Methods for Improving Seismic Performances of Bridges

by

Vivian Lai Ki Wan

B.Sc. in Civil and Structural Engineering, University of Hong Kong
(1997)

M.Eng. in Civil and Environmental Engineering, Massachusetts Institute of Technology
(1998)

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING IN CIVIL & ENVIRONMENTAL ENGINEERING

at

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 8, 1998

© Massachusetts Institute of Technology 1998
All rights reserved

Signature of Author

______________________________________________
Department of Civil and Environmental Engineering
May, 1998

Certified by


Professor Jerome J. Connor
Thesis Supervisor

Accepted by

______________________________________________
Joseph M. Sussman
Chairman, Departmental Committee on Graduate Studies
Methods for Improving Seismic Performances of Bridges

by

Vivian Lai Ki Wan

Submitted to the Department of Civil and Environmental Engineering on May 8, 1998 in partial fulfillment of the requirements for the degree of Master of Engineering in Civil and Environmental Engineering.

Abstract

The thesis examines the technical issues related to using composite materials to improve the seismic performances of bridges. Behavior of typical concrete bridge columns with substandard design details for seismic forces are investigated. The poor performance is mainly due to the lack of adequate transverse reinforcement and poor rebar details in the columns. Composite materials with high strength-to-weight and stiffness-to-weight ratios are now commonly used to reinforce existing bridge columns. The properties and mechanical behavior of composite materials are first reviewed, and thus a comparison of the seismic performance for unretrofitted and retrofitted columns is made. The results indicate that both ductility and energy absorption capability are improved significantly by adding composite reinforcement. In addition to retrofitting, the design of new composite structures is studied. Lamination theory is introduced and used to analyze a composite I-beam. Axial and bending rigidities are found. An innovative design with the top flange of the I-beam replaced by concrete is investigated. A Matlab program is written to analyze a hybrid I-beam. The program finds the neutral axis, flange strains/stresses, web shear strains/stresses, and checks the serviceability requirement for deflection and web shear buckling. Studies show that one can achieve high stiffness to weight ratios and, by suitably orientating the fibers, improve the seismic performance.

Thesis Supervisor: Professor Jerome J. Connor
Title: Professor of Civil and Environmental Engineering Department
I would like to express my profound gratitude to Professor Jerome J. Connor, my thesis advisor, for his expert and invaluable advice throughout the course of this thesis. His continuous guidelines, sincere friendship and support are greatly appreciated.

Thanks also go to Professor Oral Buyukozturk, Professor Frederick J. Mc Garry, Professor Marthinus Van Schoor, Professor Shi-Chang Wooh for their crucial comments and directions. Assistance from doctorate students, Ognz Gunes and Brian Hearing and all my friends at MIT is gratefully acknowledged.

Finally, I would like to send my deepest thanks to my family for sending me to MIT for the Master degree. Their love and encouragement constitute a priceless support.
# TABLE OF CONTENTS

ABSTRACT ................................. 2  
ACKNOWLEDGEMENTS .................... 3  
TABLE OF CONTENTS ..................... 4  
1 INTRODUCTION ......................... 6  
2 TYPICAL RECTANGULAR BRIDGES ....... 7  
   2.1 Introduction ....................... 7  
   2.2 Causes and Modes of Failures ..... 7  
      2.2.1 Design Standard .......... 7  
      2.2.2 Poor Detailing ......... 7  
   2.3 Use of Composite Materials ..... 10  
3 COMPOSITE MATERIALS ............... 12  
   3.1 Introduction ..................... 12  
   3.2 Classification and Characteristics of Composite Materials 13  
      3.2.1 Properties of Fiber/Whiskers 14  
      3.2.2 Properties of Matrices 15  
      3.2.3 Properties of Laminae/Laminates 16  
   3.3 Mechanical Behavior of Composites 17  
   3.4 Application in Bridges ......... 20  
      3.4.1 Introduction ............. 20  
      3.4.2 FRP Tendons .......... 20  
      3.4.3 GFRP reinforcing bars for beams and deck slabs 21  
      3.4.4 Pedestrian bridges using FRP structural shapes 21  
      3.4.5 FRP Composite Decks .... 21  
4 RETROFITTING BRIDGES WITH COMPOSITE MATERIALS .......... 22  
   4.1 Introduction ..................... 22  
   4.2 Comparison of the Performance between the Wrapped Column and RC Reinforced Column .................. 23  
      4.2.1 Lateral Load vs. Displacement Response 23
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.2 Load vs. Strain</td>
<td>28</td>
</tr>
<tr>
<td>4.3 Different Types of Retrofitting</td>
<td>29</td>
</tr>
<tr>
<td>4.3.1 Rectangular vs. Oval-shaped Composite Straps</td>
<td>29</td>
</tr>
<tr>
<td>4.3.2 Active vs. Passive Retrofitting</td>
<td>30</td>
</tr>
<tr>
<td>5 DESIGN OF COMPOSITE I-BEAMS</td>
<td>32</td>
</tr>
<tr>
<td>5.1 Background</td>
<td>32</td>
</tr>
<tr>
<td>5.1.1 Laminate Code</td>
<td>32</td>
</tr>
<tr>
<td>5.1.2 Stress-Strain Relationships</td>
<td>32</td>
</tr>
<tr>
<td>5.1.3 Lamination Theory</td>
<td>37</td>
</tr>
<tr>
<td>5.2 Analysis of Laminated I-Beam</td>
<td>40</td>
</tr>
<tr>
<td>5.2.1 Introduction</td>
<td>40</td>
</tr>
<tr>
<td>5.2.2 Equivalent Bending Rigidity of a Composite Beam</td>
<td>41</td>
</tr>
<tr>
<td>5.2.3 Equivalent Bending Rigidity of a Composite I-Beam</td>
<td>42</td>
</tr>
<tr>
<td>6 DESIGN COMBINED WITH CONCRETE</td>
<td>47</td>
</tr>
<tr>
<td>6.1 Introduction</td>
<td>47</td>
</tr>
<tr>
<td>6.2 Analysis of Hybrid I-Beam</td>
<td>48</td>
</tr>
<tr>
<td>6.2.1 Neutral Axis</td>
<td>48</td>
</tr>
<tr>
<td>6.2.2 Equivalent Bending Rigidity</td>
<td>50</td>
</tr>
<tr>
<td>6.2.3 Strains/Stresses in the Flanges</td>
<td>51</td>
</tr>
<tr>
<td>6.2.4 Shear Strains/Stresses in the Web</td>
<td>52</td>
</tr>
<tr>
<td>6.2.5 Ultimate Bending Moment</td>
<td>53</td>
</tr>
<tr>
<td>6.3 Design Example</td>
<td>53</td>
</tr>
<tr>
<td>6.3.1 Procedure</td>
<td>53</td>
</tr>
<tr>
<td>6.3.2 Example</td>
<td>55</td>
</tr>
<tr>
<td>7 CONCLUSIONS</td>
<td>59</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>60</td>
</tr>
<tr>
<td>APPENDICES</td>
<td>62</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

During recent earthquakes such as Northridge (1994) and Kobe (1995), many bridges failed because of poor performance of the columns, primarily due to inadequate lateral reinforcement, insufficient lap length of the starter rebars and buckling of the longitudinal reinforcements. This deficiency generated considerably research on the behavior of structures subjected to earthquake forces and the retrofitting of substandard bridges, which has resulted in the development of sophisticated methodologies and procedures for the design and repair of earthquake-resistant structures, such as steel jacketing, use of spiral reinforcements, and the use of advanced composites.

Advanced composite materials offer significant advantages in strength, stiffness, and light weight, relative to conventional metallic materials. Along with improved structural performance, there also is the freedom to select the orientation of the fibers for optimum performance. The majority applications of advanced composites are in corrosive environments to take advantage of the corrosion resistance.

In this thesis, the deficiencies of certain concrete column bridges under seismic forces are investigated. Characteristics and mechanical behavior of composite materials are introduced. Comparisons are made between unretrofitted and retrofitted columns under seismic forces. Though there is no official code available for the design of composite structures, guidelines are being drafted in Japan, Canada and United States. Based on the lamination theory, steps to analyze a composite I-beam are presented. An innovative design of an I-beam with the top flange replaced by concrete is introduced and a design example is given. A Matlab program is written to do the analysis.
2.1 Introduction

The recent earthquakes in California and Japan have caused extensive damages to highway bridge structures. Failures of these structures exposed a number of structural deficiencies in many bridges and buildings constructed before the new seismic design codes were in place. In this chapter, the causes and types of failures are discussed.

2.2 Causes and Modes of Failures

2.2.1 Design Standard

The design standards for bridges existing at the time of construction have recently been increased to reflect an improved understanding of seismic excitation. During the 1960s in Japan, the design acceleration used was 0.15g to 0.2g in conjunction with working stress design rules. In the 1980s the design acceleration was increased to 0.25g or 0.30g. However, during the Hyogo-ken Nanbu (Kobe) earthquake in 1995, in the region of heavy damage, peak accelerations were observed up to 0.8g, much greater than the design acceleration in use at the time of construction. Also, the design procedure has been changed from working stress design to ultimate stress design.

2.2.2 Poor Detailing

Reinforced concrete bridge columns, designed before the new seismic design provisions were in place, and with lapped-spliced longitudinal reinforcement in the potential plastic hinge zone, appear to fail at low ductility levels. This is due to the debonding of the lapped starter rebars, resulting from the lack of transverse reinforcement and insufficient
development length of longitudinal bars. The failure modes are brittle because strength
deterioration is very rapid following debonding of longitudinal reinforcement.

