Atmospheric contribution to the dissipation of the gravitational tide of Phobos on Mars

by

Vid Thayalan
M.Sc. University of Toronto, 2005

Submitted to the Department of Earth, Atmospheric and Planetary Sciences
in partial fulfillment of the requirements for the degree of

Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2008

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Abstract

Here, we investigate the possibility of a significant atmospheric contribution to the
tidal dissipation of the Phobos-Mars system. We apply the classical tidal theory and
we find that most of gravitational forcing is projected into the first symmetric Hough
mode which has an equivalent depth of about 57 km and it is significantly trapped in
the vertical. Therefore, no significant dissipation occurs through the vertical propa-
gation of energy and subsequent breaking of the tidal wave as the wave amplifies with
height. Alternatively, from the energy stored in the first trapped mode we estimate
that the time scale required for the dissipative mechanisms to account for the total
dissipation of the tides is of order $10^2 s$. This time scale is unrealistically short, since
it would contradict observations of propagating thermal tides in Mars atmosphere.
Therefore we conclude that the dissipation of the tidal potential that explains the
observed acceleration of Phobos most likely occurs within the solid planet.

Thesis Supervisor: Maria T. Zuber
Title: E. A. Griswold Professor of Geophysics
Acknowledgments


I thank Maria T. Zuber, Head of the Department, for introducing me to this problem and for introducing me to Professor Dick Lindzen and to Roberto Rondanelli. The idea for the subject of this thesis originated with Maria Zuber, and she is a co-author on this paper for this contribution.

Dick Lindzen, my second co-author, was the expert resource in the methodology used to solve this problem.

I dedicate this thesis to Roberto Rondanelli, my friend and third co-author on this paper. I learned the subject of atmospheric tides by watching Roberto in action. I thank him for being a kind mentor, an inspiring and well-balanced scientist, and a good human being. I would not have written this thesis, but for Roberto’s insistence.

My contribution was to perform all the calculations in the paper and to obtain all the results, with guidance from Roberto.

My friends here at MIT are what I will cherish most from my experience here. I thank Dr. Danny Abrams for being a good friend and mentor. I thank fellow students Alex Petroff, Krystle Carina Catalli, Scott Burdick, Einat Lev, and Jimmy Elsenbeck. I learned a lot from Jay Barr. I especially learned a lot from David Forney, and I thank him for it.

Finally, I thank Professor Daniel Rothman for admitting me to EAPS and giving me this wonderful opportunity. I thank Professors Bradford H. Hager and Dick K. P. Yue and the Linden Fellowship of the Earth Systems Initiative for financially supporting me through MIT. I thank Professor John W. M. Bush, my friend, for teaching me fluid mechanics, and also for being an honest, kind, friendly, and inspiring human being and a true role model to emulate. I thank him for going above and beyond his call of duty to help and guide me. I thank Maria T. Zuber again for her patience and what I can only call her maternally-instinctive guidance and advice.

I also thank Dr Vicki McKenna and Carol Sprague in the Graduate Studies Office
for all their advice and their patient help.

You don’t realize how wonderful a place MIT is until you leave it. I am blessed to have had this opportunity in my life—it has been the adventure of a lifetime—and I am grateful for it.
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Chapter 1

Introduction

The tidal exchange of angular momentum between Phobos, the inner-most natural satellite of Mars, and Mars produces an acceleration in the orbital velocity of Phobos and an increase in the rotation rate of Mars. The most recent estimate for this acceleration comes from the observation of the shadow projected on Mars by the transit of Phobos, as measured by the Mars Orbiter Laser Altimeter instrument on the Mars Global Surveyor spacecraft (Bills et al., 2005). This observed acceleration of $6.631 \times 10^{-9}$/year is consistent with an energy dissipation of 3.34 MW. Assuming that all the dissipation occurs in the solid interior of Mars, modeled as a homogeneous Maxwell viscoelastic solid, the effective viscosity of the planet was inferred to be $8.7 \times 10^{14}$ Pa s (Bills et al., 2005). The implicit assumption in this calculation is that the tidal dissipation occurs exclusively in the solid planet. In this note we deal with the possibility that significant dissipation occurs in the planetary atmosphere by investigating the response of the Martian atmosphere to the gravitational tidal forcing by Phobos.

The best known case of rapid orbit evolution in the solar system is Phobos, the inner of the two natural satellites or moons of Mars. Since its discovery by Asaph Hall in 1877, the orbit of Phobos has been intensively studied by Earth-based observers and spacecraft. Unlike the Earth's moon, Phobos is very close to Mars, at a mean distance of 9378 km, given Mars mean radius of 3395 km and has an orbit period of only 7.65 hours, well within the synchronous orbit distance. Indeed, Phobos can be
argued to be the best studied, and the most theorized about, natural satellite in the solar system.

