Search for $D^0 - \bar{D}^0$ Mixing in $D^0 \rightarrow K^+\pi^-\pi^-\pi^+$ Decays

by

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B.S. Univ. Science and Tech. of China (2001)

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Physics

at the

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Abstract

We present results of a search for $D^0$-$D^0$ mixing by analyzing $D^0 \rightarrow K^+\pi^-\pi^-\pi^+$ decays from events in 230.4 fb$^{-1}$ $e^+e^-$ data recorded by BABAR. Assuming $CP$ conservation, we measure the time-integrated mixing rate $R_M = (0.0019^{+0.016}_{-0.015} \text{ (stat.)} \pm 0.002 \text{ (syst.)})\%$, and $R_M < 0.048\%$ at 95% confidence. Using a frequentist method, we estimate that the data are consistent with no mixing at 4.3% confidence level. We present results both with and without the assumption of $CP$ conservation.

Thesis Supervisor: Richard K. Yamamoto
Title: Professor of Physics
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Contents

1 Physics of $D^0-\bar{D}^0$ Mixing ................................. 1
  1.1 Introduction to $D^0-\bar{D}^0$ Mixing .......................... 1
  1.2 $D^0-\bar{D}^0$ Mixing in the Standard Model .................. 4
    1.2.1 Short Distance Contribution .............................. 4
    1.2.2 Long Distance Contribution ............................. 5
  1.3 Experimental Techniques to Search for $D^0-\bar{D}^0$ Mixing .. 10
    1.3.1 Hadronic Method ........................................ 10
    1.3.2 Semileptonic Method .................................... 12

2 The BABAR Facility .............................................. 15
  2.1 Silicon Vertex Tracker ....................................... 17
  2.2 Drift Chamber .............................................. 18
  2.3 Detector of Internally-Reflected Cherenkov Light ............. 21
  2.4 Electromagnetic Calorimeter ................................ 22
  2.5 Instrumented Flux Return ................................... 23
  2.6 Trigger .................................................... 24

3 Data, Monte Carlo Sample and Event Selection .................. 27
  3.1 Data and Monte Carlo Samples ............................... 27
  3.2 Preselection ................................................ 28
6.4 RS Fit Results and $D^0$ Lifetime Measurement .................................. 69
6.5 Description of WS Signal Probability Density Function .......................... 75
  6.5.1 Motivation and Definition of a Functional Form ............................... 75
  6.5.2 Validation of the WS PDF on RS Data ........................................... 77
  6.5.3 Validation of the WS PDF with High Statistics Signal MC ................. 79
  6.5.4 Validation of the WS PDF with Reweighted Signal MC .................... 80
  6.5.5 Validation of the WS PDF on MC .............................................. 81

7 Mixing Measurement .................................................................................... 83
  7.1 Standard Model Fit Results ................................................................. 83
  7.2 Upper Limit and Consistency with the No-Mixing Null Hypothesis .......... 87
  7.3 Fit Results Allowing $CP$ violation .................................................... 88
  7.4 Systematic Uncertainties ....................................................................... 90
  7.5 Summary ............................................................................................... 94
List of Figures

1-1 Standard-Model quark-level box diagrams contributing to $D^0$ mixing ........ 4
1-2 Hadron-level diagram of a long-distance contribution to mixing ............... 5
1-3 Standard Model predictions for $|x|$ (open triangles) and $|y|$ (open squares). Horizontal line references are tabulated in Table 1.1 .................. 7
1-4 New Physics predictions for $|x|$. Horizontal line references are tabulated in Table 1.2. 10
1-5 Quark-level diagrams of $D^0 \rightarrow K^+\pi^-\pi^-\pi^+$ and $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ decays . . 11
1-6 History of the search for $D^0\bar{D}^0$ mixing. ................................. 13

2-1 BABAR detector longitudinal cross section [1]. ..................................... 16
2-2 Schematic view of SVT ................................................................. 17
2-3 Schematic view of SVT: transverse section. .......................................... 18
2-4 Longitudinal section of the DCH ....................................................... 19
2-5 Schematic layout of drift cells for the four innermost superlayers ............. 20
2-6 Schematics of the DIRC fused silica radiator bar and imaging region. [1]. ..... 21
2-7 A longitudinal cross section of the EMC ............................................. 22
2-8 Overview of the IFR ........................................................................ 23

3-1 $m_{D^0}$ versus $\Delta Q$ scatter plot ................................................. 30
3-2 1D projection on $m_{D^0}$ and $\Delta Q$ for $RS$ data ............................... 30
3-3 1D projection on $m_{D^0}$ and $\Delta Q$ for $WS$ data ............................... 30
The $m_{D^0}$ and $\Delta Q$ distributions for signal MC events. .................. 33

A two-dimensional histogram of $m_{D^0}$ versus $\Delta Q$ for signal MC events. .................. 33

The distribution of the signal $\Delta Q$ resolution as a function of reconstructed $m_{D^0}$ values. ................................................. 33

1D projection in $m_{D^0}$ and $\Delta Q$ for RS signal MC events .................. 36

Illustration of two-dimensional signal PDF components .................. 36

Fit result of the fake $\pi_{\text{soft}}$ background of the RS events in the $c\bar{c}$ MC sample. .................. 37

Fit result of RS $c\bar{c}$ MC combinatoric events .................. 39

Fit result of RS $c\bar{c}$ MC fake $\pi_{\text{soft}}$ and combinatoric events .................. 39

The $m_{D^0}$ and $\Delta Q$ combinatoric background distributions from the $B\bar{B}$ and $uds$ MC sample. .................. 40

Fit result of $c\bar{c}$ MC swap background events. .................. 41

Fit result of RS MC sample. .................. 44

Fit result of RS MC sample in peak region .................. 44

Fit result of RS MC sample in sideband region .................. 45

Fit result of WS MC sample .................. 46

Fit result of WS MC sample in peak region .................. 46

Fit result of WS MC sample in sideband region .................. 47

Pull distribution of 100 samples of WS MC events. .................. 47

Fit result of RS data. .................. 50

Fit result of RS data in peak region .................. 50

Fit result of RS data sample in sideband region. .................. 51

Fit result of WS data .................. 52

Fit result of WS data in peak region .................. 52

Fit result of WS data in sideband region. .................. 53
5-14 Fit result of peaking background of $D^0 \rightarrow K^+\bar{K}^0\pi^-\bar{\pi}^+$ in WS $m(\pi^-\pi^+)$ data distribution. ........................................ 53
5-15 $m_{K\pi}$ vs $m_{\pi\pi}$ histogram .......................................................... 56
5-16 $\cos(K\pi)$ vs $\cos(\pi\pi)$ and $\cos(K\pi)$ vs $\phi$. .......................... 56
5-17 Projection plots for background-subtracted data in $m(K^-\pi^+)$. .............. 57
5-18 Projection plots for background-subtracted data in $m(\pi^-\pi^+)$. .............. 57
5-19 Projection plots for background-subtracted data in $m(\pi^-\pi^+\pi^+)$. .......... 58
5-20 Projection plots for background-subtracted data in $m(K^-\pi^+\pi^+)$. .......... 58
5-21 Projection plots for background-subtracted data in $m(K^-\pi^-\pi^+)$. .......... 59
5-22 Fit to peaking background of $D^0 \rightarrow K^+\bar{K}^0\pi^-\bar{\pi}^+$ on $m(\pi^-\pi^+)$ efficiency-corrected background-subtracted sWeights plots of WS data. ........................................ 59
5-23 $R_{WS}$ versus number of bins used in the efficiency calculation. ............... 61

6-1 Measured $D^0$ decay-time uncertainty distributions .................................. 64
6-2 Projection of the maximum likelihood fit onto $t_{K\pi\pi\pi}$ for the RS truth matched signal MC ................................................................. 66
6-3 Projection of the maximum likelihood fit onto $t_{K\pi\pi\pi}$ for MC events belonging to the combinatoric background category. ........................................ 67
6-4 Projection of the maximum likelihood fit onto $t_{K\pi\pi\pi}$ for MC events belonging to the fake $\pi_{\text{soft}}$ background category. ........................................ 68
6-5 Projected RS fit to MC onto $t_{K\pi\pi\pi}$ in all and signal regions .................. 70
6-6 Projected RS fit to MC onto $t_{K\pi\pi\pi}$ in signal regions ............................... 71
6-7 Projected RS fit to MC onto $t_{K\pi\pi\pi}$ in sideband regions of both $m_{K\pi\pi\pi}$ and $\Delta Q$ .......................... 71
6-8 Projected RS fit to MC onto $t_{K\pi\pi\pi}$ in sideband region of $\Delta Q$ in the $m_{K\pi\pi\pi}$ signal region ................................................................. 72
6-9 Projected RS fit to data onto $t_{K\pi\pi\pi}$ in all and signal regions .................. 73
6-10 Projected RS fit to data onto $t_{K\pi\pi\pi}$ in signal regions ........................... 73
6-11 Projected RS fit to data onto $t_{K\pi\pi}$ in sideband regions of both $m_{K\pi\pi}$ and $\Delta Q$. 74

6-12 Projected RS fit to data onto $t_{K\pi\pi}$ in sideband region of $\Delta Q$ in the $m_{K\pi\pi}$ signal region. 74

6-13 Lifetime projection plots for RS event with ws PDF. 77

6-14 Contour plots on $a_1$ and $a_3$ for RS data fitted with WS lifetime signal PDF. 78

6-15 Contour plots on $a_2$ and $a_3$ for RS data fitted with WS lifetime signal PDF. 78

6-16 Lifetime projection plots for WS MC events. 81

6-17 Projected RS fit to MC onto $t_{K\pi\pi}$ in signal regions. 82

6-18 Projected RS fit to MC onto $t_{K\pi\pi}$ in sideband regions of both $m_{K\pi\pi}$ and $\Delta Q$. 82

7-1 Projected WS fit to data onto $t_{K\pi\pi}$ in all and signal regions. 84

7-2 Projected WS fit to data onto $t_{K\pi\pi}$ in signal regions. 84

7-3 Projected WS fit to data onto $t_{K\pi\pi}$ in sideband regions of both $m_{K\pi\pi}$ and $\Delta Q$. 85

7-4 Contours of $\Delta \log L$ levels. 86

7-5 $\Delta \log L$ as a function of $(x^2 + y^2)/2$. 86

7-6 Contours of $\Delta \log L$ levels for $D^0$ and $\bar{D}^0$ separately. 89
List of Tables

1.1 Theoretical predictions for mixing parameters (Standard Model). The notation “±” indicates the range of predictions. ...................................................... 8
1.2 Theoretical predictions for mixing parameters (New Physics). The notation ± indicates the range of predictions based on the model parameter space bounded by the data available at the time of publication. [2] ............................................. 9
2.1 PEP-II beam parameters. HER: High Energy Ring. LER: Low Energy Ring. .... 16
3.1 Summary of the data and MC samples used in this analysis. ................................. 28
6.1 Decay-time uncertainty scale factors from the likelihood fit ..................................... 69
6.2 Decay-time uncertainty scale factors from the likelihood fit ..................................... 69
6.3 Fitted $\alpha y'$ and $(x^2 + y^2)/2$ values for RS events with WS lifetime signal PDF ... 77
6.4 generated and fitted mixing values for WS signal events .......................................... 79
6.5 generated and fitted mixing values for signal WS events .......................................... 80
6.6 Fitted $a_2$ and $a_3$ values for MC WS events. ...................................................... 81
7.1 Fitted values of mixing parameters ........................................................................... 83
7.2 Values of physical quantities, determined by the fit .................................................. 83
7.3 Mixing-rate systematic uncertainties ....................................................................... 92
7.4 $\alpha y'$ systematic uncertainties .............................................................................. 92
7.5 Mixing-rate systematic uncertainties (CPV) ............................................................ 93
7.6  \( \alpha \tilde{y}' \cos \tilde{\phi} \) systematic uncertainties (CPV) ........................................ 93
7.7  \( \beta \tilde{x}' \sin \tilde{\phi} \) systematic uncertainties (CPV) ........................................ 94
7.8  \(|p/q| \) systematic uncertainties (CPV) ................................................................. 94
Chapter 1

Physics of $D^0-\bar{D}^0$ Mixing

1.1 Introduction to $D^0-\bar{D}^0$ Mixing

Mixing in the neutral meson system is the quantum-mechanical oscillation between a neutral meson and its corresponding anti-meson as a function of time. Historically, mixing was first observed in the $K^0-\bar{K}^0$ system, and later in the $B^0-\bar{B}^0$ ($B^0_s-\bar{B}^0_s$) systems, while it has not yet been observed in $D^0-\bar{D}^0$. Mixing phenomena are of great interest because the amplitudes are sensitive to weak quark couplings. $K^0-\bar{K}^0$ mixing has been studied experimentally in great detail. Gaillard and Lee [3] estimated the mass of the charm quark before its discovery, based on the calculations of the mass difference involved in $K^0-\bar{K}^0$ mixing. Moreover, the observation of abundant $B^0-\bar{B}^0$ mixing gave the first indication of a large top quark mass.

$D^0$ and $\bar{D}^0$ mesons produced in strong and electromagnetic interactions are flavor eigenstates with a well-defined quark content. The production Hamiltonian is of the form $\mathcal{H}_0 = \mathcal{H}_{\text{strong}} + \mathcal{H}_{\text{em}}$. Its eigenstates are the flavor or interaction eigenstates, $|D^0\rangle$ and $|\bar{D}^0\rangle$. $D^0$ and $\bar{D}^0$ mesons are observed by way of their decays, which are governed by the weak interaction. The states of the time evolution Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{weak}}$ are the mass or decay eigenstates $|D_1\rangle$ and $|D_2\rangle$ with the corresponding eigenvalues $m_1, m_2$ and $\Gamma_1, \Gamma_2$. They are the physically-observable states and obey an exponential decay law characterized by:
\[ |D_1(t)\rangle = |D_1(0)\rangle e^{-im_1t}e^{-(\Gamma_1/2)t}. \]  
\hspace{2cm} (1.1.1)

\[ |D_2(t)\rangle = |D_2(0)\rangle e^{-im_2t}e^{-(\Gamma_2/2)t}. \]  
\hspace{2cm} (1.1.2)

The flavor eigenstates \( |D^0\rangle \) and \( |\bar{D}^0\rangle \) are not eigenstates of the weak interaction. The time evolution of this system is described by the Schrödinger equation

\[
i \frac{\partial}{\partial t} \begin{pmatrix} |D^0(t)\rangle \\ |\bar{D}^0(t)\rangle \end{pmatrix} = \mathcal{H}_{\text{weak}} \begin{pmatrix} |D^0(t)\rangle \\ |\bar{D}^0(t)\rangle \end{pmatrix}, \quad (1.1.3)
\]

where

\[
\mathcal{H}_{\text{weak}} = \begin{pmatrix} M - \frac{i}{2} \Gamma \\ -M - \frac{i}{2} \Gamma \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}, \quad (1.1.4)
\]

\[
H_{11} = \langle D^0|M|D^0 \rangle - \frac{i}{2} \langle D^0|\Gamma|\bar{D}^0 \rangle = M_{11} - \frac{i}{2} \Gamma_{11}, \quad (1.1.5)
\]

\[
H_{12} = \langle D^0|M|\bar{D}^0 \rangle - \frac{i}{2} \langle D^0|\Gamma|D^0 \rangle = M_{12} - \frac{i}{2} \Gamma_{12}, \quad (1.1.6)
\]

\[
H_{21} = \langle \bar{D}^0|M|D^0 \rangle - \frac{i}{2} \langle \bar{D}^0|\Gamma|\bar{D}^0 \rangle = M_{21} - \frac{i}{2} \Gamma_{21}, \quad (1.1.7)
\]

\[
H_{22} = \langle \bar{D}^0|M|\bar{D}^0 \rangle - \frac{i}{2} \langle \bar{D}^0|\Gamma|D^0 \rangle = M_{22} - \frac{i}{2} \Gamma_{22}, \quad (1.1.8)
\]

where \( M \) and \( \Gamma \) are Hermitian matrices describing mixing and decay, respectively. CPT invariance requires \( M_{11} = M_{22} = M \) and \( \Gamma_{11} = \Gamma_{22} = \Gamma \). The mass eigenstates are given in terms of the flavor eigenstates,

\[
|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle, \quad (1.1.9)
\]
\[ |D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle, \]  

