

6.302 Feedback Systems

Fall Term 2002
 Problem Set 1

Issued : September 4, 2002
 Due : Monday, September 9, 2002

Problem 1: Block Diagrams

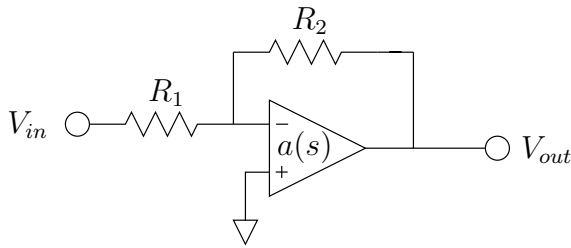


Figure 1: Op-Amp Circuit 1

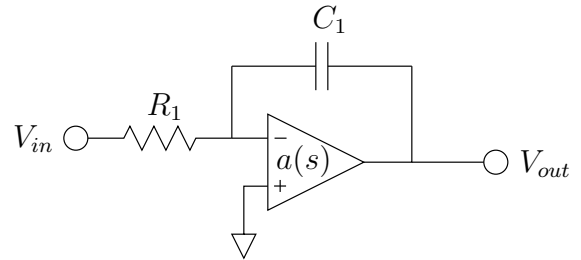


Figure 2: Op-Amp Circuit 2

Derive the block diagrams for the circuits in Figures 1 and 2. Assuming that the op-amp gain $a(s)$ is constant and goes to infinity, do the answers check with expectations?

Problem 2: Block Diagram Manipulation

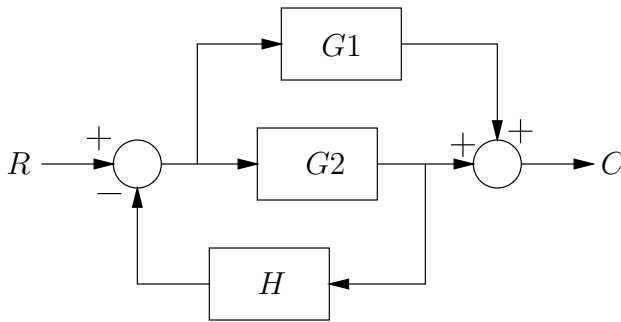


Figure 3: Problem 2, Unreduced

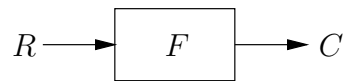


Figure 4: Problem 2, reduced

Reduce the block diagram in Figure 3 to that of Figure 4, and express F in terms of G_1 , G_2 , and H . Note that this is not the same problem that you saw on Prelab 1A.

Problem 3: Different Coordinate Systems

One of the ways which we will use to understand systems in 6.302 involves looking at them in a number of different coordinate transformations. This problem will introduce some of the methods of plotting the frequency response of a system; the transfer function of one such representative system is given below.

$$H(s) = \frac{25.25}{s^2 + s + 25.25}$$

1. Draw a plot of the poles and zeros of this transfer function.
2. By hand, draw a Bode plot for the above transfer function. Use the standard convention that you learned in 6.003; plot the magnitude measured in dB as a function of the \log_{10} frequency, and plot the linear phase as a function of the \log_{10} frequency. Use the log paper included at the end of the problem set for this purpose.
3. Plot the the real part of $H(j\omega)$ as a function of the \log_{10} frequency and plot the imaginary part of $H(j\omega)$ as a function of the \log_{10} frequency.
4. Plot the same transfer function in polar coordinates by hand. Simply put, plot the magnitude as a function of the radius, and the phase angle as a function of, well, the angle. Note that this is a parametric plot; the frequency varies along the curve. Use the polar graph paper included at the end for this plot as well.
5. Plot the transfer function on gain-phase coordinates. The construction of this type of graph follows from the observation that both magnitude and phase angle are functions of frequency; this graph represents the $10 \log_{10}$ magnitude directly as a function of the angle. When drawing this graph, make sure that you label the frequency at the points on the line which correspond to a given magnitude and angle. Use the gain-phase graph paper included at the end of this problem set.
6. Generate the above plots using MATLAB.

Problem 4: Sensitivity to Parameter Variation

For the system shown in Figure 5, determine how the fractional change in closed-loop gain depends on fractional changes in the parameters a_1 , a_2 , and f .

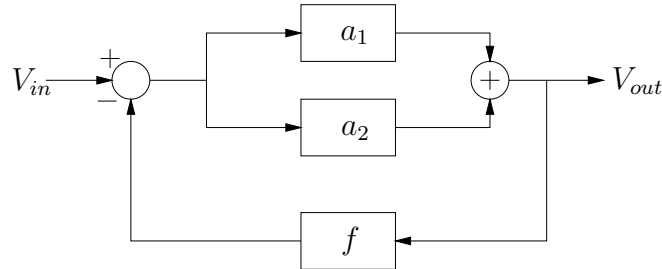


Figure 5: A sensitive system.

Computer Project 1: Introduction to Feedback Systems

This computer project should be completed using Octave, MATLAB or similar software. You may find it helpful to save your work as it may be useful for future projects. Please hand in clearly labelled printouts.

The purpose of the project is to familiarize you with transfer functions, their representation and responses. You will explore the relationship between pole-zero locations and corresponding time and frequency responses. For the following transfer functions plot (i) the pole-zero diagram, (ii) the bode plot, and (iii) the step response:

- (a) First order system

$$H_1(s) = \frac{1}{s + 1}$$

- (b) To this first order system we now add a higher frequency pole

$$H_2(s) = \frac{2}{(s + 1)(s + 2)}$$

- (c) The following two transfer functions add a right-half and a left-half plane pole, respectively

$$H_3(s) = -\frac{1}{2} \cdot \frac{s - 2}{s + 1}$$

$$H_4(s) = \frac{1}{2} \cdot \frac{s + 2}{s + 1}$$

- (d) The following transfer function is a standard second order system in canonical form with $\omega_n = 5$ and $\zeta = 0.2$.

$$H_5(s) = \frac{25}{s^2 + 2s + 25}$$

- (e) Now add a zero at the origin. Remember that s in the frequency domain corresponds to d/dt in the time domain.

$$H_6(s) = \frac{25s}{s^2 + 2s + 25}$$

- (f) For H_7 the zero is moved into the LHP.

$$H_7(s) = \frac{25(s + 1)}{s^2 + 2s + 25}$$

- (g) Now place the zero at 5 rad/s.

$$H_8(s) = \frac{5(s + 5)}{s^2 + 2s + 25}$$

- (h) For the final two transfer functions, a third pole is added to H_5 , first at a low frequency, then at a high frequency.

$$H_9(s) = \frac{25}{(s + 1)(s^2 + 2s + 25)}$$

$$H_{10}(s) = \frac{125}{(s + 5)(s^2 + 2s + 25)}$$