The newest observations of the orbit position of Phobos were made with the MOLA instrument on the Mars Global Surveyor spacecraft. The MOLA instrument observed 15 transits of the shadow of Phobos across the surface of Mars, and has directly measured the range to Phobos once. These observations provide an improved estimate of the rate of tidal dissipation within Mars. The rate of secular acceleration in the along-track motion is (136.7 ± 0.6)×10⁻⁵ deg/yr², which corresponds to a rate of change in orbital angular velocity of 6.631 ±0.029×10⁻²/yr, the highest measured for any natural satellite in the solar system (Bills et al., 2005). The energy dissipation rate is 3.34 ± 0.01MW.

It has long been understood that tides can be effective in transferring angular momentum from the spin of a planet to the orbit of a satellite, or vice versa, depending on whether the satellite is above or below the synchronous elevation, at which the satellite orbital period matches the planetary rotation period (Bills et al., 2005). Previous estimates of the secular acceleration of Phobos have been used to calculate the tidal quality factor, or $Q$, or Mars. $Q$ is a measure of the rate of tidal dissipation. $Q$ is defined as the maximum energy stored in the tide, divided by the energy dissipated per cycle. High $Q$ imply low rates of dissipation per unit forcing, that is, the establishment of a tidal field from which follows the subsequent delayed energy loss from the system in the dissipation of the established tidal field. Low $Q$ imply high rates of dissipation. $Q$ for Mars has been estimated to be 100 ± 50 (Smith and Born, 1976; Yoder, 1982).

There are many potential sources of dissipation. In the interaction between planet and satellite, there is the possibility of dissipation within the solid body of the satellite. Bills et al. (2005) assume that the dominant effect is due to tidal energy dissipation within solid Mars. They assume that Mars responds linearly to imposed gravitational potentials (potentials which will cause Mars to deform), and thus give rise to an induced potential. These potentials may be expanded in terms of spherical harmonics. The proportionality constants between the induced potential at a harmonic degree
and the gravitational potential at the same degree are called Love numbers. Love numbers depend on the internal structure and composition of Mars. Because Phobos is so near to Mars, there are nonnegligible contributions to solid planet tidal evolution from harmonic degrees 2, 3 and 4. But because the elastic tidal Love numbers for Mars are observationally constrained only at degree 2, the observed acceleration was used to model Mars as a homogeneous Maxwell viscoelastic solid. This implied an effective viscosity of $8.7 \pm 0.6 \times 10^{14} \text{Pas}$ (Bills et al., 2005). Hence, information about the orbit acceleration of Phobos was used to infer information about the bulk properties of the interior of Mars.

The bulk of tidal dissipation in the Earth-Moon system occurs in the ocean. On Mars, there are no oceans today. There is, however, a thin atmosphere. Therefore, there is the possibility of a significant atmospheric contribution to the tidal dissipation of the Phobos-Mars system.
Chapter 2

Theoretical Background

2.1 Classical tidal theory

Here, we briefly summarize the assumptions and main results of the application of classical tidal theory to a thin atmospheric shell as assumed for Mars. Detailed derivations and discussions can be found in Chapman and Lindzen (1970). The tidal fields are assumed to be small perturbations about a basic state which, to a first approximation, can be considered steady with respect to the phase speed of the wave \( c \approx 240 \text{m/s} > U \approx 10 \text{m/s} \) and in hydrostatic balance. The inviscid momentum equations, the adiabatic thermodynamic equation and the ideal gas law are linearized around this basic state.

Since we are interested in the steady state response to a forcing with a known period and magnitude, the resultant tidal fields are assumed to be periodic in time, having the same frequency as the forcing \( (\sigma = 2 \omega_L) \), where \( \omega_L = \omega - \omega_M \) is the relative rotation of the moon with respect to the planet. Here, we are interested in the response to the dominant semidiurnal component of the forcing having a wave number \( s=2 \). Under these assumptions the equations governing the atmospheric response are separable. The latitudinal variation of the fields is governed by the Laplace Tidal equation,
\[
\frac{d}{d\mu} \left( \frac{1 - \mu^2}{f^2 - \mu^2} \frac{d\Theta_n^s}{d\mu} \right) - \frac{1}{f^2 - \mu^2} \left( \frac{8 f^2 + \mu^2}{f f^2 - \mu^2} + \frac{s^2}{1 - \mu^2} \right) \Theta_n^s + \frac{4a^2 \omega^2}{gh_n^s} \Theta_n^s = 0, \tag{2.1}
\]