(1.1.10)

where \(p\) and \(q\) are complex. The corresponding eigenvalues are:

\[ \lambda_1 = m_1 - \frac{i}{2} \Gamma_1 = (M - \frac{i}{2} \Gamma) + \frac{q}{p} (M_{12} - \frac{i}{2} \Gamma_{12}) \]  

(1.1.11)

\[ \lambda_2 = m_2 - \frac{i}{2} \Gamma_2 = (M - \frac{i}{2} \Gamma) - \frac{q}{p} (M_{12} - \frac{i}{2} \Gamma_{12}) \]  

(1.1.12)

and

\[ \left( \frac{q}{p} \right)^2 = \frac{M_{12}^2 - \frac{i}{2} \Gamma_{12}^*}{M_{12}^2 - \frac{i}{2} \Gamma_{12}} \]  

(1.1.13)

If \(CP\) is conserved in this system, then \(p = q\). \((m_1, m_2)\) are the masses and \((\Gamma_1, \Gamma_2)\) are the decay widths of the two mass eigenstates \(|D_1\rangle, |D_2\rangle\) generated by the mixing dynamics. We define quantities \(x\) and \(y\) in terms of the differences in masses and widths, which are used in parameterizing the mixing process:

\[ x = 2 \frac{m_2 - m_1}{\Gamma_2 + \Gamma_1}, \quad y = \frac{\Gamma_2 - \Gamma_1}{\Gamma_2 + \Gamma_1}. \]  

(1.1.14)

The time-integrated probability of a \(D^0\) or \(\bar{D}^0\) to oscillate to \(\bar{D}^0\) or \(D^0\) first before decaying compared to decaying directly is described by the mixing rate \(R_M\). In the absence of \(CP\) violation, \(R_M\) is identical for \(D^0\) and \(\bar{D}^0\). The following expression then holds for the decay of a \(D^0\) into a final state \(f\) [4]:

\[ R_M = \frac{\mathcal{B}(D^0 \rightarrow \bar{D}^0 \rightarrow \bar{f})}{\mathcal{B}(D^0 \rightarrow f)} = \frac{(x^2 + y^2)/(2 + x^2 - y^2)}{x^2 + y^2}/2. \]  

(1.1.15)
1.2 \( D^0\xrightarrow{}\bar{D}^0 \) Mixing in the Standard Model

In the Standard Model, the mixing rate in the neutral \( D^0 \) system is expected to be very small. Due to large uncertainties in the calculation of long-distance effects, it is very difficult to estimate an upper limit for \( R_M \). Theoretical calculations within the Standard Model show that the range of possible \( R_M \) upper limits vary between \( 10^{-7} \sim 10^{-4} \).

1.2.1 Short Distance Contribution

In the standard model, the \( \Delta C = 2 \) flavor-changing neutral currents (FCNC) can occur via a box diagram as shown in Figure 1-1. The \( D^0\xrightarrow{}\bar{D}^0 \) system has an unique feature that because \( c \) and \( \bar{c} \) quarks are up-type quarks, the mixing occurs via intermediate states of down-type quarks, while in the \( B^0\xrightarrow{}\bar{B}^0 \) or \( K^0\xrightarrow{}\bar{K}^0 \) systems, the mixing proceeds via up-type intermediate state quarks. Therefore, \( D \) mixing is sensitive to the contributions of virtual down-type quarks coupled to the \( W \) boson, which might reveal physics not seen in the \( B \) or \( K \) systems. In addition, in \( D^0\xrightarrow{}\bar{D}^0 \) mixing, there is no contribution from intermediate top quarks in the box diagram. The GIM cancellation [5] produces a suppression factor that depends on the heaviest quark in the loop relative to the \( W \) boson mass. In the \( D^0\xrightarrow{}\bar{D}^0 \) system, this factor is proportional to \( m_s^2/m_W^2 \), whereas it is proportional to \( \sim m_t^2/m_W^2 (\sim m_b^2/m_W^2) \) for the \( B^0\xrightarrow{}\bar{B}^0 \) \( (K^0\xrightarrow{}\bar{K}^0) \) systems. Due to the smallness of \( V_{ub} \), the contribution from the heaviest quark \( b \)-quark in the \( D \)-mixing box diagram, \( \times[(m_b^2-m_s^2)^2/m_W^2m_c^2] \) [6] can be neglected since the suppression from Cabibbo-Kobayashi-Maskawa (CKM) [7] mixing...
factor $V_{ub}$ outweighs the potential contribution of the $b$-quark in the box. The contribution of the box diagram to the mass difference is [8]

$$\Delta m_D^{\text{box}} \simeq 1.4 \times 10^{-18} \text{ GeV} \left(\frac{m_s}{0.1 \text{ GeV}}\right)^4 \left(\frac{f_D}{0.2 \text{ GeV}}\right)^2.$$  \hspace{1cm} (1.2.16)

Using typical values of $f_D$ and $m_s$, the short-distance contribution to $x$ in the $D$ mixing box diagram $x_{\text{box}}$ is estimated to be

$$x_{\text{box}} \approx \mathcal{O}(10^{-6}) - \mathcal{O}(10^{-5}).$$ \hspace{1cm} (1.2.17)

### 1.2.2 Long Distance Contribution

![Hadron-level diagram for a long-distance physics contribution to $D^0$-$\bar{D}^0$ mixing](image)

**Figure 1-2:** Hadron-level diagram for a long-distance physics contribution to $D^0$-$\bar{D}^0$ mixing

In addition to the box diagram, it is also possible to produce flavor-changing transitions to specific mesonic intermediate states from weak interaction processes. For example, Figure 1-2 illustrates a contribution to $D$ mixing from transitions to two pseudoscalars ($K^+K^-$, $K^+\pi^-$, $K^-\pi^+$, $\pi^+\pi^-$), which are accessible to both $|D^0\rangle$ and $|\bar{D}^0\rangle$.

Recall that $D^0$-$\bar{D}^0$ mixing is an effect of SU(3) breaking [9, 10].

$$x, y \approx \sin^2 \theta_c \times [\text{SU}(3) \text{ breaking}]^2.$$ \hspace{1cm} (1.2.18)

Assuming the SU(3) breaking is perturbative, Falk, etc. [11] has shown that it is at most a
second-order effect:

\[ x, y \approx \sin^2 \theta_c \times \left( \frac{m_s}{\Lambda_{\text{hadr}}} \right)^2 \lesssim \mathcal{O}(10^{-3}), \quad (1.2.19) \]

where \( \Lambda_{\text{hadr}} \approx \mathcal{O}(1) \) GeV is a typical hadronic scale. Beyond this naive estimate, there are two basic approaches in estimating the long-distance contributions to mixing. One is the heavy-quark effective theory (HQET) inclusive approach using an operator product expansion (OPE). The other is an exclusive approach that sums over the contributions from all intermediate hadronic states, into which both \( D^0 \) and \( \bar{D}^0 \) can decay. Neither approach is perfect, and the resulting estimates of \( y \) can differ by an order of magnitude. The inclusive approach was first developed by Georgi [12] and later extended by others [13, 14]. The theory rests on the assumption that the mass of the \( c \)-quark is large compared with the typical strong interaction scale: \( m_c \gg \Lambda_{\text{hadr}} \). The result of this type of approach is summarized as following [14]:

\[ x \approx y \approx \mathcal{O}(10^{-3}). \quad (1.2.20) \]

The exclusive approach takes the contributions from all of the known hadronic states common to both \( |D^0\rangle \) and \( |\bar{D}^0\rangle \) decay. For example, one of such sets can be formed by the two charged pseudoscalar intermediate states \( \{\pi^+\pi^-, \pi^+K^-, K^+K^-, K^+\pi^-\} \) as shown in Fig 1-2.

The contributions to \( x \) from exclusive approach are not required to be on-shell, so in this case there is no symmetry breaking caused by limited phase space. If one assumes that all of the sets contribute incoherently in roughly the same amount, one concludes that [8]

\[ x \lesssim \mathcal{O}(10^{-3}). \quad (1.2.21) \]

with large uncertainty.

The estimation of the long-distance contributions requires two assumptions: phase space is the
only source of SU(3) breaking, and that the charm quark is heavy enough. Falk concludes that [11]

\[ y \lesssim O(10^{-2}). \]  

(1.2.22)

with large uncertainties.

Figure 1-3 and Fig 1-4 shows theoretical predictions from various extensions to the Standard Model as well as a number of Non-Standard Model calculations, which has been compiled by Alexey Petrov [2].

![Standard Model mixing predictions](image)

**Figure 1-3:** Standard Model predictions for $|x|$ (open triangles) and $|y|$ (open squares). Horizontal line references are tabulated in Table 1.1.
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<tr>
<th>Mass difference $x$</th>
<th>Reference Index</th>
<th>Citation</th>
</tr>
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<td>$0.9 \pm 3.7) \times 10^{-4}$</td>
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<td>Phys. Rev. D 26, 143 (1982)</td>
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<td>$(1.44 \pm 0.79) \times 10^{-6}$</td>
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<td>Z. Phys. C 27, 515 (1985)</td>
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<td>Phys. Lett. B 164, 170 (1985)</td>
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<td>$6.3 \times 10^{-4}$</td>
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<td>$(6.0 \pm 1.4) \times 10^{-3}$</td>
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<td>Phys. Rev. D 26, 143 (1982)</td>
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<td>Z. Phys. C 27, 515 (1985)</td>
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Table 1.1: Theoretical predictions for mixing parameters (Standard Model). The notation “±” indicates the range of predictions.
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<tr>
<td>$0.001 - 0.05$</td>
<td>31</td>
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</table>

Citation: Yad. Phys. 34, 435 (1981)


hep-ph/9409379

hep-ph/9409379

hep-ph/9409379

hep-ph/9508349


hep-ph/9704316


hep-ph/0110106

Table 1.2: Theoretical predictions for mixing parameters (New Physics). The notation ± indicates the range of predictions based on the model parameter space bounded by the data available at the time of publication. [2]
1.3 Experimental Techniques to Search for $D^0$-$\bar{D}^0$ Mixing

1.3.1 Hadronic Method

The hadronic method is to search for $D^0$-$\bar{D}^0$ mixing by studying wrong sign (WS) $D^0 \rightarrow K^+\pi^-(X)$ and right sign (RS) $D^0 \rightarrow K^-\pi^+(X)$ decays. WS decays can occur in two ways. 1) $D^0 \rightarrow \bar{D}^0$ by mixing first, then followed by a Cabibbo-favored (CF) decay $\bar{D}^0 \rightarrow K^+\pi^-(X)$. 2) from doubly Cabibbo-suppressed decay (DCSD) $D^0 \rightarrow K^+\pi^-(X)$. The difficulty in this method is the need to distinguish between DCSD and mixing in the entire wrong sign decay sample. There are three different ways to distinguish experimentally between DCSD and mixing decays:

1. Use the difference in the decay-time dependence between mixing and DCSD events. This method requires a highly-boosted $D^0$ and good vertex reconstruction.
2. Use the possible difference in the resonant substructure between DCSD and mixing events in \( D^0 \to K^+\pi^-\pi^0, D^0 \to K^+\pi^-\pi^+\pi^-, \) etc. modes. The Feynman diagrams of CF and DCSD decay of \( D^0 \to K^+\pi^-\pi^+\pi^- \) are shown in Fig 1-5. This method needs large statistics and good understanding of the Dalitz structure of multi-body \( D^0 \) decays.

3. Use the quantum coherence of \( D^0-\bar{D}^0 \) production at the \( \psi(3770) \) and their decays. This method requires \( e^+e^- \) annihilation in \( \psi(3770) \) threshold region. The particular advantage in this method compared with the hadronic methods 1 and 2 is that the mixing parameter \( y' \), which is related to an unkown strong phase between CF and DCSD decay can be determined by taking advantage of the quantum coherence [15].

\[
\begin{align*}
\text{Figure 1-5: Quark-level Feynman diagrams of the CF } D^0 &\to K^{-}\pi^{+}\pi^{+}\pi^{-} \text{ decay (left) and the DCSD: } D^0 &\to K^{+}\pi^{-}\pi^{-}\pi^{+} \text{ (right).}
\end{align*}
\]

The current best limit on \( D \) mixing using hadronic methods is an analysis of the decay-time distribution of \( D^0 \to K^+\pi^- \) decays. \textsc{babar} [16], Belle [17], and CLEO [18] have all performed such analyses. The advantages of this analysis strategy compared with multi-body decay channels is the clean signal from two body decays, which eliminates many systematic uncertainties in the time dependent maximum likelihood fit. A disadvantage is that there is no sensitivity to \( x \) or \( y \) individually, but only to \( x^2 + y^2 \) and \( y' \).
1.3.2 Semileptonic Method

The semi-leptonic method searches for $D^0-\bar{D}^0$ mixing by using $D^0 \rightarrow \bar{D}^0 \rightarrow X l^- \nu$ decays. One advantage of this method is DCSD is not involved. However, the missing neutrino in the decay causes this method to suffer from a large background and inferior vertex reconstruction compared with the hadronic methods. The best way to use the semi-leptonic method is using $D^0-\bar{D}^0$ production in $e^+e^-$ annihilation in the $\psi(3770)$ threshold region. This involves:

1. Searching for $e^+e^- \rightarrow \Psi'' \rightarrow D^0\bar{D}^0 \rightarrow (K^-l^+\nu)(K^-l^+\nu)$ or

2. Searching for $e^+e^- \rightarrow D^-D^{*+} \rightarrow (K^+\pi^-\pi^-)(K^+l^-\nu)\pi^+_\pi^{-}$ [19]. There is only one neutrino missing in the entire event and the threshold kinematic constraints provide a clean signal. Thus this decay channel compared to 1 might be the only one that does not suffer from large backgrounds.

E791 [20], BABAR [21] and CLEO [22] have used the semileptonic method to search for $D$ mixing. Experimental results searching for $D$ mixing (both hadronic and semileptonic) prior to B Factory are summarized in the Fig 1-6 [22].
Figure 1-6: The history of the search for $D^0\bar{D}^0$ mixing [22]. Note that the range in E691 result reflects the possible effects of interference between DCSD and mixing, and the CLEO II signal could be due to either mixing or DCSD, or a combination of the two.
Chapter 2

The $\text{BABAR}$ Facility

The data used in this analysis were collected with $\text{BABAR}$ [1], a particle detector operating with the PEP-II asymmetric storage rings at the Stanford Linear Accelerator Center (SLAC). The primary goal of the $\text{BABAR}$ experiment is the systematic study of $CP$ asymmetries in the decays of $B$ mesons. The charm pair production cross section is 1.3 nb, while the $b$ pair production cross section is 1.1 nb. Thus, PEP-II is also a charm factory as well as a $B$-factory.

PEP-II is an $e^+e^-$ storage ring system operating at a center of mass energy of 10.58 GeV, corresponding to the mass of the $\Upsilon(4S)$ resonance. An electron beam of 9.0 GeV and a positron beam of 3.1 GeV collide at PEP-II. Table 2 shows the operating parameters of these asymmetric energy storage rings. The $\text{BABAR}$ detector has several component systems. Figure 2-1 shows a longitudinal section of the detector center. The inner detector consists of a silicon vertex tracker, a drift chamber, a ring-imaging Cherenkov detector, and a cesium iodide (CsI) calorimeter. These detector components are surrounded by a superconducting solenoid providing a magnetic field of 1.5 T. The steel flux return is instrumented for muon and neutral hadron detection.
Figure 2-1: BABAR detector longitudinal cross section [1].

