Design of an Overmoded W-Band Coupled Cavity TWT
by
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Submitted to the
Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of
Master of Science
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
February 2009
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Submitted to the Department of Electrical Engineering and Computer Science on 29 August 2008, in partial fulfillment of the requirements for the Degree of Master of Science

ABSTRACT

We present the design and cold test validation of a novel, overmoded Traveling Wave Tube (TWT) capable of producing power levels in excess of 100 Watts at frequencies of 100 GHz and above. High power sources at frequencies from the W-Band (70 to 110 GHz) to the THz frequency range are needed for numerous applications including radar, DNP/NMR spectroscopy, and homeland security. The novel TWT design operates in the TM_{31} mode, of a rectangular cavity, and has transverse dimensions three times larger than a conventional TWT, thus allowing higher power handling capability and less stringent fabrication tolerances. The circuit is also amenable to multiple beam operation which will allow the use of higher beam currents. The concept of dielectric loading in a resonant cavity was utilized to suppress lower order modes and prevent parasitic oscillations. The coupling impedance of the TWT was calculated with the HFSS code and the gain with the MAGIC3D code. The results indicate that with a 0.6 mm diameter electron beam at 50 kV and 0.8 A, over 1 kW of peak output power and a few hundred watts of average output power are achievable at 99 GHz with a linear gain of 32 dB and a -3 dB bandwidth of 0.6 GHz. A cold test structure scaled to a frequency of 15 GHz was designed, built and tested with a vector network analyzer. The results proved that the dielectric loading with strips of Aluminum Nitride works to attenuate the parasitic lower order modes, thus verifying the theoretical analysis. Further cold test measurements showed dispersion and coupling impedance characteristics were accurately modeled by the computer simulations. The novel, overmoded TWT is a very promising approach to achieving high output power at W-Band and is also promising for scaling to frequencies in the 0.2 to 1.0 THz region.

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Acknowledgements

This thesis would not have been accomplished without the help of many people. I would like to thank my advisor Dr. Richard Temkin for giving me the ability to pursue this research endeavor and providing inspiration. As any member of Waves and Beams knows, a talk with Rick excites a person to do more research. He allowed me the freedom to explore several topics and encouraged my development as a scientist and engineer. I would also thank Dr. Jagadishwar Sirigiri for closely assisting and advising this thesis. His ideas and support helped me move past all the roadblocks that occurred during this research. I thank Dr. Michael Shapiro for the interesting scientific discussions and his willingness to listen to my challenge du jour.

I wish to thank Ivan Mastovsky for his help designed and manufacturing the cold tests. His ideas enable the successful experiments results. I would also thank Mark Belanger at the Edgerton center student shop who assisted me in machining a version of my cold test structure.

I fully believe the Waves and Beams division to be one of the best research groups at MIT and I am thankful to my fellow graduate student members. My officemate David Tax who rekindled my Canadian flame, taught me how to play hockey, and provided interesting conversion on any topic. He certainly made coming to work a joy everyday and helped in grammar policing this work. Roark Marsh for explaining so many accelerator concepts and teaching me that working out can be fun. Antonio Torrezan and Antoine Cerfon for listening and guiding me whenever I had a question or comment. Colin Joye and Brian Munroe for the great conversations.

I greatly relied on the support from my friends and family. My parents were always there is listen and provide advice, and my friends, Michelle, Kevin, Nate, Nareg and TJ for all the friendship and support.
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Chapter 1

Introduction

1.1 Microwave Technology

Microwave technology has become part of the backbone of modern society. Ever since the first experiments by Hertz demonstrated the existence and transmission of electromagnetic waves, scientists and engineers have sought ways to harness the electromagnetic spectrum. Today, the microwave region of the electromagnetic spectrum, 1 to 300 GHz, is the most widely used other than visible light, and with legions of iPhone users talking on Bluetooth headsets with 3G/Wi-Fi connections to the internet, the use of microwave technology is only growing. Many of the most visible applications of technology harness microwaves: cell phones, television transmission, Direct TV, GPS, radar, and microwave cooking. Each depends on an internal component that generates microwaves to perform their functions. Even designing CPUs, digital signal processors, and other high speed circuitry requires extensive knowledge of microwave theory to ensure the performance of their high-speed input/output.

The rise of microwaves coincided with the rise of technology in warfare. As air and naval forces became more a larger part of attack weaponry, a need arose for tracking and combating these forces; microwave radar was the answer. Though many microwave sources and radars existed in the 1930s; it was World War II that spurred the advancement of modern microwave sources. The Allies, in partnership with MIT’s radiation laboratory, researched and developed mass producible sources, specifically the cavity magnetron, allowing a significant technical advantage over the Axis’ counterparts. The effectiveness and widespread use of radar became so important that many believe the more advanced Allied radar systems were a major contributor to the victory over the Axis [1].

After WWII the range of available microwave sources and applications exploded. Figure 1.1 shows there are sources available for every power range of the microwave
spectrum with solid state devices in the sub-milliwatt range to megawatt-class gyrotrons. Before the invention of the transistor, all microwave sources were vacuum electron devices (VEDs). Today's microwave applications are sourced by a variety of solid state devices and VEDs. Solid state devices are used to talk with each other (cell-phones), access the internet (Wi-Fi), land our planes (radar and GPS), and predict the weather (radar). VEDs are used extensively in satellite communications, to cook our meals, as the energy source in particle accelerators, and for industrial heating of materials like ceramics and metals [2]. They will also be required for future energy sources like nuclear fusion [3] and the transmission of space solar power [4].

Like most applications of technology there are trade-offs between the technical performance (frequency, power, bandwidth, efficiency) and the cost (capital cost, maintenance, reliability, efficiency) of sources. Throughout the history of microwave systems design, these trade-offs have been at the forefront of picking the right source for each application. In many cases, applications were obvious but the source could not be found. Today, with so many microwave sources and new ideas on the horizon, the balance between performance and cost is now the most important consideration in application design.

For many microwave applications, current sources are adequate, technologically and economically. Microwave ovens are the perfect example of this. They produce a kilowatt of RF power at 70% efficiency for $10 per kW. For other applications such as communication there is still a desire to push for higher frequencies, in order to avoid spectrum competition and achieve higher bandwidths. Some applications, such as industrial heating, are achieved with current microwave technology, but could be provided more economically if more high frequency sources were available.

The ubiquity of microwave applications is based on the technical economics of microwave sources. Most consumers would never tolerate a cell phone that cost a thousand dollars, but satellite designers are more than willing to pay top dollar for the best technical achievement. Future microwave systems will require new sources capable of extreme technical performance but will gain higher and more rapid acceptance if they can be produced at moderate economic cost.
1.2 Need for High-Power Sources for W-Band and Beyond

The microwave spectrum is a broad range of frequencies and, for practicality, the spectrum has been broken up into different band designations. W-band refers to a band of frequencies between 75 and 110 GHz. The general interest in W-band sources stems from reduced atmospheric attenuation in this region [5]. By looking at microwave attenuation over a range of frequencies, as shown in Figure 1.2, we can see a window, formed by neighboring water and oxygen resonance lines. Without this window many applications, such as communication and radar, where microwave signals must travel many kilometers, would not be possible. Thus, when system designers are looking to move beyond their current designs in the Ka-band (26.5 to 40GHz), the W-band is the next logical step.

Applications capable of utilizing the W-band include: satellite data communications, power beaming and radar (including military, civilian and weather). The performance of each of these applications will improve considerably as they move to higher frequencies. Communication systems can provide larger data transfer because transfer rates are proportional to absolute bandwidth. It is easier to achieve larger absolute bandwidth with sources operating at higher frequencies. A device which can deliver 4 GHz of bandwidth at 35 GHz should be able to produce 12 GHz at 100 GHz. Power beaming systems benefit from shorter wavelengths, which lead to smaller beams and smaller receivers. For some applications, like powering a space elevator this is probably a requirement [4]. Radar's spatial resolution is inversely related to frequency; higher frequencies lead to clearer pictures [6, 7]. Each of these applications requires a large amount of microwave power, generally over 10 W, to accomplish their tasks. Clearly, there are uses for high-power (tens of Watts) W-band microwave sources, but what can fulfill it?

No single source would be expected to satisfy all of the applications proposed, but it would be advantageous to find a single source capable of economically fulfilling most of the applications. For applications like radar there is no upper limit to the amount of power desired; because applications always benefit from a larger signal to noise ratio. Considering the intense interest in THz sources we would want any design to be scalable to still higher frequencies. Figure 1.1 shows that there are several candidate sources in
the W-band, solid state devices, gyrotrons, travelling-wave tubes (TWTs), backward wave oscillators (BWOs), and extended interaction klystrons (EIKs).

1.2.1 Solid State is Not Always the Answer

Solid state microwave sources have become ubiquitous in modern society. They are the backbone of the portable communication industry (cell phones, PDA's), automotive systems (radar, navigation and cruise controls), and virtually every ground/air-based communication system. With frequencies that range from DC to 20 GHz, solid state microwave sources offer small size, low weight and moderate output power. They are an incredible enabler of technology.

Since the development of the transistor, the semiconductor industry has poured billions of dollars into developing the most technically complex processes in the world. In 1971, Intel was creating circuits with feature sizes of 10 \( \mu \text{m} \); today they are producing chips on 45 nm processes and have plans to go to 11 nm. They will soon be creating transistors on the order of an atom! \[8\] All the tools created to achieve these impressive circuits have been leveraged to enable the high performance microwave circuits we have today. Over time the continuous improvement in semiconductor processes has enabled higher frequencies and larger power densities as demonstrated in Figure 1.1 and Figure 1.3. However, for all the recent advances in solid state electronics, there are fundamental physical limitations to their future performance.

As indicated in \[9\], future high frequency sources (above 100 GHz) capable of producing power greater than one milliwatt will be limited to vacuum electron devices. The performance of microwave frequency transistors is limited by basic material physics and at higher frequencies, it will always be inferior to VEDs. Physically, maximum RF power generation is defined by the following formula:

\[ P_{RF} \text{ (max.gen)} \propto (E_c v_s)^2 \eta \]  

1.1
Figure 1.1 State of the art vacuum electron and solid state devices. In the THz region above 100 GHz there is a dearth of high power and economically practical sources. (Adapted from [10], Courtesy of Bruce Danly, NRL)
Figure 1.2 Average atmospheric attenuation versus frequency for several altitudes. The molecule responsible for each resonance is noted. At higher altitudes, where the amount of water decreases, several frequency “windows” with significantly lower attenuation can be seen. The plot was generated by implementing the millimeter-wave propagation model [5]

where $E_c$ is the critical breakdown field, $v_s$ is the saturation velocity of electrons and $\eta$ is the DC-RF conversion factor. For each of these metrics VEDs, are capable of superior performance. In order for microwave transistors to operate at a higher frequency their size must shrink significantly, reducing their breakdown voltage and power handling ability. Vacuum electron devices are capable of much larger velocities as their electrons travel ballistically, unimpeded in vacuum. In solid state devices the electrons travel through semiconductor material imparting thermal energy and reducing efficiency.

Single solid state devices cannot produce large output power. Multiple devices could be combined to create large power combining arrays and achieve a single high-power source. However, these arrays are both expensive, inefficient and for millimeter wave and higher frequencies solid state options would turn out to be expensive to build, large, and costly to run [11]. While new technologies like GaN and wide band gap semiconductors will increase performance of single devices, they will not be competitive with VEDs at power levels greater than a watt for W-band and higher frequencies [9]. It is clear that vacuum electron devices will be the answer for high output power for millimeter and terahertz frequencies.
Figure 1.3 Historical progress in average device power frequency density $P_{av} f^2$ in units of $(\text{MW})(\text{GHz})^2$ (From [10])

### 1.3 How Vacuum Electron Devices Work

The Conservation of Energy Principle states that energy is neither created nor destroyed - it only changes form - and this principle is at the heart of all vacuum electron devices. Vacuum electron devices (VEDs) generate electromagnetic power from the kinetic energy of an electron beam. It is relatively simple to create an electron beam with a large amount of kinetic energy. In most VEDs, electron beams can be generated by heating a cathode with a low work function, and then using an anode with an applied DC voltage to accelerate the electrons boiling off the cathode surface. This method is used to generate electron beams with large amounts of kinetic energy in a compact region. The generated electron beam is confined by a magnetic field to prevent the like charged electrons from repelling each other and tearing the beam apart. This confining field allows the beam to be guided into an interaction structure. It is inside this interaction structure where the kinetic energy of the electron can be partially converted into microwaves.
When charged particles are accelerated, they give off electromagnetic radiation [12]. An interaction structure, historically always metallic, can be designed to harness this property by forcing electrons into bunches which then generate coherent microwave radiation. Interaction structures can be classified into two major categories, fast wave and slow wave devices. These designations reflect whether the phase velocity of microwave propagation in the structure, in the direction of the electron beam, is faster or slower than the speed of light. The different classifications of VEDs indicate the different physical mechanisms used to develop electron bunches and extract microwave energy. The type of interaction utilized, drives the geometries of the interaction region, required electron voltage and current, and magnetic field.

In TWTs, BWOs and klystrons, a linear electron beam can be directed through a slow wave interaction region where it travels faster than the electromagnetic wave, leading to an electromagnetic shockwave. If the electron bunches travel slightly faster than the phase velocity of the circuit, the electrons will give off microwave energy in the form of Čerenkov radiation. This is the electronic equivalent of the shock wave that forms when an airplane breaks the sounds barrier.

The gyro class of devices, which includes gyrotrons, gyroklystrons and gyro-TWTs, utilizes an annular electron beam. This beam has energy both parallel and perpendicular to the magnetic field. The energy perpendicular to the magnetic field will cause the electrons to oscillate at the cyclotron frequency, \( \omega_c \):

\[
\omega_c = \frac{qB}{m_e \gamma} \approx 28 \text{ GHz} \times B(\text{Tesla})
\]

These electron oscillations give off microwave energy because of synchrotron radiation, the energy given off by the electron when it slows down while rotating around magnetic field lines.

No matter the type of interaction used, there is no way to completely transform all the kinetic energy of the electrons into microwave energy. After travelling through the interaction region they will need to be stopped. A metal collector region is used to stop the electrons and dissipate the heat resulting from the beam’s excess energy. The combination of an electron gun, magnetic field, interaction region and collector defines each VED microwave source.
1.3.1 State of the Art W-Band VED Sources

As previously stated, there is an interest in W-band microwave sources. We will take a brief survey of the state of the art sources. Figure 1.1 shows that there are several candidate sources in the W-band: solid state devices, gyrotrons, TWTs, BWOs, and EIKs. In order to satisfy the requirement for a W-band radar, a systems engineer would need large output power, a wide bandwidth, and so we will look at the state of the art with an eye for this application. We have already discussed and dismissed solid state devices; they will not be able to perform at high frequencies.

Gyro-devices, Gyro-TWT and gyrotron oscillators are more than adequate. Gyrotrons produce megawatts of output power in the W-band and beyond with over 50% efficiency [3, 13, 14]. Gyro-TWTs have produced over 500 watts of average power and over 400 kW peak with a 6 GHz bandwidth [15, 16]. However, as shown in Eq. (1.2) to generate high frequencies they require large and expensive superconducting magnets. This limits their use only to applications that need these quantities of output power, but have no need of a compact source.

Helix TWTs have demonstrated excellent bandwidth at frequencies up to 35 GHz but cannot generate high-power in the W-band. Their interaction circuits consist of a helix held by dielectric supports. The helix size reduces at high frequencies, decreasing their power handling [11, 17]. There have been no reports of helix TWTs produce in the W-band. There has been an attempt to fabricate a small 650 GHz helix BWO, but it is currently unclear how much power it could produce [18].

Extended Interaction Klystrons (EIKs), and their oscillator counter parts, EIOs, are a hybrid of a standard klystron and a short coupled-cavity slow-wave structure section. CPI, Inc. created a series of commercially available devices, including the W-band, they are capable of producing over 400W of output power at 95 GHz [19]. They also plan to increase the operating power to over 1 kW in the W-band [20]. The limitation of the EIK is bandwidth. An EIK’s instantaneous bandwidth generally does not exceed 2-3 GHz and they are often capable of frequency ranges up to a several GHz by physically tuning the devices. Thus, EIKs do meet the power levels of interest for satellite and radar but not the bandwidths for many other systems.
Coupled-Cavity TWTs (CCTWTs) operate under the same physical principles as a helix, except that their interaction circuits are solid metal cavities. These cavities are inherently larger than the helix structure and have greater power handling capabilities. When frequency increases, the interaction dimensions shrink, but coupled cavity TWTs are still able to maintain large thermal conductivities. The trade-off versus a helix TWT is bandwidth; CCTWTs utilize resonant coupled-cavities which have a more limited bandwidth. However, this bandwidth is acceptable for most radar applications. The highest frequency CCTWTs were developed in the 1980s by Bill James at CPI. These W-band CCTWTs were based on a coupled cavity ladder circuit and are capable of over 30W output power from 80-100 GHz or 900W and 1GHz bandwidth at 94 GHz [21, 22]. Thomson Tubes, now Thales, is another company that produced a 94 GHz CCTWT based on a comb delay line structure, producing 200 W of peak power over 1 GHz of bandwidth [6].

A subset of the CCTWT is the folded waveguide TWT (FWTWT). The design of TWTs is based on this interaction structure that has been very popular for the W-band due to their perceived ease of manufacture for the W-band and higher [23]. Folded waveguide designs are also considered very scalable to higher frequencies including the THz range [24]. A TWT was developed for 54GHz capable of over 100 W of output power [25], but they have yet to publish a result at 94 GHz.

Backward Wave Oscillators (BWOs) are a subset of the linear beam slow wave devices like helix TWT and CCTWTs. Slow wave structures similar to those used in CCTWTs, are capable of sustaining backward wave oscillations. The BWO is attractive to designers because it is a simple oscillation to start in many circuits. The BWO interaction has been shown to provide power at extremely high frequencies. A BWO has been reported with 52 mW output power at 650 GHz [26]. However, being an oscillator, the BWO has little instantaneous bandwidth and poor interaction efficiency.