The key detailing deficiencies and their corresponding modes of failures are listed below:

<table>
<thead>
<tr>
<th>Causes of Failures</th>
<th>Modes of Failures</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Inadequate amount of transverse reinforcement in columns.</td>
<td>The displacement of the column is large enough to yield the longitudinal steel in tension and then have it buckle on the ensuing compression part of the cycle. The insufficient confinement steel cannot prevent this buckling. Buckling cause the longitudinal steel to be ineffective and the column will fail in shear.</td>
</tr>
<tr>
<td>2 Large spacing of transverse reinforcement in columns.</td>
<td>Leading to premature buckling of the vertical column bars in the plastic hinge region.</td>
</tr>
<tr>
<td>3 Inadequate anchorage of transverse reinforcement in columns by lap splices in circular hoop reinforcement or 90° bend anchorages for rectangular hoops.</td>
<td>Loss of anchorage of the splices in the hoops reduces the effectiveness of the transverse reinforcement in preventing buckling of the vertical bars. The span on top then can slide off its bearing resulting in a drop of the superstructure.</td>
</tr>
<tr>
<td>4 Curtailment of vertical column bars in regions of anticipated plastic hinging trigger the yielding of the reinforcement at the cutoff location due to the combined effects of moment and shear, effectively raising the location of the plastic hinge into the region with</td>
<td>Column fail at the base and cover concrete spall on the tension side over almost the entire length.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>less confining reinforcement, and limiting the energy dissipating capability of the column.</td>
</tr>
<tr>
<td>5</td>
<td>Inadequate amount and detailing of localized transverse reinforcement in regions of concentrated bearing supports.</td>
</tr>
<tr>
<td>6</td>
<td>Inadequate amount of shear reinforcement in cap beams.</td>
</tr>
<tr>
<td>7</td>
<td>Inadequate amount of transverse reinforcement in joints between beams and columns.</td>
</tr>
<tr>
<td>8</td>
<td>Location of the butt welds in the longitudinal bars close to the bar cutoff location.</td>
</tr>
<tr>
<td>9</td>
<td>Inadequate support length, especially for skew supported bridges.</td>
</tr>
</tbody>
</table>

In summary, the above failures can be classified into two types, flexural and shear.

**Flexural**

Columns with starter rebars usually have flexural failures at a low ductility level. Owing to the insufficient transverse reinforcement and development length of the longitudinal...
reinforcement, concrete cover spalls followed by the debonding of the longitudinal reinforcement rebars in the lapped-spliced region. As a result, lateral load-carrying capacity reduces rapidly. The failure modes are brittle as the strength deterioration is very rapid right after the debonding of the longitudinal reinforcement. Flexural cracks spread from the bottom of the column to its mid-height.

Shear

Columns with continuous vertical rebar and full confinement over the entire height usually suffer from shear failures. The failures usually occur at higher ductility levels with a better performance in the load-displacement responses. It is due to the lack of lateral confinement. The longitudinal reinforcement buckles within the plastic hinge regions. Large shear forces, resulting from the higher longitudinal reinforcement ratio, cause extensive diagonal shear cracks. Diagonal cracking has a marked effect on the failure mechanism of bridge columns and can develop well outside the potential plastic hinge zone, particularly in columns with inadequate transverse reinforcement. When the lateral load was large enough, the formation of diagonal shear cracks occurred at the mid-height of the column. The column longitudinal bars were suddenly separated from the core concrete and a shear failure subsequently occurred.

2.3 Use of Composite Materials

Major seismic retrofit programs for concrete bridge structures have been initiated in the United States, New Zealand and Japan since the early 1970's. Methods include steel jacketing, use of composite steel-concrete columns and interlocking spirals. Seismic strengthening of concrete columns with FRP composites is one of the most popular methods and it has been shown that the flexural and shear strengths and the ductility can be improved.
In what follows, the use of composite materials, both for retrofitting and design of new structures, is discussed.
3.1 Introduction

“Composite” means two or more materials are combined on a macroscopic scale to form a useful material. It consists of two main elements: the body constituent, or matrix and structural constituents, such as fibers, particles, laminae or layers, flakes, and fillers. The properties that can be improved include:

- Strength and Stiffness
- Ductility
- Shear Capacity
- Seismic Performance
- Corrosion Resistance
- Weight
- Fatigue Life
- Thermal Insulation
- Acoustical Insulation

Composite materials have a long history of usage. Nowadays, fiber reinforced resin composites that have high strength-to-weight and stiffness-to-weight ratios have become important in weight-sensitive applications such as aircraft and motor vehicles. Also, they are starting to be considered in civil engineering applications. This recent interest by the civil engineering profession in these high-tech materials is partly the result of reducing material costs, and new applications where the inherent suitability of the materials outweighs the high material costs, which currently run anywhere between U.S. $5 and $30 per pound.
3.2 Classification and Characteristics of Composite Materials

Based on the form of the structural constituents, the composites can be grouped into five general categories as shown in Fig. 3.1.

- **Fiber composites** - composed of fibers in a matrix
- **Particulate composites** - composed of particles in a matrix
- **Laminar composites** - composed of layer, or laminar, constituents
- **Flake composite** - composed of flat flake in a matrix
- **Filled composites** - composed of a continuous skeletal matrix filled by a second materials

Figure 3.1: Classes of composites [Schwartz, 1992].

Fiber composites, such as carbon-fiber reinforced plastics (CFRP) and glass-fiber-reinforced plastics (GFRP) are the common composites used in the civil engineering industry.
3.2.1 Properties of Fiber/Whiskers

Long fibers in various forms are much stiffer and stronger than the same material in bulk form. For example, ordinary plate glass fractures at a stress level of about ten MPa, whereas glass fiber can have a strength of 5000 MPa. E-glass is the most common type of fiber used in polymer composites due to its high strength and low cost.

Strengths and stiffness of a few selected fiber materials are shown in Table 3.1. The strength-to-density and stiffness-to-density are commonly used as indicators of the effectiveness of a fiber.

<table>
<thead>
<tr>
<th>Fiber</th>
<th>Density, $\rho$ (kN/m$^3$)</th>
<th>Tensile Strength, S (GN/m$^2$)</th>
<th>$S/\rho$ (km)</th>
<th>Tensile stiffness, E (GN/m$^2$)</th>
<th>$E/\rho$ (Mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>26.3</td>
<td>0.62</td>
<td>24</td>
<td>73</td>
<td>2.8</td>
</tr>
<tr>
<td>Steel</td>
<td>76.6</td>
<td>4.1</td>
<td>54</td>
<td>207</td>
<td>2.7</td>
</tr>
<tr>
<td>E-glass</td>
<td>25</td>
<td>3.4</td>
<td>136</td>
<td>72</td>
<td>2.9</td>
</tr>
<tr>
<td>S-glass</td>
<td>24.4</td>
<td>4.8</td>
<td>197</td>
<td>86</td>
<td>3.5</td>
</tr>
<tr>
<td>Carbon</td>
<td>13.8</td>
<td>1.7</td>
<td>123</td>
<td>190</td>
<td>14</td>
</tr>
<tr>
<td>Graphite</td>
<td>13.8</td>
<td>1.7</td>
<td>123</td>
<td>250</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 3.1: Fiber properties.

Whiskers exhibit higher properties than fibers. Materials such as iron have crystalline structures with a theoretical strength of 20 GPa, yet structural steels, which are mainly iron, have strengths about several hundreds MPa. Table 3.2 shows a representative set of whisker properties.

<table>
<thead>
<tr>
<th>Whisker</th>
<th>Density, $\rho$ (kN/m$^3$)</th>
<th>Tensile Strength, S (GN/m$^2$)</th>
<th>$S/\rho$ (km)</th>
<th>Tensile stiffness, E (GN/m$^2$)</th>
<th>$E/\rho$ (Mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>87.4</td>
<td>3</td>
<td>34</td>
<td>124</td>
<td>1.4</td>
</tr>
<tr>
<td>Iron</td>
<td>87.9</td>
<td>3.9</td>
<td>44</td>
<td>215</td>
<td>2.4</td>
</tr>
<tr>
<td>C</td>
<td>16.3</td>
<td>21</td>
<td>1300</td>
<td>980</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 3.2: Whisker properties.
Fibers are relatively brittle. They lack of ductility and fail at relatively low strain as shown in Fig 3.2.

![Figure 3.2: Tensile stress-strain curves for various fibers.](image)

3.2.2 Properties of Matrices

Fibers and whiskers are of little use unless they are combined with a matrix material to form of a structural element. The function of the matrix material is to support and protect the fibers and to provide a means of distributing load to the fibers. Typically, the matrix is of considerably lower density, stiffness, and strength than the fibers or whiskers. High strength carbon fibers have tensile strengths that approach 6900 MPa, whereas the tensile strength of a typical polymer matrix may be on the order of 200 MPa. However, the combination can have high strength and stiffness, and still have low density.

The strength of fiber composites is determined by the fibers, but is influenced by the matrix. The strength of matrix-impregnated fiber bundles can be on the order of a factor
of 2 higher than the measured tensile strength of dry fiber bundles without matrix impregnation.

The stress strain curves of typical epoxy resins are plotted in Fig 3.3. This figure shows that the properties of the matrices depend on the temperature and moisture content.

![Stress strain curves of epoxy resin](image)

Figure 3.3: Stress strain curves of epoxy resin at different temperatures and moisture contents.

