

SYSTEMS OPTIMIZATION IN ULTRA SHORT HAUL AIR TRANSPORTATION

by

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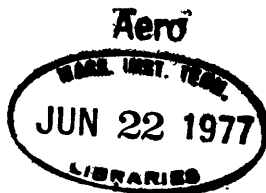


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## ABSTRACT

Systems analysis techniques are employed in an operational and financial evaluation of the potential for ultra short haul air transportation. Direct and indirect costs are modeled as functions of vehicle size and level of service. Access/egress time is analyzed using a probabilistic, random variate formulation of a total travel time model. Subdivision of point-to-point markets into region-to-point and intraregional cases is analyzed. Demand and market share sensitivities are predicted as functions of a multi-dimensional level of service quantity, where frequency of service, market subdivisions, multistep policies, and vehicle size are identified as decision variables. A network example is solved using expected values from a more general probabilistic network model. Profit-seeking and market share maximizing fare policies are examined. Extensions of the model are identified as methods of alternative transportation technology analysis.

Thesis Supervisor: René H. Miller  
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## 1.0 Motivation for Research

The suburbanization of the 1950's has had a great effect on the transportation needs of the entire community, and on the ability of existing transportation modes to serve those needs. This is not merely a question of appropriate technology, but from the standpoint of an evaluation of operational policy as well. The saturation of available transportation capability in large metropolitan areas is at hand, and worsening.

Recently, in response to problems in the related areas of energy usage and urban air pollution, a greater awareness has developed as to the history and future of this process of suburban growth and urban sprawl; in particular, in regard to the congestion problems arising from the sometimes painfully slow collection and distribution of passengers engaging in intra-regional and intercity travel under 50 miles. Of the estimated  $1.72 \times 10^{12}$  total U.S. intercity passenger miles this year, greater than 45 percent fall into this ultra-short haul category.

Several concepts have been proposed to solve the problems of congestion. Among them are high speed ground transportation. This solution, however, suffers from the large public investments required of fixed-line technology, and accentuates the problems of intermodal trip itineraries, substituting nodal congestion for link congestion. Another proposal is region to point USH air transportation. Historically, both have suffered from the economics and demographics of

short haul transportation, in that a mass market is necessary for provision of service at competitive prices.

As we shall see, only by a systematic search of operational policies and service scenarios will a great enough market penetration be expected such that the economies of scale that do exist are fully exploited. Such a systems approach will require identification and quantification of variables influencing the feasibility of the service proposal, and a similar micro-scale view of the market for such services.

Until now, analysis of ultra short haul air transportation in the United States has been limited to retrospective demand analyses in major metropolitan markets. As long as short haul market analysis is limited to such techniques the development and exploitation of the air mode is severely hindered. Were a generalized analytical model embodying more comprehensive coverage of service variables available, much of the expensive site-specific assumption formation and data development process could be avoided, making the air mode better understood and more readily integrable into the existing transportation network.

The most recent evaluations of intraregional short haul service proposals have dealt with extensions of airport feeder services to the commuter sector, with assumptions made in the areas of demand, travel time and costing criteria.

The Eastwood, Gosling and Waters study (Operational Evaluation of a Regional Air Transportation System for the San Francisco Bay Area,



September 1976) prepared at ITTE (Institute of Transportation and Traffic Engineering), University of California at Berkeley for NASA Ames Research Center analyzed an intra-regional service from a parametric standpoint, yielding data on potential usage patterns. A minimum traffic density as a function of operational policy variables such as subsidy is one of many limiting constraints. The variation of system evaluation characteristics with similar operational policy considerations is postulated. Of greatest importance in the study is the behavior of the logit demand model at the "tails" of the demand-cost-time surface. The proposed alternative modal technology (air) is quite estranged from the calibration points of the model used as it is a higher cost, lower time service to the potential user. At fare levels representative of current VTOL technology, zero-length and line haul costs are a magnitude greater than fixed-line modal technology (auto, rail). Consequently, assumptions of cost elasticities for various disaggregations of a market may require more detailed formulation.

Similarly, the access-egress or collection and distribution problem would have to more carefully formulated particularly for the more general case. The results of an MIT/FTL study published in March, 1976 (Mann, R.W. Jr., Short Haul Helicopter Service Proposal - The Feasibility of New York Airways Expansion to Nassau County, FTL R76-2) analyzing a similar intra-regional short haul service pointed out the necessity for examination of the effects on the ease of access by subdivision of the market through use of multiple stations within

a demand region.

Subsequent to the March 1976 MIT/FTL demand analysis, a financial assessment was undertaken indicating profitable operations on total costs at load factors above 45% for a 30 seat vehicle. An optimal vehicle was chosen and route generation undertaken using MIT/FTL's FA-4.5 linear programming Fleet Assignment model. An optimal fleet mix was then determined by exercising another FA-4.5 option. Relationships between vehicle size, number of terminals, and nonstop vs. load building multi-stop routings were found to profoundly affect trip time, costs (both direct and indirect) and consequently demand and market traffic density.

The importance of an operational policy framework in which to analyze the short haul air mode is of great importance. It has been shown that of the two usual basic policy formulations - maximize user benefit or maximize operator profit - the former may require subsidy levels on the order of those afforded current fixed-line technology (50-100<sup>+</sup>% of revenues). The latter is feasible in any case and operates comparatively at no net cost to society. The tradeoff of passenger-operator-community-societal benefits can in effect be read directly from the demand curves and resulting market share.

Some demographic and geographic characteristics of study areas are truly unique, rendering generalization between regions infeasible. The most important of these demographic factors - population and income distribution - plus the gross geographics - natural barriers and point-to-point vs. corridor markets - of a region can be incor-

porated within an analysis framework. The inclusion of a probabilistic, random variate analysis of the access-egress problem, plus a closed-form and parametric analysis of the network effects of market subdivision, in a demand formulation of the product form establishes the basis for analysis of multiple policy alternatives as discussed above. Alternatively a total analysis of a particular operational policy choice may be conducted.

The multiple station or market subdivision concept allows a higher frequency of service to the user at the expense of a smaller vehicle with higher unit costs, and (possibly) increased indirect costs due to smaller station volume. This is balanced, however, by the ability to build load factors (or utilize a larger vehicle) through multi-stop routings and by reduced access-egress time. By analysis of these tradeoffs in a general format, conclusions can be reached in specific cases, and in varying operational policy criteria.

In an attempt to shed new light on a problem that has existed for over a decade, this systems optimization will cover in depth quite a bit of ground in the modeling area. In chapter 2, some of the shortcomings of present demand and travel time modeling techniques will be analyzed. The validity of closed-form solutions vs heuristics derived from probability theory will be compared. Chapter 3 discusses cost modeling in USH air transportation. Direct and indirect costs and their causal attributes will be used to generate decision variables for use in chapter 4, which introduces the use of probabilistic techniques and results from queueing theory.

In chapter 5, a demand model is proposed that takes into account the decision variables uncovered in chapters 3 and 4 to build a level of service factor quantity. Using this vector, demand and market share variations may be evaluated. Operational strategies and market solution states from economic theory are presented in chapter 6 for input into the case studies evaluation in chapter 7. Both region-to-point and a general intraregional case are presented. Comparisons and proposals for areas of future research are presented in chapter 8.

While the result of this ultra short haul transportation systems analysis outlines feasible regions within policy alternatives, it is not sufficient to stop here. Transportation systems planning on paper does not carry passengers. Demonstration projects such as those proposed in chapter 8 are an opportunity to experiment and to perform market research to determine what the traveling public wants and will respond to in terms of new and innovative transportation technology. The formulation of a generalized model for alternative transportation technology analysis will enable the generation of rationalized policies for deployment of new proposed modes. A welcome spinoff to this model is its implications for other problem analysis: airport access, personal rapid transit, demand responsive "Dial-a-Ride" proposals, or generalized facilities locations problems.

The choice of introducing VTOL, STOL, compound technologies or current modes on a market by market basis is economically straightforward. By developing a viable reference model, the necessity to "invent the wheel" again and again is ended.

## 2.0 Problems with Current Short Haul Modeling

In general, the classic mathematical modelling techniques and formulations used in the analysis of the demand for transportation services are not suitable for assessment of "alternative" modal technologies. Owing to stretching of time cost frontiers by innovative transportation technology, such new modes are not well characterized by models calibrated on existing operations.

### 2.1 Demand Modelling Choices

The two most widely used demand formulations, product and logistic (logit), which are of the form

$$D \sim C^\alpha T^\beta \quad \text{product form}$$
$$D \sim \frac{1}{1 + \exp(\alpha c + \beta t)} \quad \text{logit form}$$

where  $\alpha, \beta \leq 0$

Here, alpha and beta refer to the demand elasticities with respect to cost and time respectively. These can be shown to exhibit fallacious behavior at the extremes of the cost-time frontier. Graphically, these two demand formulations appear as in figure 2.1

We may characterize the behavior of operators of transportation services as basically profit seeking in search of minimizing variable costs, and adhering to a fare selection strategy that will maximize contribution to overhead. Looking at the revenue side of such an operator's strategy in a particular market, we find that revenues are simply the product of fare and demand level at this particular point. Intuitively, we would expect that there would be some fare that would maximize revenue, and that below or above this fare,

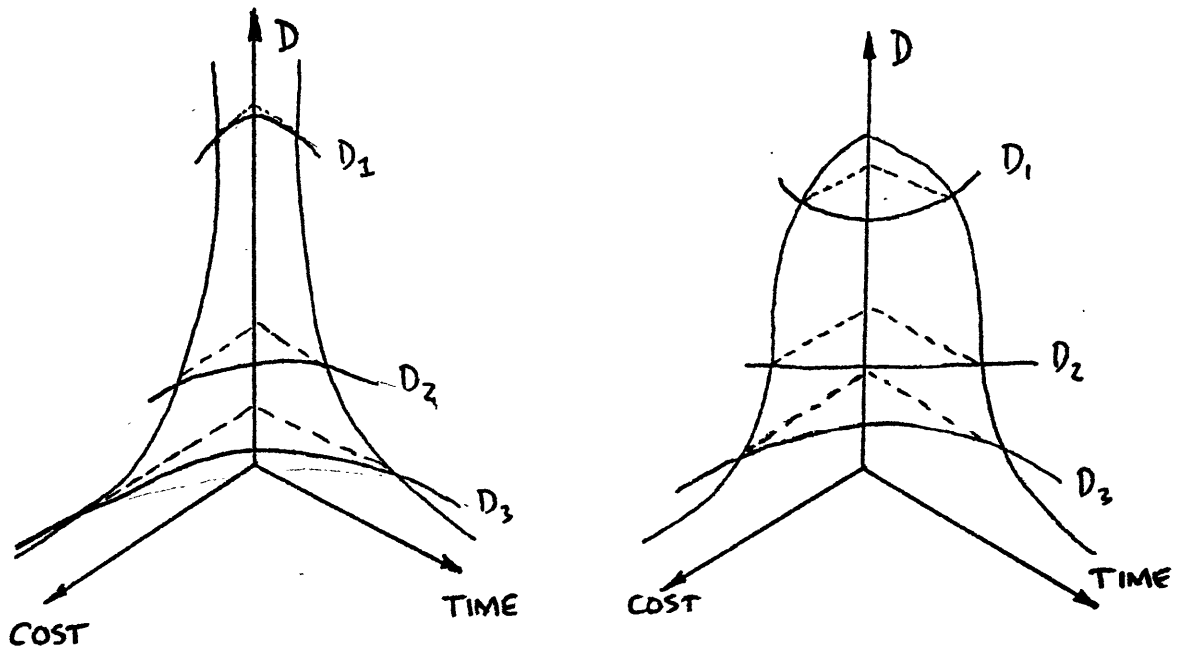
revenue would be non-optimal either owing to inadequate fare level or demand erosion. This concept is shown in figure 2.2 .

Graphically, these two demand formulations appear as in Figure 2.1 .

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Figure 2.1

Product and Logit Demand Surfaces



DEMAND ISOQUANTS

$$D_1 > D_2 > D_3$$

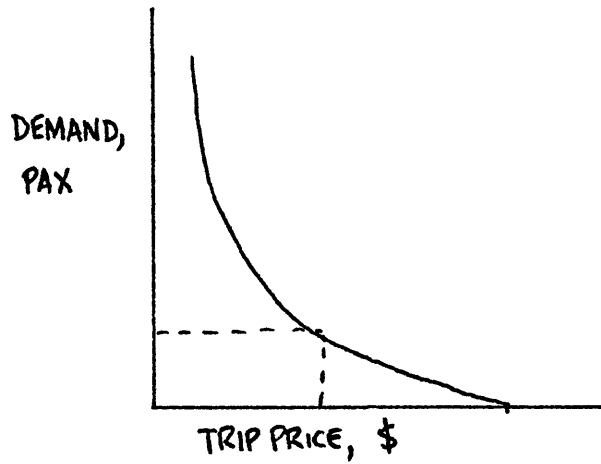
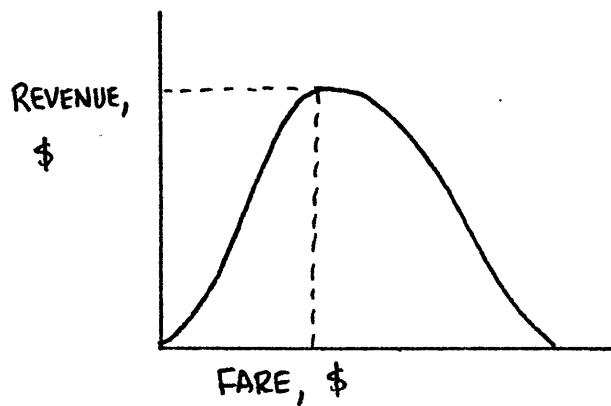


Figure 2.2



We also might expect that above a certain unreasonable fare, that no passengers would desire to travel by this particular mode. (All propensities to purchase Rolls Royce Corniche convertibles aside)

In the case of the product model, we can express the demand as a function of fare,  $F$  as:

$$D \propto K_0 C^{\alpha} T^{\beta}$$

Revenues are then:

$$R = F \cdot D$$

Or 
$$R = K_0 F^{(\alpha+1)} T^\beta$$

Maximizing R and reasonably assuming no trip time dependence on fare, we find the proportionality:

$$\frac{\partial R}{\partial F} \sim (\alpha+1) K_1 F^\alpha$$

and that  $\frac{\partial R}{\partial F}$  is always positive and revenue unbounded for  $\alpha > -1$ . This indicates  $F_{opt} = \infty$ . The case  $\alpha = -1$  corresponds to a constant revenue irrespective of fare. For  $\alpha < -1$ , which corresponds to a cost elastic market, the derivative behaves as always negative and unbounded, indicating  $F_{opt} = 0$ . Analysis of the higher derivatives gives no better interpretation of what would seem intuitively incorrect.

For the logit formulation which is extremely popular among urban transportation planners, a similar analysis shows that when demand and revenue are related as:

$$D = K_2 \left[ \frac{1}{1 + \exp(\alpha F + \beta T)} \right]$$

as before, revenue,  $R = F \cdot D$

or: 
$$R = \frac{F}{1 + \exp(\alpha F + K_3)}$$

Differentiation with respect to fare, reveals that

$$\frac{\partial R}{\partial F} = \frac{1 + \exp(\alpha F + K_3)(1 - \alpha F)}{1 + 2\exp(\alpha F + K_3) + \exp(2(\alpha F + K_3))}$$

Hence, for all  $\alpha < 0$  (which is the only logical conclusion),  $\frac{\partial R}{\partial F}$  is always greater than zero. This indicates that the maximum fare



policy is also the maximum revenue policy.

In other words, both models suggest for optimization purposes that the service provided is so valuable that as we continue to increase fare, there will always be demand consistent with maximizing revenues. This "value of service" concept, while perhaps valid for long haul operations (moon shots, for example) where there may not exist reasonable alternatives, is completely fallacious in the short haul sector, and apparently invalidates the logit demand formulation for alternative technology and financial analysis.

Here, our intuitive model would take over. We will expect to be a ceiling fare above which no demand exists for a mode at a particular level of service. Similarly, an upper bounding trip time will exist such that even at zero fare, no demand will exist for this mode. So strong are the modal cross-elasticities between price and level of service that this is in reality for short haul markets, a fair assessment of the situation.

Turning to travel time, the other demand variable, there are other areas which need to be explored on the ultra short haul "micro" scale.

## 2.2 Travel Time Modelling

In the area of travel time models, we have similar problems induced by the uncertainties of the urban or regional nature of our

service offering. Large scale modelling often fails to take into account the temporal and spatial dynamics evident in a metropolitan situation.

As has been pointed out many times previously, the importance of access/egress time as a percentage of portal-to-portal trip time is inversely proportional to stage length. Hence in a transatlantic market, 30 minutes saved in access/egress is small as compared with the seven hours required block time. By contrast, a similar thirty minute decrease afforded a short haul market will likely be the determinant of the financial viability of variously priced modes.

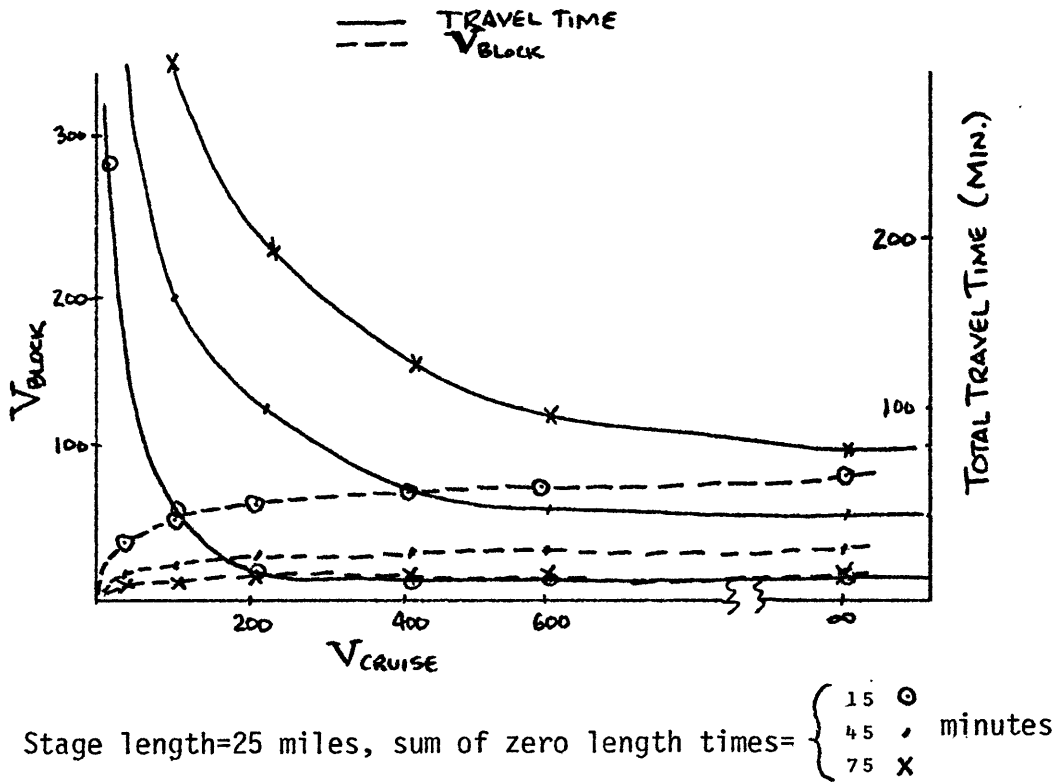
This trend, in access/ egress time importance can be seen analytically in Figure 2.3 , as we vary cruise speed or stage length. Even instantaneous transport à la Star Trek, will yield a finite (and low) block speed over any non-zero distance, increasing as stage length when saddled with an zero-length access/egress time.

The large degree to which market level variations in travel time affect the demand formulation casts doubt upon the validity of our simple travel time model. This is normally functionally represented as:

$$T = t_0 + \frac{t_1}{f} + t_2 \cdot d$$

Figure 2.3

Block speed and total travel time variation with cruise speed,  
stage length zero length times varying



where  $t_0$  = sum of access, egress, passenger processing and cycle times. "Zero Length Time".

$t_1$  = displacement (wait) time proportional to length of day, inversely proportional to frequency of service. Normally represented as one half the headway, a result that can be proved through some of the probability

modelling techniques presented later on.

$t_2 \cdot d =$  block time; inversely proportional to cruise speed of vehicle; proportional to stage length.

For this reason, the quantities of most importance to our ultra short haul example will be those portions of the portal-to-portal time that are alterable either through policy or technology choices. For the purposes of this study, these are number and distribution of transportation terminals, access/egress technology, and the service's operating policy. All of these items affect the "fixed"  $t_0$  term in our macro model of travel time. This term must, for the purposes of further analyses, be broken down into individual terms bearing functional relationships to the variables of interest.

The first concerns the degree to which certain large scale "area" markets are sub-divided into smaller (in the limiting case) "point to point" markets. An optimization methodology will be developed, using a random variate formulation of access/egress travel time to minimize a suitably weighted objective function subject to policy and technological constraints. The effects of various access/egress policy/technology considerations (Dial-A-Ride, Bulk Service, etc.) can be shown. Finally, the financial operating statistics of possible operator strategies may be evaluated by tying together cost projections and travel time models in the framework of a reasonable demand formulation and a suitable fleet assignment model.

### 3.0 Cost Modelling in Ultra Short Haul Air Transportation

For the purposes of ultra short haul air transportation, only two vehicle technologies currently exist: VTOL and STOL. The development and eventual commercial use of a compound V/STOL vehicle (tilt rotor, tilt wing, etc) will very likely hinge upon whatever operating results are managed by current technology systems. For planning purposes, however, such proposed vehicles will be included in this study.

Optimizing the cost performance of a vehicle requires detailed and exhaustive knowledge of its mission and operating environment. In the Mach .80 cruise, 3500 NM stage length, 360 available seat, 10,000 ft. paved runway regime, the Boeing 747 performs admirably, and at less than \$5.00 per vehicle mile in cruise. What, then should we expect in terms of operating costs for a 140KT cruise, 50 NM stage length, 50 seat VTOL machine? Not surprisingly, no direct scale factor is involved. In fact, the vertical take-off or short take-off device is, on a unit cost basis, considerably more expensive to operate. The compromises of aerodynamic v. powered lift v. vectored thrust; the large number of flight cycles per hour, and the short field capabilities of such a vehicle, combine to inflate direct operating cost to the level of CTOL aircraft with a much greater productivity. For some of the same reasons, STOL vehicle unit costs are also higher than corresponding CTOL unit costs, although of an intermediate magnitude.

While STOL has proved its usefulness and financial success in programs such as DeHavilland of Canada's Dash 7 service from Montreal to Toronto, it may not be suitable for true city-center operations. STOL aircraft are in reality constrained to conventional or existing airfields. In the urban setting, land acquisition costs are so high as to force adoption of a VTOL or compound V/STOL technology. Comparing land costs alone for various northeast corridor cities, a VTOL system undercuts a STOL system of similar capacity by 75 percent. This, of course, at the expense of a vehicle with higher operating costs.

### 3.1 Direct Operating Costs

In order to model the costs of a current or proposed ultra-short haul transportation system, an exhaustive analysis was made of current helicopter direct operating cost. Data was gathered on current transport category vehicles ranging in size from 5 to 40 available seats (1500 to 10,000 lbs payload). Hourly costs obtained from other than commercial transport operators were normalized where possible to reflect statutory crew sizes, equipment requirements, and standard reporting practices.

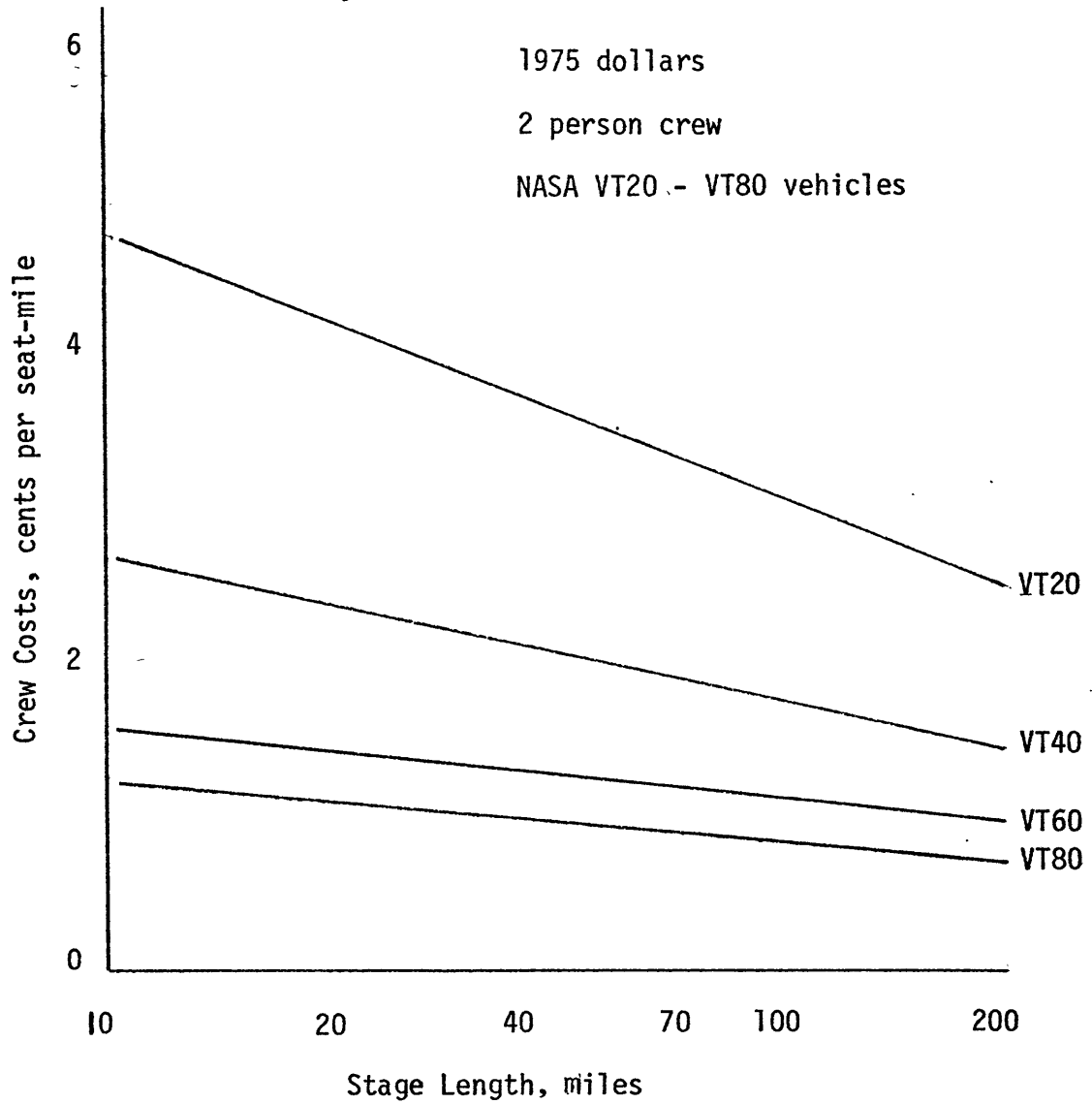
These standardized requirements and practices are detailed as follows:

- 2 person crew (per F.A.R. 90.3) in transport category aircraft. Crew costs from NASA CR-137685 based on data supplied by Douglas Aircraft Company and shown in Figure 3.1 .

- Fuel costs based on JP-4 Kerosene at 35¢ per gallon reflecting bulk contracting and projected cost increases. Non-turbine fuel at 55¢ per gallon.
- Dual IFR certification-instrumentation and hyperbolic RNAV equipment (similar to DECCA, OMEGA, LORAN-C).
- Ownership or lease costs based on seven year depreciation per CAB Form 41 standard reporting practices to residual of 10 percent.
- Utilization of 2200 hrs per year.

Figure 3.1

Crew Costs NASA CR-135872





With the exception of proposed/prototype vehicles all represent operational experience with the particular model.

Some learning-curve type variations are to be expected in these costs; the effects of familiarity with the vehicle type have not been introduced, and while it is expected that "mature" direct operating costs might be marginally lower, this affects only one data point--the Aerospatial SA.330 Puma.

A linear regression analysis was performed on the data, revealing that an excellent degree of fit was obtained with a single variable--seats available. This corresponds to the standard propellor first class "A" fare category seating density. Vehicle data and the linear regression statistics are shown in Table 3.1 and Figure 3.2 .

The effects of newer technology can be seen as orthogonal to the regression line. While a vehicle cruise speed variation of between 105 and 140 KT is present in the data, it does not correlate well with direct operating costs.

Since all VTOL vehicles are range limited by comparison with STOL and CTOL aerodynamic lifting vehicles, design range is not a significant operating cost variable. The ultra short haul mission is not one where ultimate design range enters heavily into operating cost equations for current vehicle technology. Rather, stage length is a more valid parameter. This effect is shown for several vehicle sizes in Figure 3.3 .

Table 3.1

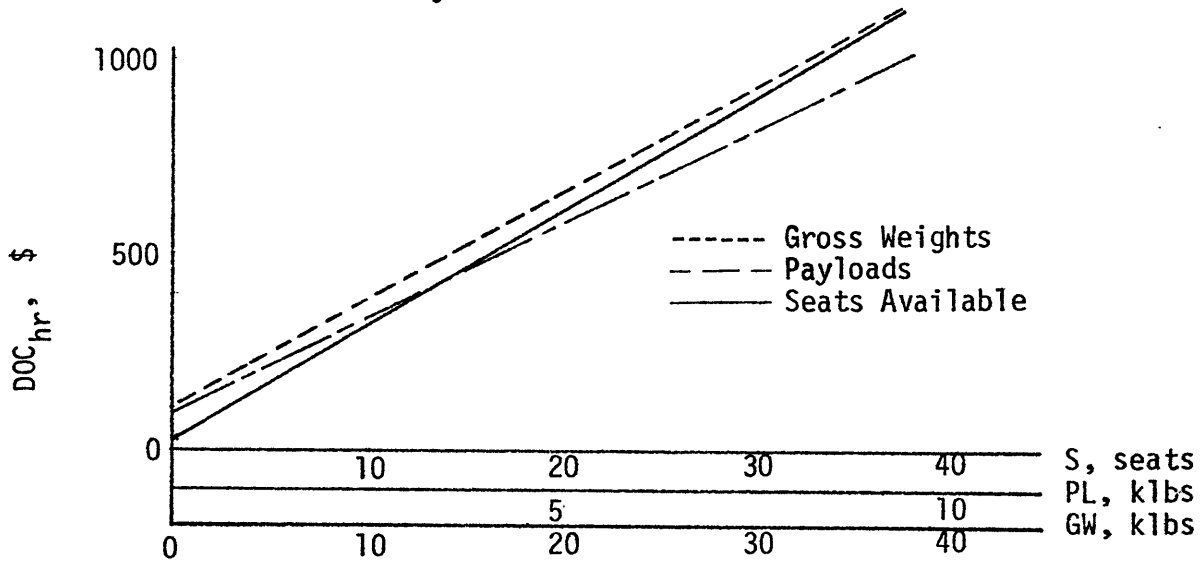
Current VTOL Technology DOC hr Figures

<u>Vehicle</u>	<u>GW</u>	<u>PL</u>	<u>Sa</u>	<u>DOC<sub>HR</sub></u>
S-58 (1 eng.)	13,000	3840	16	485
S-61L	19,000	6100	30	830
S-70	17,520	5600	20	505
S-65	36,600	7700	40	970
S-76	9,350	2550	10	293
BV-179	18,700	5657	25	615
B0-105C	5,105	1420	5	118
BV-107 (CH-46)	20,100	5940	26	868
SA.330	16,800	3700	17	710
Jet Ranger	5,950	1390	5	204

---

Figure 3.2 Current Technology VTOL Operating Costs

Linear Regression Results; Normalized  $DOC_{hr}$



Regression Equations

$$DOC_{hr}(S) = \$88.47 + \$24.32 * S$$

$$R^2 = .958$$

$$DOC_{hr}(GW) = \$92.26 + \$25.70 * (GW/1000)$$

$$R^2 = .902$$

$$DOC_{hr}(PL) = \$29.68 + \$106.80 * (PL/1000)$$

$$R^2 = .820$$

It is also apparent that while economies of scale do exist in rotorcraft DOC, they are not large as one might expect. This perhaps reflects the relative infancy of rotary wing technology. It is also interesting to note that the most current designs are sized below 20 seats, reflecting marketability projections for VTOL machines.

In order to assess the impact of state of the art on system performance, a hybrid direct operating cost formula was assembled for a vehicle of similar mission expectation. Drawing on studies by NASA and Lockheed-California, a median direct operating cost believed obtainable in the 1985 time frame is shown in Figure 3.4. By algebraic manipulation, this can be reduced to the desired function of seats available formula by assumption of average stage length alone. In recognition of the ultra short haul nature of the markets involved, this assumed average stage length was 30 miles. The formula then becomes:

$$DOC_{HR} = \$252.00 + 4.20 S_a$$

which compares favorably with similar forecasts made at MIT and elsewhere.

It is interesting to note that as a function of technology, the zero seat costs are now higher by a factor of 4 and vehicle expansion costs reduced to one sixth their current technology level. This has the effect of introducing extreme economies of scale into vehicle direct operating costs, and can be expected to have a profound effect on optimal vehicle sizing in 1990's ultra short haul air transportation markets.

Figure 3.3 NASA CR-135872

Advanced Helicopter Direct Hourly Operating Costs As Function of  
Stage Length:

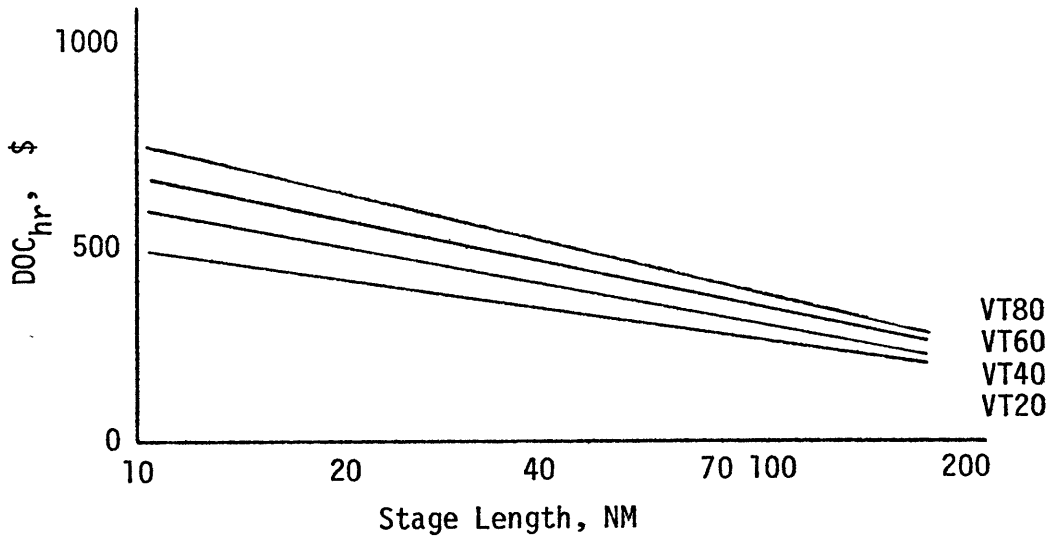
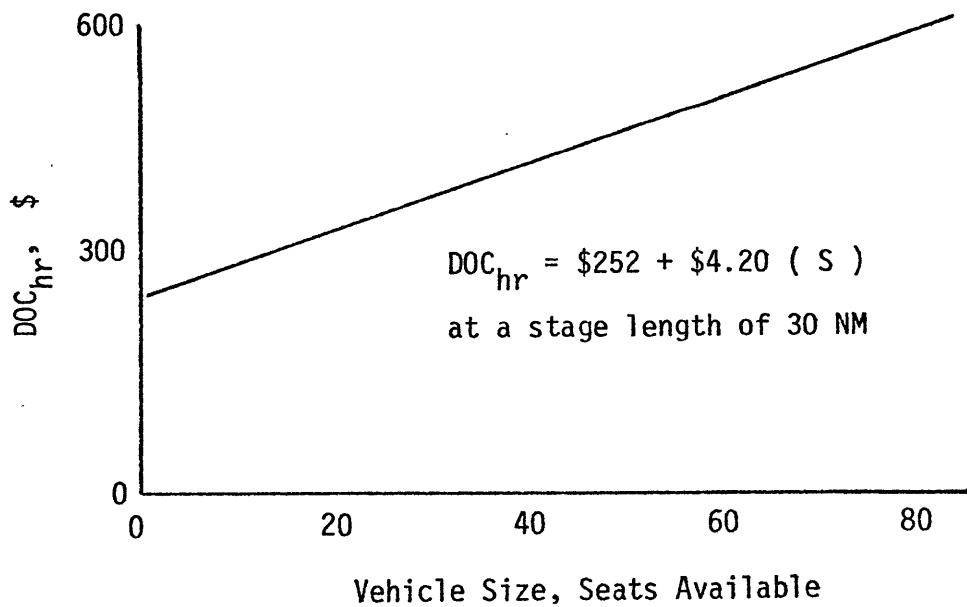


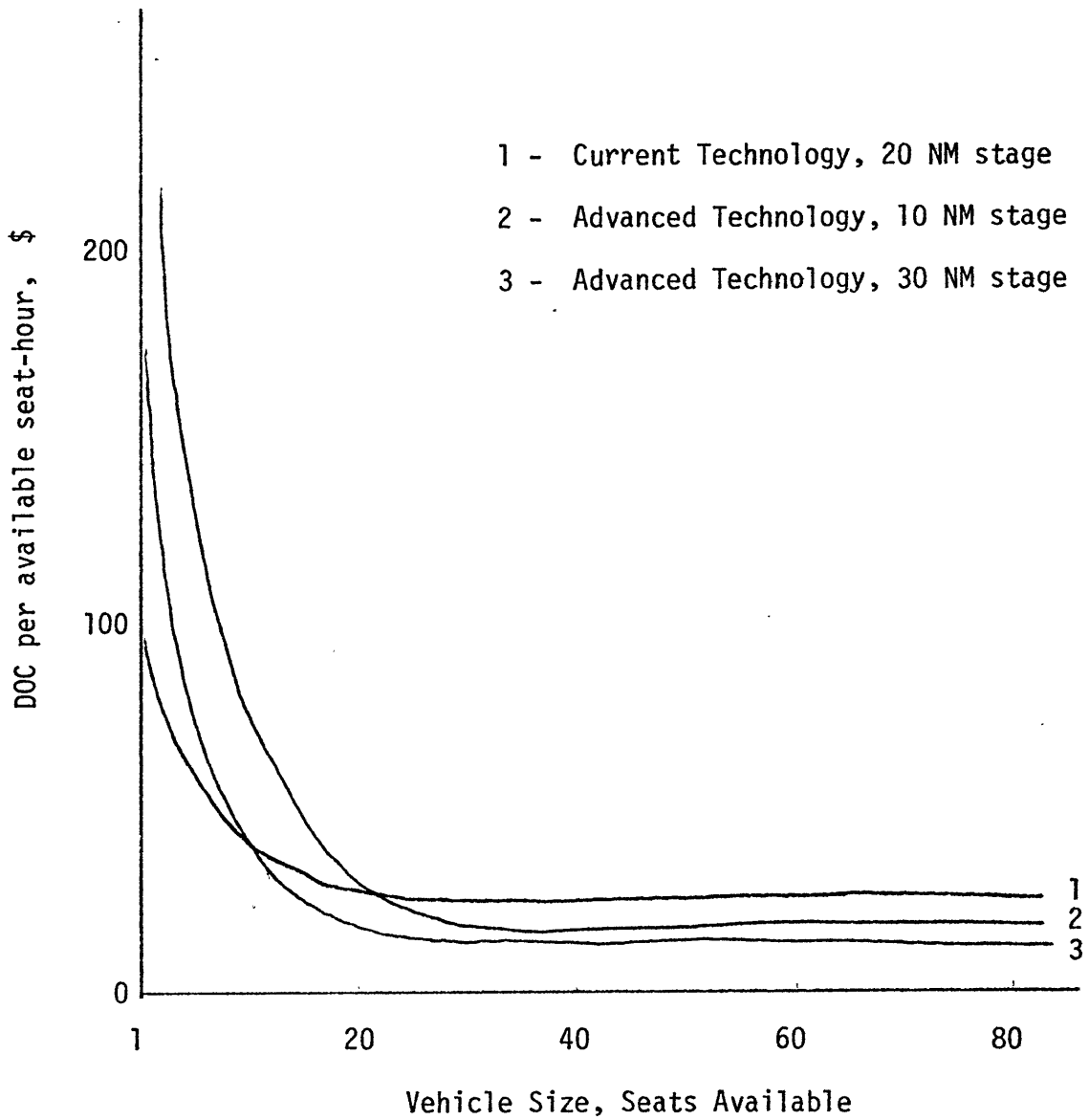
Figure 3.4 NASA CR-135872

Advanced Helicopter Direct Hourly Operating Costs, As a Function  
of Vehicle Size:



Unit seat-hour costs are compared for current and advanced helicopter technologies as a function of vehicle size for selected stage lengths in Figure 3.5 .

