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High-Frequency Cutoff Calculations

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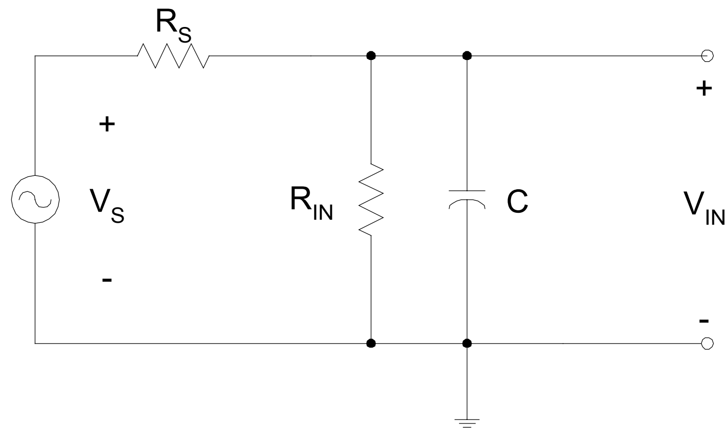


The schematic above represents a source driving an input to an amplifier. R_{IN} is the shunt input resistance associated with the amplifier input, and C represents the total shunt capacitance that includes the input capacitance and any associated connecting cable shunt capacitance. The equation for the voltage-divider transfer from source to input is:

$$\frac{V_{IN}}{V_S} = \frac{\dfrac{R_{IN} \times \frac{1}{sC}}{R_{IN} + \frac{1}{sC}}}{R_S + \dfrac{R_{IN} \times \frac{1}{sC}}{R_{IN} + \frac{1}{sC}}} = \frac{R_{IN} \times \frac{1}{sC}}{R_S \times R_{IN} + \frac{R_S}{sC} + \frac{R_{IN}}{sC}} = \frac{R_{IN}}{R_S + R_{IN} + R_S R_{IN} sC}$$

$$\frac{V_{IN}}{V_S} = \frac{1}{\frac{R_S}{R_{IN}} + 1 + R_S sC} \qquad s = j\omega; \qquad \frac{V_{IN}}{V_S} = \frac{1}{\frac{R_S}{R_{IN}} + 1 + j\omega\, R_S C} \qquad [1]$$

At low frequencies, the capacitive impedance is very large, and the simple resistive voltage divider determines the voltage transfer or “gain”:

$$\frac{V_{IN}}{V_S} = \frac{1}{\frac{R_S}{R_{IN}} + 1} = \frac{R_{IN}}{R_S + R_{IN}}; \qquad [2]$$

Let's take the example of the audiophile who wants to use his preamp outputs to drive another integrated amplifier in another room, using 30 feet of Belden 9264 single-conductor audio cable, which has a nominal capacitance of 34 picofarads per foot. Let's assume the preamp has a 600Ω output impedance, and that the integrated amplifier has line inputs with 100kΩ input impedance in parallel with another 50 pF of input capacitance. The source impedance is small enough to be ignored at low frequencies, giving a voltage-transfer ratio of 1 [equation 2]. The highest audible frequency for the average young human is 20kHz; what is the relative frequency response of the cable-source resistance-input resistance combination?

Solution: Because the ratio R_S/R_{IN} is small compared to 1, equation 1 simplifies to:

$$\frac{V_{IN}}{V_S} = \frac{1}{1 + j\omega R_S C};$$

When $\omega R_s C$ = 1, the magnitude of the denominator will equal 1.414, and the expression will have a magnitude of 0.707 which is -3dB relative to the value 1 for the purely resistive divider when R_S can be ignored. For this example, the -3dB frequency is:

$$1 = 2\pi f R_S C; f = \frac{1}{2\pi R_S C} = \frac{1}{2\pi 600 \times 1070 pF} = \frac{1}{3.77 \times 10^3 \times 1070 \times 10^{-12}} = 248 kHz$$

which is ten times higher than it needs to be! Thus, this connection is virtually flat to 20kHz.

However, suppose the preamp had a source impedance of 10,000Ω. This is still small enough so that we can ignore the ratio R_S/R_{IN} in equation 1, but now the -3dB point has dropped to about 15kHz, enough for many listeners to notice a difference.

Now let's examine the case for RF [radio frequencies]. Let's say we want to receive VHF channel 13 on a cable-TV system. This system is wired with Belden 9248 75Ω characteristic impedance cable, which has a capacitance of 16.2 pF / foot. Assuming channel 13 is the highest frequency used in this cable system, how many feet of cable can we run before high-frequency losses affect CH 13 [210-216 MHz]. In RF applications, the source impedance MUST equal the load impedance, to prevent standing waves from occurring in the transmission line. Therefore, equation [1] now becomes:

$$\frac{V_{IN}}{V_S} = \frac{1}{\frac{R_S}{R_{IN}} + 1 + j\omega R_S C} = \frac{1}{2 + j\omega R_S C} \qquad [3]$$

If we set the imaginary term equal to the real term, we will be able to find the capacitance required for the-3 dB frequency [if you wish to check this, remember that we are starting off with a resistive divider that provides a voltage "gain" of 0.5, which is -6 dB].