3.2.3 Properties of Laminae/Laminates

A lamina (ply) is a flat arrangement of unidirectional fibers or woven fibers in a matrix. Two typical laminae are shown in Fig 3.4.

![Types of laminae](image)

Figure 3.4: Two principal types of laminae.
A laminate is a stack of laminae with various orientations of principal material directions in the laminae.

3.3 **Mechanical Behavior of Composites**

Unlike most common engineering materials, composite materials are often both *non homogeneous*, i.e., their properties vary from point to point in the material, and *anisotropic*, i.e., their properties depend on the orientation of fibers.

Because composite materials are anisotropic, their stress-strain relationship is unconventional and more complicated. For example, in an isotropic material, a normal stress will cause only normal strain, and a shear stress will cause only a shear strain. However, in an anisotropic composite material, a normal stress may induce both normal as well as shear strain, and a shear stress may induce both shear and normal strain as shown in Fig. 3.5. In contrast to the thermal behavior of isotropic materials, which exhibit uniform expansion or contraction in all directions, a temperature change in an anisotropic material may induce non-uniform expansion or contraction plus shear distortion.

![Figure 3.5: Mechanical behavior of anisotropic materials.](image-url)
A crude technique used to obtain composite response in the elastic range is the *rule of mixtures*. This is a rough tool based on the simple strength-of-materials approach in which the composite properties are considered to be volume-weighted average. The following assumptions are made:

- strains in the matrix, the fibers and the composite are equal
- the load carried by the composites equals the sum of the loads carried by the fibers and the loads carried by the matrix
- there is no slip between the fibers and the matrix

These assumptions result in the following equations:

\[
\rho_c = \rho_f V_f + \rho_m V_m \quad (3.1)
\]
\[
E_{ct} = E_f V_f + E_m V_m \quad (3.2)
\]
\[
\frac{1}{E_{ct}} = \frac{V_m}{E_m} + \frac{V_f}{E_f} \quad (3.3)
\]
\[
\sigma_c = \sigma_f V_f \quad (3.4)
\]

where \( \rho_c \) = composite density
\( \rho_f \) = fiber density
\( \rho_m \) = matrix density

\( V_f \) = volume fraction of fiber
\( V_m \) = volume fraction of matrix

\( E_f \) = Young's modulus of fiber
\( E_m \) = Young's modulus of matrix
\( E_{ct} \) = Young's modulus of composite in the fiber direction
\[ E_{ct} = \text{Young's modulus of composite in the transverse direction} \]

\[ \sigma_c = \text{Strength of composite} \]

\[ \sigma_f = \text{Strength of fiber} \]

From Eqs. 3.2 and 3.4, one can see that the strength and modulus of the composite depend on the volume fraction of the fibers. The properties of some common fibers and matrices are shown in Table 3.3.

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of Elasticity</th>
<th>Ultimate Tensile Strength</th>
<th>Ultimate Strain</th>
<th>Approximate Cost, 1995</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GPa</td>
<td>MPa</td>
<td>%</td>
<td>EUC/kg</td>
</tr>
<tr>
<td>Fibers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Carbon</td>
<td>160-270</td>
<td>1400-6800</td>
<td>1.0-2.5</td>
<td>24-80</td>
</tr>
<tr>
<td>• Aramid</td>
<td>62-83</td>
<td>2800</td>
<td>3.6-4.0</td>
<td>16-24</td>
</tr>
<tr>
<td>• Glass</td>
<td>81</td>
<td>3400</td>
<td>4.9</td>
<td>2-6</td>
</tr>
<tr>
<td>• Polyethylene</td>
<td>117</td>
<td>2600</td>
<td>3.5</td>
<td>2</td>
</tr>
<tr>
<td>Resin</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Epoxy</td>
<td>2.0-4.5</td>
<td>27-62</td>
<td>4-14</td>
<td>2-4</td>
</tr>
<tr>
<td>• Vinylester</td>
<td>3.6</td>
<td>80</td>
<td>4</td>
<td>2-3</td>
</tr>
</tbody>
</table>

Table 3.3: Properties of some common fibers and matrices.

Behavior characteristics are predominately linear all the way to fracture, in the direction parallel to the fibers. The high modulus of elasticity and tensile strength rapidly decrease with increasing deviation angle between fiber orientation and loading direction. It can be easily seen by comparing Eqs. (3.2) and (3.3). Also, the deformation characteristics become increasingly dominated by the properties of the matrix.
3.4 Application in Bridges

3.4.1 Introduction

The applications of composites in bridges have taken place in two specific areas, in rehabilitation and retrofitting of bridges and in new bridges. More than fifty bridges that use FRP composites have been built throughout the world, mostly in Japan and Europe. Most of these are either pedestrian bridges or vehicular bridges. These definitely give valuable insight toward using FRP reinforcing materials for highway bridges. However, design codes or guidelines are still being drafted in Japan, Canada and United States.

Composites elements used in bridges include FRP tendons, GFRP reinforcing bars for beams and deck slabs, new pedestrian bridges using pultruded FRP structural components, and composite decks. They are summarized in the following sections.

3.4.2 FRP Tendons

FRP tendons are available in the forms of glass tendons, carbon tendons, and aramid tendons and rods. The following are the advantages of FRP tendons over steel tendons:

- high specific strength (10 - 15 times higher than steel)
- excellent fatigue resistance (which can be 3 times more resistant than steel)
- noncorrosive and nonmagnetic properties
- low thermal expansion in the case of carbon and aramid tendons
- nonconductive
- light

Several bridges in Germany and Japan were built with FRP tendons. Design provisions for using FRP tendons have been drafted for the proposed Canadian Highway Bridge Design Code [1996].
3.4.3 GFRP reinforcing bars for beams and deck slabs

GFRP reinforcing bars are now beginning to be used as reinforcement for concrete decks. Usually, the reinforcing bars are produced with E-glass fibers together with polyester or vinyl ester resins. Besides the advantages mentioned before, concrete beams reinforced with FRP bars will exhibit greater flexural strength, and deflect more (can be up to 3 times greater) than the steel-reinforced beams with wider and more closely spaced cracks at the ultimate loads.

3.4.4 Pedestrian bridges using FRP structural shapes

FRP structural shapes are available in several configurations: I-shapes, channels, angles, plates, bars and rods. Several pedestrian bridges using these configurations have been built by E. T. Techtonics of Philadelphia in Pennsylvania, California, Hawaii, Illinois, Michigan and Washington.

3.4.5 FRP composite decks

The bridge deck is always recognized as a critical component of the bridge superstructure and so FRP composites are used for the entire deck or as part of the superstructure system. The advantages are:

- low weight - reduce the inertia forces experienced during an earthquake
- high corrosion resistance - enhance the service life of the superstructure
Chapter 4

Retrofitting Bridges with Composite Materials

4.1 Introduction

Recent earthquakes in urban areas such as the one in Northridge in 1994, and in Kobe in 1995 have demonstrated the vulnerabilities of older reinforced concrete columns to seismic deformation demands. Therefore, there exist a large number of buildings and bridges that require retrofitting. FRP composite material with its high strength-to-weight and stiffness-to-weight ratios is an ideal material for the retrofitting works.

High strength FRP straps are wrapped around the column in the potential plastic hinge region, as shown in Fig. 4.1, to increase confinement and to improve its behavior under seismic forces. A layer of epoxy is brushed on the straps for interlaminar bond. After retrofitting, the ductility and energy absorption capacity are found to be improved significantly.

![Composite Straps](image)

Figure 4.1: Column retrofitted with composite straps wrapped in the potential plastic hinge region.

In this chapter, comparisons between the retrofitted and unretrofitted concrete columns will be discussed. Also, different types of retrofitting are introduced.
4.2 Comparisons between Retrofitted/Unretrofitted Concrete Columns

Composite straps will provide external confinement to the columns, prevent crushing and spalling of the concrete shell, and prevent buckling of the longitudinal reinforcement. Most of all, as mentioned in chapter 4.1, the ductility and energy absorption capacity will be increased.

In the past, much research have been carried out to investigate the performance differences of the columns before and after retrofitting under seismic excitation. In the following chapters, a summary of the investigations is given according to the following classification:

- Lateral load vs. displacement response
- Load vs. strain

4.2.1 Lateral Load vs. Displacement Response

The effect of an earthquake can be simulated by reversed cyclic loading. The load and displacement input history is divided into two phases, a load control mode and a displacement control mode. The loading sequence is shown in Fig. 4.2.

![Figure 4.2: Loading sequence.](image-url)
There are two types of longitudinal reinforcements, starter rebars (lap-splice) and continuous rebars as shown in Fig. 4.3. Their performances are different and so will be discussed separately.

**Starter Rebars**

The load-displacement curves of columns with lapped starter rebars are shown in Figs 4.4 and 4.5. The letter 'u' in the Figures is the displacement ductility factor defined as the ratio of the applied displacement over the displacement at first yielding of the longitudinal rebars.

Figure 4.3: Two types of reinforcement details.

Figure 4.4: Load vs. Displacement response of an unretrofitted column with starter rebars.
Chapter 4: Retrofitting Bridges with Composite Materials

Figure 4.5: Load vs. Displacement response of a retrofitted column with starter rebars.