Figure 3.5 Unit seat-hour costs - Current and Advanced Technologies, various stage lengths



### 3.2 Indirect and Systems Costs

The economics of facilities may be approached in a similar manner to those of vehicles, becoming more complex only in the effects of congestion. Whereas additional frequencies or larger vehicles may be utilized to deal with short term under-capacity in a market, congestion costs in terminals have no short term solution.

As passenger throughput increases, utilization of personnel and facilities become more efficient up to the point where their design point is reached. Past this point increased costs are incurred in terms of passenger inconvenience and delay. Level of service is adversely affected, and demand can be expected to erode.

As a function of design capacity, total facilities costs behave generally as our vehicle operating costs. There is a zero capacity cost, plus an incremental cost of additional capacity. Treating now the entire station operation cost--personnel plus ownership costs--we may express the cost per passenger at any individual facility as shown in Figure 3.6. Below the design capacity, there is an increasing unit cost. At operating points above the design capacity, there is an apparent everpresent asymptomatic reduction in passenger costs. In reality, delay costs, and level of service deterioration force actual incurred passenger costs to remain at or in fact increase above asymptotic costs present at the design capacity.

Figure 3.6

Station Costs As a Function of Design Capacity

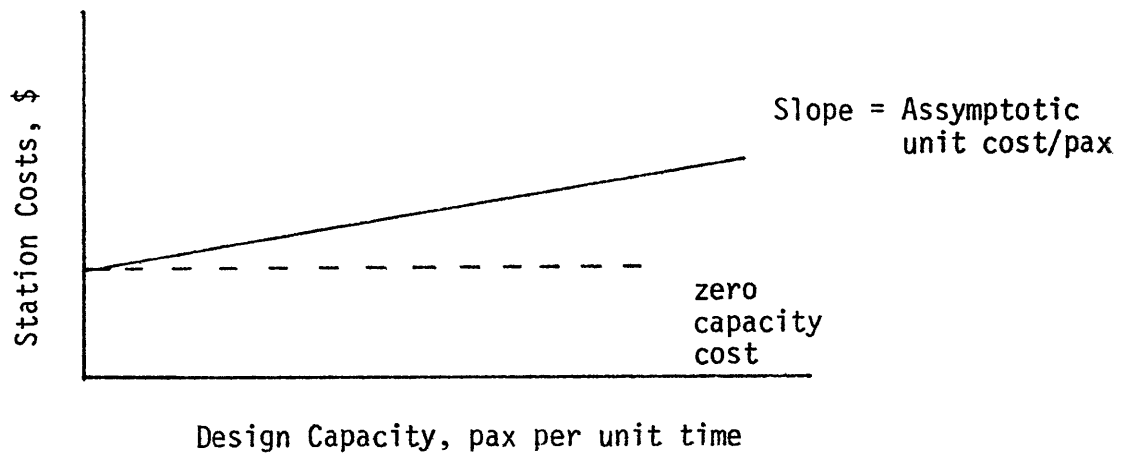
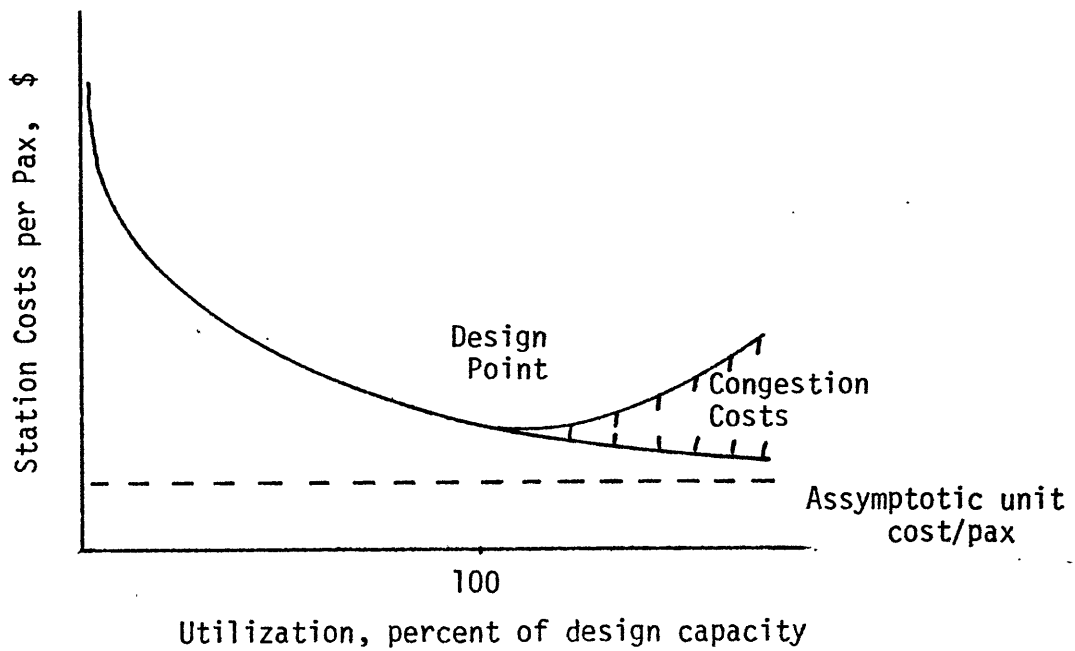


Figure 3.7

Station Unit Costs Per Passenger As a Function of Utilization

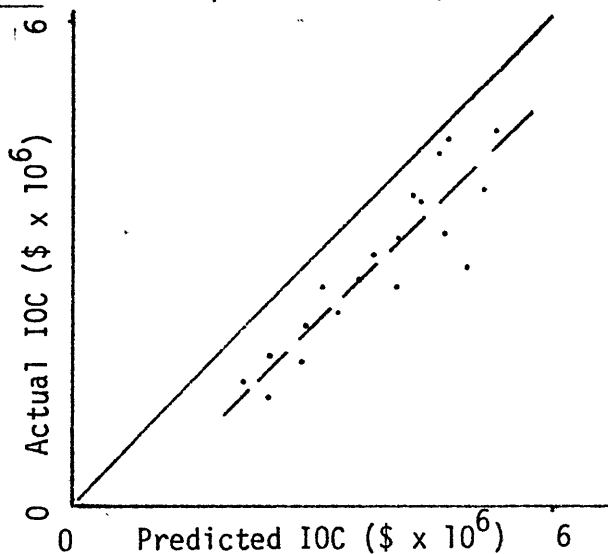




This intuitive result is an excellent area for research, and is shown schematically in Figure 3.7 . It is not clear whether these congestion costs have ever been documented, although they certainly exist.

To arrive at an accurate prediction of System Operating Costs is difficult. Practically speaking, there is no direct means of assessing instrumental increases in total system overhead. Using the work of N.K. Taneja, we can get a valid "ballpark" in which to field expected values. Using the regression equation for Local Service air carriers we can perform a basic scatter plot to see how well historical data from helicopter transport carriers (New York Airways, SFO Helicopters, and Chicago Helicopter Airlines) fits the prediction. This plot is shown in Figure 3.8 , and indicates that there is a high bias in the prediction in terms of the fixed constant, but that the variable linear component of the equation is quite valid.

Figure 3.8 Scatter plot of Helicopter v. Local Service Carrier IOC's



REGRESSION EQUATION:

$$\text{IOC (\$/YR)} = (2.002 \times 10^6) + (0.43 * R)$$

FOR LOCAL SERVICE AIR CARRIERS - FROM FTL R67-2,

"A Multi-Regression Analysis of Airline Direct Operating Costs"

A supplementary regression was performed on several variables including Revenues, RPM's, Revenue Miles (RM), Available Seat Miles (ASM). The best fit to data was obtained with Revenues again, and is of the form:

$$\text{IOC (\$/YR)} = (0.131 \times 10^6) + 0.435 * R$$

The degree of fit is described by:

$$R = .968, \text{ Standard Error} = 0.074 \times 10^6$$

This is similar in degree of fit to the equations derived by Taneja.

Neglected to this point have been the capital costs of service initiation. On the groundside, the question of terminal design and complexity is of great consequence in terms of required capital. From the incremental income statement presented above, and from the current operating statements it is clear that current operators of helicopter services have not and could not intend to invest in land or complex ground facilities.

Current policy is towards the rental of space at general aviation terminal facilities or industrial sites. At the point when access considerations require that VTOL facilities move away from airport sites, and be located optimally in city-center sites, the mode will have to

have priced itself into the market. This will be through a combination of:

- Reduction of DOC<sub>c</sub> per ASM as a function of Energy Efficiency and maintenance complexity (both technology questions)
- Greater reliability as a function of better instrumentation in reduced weather conditions (IFR flight) and mechanical complexity.

### 3.3 Decision Variables For Systems Optimization:

Having reviewed the basic economics of USH mission vehicles and representative service facilities, it is possible to postulate the existence and importance of several decision variables for use in a systems optimization of any market. The strategy is essentially one of identifying factors affecting costs of providing service, assessing impacts on level of service offered, and finally evaluating these effects on demand, market penetration and financial statistics.

Breaking total cost associated with the services in a market into direct operating costs and indirect operating costs, it is possible to quantify the above factors and observe the interrelation of many of them.

The standard definition of direct operating costs includes those portions of total cost incurred solely in the provision of service and correlated with the level of such service. For the purpose of this study, these costs may be termed variable costs. A standard profit maximization will attempt to minimize variable costs thereby maximizing

contribution to overhead. It will be shown here that by the introduction of several new degrees of freedom into portions of the demand formulation, that this variable cost minimization may not be the best tack.

Owing to certain limitations in our calculus, the assessment of variations in fare and level of service at one time are not often possible in a closed-form market solution. The author does not attempt to infer the development of a new calculus of multiple variations. Rather, an iterative technique will be developed, playing on certain factors inherent in the geometry of the product demand formulation in level of service, and the demand-level of service surface. For the moment, however, let us leave variation and demand for later.

Returning to the identification of factors intuitively affecting variance costs, several are flagged:

Vehicle size, S--direct operating cost functions were found to have the best correlation with seats available. This is intuitively correct, although somewhat surprising in light of an inferior regression fit of direct operating costs with payload. This indicates the effect of differing mission strategies in the area of design cube weights. (It must be remembered that except for relatively recent designs, VTOL vehicles have not been designed with commercial passenger transport as a primary goal). In order to reduce the unit costs of seats provided, an operator attempts to maximize vehicle size in

order to gain from the effects of economy of scale.

Frequency of service,  $f$ --direct costs attributable to a market are clearly proportional to the total number of services offered in that market. The operator has no choice but to dispatch entire vehicles, rather than full seats, hence our linear relation with flight costs and directional frequency. While vehicle size did not affect level of service or demand, frequency most definitely will affect both, and very strongly. At ultra short-haul stage lengths, frequency can be shown to be the single most important factor affecting market penetration.

Number of intermediate stops per flight,  $k$ --In dealing with short range lengths, a large portion of total block time is cycle time. This is the time associated with the taxi, takeoff, maneuver, climb, descent and landing portions of the flight. Typically, CTOL vehicles have cycle times in the over 20 minute range, far overshadowing flight time for an ultra-short-haul segment. VTOL vehicles, due to low altitude cruise and non-conventional abilities to airtaxi and circumvent CTOL air traffic control procedures, are able to reduce this by almost a magnitude, averaging 2 to 3 minutes. This time is still significant however in multistop services as it does not consider turnaround time. This is typically of the same order as cycle time. Hence, multistop flights incur cycle cost penalties and level of service (time) penalties but allow load building capability with larger, lower unit cost vehicles and/or

a larger number of facilities.

Vehicle cruise speed,  $V_{CR}$ --while important, it can be shown that although the effect of cruise speed on DOC is a direct proportional one, in the current vehicle analysis it will not be considered in favor of the various other variables mentioned previously.

Minimization of direct costs while assuming the indirect operating cost component fixed is a direct route to an operating loss. While IOC is often of the same magnitude as DOC for domestic trunk carriers, (this can be seen to result from sophisticated reservations capabilities, monumental terminal facilities, and large passenger servicing costs) the saving grace for the trunk carriers is a high vehicle productivity and large passenger volume over which to distribute these costs.

For the ultra short haul operator, a high productivity VTOL device does not currently exist. While the promise of a 1980's era VTOL vehicle with productivity comparable to or better than current STOL technology exists, the present term does not offer such a solution. Indirect costs must therefore be analyzed with the same vigor and intensity as direct costs have traditionally been.

As has been shown, facilities can be as spartan or lavish as need be, with costs commensurate with passenger appeal. Not wishing to enter into one area of behavioral or market psychology, let it merely be said that present term facilities should be designed with

austerity and function always in mind. This should be pictured as somewhere between intercity bus and local service air carrier complexity. The costs associated with these facilities vary widely, yet per passenger, they fall between \$0.50 and \$5.00. This order of magnitude appears large, but when viewed compared to the levels of service offered the passenger (which must on some quantitative--certainly qualitative--basis differ by much more than a magnitude) is not really so great.

Drawing on previous analysis, the system optimization variable with respect to station cost is:

Number of facilities,  $N$ --as some portion of facilities costs is fixed, this overhead will vary linearly with number of facilities within the demand region. Of greater importance here is the effect of number of facilities on the variable component of station costs. As the number of stations increases, level of service is increased by an area rule to be developed in a later chapter. While utilization may fall off slightly, these effective increased costs may be recouped by increased level of service or in fact increased market penetration.

The effects of market subdivision and to some extent multi-stop routings can be explained through the theory of spatially distributed queues. This will be discussed in chapter four.

#### 4.0 Travel time modelling

In the area of travel time modelling, we again require an extensive knowledge of certain quantities that characterize the regions or cities (or segments of the same) involved. In particular, it is customary to take certain demographical quantities of the studied area into account when assessing segmented or disaggregated demand potential. This data and geographic considerations will almost entirely characterize any study area, providing enough data for in depth analysis by traditional macro models. It can be shown that this same data will also provide a basis for the micro modelling postulated as necessary in the analysis of ultra short air transportation systems.

As previously identified, the area of greatest interest in discussion of travel time models will be in those areas termed fixed in the macro model. These are the zero length travel time terms comprising access, passenger processing, wait, cycle and egress times. The block time portion of the model will not be further analyzed except to the extent that it retains the distance cruise speed relationships.

Various treatments of each portion of these segments of portal-to-portal travel time have been used in previously proposed models. Among the formulations used for analysis of access/egress times are closed form and estimation (educated guess) techniques.



#### 4.1 Closed Form Techniques

In the area of closed form relationships, those proposed by Miller and Genest are the most rigorous, dealing with particularly valid geometrizations of city and regional demand areas. Each proposes a demand area geometry, a geometrically functional demand density relationship, and a travel velocity functional. From these are deduced closed-form travel time relationships that (not surprisingly) vary strongly with demand density and geometry assumptions.

That form proposed by Miller in the fourteenth memorial Lancaster Lecture is of polar form, considering a circularly symmetric demand region of radius,  $R$ . Detailed in Figure 4.1, the region consists of a central core of radius,  $r_c$  with uniform demand density,  $p_c$ . At greater than city center radius, demand density drops off geometrically with parameter,  $n$  as:

$$p(r) = p_c \left[ \frac{r_c}{r} \right]^n \quad \text{where } r \geq r_c \quad (\text{Figure 4.2})$$

The parameter is typically of the order,  $n \sim 2$ . Access is of radial-metric type, along radial and arterial highways to any of  $m$  terminals located at a common radius,  $r_t$ . Clearly, there are two possible paths involved: either  $r < r_t$  corresponding to "inside" access, or  $r_t \leq R$ , the "outside" case.

The average distance travelled is:

$$\bar{S} = \frac{\int S \cdot dp + \frac{2}{3} p_c r_c}{\int dp + p_c}$$

where  $dp = p_c r_c r^{1-n} dr d\theta$

Figure 4.1 A circularly symmetric city

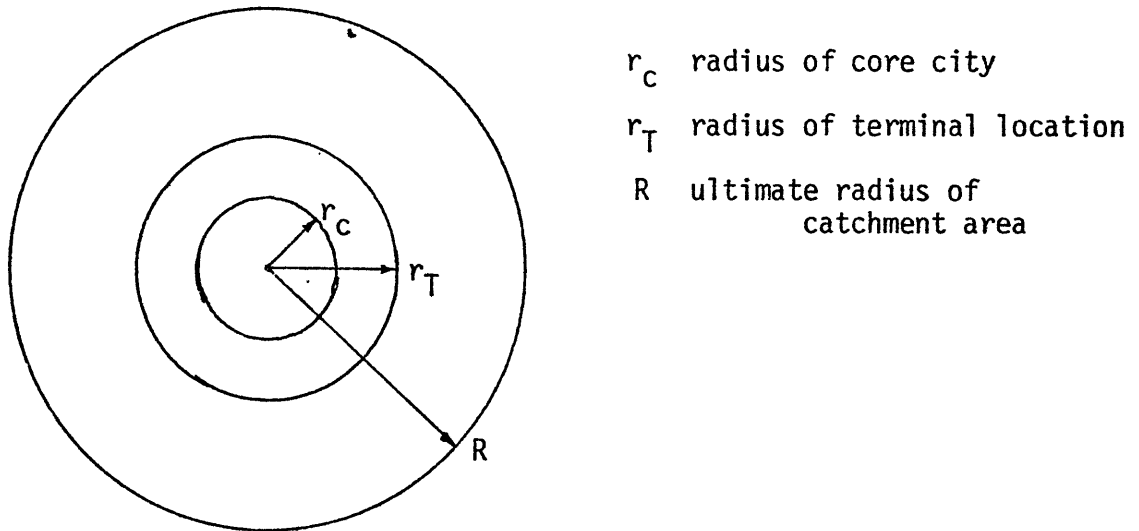
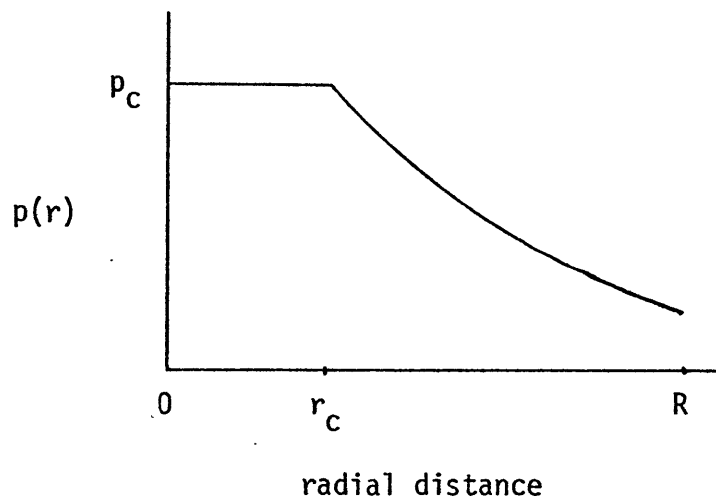


Figure 4.2 A demand profile with geometrical decay



and  $S = r_t \theta \pm (r_t - r)$  (inside, outside cases)

(This recognizes that average distance travelled in the central core region is two-thirds the central radius. The integration limits in  $\theta$  are determined by  $\theta = 2\pi/m$ , the number of equally spaced terminals.

Let us consider the uniform demand density case.

$$p(r) = p_c \left(\frac{r_c}{r}\right)^n$$

hence, the population of the region, P is:

$$\text{and } \bar{r} = \frac{\int_{r_c}^R 2\pi p(r) r^2 dr + P_c \bar{r}_c}{\int_0^R 2\pi p(r) r dr + P_c}$$

with  $\bar{r}_c$  for uniform  $p_c$  is  $\frac{2}{3} r_c$ .

The variation of  $\bar{r}/R$  with  $R/r_c$  and density decay parameter, n is:

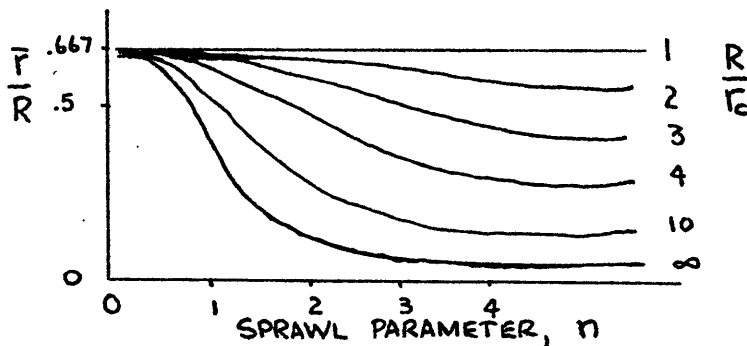


Figure 4.3

Now we can find  $\bar{r}/R$  with knowledge of decay parameter, n, and the  $R/r_c$  sprawl characteristics of the catchment area.

The integration over the two cases reduces to:

$$\begin{aligned} \bar{S} &= \frac{\int S \cdot dp + \frac{2}{3} P_c r_c}{\int dp + P_c} \\ &= \frac{\lambda \theta P_c r_c \left[ \frac{\lambda \theta}{2} (r_t - r_c) + (R - 2r_t + r_c) \right] + \frac{2}{3} P_c r_c}{P + P_c} \end{aligned}$$

where  $\lambda \theta = 2\pi/m$  as before.

Now plotting  $\frac{\bar{S}}{r_c}$  vs.  $\frac{r_t}{r_c}$  for some ultimate radius,  $r$ , we find a convenient linearization of the form  $(a + \frac{b}{m})$ . This is detailed in Figure 4.4 .

We may then express the sum of access/egress and wait times as the time lost,  $T_\ell = T \text{ access} + T \text{ egress} + T \text{ wait}$

$$= 2\bar{t} + T \text{ wait}$$

$$= 2\left(\frac{a+b}{m}\right) \frac{r_c}{v} + T \text{wait} \quad (v = \text{average access/egress speed})$$

Realizing that if previously, with one terminal location in the region,  $f_0$  frequencies were provided in the market, with  $m$  terminals the dedicated frequency at any terminal  $m$  becomes:

$$f_m = f_0/m$$

and hence,

$$T_\ell = 2\left(\frac{a+b}{m}\right) \frac{r_c}{v} + \frac{m}{2f}$$

By setting the differential  $\frac{\partial T_\ell}{\partial m}$  equal to zero, an optimum number of terminals,  $m^*$  may be determined. Similarly, an optimum dedicated frequency,  $f_m^*$  may be determined by substituting  $T_\ell$  into the demand equation.

Extensions of this model to include network effects are apparent.

This analysis assumes travel time a constant. In order to correctly model the congestion effects present in any "loaded" transportation network we incorporate a velocity function with parameter,  $q$ .

$$v(r) = v_c \left( \frac{r}{r_c} \right)^q \quad (r_c \leq r \leq R)$$

This varies with  $(r/r_c)^q$  as distance from the central core increases. Values of  $q$  are typically 0.40 but are related to  $R/r_c$ , the ultimate catchment radius ratio, so that at  $r=R$ ,  $v(R) \sim 50$  mph, while  $v(r_c) \sim 20$  mph, a characteristic maximum in the city center. Clearly,

$$q = \frac{\log [v_r/v_c]}{\log R/r_c}$$

Here,  $v_r/v_c$  describes the "far field" access velocity ratio of the suburbs with respect to the city core. This normally varies from two to five but is an intuitive function of  $R/r_c$ . As expected,  $q$  is a strong function of congestion and has an inverse relationship to the population density  $p(r)$ .

Now we can express the average access time  $\bar{t}$  directly as:

$$\bar{t} = \frac{\int \frac{r}{v} dp + \frac{2}{3} \frac{r_c}{v_c} P_c}{\int dp + P_c}$$

Again, the integration and graphical analysis reveals that there is still a linearized format with slope  $(m)^{-1}$  in the abscissa, but that

Figure 4.4 (a +  $\frac{b}{m}$ ) linearization of average access distance as a function of terminal radial location and number of terminals,  $m$

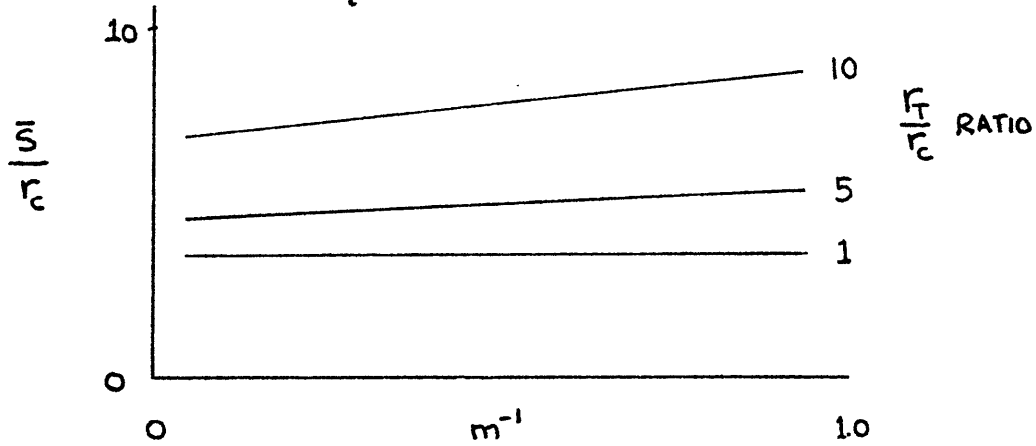
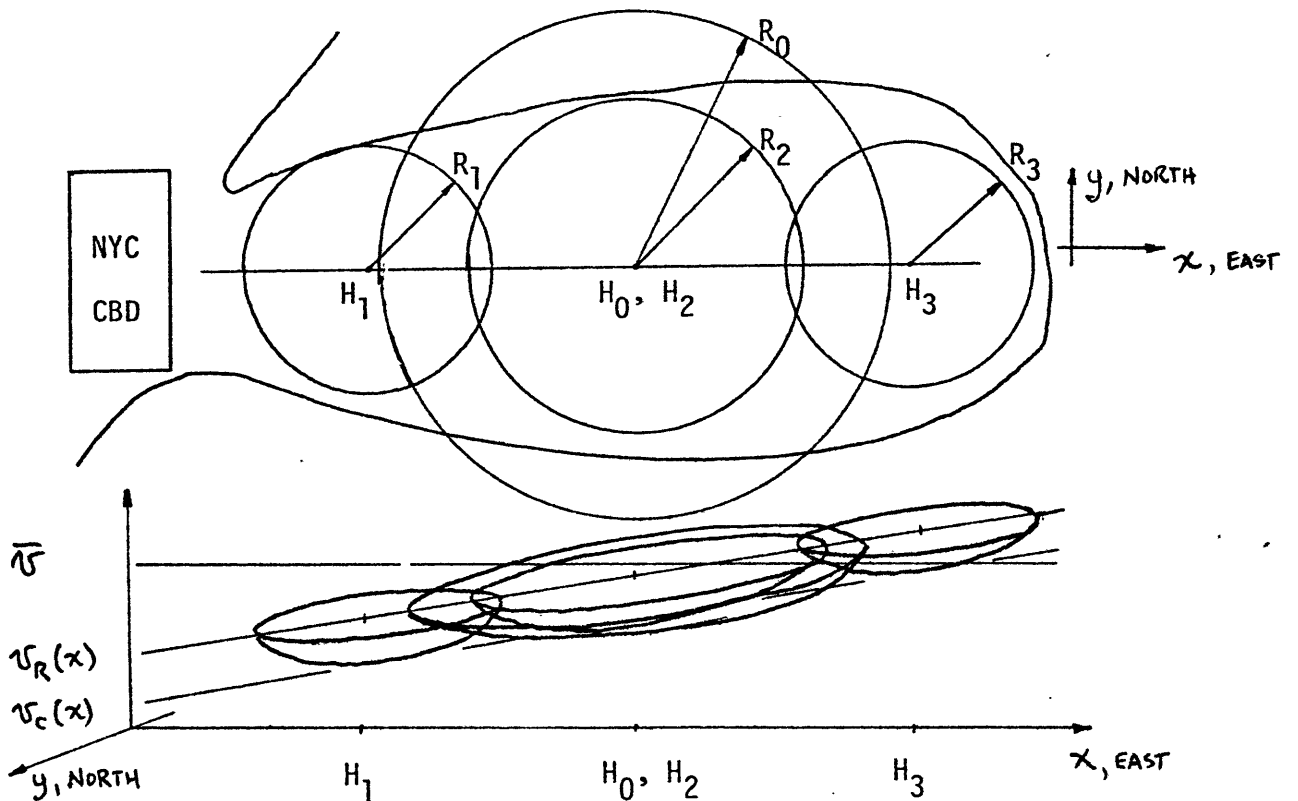


Figure 4.5 Long Island corridor market represented by multiple circularly symmetric hubs for analysis by closed-form methods; corridor access velocity profiles near hubs



access times are longer. This is expected, and even more clearly points out the necessity of city center terminal location.

Using this sort of analysis, we could also model a corridor market as several catchment radii,  $R_n$  located at hubs  $H_n$  located on a common line haul axis as shown in Figure 4.5, each with  $m$  terminals at  $r_{t,n}$ . This may be unacceptable, however, for the reasons that

--there is no truly exponential  $p(r)$  decay about the hubs,  $H_n$ .

--in order to model total service areas, catchment radii  $R_n$  will not be unique, or if  $R_n$  are unique, the model geometry does not allow the entire area to be modelled--it is not covered.

--modal split from such a formulation is difficult to envision,

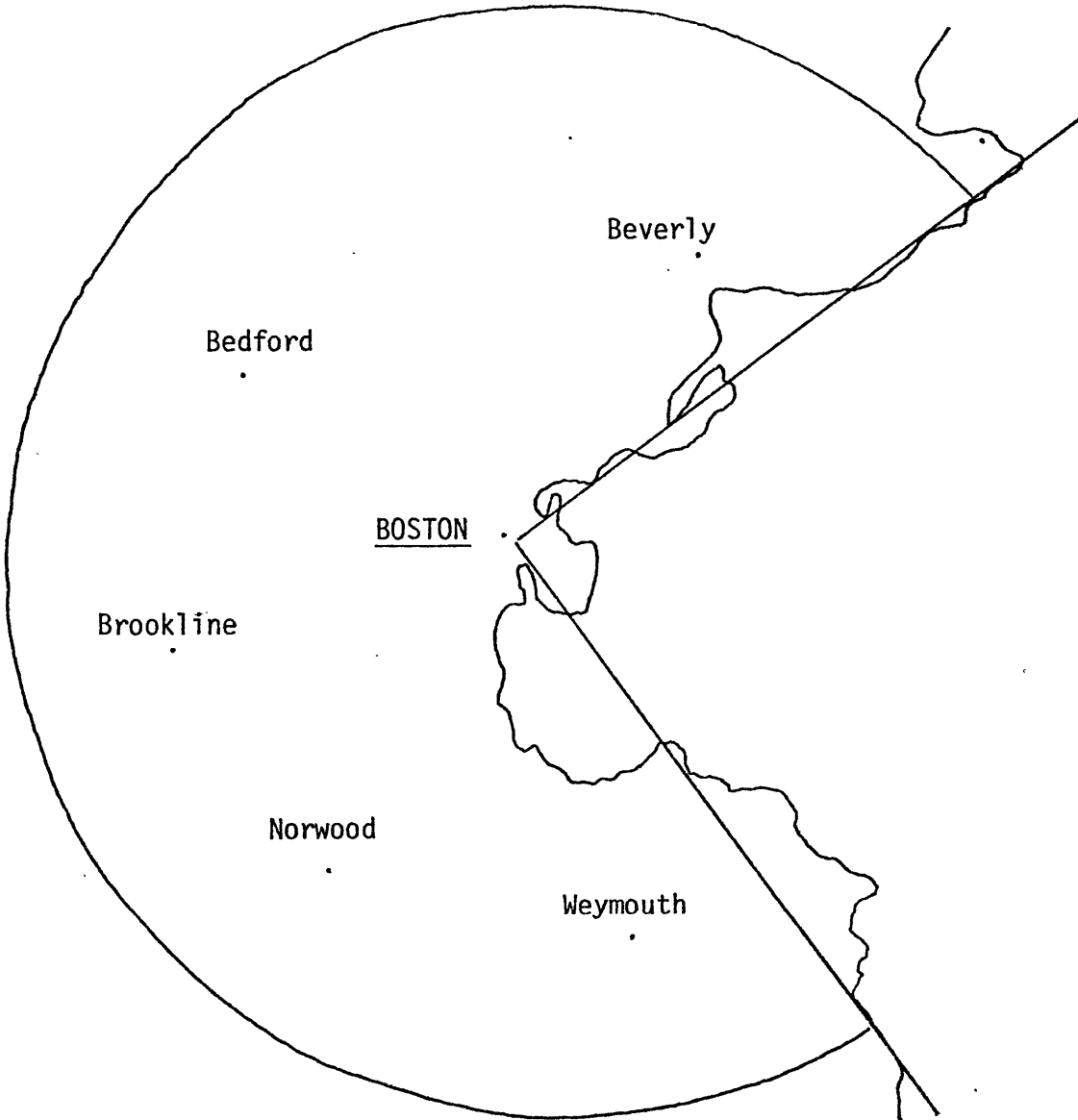
--congestion effects are hub-centered, not corridor east-west justified which clearly will be the more prominent effect.

A linearized normalization of the corridor axis velocity distribution is conceptually correct, but mathematically difficult to fathom.

Here we have touched on a geographic constraint to the model. This will become a major reason for utilization of a probabilistic random variate approach to the travel time problem.

The closed-form models of Genest consider various geometrically shaped regions; square, rhomboid, circular, and segments of circular cities. (Boston, for example, is a 270 degree city, Figure 4.6.) Yet these models suffer from extreme over-specification, and are highly

Figure 4.6 Boston, Massachusetts -- a "270 degree city"





specialized. Clearly, a more generalized--yet characteristic--modelling technique must be developed.

#### 4.2 An Alternative to Closed-Form Solutions

In order to deal with the spatial and temporal variations inherent to urban service systems, the use of a specialized mathematics of uncertainty is proposed to evaluate design criteria. Through employment of probability theory, geometrical probability, queuing theory and spatially distributed queues, the task of systems analysis in ultra-short haul air transportation is eased. In fact, this branch of mathematics and its related operations research techniques are useful in many forms of systems work.

The extent to which this area has been neglected in both the literature and in textual material amazes the author. Most information (save for a text to be published shortly by Larson and Odoni) being gleaned from various esoteric cookbook-ish compendia of theoretical and analytical techniques in probability theory. Nevertheless, exploitation of these techniques leads to breathtakingly applicable heuristic solutions for the problems facing urban systems planners.

In particular, the results of geometrical probability may be used to analyze the problems of access/egress times in multiply subdivided demand regions, or the theory of spatially distributed demand in customer-to-server systems such as ultra short haul air transportation. As mentioned previously, probability theory is responsible for the

standard transportation planning usage of expected waiting times of one half the scheduled headway and conversely the often encountered result that actual waiting times occasionally run to multiples of the scheduled headways ("clumping"). Models operating on a more micro scale still are able to predict the effects of temporal variations in demand intensity on system utilization and queues. In all, a powerful technique ideally suited to urban systems and transportation research problems.

#### 4.3 Probabilistic Techniques

Having seen the travel time modelling state of the art, we wish to be able to improve on this through the addition of some of the realistic uncertainties associated with the system being analysed. The model which will be presented here requires the reader to be somewhat familiar with functions of random variables. (The author published the preceding disclaimer as opposed to a text in probability theory). A good background in probabilistic modelling techniques is also assumed.

Perhaps of greatest importance will be the concept of the Poisson process--random incidence. In this process, events of interest are distributed randomly and uniformly along some dimension. Examples are Poisson arrivals distributed randomly and uniformly in an interval of time  $[0,t]$ , or "Poisson requests" for service distributed randomly and uniformly over a demand area  $A$ . This model has been found to be a rea-

sonable one for the generation of various events of interest.

A Poisson-type counting random variable  $N(t)$  has a probability distribution function of the form:

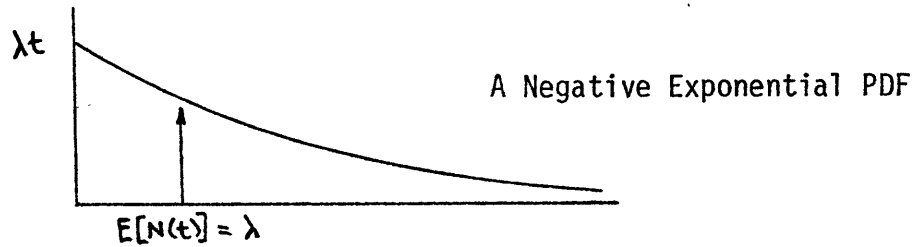
$$P[N(t)=k] = \frac{\lambda t^k e^{-\lambda t}}{k!}$$

Where  $k$  is the "intensity" parameter of the process, denoting the average rate per unit time of events occurring. It can be shown that the mean, or "expected" value of  $k$  and its variance are equal and of the form:

$$E[K] = \lambda$$

$$\sigma^2 [K] = \lambda$$

Figure (4.7) The Poisson Random Variable  $N(t)$



The process can be applied to a randomly and uniformly distributed spatial case. By substitution of an area function  $A(s)$  for  $t$  in our counting variable, this is obtained.