1.4 Future Millimeter Wave and Terahertz Sources

The electromagnetic spectrum is an intriguing continuum. It has been broken up into distinct regions such as x-rays, microwaves, visible light, and the infrared regions which provide for encapsulation, though the fundamentals of physics in each region does not
change with the stop and start of each band designation. There are also millimeter waves (MMWs – 30 GHz to 300 GHz) and terahertz waves (300 GHz – to 3 THz). The reader should note that this THz designation while probably the most common many an authors have broadened the range depending on the work.

We have discussed microwave sources specific to the W-band but the future of vacuum electron devices will not stop there. There is a strong demand for a new source in the high millimeter/terahertz wave ranges. Some potential and current uses of THz radiation are:

- Medical imaging. The photon energy of THz radiation is very low and non-ionizing. It could theoretically be used to create safer, real-time diagnostics.
- Security and imaging. Just like microwaves, THz will penetrate many materials and can be used to view the contents inside [27, 28]. Many plastic and organic materials (many explosives) have unique spectral identifiers in the THz range and powerful imaging tools could be created [29] to view them. Most proposed uses involve sensory and imaging applications. Terahertz wavelengths include 1 mm to 100 µm; in most cases well-resolved images can be formed.

Systems pushing operating frequencies into THz and higher will face significant challenges. Atmospheric attenuation from water becomes a very significant factor, limiting the useful detection range of low power sources [5]. There is a real dearth of sources for THz applications (see Figure 1.1). As discussed, the size of VEDs shrink as one attempts to generate high frequencies making generating significant output power challenging. From the other side of the frequency spectrum, lasers that work wonderfully for visible light and infrared cannot work well as the frequency gets lower. Lasers rely on material band gaps to generate photons; at lower frequencies the photon energies are too low for most materials and have very low efficiencies. The exception is the quantum cascade laser (QCL), which emits several photons for each electron. The QCL work best at higher THz frequencies and low powers (mW range).

For all the difficulty, vacuum electron devices have shown significant progress in meeting the demand for THz sources. The aforementioned 650 GHz BWO was developed as part of a program for THz imaging. Gyrotrons have been developed to
generate power up to 1 THz [30]. For VEDs operating higher than 200 GHz, conventional machining techniques such as CNC milling and electric discharge machining (EDM) cannot meet the tight tolerances. To meet this challenge, designers have turned to new manufacturing techniques such as LIGA and deep reactive ion etching (DIRE) to manufacture circuits [31-33]. These amazing manufacturing achievements have been made using standard TWT topologies scaled for higher frequency operation. There is room for new ideas.

1.5 Overmoded Interaction Structures

Achieving higher power at higher frequencies is a daunting task. Figure 1.1 shows that with every VED, after a certain point, higher frequency operation leads to exponentially lower average powers. Figure 1.3 provides a little more clarity. We see that, for most VEDs, their average power density has not been increasing over the last few decades, even with significant advances in the technology of vacuum electron devices. The two reasons for this stagnation are fundamental to microwave design. First, as frequencies scale higher, the structures shrink in size. Second, when the structures shrink they can no longer dissipate as much thermal energy and they cannot use high power electron beams without beam interception (the beam will heat the structure).

Gyrotrons have been an exception to this trend of decreasing power density. Gyrotrons are capable of being operated in higher order interaction modes [34]. The increasing power density of the gyrotron seen in Figure 1.3 is due to these devices operating in higher order modes. For a gyrotron, this
overmoded operation has two advantages: the large cavities illustrated in Figure 1.4 have more surface area and the cavities can be more easily cooled. These larger cavities also enable larger diameter electron beams, allowing more beam power without intercepting the structure.

Over the last three decades, gyrotron designers have developed devices that operate in higher and higher cavity modes. Other VEDs like the Klystron and CCTWT have always operated in their fundamental modes, meaning that as frequencies have increased their dimensions will always decrease. As cavity dimensions shrink so does surface area, and as many authors have reported, there is a generally accepted limit of 1-2 kW/cm² for cooling a copper structure [11, 35]. If it were possible to stably operate slow wave structures in a high order mode, it should be possible to increase power density. To date, there have been a few papers on overmoded linear O-beam structures with sparse details, but the general consensus is that an overmoded interaction circuit would allow higher output powers [35, 36]. Considering CCTWTs are already a good source for W-band, we can focus on trying to improve its power density. If an overmoded operation method can be developed for a CCTWT, then similar devices, such as a BWO, will be able to operate.
overmoded. It would be interesting to investigate methods for creating an overmoded CCTWT.

### 1.6 Thesis Outline

This thesis will present the design an overmoded W-band TWT. Chapters 2 and 3 will introduce material required to understand the operation of a TWT and the unique aspects of an overmoded TWT. Chapter 2 will introduce standard TWT and electromagnetic concepts. A basic linear theory of the travelling wave interaction will be introduced and an overview of non-linear TWT operation will be presented. Chapter 3 will document several ideas for enabling overmoded TWTs and analyze their practically for physical designs. The ladder TWT will be introduced as a practical solution for created an overmoded W-band TWT. Chapter 4 will present the design of an overmoded TWT. An overmoded interaction structure will be designed and integrated with the other parts of a TWT. Magic 3D simulations will be shown that indicate overmoded TWT is possible. In order ensure the interaction structure designed in Chapter 4 agrees with simulations a scaled cold test was built. Chapter 5 will present the results and design of this cold test structure. The cold test structure will verify the interaction structure. Chapter 6 will provide a summery of the thesis and propose a path for physically building an overmoded W-Band TWT.
Chapter 2

TWT Operation and Electromagnetic Physics

In order to design an overmoded CCTWT, it is important to understand the operating physics of a travelling wave tube in order to understand the engineering constraints and how overmoded interaction might affect traditional TWT physics. This chapter will give a description of basic CCTWT components and their operation. We will derive the Pierce theory of small single TWT interaction and discuss its use and the basic TWT metrics needed for the design of an interaction structure. The electromagnetic physics of resonant cavities will be developed and the relevant differences between a fundamental and overmoded interaction in a structure will be discussed. In Chapter 1, the concept of electron beam bunching was mentioned and such bunching is required to extract microwave power from a linear electron beam. Physically it can be challenging to understand the travelling wave interaction with an electron beam and we will start with velocity modulation in a klystron. Klystron physics will be used as a stepping stone to understanding the more complicated TWT interaction.

2.1 Overview of TWT Operation

Before we delve into a detailed look at travelling wave interaction we will discuss the elements of a coupled cavity TWT (CCTWT). TWTs consist of four major components generally assembled in the configuration seen in Figure 2.1. An electron gun generates an e-beam, which is confined by a magnet system into the slow wave structure’s beam tunnel. The slow wave structure (SWS) is used to interact with the e-beam and a collector stops the e-beam after travelling through the tube.
2.1.1 Electron Guns

Electron guns, for TWTs, are designed to output a beam of electrons with current $I_0$, at a voltage $V_0$ with a circular cross section. Such electron beams are often called pencil beams or O-beams for their round long shape. Electrons can be sourced in two manners, thermionic emission and field emission. With thermionic emission, a cathode made of a material with a low work function is heated, 800-1200 °C is typical, so that the electrons will have enough thermal energy to be regularly emitted from the surface. The application of an electric field from the anode will accelerate the electrons, imparting an axial velocity and sending them through the SWS. Instead of using a hot cathode, field emission can be used with a cold cathode. Cold cathodes are designed to have very large electric fields at the surface. Unlike in thermionic emission, where the surface is hot enough to force electrons out, the electric field of a cold cathode is strong enough for electrons to directly tunnel out. Currently almost all TWTs use hot cathodes. Field emission technology while attractive has not been perfected, yet. New carbon nanotube technology promises to increase the used of field emission in TWTs [11].

In high frequency TWTs, the interaction structures are small and beam tunnels less than one millimeter in diameter are very typical. Considering high power is still desired, beam currents of one Amp can be necessary. Such a beam would have a current density of 100 A/cm$^2$, and cathodes cannot provide these densities [37]. To accommodate high current densities, cathodes are made several times larger than the beam tunnel. The beam
is then focused by the anode, and magnetic field into the beam tunnel. The current generated by such an arrangement at a fixed voltage is fixed by the geometry of the gun; factors include shape and size of the cathode, shape and location of the anode and the magnetic field in the region. The relation between current and voltage is known as perveance, $P$:

$$P = \frac{I_o}{V_o^{3/2}}$$

(2.1)

Typically values range from $0.01 \times 10^6$ and $2 \times 10^6$, higher perveances are limited by space charge. For the same amount of beam power a high perveance electron gun requires lower voltage leading to reduced power supply costs and higher tube gains.

### 2.1.2 Magnets

Without a magnet system, the space charge forces of the electron beam would cause the beam to spread apart and diverge while moving through the SWS. If a large percentage of the beam were to intercept the SWS, the structure will be heated. Such heating can lead to the destruction of the SWS. To combat divergence, an axial magnetic field is applied, forcing the electrons to travel the length of the tube attached to the field lines. TWT designs utilize two types of magnets systems: periodic permanent magnets (PPM) and electromagnetic solenoids. A PPM system utilizes permanent magnets to confine electrons. These magnets create magnetic field that confine the electron beam, but are not uniform with the length of the SWS. Such a system can be more compact, and weigh less than an electromagnet and does not require a power supply making it potentially more energy efficient. However, PPMs require a more complex design to prevent additional beam instabilities, but with modern computer codes the challenge has been somewhat mitigated [11, 37]. The electromagnetic solenoid easily provides uniform axial field and has the potential for much higher field strengths. The design of an electromagnetic solenoid is much simpler, compared to a PPM magnet. The general drawbacks are the requirement of a power supply and the size of the magnet.

The minimum field required to stop the beam from diverging is known as the Brillouin field, $B$ [37]:

33
\[ B = \frac{1}{r} \sqrt{\frac{2m_e I_o}{q \pi e_o v_o}} = 0.83 \times 10^{-3} \frac{I_o^{1/3}}{r V_o^{1/4}} \text{ Tesla} \]  

(2.2)

where \( r \) is the beam radius, \( m_e \) is the electron mass, \( q \) is the electric charge, and \( v_o \) is the electron velocity. In practice this field is too small; since the current density does not remain constant while travelling through the interaction region. Beam bunching will cause local density to increase in certain regions; such increases in current density will increase the beam diameter, creating a scalloped beam. For significant bunching, which will occur in high gain TWTs, the beam diameter can increase to such an extent that it will hit the SWS. To prevent the beam from destroying the circuit, the magnetic field is operated at two to three times the Brillouin field level to minimize scalloping [37].

### 2.1.3 Slow Wave Structures

Travelling wave tubes utilize a linear electron beam, which only has significant energy in the axial direction, requiring an interaction with the axial electric fields of an SWS. The phase velocities of the electromagnetic wave and the electron must be similar in order for to have any energy transferring interaction. Such a synchronism can be most easily understood if one thinks of riding a wave, surfing, body or other. (if you haven’t been in the ocean I apologize). If one sits just outside the wave break and merely treads water, they will go nowhere. Wave after wave will pass by with no appreciable transfer of energy. To catch a wave, you need to start swimming with the wave. Swim too slowly and the wave will still pass you by; swim near the speed of the wave and you will feel fast and light as the wave starts to carry you. It is at this point that energy is being transferred from the wave to the surfer. The same goes for the travelling wave interaction, the beam and wave's phase velocities must be close to have any transfer of energy. Mathematically this can be expressed as:

\[ \omega - \beta v_e = 0 \]  

(2.3)

where \( \beta \) is the axial propagation constant, and \( v_e \) is the electron velocity. The need for circuit phase velocity to equal the electron velocity requires the use of a slow wave structure; electrons can never travel faster than the speed of light. For electrons their phase velocity is their actual velocity:
where \( m_e \) is the mass of the electron and \( V_o \) is the beam voltage. This is a non-relativistic approximation and is a good approximation for beam voltages less than 50 kV. Electrons, of course, are not capable of travelling faster than light, necessitating the need for a slow wave interaction structure. In general, phase velocity, \( v_p \), is defined as the radial frequency, \( \omega \), divided by the axial propagation constant, \( \beta \):

\[
v_p = \frac{\omega}{\beta}
\]  

Depending on the text and branch of physics, axial propagation constants are often notated as \( k_o \), we will use \( \beta \), in order to be consistent with other TWT works. For an electron beam, \( \beta_e \) is calculated, from:

\[
\beta_e = \frac{\omega}{v_e}
\]  

For a CCTWT we are concerned with the circuit’s axial propagation constant, \( \beta_c \), and defined by:

\[
\beta_c = \frac{\phi}{L}
\]  

where \( \phi \) is the axial phase advance of the electromagnetic fields per cavity and \( L \) is the length between cavities, sometimes referred to as pitch. This formulation of \( \beta_c \) is only appropriate for periodic structures, which we have in CCTWTs. \( \beta_c \) has units of \( m^{-1} \) and can be thought of as the period of the electromagnetic wave in the axial direction. In order to synchronize the phase velocities of the circuit and beam, one can either change the beam velocity, the frequency of the electromagnetic wave or the pitch. In practice, there is a limit to how much the voltage of the electron beam can be altered. For
interactions at higher frequencies, to maintain synchronism with a constant voltage electron beam, the pitch must be reduced. Thus the pitch of the interaction structure reduces as one moves to higher operating frequencies.

The interaction structure must be properly designed to have a similar phase velocity to the electron beam. In order for the structure to interact with the beam over a wide range of frequencies the interaction structure’s phase velocity must be dispersive or have changing values of phase velocity with frequency. The ideal SWS would have a phase velocity that is constant and equal to that of the electron beam. A helix TWT is as close as one can get to such a structure, but as mentioned previously will not work in W-band and higher frequencies. A dispersion diagram, or $\omega-\beta$ diagram, is used to illustrate the interaction of the electromagnetic wave and the electron beam, seen in Figure 2.2. Such a diagram can be used to find the group velocity of a wave, $v_g$: 

\[
\omega = \frac{\partial \omega}{\partial \beta} = \frac{\Delta \omega}{\Delta \beta}
\]

\[
\text{Slope } = v_p = \frac{\omega}{\beta} = v_e
\]
For an electromagnetic wave, the group velocity and its sign determine the direction and speed of energy propagation. TWTs are used as amplifiers and are required to amplify signals over a large bandwidth; they are operated in regions of forward, or positive, group velocity. A backward wave interaction can occur when the phase velocity of the electron beam intersects the electromagnetic wave in a region where group velocity \( v_g \) is negative. This backward wave interaction still has the velocity synchronism that a forward, TWT, interaction has, but with no bandwidth as the synchronous condition occurs for only one frequency. This is why the backward wave interaction is used in oscillators, BWOs, and not amplifier devices. As one can see in Figure 2.2 a BWO could be voltage tuned; if the beam voltage is changed the interaction point will change and thus oscillation frequency will change.

2.1.4 The Collector

At its simplest, the collector is a metal end cap that the electron beam will hit after the interaction structure. The electrons impart their remaining kinetic energy onto the collector, heating it. The biggest issue with designing these collectors is ensuring the beam is spread over the entire collector region by the local magnetic field, thereby preventing all the electrons from depositing their heat on a very small region of the collector. Too significant a heat load can cause damage to the collector. All the excess kinetic energy of the electron beam is considered wasted energy and reduces the efficiency of the tube.

An electron beam can have very significant energy after travelling through the circuit. All of this excess energy ends up as heat in the collector. Typically, commercial TWT collectors are depressed collectors; the voltage of the collector is not the same as the interaction circuit and is at a lower voltage. The effect of depressing the collector can either be viewed as reducing the voltage of the electron gun without change the supplied current, or recovering energy from the electron beam, either way the effect is to reduce the total energy added to the circuit. Ideally utilizing a depressed collector does not change the output power of the TWT, and thus the overall efficiency is increased. The
method effectively increases the device’s efficiency and reduces the heat load on the collector, thereby increasing reliability. There is a limit to the amount of depression that can be applied; after a certain depression voltage some electrons will reflect back into the circuit toward the electron gun.

2.2 Basic Electromagnetics

Since TWTs rely on a slow wave structure to operate, it will be advantageous to start by discussing the theory of waveguides and resonators, to better understand SWS design. Slow wave structures can be a basic waveguide, as in the case of folded wave guide TWTs, or a mix of waveguides and resonant cavities, as with traditional coupled cavity TWTs. All electromagnetic theory starts with Maxwell’s equations:

\[ \nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} + \bar{J} \quad (2.9) \]

\[ \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad (2.10) \]

\[ \nabla \cdot \bar{D} = \rho \quad (2.11) \]

\[ \nabla \cdot \bar{B} = 0 \quad (2.12) \]

Where \( E \) is the electric field strength, \( B \) is the magnetic flux density, \( H \) is the magnetic field strength, \( D \) is the electric displacement, \( J \) is the electric current density, \( \rho \) is the electric charge density and the over bar represents a vector quantity. These equations can vary spatially and temporally. Maxwell’s powerful equations are supported two constitutive relations and the continuity equation:

\[ \bar{B} = \mu \bar{H} \quad (2.13) \]

\[ \bar{D} = \varepsilon \bar{E} \quad (2.14) \]

\[ \nabla \cdot \bar{J} = -\frac{\partial \rho}{\partial t} \quad (2.15) \]

The constitutive relations relate Maxwell’s equations to the physics of real world materials. The relations presented are for are linear and isotropic permittivity, \( \varepsilon \), and permeability, \( \mu \), and will suffice for this work. In practice, the form of these equations can become far more complex, including non-linearities and anisotropic tensors. The
continuity equation is an empirical law enforcing the conservation of charge at any boundary; this equation can be derived from Eqs. (2.9) and (2.11).