After comparing the curves, we can draw the following conclusions:

<table>
<thead>
<tr>
<th>Before retrofitting</th>
<th>Cause:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- bond failure of the lapped reinforcement within the plastic hinge region of the column.</td>
</tr>
<tr>
<td></td>
<td>Results:</td>
</tr>
<tr>
<td></td>
<td>- hysteresis loops degrade rapidly after the first cycle.</td>
</tr>
<tr>
<td></td>
<td>- lateral strength drop quickly at the first cycle with very narrow energy dissipation loops.</td>
</tr>
<tr>
<td></td>
<td>lateral load carrying capacity is reduced.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>After retrofitting</th>
<th>Cause:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- presence of the confining strap prevent premature bond failure in the splice region.</td>
</tr>
<tr>
<td></td>
<td>Results:</td>
</tr>
<tr>
<td></td>
<td>- significant improvement on the lateral response with stable hysteresis loops.</td>
</tr>
</tbody>
</table>
Continuous Rebar

The load-displacement curves of columns with continuous longitudinal reinforcement are shown in Figures 4.6 and 4.7.

Figure 4.6: Load vs. Displacement response of an unretrofitted column with continuous rebars.
After comparing the curves, we can draw the following conclusions:

<table>
<thead>
<tr>
<th>Before retrofitting</th>
<th>Causes:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• large flexural shear cracks appear within and beyond the plastic hinge zone at low ductility level.</td>
</tr>
<tr>
<td></td>
<td>• due to the lack of adequate transverse reinforcement, extensive shear cracks appear along the whole height of the column and longitudinal rebars are suddenly separated from the core concrete.</td>
</tr>
<tr>
<td></td>
<td>Results:</td>
</tr>
<tr>
<td></td>
<td>• column fails in shear ultimately.</td>
</tr>
<tr>
<td></td>
<td>• lateral strength does not decay until a relatively high displacement ductility level.</td>
</tr>
<tr>
<td></td>
<td>• rapid deterioration in strength after reaching high ductility level.</td>
</tr>
</tbody>
</table>
After retrofitting

<table>
<thead>
<tr>
<th>Cause:</th>
<th>Results:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• significant improvement in strength and energy absorption capability.</td>
<td>• load-displacement responses are improved indicated by the more stable hysteresis loops.</td>
</tr>
<tr>
<td></td>
<td>• lateral strength shows no sign of structural degradation.</td>
</tr>
</tbody>
</table>

‘Pinching’ is characterized by the narrow width of the hysteresis loops near mid-cycle, resulting in less energy dissipation per cycle as compared to the loops without pinching. As shown in Figs 4.4 - 4.7, we can notice that there are more pinching in the loops with lapped starter rebars, Figs 4.4 and 4.5. It is because pinching is mainly caused by the slippage of the longitudinal bars and closing of flexural cracks in the plastic hinge zone, and this is more pronounced in the columns reinforced by starter rebars.

4.2.2 Load vs. Strain

**Longitudinal Reinforcement**

Usually, the strains in the longitudinal rebar are higher for the retrofitted case. In the unretrofitted case, there is insufficient confinement, and bond failure in the lapped starter rebar or buckling of continuous rebar occurs. However, these performances are improved after wrapping with composite straps, resulting in a higher overall ductility and energy absorption capacity as indicated by higher strains.

**Transverse Reinforcement**

Lack of adequate lateral reinforcement will result in premature yielding of the transverse hoop reinforcement, and then loss of confinement and rapid deterioration of the plastic...
zone region. As mentioned before, composite straps are efficient in providing the lateral reinforcement and so can share the hoop stresses. Generally, the hoop strains in the retrofitted column are smaller than that in the unretrofitted column, showing that the composite straps can reduce the dilation of the core concrete and share the load in the hoops.

*Composite Strap*

The strains developed in the composite straps increase with increasing lateral displacement of the columns, indicating the effectiveness of the straps in preventing the lateral expansion of the concrete core.

**4.3 Different Types of Retrofitting**

**4.3.1 Rectangular vs. Oval-shaped Composite Straps**

To retrofit rectangular columns, rectangular and oval-shaped straps, as shown in Fig. 4.8, have been used.

![Composite straps](image)

*Figure 4.8: Rectangular and oval-shaped composite straps.*

To retrofit with oval-shaped composite straps, the column is first shaped with fast curing cement and then wrapped with the composite straps.
Oval-shaped composite straps produce higher membrane confinement stresses and consequently perform better. The increase in strength of a column with oval-shaped composite straps can be up to 8% higher than that of a column with rectangular straps.

4.3.2 Active vs. Passive Retrofitting

The setup of these types of retrofitting is shown in Fig. 4.9.

Passive Retrofitting

The composite straps with fiber orientation in the circumferential direction are directly wrapped onto the columns. When the concrete expands outwards, tensile stresses are developed in the composite straps to resist this expansion and so it is called 'passive' retrofitting.
Active Retrofitting

The composite straps are made oversized for the column. The resulting gap between the column and the straps is then filled with expansive grout or pressurized epoxy resin. An active pressure is created around the column and provides the initial confinement to the column. Tensile stresses are induced in the composite straps.

Both active and passive retrofitting can provide additional confinement to the concrete column. In active retrofitting, the maximum strain reached in the composite straps is smaller and so indicates that the amount of radial dilation and cracking of the core concrete is reduced. Also, the increase in lateral load capacity is higher than the column passively retrofitted, about 10%. However, the cost of active retrofitting is much higher than that of passive retrofitting.
5.1 Background

5.1.1 Laminate Code

A laminate consists of a series of ply groups. Each ply group is made up of one material and can be specified by the angles and number of plies in the group. For example, \([0_{2}/90/90/0_{2}]_{s}\), where the subscript refers to the number of plies in each ply group and the subscript ‘s’ stands for symmetric. Therefore it is equivalent to \([0_{2}/90/90/0_{2}/90/90/0_{2}]\).

5.1.2 Stress-Strain Relationships

In this chapter, the stress-strain relationships for an individual ply are examined.

As described in chapter 3.3, an individual layer of composite has its principal material axes aligned with and transverse to the fibers and so can be considered to be an orthotropic material. Loading along these principal axes does not induce shear stresses and strains. Similarly, the applications of shear stresses does not produce normal strains.

A unidirectional layer is shown in Fig 5.1, along with the coordinate system used to establish the notation. Here direction 1 refers to the fiber direction and direction 2 ia normal to fiber direction.
Because many engineering structures made of laminates are thin, assuming plane stress is reasonable. One starts with the strain-stress relation specialized for plane stress,

$$
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
1 & -\nu_{21} & 0 \\
\frac{E_{11}}{E_{12}} & 1 & 0 \\
\frac{E_{11}}{E_{12}} & 0 & \frac{1}{G_{12}}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix}
\tag{5.1}
$$

Inverting Eq. (5.1) results in

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
\frac{E_{11}}{1 - \nu_{12}\nu_{21}} & \frac{\nu_{12}E_{11}}{1 - \nu_{12}\nu_{21}} & 0 \\
\frac{E_{22}}{1 - \nu_{12}\nu_{21}} & \frac{E_{22}}{1 - \nu_{12}\nu_{21}} & 0 \\
0 & 0 & G_{12}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}
\tag{5.2}
$$
\[
[Q] = \begin{bmatrix}
\frac{E_{11}}{1 - \nu_{12}\nu_{21}} & \frac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}} & 0 \\
\frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} & \frac{E_{22}}{1 - \nu_{12}\nu_{21}} & 0 \\
0 & 0 & G_{12}
\end{bmatrix}
\] (5.3)

The zeros in the matrices reflect the stress-strain behavior of an orthotropic material with respect to the principal material axes.

Although there appears to be five independent constants needed to describe the stress-strain response of the lamina, the \( Q \) matrix must be symmetric because the matrix describes the stress-strain relationship of an elastic material. Therefore, there are only four independent properties, and \( E_{11}, E_{22}, \nu_{12}, \nu_{21} \) are related by

\[
E_{11}\nu_{21} = E_{22}\nu_{12}
\] (5.4)

The orthotropic elasticity constants of some common composite materials are shown in Table 5.1.

<table>
<thead>
<tr>
<th>Material</th>
<th>( E_{11} ) (GPa)</th>
<th>( E_{22} ) (GPa)</th>
<th>( \nu_{12} )</th>
<th>( G_{12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon/epoxy AS4/3501-6</td>
<td>131</td>
<td>11.2</td>
<td>0.28</td>
<td>6.55</td>
</tr>
<tr>
<td>Carbon/epoxy T300/5208</td>
<td>153</td>
<td>11.2</td>
<td>0.33</td>
<td>7.1</td>
</tr>
<tr>
<td>Carbon/epoxy G30-5--/2431</td>
<td>129</td>
<td>8.6</td>
<td>0.31</td>
<td>4.1</td>
</tr>
<tr>
<td>Carbon/epoxy T300/2431 cloth</td>
<td>62.8</td>
<td>62.8</td>
<td>0.05</td>
<td>6.2</td>
</tr>
<tr>
<td>E-glass/epoxy</td>
<td>38.6</td>
<td>8.27</td>
<td>0.26</td>
<td>4.14</td>
</tr>
<tr>
<td>E-glass/epoxy cloth</td>
<td>16.6</td>
<td>16.6</td>
<td>0.05</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table 5.1: Typical orthotropic elasticity constants of some common composite materials.
Consider the x,y system that is at an angle $\theta$ with respect to the 1,2 system as shown in Fig 5.1. Using Mohr's circle transformation procedures, and working with $1/2$ the actual shear strain, the transformed equations have the following form.

