Lecture 11 - Carrier Flow *(cont.)*

September 28, 2001

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1. Majority-carrier type situations
2. Minority-carrier type situations

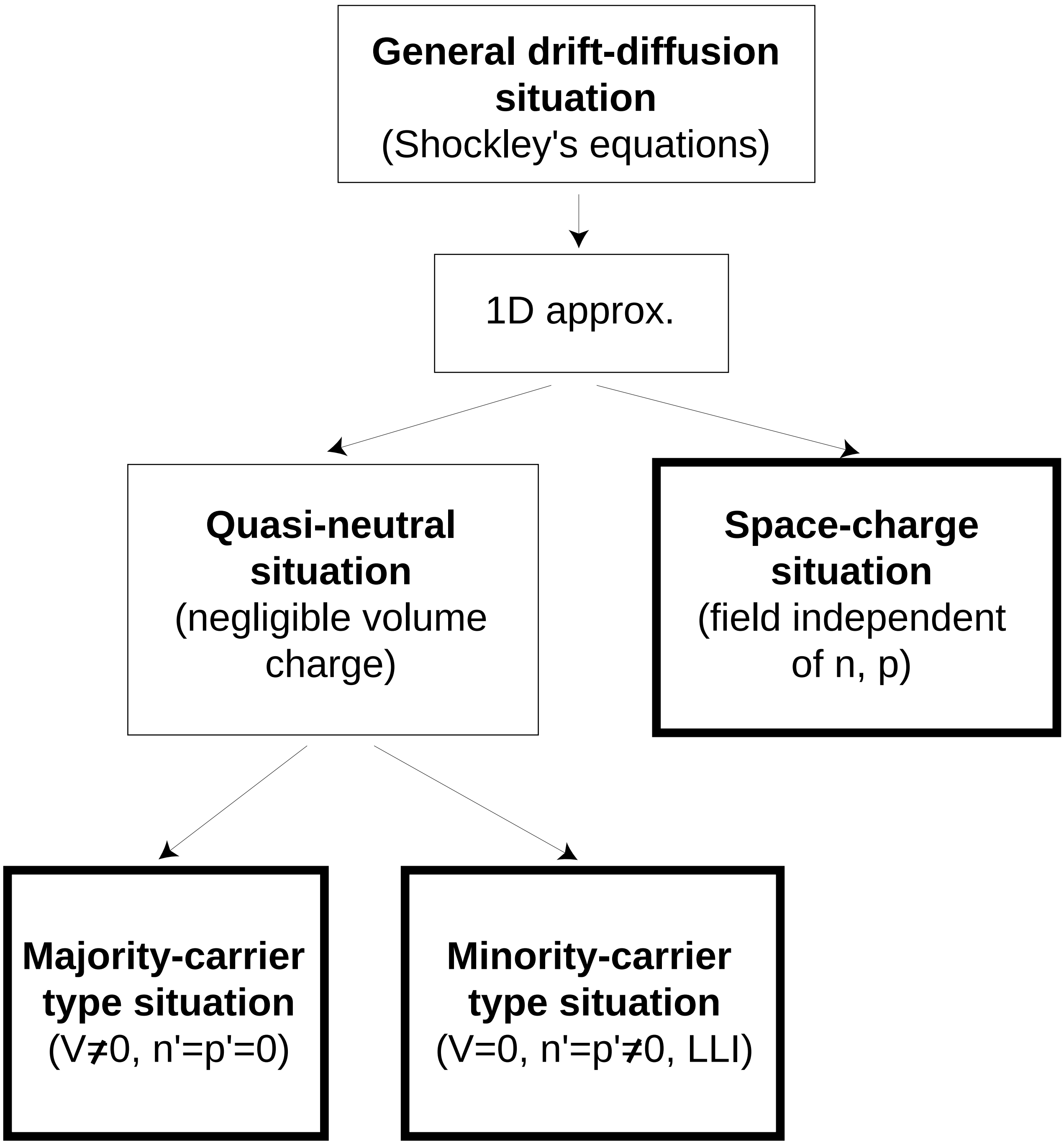
Reading assignment:

del Alamo, Ch. 5, §§5.5, 5.6 (5.6.1)

Key questions

- What characterizes *majority*-carrier type situations?
- What characterizes *minority*-carrier type situations?