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<tr>
<th>Parameters</th>
<th>Design</th>
<th>Typical</th>
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<td>9.0/3.1</td>
</tr>
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</tr>
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<tr>
<td>Luminosity (pb$^{-1}$/d)</td>
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</table>

Table 2.1: PEP-II beam parameters. HER: High Energy Ring. LER: Low Energy Ring.
2.1 Silicon Vertex Tracker

The Silicon Vertex Tracker (SVT) is the innermost subsystem of BaBar detector, which provides precise decay vertices and tracking information of the reconstructed charged particles near the interaction region. The side and transverse views of SVT are shown in Figures 2-2 and 2-3. The SVT consists of five concentric cylindrical layers of double-sided silicon detectors, as shown in Fig 2-3. The inner three layers have six detector modules and a barrel-style structure. The inner layers have strips ($z$ strips) oriented perpendicularly to the beam direction to measure the $z$ coordinate. The outer two layers consist of 16 and 18 detector modules, respectively. The modules of out layers have longitudinal strips ($\phi$ strips), which is designed for the $\phi$ coordinate measurement.

The total active silicon area is $0.96 \text{m}^2$, which provides a geometrical acceptance of 90% of the solid angle in the center of mass system. The material traversed by particles is $\sim 4\%$ of a radiation length.
2.2 Drift Chamber

The drift chamber (DCH) is the primary tracking detector. The DCH detects charged particles by collecting electrons produced when they ionize gas molecules in the tracking wires. DCH provides the detection of charged particles and the measurement of their momenta and angles with high resolution. For example, the long-lived particles, such as $K^0_S$, which decays outside of SVT, whose reconstruction is solely dependent on the DCH. Furthermore, the DCH provides good particle identification for low-momentum charged particles by measurement the energy loss, $dE/dx$ due to the ionization. A resolution of about 7% allows for $\pi/K$ separation up to 700 MeV/c.

A schematic side-view of DCH is shown in Fig. 2-4. The DCH is about 3 m long and consists of a total of 7104 small drift cells, which are arranged in 40 cylindrical layers. The 40 layers are arranged as a total of ten superlayers, each consisting of four layers. For particles with transverse momentum greater than 180 MeV/c, this provides up to 40 spatial and ionization loss measurements. The chamber is filled with 80:20 mixture of helium:isobutane, which has a radiation length.
that is five times larger than commonly used argon-based gases. This gas mixture provides a very stable ion drift velocity of 22 $\mu$m/ns. The wires in 24 of the 40 layers are designed to have small angles with respect to the $z$-axis, which provides to obtain the longitudinal position information.

![Diagram of DCH](image)

**Figure 2-4:** Longitudinal section of the DCH with principal dimensions; the chamber center is offset by 370 mm from the interaction point. [1].

The drift cell, illustrated in Fig. 2-5, are hexagonal in shape. Each drift cell consists of one sense wire surrounded by six field wires. The sense wires are made of tungsten-rhenium, while all other wires are made from aluminum. All the wires are plated with gold. A positive high voltage is applied to the sense wires and the field wires are at ground potential. The sense wires collect the electrons from the ionized gas molecules, while field wires shape the electric field around the sense wire. The typical operating voltage of 1960V and an avalanche gain of approximately $5 \times 10^4$ is obtained.
Figure 2-5: Schematic layout of drift cells for the four innermost superlayers. Lines have been added between field wires to aid in visualization of the cell boundaries. The numbers on the right side give the stereo angle of sense wires in each layer. The 1 mm-thick beryllium inner wall is shown inside of the first layer. [1].
2.3 Detector of Internally-Reflected Cherenkov Light

A new type of ring-imaging Cherenkov detector called the DIRC is used by BABAR to provide the particle identification over a wide momentum range, in a small radial dimension and with high background tolerance.

The principle of particle identification achieved in the DIRC is that particles exceeding the speed of light in a medium will emit Cherenkov radiation at a characteristic angle. This angle with respect to the momentum of the particle is well-defined, \( \theta_c = \cos^{-1}(1/\beta_n) \), where \( n \) is the refraction index of the radiator medium. The radiator material of the DIRC is fused silica. The principle is illustrated in Fig. 2-6.

![Diagram of the DIRC fused silica radiator bar and imaging region.](image)

**Figure 2-6:** Schematics of the DIRC fused silica radiator bar and imaging region. [1].

For the particles with \( \beta \approx 1 \), depending on the particle incident angle, some photons will always stay within the total internal reflection limit. Thus these photons can be transported to either one or both ends of the bar. In order to avoid the placement of photon detectors in the forward end, a mirror is placed at the forward end, which is perpendicular to the bar axis, to reflect
incident photons backward. Most of them emerge into a water-filled expansion region, called a standoff box when they arrive at the instrumented end. An array of densely packed photomultiplier tubes (PMTs) is used to detect the photons, each surrounded by reflecting cones to capture light.

2.4 Electromagnetic Calorimeter

The electromagnetic calorimeter (EMC) is a cesium-iodide crystal calorimeter and functions by measuring the energy deposited from incident particle. The EMC provides an excellent energy and angular resolution over the energy range from 20 MeV to 9 GeV. The EMC is a hermetic, total-absorption calorimeter, consisting of a finely segmented array of thallium-doped cesium iodide crystals. The silicon photodiodes are used to read out the scintillation light from cesium iodide crystals. It has full coverage in azimuth and extends in polar angle from 15.8° to 141.8° with a solid-angle coverage of 90% in the center of mass system.

Figure 2-7: A longitudinal cross section of the EMC (only the top half is shown) indicating the arrangement of the 56 crystal rings. The detector is axially symmetric around the z-axis. All dimensions are given in mm. [1].

A longitudinal cross section of the EMC is shown in Fig. 2-7. The EMC consists of a cylindrical
barrel and a conical forward endcap as shown in Fig. 2-7. The crystals have a tapered trapezoidal cross section to limit the effects of shower leakage resulting from high energy particles.

2.5 Instrumented Flux Return

The Instrumented Flux Return (IFR) is the outermost Babar detector system. It consists of multi-layers of steel and resistive plate chambers (RPC) and provides muon and neutral hadrons identification over a wide range of momenta and angles. The steel flux return of the magnet is chosen as a muon filter and hadron absorber. Single gap RPC with two-coordinate readout functions as detectors by detecting streamers from ionizing particles via capacitive readout strips. Figure 2-8 shows the overview of the IFR.

**Figure 2-8:** Overview of the IFR: Barrel sectors and forward and backward end doors; the shape of the RPC modules and their dimensions are indicated. [1].

There are several advantages of the RPC:

- Simple, low cost construction and the possibility of covering odd shapes with minimal dead
• Large signals and fast response allowing for simple and robust front-end electronics and good time resolution, typically 1-2 ns.

The IFR detectors cover a total active area of about 2000 m². There are a total of 806 RPC modules, 57 in each of the six barrel sectors, 108 in each of the four half end doors, and 32 in the two cylindrical layers.

## 2.6 Trigger

The trigger system is designed to select events of interest while rejecting background events efficiently. The beam-induced background rates are typically about 20 kHz at design luminosity while the rates for the primary physics processes are generally much lower than that, at a few hertz level. The total trigger efficiency exceeds 99% for all $B\bar{B}$ events and at least 95% for continuum events.

The trigger is implemented with two stages. The Level 1 trigger (L1) is performed in hardware using information collected from the front-end electronics. The Level 3 trigger (L3) is performed in software.

• The Level 1 trigger system is a hardware-based trigger system, which uses the information from three subsystems: DCH, EMC and IFR to make decision. The L1 trigger has an output rate of typically 1kHz during normal operation. The trigger data are processed by three specialized hardware processors. The drift chamber trigger (DCT) uses all 7104 DCH channels to assemble clusters of cell hits into track segments. The DCT then assembles track segments into tracks and check these tracks against preset transverse momentum thresholds.

The electromagnetic trigger (EMT) divides the EMC’s crystals into 280 towers each containing $7 \times 40 (\theta \times \phi)$ crystals. The EMC Calorimeter Trigger Processor Boards convert the tower information into $\phi$ coordinate-based maps in the final trigger decision. The in-
The Level 3 trigger is a software-based trigger, which receives the output from the L1 trigger, performs a full event reconstruction and classification. The trigger is performed in three stages. In first stage, events are classified according to the information collected from the Fast Control and Timing System (FCTS). In the second stage, event reconstruction algorithms are applied. The third stage creates the Level 3 output information, a set of classification information for each event.
Chapter 3

Data, Monte Carlo Sample and Event Selection

3.1 Data and Monte Carlo Samples

The analysis is based on a total luminosity of $228.8 \text{ fb}^{-1}$ collected from 1999 to 2004, which includes $207.3 \text{ fb}^{-1}$ on-resonance and $21.5 \text{ fb}^{-1}$ off-resonance data.

A Monte Carlo (MC) sample is used for this analysis. We generated the following decay modes with a flat phase space model for signal MC:

- $D^{*+} \rightarrow D^{0}\pi^{+}, D^{0} \rightarrow K^{-}\pi^{+}\pi^{-}\pi^{+}$,
- $D^{*-} \rightarrow \bar{D}^{0}\pi^{-}, \bar{D}^{0} \rightarrow K^{+}\pi^{-}\pi^{+}\pi^{-}$,
- $D^{*+} \rightarrow D^{0}\pi^{+}, D^{0} \rightarrow K^{+}\pi^{-}\pi^{+}\pi^{-}$,
- $D^{*-} \rightarrow \bar{D}^{0}\pi^{-}, \bar{D}^{0} \rightarrow K^{-}\pi^{+}\pi^{-}\pi^{+}$.

A large amount of MC is also used to understand the combinatoric background of $D^{0} \rightarrow K^{-}\pi^{+}\pi^{-}\pi^{+}$ and $D^{0} \rightarrow K^{+}\pi^{-}\pi^{+}\pi^{-}$ decays. The data and MC sample statistics are summarized in Table 3.1.
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<td>MC</td>
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<td>MC</td>
<td>$u\bar{u}, d\bar{d}, s\bar{s}$</td>
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<td>198.6</td>
</tr>
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</table>

Table 3.1: Summary of the data and MC samples used in this analysis.

3.2 Preselection

A $D^0$ candidate is reconstructed with 2 pairs of tracks with opposite charges, which are required to fit to a common vertex with $\chi^2$ probability cuts. The $D^0$ candidate is then combined with another charged track to form a $D^{*+}$ candidate, which are fitted to the interaction point (IP) with a beam spot constraint applied. At the end, the $D^{*+}$ candidates are retained if they satisfy the following loose selection criteria:

1. The kaon track from the $D^0$ decay is required to satisfy a stringent kaon identification cut.

2. One pion track from the $D^0$ decay is required to satisfy a stringent pion identification cut, the other two are required to satisfy a less stringent cut.

3. The soft pion from the $D^{*+}$ decay is required to be charged and to satisfy minimal quality cuts.

4. The $\chi^2$ probabilities of the vertex fits of the $D^0$ and the $D^{*+}$ must be greater than 0.5%.

5. We require that $1.70 \text{ GeV}/c^2 \leq m_{D^0} \leq 2.0 \text{ GeV}/c^2$, where $m_{D^0}$ is the mass of the $D^0$ candidate.

6. The momentum of the $D^0$ in the $e^+e^-$ center of mass frame, $p^*(D^0)$, must be greater than 2.2 GeV/c. This cut eliminates the background from $B^0$ decays; it is also used to reduce the
combinatoric background that tends to peak at lower momenta.

7. We require that $0.139 \text{ GeV}/c^2 \leq m(D^0\pi^+) - m_{D^0} \leq 0.160 \text{ GeV}/c^2$.

### 3.3 Analysis Selection

The full decay chains of each $D^{*+}$ candidate satisfying the preselection criteria are refitted with a beamspot constraint. They are also required to satisfy the following selection criteria:

1. All tracks must have at least 6 SVT hits to ensure good vertex resolution.

2. All tracks must have at least two $\phi$ hits and two $z$ hits in the SVT to ensure good vertex resolution, where $\phi$ and $z$ are defined in section 2.1.

3. All tracks must have at least one $\phi$ and one $z$ hit in the three inner layers of the SVT to ensure good vertex resolution.

4. The $\chi^2$ probabilities of the vertex fits of the $D^0$ and the $D^{*+}$ must be greater than 1%.

5. The momentum of the $D^0$ in the $e^+e^-$ center of mass frame must be greater than 2.4 GeV/c.

6. We require that $1.8145 \text{ GeV}/c^2 \leq m_{D^0} \leq 1.9145 \text{ GeV}/c^2$.

7. We require that $0.139 \text{ GeV}/c^2 \leq m(D^0\pi^+) - m_{D^0} \leq 0.155 \text{ GeV}/c^2$.

We perform an unbinned extended maximum likelihood fit to the two-dimensional $m_{D^0}$ and $\Delta Q$ distributions to extract the reconstructed $D^0$ signal yield.

- $m_{D^0}$: the invariant mass of the $K^\pm\pi^+\pi^-\pi^\mp$ combinations.

- $\Delta Q = m_{D^{*+}} - m_{D^0} - m_\pi$: the mass difference between the $D^{*+}$ candidate and the combination of the $D^0$ candidate and the soft pion resulting from the $D^{*+}$ decay.

Figure 3-1 shows a two-dimensional $(m_{D^0}, \Delta Q)$ scatter plot from RS and WS events. Their corresponding projected distributions on the $m_{D^0}$ and $\Delta Q$ axes are shown in Figures 3-2 and 3-3.
Figure 3-1: A two-dimensional scatter plot of $m_{D^0}$ versus $\Delta Q$ for RS (left plot) and WS (right plot) events.

Figure 3-2: Projection of the $m_{D^0}$ (left) and $\Delta Q$ (right) distribution for RS data.

Figure 3-3: Projection of the $m_{D^0}$ (left) and $\Delta Q$ (right) distribution for WS data.
Chapter 4

Maximum Likelihood Fit to $m_{K\pi\pi\pi}$ and $\Delta Q$ Distributions

4.1 Introduction

In this analysis, we perform an unbinned extended maximum likelihood fit [23] to the two-dimensional $m_{D^0}$ and $\Delta Q$ distributions to extract the reconstructed $D^0$ signal yield. Since the WS $D^0$ decay has the same decay topology as the RS, the WS signal is constrained to have the same probability density function (PDF). In this section, we give detailed descriptions of the signal and background PDFs.

4.1.1 Signal PDF

RS Signal PDF

As shown in Figure 4-1, the RS signal peaks at the $D^0$ mass in the $m_{D^0}$ distribution and at the value $m_{D^{*+}} - m_{D^0} - m_\pi$ in the $\Delta Q$ distribution. Figure 4-2 shows the two-dimensional scatter plot of $m_{D^0}$ and $\Delta Q$ from signal MC events. It clearly shows that the signal $m_{D^0}$ and $\Delta Q$ distributions (due to the detector resolutions) are slightly correlated.
In order to understand better the correlation between $m_{D^0}$ and $\Delta Q$, we divide the reconstructed signal MC events in $m_{D^0}$ into 40 bins. For the events in each bin, we fit their $\Delta Q$ distribution with a Gaussian plus a Johnson function\(^1\). We observed that the width $\sigma$ of the Gaussian function has a parabolic-like dependence on $m_{D^0}$ as shown in Figure 4-3. We repeated our studies using the data. In this case, the background $\Delta Q$ distribution is simply modeled as $\Delta Q^a \exp(b\Delta Q)$, where $a$ and $b$ are two free parameters. The data shows similar behavior in Figure 4-3.

As shown in Figure 4-3, the parabolic shape is relatively flat near the bottom. As a result, the correlation is only significant for $D^0$ candidates with large reconstructed $m_{D^0}$ values, which are presumably poorly reconstructed signal events. For the first order approximation, we can neglect the correlation for the signal candidates with small measured $m_{D^0}$ values.

