Lecture 8 - Carrier Drift and Diffusion *(cont.)*

September 21, 2001

Contents:

1. Non-uniformly doped semiconductor in thermal equilibrium

Reading assignment:

del Alamo, Ch. 4, §4.5

Key questions

- Is it possible to have an electric field in a semiconductor in thermal equilibrium?
- What would that imply for the electron and hole currents?
- Is there a relationship between mobility and diffusion coefficient?
- Given a certain non-uniform doping distribution, how does one compute the equilibrium carrier concentrations?
- Under what conditions does the equilibrium majority carrier concentration follow the doping level in a non-uniformly doped semiconductor?

1. Non-uniformly doped semiconductor in thermal equilibrium

It is possible to have an electric field in a semiconductor in thermal equilibrium $\Rightarrow$ *non-uniform doping distribution*

□ Gauss' Law: electrical charge produces an electric field:

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon}$$

$$\rho \equiv \text{volume charge density } [C/cm^3]$$

- if $\rho = 0 \Rightarrow \mathcal{E} = 0$
- if $\rho = 0 \Rightarrow \frac{d\mathcal{E}}{dx} = 0$
- $\Rightarrow$ it is possible to have $\mathcal{E} \neq 0$ with $\rho = 0$

In semiconductors:

$$\rho = q(p - n + N_D^+ - N_A^-)$$

If $N_D^+ \simeq N_D$ and $N_A^- \simeq N_A$,

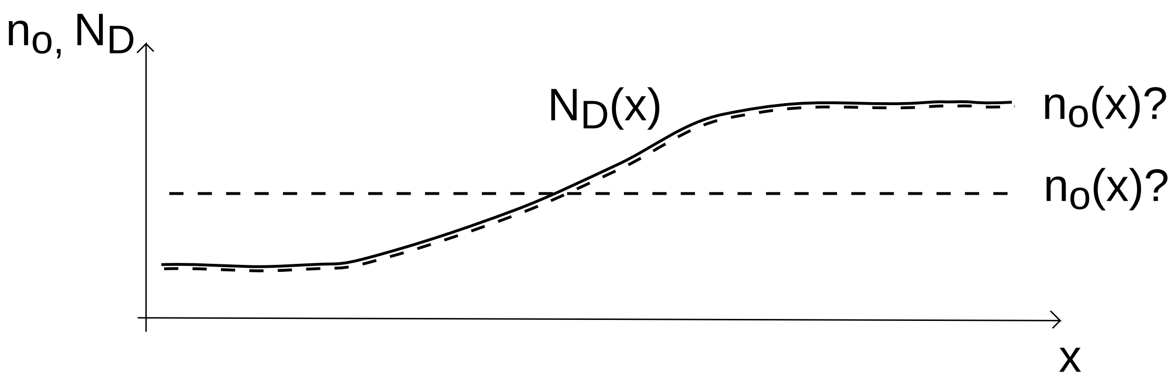
$$\frac{d\mathcal{E}}{dx} = \frac{q}{\epsilon}(p - n + N_D - N_A)$$

$$\frac{d\mathcal{E}}{dx} = \frac{q}{\epsilon}(p - n + N_D - N_A)$$

□ Uniformly-doped semiconductor in TE:

- far away from any surface ⇒ *charge neutrality*:
  $\rho_o = 0 \ \rightarrow \ \frac{d\mathcal{E}_o}{dx} = 0$
- since no field applied from the outside ⇒ $\mathcal{E}_o = 0$

