ESSAYS ON THE THEORY OF
FLEXIBLE EXCHANGE RATES

by

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M.A., Helsinki University
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ABSTRACT

The thesis consists of three essays on the macroeconomic theory of flexible exchange rates and two essays on the microeconomic theory of international investment under flexible exchange rates. The common theme of these essays is the view of the exchange rate as a relative price of monies, determined -- in the same way as interest rates and equity prices -- to clear the markets for financial assets. This is in contrast to the traditional approach which views the exchange rate as the price that equilibrates the flow demands for and supplies of foreign exchange.

Chapter I develops a monetary approach to the theory of flexible exchange rates extending the recent work of several authors on the monetary approach to balance of payments under fixed exchange rates. The paper goes beyond the existing literature by analyzing the dynamic interaction between the exchange rate, exchange rate expectations, and the balance of payments under alternative assumptions about the formation of exchange rate expectations. The problem of stability of the balance of payments adjustment process and of the exchange rate is reformulated, and analyzed in detail, in terms of the stability of relative asset prices and portfolio proportions. Particular emphasis is focused on the case of perfect foresight. It is shown that if speculators have long run foresight and if they rule out explosive price paths the short run effects of various disturbances on the momentary equilibrium values of the exchange rate and the balance of payments, as well as on the adjustment path, can be inferred from the long run, stationary state effects. The paper also compares the new approach with the traditional flow model of flexible exchange rates. It is shown that the traditional theory -- the way it is formulated in the literature -- is logically incorrect. The implications of this error are examined in some detail.

Chapter II extends the theory developed in Chapter I by analyzing explicitly the interaction between the commodity markets and the asset markets in the short run and long run determination of the exchange rate in the context of a simple model of an open economy producing traded and non-traded goods. The paper integrates the traditional theory of flexible exchange rates -- concerned with relative price elasticities -- with the asset market view advanced in this thesis. A restriction on the relative price elasticity of the excess demand for non-traded goods is shown to be a necessary but not a sufficient condition for the stability of the exchange rate. In addition, dynamic stability of the exchange rate depends on the nature of expectations and the strength of asset substitution effects. The paper also clari-
fies the difference between the conditions for a successful devaluation and the conditions for the stability of a market-determined exchange rate, showing that the former does not imply the latter.

Chapter III reformulates the theory of the effects of fiscal and monetary policies under flexible exchange rates. The differences between the traditional theory and the monetary approach developed in this thesis regarding the monetary transmission mechanism are analyzed in detail. Particular emphasis is focused on the differences between the short run and long run effects of various policies, and the role of long run speculative expectations in bridging the gap between the two.

Chapter IV develops a general equilibrium model of the international money and capital markets under flexible exchange rates. The purpose of this paper is to identify, in a testable form, the factors that explain differences in equilibrium interest rates and equity returns in different countries. The paper extends the previous analysis of Solnik (1974) by explicitly introducing money and by analyzing the effects of monetary policy on the 'beta coefficients' in the equilibrium interest rate relationships. In addition, the paper differs from the previous literature by assuming that the exchange rates satisfy the purchasing power parity equation and by analyzing the role of international investment as a means of reducing the purchasing power risk of nominal assets. It is shown that differences in nominal interest rates depend on the differences in the expected rates of inflation and on the variances and covariances of inflation rates. It is also shown that the expectations hypothesis is not in general valid. Interest rate differentials contain a currency premium which depends on the composition of asset supplies and can, therefore, be changed by monetary policy.

Chapter V develops a rigorous model of the forward exchange market in order to explain, in a testable form, discrepancies from the interest rate parity equation under flexible and fixed (but adjustable) exchange rates. The model differs from the previous contributions by dropping the assumption that domestic bonds are safe and foreign bonds are risky and by introducing a risk of default as an explicit rationale for a less than infinitely elastic covered arbitrage schedule. It is shown that the equilibrium forward premium is a nonlinear function of the interest rate parity, the expected change in the exchange rate, the variance of the exchange rate, the political risks of default, and the extent of forward market intervention; as well as the distribution of wealth amongst investors. The implications of the model for a number of issues discussed in the forward market literature are examined in detail.

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Title: Associate Professor of Economics
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## Chapter

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Chapter I

THE EXCHANGE RATE AND THE BALANCE OF PAYMENTS IN THE
SHORT RUN AND IN THE LONG RUN:
A MONETARY APPROACH

I. Introduction


The paper goes beyond the literature by analyzing the dynamic interaction between the exchange rate, exchange rate expectations, and the balance of payments under the regime of a freely floating exchange rate and under alternative assumptions about the formation of expectations. It is shown that an automatic adjustment mechanism -- similar to the Hume specie flow mechanism -- is at work under flexible exchange rates. The essence of this adjustment mechanism is the change in wealth implied by current account surpluses and deficits. If the economy is stable it will converge to a stationary state (we assume, for simplicity, no real growth) in which the stock of wealth and its composition is constant and
the current account balance is equal to zero. This corresponds to
the full equilibrium, or portfolio balance equilibrium analyzed by
McKinnon and Oates (1966). The dynamic stability of the exchange
rate and of the balance of payments is shown to depend critically
on the magnitude of asset substitution effects and the speed with
which exchange rate expectations are revised. No role is given to
relative price elasticities in contrast to the traditional models
of flexible exchange rates exemplified by J. Robinson's (1948)
classic article on the foreign exchanges, although the potential
importance of the 'elasticities conditions' for exchange rate
stability is indicated.

Another contribution of this paper is the treatment of the
dynamic adjustment path, and the link between the short run momentary
equilibrium and the long run stationary state. There is a gap in
the literature between the portfolio balance, stationary state
models, and the short run models which take the asset supplies and
expectations as given. In general, the short run impact of poli-
cies can be quite different from the long run impact depending on
the nature of expectations. It is shown in this paper that this
dilemma can be resolved by assuming that expectations are rational.
In that case, the long run impact on the exchange rate and asset
supplies correctly predicts the short run impact on the exchange
rate and the balance of payments.

The paper also compares the monetary approach to flexible
exchange rates with the traditional 'flow model' of the foreign
exchange market, which underlies practically all theoretical and
empirical work on flexible exchange rates. This flow model is
shown to be logically incorrect for two reasons. First, in general, the balance of payments flow account is no more than an ex-post identity which in no sense can be interpreted as an ex-ante equilibrium condition. In the case of perfect foresight the actual flow of capital is equal to the desired flow, but in that case the balance of payments flow account is shown to determine the change in the exchange rate, not the exchange rate itself.

The plan of this paper is as follows. Section II develops the concept of a momentary equilibrium and analyzes the short run effects of various shifts on the current account and the exchange rate with given exchange rate expectations. This section also compares the stock model of the exchange rate with the flow model. Section III analyzes the long run effects of the same shifts on the stationary state in which the current account is in balance, asset supplies constant, and exchange rate expectations correct. Section IV analyzes the dynamic stability of the exchange rate and the balance of payments under alternative assumptions about the formation of expectations. By dynamic stability, we mean the convergence of the sequence of momentary equilibria to the stationary state. Section V analyzes the dynamic response of the exchange rate and the balance of payments to various shocks, thereby connecting the short run analysis of Section II with the stationary state analysis of Section III. The concluding part discusses the importance of the new approach as well as the limitations and needed extensions of the model developed in this paper.
II. Momentary Equilibrium

In this section we develop the concept of a momentary equilibrium and analyze the short run effects of various shocks on the exchange rate and on the balance of payments. To simplify the analysis, we assume that the economy produces only traded goods, the relative price of which is fixed in the world market. We also assume that the world price level is constant and equal to one so that the domestic price level and the rate of exchange are the same thing. Labor is the only factor of production and it is fully employed. Domestic output (Y) is therefore constant. Domestic absorption is equal to the sum of private consumption (C) and government expenditure (G). Private consumption is a function of real disposable labor income (Y-T) and the stock of real financial wealth (A) in accordance with Modigliani's life cycle model of consumption. The excess of domestic output over domestic absorption equals the trade account surplus:

1. \( B = Y - C(Y - T, A) - G \)

The stock of financial wealth consists of domestic money (M) and foreign assets (F):

2. \( A = \frac{M}{P} + F \)

The demand for real balances is a function of the expected rate of inflation (exchange rate depreciation), \( \pi \), the level of real income and the stock of wealth. Since we assume that foreigners do not hold domestic paper currency, in equilibrium the domestic demand for money equals the supply of money:

3. \( \frac{M^d}{P} = L(\pi, Y, A) = \frac{M}{P} \)
The other equilibrium condition is that the demand for foreign assets equals the stock of foreign assets:

4. \( F^d = F(\pi, Y, A) = F \)

Because of the wealth constraint only one of equations (3) and (4) is independent. Substituting the definition of real wealth in equation (3) we obtain the equilibrium condition for the asset markets:

5. \( L(\pi, Y, M/P + F) = M/P \)

Given the expected rate of depreciation (\( \pi \)), the stock of foreign assets (F) and the nominal supply of money (M), this condition of equilibrium in the asset markets determines the exchange rate. Equally well we could say that the exchange rate is determined to equilibrate the demand for foreign assets with the existing stock of foreign assets.

We may solve equations (5) and (3) for the reduced form real balance and wealth equations:

6.1 \[ \bar{M} = \frac{M}{P} = H(\pi, Y, F) \]

6.2 \[ A = A(\pi, Y, F) \]

An increase in the expected rate of depreciation reduces the stock of real balances and hence the real value of financial wealth by causing the exchange rate to depreciate.

In order to complete the model we need to introduce the government budget equation and the behaviour of the money supply. For convenience, we assume that the Central Bank acquires all government debt, and does not intervene continuously in the foreign exchange market. Therefore, the nominal budget deficit is equal to the
change in the supply of money. The government can independently determine only two of the variables under its control: real tax revenue \( (T) \), real expenditure \( (G) \), and the change in the money stock \( \frac{M}{P} \). We assume that it fixes the rate of change in the nominal money stock \( (m) \) and the real tax revenue, and adjusts real expenditure accordingly. This gives us the government expenditure function:

7. \[ G = T + \frac{M}{P} \]

An increase in the price level reduces the real value of new debt issue and hence government expenditure. Substituting equation (9) into equation (8) together with the definition of real wealth we obtain the capital flow equation:

8. \[ F = B = Y - C(Y - T, \frac{M}{P} + F) - T - \frac{M}{P} = B(Y, T, F, \pi, M) \]

We assume that the rate of interest on foreign assets is equal to zero.

The last expression is the reduced form current account (capital flow) equation, obtained by substituting the reduced form real balance equation in place of \( \frac{M}{F} \). An increase in real income increases the current account surplus as long as the marginal propensity to consume out of income -- after allowing for the effect of income on the real value of financial wealth -- is less than one. We assume this to be the case. A tax financed increase in government expenditure reduces the current account surplus, as long as the private propensity to consume is less than one. An increase in the stock of foreign assets increases consumption and hence reduces the current account surplus. An increase in the expected
rate of depreciation reduces the real value of financial wealth and hence improves the current account. An increase in the rate of growth of the money stock increases government expenditure and hence reduces the current account surplus.

Equations (5) and (8) constitute the temporary equilibrium model. Equations (6.1 and 6.2) and the right hand side of equation (8) define the endogeneous variables as functions of the current stocks of assets and expectations regarding the future.

a. The Error in the Traditional Model of the Foreign Exchange Market

The traditional flow models of the foreign exchange market start from equation (4) to obtain the desired flow of capital:

\[ \dot{F}^d = F_{\pi} \pi + F_A \dot{A}^e \]

The first term represents the stock shift induced by the change in the expected rate of depreciation. The second term is the flow component of capital movements representing the proportion of expected new savings allocated to foreign assets. The error that is invariably made in the literature is to substitute the desired change in the stock of foreign assets for the actual change in the ex-post balance of payments identity:

\[ B = Y - C(Y - T, A) - G = \dot{F} = \dot{F}^d = F_{\pi} \pi + F_A \dot{A}^e \]

The error in this procedure is in assuming that the actual change in the stock of foreign assets is equal to the desired change in the stock of foreign assets. This is not, in general, the case because the change in the desired stock depends on expected changes in the stock of wealth and in the rate of depreciation. It is
only if these expectations are always correct, i.e., there is per-
fefect foresight that the two are equal. 7 The correct way to inte-
grate the capital account and the current account is to say that the current account determines the capital account:

10. \( \dot{F} = B = Y - C (Y - T, A) - G \)

Simplifying somewhat, the correct reasoning is: the stock of foreign assets determines, ceteris paribus, the exchange rate, the exchange rate determines the current account, and the current account determines the change in the stock of foreign assets. Similarly, the response pattern to an increasing desire to lend abroad (cf. Robinson (1949), Section II) is: a depreciation of the exchange rate, a current account surplus and hence an outflow of capital. This sequence is invariably and incorrectly reversed in the literature: an increase in the desire to lend abroad causes an outflow of capital, a depreciation of the exchange rate and a current account surplus. This question will be examined in more detail below.

If expectations are correct so that the actual capital outflow is equal to the desired outflow, equation (8) is a valid equilibrium condition between changes in the endogeneous variables. It does not, however, determine the exchange rate but rather the change in the exchange rate. To see why this must be the case, observe that the current account deficit \( B \) can be written as:

11. \( B = Y - C(Y - T, A) - G = Y - C - T + T - G = \Delta + (\pi - m) \frac{M}{P} \)

where we have used the household and government budget constraints. 8 Substituting this in equation (10) and using the fact that \( F_{\Pi} \) equals minus \( L_{\Pi} \) and \( F_A \) equals \( 1 - L_A \) because of the wealth constraint, we
can rewrite the balance of payments 'flow equilibrium' condition as:

12. \[ L_\pi \pi + L_A \dot{A} = (m - \pi) \frac{M}{P} = \frac{\dot{M}}{P} \]

But this equation is nothing more than the stock equilibrium condition (5) differentiated with respect to time. It determines the change in the exchange rate given \( M, F, P, \pi \), and changes in \( M, F \), and \( \pi \). The change in \( F \) is, of course, the current account balance. Even if equation (10) is correct, it is not very useful since it contains two endogeneous variables, \( \dot{P} \) and \( \dot{F} \).

A further confusion in the literature arises from the unfortunate practice of assuming that the capital account is determined largely independently of the exchange rate -- except possibly for speculative movements of capital. This is probably a major reason why there is so much concern for import and export price elasticities in the literature. In our model, the assumption that the capital account is somehow independently determined of the exchange rate and the trade account is, of course, totally unfounded.

b. The Determination of the Exchange Rate and the Balance of Payments in the Short Run

The short run determination of the exchange rate and the current account is illustrated by diagram I. The MM curve implies equilibrium in the asset markets. The DD curve (defined by equation (10)) gives domestic absorption as a function of the exchange rate. If the initial stock of foreign assets held by the private sector is \( F_0 \), the momentary equilibrium value of the exchange rate is \( P_0 \). At that exchange rate, domestic absorption equals \( P_0 B_0 \).
Diagram I

The Short Run Determination of the
Exchange Rate and the Current Account

Diagram:

- Domestic absorption and supply
- Stock of foreign assets
- Exchange Rate
- Current account
- Absorption
which is less than domestic output so that the current account is in surplus. Therefore, the economy will not stay in this momentary equilibrium position: with static expectations the economy moves to the right along the MM schedule. In consequence, the exchange rate appreciates, which, together with the increasing stock of foreign assets, increases domestic absorption until stationary state equilibrium is reached with zero current account balance and a constant stock of wealth. A stationary state obtains in the diagram when the stock of foreign assets equals $F^*$, the exchange rate is $P^*$ and the $D*D^*$ schedule intersects the supply curve at that exchange rate. (We assume here for expositional convenience that the nominal money stock is constant).

Diagram II illustrates the response of the momentary equilibrium exchange rate and the current account to two types of shifts: (i) an increase in the expected rate of depreciation (a stock shift). This causes an increase in the 'desire to lend abroad' (cf. J. Robinson (1949), Section II), without directly affecting domestic absorption. The MM curve shifts to the right causing the exchange rate to depreciate. The depreciation causes the current account to move to a surplus ($C_1B_1$ in the diagram). This surplus gradually increases the stock of foreign assets which tends to appreciate the exchange rate and cause the current account surplus to diminish. The complete dynamic analysis of this adjustment process with explicit treatment of expectations will be taken up in Section V below and analysis of the ultimate stationary state effects in Section III. (ii) A tax financed increase in government expenditure (a flow shift, cf. J. Robinson (1949), Section III).
Diagram II

The Short Run Effects of Stock and Flow Shifts

- Domestic absorption and supply
- Stock of foreign assets
- Current account surplus
- Exchange rate

Diagram showing the relationships between domestic absorption, supply, and stock of foreign assets in the context of short run effects of stock and flow shifts.
shifts the DD schedule to the left but, ceteris paribus, leaves the MM schedule unchanged. Therefore, the exchange rate remains unchanged and the current account moves to a deficit. The current account deficit reduces the stock of foreign assets, which causes the exchange rate to depreciate. Both of these effects reduce private absorption and gradually bring the economy to stationary equilibrium with a zero current account.

Thus with stationary expectations a shift in absorption that does not affect asset demands has no effect on the exchange rate in the short run. We shall see later that if expectations are rational the future effects on the exchange rate induced by the current account deficits will be reflected on the current exchange rate (see Section V below).

Diagram III illustrates the short run effects of Central Bank intervention in the foreign exchange market. The economy starts from a position of full equilibrium at $A_0$ with stock of foreign assets $F_0$ and exchange rate $P_0$. The Central Bank suddenly purchases $F_0'F_0$ of foreign exchange from the private sector with domestic money. The private sector's initial foreign asset position reduces to $F_0'$ and the MM curve shifts upwards to $M'M'$. The exchange rate jumps to $P_0'$. The sharp reduction in real financial wealth causes the current account to go to a surplus ($B_0'C_0'$ in the diagram). The surplus begins to move the economy back to a new stationary state equilibrium position. It is shown in the next section that in the new long run equilibrium position the private sector's stock of foreign assets is the same as it was initially. All that happens is that the exchange rate depreciates
Diagram III

Short Run Effects of Central Bank Intervention in the Foreign Exchange Market

Diagram showing the relationship between domestic absorption and current account balance, stock of foreign assets, and exchange rate. The graph illustrates the impact of central bank intervention on the foreign exchange market.
by exactly the same proportion that the money supply is initially increased. Note that the exchange rate initially depreciates more than in proportion to the increase in the supply of money. Subsequently, the rate appreciates down to its long run equilibrium level. One may view the impact effect of foreign exchange market intervention as a capital levy on money balances -- much in the same way as devaluation.\textsuperscript{9}

III. The Stationary State

In this section we investigate the long run effects of various shifts. The dynamic adjustment path from the short run momentary equilibrium to the long run stationary state is analyzed in Section V. In the stationary state, the stock of real wealth and its composition is constant, all nominal variables grow at the same rate and the expected rate of inflation equals the actual rate of inflation and hence the rate of change in the money stock.\textsuperscript{10}

The constancy of real wealth requires that the current account is equal to zero:

13. \( C(Y - T, \bar{M} + F) + T + \pi \bar{M} = Y \) (cf. equation 8)

Equation (13) defines the locus of \( \bar{M} \) and \( F \) that implies balance of payments equilibrium. It is illustrated by the BB schedule in diagram IV. The schedule is downward sloping -- for the trade account to remain in zero balance when the stock of foreign assets increases, the stock of real balances must fall to prevent an increase in absorption.

The second stationary state equilibrium condition requires that the stock of wealth is held in desired proportions:
14. \( L(\pi, Y, \bar{M} + F) = \bar{M} \) (cf. equation 5).

The description of the stationary state is complete once we observe that the expected rate of inflation \( (\pi) \) equals the actual rate of inflation \( \frac{\dot{P}}{P} \), which in turn equals the rate of growth in the money stock \( (m) \):

15. \( \pi = \frac{\dot{P}}{P} = m \)

The locus of the stocks of foreign assets and real balances that are consistent with portfolio equilibrium is illustrated by the MM schedule in diagram IV (cf. diagram I). It is upward sloping because an increase in the stock of foreign assets increases the demand for real balances. The intersection of the BB and MM schedules at point \( A^* \) determines the long run stationary state stock of foreign assets \( (F^*) \) and real balances \( (\bar{M}^*) \). We shall assume throughout this paper that there exists a unique stationary state for all rates of inflation greater than some negative \( \tilde{\pi} \). The long run exchange rate path (we obviously cannot talk about a long run exchange rate if that rate is steadily depreciating) is defined by:

16. \( \ln P(t) = \ln M_0 + mt - \ln \bar{M}^* \)

There is thus a one-to-one correspondence between the long run exchange rate path and the stock of real balances. The lower the stock of real balances the higher the exchange rate path. Subsequently, when we refer to a long run depreciation of the exchange rate, we mean an upward shift in the exchange rate path.

It follows immediately from equations (14) and (16) that a once-and-for-all intervention in the foreign exchange market has no long run real effects -- all that happens is that the exchange rate increases in proportion. This implies that a purchase of \( X \)
Diagram IV

The Stationary State

stock of real balances

Stock of foreign assets
units of foreign exchange by the Central Bank will be followed by a period of current account surpluses which add up to exactly X units of foreign exchange.

Fiscal policy can affect the long run equilibrium position in two ways: by changing the tax revenue and by changing the rate of growth of public debt, and hence the rate of inflation. These two methods are two alternative forms of taxation. The first is a lump sum tax on income, the second a capital levy on cash balances. They have quite different long run effects because the inflation tax changes the desired portfolio composition.

The long run effect of an increase in government expenditure financed by higher taxes can be established without ambiguity with reference to diagram IV. The BB schedule shifts down to B'B': at a given stock of foreign assets there is an increase in absorption. In order that the balance of payments remains in equilibrium, the stock of real balances must fall and reduce private absorption. Therefore, a tax financed increase in government expenditure will reduce the long run stock of foreign assets held by the private sector, and depreciate the long run exchange rate. This means that between the short run momentary equilibrium and the long run stationary state the current account must be in deficit.

The effect of an increase in the rate of inflation on the stationary state depends on the magnitude of the rate of inflation. From equations (14) and (16) we obtain:

17. $\hat{\frac{\Delta M}{\Delta}} = \frac{C\Delta \pi - L^A_M}{\Delta \pi}$
18. \[ \hat{F} = \frac{L_A \bar{M} - (C_A + \pi)L_\pi - \pi M^*}{\Delta} \]

19. \[ \hat{A} = \frac{-\pi L_\pi^* - \bar{M}}{\Delta} \]

where \( \Delta = C_A + \pi L_A \). The effect of a higher rate of inflation on the stationary state stock of real balances is unambiguously negative. Therefore, we may write the stationary state demand for money function in the form \( \frac{M}{P} = L^* (\pi, Y) \) with \( L_\pi \) negative.\(^{11}\) The effect on the stock of foreign assets and the stock of real wealth is ambiguous. If \( L_\pi \) is well behaved, the stock of wealth at first reduces with the rate of inflation reaching the minimum when the rate of inflation equals the inverse of the inflation elasticity of money demand, and thereafter increasing with the rate of inflation. The stock of real private financial wealth is minimized when the government revenue from the inflation tax is maximized.\(^{12}\) As the rate of inflation approaches infinity, the stationary state stock of wealth approaches what it would be if the rate of inflation were zero. The reason is that in both cases the average real rate of interest is the same, namely zero.\(^ {13}\) An increase in the rate of inflation at first reduces the stock of foreign assets but after a while the substitution effect begins to dominate and the stock of privately held foreign assets begins to increase. It is clear from equation (17) that this occurs before the revenue maximizing rate of inflation is reached (see diagram V).

An increase in the rate of inflation may thus be accompanied either with a period of current account deficits (when the rate of inflation is small) or a period of current account surpluses (when the rate of inflation is high). The dynamic response pattern
depends critically on how expectations are formed (see Section V).

IV. The Dynamic Stability of the Adjustment Process

In this section we investigate the dynamic stability of the balance of payments adjustment process under flexible exchange rates. By dynamic stability, we mean the convergence of the sequence of momentary equilibria to the stationary state. We shall examine three mechanisms of expectations formation:

a) static expectations: \[ \pi = \frac{P}{P} = \pi \]
b) myopic perfect foresight: \[ \pi = \frac{P}{P} = m - \dot{X} \]
c) adaptive expectations: \[ \pi = \beta \left( \frac{P}{P} - \pi \right) = \beta (m - \dot{X} - \pi) \]
where X is equal to the logarithm of the stock of real balances (lnM). The adjustment process is defined by one of the above expectations equations and the following two equations:

20. \[ \dot{F} = k(\pi, F) \] (cf. equation 8)
21. \[ \dot{X} = h(\pi, X) \] (cf. equation 6.1)

a. Static Expectations

With static expectations, \( \pi \) is constant and equation (21) becomes an ordinary differential equation in \( F \). Because \( k_F \) is negative and we assume a unique stationary state, the adjustment process is globally stable. In diagram IV, the economy moves along the asset market equilibrium line (MM). Along the static expectations path the exchange rate is continuously changing in a way that implies profit opportunities for speculators so that it hardly is an adequate representation of the adjustment process.
b. Myopic Perfect Foresight

The strong stability result that obtains in the case of static expectations suggests that the question of dynamic stability hinges on the nature of expectations formation. We show in this section that perfect foresight renders the exchange rate indetermin ate: from any initial exchange rate there is an exchange rate and foreign asset path such that expectations are continuously fulfilled and all markets are in equilibrium. This problem of indeterminacy has been raised in a different context by Black (1974), and is well known in the models of money and growth as well as in growth models with many capital goods. There is only one path along which the economy converges to the stationary state -- provided that it gets on that path in the first place.

The fact that the stationary state is a saddlepoint can be established by considering the dynamic system consisting of equations (20) and (21) with \( \dot{x} \) substituted in place of \( \pi \). It is straightforward to show that the characteristic roots of the resulting dynamic system, linearized around the stationary state, are real and of opposite sign, which is a sufficient condition for the stationary state to be a saddlepoint locally.