In terms of the travel time, however, let us return to a simpler model. A probabilistic approach to access/egress times.

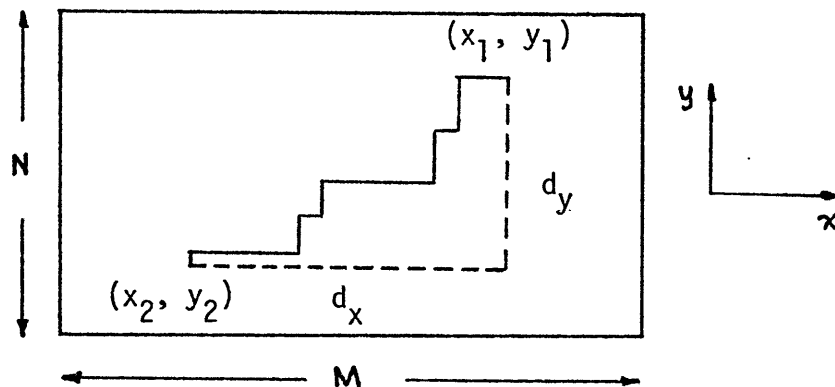
#### Mean Travel Time in a Single Sector

In this simplest case, consider a rectangular sector  $S$  Figure (4.8) of dimensions  $\bar{X}_0 = M$ ,  $Y_0 = N$ . The area is consequently  $A = M \cdot N$ .

Assume for the moment that travel distance between two random points is accomplished on a right-angle basis--normally called "Manhattan Metric"--parallel to either the X or Y axis.

Now, assume the positions of demand  $(X_1, Y_1)$  and servers  $(X_2, Y_2)$  are independent and uniformly distributed over the sector. From this, we can show that the random variables  $\bar{X}_1, \bar{X}_2$  are uniformly distributed from  $0 \rightarrow M$  and  $\bar{Y}_1, \bar{Y}_2$  similarly from  $0 \rightarrow N$ . Given these assumptions, all four variables are independent, and the probability distribution function for the X and Y components are easily found.

Figure 4.8 Travel Time in a Sector



Travel by "Manhattan Metric" (Right Angle Routing)

We have:

$$\begin{aligned}
 D_1 &= |x_1 - x_2| + |y_1 - y_2| \\
 &= d_x + d_y
 \end{aligned}$$

Letting  $\bar{X}, \bar{Y}$  be random variables representing distances in X,Y

directions respectively:

$$f_X(x) = \begin{cases} \frac{2(M-x)}{M^2} & , 0 \leq x \leq M \\ 0 & , \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{2(N-y)}{N^2} & , 0 \leq y \leq N \\ 0 & , \text{otherwise} \end{cases}$$

Since the Manhattan Metric distance is the sum of X component and Y component, we may use the scaling laws of random variables to state that the mean (expected) travel distance  $E[D_1]$  is the sum of the  $\bar{X}$  and  $\bar{Y}$  exceptions:

$$E[D_1] = E[D_x] + E[D_y]$$

Which, through evaluation is found to be:

$$E[D_1] = 1/3[M + N]$$

The distribution of  $D_1$ , should we wish to find it may be found by letting the random variable Z equal the sum of  $\bar{X}$ ,  $\bar{Y}$

$$Z = \bar{X} + \bar{Y}$$

The probability distribution function of Z may be evaluated by the convolution integral

$$f_z(z) = \int_{\bar{X}} f_{\bar{X}}(x) f_{\bar{Y}}(z-x) dx$$

From these expected distances we may derive the expected travel time for right angle metric response,  $E[T_1]$  by the assumptions of travel velocities  $v_x$ ,  $v_y$  in the x, y directions. The result is of the form:

$$\begin{aligned}
 t &= t_x + t_y \\
 &= \frac{dx}{v_x} + \frac{dy}{v_y}
 \end{aligned}$$

$$\text{and } E[T_{\perp}] = \frac{1}{3} \left[ \frac{M}{v_x} + \frac{N}{v_y} \right]$$

In case the right-angle metric is not a valid assumption, the straight line or Euclidean distance and time may be evaluated. Considering the relationship between the Manhattan Metric distance  $D_{\perp}$

$$\begin{aligned}
 \text{and the Euclidean distance, } D_e &= (X_1 - X_2)^2 + (Y_1 - Y_2)^2 \\
 D_{\perp} &= \frac{|x_1 - x_2| + |y_1 - y_2|}{D_e} \cdot D_e \\
 &= R \cdot D_e
 \end{aligned}$$

Where R is the ratio of the distances  $\frac{\text{Right Angle Metric}}{\text{Euclidean}}$ .

Again employing scaling law properties of random variables, we find:

$$E[D_{\perp}] = E[R \cdot D_e] = E[R] \cdot E[D_e]$$

Now this ratio R can be represented as follows:

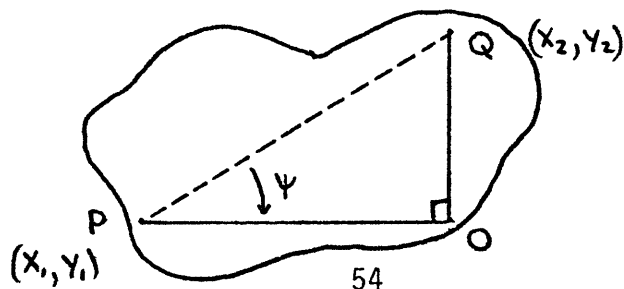


Fig. 4.9  
Any Region, R

Where

$$D_{\perp} = \widehat{PO} + \widehat{OQ} = |X_1 - X_2| + |Y_1 - Y_2|$$

$$D_e = \widehat{PQ} = (X_1 - X_2)^2 + (Y_1 - Y_2)^2$$

and random variable  $\psi$  = the angle of the X-axis with respect to the hypotenuse  $\widehat{PQ}$ .

Redrawing  $\widehat{PQ}$  as the diameter of a circle, we have:

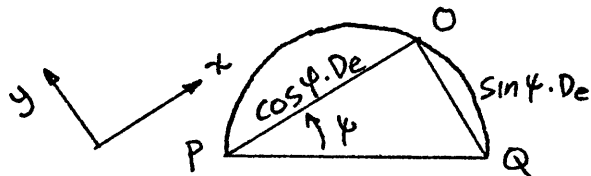


Fig. 4.10

Now as the axis system is rotated randomly and uniformly through an angle of  $\frac{\pi}{2}$ , representing all possible axis orientations,  $\widehat{PO}$  and  $\widehat{OQ}$  take on values from  $0 \rightarrow \widehat{PQ}$ . At any  $\psi$ , we have:

$$\widehat{PO} = \cos \psi \cdot D_E$$

$$\widehat{OQ} = \sin \psi \cdot D_E$$

Therefore, we may express the right angle metric distance;

$$D_{\perp} = \widehat{PO} + \widehat{OQ}$$

$$= (\cos \psi + \sin \psi) \cdot D_E$$

or,

$$R = \cos \psi + \sin \psi$$

For any particular  $\psi$ ; and utilizing the trigonometric identity, the ratio is:

$$(R|\psi) = \cos \psi + \sin \psi = \sqrt{2} \cos \left( \psi - \frac{\pi}{4} \right)$$

and the cumulative distribution function of R is:

$$F_R(r) = P\{R \leq r\} = P\left\{\sqrt{2} \cdot \cos\left(\psi - \frac{\pi}{4}\right) \leq r\right\}$$

by allowing  $\psi$  to vary uniformly over  $0 \leq \psi \leq \frac{\pi}{2}$

$$F_R(r) = 2 \int_{\text{L.L.}}^{\pi/2} \frac{2}{\pi} dr$$

Where the lower limit, L.L., of integration is:

$$\text{L.L.} = \left[\cos^{-1}\left(\frac{r}{\sqrt{2}}\right) + \frac{\pi}{4}\right]$$

We have:

$$F_R(r) = 1 - \frac{4}{\pi} \cos^{-1}\left(\frac{r}{\sqrt{2}}\right); 1 \leq r \leq 2$$

and: 
$$f_R(r) = \frac{dF_R(r)}{dr} = \frac{4}{\pi} \frac{1}{\sqrt{2-r^2}}; 1 \leq r \leq 2$$

or: 
$$E[R] = \frac{4}{\pi} \sim 1.27$$
  

$$\sigma^2[R] = 1 + \frac{2}{\pi} - \frac{16}{\pi^2} \sim 0.121$$

Hence, for future reference, the right angle metric and euclidean distances yield excellent upper and lower bounds for non-barred responses\* , and we find:

$$\frac{E[D_E]}{E[R]} = \frac{E[D_1]}{E[R]} = \frac{\pi}{\sqrt{2}} [M+N]$$

---

\*Barriers to response may be considered also, and change the result only slightly.



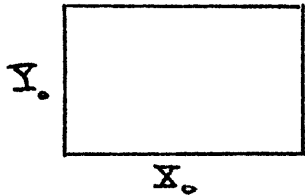
or:

$$E[T_E] = \frac{\pi}{12} \left[ \frac{M}{v_x} + \frac{N}{v_y} \right]$$

Which is the direct travel time lower bound.

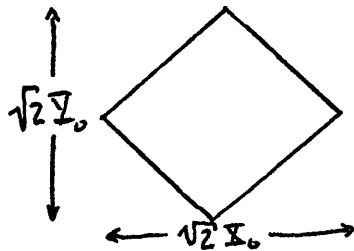
We may expand these results to variously shaped distances using other results from geometrical probability. Larson does so and finds, assuming a right angle metric travel distance and travel velocities of  $v_x$  and  $v_y$ , the travel times in the following rectangularly shaped sectors of area A:

Rectangular Sector:  $A = \bar{X}_0 \cdot \bar{Y}_0$



$$E[T_1] = \frac{1}{3} \left[ \frac{\bar{X}_0}{v_x} + \frac{\bar{Y}_0}{v_y} \right]$$

Diamond Sector:  $A = \bar{X}_0 \cdot \bar{Y}_0$



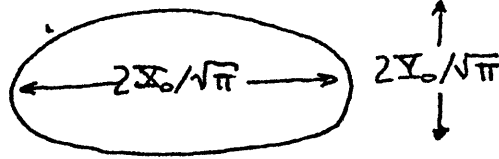
$$E[T_1] = \frac{7\sqrt{2}}{30} \left[ \frac{\bar{X}_0}{v_x} + \frac{\bar{Y}_0}{v_y} \right]$$

Other potentially desirable convex region geometries are more difficult to analyze and require special methods.

Circular (Elliptical) Sector:

We will consider a sector of Area =  $\bar{X}_0 \cdot \bar{Y}_0$  (semi-axes  $X_0/\sqrt{\pi}$ ,

$Y_0/\sqrt{\pi}$ ) as before, again, positions of passengers and terminal independent and uniformly distributed over a region such as that below:



We could solve directly for the distance between  $(X_1, Y_1)$ ,  $(X_2, Y_2)$  but it is fairly involved. Rather we prefer to use another method: Crofton's Theory of mean values. [[Appendix (A)] Consider the Euclidean ("as the crow flies") distance between two points distributed independently and uniformly over a circle of area  $A$ . By Crofton it can be shown that the expected value of the Euclidean distance,  $D_E$  is:

$$E[D_E] = \frac{128}{45\pi} \sqrt{\frac{A}{\pi}}$$

Now, consider the right angle, or metropolitan distance  $D_L$  between the points. Relating  $D_L$  and  $D_E$  we find, as before:

$$D_L = \frac{|X_1 - X_2| + |Y_1 - Y_2|}{\sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}}$$

Or:  $D_L = R \cdot D_E$  where  $R = \frac{\text{Right Angle}}{\text{Euclidean}}$

Hence,  $E[D_L] = E[R] \cdot E[D_E]$

$\frac{v_x}{v_y}$  an optimal sector dimension:

$$\bar{X}_0^* = \sqrt{\frac{v_x}{v_y} \cdot A}$$

$$\bar{Y}_0^* = \sqrt{\frac{v_y}{v_x} \cdot A}$$

This has the effect of equalizing the orthogonal response time components  $t_x, t_y$ .

Having overviewed the concept of geometrical probability, the reader may have noticed (and questioned) the assumption of both points being randomly located. This in fact is a Poisson-like situation, and is the most general result for any particular geometry. Any individual case which provides an average response for that particular sector which is greater than the random incidence result is a substantially inferior facility location. Fixing the position of either the customer or the server in the analysis is in fact easier analytically, and is in general the method employed in assessing a districting impact.

In the case of the rectangular sector, of Figure 4.8 above, what is the impact upon expected travel time of locating a facility in the center of the district at  $(\frac{M}{2}, \frac{N}{2})$ ? We have, now for the right angle metric case:

$$\begin{aligned} D_1 &= |x_1 - \frac{M}{2}| + |y_1 - \frac{N}{2}| \\ &= (dx | x_2 = \frac{M}{2}) + (dy | y_2 = \frac{N}{2}) \end{aligned}$$

And, by inspection this is

$$E[D_{\perp}] = \frac{1}{4} [M+N]$$

$$E[T_{\perp}] = \frac{1}{4} \left[ \frac{M}{v_x} + \frac{N}{v_y} \right]$$

Or, the centrally located facility cuts  $E[T_{\perp}]$  by 24 percent.

We may look at an alternate result as well. For the same value of system performance--say expected response time--how much larger can a district be when employing a central facility? Clearly, using the rectangular sector case, the area ratio of centrally located facility region  $A_c$  to the randomly oriented case,  $A$ , is:

$$A_c = \frac{4}{3} A \Rightarrow A \text{ 33\% larger district.}$$

By judicious location of  $n$  facilities in a point lattice-like array, covering a randomly shaped region,  $R$ , expected travel time may be minimized. Such problems in coverage are a direct outgrowth of geometrical probability.

Returning to our optimal sector design, with dimensions:

$$\bar{X}_0^* = \sqrt{\frac{v_x}{v_y} \cdot A}$$

$$\bar{Y}_0^* = \sqrt{\frac{v_y}{v_x} \cdot A}$$

We may note that by algebraic manipulation, we may now express the

mean travel time in the optimal sector as:

$$E[T_{\perp}^*] = 2c \sqrt{\frac{A}{v_x v_y}}$$

Where C is the same function of geometry derived previously. In the rectangular sector case, we have:

$$E[T_{\perp}^*] = \frac{2}{3} \sqrt{\frac{A}{v_x v_y}}$$

Where the assumptions here are that due to optimal sector construction,  $t_x = t_y$ . In fact, however, recent studies by Larson have shown that in actual sectors, where this assumption may not be met, the mean travel times are within a few percent of predicted times. A good example of how, (again and again and with no apparent reason) in probabilistic situations, "Nature is kind to us."

In an extension of rectangular sector design, specific to the case of large grids of individual demand regions easily quantized for machine solution, the following rules are easily developed from the preceding discussion, and the rules of so called intersector dispatch. For an arbitrary large grid of M discrete demand regions  $A_m$  of total area A divided into N catchment regions  $A_n$  acting as watershed for N facilities of comparable design capacity, (Figure 4.11 ) it follows that the expected time for a passenger desiring to travel to a particular terminal is approximately:

$$E[T] \sim \frac{2}{3} \sqrt{\frac{A}{Nv^2}}$$

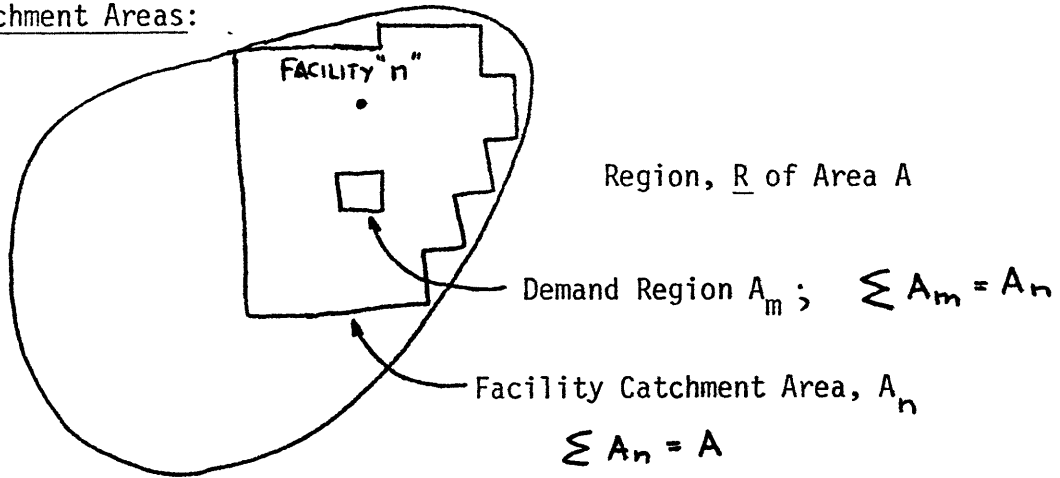
(where the previous assumption of  $V_x = V_y$  yields  $V^2$ ). This may be modified by an "inconvenience factor" which is a function of facility utilization or congestion so that:

$$E[T] \sim \frac{2}{3} \sqrt{\frac{A}{NV^2}} (1+\rho) \quad 0 \leq \rho < 1$$

the functional  $\rho$  is the probability that the particular facility designed to serve the particular demand region is unavailable or inconvenient for some reason. Alternatively,  $\rho$  can be viewed as a "congestion" term. This may be due to poor scheduling or other poor level of service quantity. If all terminal facilities provided an equal level of service then  $\rho = 0$ , due to relative uniformity of service offering. Were the primary facility unavailable for some reason and the second best terminal utilized with probability = 1, then  $\rho = 1$  and we expect an increase in the travel time to access the non-optimal facility. It can be shown through arguments related to those employed in queuing theory that below  $\rho = 0.7$ , the expected travel times vary linearly with  $\rho$ . Above this value, "congestion effects" cause a sharp rise in  $E[T]$ , invalidating the relationship.

Figure 4.11 Arbitrary Region,  $R$  with Demand Regions and Facility

Catchment Areas:



The next logical development step in the modelling technique is the assumption that the individual demand regions  $A_m$  are indivisible "atoms" of demand. That is, over each  $A_m$  the demand rate  $\lambda_m$  is a negative exponentially distributed Poisson parameter with mean  $\lambda_m$  and interdemand time  $\frac{1}{\lambda_m}$ . At this point, the model assumes a full spatially distributed queuing form analyzable by use of a hypercube queuing model. Further still, the analysis of "peaking" characteristics of urban traffic flow can be undertaken by the employment of a temporarily varying demand intensity  $\lambda_m(t)$ . The results and analytic methods of Koopman can be employed to derive time varying utilization statistics  $\rho(t)$ , and hence, the time varying components in travel times and facility queues. Temporal variations in demand accounted for, optimal time-of-day scheduling and fleet size requirements are then devised subject to operational and financial constraints.

These last areas including the hypercube queueing model are areas of current research in urban systems analysis and somewhat beyond the scope of this paper. It should be realized, however, that the orderly progression of analysis presented here should shed further light on ultra short haul air transportation systems as well.



#### 4.4 Decision Variables For Systems Optimization

Having reviewed a range of probabilistic modelling techniques, it is possible to pick out several binding policies in formulating a travel time model. From these, travel time optimization variables can be chosen. Let us return to our ultra short haul air transportation example and develop these decision variables.

In general, the ultra short haul air example can be considered either intra regional or point-to-point. The former case is best exemplified by metropolitan region, corridor, or city center operations, the latter by single market operations. Viewed at a micro level, the intra-regional case is a series of point-to-point operations with the possibility of intersector "dispatching" according to some route matrix,  $r$ . The unfortunate problem with the intra-regional case is its combinatorial mathematical nature-- the problem is in general, underspecified. With the addition of several key assumptions, though, the problem is specific enough for analysis.

Since one of the system performance measures of interest in this case is the access/egress time, we will wish one output to be  $E[T]$ . Analyzing now the single market case, the access time is seen to be a function of the market subdivision factor. In effect, how convenient (how many) are the terminal facilities at the origin and destination. From our previous analysis, we see that the expected

travel time is:

$$E[T_a] \sim \frac{2}{3} \sqrt{\frac{A_a}{N_a V_a^2}}$$

where

$A_a$  is the area of origin region

$N_a$  is the number of origin region terminal facilities

$V_a$  is the isotropic average travel velocity.

We may form a similar expression for  $E[T_e]$ , the mean egress time, by specifying  $A_e$ ,  $N_e$  and  $V_e$ . A travel time reduction by the square root of subsector area is noted.

By some device, we wish to divide the origin and destination demand region into  $N_a$  and  $N_e$  catchment areas feeding corresponding primary facilities. The actual location of these facilities may be found through application of a standard linear programming facility location solution methods, or for small  $N$  by hand. Another criterion is design capacity of each facility, which is related to station workload and utilization and the demand rate in the subregion. An intuitive result from queueing theory dictates sector design by workload (traffic density) matching, yet in the ultra short haul air case, this result may not be applicable.

The unavailability (or inconvenience) of the primary facility, leading to an increase in expected travel time will often lead the prospective demand to "balk" -- to leave the system entirely -- as opposed to enduring a subjective decrease in level of service at a

particular price. This is the basis of any modal split demand model, and must be a constantly present feedback loop for evaluation of policy considerations.

It is clear that the optimal access/egress travel times for any particular passenger may be optimized by maximizing the number of such facilities such that access/egress is reduced to a small fixed "start up" time. From several other system performance considerations, however, this is not possible. Economies of scale in vehicles and facilities dictate spreading of fixed cost components of direct and indirect operating costs over as many users as possible. This criterion yields large vehicles, large facilities and bulk service of patrons. These results may be augmented somewhat in the vehicle area by multistop load building patterns, at the expense of increases in block times due to extra cycle times and circuitry of routings.

As a result, all components of travel time must be considered, as it is relative portal-to-portal time that will determine market penetration. A total travel time model of the following form is indicated:

$$TTT = t_0 + t_1(\rho m) + t_2 / N_a + t_3 \cdot d(k) + \frac{t_4}{F} + t_5(k+1) + t_6 / N_e$$

where  $t_0$  = sum of various zero length times, such as "start up time", fixed facility-associated times, and other related non-functional time relationships.

(where)  $t_1$  = passenger processing times. A queueing result function of temporal demand rate and facility design capacity yielding  $\rho_m$ , facility utilization.

$t_2, t_6$  = origin destination region single facility access times (from geometric probability result.)

$N_a, N_e$  = number of origin destination region terminal facilities

$t_3$  = cruise time per mile  $= (V_{cr})^{-1}$

$d(k)$  = market length of haul modified by a circuitry factor based on number of intermediate stops,  $k$ .

$t_4$  = effective half-length of operating day

$f$  = market frequencies per day

$t_5$  = cycle time, a function of air traffic control constraints, but judged constant.

While this model attempts to identify and implement all of the causal relationships present in the expected total travel time, it is for the most part unmanageable in a practical sense. The facility utilization term  $\rho_m$  may be determined by queueing theory through (generally valid) assumptions of negative exponential passenger arrival and service time distributions\*, with mean arrival rates  $\lambda_m$

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\*It may be argued that due to "clumping" near departure times, arrivals are in fact a negative binomial process. In fact, passenger build-up in lounge areas or ticket lift areas supports this contention. It is not, however, a limiting assumption.

and mean service times  $\frac{1}{\mu_n}$ . Data on the number of passenger (ticket counters, etc.) processing stations,  $p$ , at the facility as a function of time allows interpretation of  $\rho(t)$  by use of an M/M/p queueing model. Were a demand oriented (vs. scheduled) service envisioned, this model and its expected wait in queue statistics would yield the demand arrival rates at the gate required for scheduling.

For preliminary analysis, this  $\rho_m$  will be assumed constant, yielding  $t_1$  constant as well. Future microscale analysis should include this term.

By assuming segment circuitry factor,  $d(k)$  to be either constant or small with respect to ultimate market stage length, the block time component,  $t_2 \cdot d$  will also be fixed for a particular market. This is reasonable for a point-to-point market analysis, but breaks down with the network considerations of an intraregional case.

The simplified model of the form

$$TTT = t_0 + t_1 / N_a + t_2 \cdot d + t_3 / f + t_4(k+1) + t_5 / N_e$$

The decision variables for the travel time optimization are then:  $N_a$ ,  $N_e$ ,  $f$ ,  $k$ . Those, with the addition of cost optimization variables will be employed in the demand and market share formulation in order to create a total picture of level of service and utility.

## 5.0 Demand As a Function of Optimization Variables

Up to this point, a range of variables influencing the costs of providing ultra-short haul air transportation and the various components of total travel times associated with such service have been identified. The task remaining is the assessment of the quantitative effects on demand and market share due to exercising these decision variables. Ultimately, analysis of particular problems will require the development of a constraint framework so that a meaningful objective function may be optimized.

In assessing these impacts, primary attention must be paid to variable cross-elasticity problems, as these affect not only the outcome, but our ability to solve the problem by reasonable analytic means. Ideally, solution by differential means or linear programming methods are a hoped for result in this form of research. Unfortunately, the combinatorial and probabilistic nature of the problem in its simplest form (and a combinatorial/probabilistic/networks problem in its general form) forces either an iterative solution in the small scale, or a machine aided mathematical programming solution in the general case.

The outline of such a solution method will be presented here, and fine tuned for specific case usage.

In assessing demand impact, we are relating factors which quantitatively characterize level of service - utility - to some abstract

behavioral psychological human reaction - propensity to travel from A to B. In the general case, by varying individual level of service components we vary the utility of the mode. We may theorize that prospective passengers attempt to minimize a weighted sum of disutilities in the choice of whether to travel or not, and if so, which mode to utilize. This brings us back to our demand model, which may or may not handle the "tails" of the problem effectively. Market calibration on the basis of existing modal technology is valid only near the existing range of trip times and costs. Alternative transportation technology, by its very definition is sure to be outside this calibration range, creating some amount of forecasting uncertainty.

Some portion of our reliance on the product form model stems from its linear format in a logarithmic plane and accompanying ease of calibration by multiple linear regression techniques. While exhibiting certain disquieting tendencies at extrema, it is mathematically tractable under analysis. As a result, it will be used to analyze the effects of the decision variable identified previously.

The formulation for a particular market is:

$$D = KC^{\alpha}T^{\beta}$$

Where  $D$ , demand is a function of level of service passengers

$C$ , the total trip cost

$\alpha$ , the cost elasticity of demand

T, the total trip time  
 $\beta$ , the time elasticity of demand  
and K, a proportionality constant.

### 5.1 The Level of Service Vector Quantity, ( $\overline{LOS}$ )

Previous development has shown that for optimization purposes, trip cost and travel time will each have a multicomponent functional relationship of the form

$$C = \text{function}(S, n, f, k)$$

$$T = \text{function}(n, f, k)$$

From these relationships, the concept of a multidimensional level of service vector is indicated. Through the calculus, we will attempt to analyze a multidimensional demand surface by differential means, iterating to an optimal tableau of level of service element values. Although convexity of the demand surface and uniqueness of solution are not assured, adequate subjective evidence of stability can be shown.

The non-linearity within the problem specification and the level of service vector does not lend itself to total differentiation, hence the use of partial derivative with respect to each vector quantity is proposed.



We wish to determine:

$$\frac{\partial D}{\partial LOS} = \begin{bmatrix} \frac{\partial D}{\partial f} \\ \frac{\partial D}{\partial n} \\ \frac{\partial D}{\partial k} \\ \frac{\partial D}{\partial s} \end{bmatrix}$$

These are easily derived from the product model and the time and cost equations.

Looking at the passenger perceived sensitivities to frequency of service, market subdivision, multistop routings and vehicle size, and assuming initially that fare is fixed, we have the following:

#### 5.1.1 Frequency Sensitivity

$$\begin{aligned} D &= KF^\alpha T^\beta \\ &= K' T^\beta \\ &= K' \left( T' + \frac{t_2}{f} \right)^\beta \\ \frac{\partial D}{\partial f} &= \beta K' \left( T' + \frac{t_2}{f} \right)^{\beta-1} \left( -\frac{t_2}{f^2} \right) \\ &= \frac{\beta D \left( -\frac{t_2}{f} \right)}{T} \end{aligned}$$

and therefore that  $(\partial D/D)/(\partial f/f) = \frac{\beta}{f} \left( -\frac{t_2}{T} \right)$

From this it may be seen that as might be expected in the limiting

case,  $\partial D/\partial f$  approaches zero at infinite frequency. A useful concept is that of saturation frequency,  $f^*$  at which a certain high percentage of the ultimate (infinite frequency) demand is generated. It has been shown by Simpson that  $f^*$  is an inverse function of length of haul, being higher in short haul "convenience" markets and lower in long haul "value of service" markets. The variation is shown in Figure 5.1 for ultra-short haul through transcontinental lengths.

### 5.1.2 Access/Egress market subdivision sensitivity

$$D = K' \left( T' + \frac{nt_2}{f} + \frac{t_1}{\sqrt{n}} \right)^\beta$$

$$\frac{\beta D}{\beta n} = \frac{\beta D \left( \frac{t_2}{f} - \frac{t_1}{2n^{3/2}} \right)}{T}$$

and therefore that  $(\partial D/D)/(\partial n/n) = \frac{\beta}{T} \left[ \frac{nt_2}{f} - \frac{t_1}{2\sqrt{n}} \right]$

The creation of submarkets within a demand region has a diminishing return to scale given a fixed daily frequency, but is a function of the ratio of access to displacement times. Minimizing the expression above leads to

$$\frac{\frac{\partial D}{\partial n}}{\frac{D}{n}} \sim n \left( \frac{t_2}{f} \right) - n^{-1/2} \left( \frac{t_1}{2} \right)$$

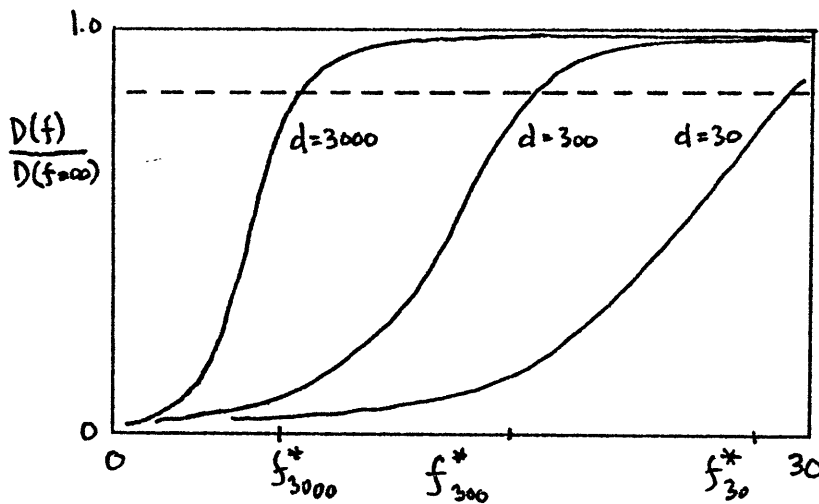
or at the minimum,  $\frac{t_1}{t_2} = \frac{2n^{3/2}}{f}$

given the ratio  $\frac{t_1}{t_2}$  and  $f^*$  we have

$$n^*\left(\frac{t_1}{t_2}, f^*\right) = \left(\frac{f^*}{2} \cdot \frac{t_1}{t_2}\right)^{\frac{2}{3}}$$

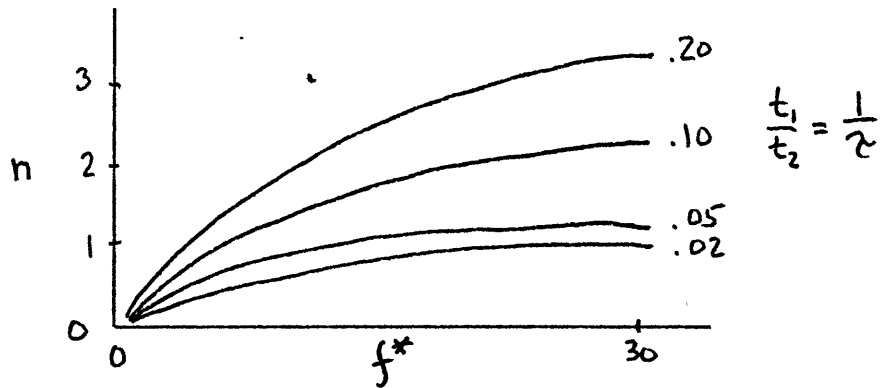
For a typical market, this is shown in Figure 5.2, and is the equivalent of a saturation market subdivision parameter. Note the significance of  $\frac{t_1}{t_2}$ . In a given market this expresses a "congestion factor",  $\alpha$ , which as it increases may justify multiple terminal facilities.

Figure 5.1 Saturation Frequency



$f^*$  Varies inversely with length of haul.

Figure 5.2 Optimum Number of Market Subdivisions



5.1.3 Multistop cycle time sensitivity

$$D = K'(T'+t_3(k+1))^\beta$$

$$\frac{\partial D}{\partial k} = \beta K'(T'+t_3(k+1))^{\beta-1} t_3$$

$$= \beta D \left[ \frac{t_3}{T} \right]$$

and therefore  $(\partial D/D)/(\partial k/k) = \beta k \left[ \frac{t_3}{T} \right]$

An increase in the number of intermediate stops is seen to have the expected negative effects on demand due to increased block to block time. The network implications, however, dictate that intermediate stops allow the use of a larger vehicle, with load building and frequency spreading in the associated sub-markets. The passenger, however, sees only the increase in total travel time, hence the deleterious effects on demand.

#### 5.1.4 Vehicle Size Sensitivity

Demand is not a function of vehicle size viewed from the passenger's viewpoint in a fare fixed scenario. Therefore,

$$\frac{\partial D}{\partial S} = 0$$

This is a subject for management/marketing psychologists, affecting some subjective "image" or "confidence" variable. Utility, however, is not affected, resulting in the above relationship.

#### 5.2 Market Share Variations With Demand

Having determined analytically the demand model's reaction to frequency of service, market subdivision, multistop operations and vehicle size, the corresponding variation in market share can be determined. The product formulation of modal split model is defined

$$D_{ab} = \sum D_{ab_j} \quad \text{for all modes, } j$$

$$= M_{ab} \sum P_j^\alpha T_j^\beta$$

or,  $MS_{ab_j} = D_{ab_j} / D_{ab}$

$$= \frac{P_j^\alpha T_j^\beta}{\sum P_j^\alpha T_j^\beta}$$

$$= \frac{D_{ab_j}}{D_{ab_j} + \sum_{j \neq i} D_{ab_j}}$$

This would appear simple to analyze, yet one of the drawbacks of the

product demand model is that it makes market share analysis tedious.

A phenomenon known as demand stimulation occurs when any existing mode increases its level of service, or a new mode is introduced. In effect, "new" patrons whose level or service requirements match those offered by the addition are created by the model. This demand is then rationalized in the process of normalizing the total market's demand. The resulting market shares incorporate the switching in passenger preference. This is shown for a hypothetical market in Table 5.1 where high speed ground transportation is added.

We have defined the variations on the demand surface as the various vector components of  $\partial D/\partial LOS$ . From this, we desire  $\partial MS/\partial LOS$ . In terms of the stimulated demand,  $\delta D_k$  due to mode k, we have

$$\begin{aligned}
 D' &= D + \delta D_k = \sum_j D_j + \delta D_k & * j \\
 D'_k &= D_k + \delta D_k \\
 MS'_k &= MS_k + \delta MS_k \\
 &= \frac{D_k + \delta D_k}{\sum_j D_j + \delta D_k} & * j \\
 &= \frac{D_k + \delta D_k}{\sum_j D_j + \delta D_k} + \frac{\delta D_k}{\sum_j D_j + \delta D_k}
 \end{aligned}$$

which, if  $\delta D_k \ll \sum_j D_j$ , yields

$$MS'_k = MS_k + \frac{\delta D_k}{\sum_j D_j + \delta D_k}$$

or 
$$\delta MS_k = \frac{\delta D_k}{D}$$

Table 5.1

Demand Stimulation in Metropolitan Corridor, Businessstown to

Capital City:

<u>Existing Modes</u>	<u>Total Travel Time</u>	<u>Out of Pocket Cost</u>	<u>Market Share</u>
Air	4	70.	56.2
Car	12	50.	22.5
Bus	16	40.	21.1
Foot	200	150.	~ 0.3
HSGT	10	60.	?

If any mode improves service then total demand increases.

Hence, normalizing for inclusion of High Speed Ground Transport

<u>Mode</u>	<u>MS</u>	<u>ΔD,%</u>	<u>ΔMS</u>
Air	48.1	-14.4	-8.1
Car	19.2	-14.4	-3.3
Bus	18.0	-14.4	-3.1
Foot	~ 0.3	-14.4	~ 0
HSGT	17.4	—	+ 14.4

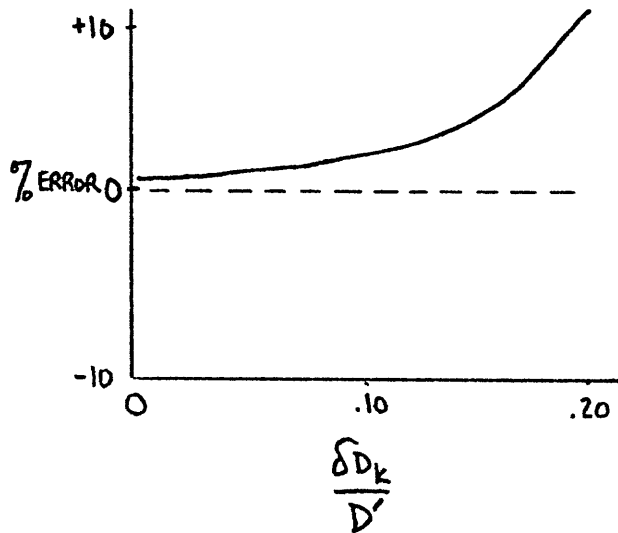
By this relation of change in demand to sum of initial and stimulated demands, the incremental market share is determined. The error in this estimate is plotted in Figure 5.3 as a function of the ratio of stimulated demand to initial demand. The small  $\delta D_k$  assumption is found to be valid within a stimulated demand percentage of  $<0.15 D$ .

From these level of service impacts on demand and market share, the effects on carrier revenues may be determined as functions of operational policy variables. In order to perform a profit or contribution maximization, however, it is necessary to know the supply cost variations with each policy alternative.

