ACCOUNTING RATES OF RETURN UNDER INFLATION

by

Michael F. van Breda

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The American economy, as the Financial Accounting Standards Board (1978) rightly points out, is characterized by privately-owned business enterprises, who rely for their funds on capital markets. The ultimate survival of such an economic system depends upon scarce resources being allocated to those enterprises that can make best use of them i.e., resources should be allocated to the most efficient producers.

This, however, presupposes an ability to measure efficiency, and to compare efficiency across firms. Financial reporting plays a major role in this. Net income is intended by the Board to be a measure of the earning power of an enterprise or, equivalently, of the enterprise's ability to generate future cash flows. This information, combined with information about the inputs that the enterprise requires, yields one measure of efficiency.

Enterprises differ in size, however, and to compare firms analysts and others typically calculate a return. This is sometimes defined as the return on equity (the earnings available for payment to common shareholders divided by common equity.) It is sometimes defined as the return on total assets (total earnings, sometimes including interest expense, divided by total assets.) Numerous other variants exist, such as the use of tangible assets only, but these need not detain us here. For all these various returns we shall use the blanket term of accounting rate of return.

The underlying notion in these return calculations appears to be the concept of an internal rate of return. This derives from the technique of discounting and is that rate that equalizes the present value of a future cash stream and its cost. In simple accept-reject decisions, the higher the internal rate, the greater the net present value of the proposed investment. Given a number of well-known caveats,
which may be found in any finance text, an investment's desirability increases with its internal rate of return. In other words, all other things being equal, one would like resources to flow to those enterprises promising to earn the highest internal rate of return.

To the extent that the accounting rate of return is an adequate surrogate for the internal rate of return, it becomes a useful measure of efficiency and a guide to resource allocation in the economy and firms. Unfortunately, it is a well-known fact that the accounting rate of return (ARR) is a very poor surrogate for the internal rate of return (IRR).

This was first emphasized by Solomon (1956, 1966). The model he used was a discrete one involving straight line depreciation only. With the enterprise in a steady state (either zero or constant growth), he demonstrated that the life of the asset, the growth of the enterprise, and the rate of inflation all had a significant effect on the ARR. He also noted that in one special case, where the rate of growth equals the "true yield," the "book yield" is equal to the "true yield." This, he claimed, without proof, held regardless of depreciation method, capitalization practice, time lags and cash flow pattern. As he pointed out:

If the findings above are valid, and there is no reason to believe that they are not, they present financial analysis with a serious dilemma. On the one hand, the ratio of net income to net book assets is not a reliable measure of return on investment. On the other hand, analysis definitely requires some measure of return on investment and there appears to be no other way in which this concept can be measured for an ongoing division or company.

The pragmatic answer is that book-yield will continue to be used, but that its use must be tempered by a far greater degree of judgement and adjustment than we have employed in the past, and in extreme cases the measure may have to be abandoned altogether in favor of an alternative measure, such as the ratio of cash flow before depreciation to gross book value.
The subject is clearly an important one. The use of accounting rates is ubiquitous. The efficient allocation of resources is fundamental to our economy. The fact that the accounting rate is such a poor substitute for the internal rate should, therefore, be a matter for concern. As a result, Solomon's paper has drawn a number of responses over the years.

At the empirical level, Mauriel & Anthony (1966) indicate that in a survey of 2,658 large companies, 60% reported that they make return-on-investment or residual-income calculations for internal evaluation purposes. Of these, 97% use the same depreciation methods as they use for external reporting. This study was replicated by Reece & Cool (1978). Of their 459 respondents using investment centers, 93% report the use of return on investment to evaluate performance. Only 7% of these companies use a depreciation figure that differs from their reported depreciation. A mere 2% adjust for inflation by using replacement cost data. As Mauriel and Anthony comment:

The reason this finding is so significant is that these accounting methods can cause serious distortions in the validity and usefulness of the divisional ROI or residual-income measures. These distortions have far reaching implications. These distortions, as we shall see, are considerably larger under inflation.