The dynamic behaviour of the stock of real balances and the stock of foreign assets with perfect foresight is illustrated in diagram V. The AA curve implies that the stock of real balances is constant (whence the rate of depreciation of the exchange rate equals the rate of growth of the nominal money stock). The BB schedule implies that the current account is zero and hence the stock of foreign assets is constant. The assumption of a unique stationary state implies that the BB curve cannot be
Diagram V

The Adjustment Process with Perfect Foresight

stock of real balances

Stock of foreign assets
steeper than the AA curve although it can be locally upward sloping. The assumption of the existence of a stationary state for positive rates of inflation also implies that the stock of foreign assets never exceeds the stationary state level of wealth with zero rate of inflation (cf. footnote 13). That is why the BB curve cuts the X-axis at a finite stock of foreign assets $F^*$. The arrows in diagram V indicate the direction of movement. Suppose that the initial stock of foreign assets is $F_0$. The initial exchange rate should be set in such a way that the initial stock of real balances is equal to $M_o$ in order that the economy converges to the stationary state. If the exchange rate is initially undervalued, the stock of real balances is too low -- $M_0'$ in the diagram, the exchange rate appreciates initially as people build up their domestic money balances faster than they accumulate foreign assets in order to restore portfolio equilibrium. After a while, the stock of real balances reaches to a point where it stops increasing (point B in the diagram.) At that point, the exchange rate begins to depreciate. The speculators catch on immediately and start the flight out of domestic money. Hyperinflation ensues and foreign money drives domestic money valueless. The opposite outcome follows if the currency is initially overvalued so that the initial stock of real balances is too high. Both the stock of real balances and the stock of foreign assets will increase at first but after a while (point C in the diagram) the substitution effect begins to dominate and the domestic residents start reducing their foreign assets because of the high yield on domestic assets. It is implausible that the boom
in the foreign exchange market could continue much beyond point D in the diagram when domestic residents no longer have any foreign assets. At that point the appreciation suddenly stops, the market collapses and speculators incur a large capital loss. If speculators have long run perfect foresight they will anticipate this outcome and will prevent the hyperdeflation from ever getting started.

It is less clear how one might rule out hyperinflation since there appears no good reason why domestic currency could not be substituted by foreign money even in domestic transactions. Of course, the society as a group loses from this since they have to give up real resources (cut down consumption) in order to accumulate foreign money. There is no self-evident competitive market mechanism which rules out the society making itself worse off by destroying the value of its money through speculation. In addition to just ruling out such possibility because it seems unreasonable (Sargent and Wallace, 1974), or because it has never happened without excessive monetary expansion, one could argue that a minimum stock of real balances is always needed to carry out some transactions -- for instance, payments of taxes. If that is the case, long run perfect foresight rules out hyperinflation as well. There remains the troublesome question of how the speculators are able to compute the initial exchange rate which will take the economy to the stationary state. We shall not attempt to tackle this question, but shall use the assumption of long run perfect foresight (rational expectations) as a convenient tool of dynamic analysis.

Diagram V shows the important property of the rational
expectations adjustment path: the stock of foreign assets and the stock of real balances always move monotonically and in the same direction. This result will prove very helpful in the next section.

c. Adaptive Expectations

With adaptive expectations the dynamic evolution of the economy is defined by equations (20) and (21) and the adaptive expectations equation (c). It is straightforward to show that a sufficient condition for the local stability of this system is that the product of the absolute value of the inflation elasticity of the demand for real balances \( (h_\pi) \) and the speed of revision of expectations \( (\beta) \) is less than one: \[ h_\pi \beta < 1 \]

If the system is stable the convergence to equilibrium is nonoscillating. This stability condition is the same as the condition of stability in Cagan's model of hyperinflation except that \( h_\pi \) is a reduced form elasticity unlike in his model. Our model of the inflationary process differs from Cagan's analysis amongst other things because equilibrium can be restored not only through price changes but also through changes in the stock of the money substitute.

V. Dynamic Response of the Exchange Rate and the Balance of Payments to Various Shocks

In this section we analyze the dynamic response of the exchange rate and the balance of payments to the various shocks considered
in Sections II and III from the short run and long run perspective. Our strategy is to use the phase-diagram introduced in the previous section and infer from that what the response of the exchange rate and the current account must be. The response of the exchange rate can be established easily from the response of the stock of real balances and the response of the current account from the direction of change in the stock of foreign assets. We shall examine separately each of the three shocks considered above and in each case compare the response pattern under the three different hypotheses about expectations formation.

a. The Dynamic Effects of An Once-And-For-All Intervention in the Foreign Exchange Market

In diagram VI the economy is initially in a stationary state with stock $F^*$ of foreign assets and $M^*$ of real money balances. The MM and the FF curves have the same interpretation as before. The Central Bank purchases $F^*$ of foreign assets. This leaves both the MM and the FF schedule unchanged. With static expectations, the economy stays on the MM schedule so that the stock of real balances reduces to $M_o$. Thereafter, the economy moves back to the same stationary state with the exchange rate appreciating and the current account in surplus. The static expectations path implies an appreciating exchange rate which the speculators persistently ignore. With foresight, this is not possible whence the initial decline in the stock of real balances is less (implying that the initial devaluation is also less). Thereafter, the behaviour of the economy is similar to that under static expecta-
Diagram VI

The Dynamic Effects of A Once-and-For-All Intervention in the Foreign Exchange Market
tions. This case illustrates that speculators with long run fore-
sight cushion the exchange rate against discrete and non-repeated
changes in the money stock. A possible response pattern of the
economy under adaptive expectations is illustrated by the $A_o^{TA}$
trajectory. The initial point is the same as with static expecta-
tions. As the exchange rate subsequently appreciates the specu-
lators revise their expectations upwards. Hence, the stock of
real balances must always be above what it is under static expect-
tations (hence the exchange rate is below what it is along the
static expectations path). Speculators may cause the stock of
real balances to go above the stationary state value which means
that the exchange rate will, after a period of appreciation, start
to depreciate. The current account cannot, however, move into a
deficit (this is implied by the stability condition).

To summarize, in all cases the exchange rate initially depre-
ciates more than in proportion to the change in the money stock.
Thereafter, the exchange rate appreciates and the current account
is in surplus until the economy has reached a new equilibrium
position with a higher exchange rate but the same values for the
real variables.

b. The Dynamic Effects of a Tax Financed
   Increase in Government Expenditure

An increase in government expenditure financed by taxes will
shift the FF schedule down and leave the MM schedule unchanged
(see diagram VII). Before the shift, the economy is at point $A$
with stock $F_o$ of foreign assets and $M_o$ of real balances. In the
Diagram VII

The Dynamic Effects of an Increase in Tax Financed Government Expenditure

stock of real balances

stock of foreign assets
new equilibrium position, both are less, $F^*$ and $M^*$, respectively. With static and adaptive expectations, the stock of real balances and hence the exchange rate remain initially unchanged. The only impact effect of the shift is that the current account moves to a deficit (cf. Section II.b.). Over time this causes the exchange rate to depreciate. Rational speculators foresee this possibility and cause the exchange rate to depreciate immediately, thereby bringing about a larger current account surplus than with static and adaptive expectations. Thereafter, the rational expectations path is similar to the static expectations path. Both of them differ from the adaptive expectations path which may cause the exchange rate to overshoot, as is illustrated by the $ATA^*$ trajectory.

Three points that emerge from this analysis merit re-emphasizing:

(i) In all cases the long run effect on the stock of foreign assets correctly predicts the short run effects on the current account.

(ii) If long run foresight is assumed, the long run effect on the exchange rate correctly predicts the short run effect on the exchange rate.

(iii) The ensuing current account deficit is temporary and in no case reverses itself.

It is also of interest to note that the short run change in the current account has informative value for speculators about the future course of the exchange rate.

We should also point out that with long run foresight every-
thing that is expected to happen in the future will have an effect on the current exchange rate.

C. The Effects of an Increase in the Rate of Monetary Expansion

We know from the stationary state analysis that two long run outcomes are possible: the stock of real balances will unambiguously decline but the stock of foreign assets may either increase or decrease depending on the strength of the substitution effect (which itself depends on the magnitude of the rate of inflation). Diagram VIII illustrates the first case. The economy is initially at point A. The new equilibrium position is A*, with stock F* of foreign assets and M* of real balances. The response to this change is radically different under adaptive and rational expectations. It is meaningless to assume static expectations in this case. With rational expectations, there is an immediate, discrete devaluation of the exchange rate, whereafter the exchange rate appreciates relative to its new trend. The sharp depreciation causes the current account to move to a surplus despite the increase in government expenditure. After the initial adjustment, the stock of real balances and the stock of foreign assets increase pari passu to the new equilibrium position. Note that the short run impact is correctly predicted by the long run stationary state impact.

With adaptive expectations there is no change in the initial exchange rate whence the increased government absorption causes the current account to move to a deficit. For a while, the stock
Diagram VIII

The Dynamic Response to An Increase in the Rate of Monetary Expansion

stock of foreign assets

stock of real balances
of real balances and the stock of foreign assets decrease *pari passu*, but once speculators catch on, the substitution effect begins to dominate, capital begins to flow out and the current account moves to a surplus. The surpluses add up to the sum of the previous deficits and the long run increase in the stock of foreign assets because of the lower rate of return on domestic assets.

The case when both the stock of real balances and of foreign assets decline in the long run can be analyzed with reference to the previous diagram VII since in both cases the new equilibrium position is to the southwest of the initial point. With rational expectations, there is an instant depreciation of the exchange rate attendant upon the (known) increase in the rate of monetary expansion. The depreciation is not, however, sufficient to cause the current account to move to a surplus. The *exchange rate will continue to depreciate faster than the new growth rate of the money stock* and the stock of real balances and of foreign assets decline *pari passu*. Note that in the previous case, the rate of depreciation was less than the rate of monetary expansion.

With adaptive expectations the path may look like trajectory $A_0 TA^*$ in diagram VII. Initially, there is no change in the exchange rate. The current account deficits cause the rate to depreciate. Speculators catch on and may cause the rate to overshoot its long run equilibrium path.

**VI. Concluding Remarks**

In this section we summarize the main principles of the monetary approach to flexible exchange rates developed in this
paper and discuss the various implications of this approach:

(i) In the long run there is a symmetry between the régime of fixed and flexible exchange rates. Under fixed exchange rates, the exchange rate is exogeneous and the supply of money endogeneous. Under flexible exchange rates, the supply of money is exogeneous and the exchange rate endogeneous. A devaluation under fixed exchange rates increases the supply of money in proportion in the long run; under flexible exchange rates, an increase in the money stock increases the exchange rate in proportion in the long run. An important long run difference between the two regimes is that under flexible rates the rate of inflation can be varied independently of the rest of the world. Changes in the rate of inflation can be interpreted as changes in capital levy on domestic money and they will have systematic effects on the long run stock of wealth as well as its composition. Other instruments of fiscal policy can be used in both regimes to alter the stationary state. Because fiscal policy and other real variables have an effect on the long run demand for money, it is not correct to say that the exchange rate can be explained by monetary factors alone.

(ii) The adjustment process is quite different under the two regimes. In both systems, the stock of wealth adjusts to its long run desired level through deficits and surpluses in the current account. Under fixed exchange rates portfolio equilibrium between domestic money and foreign assets at a given level of wealth is obtained through instantaneous capital inflows and outflows because the Central Bank supplies foreign assets at a fixed price. Under flexible exchange rates instantaneous portfolio
equilibrium is obtained through changes in the valuation of assets -- that is, through changes in the exchange rate. Whereas a desire to hold a larger proportion of foreign assets results in an immediate adjustment of private portfolios and has no long run consequences under fixed exchange rates, such a shift under flexible exchange rates will give rise to a gradual adjustment process and will have long run consequences. The exchange rate will depreciate initially and the current account will move to a surplus. This surplus increases the actual stock of foreign assets. In general, the exchange rate in the new equilibrium position will not be the same as before the portfolio shift because the long run stock of wealth will be different.

(iii) The dynamic behaviour of the exchange rate and of the balance of payments depends critically on the nature of expectations formation. The traditional theory has missed the relevant problem of instability under flexible exchange rates, namely, the problem of instability of relative asset prices, by focusing incorrectly on balance of payments flows. The crude purchasing power parity theory of exchange rates has also missed the problem of possible instability by ignoring the fact that different monies, and assets denominated in different currencies, are substitutes. The requirement of no expected profits does not rule out dynamic instability. In fact, in the case of perfect foresight the exchange rate is indeterminate -- for any initial exchange rate, there is a path along which all markets clear and expectations are continuously fulfilled. Only one of these
paths converges to the stationary state. Since hyperdeflation, or inflation, has seldom, if ever, developed by the force of speculative behaviour alone, there must be reasons why the deviant paths cannot be sustained. Some reasons are given in the paper.

(iv) This paper does not give any role to relative price effects in the adjustment process, emphasized by the traditional analysis of foreign exchange market stability. A necessary condition of stability in our model is that an increase in the stock of foreign assets reduces the current account surplus. With non-traded goods and low price elasticities, this may not happen, in which case the foreign exchange market would be dynamically unstable.

(v) If long run perfect foresight is assumed, the short run effects and the dynamic path of various disturbances can be inferred from the long run effects of these disturbances. This result greatly enhances the usefulness of the portfolio balance models of open economies.

(vi) The view of the exchange rate as a relative asset price suggests that in a world in which the underlying determinants -- monetary and real -- of the exchange rate change continuously and in a stochastic fashion, there is no reason to expect the exchange rate to be stable. In fact, the behaviour of the exchange rate is likely to resemble the behaviour of asset prices in other speculative markets, such as the stock market.

(vii) The analysis of this paper suggests a framework for analyzing the effects of monetary policy under flexible rates which departs significantly from the traditional analysis. The immediate
effect of a change in monetary policy is to change the relative price of assets -- such as the exchange rate -- and the rates of interest. These changes have effects on aggregate demand, prices and output through various channels: (a) by changing the real value of wealth and its distribution across countries, (b) by changing the rate of interest and thereby affecting the rate of investment, and (c) by possibly changing relative commodity prices and real wages. The link between monetary policy and the inflow or outflow of capital goes through the effect of monetary policy on aggregate demand and output and thereby on the current account, which determines the capital account. The direct and immediate link between monetary policy and the capital account in the traditional analysis has resulted in the false presumption that monetary policy acts fast under flexible rates because it has an immediate effect on the current account and hence on aggregate demand. The correct reasoning is, of course, that monetary policy has an immediate effect on the current account \textit{if} it has an immediate effect on aggregate demand.

(viii) Finally, the model developed in this paper can be extended in a straightforward manner to allow for rigid wages and unemployment, for changes in relative prices, and for accumulation of real capital. The extension of the model to two or more countries would bring out the point that what in the end connects the exchange rate and the current account is the transfer of wealth implied by current account deficits and surpluses and the fact that asset preferences are likely to be different in different countries.
Footnotes for Chapter I

1 Some of the most relevant literature on the monetary approach to balance of payments is collected in Frenkel and Johnson (1975). Recent contributions to the portfolio balance models of open economies under fixed exchange rates include Branson (1974), Brunner and Meltzer (1974) and Myhrman (1975). Samuelson (1971) develops a rigorous Hume-Ricardo model of trade with money. His analysis is, however, limited to the stationary state. Whilst his justification of the Marshallian partial equilibrium model of the foreign exchange market is valid in the stationary state, it is not valid when the economy is out of the stationary state for reasons elaborated in this paper.

2 Dornbusch (1973) uses an alternative formulation. In his model money is the only store of value. He assumes that the flow of saving is a function of the discrepancy between the long run demand for real balances and the current stock of real balances. This approach is identical to ours with a fixed exchange rate but has different implications if the price level is changing: it implies that investors stabilize saving and let capital gains and losses be reflected on consumption. This is both implausible and at variance with empirical evidence.

3 The equality of the excess of domestic absorption over domestic output and the current account deficit is the essence of the absorption approach to the balance of payments. See Alexander (1952) and Johnson (1958) for a discussion of this approach. Dornbusch (1973) and Mussa (1974) emphasize the similarity between the monetary approach and the absorption approach in the process of adjustment. The link between the two approaches disappears, however, once there are other assets. The excess of income over absorption represents a change in wealth, and not necessarily a change in the holdings of money balances.

4 This strong separation obtains only because we assume a small open economy producing only traded goods. In a two country model or in a model with non-traded goods, the asset market equilibrium and the commodity market equilibrium are determined simultaneously so that it would be incorrect to say that the exchange rate is determined only in the asset markets, since it depends on the relative prices of commodities which are determined simultaneously with the exchange rate (see Chapter II). But even in the more general case, it is incorrect to say that the exchange rate is determined to equilibrate the balance of payments flows.
For a useful discussion of the problem of stocks and flows in international monetary analysis, see Branson (1974). Branson makes the classic error in discussing the extension of his model to flexible exchange rate by 'allowing the foreign exchange market to determine the exchange rates so that the balance of payments . . . is equal to zero'. See Branson, *op. cit.*, page 47 and the discussion above. The same mistake is also made in a recent paper of Genberg and Kierzkowski (1975), and in just about every other previous paper on flexible exchange rates with the notable exception of Dornbusch (1974).

A classic reference on the traditional theory of foreign exchange market is Robinson (1949). For representative modern discussions, see Kindleberger (1973, Chapter 17), Sohmen (1969, especially Chapter 1), and Stern (1973, Chapter 2).

This problem is formally the same as the stock-flow distinction in closed economy monetary models. The difference between the flow model of the foreign exchange market and the stock model developed in this paper is the same as the distinction between the flow of loanable funds and the liquidity preference theories of interest. For a definitive treatment of these issues see Foley (1975). As Foley shows, there is a logically consistent way of formalizing the flow approach (stock disequilibrium) to monetary theory. No such formalization appears to exist in the literature on flexible exchange rates.

The change in household's real wealth is equal to new savings less capital losses on real money balances:

\[ A = Y - T - C - \pi \frac{M}{P} \]

where \( \pi \) is equal to the actual rate of inflation. The government budget equation states that the excess of expenditure over tax revenue equals the change in the supply of money.

\[ G - T = m \frac{M}{P} \]

where \( m = \frac{\dot{M}}{M} \) (see above). Substituting these above, we get equation (9).


The stationary state model is similar to that of McKinnon and Oates (1966), except that we assume fixed output and variable price level (exchange rate) whereas they assume a fixed price level and variable output even in the long run.

This is the Archibald-Lipsey long run demand for money function. See Archibald and Lipsey (1958). See also McKinnon (1969) for a discussion why the stock of wealth does not appear in the long run asset demand equations.
12 On the optimal inflation tax see the recent treatment of the problem by Phelps (1973).

13 With zero rate of inflation equation (12) is of the form $C(Y - T, A) + T = Y$. With an infinite rate of inflation $\pi M$ becomes zero (under appropriate conditions on the inflation elasticity of money demand) so that equation (12) is of the same form: $C(Y - T, A) + T = Y$.

14 For a discussion of this problem in models of money and growth see Hahn (1969) and references contained therein. See also the recent paper of Brock (1975). Brock resolves the problem of indeterminacy by assuming an economy of identically infinitely lived intertemporal optimizers. The transversality condition enables him to eliminate the deviant paths. It is not clear, however, what market forces enforce the transversality condition. A useful reference is also Burmeister-Dobell (1970), Chapter 6, and Stein (1970), especially Chapter I, Section E.

15 The details of the mathematical derivations are left out since they are straightforward. See, however, footnotes in Chapter II.

16 See Samuelson and Liviatan (1969) for a detailed analysis of the properties of saddlepoints in optimal growth models. The problems of local instability and multiple equilibria can also arise in our model but we assume them away.

17 For a derivation of this result in a more complicated model, see Chapter II, footnote 4.

18 If, however, speculators have long run foresight and rule out explosive price paths, speculation will cushion the exchange rate against reversible shocks as has been correctly argued by Friedman (1953). However, permanent changes in the long run determinants of the exchange rate will, even with -- and in the case of 'flow shifts' (see the text), in particular with -- rational expectations, have an accentuated effect on the spot exchange rate.
Chapter II

RELATIVE PRICES, SPECULATION, AND THE DETERMINATION
OF THE EXCHANGE RATE IN THE SHORT RUN
AND IN THE LONG RUN

I. Introduction

In this chapter we analyze the interaction between the commodity markets and the asset markets in the short run and long run determination of the exchange rate in the context of a simplified model of a small open economy. In line with the monetary models of devaluation of Dornbusch (1973), Krueger (1974), Pearce (1961), and others, we assume that the economy produces one internationally traded good and one purely domestic good. Prices are assumed to be flexible and full employment is assumed to prevail. Domestic residents hold both domestic money and foreign assets. The latter may be interpreted broadly to include foreign currency, foreign securities, and even inventories of traded goods. The allocation of wealth between the two assets depends, amongst other things, on the expected change in the exchange rate. This is how speculation affects the exchange rate, the relative price of non-traded goods, and the balance of payments.

The main purpose of this chapter is to integrate the traditional theory of flexible exchange rates -- concerned with relative price elasticities -- with the asset market view advanced in this thesis. A restriction on the relative price elasticity of the
excess demand for non-traded goods is shown to be a necessary but not sufficient condition for the dynamic stability of the exchange rate. In addition, dynamic stability hinges critically on the nature of expectations formation as well as the strength of asset substitution effects.

We also analyze the difference between the conditions for a successful devaluation and the conditions for the stability of a market determined exchange rate. The former is shown not to imply the latter. Indeed in our model, devaluation unambiguously increases the Central Bank's stock of foreign reserves.

The plan of the chapter is as follows. Section II develops the model and analyzes the short run determination of the exchange rate. Section III investigates the dynamic stability of the foreign exchange market under alternative assumptions about the formation of expectations. Section IV analyzes the response of the economy to an unanticipated, once-and-for-all devaluation. Section V investigates the effects of foreign exchange market intervention and the concluding section discusses the limitations of the analysis.
II. The Model

We assume that the economy produces traded and non-traded goods with labour as the only factor of production. The supply of the two goods is a function of the relative price:

1. \[ X_i = X_i(q), \quad i = 1, 2 \]

where \( X_i \) is the supply of domestic goods and \( X_2 \) the supply of international goods. \( q \) is the relative price of domestic goods in terms of traded goods:

2. \[ q = \frac{p_1}{p_2} \]

We assume that the world price level is constant and equal to one so that \( p_2 \) is also the exchange rate.

Consumption demand for the two commodities is a function of the two prices, nominal income and financial wealth:

3. \[ C_i = C_i(p_1, p_2, X_1 p_1 + X_2 p_2, A), \quad i = 1, 2 \]

We assume that the residents of the country hold only two types of assets -- domestic paper currency and foreign assets. The latter comprises foreign exchange, foreign securities and even storable internationally traded goods. Therefore, total nominal wealth is given by:

4. \[ A = M + F \cdot p_2 \]

where \( M \) is the nominal stock of money and \( F \) is the foreign currency value of foreign assets.

The demand for the two assets is assumed to be a function of
the two prices, the nominal stock of wealth and the expected change in the exchange rate:

5. \[ M^d = L(\pi, P_1, P_2, A) \]
   \[ (-) (+) (+) (+) \]

6. \[ F^d P_2 = F(\pi, P_1, P_2, A) \]
   \[ (+) (+) (+) (+) \]

We assume that foreigners do not hold domestic money. Therefore, equilibrium in the financial markets obtains when:

7. \[ M^d = M \]

8. \[ F^d = F \]

By the wealth constraint only one of these equilibrium conditions is independent. It helps to substitute A from the wealth constraint into equation (5) and write the equilibrium condition (7) in the form:

9. \[ L(\pi_2, q_1, F + \frac{M}{P_2}) = \frac{M}{P_2} \]

Solving this for the stock of real balances in terms of the traded good we obtain the reduced form real balance equation:

10. \[ \bar{M} = \frac{M}{P_2} = H(\pi_2, q_1, F) \]
    \[ (-) (+) (+) \]

where the signs of the partials can be easily established from equation (9). If we knew the relative price of non-traded goods equation (10) would, ceteris paribus, determine the exchange rate. In any case equation (10) tells us that if the price of non-traded goods increases the exchange rate must appreciate in order to maintain equilibrium in the asset markets.

The next step is to substitute the real balance equation (10) into the wealth constraint (4) in order to obtain the reduced form wealth equation:
11. \( \frac{A}{P_2} = A(\pi_2, q_1, F) \)
\((-) (+) (+)\)

The signs of the partials follow from those in equation (10). We
may now close the model by the condition of equilibrium in the mar-
ket for domestic goods:

12. \( C_1(q, q_1 q + x_2, \frac{A}{P_2}) - x_1(q) = E_1(q, \frac{A}{P_2}) = 0 \)
\((-)(+)\)

The model implies that \( \frac{\partial E_1}{\partial q} \) is always negative.\(^2\)

Equations (10) and (12) determine the temporary equilibrium
values of the exchange rate and the relative price of non-traded
goods, given the supplies of assets and expectations regarding the
exchange rate. The momentary equilibrium changes from one instant
to another because of changes in the stock of foreign assets and
revisions of expectations in the light of new experience.

The change in the stock of foreign assets is equal to the
trade account surplus, which in turn is equal to the excess supply
of traded goods:

13. \( F^O = B = x_2(q) - C_2(q, x_1 q + x_2, \frac{A}{P_2}) = E_2(q, \frac{A}{P_2}) \)
\((-)(-)\)

The fact that the partial of \( E_2 \) with respect to \( q \) is negative can
be proven in the same way as before (see footnote 2).

The dynamics of the formation of exchange rate expectations
is introduced later.

The Temporary Equilibrium

The determination of the temporary equilibrium values of the
exchange rate and the relative price of non-traded goods is illus-
trated in diagrams I and II. The MM schedule implies equilibrium
Diagram I

The Temporary Equilibrium (I)

relative price of non-traded goods $\phi_1$

Exchange rate

$\phi_2$

$\phi_1$

$G$

$M$

$M'$

$A_1$

$A_2$

$P_1$

$P_2$
Diagram II

The Temporary Equilibrium (II)

[Diagram showing exchange rate and relative price of non-traded goods]

Exchange rate
in the asset markets and the GG schedule an equilibrium in the market for non-traded goods. Both of them are downward sloping. Without further analysis we cannot tell which of them is steeper. Before exploring this question algebraically let us investigate the implications of the two possibilities.

An increase in the expected rate of depreciation will shift the MM schedule to the right. In diagram I where the GG curve is steeper this implies that the exchange rate will depreciate and the relative price of non-traded will increase. In diagram II where the MM schedule is steeper the exchange rate will appreciate and the relative price of non-traded goods will decrease.

