2.081J/16.230J Plates and Shells

Quiz
Monday, March 24

Problem 1 *[Must be completed in the class.]*

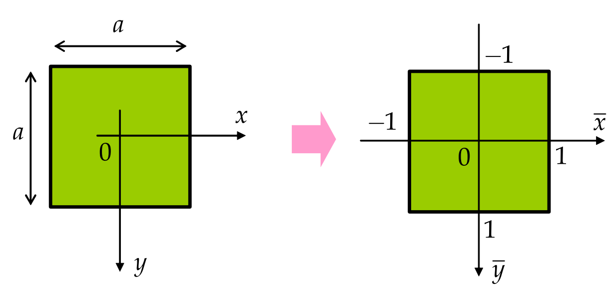
A simply supported square elastic plate $(a \times a)$ is loaded by a transervese load $q(x, y)$. The origin of the coordinate system is at the center of the plate. The shape of the deflected plate was precisely measured and then fited by the following function:

$$w(\bar{x}, \bar{y}) = w_0 \left(1 - \bar{x}^2\right)\left(1 - \bar{y}^2\right)$$

where

$$\bar{x} = \frac{x}{a/2} \quad ; \quad \bar{y} = \frac{y}{a/2}$$

and w_0 is the central deflectioin of the plate $w_0 = w(0, 0)$.



(a) Check if the kinematic boundary conditions are satisfied.

(b) Calculate the components of the bending moment tensor $M_{\alpha\beta}$ and effective shear force vector V_α .

(c) Check if the static boundary conditions are satisfied (both moments and effective shears).

(d) Determine the magnitude and the distribution of the transverse loading $q(\bar{x}, \bar{y})$ that equilibrates the plate.

(e) Is the plate resistance derived from the direct bending moments M_{xx} and M_{yy} or from the twisting moment M_{xy}?

(f) Calculate the relationship between w_0 and q and compare it with the exact solutioin to be found in the note (see Section 4.1.5). Discuss the reason for a large error.

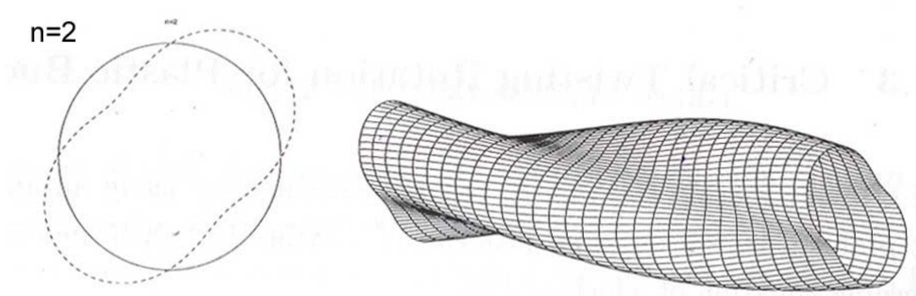
Problem 2 *[Take-home problem due to Monday 27, March 1:00pm.]*
A long elastic cylindrical shell made of mild steel ($E = 210GPa$, $\nu = 0.3$) is subjected to a twisting moment M producing a uniform field of in-plane shear membrane force $N^\circ_{x\theta}$:

$$N^\circ_{\alpha\beta} = \begin{vmatrix} 0 & N^\circ_{x\theta} \\ N^\circ_{x\theta} & 0 \end{vmatrix}$$

The buckling mode consists of a number of circumferential waves that spiral around the cylinder from one end to the other according to:

$$w = \sin\left(\frac{\bar{m}x}{a} - n\theta\right)$$

where, as in the notes[1], $\bar{m} = m\pi a/l$. It was shown in the notes that the smallest buckling force is obtained from $n = 2$. The resulting buckling mode is shown in the figure below.



(a) Derive an expression for the critical twisting moment M_{cr} causing buckling.

[1] Here, the following notations are used:

a : radius, h : thickness, l : length

(b) Keeping the radius a constant, how much should the thickness of the shell increase to double the magnitude of the twisting moment?

(c) Keeping the thickness of the shell constant, how much should the radius of the shell increase to double the magnigude of the twisting moment?

(d) Assuming $a/h = 100$, what should be the length-to-radius ratio l/a so that half of the spiral is formed ($m = 1$), as shown in the figure above?

Problem 3 [EXTRA CREDIT] *[Take-home problem due to Monday 27, March 1:00pm.]*
Derive an expression for the load-caring capacity of a clamped, circular, rigid-plastic plate under uniform transverse load using Tresca yield condition.