At the more theoretical level, the ultimate word on the subject seems to have been written by Stauffer (1971). Making use of continuous models, he demonstrates that under price certainty, but with quite arbitrary cash revenue streams, the accounting rate of return converges to the economic (or internal) rate of return as the growth converges to the economic rate. (This is a more general proof of Solomon's earlier result.) He also demonstrates, quite generally, that given economic (or interest based) depreciation the accounting rate equals the economic rate.
Prior to this Livingstone and Salomon (1970) had derived an interesting set of simulation results. Accounting rates were differentiated by the proportion of profit that was reinvested. (This corresponds to the different growth rates in Stauffer's work.) As a new level of reinvestment is established, so a new accounting rate of return is determined. The path to this new equilibrium rate was found to be oscillatory.

Van Breda & Livingstone (1976) later showed analytically that, when all profits are reinvested, the accounting rate of return converges to the economic (or internal) rate. Van Breda (1978) demonstrated that this case corresponds to the Golden Rule of classical and modern growth economics and to growth at the internal rate of return.

Harcourt (1969) examined the behavior of the accounting rate under several plausible quasi-rent streams. He concluded, finally, that

The implications of the analysis of the article are rather disheartening. It had been hoped that some rough 'rules of thumb' might be developed; and that these would allow accounting rates of profit to be adjusted for the lengths of life of machines, the patterns of quasi-rents, rates of growth, and the method of depreciation used. However, it is obvious from the calculations that the relationships involved are too complicated to allow this.

As we shall see, the prognostication should not be all that gloomy.

Two broad comments may be made about all these papers and others in the genre. First, almost without any exception they have suggested that no simple relationship exists between the ARR and the IRR. Second, with few exceptions they have simply ignored the fact of inflation. A few definitions and a single equation enable us to remedy both deficiencies. These may be found in the Appendix which generalizes the above findings to the inflation case.
Equilibrium under Inflation

Theorem 1: If the real growth of assets purchased equals the real rate of interest, then the accounting rate of return will equal the nominal rate of interest, i.e.,

\[ g = (1 + \delta) \Rightarrow \text{ARR} = i \]

\[ g = 1 + \text{real rate of growth} \]
\[ \delta = \text{real rate of interest} \]
\[ i = \text{nominal rate of interest} \]

Alternatively stated, we have the general result that if the nominal rate of growth in capital investments is equal to the nominal rate of interest, then the accounting rate of return will settle out at the nominal rate of interest. Clearly, imbedded in this theorem we have the earlier results alluded to above that in an inflation free environment the accounting rate will equal the market rate if the firm grows at the market rate of interest.

Most of the articles on the accounting rate of return have been content to leave matters pretty much at this point. The result obtained here has, typically, been categorized as a special result of little interest: a mathematical curiosity perhaps. However, it turns out that a number of comments are in order.

First, it should be noted that the result is strikingly general from an accounting point of view. Assets of different lives can be intermingled without affecting the outcome. Accrual accounting can be used. The depreciation and amortization pattern is completely general. Literally any accounting practice can be followed, provided it is followed consistently. All this is very simple to demonstrate and details may be found in Van Breda (1978).

Second, it appears that the so-called special case has some very interesting economic properties. To generate growth at the rate of
interest it is necessary to reinvest all profits. Revenue generated during year $t$ is:

$$ R_t = \sum_{s=1}^{n} g^{t+1} x^t v^{n-s+1} $$

$$ = g^{t+1} x^t p_o $$

$$ = g^{t+1} p_t $$

But, by the assumptions of the model, this is precisely the amount of capital investment the firm undertakes at the end of year $t$. Interestingly, this is true both in an inflation-free and in an inflationary environment.

In other words, to generate this case, we assume that all wages are consumed and all profits reinvested. This behavioral rule is the classical rule of economics! It has been called the Golden rule of accumulation and underlies all of the dynamic models of the economy that were constructed in the eighteenth and nineteenth century. After a fairly long eclipse by the stationary assumptions of textbook price theory, we have seen a resurgence in interest in this "special" case. Economists such as Solow (1963), Samuelson (1962), Robinson (1964), and Pasinetti (1969) have all written papers explicating the characteristics of an economy that grows at the rate of interest.