Diagrams III and IV compare the effects of an increase in the stock of foreign assets. In the first case the exchange rate will appreciate and the relative price of non-traded goods will increase. In the second case, the exchange rate will depreciate and the relative price of non-traded goods will decrease. This indicates that the second possibility implies dynamic instability since an increase in the stock of foreign assets seems to increase the current account surplus, so that there appears to be no end to reserve accumulation or decumulation. This is indeed the case as will be demonstrated below.

Let us write the model in the compact form:

14. $E_1(q, \bar{M} + F, a) = 0 \quad \text{equilibrium in the market for non-traded goods}$

15. $M = L(\pi, q, \bar{M} + F) \quad \text{equilibrium in the asset markets}$

16. $\dot{F} = B = B(q, \bar{M} + F, a) \quad \text{the current account}$
Diagram III

The Effects of an Increase in the

Stock of Foreign Assets (I)

Relative price of non-traded goods

Exchange rate
The Effects of an Increase in the Stock of Foreign Assets

Diagram IV

The Effects of an Increase in the Stock of Foreign Assets

Relative price of non-traded goods

Exchange rate
\( \bar{M} \) is the stock of real balances in terms of the traded good. \( \alpha \) is a shift parameter in the commodity demand functions. It represents a shift in consumption preferences. We may solve equations (14) and (15) for \( \bar{M} \) and \( q \) and substitute these in (16) to obtain the reduced form momentary equilibrium equations:

\[
17. \quad q = g(\alpha, \pi, F)
\]
\[
18. \quad \bar{M} = M(\alpha, \pi, F)
\]
\[
19. \quad F = b(\alpha, \pi, F)
\]

The dynamic stability of the model depends critically on the properties of these equations. To establish the signs of the partials, differentiate equations (14) and (15) totally to obtain:

\[
20. \quad \frac{\Delta q}{\Delta} = \frac{E_{1A}}{\Delta} \hat{F} + \frac{E_{LA}}{\Delta} \hat{\pi} + \frac{1-L_A}{\Delta} E_{1A} \hat{\alpha}
\]
\[
21. \quad \frac{\Delta \bar{M}}{\Delta} = \frac{L E_{1A} - L_A E_{1q}}{\Delta} \hat{F} - \frac{E_{1q} L_{\pi}}{\Delta} \hat{\pi} + \frac{L q}{\Delta} E_{1A} \hat{\alpha}
\]
\[
22. \quad \frac{\Delta F}{\Delta} = \frac{B q E_{1A} - B_A E_{1q}}{\Delta} \hat{F} + \frac{(B q E_{1A} - B_A E_{1q}) L_{\pi}}{\Delta} \hat{\pi}
\]
\[
+ \frac{B_q (1-L_A) E_{1A} + B_A L q E_{1A} + B_A L_{\alpha}}{\Delta} \hat{\alpha}
\]

where \( \Delta = -E_{1q} (1 - L_A) - L q E_{1A} \), and \( \hat{\cdot} \) denotes change. The percentage change in the exchange rate is simply minus \( \frac{\Delta \bar{M}}{\bar{M}} \) since the nominal money stock is constant. It is clear from equations (20) and (22) that everything hinges on the sign of the determinant \( \Delta \). If it is positive the MM schedule is steeper than the GG schedule and all the expected results obtain and the signs of the partials are as indicated in parentheses below equations (17) to (19). We shall summarize this as:
Condition I: $\Delta = -E_{1q}(1-L_A) - L_0 E_{1A} > 0$

What this condition says is that the price elasticity of the excess demand for non-traded goods should be greater than $[E_{1A}/(1-L_A)] \cdot L_0$, where $L_0$ is the elasticity of money demand with respect to the relative price of non-traded goods. The effects of the various shocks on the temporary equilibrium are summarized in Table I.

II. Dynamic Stability of the Foreign Exchange Market and the Balance of Payments

In this section we investigate the dynamic stability of the economy, by which we mean a convergence of the sequence of momentary equilibrium to a stationary state with constant real variables and constant or steadily growing (declining) price level.

In order for the problem to be meaningful, a stationary state must exist. We shall assume that there exists a unique stationary state.

Assumption I: The equation: $b(\alpha, \pi, F) = 0$ has a unique positive root $F^*$ for all $\pi$ greater than some negative $\tilde{\pi}$

The dynamic stability of the foreign exchange market depends critically on the nature of the formation of expectations. We shall explore three possibilities:

a. static expectations: $\pi = \pi^*$

b. perfect foresight: $\pi = \frac{\dot{P}_2}{P_2}$

c. adaptive expectations: $\pi = \beta(\frac{\dot{P}_2}{P_2} - \pi)$
The Properties of the Temporary Equilibrium
When Condition I Holds

<table>
<thead>
<tr>
<th>The Effect of</th>
<th>The Relative Price of Home Goods</th>
<th>The Effect on the Exchange Rate</th>
<th>The Current Account</th>
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<tbody>
<tr>
<td>Increase in:</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Expected Rate of Depreciation</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Stock of foreign assets</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Preference for home goods</td>
<td>+</td>
<td>-</td>
<td>?</td>
</tr>
</tbody>
</table>
a. Static Expectations

Condition I and Assumption I ensure that the foreign exchange market is globally stable. Diagram V illustrates the evolution of the economy over time. The FF schedule implies zero current account and a constant stock of foreign assets. It does not depend on the exchange rate, only on the rate of depreciation ($\pi$). The MM schedule is defined by:

$$P_2 = p(a, \pi^*, F) M$$

where $p(\ ) = \frac{1}{m(\ )}$

It implies temporary equilibrium in the asset markets and in the market for non-traded goods. The expected rate of depreciation should equal the rate of growth in the money stock in order that expectations are correct in the stationary state. (We assume throughout, for simplicity and without consequence, that the nominal money stock is constant.) The economy starts with stock $F_o$ of foreign assets. The temporary equilibrium exchange rate is $P_o$. At that point the current account is in surplus so that the stock of foreign assets is increasing. Since the temporary equilibrium always obtains the economy moves along the MM schedule with the exchange rate appreciating along the way. This adjustment path, although stable, implies profit opportunities for speculators who have foresight.

If Condition I does not hold, the real stock of wealth and the relative price of non-traded goods are unstable, even with static expectations. This is because the change in the stock of wealth is an increasing function of the actual stock of wealth. A small decrease in the stock of wealth causes the exchange rate
Diagram V

The Dynamics of the Exchange Rate and the
Stock of Foreign Assets With
Static Expectations

Stock of foreign assets
to appreciate and the price of non-traded goods to increase. This creates a current account deficit which further increases the price of non-traded goods. The economy has inflation and a current account deficit simultaneously. Both of these reduce the stock of financial wealth.

b. Perfect Foresight

In the case of perfect foresight the dynamic path of the economy is defined by equations (23) and (19), and the expectations equation:

\[ \pi = \frac{\dot{P}_2}{P_2} = \dot{X}, \text{ where } X = \ln P_2 \]

In order to study the properties of this system, linearize it around the stationary state. This yields:

\[ \dot{F} = b_o + b_\pi \pi - b_F F \]
\[ \dot{X} = P_o + p_\pi \pi - p_F F \]

Substituting from equation (24) we obtain finally:

\[ \dot{F} = b_o \frac{b_\pi}{P_\pi} X + \frac{b_\pi}{P_\pi} (\frac{P_F}{P_\pi} - b_F) F \]
\[ \dot{X} = \frac{P_o}{P_\pi} \frac{1}{P_\pi} X + \frac{P_F}{P_\pi} F \]

The characteristic roots of this system of differential equations are always real and of opposite signs. Therefore, the stationary state is a saddle point -- there is only one initial exchange rate from which the economy will converge to the stationary state.

The PP schedule implies a constant exchange rate and the FF schedule a constant stock of foreign assets. As the exchange rate approaches infinity the stock of foreign asset approaches the
steady state stock of wealth. It does not increase without limit. All that happens is that people shift from domestic money to foreign money, holding in the end only foreign assets. This also implies that the relative price of non-traded goods will converge to a constant nonzero value (see Section d below).

Suppose that the economy has initially stock $F_0$ of foreign assets (see diagram VI). If the initial exchange rate is set correctly at $P_0$ the economy will move along the TT trajectory to the stationary state. Along this path expectations are continuously realized and no speculator is either making a profit or a loss. If, however, the currency is initially undervalued at $P_0$ it will at first appreciate and then start depreciating without limit. Intuitively, the exchange rate appreciates at first because the stock of foreign assets is increasing fast because of the undervalued exchange rate. After a while, the growth of foreign assets slows down and the exchange rate begins to depreciate and the speculators immediately revise their expectations starting the flight out of domestic money. A similar story can be told for the case of an overvalued exchange rate. The explosive path is illustrated by the T"T" schedule. One might want to argue that the process of hyperdeflation must come to a stop at point Q where the economy no longer holds foreign assets, unless the foreigners are willing to lend to the residents of this country just for the purpose of them accumulating domestic money. If the process stop at point Q the domestic money is no longer as attractive because its yield suddenly falls drastically. There will be a collapse of the speculative boom -- presumably only to be followed by another one (see Section d below).
Diagram VI

The Traverse With Perfect Foresight

Stock of foreign assets
The saddle point property of the perfect foresight model raises difficult questions. What market forces are there to rule out the deviant paths? Perhaps the only answer is that in normal times people simply rule out hyperinflation or deflation as long as they have confidence that a stable long run value of the currency can be maintained. If that is so, the perfect foresight model does have the interesting property that the current exchange rate depends not only on the immediate expected changes in the exchange rate but also on everything that is expected to happen in the future.

c. Adaptive Expectations

The intermediate case between static expectations and perfect foresight is adaptive expectations with neither zero nor infinite speed of adjustment. In that case the dynamics around the steady state is defined by equations (24) and (25) and:

29. \( \dot{\pi} = \beta (\dot{x} - \pi) \)

Solving the model for \( \dot{\pi} \) and \( \dot{F} \) as functions of \( \pi \) and \( F \) we obtain:

30. \( F^0 = b_o + b_\pi \pi - b_F F \)

31. \( \frac{c}{\pi} = \frac{\beta p_F}{1-\beta p_F} b_o - \frac{\beta (1 + b p_F)}{1-\beta p_F} \pi + \frac{\beta p_F b_F}{1-\beta p_F} F \)

A sufficient condition for the stability of this dynamic system is that the product of the elasticity of the exchange rate with respect to the expected rate of depreciation and of the speed of adjustment of expectations is less than one:

32. \( \beta p_F < 1 \)

This condition looks the same as the condition of stability in
Cagan's model of hyperinflation. The only difference is that the $P_\pi$ coefficient is a reduced form elasticity rather than the inflation elasticity of money demand. It depends on all structural parameters of the model. If the model is stable, the convergence to equilibrium is nonoscillating (see footnote 4).

The translation of the model to the $(X, F)$ space is cumbersome but the same stability condition obviously holds. Furthermore, the convergence of the exchange rate must also be nonoscillating. The dynamic adjustment path is illustrated in diagram VII. The PP and the FF schedules have the same interpretation as before. The economy has initially stock $F_0$ of foreign assets. With zero expected change in the exchange rate the equilibrium exchange rate is at point $P_0$. With static expectations the economy would move along the $A_0A^*$ trajectory (corresponding to the MM schedule in diagram V) to the stationary state. This path implies profit opportunities so that if speculators have foresight there will be a revaluation of the initial exchange rate to point $P'_0$ whereafter the economy moves along the $A'_0A^*$ trajectory. Trajectory $A_0T_A^*$ illustrates a possible adjustment path with adaptive expectations. Since the exchange rate is appreciating the actual exchange rate path must always be below the static expectations path. It is quite possible, as is illustrated in diagram VII, that the exchange rate will go below the long run exchange rate by the force of expectations lagging behind actual developments. It is also possible that incorrect expectations cause the current account to be in surplus first and then in a period of deficits during which the excess
Diagram VII

The Adjustment Path With
Adaptive Expectations

Stock of foreign assets
stock of foreign assets is eliminated. This possibility is illustrated by the $A_1TA^*$ trajectory starting from initial point $A_1$ and an initial stock $F_1$ of foreign assets above the long run equilibrium stock. These adjustment paths would not be possible either with static expectations or perfect foresight.

d. The Stability of Relative Prices

The instability of the exchange rate does not necessarily imply instability of relative commodity prices. From equation (14) it follows that the stability of the relative price of non-traded goods depends on whether the total stock of real financial wealth is stable. This question can be answered by observing that the change in the real stock of wealth (in terms of traded goods) consists of new savings and capital gains or losses on existing wealth:

33. $A = X_1q + X_2 - C_1(q, A) - C_2(q, A) - \frac{P_2}{P_2} \frac{M}{P_2}$

Since $q$ is an increasing function of $A$ we may write this in the compact form:

34. $\dot{A} = C(A) - \frac{P_2}{P_2} L^*(\pi, A)$

It is immediate from equation (34) that hyperinflation will cause the second term to vanish so that the stock of real wealth will converge eventually to $A^*$ where $A^*$ is the root of $C(A) = 0$. In the long run, wealth consists of only foreign assets so that $A^*$ is equal to $F^*$ -- defined in Assumption I. This result justifies the shape of the FF schedule in diagram VI.

It is equally clear that hyperdeflation will cause the real
stock of wealth to increase without limit since the second term will explode as deflation accelerates. This implies that the relative price of non-traded goods will increase without limit and hence that the supply of traded goods will diminish to zero. Since at the same time the stock of foreign assets is declining and eventually becoming negative, it is clear that the bull market must collapse in a finite time. The speculative rally is only creating a temporary illusion of affluence which is based on nothing but groundless optimism. If speculators have foresight and are not caught in the mood of the moment, hyperdeflation can be ruled out. With myopic and adaptive expectations there is nothing in theory to prevent speculative booms and subsequent collapses in the foreign exchange market.

III. A Digression on the Effects of Devaluation

There is a confusion in the literature between the conditions for a successful devaluation and the conditions of stability of freely fluctuating exchange rate. The condition for a successful devaluation implies that a once-and-for-all change in the exchange rate reduces the current account surplus (and possibly the stock of reserves; see below). This does not imply, even in the absence of speculation, that the freely floating foreign exchange market is stable. That requires that an increase (decrease) in the stock of foreign assets reduces (increases) the current account surplus. The two conditions are not the same. The second difference is that foreign exchange market speculation is of critical importance for the stability of flexible exchange rates whereas it is of (at least
with confidence in the parity) no consequence for a fixed exchange rate regime. In our model, devaluation unambiguously improves the balance of payments.

a. The Evolution of the Economy With a Fixed Exchange Rate

With a fixed exchange rate the temporary equilibrium and the dynamics of the economy is defined by the following set of equations:

35. \( E_1(q, \frac{A}{P_2}) = 0 \) equilibrium in the market for non-traded goods (cf. equation 14).

36. \( \dot{R} = B(q, A) - F \) balance of payments = change in Central Bank reserves (cf. equation 16).

37. \( \frac{M_d}{P_2} = L(q, A) = \frac{M}{P_2} = \frac{D}{P_2} + R \) demand for and supply of money (cf. equation 15).

38. \( M_d + F_d = A = M + F \) private sector wealth constraint (cf. equation 4).

The structure of the model can be explained as follows. At any given instant, the real stock of financial wealth is given. The price of non-traded goods adjusts to clear the market for non-traded goods (equation 35). The relative price of non-traded goods and the current stock of wealth determine the rate of change in the stock of wealth. This can be seen by observing that if the Central Bank holds its stock of domestic assets constant (which we shall assume throughout) equation (36) can be written as follows:
The distribution of the private sector's stock of financial wealth is determined by equations (37) and (38). The private sector can always rearrange its portfolio at a constant exchange rate since the Central Bank supports the parity.

Before analyzing the effects of devaluation, consider the dynamic behaviour of the system defined by equations (35) and (39). This is illustrated in diagram VIII. The GG schedule implies equilibrium in the market for non-traded goods. It is upward sloping because an increase in the stock of wealth increases the demand for non-traded goods and hence their relative price. The BB curve implies a constant stock of financial wealth. It also implies a balance of payments equilibrium -- zero trade account and capital account. The economy starts at point $Q_0$ with wealth $A_0$ and relative price of non-traded goods $q_0$. At that point, the stock of wealth is below its stationary state value, and the current account is in surplus.

It is only through current account surpluses that the private sector can accumulate wealth. Assuming continuous full-employment equilibrium in the market for non-traded goods, the economy will move along the GG schedule up to the stationary state at point A.

The overall balance of payments must be in surplus to the left of point A because the demand for money is increasing: the Central Bank increases the supply of money by intervening in the foreign exchange market.

Whether the capital account is in deficit or surplus depends on the relative magnitudes of the wealth effect and the effect of
Diagram VIII

The Dynamic Behaviour of the Economy
Under Fixed Exchange Rates

Relative Price of non-traded goods

Stock of Wealth
the increasing price level. From equations (37) and (38) the capital account is:

40. $\dot{F} = [1 - L_A] A - L q \dot{q}$

The sign of $\dot{F}$ is ambiguous without further information. Differentiating equation (35) totally with respect to time, solving for $q$ and substituting back in equation (40) we obtain:

41. $\dot{F} = [(1 - L_A) + L q \frac{E_1 A}{E_1 q}] \dot{A}$

The expression in the brackets is equal to $-\Delta/E_1 q$, where $\Delta$ is defined in Condition I on page 10. Therefore, if $\Delta$ is positive -- which implies that the flexible exchange rate regime is stable with static expectations the capital account is in deficit as long as the stock of wealth is below the stationary state value (and in surplus thereafter).

b. The Effects of Devaluation

This section investigates the short run and the long run effects of an unanticipated devaluation. Suppose that the private sector's initial wealth composition is $\frac{M}{P_2}$ in money and $F$ in foreign assets. A devaluation of $X$ percent will reduce wealth by $\frac{M}{P_2} X$ in terms of the traded good. From equation (35) the increase in the price of non-traded goods is:

42. $\dot{q} = \frac{E_1 A}{E_1 q} \dot{A}$, where $\dot{A} = -\frac{M}{P_2} X$

which is unambiguously positive. From equation (36) the effect on the current account is:

43. $\dot{B} = [-B \frac{E_1 A}{E_1 q} + B_A] \dot{A}$

The expression in the parenthesis is negative so that a devaluation
unambiguously improves the current account.

The effect of devaluation on the capital flow account is ambiguous. Since the current account improves, the rate of accumulation of wealth will increase. We know from the analysis on page that this may either reduce or increase the capital (flow) account deficit. If Condition (I) holds (normal case) the capital flow account will be in deficit after the devaluation as people keep up the increase in their foreign assets in pace with their increasing wealth. If Condition (I) does not hold the capital flow account will go into a surplus (if it was in balance before). This is because foreign assets are inferior assets and their holdings actually decline as the economy accumulates more wealth.

The effect of devaluation on the overall balance of payments is unambiguously positive. This is because the accumulation of wealth increases the demand for money which can be satisfied only through surpluses in the balance of payments.

The devaluation also has an impact effect on the stock of foreign reserves of the Central Bank because of its effect on the private sector's portfolio of assets. The change in reserves is equal to the change in the nominal stock of money held by the private sector \((M/P_2)\) in foreign currency. The algebraic formula for the discrete change in the stock of Central Bank reserves is:

\[ \hat{R} = \hat{M} - \hat{P}_2 = [(1-L_A) + L_q \hat{F}_{1q}] \hat{A} = \frac{\Delta}{\hat{E}_{1q}} \hat{A} \]

If Condition (I) holds, the devaluation increases the stock of foreign assets immediately because of the portfolio effect. If Condition (I) does not hold, devaluation reduces at first the Central Bank's stock of foreign reserves.
The long run effect of the devaluation on the stock of reserves is the same in both cases -- the price level and the stock of money will increase in the same proportion as the currency is devalued.

The short run and long run effects of devaluation are illustrated in figures I and II. Figure I illustrates the effects on balance of payments flows and figure II the effects on the stock on Central Bank reserves.

Figure I-A illustrates the response of the balance of payments flows to devaluation. The economy is initially in a stationary state equilibrium. At time $t_0$ the currency is devalued. The current account will go to a surplus ($B_o$), the capital flow account to a deficit ($F_o$), and the overall balance of payments to a surplus ($R_o$). The impact effect of the devaluation on the stock of reserves cannot be seen in diagram IX (see diagram X). The abnormal case is illustrated in diagram IX-B which is self-explanatory.

The effect of devaluation on the stock of Central Bank reserves is shown in figure II. In the normal case the stock of reserves increases immediately from $R_o$ to $R_o'$ as domestic residents sell their foreign assets for domestic money. Thereafter the stock of reserves increases steadily through overall surpluses in the balance of payments, converging asymptotically to $R^*$. The total increase in reserves is equal to $\frac{X}{P_2^e}M$ where $X$ is the percentage of devaluation and $\frac{M}{P_2^e}$ the stationary state stock of real balances (measured in foreign currency).

In the abnormal case, the stock of Central Bank reserves
Figure I
The Effects of Devaluation on the Balance of Payments

A. Normal Case

B. The Abnormal Case
The Effects of Devaluation on the Central Bank's
Stock of Foreign Exchange Reserves

A. Normal Case

B. Abnormal Case
initially falls (from $R_0$ to $R'_0$ in Figure II-B. This is because a reduction of wealth increases the demand for foreign assets. After the initial adverse effect, the stock of reserves begins to increase by force of the surpluses in the overall balance of payments, reaching eventually the same stationary state level ($R^*$) as in the previous case.

The above analysis suggests that devaluation can be interpreted as a capital levy on financial wealth. As long as the private sector has nominally fixed assets, devaluation will initially reduce the real value of wealth. Furthermore, it will reduce the stock of real balances in relation to wealth. In the long run, neither real wealth nor the stock of real balances is affected by devaluation. Therefore, after the devaluation, the current account and the overall balance of payments must be in surplus for the private sector to be able to reconstitute its asset position. The condition of stability of the freely floating foreign exchange market is relevant only for the impact effect of devaluation, not for the total effect which is unambiguous. It should also be emphasized that the process of adjustment to new equilibrium after the devaluation will involve changes in relative commodity prices and reallocation of resources from the home sector to the external sector of the economy.

The view of devaluation as a capital levy implies that devaluation must be unanticipated in order to be successful. For if the private sector anticipates devaluation they will sell their domestic assets as much as they can. Indeed, it may be even possible that they become net debtors in domestic currency in which case
the effect of devaluation is to increase private sector wealth -- the Central Bank or government would impose the capital levy on itself.

V. The Effects of Foreign Exchange Market Intervention and Other Policies

If the Central Bank wishes to increase its stock of reserves under a regime of freely floating exchange rate, it can do so by buying foreign assets with money. The effects of such policy are illustrated in diagram IX. The economy is initially at the stationary state with exchange rate $P_o$ and stock of private foreign assets $F_o$. The Central Bank buys $F'_oF'_o$ of foreign exchange from the private sector. The ultimate effect of this policy is to increase the exchange rate in proportion and to leave the stock of foreign assets held by the private sector unchanged. The short run impact and the adjustment path depend on the nature of expectations. In all cases the exchange rate initially overshoots. With static expectations (cf. Section IIIa) the exchange rate will jump to $P'_o$ and then appreciate along the MM line to the new stationary state value. With perfect foresight the exchange rate will jump less, to point $P''_o$, and then monotonically appreciate to the same stationary state value (cf. Section IIIb). With adaptive expectations, a possible adjustment path is illustrated by the $A'_oTA_1$ trajectory (cf. Section IIIc). The speculators incorrectly expect the exchange rate to stay at the initial value; gradually revise their expectations in such a way that the exchange rate goes below
Diagram IX

The Effects of Foreign Exchange Market Intervention

Stock of foreign assets
the long run value. With non-traded goods, speculative behaviour not only affects the price level and the balance of payments (intertemporal allocation of consumption) but also the relative price of non-traded goods and the allocation of resources between the two sectors. If the market is stable, these effects are only temporary -- the long run equilibrium position will be the same as before except for a higher price level.

In the long run, devaluation and a direct intervention in the foreign exchange market with a market determined exchange rate are alternative methods for obtaining exactly the same result. The short run effects and the nature of the implied adjustment paths is, however, quite different.

A reduction in the stock of the Central Bank's domestic assets would be another alternative to devaluation. Since we do not have a domestic bond market in the model, the only way that the supply of money may change, apart from foreign exchange market intervention, is through fiscal policy. In the model, a capital levy of X percent on private holdings of domestic assets has exactly the same effects as a devaluation of X percent, except for the level of prices. Other fiscal measures, such as a temporary tax on incomes, the proceeds of which are used to buy foreign assets, will have identical long run effects but the time path will obviously be quite different for each fiscal policy.

VI. Concluding Remarks

The analysis of this chapter can be extended in a number of directions. Perhaps the most important modification would be to
allow for rigidity of nominal wages and existence of unemployment. Such extension would be straightforward to incorporate into the framework developed in this paper. If one assumed, for example, that the price of non-traded goods and the nominal wage rate were fixed in domestic currency equation (12) would determine the level of output in the home goods sector. Furthermore, the output of traded goods would now be an increasing function of the exchange rate -- the latter would determine the level of profitable production given the nominal wage rate. The assumption of rigid wages and domestic prices is hard to justify in an analysis of the adjustment process, which is why we have assumed price flexibility even in the short run.

The introduction of real capital and bond markets would not alter the analysis significantly although it would make it much more involved. Such extensions would, however, be necessary should one wish to assume away the real balance effect. With only inside money (and no outside debt) changes in the exchange rate affect the economy by changing relative prices and real wages, and/or by changing the real rate of interest. These two channels are absent in the model developed in this chapter.

The extension of the model to two or more countries would bring out the point that in the end what links the current account with the exchange rate is the transfer of wealth implied by a nonzero current account and the fact that asset and consumption preferences are likely to be different in different countries.⁶
Footnotes for Chapter II

1 Only the expected change in the exchange rate is included in the asset demand equations in order to keep the model manageable. A rigorous justification can be given in the case of a homothetic utility function. Let $\alpha$ be the proportion of expenditure on domestic goods. Then the price index is:

\[ p = P_1^{\alpha} P_2^{1-\alpha} \]

and the real return on money is $\pi_1 - (1-\alpha)\pi_2$. The real return to foreign assets is the percentage change in $P_2/P$ which is $\alpha(\pi_2 - \pi_1)$. The difference in real returns is therefore $\pi_2$. In the stationary state the relative price of non-traded goods is constant so that $\pi$ is the rate of inflation.

