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ALFRED P. SLOAN SCHOOL OF MANAGEMENT

ADVERTISING BUDGETING AND GEOGRAPHIC ALLOCATION

Glen L. Urban

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139
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ABSTRACT

This paper presents a model to aid in determining a total advertising budget and allocating it to a national media and local expenditure in each of a number of heterogeneous geographic areas. The model includes consideration of area differences in advertising response, media efficiency and impact, distribution levels in areas, and carryover effects. A "good" and hopefully "best" budget and allocation is found by a multi-stage, on-line heuristic. An application of this model indicates that it has the potential to improve decisions and lead to implementation.

INTRODUCTION

When an organization views its market as a group of heterogeneous geographic segments, the problem of determining how much to spend on advertising becomes a problem of simultaneously budgeting and allocating funds. The solution to this problem is a local advertising strategy. The purpose of this paper is to develop and demonstrate a model to aid in determining local marketing strategy.

The problem of overall budgeting for advertising has received much attention, and some game theoretical aspects of allocation have been discussed. But there is evidence to indicate that this theory is not being utilized in allocating promotional expenditures. The first criteria for this author's model is that it lead to application. The second criteria

1The author gratefully acknowledges that the programming for the system was done by Jay Wurts.

2See [4], pp. 115-137 for discussion of existing theory.

3See [3].
is that the model include the major theoretical and behavioral phenomena important in this problem. These include differences in area potential, growth rates, advertising responsiveness, media costs and efficiencies, profit margins, competition, and carryover. The third criteria is that the model be a starting point for an evolutionary system that would lead from the initial aggregate one variable model to a more detailed multivariate model. The final criterion is that the model structure lead to data based parameter estimates.

In defining the specific problem to be attacked by the model a clear distinction must be drawn between the budgeting decision and other advertising decisions. First, it is assumed that the appeal has been selected and that a reference state defined by a starting point forecast of sales for a starting point budget have been determined. The problem to be studied is how to find the best total budget and geographic allocation of that budget given the appeal and reference state conditions. After these budget decisions have been made, a media schedule would be developed and executed. A media model would not solve the budgeting and allocation problem since most media models do not link their schedules to the sales and profit criteria of budgeting. A model such as MEDIAC which makes this linkage also would be unsuitable since in a market with say 30 local segments, the number of media options would be much beyond the capacity of the media model. If a simplification of one media per area were made, the media model might be used to examine allocations, but the media model overhead in considering exposure and duplication would make this procedure inefficient.

^See [2].
The interaction between media models and the budgeting model developed in this paper would be iterative. The budgeting model would specify a dollar allocation for each area assuming an average set of media performance characteristics. Then the media model would select a best schedule subject to this budget constraint. If the schedule had better average characteristics than the budgeting model had assumed, the budgeting model would be re-run with the new characteristics and the iteration would continue until the budgeting assumptions and media schedule results agreed.

This paper will develop the model for local advertising response, specify the model's input needs, discuss the heuristic procedure used to find the best strategy, outline the model's limitations and extensions, and present a real application of the model.

LOCAL STRATEGY MODEL

Local Advertising Response:

The first step in structuring local advertising response is to define an operational measure of the advertising pressure that exists in an area. Consider the basic flow of advertising expenditure. Part of the total budget goes to national media with some share of this falling on a local area and part directly to a local area. See Figure One. The advertising that is seen in a local area is either from national or local media. The concept of weighted advertising is used to define a common measure for advertising pressure on an area. Weighted advertising is defined as the share of national expenditure in an area plus the local expenditure weighted for media exposures, impact, and efficiency. The
FIGURE 1: Advertising Flow
weighting is based on a ratio of the rated exposure value per dollar in the local area to the rated exposure value per dollar in national media.

The rated exposure value is the sum of the exposures in each media multiplied by a media rating which reflects its impact given someone is exposed. The rated exposure value per dollar is the rated value divided by the advertising expenditure required to obtain it. The weighted advertising in a local area is:

\[
\text{WTADVL}_{a,r,t} = \text{ADVNAV}_{t} \cdot \text{SHRLOC}_{a,r,t} \cdot \text{EFFNAV}_{t} + \text{ADVLOC}_{a,r,t} \cdot \frac{\text{REVLD}_{a,r,t} \cdot \text{RIMPCT}_{a,r,t}}{\text{REVND}_{t}}
\]

\(\text{WTADVL}_{a,r,t}\) = weighted advertising in local area \(a\) of region \(r\) in time period \(t\)

\(\text{ADVNAV}_{t}\) = dollar expenditure on national media in period \(t\)

\(\text{SHRLOC}_{a,r,t}\) = share of national media expenditure that falls in area \(a\) of region \(r\) in period \(t\)

\(\text{EFFNAV}_{t}\) = effectiveness of national media at time \(t\). (Set to 1.0 initially).