Maxwell's equations can be reformulated to give the Helmholtz wave equations. These equations describe the propagation of fields inside a material and will be used in the following sections to solve for the rectangular waveguide and resonant cavity modes:

\[
\begin{align*}
\nabla^2 + \mu \varepsilon \frac{\partial^2}{\partial t^2} \bar{E} &= 0 \\
\nabla^2 + \mu \varepsilon \frac{\partial^2}{\partial t^2} \bar{H} &= 0
\end{align*}
\]

(2.16) (2.17)

In all of these equations, there are only linear terms, allowing the use of the superposition principle. Superposition allows one to separate solutions into orthogonal components and then linearly recombine to find the complete solution. For the following problems these solutions can be decomposed into two orthogonal solutions, TM and TE modes. TM modes are characterized as having all transverse magnetic fields, no magnetic fields in the direction of propagation, \( H_z = 0 \). TE modes have all transverse electric fields, \( E_z = 0 \).

### 2.2.1 Rectangular Waveguide Modes

We are interested in the modes of a rectangular waveguide for two reasons: understanding waveguide dispersion instills an understanding of the dispersion curves used in SWSs, and they are a conceptual stepping stone to resonant cavities.

Typically, when discussing the theory of waveguides, the axes are chosen such that the direction of propagation is in the \( z \) direction, also referred to as the axial direction. This is the common nomenclature and is true in TWT literature where the electron beam and electromagnetic power is considered to always travel in the \( z \) direction of the interaction circuit. We will assume cross-sectional waveguide dimensions of \( a \) and \( b \) in the \( x \) and \( y \) directions respectively, shown in Figure 2.3. Since this is a waveguide, all
solutions should propagate in the $z$ direction, thus giving the fields a dependence proportional to $e^{\pm jkz}$. Additionally, we are interested in periodic frequency solution, making the time dependence of the solutions, $e^{-j\omega t}$. Including superposition of the modes, we can now rewrite the Helmholtz Eqs. (2.16), (2.17) as:

$$\left(\nabla^2 + \mu\varepsilon \omega^2 - \beta^2\right)E_z = 0 \quad (2.18)$$
$$\left(\nabla^2 + \mu\varepsilon \omega^2 - \beta^2\right)H_z = 0 \quad (2.19)$$

Where (2.18) is the TM solution and (2.19) is the TE solution. The Laplacian contains only the transverse components as indicated by the subscript $t$. To solve for the dispersion and fields in the waveguide we must apply the boundary conditions of the waveguide:

$$\overline{E}_1^t - \overline{E}_2^t = 0 \quad (2.20)$$
$$\overline{H}_1^t - \overline{H}_2^t = \overline{J}_f \times \hat{n} \quad (2.21)$$
$$\left(\overline{B}_1^t - \overline{B}_2^t\right) = 0 \quad (2.22)$$
$$\overline{D}_1^t - \overline{D}_2^t = \rho_f \quad (2.23)$$

We are assuming perfectly conducting walls, which would not support any free currents $J_f$, or free charges $\sigma_f$. The electric field inside the metal waveguide walls must be zero, which requires that the electric field along the waveguide wall is zero. The transverse magnetic field outside the waveguide wall is also zero, requiring the transverse field inside the waveguide to be zero. Applying these conditions to (2.19) yields the TE solution to the waveguide, from [38] the solution is:
\[
\begin{align*}
E_x &= -k_x e^{jke} \cos k_x x \sin k_y y \\
E_y &= k_x e^{jke} \sin k_x x \cos k_y y
\end{align*}
\] (2.24)

The magnetic fields are of similar form. Since these solutions are spatially periodic, the solutions can satisfy the boundary conditions for multiple values of \(k_x\) and \(k_y\):

\[
k_x a = m\pi
\]
(2.25)

\[
k_y b = n\pi
\]
(2.26)

where \(m\) and \(n\) are integers used to classify the mode. From Eq. (2.19) we also get the following dispersion relation.

\[
\beta_z = \sqrt{\omega^2 \mu \varepsilon - k_x^2 - k_y^2} = \sqrt{\omega^2 \mu \varepsilon - (m\pi/a)^2 - (n\pi/b)^2}
\]
(2.27)

From hereon, we will be referring to these modes in the form \(\text{TE}_{mn}\). This relation has been plotted for three TE modes in Figure 2.4. These modes have phase velocities greater than the speed of light in all cases and cannot be used for an interaction with a linear electron beam. It should be noted that the axial propagation term \(\beta_z\) is the same as \(k_z\) used by many electromagnetic theory books, and can be used interchangeably. The choice of \(\beta\) is one used in most TWT texts and will be kept.

![Figure 2.4](image)

**Figure 2.4** The dispersion of a rectangular waveguide for some \(\text{TE}_{mn}\) modes is shown. High order modes have higher cutoff frequency, but asymptotically approach the \(\text{TE}_{10}\) mode for large axial propagation values. The red line shows the speed of light, and illustrates how the mode’s group and phase velocities approach the speed of light for high axial wave numbers.
2.2.2 Rectangular Resonators

A rectangular resonator can be thought of as a rectangular waveguide with metal plates on its ends, shown in Figure 2.5. The fields and resonant frequencies of a rectangular cavity can be solved from the waveguide solutions by considering cavity solutions as a superposition of the $+z$ and $-z$ waveguide solutions. Considering TE modes have no electric field in the $z$ direction and a linear electron beam will only interact with fields parallel to its motion; we shall only concern ourselves with TM modes. The field solutions for a TM cavity are given in [38]:

\[
E_z = E_o \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \cos \frac{p \pi z}{d}
\]

\[
E_x = - \frac{E_o}{(m \pi / a)^2 + (\pi / b)^2} \frac{m \pi}{a} \frac{p \pi}{d} \cos \frac{m \pi}{a} \sin \frac{n \pi}{b} \sin \frac{p \pi}{d}
\]

\[
E_y = - \frac{E_o}{(m \pi / a)^2 + (\pi / b)^2} \frac{n \pi}{b} \frac{p \pi}{d} \sin \frac{m \pi}{a} \cos \frac{n \pi}{b} \sin \frac{p \pi}{d}
\]

\[
H_x = - \frac{i \omega E_o}{(m \pi / a)^2 + (\pi / b)^2} \frac{n \pi}{b} \frac{m \pi}{a} \cos \frac{n \pi}{b} \cos \frac{p \pi}{d}
\]

\[
H_y = \frac{i \omega E_o}{(m \pi / a)^2 + (\pi / b)^2} \frac{m \pi}{a} \cos \frac{m \pi}{a} \sin \frac{n \pi}{b} \cos \frac{p \pi}{d}
\]

With a dispersion equation for the frequency of each mode given by:
Figure 2.6 The mode patterns of the electric field for several TMmn of a rectangular resonator are presented. (a) TM11, (b) TM21, (c) TM31, and (d) TM12. Red areas are regions of highest field strength, blue regions of lowest field strength.

$$\omega^2 \varepsilon \mu = \left( m\pi / a \right)^2 + \left( n\pi / b \right)^2 + \left( p\pi / d \right)^2$$

These solutions are valid for any rectangular metal box, and the resonant frequencies are referred to as TM\(_{mn}\) modes. If \(p \neq 0\), \(E_x\) and \(E_y\) will not be zero; these off-axis fields would bring an unwanted element to analyzing the electron beam in TWT design. Going forward we will assume \(p = 0\) and TM\(_{mno}\) is equivalent to TM\(_{mn}\). Figure 2.6 illustrates the electric field patterns for the first four TM modes in a rectangular resonator. Generally, for a standard ladder TWT, where \(a > b > d\) is a reasonable assumption, the operating mode will be the fundamental TM\(_{11}\). All other modes will then be higher order modes of the cavity. Clearly, from Eq. (2.29), if two cavities are both designed for a single frequency, with one operating in the fundamental mode and the other operating in a higher order mode (HOM), the HOM cavity will have larger dimensions.

### 2.2.2.1 Energy Loss in Resonators

We have claimed higher order modes have lower losses and we will quickly quantify this statement. In a cavity resonator, the quality factor \(Q\) is related to the losses. \(Q\) is defined as the ratio of energy stored in a resonator to the energy dissipated per cycle.

$$Q = \frac{\omega_b U}{P_d}$$

\(\omega_b\) is the frequency of the resonant mode and \(U\) is the stored energy. The energy stored is given by the well known formula
\[ U = \frac{1}{4} \text{Re} \iint (|E|^2 + \mu |H|^2) \]  

(2.31)

So far we have assumed the cavity walls are perfectly conducting. This was only an approximation, and the walls will have a conductivity \( \sigma \). For materials like copper which have large conductivities, \( \sigma \approx 5.8 \times 10^7 \) ohm-m, the addition of conductivity will not change the fields of the cavity, and the losses will be due to the ohmic loss from wall currents generated by the cavity’s magnetic field [38]. The total power dissipated can be found by integrating the square of the magnetic field over the entire cavity surface:

\[ P_d = \frac{1}{2} \text{Re} \iint S \mathbf{n} \cdot (\mathbf{E}_{\text{wall}} \times \mathbf{H}_{\text{wall}}^*) = \frac{1}{2} \sqrt{\frac{\omega \mu}{2\sigma}} \iint S |\mathbf{H}_{\text{wall}}|^2 \]  

(2.32)

Using Eq. (2.32) and the magnetic fields of a rectangular resonator from the previous section we can calculate the Q for different modes. For a rectangular cavity like those used in a ladder CCTWT, where \( a > b > d \) and \( p = 0 \), higher-order modes will have higher Q than the fundamental mode. One might assume the Q of a higher order mode would decrease if the resonator were increased in size. If one had two cavities of different sizes, where one cavity’s fundamental, TM\(_{11}\), mode was at the same frequency as the other’s higher order mode, TM\(_{31}\) mode, then one might assume the Q’s of these two modes would be equal. This is not the case, and in fact Q of TM\(_{31}\) is twice that of Q TM\(_{11}\). For structures that rely on resonant cavities, like CCTWTs, this result suggests not only would operating in a higher order mode lead to larger cavities, but the wall losses, or ohmic losses, should be lower.

### 2.3 Klystrons

In order to better understand the interaction of a TWT cavity with an electron beam, we will first take a simple look at the operation of a klystron. Klystrons and other linear beam microwave devices, like TWTs, extract microwave power by modulating the velocity of the electron beam to create electron bunches. Velocity modulation can be achieved by placing the electron beam in an electric field parallel to the motion of the electrons. As shown in Figure 2.7, electrons can either be accelerated or decelerated, depending on the phase of the electric field. Thus, a non-time varying (referred to as a dc)
electron beam subject to a time-varying electric field will gain a space and time varying velocity component.

The two-cavity klystron is a simple VED consisting of an electron gun, magnetic field, two resonant cavities, and a collector. The electron-wave interaction is illustrated by an Applegate diagram, Figure 2.8. The first klystron cavity applies a small rf electric field to the electron beam. The small perturbation applied to the beam is allowed to grow in the drift space between both klystron cavities. By the time the beam has travelled to the location of the second cavity, a significant bunch can be seen. The dc electron beam now has a very large time-varying, or ac, component. An ac beam current will induce an ac electric field in the cavity and transfers the beams kinetic energy. These fields will correspond to the resonant modes of the cavity.

We see for a klystron that a low energy input was able to modulate a higher energy electron beam, which created a much higher energy output. In other words, klystrons are capable of generating gain and being used as an amplifier. The klystron’s use of resonant cavities means it has inherently narrow bandwidth. A TWT can overcome this with the use of an interaction structure interacting over a much longer length, not just with two separate cavities.

2.4 Pierce Theory of Travelling Wave Interaction

Several authors have broached the subject of the interaction of an electron beam with a travelling wave structure. The classical small-signal theory that will be presented was developed by J.R. Pierce [39]. The parameters he derived are now considered standard metrics to be discussed with any TWT design and will be used in the design of this TWT. Derivations of Pierce’s theory is described in several sources [17, 37, 39], each varies slightly in details and nomenclature, all have been used to guide this derivation.

As a small signal theory all physical parameters such as current, and electron velocity can be rewritten in terms of static, dc, and time varying, ac, components. The terms dc and ac are a throwback to linear perturbation circuit analysis techniques, which
Figure 2.7 In an RF electric field the force applied to an electron varies based on the phase of the electric field. Dependent on the current phase of the electric field some electrons will accelerate, others will be slowed.

Figure 2.8 A two cavity Klystron and Applegate diagram. This diagram shows the density of the electron beam over time while travelling between the two cavities. As the electron beam travels from the bottom to the top, it passes through the first cavity which modifies the beam’s velocity. As the beam drifts, this slight velocity modulation leads to electron bunches. (From [11])
this theory grew out of. Pierce’s theory is built up by finding the ac current, \( i_z \), caused by an axial electric field, \( E_z \), referred to as the electronic equation. The microwave circuit is then modeled as a transmission line, this model is used to determine the field in the circuit due to an ac current in the beam, referred to as the circuit equation. The circuit equation and electronic equation can be combined to solve for the axial wave solutions. The theory starts without accounting for the effects of circuit loss and space charge. Since the Pierce theory is a small-signal theory all quantities will be assumed to vary as \( e^{j(\omega t - \gamma z)} \). In Pierce’s original theory he used \((j \omega t - I z)\). Most authors have opted for the former notation which is more commonly used in electromagnetic theory.

2.4.1 The Electronic Equation

We are interested in determining the relationship between the slow wave circuit and the electron beam. We will start with finding the ac current of the electron beam, \( i \), from the electric field due to the space charge effect of electron bunching. Physically, there are three equations we can use: the definition of current density, the force of an electric field on an electron beam, and the continuity equation. It is assumed the electron velocity, \( v \), the charge density, \( \rho \), the current density, \( J \), and the axial electric field, \( E_z \) will vary around their average or dc values:

\[
\begin{align*}
\rho &= \rho_o + \rho_1 e^{j(\omega t - \gamma z)} \\
J &= -J_o + J_1 e^{j(\omega t - \gamma z)} \\
E_z &= E_1 e^{j(\omega t - \gamma z)} 
\end{align*}
\]  

(2.33)

The spatial propagation constant is \( \gamma = j\alpha + \beta_c \), where \( \alpha \) is the attenuation of the microwaves and the \( \beta_c \) is phase velocity of the microwave signal. This should not be confused with the electron beam’s phase velocity, \( \beta_e \equiv \omega / v_o \).

Electron current density is defined as the product of the electron density and velocity of the electrons, \( J = \rho v \). We are assuming a small value for ac terms and \( \rho_1 v_1 \) can be ignored.

\[
J = \rho v = (\rho_o + \rho_1) (v_o + v_1) \approx -J_o + J_1 e^{j(\omega t - \gamma z)} 
\]  

(2.34)

Expanding out the terms will give us \( J_o = -\rho_o v_o \) and:

\[
J_1 = \rho_1 v_o + \rho_o v_1
\]  

(2.35)
The force equation, \(-qE = ma\), is used to relate electric field to beam velocity. The force is negative for an electron beam:

\[
\frac{d\mathbf{v}}{dt} = \left( \frac{\partial}{\partial t} + \frac{\partial}{\partial z} \right) \mathbf{v} = -qE_i e^{i\omega t - \gamma z} = (j\omega - j\gamma v_o) \mathbf{v}_1 e^{i(\omega t - \gamma z)}
\]

(2.36)

where \(q\) is the electric charge constant, and \((dz/dt)\) is the dc electron velocity, \(v_o\). Algebraic manipulation gives:

\[
\mathbf{v}_1 = \frac{-q/m}{j\omega - j\gamma v_o} E_i
\]

(2.37)

Conservation of electric charge across a boundary using the continuity equation gives the last unknown:

\[
\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} = (-j\gamma \mathbf{J} + jk\omega \rho) \Rightarrow \rho_1 = -\frac{\gamma J_1}{\omega}
\]

(2.38)

Substituting Eqs. (2.35) and (2.37) into (2.38) yields:

\[
J_1 = j \frac{q\omega J_o}{v_o m(j\omega - \gamma v_o)} E_i
\]

(2.39)

By assuming there are no non-axial fields and the magnitude of the electric field is uniform over the electron beam, the current densities, \(J_o\) and \(J_l\), will be proportional to the current, \(I_o\) and \(I_l\), which is also \(i_z\). We also assume the dc electron beam is not relativistic, then \(v_o = \sqrt{(2q/m)v_0}\). We then get the final form of the electronic equation:

\[
i_z = j \frac{\beta_e I_o}{2v_o (\beta_e - \gamma)^2} E_z
\]

(2.40)

2.4.2 The Circuit Equation

Pierce’s theory models the microwave circuit as a transmission line. It is assumed that the propagation velocity on this transmission line is less than the speed of light. The exact form of the transmission line used is not relevant in the end; it only serves as a theoretical basis for the analysis.