2 This is because:

\[
\frac{dc_1}{dq} = \frac{\partial c_1}{\partial q} + \frac{\partial c_1}{\partial Y} \cdot X_1 + \frac{\partial c_1}{\partial Y} (dx_1q + dx_2)
\]

\[
= [\frac{\partial c_1}{\partial q} - \frac{\partial c_1}{\partial Y} \cdot X_1] + \frac{\partial c_1}{\partial Y} \cdot X_1 = \frac{\partial c_1}{\partial q} < 0
\]

The expression in the first bracket is equal to zero by the condition of profit maximization. The expression in the second bracket is the Slutsky decomposition. The income effect of the price change is cancelled out. I am indebted to Hal Varian for a clarifying discussion on this point.

3 The characteristic roots are:

\[
m_1, m_2 = \frac{1}{p\pi} + \left( \frac{b\pi}{p\pi} p_F - b_F \right) \pm \sqrt{\left( -\frac{1}{p\pi} + \frac{b\pi}{p\pi} p_F - b_F \right)^2 + 4 \frac{b\pi p_F}{p\pi}}
\]

\[
= C \pm \sqrt{C^2 + 4 \frac{b_F}{p\pi}}, \text{ where } C = \frac{1}{p\pi} + \left( \frac{b\pi}{p\pi} p_F - b_F \right)
\]

The roots are real and of opposite sign.

4 The characteristic roots are, after a little algebra:

\[
m_1, m_2 = C \pm \sqrt{C^2 - 4 \frac{b_F}{1-\beta p\pi}}, \text{ where } C = -b_F - \beta \frac{1 + b\pi p_F}{1 - \beta p\pi}
\]

Clearly $C$ is negative if $1 - \beta p\pi > 0$. Furthermore, the discriminant can be shown to be always positive so that if the model is stable, convergence to equilibrium is nonoscillating.

For a derivation and interpretation of a similar stability condition in a different model, see Mussa (1973).

5 cf. Dornbusch (1973). It should be emphasized that the assumption of a unique stationary state equilibrium is critical to the analysis. The existence of multiple equilibria would, of
course, imply instability even in the fixed rate model and in that situation devaluation could well have an adverse effect on the stock of foreign reserves (see Dornbusch (1973a)). The assumption of a 'well-behaved' economy is also critical to the analysis. Complications that arise from wage-price rigidities and various distortions are analyzed in detail in Bhagwati (1975).

The implications of these differences for the short run effects of devaluation are examined in Dornbusch (1973a).
Chapter III
EXCHANGE RATE EXPECTATIONS, AND THE SHORT RUN
AND THE LONG RUN EFFECTS OF FISCAL AND
MONETARY POLICIES UNDER FLEXIBLE EXCHANGE RATES

I. Introduction

This chapter analyzes the short run and the long run effects of various fiscal and monetary policies on the exchange rate and the balance of payments in a small open economy with flexible prices and full employment. Apart from this simplifying assumption, the paper differs from the extensive literature on the subject in several important respects. First, in the short run analysis we view the exchange rate as a relative price of monies which is determined, in the same way as interest rates and equity prices, to clear the financial markets. This is in contrast to the traditional approach, exemplified by the seminal contributions of Fleming (1962) and Mundell (1968),¹ which views the exchange rate as a price that equilibrates the 'balance of trade' with the largely independently determined 'balance of lending'. This flow model of the exchange rate was shown to be logically incorrect in Chapter I. This chapter will examine the consequences of the mistake for macroeconomic analysis.

The second difference from the literature is the explicit treatment of exchange rate expectations. Argy and Porter (1972) show that the short run effects of fiscal and monetary policies
under flexible exchange rates depend critically on the way speculators form their expectations about the future. They do not, however, examine the implications of various mechanisms of expectations formation on the dynamic adjustment path. In this paper, we investigate this question in great detail both in terms of the dynamic stability of the balance of payments adjustment process, and in terms of its qualitative properties. We shall focus in particular on the implications of rational exchange rate expectations. It is commonly argued, following Friedman (1953), that in a well functioning foreign exchange market there can be no expected, systematic profit from foreign exchange market speculation. Yet, the macroeconomic implications of this 'efficient market' hypothesis have not been worked out in the literature. It is shown in this paper that the assumption of rational expectations implies radically different adjustment patterns from, for example, the dynamic response patterns that obtain when expectations are formed adaptively. The assumption of rational expectations (perfect foresight) implies that the exchange rate is unstable unless the initial exchange rate is set just right. This problem is well known in models of money and growth (Black (1974), Brock (1975), Hahn (1969)) and has been typically assumed away in the recent rational expectations monetary models (for example, Sargant and Wallace (1973)). No resolution of the problem is offered in this paper beyond some suggestions.

A third contribution of this paper is a detailed analysis of the adjustment process from the short run momentary equili-
brium to the long run stationary state. There is a gap in the literature on this question. Several authors have recently examined the long run stationary state effects of various policies whilst the earlier literature dealt exclusively with the short run effects in an extended IS-LM model of the open economy. It is shown in this paper that with long run foresight (rational expectations) the short run impact effects of various shocks on the exchange rate and the balance of payments can be inferred from the long run effects of these shocks in the stationary state exchange rate and asset supplies. The short run effects with rational expectations are shown in many cases to be radically different from the short run effects implied by exogeneous expectations.

Finally, the analysis of this paper has several interesting implications for the behaviour of market determined exchange rates and for the nature of the monetary transmission mechanism under flexible exchange rates. Those are taken up in the concluding section.

The structure of the model developed in this paper is very simple. We assume flexible prices and wages, full employment and continuous equilibrium (momentary equilibrium) in the financial markets, as well as exogenously given relative prices. We assume, too, that there is no real growth nor accumulation of real capital. These assumptions sharpen the analysis but the general principles elucidated in this paper go beyond this or any other simplified model.

The paper proceeds as follows. Section II analyzes the
short run determination of the exchange rate and the balance of payments and investigates the effects of various policies and shocks with exogeneous expectations. Section III analyzes the effects of the same shifts on the stationary state. Section IV examines the problem of dynamic stability of the balance of payments adjustment process, and Section V investigates in detail the adjustment path from the short run momentary equilibrium to the long run stationary state. The concluding section summarizes the results and compares them with the traditional analysis.
II. Asset Market Equilibrium and the Balance of Payments in the Short Run

In this section we analyze the simultaneous determination of the exchange rate, the interest rate and the current account in the short run with given expectations and asset supplies. We shall also examine the short run effects of various monetary policies and shocks.

To simplify the analysis, we assume that the economy produc-es only traded goods, the relative prices of which are determined in the world market. We also assume flexible wages and full em-ployment of labour, which is the only factor of production. There is no accumulation of real capital.

a. Asset Market Equilibrium

The residents of the economy hold three types of assets: domestic money (M), domestic securities (B), and foreign securi-ties (F). The demand for real balances is a function of real income (Y) and the nominal rate of interest (r). The supply of money is determined by the Central Bank. The money market equil-ibrium condition is then:

1. \[ \frac{M^d}{P} = m(r)Y = \frac{M}{P} \]

The supply of money is equal to the stock of domestic bonds (B^{cb}) and of foreign bonds (F^{cb}) acquired by the Central Bank since the money supply can change only as a result of open mar-ket operations (\( \Delta B^{cb} \)) or foreign exchange market intervention (\( \Delta F^{cb} \)).
2. \[ M = B^{Cb} + F^{Cb} \]

We assume that changes in the valuation of the Central Bank's stock of foreign reserves have no effect on the money supply.

Using the simplification justified in chapter IV we write the demand for domestic bonds as the difference between the total demand for domestic assets -- money and bonds -- and the demand for money.

The total demand for domestic assets in turn is a function of the difference between the domestic interest rate \( r \) and the foreign interest rate \( r^* \) adjusted for expected change in the exchange rate \( \pi \), and proportional to real financial wealth \( A \). We ignore foreign holdings of domestic assets.

The equilibrium condition for the market of domestic bonds is:

3. \[ \frac{B_d}{P} = d(r - r^* - \pi) A - \frac{M_d}{P} = \frac{B}{P} \]

where \( B \) is the supply of domestic bonds to the private sector. We assume that all bonds are of short term maturity. \( B \) is equal to the total stock of public debt \( B^g \), less the amount held by the Central Bank \( B^{Cb} \). Using the Central Bank balance sheet given in equation (2) we may rewrite equilibrium condition (3) in the form:

4. \[ \frac{D_d}{P} = d(r - r^* - \pi) A = \frac{B^g + F^{Cb}}{P} = \frac{D}{P} \]

where \( D \) is the total supply of domestic assets. Equation (4) assumes that the money market equilibrium condition obtains. Note that open market operations have no effect on the supply of domestic assets. The Central Bank changes the total supply of domestic assets only by exchanging domestic money, or bonds
for foreign securities with the private sector. This it can accomplish in two ways:

(i) by buying foreign securities with domestic money. This corresponds to intervention in the spot foreign exchange market.

(ii) by buying foreign securities with domestic money and selling domestic securities in the open market in the same amount. In the special case when the exchange rate risk is the only risk in international investment, exchange of domestic bonds for foreign bonds is identical in its effects to intervention in the forward exchange market. Although the forward market is not explicit in the model, we shall frequently refer to forward market intervention when we mean an exchange of domestic bonds for foreign bonds.

Since there are only three assets the demand for foreign assets is given by:

5. \( F^d = f(r - r^* - \pi) A = [1 - d(r - r^* - \pi)] A = F \)

The supply of foreign assets \((F)\) to the private sector is equal to the total stock of foreign exchange reserves held by the private sector and the Central Bank \((F_{cb})\) less the stock of foreign assets held by the Central Bank \((F_{cb})\):

6. \( F = F^t - F_{cb} \)

There are three assets and three conditions of equilibrium. One of them is redundant by the wealth constraint:

7. \( M^d + B^d + F^d = D^d + D^d = A = D + F \)
Given the asset supplies and the level of real income the remaining two equilibrium conditions determine the exchange rate (price level) and the nominal rate of interest. Without loss of generality we assume that the world price level is constant and equal to one. The price level and the exchange rate are then the same thing.

It is convenient to write the two equilibrium conditions in the form:

8. \( m(r)Y = \phi \bar{D} \) \hspace{1cm} money market equilibrium
9. \( f(r - r^* - \pi)(\bar{D} + F) = F \) \hspace{1cm} equilibrium in the market for foreign assets

where \( \phi \) is the ratio of the money stock to the total supply of domestic assets and \( \bar{D} \) is the real value of domestic assets held by the private sector. The Central Bank changes \( \phi \) by domestic open market operations and \( F \) by exchanging domestic assets for foreign assets. The exchange rate is equal to \( D/\bar{D} \).

**Diagram I illustrates the simultaneous determination of the rate of interest and the real value of the stock of domestic assets (and hence the exchange rate).** The MM schedule is defined by equation (8), and it implies equality of the demand for and supply of money. It is downward sloping because the demand for real balances is a decreasing function of the rate of interest. The FF curve implies equilibrium in the market for foreign assets (foreign exchange market). It is upward sloping because an increase in the domestic interest rate increases the total demand for domestic assets.
Diagram I

Momentary Equilibrium in the Asset Markets

interest rate

real value of the stock of domestic assets
The effects of various policies and shifts can be easily analyzed with the aid of the diagram. The MM schedule is moved by changes in real income and liquidity preference, and by open market operations in the domestic bond market. The FF schedule is moved by shifts in asset preferences between domestic and foreign assets, by changes in the foreign interest rate and exchange rate expectations, and by an exchange of domestic assets for foreign assets.

An increase in the money supply brought about by domestic open market operations will shift the MM schedule to the left. Ceteris paribus, the interest rate will decline and the exchange rate will depreciate reducing the real value of domestic assets (to r' and D' respectively in diagram I). Because the velocity of money will decline the exchange rate will not depreciate in proportion to the change in the money stock.

An exchange of domestic assets (with no change in the money-debt ratio) for foreign assets with the private sector will shift the FF schedule to the left (to F'F' in the diagram). The interest rate will increase and the exchange rate will depreciate implying a reduction in the real value of domestic assets. Because the velocity of circulation increases the exchange rate will depreciate by a larger percentage than the increase in the supply of domestic assets.

A spot market intervention involves an exchange of money for foreign assets. A Central Bank purchase of foreign exchange shifts the MM schedule to the left (M'M' in diagram II) and the FF schedule to the left (F'F' in the diagram). The exchange rate
Diagram II

The Effects of Spot Market Intervention

real value of the stock of domestic assets
will depreciate and the interest rate will fall. Because of
the decline of the interest rate the exchange rate will depreci-
ate by less than the percentage change in the money stock.

The effects of an exchange of domestic securities for for-
eign securities (corresponding to forward market intervention
in the special case) are illustrated in diagram III. Because
the proportion of money in the total stock of domestic assets
decreases the MM schedule shifts to the right (to M'M' in the
diagram). The FF schedule shifts to the left because the supply
of foreign assets is reduced. The interest rate increases unam-
biguously and the real value of domestic assets declines implying
that the exchange rate depreciates.

The analysis implies that 'managed floating' does not re-
quire intervention in the spot exchange market. In the special
case when the interest rate parity obtains the spot exchange rate
can be managed, together with the domestic interest rate, by
simultaneous open market operations and forward market inter-
tervention. If the interest rate parity does not obtain, forward
market intervention alone is not sufficient but has to be sup-
plemented by government borrowing and lending.

The effect of an increase in the foreign interest rate or
in the expected rate of depreciation is to shift the FF schedule
to the left and thus to cause an increase in the domestic in-
terest rate and a depreciation in the exchange rate. Unless the
velocity of circulation is constant, the domestic interest rate
will not increase by as much as the foreign interest rate. An
Diagram III

The Effects of Forward Market Intervention

real value of domestic assets
increase in domestic income shifts the MM schedule to the right causing an increase in the interest rate and an appreciation in the exchange rate.

b. The Effects of Monetary Policy

Under Pegged Exchange Rates

The model developed above can be easily adapted to the analysis of monetary policy under fixed exchange rates. The money market equilibrium condition corresponding to equation (8) is now:

10. \( m(r)Y = \frac{B_{cb}}{P} + F^{cb} \)

where \( B^{cb} \) is controlled by open market operations and \( P \), the exchange rate is a fixed parameter.

The other equilibrium is that the total demand for domestic assets equals the supply of domestic assets:

11. \( d(r - r^* - \pi) A = \frac{p^g}{P} + F^{cb} \) (cf. equation 4)

The real value of financial wealth is now exogeneous because its valuation does not change.

The simultaneous determination of the rate of interest and the Central Bank's stock of reserves is illustrated in Diagram IV. The MM schedule is defined by equation (10) and the FF schedule by equation (11). A domestic open market purchase will shift the MM schedule to the left causing a decline in the domestic interest rate and in the Central Bank's stock of foreign assets. The magnitude of the offsetting decline in foreign assets depends on the slope of the FF schedule. Empirical evidence provided in Kouri and Porter (1974) and Kouri (1975) for a number of industrial countries suggests that the FF schedule was quite flat during the period of fixed exchange rates.
Diagram IV

The Interest Rate and the Stock of Reserves

Under a Pegged Exchange Rate Regime

stock of Central Bank's foreign assets
c. The Balance of Payments in the Short Run

We have analyzed the determination of the exchange rate without any reference to the balance of payments. The traditional theory of the foreign exchange market incorrectly views the exchange rate as a price that equilibrates the flow demands for and supplies of foreign exchange. As will be demonstrated in this paper, the link between the current account and the exchange rate is that the exchange rate determines, ceteris paribus, the current account and the current account determines the inflow or outflow of capital thereby affecting the rate of change of the exchange rate.

The trade account is equal to the difference between domestic output and total absorption, and the current account is equal to the sum of the trade account and the interest service account. We assume that the domestic output is fixed by the constant labour force which is fully employed. Private absorption equals private consumption since there is no accumulation of real capital. We assume that consumption (C) is a function of real disposable labour income and the real stock of financial wealth in accordance with Modigliani's life cycle model of consumption:

\[ C = C(Y - T, A) \]

where \( T \) is the real value of taxes. The current account is therefore:

\[ B = Y - C(Y - T, A) - G + r^*(F + F^{cb}) \]

To complete the model we need to specify the behaviour of the government. Government expenditure consists of the purchase of goods and services (G) and the service of public debt. Since
proportion $\phi$ of public debt is held by the Central Bank, the interest service is $r(1 - \phi)D$. Government revenue consists of tax revenue ($T$) and interest income on international reserves ($r* F^{cb}$). The difference between expenditure and receipts is the budget deficit which is financed by new debt issue:

14. $D = G \cdot P + r(1 - \phi)D - T \cdot P - r* F^{cb}$

The government can choose independently any two of the following three variables: real expenditure ($G$), net tax revenue ($T$), and the nominal deficit. We assume that the government fixes the rate of growth of public debt ($\dot{D} = \mu D$), and the level of real expenditure net of interest earnings on foreign exchange reserves. We assume for convenience that the government spends all foreign interest earnings. Net tax revenue is adjusted to maintain budget balance. The endogeneous tax revenue function is therefore:

15. $T = G' - [\mu - r(1- \phi)] \overline{D}$

where $G' = G - r* F$.

Substituting this in equation (13) we obtain the current account or capital flow equation:

16. $\dot{F} = B = Y - C(Y - G' + [\mu - r(1- \phi)] \overline{D}, F + \overline{D}) - G' + \mu* F$

Equation (16) together with equations (8) and (9), constitute the temporary equilibrium model. The equilibrium is temporary because the stock of foreign assets, the exchange rate and exchange rate expectations are changing.

Whilst the effects of various shifts on the interest rate and exchange rate can be established without ambiguity, this is not the case for the effects on the current account. Consider
first the effect of an increase in the stock of foreign assets (F) on the capital flow account. At a constant interest rate, and hence constant $\bar{D}$, the effect on the capital flow account is $(-C_A + r^*)dF$. If the propensity to consume out of wealth is less than the foreign rate of interest an increase in the stock of foreign assets will increase the current account surplus; which implies that the balance of payments adjustment process is globally unstable, even with static expectations, at a constant interest rate. We shall state this as:

*Condition I:* $C_A - r^* > 0$

Condition I does not rule out instability because of the effects of changes in F on the rate of interest and the real value of domestic assets. Since a decrease in the rate of interest at a constant real value of domestic assets reduces the current account surplus, thus reinforcing the stabilizing effects, the problem lies with the effect of changes in the stock of domestic assets on the current account. At a constant interest rate the effect of changes in $\bar{D}$ on the current account is

$$(-C_A + C_Y [(1-\phi)(r - \mu)] d\bar{D}.$$  

If the expression in the brackets is negative, the wealth effect of changes in $\bar{D}$ dominates the effect on consumption through induced changes in disposable income. We state this as:

*Condition II:* $C_A - C_Y[(1-\phi)r - \mu] > 0$

If both condition (I) and (II) hold the effects of all disturbances on the current account can be established without ambi-
guity. All the relevant multipliers are collected in Table I. The signs for the effects on the current account assume that conditions (I) and (II) hold.

It is important to distinguish between two types of shifts: (i) shifts that affect either asset demands or current supplies (stock shifts), and (ii) shifts that only affect absorption (flow shifts). In the context of the model the latter shift -- exemplified by an increase in government expenditure, whether financed by taxes or an increase in the growth rate of public debt -- will have no effect on the current exchange rate with exogeneous expectations. In time such shifts will, of course, have an effect because the current account deficit or surplus will change the supply of foreign assets. If speculators have foresight these future changes in the exchange rate will be immediately reflected on the current exchange rate, as will be demonstrated below.

Another point that deserves emphasis is the distinction between the effects of various disturbances on stock demands for assets and their effect on the capital flow account. The literature on flexible exchange rates invariably makes the mistake of reasoning, for example, that an increase in the domestic interest rate brought about by domestic monetary policy generates an inflow of capital, thereby appreciating the exchange rate and forcing the current account into a deficit. The deficit in the current account is a major channel through which monetary policy affects aggregate demand in Keynesian models of flexible exchange
<table>
<thead>
<tr>
<th>Endogeneous Variables</th>
<th>dY</th>
<th>dG</th>
<th>dμ</th>
<th>Exogeneous Variables</th>
<th>dφ</th>
<th>dπ</th>
<th>dr*</th>
<th>dF</th>
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<tbody>
<tr>
<td>dr</td>
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<td></td>
<td>$\frac{mf}{A}$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{\phi D}{A}$</td>
<td>$-\frac{\phi r}{A}$</td>
<td>$-\frac{\phi r}{A}$</td>
<td>$-\frac{(1-f)}{A}$</td>
<td></td>
</tr>
<tr>
<td>dD</td>
<td>$\frac{-mf_r}{A}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{frD}{A}$</td>
<td>$-\frac{mrfr}{A}$</td>
<td>$-\frac{mrfr}{A}$</td>
<td>$-\frac{m_r(1-f)}{A}$</td>
<td></td>
</tr>
<tr>
<td>dF</td>
<td>$(1-Cy) + \frac{Amf}{A}$</td>
<td>$-1+C_y$</td>
<td>$-C_yD$</td>
<td>$-C_yrD - \frac{A\phi D}{A}$</td>
<td>$-\frac{A\phi r}{A}$</td>
<td>$-\frac{A\phi r}{A}$</td>
<td>$(C_A - r*) - \frac{A\phi (1-f)}{A}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-B \frac{mr}{A}$</td>
<td>$+ Bf \frac{D}{rA}$</td>
<td>$-Bmfr \frac{r}{A}$</td>
<td>$-Bmfr \frac{r}{A}$</td>
<td>$-Bm_r(1-f)$</td>
<td></td>
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</table>

Note: $\Delta = -fm_r - \phi r > 0$, $A = Cy(1-\phi)D > 0$, $B = Cy(\mu - (1-\phi)\mu) + C_A \geq 0$
rates. The error in this analysis is that it is not the capital flow account that determines the current account but exactly the reverse. The correct reasoning is: monetary tightening increases the interest rate and simultaneously appreciates the exchange rate (with given expectations) to a point where financial markets are again in equilibrium. The appreciation of the exchange rate increases real wealth and thereby, via an increase in absorption (if Condition II holds), causes the current account to move to a deficit. This reversal of the chain of causation implies that it is incorrect to argue that monetary policy affects aggregate demand fast under flexible exchange rates because of its immediate effect on the current account via capital movements. The effect on the current account is immediate if the effect on aggregate demand is immediate. It is true that monetary policy has an immediate effect on the exchange rate and thereby on commodity prices.

Finally, once the effect of policy changes on exchange rate expectations is allowed for, it is quite possible, for example, that a tight monetary policy will cause the current account to move to a surplus rather than a deficit. This occurs when the long run wealth effect of the higher interest rate dominates the substitution effect, and when speculators have foresight (see Section d. The Reduced Form Equations

It is convenient to summarize the results of this section
by writing the momentary equilibrium values of the endogeneous variables as functions of the exogeneous variables:

17. \( r = r(\phi, r^* + \pi, Y, F) \) 
   \((-) (+) (+)(-)

18. \( D = h(\phi, r^* + \pi, Y, F) \) 
   \((-) (-) (+)(-)

19. \( F = k(\phi, r^* + \pi, Y, G, \mu, F) \) 
   \((+) (+) (+)(-)(-)(-)

The signs of the partials are established in Table I, those for the capital flow equation assume that Conditions (I) and (II) hold.

III. The Stationary State

In the short run the stock of foreign assets is given, only its domestic valuation can change. In the long run, it adjusts through deficits and surpluses in the current account. In this section we examine the determinants of the long run asset positions and the long run exchange rate path. Section V analyzes the process of adjustment from the short run momentary equilibrium to the long run stationary state.

The stationary state model consists of the following three equations:

20. \( \frac{M^d}{P} = m(r)Y = \frac{M}{P} = \bar{D} \) money market equilibrium (cf. equation 8)

21. \( F^d = f(r - r^* - \pi)(F + \bar{D}) = F \) equilibrium in the market for foreign assets (cf. equation 9)
22. \[ Y - C(Y - G + (\pi - (1-\phi)r)D) = \text{balance of payments equilibrium} \]
\[ F + D - G = r^*F \]  
(cf. equation 16)

These equations differ from the momentary equilibrium equations only in that the rate of growth of public debt (\(\mu\)) and the expected rate of inflation (\(\pi\)) are now equal, and the rate of change in the stock of foreign assets is set equal to zero. Assuming again that the nominal supply of money is a constant proportion (\(\phi\)) of the nominal stock of public debt, we can solve the money market equilibrium condition for the rate of interest, and substitute that in equations (21) and (22) to get two equilibrium conditions in terms of the stocks of domestic and foreign assets:

23. \[ \bar{D} = h(\phi, r^* + \pi, Y, F) \]
\[ (-) (-) (+) (+) \]

24. \[ \bar{D} = f(\phi, r^*, \pi, G, Y, F) \]
\[ (-) (-) (-) (-) (+) (-) \]

Equation (20) defines the locus of \(\bar{D}\) and \(F\) that imply portfolio equilibrium. It is the same as before and it is illustrated by the MM schedule in diagram V. Equation (21) defines the locus of \(\bar{D}\) and \(F\) that imply balance of payments equilibrium. It is downward sloping if Conditions I and II hold -- an increase in the stock of foreign assets, \textit{ceteris paribus}, increases absorption whence the real value of domestic assets must decline in order for the balance of payments to remain in equilibrium. The signs of the partials follow from Conditions (I) and (II). We assume that equations (23) and (24) have a unique solution.

The intersection of the MM and the FF schedules (see diagram
V) determines the stationary state values of the stock of foreign assets and of the real value of domestic assets. The long run exchange rate path is uniquely determined by the long run stock of domestic assets:

25. $\ln P(t) = \ln D_0 + \pi t - \ln D$

Inspection of equations (20) and (22) reveals that the long run equilibrium values of the stock of foreign assets and the real value of domestic assets do not depend on the (constant) stock of foreign assets held by the Central Bank. This means that a purchase of foreign assets by the Central Bank in the amount $X$ with domestic assets, and with no change in the composition of domestic assets, will be followed by a period of current account surpluses adding up to exactly $X$ units of foreign assets. The only effect of the policy is to increase the exchange rate in proportion.