Figure 5.3

Error Analysis:

Small  $\delta D_k$  Assumption in Calculation of  $\delta MS_k$





## 6.0 Marketing Strategies

Due to the nature of the problem at hand -- that is, a quasi-stable relationship between demand, costs and prices -- it is necessary to look to the operator's perspective view of level of service in the marketplace. The implementation of any policy affecting quality of service -- utility to the public-- profoundly affects his costs of providing a supply of transportation services. This concept is easy to grasp in the direct operating cost regime, but rather more difficult to assess in the indirect costs area. It was shown that for the ultra-short haul operator, these indirect operating costs and associated passenger oriented costs account for a large portion of total costs. It remains now to relate supply costs to operational policy, and demand to supply of transportation services.

## 6.1 Operational Policies

The analysis of supply costs for the ultra-short haul operator follows a similar line to that of the demand analysis, but addresses the operator perceived variations in operating costs due to level of service and quantity of service changes. Their costs may be expressed as a sum of direct and indirect costs:

$$C = \text{Direct Operating Cost} + \text{Indirect Operating Costs} + \text{System Overhead}$$

$$C = (C_0 + C_1 \cdot S_a) t_{\text{flight}}^f + N \cdot \text{SOC}$$

$$\text{when } t_{\text{flight}} = t_{\text{block}} + (k+1)t_{\text{cycle}}$$

(The overhead will not enter into our cost functional. It is generally a straight percentage of revenues and may be ignored for now.) .

By analysis of this equation, it is possible to find the operator's percentage and marginal costs of implementing changes in service frequency, vehicle size, market subdivision, and load building multistop policy.

Differentiating with respect to the operational policy decision variables defined previously, the following relationships are developed.

### 6.1.1 Frequency Sensitivity

$$\begin{aligned} \frac{\partial C}{\partial f} &= (C_0 + C_1 \cdot S) t_f = \text{DOC}_{\text{hr}} \cdot t_f \\ &= \text{Marginal Cost per Frequency} \\ &= \text{MC}_f \end{aligned}$$

Although some of the indirect cost components relate to station utilization and passenger volume, in marginal frequency terms these are treated as fixed, as they are not a variable cost component. This is the operator's incremental cost of a frequency

and can be seen to vary with vehicle size. This is shown in Figure 6.1 .

### 6.1.2 Vehicle Size Sensitivity

$$\begin{aligned} \frac{\partial C}{\partial S} &= C_1 \cdot t_{\text{flight}} \\ &= \text{Marginal cost per seat} \\ &= MC_s \end{aligned}$$

In response to changing market conditions, an operator may desire to employ a vehicle of different capacity.  $S$ . From the derivation of the direct operating costs model, the marginal cost of "adding a seat" may be determined. This should not, however, be construed as an operative policy. The unit of supply is the frequency, hence the variation with vehicle size is a planning era exercise. The operator may only exploit economies of scale in vehicle size before the fact or with "inflatable vehicles."

### 6.1.3 Market Subdivision

$$\begin{aligned} \frac{\partial C}{\partial M} &= SOC \\ &= \text{Station Creation Cost} \end{aligned}$$

The cost of subdividing the market is just the cost of station creation and operation, assuming this analysis is undertaken in the initial planning stages.

### 6.1.4 Multi-Stop Sensitivity

$$\frac{\partial C}{\partial k} = t_{\text{cycle}} \cdot DOC_{\text{hr}} + [\text{circuitry factor}]$$

This is merely the cost associated with the cycle time plus some circuitry term. On longer haul operations, circuitry is treated as a higher order term in  $k$ , but this may be overly simple for ultra short haul operations. As segment additions increase  $d$ , there may also be a  $t_{\text{flight}}$  increment to assess. In any case the causal variable is time, leading to the above relationship.

From these sensitivities, it is possible to show that at the market level, economies of scale exist. As demand, and the quantity of services increase, the unit supply costs are lower. Looking at our point-to-point market, variable cost reductions are attributable to larger aircraft. Pure economic efficiency corresponds with a fewer number of frequencies with larger passenger loads.

While this is true in cases where indirect costs are not a high percentage of total cost, it ignores the effects of changing demand with level of service variation.  $N$  large vehicle frequencies supplying  $q$  seats do not provide the same level of service as  $2N$  smaller vehicle frequencies supplying the same  $q$  (or likely a greater number of) seats. Variable ground costs per passenger due to differences in station loading (again a characteristic of ultra short haul air transportation) are also neglected in this classic economic efficiency formulation.

The network basis of the intraregional problem injects further difficulty into the determination of supply costs, making it impossible to analyze quality variations in marginal costs unless

the level of service vector quantities are held constant. Unit supply incremental costs also vary with time of day and routing factors due to the inability to correctly account for and allocate costs. The calculus of variations is not up to the task created by such a problem. Determination of unit quantities for further economic analysis requires that a scheduling function which optimizes profit be constructed; only in this manner may supply quantity be altered and marginal costs and revenues be found. As a result, only in the simplest cases will unique marginal costs exist. For other than these cases, a cut and try with recursive parametrization will be required.

Before embarking on a case study, however, the issue of operational policy and demand-supply must be clarified.

## 6.2 Market Solutions

It has been shown that for our demand model, the demand level of service/cost relationships are similar to those in Figure . This consumer model has its analogue in the operator's supply/LOS/cost relationships. Supply quantities available at particular level of service and variable costs are shown in figures 6.1, 6.2. It may be seen that at a given level of service and vehicle size, as output quantities are increased, average variable costs per seat asymptotically approach the expected marginal cost per seat.

Figure 6.1 Variable costs based on frequencies offered

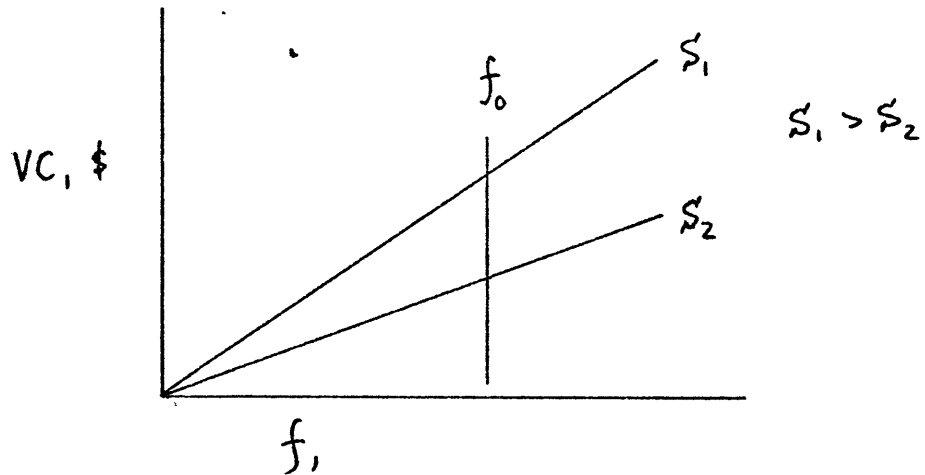
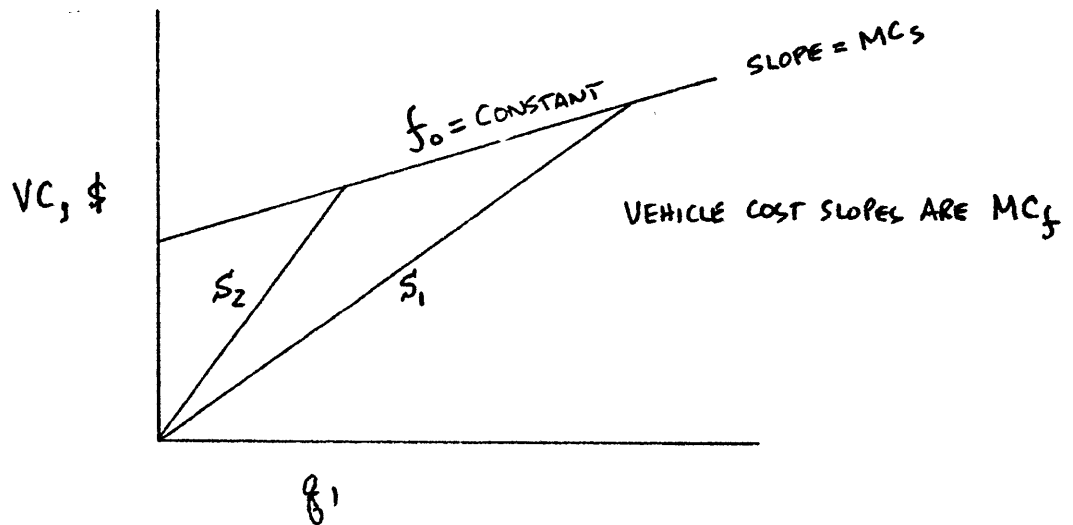


Figure 6.2 Variable costs based on quantity of seats offered



where  $g = f \cdot S$ , and

$$\lim_{S \rightarrow g} MC_f = MC_s$$

These variable operational costs may be derived from the direct operational cost per hour equations:

$$TVC(q) = (C_0 + C_1 \cdot S) f = \left(\frac{C_0}{S} + C_1\right) q$$

$$AVC(q) = \frac{TVC(q)}{q} = \left(\frac{f \cdot C_0}{q} + C_1\right) \quad (\text{Given L.O.S.})$$

The marginal operating costs are expressed:

Per Frequency:

$$MC_f = \frac{C_0}{S} + C_1$$

Per Seat:

$$MC_s = C_1 \quad (\text{Given L.O.S.})$$

Hence, for a given vehicle size,  $MC_s$  and  $MC_f$  are constants, with  $MC_s < MC_f$  for finite vehicle size.

In relating the demand and supply curves, the ratio of demand to seats offered is termed load factor, LF. Unfortunately, if the non-uniqueness of a particular cost associated with a service offering has proven discomforting, a similar non-uniqueness in the basic "breakeven" load factor is not likely to elicit confidence. This ratio,  $LF_{DE} = \left(\frac{\text{cost}}{\text{price}}\right)$  normalizes the price (consumer) and cost (operator) portions of the demand and supply curves and exists in total, variable and marginal formats.

Equating net yield per passenger to price allows investigation of profit maximizing behavior as the operator varies supply quan-

tity,  $q$  while maintaining a given level of service. The profit relationship is simply excess of revenues over costs:

$$\Pi = R - C$$

Varying the supply quantity yields:

$$\frac{\partial \Pi}{\partial q} = \frac{\partial R}{\partial q} - \frac{\partial C}{\partial q} = MR - MC$$

Maximizing,  $MR=MC$  at optimal supply quantity. Hence, marginal cost pricing maximizes the profit at a given level of service. Now, the revenues are:

$$R = NY_p \cdot D = P \cdot D$$

and 
$$MR = p \cdot \frac{\partial D}{\partial q}$$

It remains, then, to find the variation of demand with supply quantity. One may theorize various functional relationships, but these are best explored through specific examples.

At this point, a general case may be analyzed: a monopolist supplier of ultra short haul transportation services in the metropolitan market AB. The carrier will control price (net yield per passenger)\*, supply quantity (in the dimensions frequency and vehicle size) and level of service (in the dimensions frequency, market subdivision and intermediate stop policy) in order to establish a market solution maximizing profit. One may argue these various strategies by allowing a minimal number of degrees of freedom, and



analyzing through the calculus.

### 6.2.1 Operator Varies S, Vehicle Size (f, P, Level of Service fixed)

This case has been discussed previously. The passenger does not perceive vehicle size as a demand variable as long as seat is available for him/her. The resulting solution is:

$$\frac{\partial D}{\partial q} = 0$$

and  $\frac{\partial C}{\partial q} = 0$  For  $\Pi^*$

This, however is an inconsequential result, as the operator attempts to minimize variable costs by selling his aircraft and investing the capital. The result is an infinite load factor,  $LF^* = \infty$ . If the maximum load factor is constrained such that

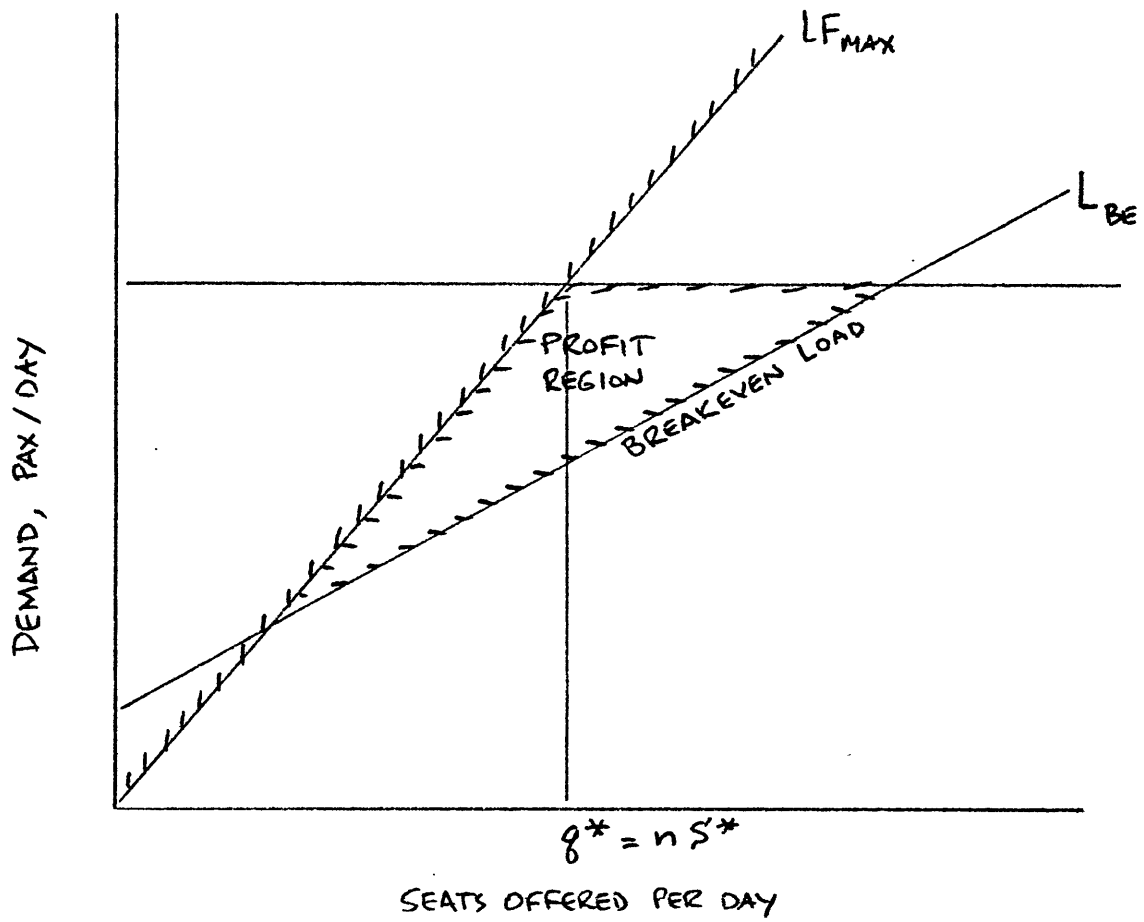
$$0 \leq LF_{\max} \leq 1.0$$

The result is that  $LF^* = LF_{\max}$ . This is diagrammed in Figure 6.3  $S^*$  is shown to be defined by the intersection of the  $LF_{\max}$  and constant demand loci. This maximizes profit at  $\Pi^*$ . The optimal value of  $LF_{\max}$  is determined by trading off passenger access to any particular flight versus profitability through application of a

---

\* The yield,  $NY_p$  will be found to vary as a function of demand and load factor. For the time being, though, the carrier controls it.

Figure 6.3 Optimizing vehicle size for a monopolist carrier



reservations policy. Clearly, as  $LF_{\max}$  rises towards 1, the probability of a random passenger finding a seat available drops.

$LF_{\max}$  varies with the degree to which demand is stochastic as opposed to deterministic.\*\*

Neuve Église and Simpson solve analytically for  $S^*$  through the technique of the Lagrangian multiplier,

$$\Pi = R - C + \Lambda \left( \frac{D}{f \cdot S} - LF_{\max} \right)$$

Where  $\Lambda$  is a weighting function of the "value" of load factor increase.

$$\frac{\partial \Pi}{\partial S} = MC_s \cdot f - \frac{\Lambda}{f S^2} = 0 \text{ at } S^*$$

$$\frac{\partial \Pi}{\partial \Lambda} = \frac{D}{f \cdot S} - LF_{\max}$$

or,  $LF^* = LF_{\max}, S^* = \frac{D}{f \cdot LF_{\max}}$

This analysis yields the optimal vehicle size as a function of projected demand and maximum vehicle loading. In reality, however, the operator does not have the capability to vary  $S$ , rather he fine tunes the market by adjusting daily frequencies. This case is taken next.

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\*\* For purely deterministic demand  $LF_{\max} = 1$ . For  $\sigma^2=0$ ,  $LF_{\max}$  is lower.

6.2.2 Operator Varies f, Daily Frequencies (S,P, other level of Service Variables Fixed)

With an existing fleet, and a regulated price, the operator varies level of service through its frequency of service component. This alters the demand function and affects the total demand, D. Hence; through variation of f:

$$\frac{\partial D}{\partial q} = \frac{\partial D}{\partial f} \cdot S^{-1}$$

and the marginal revenue expression from before becomes:

$$MR = P \cdot \frac{\partial D}{\partial q} = p \frac{\partial D}{\partial f} \cdot S^{-1}$$

The profit equation then is:

$$\frac{\partial \Pi}{\partial f} = \frac{P}{S} \cdot \frac{\partial D}{\partial f} - MC_f \cdot S$$

and:  $\frac{\partial D}{\partial f} = \frac{MC_f \cdot S}{p}$  , the marginal breakeven load

or:  $\frac{\partial D}{\partial q} = \frac{MC_f}{p} = LF_{BEM}$ , the marginal breakeven load factor.

Notice that in varying frequency the load factor drops to breakeven marginal in the last additional service. This is as opposed to  $LF_{max}$  with seats as a variable. Wherever vehicle size is left free, analysis by Lagrangian multiplier method yields this result. The optimal frequency,  $f^*$  is found by deduction from the

assumed demand profile, where:

$$D = KP^\alpha T^\beta$$

Where  $T = T' + \frac{t_2}{f}$   
 and  $T' =$  sum of other time components

$$\frac{\partial D}{\partial f} = K' \beta T^{\beta-1} \left( -\frac{t_2}{f^2} \right)$$

and  $\left\{ f^* \mid \frac{\partial D}{\partial f} = \frac{MC_f \cdot S}{p} \right\}$

Therefore  $f^* = \left[ \frac{-PK' \beta t_2}{MC_f S T^{\beta-1}} \right]^{1/2}$

### 6.2.3 Operator Varies N, Number of Terminals (q, P, other Level of Service Fixed)

Here, the operator explores the effects of reduced access time as a level of service variable. In that the only meaningful M is integer, the results are discrete in M. No reasonable differential format is possible. A graphical or cut-and-try method is preferable.

### 6.2.4 Operator Varies k, Number of Intermediate Stops

Again, as a trade-off between load building and reduced block time, the operator varies k. k is integer, and results are discrete in k. The graphical method will be employed.

### 6.2.5 Operator Varies f, S: (P, other level of service fixed)

As we are leaving S free, LF rises to  $LF_{\max}$  at  $f^*$  and  $S^*$ . Using the Lagrange multiplier,

$$\Pi = R - C + \Lambda \left( \frac{D}{f \cdot S} - LF_{\max} \right)$$

$$\frac{\partial \Pi}{\partial f} = P \frac{\partial D}{\partial f} - MC_f \cdot S - \Lambda \left[ \frac{D}{f^2 S} - \frac{1}{f S} \cdot \frac{\partial D}{\partial f} \right]$$

$$\frac{\partial \Pi}{\partial S} = -MC_s \cdot f - \Lambda \left[ \frac{D}{f S^2} \right]$$

Now  $\frac{\partial \Pi}{\partial f} = \frac{\partial \Pi}{\partial S} = 0$ , and

$$\frac{\partial \Pi}{\partial \Lambda} = \frac{D}{f \cdot S} - LF_{\max}$$

Which gives the expected  $LF^* = LF_{\max}$

Then:  $\left[ P + \frac{\Lambda}{f \cdot S} \right] \frac{\partial D}{\partial f} = MC_f \cdot S + \Lambda \cdot \frac{D}{f^2 \cdot S}$

Or  $\Lambda = \frac{-MC_s \cdot S^2 \cdot f^2}{D}$

And  $\frac{\partial D}{\partial f} = \frac{(MC_f - MC_s) S}{P - MC_s / LF_{\max}} = L_{BE_{f,s}}$

This  $L_{BE_{f,s}}$  is smaller than  $L_{BE_f}$  due to the higher degree of freedom.

Using the demand formulation method employed previously with

$T = T' + \frac{t_2}{f}$  we have:

$$D = K' T^\beta$$

$$\frac{\partial D}{\partial f} = K' \beta T^{\beta-1} \left( -\frac{t_2}{f^2} \right)$$

$$\text{and } \left\{ f^* \left| \frac{\partial D}{\partial f} = \frac{(MC_t - MC_s)S}{P - MC_s / LF_{\max}} \right. \right\}$$

$$\text{therefore, } f^* = -K' t_2 \left( \frac{\partial D}{\partial f} \right)$$

### 6.2.6 Operator Varies P, f, S:

Here, operator attempts to maximize profit while varying both price and supply of seats. This, of necessity, requires a correction to the yield per passenger term:

$$F = P + GC_p$$

Where  $GC_p$  = Ground costs per passenger = constant (for now).

$$\Pi = R - C + \Lambda \left( \frac{D}{f \cdot S} - LF_{\max} \right)$$

$$\frac{\partial \Pi}{\partial f} = \frac{\partial \Pi}{\partial p} = (D + P) \frac{\partial D}{\partial p} + \frac{\Lambda}{f \cdot S} \frac{\partial D}{\partial p}$$

Using this last equation, and substituting for  $\Lambda = \frac{-MC_s f^2 S^2}{D}$

$$\frac{\partial D}{\partial p} = \frac{-D}{\left( \frac{P - MC_s}{LF_{\max}} \right)}$$

now, returning to the demand formulation,

$$D = K F^\alpha T^\beta$$

$$= K'' F^\alpha$$

$$\frac{\partial D}{\partial p} = K'' F^{\alpha-1} = \left[ \frac{\alpha D}{F} \right]$$

Substituting this expression for  $\frac{\partial D}{\partial p}$ ,

$$\frac{\alpha D}{F} = \frac{-D}{(P - MC_S^*)} \cdot \frac{1}{LF_{\max}}$$

and  $F = \alpha \left[ p - \frac{MC_S}{LF_{\max}} \right]$

but  $F = P + GC_p$

or 
$$P^* = \frac{\alpha}{\alpha+1} \left[ \frac{MC_S}{LF_{\max}} - \frac{GC_p}{\alpha+1} \right]$$

$$= F^* - GC_p$$

[NOTE: That this is only meaningful for  $\alpha \leq 1$ , denoting a price elastic market.]

and: 
$$F^* = \frac{\alpha}{\alpha+1} \left[ \frac{MC_S}{LF_{\max}} + GC_p \right]$$

Now, substitute  $P^*$  into the optimal frequency and vehicle size solution to find the breakeven load,  $(\frac{\partial D}{\partial F})^*$ . This yields  $f^*$  and  $S^*$ .

From these results, an optimal "neighborhood" may be searched numerically such that those discrete variables,  $N$  and  $k$  are also optimized. This will be the subject of the case studies of Chapter 7.



## 7.0 Case Studies and Results

The preceding results, drawn from urban service systems analysis and transportation economics, may be tied together in a purposeful analysis on the case study level. While taken individually, these disciplines permit great insight into particular areas of interest in transportation research. This approach does not fully exploit the information uncovered in such an analysis. By employing a systems approach to such a study, the areas of subtle interrelation between decision variables are fully explored. This is of greatest importance in the urban sector due to the "randomness" of the study's operational environment. Deterministic behavior does not, however, diminish the suitability in other areas of analysis.

Two case studies will be presented in this chapter: The first, an analysis of a "corridor market"; the second illustrating the complexity of a general intra-regional analysis. Both will proceed in the fashion laid out in the preceding background. While the first case study is more or less determinate, the generalized case will point out several areas for research in alternative transportation analysis.

The term "Optimization" requires that there be some objective measure of performance to maximize. What, exactly, this measure should be is dependent on the operational policy the system is run under-- public or private sector.

Clearly, the initial operating policy under which ultra short haul air transportation functions will be to a large extent the determinant of its eventual role in the total regional transportation system. To the extent that the system is competitively priced with regard to operating costs, value and level of service provided, it will at some point maximize not only its financial return to the operator, but perhaps its market share as well.

From the picture of vehicle and facilities costs, it is possible to characterize policies ranging from maximum passenger benefit through maximum operator benefit, with associated profit (or subsidy) and market penetration statistics. While maximizing operator benefit is easily stated mathematically, maximizing passenger benefit is not. The "zero profit" condition can be used as a maximum marketshare indicator, but does not very well argue the point of maximizing passenger utility. A service operating in the public sector (e.g. mass transit) may tend to surrender the inherent advantages possessed by ultra short haul air and thereby degrade the quality of service provided. This has occurred in each case where the public sector has attempted an optimization (revitalization?) of existing transportation technology whether it be mass transit, rail, barge or the proposed regulatory reform of the domestic air transportation industry.

For this reason, the analysis will maintain a private sector basis for the operation of the ultra short haul service.

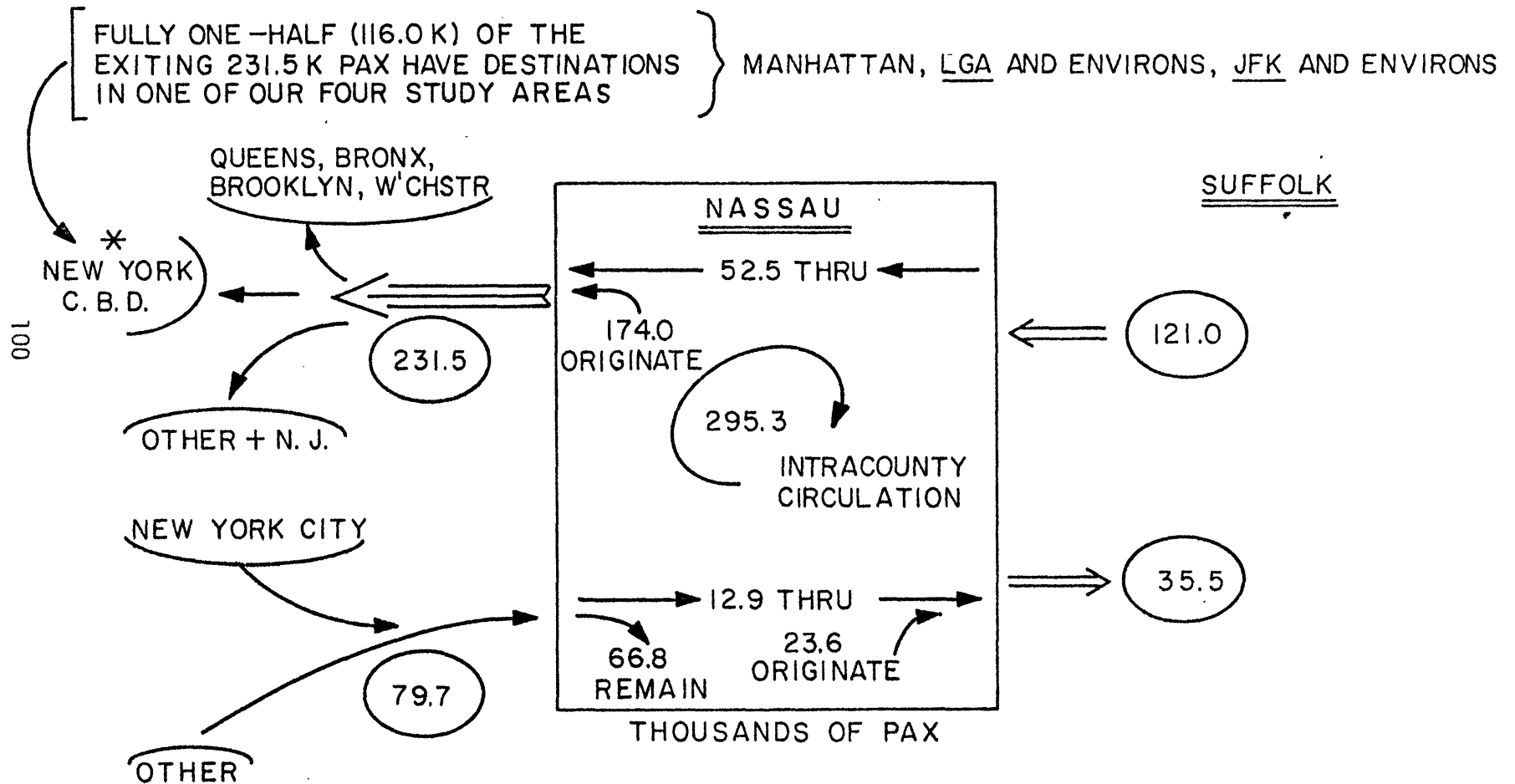
#### 7.1 Long Island Service Study

The first study evaluates the market impact of the addition of ultra short haul air service to the Long Island/New York City CBD corridor. (Figure 7.1 ). This is an extension of the previous project undertaken by the author in the early days of this research. At that time, a captive volume of journey-to-work trips was used to assess the feasibility of such a project. By analyzing this particular travel group -- commuters -- the problems presented by demand stimulation were removed. This portion of the total regional trip volume is relatively insensitive to level of service in the sense of demand stimulation. In terms of modal choice, this group is extremely sensitive, and it was in this light that the analysis was conducted.

The Long Island corridor is a classic example of the growth and development of a suburban labor force causing a dichotomy between the transportation technology to satisfy those needs. As suburbanization and "sprawl" occurred, a developing network of arterial and radial highways served the commuters' needs for transportation services to the city center central business districts. At the point when further expansion meant population displacement,

Figure 7.1

PICTORIAL OF DAILY ONE-WAY PASSENGER MOVEMENTS [JOURNEY-TO-WORK],  
 NASSAU AND SUFFOLK COUNTIES



REFERENCES: 1970 CENSUS, 1973 TRISTATE REGIONAL PLANNING COUNCIL "J-T-W" REPORT,  
 E.C.I. SYSTEMS STAFF.

Table 7.1

## ORIGINS - PLACE OF RESIDENCE

	W. Nassau County	E. Nassau County	W. Suffolk County	Totals
Nassau Co.	125858	122401	28074	276333
Suffolk Co.	3726	9733	101530	114989
Queens Co.	24139	17728	8527	50394
Brooklyn	17458	11258	5188	33904
Manhattan 59th St. So.	57726	26684	15168	99570
Manhattan 59th St. No.	8306	4429	1924	14659
Bronx	2060	1466	641	4167
Westchester	934	397	253	1584
Conn., N.J.	3201	1968	1163	6332
<b>Totals</b>	<b>243408</b>	<b>196074</b>	<b>162458</b>	<b>601940</b>

FROM: "Long Island Journey to Work Report," New York Office of Transportation, 1970

development of this highway network was curtailed.

As suburban employment centers developed, worktrips involving intra-suburban area transport quickly overtaxed existing suburban transit facilities and sent suburbanites heading for the highways. Large numbers of suburban residents continued to commute to city center central business district areas, while increasing numbers of ultra short intra suburban trips were generated.

Illustrative of this situation is an analysis of the worktrip passenger movements in a portion of the Long Island corridor presented in Figure 7.1, Nassau and Suffolk Counties comprise the outlying two-thirds of the Island corridor, and represent the demarcation of the "suburban" region spoken of above. Table 7.1 shows the origin-destination matrix for Nassau and Suffolk Counties to the New York standard metropolitan statistical area. The magnitude of these flows regularly saturate existing transportation services. The low level of service offered in fixed-line technologies (rail, subway, bus) induces intermodal trip itineraries and shifts towards automobile usage. Congestion effects yield unacceptably long trip times as shown in Table 7.2 -- uncharacteristically independent of mode or technology level. In short, transportation services in the Long Island corridor are operating in saturation. There is a need for an evaluation of transportation alternatives.

Table 7.2  
 BASE CASE  
 REPRESENTATIVE TOTAL TRIP TIMES (MINUTES) AS A  
 FUNCTION OF TRAVEL MODE

		To	Wall Street	Upper Manhattan	LGA	JFK
		From				
Auto	W. Nassau		56.9	63.1	46.3	43.2
	E. Nassau		68.3	74.5	57.7	55.0
	W. Suffolk		87.3	93.5	76.7	74.0
		To	Wall Street	Upper Manhattan	LGA	JFK
		From				
Rail	W. Nassau		44.5	45.4	55.3	N/A
	E. Nassau		63.0	63.7	73.8	N/A
	W. Suffolk		77.5	83.1	93.2	N/A
		To	Wall Street	Upper Manhattan	LGA	JFK
		From				
Park/ Ride	W. Nassau		45.5	46.5	65.3	N/A
	E. Nassau		63.8	64.5	74.6	N/A
	W. Suffolk		83.8	89.4	99.5	N/A

One of these alternatives is ultra short haul air transportation.

An ultra short haul air transportation system will be defined as serving intraregional and inter urban trips by passengers and freight over distances of less than 50 miles. The advantages of air are many, and include higher block speed and freedom from geographical constraints. An air system uses a smaller percentage of the land required by fixed line technology, and has the potential of having a much smaller noise impact. New "high technology" ground systems require large, high risk initial public investment yet are by comparison far less flexible and demand responsive than air systems. The "blight" and dislocation created by surface or subsurface technology is not created by an air system -- it is readily integrable into existing facilities. Finally, air systems offer better travel services to the passenger, and have the potential of being a fuel efficient alternative to the private automobile.

In this light, a feasibility study -- an alternative technology analysis -- was undertaken. In order to view the situation in the most objective manner possible, the current state of transportation systems on Long Island was assessed, then modelled. The proposed alternative, ultra short haul air transportation, was then added to the model in order to evaluate its impact on a captive volume of travel -- the journey to work trips.

The study area -- essentially the eastern portion of the New York standard metropolitan statistical area -- consists of Manhattan



(New York City central business district), Nassau County and Suffolk County. These portions of the corridor were chosen on the basis of commuting trip volumes, income levels and thoughts towards interfaceability with an existing ultra-short haul air network. Queens County was eliminated from the analysis due to ready availability of high frequency surface modes and its proximity to the central business district. This proximity argument is a strong determinant of modal split. Considering the journey to work trip, the O-D matrix is primarily from out-Island to central business district during the morning commute. The return from work flow (decommute) would in general be similar with respect to flow magnitudes and directions. The temporal variations of congestion effects are also analogous during the morning and evening peak periods of travel.

Subregions within the two counties and the central business district were linked into an existing transportation network demand model, and the model calibrated\* to fit available flow and modal split data. Four modes were identified -- auto, rail, park-ride, and the ultra short haul air alternative. Reported total travel times by mode matched those output by the model primarily due to the existence of a "congestion function" based on the ratio of actual to design link volumes.

The demand forecast was performed for several operational policy/

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\*Elasticities and individual modal "image" variables were found via regression analysis and are listed in Table 7.4, page 118.

scenario combinations. The results indicated that there was sufficient demand to support a multiple rotorcraft fleet at acceptable levels of utilization. The ultra-short haul air mode market penetration ranged from 1.9 percent to 6.0 percent over the range of policies/scenarios investigated. Sensitivity analyses were performed on level of service parameters including fare, frequency, service patterns and heliport location. Fleet requirements for the Sikorsky S-61 Rotorcraft were calculated as a function of overall level of service and a sample fleet assignment and schedule generated. System planning for off-peak use was not examined, but would add to equipment utilization and latent demand stimulation in the non-business sector. [For a full examination of this preliminary work, the reader should refer to M.I.T. Flight Transportation Laboratory Report FTL R76-2 "Short Haul Helicopter Demonstration Program."]

This analysis pointed out several areas for research. Among them:

1. A better treatment of access/egress distance and time.
2. Terminal "coverage" of demand regions.
3. Does an optimum number of sub-regions within a demand region exist?
4. Limiting criteria for use of ultra-short haul air transportation.
5. The network considerations of "randomness" in the analysis of access/egress and urban problems in general.

Using the data aggregated for the previous study and developing the costing and travel time analysis technologies of Chapters 3 and 4, the Long Island problem was again analyzed. This time, however, with a "clean slate" and from the standpoint of a systems optimization.

With Queens and Kings County excluded from the demand region for reasons previously mentioned, the "corridor" market took on more of a point-to-point nature. The system could now be modelled as an A-B market with the characteristics as shown in Figure 7.3. The Nassau-Suffolk County region was quantized into cells of work trip demand, corresponding to census reporting regions compiled by the Tri-State Regional Planning Commission. From this data and from that of the previous study, the mean access time between random orientations of demand cells and a single facility were found. In order to comply with the homogeneity requirements of the probabilistic analysis, the region was remodelled in this manner. Interestingly, by normalizing demand density with income level, a nearly homogenous demand/income density was created. The only real requirement was the omission of the very sparsely populated and remote eastern portion of Suffolk County. This resulted in a roughly rectangular homogenous demand region with dimensions 13 by 35 miles containing 20,000 journey to work trips to the New York central business district.