\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix} = \left[ T^{-1} \right] \begin{pmatrix} Q \end{pmatrix} \begin{pmatrix} \epsilon_x \\
\epsilon_y \\
\gamma_{xy} \end{pmatrix} = \begin{pmatrix} \bar{Q} \end{pmatrix} \begin{pmatrix} \epsilon_x \\
\epsilon_y \\
\gamma_{xy} \end{pmatrix} 
\] (5.5)

where

\[
[T] = \begin{bmatrix}
\cos^2 \theta & \sin^2 \theta & 2 \sin \theta \cos \theta \\
\sin^2 \theta & \cos^2 \theta & -2 \sin \theta \cos \theta \\
-\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta
\end{bmatrix} 
\] (5.6)

\[
[R] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix} 
\] (5.7)

\[
\bar{Q}_{11} = Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{33}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\
\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{33}) \sin^2 \theta \cos^2 \theta + Q_{12} (\sin^4 \theta + \cos^4 \theta) \\
\bar{Q}_{22} = Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{33}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\
\bar{Q}_{13} = (Q_{11} - Q_{12} - 2Q_{33}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{33}) \sin^3 \theta \cos \theta \\
\bar{Q}_{23} = (Q_{11} - Q_{12} - 2Q_{33}) \sin \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{33}) \sin \theta \cos^3 \theta \\
\bar{Q}_{33} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{33}) \sin^2 \theta \cos^2 \theta + Q_{33} (\sin^4 \theta + \cos^4 \theta) 
\] (5.8)

Note that $\bar{Q}_{13}$ and $\bar{Q}_{23}$ terms are no longer 0 since the axes are not principal material axes.
Graphs of the various terms of the $\bar{Q}$ matrix as functions of the angle $\theta$ between the fiber axis and the x axis are shown in Figs 5.2 and 5.3. From Fig 5.2, we can notice that composite materials are stiffest in the direction of the fibers and weakest in the direction transverse to the fibers. From Fig 5.3, the $3,3$ shear modulus term increases until reaches maximum at $45^\circ$ which is the direction that can provide maximum shear stiffness.

![Graph 5.2](image1.png)

**Figure 5.2:** Change in the $Q_{11}$ and $Q_{22}$ stiffness coefficients with angle of rotation.

![Graph 5.3](image2.png)

**Figure 5.3:** Change in the $Q_{12}$, $Q_{33}$, $Q_{13}$ and $Q_{23}$ stiffness coefficients with angle of rotation.
5.1.3 Lamination Theory

In this Chapter, the relationships between the applied in-plane loads/bending moments and the stresses within the individual layers are first set up. Then the relationships between the stress/moment resultants and the strains for the whole structure are developed.

**Flat Plate**

Consider a flat plate as shown in Fig 5.4.

![Figure 5.4: Plate deformation - extension and bending.](image)

It is assumed that normal to the center line remains normal after deformation. Also, the plate displaces in the z direction only because of the bending motion, and that no variation of $w$ through the thickness takes place. The result is as follows:

\[
\begin{align*}
\varepsilon_z &= \frac{\partial u_0}{\partial x} \\
\varepsilon_y &= \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z \left( -\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 w}{\partial x \partial y} \right)
\end{align*}
\]
\[
\{ \varepsilon \} = \{ \varepsilon^0 \} + z\{ \kappa \} \quad (5.10)
\]

where \( \{ \varepsilon \} \) is the center-line strains and \( \{ \kappa \} \) is the curvature.

**Laminate**

Consider the laminate as shown in Fig 5.5. After assuming that the individual layers are perfectly bonded together, i.e. all the layers displace together under the action of the applied loads, the following equations can be obtained.

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} = \sum_{k=1}^{N} h_k \int_{-h_{k-1}}^{-h_k} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} dz
\quad (5.11)
\]
\[
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = \sum_{k=1}^{N} \frac{h_k}{h_{k-1}} \int \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} \, dz
\] (5.12)

Note that the resultant forces and moments are calculated on a unit width basis and so the total forces and moments should be obtained by multiplied them by the width of the structural elements. Combining with Eqs. (5.5) and (5.10), a relationship between stress resultants, moment resultants, center-line strains and curvatures can be obtained.

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} = \begin{bmatrix}
A & B \\
B & D
\end{bmatrix} \begin{bmatrix}
\varepsilon^0 \\
\kappa
\end{bmatrix}
\] (5.13)

where the \( A \), \( B \) and \( D \) matrices are each 3 by 3 matrices defined as

\[
A_j = \sum_{k=1}^{N} \frac{Q_{y_k}}{h_k} (h_k - h_{k-1})
\] (5.14)

\[
B_j = \sum_{k=1}^{N} \frac{Q_{y_k}}{h_k} \frac{h_k^2 - h_{k-1}^2}{2}
\] (5.15)

\[
D_j = \sum_{k=1}^{N} \frac{Q_{y_k}}{h_k} \frac{h_k^3 - h_{k-1}^3}{3}
\] (5.16)

Under the assumption that all the layers are perfectly bonded, \( \{\varepsilon^0\} \) and \( \{\kappa\} \) are constants for each ply group. From Eq. (5.13), we can notice that there is a coupling between the in-plane behavior and the bending behavior because of the presence of the \( B \) matrix. However, if the laminate is symmetric with respect to the midplane, the \( B \) matrix vanishes and the coupling does not occur. The matrices \( [A] \) and \( [D] \) are axial stiffness and bending stiffness respectively. Follows are the steps to find these stiffness values:
1. Determine the properties of the composites materials used, i.e. $E_{11}$, $E_{22}$, $G_{12}$ and $\nu_{12}$ from Table 5.1. Calculate the matrix $[Q]$ from the Eq. (5.3). If different materials are used for different ply groups, then each ply will have its own $[Q]$ matrix.

2. Determine the angles between the loading applied and the direction of fibers i.e. $\theta$ for each ply groups. Calculate the matrix $[\bar{Q}]$ from the Eq. (5.8). Different ply groups will have different $[\bar{Q}]$ matrixes.

3. Determine the position of the mid-plane and then calculate the distances of each ply groups from the mid-plane, i.e. $h_k$, Fig. 5.5. Calculate $[A]$ and $[D]$ from Eqs. (5.14) and (5.16).

5.2 Analysis of Laminated I-Beam

5.2.1 Introduction

An I-beam is one of the commonly used structural forms for bridge decks and so in the following chapters, the discussion is focused on this type of structural member.

The wide flanges at the top and bottom carry mainly the bending stresses and so the plies there are parallel to the neutral axis and most of the fibers are $0^\circ$ fibers to increase the stiffness in this direction. However, as the composites are extremely weak in the directions transverse to the fibers, fibers must be placed in more than one direction, otherwise even secondary loads in the transverse directions can cause failure. Since the function of the web is mainly to carry shear, the plies are oriented normal to the neutral axis and the fibers are mostly $\pm 45^\circ$ fibers.
5.2.2 Equivalent Bending Rigidity of a Composite Beam

The conventional beam formulation provides the basis for obtaining the equivalent bending rigidity of a composite. Therefore, before analyzing the I-beam, the equivalent bending rigidity of a composite beam is derived first.

The aspect ratio of a beam is defined as the ratio of cross-section width to height. The analyses for narrow beams and wide beams are different because the distortions in the transverse directions for each cases are different, as shown in Fig 5.6.

![Narrow-beam cross-section](image)

![Wide-beam cross-section](image)

Figure 5.6: Distortions of the cross sections in the transverse directions for both narrow and wide beams.

Here are the steps to find $EI$:

1. Basic Equation [extracted from Eq. (5.13)]

$$
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} =
[D]
\begin{bmatrix}
\kappa_x \\
\kappa_y \\
\kappa_{xy}
\end{bmatrix}
$$

2. Mathematical differences between narrow and wide beams

Narrow beams: $M_y = M_{xy} = 0$

Wide beams: $\kappa_y = \kappa_{xy} = 0$
3. Equivalent bending rigidities $\overline{EI}$ (derived by combing steps 1 and 2)

Narrow beams: $\overline{EI} = \frac{b}{D^{-11}}$ \hfill (5.18)

Wide beams: $\overline{EI} = bD_{11}$ \hfill (5.19)

where $b$ is the width of the cross section and $D_{11}$ can be obtained by following the steps in chapter 5.1.3.

5.2.3 Equivalent Bending Rigidity of a Composite I-Beam

We analyze an I-beam by separating it into the web and two flanges as shown in Fig 5.7. The web and the flanges are treated as beams. The mid-planes of each parts are parallel to the lamination directions. The lamination constants, such as $[A]$ and $[D]$, for each part are computed about their own mid-planes.

![Figure 5.7: The mid-planes for the web and two flanges of an I-beam.](image)
The coordinate system used is shown in Fig 5.8.

Important points that need to be considered are:

- The strain distribution is taken to be linear, but as the fibers in different layers will have different orientations, the stiffness are different and so the stress distributions will vary linearly with strains.

- Stress and strain distributions within each individual plies are two dimensional.

- The plies are usually aligned symmetrically within each parts to avoid distortion owing to the thermal effects, and also eliminate the effect of coupling between the in-plane behavior and bending behavior. The $[B]$ matrix is zero.

- Axial strain distributions are determined based on the assumption: plane section remains plane after deformation.