The fundamental property of a firm that grows at the rate of interest is that it permits maximum consumption to the wage earners. At the economic level the Golden rule implies maximum consumption for all members of the economy. Whether a firm, or the economy as a whole, moves onto this particular growth path is a matter for empirical investigation. From a purely theoretical point of view though, this growth path provides some very clean cut results, which may be used as a basis of comparison with other paths involving other behavioral assumptions.
This is the essence of the equilibrium method. Interpreted in one way equilibrium is a state to which a system is tending. Interpreted in another way, as Shackle (1961) reminds us, equilibrium is a state with well-known and easily derivable properties, from which divergencies may be measured. This is its major use here.

Restated, the term equilibrium is used in this paper to describe a methodology rather than an empirical state. There is no claim made here that the firm tends to a growth rate equal to the rate of interest. The claim is made, however, that this growth path is analytically interesting. It forms the departure point for an analysis of all other growth paths. In short, it, rather than the zero growth path, is the base case.

**Some Disequilibrium Results**

Using the equilibrium methodology, one is is able to arrive at several interesting results that capture the behavior of the ARR for growth rates other than that of the base case. Formally, these may be stated as two theorems. Proofs of each may be found in the Appendix.

**Theorem 2:** For all methods of accounting, for all asset lives, and for all rates of inflation, we have

\[ \text{ARR}_t \gtrless i \quad \text{whenever} \quad g \lesssim 1 + \rho \]

\[ \text{ARR}_t \lesssim i \quad \text{whenever} \quad g \gtrsim 1 + \rho \]

The divergence of the accounting rate from the nominal rate of interest increases with the divergence of the growth rate from the rate of interest.

**Theorem 3:** The divergence of the accounting rate from the nominal rate of interest increases with the life of the assets. This holds for all rates of growth.
Thus, in contradistinction to Harcourt, who claimed that no useful relationships exist, we do indeed have several most interesting relationships captured in Theorems 2 and 3. These relationships are illustrated in Figure 1. The wide disparities between accounting rates of return engendered by differing depreciation methods, inflation, and varying asset lives, all disappear at the Golden rule growth point, and then reappear reversed as growth rates become very high.

Graphs such as these could be used in company evaluations. Given a suitable market rate of interest the firm's nominal growth rate, and the life of its assets, one could read off the graph, the resulting accounting rate of return. A firm whose accounting rate was above this would be making more than the rate of interest and one whose actual rate plotted below this would be making less.

Clearly, we could do the same by using an equation. The advantage of the graph is its visual demonstration of a sliding-down phenomenon in rates as growth increases. It emphasizes too the fact that since most companies have growth rates which are below the nominal rate of interest, that accounting rates are in general biased upwards from the true rate.
Furthermore, it is apparent that inflation biases all accounting rates upwards. Moreover, with low rates of growth, the dispersion of accounting rates is larger the higher the rate of inflation. For example, given a real interest rate of 3%, and zero-growth companies we have the following steady state accounting rates:

<table>
<thead>
<tr>
<th>Asset life</th>
<th>2%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5.20</td>
<td>8.42</td>
<td>13.79</td>
</tr>
<tr>
<td>10</td>
<td>5.39</td>
<td>8.78</td>
<td>14.47</td>
</tr>
<tr>
<td>20</td>
<td>5.77</td>
<td>9.57</td>
<td>16.00</td>
</tr>
<tr>
<td>50</td>
<td>7.08</td>
<td>12.43</td>
<td>21.77</td>
</tr>
</tbody>
</table>

Nominal rate of interest: 5% 8% 13%

Accounting rates vary here from a low of 5.20% to a high of 21.77% while the real internal rate remains a steady 3%. The range is 1.88% at 2% inflation. This rises to 7.97% at 10% inflation.