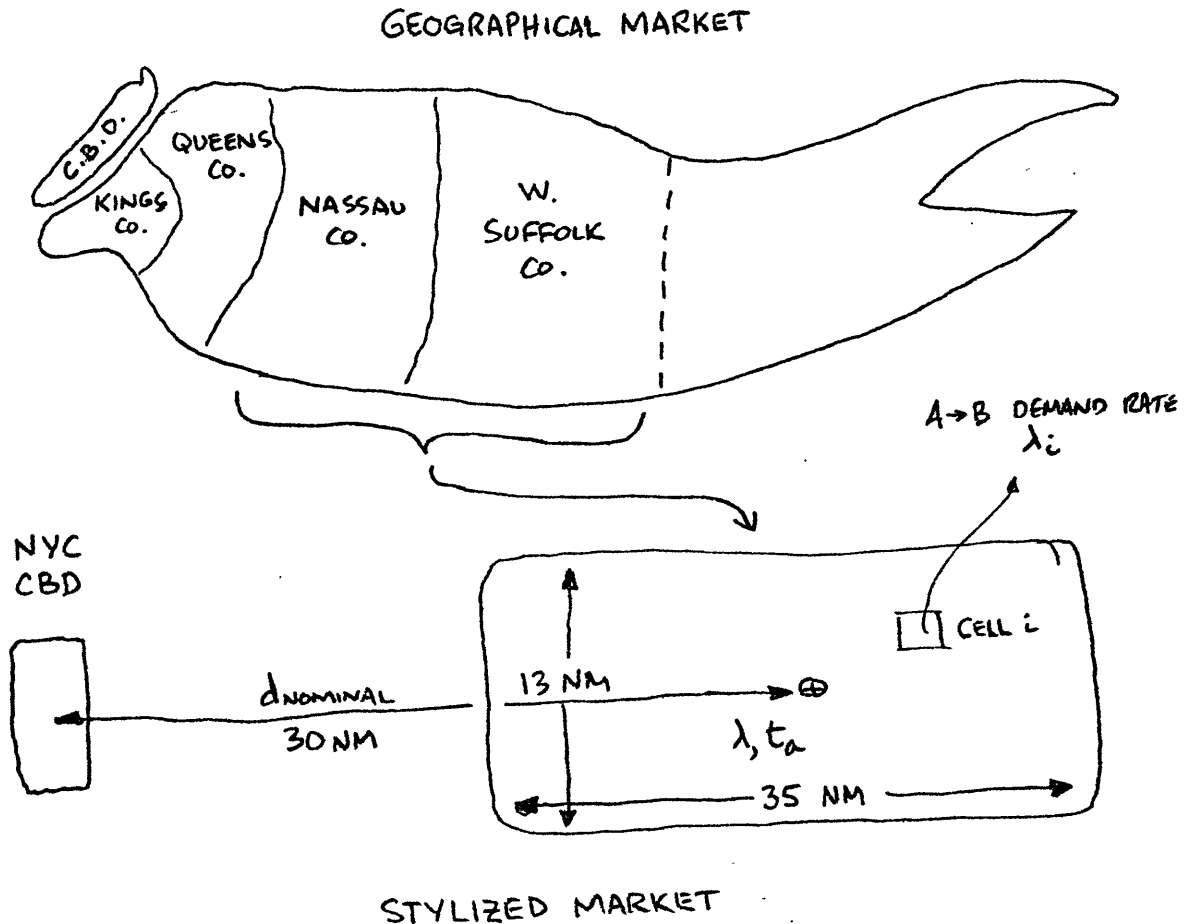
Travel Time:

A simple application of the material in Chapter IV shows that for right angle metric travel, the expected travel distance to a single, randomly located facility would be 16 miles, and that central location would yield a 12 mile expectation. The previous analysis based on demand cells, network travel and a single facility had yielded an average access distance of 13.4 miles, with link congestion showing an average travel speed of 34 MPH en route.

Figure 7.3

Long Island -- New York City Central Business District Market

Characteristics



Remembering the upper and lower bounding relationships of right angle metric distance and Euclidean distance, and taking the isotropic travel speed of 34 MPH average, developed in FTL R76-2 (March 1976), we have the following bounding criteria for access travel distance and time to a single facility.

	Random Location		Central Location	
	<u>Distance</u>	<u>Time(min)</u>	<u>Distance</u>	<u>Time(min)</u>
Right Angle Metric	16.0 mi	28.2	12.0 mi	21.18
Euclidean	12.6	22.2	9.42	16.63
Model Estimate	13.6 miles/ 24.0 min			

As a conservative estimate, the baseline (single facility) access time was chosen as 24 min (0.40 hrs) reflecting the belief that access routes, even if basically Euclidean in nature, may be to a certain extent (10 to 12 percent) circuitous, and that facility location is rarely "optimal" (or in this case central) in practice, but must be formally integrated into the community. This 0.40 hour figure was then used in the travel time model as the base coefficient to normalize the area rule term showing the influence of market subdivision on total travel time.

The in-vehicle component of travel time can be broken down into block time  $t_b$  and cycle time  $t_c$  components. The block time is related

only to segment length and cruise speed, while the cycle time is incurred in multiples of climb-maneuver-descent operations. Hence, nonstop flights incur a single cycle time. In general, a passenger on a flight with  $k$  intermediate stops incurs  $[(k + 1)t_c + t_b]$  in-vehicle travel time. Cycle times for rotorcraft at major metropolitan airports where VTOL aircraft are often treated like CTOL aircraft for air traffic control purposes are on the order of 3 -- 4 minutes. Cycle times at outlying metroports are shorter, due to the absence of air traffic control restrictions. Poor weather operations would raise these figures (especially at major airports) and an efficient VTOL air traffic control strategy could lower them.

For modelling purposes, cycle time has been set at 3 minutes. This is conservative, but takes poor weather delays into account. Block time is figured on the basis of a cruise speed of 140 KT and a nominal stage length of 30 NM. This distance is again dependent upon actual facility location, but is easily derived on a probabilistic basis. In a case such as this, where intraregional distance is large by comparison to the market point-to-point distance, careful consideration of routings as to circuitry is required.

Displacement, or wait time,  $t_w$  is that portion of the total travel time incurred at the originating facility waiting for the next scheduled service. This is generally regarded (and was shown

probabilistically by Simpson) to be one half the interarrival time or headway. Knowing the frequency of service,  $f$ , the displacement time is then proportional to  $1/2 f^{-1}$ . The modifying coefficient to this term is normally of the order "hours of operation." Strictly, then, if  $f$  services are offered, during  $H$  hours a day, expected wait time is:  $\frac{H}{2f}$ . More reasonably, service is assumed demand responsive and hence during morning and evening peak hours, local headways are lower. Based on two four hour peak periods per day, and ten off-peak hours per 18 hour day, with a total daily frequency  $f$  weighted in response to demand, expected peak hour wait time is more on the order to  $2/3$  the long term average. Hence, the analysis has used  $t_w = 2/3 \cdot \frac{18}{2f} = \frac{6}{f}$  as the frequency related portion of total travel time. This is conservative for peak hour travellers and not unreasonable for less time sensitive off peak travellers.

Passenger processing times  $t_p$  are assumed fixed in this analysis and function as a catch-all for such terms as parking lot to terminal time, ticketing (assumed simplified--more along the lines of purchasing rail tickets), baggage claim (assumed primarily carry-on/off) and interline time to egress mode. Logically, some of these items should bear functional relationships to facility traffic volume or other system variable. Realistically, an entire system stimulation is not practical, as these items are by comparison cause

and effectively incidental to the big picture when viewed in the macro sense. The sum of the processing times was taken as 12 minutes (0.20) hours.

Egress time is generally treated the same as access time, with an area rule governing segmentation of the destination region. For the purposes of the model, however, the destination egress time will be considered fixed so as not to cloud the issue of demand stimulation by market segmentation. Thus, egress time is then from the single facility time in the March 1976 report, and it is assumed  $t_e = 0.20$  hr.

The total travel time model is formulated as follows:

$$\begin{aligned} TTT &= t_a + t_w + t_b + t_c + t_e + t_p \\ &= 0.40n^{-1.2} + \frac{6}{f} + \frac{d_{nom}}{140} + 0.05(k+1) + 0.40. \end{aligned}$$

Where  $n$  = number of origin region subregional facilities = 1,2,3...

$f$  = daily frequency of service  $f \geq 20$

$d_{nom}$  = nominal point-to-point market distance  $d \sim 30$  NM

$k$  = number of intermediate stops experienced  $k = 0,1,2,3...$

## 7.2 Costs of Providing Service

Direct operating cost per hour as a function of seats available and stage length are drawn from the material of chapter 3. Based on nominal stage length of 30 NM the NASA cost equation becomes:



$$DOC_{hr} = \$252 + \$4.20(S)$$

From regression of current equipment operating costs, the corresponding cost equation is

$$DOC_{hr} = \$88.50 + \$24.30(S)$$

These projections are then converted to variable frequency costs given a vehicle size through multiplication by flight time, the sum of block and cycle times. Based on the 30 NM nominal distance,

$$t_b = 0.21 \text{ hours}$$

$$t_f = 0.26 + 0.05 k$$

The resulting frequency costs are then:

$$MC_f = t_f(\text{Direct Operating Costs/hour})$$

Indirect costs of a variable nature lie basically in the area of station costs, and passenger servicing costs, traffic and aircraft serving costs, promotion and sales and general and administrative overhead. With the exception of station costs and general and administrative costs, the rest of the indirect operating cost components are more or less proportional to passengers boarded. The most generally regarded figures for the local service carriers were estimated by Douglas Aircraft at:

Passenger, Aircraft, Traffic Servicing:

$$SVC = \$4/\text{Boarding} + \$100/\text{Departure} + \frac{\$150}{\text{Gross Wt. Ton}}$$

$$\text{Sales} = .12 * \text{Yield}$$

$$\text{Overhead} = .05 * (\text{SVC} + \text{Sales})$$

These servicing costs are predicated on cycle times and manning requirements that are an order of ten higher than those envisioned for the ultra short haul operator. Sales and general and administrative however, are reasonable. As such, these traffic associated costs are subtracted directly from the net yield per passenger. The number of stations, however, will be one of the level of service variations in the analysis. As such, a separate break out of this cost will be required.

The facilities envisioned are assumed rather functional, and the capital costs paid for primarily by concessionaire rental such as parking and on-site rentals. A passenger rebillable station operation cost of approximately \$1,000/day, of which 5 percent, or \$250/day is a fixed capital cost. The remainder of this cost is quite reasonably associated with the previously mentioned traffic statistics. These station related ground costs are averaged over passengers boarded. At low frequencies of service, demand will exceed capacity for all but large vehicle sizes, and indirect operating costs per passenger is a large percentage of the ticket price. As capacity catches up to demand, and load factor drops below  $LF_{\max}$ , variable indirect costs are a much smaller percentage of the total fare. In any case, the model subtracts ground costs from the ticket yield such

that:

$$NY_p = F - GC_p$$

where  $GC_p = f(SOC, LF)$

### 7.3 The Demand Model

The demand model used is of the product form and assumes two demand groups, roughly described as business and pleasure. These models were fit to the traffic base based on existing travel volume and modal splits. The result was a parametric program that exercised each of the portions of the level of service vector, and described the demand surface in several dimensions. The changes in demand level may be expressed in terms of market share by the assumption of other modal utilities remaining constant.

The characteristics of the two travel groups differ mainly in respect to their time and cost demand elasticities. On an aggregate basis, the two country total travel demand and modal split to the New York City central business district is well modelled by "gravity" elasticities. ( $\alpha=\beta=-2.0$ ). Using time-series home interview survey data provided by the Tri-State Planning Commission, the travel volumes for both journey-to-work and supplementary travel were analyzed via regression analysis. Considering only those regions of interest in the Long Island Corridor, these figures provide an excellent reference point. Disaggregating the market groups, the journey-to-work portion of the flow, which accounts for 46 percent of total

corridor travel and 84 percent of total trips to the CBD is slightly biased towards time elasticity (although remaining fully cost elastic) with convenience and modal "attractiveness" accounting for a shift to the public transit mode. The model used for the journey to work portion of the flow used a time elasticity of -2.50 with a cost elasticity of -1.50. Modal attractiveness coefficients developed in the regression favored "non-participatory" modes, which would bias the modal split of the line haul portion of the trip towards rail and park-ride. The attractiveness coefficient for the ultra-short haul air alternative was set the same as that for park-ride. Although unquestionably more aesthetically and technically attractive, a reduce reliabiltiy (in the present term, at any rate) would tend to force ultra-short haul air to compete on a time savings basis alone. Advancements in the reliability of all-weather operations would of course raise its attractiveness proportionally. These regression developments are summarized in Table 7.4.

#### 7.4 Fare Policy:

Here, two cases were run corresponding to the vehicle technology and costing model employed. Each fare schedule used the results of Chapter VI and was linearized to a boarding charge plus cost per line haul mile basis. In that the vehicle costs differ extremely, so do the fare policies and the resulting demand and

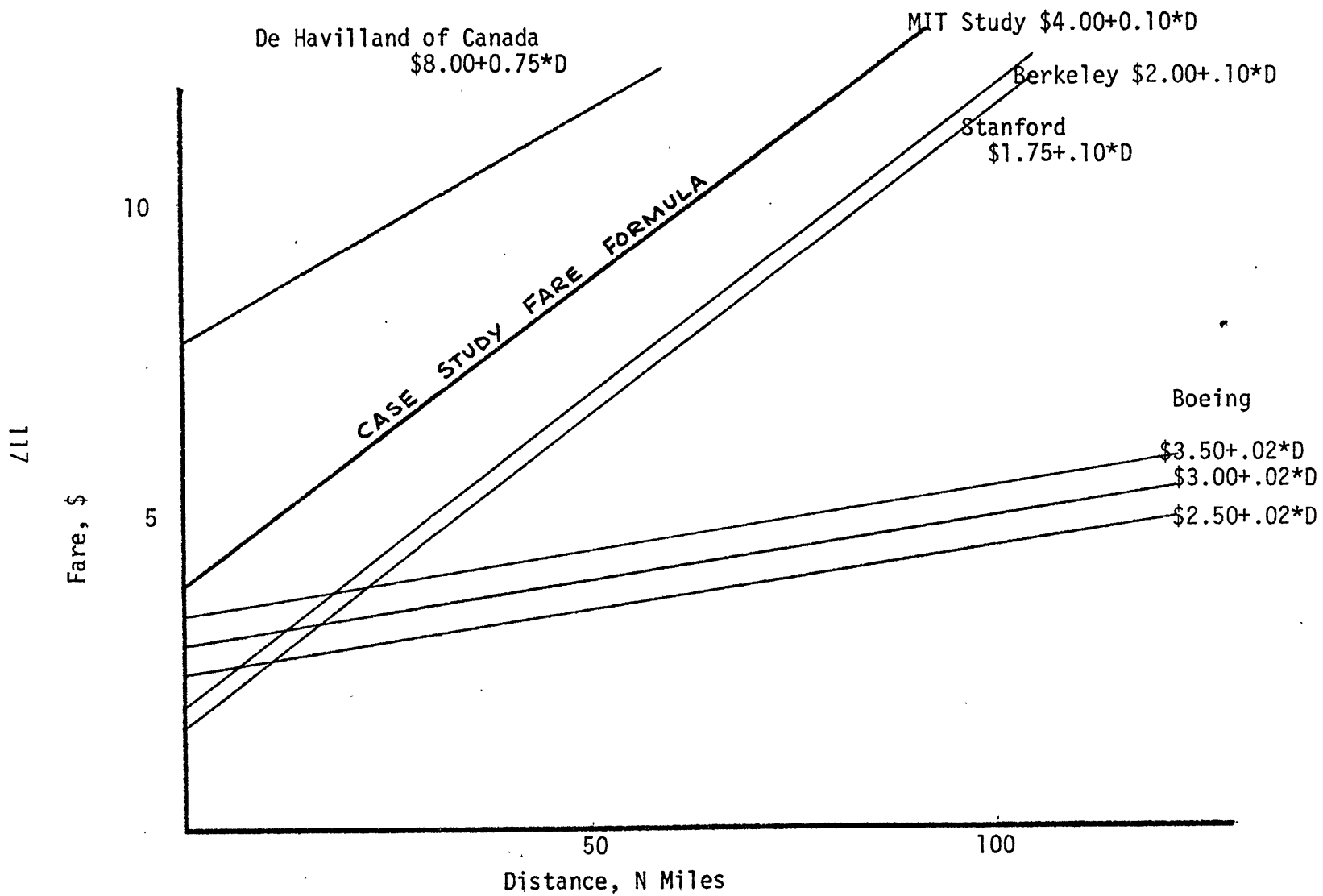


Figure 7.4

Various Fare Formulae Employed in Short Haul Studies

market compositions.

Table 7.4

$$D_j = M I_j P^{\alpha} T^{\beta}$$

$$D = \sum D_j \quad \alpha_j$$

	<u>Auto</u>	<u>ParkRide</u>	<u>Rail</u>	<u>Air</u>
Attractiveness Modifier	1.74	2.67	1.24	2.67
Price	1.75	2.30	2.05	(7.00~28.50)
Total Travel Time	1.75	1.25	1.30	0.90
Market Share	.265	.463	.247	?

<u>Travel Group</u>	<u>(<math>\alpha, \beta</math>)</u>	<u>Volume</u>	<u>%Total Travel</u>
Journey to Work	(-1.5, -2.5)	116,000	0.807
Other Hub Bound Travel	(-2.25, -1.75)	27,800	0.193
Aggregate	(2.0, -2.0)	143,800*	100

\*Prior to demand stimulation

The distance-linearized fare formula for each vehicle is presented below. This is a composite of the NASA, Lockheed-California, Stanford, De Havilland of Canada and McDonnell Douglas fares shown in Figure 7.4.

Fare = Boarding Charge + (Line Haul Mile charge) \* Distance

For the existing vehicle family, this is:

$$\text{Fare} = 15.00 + 0.55 * \text{Distance}$$

For the NASA vehicle family:

$$\text{Fare} = 4.00 + 0.10 * \text{Distance}$$

The differences in the fare policies reflect the belief that greater economies of scale may be exploited in both productivity per direct operating cost dollar, and in efficiency or cost-sharing in the indirect cost area. The current vehicle family fare policy also reflects the very real cost of operations, which when viewed in comparison to short haul CTOL or STOL vehicles are a magnitude higher in the line haul.

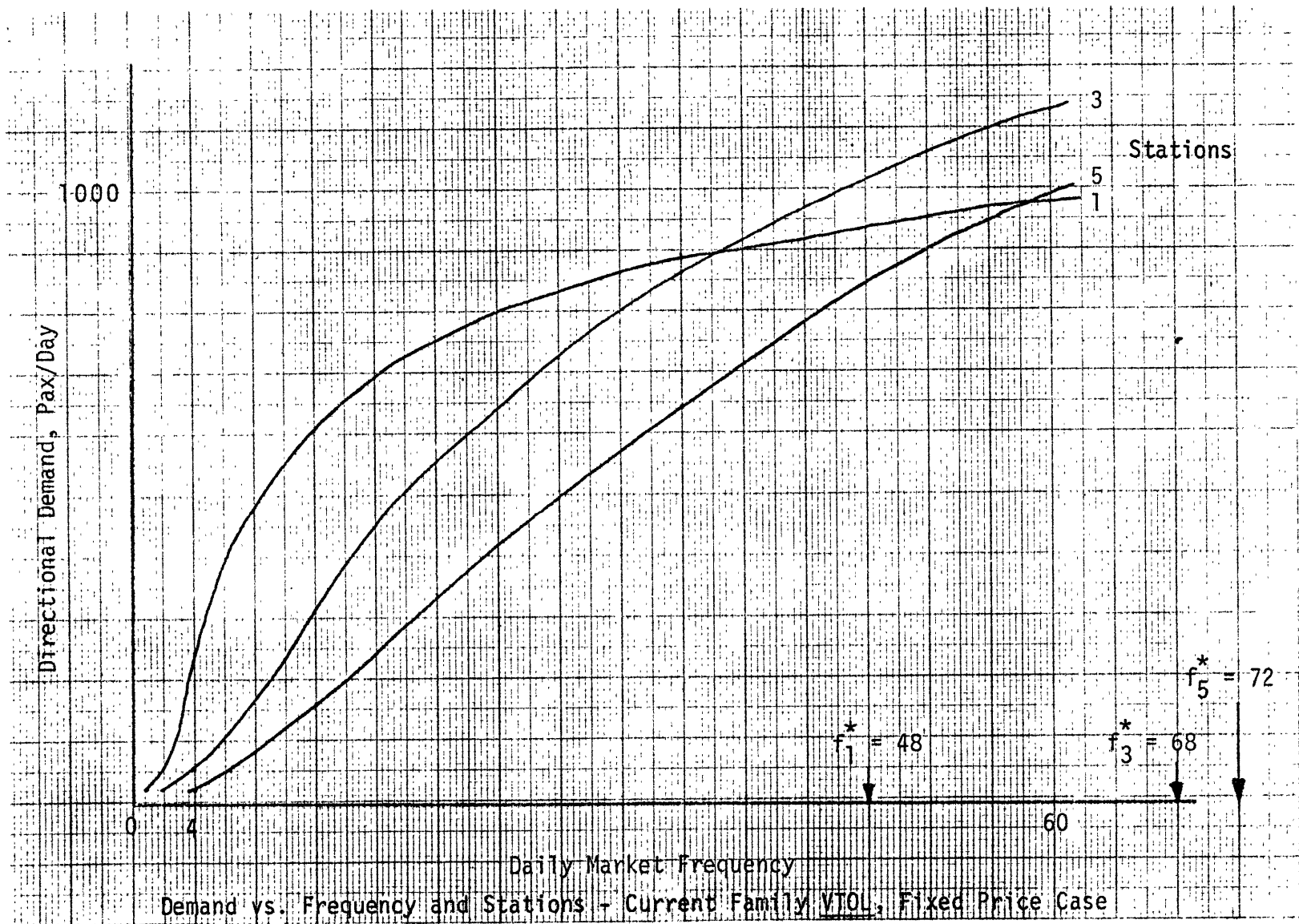
#### 7.5 Case Study Results: 7.5.1 Maximize Contribution Policy

Based on the sensitivities and heuristics discussed in Chapter 6 and results from actual operations, areas of feasibility with respect to each of the level of service variables were defined. A recursive parametric program was used to assess the various combinations and resulting levels of service. The outputs of the program

were demand, overall financial statistics and various operating ratios such as ratio of direct to indirect costs, load factors. From these figures, an excellent view is provided of the feasibility and practicability of the proposed service.

Figures 7.5, 7.6 show the effects of demand stimulation by improved frequency of service and the subdivision of the demand region. It is noted that for the commuter demand group, these effects are greater due to a greater time elasticity. The saturation frequencies,  $f^*$  corresponding to the 90th percentile of the infinite frequency demand, are seen to be in the range of 72 to 36 dependent upon degree of market subdivision. This is interesting in that market subdivision with a fixed price policy has the same effect as making the market in question a "value of service" market or increasing its effective stage length by reducing the access to total time ratio. This is a far reaching effect, as it allows multistop load building routing, and larger vehicles while retaining the attractiveness of a minimized access time. While there is no "weighting" of in-vehicle vs. out-of-vehicle times, this serves to indicate a bias in favor of in-vehicle vs. out-of-vehicle time as being less objectionable to the passenger. This result, if true, would alter certain thoughts regarding the value of vehicles possessing high block speeds vs. improvements to out-of-vehicle services -- a trade-off of indirect operating costs vs. direct operating costs.





Demand vs. Frequency and Stations - Current Family VTOL, Fixed Price Case

Figure 7.5

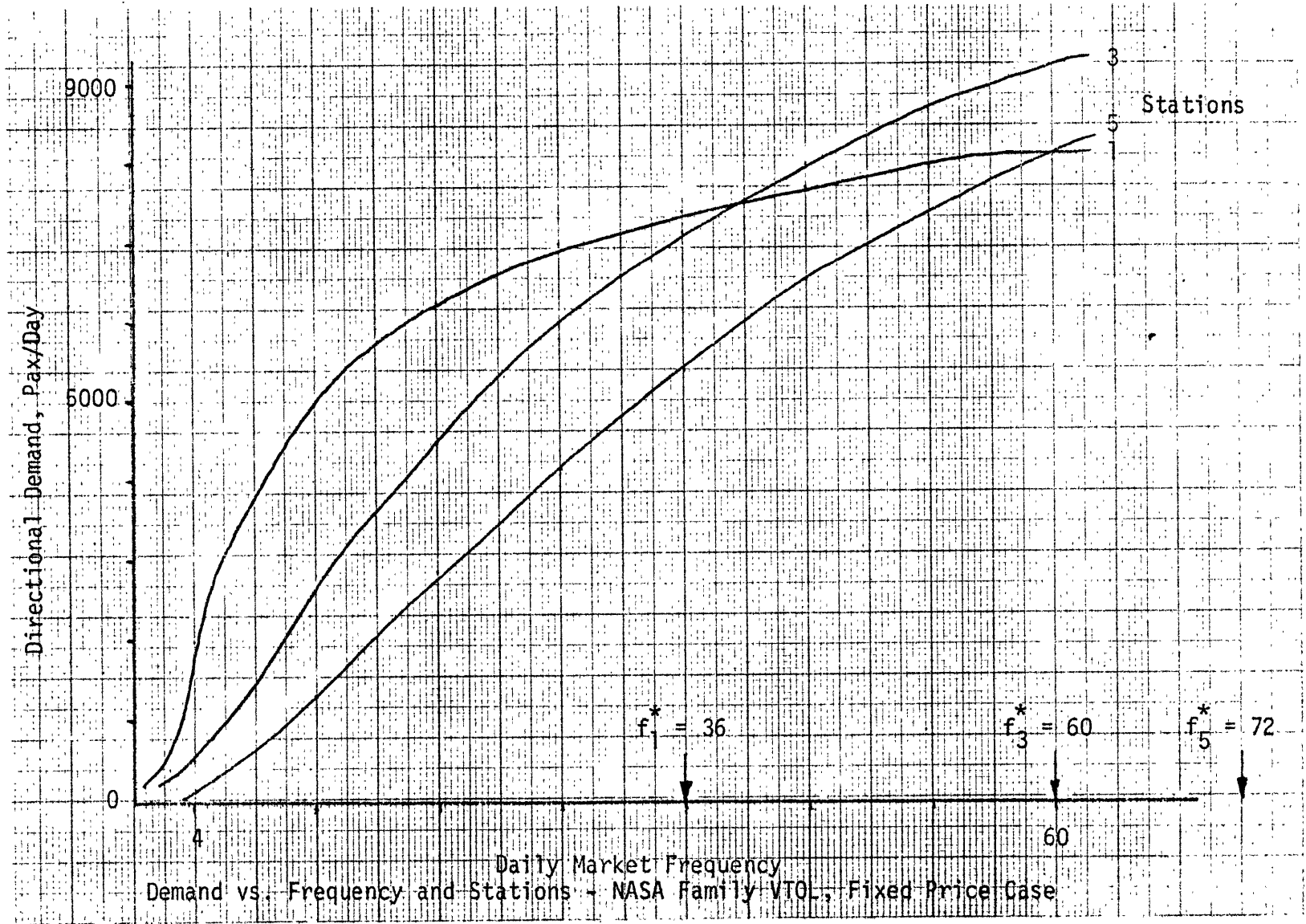


Figure 7.6

Financial statistics of various operational policies and scenarios are presented in Figures 7.7 through 7.12. These indicate profit (or loss) on total costs and revenues based on various vehicle sizes, levels of station operating costs, market subdivision, and fare policy. It can be seen that with smaller vehicle sizes and at lower daily frequencies, operations are capacitated by maximum load factor in the analyses, this is set at 80 percent, a figure which for scheduled service with greater than four frequencies per hour provides an excellent "second chance" facility, even during the peak hours of usage. As vehicle size increases, LF drops, and operations at smaller breakeven loads remain profitable. At higher frequencies of service with a particular vehicle size, market subdivision allows maintenance of a profitable operation through demand stimulation and increased modal utility. As station operation costs increase, the net yield per passenger drops, increasing the breakeven passenger load. This has the effect of squeezing the feasible frequency/vehicle/size/multistop/multistation region, narrowing the range of policy choices over which a profitable operation is possible. It can be shown by varying station cost over a rather narrow range corresponding to short versus long term retirement of capital costs that indirect operating costs per passenger will increase to a critical percentage of total revenues [Figure 7.13].

