems shall be designed with appropriate limits on the potential amount and rate of reactivity increase to assure that the effects of postulated reactivity accidents can neither (1) result in damage to the reactor coolant pressure boundary greater than limited local yielding nor (2) sufficiently disturb the core, its support structures or other reactor pressure vessel internals to impair significantly the capability to cool the core. These postulated reactivity accidents shall include consideration of rod ejection (unless prevented by positive means), rod dropout, steam line rupture, changes in reactor coolant temperature and pressure, and cold water addition.**

Standard Review Plan: 4.3 Nuclear Design 2

The areas concerning reactivity coefficients include:

The applicant's presentation of calculated nominal values for the reactivity coefficients such as the moderator coefficient, which involves primarily effects from density changes and takes the form of temperature, void, or density coefficients; the Doppler coefficient; and power coefficients.

Core Composition Changes – Fission Product Poisoning

- Some fission products have large thermal absorption cross section. The poisoning effect is insignificant for fast reactors.
- Most important products: $^{135}_{54}\mathrm{Xe}$ with $\sigma_a^X \cong 3\text{x}10^6\,\mathrm{b} = 3\text{x}10^{-22}\,\mathrm{m}^{-2}$ and $^{149}_{62}\mathrm{Sm}$ with $\sigma_a^S \cong 5\text{x}10^4\,\mathrm{b} = 3\text{x}10^{-24}\,\mathrm{m}^{-2}$
- We measure the impact of a poison by calculating the reactivity decrease it causes.

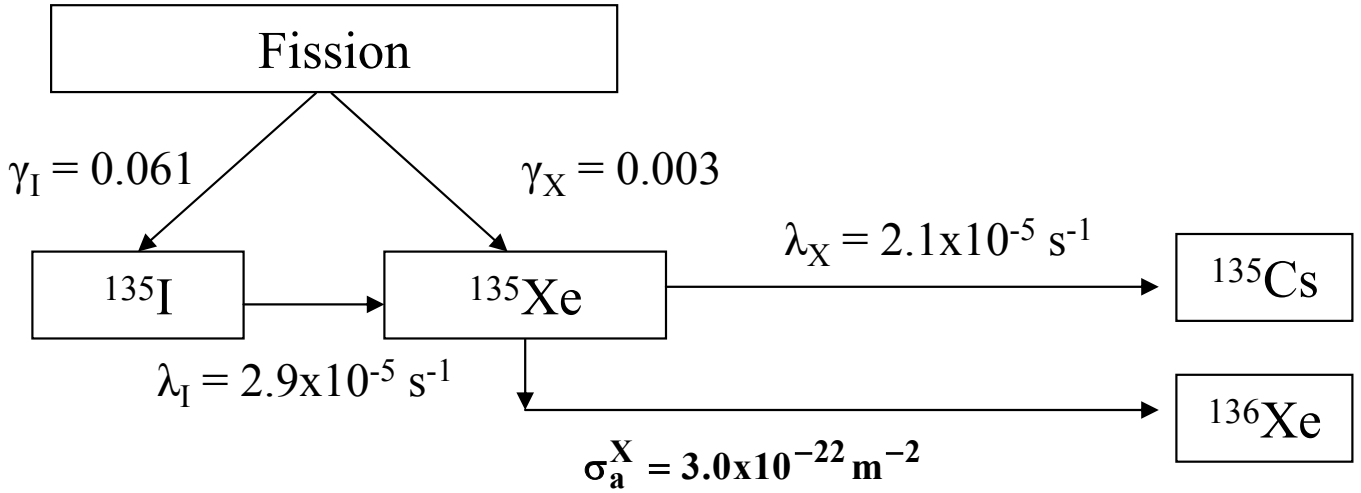
$$\Delta\rho = \rho' - \rho = \frac{k'-1}{k'} - \frac{k-1}{k} = \frac{1}{k}\left(1 - \frac{k}{k'}\right)$$

- The thermal utilization $f \equiv \left(\frac{\Sigma_{a2}^F}{\Sigma_{a2}}\right)$ is the only factor that is appreciably affected by the poison.

$$\Delta\rho = \frac{1}{k}\left(1 - \frac{k}{k'}\right) \cong \left(1 - \frac{f}{f'}\right) = 1 - \frac{\Sigma_a + \Sigma_{aP}}{\Sigma_a} = -\frac{\Sigma_{aP}}{\Sigma_a}$$

$$\boxed{\Delta\rho = -\frac{\Sigma_{aP}}{\Sigma_a}}$$

Xenon Poisoning



$$\frac{\partial I(\underline{r},t)}{\partial t} = \gamma_I \Sigma_f \varphi(\underline{r},t) - \lambda_I I(\underline{r},t)$$

$$\frac{\partial X(\underline{r},t)}{\partial t} = \gamma_X \Sigma_f \varphi(\underline{r},t) + \lambda_I I(\underline{r},t) - \lambda_X X(\underline{r},t) - \sigma_a^X \varphi(\underline{r},t) X(\underline{r},t)$$

$$I_\infty = \frac{\gamma_I \Sigma_f \varphi_0}{\lambda_I} \qquad X_\infty = \frac{(\gamma_I + \gamma_X)\Sigma_f \varphi_0}{\lambda_X + \sigma_a^X \varphi_0}$$

$$\Delta\rho = -\frac{\sigma_a^X (\gamma_I + \gamma_X)\Sigma_f \varphi_0}{(\lambda_X + \sigma_a^X \varphi_0)} \qquad \Delta\rho = -0.023 \quad \text{for} \quad \varphi_0 = 10^{17}\ n/m^2 s$$

Xenon and Reactor Shutdown

$$\frac{\partial I(\underline{r}, t)}{\partial t} = -\lambda_I I(\underline{r}, t)$$

$$\frac{\partial X(\underline{r}, t)}{\partial t} = \lambda_I I(\underline{r}, t) - \lambda_X X(\underline{r}, t)$$

A reactor operating at a flux of 2×10^{18} n/m^{2}s will have a negative insertion of reactivity of about -0.33, a sizable amount.

Graph removed for copyright reasons.

Source: Glasstone & Sesonske, *Nuclear Reactor Engineering,* Chapman & Hall, 1994