If we maintain the real interest rate at 3%, but now allow for real growth of 5% we have the following steady state accounting rates:

<table>
<thead>
<tr>
<th>Asset life</th>
<th>2%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4.97</td>
<td>7.98</td>
<td>12.99</td>
</tr>
<tr>
<td>10</td>
<td>4.86</td>
<td>7.77</td>
<td>12.61</td>
</tr>
<tr>
<td>20</td>
<td>4.67</td>
<td>7.39</td>
<td>11.88</td>
</tr>
<tr>
<td>50</td>
<td>4.26</td>
<td>6.56</td>
<td>10.33</td>
</tr>
</tbody>
</table>

Nominal rate of interest: 5% 8% 13%

The accounting rates are all smaller now than above. For example, with a life of 20 years and an inflation rate of 5%, real growth of 5% causes the accounting rate to fall from 9.57% to 7.39%. This is below the
nominal rate of interest of 8% because the real growth rate of 5% now exceeds the real interest rate of 3%. This has the additional effect of causing accounting rates to fall as the life of the asset increases. This fall should be compared with the increase in rates above, due to a zero rate of growth. Finally, it should be noted that while the range of rates increases with increasing inflation, the range decreases as one approaches closer to the Golden rule equilibrium point.

The disturbing feature of this analysis is that low-growth companies will show higher accounting rates of return than fast-growing firms, ceteris paribus. This, by itself, is not necessarily bad. Often, however, growth is accompanied by higher economic returns, which may not be sufficient to offset the effect of growth on the accounting rate. As a result, capital might flow to those companies (or divisions of companies) that are earning lower economic rates of return.

**Current Cost Adjustments**

This problem is only partly mitigated by the use of replacement cost data - at least, in the manner in which it is usually calculated. This entails establishing the current cost of each asset. This cost is then depreciated by one of the usual accounting methods to yield the net book value at current cost and the current cost of depreciation.

It is trivial to demonstrate that the resulting model is identical to that derived by Stauffer and others for the non-inflation case. Details are in the Appendix. They reveal that the accounting rate as adjusted for inflation will tend to the real rate of interest as real growth converges to the real rate of interest. Equality will only be achieved when the growth rate equals the real interest rate (or if economic depreciation is used). For all other points, the accounting rate will be biased, with the degree of bias a function of the growth rate (and the
life of the assets).

Clearly, once the asset values have been adjusted to replacement costs, one should compare the accounting rate with the real rate of interest. This is probably of the order of 3%. By this token, Ford Motor Company earned 4.4% in 1976, which is, very approximately, allowing for growth and the life of their assets, where one would expect them to be. What is not clear, is whether analysts, who do this adjustment, realize that the figure for comparison is the real rate of interest, and no longer the nominal rate of interest.

The entire problem vanishes when one uses the market price of old assets. The book value would then be the market value and the depreciation (or appreciation) would be the change in this value over the year. The result would be a formula of the form

\[
ARR_n = \sum_{s=1}^{n} x^n g^s - \sum_{s=1}^{n} (u_{n-s} - u_{n-s+1})g^s \\
\sum_{s=1}^{n} u_{n-s} g^s
\]

where \( u_s = \) market value of an asset with life \( S \) at the end of year \( n \).

Note that we are using start-of-period assets at end of year costs.

It is well-known that

\[
u_{n-s} - u_{n-s+1} = x^n - u_{n-s}
\]

and, therefore,

\[
ARR_n = \sum_{s=1}^{n} x^n g^s - \sum_{s=1}^{n} (x^n - u_{n-s})g^s \\
\sum_{s=1}^{n} u_{n-s} g^s
\]

\[= f\]
This assumes, of course, that there is no technological obsolescence or other unexpected events which would yield windfall gains or losses. Unfortunately, the method is probably infeasible since so few used assets are traded. This leaves us with our earlier formulations and all the problems that have been noted in the past.

Conclusion: This paper set out to add two results to the literature on accounting rates of return. First, it demonstrated the impact of inflation on the ARR. One of its effects is to widen the dispersion of accounting rates, particularly for slow-growing firms. Its other effect is to cause the accounting rate to converge to the nominal, as opposed to the real, rate of interest, as the growth rate tends to the rate of interest. This result does not appear to have been noted in the literature.

The second thrust of the paper was to point out that a crude visual relationship can be established between the accounting rate and the internal rate. While this does not obviate the problem, it does highlight, graphically, the effect of accounting on the underlying rate of profit. It is quite apparent, for instance, that there is a tradeoff between the accounting rate of return and growth, quite independently of the underlying economic rate. This does not appear to have been noted in the literature either.

