MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
Department of Mechanical Engineering

**6.050J/2.110J** **Information and Entropy** **Spring 2003**

# Final Exam Solutions

## Solution to Problem 1: Knowin' the Nomenclature (9%)

### Solution to Problem 1, part a.

______Boltzmann's Constant or entropy______ can have units of Joules/Kelvin.

### Solution to Problem 1, part b.

______ASCII______ is a fixed length code.

### Solution to Problem 1, part c.

The part of the heat-engine cycle without change in entropy is ______adiabatic______.

### Solution to Problem 1, part d.

______GIF______ is a reversible image compression technique.

### Solution to Problem 1, part e.

The ___Hamming distance___ is calculated by taking the number of ones in the XOR of two binary strings.

### Solution to Problem 1, part f.

The energy values of an electron wavefunction in a square well are ______discrete______, not ______continuous______.

### Solution to Problem 1, part g.

A thermodynamic quantity that is the same for two systems in contact is ______intensive______.

### Solution to Problem 1, part h.

The maximum efficiency of a heat engine is named after ______Carnot______.

### Solution to Problem 1, part i.

______Finlayson______ was the most interesting aspect of this course.

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# Solution to Problem 2: Behaving Badly (10%)

## Solution to Problem 2, part a.

The truth table and transition diagram for the NANNY gate are:

| AB | Out |
|---|---|
| 00 | 0 |
| 01 | 0 |
| 10 | 1 |
| 11 | 0 |

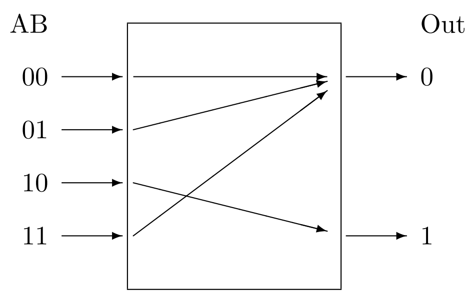


Figure F–3: Solution to NANNY Gate Truth Table and Transition Diagram

## Solution to Problem 2, part b.

Since the nanny always reminds the child, A=1 always. Thus the remaining probability is split evenly between AB=10 and 11.

AB=00: 0 AB=01: 0 AB=10: 0.5 AB=11: 0.5

Out=0: 0.5 Out=1: 0.5

## Solution to Problem 2, part c.

Is it reversible? no Is it universal? yes Is it deterministic? yes

## Solution to Problem 2, part d.

How many NANNY gates would you need to make a NOT gate? 1 How would you do it?

Set A=1 and use B as the input.

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# Solution to Problem 3: Parity, Code-rates, and Framing Errors (10%)

## Solution to Problem 3, part a.

The code rate of your scheme is

code rate: 8/9

### Solution to Problem 3, part b.

With your error checking scheme it is not always possible to detect this sort of error.

> No, consider the message "000000000." The first eight bits are the data, the ninth bit is the parity. If you lose any bit, the result, "00000000," is still a valid message. Thus you do not detect the error.

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## Solution to Problem 4: Architecture – Building Of Major Buildings (15%)

### Solution to Problem 4, part a.

$I_{cat}$ in bits and the three probabilities $I$, $B$, and $U$ are:

$I =$ 1/3 $B =$ 1/3 $U =$ 1/3 $I_{cat} =$ 1.58 bits

### Solution to Problem 4, part b.

The bounds on the probabilities are:

$I_{min}$ 0.05 $I_{max}$ 0.24 $B_{min}$ 0 $B_{max}$ 0.95 $U_{min}$ 0 $U_{max}$ 0.76

### Solution to Problem 4, part c.

Larger/smaller than in 1950?: Larger

Reasoning: Raising the average aesthetic index is analogous to raising the temperature, which will increase the entropy as long as the average index is less than 2.

### Solution to Problem 4, part d.

Aesthetic index up/down?: Down

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## Solution to Problem 5: Green Eggs and Hamming (15%)

### Solution to Problem 5, part f.

How long a binary string (in bits) is needed to encode one order?

bits: 2

### Solution to Problem 5, part f.

Give a suitable code using this number of bits.