where \(\mu = \cos \theta\), \(\theta\) is the colatitude, \(f = \sigma / 2\omega\). \(\Theta_n^s\) are the solutions of the equation, known as Hough functions and \(h_n^s\) are the equivalent depths which represent the eigenvalues of the problem. The solutions \(\Theta_n^s\) and the corresponding \(h_n^s\) are found by writing \(\Theta_n^s\) as an infinite series of (normalized) associated Legendre polynomials. When the series expansion is substituted into equation 2.1, the problem reduces to finding the solution of an infinite set of linear equations for the coefficients (Chapman and Lindzen, 1970). The series expansions are believed to be a complete asymptotic representation of the solutions and have an optimal truncation depending on the number of the mode (Ioannou and Lindzen, 1993).

The vertical structure of the modes is found through the solution of the equation

\[
\frac{d^2 y_n}{dx^2} + \left( \frac{N^2 H^2}{gh_n} - \frac{1}{4} \right) y_n = 0, \tag{2.2}
\]

where \(y_n\) is related to the rest of the tidal fields through the divergence field \(\chi_n\) as \(y_n = \chi_n e^{(x/2)}\), where \(x = \int^z \frac{dz}{H}\) is the log-pressure vertical coordinate. Assuming a perfectly spherical planet, the lower boundary condition \(w = 0\) can be written as,

\[
\gamma h_n \left[ \frac{dy_n}{dx} + \left( \frac{H}{h_n} - \frac{1}{2} \right) y_n \right]_{x=0} = \frac{i\sigma \Omega_n}{g} \tag{2.3}
\]

At the upper boundary, we impose the radiation condition, which for propagating modes is equivalent to choosing the solution that has an upward flux of energy (Wilkes, 1949). Once the gravitational forcing and the eigenvalues are known, equation 2.2 can be numerically integrated following a procedure similar to the one described by Lindzen (1990, p. 297). In sum, any given field \(f\), can be written as,

\[
f(\theta, \phi, z, t) = \sum_n f_n(z) \Theta_n^s(\theta) e^{i(\sigma t + \phi)}. \tag{2.4}
\]

For the case of the horizontal velocities it is customary to write the fields in terms
of associated functions $U_n^a$ and $V_n^a$ which contain all the $\theta$-dependence of the fields.

For simplicity, we have assumed a perfectly spherical Mars, without including any energy dissipation in the form of frictional drag with the surface topography, despite the degree 2,2 equatorial features that includes the Tharsis bulge (Smith et al., 1999).

### 2.1.1 Tidal Potential

The gravitational tidal potential exerted by Phobos at an arbitrary point $\mathbf{a}$ over the surface of the planet, measured from the center of Mars can be written as

$$\Omega = -\frac{GM_s}{D - \mathbf{a}} = -\frac{GM_s}{D} \sum_{n=0}^{\infty} \left( \frac{a}{D} \right)^n P_n(\mu),$$

where $G$ is the gravitational constant, $M_s$ the mass of the satellite, and $D$ is the position vector of the satellite in a coordinate system centered at the center of mass of the planet. The first two terms in the series expansion 2.5 are a constant potential with no consequence in the forcing, and a term which gives an homogeneous forcing equivalent to the acceleration experienced by the center of mass of the planet. For the purpose of specifying the tidal forcing we will be concerned only with the term of order $\left( \frac{a}{D} \right)^3$. Then, the semidiurnal component of the gravitational potential due to satellite of mass $M_s$ has the form,

$$\Omega \approx -\sqrt{\frac{3}{5}} \frac{GM_s a^2}{D^3} \overline{P_2}(\cos \theta)(1 - \frac{\epsilon^2}{2}),$$

where $\overline{P_2}(\mu)$ is the normalized associated Legendre polynomial of degree (2,2) and $\epsilon$ is the orbital inclination of the satellite. Despite Phobos being relatively close to the surface of Mars compared to the Moon, the smaller mass of Phobos dictates that the gravitational potential of the Moon over the Earth is still about two orders of magnitude larger than the gravitational potential of Phobos over Mars.
Chapter 3

Results

3.1 Characteristics of the response

As expected from the relatively large value of $f$, the expansion of the $\Theta_n^p$ are dominated by the contribution of the corresponding $P_n^2$, that is, only a small correction from the spherical harmonics appears as a result of the rotation of the planet relative to the rotation of the moon. As a consequence, since the gravitational forcing can be expressed in terms of $P_2$ most of the excitation goes into the first symmetric Hough mode $-\Theta_2^2$. From table 3.1 we see that the contribution of each of the higher modes to the expansion of the gravitational potential is reduced by about two orders of magnitude relative to the preceding mode.