These results are quite general. The accounting rate is always equal to the internal rate at the point that the growth rate equals the interest rate, regardless of the accounting methodology used. Assets of different lives may be comingled and any form of depreciation, inventory accounting, or capitalization may be used, provided they are used consistently through time.
The results do not extend, however, to the dynamic case. Everything that has been said in this paper, and those referenced, deals with long-run, steady-state situations. A companion piece to this article deals with the behavior of the accounting rate of return under unexpected inflation and changes in growth rates. This paper had the more limited purpose of adding to our knowledge on steady states.
Mathematical Appendix

Theorem 1: Assume we have a firm that is in a steady-state growth situation. Assume further a steady, constant rate of inflation. Allow the life of the assets to be fixed and the quasi-rent generated from these assets to be uniform. Then we may define the following symbols:

- \( p_s \) = initial cost of an asset in year \( s \)
- \( q_t \) = quasi-rent generated by an asset in an inflation free environment
  
  \[ = 1 \text{ by assumptions above} \]
- \( \rho \) = real market rate of interest
  
  \[ = \text{real internal rate of return under perfect competition and free entry} \]
- \( x \) = 1 + rate of inflation
- \( i \) = nominal rate of interest
  
  \[ = (1 + \rho)x - 1 \]
- \( n \) = life of the asset

Using these symbols we have:

\[
P_o = \sum_{s=1}^{n} x^s (1+i)^{-s} = \sum_{s=1}^{n} (1+\rho)^{-s} \quad \ldots \quad (1)
\]

We also have

\[
P_t = \sum_{s=1}^{n} x^t \cdot x^s (1+i)^{-s} \quad \ldots \quad (2)
\]

Define further

- \( g = 1 + \text{rate of real growth in capital investments} \)
- \( w_{i+1} = \text{book value in year } (t+1) \text{ of the asset purchased in year } t \text{ where } w_o = 1 \text{ and } w_n = 0 \)
- \( D_t = \text{annual depreciation in year } t \)
- \( B_t = \text{book value at the start of year } t \)
- \( P_{i+1} = \text{initial cost in year } (t+1) \)
It follows that for $t \geq n$ we have by definition

$$R_t = \sum_{s=1}^{n} g^{t-n} x^t g^s$$

$$D_t = \sum_{s=1}^{n} g^{t-n} (w_{n-s} - w_{n-s+1}) p_{s-1} g^s$$

$$B_t = \sum_{s=1}^{n} g^{t-n} w_{n-s} p_{s-1} g^s$$

Combining these we have

$$\text{ARR}_t = \frac{\sum_{s=1}^{n} x^w g^s - \sum_{s=1}^{n} (w_{n-s} - w_{n-s+1}) p_{s-1} g^s}{\sum_{s=1}^{n} w_{n-s} p_{s-1} g^s}$$

It should be noted that this is independent of $t$ i.e., once $t \geq n$ the accounting rate of return settles down to a steady rate. This expression can be further simplified since

$$\sum_{s=1}^{n} w_{n-s+1} p_{s-1} g^s = \sum_{s=2}^{n} w_{n-s+1} p_{s-1} g^s = \sum_{s=2}^{n+1} w_{n-s+1} p_{s-1} g^s - p_n g^{n+1} = xg \sum_{s=1}^{n} w_{n-s} p_{s-1} g^s - p_n g^{n+1}$$

whence

$$\text{ARR}_t = \frac{x^n (\sum_{s=1}^{n} g^s - p_0 g^{n+1}) - (1 - xg) \sum_{s=1}^{n} w_{n-s} p_{s-1} g^s}{\sum_{s=1}^{n} w_{n-s} p_{s-1} g^s}$$
The theorem follows easily and simply from (7) since with \( g = v^{-1} \) we have

\[
\text{ARR}_t = x^n g^{n+1} \left( \sum_{s=1}^{n} v^{n-s+1} - p_o \right) - (1-xg) \sum_{s=1}^{n} w_{n-s} p_{s-1} g^s \\
\sum_{s=1}^{n} w_{n-s} p_{s-1} g^s
\]

where \( v = (1 + f)^{-1} \)