Detailed studies using MC events show that the RS signal PDF $P_{rs}^{\text{sig}}$ can be well described as a sum of three components:

$$P_{rs}^{\text{sig}} = f_{\text{core}} \cdot S_{\text{core}} + f_{\text{wide}} \cdot S_{\text{wide}} + (1 - f_{\text{core}} - f_{\text{wide}}) \cdot S_{\text{jsu}}, \quad (4.1.1)$$

where the variables $f_{\text{core}}$ and $f_{\text{wide}}$ are free parameters determined from fit. The components $S_{\text{core}}$, $S_{\text{wide}}$ and $S_{\text{jsu}}$ are defined as:

- **Core component:**

  $$S_{\text{core}} = f_1 \cdot g_1(m_{D^0}) \cdot g_1(\Delta Q) + (1 - f_1) \cdot g_2(m_{D^0}) \cdot g_2(\Delta Q), \quad (4.1.2)$$

  where $g(x)$ is a Gaussian function of the variable $x$. In the fit, the means and widths of the four Gaussian functions as well as the fraction parameter $f_1$, are allowed to vary.

---

\(^1\)The Johnson function will be explained in detail in later sections.
Figure 4-1: The $m_{D^0}$ and $\Delta Q$ distributions for signal MC events.

Figure 4-2: A two-dimensional histogram of $m_{D^0}$ versus $\Delta Q$ for signal MC events.

Figure 4-3: The distribution of the signal $\Delta Q$ resolution as a function of reconstructed $m_{D^0}$ values. The left plot is for the signal MC; the right plot is for the data.
• Wide Gaussian component:

\[
S_{\text{wide}} = \exp \left\{ \frac{(m_{D^0} - \overline{m}_{D^0})^2}{2\sigma_{m_{D^0}}^2} \right\} \cdot \exp \left\{ \frac{(\Delta Q - \overline{\Delta Q})^2}{2(\sigma_{\Delta Q} + r_{\text{corr}}((m_{D^0} - \overline{m}_{D^0})/\sigma_{m_{D^0}})^2)\right\},
\]

(4.1.3)

where the parameters \( \overline{m}_{D^0}, \sigma_{m_{D^0}}, \overline{\Delta Q}, r_{\text{corr}}, \) and \( \overline{m}_{D^0} \) are all allowed to vary in the fit. Note that for this component the width of the \( \Delta Q \) Gaussian depends on the value of the reconstructed \( m_{D^0} \) value. This term is introduced to account for the correlation between \( m_{D^0} \) and \( \Delta Q \) for poorly-reconstructed \( D^0 \) candidates.

• Jsu component:

\[
S_{\text{jsu}} = (f_1 \cdot g_1(m_{D^0}) + (1 - f_1) \cdot g_2(m_{D^0})) \cdot J;
\]

(4.1.4)

where \( J \) is known as the Johnson SU function defined as:

\[
J(\Delta Q; \overline{\Delta Q}, \sigma_{\Delta Q1}, \delta, \gamma) = \\
\frac{\delta}{\sigma_{\Delta Q_i} \sqrt{2\pi} \sqrt{1 + \left( \frac{\Delta Q - \overline{\Delta Q}}{\sigma_{\Delta Q1}} \right)^2}} \cdot \exp \left\{ -\frac{1}{2} \left[ \gamma + \delta \text{Argsh} \left( \frac{\Delta Q - \overline{\Delta Q}}{\sigma_{\Delta Q1}} \right) \right]^2 \right\},
\]

(4.1.5)

and

\[
\text{Argsh}(x) = \log(x + \sqrt{1 + x^2}).
\]

(4.1.6)

In this case, the variables \( \overline{\Delta Q}, \sigma_{\Delta Q1}, \delta \) and \( \gamma \) are free parameters in the fit. The two Gaussian functions \( g_1(m_{D^0}), g_2(m_{D^0}) \) and parameter \( f_1 \) are the same functions and parameter defined in the Eq. 4.1.2.

To validate the signal PDF parametrization, we perform an unbinned maximum likelihood fit to the signal MC events. As shown in Figures 4-4 and 4-5, the PDF describes the signal distribution
very well.

**WS Signal PDF**

The parametrization of the WS signal is the same as the RS signal (4.1.1), and one of the fitting strategies is to use the large statistics of RS signals to constrain the lower statistics of the WS signal. This implies:

$$P_{ws}^\text{sig} = P_{rs}^\text{sig}$$  \hfill (4.1.7)

### 4.1.2 $D^{*+}$ Backgrounds Reconstructed from a Real $D^0$ with a Random $\pi_{soft}$

This background is produced when a $D^{*+}$ is reconstructed with a good $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ decay combined with a random slow pion, $\pi_{soft}$. Since these candidates include a genuine $D^0$, they peak in the $m_{D^0}$ distribution, but not in $\Delta Q$. Moreover, these events have lifetime distribution similar to legitimate $D^0$ events since the random $\pi_{soft}$ does not affect vertex fit significantly.

Since this category comes from a real $D^0$ with a random $\pi_{soft}$, we expect it to have the identical distribution as the real $D^0$ signal in $m_{D^0}$. Thus its PDF is simply the remaining $m_{D^0}$ part of equation 4.1.1 after integrating over the $\Delta Q$ variable, and can thus be written as:

$$F(m_{D^0}) = f_{wide} \cdot g_{m_{D^0}wide} + (1 - f_{wide}) \cdot (f_1 \cdot g_1(m_{D^0}) + (1 - f_1) \cdot g_2(m_{D^0})),$$  \hfill (4.1.8)

where the Gaussian functions $g_{m_{D^0}wide}$, $g_1(m_{D^0})$, $g_2(m_{D^0})$ and the parameters $f_{wide}$, $f_1$ are all identical to the corresponding ones in the signal PDF.

The background distribution in $\Delta Q$ is modeled as:

$$K(\Delta Q; c) = (\Delta Q + m_\pi) \sqrt{t} \exp(-ct); \quad t = ((\Delta Q + m_\pi)/m_\pi)^2 - 1.0,$$  \hfill (4.1.9)

where $c$ is a free parameter. The total PDF $P_{fake\pi}$ for this category is the product of the above
Figure 4-4: Projection plots of the fit to $m_{D^0}$ and $\Delta Q$ distribution for RS signal MC events

Figure 4-5: Three components of two-dimensional signal PDF from the fit to RS signal MC events.
Regarding the background due to the combination of a real $D^0$ with a random $\pi_{\text{soft}}$, the parameter $c_{rs}$ of its PDF is fixed to the value extracted from the MC events, since its contribution for the RS decays is very small compared to the large number of signal events. However, for WS events, the parameter $c_{ws}$ is allowed to vary in the fit.

The result of fitting the fake $\pi_{\text{soft}}$ background in the MC sample is shown in Figure 4-6. It is clear that our PDF parametrization describes the distribution very well.
4.1.3 Combinatoric Background

The combinatoric background neither peaks in $m_{D^0}$ nor in $\Delta Q$. We describe the $m_{D^0}$ distribution as a falling exponential. The background shape in $\Delta Q$ is described by the same $\mathcal{K}$ used for the real $D^0$ fake $\pi_{\text{soft}}$ background in equation 4.1.10.

A small Gaussian component is added to the $\Delta Q$ component in addition to the $\mathcal{K}$ function, for the RS events. This is because some RS combinatoric events come from decays $D^{*+} \rightarrow D^0\pi^+$, $D^0 \rightarrow$ other than $(K^-\pi^+\pi^+\pi^-)$. The shape of these events cannot be modeled precisely by the single $\mathcal{K}$ function since the DCSD decay is rare, a single $\mathcal{K}$ can adequately describe the combinatoric background in $\Delta Q$ for the WS events. Therefore the PDF for the combinatoric background, $p_{\text{comb}}$, can be written as:

$$p_{\text{comb}} = \exp(m_{D^0}) \cdot (f_{\text{comb}} \cdot \mathcal{K}(\Delta Q; c_{\text{comb}}^\text{rs}) + (1 - f_{\text{comb}}) \cdot g_{\text{comb}}(\Delta Q))$$  \hspace{1cm} (4.1.12)

$$p_{\text{comb}} = \exp(\Delta Q) \cdot \mathcal{K}(\Delta Q; c_{\text{comb}}^\text{ws})$$  \hspace{1cm} (4.1.13)

The mean and width of the Gaussian function $g_{\text{comb}}(\Delta Q)$, as well as the parameters $c_{\text{comb}}^\text{rs}$, $c_{\text{comb}}^\text{ws}$, and $f_{\text{comb}}$ are all allowed to vary in the fit.

Figure 4-7 shows the fit result of the combinatoric background using the MC sample. Figure 4-8 shows the fit result on the combination of truth-matched fake $\pi_{\text{soft}}$ background and combinatoric background events from the $c\bar{c}$ MC sample. We see that our background parametrizations describe the MC sample very well. The combinatoric background from $B\bar{B}$ and $uds$ MC samples are shown in Figure 4-9. Its contribution is much smaller than the combinatoric background from $c\bar{c}$.

4.1.4 Swapped $D^0$ Background

This background occurs when a pair of oppositely-charged kaon and pion tracks are mis-identified as a pion and a kaon respectively in a $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ decay. Such decays are reconstructed
Figure 4-7: Fit result of RS $c\bar{c}$ MC combinatoric events

Figure 4-8: Fit result of RS $c\bar{c}$ MC fake $\pi_{soft}$ and combinatoric events
Figure 4-9: The $m_{D^0}$ and $\Delta Q$ combinatoric background distributions from the $B\bar{B}$ and $uds$ MC sample. The red histogram is the contribution from the $uds$ MC sample, and the open histogram is the contribution from the $B\bar{B}$ MC sample. Both contributions are reweighted to the corresponding data luminosities.
like a fake $\bar{D}^0 \rightarrow K^-\pi^+\pi^+\pi^-$ WS event. We consider the swapped background for the WS likelihood and neglect the contribution of this background in the RS likelihood as it is unimportant due to small size of WS sample. Note that there is a similar background if a pair of charged kaon and pion tracks with the same sign are mis-identified as a pion and kaon respectively in a $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ decay. However, such background will not switch the $\pi_{soft}$ tagging for the $D^0$ candidates. MC studies show that this background is negligible for the RS and WS signals due to the extremely small double mis-identification probability.

![Fit result of $c\bar{c}$ MC swap background events.](image)

**Figure 4-10**: Fit result of $c\bar{c}$ MC swap background events.

We modeled these events with a single Gaussian in the $m_{D^0}$ distribution and the sum of a Gaussian and Johnson SU function $J$ defined in Eq. (4.1.5) of the $\Delta Q$ distribution. The parameters of this category are determined by the signal MC with oppositely-charged kaon and pion mass exchanged.

$$
B^{\text{swap}D^0}_{\text{ws}} = g_{\text{swap}}(m_{D^0}) \cdot (f_{\text{coreswap} \cdot g_{\text{swap}}(\Delta Q)} + (1 - f_{\text{coreswap} \cdot J(\Delta Q)})
$$

Figure 4-10 shows the fit result on the swapped events which were manually generated from $c\bar{c}$ M.C. by exchanging the mass hypothesis of oppositely-charged pairs of kaons and pions.
4.1.5 Likelihood Functions

We perform the extended maximum likelihood fit on the RS and WS events simultaneously. The extended likelihood is modeled as:

\[
\mathcal{L}_{\text{ext}} = \frac{e^{-\sum n_j}}{N!} \prod_{i=1}^{N} \sum_{j=1}^{m} n_j \mathcal{P}_j(x_i).
\]  

(4.1.15)

where \(N\) is the number of events in a given category, \(n_j\) the number of events for hypothesis \(j\) and \(\mathcal{P}_j(x_i)\), which is the value of the PDF as a function of \(x\) for hypothesis \(j\) in event \(i\).
Chapter 5

Measurement of $D^0 \rightarrow K^+ \pi^- \pi^- \pi^+$

Branching Ratio

5.1 Fit Results

5.1.1 MC Fit Result

We perform an unbinned maximum likelihood fit on a MC data set that is produced by mixing 1701205 $c \bar{c}$ MC events containing 1158299 $RS D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$ signals events and 3382 WS signal MC events in order to validate our fitting model. The fit yields:

$$N_{RS} = (1.175225 \pm 0.001427(stat.)) \times 10^6$$

(5.1.1)

$$N_{WS} = 3409 \pm 75(stat.)$$

(5.1.2)

The corresponding $R_{WS}^{fit} = (0.2901 \pm 0.0064) \times 10^{-2}$, which is in good agreement with the generated value $R_{WS}^{gen} = 0.2920 \times 10^{-2}$. The projected fit results are shown in Figures 5-1–5-6.
**Figure 5-1:** Fit result of RS MC sample.

**Figure 5-2:** Fit result of RS MC sample. Here we select the peak region in $m_{D^0}$ with $1.83 \text{ GeV}/c^2 < m_{D^0} < 1.89 \text{ GeV}/c^2$. 
Figure 5.3: Fit result of RS MC sample. Left: upper sideband of $m_{D^0}$ with $1.89 \text{ GeV}/c^2 < m_{D^0} < 1.9145 \text{ GeV}/c^2$. Right: lower sideband of $m_{D^0}$ with $1.8145 \text{ GeV}/c^2 < m_{D^0} < 1.83 \text{ GeV}/c^2$.

5.1.2 WS MC Studies

The uncertainty in the measurement of $R_{WS}$ is dominated by the statistical error on the number of WS signal events. In order to understand the possible biases of the measured event number and corresponding error, we repeat our measurements on 100 WS MC samples. Each is constructed as follows:

- 1000 WS signal MC events consisting of 500 $\bar{D}^0 \rightarrow K^−\pi^+\pi^−\pi^+$ events, and 500 $D^0 \rightarrow K^+\pi^−\pi^+\pi^−$ events. These 1000 events are selected from a total of 1049000 WS signal MC events divided into 100 statistically independent samples.

- Randomly select 40000 WS background events from a total of 54212 events from the $c\bar{c}$ MC for each sample. Events may overlap due to limited statistics.

The pull plot for fitting these 100 samples is shown in Figure 5-7 with mean equal to $-0.055 \pm 0.078$ and width equal to $0.70 \pm 0.072$. We see no bias in the measured WS signal event number. However, it seems that the corresponding error is slightly overestimated. On the other hand, this
Figure 5-4: Fit result of WS MC sample

Figure 5-5: Fit result of WS MC sample. Here we select the peak region in $m_{D^0}$ with $1.83 \text{ GeV}/c^2 < m_{D^0} < 1.89 \text{ GeV}/c^2$. 
Figure 5-6: Fit result of WS MC sample. Left: upper sideband of $m_{D^0}$ with $1.89 \text{ GeV}/c^2 < m_{D^0} < 1.9145 \text{ GeV}/c^2$. Right: lower sideband of $m_{D^0}$ with $1.8145 \text{ GeV}/c^2 < m_{D^0} < 1.83 \text{ GeV}/c^2$.

Figure 5-7: Pull distribution of 100 samples of WS MC events.
overestimate is expected due to the overlap of WS background events. As a result, we do not
correct for any possible overestimate of the statistical uncertainty on the measured number of WS
signal events.

### 5.1.3 Data Fit Result

We perform a simultaneous two-dimensional fit in \( (m_{D^0}, \Delta Q) \) to the RS and WS data sets. The fit yields:

\[
N_{RS} = (7.457 \pm 0.002) \times 10^5 
\]

\[
N_{WS} = 2354 \pm 84
\]

The projected fit results are shown in Figures 5-8–5-13.