One can get a rough idea of the vertical propagation of the modes by considering an isothermal version of equation 2.2. In that case the quantity $N^2H^2$ becomes simply $\kappa H$, where $\kappa = (\gamma - 1)/\gamma$ and $\gamma$ is the ratio of the heat capacity at constant pressure and the heat capacity at constant volume. Therefore, when $4\kappa H/h_n$ is less than one the corresponding mode propagates in the vertical, whereas when it is more than one the mode is vertically trapped. From table 3.1 we see that at least the first two modes, which are the ones that concentrate the bulk of the gravitational forcing, are vertically trapped. The consequence for the problem at hand is that the evaluation of the tidal dissipation cannot be done directly by calculating the upward energy flux $\overline{p_w}$, where $p$ and $w$ are the pressure and vertical velocity tidal fields, respectively. We
Table 3.1: Coefficients of the expansion of the first four Hough modes in terms of normalized associated Legendre polynomials. Also shown are the equivalent depths corresponding to each mode (exact and approximated) and the parameter $4\kappa H/h_n$, indicative of the vertical propagation of each mode. The data is for Mars and Phobos. $f = -2.2152$ and $s = 2$

<table>
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<th>4</th>
<th>6</th>
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<td>3.91756</td>
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<td>$\sigma^2 \omega^2 \frac{n(n+1)+f}{n(n+1)+2f}$</td>
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<td>15.5534</td>
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<td>0.6203</td>
<td>1.3566</td>
<td>2.3636</td>
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</table>

$\Theta_2$ | $\Theta_4$ | $\Theta_6$ | $\Theta_8$ |
<table>
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<tr>
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<td>-0.05445</td>
<td>0.99463</td>
</tr>
<tr>
<td>$P_{8,2}$</td>
<td>-0.00000</td>
<td>0.00127</td>
<td>-0.08799</td>
</tr>
</tbody>
</table>

Figure 3-1: The magnitude of some tidal fields for the first Hough mode ($n = 2$) and the basic temperature profile for the two idealized soundings, Viking 1 (solid line) and Viking 2 (dashed line) (Profiles are idealizations from the Viking soundings (Seiff and Kirk, 1977)). a) Basic vertical temperature profiles. b) $Q^2$, index of refraction squared. c) Magnitude of the zonal velocity perturbation $|v|$. d) Magnitude of the pressure perturbation $|p|$. 

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know that for vertically propagating modes the tidal fields grow with height as $\rho_0^{-\frac{1}{2}}$ and therefore it is expected that at some height the waves will become unstable and will break, dissipating the upward energy flux through the generation of turbulence. Such an explicit calculation can still be done for the higher modes which propagate in the vertical, that is $n \geq 6$; however, as we will see, we find very modest contributions through this mechanism.

A more realistic approach is to consider a a varying vertical profile and evaluate numerically equation 2.2. We use as a reference the vertical profiles taken during the descent of the Viking landers (Seiff and Kirk, 1977). It is evident from the Viking observations that these profiles show not only the background state of the atmosphere but also wave activity superimposed. We construct two profiles V1 and V2 (see Fig. 3-1.a) by smoothing the original Viking soundings and taking only the mean vertical lapse rate over five or six layers of about 40 km in height. We use 1000 vertical levels over a vertical domain of 200 km using the numerical equivalents of the boundary conditions. Note that the magnitude of the tidal fields is not particularly sensitive to the characteristics of the basic temperature profile.

We see in Fig.3-1.b that the refractive index $Q^2$ remains negative over the entire
atmosphere. As a consequence the magnitude of the velocity and temperature tidal fields grow only slowly with height, the slow growth is due to the competing effect of the decrease in density with height which amplifies the response and the decaying nature of the solution. In turn these tidal fields remain within the same order of magnitude throughout the atmosphere (Fig.3-1.c). Our model does not consider explicitly any dissipative mechanism that in the real atmosphere will likely damp any slow grow with height. The pressure perturbation has a different behavior showing a strong decay in amplitude with height (Fig. 3-1.d).