The point of the transmission line model is that it becomes a simple way to reconcile that an ac current on the electron beam will induce a voltage and current in the microwave circuit. This is similar to a bunched electron beam inducing wall current in the real microwave circuit. The transmission line is assumed to have a distributed inductance, \(L\), and capacitance, \(C\), per unit length. For this section only, \(C\) will refer to a capacitance. We can start by assuming an electron beam with ac current, \(i_o\), passes close to the circuit inducing a current, \(\mathcal{M}i_o\), onto the transmission line. This
current is injected into the transmission line at each node between inductors, marked

Figure 2.9, at point A. Circuit techniques like Kirchoff's voltage and current laws can then be applied. The voltage and current changes between each node can be written as \( \frac{\partial V}{\partial z} \Delta z \), respectively \( \frac{\partial I_L}{\partial z} \Delta z \). Now using KCL at node A and KVL around ABCD, assuming \( \Delta z \to 0 \) we get:

\[
\frac{\partial i_z}{\partial z} = -j \omega CV + \frac{\partial I_L}{\partial z} \quad (2.41)
\]

\[
\frac{\partial V}{\partial z} = j\omega I_L \quad (2.42)
\]

Since all quantities are still small signal they vary as \( e^{j(\omega t - \gamma z)} \). Eqs. (2.41) and (2.42) become:

\[
j\gamma i_z = j\omega CV + j\gamma I_L \quad (2.43)
\]

\[-j\omega V = i\omega LI_L \quad (2.44)
\]

Substituting for \( I_L \):

\[
V = \frac{\gamma \omega L i_z}{\gamma^2 - \omega^2 LC} \quad (2.45)
\]

We now assume as before that the electrons pass close to the transmission line, then the change in voltage between nodes of the transmission line \( V \) will be an electric field on the electron beam.
Figure 2.9 Transmission line model for Pierce's circuit analysis

\[ E_z = -\nabla \cdot V = -\frac{\partial V}{\partial z} = j \frac{\gamma^2 \omega L}{(\gamma^2 - \omega^2 LC)} i_z \]  

(2.46)

L and C can be eliminated by defining the phase velocity and the interaction impedance of the circuit:

\[ v_p = \left( \frac{1}{\sqrt{LC}} \right)^{1/2} \quad \text{and} \quad K = \left( \frac{L}{C} \right)^{1/2} \]  

(2.47)

By rewriting the electric field Eq. (2.46) we get the following form.

\[ E_z = j \frac{\gamma^2 \beta_c K}{\gamma^2 - \beta_c^2} i_z \]  

(2.48)

The form of this equation is referred to as the circuit equation and now we have two equations for solving \( E_z \) and \( i_z \). Please note that the interaction impedance \( K \) has been referred to by many names by many authors over the last 50 years: interaction impedance, coupling coefficient, coupling impedance, coupled impedance; we will stick to interaction impedance. Phase velocity, dispersion, and the interaction impedance, \( K \), are the most used metrics in TWT cavity design. The equations of (2.47) can be used to find L and C values from a given phase velocity and \( K \). Interaction impedance can be tricky to understand in the literature because of the number of different names and forms it goes by. Many textbooks do not even notate it coupling impedance as \( K \), preferring the notation of \( Z_o \). Another common formulation for \( K \) is to assume the circuit has no loss, which means, \( K \), will be a real valued resistance. For microwave circuits the circuit impedance is related to power travelling on the circuit by
\[ Z = \frac{|V|^2}{2P} \quad (2.49) \]

And from (2.46):
\[ E = V\beta_c \quad (2.50) \]

We get the following for the interaction impedance:
\[ K = \frac{|E_n|^2}{2\beta_c^2 P} \quad (2.51) \]

where subscript \( n \) refers to the space harmonic of the electric field, the fields that would be by the electron beam. This form for the interaction impedance is very useful in determining, \( K \). In some cases, such as the Curow-Gittins CCTWT circuit [17], it is possible to get formulas for the inductance and capacitance for a unit cell of the circuit using analytic theory. However, in many cases it is very challenging to determine the interaction impedance analytically, in which case field data must be used in conjunction with Eq. (2.51) to find the interaction impedance. This method will be used in future chapters when evaluating the performance of TWT structures.

### 2.4.3 Determining the Propagation Constants

We can now combine the electronic equation (2.40) and circuit equation (2.48) to determine the actual growth of the small signal wave we started with:
\[ 1 = \frac{\beta_c I_e}{2V_o (\beta_c - \gamma)^2} \frac{\gamma^2 \beta_c K}{(\gamma^2 - \beta_c^2)} \quad (2.52) \]

Pierce defines a quantity \( C \), which encapsulates the beam impedance and coupling coefficient. It will turn out that gain is proportional to this constant \( C \):
\[ C \equiv \left( \frac{I_o K}{4V_o} \right)^{1/3} \quad (2.53) \]

After substitution for \( C \), we get the final version of the determinantal equation for calculating the growth rates of the propagation constants.
\[ \frac{\beta_c}{(\beta_c - \gamma)^2} \frac{\gamma^2 \beta_c}{(\gamma^2 - \beta_c^2)} 2C^3 +1 = 0 \quad (2.54) \]

This is now a fourth-order equation for \( \gamma \). There will be 4 solutions for \( \gamma \), corresponding to the four modes of the circuit. With the exception of synchronous operation, when
\( \beta_c = \beta_e \), there are no other easily obtained analytic solutions. However, a computer can be easily used to numerically calculate the values of \( \gamma \). So far, we have been very liberal with the use of \( \beta \) and phase velocity and propagation in our notation. As mentioned previously, \( \beta_e \) is the electron beam’s propagation constant, it is treated as a constant for this analysis. \( \beta_c \) is the propagation constant for the microwave circuit and can be complex if the circuit is lossy. The circuit’s propagation constant is fixed by the geometry of the structure, and is determined purely from the dispersion characteristic of the structure.

In order to understand each of the four \( \gamma \) solutions we will look at the analytic solution in the case of synchronous operation. If one assumes the solutions in the synchronous case only differ from \( \beta_e \) by a small amount, it can be shown that the solutions are {{7 Gilmour, A.S. 1994}[37]}}:

\[
\begin{align*}
\gamma_1 &= \beta_e + \frac{\beta_e C}{2} + j \frac{\sqrt{3}}{2} \beta_e C \\
\gamma_2 &= \beta_e + \frac{\beta_e C}{2} - j \frac{\sqrt{3}}{2} \beta_e C \\
\gamma_3 &= \beta_e - \frac{\beta_e C}{2} \\
\gamma_3 &= -\beta_e + \frac{\beta_e C^3}{4}
\end{align*}
\]

(2.55)

Since we defined the electric field as varying as \( e^{i(\omega t - \gamma z)} \), the first solution will be a growing exponential and the second solution will be a decaying exponential, both travelling slower than the electron beam. The other two solutions represent waves travelling forwards and backwards at the speed of the electron beam. By solving the boundary and initial conditions, Pierce shows the gain in a TWT can be approximated as:

\[
G = -9.54 + 47.3 C \frac{\beta_e z}{2\pi} \text{ dB}
\]

(2.56)

Where \( C \) is the gain from before and \( z \) is the interaction length. Now we can see why \( C \) is the gain parameter and why, when designing a TWT, it is advantageous to have high efficiency electron guns and large interaction impedances. Larger values of \( C \) lead to high TWT gains, allowing shorter structures and limiting the potential for parasitic oscillations. For many practical tubes \( C \) is between 0.1 and 0.01 [37]. There are
fundamental limits to the length of the structure due to the effects of saturation, and backward wave oscillations.

2.4.4 The Effect of Space Charge, Circuit Loss, and Non-Synchronous Operation

The asymptotic gain equation, Eq. (2.56) provides insight into TWT gain, but it is only valid in lossless circuits without space charge and in synchronous operation. In practice, none of these assumptions are true. TWTs have large operating bandwidths, and beam voltage does not change, so the tube will always be operating in non-synchronous conditions. In order to generate significant output power, it will be necessary to have large beam power and thus large currents. At high frequencies, interaction structures and their beam tunnels are small which leads to significant space charge effects. There will always be losses in practical circuits, and it is especially true for our TWT design where most suppression techniques lead to increased loss in the operating mode. As mentioned before the determinantal equation can be solved numerically when including the effects of non-synchronous operation, space charge, and including lossy circuits.

Loss can be easily added to Eq (2.54) by adding an imaginary loss term to the circuit propagation velocity. Non-synchronous operation can be accounted for by altering the circuit propagation velocity relative to the electron velocity. Pierce used the normalized velocity term $b$ to account for asynchronous operation. Loss and non-synchronous operation are encapsulated by:

$$P_c = e (1 + Cb) - j \beta_cCd$$

(2.57)

where

$$b = \frac{v_o - v_p}{Cv_p}$$

(2.58)

$$d = 0.0148 \frac{L}{C}$$

(2.59)

$v_o$ is the electron velocity and $v_p$ is the phase velocity of the circuit. $L$ is circuit loss in dB/wavelength. These terms are all normalized to $C$, Eq. (2.53), and the axial propagation constant of the electron beam, to make it easier to understand the theoretical implications of each term.
To account for space charge, most authors use the following equation:

\[ 4QC = \left( \frac{\beta_p}{\beta_e C} \right)^2 = \left( \frac{\omega_p}{\omega C} \right)^2 \]  
(2.60)

\[ \omega_p = \sqrt{\frac{n_e q^2}{m_e e_o}} \]  
(2.61)

The space charge parameter QC is related to the plasma frequency \( \omega_p \) of the electron beam. The plasma frequency of an electron plasma is related to the electron density of the beam \( n_e \). Thus, a high current electron beam passing through a small beam tunnel will have a large space charge, QC. One would expect significant space charge effects for high power, high frequency TWTs, as beam tunnels in high frequency TWTs are often very small. This modifies the determinantal equation as such:

\[ \frac{\beta_e}{(\beta_e - \gamma)^2} \left( \frac{\gamma^2 \beta_e}{\nu^2 - \beta_e^2} \right) \left( 2C^3 - \frac{\beta_e^2}{\beta_e} \right) + 1 = 0 \]  
(2.62)

Now we can use numerical solvers to calculate the effects on the growing travelling wave for realistic TWT operating conditions.

Traditionally, instead of reporting the exact values of the propagation constant, one again looks at normalized terms to remove the effects of \( C \) and \( \beta_e \) from stifling understanding. We define the calculated propagation constant to be:

\[ \beta = \beta_e (1 - Cy) + j\beta_e Cx \]  
(2.63)
Using this terminology, \( x \) and \( y \) will be the normalized gain and phase per length. Thus, larger values of \( x \) mean a high gain for an interaction structure of a given length. Figure 2.10 illustrates the effects of the space charge parameter QC on the gain of a TWT. Clearly, for larger values of QC the gain is lower and the bandwidth, represented by \( b \) is smaller.

\[ QC = 0.25 \]
\[ 0.5 \]
\[ 1.0 \]

\[ QC = 0.25 \]
\[ 0.5 \]
\[ 1.0 \]

\[ x_1 \]
\[ y_1 \]

Figure 2.10 Effect of space charge on the normalized propagation constants for the growing wave. (From [37])

### 2.4.5 Calculation of Gain

With this Pierce theory, the basic performance of a TWT can be approximated. What is needed is knowledge of the electron beam’s voltage, current, and radius, and the interaction structure’s coupling impedance, dispersion, and loss. Since the gain of the TWT is proportional to the cube root of the coupling impedance, and it is necessary to know coupling impedance to make estimates of the tube’s gain. The gain parameter \( C \) is also related to the conductance of the electron beam; it is preferable to have electron beams with high currents and low voltages, the type of beam generated by an electron gun with high perveance. Knowledge of an interaction structure’s dispersion will drive the required beam voltage and control the bandwidth of the TWT. It has been found that the dispersion is the more important parameter to accurately know when calculating the gain of a TWT. Change in circuit phase velocity as little as 0.5% leads to multi-dB
changes in gain over frequency. Whereas it took a change of 10% in the interaction impedance to effect gain similarly [40].

2.4.6 Large Signal Considerations

The small signal description of TWT operation only works up to a certain point, where the small signal gain assumption no longer holds. Figure 2.11 shows the complete TWT gain cycle. We start with a small signal and a dc electron beam. The small electric field starts to bunch the beam, increasing local electron density, and starting the bunching process. The beam continues to bunch and at a certain point the signal grows exponentially. The growth cannot continue forever and, at the onset of saturation, the electron density is great enough that space charge forces cause some electrons to slow down and others to speed up relative to the bunch. This process effectively breaks up the electron bunch and the signal can no longer be amplified, the electric field starts decreasing and the bunching is no longer as efficient as it was in the exponential growth stage.

2.5 Summary and Conclusions

We have presented an overview of the knowledge required to understand the operation and design of a TWT. This chapter introduced the required components and discussed
Figure 2.11 Diagram demonstrating basic travelling wave interaction with the electron beam

some of the interplay between the separate components. Some of the material presented in this chapter will be referenced and utilized in designing the overmoded TWT. In general, from this point on familiarity with CCTWTs and the concepts presented here will be assumed.
Chapter 3

Enabling Overmoded Slow Wave Structures

In order to operate a microwave tube in a higher order mode it is necessary to develop methods for suppressing interactions with both lower and higher frequency non-operating modes. The need to suppress higher order modes has long been discussed in the design of TWTs [37, 41]. These modes need to be suppressed in order to avoid parasitic oscillations which degrade tube performance and efficiency. In a traditional TWT, parasitic oscillations, such as backward wave or $\pi$–mode (where group velocity is zero) oscillations, are always at higher frequencies than the desired fundamental operating mode. For an overmoded structure the same methods used in a standard TWT will not necessarily succeed in allowing operation in a single higher order mode. This chapter will discuss reasons for suppressing non-operating modes, and provide potential methods for integrating mode suppression into a TWT. Analysis of the effectiveness of suppression and potential for manufacture will be presented.

3.1 Coupled Cavity Structures

A slow wave structure, as the name would suggest, is designed for one purpose; to keep travelling electromagnetic waves slower than the speed of light. Over the years, engineers have come up with a tremendous number of geometries and materials to create these slow wave structures, and various textbooks provide plenty of insight into their operation [17, 37, 42]. However, for high-power TWTs fundamental backward wave structures are popular for producing CCTWTS. A backward fundamental structure is one whose dispersion curve has a negative group velocity for values of $\beta L$ between 0 and $\pi$. A typical structure can be seen in Figure 3.1. This particular design by the Hughes aircraft company, sometimes referred to as a Curow-Gittins circuit, contains all the of basic coupled cavity elements.
The cavities and coupling slots are labeled and a beam tunnel is provided between the cavities. The shape of each cavity and the width and angle of the coupling slots determine the structure's dispersion and interaction impedance. This style of coupled cavity structure has been very successful in creating high power TWTs.

The Gittins structure does have a problem when scaled to higher frequencies. This type of coupled cavity structure is made by stacking multiple cavities and brazing them together. The process illustrated in Figure 3.2(a) requires that each cavity be precisely fabricated individually and then stacked on top of each other. The error in machining and aligning each cavity is multiplied by the number of cavities in the structure. In a high frequency, high gain structure with tens, and perhaps hundreds of cavities these errors add up. For a small, high-frequency TWT the error introduced in manufacturing will become unacceptable. Bill James, of Varian Inc. (now CPI), proposed a new method of manufacturing a TWT that greatly reduced these errors. His method, shown in Figure 3.2(b), allows all the cavities to be machined into just two metal plates. Now only four parts need to be brazed together, which greatly reduces the work required to machine the whole structure. Also the errors in machining each cavity will no longer sum together and the part can be toleranced by the machining method.

This TWT structure is known as a ladder structure and is used to describe all slow wave structures which contain rectangular cavities coupled by rectangular coupling slots. As mentioned in Chapter 1, this ladder structure has been successfully used in W-band TWTs and thus would be an ideal candidate for potential overmoded operating. The Bill
Figure 3.2 Manufacturing techniques of two different CCTWTs. (a) shows the standard coupled cavity stack where each cavity is made by stacking metal disks together. (b) shows the single staggered ladder circuit pioneered by Bill James. Cavities and coupling slots are machine into two plates reducing the required number of parts to four. ((a) adapted from [22]. (b) from [37])

James TWTs all operated in the fundamental, TM$_{11}$ mode, of their rectangular cavities. In the rest of this chapter we will look at potential methods to enable the overmoded operation of the ladder structure. Considering the past successes of the Gittins TWT circuit methods for its overmoded operation will also be discussed.

### 3.2 Operation in a Higher Order Mode

An electron beam that interacts with a higher order mode is guaranteed to interact with lower modes, as seen in Figure 3.3. When the electron beam intersects these modes an interaction can occur. Since we are using structures that operate in a fundamental backward mode, interaction with lower order modes will almost certainly be through either backwards wave or $\pi$-mode oscillation. One can calculate the interaction impedance of the electron beam for each of these modes where the beam intersects the mode on the dispersion curve. Several authors have tackled the subject of start currents for these parasitic oscillations [17, 43]. To prevent these oscillations, in the case of a backwards wave oscillation, either the beam current
Figure 3.3 Dispersion diagram illustrating the TWT interaction of an electron beam with a higher-order operating mode. In order to interact in first forward space harmonic the electron beam passes through the two lower modes and can have interactions with them.

should be small, the interaction impedance low or the circuit be short, all potentially restrict the gain of the forward wave interaction. In the case of a $\pi$-mode oscillation the loss in the circuit must be large to prevent oscillation.

In many cases of standard TWT geometries, such as the ladder circuit, the interaction impedance of lower modes is greater than the impedance of higher order modes. We have explicitly found this to be true for the ladder TWT geometry that will be presented in Chapter 4. For a travelling wave interaction at a phase advance per cavity, $\beta L$, of $2\pi/3$, the TM$_{11}$ mode's interaction impedance is 5 times larger than the TM$_{31}$ mode's. This significantly higher interaction impedance will be a major issue of designing an overmoded tube. There is a need to develop methods to suppress parasitic oscillations to enable a high gain design.

3.3 Suppression of Non-Operating Modes

The gyrotron is the most common microwave device to utilize overmoded operation and has been studied as a base case for accomplishing non-operating mode suppression. Gyrotrons utilize many techniques to ensure single mode operation: beam placement,
magnetic field, start-up scenarios, and an interaction structure design with selective loss [34]. Through a combination of these devices, gyrotrons are capable of highly overmoded operation; one tube operating in the TE$_{22,6}$ mode, which has 100s of lower order cavity modes [13].

Unfortunately, not all the beneficial features of a gyrotron apply to TWT design. Changing a TWT's magnetic field is not an option, as the frequency of operation is independent of the magnetic field and thus would have no effect. Start-up scenarios are not relevant for an amplifier, such as a TWT. Amplifiers need to be stable in the absence of an input. Beam placement is the only relevant design parameter for improving TWT performance.

By changing the position of the beam relative to the cavity field one can change the interaction impedance between the circuit and beam. For instance, the electron beam normally passes through the symmetrical center of the resonant cavity. Figure 2.6 depicts the mode patterns for the four lowest resonant frequencies of a rectangular cavity. Modes such as the TM$_{11}$ and TM$_{31}$ have a maximal field on center, whereas other modes such as TM$_{21}$ and TM$_{12}$ have nulls at this location. As a result of placing the beam in the center of cavity, the interaction impedance across the electron beam’s cross-section is much greater for a TM$_{11}$ or TM$_{31}$ mode since mode competition is reduced. If the location of the beam is changed to be at a maximum for the TM$_{21}$ mode, a similar situation occurs and the interaction impedance of the TM$_{11}$, TM$_{31}$ modes is reduced. However, because the TM$_{11}$ is the fundamental cavity mode, and has large fields throughout the cavity, it is impossible to significantly reduce its interaction impedance purely by beam placement. Another method must be used to reduce this mode’s interaction impedance. If the TM$_{11}$ experienced higher losses than the desired operating mode, mode selectivity would occur.