\(\text{ADVLOC}_{a,r,t}\) = local advertising dollar expenditure in area \(a\) in region \(r\) in period \(t\)

\(\text{REVLD}_{a,r,t}\) = rated exposure value locally per dollar in local area \(a\) of region \(r\) in period \(t\)

\(\text{REVND}_{t}\) = rated exposure value nationally per dollar in period \(t\)

\(\text{RIMPCT}_{a,r,t}\) = impact of local media relative to national media in area \(a\) of region \(r\) in period \(t\). (Set initially to 1.0)

\(\text{EFFNAV}\) is an index which can be used to change the effectiveness of national media expenditure and \(\text{RIMPCT}\) is an index which can be used to change the impact of local expenditure relative to national. For example, these indexes may be used if new media opportunities appear. The regional subscripts are defined so that allocations can be conducted on only specific regions if this is desired.
Having defined a measure of advertising pressure in a local area, the next task is to specify how it affects sales. A general function is used to relate weighted advertising and sales. Initially, sales are specified by:

\[ (2) \quad T_{SALEL_{a,r,t}} = R_{SALEL_{a,r,t}} \cdot A_{DRSP_{a,r}} \left( \frac{W_{ADV_{a,r,t}}}{W_{ADV_{L_{a,r,t}}}} \right) \]

- \( T_{SALEL_{a,r,t}} \) = temporary forecast of sales in local area \( a \) of region \( r \) in period \( t \)
- \( R_{SALEL_{a,r,t}} \) = reference sales in local area \( a \) of region \( r \) in period \( t \) if reference advertising plan is carried out
- \( A_{DRSP_{a,r}} \) = function describing proportionate changes in sales from reference \( \left( R_{SALEL_{a,r,t}} \right) \) for a change in weighted advertising \( \left( W_{ADV_{a,r,t}} \right) \) relative to the weighted advertising implied by the reference plan \( \left( W_{ADV_{L_{a,r,t}}} \right) \)

The function \( A_{DRSP} \) will generally experience diminishing returns, but must be determined for each area. Given this curve, the change in sales from reference can be predicted for changes of advertising from the reference. The forecasted sales level \( (T_{SALEL}) \) is next adjusted for distribution effects and carryover effects.

Although the model is primarily an advertising model, the distribution differences between areas are included. This is a first step in the evolutionary development that will eventually lead to a full marketing mix local strategy model. With distribution included, changes in distribution can be reflected in the best advertising strategy. Distribution is included in the model by adjusting sales to full distribution sales and then multiplying by the percent of the market covered by an alternate distribution plan. The adjusted sales level \( (A_{SALEL}) \) is:
The distribution level is defined as the percent of outlets that carry
the product and is functionally related to the percentage coverage by AVAIL.

The phenomenon of carryover effects is included in the model by
specifying how much of the change in sales from reference in one period is
retained in the next period. The model considers the carryover in terms of
changes since the starting point plan implies the carryover for the reference
allocation. The change in sales in area \( a \) in region \( r \) and period \( t-1 \) is:

\[
DSALEL_{a,r,t-1} = SALEL_{a,r,t-1} - RSALEL_{a,r,t-1}
\]

\[
SALEL_{a,r,t-1} = \text{sales in area } a \text{ in region } r \text{ in period } t-1. \quad (\text{See Eq. 5}).
\]

\[
RSALEL_{a,r,t-1} = \text{reference sales in local area } a \text{ in region } r \text{ in period } t-1 \text{ if reference advertising plan is carried out.}
\]

The amount of this sales change that is retained from the last period is:

\[
RETSLL_{a,r,t-1} = DSALEL_{a,r,t-1} \cdot RETRT_{a,r}
\]

\[
RETSLL_{a,r,t-1} = \text{retained sales in area } a \text{ of region } r \text{ from period } t-1
\]

\[
RETTR_{a,r} = \text{retention rate for area } a \text{ of region } r
\]

The retention rate reflects the brand loyalty of buyers and the competitive
conditions in the local area. The retained sales will effect the sales of
the next period. The model reduces this period's sales by the proportion of
retained sales to last period's sales to obtain the final sales level (SALEL):
This proportionate change assumes that the original seasonal pattern of reference sales is maintained since it presumes the retained sales follow the seasonal pattern.