The only other shift the effect of which can be established without ambiguity is a tax financed increase (or decrease) in government expenditure, since it only shifts the FF schedule (to $F'F'$ in diagram V) causing both the stock of foreign assets and the real value of domestic assets to decline.

For all other disturbances, the diagram is not a very useful tool for comparative static analysis because the disturbances shift both schedules in such a way that the end result is ambiguous. To get more structure into the model, we first examine the effects of various disturbances on the stationary state when the Central Bank pegs the nominal interest rate by open market opera-
Diagram V

The Stationary State Equilibrium

stock of foreign assets
tions. In that case, we may solve the model for $\bar{D}$ and $F$ in the form:

26. $\bar{D} = d^*(r, r^*, \pi, Y, G)$
   \[(+)(?)(-)(+)(-)

27. $F = f^*(r, r^*, \pi, Y, G)$
   \[(?)(+)(?)(+)(-)

These reduced form equations can be interpreted as the long run asset demand functions. The problem of indeterminacy of the signs of the long run effects arises from the fact that domestic and foreign assets need not be substitutes in the long run. The reason for this is that an increase in the foreign interest rate, for example, will have an effect on the long run stock of wealth as well as on the division of that wealth between domestic and foreign assets. If the elasticity of substitution is very low, the wealth effect will dominate and an increase in the foreign interest rate will increase the long run demand for domestic assets. If the elasticity of substitution between domestic and foreign assets is sufficiently high, domestic and foreign assets become substitutes and the long run effects of all disturbances can be established without ambiguity. In order not to get involved in an unproductive taxonomic exercise, we shall assume that this is the case:

*Condition III:*

\[d^*_{r^*} < 0, f^*_r < 0, f^*_\pi > 0\]

Note that these conditions are not implied by the conditions for stability under static expectations.

Assuming that Condition III as well as Conditions I and II
hold, we may write the long run equilibrium interest rate, and the stocks of domestic and foreign assets as functions of the exogeneous variables:

28. \[ r = l^*(\phi, r^*, \pi, Y, G) \quad (\text{cf. equation 17}) \]

29. \[ D = h^*(\phi, r^*, \pi, Y, G) \quad (\text{cf. equation 18}) \]

30. \[ F = k^*(\phi, r^*, \pi, Y, G) \quad (\text{cf. equation 19}) \]

The only ambiguity is in the effect of an increase in real income on the domestic interest rate: there will be an increase both in the demand for real balances and in the supply of real balances (see equation 8). Depending on the relative strengths of these two effects, the interest rate will either decrease or increase.

Note that the 'flow variables' such as government expenditure enter the long run reduced form equations whereas they do not enter the short run reduced form equations.

IV. Exchange Rate Expectations and Dynamic Stability

In this section we analyze the dynamic stability of the balance of payments adjustment process under alternative assumptions about the formation of expectations.

a. Static Expectations

If people expect a constant rate of depreciation or appreciation (equal to the rate of growth in public debt) Conditions I and II guarantee the global stability of the stationary state
since equation (21) becomes an ordinary differential equation in $F$ with $\frac{\partial F}{\partial F}$ everywhere negative. The economy is always on the asset market equilibrium schedule (MM in diagram V). In diagram V the arrows indicate the adjustment path in response to an increase in government expenditure.

Whilst the static expectations path is stable, it is unlikely that the economy will stay on it since the path implies that speculators persistently ignore the deceleration or acceleration in the rate of depreciation.

b. Myopic Perfect Foresight

The strong stability result that obtains in the case of static expectations indicates that the question of dynamic stability hinges on the nature of speculative expectations. We show in this section that myopic perfect foresight makes the exchange rate indeterminate: from any initial exchange rate there is an exchange rate and foreign asset path along which expectations are continuously realized. No speculator is either losing or making money. This problem has been raised by Black (1974) in a somewhat different context. There is one path along which the economy converges to the stationary state with expectations continuously fulfilled provided that it gets on that trajectory in the first place.

The dynamics of the adjustment process with perfect foresight is defined by:

31. $\dot{F} = k(r^* + \dot{X} - m, F)$ (cf. equation 21)
32. \( X = h(r^* + \dot{X} - m, F) \) (cf. equation 20)

where \( X = \ln D \) and the rate of change in the exchange rate is \( \dot{X} - m \). Solving this system of two equations for \( \dot{X} \) and \( F \) as functions of \( X \) and \( F \) we obtain:

33. \( \dot{F} = a(X, F) \)
34. \( \dot{X} = b(X, F) \)

where we have suppressed the other variables for expositional convenience.

A sufficient condition for the stationary state \((X^*, F^*)\) to be a saddlepoint locally is that the eigen values of the Jacobian are of opposite sign. That can be easily established to be the case.

The dynamic behaviour of the real value of domestic assets and of the stock of foreign assets with perfect foresight can be analyzed with reference to phase diagram VI. The BB schedule is defined by \( b(X, F) = 0 \). The stock of real balances is constant on this curve, increasing to the left of it and decreasing to the right of it. The AA schedule is defined by \( a(X, F) = 0 \), and it implies a zero current account. The stock of foreign assets is increasing to the left and decreasing to the right of this curve. The fact that the FF curve is downward sloping is implied by Conditions (I) and (II), and the assumption of a unique stationary state. The FF schedule cuts the X-axis at point \( F^* \), where the stock of foreign assets equals the stationary state stock of wealth.

The heavy arrow in diagram VII shows the trajectory along which the economy converges to the stationary state.
If the initial stock of reserves is \( F_0 \) the correct initial value for the stock of domestic assets is \( D_0 \). If the exchange rate is initially undervalued so that the real value of domestic assets is too low (\( D'_o \)) the exchange rate will, after a while, start to depreciate without limit along the \( T'T' \) trajectory. The stock of foreign assets will converge asymptotically to \( F = A^* \) where \( A^* \) is the stationary state stock of real wealth. The stock of foreign assets does not increase without limit because consumption is an increasing function of wealth. All that happens is that people shift from domestic assets into foreign assets in their portfolios. If the exchange rate is initially overvalued so that the real value of domestic assets is too high (\( D''_o \)) the exchange rate will begin to appreciate without limit. Both the stock of foreign and domestic assets will increase at first but after a while the substitution effect induced by the appreciation begins to dominate and the stock of foreign assets will go to zero in a finite time.

It is clear from this discussion that the assumption of myopic perfect foresight is not alone sufficient to determine the exchange rate path. It seems reasonable to assume that hyperdeflation must come to an end in a finite time when the private sector runs out of foreign assets (point \( Q \) in diagram VI). At that point, the speculative boom in the foreign exchange market will suddenly collapse and the speculators will incur a capital loss. If they have long run foresight, they will anticipate the eventual outcome and prevent the hyperdeflation from ever getting started. It is less obvious how hyperinflation can
Diagram VI

Dynamic Stability With Perfect Foresight

real value of domestic assets

stock of foreign assets
can be ruled out. One possible line of argument is that there is always a minimum below which the stock of real money balances cannot go -- for instance, because taxes must be paid in domestic currency. If this is the case, speculators with long run foresight will also prevent hyperinflation from developing by the force of speculative expectations alone.

This question remains open. We shall assume in this paper that speculators have long run foresight and rule out explosive price paths. This assumption -- which is made in the recent literature on rational expectations (cf. Sargent and Wallace (1973)) -- enables us to endogenize exchange rate expectations and link the short run momentary equilibrium with the long run stationary state.

c. Adaptive Expectations

With adaptive expectations, the dynamic evolution of the economy is defined by equations (20), (21), and the expectations equation:

35. \( \pi^* = \beta(-\dot{X} + m - \pi) \)

where \( \dot{X} \) and \( m \) are the same as before. If Conditions (I) and (II) a sufficient condition for the local stability of the stationary state is:

36. \( \beta\varepsilon_\pi < 1 \)

where \( \varepsilon_\pi = -\partial \ln D / \partial \pi \) is the inflation elasticity of the reduced form demand for domestic assets. If this condition holds, the convergence to equilibrium is nonoscillating.
V. The Adjustment Path

In this section we investigate the dynamic response of the exchange rate and the balance of payments to monetary and fiscal policy. We shall consider five types of shifts and in each case compare the response pattern under the three alternative mechanisms of expectations formation: static, adaptive and long run perfect foresight. The five shifts that we consider are:

(a) an once-and-for-all purchase of foreign assets by the Central Bank with no change in the composition of domestic assets between money and bonds.

(b) an increase in the proportion of money in total debt.

(c) a tax financed, permanent increase in government expenditure.

(d) an increase in the foreign rate of interest.

(e) an increase in the rate of growth of the stock of public debt.

The long run and the short run effects with exogeneous expectations of these shifts have already been established in Sections II and III. The effects of various other shifts can be inferred from the response to these five shocks.

a. Once-And-For-All Purchase of Foreign Assets

The effects of this policy are demonstrated in diagram VII. The shift leaves the MM and FF schedules unchanged. The private sector's stock of foreign assets is suddenly reduced to $F_0'$. With static expectations short run equilibrium is reached at point $A_0'$. 

Diagram VII

The Dynamic Effects of Monetary Policy

[Diagram showing the relationship between real value of domestic assets and stock of foreign assets]
with a lower stock of domestic assets. Thereafter the stocks of foreign and domestic assets are increasing *pari passu* along the MM schedule. The current account is in surplus and the exchange rate is appreciating. Rational speculators foresee this possibility and cause the exchange rate to appreciate immediately from the static expectations point A'_0'. Thereafter the economy moves much in the same way as with static expectations. It is worthwhile to note that if speculators have foresight they will cushion the exchange rate against reversible shocks. With adaptive expectations, the adjustment path might look like trajectory T'T' in diagram VII. It is quite possible that the persistence of incorrect expectations will cause the exchange rate to overshoot its long run path and then depreciate back to the correct level. Such reversals are not possible either with static or rational expectations.

In all cases, the exchange rate initially *depreciates by more than in proportion* to the increase in the supply of domestic assets. At the same time the *domestic interest rate increases*. With rational expectation the increase is lessened by the expected appreciation of the exchange rate.

The response pattern of the exchange rate would be reflected in the term structure of forward prices -- the prices of short term contracts would increase by more, and the price of very long term contracts by the same proportion that the exchange rate will eventually depreciate. This means that if the economy is constantly and stochastically disturbed by reversible shocks of this kind the spot rate will be unstable and the long term forward prices quite stable.
b. An Increase in the Money-Debt Ratio

This shift will change the long run equilibrium position to A'. With static expectations, the exchange rate will depreciate sharply to force the real value of domestic assets down to B_0'. This forces the current account to a surplus. Gradually, equilibrium is restored through exchange rate appreciation and current account surpluses which increase the stock of foreign assets. Again rational speculators will cushion the impact effect -- the stock of domestic assets reduces only to B_0". A possible response pattern with adaptive expectations is illustrated by the B_0'TA' trajectory.  \(^8\)

c. A Tax-Financed, Permanent Increase in Government Expenditure

The response to an increase in government expenditure, financed by taxes is illustrated in diagram VIII. With static and adaptive expectations, there is no immediate effect on the exchange rate because this flow shift does not affect the asset market equilibrium. It is only gradually that the deficits in the current account -- and, in the case of adaptive expectations, the revision of expectations -- cause the exchange rate to depreciate. With foresight, speculators capitalize immediately on the future depreciation and cause an immediate devaluation of the rate. The devaluation is not, however, large enough to turn the current account into a surplus.
Diagram VIII

The Dynamic Effects of Tax-Financed Increase in Government Expenditure

real value of domestic assets

stock of foreign assets
d. An Increase in the Foreign Interest Rate

The effects of an increase in the foreign interest rate are qualitatively identical with those of an increase in the money-debt ratio and can be inferred from diagram VII and the discussion in Section (b).

e. An Increase in the Rate of Growth of Nominal Debt

In the long run, an increase in the rate of growth of public debt will increase the rate of inflation by the same amount, reduce the real value of domestic assets and increase the stock of foreign assets (if Condition III holds). The adjustment path is radically different between the cases of adaptive and rational expectations (assumption of static expectations obviously makes no sense in this case).

The differences are illustrated in diagram IX. The stationary state equilibrium position shifts from point A to point A'. If expectations are rational, the speculators force an immediate devaluation of the exchange rate, which reduces the real value of domestic assets to A'. This sharp decline in wealth outweighs the expansionary effect of higher rate of government transfer payments (the mechanism, we should recall, whereby the stock of public debt is increased) on absorption. Thereafter the stock of domestic assets and of foreign assets increase \textit{pari-passu} with the exchange rate appreciating and the current account in surplus.

With adaptive expectations, the response pattern is
Diagram IX

Dynamic Response to a Higher Rate of Monetary Expansion

real value of domestic assets

stock of foreign assets
radically different. Initially, the higher rate of monetary expansion has no effect on the exchange rate because it does not affect the asset market equilibrium. The current account moves to a deficit. The deficit depreciates the exchange rate and causes speculators to revise their expectations. It is only when the substitution effect begins to dominate and the stock of wealth has declined sufficiently that the current account moves to a surplus and the exchange rate begins to appreciate. Because the total effect of the shift is the same in both cases, the period of surpluses must last long enough for the past deficits to be made up.

The differences in the adjustment processes are further illustrated in Figure I. The higher rate of monetary growth is introduced at time $t_0$. With rational expectations, the exchange rate jumps to $P_0'$ and thereafter at first appreciates back to the trend and then increases steadily at the rate of growth of the stock of domestic debt. With adaptive expectations, there is no jump in the exchange rate. Instead, the rate of change accelerates; the rate goes above the trend and appreciates back to it. The same difference is reflected in the behaviour of the current account (see Figure I (b), which is self-explanatory.)

VI. Concluding Remarks

In this section we summarize the main principles of the approach to the monetary transmission mechanism under flexible exchange rates advanced in this paper, and compare it with the traditional analysis.
A. The Exchange Rate

B. The Current Account

Figure I

Dynamic Response to a Higher Rate of Monetary Expansion
(i) In the short run the exchange rate is one of the prices that adjusts to clear the international financial markets. The immediate effect of changes in monetary policy is to change the interest rate and the relative prices of existing assets -- such as the exchange rate. These changes have effects on aggregate demand, prices, output, and balance of payments through various channels: (a) by changing the real value of financial assets (this is the sole effect in our simplified model); (b) by changing the rate of interest, and the relative price of reproducible capital, and thereby the level of investment and possibly (c) by changing relative commodity prices and real wages (this is the only effect of exchange rate changes in the traditional models).

(ii) The link between monetary policy and the capital flow account goes through the effect of monetary policy on aggregate demand and output via the channels described above; and thereby (in line with the absorption approach) on the current account, which determines the capital account. This chain of reasoning is invariably and incorrectly reversed in the traditional analysis, which argues as follows: a tightening of monetary policy will increase interest rates. This causes an inflow of capital, which forces an equal deficit in the current account by appreciating the exchange rate. The change in the current account is the main channel through which tight monetary policy affects the level of income on the Keynesian models of an open economy under flexible exchange rates. This interpreta-

of the transmission process has given rise to the false presump-
tion that monetary policy affects aggregate demand fast because of its immediate effect on the capital account -- and hence the current account. The correct reasoning is, of course, that monetary policy has an immediate effect on the current account if it has an immediate effect on absorption or output. 3

(iii) The flow model of the foreign exchange market has also given rise to a number of other confusions. One of these is 'elasticity pessimism' which arises from the assumption that the 'balance of lending' is determined independently of the current account and the exchange rate. This problem disappears once it is recognized that the exchange rate is determined in the asset markets and that the current account determines the capital flow account. 10

Another confusion which disappears with the new view of exchange rate determination is that the 'stock shift' component of capital movements will cause large reversible variations in the exchange rate and the current account.

(iv) The view of the exchange rate as a relative price of monies suggests that in a world in which the underlying determinants -- monetary and real -- of the exchange rate are continuously and stochastically changing, there is no reason to expect the exchange rate to be stable. In fact, its behaviour is likely to resemble the behaviour of asset prices in other speculative markets, such as the stock market. The model developed in this paper suggests that the spot exchange rate will contain a transitory component reflecting the short run adjustment process. This transitory component has less and less effect on long term forward
prices which, if the theory is correct, should show much less variability.

(v) On the methodological side, this paper has shown that with long run foresight, the effects of various shifts on the exchange rate and the current account in the short run, as well as the nature of the adjustment process, can be inferred from the long run stationary state effects.

(vi) Finally, the model can be extended in a straightforward manner in a number of directions. The only point that needs to be made here is that with more than one country and with many commodities, the asset markets and the commodity interact in determining the exchange rate. Furthermore, the link between the current account and the exchange rate in a world model would be the redistribution of wealth implied by current account surpluses and deficits and the fact that asset sumption preferences are likely to differ between countries.
Footnotes for Chapter III

1 See also Sohmen (1969), especially Chapter V and a representative textbook treatment in Stern (1973), Chapter 10.

2 The seminal contribution on the portfolio balance approach is in McKinnon and Oates (1966). See also McKinnon (1969) and the recent papers of Branson (1974) and Myhrman (1975). Swoboda (1972) discusses the differences between the short run and the long run effects of policies under fixed exchange rates. He calls the short run equilibrium position a "quasi-equilibrium." This corresponds to what we call a momentary equilibrium. Useful references are also Dornbusch (1975), and Frenkel and Rodriguez (1975) which analyze the short run, and the long run effects and the adjustment process under fixed exchange rates. There is an analogue to the short run and long run models of open economies in closed economy models of fiscal policy. For a recent paper on this, see Blinder and Solow (1973). Brunner and Meltzer (1974) analyze the interaction between budget deficits and balance of payments deficits.

3 This result can be established from Table I by setting $\bar{d}d\phi$ equal to minus $dF$.

4 This result can also be established from Table I by setting $d\bar{F}$ equal to $(1-\phi)d\bar{D} - \bar{D}d\phi$.

5 Our analysis of the effects of forward market intervention differs from that in the traditional literature, exemplified by Tsiang's (1958) seminal contribution, in that the forward premium (which is equal to the interest rate differential) as well as the spot exchange rate is determined in the asset markets rather than in the 'flow' market for foreign exchange. The flow model of the foreign exchange market is particularly unsuitable for the analysis of external monetary policy, in that one does not usually think of monetary policy in terms of rates of change of Central Bank's open market portfolios.

6 The distinction between the short run and long run asset demand functions is the same as the distinction between the short run and long run demand for money functions emphasized by Archibald and Lipsey (1958).

7 This condition differs from the stability condition in Cagan's model of hyperinflation only in that $\varepsilon$ is a reduced form elasticity, rather than the elasticity of money demand with respect to inflation. On this point, see also Mussa (1973).

8 If condition III does not hold, the current account may instead go to a deficit in response to expansionary monetary policy. This is because the long run stock of foreign assets will be less, and this requires a period of current account deficits in the transition to new equilibrium.
Models that assume 'infinite capital mobility' get away from the stock flow problem of exchange rate determination by assuming, in Keynesian models, that the exchange rate does not effect asset demands. In that case, the money supply determines output (velocity being exogeneous) and the exchange rate equilibrates aggregate demand with aggregate supply. There is no difference between the short run and the long run in these models because the change in asset supplies implied by current account surpluses or deficits has no effects on anything.

A condition on elasticities is still required for the dynamic adjustment process to be stable in the long run. If price elasticities are very low it is possible that an increase in the stock of foreign assets will improve the current account, in which case the economy would never converge to a stationary state. Short run inelasticities of commodity demands are not a source of concern once the asset market nature of exchange rate determination is recognized.
Chapter IV
INTERNATIONAL INVESTMENT AND INTEREST RATE LINKAGES
UNDER FLEXIBLE EXCHANGE RATES

I. Introduction

This chapter investigates the interdependence of national financial markets under the regime of flexible exchange rates in the framework of a general equilibrium model of the international capital markets. It is assumed that all financial assets, except money, are traded in the international financial markets in which transactions costs and other impediments to trade are small enough to be ignored. Money is used only in the country in which it is issued. There is no international money in the narrow sense of a medium of exchange, nor are different monies substitutable in domestic transactions. It is implicitly assumed that international transactions are largely settled by credit amongst trading partners so that the net demand for transactions balances in international trade is insignificant. This assumption also assumes away the potentially important problem of Gresham's Law -- the problem of coexistence of more than one money. This problem is deep enough to warrant a separate investigation. Furthermore, we are forced to use a rather crude method of deriving the demand for money -- that approach would be much more difficult to apply with many monies. Investors are, however, free to borrow and lend in any currency of their choice -- the holdings of foreign assets thus reflect portfolio motives rather than transactions
demand for international reserves.

The model assumes that the exchange rates between currencies satisfy the purchasing power parity equation. The relative prices of commodities are assumed to be fixed. Underlying this is the assumption of full employment and flexible prices.

It is further assumed that there is one country -- the center country -- that has settled on a fully anticipated path of inflation. The real rate of interest in that country is assumed exogenous. International arbitrage ensures that the same real rate will prevail in all countries -- in this respect there is no autonomy despite flexibility of exchange rates. The interest rates on bonds denominated in different currencies may, however, differ because of differences in inflation rates. If inflation rates were deterministic, the nominal interest rates on bonds denominated in different currencies could differ only by the expected change in the exchange rate (with transactions costs there would be a neutral band of interest rates within which no arbitrage opportunities would arise). These interest rate parity equations no longer hold with uncertainty and risk aversion. It is shown in the paper that in addition to the expected change in the exchange rate, the interest rate of a periphery country differs from that of the center country by a variance term and a currency premium that is a compensation for the systematic inflation risk of the currency of that country. It is shown that this currency premium can be changed by monetary policy and by changes in the distribution of wealth between countries. Furthermore, if the rates of inflation are correlated with the real returns on equity the difference in
interest rates will also depend on the average real return on capital assets. These implications of the analysis demonstrate that changes in nominal interest rates between countries in excess of changes in exchange rate expectations are not in any way inconsistent with complete integration of the financial markets. It is also shown that unless the demand for money is completely interest-inelastic, interest rate differentials will not completely adjust to changes in the expected rate of inflation.

Forward markets in currencies are not introduced explicitly -- there is no need to do so since there are bond markets in every currency. As Kindleberger (1975, Appendix F to Chapter 17) has emphasized, the forward market is related to the spot market by arbitrage once the interest rates are given; except for transactions costs or other risks, the forward premium and the interest rate differential must be equal.² The results of the analysis can be directly applied to some questions discussed in connection with forward markets -- for instance, that the forward premium is not related in any simple way to the expected change in the exchange rate.

Another important implication of the analysis is that if there is no correlation between inflation rates and real equity returns, the control of nominal interest rates does not enable the periphery countries to have any effect on the required real returns on risky capital (the cost of capital) which only depend on the exogeneous real interest rate and on the real returns on other capital assets.
The plan of the paper is as follows. The next section develops the underlying model of portfolio selection and money demand by using the approach developed by Merton (1969, 1971, 1973) and applied to international capital markets by Solnik (1973, 1974). The only novelty there is the explicit introduction of money in a manner suggested by Sidrauski (1967) amongst others in a deterministic context. A reader who is not interested in techniques can go directly to Section III that derives the equilibrium interest rate relationships. Section IV analyzes the effects of monetary policy and other shifts in more detail in a two country version of the model. The concluding section discusses the limitations of the model and suggests directions for further research. Whilst no empirical testing is undertaken in this paper, there are a number of testable implications and the results do resolve some puzzles observed by other researchers.
II. The Model

The world economy consists of $n$ countries each with its own currency. Currency is used only in the country in which it is issued -- it is thus a nontraded asset. The implicit assumption is that international transactions are settled by credit amongst the trading partners -- a feature not much unlike actual practice. Each investor is, however, able to lend and borrow in any currency of his choice. In line with the standard assumptions of the models of capital market equilibrium I shall assume away transactions costs or other causes of differences in borrowing and lending rates; I shall also assume away the problem of default. These assumptions imply that the only attribute that distinguishes one bond from another is its currency denomination. An example that approximates this assumption might be the deposits and loans in the Eurocurrency markets. In addition to bonds investors may buy and sell equity in every country. For convenience but with no loss of generality it is assumed that there is only one equity in each country. Short sales in equity are allowed. Equity can be interpreted quite broadly in the analysis -- for instance, it can be thought of as holdings of gold or inventories of primary commodities.

There are altogether $3n$ assets that are traded in the financial markets and $3n-1$ relative prices that are determined by the conditions of asset market equilibrium, namely, $n$ interest rates, $n$ equity prices and $n-1$ exchange rates between currencies. It is assumed that the asset markets are always in equilibrium.

The only novel feature in the analysis of investor behaviour
is the introduction of money, although admittedly in a rather crude way. It is clear that it does not make much sense to talk about exchange rates in a model in which money does not appear, since the exchange rate is a relative price of one money with respect to another. It is assumed that investors derive utility from the stock of real balances they hold as well as from consumption. The approach is the same as that used in a deterministic context by Sidrauski (1967) and in a stochastic model by Dixit and Goldman (1970). The aim of each investor is to maximize the expected value of discounted utility over an infinite time horizon subject to the wealth constraint and stochastic returns to be specified below.

II-1. Price and Exchange Rate Dynamics

It is assumed that the center country, taken as the nth country, has a certain rate of inflation ($\pi_n$) fully anticipated by everybody. The reason why $\pi_n$ is assumed to be deterministic is to have one asset with a known real rate of return. This assumption simplified the analysis although it is not critical for the main results.