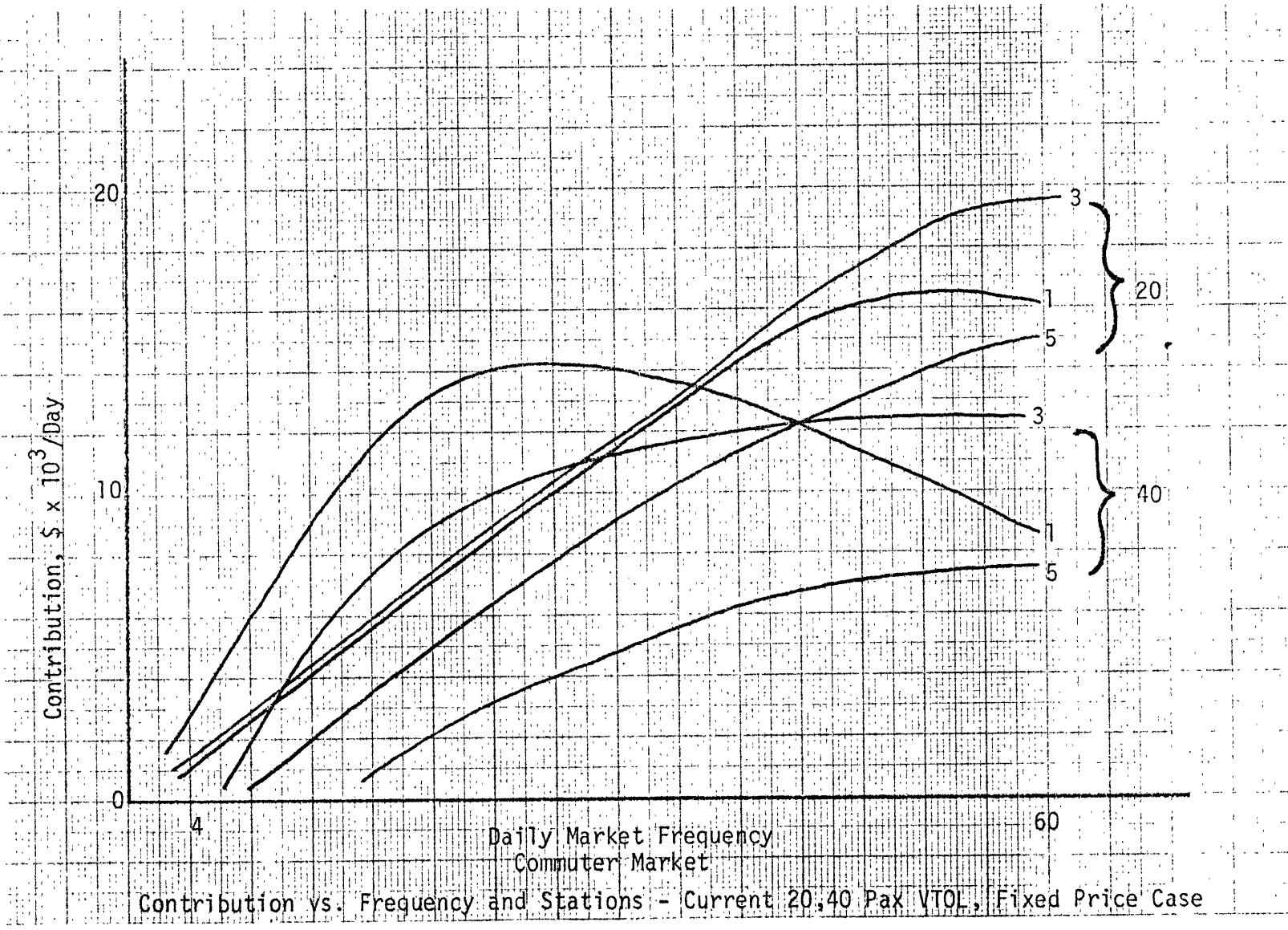


Figure 7.7

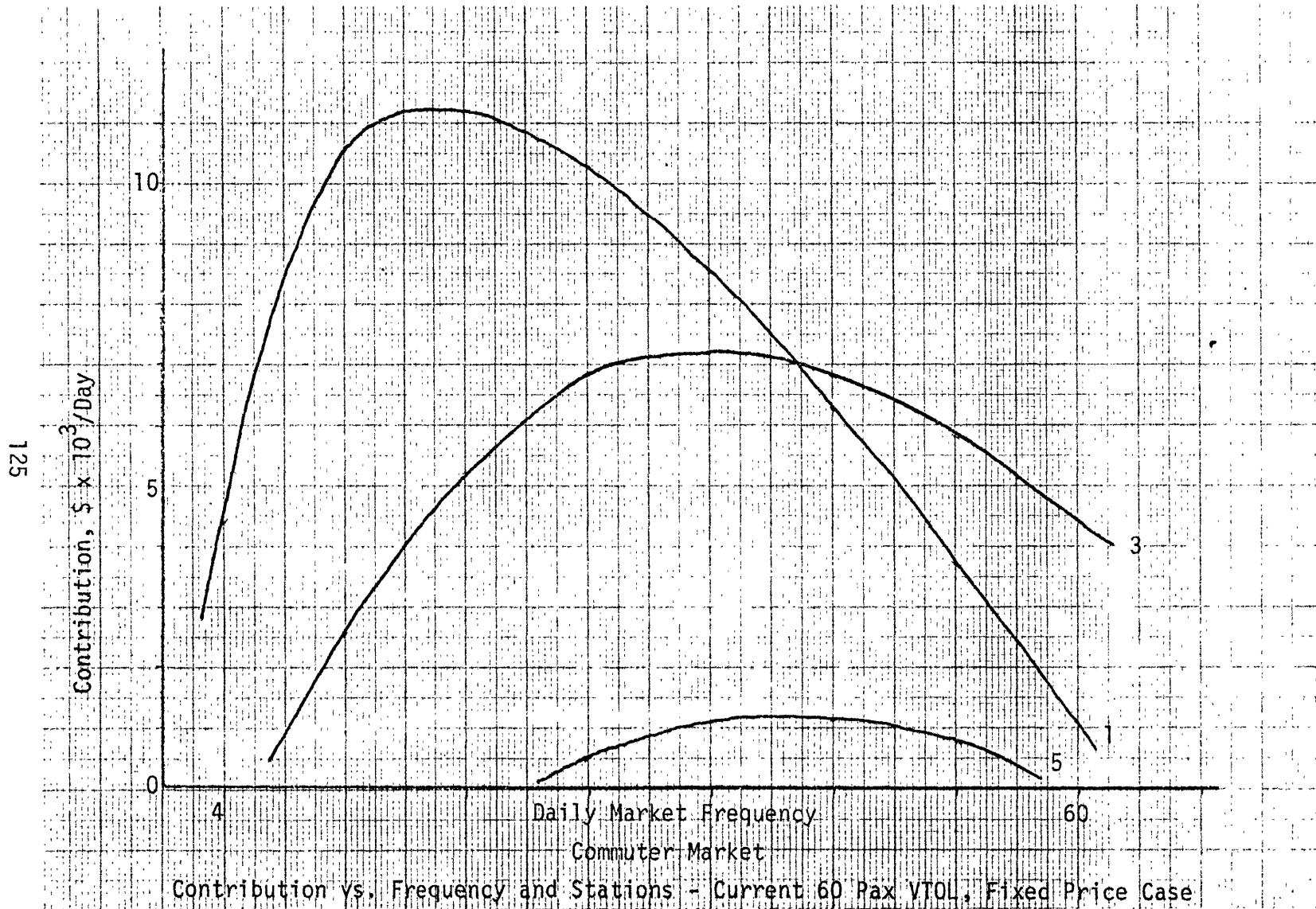


Figure 7.8

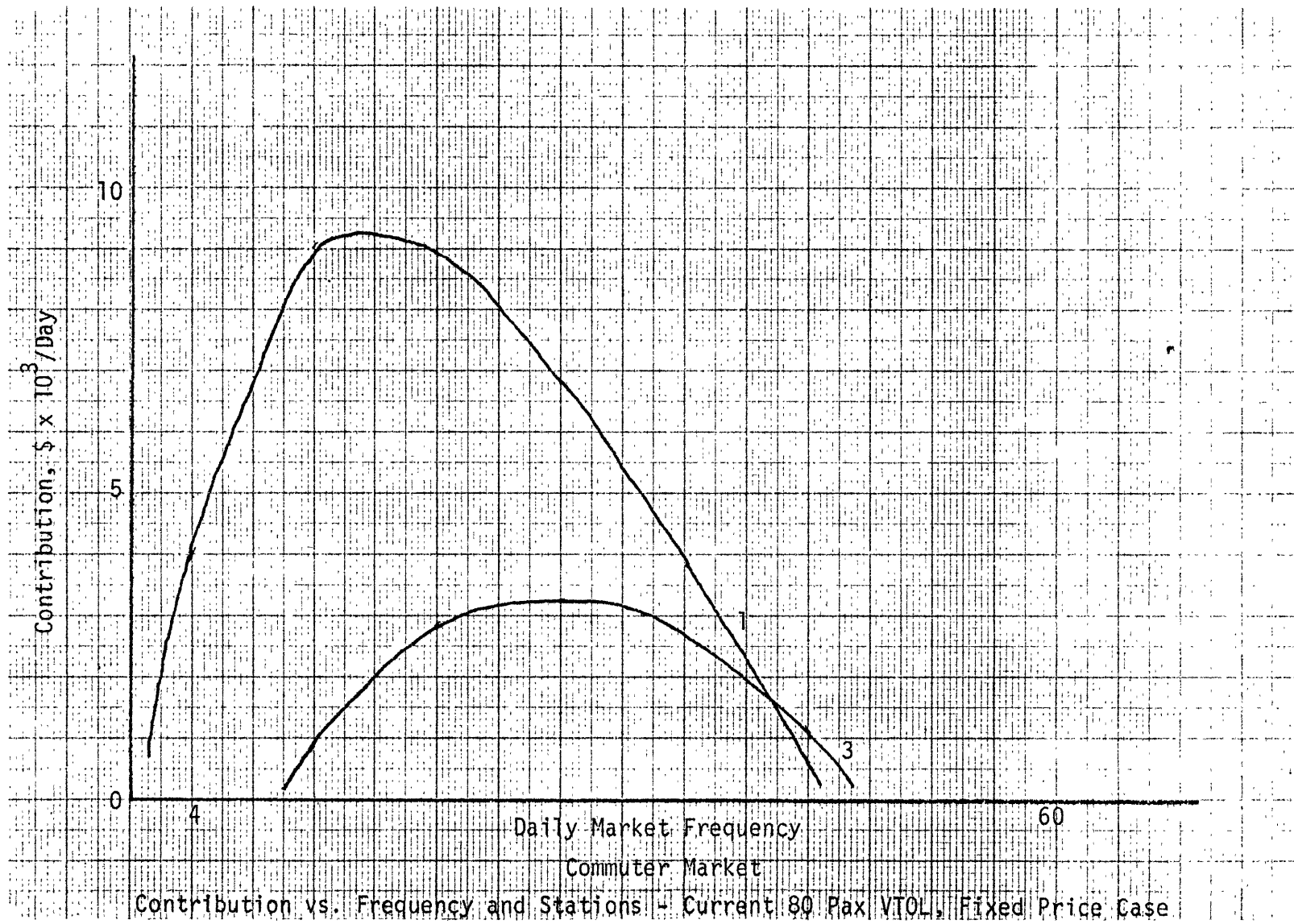
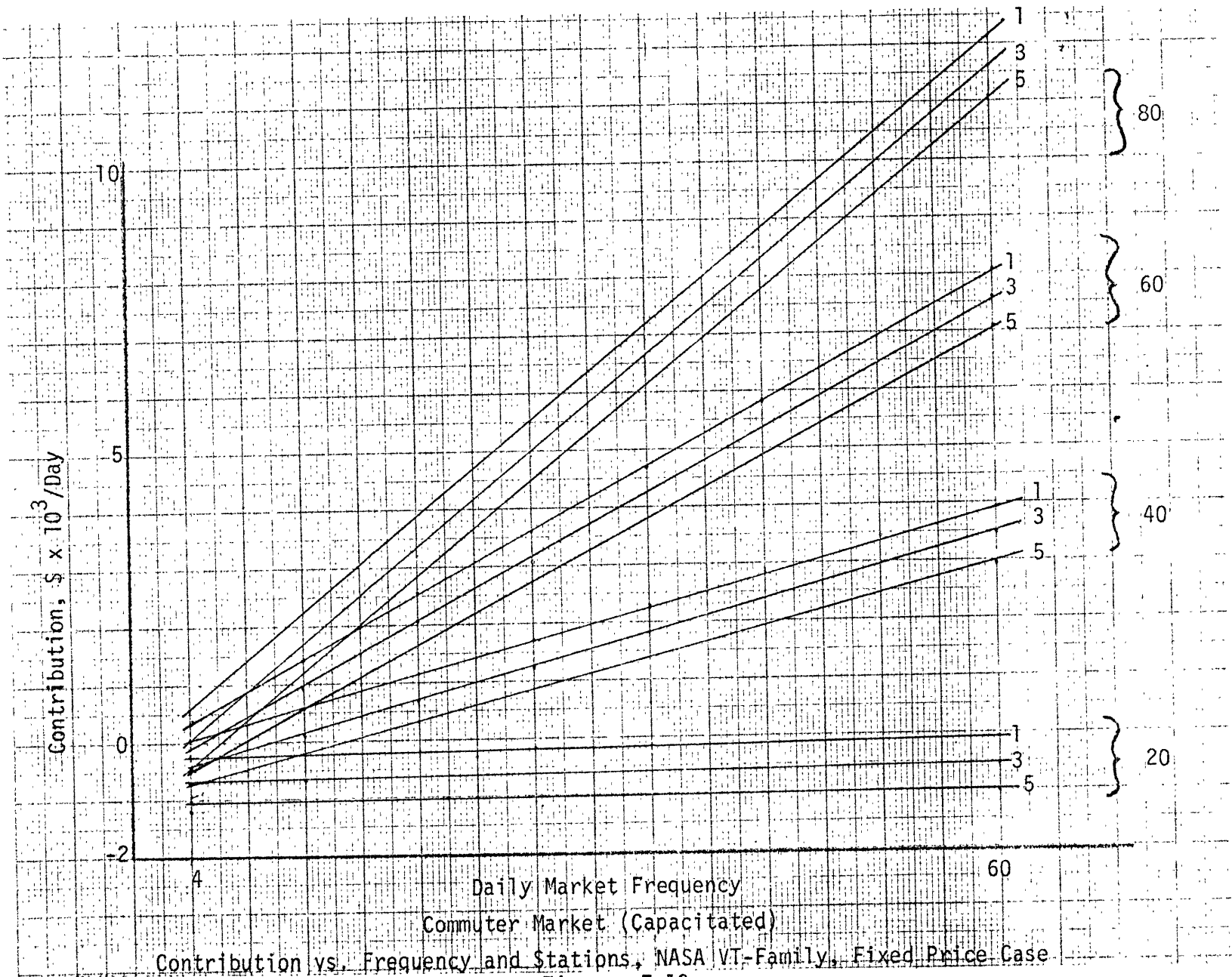
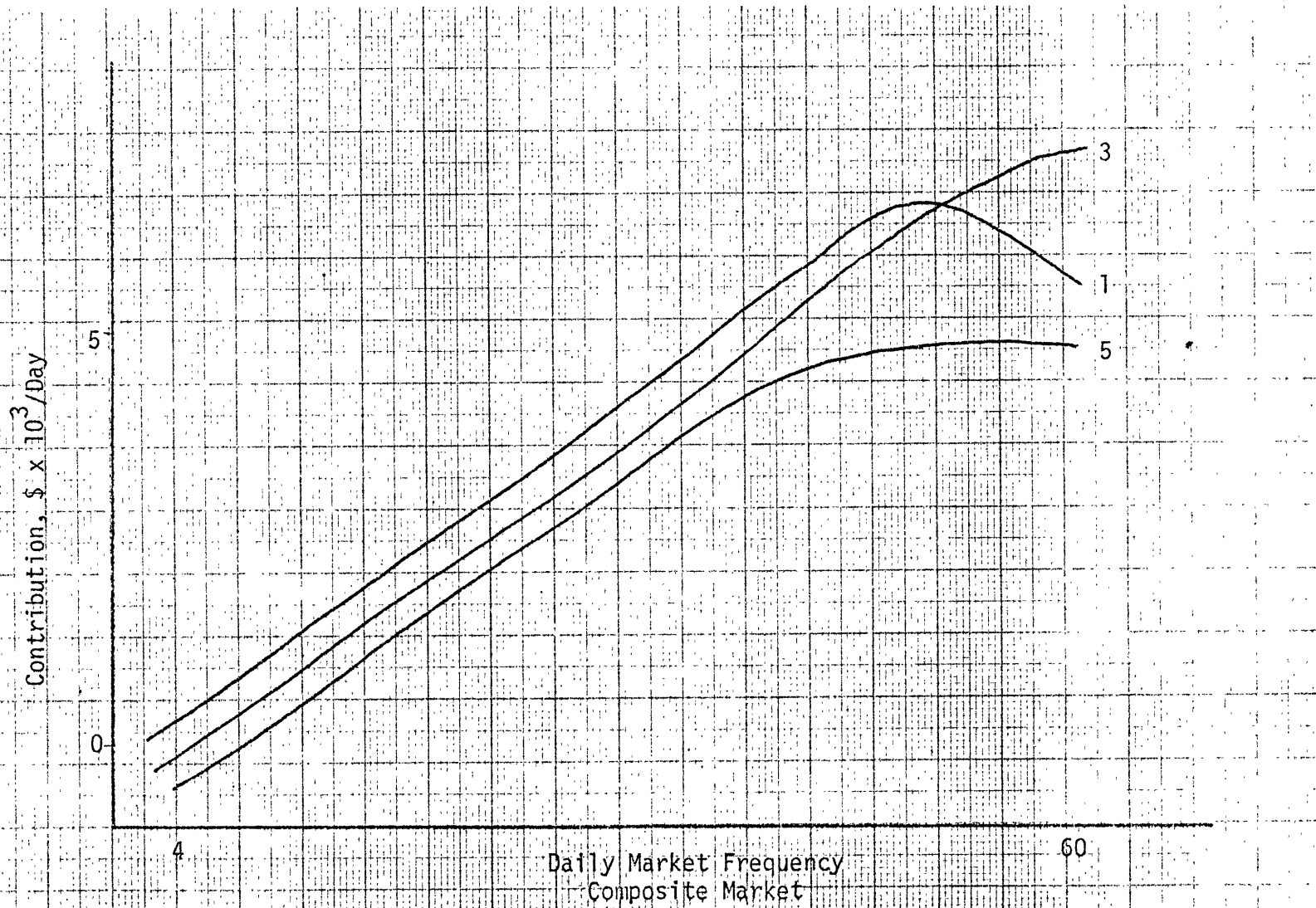


Figure 7.9



Contribution vs. Frequency and Stations, NASA VT-Family, Fixed Price Case  
Figure 7.10



Contribution vs. Frequency and Stations - NASA VT60, Fixed Price Case

Figure 7.11



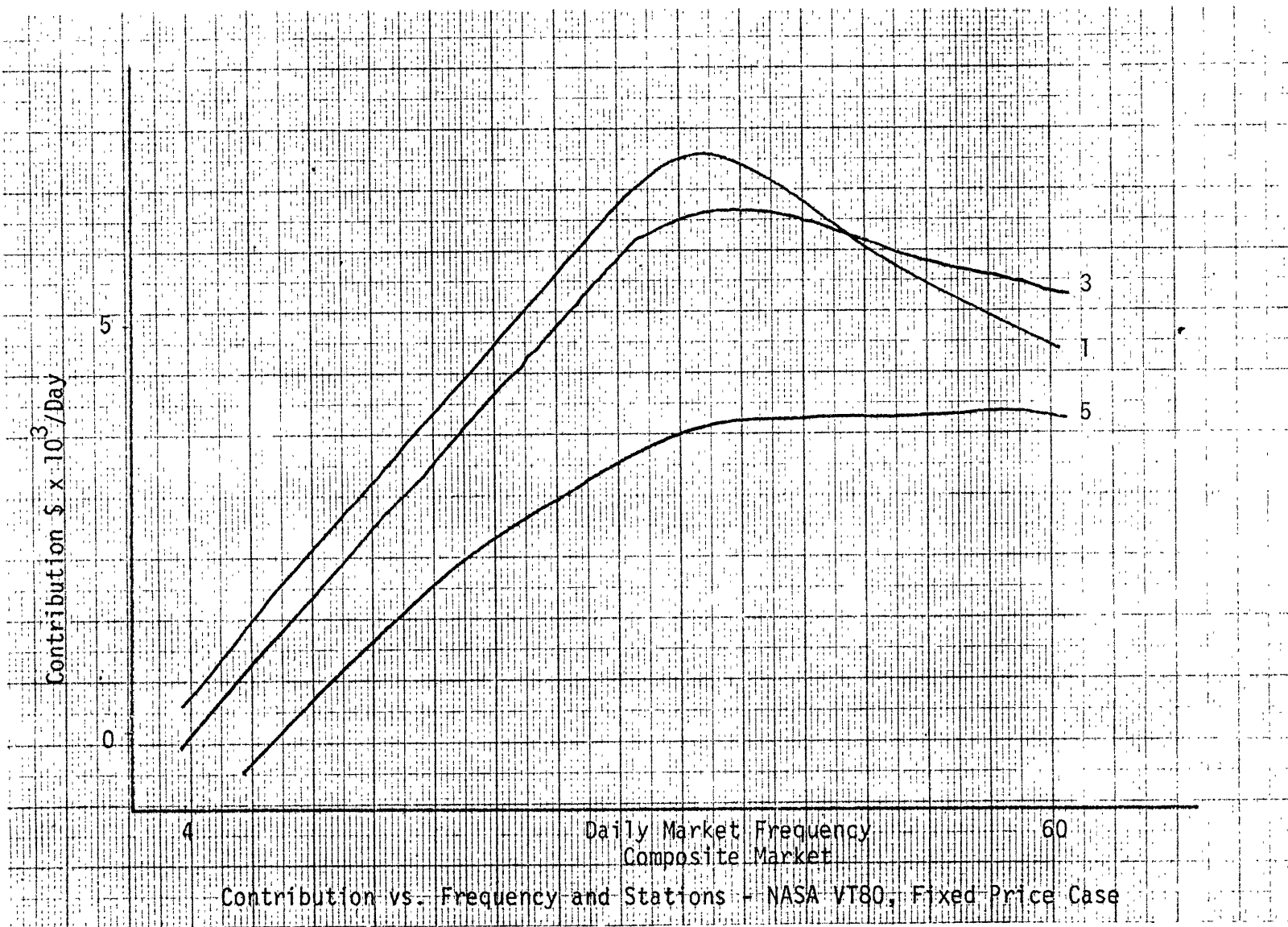


Figure 7.12

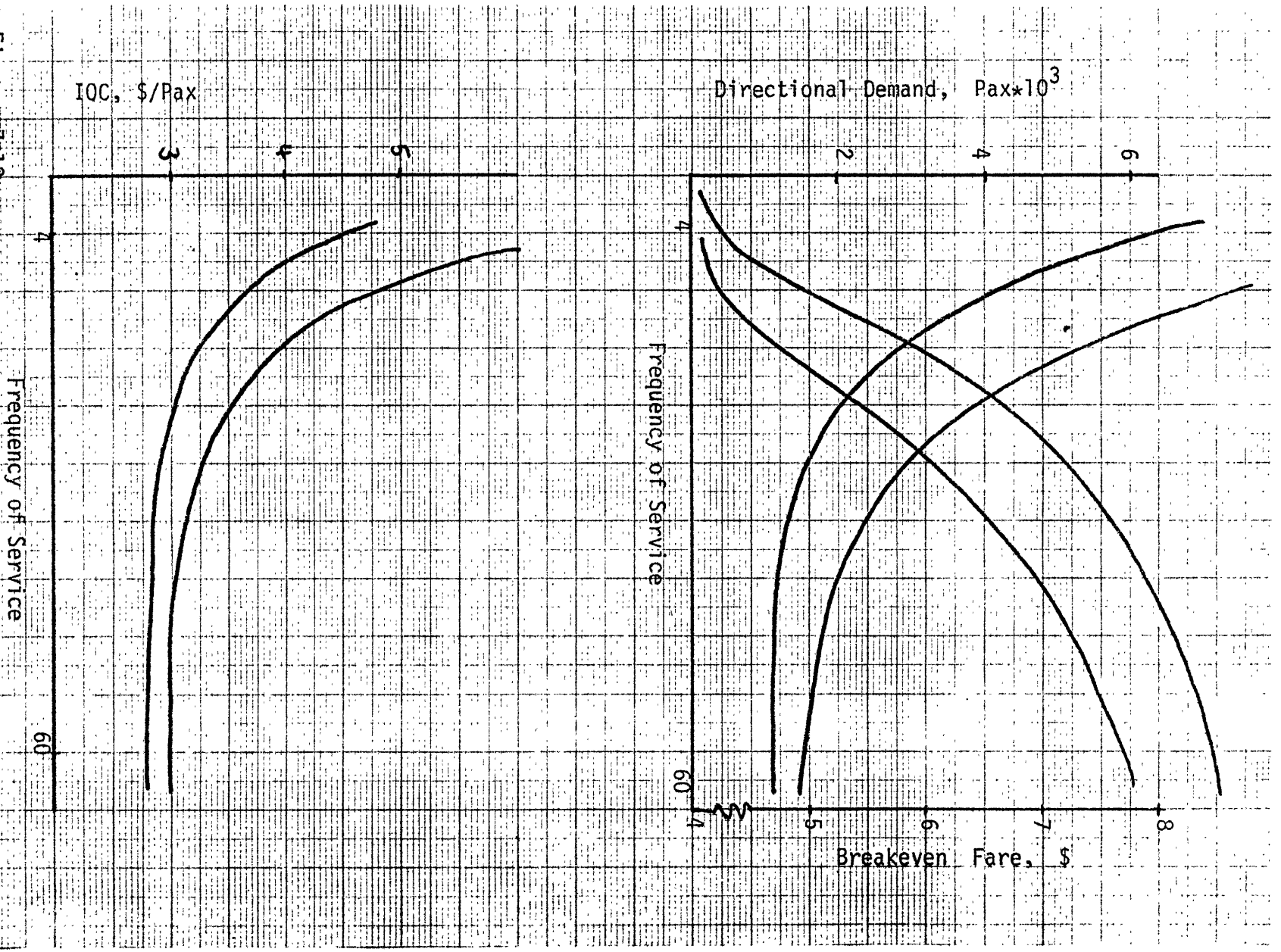
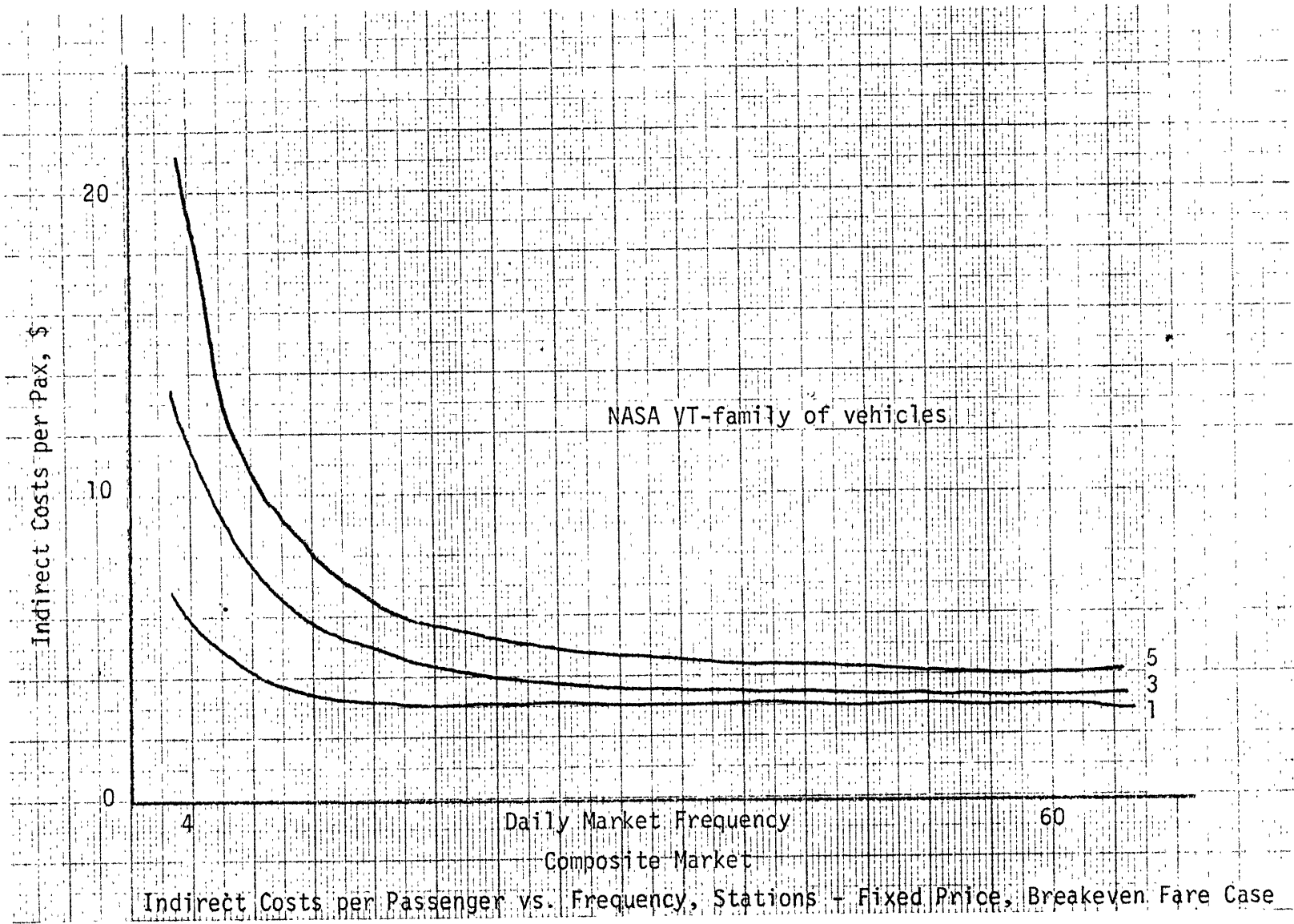


Figure 7.13  
 Influence of Station Costs on Demand, Breakeven Fare and IOC [Maximum Market Share Case] NASA VI-60 Vehicle, 3 Stations. SOC = \$250, \$750/Day

The variation of indirect operating costs per passenger in capacitated operations is detailed in Figures 7.14 and 7.15. Economies of scale are evident in all but the traffic and passenger servicing areas of indirect costs, leading to the asymptotic decline in indirect operating costs per passenger towards this figure. In cases where traffic volume is not heavy enough to support the overhead structure, the indirect operating costs will rise, eventually exceeding the fare charged. This is shown for the composite market group in Figure 7.14. As a "mature" operation is evolved, it is possible that even the passenger and traffic servicing component may be subject to economies of scale in terms of employee productivity and management advances. History, however, does not bear out this possibility.

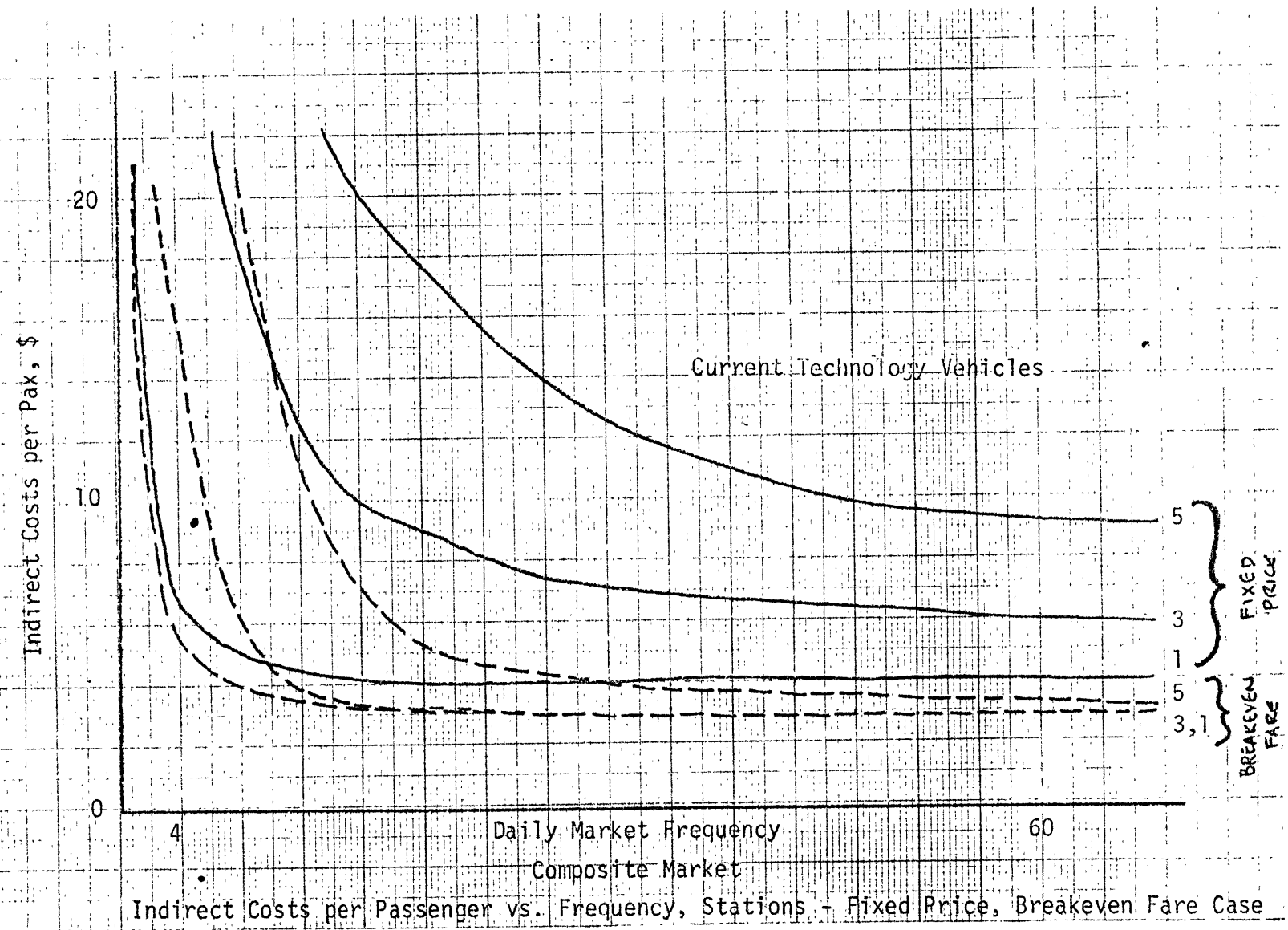
The effects of multistop routings are difficult to analyze in the single market case. The extra delays incurred due to the increased cycle time component of total time will diminish demand for the through segments of such a flight. On the other hand, the ability to offer service at a higher daily frequency with a larger vehicle is provided. This may or may not outweigh the delay costs. The analysis of Figures 7.16 and 7.17 is based on an effective frequency term of:

$$f'_k = f * (1 + 9 * k) \quad k = 0, 1, 2, \dots$$



Indirect Costs per Passenger vs. Frequency, Stations - Fixed Price, Breakeven Fare Case

Figure 7.14



Indirect Costs per Passenger vs. Frequency, Stations - Fixed Price, Breakeven Fare Case

Figure 7.15

where  $f$  = nonstop frequency

$k$  = number of intermediate stops experienced by passengers  
at the originating station.

The multistop frequency weighting value of .9 is derived from the demand function as representing an equivalent disutility (hence demand diminution). Combining the effects of multistation operations and the previous heuristic for  $f_k$ , we have for each specific  $N, k$ ,

$$f'_{N,k} = f_* \frac{(1+0.9*k)}{N} \quad \begin{array}{l} k=0,1,2\dots \\ N=1,2,3 \end{array}$$

where now,  $f$  = nonstop daily total market frequency

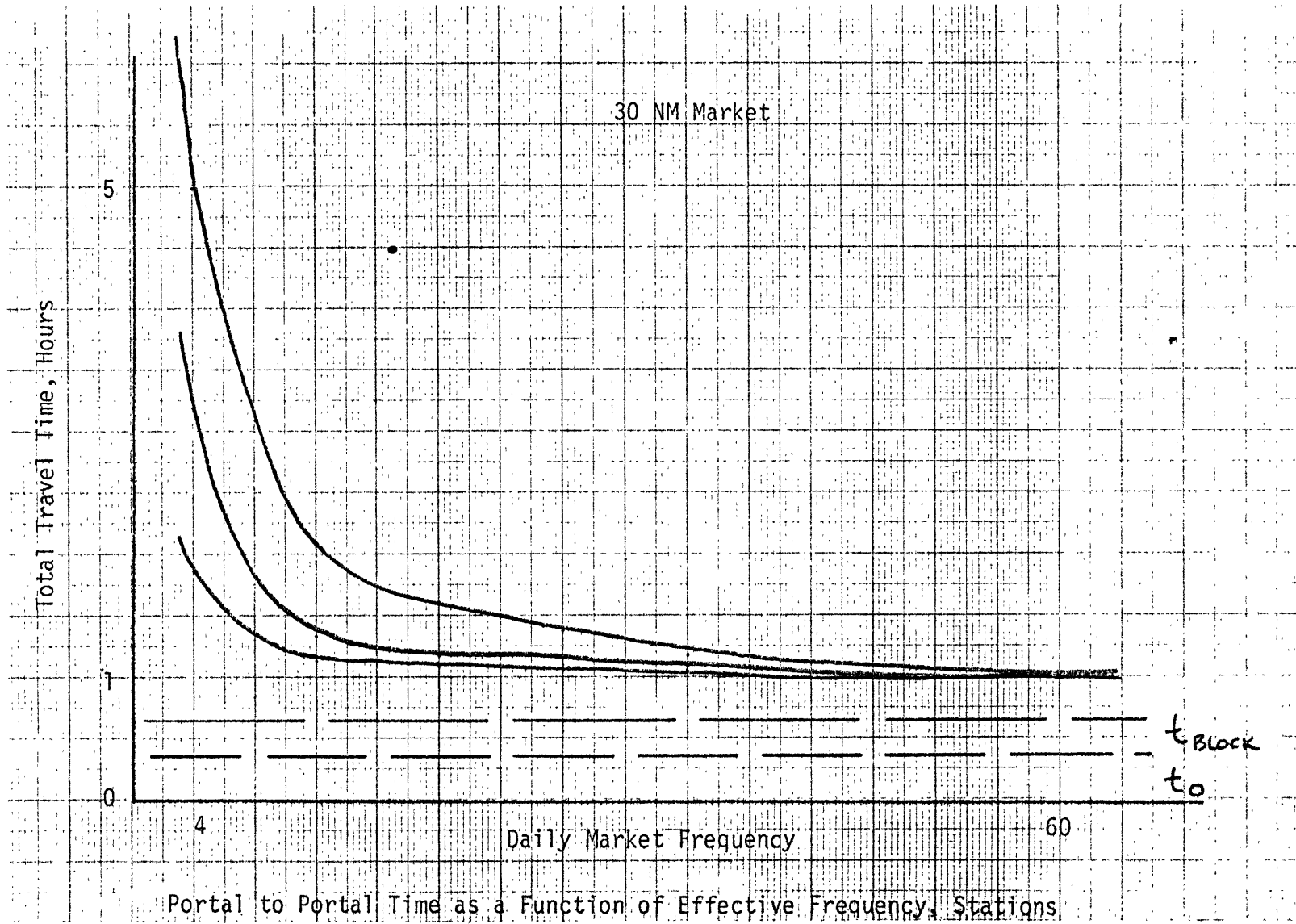
$k$  = number of intermediate stops

$N$  = number of equal workload submarkets (stations)

The demand model's response and the magnitude of station operation costs SOC, will influence the width of the feasibility region. By introducing quite a bit of added complexity into the analysis, this formulation of effective frequency heavily influences the network aspects of the model. It is exactly this problem that must be addressed in the general intraregional problem considered in the next chapter.

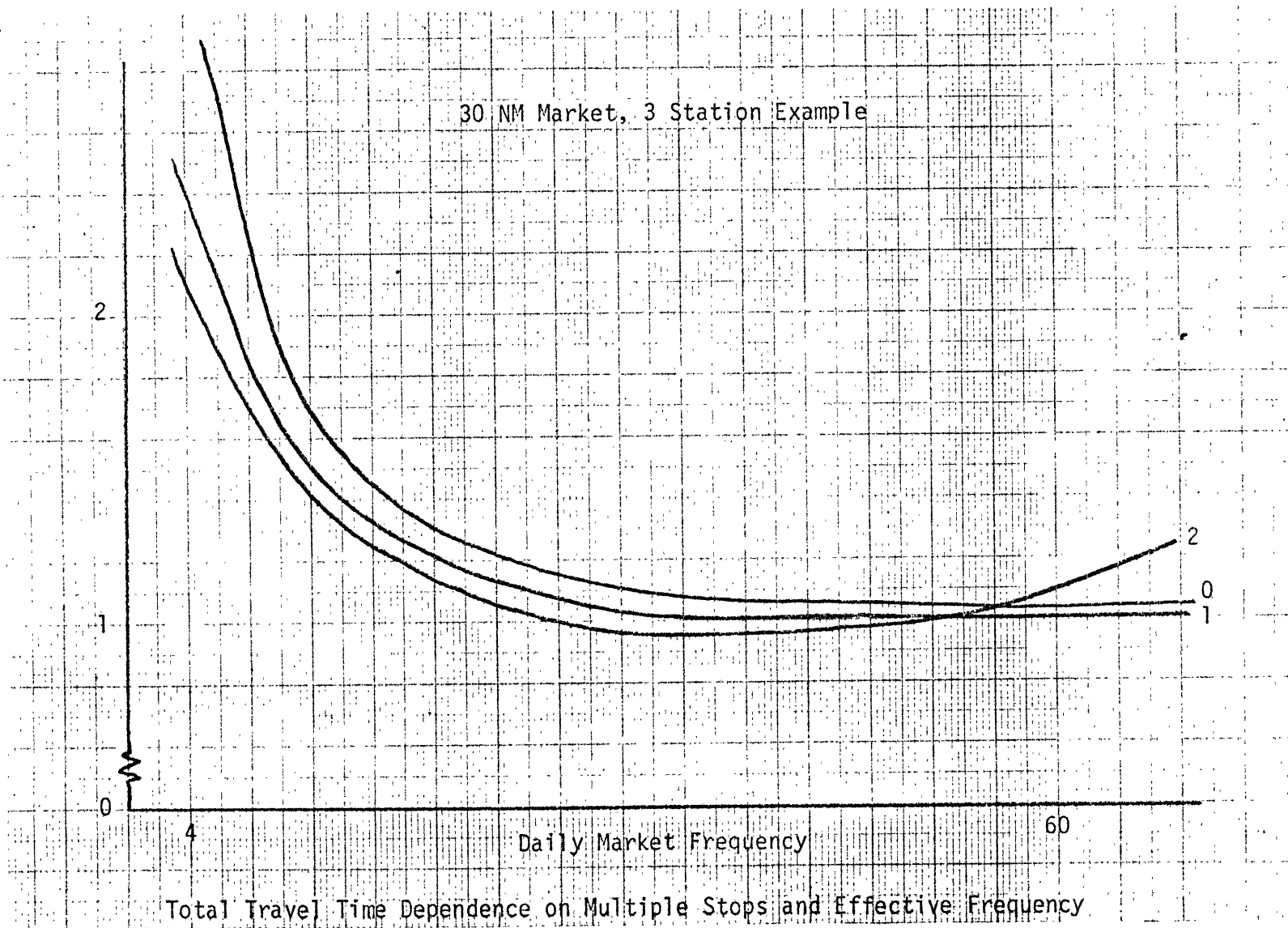
### 7.5.2 Maximize Market Share Policy

This related case used the results of Chapter 6 and optimum pricing policy in order to maximize market share with the additional constraint that revenues must equal or exceed total costs. This is



Portal to Portal Time as a Function of Effective Frequency, Stations

Figure 7.16



Total Travel Time Dependence on Multiple Stops and Effective Frequency

Figure 7.17



achieved in the parametric model by a recursive method similar to Newton-Raphson approximation. The convexity of the demand - level of service surface is exploited in order to guarantee convergence within six to eight iterations. Demand and price are recalculated within each iteration as a check on the degree of stability of the solution. Breakeven solutions are found in 92 percent of all cases. The remaining cases run into trouble with the double curvature found in the low ranges of the demand-frequency curves. Final statistics are output for each parametrization

Breakeven fare based on total costs is shown in Figures 7.18, 7.19 for the current and NASA vehicles. Demand at the given fare is presented in Figures 7.20 through 7.27 for the composite market and various vehicles.