$$2 = 2\pi\, f\, R_S C; \quad C = \frac{1}{\pi\, R_S f} = \frac{1}{\pi 75 \times 216 \times 10^6} = \frac{1}{236 \times 216 \times 10^6} = 19.6\, pF$$

So 1.2 feet of this cable have a 3 dB loss at 216 MHz! How does a cable system work when the losses are so great, you might ask? Well, they use lots of amplifiers, lots of high-frequency boost, and fortunately TV receivers are designed to give a good picture with inputs that vary in level over a sixty decibel range! It's actually a pretty complex system. It's also not fair to characterize a cable at these frequencies as a zero resistance series element with a lumped parallel capacitance. The cable actually looks like a long distributed series of series inductances and shunt capacitances, and the analysis is slightly more complex than above. But you get the idea!

Now let's look at another classical example of high frequency losses due to shunt capacitance, the 10:1 oscilloscope probe. Most of you know that the standard oscilloscope vertical amplifier input impedance is 1 Megohm in parallel with 20 pF shunt input capacitance [it's printed on the scope faceplate]. If we used a 1:1 probe on high impedance circuits, we might load down high-impedance circuits with the input impedance of the 'scope, or affect the high frequency response of the circuit under test with the extra 20 pF of input capacitance, so the 10:1 probe has become the defacto standard. This requires that R_S becomes 9 Megohms. Now equation [1] becomes:

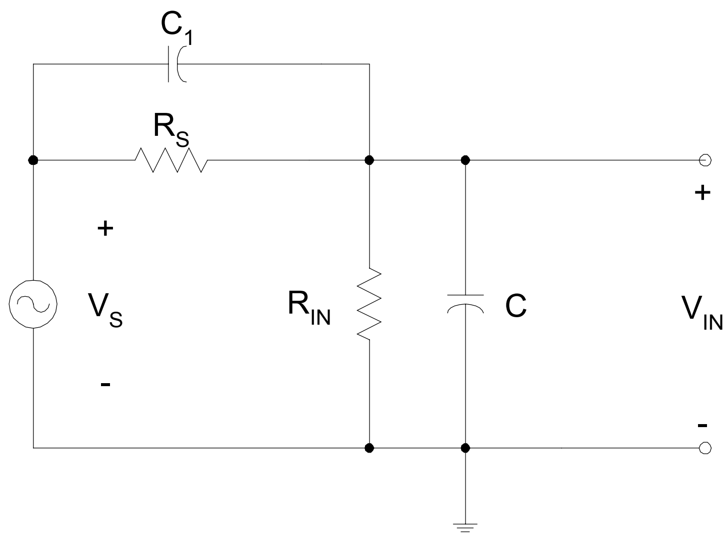
$$\frac{V_{IN}}{V_S} = \frac{1}{\frac{R_S}{R_{IN}} + 1 + j\omega\, R_S C} \qquad = \qquad \frac{1}{9 + 1 + j\omega\, R_S C} = \frac{1}{10 + j2\pi f\, 9 \times 10^6 \times 20 \times 10^{-12}} \qquad [4]$$

Again, setting the real part equal to the imaginary part and solving for f:

$$10 = 2\pi f\, 9 \times 10^6 \times 20 \times 10^{-12}; \qquad f = \frac{10}{2\pi 9 \times 10^6 \times 20 \times 10^{-12}} = 8.84 kHz$$

Thus, the 10:1 probe in combination with the input capacitance of the scope is down 3 dB at 8.84 k Hz!! Since we hope to use our oscilloscopes to view frequencies up to 150 MHz, this is a depressing development!

However, we can compensate for this situation by adding a compensating capacitor C_1 in parallel with R_S, as shown in the modified figure below.



The equation for this voltage divider is:

$$\frac{V_{IN}}{V_S} = \frac{\dfrac{R_{IN} \times \dfrac{1}{sC}}{R_{IN} + \dfrac{1}{sC}}}{\dfrac{R_S \times \dfrac{1}{sC_1}}{R_S + \dfrac{1}{sC_1}} + \dfrac{R_{IN} \times \dfrac{1}{sC}}{R_{IN} + \dfrac{1}{sC}}} \qquad [5]$$

Equation [5] expands to:

$$\frac{V_{IN}}{V_S} = \frac{R_{IN}\left(sC_1R_S + 1\right)}{R_S\left(sCR_{IN} + 1\right) + R_{IN}\left(sC_1R_S + 1\right)} \qquad [6]$$

If we choose: $C_1R_S = CR_{IN};\ or\ C_1 = C\dfrac{R_{IN}}{R_S};\quad C_1 = 20\,pF\,\dfrac{1\,M\Omega}{9\,M\Omega} = 2.22\,pF$ [7]

then equation [6] reduces to:

$$\frac{V_{IN}}{V_S} = \frac{R_{IN}}{R_S + R_{IN}} \qquad [8]$$

which is independent of frequency. In real life, C_1 is usually made adjustable so that it may be adjusted to exactly compensate for the input capacitance C. The scope probe is attached to a calibration square wave signal generated by the oscilloscope and the adjustable capacitor in the probe is adjusted to produce a flat top on the square wave.