1st problem set,

1. Page 308, #3. Sketch the graphs <u>**very carefully**</u>.
2. Page 308, #4.
3. On page 303 of DeGroot & Schervish, we read that

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$$

   This identity is true if α and β are <u>**any**</u> positive numbers.
   $\alpha+3$ is a positive number.
   Therefore this identity is true if $\alpha+3$ is put in place of α.

   Use that fact to find $E\left(X^3\right)$ if $X \sim \text{Beta}(\alpha, \beta)$. By using identities satisfied by the gamma function, simplify the result so that the gamma function is not mentioned in your bottom-line answer.

4. Page 320, #1.
5. On page 314 of DeGroot & Schervish, the authors appear to assume the reader does not know matrix algebra. For any matrix A, let A' be its transpose, so that in particular

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} x_1, x_2 \end{bmatrix}.$$

   Let $f(x_1, x_2)$ be just as on page 313 of DeGroot & Schervish. Let $V = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$; this 2×2 matrix can be considered the variance of the random vector $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$.

   (a) Show that $f(x_1, x_2) = \dfrac{1}{2\pi} \cdot \dfrac{1}{\sqrt{\det V}} \cdot \exp\left\{\dfrac{-1}{2} \cdot (x-\mu)'V^{-1}(x-\mu)\right\}$ where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$.

   (b) Rely on **(5.12.8)** on page 317; use the same notation and assume that what it says there is true; don't try to prove it from scratch. Let $A = [a_1, a_2]$. Let $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$. Show that $Y = AX + b$. Show that $E(Y) = AE(X) + b$. Show that $\text{Var}(Y) = AVA'$.

6. Page 318, # 1.
7. In **Example 5.12.4** on pages 317-318, find the probability that the sum of the heights of the husband and the wife exceeds 140 inches.