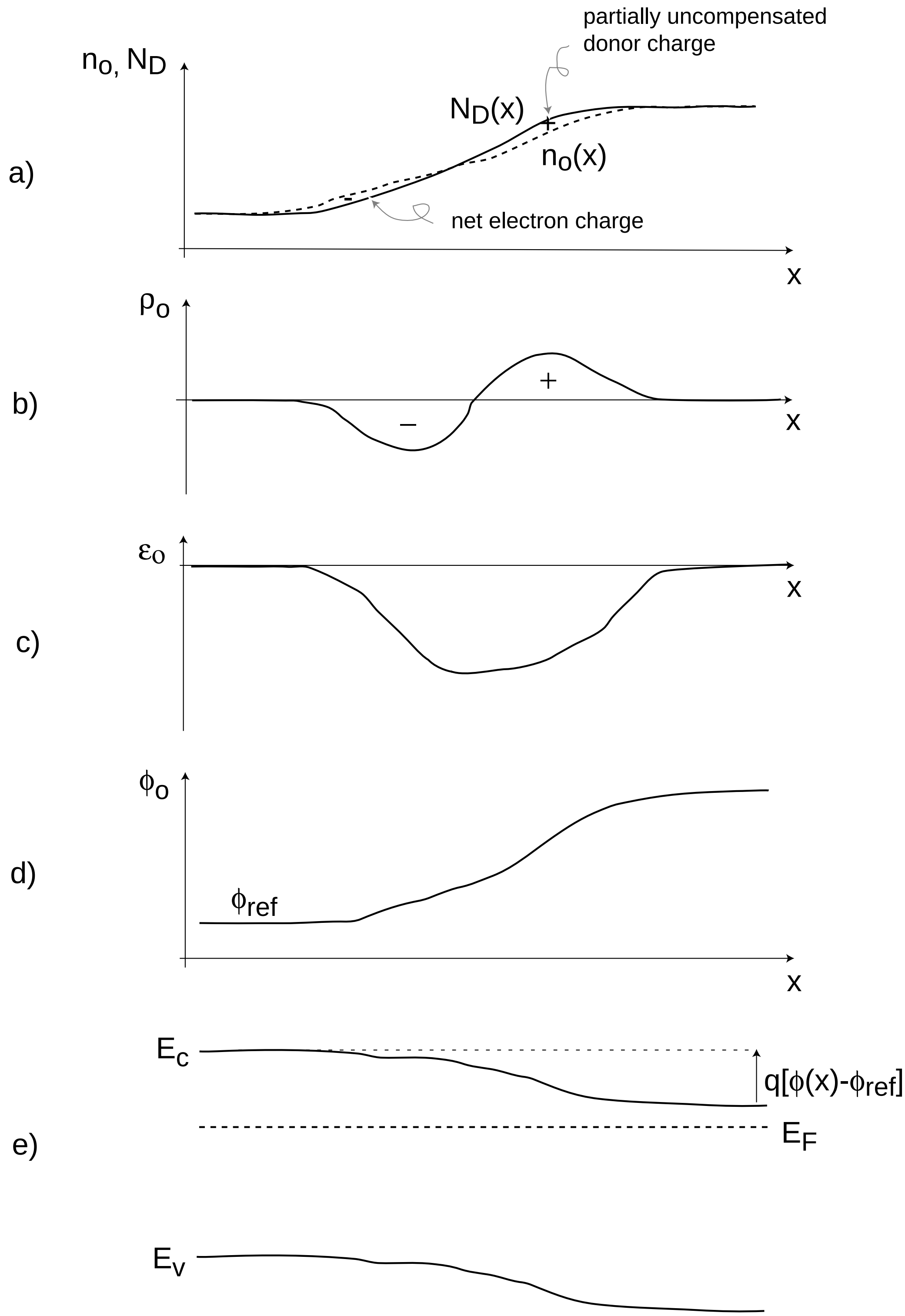
□ Non-uniformly doped semiconductor in TE (n-type):



$$\frac{d\mathcal{E}_o}{dx} = \frac{q}{\epsilon}(N_D - n_o)$$

Three possibilities:

- $n_o(x) = N_D(x) \Rightarrow$ net diffusion current
- $n_o(x)$ uniform $\Rightarrow$ net drift current
- $n_o = f(x)$ but $n_o(x) \neq N_D(x)$ in a way that there is no net current


partially uncompensated
donor charge
n_o, N_D
$N_D(x)$
$n_o(x)$
a)
net electron charge
x
ρ_o
b)
x
ε_o
x
c)
ϕ_o
d)
ϕ_{ref}
x
E_c
$q[\phi(x)-\phi_{ref}]$
E_F
e)
E_v