Overview of simplified carrier flow formulations



Simplified set of Shockley equations for 1D quasi-neutral situations

$$p - n + N_D - N_A \simeq 0$$

$$J_e = -qnv_e^{drift} + qD_e\frac{\partial n}{\partial x}$$

$$J_h = qpv_h^{drift} - qD_h\frac{\partial p}{\partial x}$$

$$\frac{\partial n}{\partial t} = G_{ext} - U + \frac{1}{q}\frac{\partial J_e}{\partial x} \quad \text{or} \quad \frac{\partial p}{\partial t} = G_{ext} - U - \frac{1}{q}\frac{\partial J_h}{\partial x}$$

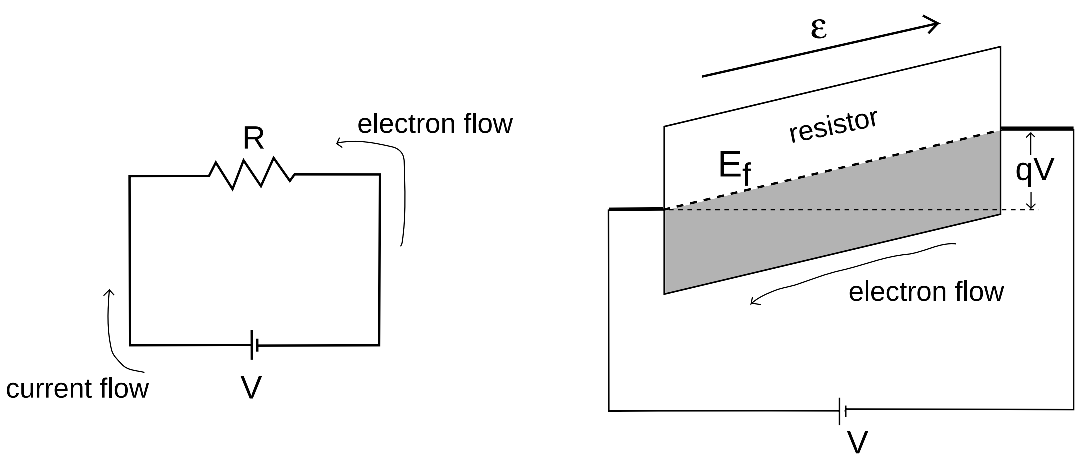
$$\frac{\partial J_t}{\partial x} \simeq 0$$

$$J_t = J_e + J_h$$

1. Majority-carrier type situations

Voltage applied to extrinsic quasi-neutral semiconductor without upsetting the equilibrium carrier concentrations.

□ Remember what a battery does:



circuit view **energy view**

• Battery picks up electrons from positive terminal, increases their potential energy and puts them at the negative terminal.

• If provided with a path (resistor), electrons flow.

□ Characteristics of majority carrier-type situations:

- electric field imposed from outside
- electrons and holes drift
- electron and hole concentrations unperturbed from TE

Simplifications:

- neglect contribution of minority carriers
- neglect time derivatives of carrier concentrations

⇒ problem becomes completely *quasi-static*

□ Simplification of majority carrier current (n-type):

Must distinguish between internal field in TE ($\mathcal{E}_o$) and total field outside equilibrium ($\mathcal{E}$).

In equilibrium:

$$J_{eo} = -qn_o v_e^{drift}(\mathcal{E}_o) + qD_e \frac{dn_o}{dx} = 0$$

Out of equilibrium:

$$J_e \simeq -qn_o v_e^{drift}(\mathcal{E}) + qD_e \frac{dn_o}{dx}$$

Hence:

$$J_e = -qn_o[v_e^{drift}(\mathcal{E}) - v_e^{drift}(\mathcal{E}_o)]$$

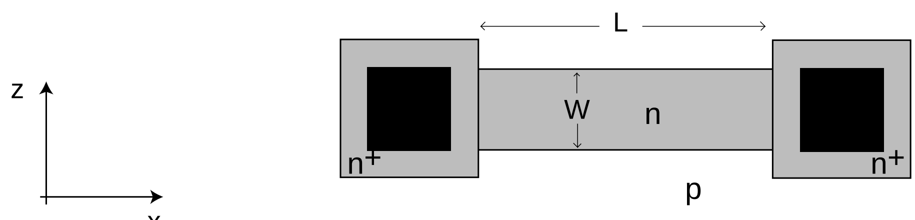
For low fields,

$$J_e = qn_o\mu_e(\mathcal{E} - \mathcal{E}_o) = qn_o\mu_e\mathcal{E}'$$

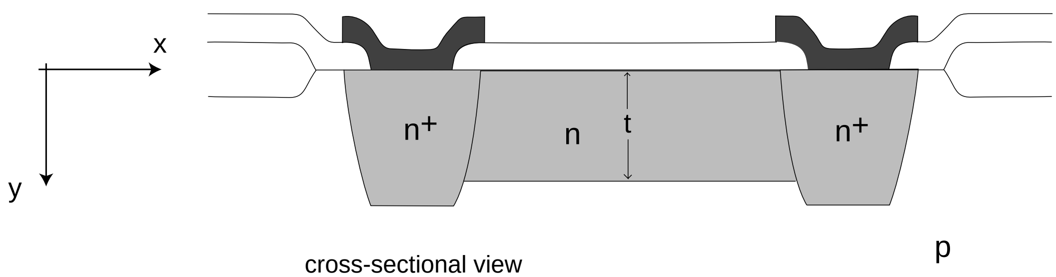
□ Equation set for 1D majority-carrier type situations:

n-type	p-type
$n \simeq n_o \simeq N_D$	$p \simeq p_o \simeq N_A$
$J_e = -qn_o[v_{de}(\mathcal{E}) - v_{de}(\mathcal{E}_o)]$	$J_h = qp_o[v_{dh}(\mathcal{E}) - v_{dh}(\mathcal{E}_o)]$
$\frac{dJ_e}{dx} \simeq 0,\ \frac{dJ_h}{dx} \simeq 0,\ \frac{dJ_t}{dx} \simeq 0$	
$J_t \simeq J_e$	$J_t \simeq J_h$

☐ Example 1: *Integrated Resistor* with uniform doping (n-type)



top view



cross-sectional view

Uniform doping $\Rightarrow \mathcal{E}_o = 0$, then:

$$J_t = -qN_D v_e^{drift}(\mathcal{E})$$

- If $\mathcal{E}$ not too high,

$$J_t \simeq qN_D \mu_e \mathcal{E}$$

I-V characteristics:

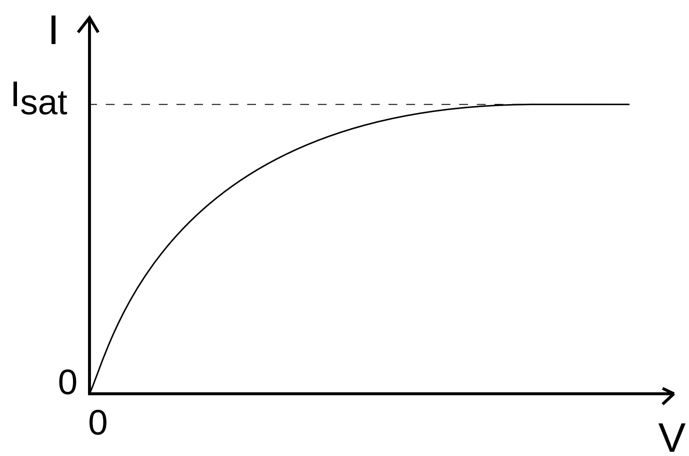
$$I = WtqN_D\mu_e \frac{V}{L}$$

• In general (low and high fields):

$$I = WtqN_D \frac{v_{sat}}{1 + \frac{v_{sat}}{\mu_e}\frac{L}{V}}$$

which for high fields saturates to:

$$I_{sat} = WtqN_D v_{sat}$$



2. Minority-carrier type situations

Situations characterized by:

- excess carriers over TE
- no external electric field applied (but small internal field generated by carrier injection: $\mathcal{E} = \mathcal{E}_o + \mathcal{E}'$)

Example: electron transport through base of npn BJT.