There is a peaking background in \( (m_{D^0}, \Delta Q) \) due to real WS signal events coming from the
singly Cabibbo-suppressed decays \( D^0 \rightarrow K^+ \bar{K}^0 \pi^- \) followed by \( \bar{K}^0 \rightarrow K_s^0 \rightarrow \pi^+\pi^- \). Since
the \( K_s^0 \) lifetime is relatively long, its corresponding \( \pi^+\pi^- \) decay vertex is usually displaced away
from the \( D^0 \) decay vertex, and therefore, these events fail the vertex probability requirement since
all charged tracks are required to originate from the same vertex position. Nevertheless, a small
fraction of these events can still pass our selection criteria if their \( K_s^0 \) decay vertex is close to the
\( D^0 \) decay vertex. As shown in Figure 5-14, we can clearly see a peaking structure at the \( K_s^0 \) mass.
Since the real signal (DCSD/mixing) decays do not peak at the \( K_s^0 \) mass, we can then estimate the
number of events of this peaking background by fitting the \( m_{\pi^+\pi^-} \) distribution, which is shown in
Figure 5-14.

In principle we can eliminate this kind of peaking background by imposing a mass window cut
to reject the \( D^0 \) candidate if one of its oppositely charged \( \pi\pi \) pairs has an invariant mass within
\( \pm 20 \text{ MeV}/c^2 \) of the nominal \( K_s^0 \) mass. However, since the RS and WS events may not have the
same \( m_{\pi^+\pi^-} \) distribution, the percentage of the signal events being rejected by such a cut may
not be the same for RS and WS signals. Thus it can potentially cause a bias in the branching ratio measurement. This possible bias can be avoided by extracting the number of the RS and WS signal events being subtracted using the signal events in the nearby \( m_{\pi^+\pi^-} \) distribution. However, in our analysis we select a different approach without imposing any cut on the \( m_{\pi^+\pi^-} \) distribution. The method will be explained in the next two sections.

Similarly, there is a peaking background in \((m_{D^0}, \Delta Q)\) due to real RS signal events coming from singly Cabibbo-suppressed decay \( D^0 \to K^- K^0 \pi^- \) followed by \( K^0 \to K_s^0 \to \pi^+\pi^- \). However, its contribution is tiny compared to the large amount of RS signal events. In fact, given that the \( \mathcal{B}(D^0 \to K^- K^0 \pi^-) = (6.9 \pm 1.0) \times 10^{-3} \) is close to \( \mathcal{B}(D^0 \to K^+ K^0 \pi^-) = (5.3 \pm 1.0) \times 10^{-3} \), the expected peaking background from \( D^0 \to K^- K^0 \pi^- \) for the RS decay is around a few hundred. It is much smaller than the RS signal yield. As shown in Figure 5-14, we see no significant peak at the \( K_s^0 \) mass for the RS data.
Figure 5-8: Fit result of RS data.

Figure 5-9: Fit result of RS data. Here we select the peak region in $m_{D^0}$ with $1.83 \text{ GeV/c}^2 < m_{D^0} < 1.89 \text{ GeV/c}^2$. 
Figure 5-10: Fit result of RS data sample. Left: upper sideband of $m_{p\theta}$ with $1.89 \text{ GeV/c}^2 < m_{p\theta} < 1.9145 \text{ GeV/c}^2$. Right: lower sideband of $m_{p\theta}$ with $1.8145 \text{ GeV/c}^2 < m_{p\theta} < 1.83 \text{ GeV/c}^2$. 
Figure 5-11: Fit result of WS data

Figure 5-12: Fit result of WS data. Here we select the peak region in $m_{D^0}$ with $1.83 \text{ GeV/c}^2 < m_{D^0} < 1.89 \text{ GeV/c}^2$. 

52
Figure 5-13: Fit result of WS data. Left: upper sideband of $m_{D^0}$ with $1.89 \text{ GeV/c}^2 < m_{D^0} < 1.9145 \text{ GeV/c}^2$. Right: lower sideband of $m_{D^0}$ with $1.8145 \text{ GeV/c}^2 < m_{D^0} < 1.83 \text{ GeV/c}^2$.

Figure 5-14: Left plot: Fit to peaking background of $D^0 \rightarrow K^+ \bar{K}^0 \pi^-$ in the WS $m(\pi^- \pi^+)$ data distribution. The peaking structure is modeled as a Gaussian with mean and width allowed to vary in the fit. The remaining $m(\pi^- \pi^+)$ distribution for the signal events is modeled as a first order polynomial. Right plot: $m(\pi^- \pi^+)$ distribution of RS data. For both distributions, we require that $|m_{D^0} - 1.8645| \leq 20 \text{ MeV/c}^2$ and $|\Delta Q - 0.00584| \leq 2 \text{ MeV/c}^2$. 
5.2 Efficiency Study

The efficiency as a function of the kinematic variables is shown in Figures 5-15-5-21. In order to estimate the efficiency variation in the four-body decay $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$, we take the following five independent kinematic variables, known as Cabibbo-Maksymowicz variables [24], to parameterize the phase space: $m_{K^-\pi^-}$, $m_{\pi^+\pi^+}$, $\theta_{K^-\pi^-}$, $\theta_{\pi^+\pi^+}$ and $\phi$. $\theta_{K^-\pi^-}$ is the angle between the $K^-$ momentum in the rest frame of the $K^-\pi^-$ system and the momentum of the $K^-\pi^-$ system in the $D^0$ meson’s rest frame. $\theta_{\pi^+\pi^+}$ is the angle between the pion momentum in the rest frame of the $\pi^+\pi^+$ system and the momentum of the $\pi^+\pi^+$ system in the $D^0$ meson’s rest frame. $\phi$ is the angle between the $K^-\pi^-$ decay plane and the $\pi^+\pi^+$ decay plane in the rest frame of the $D^0$ meson.

We calculate efficiencies for each data point by taking the ratio between the number of reconstructed signal MC events and the number of events that are generated using a flat phase space model.

$$
\epsilon_k = \frac{N_{\text{rec}}}{N_{\text{gen}}},
$$

(5.2.5)

where $N_{\text{gen}}$ and $N_{\text{rec}}$ are counted as events flowing into the five-dimensional volume that is denoted by the variables above. The five-dimensional volume is defined as:

$$
x_{n,i} \leq x_n \leq x_{n,i+1} \quad n = 1, 2, 3, 4, 5
$$

(5.2.6)

$$
x_{n,i+1} - x_{n,i} = \Delta x_n = \frac{x_{n,max} - x_{n,min}}{N_{\text{bins}}} \quad n = 1, 2, 3, 4, 5
$$

(5.2.7)

The volume size is varied by changing the total number of bins, $N_{\text{bins}}$, from 1 to 300. The systematic error dependence on the binning is evaluated in section 5.3.1.

The final DCSD WS ratio with efficiency-corrected yields $N'$ using the sWeights method [25]
is:

$$R_{WS} = \frac{N'(D^0 \rightarrow K^+\pi^-\pi^+\pi^-)}{N'(D^0 \rightarrow K^-\pi^+\pi^-\pi^+)}$$  \hspace{1cm} (5.2.8)$$

5.3 Measurement of the WS Ratio $R_{WS}$

As discussed above, there is a peaking background in $(m_{D^0}, \Delta Q)$ due to the real WS signal coming from the singly Cabibbo-suppressed decay $D^0 \rightarrow K^+K^0\pi^-$ followed by $K^0 \rightarrow K_S^0 \rightarrow \pi^+\pi^-$. Since the real signal (DCSD/mixing) decays do not peak at the $K_S^0$ mass, we can use the sWeight method to extract this peaking background yield $N_{Wbkgpeak}$ by fitting the efficiency-corrected and background-subtracted $m_{\pi^+\pi^-}$ distribution, which is shown in Figure 5-22.

Similarly, there is also a peaking background on $(m_{D^0}, \Delta Q)$ due to real RS signal events coming from the singly Cabibbo-suppressed decay $D^0 \rightarrow K^-K^0\pi^-$ followed by $K^0 \rightarrow K_S^0 \rightarrow \pi^+\pi^-$. However, its contribution is tiny compared to the large number of RS signal events. Nevertheless, we follow the same strategy to estimate its contribution $N_{Rbkgpeak}$ to the RS events by fitting the efficiency-corrected and background-subtracted $m_{\pi^+\pi^-}$ distribution, which is shown in Figure 5-22.

After the peaking background yield is extracted, the final WS decay rate is calculated as:

$$R_{WS} = \frac{N'(D^0 \rightarrow K^+\pi^-\pi^+\pi^-) - N_{Wbkgpeak}}{N'(D^0 \rightarrow K^-\pi^+\pi^-\pi^+) - N_{Rbkgpeak}}$$  \hspace{1cm} (5.3.9)$$

where $N'$ represents the efficiency-corrected yield using the sWeights method [25].

The final WS decay ratio of the $D^0 \rightarrow K^-\pi^+\pi^-\pi^-$ decay is found to be:

$$R_{WS} = (0.3228 \pm 0.0120 \text{ (stat)}) \times 10^{-2}$$  \hspace{1cm} (5.3.10)$$
Figure 5-15: A two-dimensional histogram of $m_{K\pi}$ versus $m_{\pi\pi}$ for data.

Figure 5-16: Efficiency variables $\cos(K\pi)$ vs $\cos(\pi\pi)$ and $\cos(K\pi)$ vs $\phi$. 
For the WS distribution, the peak at the $K_S^0$ mass value is the peaking background from singly Cabibbo-suppressed decays $D^0 \to K^+ K^0 \pi^-$ followed by $K^0 \to K_S^0 \to \pi^+ \pi^-$. 

**Figure 5-17:** Projection plots for background-subtracted data in $m(K^-\pi^+)$. Left: RS. Right: WS.

**Figure 5-18:** Projection plots for background-subtracted data in $m(\pi^-\pi^+)$. Left: RS. Right: WS.
Figure 5-19: Projection plots for background-subtracted data in $m(\pi^{-}\pi^{+}\pi^{+})$. Left: RS. Right: WS.

Figure 5-20: Projection plots for background-subtracted data in $m(K^{-}\pi^{+}\pi^{+})$. Left: RS. Right: WS.
Figure 5-21:
Projection plots for background-subtracted data in $m(K^-\pi^-\pi^+)$.
Left: RS. Right: WS.

Figure 5-22: Left plot: Fit to peaking background of $D^0 \rightarrow K^+K^0\pi^-$ in $m(\pi^-\pi^+)$ efficiency-corrected, background-subtracted sWeights plots of WS data. The peaking structure is modeled as a Gaussian with mean and width are allowed to vary in the fit. The $m(\pi^-\pi^+)$ efficiency-corrected distribution for the signal events near the $K^0_S$ mass region is modeled as a first order polynomial.
Right plot: Fit result of peaking background of $D^0 \rightarrow K^-K^0\pi^+$ in $m(\pi^-\pi^+)$ efficiency-corrected background-subtracted RS data. The fit model used here is the same as the one used in the WS data. However, since the relative fraction of peaking structure is tiny, the mean and width of the $K^0_S$ Gaussian is constrained to have the same values as the ones of the fit to the WS data.
5.3.1 Systematic Uncertainties

We are measuring a branching ratio in which the decay kinematics of the RS and WS $D^0$ decays are very similar. As a result, the systematic uncertainties due to charged tracking efficiency, particle identification efficiencies, etc., all cancel approximately to first order. In this section, we summarize the dominant systematic uncertainties.

5.3.2 Fit Model Systematics

Parametrization Systematics

To estimate the swapped $D^0$ background, we fixed the PDF based on our study in section 4.1.4. There are 9 fixed parameters in the fit. We varied each fixed parameter by ±σ from the original fixed fitted value. We then compared the difference between the results from the variation and the original value. By summing the largest difference for each variation, we assigned for $R_{WS}$ a systematic error from parameterization of $1.14 \times 10^{-5}$.

Fitting Model

We estimate the error for the fit model by using the MC sample. We compare the ratio $R_{gen} = 0.2920 \times 10^{-2}$ used to generate events with the fitted value $R_{fit} = (0.2901 \pm 0.0064) \times 10^{-2}$. As a result, we take this difference as the systematic error for the fit model.

Efficiency Method

Figure 5-23 shows the final efficiency-corrected $R_{WS}$ as a function of the number of bins, 1 to 300, used in the efficiency calculation. A flat line fitted to $R_{WS}$ shows that the systematic uncertainty from the binning is almost negligible. Since the largest variation is about 0.000009, we assign this to the corresponding systematic uncertainty.
Peaking Background

As we discussed before, the peaking background is estimated by fitting the $K^0_s$ peak in the $m(\pi^+\pi^-)$ distribution from $RS$ and $WS$ efficiency-corrected signal yields. The statistical uncertainty of the fitted peaking background yield is taken as the corresponding systematic uncertainty in the calculation of $R_{WS}$.

5.3.3 $R_{WS}$ Result

After adding the systematic uncertainties in quadrature, we obtain:

$$R_{WS} = (0.3228 \pm 0.0120 \text{ (stat)} \pm 0.0033 \text{ (syst)}) \times 10^{-2}. \quad (5.3.11)$$
Chapter 6

Analysis of the $D^0$ Decay-Time Distribution

6.1 Introduction

A maximum likelihood fit to the $D^0$ decay-time distribution is performed to search for $D^0$-$\bar{D}^0$ mixing. This involves fitting both the RS $D^0 \rightarrow K^-\pi^+\pi^+\pi^-$ and the WS $D^0 \rightarrow K^+\pi^-\pi^-\pi^+$ decay-time distributions.

The fit is a three-dimensional likelihood fit to $\{m_{D^0}, \Delta Q, t_{K\pi\pi}\}$. The two-dimensional fit to $\{m_{D^0}, \Delta Q\}$ is performed first in order to separate signal from the background for RS and WS events.

With the mass related parameters fixed from the $\{m_{D^0}, \Delta Q\}$ fit, a three-dimensional fit is performed with different signal PDFs for the RS and WS decays. The high-statistics of RS decays serves to determine precisely the parameters of the exponential form of the DCSD time distribution and provide resolution models for the lower-statistics WS decays. Any deviation from this well determined form indicates $D$ mixing in the WS sample.
6.2 Measured Decay-Time Uncertainties

The $D^0$ decay time, $t_{K\pi\pi}$, and its associated uncertainty, $\sigma_t$, are calculated from the simultaneous fit to both the $D^{*+}$ and $D^0$ decay vertices in the $D^{*+} \rightarrow D^0\pi^+, D^0 \rightarrow K^{\mp}\pi^\pm\pi^\pm\pi^\mp$ decay chain. The decay-time is

$$t_{K\pi\pi} = \left(\frac{m_{D^0}}{p}\right) \Delta x,$$  \hfill (6.2.1)

where $p$ is the $D^0$ momentum and $\Delta x$ is the distance in the lab frame between the $D^{*+}$ and $D^0$ decay vertices. The uncertainty is calculated using the decay-fit covariance matrix that includes the correlations between the two vertex positions. In fitting to the decay-time distribution, we exclude candidates with poorly measured decay times by requiring

$$\sigma_t < 0.5 \text{ ps}.$$  \hfill (6.2.2)

Figure 6-1 illustrates both the relationship between $\sigma_t$ and $t_{K\pi\pi}$ and the overall $\sigma_t$ distribution. The mean $\sigma_t$ for signal events is near 0.29 ps. The $\sigma_t$ of individual events is used in the decay-time
distribution fit, despite the fact that we do not fit a PDF to the $\sigma_t$ distribution.

6.3 Description of RS Probability Density Functions

6.3.1 Signal

The signal PDF for the decay time is based on an exponential shape convolved with a Gaussian resolution function to account for the uncertainty in the decay-time measurement:

$$
\rho_{RS,i}(t; \tau, k) = e^{-t/\tau} \otimes \exp \left(-\frac{(t + t_b)^2}{2(k\sigma_i)^2}\right)
= \exp \left(-\frac{t + t_b}{\tau} + \frac{(k\sigma_i)^2}{2\tau^2}\right) \text{erfc} \left(-\frac{t + t_b}{\sqrt{2(k\sigma_i)}} + \frac{k\sigma_i}{\sqrt{2\tau}}\right)
$$

(6.3.3)

The index $i$ in this PDF spans the events, representing the fact that the uncertainty $\sigma_i$ is measured for each event as described in Section 6.2. The resolution model depends on the parameter $k$, which is a scale factor $\approx O(1)$ by which the event uncertainty is multiplied. The assumption underlying this choice of resolution model is that the uncertainties are relatively accurate, but they may be systematically under- or over-estimated by a global scale factor.