□ Goal: understanding physics of non-uniformly doped semiconductors in TE

- principle of detailed balance for currents in TE
- Einstein relation
- Boltzmann relations
- general solution
- *quasi-neutral* solution

□ In thermal equilibrium:

$$J = J_e + J_h = 0$$

Detailed balance further demands that:

$$J_e = J_h = 0$$

[study example of §4.5.2]

□ Einstein Relation

μ relates to "ease" of carrier drift in an electric field.

D relates to "ease" of carrier diffusion as a result of a concentration gradient.

Is there a relationship between μ and D?

Yes, Einstein relation:

$$\frac{D_e}{\mu_e} = \frac{D_h}{\mu_h} = \frac{kT}{q}$$

Relationship between D and μ only depends on T.

[study derivation and restrictions in §4.5.2]

□ Boltzmann Relations

In TE, *diffusion* = *drift* ⇒ there must be a conexion between electrostatics and carrier concentrations.

In TE, $J_e = 0$:

$$J_e = q\mu_e n_o \mathcal{E}_o + qD_e \frac{dn_o}{dx} = 0$$

Then, relationship between $\mathcal{E}_o$ and equilibrium carrier concentration:

$$\mathcal{E}_o = -\frac{kT}{q}\frac{1}{n_o}\frac{dn_o}{dx} = -\frac{kT}{q}\frac{d(\ln\, n_o)}{dx}$$

Express in terms of ϕ:

$$\frac{d\phi}{dx} = \frac{kT}{q}\frac{d(\ln\, n_o)}{dx}$$

Integrate:

$$\phi(x) - \phi(ref) = \frac{kT}{q}\ \ln\frac{n_o(x)}{n_o(ref)}$$

Relationship between the *ratio* of equilibrium carrier concentration and *difference* of electrostatic potential at two different points.

At 300 K: factor of 10 in $n_o \rightarrow\ \frac{kT}{q} \ln\, 10 = 60\ mV$ of $\Delta\phi$

⇒ "*60 mV/decade rule*"

If $\phi(ref) = 0$ where $n_o = n_i$:

$$n_o = n_i \exp \frac{q\phi}{kT}$$

Similarly:

$$p_o = n_i \exp \frac{-q\phi}{kT}$$

Note: $n_o p_o = n_i^2$

□ **Equilibrium carrier concentration**: general solution in the presence of doping gradient (n-type)

$$\frac{d\mathcal{E}_o}{dx} = \frac{q}{\epsilon}(N_D - n_o)$$

Relationship between $\mathcal{E}_o$ and n_o in thermal equilibrium:

$$\mathcal{E}_o = -\frac{kT}{q}\frac{d(\ln\, n_o)}{dx}$$

Then:

$$\frac{\epsilon kT}{q^2}\frac{d^2(\ln n_o)}{dx^2} = n_o - N_D$$

One equation, one unknown. Given $N_D(x)$, can solve for $n_o(x)$ (but in general, require numerical solution).

□ Quasi-neutral situation

$$\frac{\epsilon kT}{q^2}\frac{d^2(\ln n_o)}{dx^2} = n_o - N_D$$

If $N_D(x)$ changes slowly with $x \Rightarrow n_o(x)$ will also change slowly, then:

$$n_o(x) \simeq N_D(x)$$

The majority carrier concentration closely tracks the doping level.