But \( p_o = \sum_{s=1}^{n} v^s \)

\[
= \sum_{s=1}^{n} v^{n-s+1}
\]

and, therefore

\[
\text{ARR}_t = (xg-1) \sum_{s=1}^{n} w_{n-s} p_{s-1} g^s \\
\sum_{s=1}^{n} w_{n-s} p_{s-1} g^s
\]

\[
= xg-1
\]

\[
= (1 + \text{rate of inflation}) (1 + \text{real rate of interest}) - 1
\]

\[
= \text{nominal rate of interest } i
\]

Theorem 2: This is a simple extension of (7) since

\[
\text{ARR}_t = x^n \left( \sum_{s=1}^{n} g^s - p_o g^{n+1} \right) + (xg-1) \\
\sum_{s=1}^{n} w_{n-s} p_{s-1} g^s
\]

\[
= x^n g^{n+1} \left( \sum_{s=1}^{n} g^{s-n-1} - p_o \right) + (xg-1) \\
\sum_{s=1}^{n} w_{n-s} p_{s-1} g^s
\]
Theorem 3: A general proof of this theorem does not appear to be possible.

Proofs for each type of depreciation are not difficult, although tedious.

The cleanest case involves straight-line depreciation and an inflation-free environment. A sketch of this proof is, therefore, provided. We have for this case

\[ x = 1 \]

and \( w_{n-s} - w_{n-s+1} = 1/n \)

Substituting these into (6), and simplifying, we have

\[
 r_n = \frac{w - p}{q} n \cdot \frac{L_s^n g_s}{L_s^n q_s}
\]

where \( p_n = \sum_{s=1}^{n} v^s \)

i.e., the price of an asset whose life is \( n \) years, and \( r_n \) is the accounting rate of return on assets with lives of \( n \) years.

But, \[ \frac{n - p}{q} n = \frac{(1 - v)}{v} \sum_{s=1}^{n} \theta_s \]

where \( \theta = (1 + \rho) \)

whence (12) reduces to

\[
 r_n = \frac{\rho \sum_{s=1}^{n} s \theta^s}{\sum_{s=1}^{n} s \theta^s} \cdot \frac{\sum_{s=1}^{n} g^s}{\sum_{s=1}^{n} \theta^s}
\] ...

(13)

One can now use the notion of equilibrium to define

\[ (1 + \rho) = (1 + \delta) g \]

The theorem then reduces to

\[ r_n > r_{n-1} \quad \text{for all } \delta > 0 \]

\[ r_n < r_{n-1} \quad \text{for all } \delta < 0 \]
It is immediately apparent, incidentally, that if \( \delta \) is zero, we have the result of Theorem 1 again. If one explodes (13) in terms of \( \delta \) using the method of undetermined coefficients and relying on \( \delta \) declining in orders of magnitude, the proof of Theorem 3 is quite straightforward. Cleaner proofs exist, no doubt. These rather dirty proofs have the sole virtue of working.

Current cost analysis: Analytically, all that is entailed as one switches from historical to current cost is the substitution of \( p_n \) for \( p_{s-1} \) in (6). This yields

\[
\text{ARR}_t = \frac{\sum_{s=1}^{n} x \cdot g^s - \sum_{s=1}^{n} (w_{n-s} - w_{n-s+1}) p_n \cdot g^s}{\sum_{s=1}^{n} w_{n-s} \cdot p_n \cdot g^s} = \frac{\sum_{s=1}^{n} g^s - p_o \sum_{s=1}^{n} (w_{n-s} - w_{n-s+1}) \cdot g^s}{p_o \sum_{s=1}^{n} w_{n-s} \cdot g^s} \quad \ldots \quad (14)
\]

By analogy with equation (7) this simplifies to

\[
\text{ARR}_t = \left( \sum_{s=1}^{n} g^s - p_o \cdot g^{n+1} \right) \cdot \frac{1}{p_o \sum_{s=1}^{n} w_{n-s} \cdot g^s} + (g-1) \quad \ldots \quad (15)
\]

This is the result derived by Stauffer and others for the non-inflation case.
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ACCOUNTING RATES OF RETURN

$L = $BOOK LIFE OF FIXED ASSETS

ASSUMED RATE OF INFLATION = 5%

NOMINAL RATE OF GROWTH IN ASSETS