Problem: Bode Obstacle Course

You are given a set of performance specifications for a unity-gain feedback system. They are as follows:

- Crossover frequency of \( \omega_c = 100 \text{ rps} \) with a slope of -1 at crossover.
- Zero steady state error for a ramp input.
- \( \left| \frac{E(j\omega)}{R(j\omega)} \right| < 0.005 \) for sinusoidal inputs over the range \( 0 < \omega < 1 \text{ rps} \).
- For sensor noise variations in the frequency range \( \omega > 1000 \text{ rps} \), \( \left| \frac{C(j\omega)}{N(j\omega)} \right| < 0.02 \).
- To obtain an adequate margin of stability, the frequency range of the -1 slope around crossover should be at least 2 decades.
- The phase margin should be at least 60°.

1. Draw the Bode obstacle course for these specifications.

2. Construct an asymptotic Bode magnitude curve which will meet these specifications with as little extra magnitude as possible. At the corners of your diagram label the corner frequencies and the values of the magnitudes of \( L(j\omega) \) at each corner. Give an expression for a rational function \( L(s) \) which will exhibit the same asymptotic curve that you have just constructed.

3. Based upon the asymptotic magnitude curve of Part 2, draw the approximate asymptotic Bode magnitude curve of the closed loop response \( \left| \frac{E(j\omega)}{R(j\omega)} \right| \).

Computer Project 6: Nichols, Stability Margins and the Bode Obstacle Course

This computer project should be completed using Octave, MATLAB or similar software. You may find it helpful to save your work as it may be useful for future projects. Please hand in clearly labelled printouts. The purpose of this project is to investigate the utility of the Nichols chart, stability margins, and the Bode obstacle course.

1. Nichols

Using the software, produce Nichols plots of the following transfer functions assuming unity feedback.
(a) \[ L_1(s) = \frac{1}{s+1} \]

(b) \[ L_2(s) = \frac{1}{(s+1)^2} \]

(c) \[ L_3(s) = \frac{K}{(s+1)^3} \]

i. What is the gain margin of the system \((K = 1)\)?

ii. What is the phase margin for \(K = 4\)?

iii. Are these values the same as those produced using the Nyquist plot in the last project?

iv. On the same axis produce labelled plots for \(K = 1, 2, 4, 8\) and 10.

(d) Plot the Nichols chart for \(K = 1\) and 10 for \[ L_4(s) = \frac{25K}{s^2 + 2s + 25}. \]

(e) As you begin to design compensators you may find that you use phase margin considerations more than gain margin. To demonstrate the importance of gain margin, consider a canonical second-order system with \(\omega_n = 1000\) rps and \(\zeta = 0.1\) in series with an integrator, in unity feedback configuration.

\[ L_5(s) = \frac{K\omega_n^2}{s(s^2 + 2\zeta\omega_n + \omega_n^2)} \]

i. On the same axes plot the step response for the closed-loop system with \(K = 10, 100,\) and 200.

ii. On the same axes plot the Nichols charts for the same values of \(K\). Construct a table with the gain and phase margins for each value of \(K\).

(f) \[ L_6(s) = \frac{1}{s^2 + 1} \]

Is the plot as expected? If not, submit a hand sketch of the correct Nichols chart.

2. Compensation

Having looked both at the root locus and Nyquist methods for analyzing the triple-integrator compensation problem, you now turn to the Nichols chart since the problem is one of gain-setting.

\[ L(s) = \frac{K(s+2)^2}{s^8(s+100)^2}, \]
(a) Plot the Nichols plot of the system, with $K = 1$.
(b) Find the range of $K$ for which the closed-loop system is stable.
(c) By choosing the appropriate value of $K$ maximize the phase margin.
   i. Plot the new Nichols plot on the same axes.
   ii. Mark the phase margin and gain margin on the graph.
(d) Plot the step response of the closed-loop system with your value of $K$. What is the peak overshoot?
(e) Produce the bode diagram of the system with your value of $K$ and mark the gain and phase margin on the plot.

3. Bode Obstacle Course
In this problem you will be designing a series compensator for our DC Motor. As a reminder the motor parameters are: $J = 3 \times 10^{-5}$ kg-m$^2$, $R_m = 8$ Ω, $L_m = 5$ mH, $K_t = 0.025$ N-m/A, and $K_e = 0.025$ V-s-rad.
You are designing a position-servo for the motor. Position sensing is via a potentiometer for which $K_{pot} = 1$. The specifications that you must meet are:
(a) Zero error to a unit-step input.
(b) No more than 0.01V error to a unit-ramp input.
(c) Percent overshoot of no more than 20%
(d) Crossover frequency 10 rps
(e) Noise rejection above 1000rps $\leq 10^{-7}$
(a) Plot the Bode Diagram of the uncompensated system.
(b) Sketch the Bode obstacle course on the plot, either on the computer or by hand on a printout.
(c) Design a series compensator $G_c$ to meet the specifications.
(d) To check that you have met the design specifications plot the following:
   i. Step response and error to step on the same axes.
   ii. Ramp response and error to a ramp on the same axes.
   iii. Bode diagram.