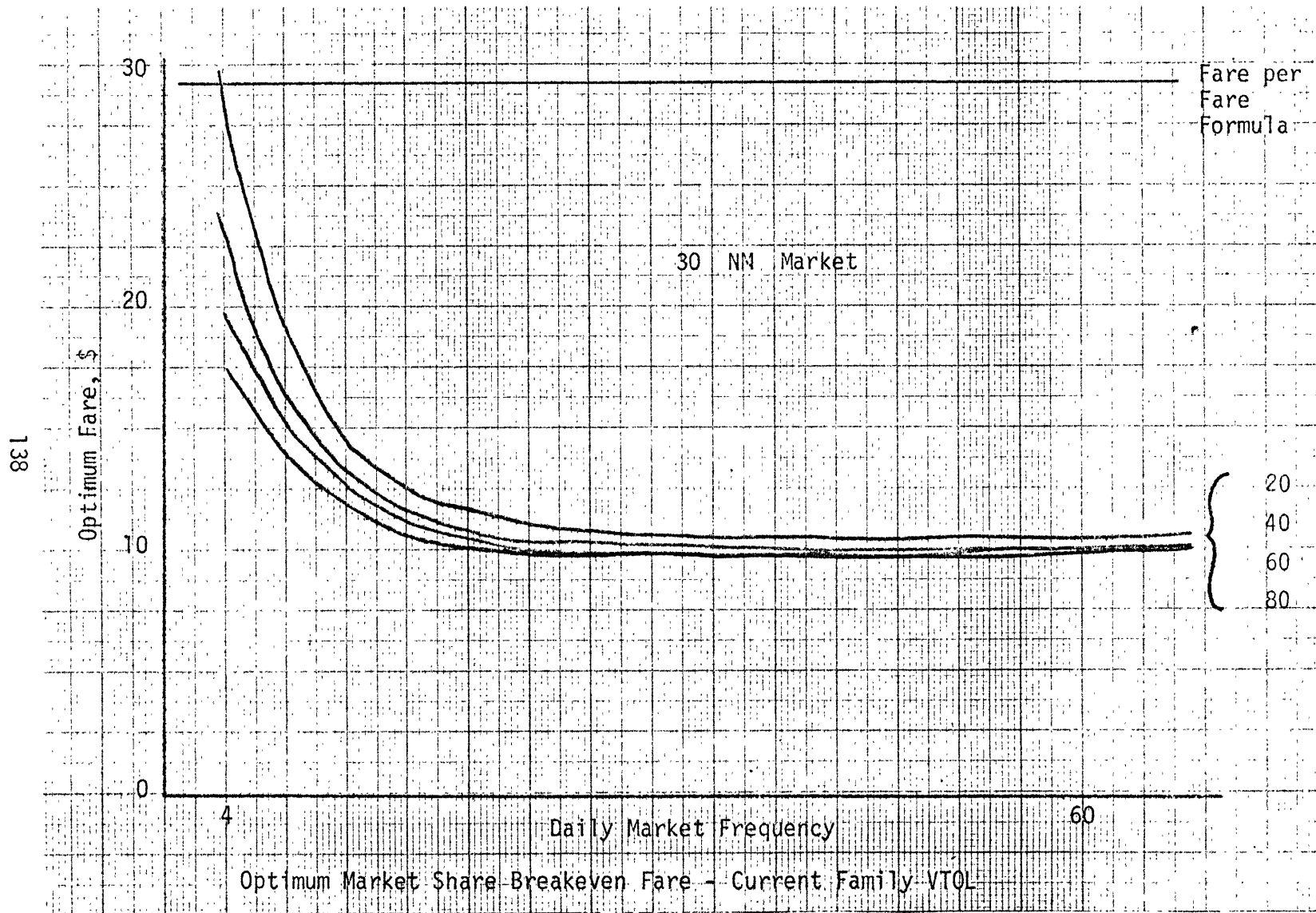


Figure 7.18

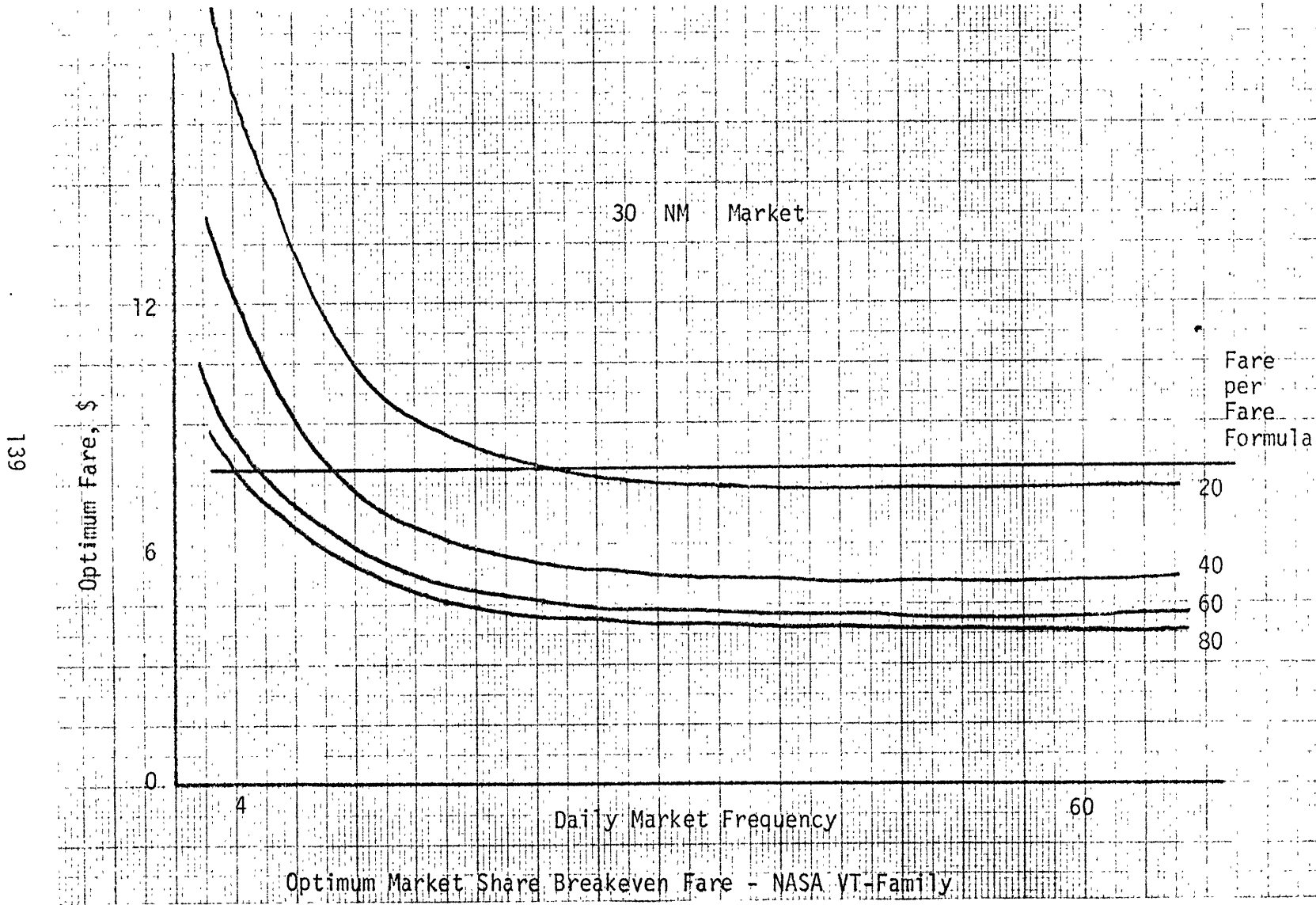
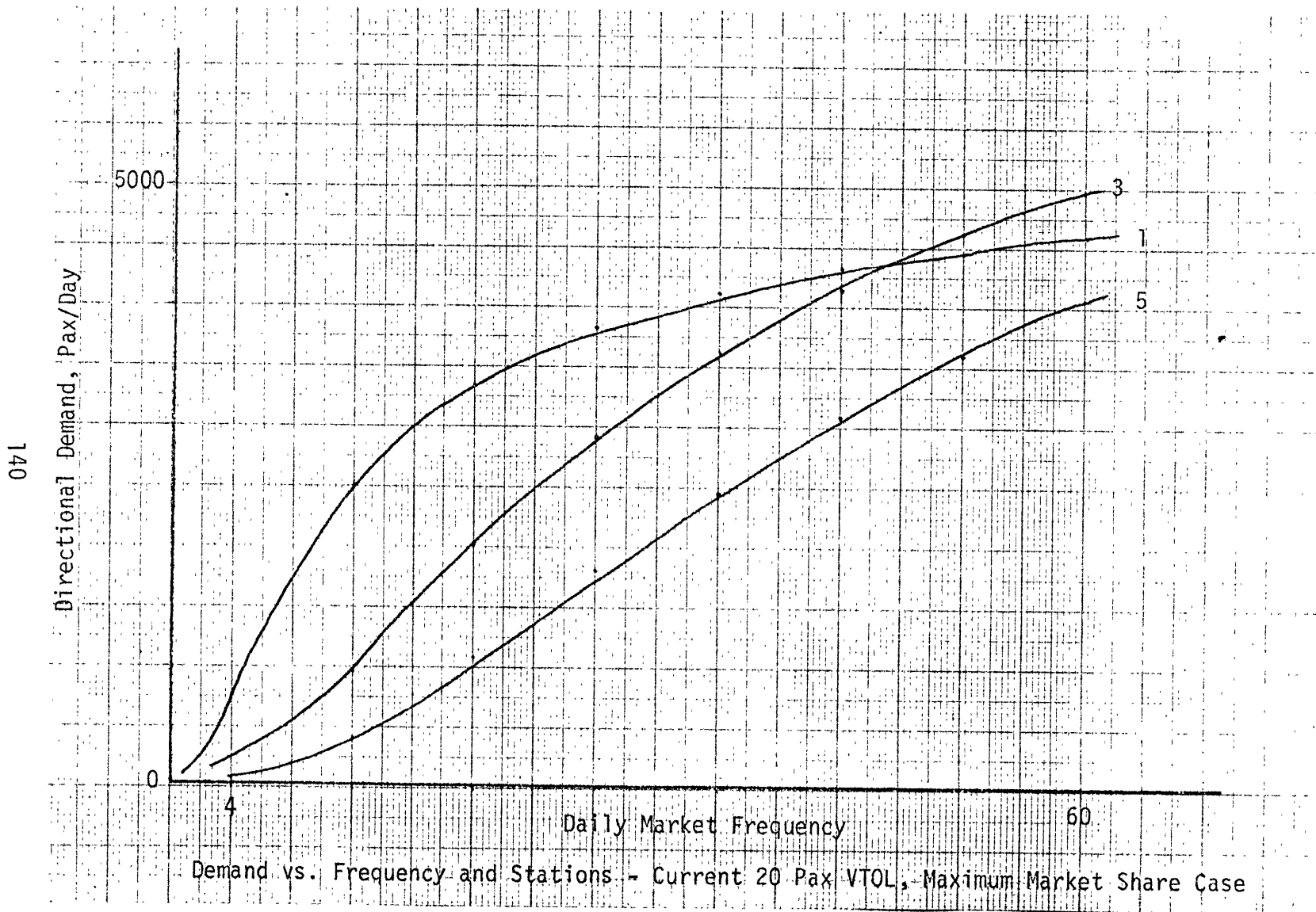
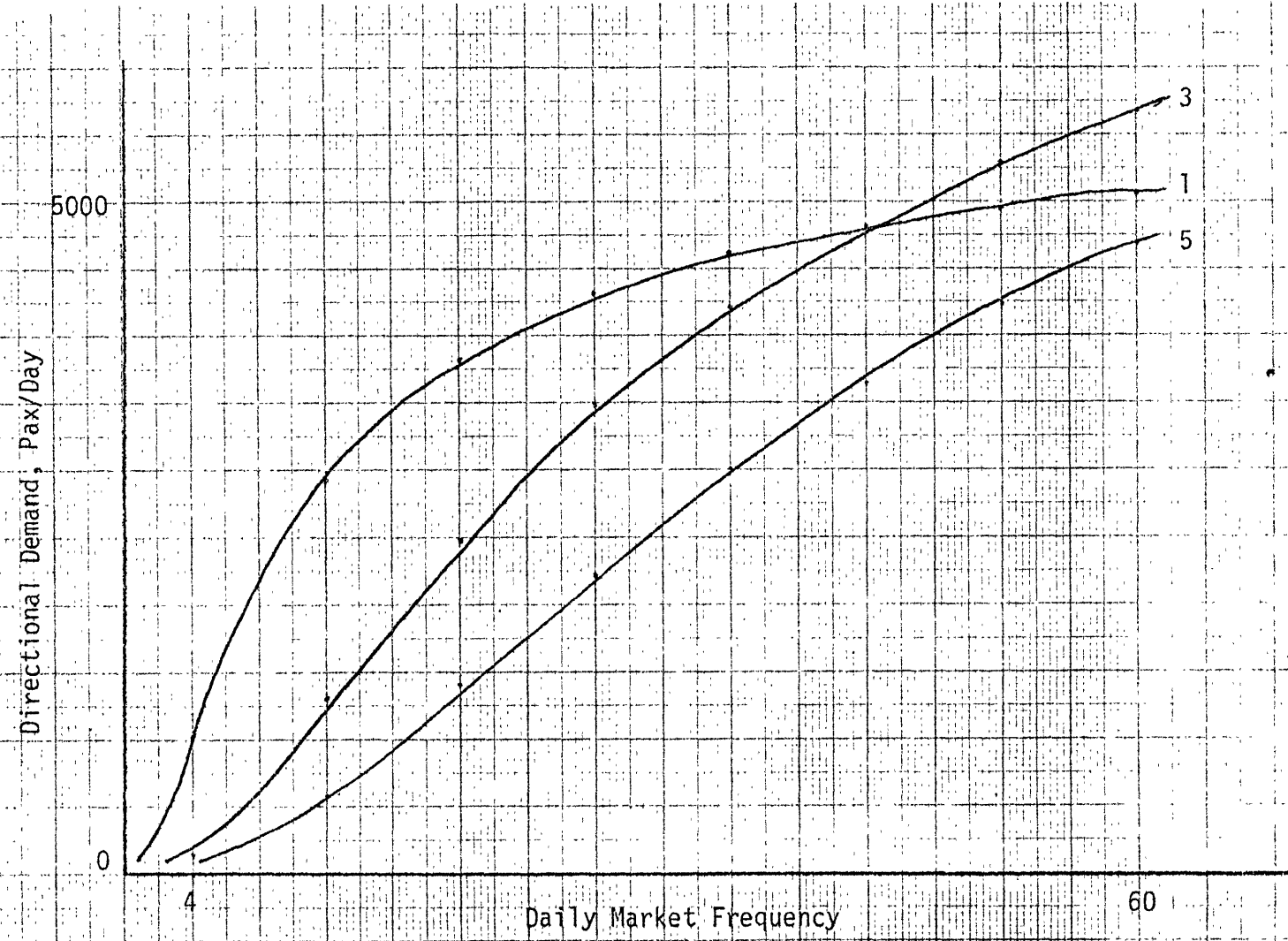


Figure 7.19



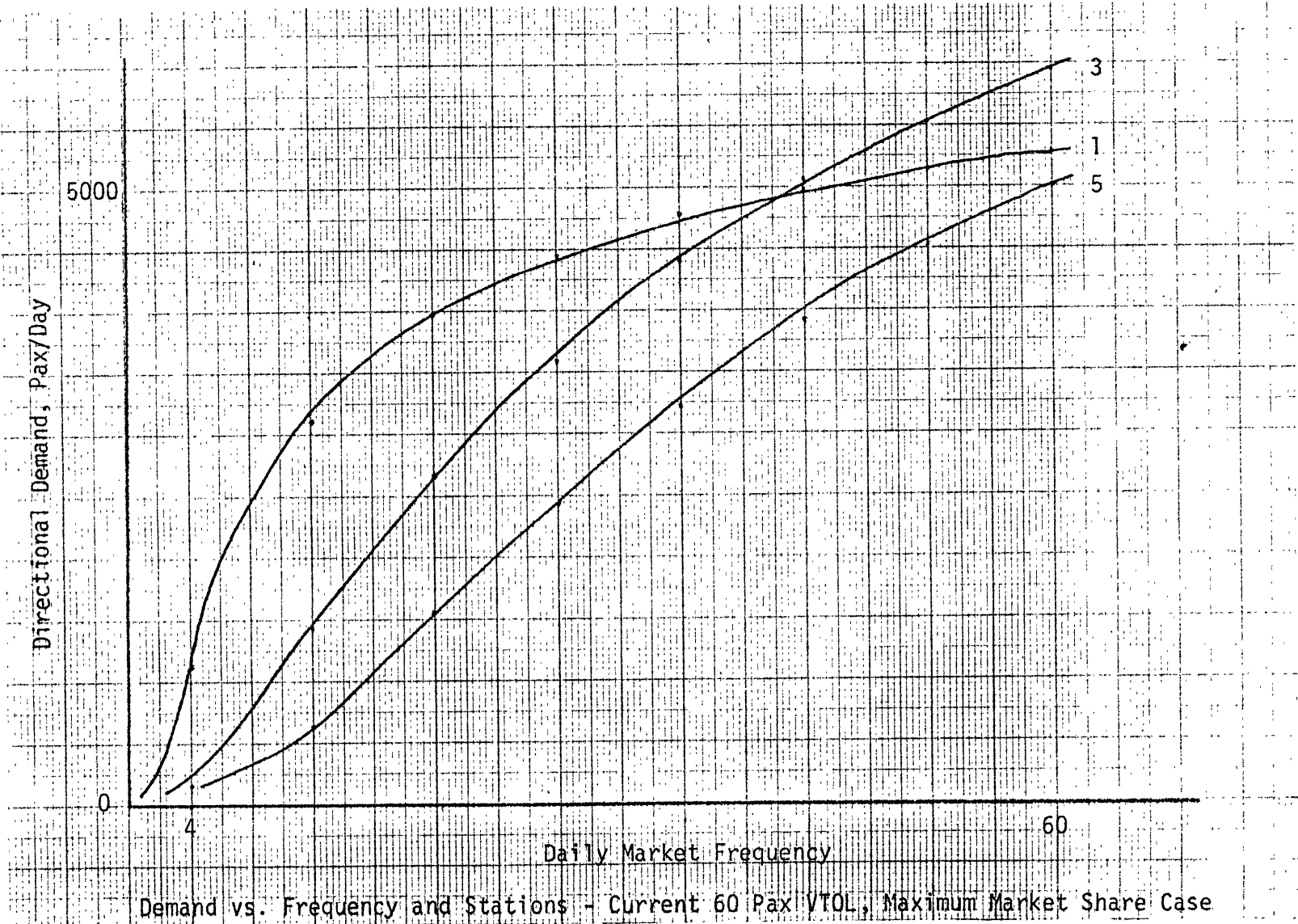
Demand vs. Frequency and Stations - Current 20 Pax VTOL, Maximum Market Share Case

Figure 7.20



Demand vs. Frequency and Stations - Current 40 Pax VTOL, Maximum Market Share Case

Figure 7.21



Demand vs. Frequency and Stations - Current 60 Pax VTOL, Maximum Market Share Case

Figure 7.22

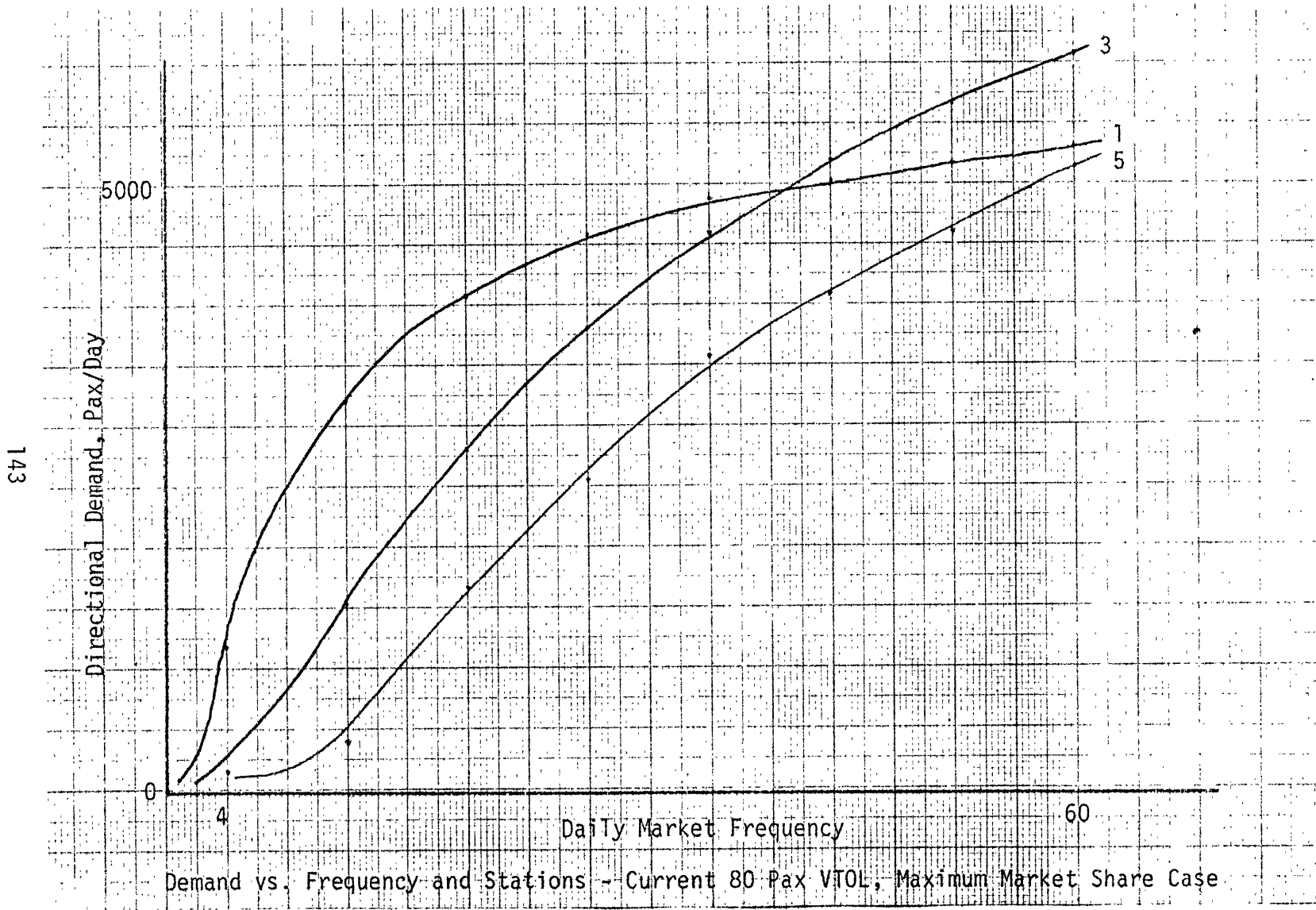
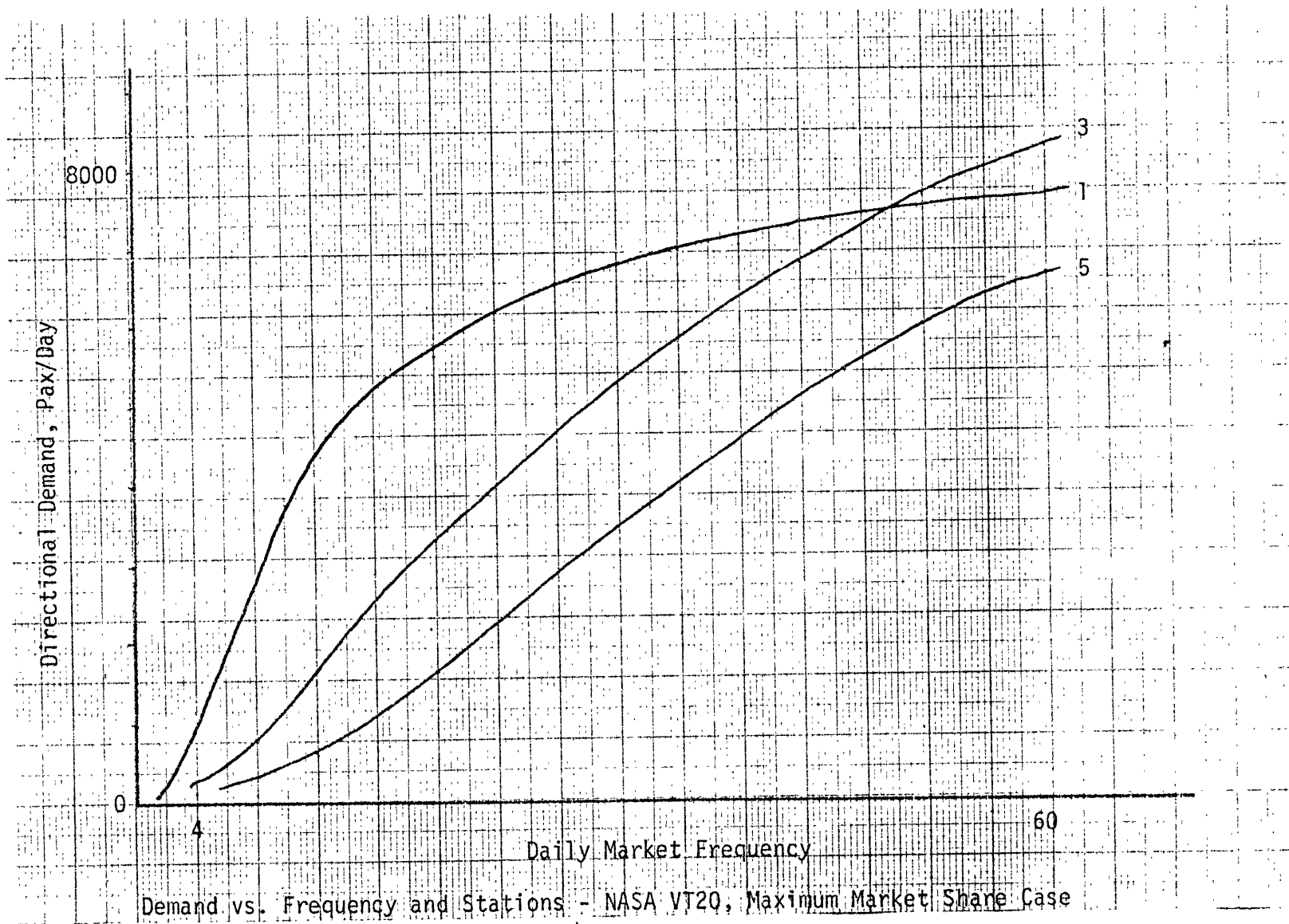


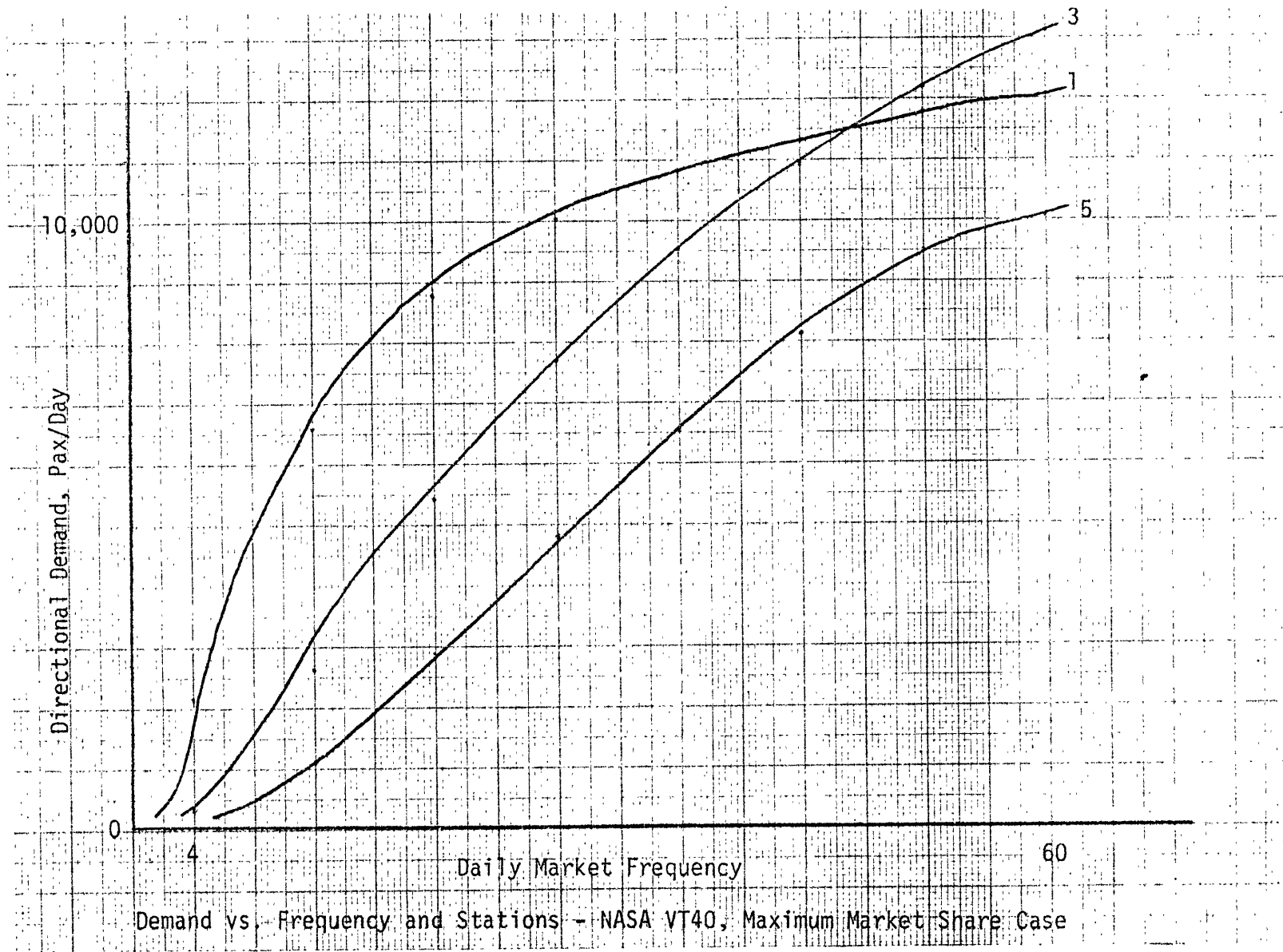
Figure 7.23



Demand vs. Frequency and Stations - NASA VT20, Maximum Market Share Case

Figure 7.24





Demand vs. Frequency and Stations - NASA VT40, Maximum Market Share Case

Figure 7.25

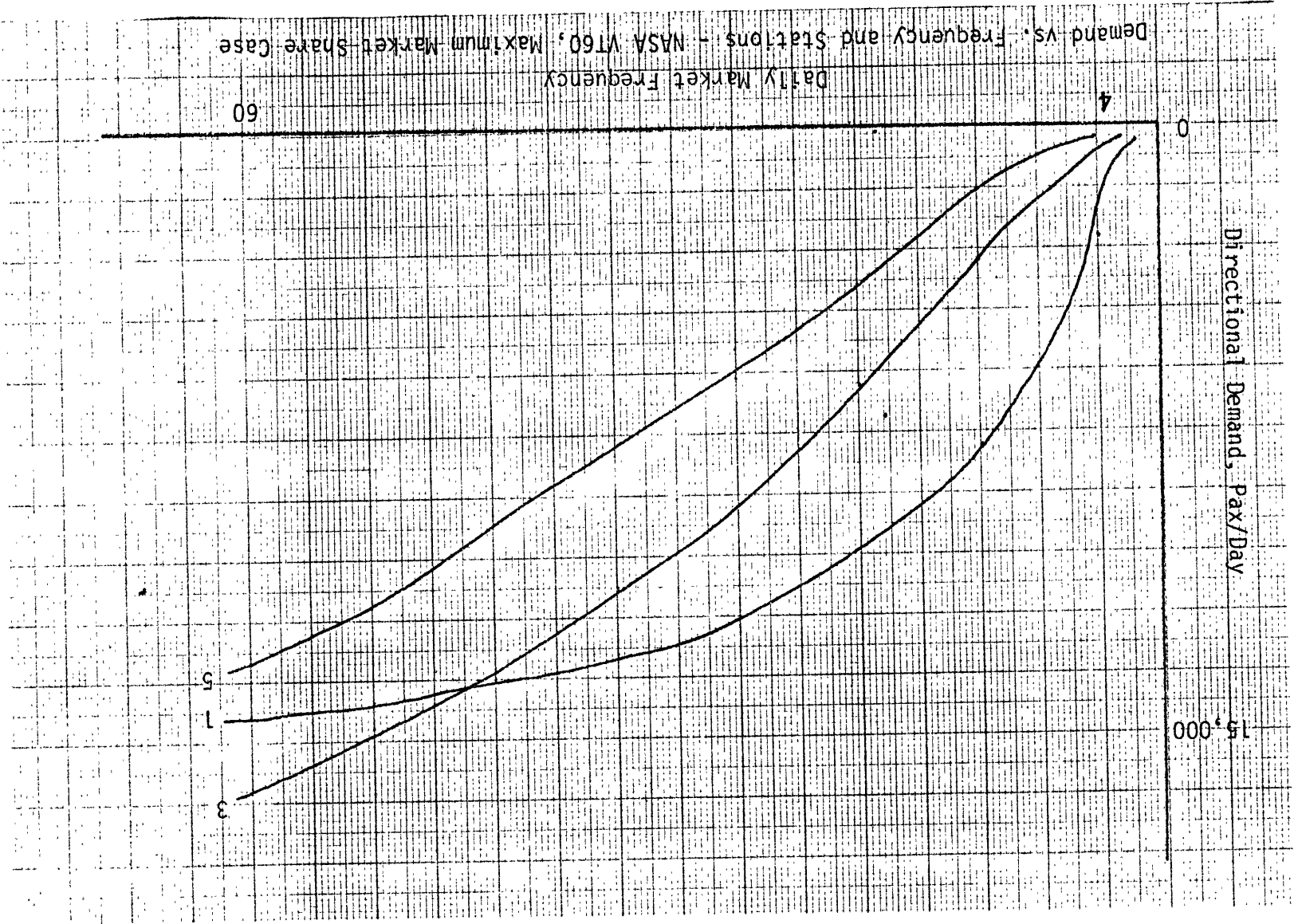
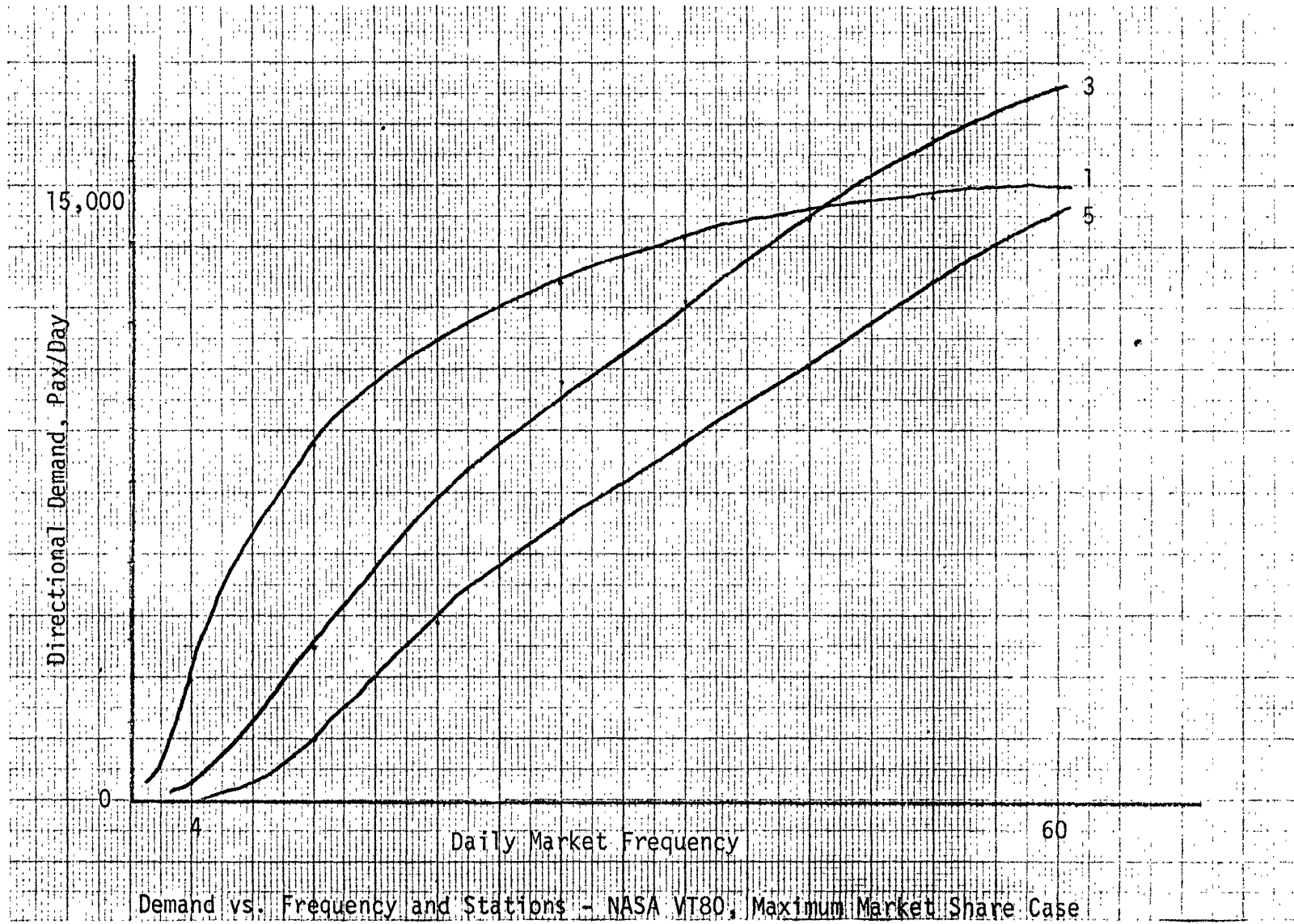


Figure 7.26



Demand vs. Frequency and Stations - NASA VT80, Maximum Market Share Case

Figure 7.27

### 7.5.3 Intraregional Service Study

If the results of the Long Island example shed light on the interrelationships among various levels of service variables, it also brought some questions to the fore. Among them:

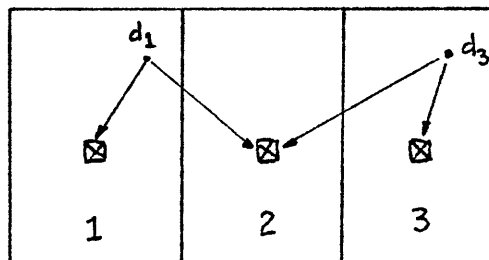
- The limitations inherent in a single market analysis of the various factors affecting level of service as opposed to a network analysis. (e.g., the intraregional transportation problem)
- The probabilistic nature of travel times in a network
- The temporal and spatial distribution of demand, and its effects on station workload and other queuing results.

The problem of single market vs. network analysis is one that is easily treated by making assumptions about travel times and about demand distributions in time and space. Using the heuristics and results of probability theory developed in Chapter 6, a network model may be synthesized.

Knowing the relationships of demand to frequency, market subdivision and multistop service, it is possible to formulate a linear programming problem to solve for best generalized equipment utilization and routing policies. We must, however, abandon the idea of probabilistic and temporal variations in travel times in favor of analyzing the scenario during various locales during the operating day and using instead these expectations. Spatial distribution of

demand must be assumed such that workload sharing occurs and such that no queuing occurs. Those passengers who cannot be accommodated in their own district and find no seats or reservations available are (quite reasonably) lost to the system.

Another problem not currently encountered in the model that is termed "the burden of central location". Well documented in the literature and an intuitively correct result - this documents the tendency for the workload in a centrally located facility to be higher than average due to its greater probability of being a "second best choice" to several remote facilities. This is easily shown for a three sector corridor example in Figure 7.26. The assumption of workload averaging and sector-dedicated demand allows a simpler formulation of the problem, yet it begins to deviate from the intuitive "real world" theory of operation. It can be



Facility 2 functions as a "second best" choice for demands  $d_1, d_3$ , in regions 1 and 3. Workload at facility 2 is therefore higher than system average.

Figure 7.26

shown, however, that for cases where a steady state results exists, that the modelled result is very close to the actual situation.

The network analysis of operational and financial feasibility was performed using the IBM linear programming package MPSX with the Flight Transportation Laboratory's FA-4.5 fleet assignment model (Figure 7.27). FA-4.5 preprocesses and postprocesses matrices of input data to and from the MPSX solution package. It was altered slightly in order to function as a demand assignment model, and by varying costs, routing and fleet mix was allowed to solve for local optima within scenarios. The man-machine interface then "globalized" the problem, which although combinatorial in nature, may be radically reduced in size.

From the results of the Long Island market example, demand responsiveness to level of service variables is known. For a generalized intraregional case, we may alter the relationships with reference to a random station:

- access and egress times are proportional to  $(n)^{-1/2}$
- effective frequencies are again normalized with respect to  $k$  and  $n$ , but must include normalization with respect to  $n_a$  and  $n_e$ .

$$f''_{n_a n_e k} = f^* \frac{(1+0.9*k)}{N_a N_e}$$

- demand is responsive to effective frequency

The problem is then solved with variables ranging similar to those

Fig. 7.27

FLEET ASSIGNMENT MODEL

FA-4.5

OBJECTIVE FUNCTION:

$$\text{MAX} \left\{ \sum_{PQ} \sum_{R \in R_{PQ}} Y_{PQ} * P_{PQR} - \sum_R \sum_A DC_{RA} * N_{RA} - IC_T \right\}$$

SUBJECT TO THE FOLLOWING CONSTRAINTS:

1A) LOAD FACTORS

$$\sum_A LF_{RA} * S_A * N_{RA} - \sum_{\substack{P < J \\ I < Q}} P_{PQR} \geq 0 \quad \forall IJ \in R$$

1B) TRAFFIC-FREQUENCY RELATIONSHIPS

$$\sum_K P_{PQ}^K * N_{PQ}^K - \sum_{R \in R_{PQ}} P_{PQR} \geq 0 \quad \forall PQ$$

1C) DAILY FREQUENCIES SUM OF SEGMENT FREQUENCIES

$$\sum_M K_M * \sum_A \sum_{R \in R_{PQ}} N_{RA} - \sum_K N_{PQ}^K = 0 \quad \forall PQ$$

# FLEET ASSIGNMENT MODEL

## FA-4.5 (CONT'D)

### 2) FLEET AVAILABILITY

$$\sum_R T_{\text{BLOCK}_{RA}} * N_{RA} \leq U_A * A'_A \quad \forall A$$

### 4) LEVEL OF SERVICE

$$\sum_{M=1, M} \sum_{R \in R_{PQ}} \sum_A N_{RA} \geq NM_{\text{MIN}_{PQ}} \quad \forall PQ$$

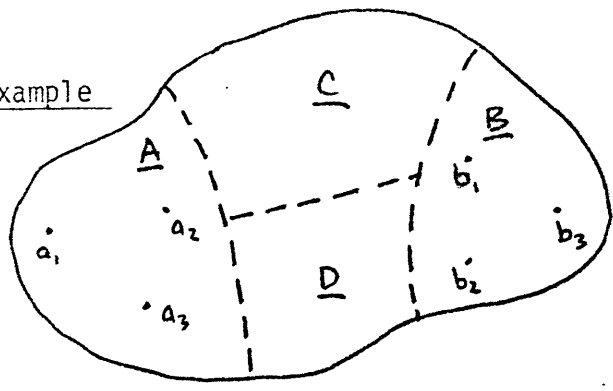


of the Long Island study and projected indirect operating costs, fare levels and vehicle sizes. Expected minimum utilization may be input to solve for required fleet size.

The example uses a generalized intraregional market similar to that of the Long Island case. The region-to-point market is expanded into a true region-to-region case. The region,  $R$  (Figure 7.28) consists of a number of demand sub-regions A, B, C, etc. Sub-regions A and B are employed to illustrate the problem. Each subregion  $Q$  is served by several facilities  $q_1, q_2, q_3, \dots, q_n$  each with a dedicated local demand. A market PQ experiences a level of service - primarily defined by frequency of service,  $f$  - according to  $f''_{n_a n_e k}$ , the effective frequency given  $N_a$  and  $N_e$  for P and Q. A three by three station example is used due to its apparent optimality in the region-to-point profit optimization. Hence, each local market experiences a frequency  $f''_{3,3,k}$ . The total demand may be divided into three equal workloads, and the model is run for nonstop, onestop and twostop routings with the VT20 - VT80 family of NASA rotorcraft.