The assumption implies that the price level ($P_n$) in the n'th country behaves according to:

$$\frac{dP_n}{P_n} = \pi_n \, dt$$

The other countries can maintain an inflation rate different from $\pi_n$ by allowing their exchange rate vis'a vis the currency of the n'th country to change. Ignoring the problems that arise from changes in relative prices, in particular between tradable and non-
tradable goods, we assume that the price level in country i, \( P_i \), is determined by the purchasing power parity equation:

\[ P_i = E_{ni} \cdot P_n \]

The dynamics of the exchange rate is given by:

\[ \frac{dE_{ni}}{E_{ni}} = e_i \, dt + \sigma_i \, du_i, \quad i = 1, \ldots, n-1 \]

where \( du_i \) is a Wiener process or Brownian motion. The meaning of continuous stochastic processes like (3), known as Ito processes, in the context of portfolio theory is discussed in Merton (1971) and Fischer (1975). Briefly, (3) implies that over a short time interval, the proportionate change in the exchange rate is normal with mean \( e_i \, dt \) and variance \( \sigma_i^2 \, dt \). Also, (3) implies that the exchange rate follows a random walk and is log-normally distributed. Furthermore, the stochastic component of (3) is serially uncorrelated no matter how short the time interval.

Using Ito's Lemma we can obtain the dynamics of inflation in country i:

\[ \frac{dP_i}{P_i} = (\pi_n + e_i) \, dt + b_i \, du_i, \quad i = 1, \ldots, n. \]

The economic rationale of these equations is the following. Each country has settled on a certain rate of inflation, which varies stochastically around its mean. The market participants have learned the mean rate of inflation as well as its variance. They are right on an average, although they may be in error in any given period. These assumptions correspond to what McKinnon (1971) calls the Fisherian model. The exchange rate adjusts continuously to enable the countries to maintain divergent rates of inflation.
II-2. Asset Return Dynamics

The nominal return on bonds is known with certainty. It is assumed that market participants expect the nominal interest rate to remain constant -- otherwise we would have to deal with the difficult problem of term structure of interest rates. Let $B^i_j$ be the nominal value of bonds of country $j$ held by an investor in country $i$. The real value of this investment is $B^i_j$ multiplied by the exchange rate ($E_{ji}$) and divided by the price level in country $i$ ($P_i$). By the purchasing power parity equation, this is simply $B^i_j$ divided by the price level in country $j$ ($P_j$). Thus the real return on a bond denominated in currency $j$ is the same for all investors; it is the nominal return plus the expected change in the price of money (inverse of the price level) in country $j$. In addition, there is a stochastic component reflecting unanticipated inflation in country $j$. More formally, if $\bar{B}_j$ is the real value of bonds denominated in currency $j$ it changes stochastically according to:

$$\frac{dB^i_j}{B^i_j} = (R^i_j - \pi^i_j + \sigma^2_j) \, dt - \sigma_j \, du_j, \quad j = 1, \ldots, n$$

Since there is no unanticipated inflation in the $n'$th country, $\sigma_n$ is equal to zero.

Money differs from bonds only in that its nominal return is equal to zero. Therefore, the real value of money of country $j$ ($\bar{M}_j$) changes stochastically according to:

$$\frac{d\bar{M}_j}{\bar{M}_j} = (-\pi^j_j + \sigma^2_j) \, dt - \sigma_j \, du_j, \quad j = 1, \ldots, n$$
The similarity between (5) and (6) will prove useful below.

It is assumed that the return to equity accrues in the form of capital gains or losses. A detailed discussion of this approach is given in Merton (1973); it amounts to assuming that the firms distribute dividends by buying back their shares. The real price of equity, \( q_i \), \( i = 1, \ldots, n \), is assumed to change according to:

\[
\frac{dq_i}{q_i} = \alpha_i dt + s_i dz_i, \quad i = 1, \ldots, n
\]

II-3. The Budget Equation

At each moment an investor in country \( i \) is constrained by the wealth constraint:

\[
W^i = M^i + \frac{\sum B^i_j}{P^i_j} + \sum K^i_j q_j
\]

where \( W^i \) is the real stock of wealth and \( M^i \) is the nominal stock of money, and all other variables are as defined above. At each instant the investor chooses the composition of his portfolio and the rate of consumption. Once these decisions are made the stock of wealth changes stochastically according to:

\[
dW^i = \frac{M^i}{P^i_i} d\left(\frac{1}{P^i_i}\right) + \sum \frac{B^i_j}{P^i_j} d\left(\frac{1}{P^i_j}\right) + \sum B^i_j R^i_j dt + \sum K^i_j dq^i_j - C dt
\]

A detailed derivation of this equation is left out, since its interpretation is obvious. The first part on the right hand side represents income accruing to wealth; that income takes the form of capital gains and interest payments on bonds. The difference between income and consumption is saving which is exactly what the equation says. Substituting from above, we may write equation (10) in the more convenient form:
11.  \[ \frac{dW}{dt} = \left[ \frac{M_i}{P_i} \right] \left( -\pi_j + \sigma_i^2 - \gamma_n \right) + \Sigma \left[ \frac{B_j}{P_j} \right] (r_j - r_n) + \Sigma K_j (a_j - r_n) \\
+ \left[ \frac{M_i}{P_i} \right] \left[ r_n \right] dt - C dt - \left[ \frac{M_i}{P_i} \right] \sigma_i du_i - \Sigma \left[ \frac{B_j}{P_j} \right] \sigma_j du_j + \Sigma K_j s_j dz_j \]

where \[ r_j = R_j - \pi_j + \sigma_j^2, \quad (j = 1, \ldots, n-1) \]
and \[ r_n = R_n - \pi_n \]

II-4. The Investor's Choice Problem

It is assumed that each investor chooses that consumption and portfolio strategy that maximizes the expected value of utility from the program over an infinite time horizon. The instantaneous utility is assumed to be a strictly concave function of consumption and the stock of real balances. The choice problem for a typical investor in country \( i \) is thus:

12. \[ \text{Max} \int_t^\infty U(C^i, \frac{M_i}{P_i}, \frac{M_i}{P_i}) \left( \frac{M_i}{P_i} \right) e^{-\rho \tau} dt \]
subject to the budget constraint (9).

The problem is solved by the technique of stochastic dynamic programming. Define:

13. \[ J(W_j, t) \equiv \text{Max} \int_t^\infty U(C^i, M_i) \left( \frac{M_i}{P_i} \right) e^{-\rho \tau} dt \]
subject to the same constraint as above. The detailed derivation of the fundamental equation of optimality is left out. After some manipulations, it can be written in the form (the subscripts and superscripts are left out for clarity):

14. \[ 0 \equiv \text{Max} \left\{ U[C, \frac{M_i}{P_i}] e^{-\rho \tau} + J_t + J_w [ -mR_i + b'r + k'a + r_n ] - J_w C \right. \\
+ \left. \frac{1}{2} J_{ww} W^2 \left[ b'\Omega b + k'Sk + 2b'Tk \right] \right\} \]

where

\[ m = \text{proportion of wealth invested in money} \left[ \frac{M_i}{P_i} \right] \]

\[ b = \text{column vector whose i' th component is } b_i + m_i \quad \text{and other components } b_j, \text{ the proportion of wealth invested in } j' \text{th} \]
bond, \( j = 1, \ldots, n-1 \).

\( k \) = column vector whose typical element is \( k_j \); proportion of wealth invested in \( j \)'th equity, \( j = 1, \ldots, n \).

\( r \) = column vector of excess returns on bonds whose typical element is \( R_j - \pi_j + \sigma_j^2 - \rho_n + \pi_n \).

\( \alpha \) = column vector of excess returns on equity whose typical element is \( \alpha_j - \rho_n + \pi_n \).

\( \Omega = (n-1) \times (n-1) \) variance-covariance matrix of inflation rates.

\( S = n \times n \) variance-covariance matrix of equity returns.

\( \Gamma = (n-1) \times n \) variance-covariance matrix of inflation rates and equity returns.

This is now a standard optimization problem which is unconstrained because the safe asset has been eliminated by the wealth constraint.

The first order conditions are:

15.1 \[ U_C = J_w \]

15.2 \[ U_M + R_i J_w \]

15.3 \[ J_w W + J_{ww} W^2 [\Omega b + \Gamma k] = 0 \]

15.4 \[ J_w W \alpha + J_{ww} W^2 [S k + \Gamma' b] = 0 \]

These conditions are also sufficient because of the assumed strict concavity of \( U(C, \frac{M}{P}) \).

II-5. The Optimal Portfolio

From 15.1 to 15.2 we obtain immediately the result:

16. \[ U_C = \left( \frac{1}{R_i} \right) U_M \]

This equation simply says that the marginal rate of substitution between money services and consumption \( \frac{U_M}{U_C} \) should equal the relative price, namely, the rate of interest. Thus the demand for
money can be written as a function of only the desired consumption and the nominal rate of return on short term bonds. The intuitive reason for this result is that money and bonds have identical risk characteristics. The result does not depend on the assumption that there is no (unanticipated) inflation. In particular, if we assume that \( U(C, M) \) is a homogeneous function of \( C \) and \( M \), the demand for money function can be written in the familiar liquidity preference form:

\[
\frac{M_i}{P_i} = v(R_i) C
\]

Consider first the case in which equity returns are not correlated with the rates of inflation. In that case the equilibrium portfolio proportions are given by:

\[
\begin{align*}
b &= a\omega^{-1}r \\
k &= a\omega^{-1}a
\end{align*}
\]

where \( a = -J_W/WJ_{WW} \) as the Arrow-Pratt measure of relative risk aversion. The noticeable feature of these results is that the proportion of wealth invested in equity does not depend on exchange rate (inflation) risk. Similarly, the composition of the bond portfolio only depends on the real returns and risk attributes of bonds. Also, the usual separation theorem holds -- the composition of the portfolio of risky assets is the same for all investors who have homogeneous expectations. The other interesting feature of these results is that for investors of country \( i \), money and bonds can be aggregated into a single asset the demand for which depends on the same variables as that for any other asset. This means that domestic residents always "hedge" their holdings of money balances by borrowing an equal amount in local currency. Complete hedging is
possible because money and bonds have identical risk characteristics. This result would not obtain -- nor would equation (16) above -- if there were no short term asset with a riskless nominal return.

II-6. The Consumption Function and the Demand for Money Function

A complete solution to the choice problem is obtained by specifying the utility function and solving for the consumption and demand for money functions. Whilst this can be done in more general cases, I shall assume that the utility function is of the form:

19. \( U(C, M) = \alpha \log C + (1-\alpha) \log \left( \frac{M}{P} \right) \)

Using this and the first order conditions, one can rewrite (14) as a partial differential equation in \( J \) which can be easily solved. Using the first order condition (15.1) and equations (16) and (19) one obtains the simple and familiar result that consumption is a constant proportion of wealth:

20. \( C = cW \)

where

\[ c = \alpha \rho \]

The consumption propensity depends only on the rate of time preference and parameter \( \alpha \). The economic meaning of \( \alpha \) is the following. At any given instant total consumption is the sum of the consumption of commodities and the consumption of 'money services', which is assumed to be proportional to the stock of money (there is no harm in making that proportion equal one). Total consumption
expenditure is then \( C + \frac{R}{P} \). The proportion of the consumption of commodities is \( \frac{C}{C + \frac{R}{P}} = C \cdot \frac{1}{\alpha} C = \alpha \). Thus equation (20) also implies that the total consumption propensity is equal to the rate of time preference.

From (16) and (20) we get the demand for money function:

\[ \frac{M_d}{P_i} = m(R_i)W = \left( \frac{1}{R_i} \right) cW \]

The demand for money depends on permanent income or wealth rather than current income, the reason for this being that consumption depends on wealth rather than income.

III. The Equilibrium Yield Relationships Amongst Assets

III-1. Equilibrium Returns on Equity

Assuming that investors have identical expectations we can now derive the equilibrium relationship between expected returns on equity in different countries. The total demand for equity for country \( i \) is given by:

\[ K_i^d = (\sum h_i^h) \sum_{j=1}^{n} -1 (a_j - R_n - \pi_n) = K_i q_i \]

where \( K_i q_i \) is the market value in real terms of the existing supply of equity. Let \( V \) be the market value of all stocks and let \( V_i \) be defined by \( K_i q_i = v_i V \). Substituting this above we obtain:

\[ a_i - R_n - \pi_n = KA \sum_{j=1}^{n} S_{ij} = KAS_i M \]

where

\[ A = \frac{1}{(\sum h_i^h)} \]

\( S_{iM} \) is the covariance of the return on the \( i \)'th security with the return on a portfolio consisting of all stocks with the share of each security corresponding to its share of the total market value.
(the so-called market portfolio). Multiplying both sides of equation (23) by \( w_j \) and adding up we get:

24. \( \alpha_M - R_n - \pi_n = K \alpha M \)

where \( s_M^2 \) is the variance of the return on the market portfolio.

Combining these two equations we obtain:

25. \( \alpha_i - r_n = \frac{s_{iM}^2}{s_M^2} (\alpha_M - r_n) = \beta_i (\alpha_M - r_n) \)

where

\[ r_n = R_n - \pi_n \]

This is the standard security market line equation which states that investors are compensated in terms of expected return for bearing systematic (market) risk. \( (\alpha_M - r_n)/s_M^2 \) is often called the market price of risk and accordingly \( s_{iM}^2 \) is defined as the systematic risk of security \( i \). The important implication of this result from the viewpoint of the theory of flexible exchange rates is that as long as the purchasing power parity holds and there is no systematic relationship between inflation rates and equity returns the flexibility of the exchange rates in no way alters the equilibrium relationships between returns on risky capital. In this sense, there is no more monetary autonomy under flexible exchange rates than there is under fixed exchange rates under similar circumstances.

In particular, a small economy that is extensively integrated into the world commodity and capital markets (for instance a state in the United States) cannot insulate itself by simply having its own money. The relative prices of commodities continue to be determined in the world market. Similarly, the real
interest rate and the required rates of returns on risky capital are given exogenously.

If there is a systematic relationship between equity returns and the rates of inflation, the effect of changes in nominal interest rates on equity returns can be of either sign depending on the signs and magnitudes of the covariance terms. For example, it is quite possible that domestic bonds and equity are complements rather than substitutes in the portfolio of investors.⁹

III-2. Equilibrium Relationships Between Interest Rates

Flexibility of exchange rates permits, however, differences in nominal interest rates between various countries. A common hypothesis about interest rate differentials is that the interest rate differential can be attributed to the expected percentage change in the exchange rate. There is one case in the model developed above in which the expectations hypothesis is (almost) correct. This is when there is no net outside supply of bonds in any currency. To demonstrate this, consider the total demand for bonds denominated in currency i:

\[ \frac{B^d_i}{\bar{P}_i} = (\Sigma a^h_w^h) \sum_{ij} r_{ij} - \frac{M^d_i}{\bar{P}_i} = \frac{B^s_i}{\bar{P}_i}, \quad i = 1, \ldots, n-1 \]

The right hand side is the net supply of bonds to the private sector. It is equal to the outstanding stock of government debt \((B^G_i)\) less the amount held by the Central Bank. Assuming that the Central Bank changes the money supply by open market operations in bonds that is equal to the nominal supply of money. We can therefore rewrite (26) in the form:
An immediate implication of this is that if the supply of government debt is zero, or if the government chooses the composition of its debt according to the same criterion as the private sector, the expectations hypothesis is valid. In other words:

$$28. R_i - R_n = \pi_i - \pi_n - \sigma_i^2 = e_i - \sigma_i^2, \quad i = 1, \ldots, n-1$$

The difference in interest rates between countries $i$ and $n$ is equal to the expected change in the exchange rate corrected for the variance term. The reason why the variance term appears in equation (28) is that the expected change in the purchasing power of money (inverse of the price level) is not the same as minus the rate of inflation. As an illustration, consider the following example. The price level today is equal to one, and it is 1.6 with probability 0.5 and 0.4 with probability 0.5 tomorrow. The expected rate of inflation is obviously zero (the expected price level being $1.6 \times 0.5 + 0.4 \times 0.5 = 1$). However, the expected real return on money is the expected value of $1/P_2 - 1$ where $P_2$ is the price level in the second period. That is equal to $1/0.64 - 1 = 53\%$. There is no expected inflation but yet money has a large expected positive return. It turns out that in the continuous time case the difference between the mean rate of inflation and the mean rate of change in the purchasing power of money is equal to minus the variance of inflation.

In general, however, the expectations hypothesis does not hold. If there exist net supplies of bonds exogenously given to the private sector, interest rates on assets will contain a
premium (or discount) in compensation for the systematic risk that the holders of these bonds must bear. This currency premium can be interpreted as a 'habitat' effect on interest rates in that there are some market participants who have a 'strong preference' for particular currencies, e.g., governments who issue debt in their own currency. The exogeneous supply could also be interpreted as the net supply (demand) of bonds by investors who only hold a few assets and do not operate in the international capital market in the same way as the rational investor described in the model.

In the same way as in the previous section, we can derive the required compensation in terms of expected return for investors to hold the existing supply of bonds denominated in various currencies:

\[
29. \quad r_i - r_n = b_i (r_M - r_n) = \frac{\sigma_{iM}}{\sigma_M^2} (r_M - r_n), \quad i = 1, \ldots, n-1
\]

where \( r_i \) is the expected real return on bonds denominated in the \( i \)'th currency, \( r_M \) is the weighted average real return on all bonds (each bond yield weighted by the market share; in the standard terminology \( r_M \) is the expected real return on a market portfolio of bonds). As before, \( \sigma_{iM} \) is the covariance between the expected return on the \( i \)'th bond and that on the market portfolio of bonds, and \( \sigma_M^2 \) is the variance of the return on the market portfolio.

The difference in nominal interest rates is accordingly:

\[
30. \quad R_i - R_n = e_i - \sigma_i^2 + b_i (r_M - r_n), \quad i = 1, \ldots, n-1
\]

In addition to the expectations term, the interest rate differential contains a premium for the systematic inflation risk. This premium is neglected in the traditional interest rate parity
calculations. Since the forward premium on the i'th currency is equal to the difference in nominal interest rates in the absence of transactions costs equation (30) provides yet another demonstration that the forward premium is not an unbiased predictor of the change in the exchange rate. Equation (30) also implies the possibility that a country with a depreciating exchange rate will have a lower interest rate than that in the center country -- this happens when the inflation rate in country i is negatively correlated with the average inflation rate in the world (which implies that b_i is negative).

IV. Monetary Policy, Expected Inflation and the Rate of Interest

This section analyses the effects of monetary policy and exchange rate expectations in more detail. Since equation (34) obtains in general the effect of monetary policy is to change the 'currency premium' via changes in the b coefficient and the average return on the market portfolio of bonds. For simplicity, it is assumed that there are only two countries -- a center country (denoted by an asterisk) and a 'periphery' country. The real rate of interest in the center country is exogeneously given as before.

There are three assets in the periphery: money, bonds, and capital, the latter two of which are internationally traded. The first equilibrium condition is that the demand for and the supply of money are equal:

31. \( \frac{M^d}{P} = m(R)W = \frac{M}{P} \)

The second is that the net demand for government debt equals the
outstanding stock:

\[ \frac{B^d}{P} = d(R - R^* - \pi + \sigma^2, \sigma^2) (W + W^*) = \frac{B^G}{P} \]

where

\[ d(R - R^* - \pi + \sigma^2, \sigma^2) = \frac{1}{\sigma^2} (R - R^* - \pi + \sigma^2) \]

It is assumed for simplicity that the rate of inflation in the center country is equal to zero. The demand functions can be aggregated since in the framework of the model the portfolio proportions are the same for investors in the two countries. This implies that the distribution of wealth between the two countries does not depend on current asset values although it may change over time if the saving propensities differ between the two countries. The underlying assumptions are that investors have identical expectations and the same degree of risk aversion, namely, one. In general, the distribution of wealth will obviously be endogeneous, and a complete macroeconomic model is required to establish the determination of the aggregate stock of wealth and its distribution. The following analysis would still apply except that the distribution of wealth must be interpreted as an endogeneous variable. Let then \( w \) be the proportion of world wealth owned by investors of the periphery:

\[ W = w(W + W^*) \]

We can combine equations (31) and (33) to yield:

\[ m(R) = w \phi d(R - R^* - \pi + \sigma^2, \sigma^2) \]

where \( \phi \) is the ratio of the money stock to the total supply of government debt. It is the variable that the Central Bank of the periphery country controls by open market operations. The left hand side gives the desired ratio of the money stock to wealth
as a function of the rate of interest. The right hand side gives the ratio of the money stock to wealth that is consistent with equilibrium in the bond market. The two loci are depicted in diagram I (MM and BB schedules respectively).

Consider now the effect of an increase in the supply of money relative to the supply of government debt. The BB schedule will shift to the right while the MM schedule will remain unchanged. Assuming that there is no change in the expected rate of inflation, the nominal interest rate will decline from \( R \) to \( R' \) and the proportion of wealth in the form of money will increase from \( m \) to \( m' \).

Of particular interest is the effect of an increase in the expected rate of inflation. It is often implied in the literature that this should increase the difference in nominal interest rates by the same amount. It is clear from the diagram that unless the demand for money is completely interest inelastic, this is not the case. The shift of the BB schedule to the left results in a higher interest rate and a lower proportion of wealth held in the form of money. The magnitude of the decline in the real return on domestic bonds depends on the interest elasticity of the demand for money. This failure of the Fisher parity is well known in the domestic monetary theory literature (see, for example, Mundell [1971], Chapter 2).

An increase in the variance of inflation leaves the MM schedule unchanged but shifts the BB schedule upwards. Therefore, both the nominal and the real interest rate on the bonds of the periphery country will rise.
Diagram I

The Determination of the Rate of Interest

Ratio of Money to Wealth
To complete the analysis we need to consider the determination of the price level in the periphery country. With the nominal interest rate given by equation (33) the price level has to adjust to equilibrate the demand for and the supply of money in equation (30). The only open question is what determines the world stock of wealth. In the special case of identical investors we can answer this question without developing the complete macroeconomic model. Consider the condition of equilibrium in the market for consumer goods:

\[ C + C^* = cW + c^*W^* = [c_w + c^* (1 - w)] (W + W^*) = \bar{c}(W + W^*) = C^S \]

where \( \bar{c} \) is the average propensity to consume in the world, and \( C^S \) is the total supply of consumer goods. Assuming that both countries produce two commodities -- the internationally traded consumer good and a non-traded capital good -- the world supply of consumer goods is a function of the relative prices of capital goods as well as the stocks of capital in the two countries. But we have already shown before that the demand for capital does not depend on the interest rate in the periphery country. This means that the prices of capital goods and hence the supply of consumer goods are exogeneous to the small country at any given instant. Therefore, by equation (35) the stock of wealth is also exogeneously given.

We can, therefore, easily establish the effects of the three shifts considered above on the price level (exchange rate) by considering equation (31). An open market operation will increase the price level but not in proportion because the nominal interest
rate and hence the velocity of circulation will change. This case of non-neutrality is, of course, well known in the literature (see, for example, Tobin [1972], page 861). An increase in the expected rate of inflation will raise the nominal interest rate and hence the velocity of circulation resulting in a jump in the price level. An increase in the variance of inflation will have the same effect since it increases the nominal rate of interest.

All of these disturbances leave the relative price of capital and hence the rate of investment unchanged. Furthermore, the trade account will also remain unchanged since it is the difference between domestic consumption and the domestic supply of consumer goods, both of which are unaffected by the shifts considered above.

In summary, a country that is integrated into the world commodity and capital markets and is not large enough to affect relative prices in these markets (in particular the real rate of interest) will not increase its autonomy by allowing the exchange rate to fluctuate except in terms of nominal magnitudes. Regarding these we have shown that various neutrality postulates do not in general hold: open market operations do not change the exchange rate and hence the price level in proportion, an increase in expected inflation will lower the real interest rate on domestic bonds. In addition we have, in this section and in the previous sections, identified the variances and covariances of inflation rates, the distribution of wealth and the composition of government debt as important determinants of differences in interest rates.
It is also of interest to note that the above analysis can be applied almost without alternation to the problem of indexation if the real interest rate in the center country is interpreted as the return on indexed bonds. In terms of that problem, the analysis extends the work of Fischer (1974) by incorporating money in an explicit way and also has a bearing on the issue raised by Tobin (1971, Chapter 21, Section 4). Tobin points out that open market operations in indexed bonds have a much more powerful effect since real bonds are much closer substitutes to real capital assets. In the above analysis, open market operations in nominal bonds -- which are close substitutes to money -- have no effect on the prices of real capital assets. If the Central Bank of a periphery country would carry out open market operations in indexed bonds there would be complete offsetting in the same way as under the regime of fixed exchange rates with open market operations in nominal bonds. The reason for this offsetting is the fact that the periphery country is not large enough to affect the world interest rate. In this sense there is a complete symmetry between fixed and flexible exchange rates.

V. Concluding Remarks

V-1. Limitations of the Model

The analysis of this paper was built on a set of simplifying assumptions which one obviously has to modify when analyzing the actual behaviour of international financial markets. The assumption of a given real rate of interest in the center country is one of these. A complete general equilibrium analysis would
allow for the endogeneous determination of the real interest rate. Another problem arises if there is no riskless asset. In that case the analysis of the paper, suitably modified, would still apply. As is shown by Black (1972) and Merton (1972), the equilibrium relationships between expected real returns would be modified only in that the real rate of interest would be replaced by the expected return on a portfolio of all risky assets constructed in such a way that its rate of return is uncorrelated with the average market rate of return.

The effects of taxes on foreign investments could be incorporated into the analysis along the lines suggested by Black (1974). If some financial assets were nontraded internationally, again the analysis could be suitably modified. In that case, the asset demand functions of investors in different countries would obviously be different. Completely nontradable assets -- such as human capital -- could also be incorporated into the model as is shown in a different context by Fischer (1974). Mayers (1972) analyzes the case of non-marketable assets in the context of the standard mean variance model.

Aliber (1973, 1974a) has emphasized the political risk as an important attribute of assets traded in the international financial markets. The risk of default because of bankruptcy is another obvious factor explaining differences in interest rates.

Merton (1973) has shown how stochastic changes in the interest rate and in the expected returns on assets can be handled in the framework of the type of model and in this paper. The term structure of interest rates and its relationship to the term
structure of exchange rate expectations is obviously an important area of research. Porter (1971) develops a model of the term structure of interest rates in an open economy that is based on the expectations hypothesis.

In this context one might also want to replace the random walk hypothesis with alternative assumptions. For instance, a more reasonable assumption about exchange rates might be that there is some level or path around which the exchange rate fluctuates. A treatment of portfolio problems with regressive and adaptive expectations is given in Merton (1971, Section 9) and a general discussion of the nature of speculative price processes is given in Samuelson (1972, especially section 8).