Other methods have been utilized to ensure single mode operation in gyrotrons. Here at MIT, a gyrotron oscillator utilizing photonic band gap (PBG) structures was built to provide loss to the non-operating modes [44, 45]. It may be possible to adapt the technique used for the gyrotron oscillator to create an overmoded TWT. TWTs traditionally have also been designed to suppress higher-order modes and one of the methods used involves adding lossy dielectrics to the interaction circuit. This loading can
Figure 3.4 The mode patterns for several TMmn of a rectangular resonator are presented. (a) TM11, (b) TM21, (c) TM31, and (d) TM12 be used to increase the loss in unwanted modes. We will take a closer look at exploiting PBGs and dielectric loading to suppress unwanted modes for an overmoded CCTWT.

3.4 Photonic Band Gap Structures

Photonic band gap structures (PBGs), also known as photonic crystals, are one-, two-, and three-dimensional devices made of regularly arranged metal or dielectric materials (i.e. rods or spheres) capable of acting as reflectors of electromagnetic energy over specific frequencies bands [46, 47]. Figure 3.5 shows example geometries of PBG structures. In theory, PBGs could bring great changes to the world of vacuum electron devices. The concept of overmoded operation is mentioned often in this thesis. In general, vacuum electron devices should not be operated in multiple modes. Such multi-mode operation can lead to unwanted frequency content, power inefficiencies and absolute instabilities. Unwanted operating modes are commonly not at the desired frequency of operation, however, a structure with frequency selective reflection could be designed to allow only a single mode. Experiments at MIT have demonstrated the effectiveness of creating a PBG with frequency selective reflection.

The concept of such frequency selective structures is not new. The one-dimensional PBG structure has been well known for over a century as a Bragg filter. What has changed is the realization of 2-D and 3-D PBGs, whose theory has been developed over
the last 20 years. The added dimensions of these new structures provide access to much more interesting geometries not available in one-dimension. This work focuses on one- and two-dimensional PBG structures. Three-dimensional PBG structures are complicated to manufacture and they may have promise for future vacuum electron devices, but we should investigate similar geometries first. Over the last 20 years significant progress has been made in the theory of PBG structures; many books can be found to serve as a reference for the theory [45, 47]. We will give an overview and analysis techniques of the important characteristics of PBG structures needed designing for vacuum electron devices.

3.4.1 Theory of Photonic Band Gap Structures

In solid state physics, the term band gap refers to energy bands that particles, such as electrons, cannot reside in. The photonic band gap is a similar device frequency band that electromagnetic waves, photons, cannot propagate through. The PBG property arises from the regular spacing of a PBG lattice. Figure 3.6 shows what the lattice looks like for a two dimensional PBG structure, with rods of radius r, organized in either a triangular or square array. These lattices are assumed to extend infinitely, giving the structure translational symmetry. Since the lattice extends equally in all directions, if one were to move to any other rod of the lattice it would still be surrounded by the same infinite lattice, thus the lattice has translational symmetry.

Translational symmetry gives us a powerful tool in analyzing PBG structures. If a structure has translational symmetry, any electromagnetic fields propagation through the
structure must share this symmetry. Mathematically, a function with translational symmetry can be expressed by:

$$x(\vec{r} + \vec{T}) = x(\vec{r})$$

(3.1)

where $\vec{T}$ is the translational vector. In the case of the 2-dimensional PBG $\vec{T}$ is:

$$\begin{align*}
\text{SquareLattice} : & \quad \vec{T} = n\hat{x} + m\hat{y} \\
\text{TriangularLattice} : & \quad \vec{T} = \left( n + \frac{m}{2} \right)\hat{x} + m\frac{\sqrt{3}}{2}\hat{y}
\end{align*}$$

(3.2)

where $m$ and $n$ are integers that represent translations along discrete points on the lattice and $a$ is the lattice constant. The electromagnetic fields must be equal when translated by this vector, and due to the freedom in the $z$ axis they will vary as $e^{j(\omega t - k_z z)}$. If the fields are represented by $H$:

$$H(\vec{r} + \vec{T}) = H(\vec{r})e^{j\vec{k}_z \cdot \vec{T}}$$

(3.3)

where $k_z$ is the transverse wavenumber in the plane of the PBG structure. The Helmholtz wave equation becomes:

$$\left( \nabla_i^2 + \frac{\omega^2}{c^2} - k_z^2 \right) H(\vec{r}) = 0$$

(3.4)

We are interested in solving this equation to find the frequencies, $\omega$. This is a two-dimensional problem, and just like the solutions to the rectangular waveguide there will be TE and TM modes. These modes correspond to the boundary conditions in the $z$
Figure 3.7 Reciprocal lattice and Brillouin zones (a) square lattices and (b) triangular lattices. The irreducible Brillouin zones for each lattice time has been annotated (From [45])

direction. TM mode will have no axial magnetic fields and TE modes will have no axial electric fields. Since we have a regular lattice and not an infinite number of $k_L$ to consider, it will suffice to calculate the fields for a finite region of $k$ space called the irreducible Brillouin zone, shown in Figure 3.7. By analyzing the symmetries of the problem, it can be shown all unique solutions to the wave equation are restricted to values of $k_L$ on the irreducible Brillouin zone [47]. Special symmetry values for $k_L$, $\Gamma$, $X$, $J$, and $M$ have been labeled and correspond to the following:

$$\Gamma : k_\perp = 0$$

**SquareLattice**:  

$$X : k_\perp = \frac{\pi}{a} \hat{x}$$

$$M : k_\perp = \frac{\pi}{a} \hat{x} + \frac{\pi}{a} \hat{y}$$

$$\Gamma : k_\perp = 0$$

**TriangularLattice**:  

$$X : k_\perp = \frac{2\pi}{\sqrt{3}a} \hat{x}$$

$$M : k_\perp = \frac{2\pi}{3a} \hat{x} + \frac{2\pi\sqrt{3}}{3a} \hat{y}$$

(3.5)

By sweeping $k_\perp$ from $\Gamma$, $X$, $J$, and back to $\Gamma$, a dispersion diagram of frequency vs wavenumber can be created, shown in Figure 3.8. All frequencies for which there are no real solutions correspond to frequencies without propagation through the PBG lattice,
corresponding to a bandgap. To calculate all possible frequencies of propagation it is only a matter of calculating the frequencies for values of the prescribed values of $k_{\perp}$ [47].

Generally, the fields are solved for with the help of numerical codes. When designing dielectric PBGs one should take advantage of MIT photonic bands (MPB) [48]. This modeling package is a very powerful and efficient solver simulating many dimensional dielectric PBG structures. For metallic PBGs, more generic electromagnetic eigenmode solvers such as HFSS or Microwave Studio should be used.

### 3.4.2 Band Gap Maps

Using the numerical method described in the previous section it is possible to generate global band gap maps. The band gaps maps can be used to quickly determine the existence if a global band gap for any variation of $r/a$ for a particular PBG geometry. The band gaps maps for metallic PBGs with TE and TM modes in triangular and square lattices are shown in Figure 3.9 and Figure 3.10.

To create a band gap map using HFSS, one only needs to model the rod and unit cell outlined in Figure 3.7. Appropriate values of $k_{\perp}$ can be imposed by master/slave phase boundaries on the transverse edges of the structure. Using this, one should have a thin
structure in the axial direction where the boundaries should be perfect H or E planes if either TE or TM solutions are desired. A dispersion curve for each value of $r/a$ can then be created using the eigenmodes calculated by HFSS. The global band gap map is generated by calculating all possible $r/a$ values. This method is quite useful as both dielectric and metal structures can be calculated.

Regions with a global band gap have zero transmission through the PBG lattice. For a vacuum electron device it is desirable to have an operating frequency inside a global gap and other unwanted modes outside the band gap.
At the start of our PBG theory, we assumed we would have an infinite lattice of rods to work with. This is a requirement of the analytic theory, however, in practice, when operating inside of a band, only a few (3-5) rows of rods are necessary.

### 3.4.3 Metallic PBG Structures

PBG theory has demonstrated photonic crystals can be used to make a frequency selective structure. Ideally a PBG structure would be made out of copper. A copper structure would be vacuum compatible and have few reliability issues. However, as will be investigated, it is unlikely a metallic PBG structure can be used to make a TWT slow wave structure without a fundamental mode present.

Since we only consider the design of a TWT SWS, we are only interested in modes with electric fields on axis, TM modes. As shown by the TM mode global band gap map in Figure 3.9 a two dimensional metallic PBG, regardless of geometry, will always have a low frequency region where there is a global band gap. This guarantees there will be a confined mode which will look like the fundamental mode of the cavity. This feature of the PBG structure was used to make the 17 GHz accelerator structure here at MIT [45]. In the case of the accelerator structure it was desirable to confine the lowest order mode and allow all higher order modes to escape.

The MIT PBG gyrotron experiment used a cavity made of metal rods that would selectively confine a TE\(_{04}\)-like cylindrical cavity mode exclusively. The TE mode global band gap map in Figure 3.10 demonstrates that there is no global band gap at low frequencies and that lower order modes are allowed to transmit, while allowing confinement of a single higher order mode. The PBG gyrotron structure has the same properties we are looking for in our TWT structure. Is there some way to implement a metallic PBG lattice to confine only one higher order mode?

On the surface it would appear there is a way. We start by looking at the definitions of TM and TE modes and how they relate to PBG structures. Figure 3.11 shows the orientation of the electric fields for TE and TM mode relative to the lattice. For TM modes the electric fields are parallel to the lattice rods and for the TE modes the electric fields are perpendicular to the lattice rod. We know for a TWT the electric fields need to
be parallel to the electron beam in order to interact. Thus, one would think to utilize the PBG concept we could take a standard TWT structure, like the ladder TWT, and add a 2-D PBG lattice to the side.

The size of this PBG structure is a concern, for a W-band TWT the band gap should be located at 94 GHz. A quick calculation of the size of the metal PBG rods in a triangular lattice yields a lattice constant $a \sim 1$ cm. To make the PBG concept work there must be at least one rod bordering each cavity with the cavity spacing $L \sim 1$ cm. The electron velocity would have to be greater than the speed of light to interact with the structure, which is not physically possible.

One might wonder if only part of the rods are needed to form the walls of the interaction structure. This concept of substituting square rods for round rods is illustrated in Figure 3.12. This configuration is known as a Bragg filter and the reflection of this structure is given in Figure 3.13. Clearly, with this structure it is 100% reflecting at low frequencies, and would guarantee interaction with a lower order mode. So why did the TE mode band gap map suggest we might be able to accomplish this task?

The global band gap calculations for TE modes have one major approximation. The surfaces in the axial direction are perfect H boundaries, they force the magnetic fields in the axial direction to zero. This approximation would be good for an infinitely long waveguide or perhaps an open ended resonator, not for a TWT. In a TWT the roof of the cavities are metal, certainly not perfect H planes. Thus the structure will always have the
Figure 3.12 Presented here are basic views of the top coupled ladder structure with a Bragg reflector added. This reflector can be designed to be transparent to certain bands of frequency while 100% reflecting to others, at low frequencies it will always be reflecting.
Figure 3.13 Reflection through the Bragg structure shown in Figure 3.12. The structure is frequency selective allowing transmission for some frequencies. At lower frequencies a metal bragg structure will always have 100% reflection, generated by code derived from [50]

potential for confined lower mode. The metallic PBG does not look like a solution for an overmoded TWT interaction structure.

3.4.4 Dielectric PBG Structures

The PBG concept might not work with metallic structures, but could still work with dielectrics. A global band gap map for dielectric rods in air and holes in a dielectric is depicted in Figure 3.14. There are global band gaps for TM modes without a low frequency global band gap. If engineered correctly, it would be possible to create an overmoded TWT in this fashion. A standard TWT geometry, like the Gittins design shown in Figure 3.15 or ladder circuit with its outer cavity wall removed, could be placed inside a dielectric PBG structure. The PBG would act as the walls for the desired operating mode, while preventing non-operating modes from coupling with the beam. The geometry of the PBG lattice should be chosen to have a global gap at the desired operating frequency.

When using dielectrics it is still possible to go with a 1-D PBG structure, commonly known as the dielectric Bragg filter. Further HFSS simulations have confirmed that this concept would work in eliminating undesirable modes without attenuating the desired operating mode.
Figure 3.14 Global band gap map for a square lattice of holes in a dielectric material (a) and rods in air (b) [47]

Figure 3.15 Curow-Gittins style TWT structure with dielectric PBG walls. This structure would be able to have confined modes.

For practical TWTs some dielectric ideas are problematic. Dielectric materials such as sapphire and diamond have excellent thermal conductivities and large dielectrics. Clearly these dielectric PBG concepts would increase the complexity of the TWT circuit. It also remains to be seen as to whether these structures could be practical with the small interaction structure at the W-band, and beyond. These dielectric concepts were not heavily pursued due to outstanding questions with using dielectric materials in close proximity to an electron beam. Alumina and diamond are very good insulators. As such electrons that impact on these materials will not go anywhere slowly charging up the surface. This charge up can interfere with the electron beam, and prevent it from making
it through the structure. Given these concerns, while an all dielectric PBG structure is interesting, dielectric loading was considered to be a less risky option.

3.5 Dielectrically Loaded TWTs

We have repeatedly discussed increasing attenuation or loss of an unwanted mode. By placing an absorbing, or lossy, dielectric material in a region of high electric field, loss will be introduced into the mode. Since some resonant modes have nulls where other modes have maxima, placing such lossy material in the proper location will alter one mode while leaving the other unchanged. Figure 3.16 illustrates this loading concept; the fields of the TM$_{11}$ mode are affected by the presence of a lossy dielectric while the fields of the TM$_{31}$ are unchanged. We can look at the $Q$ of a mode to discover how much loss has been introduced by dielectric loading. As mentioned in Chapter 2, the $Q$ of a mode is defined by its loss per cycle; a mode with low $Q$ will have significant losses. Figure 3.17 analyzes the effect of dielectric losses on the $Q$ of a cavity, as the width of the dielectric material shown in fig 10 is increased as a percentage of the total cavity width. Up to a dielectric width percentage of 18% one mode, the TM$_{31}$ mode, has had little change to its $Q$, while the others are reduced by multiple orders of magnitude. Clearly, dielectric loading can be used to create a mode selective device.

For a vacuum electron device there are some practical material considerations. Any dielectric material to be used will have to be vacuum compatible. Additionally, the dielectric material must have a significant loss across the frequencies of interest. Loaded AlN ceramics have been found to have significant loss. Several ceramics have been engineered to meet the challenge of lossy materials in vacuum electron devices. One such dielectric has a dielectric constant of 20 and a loss tangent of 0.25 at 95 GHz [51]. As a ceramic AlN is vacuum compatible and has a suitable lossy dielectric for this application.

These lossy dielectrics can be easily integrated in the top and bottom plates of a ladder structure. This device design is both simple and easy to manufacture. Although lossy ceramics are expensive, the contribution to the overall cost of the structure is low. Dielectric loading will be the method pursued in the following chapter for creating an overmoded W-band TWT.
Figure 3.16 The addition of dielectric loading for the (a) TM11 and (b) TM31 modes. Electric field patterns for the TM11 mode has been altered; field energy can be seen moving into the dielectric region. The fields are the same for the TM31 mode without dielectric loading.

Figure 3.17 Q of a rectangular resonant cavity where lossy dielectric have been placed in the nulls of the TM31 mode. As the width of the electric is increased as a percentage of the total cavity width the Qs of the other mode are significantly lowered.
3.6 Summary and Conclusions

This chapter presented several methods for enabling overmoded operation of a TWT. These methods focus on attenuation of the non-operating modes of a TWT, while allowing a single operating mode to be unperturbed. Two methods, dielectric PBGs and dielectric loading, were found to meet these criteria. Due to the far increased complexity of fabricating and integrating a dielectric PBG with a TWT, dielectric loading was proposed and will be used in the following chapters in the design of an overmoded TWT.
Chapter 4

Design of a W-Band CCTWT

In the preceding chapters, the introductory material for an overmoded W-band CCTWT was presented. In this chapter we will introduce the design of such a device. A case has been made for designing a CCTWT based on an overmoded dielectrically loaded ladder structure, a structure which should succeed. Multiple ladder circuit geometries will be discussed, with the final design utilizing a single staggered ladder circuit. Ansoft’s HFSS was used to calculate dispersion and coupling impedance of this slow wave structure, and Magic 3D to fully simulate the electron beam/electromagnetic field interaction, and calculate output power and bandwidth. Results will show an overmoded dielectrically loaded interaction structure can be used as a W-band TWT.

4.1 Design Goals

The foremost design goal is to present a CCTWT design, which utilizes an overmoded interaction structure that produces TWT amplification of a single input frequency. There were no other hard specifications for the design of the TWT. There was some flexibility with respect to the design frequency; though there is a real interest in sources at or near 94 GHz. The design structure will be at 99 GHz, however it should be noted, a 94 GHZ structure could be easily generated with a slight change to the cavity geometry and beam voltage. With respect to the structure’s gain, given that this is a TWT amplifier, a gain of over 30 dB would be a significant achievement, and would be required of any practical TWT. In addition, an output power of over 1 kW would equal the best performing W-band TWT to date [22].

4.2 Types of Ladder Structures

The ladder structure, as mentioned in Chapter 3, consists of coupled rectangular cavities. There are several ways these rectangular cavities can be coupled between each other: inline coupling, staggered coupling and double staggered coupling [22]; with each having
its own advantages. Inline coupling promises cavities capable of handling the most power, but with lower bandwidths. The inline design created a coalesced mode circuit and can have issues with backward wave oscillations [37], thus it will not be pursued. Double staggered coupling can generate very large bandwidths, as large as 20% of the operating frequency [22], but has reduced power handling compared to an inline coupling circuit. Single staggering has a lower bandwidth and comparable power handling to the double staggered circuit. A single staggered circuit, shown in Figure 4.1, has coupling slots whose position changes 180° relative to each between cavities. The double staggered circuit is more complicated to manufacture, since it requires two coupling slots whose position changes 180° relative to each other between cavities. As a result it was not investigated.