When does this simple result apply?

$$\frac{\epsilon kT}{q^2}\frac{d^2(\ln n_o)}{dx^2} \ll N_D$$

or

$$\frac{\epsilon kT}{q^2 N_D}\frac{d^2(\ln n_o)}{dx^2} \ll 1$$

Alternatively:

$$\frac{n_o - N_D}{N_D} \ll 1$$

Define *Debye length*:

$$L_D = \sqrt{\frac{\epsilon kT}{q^2 N_D}}$$

Rewrite condition:

$$L_D^2 \left|\frac{d\mathcal{E}_o}{dx}\right| \ll \frac{kT}{q}$$

Note:

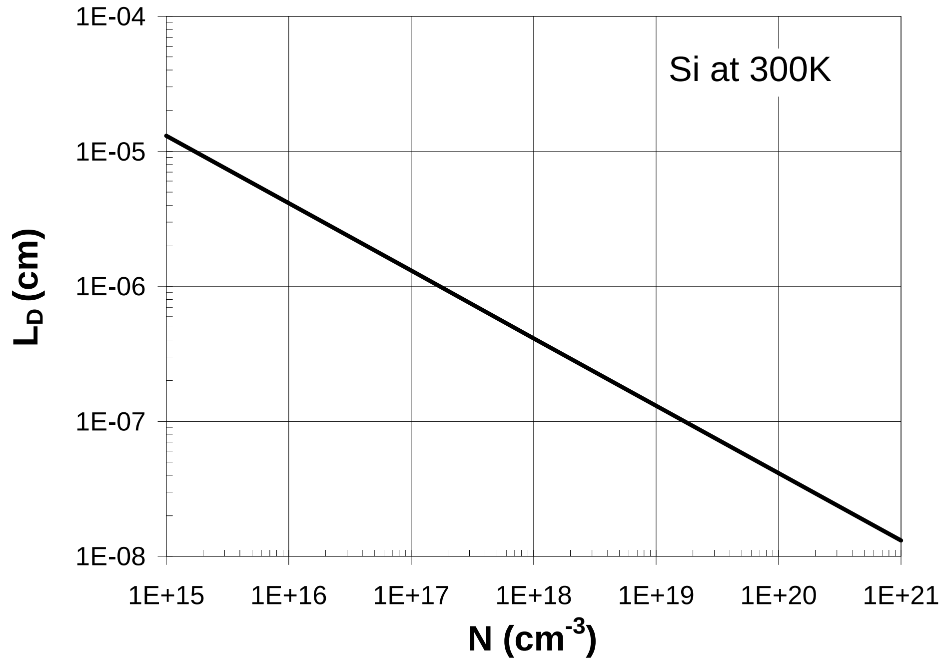
- $L_D|\frac{d\mathcal{E}_o}{dx}|$ is change in electric field over a Debye length
- $L_D^2|\frac{d\mathcal{E}_o}{dx}|$ is change in electrostatic potential over a Debye length

For a non-uniformly doped profile to be quasi-neutral in TE, the change in the electrostatic potential over a Debye length must be smaller than the thermal voltage.

$$L_D = \sqrt{\frac{\epsilon kT}{q^2 N_D}}$$

L_D characteristic length for electrostatic problems in semiconductors.

Similar arguments for p-type material.



Since $L_D \sim 1/\sqrt{N}$:

$$N \uparrow \Rightarrow L_D \downarrow \Rightarrow \text{QN condition easier to fulfill.}$$

Key conclusions

- Detailed balance in TE demands that $J_e = J_h = 0$ everywhere.
- *Einstein relation*:

$$\frac{D}{\mu} = \frac{kT}{q}$$

- *Boltzmann relations*: in TE, difference of electrostatic potential between two points is proportional to ratio of carrier concentration at same points:

$$\phi(x) - \phi(ref) = \frac{kT}{q} \ln \frac{n_o(x)}{n_o(ref)} = -\frac{kT}{q} \ln \frac{p_o(x)}{p_o(ref)}$$

- In *quasi-neutral* non-uniformly doped semiconductor in TE:

$$n_o(x) \simeq N_D(x)$$

- *Debye length*: key characteristic length for electrostatics in quasi-neutral semiconductor.
- Order of magnitude of key parameters for Si at 300K:
  - Debye length: $L_D \sim 10^{-8} - 10^{-5}\ cm$ (depends on doping)

Self-study

- Derive Einstein relation.
- Study example supporting detailed balance for J_e and J_h.
- Work out exercises 4.5-4.7.