The complete RS three-dimensional signal PDF is

$$
\mathcal{P}_{RS,i}(m_{D^0}, \Delta Q, t) = S(m_{D^0}, \Delta Q) \times \{(1 - f_{11}) \cdot \rho_{2,i} + f_{11} \cdot \rho_{1,i}\}
$$

(6.3.4)
\[ p_{1,i} = p_{RS,i}(t; \tau, k_1, t_{b1}) \]
\[ p_{2,i} = p_{RS,i}(t; \tau, k_2, t_{b2}) \]

where \( S(m_{D^0}, \Delta Q) \) is defined in section 4.1.1 and \( f_{i1} \) is the parameter representing the fraction of events described by each Gaussian resolution function. Figure 6-2 shows the projection plot in \( t_{K\pi\pi} \) for the truth-matched signal MC.

![Figure 6-2: Projection of the maximum likelihood fit onto \( t_{K\pi\pi} \) for the RS truth matched signal MC.](image)

### 6.3.2 Combinatoric Background

This background is described in section 4.1.3. We used a Gaussian with the mean slightly shifted towards positive decay times and a Crystal Ball function [26] similar to Equation 6.3.5 to describe the tail in the positive direction.
\[ cb(x; \bar{x}, \sigma, \alpha, n) = \begin{cases} 
\exp \left( -\frac{(x-\bar{x})^2}{2\sigma^2} \right) & \text{if } \frac{(x-\bar{x})}{\sigma} < \alpha \\
\alpha \left( b + \frac{(x-\bar{x})}{\sigma} \right)^{-n} & \text{if } \frac{(x-\bar{x})}{\sigma} \geq \alpha 
\end{cases} \]  
(6.3.5)

\[ \alpha > 0, \quad a = (1/\alpha) \exp(\alpha^2/2), \quad b = (1/\alpha) - \alpha \]

The complete three-dimensional PDF for this background category is

\[ \mathcal{P}_{\text{comb}}(m_{D^0}, \Delta Q, t) = \]

\[ C(m_{D^0}, \Delta Q) \times \left\{ f_{c2} \cdot g(t; \tilde{t}_2, \sigma_{c2}) + (1 - f_{c2}) \cdot cb(t; \tilde{t}_1, \sigma_{c1}, \alpha_c, n_c) \right\} \]  
(6.3.6)

where \( C(m_{D^0}, \Delta Q) \) is the same as \( \mathcal{P}_{\text{comb}}^{\text{MC}} \) defined in section 4.1.3. \( g(t; \tilde{t}_2, \sigma_{c2}) \) is a Gaussian, and \( f_{c2} \) is a parameter representing the fraction of events described by the Gaussian. Figure 6-3 shows the projection plot in \( t_{K\pi\pi} \) for truth-matched combinatoric background MC.

**Figure 6-3:** Projection of the maximum likelihood fit onto \( t_{K\pi\pi} \) for MC events belonging to the combinatoric background category.
6.3.3 Fake $\pi_{\text{soft}}$ Background

This background, described in section 4.1.2, is due to real $D^0$ decays with random $\pi^\pm_s$ candidates. Because the $D^{*+}$ decay vertex is primarily determined by the intersection of the $D^0$ flight direction projected backward with the beamspot, and the $D^0$ candidate is real, this background has the same shape as the signal with the same lifetime. Thus the decay-time PDF is given by Eq. 6.3.4, and we have

$$P_{\text{fake},i}(m_{D^0}, \Delta Q, t) =$$

$$\mathcal{F}(m_{D^0}, \Delta Q) \times \{f_{t1} \cdot \rho_{1,i} + (1 - f_{t1}) \cdot \rho_{2,i}\}$$

(6.3.7)

where $\mathcal{F}(m_{D^0}, \Delta Q)$ is the same as $P_{t_{\text{ts}}}$ defined in section 4.1.2. Figure 6-4 shows the projection plot in $t_{\text{K}\pi\pi}$ for the truth-matched fake $\pi_{\text{soft}}$ background MC.

\begin{figure}[h!]
\centering
\includegraphics[width=0.5\textwidth]{fig6-4.png}
\caption{Projection of the maximum likelihood fit onto $t_{\text{K}\pi\pi}$ for MC events belonging to the fake $\pi_{\text{soft}}$ background category.}
\end{figure}
6.4 RS Fit Results and $D^0$ Lifetime Measurement

The unbinned maximum likelihood fit described above is performed in two steps. First, the two-dimensional $\{m_{K\pi\pi}, \Delta Q\}$ fit is performed to adjust all the parameters and yields to the data set with the requirement $\sigma_t < 0.5$ ps. The signal yield after performing this step is $7.005 \times 10^5 \pm 9.15 \times 10^2$. Second, the parameters of the two-dimensional fit are fixed, and the three-dimensional fit is performed to find the best values of the parameters in the decay-time PDFs. The $\{m_{K\pi\pi}, \Delta Q\}$ PDF parameters are fixed because the decay-time distribution is not capable of separating the various categories as well as the mass distribution does. The final fit is shown projected onto the $t_{K\pi\pi}$ axis for various regions of $\{m_{K\pi\pi}, \Delta Q\}$ in Figures 6-9–6-11.

The scale factors $\{k_1, k_2\}$ are initially allowed to vary in the fit, and their final values are given in Table 6.2. Because the resolution parameters are highly correlated, $t_{b1}$ is fixed at 0 while $t_{b2}$ is allowed to vary in the fit. The fraction of the $k_1$ component from MC is 9.75% while it is 13.6% from the data. The fitted parameters are shown in Table 6.1 and 6.2 respectively. The projected plots in $t_{K\pi\pi}$ for MC and data are shown in Figs. 6-5–6-12.

Table 6.1: Decay-time uncertainty scale factors from the likelihood fit for MC.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>1.373</td>
<td>$t_{b1}$</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.969</td>
<td>$t_{b2}$</td>
</tr>
</tbody>
</table>

Table 6.2: Decay-time uncertainty scale factors from the likelihood fit for data

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>1.295</td>
<td>$t_{b1}$</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.989</td>
<td>$t_{b2}$</td>
</tr>
</tbody>
</table>

The fitted lifetime for MC is

$$\tau = (0.4105 \pm 0.0006 \text{ (stat)}) \text{ fs} \quad (6.4.8)$$
The fitted lifetime for data is:

\[ \tau = (0.4092 \pm 0.0006 \text{ (stat)}) \text{ fs} \]  

(6.4.9)

where the systematic uncertainty is omitted.

Comparing these values with each other and with the accepted lifetime of

\[ \tau_{\text{RPP}} = 0.4103 \pm 0.0015 \text{ fs} \]  

(6.4.10)

reported in the Review of Particle Physics [27], we believe that the lifetime fit and the data are sufficiently well understood to proceed with an analysis of the WS decay-time distribution.

**Figure 6-5**: Projection of the maximum likelihood fit to the RS MC onto \( t_{K\pi\pi} \) for all the events (left) and for the signal region (right). The signal region is \( 1.845 \text{ GeV}/c^2 < m_{K\pi\pi} < 1.885 \text{ GeV}/c^2 \) and \( 0.053 \text{ GeV}/c^2 < \Delta Q < 0.063 \text{ GeV}/c^2 \).
Figure 6-6: Projection of the maximum likelihood fit to the RS MC onto $t_{K\pi\pi}$ for the $m_{K\pi\pi}$ signal region (left) and for the $\Delta Q$ signal region (right). The $m_{K\pi\pi}$ signal region is $1.845 \text{GeV}/c^2 < m_{K\pi\pi} < 1.885 \text{GeV}/c^2$ and $0.0 \text{GeV}/c^2 < \Delta Q < 0.012 \text{GeV}/c^2$; the $\Delta Q$ signal region is $1.8145 \text{GeV}/c^2 < m_{K\pi\pi} < 1.9145 \text{GeV}/c^2$ and $0.053 \text{GeV}/c^2 < \Delta Q < 0.063 \text{GeV}/c^2$.

Figure 6-7: Projection of the maximum likelihood fit to the RS MC onto $t_{K\pi\pi}$ for sideband regions emphasizing the combinatoric background in the lower $m_{K\pi\pi}$ region (left) and in the upper $m_{K\pi\pi}$ region (right). The lower $m_{K\pi\pi}$ region is $1.8145 \text{GeV}/c^2 < m_{K\pi\pi} < 1.83 \text{GeV}/c^2$ and $0.0 \text{GeV}/c^2 < \Delta Q < 0.012 \text{GeV}/c^2$; the upper $m_{K\pi\pi}$ region is $1.89 \text{GeV}/c^2 < m_{K\pi\pi} < 1.9145 \text{GeV}/c^2$ and $0.0 \text{GeV}/c^2 < \Delta Q < 0.012 \text{GeV}/c^2$. 
Figure 6-8: Projection of the maximum likelihood fit to the RS MC onto $t_{K\pi\pi}$ for a side-band region emphasizing the fake $\pi_{soft}$ background. The region is $1.845 \text{ GeV}/c^2 < m_{K\pi\pi} < 1.885 \text{ GeV}/c^2$ and $0.075 \text{ GeV}/c^2 < \Delta Q < 0.012 \text{ GeV}/c^2$. 
Figure 6-9: Projection of the maximum likelihood fit to the RS data onto $t_{K\pi\pi\pi}$ for the all the events (left) and for the signal region (right). The signal region is $1.845 \text{ GeV}/c^2 < m_{K\pi\pi\pi} < 1.885 \text{ GeV}/c^2$ and $0.053 \text{ GeV}/c^2 < \Delta Q < 0.063 \text{ GeV}/c^2$.

Figure 6-10: Projection of the maximum likelihood fit to the RS data onto $t_{K\pi\pi\pi}$ for the $m_{K\pi\pi\pi}$ signal region (left) and for the $\Delta Q$ signal region (right). The $m_{K\pi\pi\pi}$ signal region is $1.845 \text{ GeV}/c^2 < m_{K\pi\pi\pi} < 1.885 \text{ GeV}/c^2$ and $0.0 \text{ GeV}/c^2 < \Delta Q < 0.012 \text{ GeV}/c^2$; the $\Delta Q$ signal region is $1.8145 \text{ GeV}/c^2 < m_{K\pi\pi\pi} < 1.9145 \text{ GeV}/c^2$ and $0.053 \text{ GeV}/c^2 < \Delta Q < 0.063 \text{ GeV}/c^2$. 
Figure 6-11: Projection of the maximum likelihood fit to the RS data onto $t_{K^{\pi\pi}}$ for sideband regions emphasizing the combinatoric background in the lower $m_{K^{\pi\pi}}$ region (left) and in the upper $m_{K^{\pi\pi}}$ region (right). The lower $m_{K^{\pi\pi}}$ region is $1.8145 \text{GeV}/c^2 < m_{K^{\pi\pi}} < 1.83 \text{GeV}/c^2$ and $0.0 \text{GeV}/c^2 < \Delta Q < 0.012 \text{GeV}/c^2$; the upper $m_{K^{\pi\pi}}$ region is $1.89 \text{GeV}/c^2 < m_{K^{\pi\pi}} < 1.9145 \text{GeV}/c^2$ and $0.0 \text{GeV}/c^2 < \Delta Q < 0.012 \text{GeV}/c^2$.

Figure 6-12: Projection of the maximum likelihood fit to the RS data onto $t_{K^{\pi\pi}}$ for a sideband region emphasizing the fake $\pi_{\text{soft}}$ background. The region is $1.845 \text{GeV}/c^2 < m_{K^{\pi\pi}} < 1.885 \text{GeV}/c^2$ and $0.075 \text{GeV}/c^2 < \Delta Q < 0.012 \text{GeV}/c^2$. 

74
6.5 Description of WS Signal Probability Density Function

6.5.1 Motivation and Definition of a Functional Form

The time-dependent decay rate including $D$ mixing for a two-body $D^0 \to K^+ \pi^-$ decay is:

$$\Gamma_f(t) = |\bar{A}_f|^2 e^{-\Gamma t} \left( R_D + \sqrt{R_D} y_f'(\Gamma t) + \frac{x^2 + y^2}{4} (\Gamma t)^2 \right). \quad (6.5.11)$$

The time-dependent decay rate including $D$ mixing for a multi-body decay is:

$$\Gamma_f(t) = \bar{A}_f^2 e^{-\Gamma t} \left( R_I + \alpha \sqrt{R_I} y_f'(\Gamma t) + \frac{x^2 + y^2}{4} (\Gamma t)^2 \right), \quad (6.5.12)$$

$$0 \leq \alpha \leq 1.$$ 

It is be noted that $\alpha$, $R_I$ and $y_f'$ are the parameters depending on the Dalitz plot region while the mixing term $(x^2 + y^2)$ is independent of the Dalitz plot variables.

Previous analyses of the $D^0 \to K^+ \pi^-$ lifetime have fitted for $\{x^2, y'\}$, where $\{x', y'\}$ are the quantities $\{x, y\}$ rotated by a strong phase shift $\delta$,

$$x' \equiv x \cos \delta + y \sin \delta \quad (6.5.13)$$

$$y' \equiv y \cos \delta - x \sin \delta, \quad (6.5.14)$$

and

$$(x'^2 + y'^2) = (x^2 + y^2). \quad (6.5.15)$$

We cannot directly fit the form given by Eq. 6.5.11, because the multi-body decay rate has an additional factor $\alpha$ to which we have no direct sensitivity. Therefore, we consider the function

$$\Gamma(t) \propto e^{-\Gamma t} \left( a_1^2 + \frac{a_1 a_3}{1 + a_2} \Gamma t + \left( \frac{a_3}{4} \right) (\Gamma t)^2 \right) \quad (6.5.16)$$
We can relate physically meaningful quantities to the parameters \{a_1, a_2, a_3\}:

\[
R_I = a_1^2
\]
(6.5.17)

\[
\sqrt{R_I a y_I'} = \frac{a_1 a_3}{1 + a_2^2}
\]
(6.5.18)

\[
(x^2 + y^2)/2 = \frac{a_3^2}{2}
\]
(6.5.19)

In this form, the amplitude of the mixing rate, \((x^2 + y^2)/2\) and the sign and size of the interference term are determined by \(a_3\) and \(a_2\). The sign of \(a_1\) is stable because the value of \(R_I\) is well-determined and not near zero.

The signal PDF for the WS decay time is based on Eq. 6.5.16 convoluted with a Gaussian resolution function:

\[
\rho_{WS,i}(t; \tau, a_1, a_2, a_3, k) = \exp\left(-\frac{t^2}{2(k\sigma_i)^2}\right).
\]
(6.5.20)

The complete, three-dimensional WS signal PDF is

\[
\mathcal{P}_{WS,i}(m, \Delta Q, t) = S(m, \Delta Q) \times \{f_{i1} \cdot \rho'_{1,i} + (1 - f_{i1}) \cdot \rho'_{2,i}\}
\]
(6.5.21)

\[
\rho'_{1,i} = \rho_{WS,i}(t; \tau, a_1, a_2, a_3, k_1)
\]
\[
\rho'_{2,i} = \rho_{WS,i}(t; \tau, a_1, a_2, a_3, k_2)
\]

where \(S(m, \Delta Q)\) is defined in section 4.1.1 and \(f_{i1}\) is a parameter representing the fraction of events described by the various Gaussian resolution functions.
6.5.2 Validation of the WS PDF on RS Data

We randomly selected 3000, 300000 and all RS events from data, which include 2100, 210000 and 700000 RS signal events. We fit this dataset using the 3D RS PDF except that the RS lifetime signal PDF is replaced by the wrong sign lifetime signal PDF. The fitted $\alpha y'$ and $(x^2 + y^2)/2$ are shown in Table 6.3 and the projection plot of $t_{K\pi\pi}$ is shown in Fig 6-13. Figure 6-14 shows the contour plots of $a_1$ vs. $a_3$ and $R_D$ vs. $R_{mix}$. Figure 6-15 shows the contour plots of $a_2$ vs. $a_3$ and $\alpha y'$ vs. $R_{mix}$.