Two approximations:

1. $\mathcal{E}$ small $\Rightarrow |v^{drift}| \propto |\mathcal{E}|$
2. Low-level injection $\Rightarrow$ for n-type:
    - $n \simeq n_o$
    - $p \simeq p'$
    - $U \simeq \frac{p'}{\tau}$
    - negligible minority carrier drift due to $\mathcal{E}'$
      (but can't say the same about majority carriers)

Shockley equations for 1D quasi-neutral situations

$$p - n + N_D - N_A \simeq 0$$

$$J_e = -qnv_e^{drift} + qD_e\frac{\partial n}{\partial x}$$

$$J_h = qpv_h^{drift} - qD_h\frac{\partial p}{\partial x}$$

$$\frac{\partial n}{\partial t} = G_{ext} - U + \frac{1}{q}\frac{\partial J_e}{\partial x} \quad \text{or} \quad \frac{\partial p}{\partial t} = G_{ext} - U - \frac{1}{q}\frac{\partial J_h}{\partial x}$$

$$\frac{\partial J_t}{\partial x} \simeq 0$$

$$J_t = J_e + J_h$$

□ Further simplifications for n-type minority-carrier-type situations

- Majority-carrier current equation:

$$J_e \simeq q(n_o + n')\mu_e(\mathcal{E}_o + \mathcal{E}') + qD_e(\frac{\partial n_o}{\partial x} + \frac{\partial n'}{\partial x})$$

but in TE:

$$J_{eo} = qn_o\mu_e\mathcal{E}_o + qD_e\frac{\partial n_o}{\partial x} = 0$$

Then:

$$J_e \simeq qn_o\mu_e\mathcal{E}' + qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$$

- Minority-carrier current equation:

$$J_h \simeq q(p_o + p')\mu_h(\mathcal{E}_o + \mathcal{E}') - qD_h(\frac{\partial p_o}{\partial x} + \frac{\partial p'}{\partial x})$$

In TE, $J_{ho} = 0$, and:

$$J_h \simeq qp'\mu_h\mathcal{E}_o + qp'\mu_h\mathcal{E}' - qD_h\frac{\partial p'}{\partial x} \simeq qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$$

- Minority-carrier continuity equation:

$$\frac{\partial p'}{\partial t} = G_{ext} - \frac{p'}{\tau} - \frac{1}{q}\frac{\partial J_h}{\partial x}$$

Now plug in J_h from above:

$$D_h\frac{\partial^2 p'}{\partial x^2} - \mu_h\mathcal{E}_o\frac{\partial p'}{\partial x} - \frac{p'}{\tau} + G_{ext} = \frac{\partial p'}{\partial t}$$

One differential equation with one unknown: p'.

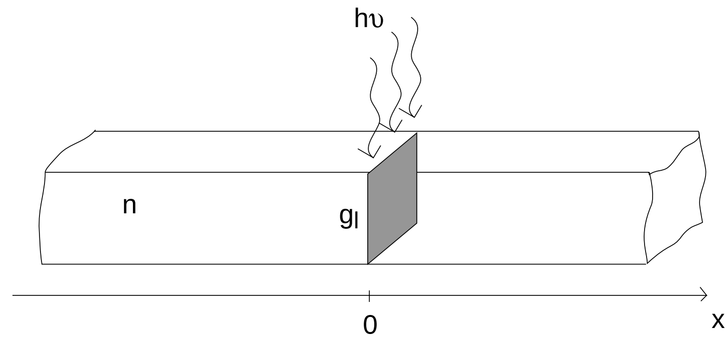
If G_{ext} and BC's are specified, problem can be solved.