To illustrate the relative importance of the higher order terms in the expansions Figs. 3-2 and 3-3 show the latitudinal variation of the magnitude of the surface perturbation of $u$ and $v$ respectively. It can be seen that the magnitude of the sum of the first four modes is about the same order of magnitude ($\sim 10^{-5}$ m/s) and it has almost the same latitudinal distribution as the first Hough mode. We will see next that for the purpose of calculating the order of magnitude of the energy stored in the tide, it suffices to consider only the first Hough mode.

**Energy calculations**

Here, instead of explicitly calculating the dissipation in the atmosphere as the vertical flux of energy of the wave, we explore the possibility that the known value of the
dissipation required to explain the orbital acceleration of Phobos occurs through the
dissipation of the tidal energy exclusively in the atmosphere. To this end we calculate
the total energy of the tide per unit volume $E$, which can be expressed as the sum of
a kinetic and a available potential contribution (see e.g. Gill, 1982),

$$E = \frac{1}{2} \rho_0 (u^2 + v^2 + w^2) + \frac{1}{2} \rho_0 \frac{g^2}{N^2} \left( \frac{\rho}{\rho_0} \right)^2$$  \hspace{1cm} (3.1)

We integrate this expression for each of the Hough modes in which the response
has been decomposed. As before, the energy is concentrated in the gravest mode,
$n = 2$ and the higher order contributions are at least two orders of magnitude smaller
than for $n = 2$. We look at the quantity

$$\tau_{\text{diss}} = \int_V \frac{E dV}{dt},$$  \hspace{1cm} (3.2)

where the integral is taken over the volume of the atmosphere, $V$. When $dE/dt$
is $3.34 \times 10^6 W$, the dissipation consistent with the acceleration of Phobos orbit, then
$\tau_{\text{diss}}$ is the time scale required to account for such a dissipation within the atmosphere.
The value of $\tau_{\text{diss}}$ for the first mode ranges from $1.01 \times 10^2$ to $2.33 \times 10^2$ s, depending
on whether the temperature vertical profile used is $V2$ or $V1$. The contribution of
the higher order modes to the total energy (or to the total dissipation time scale) is
negligible.
Chapter 4

Concluding Remarks

The dissipation time scale obtained in the previous section is extremely short compared to known dissipative processes in the Martian atmosphere such as the radiative damping time constant in the lower atmosphere ($\sim 10^5$ s) (Zurek et al., 1992). As discussed by Chapman and Lindzen (1970), a local condition for neglecting the dissipation in the formulation of the classical tidal theory is that $\tau_{\text{diss}} \gg \tau_{\text{tide}}$. For the Phobos-Mars system $\frac{\tau_{\text{tide}}}{2\pi} = 3.18 \times 10^3$ s and therefore our assumed source of dissipation would not only be important in calculating the tidal response but would be a dominant term in such a way as to effectively damp the tidal response. We can be confident that dissipation time scales as short as that required to explain the tidal dissipation resorting exclusively to the Martian atmosphere are not realistic. Solar diurnal and semidiurnal tides have an even larger forcing period, yet still they have been observed in the surface pressure record, and also the wave structure of the Viking landing profiles has been attributed to vertically propagating thermal tides. No such observations would be possible if the $\tau_{\text{diss}}$ were as small as required to match the Phobos orbital acceleration. Therefore our calculations give confidence that the source of the tidal friction most likely resides in the planetary interior.

All effects of the surface topography of Mars on atmospheric flow fields are neglected. Dissipative processes like turbulence and viscosity are also ignored.

We can calculate the time that takes the tide to establish itself $\tau$, by calculating the vertical component of the group velocity, which is the speed at which energy prop-
agates vertically. We approximate $C_{gz} = -\frac{N\kappa_h}{m^2}$, where $N$ is the buoyancy (natural frequency of the system and $\kappa_h$ and $m$ are the vertical and horizontal length scales respectively. This approximation holds since the vertical lengthscale is much smaller than the horizontal lengthscale, both of which are much smaller than $N\kappa_H/f$, that is the wave is fast compared to the rotation of the planet. For the $n = 2$ mode, the horizontal wavelength is half the circumference of Mars. We take the vertical lengthscale to be the distance over which the magnitude of the pressure perturbation $p_2(z)$ for the $n = 2$ mode decays by about two orders of magnitude with respect to the surface. That gives a vertical scale of about 50km. Using these values, we obtain $\tau$ approximately equal to $3.5 \times 10^4$ s. Therefore, for the Phobos-Mars system, the atmospheric dissipation required to explain the orbital decay acts faster than the time that the tide takes to establish itself. Therefore, the atmospheric mechanism cannot provide the dissipation required.
Bibliography