Table 6.3: Fitted $\alpha y'$ and $(x^2 + y^2)/2$ values for RS events with WS lifetime signal PDF

<table>
<thead>
<tr>
<th>RS events</th>
<th>$\alpha y'$</th>
<th>$(x^2 + y^2)/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000</td>
<td>0.001 ± 0.001</td>
<td>0.000000 ± 0.000001</td>
</tr>
<tr>
<td>300000</td>
<td>-0.0003 ± 0.0002</td>
<td>0.000001 ± 0.000001</td>
</tr>
<tr>
<td>1030457</td>
<td>0.00004 ± 0.00008</td>
<td>0.000001 ± 0.000002</td>
</tr>
</tbody>
</table>

Figure 6-13: Projections of the maximum likelihood fit to the 3000 RS events with WS lifetime signal PDF onto $t_{K\pi\pi}$ for the all the events (left) and for the signal region (right). The signal region is $1.845 \text{ GeV}/c^2 < m_{K\pi\pi} < 1.885 \text{ GeV}/c^2$ and $0.053 \text{ GeV}/c^2 < \Delta Q < 0.0063 \text{ GeV}/c^2$.  

77
**Figure 6-14**: Contour plots of $a_1$ vs. $a_3$ (left) and $R_D$ vs. $R_{\text{mix}}$ (right) for RS data fitted with WS lifetime signal PDF. Left plot shows contours of $\Delta \log \mathcal{L} = 0.5, 2.0, 4.5$, representing $1\sigma, 2\sigma, \text{and } 3\sigma$ contours in black, red, and green, respectively. Right plot shows the contours of $\Delta \log \mathcal{L} = 2.0, 4.5$ in solid and dash line respectively.

**Figure 6-15**: Contour plots of $a_2$ vs. $a_3$ (left) and $\alpha y'$ vs. $R_{\text{mix}}$ (right) for RS data fitted with WS lifetime signal PDF. One left plot are contours of $\Delta \log \mathcal{L} = 0.5, 2.0, 4.5$, representing $1\sigma, 2\sigma, \text{and } 3\sigma$ contours in black, red, and green, respectively. On the right plot are the contours of $\Delta \log \mathcal{L} = 2.0, 4.5$ in solid and dash line respectively.
6.5.3 Validation of the WS PDF with High Statistics Signal MC

The samples with different $\alpha y'$ and $(x^2 + y^2)/2$ combinations are produced by generating high statistics WS signal events. One million events are generated for each combination. Each combination has a value of $(x^2 + y^2)/2$ around or larger than the current mixing search sensitivity of about 0.00005. The generated and fitted values are shown in Table 6.4.

**Table 6.4:** Generated and fitted mixing parameters for WS signal events.

<table>
<thead>
<tr>
<th>events</th>
<th>$\alpha y'$</th>
<th>$\alpha y'_{fit}$</th>
<th>$x^2 + y^2/2$</th>
<th>$x^2 + y^2/2_{fit}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000000</td>
<td>-0.0061538</td>
<td>-0.006280 ± 0.000196</td>
<td>0.00020</td>
<td>0.000204 ± 0.000005</td>
</tr>
<tr>
<td>1000000</td>
<td>0.0061538</td>
<td>0.006524 ± 0.000537</td>
<td>0.00020</td>
<td>0.000194 ± 0.000008</td>
</tr>
<tr>
<td>1000000</td>
<td>-0.0053292</td>
<td>-0.005337 ± 0.000194</td>
<td>0.00015</td>
<td>0.000153 ± 0.000005</td>
</tr>
<tr>
<td>1000000</td>
<td>0.0053292</td>
<td>0.005579 ± 0.000279</td>
<td>0.00015</td>
<td>0.000143 ± 0.000007</td>
</tr>
<tr>
<td>1000000</td>
<td>-0.0043513</td>
<td>-0.004363 ± 0.000190</td>
<td>0.00010</td>
<td>0.000099 ± 0.000005</td>
</tr>
<tr>
<td>1000000</td>
<td>0.0043513</td>
<td>0.004264 ± 0.000261</td>
<td>0.00010</td>
<td>0.000103 ± 0.000007</td>
</tr>
<tr>
<td>1000000</td>
<td>-0.0030769</td>
<td>-0.002959 ± 0.000191</td>
<td>0.00005</td>
<td>0.000048 ± 0.000005</td>
</tr>
<tr>
<td>1000000</td>
<td>0.0030769</td>
<td>0.003206 ± 0.000242</td>
<td>0.00005</td>
<td>0.000047 ± 0.000006</td>
</tr>
</tbody>
</table>
6.5.4 Validation of the WS PDF with Reweighted Signal MC

The samples with different $a_2, a_3$ combinations are produced by reweighting the signal MC events. The true lifetime without mixing has a distribution $f(t)$ shown in Eq. 6.5.22, while the distribution with mixing $g(t)$ is given in Eq. 6.5.23. For each signal MC event, we select the event that satisfies $w < \frac{g(t)}{f(t)}$, where $w$ is a random number with uniform distribution between 0 and 1. In order to get the largest statistics, $g(t)$ could be multiplied by a constant scale provided $\alpha g(t) < f(t)$ for all true lifetimes $t$.

\[
f(t) \propto e^{-\Gamma t} \quad (6.5.22)
\]

\[
g(t) \propto e^{-\Gamma t} \left( a_1^2 + \frac{a_1 a_2}{1 + a_2^2} \Gamma t + \left( \frac{a_3^2}{4} \right) (\Gamma t)^2 \right) \quad (6.5.23)
\]

We fit these mixing samples using the WS lifetime signal PDF with lifetime and resolution parameters fixed from the fit on RS MC events given in Table 6.1. The generated and fitted mixing parameters are shown in Table 6.5:

<table>
<thead>
<tr>
<th>Events</th>
<th>$\alpha y'$</th>
<th>$\alpha y'_{fit}$</th>
<th>$x^2 + y^2/2$</th>
<th>$x^2 + y^2/2_{fit}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1072429</td>
<td>0</td>
<td>-0.0006 (\pm) 0.0002</td>
<td>0</td>
<td>0.000018 (\pm) 0.000005</td>
</tr>
<tr>
<td>793673</td>
<td>0.001</td>
<td>0.0019 (\pm) 0.0002</td>
<td>0.000002</td>
<td>0.000027 (\pm) 0.000006</td>
</tr>
<tr>
<td>766665</td>
<td>-0.001</td>
<td>-0.0017 (\pm) 0.0002</td>
<td>0.000002</td>
<td>0.000016 (\pm) 0.000005</td>
</tr>
<tr>
<td>343058</td>
<td>0.005</td>
<td>0.0040 (\pm) 0.0004</td>
<td>0.00005</td>
<td>0.000085 (\pm) 0.000011</td>
</tr>
<tr>
<td>723651</td>
<td>-0.005</td>
<td>-0.0053 (\pm) 0.0002</td>
<td>0.00005</td>
<td>0.000062 (\pm) 0.000004</td>
</tr>
<tr>
<td>144520</td>
<td>0.01</td>
<td>0.0086 (\pm) 0.0008</td>
<td>0.0002</td>
<td>0.000024 (\pm) 0.000021</td>
</tr>
<tr>
<td>172404</td>
<td>-0.01</td>
<td>-0.0104 (\pm) 0.0003</td>
<td>0.0002</td>
<td>0.000021 (\pm) 0.000010</td>
</tr>
</tbody>
</table>
6.5.5 Validation of the WS PDF on MC

We also use a MC sample generated with no mixing that is a combination of signal and background WS events with statistics similar to real data. We perform the mixing fit on this dataset using exactly the same procedure as that on the real data. The fitted $a_2$ and $a_3$ are listed in Table 6.6. Figures 6-16–6-18 show the projection plots of fit result in $t_{K\pi\pi}$.

Table 6.6: Fitted $a_2$ and $a_3$ values for MC WS events.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_2$</td>
<td>$0.0016 \pm 0.0016$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$0.0001 \pm 0.007$</td>
</tr>
</tbody>
</table>

Figure 6-16: Projections of the maximum likelihood fit to the WS MC events onto $t_{K\pi\pi}$ for all the data (left) and for the signal region (right). The signal region is $1.845 \text{ GeV}/c^2 < m_{K\pi\pi} < 1.885 \text{ GeV}/c^2$ and $0.053 \text{ GeV}/c^2 < \Delta Q < 0.0063 \text{ GeV}/c^2$. 

81
Figure 6-17: Projections of the maximum likelihood fit to the RS MC onto $t_{K^{\pi\pi}}$ for the $m_{K^{\pi\pi}}$ signal region (left) and for the $\Delta Q$ signal region (right). The $m_{K^{\pi\pi}}$ signal region is $1.845 \text{ GeV}/c^2 < m_{K^{\pi\pi}} < 1.885 \text{ GeV}/c^2$ and $0.0 \text{ GeV}/c^2 < \Delta Q < 0.012 \text{ GeV}/c^2$; the $\Delta Q$ signal region is $1.8145 \text{ GeV}/c^2 < m_{K^{\pi\pi}} < 1.9145 \text{ GeV}/c^2$ and $0.053 \text{ GeV}/c^2 < \Delta Q < 0.063 \text{ GeV}/c^2$.

Figure 6-18: Projections of the maximum likelihood fit to the RS MC onto $t_{K^{\pi\pi}}$ for sideband regions emphasizing the combinatoric background in the lower $m_{K^{\pi\pi}}$ region (left) and in the upper $m_{K^{\pi\pi}}$ region (right). The lower $m_{K^{\pi\pi}}$ region is $1.8145 \text{ GeV}/c^2 < m_{K^{\pi\pi}} < 1.83 \text{ GeV}/c^2$ and $0.0 \text{ GeV}/c^2 < \Delta Q < 0.012 \text{ GeV}/c^2$; the upper $m_{K^{\pi\pi}}$ region is $1.89 \text{ GeV}/c^2 < m_{K^{\pi\pi}} < 1.9145 \text{ GeV}/c^2$ and $0.0 \text{ GeV}/c^2 < \Delta Q < 0.012 \text{ GeV}/c^2$. 
Chapter 7

Mixing Measurement

7.1 Standard Model Fit Results

The final fit for the wrong sign data is shown in Figures 7-1–7-3 projected onto the $t_{K\pi\pi}$ axis for various regions of $\{m_{K\pi\pi}, \Delta Q\}$. The fitted parameters and calculated mixing parameters are shown in Table 7.1–7.2.

Table 7.1: Fitted values of mixing parameters, corresponding to the likelihood maximum with statistical uncertainty only.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0.0584 ± 0.0021</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.5 ± 0.6</td>
</tr>
<tr>
<td>$a_3$</td>
<td>−0.0193 ± 0.0084</td>
</tr>
</tbody>
</table>

Table 7.2: Values of physical quantities, calculated from the fit parameters with statistical uncertainty only.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_D$</td>
<td>(0.341 ± 0.025)%</td>
</tr>
<tr>
<td>$\alpha q'$</td>
<td>−0.0059 ± 0.0059</td>
</tr>
<tr>
<td>$(x^2 + y^2)/2$</td>
<td>(0.019 ± 0.016)%</td>
</tr>
</tbody>
</table>

Contours of $\Delta \log \mathcal{L}$ levels, constructed in terms of physical quantities, are shown in Figure 7-
Figure 7-1: Projection of the WS maximum likelihood fit onto $t_{K^{+}\pi^{-}\pi^{0}}$ for all the data (left) and for the signal region (right), which is defined as: $1.845 \text{ GeV/c}^2 < m_{K^{+}\pi^{-}\pi^{0}} < 1.885 \text{ GeV/c}^2$ and $0.053 \text{ GeV/c}^2 < \Delta Q < 0.063 \text{ GeV/c}^2$.

Figure 7-2: Projections of the WS maximum likelihood fit onto $t_{K^{+}\pi^{-}\pi^{0}}$ for the $m_{K^{+}\pi^{-}\pi^{0}}$ signal region (left) and for the $\Delta Q$ signal region (right). The $m_{K^{+}\pi^{-}\pi^{0}}$ signal region is $1.845 \text{ GeV/c}^2 < m_{K^{+}\pi^{-}\pi^{0}} < 1.885 \text{ GeV/c}^2$ and $0.0 \text{ GeV/c}^2 < \Delta Q < 0.012 \text{ GeV/c}^2$; the $\Delta Q$ signal region is $1.8145 \text{ GeV/c}^2 < m_{K^{+}\pi^{-}\pi^{0}} < 1.9145 \text{ GeV/c}^2$ and $0.053 \text{ GeV/c}^2 < \Delta Q < 0.063 \text{ GeV/c}^2$. 
Figure 7-3: Projections of WS maximum likelihood fit onto $t_{K\pi\pi}$ for sideband regions emphasizing the combinatoric background in the lower $m_{K\pi\pi}$ region (left) and in the upper $m_{K\pi\pi}$ region (right). The lower $m_{K\pi\pi}$ region is $1.8145 \text{ GeV}/c^2 < m_{K\pi\pi} < 1.83 \text{ GeV}/c^2$ and $0.0 < \text{ GeV}/c^2 \Delta Q < 0.012 \text{ GeV}/c^2$; the upper $m_{K\pi\pi}$ region is $1.89 \text{ GeV}/c^2 < m_{K\pi\pi} < 1.9145 \text{ GeV}/c^2$ and $0.0 \text{ GeV}/c^2 < \Delta Q < 0.012 \text{ GeV}/c^2$.

4. The contours correspond to $\Delta \log \mathcal{L} = 1.15, 3.0$, corresponding to the two-dimensional 68.3% and 95.0% confidence intervals, respectively. The likelihood is approximately a Gaussian, which is described by a function of the signed quantity $\text{sgn}(\alpha y') \cdot (x^2 + y^2)/2$, especially for values of mixing with negative interference. Figure 7-5 illustrates this, showing that $\Delta \log \mathcal{L}$ is approximately parabolic except for very small values of the mixing rate.

The results with statistical uncertainties only:

\[ R_M = \left( \frac{x^2 + y^2}{2} \right) = (0.019^{+0.016}_{-0.015})\% \]  \hspace{1cm} (7.1.1)

\[ R_D = (0.341^{+0.019}_{-0.022})\% \]  \hspace{1cm} (7.1.2)

\[ \alpha y' = -0.006^{+0.005}_{-0.005} \]  \hspace{1cm} (7.1.3)

These statistical uncertainties are consistent with the estimates given by MINUIT [28] in Table 7.2.
Figure 7-4: Contours of $\Delta \log L = 1.15, 3.0$, for the size of the interference term as a function of the integrated mixing rate (left) and the DCSD rate as a function of the integrated mixing rate (right).

Figure 7-5: $\Delta \log L$ as a function of the signed mixing rate, for the fit assuming CP conservation.
The measurement of \( R_D \) has not been corrected for efficiency differences between DCSD and CF decays. However, the mixing rate \( (x^2 + y^2)/2 \) is independent of any efficiency correction.

### 7.2 Upper Limit and Consistency with the No-Mixing Null Hypothesis

We estimate a 95\% confidence level upper limit for \( R_M \) from the \( \Delta \log L \) curve in Figure 7-5. This likelihood curve is generated in the same general manner as the contours in Figure 7-4; for a given value of \( \text{sign}(\alpha y') \cdot (x^2 + y^2)/2 \), the two remaining degrees of freedom are varied in order to find the maximal likelihood.