Shockley equations for 1D minority-carrier type situations

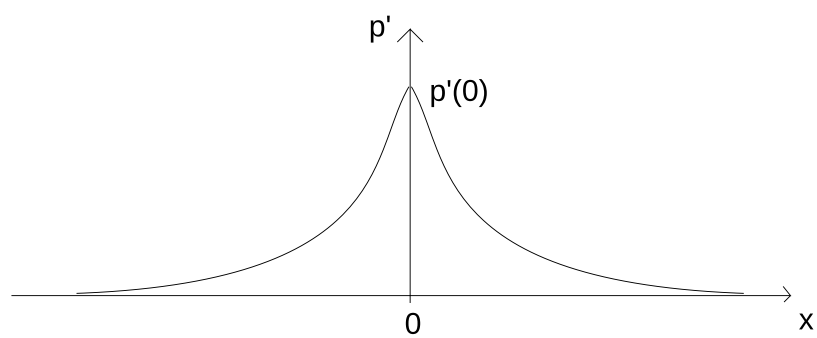
n-type	p-type
$p_o - n_o + N_D - N_A \simeq 0$	
$p' \simeq n'$	
$J_e = qn_o\mu_e\mathcal{E}' + qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$	$J_e = qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$
$J_h = qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$	$J_h = qp_o\mu_h\mathcal{E}' + qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$
$D_h\frac{\partial^2 p'}{\partial x^2} - \mu_h\mathcal{E}_o\frac{\partial p'}{\partial x} - \frac{p'}{\tau} + G_{ext} = \frac{\partial p'}{\partial t}$	$D_e\frac{\partial^2 n'}{\partial x^2} + \mu_e\mathcal{E}_o\frac{\partial n'}{\partial x} - \frac{n'}{\tau} + G_{ext} = \frac{\partial n'}{\partial t}$
$\frac{\partial J_t}{\partial x} \simeq 0$	
$J_t = J_e + J_h$	

Example 1:
Diffusion and bulk recombination in a "long" bar

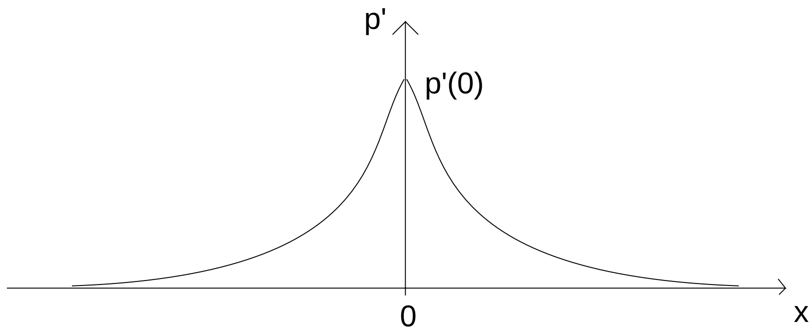
Uniform doping: $\mathcal{E}_o = 0$; static conditions: $\frac{\partial}{\partial t} = 0$



Minority carrier profile:



Majority carrier profile? $n' = p'$ exactly?