4.3 Single Staggered Ladder Circuits

The single staggered ladder circuit is a backward fundamental structure. Thus we are interested in a travelling wave interaction operating near the $3\pi/2$, or 270°, of phase advance per cavity. This point is chosen to ensure the interaction will have approximately the maximum bandwidth that the interaction structure will allow. In order for the beam to interact at the $3\pi/2$ operating point, the phase velocities of the beam and circuit should be equal:

$$v_e = v_p = \frac{\omega}{\beta_c}$$  \hspace{1cm} (4.1)

For a circuit with a fixed operating frequency the electron beam voltage is inversely proportional to circuit pitch squared. This is a fundamental trade-off since a larger circuit pitch, $L$, will require higher beam voltages. The CPI W-band Millitron TWT uses a 50 kV
Figure 4.1 Views of a side coupled single staggered ladder structure.
Figure 4.2 Views of a top coupled single staggered ladder structure.
electron gun [22], and this electron gun could be used in the design of this TWT. A 50 kV beam requires a pitch of ~1 mm to interact at 99 GHz. From here on the pitch of the structure will be 1 mm.

There are two ways that the circuit can be staggered: side coupled, with coupling slots on the long sides of the cavity as shown in Figure 4.1; and top coupled, with coupling slots on the short side of the cavity as shown in Figure 4.2. The top and side terminology comes from the assumption that the cavities will be manufactured such that the longer side will be horizontal to the ground. With side coupling, there is no difference between the slots of an overmoded or fundamental structure. However, this is not the case for the top coupled structure, a fundamental mode top coupled structure would have one coupling slot across the top [21]. The overmoded top coupled structure has three slots, corresponding to the maxima of the TM_{31} mode. Such an arrangement was found to dramatically increase the bandwidth of the structure. The performance differences between the two structures will be discussed later.

4.3.1 Using HFSS to Determine Dispersion and Coupling Impedance

In order to analyze these single staggered structures for the design of a TWT, we require a method for calculating the structure’s dispersion and coupling impedance. Unlike with other TWT slow wave structures, as in the folded waveguide TWT, there is no published theory to analytically calculate dispersion and coupling impedance for the staggered ladder structure. Instead, we can use Ansoft’s HFSS [52], a numerical electromagnetic field solver to simulate our structure. From these simulations, we can determine dispersion characteristics and coupling impedance for ladder structures of different geometries.

This section will go into the details for implementing dispersion and coupling impedance calculations with HFSS. We verified the following modeling technique and calculations by benchmarking our results. The work of other authors, a folded waveguide TWT whose geometry, dispersion are published [25]. Another paper used the same folded waveguide TWT structure to analytically and computationally calculate the structure’s dispersion and coupling impedance [40]. We confirmed that our technique yields identical results to those of previously published results.
4.3.2 Dispersion

Theoretically HFSS can be used to simulate an entire TWT structure and derive the dispersion and coupling impedance from the solved electric field data and S-parameters. However, an entire TWT circuit will consist of many cavities, often 30 or more. Though such a model is solvable with modern computers, it would still take considerable time to solve. In order to efficiently study the effects of differing cavity geometries rapid simulations are imperative. Thankfully, we can extract the necessary information from a model comprised of just a few coupled cavities. In order to produce accurate results, the model must be symmetric such that copies of the model when joined together yield the same geometry as a full length circuit.

We are interested in investigating the effect of different cavity geometries in the single staggered ladder. Since the coupling slot will rotate by $180^\circ$ from cavity to cavity, at least two cavities will be required to make a symmetric structure. Figure 4.3 shows a two cavity model of the side coupled ladder structure, this model was used for modeling our ladder circuits.

Figure 4.3 Two cavity structure used for HFSS simulation of dispersion and coupling impedance. Locations of master/slave boundaries are shown. The coupling impedance is calculated on the polyline through the beam tunnel.
Calculating dispersion requires finding the frequency associated with each phase advance per cavity. HFSS allows one to force the phase advance of the structure through the use of master/slave boundary conditions. These boundary conditions force all solutions to have the specified phase relation between the two boundaries. Since we are modeling two cavities, the specified phase advance at the boundaries will not be equivalent to the phase advance per cavity.

With the master/slave boundaries in place an eigenmode solver is used to calculate the frequencies. A dispersion curve is generated by solving for multiple phase advances and plotting the resulting frequencies. For each cavity mode and every specified phase advance HFSS will find two frequencies which correspond to two different phase advances per cavity. With two cavities between the boundary conditions, there are two phase advances per cavity that will satisfy the boundary conditions. For example, a master/slave boundary of 0°, would have solutions of 0° and 180° per cavity. The 180° phase advance would yield a net advance of 360° after two cavities, which is equivalent to 0° phase. For a master/slave boundary of 180°, there are solutions of 90° and 270° phase advance per cavity, but in our case the corresponding frequency for each of these two solutions is equal.

The frequencies calculated by HFSS are plotted in Figure 4.4, this data is generated

![Figure 4.4 Dispersion frequency data from HFSS with master/slave boundary conditions. These data needs to be “unfolded” in order to generate a useful dispersion curve.](image)
Figure 4.5 Electric field vectors for the two solution of the TM$_{31}$ with a 0° master/slave boundary condition. These fields show that the lower frequency indicated in Figure 4.4 corresponds to (a) 0 or 360 degrees and the higher frequency corresponds to (b) 180 degrees per cavity directly by HFSS. Each pair of curves corresponds to a cavity mode starting with the TM$_{11}$ mode. We see that solutions meet at a master/slave phasing of 180°, this is due to the symmetry in the dispersion curve about the 90° and 270° phase point. By looking at the electric fields in the cavity for each mode, shown in Figure 4.5, we can determine the actual phase advance per cavity of each solution. The phase advance of the higher frequency corresponds to either a 0° or 360° advance per cavity and the lowest frequency corresponds to a 180° phase advance. With this knowledge the entire dispersion curve can be constructed.

4.3.3 Coupling Impedance

From Chapter 2, we defined the coupling impedance as:

$$K = \frac{\left| E_n \right|^2}{2\beta_n^2 P}$$

(4.2)

where $E$ is the electric field, $P$ is power travels through the circuit, $\beta_n$ is the circuit’s axial propagation constant and $n$ is the $n^{th}$ space harmonic. As a reminder:

$$\beta_n = \frac{\phi}{L}$$

(4.3)

where $L$ is cavity pitch and $\phi$ is the axial phase advance between cavities. As previously mentioned, the dispersion diagram extends infinitely, in a series of space harmonics of the cavity mode. The electron beam does not interact with the full magnitude of the
electric field. Rather it will only interact with the spatial harmonic of the electric field. $E_n$ becomes:

$$E_n = \frac{1}{p} \int E_z(z) e^{i\beta_n z} \, dz$$  \hspace{1cm} (4.4)$$

where $p$ is the total length of integration. Coupling impedance is only based on the axial electric fields travelling with the electron beam, $E_n$, found by taking the spatial Fourier transform. For these calculations it is assumed that $\beta_n$ is the axial velocity of the circuits, first space harmonic. The same model and its corresponding simulations were used to calculate both dispersion and coupling impedance.

We are interested in calculating coupling impedance at specific beam tunnel radii. In order to get access to the electric field data, a polyline will need to be added to the model in HFSS, shown in Figure 4.3, for on-axis coupling impedance calculations. For these calculations, we already know $\beta_n$ from the phase advance of the master/slave boundaries. To calculate the axial power flowing through the structure one needs to integrate the real part of the Poynting vector in the axial direction over an entire plane normal to the axis of the beam tunnel. Calculating $E_n$ in HFSS is not straight-forward. Eq. (4.4) shows the integration of the total electric field in the $z$ direction multiplied by a phase, where both terms will have both real and imaginary parts. The HFSS calculator does not allow one to integrate non-real numbers; therefore the following formulas are used to find $|E_n|^2$:

$$E_n = \frac{1}{p} \int \text{Re}(E_z(z) e^{i\beta_n z}) \, dz + j \frac{1}{p} \int \text{Im}(E_z(z) e^{i\beta_n z}) \, dz$$  \hspace{1cm} (4.5)$$

These equations are mathematically equivalent to Eqs. (2.51) and Eq. (4.4), but easily implemented in HFSS. Using these formulas it is possible to calculate the coupling impedance along the polyline.

### 4.3.4 Calculations in Dielectrically Loaded Structures

All of the dispersion calculations were accomplished using models without dielectric loading. It was found that the addition of dielectric loading had no effect on the dispersion and coupling impedance of the desired operating mode. This was expected
since the loading was specifically designed not to affect the operating mode. When dielectric loading is added to the structure it significantly increases the simulation time of the eigenmode solver; thus, it is preferable to simply model the interaction structure without dielectric loading, which has no impact on the accuracy of the solutions.

### 4.4 The Side and Top Coupled Ladder Structure

The dispersion and coupling impedance characteristics of a slow wave structure will determine the gain of a TWT. We can analyze its dispersion and coupling impedance for the two different coupled slot geometries. Figure 4.6 shows the terminology that will be used to describe the different single staggered geometries and we observe there is only one difference between the definitions of the side coupled and top coupled geometries. For the side coupled geometry, slot width refers to the single slot, as indicated in Figure 4.6; while for the top coupled structure, slot width refers to the width of each individual coupling slot.

![Figure 4.6 View of two cavity side coupled ladder structure illustrating various geometric parameters.](image)

Figure 4.6 View of two cavity side coupled ladder structure illustrating various geometric parameters.
The size of the cavity will determine the frequency of the TM$_{31}$ mode. Since we want a 99 GHz TWT structure having a rectangular cavity, the cavity width and height were chosen to give a central passband of 99 GHz. The cavity pitch was previously locked at 1 mm, due to the selection of a 50 kV beam voltage. Using the methods described in Section 4.3, dispersion and coupling impedance were calculated for various cavity geometries. Figure 4.7 shows typical dispersion and coupling impedance curves for the TM$_{31}$ mode of the single staggered structure. In the end, to compare the different geometries it is not important to know dispersion or coupling impedance for every phase advance. In order to save computational resources, we need only to calculate dispersion for a $0^\circ$ phase of the master/slave boundaries. This point will give the maximum ($\beta L=0$ or $2\pi$) and minimum ($\beta L=\pi$) frequencies. The difference in these frequencies will be the entire passband of the TM$_{31}$ mode and should be proportional to the structure’s bandwidth, meaning a large passband should lead to a large bandwidth in a real TWT. Since our TWT will operate at values of around $\beta L=3\pi/2$, it is not necessary to calculate coupling impedance over the entire band.

We will calculate coupling impedance for only one point and assume this point reflects on the value of coupling impedance for other phase advances. For these tests, a $175^\circ$ phase of the master/slave boundary was used. The $180^\circ$ phase point was not used because of inaccuracies in HFSS calculations of coupling impedance at this point. Several simulations using HFSS and comparisons to previously published results [40]
have shown this to be an acceptable approximation. The results of these geometry scans are shown in the next two sections.

4.4.1 Characteristics of Side Coupled Structure

The cold circuit bandwidth and coupling impedance of the side coupled single staggered structure are shown in Figure 4.8 and Figure 4.9 respectively. These figures show the relation of cold circuit bandwidth and coupling impedance for different values of slot width and beam tunnel radius. Cold circuit bandwidth increases with slot width, but is relatively unaffected by change in the width of the beam. We see that for large slot widths, a cold circuit bandwidth of over 10 GHz is possible. While bandwidth may have increased with larger slot widths, Figure 4.9 shows that the coupling impedance will decrease. For equal slot widths a smaller beam tunnel leads to larger coupling impedance. The travelling wave growth rate, and thus gain, is related to the cube root of the coupling impedance. This suggests an increased gain bandwidth product when operating with large slot widths and a small beam tunnel. In practice, such a small beam tunnel would prohibit the use of larger currents, and such a reduction of current would in turn lower the gain, while operating with a larger beam tunnel would lead to larger gains if the current is also increased. One issue that arises with larger slot widths is that the bandwidth for the operating in the TM$_{31}$ mode translates into large bandwidth for other modes, which can thus bring the passbands of other higher order modes into the passband of the operating mode. It is unclear what the exact effect would be on tube operation, but we believe that it is a scenario that should be avoided. The effect of varying the slot height was not looked at closely, since there did not appear to be any easily understood trends. Nonetheless, the effect of slot height should be examined once the other geometries have been fixed.
Figure 4.8 Cold circuit bandwidth vs. slot width for Side coupled single staggered structure. The general trend shows that bandwidth is larger for wider slot widths.

Figure 4.9 Coupling impedance vs. slot width for Side coupled single staggered structure. Coupling impedance drops as slot width and beam tunnel radius are increased.
Figure 4.10 Cold circuit bandwidth vs. slot width for Top coupled single staggered structure. The general trend shows that bandwidth is larger for wider slot widths.

Figure 4.11 Coupling impedance vs. slot width for Top coupled single staggered structure. Coupling impedance drops as slot width and beam tunnel radius are increased.
4.4.2 Characteristics of Top Coupled Structure

The cold circuit bandwidth and coupling impedance of the top coupled single staggered structure are shown in Figure 4.10 and Figure 4.11 respectively. These figures show the relation of cold circuit bandwidth and coupling impedance for different values of slot width, beam tunnel radius, and slot height. As with the side coupled circuit, the cold circuit bandwidth increases with slot width; however, in this case, larger beam tunnels lead to smaller cold circuit bandwidths. We see that for large slot widths, a cold circuit bandwidth of over 10 GHz is possible. Again, similar to the side coupled circuit, larger beam tunnels and slot widths lead to smaller values of coupling impedance. The gain-bandwidth product for the top coupled structure appears conserved as it was for the side coupled structure.

4.5 W-band Interaction Structure

The figures in the previous sections have given information on the geometry of the TWT; however these results were all based on simulating cavities without the addition of dielectric loading. In order to have proper mode selection, the cavities will have to be resonant for the TM$_{31}$ mode. Although the side coupled structure has very large bandwidth for a slot width length nearing the cavity’s height, such wide slots would not create a resonant cavity and dielectric loading would not be effective as a mode selector.

From the bandwidth and coupling impedance data there does not appear to be a reason to choose one structure over the other. The top and side coupled structures have comparable bandwidth and coupling impedance; however building a coupler for the top coupled structure would be more challenging since a coupler would need to be made that excites each of the TM$_{31}$ mode’s field maxima. A side coupled structure could be excited by a TE$_{10}$ mode waveguide coupled to the side of the structure. Since a side coupled structure has a simple coupler, it was chosen.
Figure 4.12 Transmission through the 99 GHz single staggered side coupled ladder circuit with and without dielectric loading. Transmission for each mode is indicated on the chart. There is clearly significant attenuation of the non-operating mode when dielectric loading is added to the simulation. A 9 cavity section of the interaction structure was modelled.

4.5.1 Multiple Beam Structure

The top coupled structure has three beam tunnels, one for each maxima of the TM\(_{31}\) mode. This interaction circuit can support multiple electron beams. Such an approach would allow the use of a lower beam current density to achieve the same total current. It also helps in further suppressing mode competition in an overmoded circuit because the selective placement of the beams favors coupling with the design mode while reducing coupling to the competing modes. Such a top coupled structure might be better suited for a TWT; however due to the many unknowns in the design of such a structure, it was advisable to simply remain with the design of the of the single beam structure. In the future, a multiple beam interaction structure should be investigated as it shows great promise in increasing the operating power of an overmoded TWT.
4.5.2 Interaction Structure Design

As mentioned in previous sections, the rectangular cavities were sized to give a passband centered around 99 GHz, and the cavity pitch was selected to interact with a 50 kV electron beam. For an ideal high-power TWT, the beam tunnel would be as large as possible. A larger beam tunnel makes it easier to put more current through the structure. For the same current, a larger diameter electron beam will have lower current density, lower space charge and less interception of the beam. The downside to larger beam tunnels is a drastic decrease in coupling impedance. A slot width of 1.2 mm and beam tunnel diameter of 0.8 mm were chosen as a compromise.

Since dielectric loading is being used to add mode selectivity to the structure, care was taken in designing the size of the dielectrics. From Chapter 3, in Figure 3.17, we saw that making the dielectric loading too wide would attenuate the operating TM$_{31}$ mode, while making the dielectrics too small wouldn’t attenuate non-operating modes enough. Also to be vacuum compatible Aluminum Nitride ceramics were used as the lossy dielectrics, with $\varepsilon \sim 20$ and $\tan \delta \sim 0.25$. Dielectric loading was adding by adding rectangular AlN rods, 0.6 mm wide and 0.4 mm high, to the structure to achieve mode suppression. HFSS was used to simulate the loss in the interaction structure for different modes. As seen in Figure 4.12 non-operating modes are heavily attenuated, while the operating mode is barely affected.

<table>
<thead>
<tr>
<th>Cavity Geometry</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity Width</td>
<td>5.6 mm</td>
</tr>
<tr>
<td>Cavity Height</td>
<td>2.4 mm</td>
</tr>
<tr>
<td>Cavity Pitch</td>
<td>1 mm</td>
</tr>
<tr>
<td>Slot Width</td>
<td>1.2 mm</td>
</tr>
<tr>
<td>Slot Depth</td>
<td>0.6 mm</td>
</tr>
<tr>
<td>Dielectric Width</td>
<td>0.6 mm</td>
</tr>
<tr>
<td>Dielectric Height</td>
<td>0.4 mm</td>
</tr>
<tr>
<td>Beam Tunnel Diameter</td>
<td>0.8 mm</td>
</tr>
</tbody>
</table>

Table 4.1 Proposed design values for a 99 GHz CCTWT side coupled single staggered interaction structure.
A single staggered slot structure, whose design values are shown in Table 4.1, was selected for a more complete design. These structure geometries represent a compromise between various geometries, and while not a completely optimized choice it will work in demonstrating an overmoded TWT.