- The lateral stress $N_y$ and $N_{xy}$ are assumed to be zero. $M_{xy}$ is neglected.
Web

For the Web, the bending moment calculated using \([D]\) matrix is not the transverse moment because the plies are aligned in the vertical direction. However, the analysis can be done as follows:

According to Eq. (5.13),

\[
\varepsilon^0 = A^{-1}_{11(web)} N_x
\]

(5.20)

where \(\varepsilon^0 = \xi \kappa_x\) (See Fig 5.7 for the definition of \(\xi\).)

\[
M_{(web)} = \int_{-h/2}^{h/2} N_x \xi d\xi = \int_{-h/2}^{h/2} (1 / A^{-1}_{11(web)}) \xi^2 \kappa_x d\xi
\]

(5.21)

\[
= \frac{h^3}{12 A^{-1}_{11(web)}} \kappa_x
\]

Flanges

Same as beams, the analysis of flanges will be done for two cases, narrow and wide flanges.

1. Basic Equations (inverse of Eq. (5.13))

\[
\begin{bmatrix}
\varepsilon^0 \\
\kappa
\end{bmatrix} =
\begin{bmatrix}
A^{-1}_{(flange)} & 0 \\
0 & D^{-1}_{(flange)}
\end{bmatrix}
\begin{bmatrix}
N \\
M
\end{bmatrix}
\]

(5.22)
2. **Assumptions:**

\[ N_y = N_{xy} = 0 \]

Narrow flanges: \[ M_y = M_{xy} = 0 \]

Wide flanges: \[ \kappa_y = \kappa_{xy} = 0 \]

3. **Bending Moments of the Flanges:**

\[ M_{(flange)} = b_f (N_x \xi_1 + M_x) \quad (5.23) \]

Narrow flanges:
\[ N_x = \frac{\xi}{A^{-1}_{11(flange)}} \kappa_x \quad (5.24) \]
\[ M_x = \frac{1}{D^{-1}_{11(flange)}} \kappa_x \quad (5.25) \]
\[ M_{(flange)} = b_f \left( \frac{\xi^2}{A^{-1}_{11(flange)}} + \frac{1}{D^{-1}_{11(flange)}} \right) \kappa_x \quad (5.26) \]

Wide flanges:
\[ N_x = \frac{\xi}{A^{-1}_{11(flange)}} \kappa_x \quad (5.27) \]
\[ M_x = D^{-1}_{11(flange)} \kappa_x \quad (5.28) \]
\[ M_{(flange)} = b_f \left( \frac{\xi^2}{A^{-1}_{11(flange)}} + D_{11(flange)} \right) \kappa_x \quad (5.29) \]

**Equivalent bending rigidities of the I-Beam**

The total moment for the beam can be obtained by summing the moments for the web and two flanges:
\[ M_{(beam)} = M_{(web)} + 2M_{(flange)} = EI_{eff} \kappa_x \]  \hspace{1cm} (5.30)

Therefore the effective \( EI_{eff} \) for narrow and wide flanges are as follows:

**Narrow flanges I-Beam:**

\[ EI_{eff} = \frac{h^3}{12A^{-1}_{11(web)}} + 2b_f\left(\frac{\xi_1^2}{A^{-1}_{11(flange)}} + \frac{1}{D^{-1}_{11(flange)}}\right) \]  \hspace{1cm} (5.31)

**Wide flanges I-Beam:**

\[ EI_{eff} = \frac{h^3}{12A^{-1}_{11(web)}} + 2b_f\left(\frac{\xi_1^2}{A^{-1}_{11(flange)}} + D_{11(flange)}\right) \]  \hspace{1cm} (5.32)
6.1 Introduction

In this chapter, the uses of composites combined with concrete will be described. FRP has higher tensile strength than compressive strength (about two times) and so the compressive flange is comparatively weaker than the tensile flange. Also, owing to the local buckling in the compressive flange, a design with the top flange replaced by concrete is proposed. It reduces the material cost and increases the stiffness of the I-Beam. It’s section is shown in Fig. 6.1.

![Figure 6.1: Beam sections of FRP combined with concrete.](image-url)
Before setting any equations, certain assumptions have been made:

- Plane sections remain plane in the bending analysis
- Shear stresses are constant along the width of the GFRP webs and concrete
- Debonding at the concrete-GFRP is neglected. At the concrete-GFRP interface, bond failure is unlikely to occur as the shear strength of epoxy is often several times higher than that of concrete and also shear connectors can be used.

In the flange, the stresses are mainly normal and unidirectional, and so almost all fibers are 0° fibers as in laminated I-Beam described in chapter 5.2. As for the webs, as they mainly carry shear stresses, the fibers are ±45° fibers.

In this chapter, the neutral axis, equivalent bending rigidity, stress and strain in the flange, shear in the web and also the moment capacity of the I-beam will be derived. A design example is given in the section 6.3.

6.2 Analysis of Hybrid I-Beam

6.2.1 Neutral Axis

To find the neutral axis, the tension and compression forces provided by the section are set to be equal as there is no axial force applied.

*Tension*

Flange -

\[ N_x = \frac{1}{A^{-1}11_{flange}} \left( h_f \xi \right) \kappa_x \]  

(6.1)

where \( \xi = h_v - y + \frac{h_f}{2} \)
Web - \[ \varepsilon_x^0 = A^{-1}_{1\text{ (web)}}N_x \]

where \( \varepsilon_x^0 = \xi \kappa_x \)

\[
N_x = \int_{0}^{h_w-y} \frac{1}{A^{-1}_{1\text{ (web)}}} \xi \kappa_x d\xi
\]

\[= \frac{(h_w - y)^2}{2A^{-1}_{1\text{ (web)}}} \kappa_x \tag{6.2} \]

**Compression**

Flange - \[\sigma = E_c (\xi \kappa_x)\]

\[
N_x = \int_{y}^{y+h_f} b_f E_c \kappa_x d\xi
\]

\[= b_f E_c \kappa_x \int_{y}^{y+h_f} \xi d\xi
\]

\[= \frac{b_f E_c \kappa_x}{2} [(y + h_f)^2 - y^2] \tag{6.3} \]

Web - \[N_x = \frac{y^2}{2A^{-1}_{1\text{ (web)}}} \kappa_x \tag{6.4} \]

Rearranging the equations, we get:

\[y = \frac{h_w^2 A^{-1}_{1\text{ (flange)}} + b_f A^{-1}_{1\text{ (web)}}(2h_w + h_f - b_f E_c h_f^2 A^{-1}_{1\text{ (web)}} A^{-1}_{1\text{ (flange)}})}{2(b_f E_c h_f A^{-1}_{1\text{ (web)}} A^{-1}_{1\text{ (flange)}} + h_w A^{-1}_{1\text{ (flange)}} + b_f A^{-1}_{1\text{ (web)}})} \tag{6.5} \]
6.2.2 Equivalent Bending Rigidity

The bending rigidity of the I-Beam is provided by the concrete flange, web and the bottom FRP flange. Their effects are given as follows:

**Concrete Flange**

\[
E I_{\text{eff}} = E c b_f h_{f1} \left( \frac{h_{f1}^2}{12} + \xi_2^2 \right) \tag{6.6}
\]

**Web**

Refer to Eq. (5.21)

\[
M_{(\text{web})} = \int_{-(h_w-y)}^{y} N \xi d\xi
\]

\[
= \int_{-(h_w-y)}^{y} \frac{1}{A^{-1}_{11(\text{web})}} \xi^2 \kappa d\xi
\]

\[
= \frac{1}{3 A^{-1}_{11(\text{web})}} \left[ y^3 + (h_w - y)^3 \right] \kappa
\tag{6.7}
\]

**FRP Flange**

Narrow flange -

\[
M_{(\text{flange})} = b_f \left( \frac{\xi_1^2}{A^{-1}_{11(\text{flange})}} + \frac{1}{D^{-1}_{11(\text{flange})}} \right) \kappa
\tag{5.26}
\]

Wide flange -

\[
M_{(\text{flange})} = b_f \left( \frac{\xi_1^2}{A^{-1}_{11(\text{flange})}} + D_{11(\text{flange})} \right) \kappa
\tag{5.29}
\]
Overall I-Beam

The effective $\bar{EI}$ is an overall effective stiffness for the beam and is given by:

Narrow flange -

$$\bar{EI} = E_c b_f h_f \left( \frac{h_f^2}{12} + \varepsilon_z^2 \right) + \frac{1}{3A_{11(web)}} \left[ y^3 + \left( h_w - y \right)^3 \right] + b_f \left( \frac{\varepsilon_y^2}{A_{11(flange)}} + \frac{1}{D_{11(flange)}} \right) \quad (6.8)$$

Wide flange -

$$\bar{EI} = E_c b_f h_f \left( \frac{h_f^2}{12} + \varepsilon_z^2 \right) + \frac{1}{3A_{11(web)}} \left[ y^3 + \left( h_w - y \right)^3 \right] + b_f \left( \frac{\varepsilon_y^2}{A_{11(flange)}} + D_{11(flange)} \right) \quad (6.9)$$

6.2.3 Strains/Stresses in the Flange

To find the flange strains and stresses, the curvature of the I-beam has to be determined first. Then the strains in the individual plies can be obtained by multiplying the curvature with the distance of that ply to the neutral axis. By the stress-strain relationship [Eq. (4.5)], the stresses can then be found.

1. Curvature

$$\kappa_x = \frac{M_{beam}}{EI} \quad (6.10)$$

2. Strains

Narrow beams:

$$\varepsilon_x = z \kappa_x \quad (6.11)$$

$$\varepsilon_y = z \kappa_y = z \left( \frac{D_{11}^{-1}}{D_{11}^{-1}} \kappa_x \right) \quad (6.12)$$
where $z$ is the distance between the ply concerned and the neutral axis.