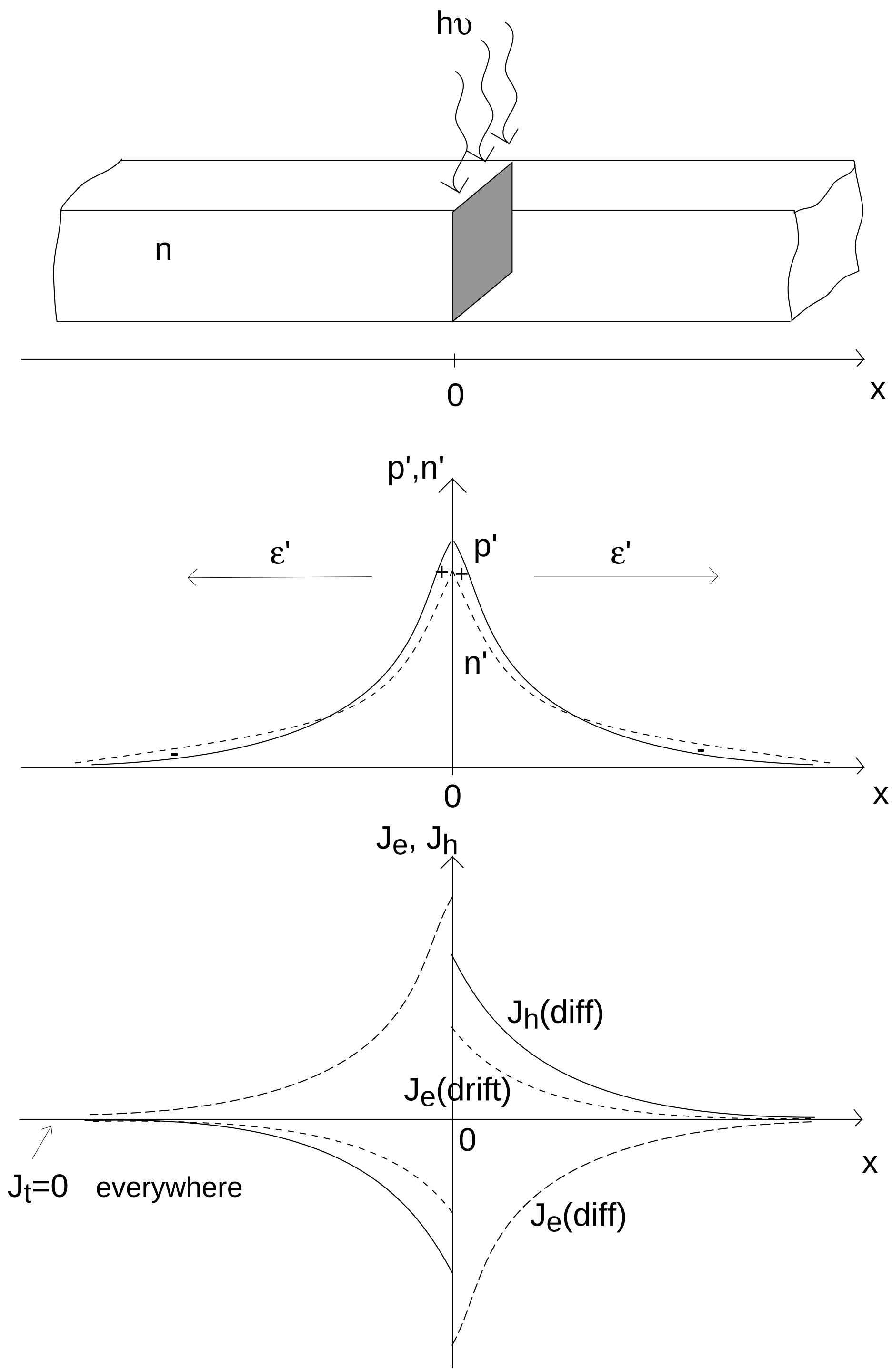


Far away $J_t = 0 \Rightarrow J_t = 0$ everywhere.

$$J_t = J_e + J_h \simeq q n_o \mu_e \mathcal{E}' + q(D_e - D_h)\frac{dp'}{dx} = 0$$

If $D_e = D_h \Rightarrow$ diffusion term $= 0$
$\Rightarrow$ drift term $= 0$
$\Rightarrow \mathcal{E}' = 0$
$\Rightarrow n' = p'$

But, typically $D_e > D_h \Rightarrow$ diffusion term < 0 (for $x > 0$)
$\Rightarrow$ drift term > 0
$\Rightarrow \mathcal{E}' > 0$ (for $x > 0$)
$\Rightarrow$ *and* $\mathcal{E}' \propto D_e - D_h$
$\Rightarrow n' \neq p'$ (but still $n' \simeq p'$)
$\Rightarrow$ *and* $|n' - p'| \propto D_e - D_h$


hυ
n
0
x
p',n'
ε'
p'
ε'
n'
0
x
Je, Jh
Jh(diff)
Je(drift)
0
x
Jt=0 everywhere
Je(diff)

Key conclusions

- *Majority carrier-type situations* characterized by application of external voltage without perturbing carrier concentrations.
- Majority-carrier type situations dominated by drift of majority carriers.
- Integrated resistor:
  - for low voltages, current proportional to voltage across
  - for high voltages, current saturates due to v_{sat}
- Minority-carrier type situations dominated by behavior of minority carriers: diffusion, recombination and drift.

Self-study

- Non-uniformly doped resistor
- Sheet resistance