4.6 Design and Simulation of a W-band TWT

HFSS generates accurate dispersion and coupling impedance data, which can be used to approximate the gain and bandwidth of a TWT structure using the Pierce calculations from Chapter 2. Dispersion and coupling impedance data from HFSS can also be used in conjunction with more advanced codes like LATTE [53] or Christine 1-D [54]. These codes model the non-linear effects in a TWT, such as saturation, velocity spread, and harmonic generation. These codes have been shown to accurately model traditional fundamental mode TWTs; however there was some uncertainty in their ability to properly model an overmoded TWT interaction. In order to be confident that proper overmoded TWT operation is achievable, a 3D particle-in-cell code Magic 3D [55], was used to simulate the full TWT structure. Magic 3D has been shown to have good agreement with produced TWTs [40].

Magic 3D simulates the interaction between an electron beam and the interaction structure. The code simulates the electron beam as a number of charged particles moving with an initial velocity dictated by the beam voltage. These simulated particles interact with the electromagnetic fields of the interaction structure. The advantage of such a particle-in-cell code is that it is the closest thing to actually simulating the TWT interaction. The only information required by Magic 3D is the beam parameters and the geometry of the interaction structure. No pre-derived metrics such as dispersion or coupling impedance need to be feed into the program. Additionally, if the results from Magic 3D agree with the results from a 1-D code like Latte, we can have additional confidence that the same modeling efforts that work for traditional TWTs will work for an overmoded TWT.
4.6.1 Magic Modeling

Magic 3D simulations require the user to provide a text file detailing the geometries of the interaction circuit, the properties of the electron beam, and the input and output ports. The 99 GHz structure design values, in Table 4.1, were used in the construction of the Magic 3D model. The electron beam was modeled as a 49.5 kV, 830 mA electron beam with a radius of 0.32 mm and no velocity spread. Even though the beam tunnel is 0.4 mm the electron beam needs to have a smaller radius to avoid structure interception once charge bunching starts to occur. Magic 3D allows the user to specify a magnetic field in order to constrain the electron beam. A perfect axial solenoid magnetic field was assumed where field strength of 0.70T was used, which is 4 times the Brillouin field. Magic 3D does not take any thermal effects into account so the last cavity of the modeled structure was used as the electron collector. Magic does not allow the user to explicitly specify a lossy dielectric; instead the user must model a lossy dielectric as a normal dielectric inside a conductive volume. The conductivity of the volume should be chosen to equal the imaginary component of the material’s dielectric constant. The required conductivity, $\sigma$, is given by [2]:

$$\sigma = \omega \varepsilon \varepsilon \tan \delta$$  \hspace{1cm} (4.7)

where $\omega$ is the frequency being modeled, $\varepsilon$ is the nominal dielectric constant, and $\tan \delta$ is the loss tangent to the material. We assumed a material similar to a Ceradyne Aluminum Nitride composite with an $\varepsilon$ of 20 and $\tan \delta$ of 0.25 [51].

In order to achieve the design goal of over 30 dB of gain 46 coupled cavities were required. These simulations require a modest amount of computational resources. It takes one core of an Intel core 2 duo processor over 12 hours to simulation 10 ns of a TWT interaction. Generally, for the 46 mm long model we are simulating, it takes 3-4 ns for the electron beam and forward wave to propagate down the tube and an additional 5-15 ns to be sure that there are no parasitic oscillations or instabilities present. In order to reduce simulation time, a parallel version of Magic was used on an 8 core machine. The use of this machine reduced simulation time by a factor of four.

Initial magic simulations were run on shorter structures, between 10 and 20 total coupled cavities. During these trials runs it was found that the reflection of the amplified
wave would lead to large instabilities. The reflected output would travel back to the input of the structure and be reflected again. This re-reflected wave was then amplified as if it was part of the input. Since the TWT has a high gain, this re-reflected signal was amplified to a larger amplitude than the original amplified signal input. This is one reason that all high power TWTs have a sever isolating the input and output stages [37]. A sever was added to Magic 3D simulations to create stability. This sever was implemented by increasing the width of the dielectric loading, thereby creating a high loss section of the TWT. This high loss sever was found to increase the TWT's stability. The Magic 3D model is similar to Figure 4.13, where the only difference is that the magic model did not use the waveguide up tapers in order to reduce the simulation domain.

4.6.2 Latte Modeling

As previously mentioned, a single Magic 3D simulation can take hours to generate just one data point for either frequency or input power. It would be extremely advantageous to use a 1D code like Latte. Latte is capable of generating entire bandwidth and saturation curves in seconds; however Latte requires the user to input dispersion and coupling impedance characteristics. Additionally, the program requires the user to input the cold circuit loss in $dB/cm$ as well as the electron beam parameters, voltage, current.

Figure 4.13 Complete proposed W-Band interaction circuit. Included are waveguide uptapers, and a sever to prevent oscillations and reflections from the outputs.
and radius. With this information, Latte is capable of calculating the gain of a TWT by axial distance and determining the onset of saturation for a given circuit length. The dispersion and coupling impedance values input into Latte were generated by HFSS. In order to determine the cold circuit loss, Magic 3D was run without an electron beam.

### 4.6.3 Magic and Latte Results

This section presents results of both Magic and LATTE modeling for the 46-cavity dielectrically loaded CCTWT. This CCTWT has over 600 MHz of bandwidth, determined from Figure 4.14; this bandwidth curve was generated with an input power of 120 mW. This CCTWT has over 30 dB of linear gain and as shown in Figure 4.15, gain is linear for input power less than 1 W and the tube has a maximum saturated output power of 1 kW. A 99.4 GHz input signal was used for simulating the saturation curve.

![Figure 4.14 Simulations results from Magic 3D and Latte showing bandwidth. The beam voltage is 49.5 kV, beam current is 0.83 A, and a constant drive power of 120 mW is used.](image)
Figure 4.15 Magic 3D and Latte results showing the saturation characteristics of the device.

Figure 4.16 Magic results for a 46-cavity structure with and without dielectric loading. Beam voltage is 49.5 kV, beam current is 830 mA, and the circuit is driven by a 1 W, 99.4 GHz signal. With dielectric loading the fundamental mode of the circuit near 60 GHz is suppressed by 60 dB. Without dielectric loading the fundamental mode is only 16 dB lower.
Both Figure 4.14 and Figure 4.15 show good agreement between LATTE and Magic 3D simulations. This result shows that future overmoded TWT design can be more rapidly analyzed used faster 1D codes. Magic 3D could then be used to verify a few final designs, saving the designer countless hours.

In order to prove the necessity of suppressing non-operating modes, two Magic 3D simulations were run, one with dielectric loading and the other without dielectric loading. For a 1 W, 99.4 GHz input, the output of both simulations was Fourier transformed and the results are shown in Figure 4.16. This result shows that without dielectric loading there is significant frequency content in many other cavity modes. The most significant of these modes is around 60 GHz and corresponds to the fundamental TM_{11} mode. With dielectric loading the power in non-operating modes is reduced by over 40 dB and the only significant signal is a harmonic of the input. Clearly, without the addition of dielectric loading this TWT would not be practical as an amplifier.

A demonstration of the TWT’s stability is shown in Figure 4.17. We see the output of the tube in response to a square wave input. Once the input is turned off there is no longer any output power, thus this is a stable circuit in the absence of input.

4.7 Summary and Conclusions

This chapter introduced the design of an overmoded TWT. This TWT utilizes a dielectrically loaded ladder circuit interaction structure to operate in the TM_{31} mode of the structure. As proposed the structure has 30 dB of gain with a 600 MHz bandwidth and a saturation output power of over 1 kW. Magic 3D simulations were run and show that it is possible to operate a TWT in a high order mode, as long as the non-operating modes are suppressed. Magic 3D simulations agreed with Latte simulations showing that 1-D TWT codes can be used as a first step in future overmoded TWT designs.
Figure 4.17 The step response of the 99 GHz TWT. Beam voltage is 49.5 kV, beam current is 830 mA, and the circuit is driven by 1 W, 99.4 GHz signal. Once the step turns off the output power falls showing the stability of the tube.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Voltage</td>
<td>49.5 kV</td>
</tr>
<tr>
<td>Beam Current</td>
<td>830 mA</td>
</tr>
<tr>
<td>Magnetic Field</td>
<td>0.7 T</td>
</tr>
<tr>
<td>Total CCTWT length</td>
<td>4.6 mm</td>
</tr>
<tr>
<td>Beam Tunnel Diameter</td>
<td>0.8 mm</td>
</tr>
</tbody>
</table>

Table 4.2 Proposed parameters for a 99 GHz TWT.
Chapter 5
Design and Measurement of Cold Test Structure

5.1 Theory of Cold Test Measurement

The goal of a cold test is to determine the dispersion, coupling impedance and loss though the structure. The loss of each mode in the structure can be easily determined by measuring the S parameters of a cold test structure. The other two parameters are more difficult to obtain. We will go over the theory used for calculating dispersion and coupling impedance. As will be shown, knowledge of the structure’s group velocity and thus dispersion is required to calculate coupling impedance. We will then discuss the measurement and calculation of the dispersion, $\omega$-$\beta$ diagram and then discuss the measurement of coupling impedance.

5.1.1 Dispersion

When we were calculating the dispersion characteristics in Chapter 4 by using the master/slave boundary conditions we were effectively assuming we had an infinite number of cavities in our TWT. In practice we can only build a finite number of cavities. The following method has been used by authors [17, 56-58] to determine the dispersion diagram for a coupled cavity structure. It is a general property of coupled resonant cavities that if one has $N$ coupled cavities then the system will have $N+1$ resonances and these resonances we will be spaced equally apart in axial phase [59]. Thus, if we were to build a coupled cavity structure with $N$ cells one should be able to measure $N+1$ resonances. According to Gittins, in a coupled cavity structure terminated by either a perfect open or short there will be a standing wave in the structure at each one of these resonances. Each of these resonances will correspond to a mode with an electric field strength that varies with an integer number of half-wavelengths along the structure. From the number of half-wavelengths along the structure one can then find $\beta$ [17]:
\[ \beta = \frac{n \pi}{l} \]  \hspace{1cm} (5.1)

where \( n \) is the integer number of half-wavelengths of the resonant mode on the structure and \( l \) is the total axial length of the resonant circuit.

Is it known that if one places a small dielectric or metal object within a resonant cavity one will perturb the electric fields of the cavity and slightly alter the resonant frequency [38]. If one assumes the electric field is constant over the perturbing object then the change in resonant frequency will be proportional to the magnitude of the electric field squared:

\[ \Delta \omega \propto |E|^2 \]  \hspace{1cm} (5.2)

One could then take a small dielectric bead and pass it though the beam tunnel of a coupled cavity structure. Measurements of the change in the structure’s resonant frequencies for each position of the bead allow one to determine the electric field strength along the beam tunnel as a function of position. Such a test is known as a “bead pull” test and is commonly used by the accelerator and TWT community. If the structure is properly terminated there will be an exact number of half wavelengths along the structure for each resonance. The number of half wavelengths along the structure will be equal to the number of perturbations as a dielectric bead is pulled. These perturbations are related to the strength of the electric field and can be measured as the number of local frequency minima seen by each frequency as the bead is pulled through the structure. Once one knows the number of half wavelengths or perturbations along the length of the structure, Equation (5.1) can then be used to calculate \( \beta \) for each resonant frequency. A dispersion diagram can then be constructed. Figure 5.1 shows how these resonances will look on a dispersion diagram.

Although it is theoretically possible to find \( N \) resonances for an \( N \) cell structure, some point out that in practice it is not possible to measure every resonance[17]. In a practical structure each resonance will have a finite \( Q \)
Figure 5.1 Dispersion diagram illustrating the resonances of a 9 cavity structure. Each circle represents a resonant point and each resonance in spaced equal distantly, $\pi/9$, apart from each other. corresponding to a certain width in frequency. One must not create a cold test structure with too many cells or else it will be impossible to measure the resonances as their Q-broadening will cause the resonances to overlap. It is especially hard to measure the resonance corresponding to values of $\beta L = n\pi$, where $n$ is an integer. In many coupled cavity circuits, including our structure, the group velocity for these resonances is near zero. As a consequence the difference in resonant frequency between these modes will be small and it may then be impossible to distinguish the modes.

5.1.2 Coupling Impedance

From Chapter 2 we know that coupling impedance is a measure of the axial electric field strength due to an amount of power travelling along the circuit:

$$K = \frac{|E_n|^2}{2\beta_n^2 P}$$  (5.3)

We can use the dielectric perturbation technique mentioned in the previous section to measure the electric field and determine the coupling in impedance. It can be shown that for a resonator changing the stored energy of the resonator by $\Delta U$ leads to a small change in the resonant frequency $\Delta\omega$:

$$\frac{\Delta\omega}{\omega} = -\frac{\Delta U}{2U}$$  (5.4)
We assume that a small cylindrical dielectric bead perturbs the field. If the bead is small enough the electric field over the dielectric perturber is constant and the change of energy due to the perturber is:

$$\Delta U = -\int_0^l \pi b^2 (\varepsilon - \varepsilon_o) E^2 dz = -\frac{1}{2} \pi b^2 (\varepsilon - \varepsilon_o) E^2 l$$

(5.5)

where $b$ is the radius of the dielectric bead, $\varepsilon$ the dielectric constant of the bead, and $l$ the length of the bead and assuming the electric field is constant over the length of the bead. The power flow $P$ is the product of the energy stored per unit length, $W$, and the group velocity, $v_g$:

$$P = Wv_g = \frac{U v_g}{l}$$

(5.6)

where $U$ is the energy in the cavity, $l$ the length of the bead and $v_g$ the group velocity. We can determine the group velocity from the slope of the measured dispersion using the methods of the previous section:

$$v_g = \frac{\partial \omega}{\partial \beta} = \frac{\Delta \omega}{\Delta \beta}$$

(5.7)

Substituting Eqs. (5.6) and (5.5) into (5.4) we get:

$$K = \frac{|E|^2}{2 \beta^2 P} = \frac{2 \Delta \omega}{\omega \pi b^2 (\varepsilon - \varepsilon_o) v_g \beta^2}$$

(5.8)

The relation of coupling impedance to a shift in resonant frequency from the presence of a small dielectric bead. When performing this experiment it is important to ensure that any bead used is small enough to justify the constant field approximations. Most experimenters use materials with high dielectric constants to create larger perturbations of the equivalent sized bead thereby making measurements easier.

### 5.2 Scaling an Electromagnetic Structure

In order to have confidence in the structure proposed in Chapter 4 it is necessary to build and test a physical structure. The dimensions of the 99 GHz structure are small; the pitch is 1 mm, and each cavity is only 0.5 mm in length. Although a structure of this size is certainly within the realm of modern manufacturing techniques it still costs a significant amount of time and money to build. It would be advantageous to test the new structure
without the expense of high precision machining. Thanks to the beauty of electromagnetic physics there is a way. A closer look at Maxwell’s equations from Chapter 2 shows the equations are scale invariant. If one were to create an electromagnetic structure with dimensions \( x \) and scale them by \( s \) such that \( x' = x \times s \), the electromagnetic properties of the structure would also scale by \( s \). Take the resonant cavity discussed in Chapter 2; if we increase the dimension of the cavity walls by a factor \( s \) then the resonant frequency would also be reduced by \( s \):

\[
\omega^2 \varepsilon \mu = \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 + \left( \frac{p \pi}{d} \right)^2
\]

\[
\downarrow
\]

\[
\left( \frac{m \pi}{sa} \right)^2 + \left( \frac{n \pi}{sb} \right)^2 + \left( \frac{p \pi}{sd} \right)^2 = \left( \frac{\frac{\omega_0}{s}}{s} \right)^2 \varepsilon \mu
\]

(5.9)

This gives us a very powerful tool in cold testing a new structure; we can scale the TWT structure to a lower frequency, increasing the structures dimensions and easing manufacturing. In this larger scaled structure the tolerances will be relaxed and cheaper techniques can be used in the manufacture. The experiment will become significantly easier to perform as the larger device is easier to handle and will require less precise measurement equipment. The scaled cold test experiment will yield data that can be directly related to the 94 GHz structure’s performance; it will be used to verify HFSS results.

**5.3 A Scaled 15 GHz TM\textsubscript{31} Structure**

This section will look at the design and manufacturing of a scaled version of the dielectrically loaded overmoded CCTWT interaction structure proposed in Chapter 4.

**5.3.1 Scaling Factor**

The question becomes by what amount the structure should be scaled. Maxwell’s equations do not care what factor we scale the structure by, but there are realities in setting up the experiment that will dictate the dimensions of the cold test structure. When performing the cold test several factors need to be considered. The structure is overmoded and in order to prove the attenuation of unwanted modes the experiment will need to have the ability to measure the TM\textsubscript{11} mode of the structure. There are also not an
infinite number of waveguide sizes; each band designation, like the W-band, has a fixed corresponding waveguide dimension. These dimensions have been standardized to allow interoperability between microwave systems. Any experiment will have to conform to standard waveguide sizes and the experiment will need to be performed within one of these bands; building custom waveguide tapers would be an unnecessary step.

The Millimeter Wave Laboratory at MIT has the capability to measure network parameters in the frequency range of 10MHz to 40GHz with the use of an Agilent E8363B VNA. While the VNA is capable of operation up to 40 GHz in the lab there are calibration kits and waveguide for the Ku band (12-18 GHz). The lab also has plenty of X-Band (8-12GHz) waveguide that could be used. If the X-band were used this would represent a scaling of ~10 times. The dimensions of each cavity would be about 5.6 cm x 2.4 cm x 1 cm. Considering a cold test structure would consist of several cavities, X-band operation makes the whole structure rather large. A Ku-band structure on the other hand would give a moderate size that could still be easily built with conventional machining and used for the cold test.