Green Eggs: 00 Pancakes: 01 Greencake Combo: 10

### Solution to Problem 5, part g.

What is the Hamming distance required between any pair of codewords to achieve single error-correction?

Minimum Hamming Distance: 3

### Solution to Problem 5, part d.

A suitable code with this Hamming distance using five bits per order is:

Green Eggs: 00000 Pancakes: 00111 Greencake Combo: 11100

### Solution to Problem 5, part g.

average bit length is less/equal/more: less

### Solution to Problem 5, part f.

A Huffman code that takes advantage of these probabilities is:

Green Eggs: 00 Pancakes: 1 Greencake Combo: 01

### Solution to Problem 5, part g.

The average code length of this code in bits is:

average length: 1.5

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## Solution to Problem 6: Rational Spammers (15%)

### Solution to Problem 6, part a.

The process-flow diagram below

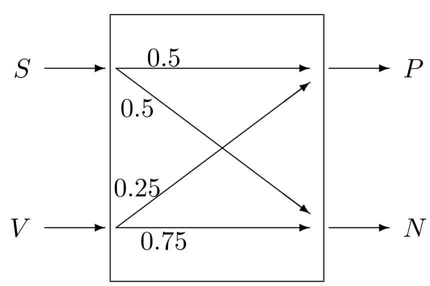


Figure F–4: Solution to Spam Filter Transition Diagram

### Solution to Problem 6, part b.

What are $P$ and $N$?

$P =$ 0.3 $N =$ 0.7

### Solution to Problem 6, part c.

Continuing with this model of the spam filter as a channel, determine the input information $I_{in}$, the output information $I_{out}$, the noise $N$, the loss $L$, and the mutual information $M$, all expressed in bits.

$I_{in} =$ 0.72 bits $I_{out} =$ 0.88 bits $N =$ 0.848 bits $L =$ 0.688 bits $M =$ 0.032 bits

### Solution to Problem 6, part d.

Each spam message you have to read costs you one minute, and half of them get through the filter. Each valid message takes one minute to read, and three-quarters of those get through the filter. One-quarter of the valid messages are lost, resulting in an extra five minutes apiece. In the absence of the spam filter it takes you one minute per message. Thus, the point after which the spam filter starts to save you time is represented by the solution to the following equation:

$$S(1 \text{ min} \times 0.5) + (1 - S)(1 \text{ min} \times 0.75 + 5 \text{ min} \times 0.25) = 1 \text{ min} \tag{F–1}$$

This occurs at:

spam rate: 2/3

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## Solution to Problem 7: I Am... Getting... So Hot, I Want to Open the Refrigerator Door (10%)

### Solution to Problem 8, part a.

Will the room cool down as a result?

yes/no: no

### Solution to Problem 7, part b.

Why or why not?

Since the refrigerator is moving heat to and from the same space, at best the room temperature cannot change. Since the refrigerator actually generates heat in the process of doing its work, the room will heat up.

### Solution to Problem 7, part c.

Will the area of his bed cool down as a result?

yes/no: yes, in the short term

### Solution to Problem 7, part d.

Why or why not?

In the short term, the area near his bed will cool down. But over time, the area near his bed will equilibrate with the rest of the room, returning us to part (b)

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# Solution to Problem 8: A Dirty Sort of Business (15%)

## Solution to Problem 8, part a.

The Carnot efficiency for MUD is:

$\eta$ for MUD: 0.5

## Solution to Problem 8, part b.

| | MUD | CRUD | Total |
|---|---|---|---|
| Heat into car interior | 1 | -3 | -2 |
| Heat extracted from Muffler | 2 | -4 | -2 |
| Work input | -1 | 1 | 0 |

Table F–4: Solution to Product X heat extraction data, in Watts

## Solution to Problem 8, part c.

Explain what is wrong: As can be seen from the table above, Bitdiddle's design does not work as intended. While the car interior is cooled as expected, the muffler heats up. Thus we realize that Bitdiddle's design as conceived violates the second law of thermodynamics. The error originated in the assumption that a reversible heat engine could have less than the Carnot efficiency. If a heat engine has lower efficiency it must increase the entropy of the universe.