Rather than assume given stochastic processes for exchange rates and asset prices it would be desirable to derive these from the stochastic processes of the exogeneous variables -- such as money supply, productivity growth and so forth. For instance, one might start by specifying some rules for monetary policy and then examine the effects of these rules on the stochastic behaviour of the exchange rates, assuming that expectations are rational.

Transactions costs have been introduced as an explanation of deviations from the interest rate parity by Branson (1969), Prochowny (1970), Frenkel (1973) and Frenkel and Levich (1975). Transactions costs between countries introduce a "neutral band" within which interest rate differentials do not give rise to arbitrage. The available evidence suggests, however, that these transaction costs are not very large in the foreign exchange markets or in the short term money markets. Aliber (1974, b) pro-
vides evidence that these costs have risen recently with the increased variability in exchange rates.

The assumptions of full employment, flexible prices and the purchasing power parity need to be changed when analyzing short run movements in exchange rates. When the purchasing power parity does not hold or when there are changes in relative prices and consumption patterns differ between countries, the same asset will yield different real returns to investors in different countries. There would be thus exchange risk in addition to inflation risk.¹⁴

V-2. Summary of the Analysis

This paper has identified the various determinants of differences in interest rates between countries under flexible exchange rates. It has shown how these differences depend on the expected rates of inflation and on the variances and covariances of inflation rates. It has also shown that the expectations hypothesis is not, in general, valid. Interest rate differentials contain a currency premium which depends on the composition of asset supplies and may, therefore, be changed by monetary policy.

The model was constructed in such a way that the central banks of small economies had little freedom except with regard to nominal magnitudes. Further research can proceed along two lines. One is to recognize the various limitations discussed in the previous section and to develop a complete macroeconomic model that would allow for changes in relative prices of assets and commodities and also would allow for unemployment. Another line of research is to pur-
sue further the theme of integration of this paper by allowing for the substitutability of monies. A complete substitutability of monies would mean, of course, that the central banks would not even have the freedom of pursuing different rates of inflation -- the Gresham's law would obtain and monies of inferior quality would completely lose their value.
Footnotes for Chapter IV

1 The assumption of full employment and fixed relative prices is not necessary for the purchasing power parity. The purchasing power parity would also hold if all commodities were internationally traded and transportation costs were negligible. For a discussion of the purchasing power parity (PPP) doctrine see Yeager (1958), Balassa (1964), Samuelson (1964), Gaillot (1970), and Holmes (1967). Holmes gives a favourable interpretation of Cassel's pioneering work on the purchasing power parity. The empirical evidence on the PPP is mixed. It certainly does not hold in the short run except in the trivial sense that prices of standardized traded commodities in different currencies obey the PPP (in a competitive market). Gaillot concludes from his empirical study that PPP is a good approximation when explaining long run movements in the exchange rate.

2 In practice the forward premium (or discount) does not quite equal the interest rate differential. One reason for the observed differences is the cost of arbitrage: Frenkel and Levich (1974) show that most observations on forward premiums fall in the neutral band introduced by transactions costs. Another reason is the fact that the supply of arbitrage funds may be constrained by restrictions on borrowing. Also, if bonds of different countries have different risk characteristics apart from the currency risk one would not expect the interest parity equation to hold (e.g., Aliber (1974)). A detailed discussion of the interest parity equation is provided in Officer and Willet (1970). See also Chapter V.

3 It has been commonly assumed that the U.S. dollar has served as an international medium of exchange in the post-war period. This view has been challenged by S. Grassman (1973) who provides empirical evidence that in the case of Sweden, Denmark and more tentatively of the Federal Republic of Germany, "most transactions are settled in the seller's or the purchaser's currency." (Grassman (1971), p. 106).

4 More precisely, the mean and variance of the logarithmic change in the exchange rate are given by:

\[
E \log \left( \frac{E_n(t)}{E_n(0)} \right) = (e_1 - \frac{1}{2} \sigma_1^2)t, \quad \text{and}
\]

\[
E_0 \left[ \left( \log \left( \frac{E_n(t)}{E_n(0)} \right) - E_0 \log \left( \frac{E_n(t)}{E_n(0)} \right) \right)^2 \right] = \sigma_1^2t, \quad \text{respectively.}
\]
Ito's Lemma is sometimes called the Fundamental Theorem of Stochastic Calculus. Let \( Y = F(P_1, \ldots, P_n, t) \) be twice continuously differentiable, defined on \( \mathbb{R}^n \times [0, \omega] \) where \( P_i \)'s satisfy the stochastic process:

1. \( \frac{dP_i}{P_i} = \alpha_i dt + \sigma_i dz_i, \ i = 1, \ldots, n \)

Ito's Lemma states that the stochastic differential of \( Y \) is given by:

2. \( dY \approx \sum \frac{\partial F}{\partial P_i} dP_i + \frac{\partial F}{\partial t} dt + \frac{1}{2} \sum \sum \frac{\partial^2 F}{\partial P_i \partial P_j} dP_i dP_j \)

The product \( dP_i dP_j \) is defined by:

3.1 \( dz_i dz_j = \rho_{ij} dt, \ i, j = 1, \ldots, n \)
3.2 \( dz_i dz_j = 0, \ i = 1, \ldots, n \)

where \( \rho_{ij} \) is the correlation coefficient between the Wiener processes \( dz_i \) and \( dz_j \). A further discussion of Ito's Lemma is given in Merton (1971).

The general solution is:

\[ [b \ k]' = aE^{-1}(\Gamma a)' \]

where \( E \) is defined as:

\[ E = \begin{pmatrix} \Omega & \Gamma \\ \Gamma' & s \end{pmatrix} \]

Since \( E_{ij} \) can be of any sign domestic bonds and equity, for instance, can be either complements of substitutes.

Notice that this property is quite different from that usually assumed in macroeconomic models of the IS-LM variety that aggregate bonds with equity. If the short term interest rate moved randomly in the model then the return to long term bonds would be stochastic, and they should be included in the aggregate of risky capital assets.

See Merton (1969, 1971) for a detailed derivation of the optimal consumption-portfolio policy.

If the rates of inflation are correlated with the real returns on equity the required expected return on equity depends also on the returns to bonds. In that case equation (25) is replaced by 25':

\[ \alpha_i - r_n = \beta_i (\alpha_M - r_n) + \gamma_i (r_M - r_n), \ i = 1, \ldots, n \]
where \( r_M \) is the expected real return on the market portfolio of bonds. The \( \beta_i \) and \( \gamma_i \) coefficients are defined by:

\[
\beta_i = \frac{1}{1-\rho^2} \frac{s_{iK}}{s_K^2} - \frac{\rho^2}{1-\rho^2} \frac{s_{iB}}{\sigma_{BK}}, \text{ and} \\
\gamma_i = \frac{1}{1-\rho^2} \frac{s_{iB}}{\gamma_B^2} - \frac{\rho^2}{1-\rho^2} \frac{s_{iK}}{\sigma_{BK}}, \text{ where}
\]
\[
\rho = \text{correlation coefficient between the real return on the market portfolio of equity and bonds.} \\
\sigma_{BK} = \text{covariance between the same.} \\
s_K^2 = \text{variance of the real return on the market portfolio of equity.} \\
\sigma_B^2 = \text{variance of the real return on the market portfolio of (risky) bonds.} \\
s_{iK} = \text{covariance between the real return on equity i and the market portfolio of equity.} \\
s_{iB} = \text{covariance between the real return on equity i and the market portfolio of bonds.}
\]

If \( s_{iB} \) and \( \rho \) are equal to zero, 25' reduces to equation 25.

If there is no safe asset the safe return \( r_n \) in equations (25) and (25') is replaced by the real return on a portfolio that has zero correlation with the market portfolio of all assets (say \( r^* \)), the so-called 'zero-beta' portfolio (see Black (1972) and Merton (1972)).

Solnik's equation for equity returns, (Solnik, 1973, page 32), differs from equation (25) in that in his model the domestic bond is a safe asset for domestic investors while it is not safe for foreign investors. This is because Solnik assumes that there is no unanticipated inflation despite the unanticipated movements in the exchange rate. A result of this assumption is that \( r_n \) is replaced by \( r_i \) on the left hand side of equation (25) and by \( r_M \) on the right hand side.

I am indebted to S. Fischer for suggesting this illustration.

If the rates of inflation are correlated with equity returns equation (29) must be replaced by (29'):
\[ r_i - r_n = b_i' (r_M - r_n) + c_i (\alpha - r_n), \quad i = 1, \ldots, n-1 \]

Coefficients \( b_i' \) and \( c_i \) are defined by:

\[
b_i' = \frac{1}{1-\rho^2} \frac{\sigma_{iB}}{\sigma_B^2} - \frac{\rho^2}{1-\rho^2} \frac{\sigma_{iK}}{\sigma_{BK}}
\]

\[
c_i = \frac{1}{1-\rho^2} \frac{\sigma_{iK}}{s_K^2} - \frac{\rho^2}{1-\rho^2} \frac{\sigma_{iB}}{\sigma_{BK}}
\]

where \( \rho, \sigma_B^2, \sigma_{BK} \) and \( s_K^2 \) are as defined in footnote (10) above and:

\( \sigma_{iK} \) = covariance of the real return on bond \( i \) with the real return on a market portfolio of equity.

\( \sigma_{iB} \) = covariance of the real return on bond \( i \) with the real return on the market portfolio of (risky) bonds.

(29') collapses to (29) if \( \sigma_{iK} \) and \( \rho \) are equal to zero. If there is no riskless asset \( r_n \) is replaced by \( r^* \) as defined in footnote (10) above. In that case, equation 29' (or 29) applies to the center country as well.

Solnik's equation (25), (Solnik 1973, page 37) appears similar to equation (29) in our model in that it also contains 'a currency premium' term. In his model the currency premium has to do with the hedging of equity investments by borrowing in the currency in which the equity is denominated rather than with the hedging of the purchasing power risk of bonds.

12 Aliber (1974(a)) provides evidence that the forward premium is not an unbiased estimate of the change in the exchange rate. He interprets the difference as a measure of 'currency preference.' In terms of our model, this is not quite correct for the difference also contains the variance term. Aliber's data seems to indicate that the departure from the Fisher parity is greatest for currencies with the most variable exchange rate. Irving Fisher (1930, Chapter XIX) also analyzes the differences in interest rates on bonds of different currency denomination. It would be interesting to re-examine Fisher's long time series data on the basis of our model.

13 The structure of a two sector macroeconomic model of this type is thoroughly explained in Foley and Sidrauski (1971).
Solnik's (1973, 1974) model contains no inflation risk, although the exchange rate moves stochastically. This is strictly speaking acceptable only if the investors only consume domestic goods, the prices of which change in a foreseen way. In our model, all goods are traded and hence a domestic currency bond is not necessarily any safer than a foreign currency bond.
Chapter V
THE DETERMINANTS OF THE FORWARD PREMIUM

I. Introduction

This chapter develops a model of the forward exchange market that departs in several respects from the 'modern theory' of forward exchange, first formalized by S. C. Tsiang (1958). The main difference is that we view the forward contract as a financial asset, the rate of return of which is determined in the same way as the rates of return of other assets, such as short term bonds. There is an error in the existing literature on the subject in that the forward market is linked, through covered interest arbitrage flows, to the flow supply of and demand for spot foreign exchange. In our model, the forward premium is determined by the condition that the stock excess demand for forward exchange equals zero. Although we do not, in this paper, develop a complete general equilibrium model of the asset markets, we indicate how the model of the forward market developed in this chapter can be integrated into such a model.

Another difference from the existing literature is that we abandon the institutional approach and derive the demand for and supply of forward exchange contracts and covered assets from a model of optimal portfolio choice over time. The resulting solution is then interpreted in terms of covered interest arbitrage and forward market speculation. We do not have explicit equations
for the demand for forward exchange by trade hedgers. The reason is that we treat a receipt of foreign exchange as a discount bond in the currency in which it is denominated and assume that it is tradable like all other financial assets.

The analysis of this paper also extends the work of Feldstein (1968), Leland (1971), and Solnik (1974) by allowing domestic residents to consume foreign as well as domestic goods. This implies that even if the domestic currency price of domestic goods is constant, the domestic bond is not a safe asset, as is typically assumed. The paper also derives an explicit theory of why the covered interest arbitrage schedule is downward sloping, and is not infinitely elastic. We assume that there is a risk of default, which implies that covered interest arbitrage is no longer riskless. This gives us an explicit theory of the departures from the interest rate parity. It is shown that the forward premium depends not only on the interest rate differential but also on the expected change in the exchange rate, the variance of the exchange rate, the risks of default, and the extent of intervention in the forward exchange market.

The model developed in this paper has several testable implications. Those are pointed out in various parts of the paper. Section II develops the model of forward exchange speculation and Section III analyzes the determinants of the equilibrium forward premium. The concluding section discusses the empirical implications of the analysis and suggests various extensions of the model.
II. The Determinants of Foreign Exchange Speculation and Covered Interest Arbitrage

a. The Basic Model

In line with the previous contributions we assume that there are two countries, the United States (U.S.) and the United Kingdom (U.K.). In addition to the two national monies, there are two short term securities: the U.S. security denominated in dollars and the U.K. security denominated in sterling. The analysis may be easily extended to cover external currency assets, such as the Eurocurrency deposit and loans, as well as equity claims on real capital. Investors may also buy and sell the dollar and the pound forward in the forward exchange market, assumed to be a short term market. For simplicity, we ignore holdings of money balances (currency) in the analysis of investor behaviour. Currency transactions are not important in international arbitrage and foreign exchange speculation, and in any case the introduction of money holdings would not substantially affect the analysis.

b. The Behaviour of Prices and the Exchange Rate

As in the previous paper we assume that the exchange rate follows a continuous time stochastic process of the form:

1. \[ \frac{dS}{S} = \pi(Y) \ dt + \sigma(Y) \ du \]

where \( S \) is the dollar price of sterling, \( Y \) is a vector of state variables that comprise all the variables that affect the exchange rate, and \( du \) is Brownian motion. This specification implies that the exchange rate behaves 'smoothly' -- its sample path is contin-
uous with probability one. Discrete jumps in the exchange rate will be considered in Section 9 below. Equation (1) implies that the proportionate change in the exchange rate over a short time interval $\Delta t$ is approximately normally distributed with mean $\pi(Y)\Delta t$ and variance $\sigma^2(Y)\Delta t$. Furthermore, the unanticipated changes in the exchange rate are non-serially correlated however short the time interval. The specification is therefore consistent with the efficient market hypothesis of Fama and Samuelson. In the special case when $\pi$ and $\sigma^2$ are constant, the exchange rate does follow a random walk and is lognormally distributed.

Because the exchange role moves stochastically, so do the price deflators of expenditure in the two countries. We assume that the shares of expenditure on domestic and imported goods are constant. We also assume that the local currency prices of American and British goods are constant. (It is sufficient to assume that their rate of change is fully foreseen.) Therefore, the price deflators for consumption expenditure in the U.S. and in the U.K. are given by:

2. $\text{CPI}_A = \frac{P_A}{P_A/S} = P_A^{\alpha} P_B^{1-\alpha} S^{1-\alpha}$

3. $\text{CPI}_B = \frac{P_B}{P_A/S} = P_B^{\beta} P_A^{1-\beta} S^{1+\beta}$

where

$\text{CPI}_A$ (CPI$_B$) = consumer price index in the U.S.

$P_A$ = dollar price of American goods

$P_B$ = sterling price of British goods

$\alpha(\beta)$ = share of domestic goods in American (British) consumption
From equations (1) to (3) we can derive the stochastic behaviour of price deflators of consumption in the two countries:

4. \[
\frac{d\text{CPI}_A}{\text{CPI}_A} = [(1-\alpha) \pi - \frac{1}{2} \alpha (1-\alpha) \sigma^2] \, dt + (1-\alpha) \sigma du
\]

5. \[
\frac{d\text{CPI}_B}{\text{CPI}_B} = [-(1-\beta) \pi + \frac{1}{2} (1-\beta) (2-\beta) \sigma^2] \, dt - (1-\beta) \sigma du
\]

The only counterintuitive result in these equations is that the mean rate of inflation depends also on the variance of the exchange rate. Roughly speaking, this is because the price level is a concave function of the exchange rate, and by Jensen's inequality, the mean of a concave function is less than the value of the function at the mean. This point is illustrated by the following example. Suppose that \( \alpha \) is 0.5 and \( P_A \) and \( P_B \) are equal to one. Then \( \text{CPI}_A = \sqrt{S} \). Suppose that the current exchange rate is one and the next period's rate is 1.6 with probability 0.5 and 0.4 with probability 0.5. There is therefore no expected change in the exchange rate. The expected price level in the next period is \( 0.5 \sqrt{1.6} + 0.5 \sqrt{0.4} = 3/\sqrt{10} \), which is less than one, whence the expected rate of inflation is negative.

c. Real Returns on Securities

Consider now an investment of \( B_A \) dollars by an American investor in U.S. bonds. The real value of this investment is \( B_A/\text{CPI}_A \). Over the next instant the real value changes because of capital gains or losses due to inflation and because of interest payments on the principal.
7. \[ \frac{dB_A}{B_A} = \bar{B}_A \frac{d(\frac{1}{CPI_A})}{(\frac{1}{CPI_A})} + \bar{B}_A R_A \, dt \]

where \( R \) is the nominal rate of interest in the United States.

Using Ito's Lemma we can rewrite (7) in the form:

8. \[ \frac{dB_A}{B_A} = [R - (1-\alpha)\pi + 1/2(1-\alpha)(2-\alpha)\sigma^2] \, dt - (1-\alpha)\sigma du \]

\[ \equiv r_A \, dt - (1-\alpha)\sigma du \]

In the absence of a risk of default the mean real return on U.S. bonds for an American investor is \( r_A \), which is equal to the nominal rate adjusted for expected inflation due to exchange rate change, and for the variance of the exchange rate, for reasons already discussed.

We assume that there is a possibility, however unlikely, that an investor in American short term bonds will lose everything. In reality, of course, the risk of default on U.S. treasury bills, for example, is extremely small, particularly for American investors. The risk is much greater for foreign investors whose investments in the U.S. may be frozen for political reasons, or may lose value because of capital controls.

The risk of default is handled mathematically in the following way. The occurrence of the event 'default' is assumed to follow a Poisson process, where the probability of default in the next instant is \( \lambda dt \), and the probability of the default occurring \( t \) units of time from now is \( (1-e^{-\lambda t}) \). There is a theory of stochastic differential equations of the Poisson type similar to that of the Wiener processes.\(^5\) The stochastic differential equation corresponding to (6) with the possibility of default is:
where \( q(t) \) is a Poisson process. The expected rate of return is now \( r_A - \lambda_A \) rather than \( r_A \), and the instantaneous variance conditional on no default is \((1-\alpha)\sigma^2\) as before.

The return dynamics of sterling bonds for an American investor, and of dollar and sterling bonds for a British investor can be similarly derived. For convenience these are collected in Table I.

Table I reveals some interesting facts. First, even ignoring the risks of default, the same asset has completely different risk-return characteristics for investors in different countries or for investors with different tastes. Exchange rate changes give rise to this distortion because they change relative prices. In our model, the asymmetry disappears only if both \( \alpha \) and \( \beta \) are equal to one half, in which case consumption expenditure is divided equally between domestic and imported commodities by all investors.

The second interesting implication of Table I has to do with the question whether the forward premium is an unbiased predictor of the change in the exchange rate. This question was raised by J. Siegel (1972) who assumed that investors consume only domestic products, in which case both \( \alpha \) and \( \beta \) are equal to one. In that case a risk-neutral American investor will hold both domestic and foreign bonds only if \( R \) is equal to \( R^* + \pi \) while a British investor will hold both securities only if \( R \) is equal to \( R^* + \pi - \sigma^2 \). Siegel (1972) concluded that the forward premium is not an unbiased predictor of the change in the forward premium, when he should have
### Table I

The Real Risks and Returns to Bonds

<table>
<thead>
<tr>
<th>Denomination</th>
<th>American</th>
<th>British</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dollar</strong></td>
<td>$r_A - \lambda_A = R - (1-\alpha)\pi + \frac{1}{2}(1-\alpha)(2-\alpha)\sigma^2$</td>
<td>$r_B - \lambda_B = R - \beta\pi + \frac{1}{2}\beta$</td>
</tr>
<tr>
<td><strong>Sterling</strong></td>
<td>$r_A^* - \lambda_A^* = R^* + \alpha\pi = \frac{1}{2}(1-\alpha)\sigma^2$</td>
<td>$r_B^* - \lambda_B^* = R^* + (1-\beta)\pi - \frac{1}{2}\beta (1-\beta)\pi$</td>
</tr>
</tbody>
</table>

1 The corresponding stochastic differential equation can be read from the table according to:

$$\frac{dA}{A} = rdt + rdu - dq.$$
concluded, as was pointed out by Roper (1975), that the question is meaningless since the expected change in the exchange rate is different for investors in two different countries ($\pi$ for an American and $\pi-\sigma^2$ for a British investor). An implication of this is that risk neutrality is inconsistent with the existence of equilibrium unless there are limits on the size of forward market positions. In that case, the observed forward premium depends on the mean rate of change in the exchange rate ($\pi$), its variance ($\sigma^2$) and the distribution of wealth amongst investors.

In general, this problem disappears only if $R = R^* + \pi - (1-\alpha)\sigma^2$ $R = R^* + \pi - \beta\sigma^2$ hold simultaneously. Apart from the case of zero variance in the exchange rate this obtains when $\alpha + \beta = 1$. It is interesting, although not particularly important, that in this case the exchange rate satisfies the purchasing power parity, for $CPI_A/CPI_B = P_A^\alpha P_B^{1-\alpha} / P_A^{1-\beta} P_B^{\beta} s^{-1+\beta} = S^{2-\alpha-2} = S$. This condition is very unlikely to be met in practice since in general the share of domestic goods in total consumption is much greater than one-half.

c. Instantaneous Forward Contracts

Consider an investor who buys $X$ pounds forward to be delivered at time $t+\Delta t$ (at a price $F(t+\Delta t)$) and immediately sells the receipt of pounds at a (stochastic) spot price $S(t+\Delta t)$. The profit from this operation per unit of time is $\frac{X[S(t+\Delta t) - F(t+\Delta t)]}{\Delta t}$ or $X S \frac{\Delta S}{\Delta t F(t)} - \frac{F(t+\Delta t) - F(t)}{\Delta t F(t)}$. Consider the limit of this expression as the time interval approaches zero. Obviously, $F(t)$ must equal $S(t)$. Therefore, the second term is simply $\frac{[F(t+\Delta t) - F(t)]}{\Delta t F(t)}$. 
We shall define the limit of this expression as the *instantaneous forward premium on the pound*:

10. \( f = \lim_{\Delta t \to 0} \frac{F(t + \Delta t) - F(t)}{\Delta t F(t)} \)

The first term is the stochastic change in the exchange rate over time interval \( \Delta t \). As \( \Delta t \) approaches zero it converges in probability to the stochastic process defined by equation (1) above. Therefore, the change in dollar value of wealth over a brief time interval \( dt \) from the forward market transaction is:

11. \( dW_1 = XS[\pi - f] \, dt + \sigma du \)

Alternatively, consider an investor who borrows \( Z \) dollars and buys sterling securities in the amount of \( X \) pounds, where \( X = Z/S \).

The change in the value of this portfolio per unit of time is:

\[
\frac{Z(t + \Delta t)}{\Delta t S(t + \Delta t)} - \frac{X(t + \Delta t)}{\Delta t} = XS \left[ \frac{\Delta Z}{\Delta t Z} + \frac{\Delta S}{\Delta t S} - \frac{\Delta X}{\Delta t X} \right] \text{ since the current value is equal to zero.} \]

As \( \Delta t \) approaches zero, the first term in the brackets in the last expression approaches the force of interest on sterling securities (\( R^* \)) and the last term the force of interest on dollar securities (\( R \)). The second term approaches the continuous time stochastic process \( dS/S \) as above. Therefore, the change in the dollar value of wealth from this investment strategy is given by:

12. \( dW_2 = XS[(R^* + \pi(Y) - R) \, dt + \sigma(Y)du] \)

Both investment strategies require no current resources and involve identical exchange rate risk. Therefore, assuming that the exchange rate risk is the only risk in international investment, the mean rates of return to the two investment strategies must be equal to avoid arbitrage, whence:

\[ \pi - f = R^* + \pi - R \]
or $R - R^* = f$. This is simply the interest rate parity equation expressed in terms of instantaneous rates of return. It is evident from this result that the forward market adds little to the analysis if the exchange rate risk is the only relevant risk. In the next sections, we introduce the risk of default which invalidates the above arbitrage condition since the simultaneous borrowing and lending in domestic and foreign bond markets involves the risk of default as well as the risk of exchange rate changes.

Finally, the change in the real value of wealth from a forward position in pounds is given by:

13. $d\bar{W}_1 = (X/S/CPI_A) [\pi - f - 1/2(1-\alpha)\sigma^2] \ dt + \sigma du$

d. Optimal Portfolio Rules and Forward Positions

We assume that each investor maximizes the expected value of the discounted utility from consumption of domestic and imported goods over an infinite time horizon. Consider first an American investor whose current wealth is $W_A/CPI_A$ ($\bar{W}_A$) in real terms. He can allocate this wealth into domestic bonds ($B_A/CPI_A$) and foreign (sterling) bonds ($B_A^*S/CPI_A$):

14. $\frac{W_A}{CPI_A} = \bar{W}_A = \frac{B_A}{CPI_A} + \frac{B_A^*S}{CPI_B} \equiv \bar{B}_A + \bar{B}_A^*$

Note that because we only consider instantaneous forward contracts the current forward position does not enter the wealth constraint. However, if we allowed for long term forward contracts the current market value of forward contracts maturing some time in the future would enter the wealth constraint. As was shown in
the previous section the forward position does affect the change in wealth.

Once the investor has chosen the stocks of domestic and foreign securities and the net forward position, the change in wealth is stochastic, consisting of four parts: the nominal interest receipts on domestic and foreign securities, capital gains or losses due to exchange rate changes and inflation on securities as well as on the forward position, and capital losses due to default; less the purchase of domestic and imported consumer goods.