Wide beams:

\[ \varepsilon_x = z \kappa_x \]  
\[ \varepsilon_y = z \kappa_y = 0 \]  

The strains then can be transformed into the fiber directions for the plies of interest and are given as:

\[ \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = [R][T][R^{-1}] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ 0 \end{bmatrix} \]  

3. **Stresses**

\[ \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [Q] \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \]  

6.2.4 **Shear Strains/Stresses in the Web**

The shear force is carried primarily by the web in an I-Beam with thin flanges. Dividing the shear force by the height of the web gives an estimate of $N_{xy}$ that then can be applied to the web layup. To resist the shear force, the web may have fibers placed at angles to the axial direction, mainly $\pm 45$ degrees. The resulting strains and stresses are then transformed to the fiber directions.

\[ N_{xy} = \frac{V}{h_w} \]
The strains can then be converted into fiber directions by Eq. (6.15) and the stresses can be obtained by Eq. (6.16).

6.2.5 Ultimate Bending Moment

In the concrete compression flange, when the maximum compressive strain equals the ultimate failure strain of concrete $\varepsilon_c^u$, the concrete crushes and it defines the ultimate bending moment of the I-beam section.

The curvature of the I-beam at the moment of ultimate bending moment:

$$
\kappa_{x(ult)} = \frac{\varepsilon_c^u}{y + h_f}
$$

$$
M_{ult} = E I \kappa_{x(ult)}
$$

6.3 Design Example

In this section, the procedure and an example of the analysis are presented. A Matlab program has been written to analyze the I-beam.

6.3.1 Procedure

To analyze a hybrid composite I-beam, one has to know the properties of the composite used and the geometry of the I-beam. After the load and size of the section is specified, the analysis can be done and serviceability checked according to the deflection.
requirement and web shear buckling constraint. The procedure is described given in the following flow charts.

*Properties of the Composite*

- Obtain lamina properties $E_{11}, E_{22}, G_{12}$ and $\nu_{12}$
- Determine lamina stiffness matrix $Q$
- Obtain angle of orientation for each lamina and calculate the transformation matrices
- Determine lamina transformed stiffness matrices $\bar{Q}$ for each lamina
- Determine coordinates $h_i$ for each lamina from the mid-planes of the web or flange
- Calculate the axial and bending rigidities of the web and flange, $A_{\text{web}}, D_{\text{web}}, A_{\text{flange}},$ and $D_{\text{flange}}$

*Geometry of the I-Beam*

- Obtain the geometry of the I-beam, $h_{f1}, h_{f2}, b_f, h_w,$ and $t_w$
- Determine the properties of the concrete used $E_c$ and $\varepsilon_c^u$
- Obtain the neutral axis and equivalent bending rigidity of the section
- Determine the stresses and strains in the flanges
- Determine the shear stresses and strains in the web
Serviceability Check

Determine the allowable deflection by the equation
\[ \Delta_{\text{allowable}} = \frac{L}{200} \]

Compare the above value to the maximum deflection of the beam

Calculate equivalent laminate elastic properties \( E_x, E_y, G_{xy}, v_{xy} \)
and \( v_{yx} \)

Calculate the average web shear buckling stress with the use of Timoshenko equations

6.3.2 Example

The cross section of the hybrid I-beam is shown in Fig. 6.2. The beam is simply supported over span 2m with distributed load 10 kN/m. The top flange is made of concrete with strength 30 MPa, while the web and bottom flange are made of laminated composite materials. The layup of the web is \([\pm 45, / 90, / 0, / 0]_s\), with the \(\pm 45\) plies carried into the flange. The flange has 8 additional plies of 0 (axial) and another set of \(\pm 45\) plies to give it a symmetric layup. The thickness of each plies is normally 0.13208 mm. The dimensions and loading are as follows:

![Cross section of the hybrid I-beam](image)

Figure 6.2: Cross section of the hybrid I-beam.
Properties of the materials:

Composite - AS4/3501-6 carbon/epoxy prepreg
- $E_{11}$ - 131 GPa
- $E_{22}$ - 11.2 GPa
- $G_{12}$ - 6.55 GPa
- $\nu_{12}$ - 0.28

Concrete:
- Young Modulus $E_c$ - 30 GPa
- Maximum failure strain $\varepsilon_c^u$ - 0.003

Geometry of the I-beam:
Top concrete flange height $h_{f1}$ - 50 mm
Bottom FRP flange height $h_{f2}$ - 3.17mm
Flange width $b_f$ - 80 mm
Web height $h_w$ - 200mm
Web width $t_w$ - 3.17mm
Web layup - $[\pm45/90_2/0_2]_s$
Flange layup - $[\pm45/0_8]_s$

Loading:
Length - 2000mm
Distributed load - 10 kN/m

As the analysis of composite materials involves the manipulation of matrices, all the calculations are done using a Matlab program. The program is included in the Appendix. Some important results are listed below.
Analysis Results

As the aspect ratio of the bottom flange is about 30, it can be treated as a wide flange. Below are the results obtained following the steps as described in chapter 6.3.1.

Neutral axis: 19.3 mm below the bottom part of the concrete flange
Equivalent bending rigidity: 1.12 MNm
Maximum curvature: 0.45%
Maximum flange strain: 0.08% (0° fibers)
Maximum flange stress: 108 MPa (0° fibers)
Web shear strain: 0.03% (45° fibers)
Web shear stress: 31 MPa (45° fibers)

Serviceability Check

Deflection -
The deflection is calculated by means of the usual beam formula with EI replaces by the effective $\overline{EI}$.

\[
\Delta = \frac{wL^4}{\overline{EI}} = 17\text{mm}
\]

\[
\Delta_{\text{allowable}} = 10\text{mm}
\]

Web shear buckling -
The average web shear buckling stress $r_{bw}$ can be determined using the formulas suggested by Timoshenko.

\[
r_{bw(\text{allowable})} = 44\text{MPa}
\]

\[
r_{bw(\text{calculated})} = 16\text{MPa}
\]
Comments

1. The calculated parameters, such as strains and stresses are reasonable values compared to the material strengths as shown in Table 5.1.

2. The deflection of the I-beam is slightly greater than the allowable value. This can be amended by adding additional 0 plies to the flange, or replacing the ±45 plies in the flange with 0 plies to increase the stiffness. The latter choice will cause no weight penalty, but will make the flange layup to be unsymmetrical. Though the formulas derived in this thesis are based on symmetry of the composite layup, their use in unsymmetrical cases will lead to a minor errors only.

3. As the thickness of composite plies are very thin, buckling of the flanges and torsional stiffness are important problems associated with composite structures. This situation can be improved by additional foam bonded to the beam, which is not discussed in this thesis.
Chapter 7
Conclusions

Many bridge failures during the recent earthquakes were caused by poor performance of concrete columns, mainly due to inadequate lateral reinforcement and insufficient lap length of the starter rebars and buckling of the longitudinal continuous reinforcement. The use of composite materials in retrofitting the substandard bridges has significantly increased in these years.

Fiber composites are made of fibers such as carbon and glass embedded in a resin matrix. Fiber reinforced plastics are recognized for their high strength-to-weight and stiffness-to-weight ratios, also their good fatigue life, light weight, ease of transportation and handling, and low maintenance costs. Seismic resistance of retrofitted concrete columns improves significantly as a result of the confining action of the FRP composite straps. The straps are highly effective in confining the core concrete and preventing the longitudinal reinforcement bars from buckling under cyclic loading. This finding is verified by the larger lateral load capacities and the improved lateral-displacement hysteresis loops observed.

In the thesis, lamination theory is introduced. It is the core for the design of composite materials. A laminated I-beam is analyzed and it is found that the stiffness of the beam greatly depends on the angles of fiber placement with respect to the loads in the individual plies, and the volume fractions of the fiber in the composites. A hybrid I-beam design with the top flange replaced by concrete is evaluated. Results show that the beam has a high stiffness to weight ratio.

Although composites are excellent construction materials, their high initial cost is a significant barrier to their adoption for traditional structures at this time.
References


Appendices

Appendix A: Matlab Program  63

Appendix B: Outputs from the Program  67
% This program is written to analyse a hybrid I-beam made of FRP with concrete
% replace the top flange.

% **********************************************************************************************
% Vivian Lai Ki Wan
% MENG (Course 1, HPS)
% 979384972

% Method for Improving Seismic Performance of Bridges
% **********************************************************************************************
disp('Enter the Properties of the FRP Material and Define Q ')

disp(' Enter the Properties of the FRP Material and Define Q ')
E11=input(' Enter E11 : ');
E22=input(' Enter E22 : ');
v12=input(' Enter v12: ');
G12=input(' Enter G12: ');
v21=E22*v12/E11;
Ec=input(' Enter the Young Modulus of Concrete: ');
Q=[E11/(1-v12*v21) v21*E11/(1-v12*v21) 0; v12*E22/(1-v12*v21) E22/(1-v12*v21) 0; 0 0 G12]
disp('Transformation Matrix and BarQ')

% Transformation Matrix for x,y system at 45 degree with respect to 1,2 system.
theta45=45*pi/180;
T45=[(cos(theta45))^2 (sin(theta45))^2 2*sin(theta45)*cos(theta45); ...
     (sin(theta45))^2 (cos(theta45))^2 -2*sin(theta45)*cos(theta45); ...
     -sin(theta45)*cos(theta45) sin(theta45)*cos(theta45) (cos(theta45))^2-(sin(theta45))^2]