We are interested in finding the interaction impedance, $K$, dispersion, and $\omega-\beta$ diagram for the TM$_{31}$ mode. We are also interested in looking at the attenuation of the TM$_{11}$, TM$_{21}$, and TM$_{12}$ modes. In this structure, geometry of the cavities, the frequency of the TM$_{31}$ mode is ~55% larger than the TM$_{11}$ mode. It is not possible to get all of the relevant modes into the Ku band. However, the Ku band waveguide is not cutoff at 12 GHz and can be used down to 10 GHz albeit with increased attenuation. Since it is not desirable to create a special waveguide coupler for the cold test, the Ku band waveguide would have to directly couple to the structure. This means that the height of the cavities should be the same as the width of the waveguide. The ratio of 99 GHz height to the width of Ku band waveguide is ~6.58, and the rest of the dimensions were scaled by this amount. Such a scaling gives a TM$_{31}$ mode of ~15 GHz. The relevant dimensions of the
<table>
<thead>
<tr>
<th>Dimension</th>
<th>94 GHz structure scaled 6.58 x</th>
<th>Dimensions of Final Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity Width</td>
<td>36.96 mm</td>
<td>36.96 mm</td>
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<td>Cavity Height</td>
<td>15.80 mm</td>
<td>15.79 mm</td>
</tr>
<tr>
<td>Cavity Length - Pitch</td>
<td>3.3 mm</td>
<td>3.31 mm</td>
</tr>
<tr>
<td>Coupling Slot Height</td>
<td>3.96 mm</td>
<td>3.93 mm</td>
</tr>
<tr>
<td>Coupling Slot Width</td>
<td>8.58 mm</td>
<td>8.56 mm</td>
</tr>
<tr>
<td>Beam Tunnel Diameter</td>
<td>6.3 mm</td>
<td>6.25 mm</td>
</tr>
</tbody>
</table>

Table 5.1 Table of cold test Structure specified to machine shop and measured on the finished structure. Measurements have an error of ±0.05mm

cold test structure can be seen in Table 5.1. The structure is a scaled copy of the 99 GHz structure to within a few thousandths of an inch. The small differences in dimensions from a 6.58x true scaled structure are due to rounding the English unit dimensions used in manufacturing.

### 5.3.2 Manufacture of Cold Test Structure

How many cells are enough for a cold test? As pointed out in the theory section having too many cavities can hinder the ability to measure results. Gittins recommends a seven cavity structure [17], but we want more resolution and a nine cell structure was finally decided on. In order to perform measurements on the structure we need to integrate a method of exciting and measuring the structure’s resonances. Many authors choose to create a closed cavity structure without waveguides and then excite modes with small coaxial antennas [17, 56]. This method would work for calculating dispersion and coupling impedance, but would not be useful for the determining the loss from dielectric loading. To measure the circuit the two end cavities will need to be joined with a TE\(_{10}\) fundamental mode waveguide. Utilizing this waveguide allows a connection to the lab VNA, and the ability to measure S-parameters. The resonances can be measured from dips in S11 measurements and the loss due to the dielectric can be directly measured by taking S21.

In Chapter 3 we mentioned the ladder circuit is attractive for high frequency TWTs because it can be made with only a few pieces instead of stacking multiple cavities.
together to make a full structure. Our circuit can be built from four stacked metal plates, as shown in Figure 5.2. In order to simplify construction the height of the waveguide coupler should be the same as the cavity height. The two middle plates will have the cavities, coupling slots, waveguides, and beam tunnels machined into them. The cavities will be open on top and bottom, two other plates will sandwich the cavity plates and finish enclosing the resonant cavities. These outer plates will also have grooves machined into them where the dielectric loading material can be added. Six holes are drilled into each plate. Four of these holes are precision reamed and along with four precision steel dowels keep the plates aligned laterally. The other two holes are for a pair of thru bolts used to apply pressure to the stack. A significant pressure is required to ensure electric conductivity between plates and give the resonant modes a high Q. This cold test structure is not designed for vacuum compatibility. As such, the outer plates are machined from aluminum. The inner cavity plates are made out of copper in order to take

Figure 5.2 Exploded view of cold test structure. The structure is made of four plate, two top and bottom plates made from aluminum and two middle cavity plates made from copper. When stacked on each other the thickness of the middle two plates is the same as the width of Ku band waveguide.
advantage of copper's high conductivity. The cavity was machined by wire EDM and all other geometries of the structure were machined with a CNC mill. The actual structure can be seen in Figure 5.3. To ensure ladder circuit has a high level of manufacturability a second structure created using CNC milling only. Measured results from both structures were in excellent agreement demonstrating the ladder circuit is indeed easy to manufacture.

5.4 Experimental Setup

In order to implement the proposed measurements an experimental test stand needed to be developed at the MIT’s Millimeter Wave Laboratory. In order to implement the proposed cold tests an accurate method for measuring resonances in transmission through the structure was needed. Additionally, an accurate method for pulling a dielectric bead through the structure was required.

An Agilent E8363B VNA is available and can be used to collect the required S-parameter data: S11 for the bead pull tests and S21 for measuring the loss thought the circuit. An Agilent P11644A Ku-band was used to connect the cold test structure. The VNA only has SMA ports on it and the calibration kit contains SMA to Ku waveguide adapters. A precision calibration could be obtained from the VNA to the end of the waveguide flanges over the Ku band (12 to 18 GHz). This allowed for the cold test structure to be the device under test and removed any significant measurements errors associated with the VNA and the connections to the structure. To measure the loss through the structure only the calibrated VNA is required and no additional apparatus is needed. The bead pull test requires additional equipment

Conceptually, performing a bead pull test is very simple. One needs to take a small dielectric bead, attach it to a wire and pull it through the center of the beam tunnel while measuring resonant frequencies at fixed steps. The bead must be small and as such have a large dielectric constant to create a large resonant shift. Dr. Jake Haimson, an expert on the cold testing of accelerator and TWT components, suggested alumina would be an appropriate material. The dielectric constant of alumina is \( \approx 8.9 \) at 15 GHz [60].
Figure 5.3 (a) Final assembly of 15 GHz cold test structure. (b) Exploded view of the cavity plates.
A 2.2mm diameter and 1/8” long alumina bead was available and was used for the bead pull tests.

Only the dielectric bead is supposed to interact with the electric fields; any string used to pull the bead should be small enough to not alter the resonant frequencies. For the bead pull experiment a 0.004” monofilament nylon fishing line was used to support the bead. An experiment with just the fishing line and no bead showed a resonant frequency shift 1/100th of the bead pull test, well below the experimental error due to other sources.

The Millimeter Wave Lab contains a 3-axis precision scanner, capable of automated moving of a rigid mount in precise 0.025 mm steps. This device is more than adequate for a bead pull test where the cavity pitch is 6.6 mm. By using only one axis of the scanner the bead could be precisely pulled through the beam tunnel. The monofilament wire with bead was attached the scanner and kept taut by a counterweight dangling from a low friction pulley. The pulley was implemented using an optical mount and Lego®. A computer program was used to step the scanner in 0.2 mm distances; using the VNA to

Figure 5.4 Experimental setup used for measuring properties of the cold test structure
take the 2-port S-parameter data at every step. The complete experimental setup can be seen in Figure 5.4

5.5 Experimental Results

This section will report the experimental results taken on the 15 GHz cold test structure. A discussion of data analysis techniques will be given and comparison of results to HFSS simulations will be shown. The results will show excellent agreement with HFSS and prove a dielectric loaded TWT structure can be built and attenuate unwanted modes without disrupting the dispersion or coupling impedance characteristic of the operating mode.

5.5.1 Transmission – S21

A VNA was used to measure the transmission, or S21, of the structure. In order to determine the effects of dielectric loading on the structure two tests were performed. First, the transmission of the structure was measured before slots for dielectric loading had been cut into the top and bottom plates. Then the slots were cut into the plates and bars of Aluminum Nitride, AlN - a lossy dielectric, were placed in the slots of the structure. While performing the second measurement it was found that the Q of the structure was far too low and no measurements were possible. The addition of the dielectric loading had created small gaps between the cavity plates and the outer plates. The effect was a structure without resonant modes. A silver loaded conductive lubricating paste, CONDUCTOLube, was added between the outer plates and the cavity plates. The paste served to close the gaps and ensure high conductivity throughout the structure enabling high Q resonant modes. If the structure had been brazed and not bolted such steps would not have been necessary. The additional cost and lack of experimental flexibility made brazing the structure unattractive.

A second measurement was taken with dielectric loading installed and Figure 5.5 shows both results. Compared to the other modes the TM31 operating mode has significantly lower additional losses. While the TM31 mode is attenuated by 5 dB on average, the TM11, TM21, and TM12 modes are attenuated by an additional 35 dB, 30dB
Figure 5.5 Transmission results from the 15 GHz cold test structure. Transmission is show before (red) and after (black) dielectric loading was added to the cold test structure.

and 8 dB respectively. The additional attenuation of the TM$_{11}$ is likely greater than 35 dB as the hump is likely not a true mode but imperfections due to the bolted structure. These experimental results compare favorably with those of Chapter 4. The major difference between HFSS and experiment is that the overall loss is much higher than it was in simulation. Without dielectric loading the TM$_{31}$ still has 6 dB of loss; these losses are due to the lower Q of the actual structure and the fact that the circuit is not matched to the waveguide. The experiment shows that even with dielectric loading the TM$_{12}$ mode is not well attenuated, verifying a similar HFSS result. The results demonstrate that the two modes, TM$_{11}$ and TM$_{21}$, most capable of interfering with the operation of a TM$_{31}$ overmoded TWT, are heavily suppressed.

5.5.2 Dispersion

A bead pull test was performed in order to measure dispersion. As mentioned in the theory section, in order determine the value of $\beta$ at each resonant frequency, one needs to
know how many local minima there are in the electric field. Figure 5.6 shows a typical 
$|S_{11}|$ resonant curve for the cold test structure. Seven resonances can clearly be seen. 
When the bead pull test was performed S-parameters were taken at each step of the bead. 
A computer program written in MATLAB® was used to measure the resonant frequency 
of each peak for each position of the bead. The results of this program are shown in 
Figure 5.7, and demonstrate the variation of the electric field for each mode.

In Figure 5.7 certain resonant modes are more clearly distinguishable over others. 
The reason for this is our lack of a perfect open or shorted resonator as assumed in the 
theory section. One can see that the perturbation of the mode labeled seven could be 
viewed as a mode with only one perturbation. The same goes for mode 6; there are 
clearly 6 smaller perturbations on top of two larger perturbations. The only mode this is 
not true for is the n=4 mode, where the mode is symmetric. The reason for this is the 
excitation of the mode corresponding to the other half of the dispersion curve, where 
frequency is decreasing as $\beta$ increases. We are interested only in the part of the 
dispersion diagram where frequency is increasing versus beta, and the modes have been 
labeled as such. Now using Eq. (5.1) and assuming the length of the structure is eight 
and half full 6.6 mm cavities, dispersion can be calculated and is plotted in Figure 5.8.

![S11 curve](image)

**Figure 5.6** A typical $|S_{11}|$ curve for the sold test structure. Seven resonant frequencies can be clearly 
seen. This curve is from the structure without dielectric loading.
Figure 5.7 The frequency of each resonance as a function of bead position. Each resonant frequency has a specific number of local minima as a function of position; the number of minima for the backward fundamental mode is shown. The resonant modes with 3, 4, and 5 perturbation minima are illustrated.

The experimentally measured dispersion appears to match exactly with the dispersion calculated by HFSS. The HFSS simulation uses the exact dimensions of the cold test structure shown in Table 5.1, and used the 2-cavity cell method discussed in Chapter 4. The simulation also included the effect of air as a dielectric and the filleted corners to account for the 0.093” diameter tolling radius. It should be noted that the HFSS results are no more accurate than 0.01%, which is the width of the markers annotating the results.

5.5.3 Coupling Impedance

From the dispersion graph shown in Figure 5.8 we have the information we need to calculate the group velocity which is in turn needed to calculate the coupling impedance. The dispersion results from HFSS will be used to calculate group velocity. HFSS can determine group velocity much more precisely since far more points of the dispersion
curve can be generated. Considering how accurately the experimental dispersion results aligned with HFSS simulations this should introduce far less error than calculating the group velocity between the 7 measured points.

To measure the coupling impedance we used the same bead pull data used in calculating dispersion. Instead of measuring the number of local minima we are now interested in the magnitude of the frequency perturbation, as illustrated in Figure 5.9. Once again the mixing of the forward and backward harmonics is interfering with the measurement. By measuring just the local frequency perturbation and then averaging over all the local minima at each individual resonance a clear measurement of $\Delta \omega$ was possible. Then using Eq. (5.8) the coupling impedance at each unperturbed resonant frequency could be calculated. The experimental results, shown in Figure 5.10, are in good agreement with HFSS simulation. Error bars have been added based upon the overall uncertainty in the measurement. These error bar account for uncertainty in measuring $\Delta \omega$, group velocity, dielectric constant, and the bead radius. The error in measuring $\Delta \omega$ and calculating group velocity account for most of the error bar.
Figure 5.9 Illustrating the measurement of $\Delta \omega$ from bead pull data. $\Delta \omega$ is used to find the value of coupling impedance at each resonant frequency.

Figure 5.10 Measured coupling impedance for the 15 GHz cold test structure. The theoretical curve is calculated from an HFSS model of the cold test structure.
5.6 Summary and Conclusions

We have built a scaled cold test of the single staggered cavity ladder structure, proposed in Chapter 4. The structure was successfully machined, assembled and measurements have demonstrated excellent agreement with HFSS simulations. It was shown, HFSS modeling can predict the performance of a cold test structure with high accuracy, and HFSS has decent ability to model lossy dielectrics. The success of the cold test has demonstrated that the overmoded W-band TWT, proposed in Chapter 4, would succeed if built.
Chapter 6

Conclusions and Future Direction

6.1 Conclusions

In the coming years it is clear there is a need for new vacuum electron devices capable of hundreds of watts of output power at frequencies greater than 100 GHz. It was believed that an overmoded CCTWT would be capable of fulfilling this need and enhancing the performance of CCTWTs. The computational and experimental results reported in this thesis demonstrate the possibility of developing an overmoded W-band CCTWT. We demonstrated single mode operation of an overmoded CCTWT, relying on vacuum compatible dielectric loading to suppress beam interaction with unwanted modes. The results also indicate that standard TWT analysis techniques can be used to accurately predict the performance of an overmoded CCTWT. We have shown that dielectric loading, or some other form of mode selectivity, is required to ensure a stable operation of an overmoded CCTWT. Significant work was done to accurately model the interaction structure and to verify the model through the use of several of several computer codes, including HFSS, Magic 3D, and Latte.

Progress was made in developing efficient, fast, and accurate techniques for analyzing overmoded CCTWTs. We demonstrated that the results extracted from HFSS and imported into Latte could accurately predict the outcome of Magic 3D simulations. The agreement between Magic 3D and Latte provides evidence that our HFSS modeling technique was accurately calculating dispersion and coupling impedances. Even though our method for calculating dispersion and coupling impedance with HFSS was verified by comparison to other works and a cold test, it is always important to have additional confirmation. It was shown that computer simulations can be relied upon to guide design efforts and produce good agreement with cold test structures.

In order to verify the ladder circuit manufacturing technique and the effect of dielectric loading, a scaled cold test structure of the 99 GHz design was constructed and measured. Measurements verified the results from HFSS used in designing the CCTWT
interaction structure. Measurements demonstrated that dielectric loading could be
designed to not significantly affect the operating mode, while significantly attenuating
unwanted modes; a result that agreed with HFSS. The cold test showed excellent
agreement with values of dispersion and coupling impedance from HFSS simulations.
The excellent agreement between HFSS and cold test results, prove HFSS is an accurate
tool that should be used in designing slow wave structures for overmoded TWTs.

6.2 Future Direction

The design presented in Chapter 4 looks promising, but additional efforts could be made
before producing a full-fledged dielectrically loaded overmoded TWT. There are some
potential drawbacks that will make it hard to realize the current design.

The electron gun required by the design might be difficult to procure and/or
manufacture. The design assumed the use of a 50 kV, 0.83 A electron gun, producing a
0.64 mm diameter electron beam. An electron gun with these parameters has not been
built, and it has been suggested that such high voltage, high current, and small electron
beam radius would be very challenging to engineer. There are linear beam electron guns
available at lower voltages and currents. A more practical design would require the use
of a lower electron voltage, and thus smaller cavity pitch, and a lower electron current.
Only power handling and manufacturing techniques will limit how small a pitch can be
manufactured. There are also concerns about confining a 0.83 A beam through such a
small beam tunnel. Lowering the current to 0.2 A but reduce tube gain, which this could
be countered by increasing the length of the tube. However, this lower current will reduce
the saturation power level from over 1 kW to 200 W.

The design presented was not fully optimized and there is certainly room for
improvements. While 600 MHz of bandwidth on a saturation power of over 1 kW were
achieved, there is no obvious reason the single staggered structure is not capable of a
better performance. Without the good agreement between Magic 3D and Latte it was not
very practical to attempt an optimized design. Using Magic 3D to do an optimization
study would have required months of computer time. Now that Latte can be utilized in
conjunction with HFSS an optimization study, analyzing the effects of interaction
structure geometry on overall TWT performance can be expedited.
This thesis did not take an in depth look at thermal analysis of a CCTWT. A future design should take a closer look at the thermal effects of beam interception and ohmic heating to determine the true maximal output power of an overmoded CCTWT.

Through the design process, several promising ideas were put aside to focus on creating a working design, and proving the overmoded CCTWT concept. The ideas of multiple electron beams and utilizing a double staggered ladder circuit were mentioned but not thoroughly investigated.

The use of multiple electron beams could lead to a serious increase in performance. One could put 0.6 A of current through a TM31 mode interaction structure using three 0.2 A beams. The top coupled single staggered ladder circuit has three maxima for electron beams to interact with, and the coupling slots would not interfere with multiple electrons beam. With the side coupled structure, the coupling slots would interfere with the electron beams. Since the beam tunnel radius is the ultimate limitation on current, a three beam CCTWT could be capable of up to three times the output power. It will be quite challenging to transmit and align a multiple beam electron gun.

The proposed CCTWT design used a single staggered ladder and achieved 600 MHz of hot bandwidth. The double staggered ladder circuit is known to have wider bandwidth characteristics than the single staggered ladder circuit, and this could potentially be used to create a much wider bandwidth CTWT. It is unknown how much more challenging the manufacture of a double staggered ladder structure would be.

These concepts are certainly worthy of further examination and could lead to impressive improvements in CCTWT operating performance.
References


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