Using Ito's Lemma and the previous results, we obtain:

\[ d\bar{W}_A = \left[ \bar{E}_A r_A + \bar{E}_A^* r_A^* + X(p - f - (1-\alpha)\sigma^2) \right] dt - (C_A p_A + C_A^* p_A^*) dt + \left[ -(1-\alpha) \bar{E}_A + \alpha \bar{E}_A^* + \bar{X} \right] \sigma du - \bar{E}_A dq_A - \bar{E}_A^* dq_A^* \]

where \( \bar{X} \) is the real value of the forward position in pounds, \( p_A \) is the relative price of domestic goods and \( p_A^* \) the relative price of imported goods; and \( q_A(t) \) and \( q_A^*(t) \) are the Poisson processes for the occurrences of default on domestic and foreign securities respectively.

It is convenient to rewrite (14) in the form:

\[ d\bar{W}_a = \bar{W}_A \left[ (b_A r_A + b_A^* r_A^*) + X [p - f - (1-\alpha)\sigma^2] \right] dt \\
- \left( C_A p_A + C_B p_B \right) dt + \bar{W}_A \left[ -(1-\alpha) b_A + ab_A^* - X \right] \sigma du \\
- \bar{W}_A \left[ b_A dq_A + b_A^* dq_B^* \right] \]

where \( b_A \) and \( b_A^* \) are the proportions of domestic and foreign securities in the total portfolio and \( X \) is the ratio of the forward position to the total value of wealth. The portfolio proportions \( b_A \) and \( b_A^* \) must satisfy the wealth constraint:
17. \( b_A + b_A^* = 1 \)

The investor's objective is to maximize the expected utility of consumption over an infinite time horizon subject to (16) and (17). Define \( J(W_A, S, t) \) as the derived utility function of initial wealth, the exchange rate, and time -- the only state variables of the choice problem:

18. \( J(W_A, S, t) \equiv \max \mathbb{E}_t \int_t^\infty u(C_A, C_A^*) \ e^{-st} \ dt \) (cf. (16) and (17))

In order to be consistent with the previous assumption of constant expenditure shares we shall assume that the instantaneous felicity function is of the form:

19. \( u(C_A, C_A^*) = \frac{1}{\gamma} (c_A c_B)^{1-\alpha} \gamma, \gamma < 0, 0 < \gamma < 1; \)

\( \equiv \alpha \ln c_A + (1-\alpha) \ln c_B, \gamma = 0 \)

where \( 1 - \gamma \) is the Arrow-Pratt measure of relative risk aversion.

In this paper we consider only the case of Bernoulli utility (\( \gamma = 0 \)).

The solution to this optimizing problem is found by applying stochastic dynamic programming as explained by Samuelson (1969) and in the continuous time case by Merton (1969). Using the method described by Merton (1971) we get in the end a deterministic equation of optimality:

20. \( 0 = \max \left\{ u(C_A, C_B) - \omega J^* + J_W^* W_A \ [r_A b_A + r_A^* b_A^* \]

\[ + [\pi - f - 1/2 (1-\alpha) \sigma^2] x] - J_W^* (C_A^P A + C_B^P A^*) + 1/2 J_W^* W_A^2 \]

\[ - (1-\alpha) b_A + \alpha b_A^* \ x] + \lambda_a \left[ J^*(b_A W_A, S) - J^*(W_A, S) \right] + \lambda_B \left[ J^*(b_A^* W_A, S) - J^*(W_A, S) \right] + J^*_S + 1/2 J^*_S \sigma^2 \right\} \text{ s.t. (17)} \)
where \( J^*(W, S) = e^{-pt} J(W, S, t) \). We are mainly interested in the optimal portfolio proportions and in the optimal forward position.

e. The Demand for Forward Exchange

After some algebra we obtain the optimal forward position as:

\[
X_A^d = (1-a) - b_A^* + \frac{\pi-(1-a)\sigma^2}{\sigma^2} - f
\]

hedging
demand for
forward exchange
demand for
forward exchange by
covered interest
speculative
demand for
forward exchange
arbitrageurs

Equation (21) has considerable intuitive appeal and it is similar to the traditional specification except for some obvious differences. The first term is the demand for forward exchange arising from the hedging of the consumption of imported goods. Even if investors had infinite risk aversion, or even if the variance of the exchange rate were infinite the investors would have an open position in forward exchange as long as they consume imported goods. The second term is the demand for forward pounds by covered interest arbitrageurs, as in the traditional model of the forward exchange market. The third term represents the speculative demand for forward exchange. It differs from the traditional specification in that it depends on the difference between the forward premium and the expected change in the exchange rate adjusted for the variance term for reasons discussed above. The slope of the speculative schedule is determined by the variance of the exchange rate -- the smaller the variance the more sensitive is the speculative demand for forward exchange to the discrepancy between the forward premium and the expected change in the exchange rate.
Inspection of equation (21) shows that in this case of Bernoulli utility function the result is simplified even further, for the total demand for forward exchange is independent of consumption preferences, because any change in the hedging demand is completely offset by an opposite change in the speculative demand for forward exchange, at given interest rates and exchange rate expectations. The reason for this is that an increase in the expenditure share of imported goods \((1-\alpha)\), for instance, will reduce the real return of sterling securities for Americans and thereby the speculative demand for sterling balances. We may thus rewrite equation (21) in the simple form:

\[
X_A^{d} = -b_A^* + \frac{\pi - f}{\sigma^2}
\]

Assuming that American investors have identical preferences and exchange rate expectations the total demand for forward pounds, in pounds is given by:

\[
X_A^d = B_A^{d} + \frac{\pi - f}{\sigma^2} \frac{W_A}{S} = -B_A^* + \frac{\pi - f}{\sigma^2} \left[ \frac{B_A^*}{S} + \overline{B_A}^* \right]
\]

where \(B_A^{d}\) is the demand for sterling securities, and \(\frac{W_A}{S}\) the sterling value of wealth of American investors, consisting of \(\frac{B_A^*}{S}\) in dollar securities and \(\overline{B_A}^*\) of sterling securities. It is important to point out that the demand for forward pounds is a decreasing function of the spot price of sterling. An increase in the price of sterling will reduce the sterling value of financial assets and therefore the demand for sterling assets which is proportional to wealth.

The supply of forward pounds by British investors -- identically equal to their demand for forward dollars can be similarly
derived. Assuming again homogeneous preferences, it is given by:

24. \[ x_B^d = (1-\beta) W_B^d - B_B^d - \frac{\pi - \beta - f}{\sigma^2} W_B \]

where \( W_B \) is the sterling value of financial wealth of British investors, and \( B_B^d \) the demand for dollar securities by British investors. We can simplify this equation into:

25. \[ x_B^d = B_B^*d - \frac{\pi - f}{\sigma^2} W_B = B_B^*d - \frac{\pi - f}{\sigma^2} (\bar{B}_B + \bar{B}_B^*) \]

where \( B_B^*d \) is the demand for sterling securities by British investors. The total world demand for forward pounds is the sum of \( x_A^d \) and \( x_B^d \):

26. \[ x^d = -B^*d + \frac{\pi - f}{\sigma^2} W = -B^*d + \frac{\pi - f}{\sigma^2} (\bar{S} + B^*) \]

f. The Covered Arbitrage Schedule

The remaining step is to derive the holdings of domestic and foreign securities. The details of the derivation are left out. After some algebra, we obtain the condition: 6

27. \[ \lambda_A \frac{1}{1 - b_A^*} - \lambda_A^* \frac{1}{b_A^*} = R^* - R + f \]

where \( b_A^* \) is the proportion of wealth invested in sterling securities. This equation implicitly defines the covered arbitrage schedule:

28. \[ B_A^*d = b_A^*(R^* - R + f; \lambda_A, \lambda_A^*) \frac{W_A}{S} = b_A(I, \lambda_A, \lambda_A^*) \frac{W_A}{S} \]

where \( I = R^* - R - f \) is the covered interest differential. Most empirical and theoretical studies of short term capital movements have postulated a downward sloping covered arbitrage schedule looking like equation (28) without really justifying its form.

Although (27) cannot be solved in a closed form all of its
properties can be easily established. The shape of the \( b() \) function is shown in Figure I. When the covered interest rate differential is equal to zero the ratio of the proportion of domestic securities \( (b_A) \) to the proportion of foreign securities \( (b_{A^*}) \) equals the ratio of the probabilities of default \( (\lambda_A/\lambda_B) \). The slope of the covered arbitrage schedule is given by:

\[
\frac{\partial b_{A^*}}{\partial I} = 1/(\frac{\lambda_A}{(1-b_{A^*})^2} + \frac{\lambda_B}{b_{A^*}^2})
\]

At the interest rate parity, the slope is \( \lambda_A\lambda_B/(\lambda_A + \lambda_B)^3 \). For small probabilities of default the slope is very high, approaching infinity as either of \( \lambda_A \) and \( \lambda_B \) approach zero. As the covered differential increases (or decreases) the slope become increasingly steep, reflecting the increasing riskiness of a portfolio concentrated on a single asset. The premium that investors require to invest more in a foreign or domestic asset can be interpreted as a premium for bearing the 'political risk' of default, or as a premium for the increasing illiquidity of a concentrated portfolio. Both of these factors provide an economic rationale for the inelastic supply of arbitrage funds after a point.

In the case that domestic securities are riskless, investors in America will never invest abroad unless the foreign interest rate, covered against exchange rate risk, is greater than the domestic interest rate. The covered arbitrage schedule in this case looks like the \( A_2A_2 \) schedule in Figure I.

The demand for sterling securities by British investors can be derived in the same way:
Figure I

The Covered Arbitrage Schedule

the covered interest differential

the proportion of British securities
30. \( B_B^{\text{d}} = b_B^* (R^* - R + f, \lambda_B, \lambda_B^*) \ W_B = b_B^* (I, \lambda_B, \lambda_B^*) \ W_B \)

In general, we would expect that foreign investments are riskier than domestic investment, whence \( \lambda_A < \lambda_B \) and \( \lambda_A^* > \lambda_B^* \). This type of asymmetry has the effect of reducing the level of international investment. In the special case when domestic assets are riskless and foreign assets risky, (both \( \lambda_A \) and \( \lambda_B^* \) are zero and \( \lambda_A^* \) and \( \lambda_B \) are nonzero) there exists no equilibrium in the international financial markets: the American investors will invest in Britain only if the covered interest rate differential is negative -- hence no mutually advantageous trade in securities is possible. This shows that political uncertainties, capital controls and other factors which give rise to asymmetric risks have a much greater impact on international investment than exchange rate uncertainty which can be handled by adjustments in relative yields.

g. Discrete Changes in Parity

In this part we investigate the effects of anticipated devaluations or revaluations on forward positions and portfolio choice. We assume that the percentage of devaluation (revaluation) is known but its time is uncertain. In particular, the event of devaluation is assumed to be Poisson distributed. In other words:

31. \( \text{prob} \{ \text{devaluation occurs in the interval } [t, t + \Delta t] \} = \mu \Delta t + O(\Delta t) \)

\( \text{prob} \{ \text{devaluation does not occur in the interval } [t, t + \Delta t] \} = 1 - \mu \Delta t + O(\Delta t) \)
prob \{ \text{devaluation occurs more than once in the time interval} \\
[t, t + \Delta t] \} = 0(\Delta t)

where \( \mu \) is the mean number of occurrences of the event of devaluation per unit of time and \( 0(\Delta t) \) is such that \( \lim_{\Delta t \to 0} 0(\Delta t)/\Delta t = 0 \).

In the event of devaluation, the exchange rate is assumed to jump by \( y \) percent. To simplify the analysis, we assume that apart from the uncertain time of devaluation, the exchange rate is deterministic. This enables us to highlight the effects of speculative attacks that characterized the Bretton Woods system of 'fixed but adjustable exchange rates.' We also assume for simplicity and unrealistically, that despite the devaluation domestic currency prices remains fixed. The stochastic behaviour of the exchange rate is now given by:

32. \[ ds = Sdz \]

where \( z(t) \) is a Poisson process defined by (31) above.

The main difference from the previous case of smooth changes in the exchange rate is that forward market speculation now entails the possibility of a large discrete increase in wealth. The change in wealth from forward market speculation for an American investor corresponding to equation (11) is:

33. \[ dW_1 = -Xsf \, dt + XSdz \]

where \( X \) is the forward position in pounds. The expected change in the real value of wealth is accordingly:

34. \[ E_dW_1 = -\overline{Xf} + \mu \overline{X}[y^\alpha - y^{-1+\alpha}] \]

where \( \mu \) is the probability of devaluation and \((y-1)\) the percentage of change in the parity. If \( y > 1 \) the sterling is revalued
and if \( y < 1 \), it is devalued.

In the case of constant relative risk aversion we obtain, by using the same method as before, the optimal forward position as:

\[
x^d = -\frac{1}{y - 1} - b_A^* + k\left(\frac{\mu}{f}\right) \frac{1}{1 - \gamma} (y^d - y^{-1+\alpha}) \frac{Y}{1 - \gamma}
\]

where \( k \) is a parameter that depends on all the structural parameters.

Two special cases are of particular interest. First, in the case of infinite risk aversion the investor will still hold an open spot and forward position in foreign currency, in proportion \((1 - y^{-1+\alpha})/(y^\alpha - y^{-1+\alpha})\) of total wealth. It is straightforward to show that the real value of this portfolio is unaffected by devaluation.

We shall examine the case of Bernoulli utility function in greater detail. In that case, equation (34) simplifies to:

\[
x^d = -b_A^* + \frac{\mu(y-1) - f}{f(y-1)}
\]

The second term can be thought of as the supply of forward pounds by speculators.

The speculative function is illustrated by the SS and S'S' schedules in Figure II. In the first case a revaluation of the pound of \((y_1-1)\) percent is expected with probability \( \mu \). If the cost of forward cover equals the expected profit from revaluation, the speculative supply equals zero. Since the demand for forward pounds by covered interest arbitrageurs is almost certain to be positive, we can conclude that in the absence of forward market intervention the forward premium is less than the expected change in the parity. The S'S' schedule illustrates the supply of
Figure II

The Speculative Supply of Forward Pounds

$S$

forward premium

$\mu(1-\gamma)$

speculative supply

$S'$

forward discount

$\mu(\gamma-1)$
pounds when a devaluation of \( y_2 \) percent is expected with the probability \( \mu \). Again, the equilibrium forward discount must be less than the expected devaluation in order that speculators hold a positive forward position in sterling.

Another interesting property of the speculative schedule is that even if the forward premium (or discount) becomes infinitely large, the speculators will not hold an infinite position in forward pounds. In the case of expected revaluation, their holdings of forward sterling are bounded below by minus \( 1/(y_1 - 1) \) and, in the case of expected devaluation, above by \( 1/(1-y_2) \).

The covered arbitrage schedule is the same as in the previous section. The demand for forward exchange and securities by British investors can be derived in the same way. In the case of Bernoulli utility function and identical expectations, these demand functions are the same as above.

**III. The Determinants of the Forward Premium**

In this section we shall investigate the determinants of the forward premium assuming that the rates of interest in the U.S. and the U.K. are pegged by monetary policy. We shall consider the case of symmetric risks, which implies, as we have seen, that the demand for forward exchange and for dollar and sterling securities is independent of the distribution of wealth between the two countries.

**a. Flexible Exchange Rates**

The forward market equilibrium condition requires that the
supply of forward exchange by speculators and by the Central Bank(s) forward market intervention equals the demand for forward exchange by covered interest arbitrageurs:

\[ \frac{\pi - f}{\sigma^2} - q = b^d(R^* - R + f, \lambda, \lambda^*) \]

where \( q \) is the supply of forward exchange by Central Banks in proportion to world wealth. Equation (31) shows that when the covered interest arbitrage schedule is not infinitely elastic the forward premium is not determined by the interest rate parity alone, but depends also on exchange rate expectations \( \pi \), exchange rate risk \( \sigma^2 \), and the amount of intervention in the forward market by the Central Banks \( q \). In the more general case, the forward premium also depends on the distribution of wealth between countries.

The determination of the forward premium is illustrated in Figure III. The AA schedule is the same as in Figure I. An increase in the forward premium makes covered holdings of sterling assets more attractive although the marginal gain becomes smaller and smaller as the proportion of wealth invested in Britain increases. The speculative supply of forward exchange is a decreasing function of the forward premium as is illustrated by the downward sloping SS schedule in Figure I. When the forward premium equals \( \pi \), the expected rate of change in the dollar price of sterling, the speculative supply of sterling is equal to zero. On the other hand, when \( \pi \) equals \( \pi - \sigma^2 \), the expected change in the sterling price of dollar, the speculative supply of forward exchange in proportion to wealth is equal to one. Therefore, in the absence of forward market intervention the forward premium
is bounded above by the dollar parity \((\pi)\) and below by the sterling parity \((\pi - \sigma^2)\). The margin of error made by assuming that the forward premium is determined by exchange rate expectations along \((\pi)\) is thus at most equal to the variance of the exchange rate.

The SS schedule shifts as a result of changes in exchange rate expectations \((\pi)\), exchange risk \((\sigma^2)\) and forward intervention \((q)\). The AA schedule shifts as a result of changes in political and other such risks \((\lambda, \lambda^*)\), and changes in the interest rate differential. An increase in the expected rate of appreciation of sterling will shift the SS schedule upwards by the same amount, as shown by schedule S'S' in Figure III. The magnitude of the increase in the forward premium depends on the slope of the covered arbitrage schedule at the initial point \(P\). In the figure, the share of sterling assets of total wealth is already so large that there is only a small increase in the demand for sterling balances -- the forward premium will adjust almost completely to the change in exchange rate expectations.

The effect of forward market intervention on the forward premium can be analyzed in exactly the same way. An increase in the support of the dollar in the forward market will shift the SS schedule to the left (to schedule S'S" in Figure III). In the figure, the covered arbitrage schedule is elastic to the left at the initial point \(P\). Therefore, forward market intervention is quite successful in reducing the desired holdings of sterling assets and increasing the desired holdings of dollar assets. Only a small adjustment in the forward premium is required.

An increase in the variance of the exchange rate will rotate
Figure III

The Determination of the Forward Premium

dollar parity

forward premium

sterling parity

demand for and supply of forward pounds
the SS schedule downwards at the dollar parity (see Figure III) and thus reduce the forward premium and the desired holdings of sterling balances. An increase in the sterling rate of interest will shift the AA schedule downwards by the same amount (to A'A' in Figure III). When the speculative schedule is quite elastic, there will be only a small offsetting decline in the forward premium as is illustrated by points P and Q in Figure III. On the other hand, when the speculative schedule is steep, there will be an almost completely offsetting reduction in the forward premium (cf. points P₁ and Q₁ in Figure III).

The effect of an increase in the risk of covered investments in Britain is to shift the covered arbitrage schedule to the left and make it less elastic.

In conclusion, the forward premium is a nonlinear function of the interest rate differential \( (R^* - R) \), the expected change in the exchange rate \( (\pi) \), the variance of the exchange rate \( (\sigma^2) \) the amount of intervention in the forward market \( (q) \), and the parameters of political risk:

\[
37. \, f = f(R^* - R, \pi, \sigma^2, q, \lambda, \lambda^*)
\]

\[
(-) \, (+)(-) \, (+)(+) \, (-)
\]

where the \( f \) function is implicitly defined by:

\[
38. \, \frac{\lambda}{1 - \frac{\pi - f}{\sigma^2} + q} - \frac{\lambda^*}{\frac{\pi - f}{\sigma^2} - q} = R^* - R + f
\]

Equation (32) is a generalization of the interest rate parity equation to which it collapses when the political or other risks of covered arbitrage are absent (\( \lambda \) and \( \lambda^* \) are zero).
By expanding the left hand side of equation (33) around the mean \((\bar{\lambda}, \bar{X}, \bar{\pi}, \bar{f}, \bar{\sigma}^2, \bar{q})\) and solving the resulting linear equation for \(f\) we obtain:

\[ f = a_0 + a_1(R - R*) + a_2\pi + a_3\sigma^2 + a_4q + a_5\lambda + a_6\lambda^* \]

where the coefficients \((a_i)\) are functions of the parameters and are evaluated at the mean. This linear approximation resembles the equations that have been used by various authors to test the 'modern theory' of the forward market. If the theory developed in this paper is correct such linear approximation is likely to be satisfactory only if there is very little movement in the right hand side variables around their sample means.

b. Expected Devaluation and the Forward Premium

Under the Bretton Woods system of fixed but adjustable exchange rates (fixed within a narrow band) the major source of exchange rate uncertainty was the possibility of parity changes. We have already indicated that the forward premium is not an unbiased predictor of the expected rate of devaluation. The determinants of the discrepancy are analyzed in this section. The equilibrium condition for the forward market is now:

\[ S(f, \mu, y) + q = b^d(R^* - R - f, \lambda, \lambda^*) \]

where \(S = \frac{1}{P} - \frac{1}{1-Y}\), and \(1-y\) is the expected rate of devaluation of the sterling parity. Using this equilibrium condition, we can obtain a rough estimate of the bias of the forward premium as the predictor of devaluation. Let \(\gamma\) be the supply of sterling securities less the supply of forward exchange by the Central Bank in proportion to total wealth. Then the forward premium is given
by: $f = \mu (1-y) / (1+r(1-\gamma))$. The bias is the greater the greater is the stock of sterling securities in relation to total wealth.

IV. Concluding Remarks

The results of this paper have several interesting implications for empirical analysis.

(i) The forward premium is, in general, a function of the interest rate parity and the expected change in the exchange rate, as well as foreign exchange risk and risks of default on domestic and foreign securities. This relationship is highly nonlinear whence linear approximations are likely to be good only if there is little variation in the explanatory variables. The nonlinearity is due to the fact that the covered arbitrage schedule is very elastic in the 'normal' range of asset holdings but becomes increasingly inelastic as portfolios become concentrated in any particular security.

(ii) Whether under flexible exchange rates or under fixed exchange rates with an expected parity change the forward premium is not an unbiased predictor of the expected change in the exchange rate. This is not true even if investors are risk neutral as long as the exchange rate affects relative prices and consumption patterns differ between countries. In fact, risk neutrality in the absence of restrictions on forward positions is inconsistent with the existence of equilibrium.

(iii) Under flexible exchange rates the forward premium can differ from the expected change in the exchange rate (both measured in terms of the same currency), in the context of our model,
at most by the variance of the exchange rate. This means that
large exogeneous changes in the interest rate differential do not
cause an equally large offsetting change in the forward premium
unless they affect the expected change in the exchange rate.
This implies that independent interest rate policy will have a
large effect on the spot exchange rate by causing large changes
in desired portfolios.

(iv) The effect of forward market intervention on the for-
ward premium depends on the slopes of the speculative and covered
arbitrage schedules. If the variance of the exchange rate is
small, forward market intervention will have little effect on
the forward premium and hence on asset demands. If the covered
arbitrage schedule is very steep, ceteris paribus, the change in
the forward premium induced by forward market intervention will
have little effect on asset demands.

(v) One should be careful not to make the mistake, which is
often made, of concluding that if the covered interest differential
is always zero, no changes in asset positions can take place.
When there are no risks to covered interest arbitrage, the for-
ward premium is identically equal to the interest rate differen-
tial except for small discrepancies due to transaction costs.
But in this extreme case, the demand for domestic and foreign
assets can no longer be expressed as a function of the covered
interest differential as can be easily established from the model
developed in this paper. Instead, the asset demands become
functions of the difference between the interest rate differen-
tial and the expected change in the exchange rate. The interest
sensitivity of asset demands depends now solely on the variance of the exchange rate (as well as the degree of risk aversion). An increase in the domestic interest rate can have large effects as asset demands and hence on the exchange rate without any observed change in the forward premium.

(vi) Finally, the forward market model developed in this paper could be easily integrated into a general equilibrium model of asset markets. The link between these markets and the forward market would be provided by covered interest arbitrageurs. Since the spot exchange rate is determined in the asset markets, that is how the spot rate and the forward premium would be jointly determined in our model.

On the theoretical side, the treatment of long term forward contracts, the problem of the link of the term structures of interest rates, and the hedging of uncertain income and payment streams are both very difficult, and fruitful problems for further study.
Footnotes for Chapter V

1 See also Sohmen (1968). The most recent contributions, and most relevant for the purposes of this paper, are Feldstein (1968) and Leland (1971). Feldstein provides an extensive bibliography of the earlier literature. The literature tends to be very institutional in its approach. Stein's (1962) model of the forward market analyzes carefully the behaviour of the various participants in the forward exchange market. The most recent empirical studies on the forward exchange market are those of Argy and Hodjera (1973), and Frenkel and Levich (1975). An excellent survey of the theoretical and empirical work in this area is provided by Hodjera (1973).

2 For a critical survey of the various explanations of the downward sloping covered arbitrage schedule, see Officer and Willet (1970). See, too, Aliber (1973) who explains the departure from interest rate parity in terms of 'political risk'. Aliber's political risk can probably be formalized very similarly to the risk of default. Tsiang (1958) explained the downward sloping covered arbitrage schedule in terms of increasing illiquidity of concentrated portfolios. Stoll (1968) argues that the risk of default is the only reason for the downward sloping covered arbitrage schedule.

3 See Fama (1970) and Samuelson (1973).

4 See Samuelson (1973).

5 See Merton (1971), Section 8.

6 The way to obtain this equation is to guess that the derived utility function is of the form $A + B \ln W$, substitute that in the equation of optimality and optimize the resulting expression with respect to $b^*_A$ in the ordinary way.
BIBLIOGRAPHY


BIBLIOGRAPHICAL NOTE

The author was born in Kemijarvi, Finland on February 12, 1949. He received his schooling in Kemijarvi and from 1966 to 1970 at the Atlantic College, Glamorgan, Great Britain where he took the GCE advanced level examinations in economics, history, and mathematics. He obtained his MA degree in economics from Helsinki University in 1970. From 1970 to 1972 he worked at the International Monetary Fund as a Young Professional economist, becoming an economist in the Research Department in 1972. He is currently on leave of absence from that position. In September, 1975, he will join the Department of Economics at Stanford University as an Assistant Professor of Economics. He will be on leave from that position during the fall semester of 1975 at the Institute for International Economic Studies at Stockholm University.

The author is married to Marja Kristiina (Katona) Kouri and is the father of Janne Pentti, born on April 3, 1975 in Cambridge, Massachusetts.