Interpreting the likelihood \( L \) as a probability density function, we choose the limit \( L \) such that

\[
\int_{-L}^{L} L(z)dz = 0.95 \tag{7.2.4}
\]

\[
z = \text{sign}(\alpha y') \cdot (x^2 + y^2)/2
\]

In this manner, we find the following Standard Model upper limit on the mixing rate:

\[
\left( \frac{x^2 + y^2}{2} \right) < 0.048\% \text{ at } 95\% \text{ confidence level.} \tag{7.2.5}
\]

We estimate the consistency of the data set with no-mixing (i.e., the signal significance) using a frequentist method. An ensemble of 1,000 Monte Carlo data sets are generated assuming the no-mixing hypothesis. Each data set has 76336 events (corresponding to the total number of events in the observed WS data set), with each event randomly chosen to be in a particular signal or background category. For each event, values of \( \{m_{K\pi\pi}, \Delta Q, t_{K\pi\pi}\} \) are generated according to the PDF of the particular category. The parameter values of the PDFs (i.e., the shapes) are determined by setting \( \{a_2, a_3\} = \{1, 0\} \) (i.e., no-mixing) and fitting the remaining parameters to
the observed data. This yields an optimal value of $a_1$ for the observed data set under the no-mixing hypothesis. When a simulated data set is generated, the WS PDF is fit to it, allowing $\{a_1, a_2, a_3\}$ to vary. The most likely value of $a_3$ for that simulated data set is recorded.

Using this technique, we find that 43 out of 1000 data sets have best-fit values of $a_3$ that are further from zero than the value obtained from the observed data set. We conclude that the consistency of this data set with no-mixing is 4.3%.

### 7.3 Fit Results Allowing CP violation

To allow for CP violation from sources beyond the Standard Model, the mixing fit is performed separately to the $D^0 \rightarrow K^+\pi^-\pi^-\pi^+$ and $\bar{D}^0 \rightarrow K^-\pi^+\pi^+\pi^-$ samples. The time-dependent decay rate in Equation 6.5.12 is modified according to the transformations

$$
\alpha y' \rightarrow \left| \frac{p}{q} \right|^{\pm 1} (\alpha y' \cos \phi \pm \beta x' \sin \phi)
$$

$$
(x^2 + y^2) \rightarrow \left| \frac{p}{q} \right|^{\pm 2} (x^2 + y^2),
$$

using $+$ for $\Gamma(\bar{D}^0 \rightarrow K^-\pi^+\pi^+\pi^-)/\Gamma(D^0 \rightarrow K^-\pi^+\pi^+\pi^-)$ and $-$ for the charge-conjugate rate. The parameter $\beta$ is defined similarly to the parameter $\alpha$ in Equation 6.5.12, $\phi$ is a CP-violating weak phase, and the complex numbers $p$ and $q$ come from the definition of the mass eigenstates

$$
|D_{1,2} = p|D^0\rangle \pm q|\bar{D}^0\rangle.
$$

Contours of $\Delta \log \mathcal{L} = 1.15, 3.0$, are shown in Figure 7-6 for $D^0 \rightarrow K^+\pi^-\pi^-\pi^+$ and $\bar{D}^0 \rightarrow K^-\pi^+\pi^+\pi^-$ separately. We do not look for direct CP violation in the doubly Cabibbo-suppressed rate because it is a tree-diagram process in the Standard Model. Therefore, any discrepancy between the two samples is attributed to different efficiencies for soft $\pi^+$ compared to $\pi^-$ and to statistical fluctuation. Combining the fit results of the two samples in $\{a_2^+, a_3^+\}$ for $D^0$
Figure 7-6: Contours of $\Delta \log L$ levels for $D^0$ and $\bar{D}^0$ separately. The hatched regions are physically disallowed, as they would imply a level of interference without a correspondingly large level of mixing.

and $\{a_2^-, a_3^-\}$ for $\bar{D}^0$, the following physical quantities are constructed:

\[
\frac{(x^2 + y^2)}{2} = \left| \frac{a_3^+ a_3^-}{2} \right| 
\]

\[
\alpha y' \cos \phi = \left( \frac{1}{2} \right) \left( \sqrt{\frac{a_3^+}{a_3^-}} \left( \left( \frac{a_3^-}{1 + (a_2^-)^2} \right) + \sqrt{\frac{a_3^+}{a_3^-}} \left( \frac{a_3^+}{1 + (a_2^+)^2} \right) \right) \right) 
\]

\[
\beta x' \sin \phi = \left( \frac{1}{2} \right) \left( \sqrt{\frac{a_3^+}{a_3^-}} \left( \left( \frac{a_3^-}{1 + (a_2^-)^2} \right) - \sqrt{\frac{a_3^+}{a_3^-}} \left( \frac{a_3^+}{1 + (a_2^+)^2} \right) \right) \right) 
\]

\[
|p/q| = \sqrt{\left| \frac{a_3^-}{a_3^+} \right|} 
\]

Uncertainties on the four quantities above are estimated by scanning the six-dimensional parameter space to determine the maximal deviations of the quantities from their best-fit values at
\[ \Delta \log L = 0.5. \] The results, with statistical uncertainties only, are

\[
\left( \frac{x^2 + y^2}{2} \right) = (0.017 \pm 0.017)\% \quad (7.3.13)
\]

\[
\alpha y' \cos \phi = -0.006 + 0.008 \quad (7.3.14)
\]

\[
\beta x' \sin \phi = 0.002 + 0.005 \quad (7.3.15)
\]

\[
|p/q| = 1.1 + 0.040 \quad (7.3.16)
\]

### 7.4 Systematic Uncertainties

The systematic uncertainties in the decay-time fit to the WS sample come from quantities that are determined by the RS sample. This includes the decay-time resolution function and the measured lifetime. The decay-time uncertainty selection of

\[ \sigma_t < 0.5 \text{ ps} \quad (7.4.17) \]

is a criterion that may also bias the decay-time distribution, therefore it is investigated. Finally, the mixing results may be biased to the extent that the two-dimensional fit in \( \{m_{K\pi\pi}, \Delta Q\} \) does not correctly separate signal from background. The background shape in \( m_{K\pi\pi} \) is varied in an attempt to quantify this possible bias.

The systematic uncertainty estimation for \( R_D \) and \( \alpha y' \) should also include phase space efficiency differences between the DCSD and the CF decays. This has been done for \( R_D \) by considering the effect of the efficiency correction on the branching ratio. However, as the immediate goal of this work is to measure the mixing rate, efficiency considerations for \( \alpha y' \) are not considered at this time.

The systematic uncertainties are estimated by repeating the analysis with variations designed to quantify possible systematic bias, and taking the uncertainties to be the differences between the
results before and after the each variation. The estimated uncertainties are listed in Tables 7.3–7.4. Overall systematic uncertainties are obtained by adding the individual systematic uncertainties in quadrature. The variations are summarized as following:

- The resolution function is a sum of double Gaussians, where the width of each Gaussian is given by a scale factor multiplied by the per-event estimated decay uncertainty. The two scale factors \( \{k_1, k_2\} \) shown in Eq 6.3.1 are initially determined in a fit to the RS sample, and then they are fixed for the completion of the fit to the decay time distribution. We estimate the uncertainty associated with the resolution function by forcing one of the scale factors \( k_1 \) to be unity, and finding best-fit values for another scale factor \( k_2 \).

- The fitted lifetime is not as accurate as the value listed in the Review of Particle Physics (RPP) [27]. This systematic uncertainty is estimated by setting the lifetime to the value given in RPP.

- The selection on the per-event decay-time uncertainty is investigated by moving the selection criterion from 0.5 ps to 0.6 ps.

- The PDF used to describe the background contribution in the \( m_{K\pi\pi} \) distribution is changed from an exponential to a second-order polynomial. This change allows some fraction of events to be weighted toward background, and so affects the number of events contributing to the mixing signal.

The contributions of each variation to the systematic uncertainties are shown in Table 7.3–7.8. The overall systematic uncertainties are very small compared to the uncertainties derived from the \( \Delta \log \mathcal{L} \) levels, and they contribute insignificantly to the total uncertainty when combined with the statistical uncertainties in quadrature.
Table 7.3: Systematic checks and uncertainties of the mixing-rate measurement.

<table>
<thead>
<tr>
<th></th>
<th>( \frac{x^2 + y^2}{2} )</th>
<th>Statistical uncertainty</th>
<th>Difference from primary result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary result</td>
<td>0.019%</td>
<td>+0.016%</td>
<td>-0.013%</td>
</tr>
<tr>
<td>Decay-time resolution function</td>
<td>0.020%</td>
<td>0.016%</td>
<td>0.001%</td>
</tr>
<tr>
<td>Set ( \tau ) to RPP value</td>
<td>0.018%</td>
<td>0.016%</td>
<td>-0.001%</td>
</tr>
<tr>
<td>Selection requirement ( \sigma_t &lt; 0.6 \text{ ps} )</td>
<td>0.020%</td>
<td>0.017%</td>
<td>0.001%</td>
</tr>
<tr>
<td>2nd-order polynomial ( m_{K\pi\pi} ) background</td>
<td>0.020%</td>
<td>0.016%</td>
<td>0.001%</td>
</tr>
</tbody>
</table>

Table 7.4: Systematic checks and uncertainties of the \( \alpha y' \) measurement.

<table>
<thead>
<tr>
<th></th>
<th>( \alpha y' )</th>
<th>Statistical uncertainty</th>
<th>Difference from primary result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary result</td>
<td>-0.00590</td>
<td>+0.0050</td>
<td>-0.0005</td>
</tr>
<tr>
<td>Decay-time resolution function</td>
<td>-0.00624</td>
<td>0.00562</td>
<td>-0.0003</td>
</tr>
<tr>
<td>Set ( \tau ) to RPP value</td>
<td>-0.00585</td>
<td>0.00595</td>
<td>0.00005</td>
</tr>
<tr>
<td>Selection requirement ( \sigma_t &lt; 0.6 \text{ ps} )</td>
<td>-0.00637</td>
<td>0.00625</td>
<td>-0.0005</td>
</tr>
<tr>
<td>2nd-order polynomial ( m_{K\pi\pi} ) background</td>
<td>-0.00637</td>
<td>0.00581</td>
<td>-0.0004</td>
</tr>
</tbody>
</table>
Table 7.5: Systematic checks and uncertainties of the mixing-rate measurement allowing possible \( CP \) violation.

<table>
<thead>
<tr>
<th></th>
<th>( \frac{x^2 + y^2}{2} )</th>
<th>Difference from primary result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary result</td>
<td>0.017%</td>
<td></td>
</tr>
<tr>
<td>Decay-time resolution function</td>
<td>0.019% 0.002%</td>
<td></td>
</tr>
<tr>
<td>Set ( \tau ) to RPP value</td>
<td>0.017% 0.000%</td>
<td></td>
</tr>
<tr>
<td>Selection requirement ( \sigma_t &lt; 0.6 \text{ ps} )</td>
<td>0.019% 0.002%</td>
<td></td>
</tr>
<tr>
<td>2\textsuperscript{nd}-order polynomial ( m_{K\pi\pi} ) background</td>
<td>0.018% 0.001%</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.6: Systematic checks and uncertainties of \( \alpha \bar{y}' \cos \bar{\phi} \) allowing possible \( CP \) violation.

<table>
<thead>
<tr>
<th></th>
<th>( \alpha \bar{y}' \cos \bar{\phi} )</th>
<th>Difference from primary result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary result</td>
<td>-0.0055</td>
<td></td>
</tr>
<tr>
<td>Decay-time resolution function</td>
<td>-0.0116 -0.0061</td>
<td></td>
</tr>
<tr>
<td>Set ( \tau ) to RPP value</td>
<td>-0.0055 0.0000</td>
<td></td>
</tr>
<tr>
<td>Selection requirement ( \sigma_t &lt; 0.6 \text{ ps} )</td>
<td>-0.0060 -0.0005</td>
<td></td>
</tr>
<tr>
<td>2\textsuperscript{nd}-order polynomial ( m_{K\pi\pi} ) background</td>
<td>-0.0059 -0.0004</td>
<td></td>
</tr>
</tbody>
</table>
Table 7.7: Systematic checks and uncertainties of $\beta \bar{z}' \sin \bar{\phi}$ allowing possible $CP$ violation.

<table>
<thead>
<tr>
<th></th>
<th>Difference from primary result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta \bar{z}' \sin \bar{\phi}$</td>
</tr>
<tr>
<td>Primary result</td>
<td>0.0023</td>
</tr>
<tr>
<td>Decay-time resolution function</td>
<td>0.0080</td>
</tr>
<tr>
<td>Set $\tau$ to RPP value</td>
<td>0.0023</td>
</tr>
<tr>
<td>Selection requirement $\sigma_t &lt; 0.6$ ps</td>
<td>0.0021</td>
</tr>
<tr>
<td>$2^{nd}$-order polynomial $m_{K\pi\pi^0}$ background</td>
<td>0.0023</td>
</tr>
</tbody>
</table>

Table 7.8: Systematic checks and uncertainties of $|p/q|$ allowing possible $CP$ violation.

<table>
<thead>
<tr>
<th></th>
<th>Difference from primary result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$</td>
</tr>
<tr>
<td>Primary result</td>
<td>1.06</td>
</tr>
<tr>
<td>Decay-time resolution function</td>
<td>1.02</td>
</tr>
<tr>
<td>Set $\tau$ to RPP value</td>
<td>1.06</td>
</tr>
<tr>
<td>Selection requirement $\sigma_t &lt; 0.6$ ps</td>
<td>1.05</td>
</tr>
<tr>
<td>$2^{nd}$-order polynomial $m_{K\pi\pi^0}$ background</td>
<td>1.05</td>
</tr>
</tbody>
</table>

### 7.5 Summary

Combining the central values with the estimated uncertainties, and assuming $CP$ conservation, we find

\[
\left( \frac{x^2 + y^2}{2} \right) = (0.019 \pm (0.016/0.015) \text{ (stat)} \pm 0.002 \text{ (syst)}) \% \quad (7.5.18)
\]

\[
< 0.048\% \text{ at } 95\% \text{ confidence level} \quad (7.5.19)
\]

\[
\alpha y' = -0.006 \pm (0.005/0.005) \text{ (stat)} \pm 0.001 \text{ (syst)} \quad (7.5.20)
\]
The data are consistent with the no mixing hypothesis at a confidence level of 4.3%. Allowing the possibility of $CP$ violation, we find

$$\left( \frac{x^2 + y^2}{2} \right) = (0.017^{+0.017}_{-0.016}) \pm 0.003\%$$

$$\alpha \tilde{y}' \cos \tilde{\phi} = -0.006^{+0.008}_{-0.006} \pm 0.006$$

$$\beta \tilde{x}' \sin \tilde{\phi} = 0.002^{+0.005}_{-0.003} \pm 0.006$$

$$|p/q| = 1.1^{+4.0}_{-0.6} \pm 0.1$$

The result is compatible with $D^0 \rightarrow K\pi\pi^0$ result [29] from BABAR, which shows the time-integrated mixing rate $R_M = (0.023^{+0.018}_{-0.014} \text{ (stat.)} \pm 0.004 \text{ (syst.)})\%$, and $R_M < 0.054\%$ at the 95% confidence level, assuming $CP$ invariance. The latest $D^0 \rightarrow K\pi$ result from BABAR [30] shows a evidence for $D^0$ mixing with a result inconsistent with the no-mixing hypothesis with a significance of 3.9 standard deviations using 384 fb$^{-1}$ of $e^+e^-$ colliding-beam data. It will be very interesting to combine these two modes with $D^0 \rightarrow K^+\pi^-\pi^-\pi^+$ to provide a more precise limit for $D^0$ mixing in the future.
Bibliography