% Transformation Matrix for x,y system at 90 degree with respect to 1,2 system.
theta90=90*pi/180;
T90=[(cos(theta90))^2 (sin(theta90))^2 2*sin(theta90)*cos(theta90); ...
     (sin(theta90))^2 (cos(theta90))^2 -2*sin(theta90)*cos(theta90); ...
     -sin(theta90)*cos(theta90) sin(theta90)*cos(theta90) (cos(theta90))^2-(sin(theta90))^2]

% Transformation Matrix for x,y system at 0 degree with respect to 1,2 system.
theta0=0*pi/180;
T0=[(cos(theta0))^2 (sin(theta0))^2 2*sin(theta0)*cos(theta0); ...
     (sin(theta0))^2 (cos(theta0))^2 -2*sin(theta0)*cos(theta0); ...
     -sin(theta0)*cos(theta0) sin(theta0)*cos(theta0) (cos(theta0))^2-(sin(theta0))^2]
Appendix A: Matlab Program

R=[1 0 0; 0 1 0; 0 0 2];
BarQ45=inv(T45)*Q*R*T45*inv(R)
BarQ90=inv(T90)*Q*R*T90*inv(R)
BarQ0=inv(T0)*Q*R*T0*inv(R)

disp('******************************************************************

disp(' Define A and D ')
disp('WEB')
Aweb=BarQ45.*(2*1.05664e-3)+BarQ90.*(2*.26416e-3)+BarQ0.*(2*.26416e-3)
temp45=((-.52832e-3)^3-(-1.58496e-3)^3+(1.58496e-3)^3-(.52832e-3)^3)/3;
temp90=((-.26416e-3)^3-(-.52832e-3)^3+(.52832e-3)^3-(.26416e-3)^3)/3;
temp0=((.52832e-3)^3-(-1.05664e-3)^3+(1.05664e-3)^3-(.52832e-3)^3)/3;
Dweb=BarQ45.*temp45+BarQ90.*temp90+BarQ0.*temp0

disp('FLANGE')
Aflange=BarQ45.*(2*.52832e-3)+BarQ0.*(2*.79248e-3)+BarQ90.*(2*.26416e-3)
temp45 = ((-1.05664e-3)^3-(-1.58496e-3)^3+(1.58496e-3)^3-(1.05664e-

temp0=((0)^3-(-1.05664e-3)^3+(1.05664e-3)^3-(0)^3)/3;
Dflange=BarQ45.*temp45+BarQ0.*temp0

disp('******************************************************************

disp(' Enter the Geometry of the Section ')
disp('')

hf1=input(' Enter the concrete flange height: ');
bf=input(' Enter the flange width: ');
hw=input(' Enter the web height ');
hf2=input(' Enter the bottom FRP flange height: ');
tw=input(' Enter the width of the web: ')

disp('******************************************************************

disp(' Enter the Force and Length of the Beam ')

L=input(' Enter the length of the beam: ');
w=input(' Enter the uniform distributed load: ');

Moment=w*L^2/8
Shear=w*L/2

disp('******************************************************************

disp(' NEUTRAL AXIS ')

inv_Aweb=inv(Aweb);
inv_Dweb=inv(Dweb);
inv_Aflange=inv(Aflange);
inv_Dflange=inv(Dflange);

Denominator=2*(bf*Ec*hfl*inv_Aweb(1,1)*inv_Aflange(1,1)+hw*inv_Aflange(1,1)+bf*inv_Aweb(1,1));
Nominator=hw^2*inv_Aflange(1,1)+bf*inv_Aweb(1,1)*(2*hw+hfl2)-
bf*Ec*hfl^2*inv_Aweb(1,1)*inv_Aflange(1,1);
y=Nominator/Denominator

disp(' ***************************************
I)
disp(' ')
disp(' EQUIVALENT BENDING RIGIDITY ')
disp(' ')
disp(' The flange is considered to be wide beam. ')

X1=hw-y+hfl2/2;
X2=y+hfl1/2;

EI=Ec*bf*hfl*(hfl^2/12+X2^2)+1/3/inv_Aweb(l,l)*(y^3+(hw-y)^3)+bf*(X1^2/invAflange(l,1)+Dflange(1,l))

disp(' ***************************************
I)
disp(' ')
disp(' Strains and Stresses in the Flange ')

Curvature=Moment/EI
% z is the distance from the neutral axis to the bottom 0 ply as the strains
% and stresses should be the largest there.
z=X1+.79248e-3;
Epsilonx=z*Curvature;
Epsilony=0;
Epsilonxy=[Epsilonx; Epsilony; 0];
Strain12=R*TO*inv(R)*Epsilonxy
Stress12=Q*Strain12

disp(' ***************************************
I)
disp(' ')
disp(' Shear Strains/Stresses in the Web ')

Nxy=Shear/hw;
Epsilonxy=inv_Aweb*[0; 0; Nxy];
Shear_strain12=R*T45*inv(R)*Epsilonxy
Shear_stress12=Q*Shear_strain12

disp(' ***************************************
I)
disp(' ')
disp(' Ultimate Bending Moment ')
Max_concrete_strain=input(' Enter the ultimate failure strain of concrete: ') 
Curvature_failure=Max_concrete_strain/(y+hfl)
Mult=EI*Curvature_failure

disp(' ************************************************
disp('')
disp(' Check Serviceability: Deflection ')

Allowable_deflection=L/200
Deflection=w*L^4/8/EI

disp(' ************************************************
disp('')
disp(' Calculation of the Equivalent Elastic Properties in x,y Coordiantes ')
disp('')
disp(' WEB')

Ex=12/tw^3/inv_Dweb(2,2)
Ey=12/tw^3/inv_Dweb(1,1)

vxy=-inv_Dweb(1,2)/inv_Dweb(2,2)
vyx=-inv_Dweb(1,2)/inv_Dweb(1,1)

disp(' ************************************************
disp('')
disp(' Check Serviceability: Web Shear Buckling ')

Dl=Ex*tw^3/12/(1-vxy*vyx);
Dt=Ey*tw^3/12/(1-vxy*vyx);
H=.5*(vxy*Dt+vyx*Dl)+1.058e10*tw^3/6/(1-vxy*vyx);
theta=sqrt(Dl*Dt)/H

K=22.2;

Allowable_thau=4*K*sqrt(Dt*H)/tw/hw^2

thau=Shear/tw/hw
Enter the Young Modulus of Concrete: 30e9

Enter the Properties of the FRP Material and Define Q

Enter E11 : 131e9
Enter E22 : 11.2e9
Enter v12: .28
Enter G12: 6.55e9

Enter the Young Modulus of Concrete: 30e9

Q =

1.0e+11 *
1.3188 0.0316 0
0.0316 0.1128 0
0 0 0.0655

Transformation Matrix and BarQ

T45 =

0.5000 0.5000 1.0000
0.5000 0.5000 -1.0000
-0.5000 0.5000 0.0000

T90 =

0.0000 1.0000 0.0000
1.0000 0.0000 -0.0000
-0.0000 0.0000 -1.0000

T0 =

1 0 0
0 1 0
0 0 1

BarQ45 =

1.0e+10 *
4.3918 3.0818 3.0152
3.0818 4.3918 3.0152
3.0152 3.0152 3.4211

BarQ90 =

1.0e+11 *
0.1128 0.0316 0.0000
0.0316 1.3188 0.0000
0.0000  0.0000  0.0655

BarQ0 =
1.0e+11 *
  1.3188  0.0316   0
  0.0316  0.1128   0
  0       0       0.0655

******************************************************************************
Define A and D
WEB
Aweb =
1.0e+08 *
  1.6845  0.6846  0.6372
  0.6846  1.6845  0.6372
  0.6372  0.6372  0.7922

Dweb =
  114.8495  79.0847  77.0710
  79.0847  123.7423  77.0710
  77.0710  77.0710  88.0906

FLANGE
Aflange =
1.0e+08 *
  2.6139  0.3924  0.3186
  0.3924  1.3395  0.3186
  0.3186  0.3186  0.4999

Dflange =
  185.7600  60.0489  56.3211
  60.0489  90.9034  56.3211
  56.3211  56.3211  69.0548

******************************************************************************
Enter the Geometry of the Section

Enter the concrete flange height: 50e-3
Enter the flange width: 80e-3
Enter the web height 200e-3
Enter the bottom FRP flange height: 3.16992e-3
Enter the width of the web: 3.16992e-3

tw = 0.0032

Enter the Force and Length of the Beam
Enter the length of the beam: 2
Enter the uniform distributed load: 10e3

Moment = 5000

Shear = 10000

NEUTRAL AXIS

y = 0.0193

EQUIVALENT BENDING RIGIDITY

The flange is considered to be wide beam.

EI = 1.1188e+06

Strains and Stresses in the Flange

Curvature = 0.0045

Strain12 = 1.0e-03 * 0.8181

Stress12 =
Shear Strains/Stresses in the Web

Shear_strain12 =
1.0e-03 *
0.2570
-0.8555
0.0000

Shear_stress12 =
1.0e+07 *
3.1198
-0.8835
0.0000

Ultimate Bending Moment

Enter the ultimate failure strain of concrete: .003

Max_concrete_strain =
0.0030

Curvature_failure =
0.0433

Mult =
4.8413e+04

Check Serviceability: Deflection

Allowable_deflection =
0.0100

Deflection =
0.0179
Calculation of the Equivalent Elastic Properties in x,y Coordinates

WEB

Ex =
2.0136e+10

Ey =
1.6956e+10

vxy =
0.2458

vyx =
0.2070

***************************************************************

Check Serviceability: Web Shear Buckling

theta =
0.7295

Allowable_thau =
4.0588e+07

thau =
1.5773e+07

>> diary off