Figure 7.28 The Intraregional Example

pt-pt.  $\widehat{AB}$  experiences  $f = f$   
 Reg-pt.  $\widehat{a_n B}$  exp.  $f' = f_{n,k}$   
 Reg'l.  $\widehat{a_n b_n}$  exp.  $f'' = f_{n,m,k}$



Particular variable values used in the network solution

were:

Vehicle and Direct Costs

•The VT series of NASA costs were used, hence

$$DOC_{HR} = 252. + 4.20 (S)$$

For the individual vehicles, these are

VT20 @ \$336./HR

VT40 @ \$420./HR

VT60 @ \$504./HR

VT80 @ \$588./HR

Cruise speeds conservatively chosen as 140 KT

Maximum utilization (rotors on) of 8 hours per day

Fares

•The NASA/Lockheed composite fare of  $\$4.00 + \$0.10*(d)$

Network data

•Six stations - A1, A2, A3, B1, B2, B3 - three in each demand subregion.

•15 city pairs, eight linehaul and six local (intra A,B).

•Traffic-frequency curves normalized from the Long Island example.

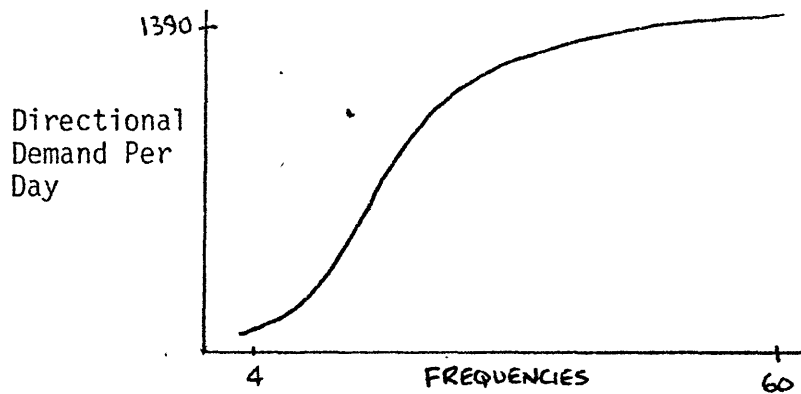


Fig. 7.29  
Traffic-Frequency  
Curve

•Multistop frequency weightings

$$\text{nonstop} = 1.0$$

$$\text{onestop} = 0.9$$

$$\text{twostop} = 0.8$$

•72 possible routings of two stops or less

In general, with N and M stations in origin and destination zones, the upper bound on the number of k-stop or less flights is:

$$[r] = \sum_k^{N+M} P_{k+1} = \sum_k \binom{N+M}{k+1}; (k+1) < (N+M)$$

$$= \sum_k \frac{(N+M)!}{(N+M-(k+1))!}$$

Symmetry pares this down by a factor of two, and rationalized routing reduces this further.

### Indirect Costs

Boarding charge is \$2.50 per passenger plus an indirect charge of \$0.05 per mile in cruise.

Overhead is (N+M)\*station capital costs

OVH = \$3000 / day for six stations

### Financial Statistics

- Revenue is the total income from fares.
- Yield is  $0.96 * \text{Revenues}$
- Net Yield per passenger is Fare minus Passenger Costs

$$NY_p = (F - GC_p)$$

• Direct operating costs are the sum of fleet  $DOC_{HR}$  times utilization.

• Systems costs are  $(0.161 * 10^6) + 0.43 * \text{Revenues}$

• Contribution is the objective function of the linear program and is excess of revenues over total costs.

### Geographical Data

• Interregional stage length of 30 NM.

• Intraregional stage length equal to 9 NM. This was derived from a  $200 \text{ NM}^2$  assumed circular capture area for each subregion, with three stations per subregion. Hence  $66.7 \text{ NM}^2$  per station capture area yielding interstation distance of 9 NM.

The results of the network analysis are presented in FA-4.5 postprocessor output format in Table 7.5. The optimal solution yields a contribution to overhead of \$18,283 per day on the basis of 371 flights and 519 segments flown. The market penetration is 8.07 percent with 17,600 passengers and 527,000 revenue passenger

miles. System average load factor is 78 percent. Fleet requirements for the optimal three by three intraregional example are 14 aircraft, of which 8 are the VT80 and 6 VT20. No VT40 or VT60 aircraft are used.

Greater than 81 percent of the passengers travel nonstop, and none travel via twostop flight. Although fully 40 percent of the total system flights are onestop, these are generally flown with the VT20 aircraft and are flown for positioning purposes. These tag ends account for the system wide load factor being less than the 80 percent maximum.

Note that the system total effective frequency in the AB market is 129 daily round trips, and that on the average during the 16 hour operating day this is 8.05 flights per hour A→B. This provides a system mean headway of 7.5 minutes and an expected wait time of less than 4 minutes. The dedicated submarket round trip frequency is 21.5 with a headway of 44.6 minutes and an expected wait time of less than 23 minutes.

With a rolling scheduled, zone fare system, these dedicated submarket statistics are quite reasonable. Realizing that in fact the "second best" local facility may be an adequate substitute doubles the effective frequency, although perturbing the localized demand model. A steady state argument may be advanced, however, invoking a derivative of the Law of Detailed Balance from probability

theory to assume a net zero sum from those passengers crossing localized facility boundaries. This modifies any queueing argument to be drawn from the data from M/M/1 to M/M/n, but adds a realistic bit of behavioral and marketing psychology to the model.

\*\*\* ULTRA SHORT HAUL NETWORK EXAMPLE -- 1X3 STATIONS \*\*\*

INPUTS SHOULD BE VALUES FOR THE SYSTEM TOTALS PER DAY .  
 BECAUSE THE SOLUTION ASSUMES A SYMMETRICAL SYSTEM, DEMANDS AND ROUTES ARE ENTERED FOR ONE WAY ONLY.  
 OUTPUTS WILL BE THE FULL SYSTEM TOTAL PER DAY MULTIPLIED BY 1,000 AS SPECIFIED IN THE INPUT DATA.  
 THE OUTPUT WILL BE ADJUSTED TO ELIMINATE FREQUENCIES LESS THAN 0.50 FLIGHTS PER DAY IF POSSIBLE.

IOC'S:  
 BASIC OVERHEAD PER DAY =\$ 1500.000  
 IOC PER REVENUE PASSENGER MILE=\$ 0.0  
 IOC PER AIRCRAFT MILE=\$ 0.0500  
 IOC PER PASSENGER HOURLY=\$ 2.500  
 IOC PER AIRCRAFT DEPARTURE=\$ 0.0  
 IOC PER PASSENGER CONNECTION=\$ 0.0

AIRCRAFT DATA:

NAME	QUANTITY	HRS/ DAY	HOURS/TC	COST/TO	SPEED	COST/MI	COST/HR	SEATS	IOC/TO	IOC/MI	RANGE (MI)
VT20	10.0	8.000	0.050	0.0	140.0	0.0	376.00	20.0	0.0	0.050	200.
VT40	10.0	8.000	0.050	0.0	140.0	0.0	420.00	40.0	0.0	0.050	200.
VT60	10.0	8.000	0.050	0.0	140.0	0.0	504.00	60.0	0.0	0.050	200.
VT80	10.0	8.000	0.050	0.0	140.0	0.0	588.00	80.0	0.0	0.050	200.

FARE STRUCTURE=\$ 4.00/DEPARTURE + \$0.1000/MILE. (ANY FARES GIVEN IN THE CITY PAIR DATA WILL BE REPLACED)  
 YIELD IS 0.960 TIMES THE FARE

FREQUENCY COEFFICIENTS: NONSTOP=1.0000 ONESTOP=0.9000 TWOSTOP=0.8000

STATION DATA: NAME MAX DEP LANDING FEES BY A/C

NAME	MAX DEP	L1	L2	L3	L4	L5	L6	L7	L8
A1	0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0
A2	0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0
A3	0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0
B1	0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0
B2	0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0
B3	0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0	\$ 0.0

THERE ARE 6 STATIONS IN THE LIST

=====  
 CITY PAIR DATA: =====  
 LOAD FACTOR VALUES WILL BE MULTIPLIED BY 1.000 (THE LOAD FACTORS PRINTED BELOW ARE THE INPUTS BEFORE ADJUSTMENT)

A1 -B1 LP=0.8000 FARE=\$ 7.00 DISTANCE= 30.0  
 TRAFFIC CURVE: ( 12, 496.00) ( 24, 976.00) ( 44, 1233.00) ( 60, 1390.00) (

A1 -B2 LP=0.8000 FARE=\$ 7.00 DISTANCE= 30.0  
 TRAFFIC CURVE: ( 12, 496.00) ( 24, 976.00) ( 44, 1233.00) ( 60, 1390.00) (

A1 -B3 LP=0.8000 FARE=\$ 7.00 DISTANCE= 30.0  
 TRAFFIC CURVE: ( 12, 496.00) ( 24, 976.00) ( 44, 1233.00) ( 60, 1390.00) (

A2 -B1 LP=0.8000 FARE=\$ 7.00 DISTANCE= 30.0  
 TRAFFIC CURVE: ( 12, 496.00) ( 24, 976.00) ( 44, 1233.00) ( 60, 1390.00) (

A2 -B2 LP=0.8000 FARE=\$ 7.00 DISTANCE= 30.0

FA-4.5 Postprocessor Output  
 Intraregional Service Case

Table 7.5

PREF COEF: (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (

A2 -B1 LP=0.8000 FARE=\$ 7.00 DISTANCE= 30.0  
 TRAFFIC CURVE: ( 12, 496.00) ( 28, 976.00) ( 44, 1233.00) ( 60, 1390.00) (

PREF COEF: (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (

A3 -B1 LP=0.8000 FARE=\$ 7.00 DISTANCE= 30.0  
 TRAFFIC CURVE: ( 12, 496.00) ( 28, 976.00) ( 44, 1233.00) ( 60, 1390.00) (

PREF COEF: (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (

A3 -B2 LP=0.8000 FARE=\$ 7.00 DISTANCE= 30.0  
 TRAFFIC CURVE: ( 12, 496.00) ( 28, 976.00) ( 44, 1233.00) ( 60, 1390.00) (

PREF COEF: (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (

A3 -B3 LP=0.8000 FARE=\$ 7.00 DISTANCE= 30.0  
 TRAFFIC CURVE: ( 12, 496.00) ( 28, 976.00) ( 44, 1233.00) ( 60, 1390.00) (

PREF COEF: (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (1.00,0.90,0.80) (

A1 -A2 LP=0.8000 FARE=\$ 4.50 DISTANCE= 5.0  
 A1 -A3 LP=0.8000 FARE=\$ 4.50 DISTANCE= 5.0  
 A2 -A3 LP=0.8000 FARE=\$ 4.50 DISTANCE= 5.0  
 B1 -B2 LP=0.8000 FARE=\$ 4.50 DISTANCE= 5.0  
 B2 -B3 LP=0.8000 FARE=\$ 4.50 DISTANCE= 5.0  
 B1 -B3 LP=0.8000 FARE=\$ 4.50 DISTANCE= 5.0  
 THERE ARE 15 CITY PAIRS IN THE LIST

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\*\*\*\*\* ROUTE DATA: \*\*\*\*\*  
 COSTS COMPUTED FROM THE TIMES USING COST/HR VALUE  
 IF THE TIME IS NEGATIVE, THE CORRESPONDING AIRCRAFT IS NOT PERMITTED TO FLY THE ROUTE.

ROUTE	FROM	TO	COSTS	TIMES	FARE	DISTANCE	COEF	FARE	DISTANCE	COEF
ROUTE 1	A1	A2	\$ 28.80	36.00	\$ 43.20	\$ 50.40	0.086	\$ 0.086	0.086	0.086
ROUTE 2	A1	B2	\$ 88.80	111.00	\$ 133.20	\$ 155.40	0.264	\$ 0.264	0.264	0.264
ROUTE 3	A1	B3	\$ 88.80	111.00	\$ 133.20	\$ 155.40	0.264	\$ 0.264	0.264	0.264
ROUTE 4	A2	B1	\$ 88.80	111.00	\$ 133.20	\$ 155.40	0.264	\$ 0.264	0.264	0.264
ROUTE 5	A2	B2	\$ 88.80	111.00	\$ 133.20	\$ 155.40	0.264	\$ 0.264	0.264	0.264
ROUTE 6	A2	B3	\$ 88.80	111.00	\$ 133.20	\$ 155.40	0.264	\$ 0.264	0.264	0.264
ROUTE 7	A3	B1	\$ 88.80	111.00	\$ 133.20	\$ 155.40	0.264	\$ 0.264	0.264	0.264
ROUTE 8	A3	B2	\$ 88.80	111.00	\$ 133.20	\$ 155.40	0.264	\$ 0.264	0.264	0.264
ROUTE 9	A3	B3	\$ 88.80	111.00	\$ 133.20	\$ 155.40	0.264	\$ 0.264	0.264	0.264
ROUTE 10	A1	B1	\$ 117.60	147.00	\$ 176.40	\$ 205.80	0.350	\$ 0.350	0.350	0.350
ROUTE 11	A1	B1	\$ 117.60	147.00	\$ 176.40	\$ 205.80	0.350	\$ 0.350	0.350	0.350
ROUTE 12	A1	B2	\$ 117.60	147.00	\$ 176.40	\$ 205.80	0.350	\$ 0.350	0.350	0.350
ROUTE 13	A1	B2	\$ 117.60	147.00	\$ 176.40	\$ 205.80	0.350	\$ 0.350	0.350	0.350
ROUTE 14	A1	B3	\$ 117.60	147.00	\$ 176.40	\$ 205.80	0.350	\$ 0.350	0.350	0.350



	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 16	A2 B1 B2		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 17	A2 B1 B3		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 20	A2 B3 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 21	A2 B3 B2		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 22	A3 B1 B2		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 23	A3 B1 B3		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 24	A3 B2 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 25	A3 B2 B3		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 26	A3 B3 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 27	A3 B3 B2		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 30	A1 A2 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 31	A1 A2 B2		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 32	A1 A2 B3		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 33	A1 A3 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 34	A1 A3 B2		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 35	A1 A3 B3		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 36	A2 A1 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 37	A2 A1 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 38	A2 A1 B3		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 39	A2 A3 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 40	A2 A3 B2		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 41	A2 A3 B3		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 42	A3 A1 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 43	A3 A1 B2		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 44	A3 A1 B3		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 45	A3 A2 B1		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 46	A3 A2 B2		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	
ROUTE 47	A3 A2 B3		TIMES=	0.350	0.350	\$	0.350
	COSTS=	\$ 117.60	\$ 147.00	\$ 176.40	\$ 205.80	\$	

THERE ARE 43 ROUTES IN THE LIST

1. ROUTE DATA

THE VALUES GIVEN ARE FOR ONE WAY TRAFFIC, THAT IS FOR ONE HALF OF A SYMMETRICAL SYSTEM.

ROUTE 1001 A1 TO A2  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1002 A1 TO B2  
834.49 PASSENGERS FROM A1 TO B2  
13.04 TRIPS PER DAY  
13.04 TRIPS USING VT80

ROUTE 1003 A1 TO B3  
662.15 PASSENGERS FROM A1 TO B3  
10.35 TRIPS PER DAY  
10.35 TRIPS USING VT80

ROUTE 1004 A2 TO B1  
976.00 PASSENGERS FROM A2 TO B1  
15.25 TRIPS PER DAY  
15.25 TRIPS USING VT80

ROUTE 1005 A2 TO B2  
976.00 PASSENGERS FROM A2 TO B2  
15.25 TRIPS PER DAY  
15.25 TRIPS USING VT80

ROUTE 1006 A2 TO B3  
976.00 PASSENGERS FROM A2 TO B3  
15.25 TRIPS PER DAY  
15.25 TRIPS USING VT80

ROUTE 1007 A3 TO B1  
976.00 PASSENGERS FROM A3 TO B1  
15.25 TRIPS PER DAY  
15.25 TRIPS USING VT80

ROUTE 1008 A3 TO B2  
808.90 PASSENGERS FROM A3 TO B2  
13.89 TRIPS PER DAY  
13.89 TRIPS USING VT80

ROUTE 1009 A3 TO B3  
831.15 PASSENGERS FROM A3 TO B3  
12.99 TRIPS PER DAY  
12.99 TRIPS USING VT80

ROUTE 1010 A1 TO B1 TO B2  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1011 A1 TO B1 TO B3  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1012 A1 TO B2 TO B1  
0.0 PASSENGERS FROM B2 TO B1  
99.78 PASSENGERS FROM A1 TO B1  
0.0 PASSENGERS FROM A1 TO B2  
5.61 TRIPS PER DAY  
5.61 TRIPS USING VT20

ROUTE 1013 A1 TO B2 TO B3  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1014 A1 TO B3 TO B1  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1015 A1 TO B3 TO B2  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1016 A2 TO B1 TO B2  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1017 A2 TO B1 TO B3  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1020 A2 TO B3 TO B1  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1021 A2 TO B3 TO B2  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1022 A3 TO B1 TO B2  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1023 A3 TO B1 TO B3  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1024 A3 TO B2 TO B1  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1025 A3 TO B2 TO B3  
0.0 PASSENGERS FROM B2 TO B3  
144.85 PASSENGERS FROM A3 TO B3  
0.0 PASSENGERS FROM A3 TO B2  
9.05 TRIPS PER DAY  
9.05 TRIPS USING VT20

ROUTE 1026 A3 TO B3 TO B1  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1027 A3 TO B3 TO B2  
NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1030 A1 TO A2 TO B1  
0.0 PASSENGERS FROM A2 TO B1  
682.22 PASSENGERS FROM A1 TO B1  
0.0 PASSENGERS FROM A1 TO A2  
12.75 TRIPS PER DAY  
2.79 TRIPS USING VT20  
9.96 TRIPS USING VT80

ROUTE 1031 A1 TO A2 TO B2  
0.0 PASSENGERS FROM A2 TO B2  
141.51 PASSENGERS FROM A1 TO B2  
0.0 PASSENGERS FROM A1 TO A2  
8.84 TRIPS PER DAY  
8.84 TRIPS USING VT20

ROUTE 1032 A1 TO A2 TO B3  
0.0 PASSENGERS FROM A2 TO B3  
204.00 PASSENGERS FROM A1 TO B3  
0.0 PASSENGERS FROM A1 TO A2  
12.75 TRIPS PER DAY  
12.75 TRIPS USING VT20

ROUTE 1033 A1 TO A3 TO B1  
0.0 PASSENGERS FROM A3 TO B1  
204.00 PASSENGERS FROM A1 TO B1  
0.0 PASSENGERS FROM A1 TO A3  
12.75 TRIPS PER DAY  
12.75 TRIPS USING VT20

ROUTE 1034 A1 TO A3 TO B2  
 24.71 PASSENGERS FROM A3 TO B2  
 0.0 PASSENGERS FROM A1 TO B2  
 0.0 PASSENGERS FROM A1 TO A3  
 1.54 TRIPS PER DAY  
 1.54 TRIPS USING VT20

ROUTE 1035 A1 TO A3 TO B3  
 0.0 PASSENGERS FROM A3 TO B3  
 109.85 PASSENGERS FROM A1 TO B3  
 0.0 PASSENGERS FROM A1 TO A3  
 6.87 TRIPS PER DAY  
 6.87 TRIPS USING VT20

ROUTE 1036 A2 TO A1 TO B1  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1037 A2 TO A1 TO B1  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1038 A2 TO A1 TO B3  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1039 A2 TO A3 TO B1  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1040 A2 TO A3 TO B2  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1041 A2 TO A3 TO B3  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1042 A3 TO A1 TO B1  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1043 A3 TO A1 TO B2  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1044 A3 TO A1 TO B3  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1045 A3 TO A2 TO B1  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

ROUTE 1046 A3 TO A2 TO B2  
 0.0 PASSENGERS FROM A2 TO B2  
 62.49 PASSENGERS FROM A3 TO B2  
 0.0 PASSENGERS FROM A3 TO A2  
 3.91 TRIPS PER DAY  
 3.91 TRIPS USING VT20

ROUTE 1047 A3 TO A2 TO B3  
 NO FLIGHTS ARE FLOWN ON THIS ROUTE

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2. ROUTE SEGMENT DATA  
THE VALUES GIVEN ARE FOR ONE WAY TRAFFIC, THAT IS FOR ONE HALF OF A SYMMETRICAL SYSTEM.

ROUTE SEGMENT A2 TO A1  
1027.74 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
682.22 PASSENGERS FROM ROUTE 1030  
141.51 PASSENGERS FROM ROUTE 1031  
204.00 PASSENGERS FROM ROUTE 1032

ROUTE SEGMENT A3 TO A1  
313.85 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
204.00 PASSENGERS FROM ROUTE 1033  
109.85 PASSENGERS FROM ROUTE 1035  
NO PASSENGERS FROM ROUTE(S) 1034,

ROUTE SEGMENT A3 TO A2  
62.49 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
62.49 PASSENGERS FROM ROUTE 1046

ROUTE SEGMENT B1 TO A1  
NO SERVICES FROM ANY ROUTES

ROUTE SEGMENT B1 TO A2  
1658.22 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
976.00 PASSENGERS FROM ROUTE 1004  
682.22 PASSENGERS FROM ROUTE 1030

ROUTE SEGMENT B1 TO A3  
1180.00 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
976.00 PASSENGERS FROM ROUTE 1007  
204.00 PASSENGERS FROM ROUTE 1033

ROUTE SEGMENT B2 TO A1  
924.26 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
834.49 PASSENGERS FROM ROUTE 1002  
89.76 PASSENGERS FROM ROUTE 1012

ROUTE SEGMENT B2 TO A2  
1180.00 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
976.00 PASSENGERS FROM ROUTE 1005  
141.51 PASSENGERS FROM ROUTE 1031  
62.49 PASSENGERS FROM ROUTE 1046

ROUTE SEGMENT B2 TO A3  
1058.37 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
888.80 PASSENGERS FROM ROUTE 1008  
144.85 PASSENGERS FROM ROUTE 1025  
24.71 PASSENGERS FROM ROUTE 1034

ROUTE SEGMENT B2 TO B1  
89.78 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
89.78 PASSENGERS FROM ROUTE 1012

ROUTE SEGMENT B3 TO A1  
662.15 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
662.15 PASSENGERS FROM ROUTE 1003

ROUTE SEGMENT B3 TO A2  
1180.00 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
976.00 PASSENGERS FROM ROUTE 1006  
204.00 PASSENGERS FROM ROUTE 1032

ROUTE SEGMENT B3 TO A3  
440.99 PASSENGERS. LOAD FACTOR=0.80 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80

ROUTE SEGMENT B1 TO B1  
NO SERVICES FROM ANY ROUTES

ROUTE SEGMENT B3 TO B2  
144.85 PASSENGERS, LOAD FACTOR=0.60 MAXIMUM PERMISSIBLE LOAD FACTOR=0.80  
144.85 PASSENGERS FROM ROUTE 1025

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3. CITY PAIR ORIGIN AND DESTINATION DATA  
THE VALUES GIVEN ARE FOR ONE WAY TRAFFIC, THAT IS FOR ONE HALF OF A SYMMETRICAL SYSTEM.

CITY PAIR B1 TO A1  
976.00 PASSENGERS, 31.11 SERVICES  
976.00 PASSENGERS, 31.11 ONESTOP SERVICES FROM ROUTE(S) 1012, 1030, 1033,

CITY PAIR B1 TO A2  
976.00 PASSENGERS, 28.00 SERVICES  
976.00 PASSENGERS, 28.00 NONSTOP SERVICES FROM ROUTE(S) 1004, 1030,

CITY PAIR B1 TO A3  
976.00 PASSENGERS, 28.00 SERVICES  
976.00 PASSENGERS, 28.00 NONSTOP SERVICES FROM ROUTE(S) 1007, 1033,

CITY PAIR B2 TO A1  
976.00 PASSENGERS, 29.04 SERVICES  
834.49 PASSENGERS, 18.05 NONSTOP SERVICES FROM ROUTE(S) 1002, 1012,  
141.51 PASSENGERS, 10.39 ONESTOP SERVICES FROM ROUTE(S) 1031, 1034,

CITY PAIR B2 TO A2  
976.00 PASSENGERS, 28.00 SERVICES  
976.00 PASSENGERS, 28.00 NONSTOP SERVICES FROM ROUTE(S) 1035, 1031, 1046,

CITY PAIR B2 TO A3  
976.00 PASSENGERS, 28.79 SERVICES  
913.51 PASSENGERS, 24.49 NONSTOP SERVICES FROM ROUTE(S) 1008, 1025, 1034,  
62.49 PASSENGERS, 3.91 ONESTOP SERVICES FROM ROUTE(S) 1046,

CITY PAIR B3 TO A1  
976.00 PASSENGERS, 29.96 SERVICES  
662.15 PASSENGERS, 10.35 NONSTOP SERVICES FROM ROUTE(S) 1003,  
313.85 PASSENGERS, 19.62 ONESTOP SERVICES FROM ROUTE(S) 1032, 1035,

CITY PAIR B3 TO A2  
976.00 PASSENGERS, 28.00 SERVICES  
976.00 PASSENGERS, 28.00 NONSTOP SERVICES FROM ROUTE(S) 1006, 1032,

CITY PAIR B3 TO A3  
976.00 PASSENGERS, 28.91 SERVICES  
831.15 PASSENGERS, 19.85 NONSTOP SERVICES FROM ROUTE(S) 1009, 1035,  
144.85 PASSENGERS, 9.05 ONESTOP SERVICES FROM ROUTE(S) 1025,

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ALL VALUES IN SECTIONS 4 THROUGH 7 ARE FOR TWO WAY TRAFFIC, THAT IS FOR THE TOTAL ACTIVITIES OF THE SYMMETRICAL.  
A MULTIPLIER OF 1.000 HAS BEEN APPLIED TO THESE PER DAY VALUES, AS REQUESTED IN THE INPUT DATA.

4. SYSTEM ECONOMICS IN DOLLARS PER DAY

REVENUE = 118056.62  
 DOC = 53760.91  
 IOC = 46012.90  
 LANDING FEES = 0.0  
 CONTRIBUTION = 18282.78  
 OBJECTIVE FN = 16562.34

5. SYSTEM TRAFFIC PER DAY

PASSENGERS = 17567.945  
 81.34% TRAVELED WITH 0 STOPS. AVERAGE TRIP LENGTH= 30.00 MILES  
 18.66% TRAVELED WITH 1 STOPS. AVERAGE TRIP LENGTH= 30.00 MILES  
 0.0% TRAVELED WITH 2 STOPS. AVERAGE TRIP LENGTH= 0.0 MILES  
 RPM = 527038.875  
 RM = 11860.730  
 TAKE OFFS = 518.814  
 FLIGHTS = 370.666  
 LP(PAX/SFATS) = 0.798  
 LP(RPM/RSM) = 0.770

6. FLEET USAGE DATA PER DAY

AIRCRAFT	HRS ACTIVE	HRS IDLE	TAKEOFFS PER HR	AVERAGE STAGE LENGTH	HRS GAINED	HRS LOST
VT20	44.88	35.12	5.714	17.50	0.0	0.0
VT40	0.0	80.00	0.0	0.0	0.0	0.0
VT60	0.0	80.00	0.0	0.0	0.0	0.0
VTRJ	66.79	14.21	3.988	28.10	0.0	0.0

7. AIRPORT ACTIVITY DATA PER DAY

NAME	LANDINGS
A1	94.50
A2	122.25
A3	97.40
B1	61.61
B2	85.80
B3	67.25

\*\*\*\*\* ULTRA SHORT HAUL NETWORK EXAMPLE -- 3X3 STATIONS

\*\*\*\*\*

## 8.0 Towards Alternative Transportation Technology Analysis

Having seen the results of the single market and network analyses, it is possible to draw some conclusions regarding systems optimization in ultra short haul air transportation. Several points may be advanced when treating decision variables and looking at the operational policy/scenario interface beyond which the system operates. Other questions exist in the areas of operational feasibility and social implications.

### 8.1 Operational Feasibility

Vehicle size is only a demand stimulus if the operator is allowed a flexible pricing policy permitting vehicle-specific costs to be recovered. In a regulated price environment, the operator chooses vehicle size to maximize contribution to overhead while maintaining a breakeven load on the last additional frequency. There is no incremental level of service attribute under such a constant price policy, hence no demand stimulation accountable to vehicle size. In order to give an ultra short haul operator the flexibility required to maximize his market share he must be allowed pricing flexibility in order to comply with demand fluctuations.

Market subdivision, while clearly a reasonable option for the profit maximizing operator, may not be a valid concept for the operator in search of maximum market share. The relatively higher indirect costs per passenger associated with the multiple station operations exceed the weighted level of service increases offered



by such a policy. The maximum market share occurs at the minimum disutility consistent with breakeven operations. For this reason, the high cost of station operation vs. other forms of equivalent level of service increase is the determinant in the maximum market share case.

The advantages of multistop service are universal. Its ability to allow load building with larger vehicles (thus exploiting economies of scale in large vehicle operating costs) and provision of service at higher effective frequencies makes it a useful tool in either profit or market share maximization. In the intraregional example, it is multistop service (and associated low turnaround times) that yield demands close to those of nonstop service, yet at appreciably lower fully allocated costs to both operator and consumer.\* The higher effective frequencies provided by multistop service also allow higher equipment utilization or a smaller fleet requirement at a given level of service.

A historical trend of lower than average equipment utilizations due to the highly directional and peaked traffic flows is an area that bears close study. Were the higher local utilization levels obtained during the peak hours able to be maintained throughout the operating day, an operation such as that proposed would be an unquestioned success. To this date, no

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\*Indirect costs, however are higher due to fewer passengers enplaned per stop. Breakeven fares are therefore higher.

method short of cross-subsidization of off-peak travel has been proposed, either through peak hour pricing or off-peak subsidy. [The premises of this paper has been that ultra short haul air transportation can pay its own way; as such no subsidy has been proposed.] The stimulation of off-peak travel is a market research area of potential interest to ground and air mode transportation planners alike. This area is an absolute priority for any profit-seeking mode, as equipping for peak period demands will require a much larger capital investment in fleet than can be supported on low average utilization and off peak demand.

## 8.2 Policy Implications

Another area affecting operational feasibility is the air traffic control environment and the reliability of the ultra-short haul air service. While cancellations and delays due to mechanical failures have been for the most part eliminated in rotorcraft, (New York Airways reports less than two percent mechanicals on a yearly basis) the problems inherent in VTOL all-weather operations have not been as fully solved. Experiments with various forms of navigation aids have yielded a large body of operational experience. The present primary nonprecision approach aid continues to be the VOR/DME. While highly accurate VLF hyperbolic and RNAV systems are available and deployed, their in-service

reliability has been only fair and they are not approved for precision approaches. The ILS and MLS are the approved precision approach devices and while both reliable and accurate, they do not cater to the particular capabilities and needs of VTOL vehicles. Including VTOL aircraft in a group with CTOL aircraft for the purposes of terminal procedural compliance completely ignores the special capabilities of the VTOL device. The relatively large body of airspace reserved for CTOL procedures is not consistent with the ability of the VTOL aircraft to make an approach to a point in space and hold. By segregation of VTOL and CTOL terminal approaches and adoption of relaxed weather reporting and approach minimum criteria, the operational capabilities and reliability of rotor craft operations may be maximized.

### 8.3 Social Questions

The final area of interest in this systems look at ultra short haul air transportation is one that is not readily quantizable in any case - social costs and community acceptance. In demonstrating the air service to the travelling public, the expectations and desires of the non-travelling public in these social areas must be considered. An equally important aspect of any demonstration is the illustration of the improved environmental aspects of lower noise, pollution and better local access traffic patterns to the

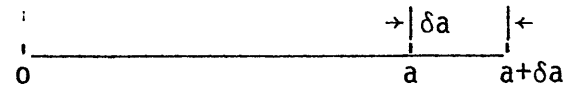
public at large for the purpose of obtaining the required levels of community support and acceptance. Only with careful planning - and this must include the public at all levels, not just in the post facto stages - will ultra short haul air transportation receive enough exposure so that the needed advances in the state of the art of tilt-wing/rotor and other VTOL technologies can occur.

APPENDIX A: Crofton's Method for Mean Values

Linear Crofton Case:

$X_1, X_2$  Distributed uniformly and independently

$$\mu = E[|X_1 - X_2|]$$



Define

$$\overline{|X_1 - X_2|} \text{ over } [0, a + \delta a] \triangleq \mu + \delta\mu$$

Consider 4 partitioning events (mutually exclusive)

$$E_1: (0 \leq x_1 \leq a, 0 \leq x_2 \leq a) \quad P(E_1) = \frac{a^2}{(a + \delta a)^2}$$

$$\left. \begin{array}{l} E_2: (0 \leq x_1 \leq a, a \leq x_2 \leq a + \delta a) \\ E_3: (a \leq x_1 \leq a + \delta a, 0 \leq x_2 \leq a) \end{array} \right\} P(E_2 E_3) = \frac{a \cdot \delta a}{(a + \delta a)^2}$$

$$E_4: (a \leq x_1 \leq a + \delta a, a \leq x_2 \leq a + \delta a) \quad P(E_4) = \frac{(\delta a)^2}{(a + \delta a)^2}$$

Now

$$\begin{aligned} \mu + \delta\mu &= \sum_{i=1}^4 E[|X_1 - X_2| | E_i] P(E_i) \\ &= \mu \frac{a^2}{(a + \delta a)^2} + 2\mu_1 \frac{a \delta a}{(a + \delta a)^2} + O(\delta a^2) \end{aligned}$$

Where  $\mu_1 \equiv E[|X_1 - X_2|] \mid \text{one point in } [a, a + \delta a]$

(We calculate  $\mu_1$  due to its ease compared to  $\mu$ .)

$$\text{By inspection, } \mu_1 = \frac{a}{2} + \text{H.O.T.}$$

we simplify by:

$$\delta\mu = \frac{2|\mu_1 - \mu|}{a} (\delta a)$$

substitute

$$\mu_1 = \frac{a}{2}, \quad \delta\mu = \left( \frac{a - 2\mu}{a} \right) \delta a$$

or,

$$\frac{d\mu}{da} + \frac{2}{a} \mu = 1$$

which gives

$$\mu = \frac{a}{3}, \text{ the expected result.}$$

This may be extended to a 2 dimensional case, and is very useful in the analysis of mean distances in a circular region.

Travel Times (Distances) In A Circle Using Crofton's Theorem on Mean Values:

Apply Crofton's method for finding mean Euclidean distance

$$E[D_E] = E[\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}]$$

Where  $(x_1, y_1), (x_2, y_2)$  are assumed uniformly and independently distributed over a circle of radius R.

Now, let  $\mu = E[D_E]$

$$\mu_1 = E[D_E | \text{one point in } \delta R \text{ Ring}]$$

$$\mu + \delta\mu = \text{Mean Travel Distance in Circle of Radius } R + \delta R$$

Consider 4 mutually exclusive events and their probabilities:

$E_1$ : (Both points in circle, R)	$P(E_1) = \frac{R^4}{(R+\delta R)^4}$
$E_2$ : (Pt. 1 in circle, Pt. 2 in $\delta R$ Ring)	$P(E_2) = \frac{2R^3\delta R}{(R+\delta R)^4}$
$E_3$ : (Pt. 2 in circle, Pt. 1 in $\delta R$ Ring)	$P(E_3) = \frac{2R^3\delta R}{(R+\delta R)^4}$
$E_4$ : (Both points in $\delta R$ Ring)	$P(E_4) = \frac{4R^2(\delta R)^2}{(R+\delta R)^4}$

And the four events partition the assumptions on the 2 points.

Hence, we know that:

$$\mu + \delta\mu = \sum_{i=1}^4 E[D_E | E_i] \cdot P\{E_i\}$$

$$= \mu \frac{R^4}{(R+\delta R)^4} + 2 \mu_1 \frac{2R^3 \delta R}{(R+\delta R)^4} + \text{H.O.T.}$$

Expanding  $\frac{1}{(R+\delta R)^4}$  gives:

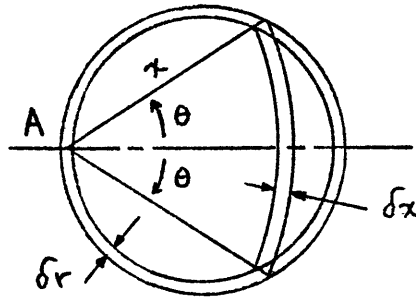
$$(R+\delta R)^{-4} \approx R^{-4} - 4R^{-5} \delta R + \text{H.O.T.}$$

$$\text{Or } \mu + \delta \mu = \mu \left( 1 - 4 \left( \frac{\delta R}{R} \right) \right) + 4 \mu_1 \left( \frac{\delta R}{R} \right) + \text{H.O.T.}$$

$$\text{Hence, } \delta \mu = \frac{4(\mu_1 - \mu)}{R} \delta R$$

Now, to find  $\mu_1$  we fix a point, A in the  $\delta R$  Ring

of the circle, and define a  $\delta x$  Ring which is a distance  $x$  to A; its width is  $\delta x$ .



$$\text{Now, area of } \delta x \text{ Ring is: } 2\theta \delta x = 2x \left( \cos^{-1} \frac{x}{2R} \right) \delta x$$

From above,  $\mu_1 = E[D_E | \text{Given a point in } \delta x]$

And by varying  $x$  from  $0 \rightarrow 2R$  we can cover the entire circle.

If  $\mu_1 = E[D_E | 1 \text{ point in } \delta x]$

$$= \frac{1}{\pi R^2} \int_0^{2R} x \cdot 2x \left( \cos^{-1} \frac{x}{2R} \right) dx$$

(i.e. Total area in R)



Now, substitute for  $\frac{x}{2R} = y$  in above integral,

and

$$\begin{aligned}\mu_1 &= \frac{16R}{\Pi} \int_0^1 y^2 \cos^{-1} y \, dy \\ &= \left\{ \frac{y^3 \cos^{-1} y}{3} + \frac{1}{3} \left[ -\frac{2}{3} (1-y^2)^{3/2} - y(1-y^2)^{1/2} \right] \right\} \frac{16R}{\Pi} \Big|_0^1 \\ \mu_1 &= \frac{2}{9} \cdot \frac{16R}{\Pi} = \frac{32R}{9\Pi}\end{aligned}$$

Now, returning to  $\mu$  :

$$\begin{aligned}\delta\mu &= \frac{4(\mu_1 - \mu)}{R} \delta R = 4 \left( \frac{32R}{9\Pi} - \mu \right) \frac{\delta R}{R} \\ &= \left( \frac{128}{9\Pi} - \frac{4\mu}{R} \right) \delta R\end{aligned}$$

let  $\mu = k \cdot R$ , find  $k$ :

$$k = \frac{128}{9\Pi} - 4k; \text{ Hence } k = \frac{128}{45\Pi} \text{ and } \mu = \frac{128R}{45\Pi}$$

In summary, we have:

2 points independently and uniformly distributed over space.

We wish to calculate the expectation of a function of their relative position. We expand the space by a small increment,  $\delta$  and relate the desired expectation to the expectation of the same function conditioned on the assumption that one event lies within  $\delta$ . (The conditional mean is easier to calculate than using the elliptical Tables). The desired expectation is then found by solving a (usually simpler) differential equation.

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