Given the above relationships the model will determine the sales in an area given an advertising and distribution strategy. The profit in a local area is:

\[
PROFL_{a,r,t} = SALEL_{a,r,t} \cdot PMARGN_{a,r,t} - ADVLOC_{a,r,t} - SFC_{a,r,t}
\]

\[
PMARGN_{a,r,t} = \text{contribution margin in area } a \text{ of region } r \text{ and time } t
\]

\[
SFC_{a,r,t} = \text{semi-fixed costs in area } a \text{ of region } r \text{ in period } t
\]

The national profit is the total of the profit in each area less national media expenditure (ADVNAT) and the total profit over a planning period is obtained by summing national profit.

The model will determine sales and profits for alternate advertising strategies when given a starting point sales-advertising plan and the response measures.

**Input to Model:**

The parameters required by the model are given in Figure Two, along with the source of the input. This data is stored in the data bank and processed by statistics to determine model input. It is beyond the scope of this paper to discuss the statistical issues of estimation, but it is clear that careful experimental design and statistical regression
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*Equation numbers

FIGURE 2: Model Input
are required. Even after the most careful statistics, subjective judgments may also be required in specifying the best set of input.

Heuristic Solution Procedure:

Since the proposed model is a multiperiod model with carryover effects and has a complex response mechanism (i.e. dollars to weighted advertising to sales) the algorithmic procedures of linear and non-linear programming are not applicable to the problem of finding the best budget and allocation. To attempt to find the "best" budget and allocation, a multi-stage interactive heuristic is used. The heuristic solution procedure exists at three levels: "allocate", "split" and "search". At each level the man interacts with the procedure so that the knowledge of the manager and the power of the search rule can be combined to efficiently find the best strategy.

"Allocate" takes the total budget and national advertising as given and allocates the total local expenditure to each area in an attempt to maximize the total profit over the planning period. The steps of "allocate" are

1. Calculate the marginal rewards to local advertising (ADVLOC in Eq. 1) in each area. That is, additional profit generated by changing local advertising expenditure. This is generated by removing a percent of advertising (DELALL) in an area in each time period, calculating the change in total profits over the planning period in that area, and dividing by the total value of the advertising taken out. The amount of advertising change is summed over the planning period and is called TMOVED.

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5 See application section of this paper for an example of the estimation procedure.
2. In each area check to see if moving on "S" shaped response curve by evaluating marginal reward for 2 x DELALL reduction in local advertising. If marginal for 2 x DELALL is less than marginal for 1 x DELALL, remove area from consideration unless management is willing to abandon local area. If marginal for 2 x DELALL is greater than 1 x DELALL, add area to list for consideration.

3. Find the areas with the largest and smallest marginals on list of areas to be considered. Define $T_{MOVED_L} = T_{MOVED}$ for low area (L) and $T_{MOVED_H} = T_{MOVED}$ for high area (H).

4. If the high marginal minus the low marginal as a percent of the absolute value of the high marginal is greater than the tolerance (TOLALL), go to 5. If not, stop.

5. Is new high area the same as the low area at last iteration, and is the new low area the same as the high area at the last iteration? If yes, stop. If no, go to 6.

6. If $T_{MOVED_L} \leq T_{MOVED_H}$, MOVE = $T_{MOVED_L}$ and go to 7. If $T_{MOVED_L} > T_{MOVED_H}$, MOVE = $T_{MOVED_H}$ and go to 7.

7. Change advertising levels by removing MOVE from low area (L) and adding MOVE to high area (H). Revise levels in each period in proportion to MOVE relative to total local advertising in area over planning period.

8. Calculate new marginals based on DELALL for areas where local advertising has been changed. Go to 2.
The allocation heuristic is controlled by the amount of advertising moved (DELALL) and the tolerance (TOLALL). A good procedure is to start with a large DELALL (e.g. 25%) and TOLALL (e.g. 25%) and then after finding a solution repeat with a lower DELALL and TOLALL. Step two assures that if current advertising is on an upward concave section of an "S" shaped advertising response curve, the heuristic will not lead to removing all advertising from the area unless this is acceptable. An "S" curve may reflect short run considerations in the area and not the long run possibilities for changes in the area that would warrent its maintenance. For example, a low distribution level might be increased in the future. Step 5 prevents a cycling in the heuristic. Step 6 assures that the amount moved from one area does not become an overbearing amount. This is important if budgets vary widely between areas. Step 7 changes the advertising levels proportionately so that the seasonal pattern of expenditure in the area will be maintained. The heuristic continues until a best allocation of local expenditure is found within the specified tolerance.

"Split" takes the total budget as given and determines how much is split to national media and how much to local media (see Figure 1). Split begins moving expenditures from local to national. If it finds this successful, it continues until insufficient improvement occurs. If split finds the initial movement unprofitable, it moves expenditure from national to local and continues in this direction if there is sufficient improvement. The steps of "split" are:
1. Move a percent of local expenditure (PSPLIT) to national, use "allocate" to find best local allocation, and determine the change in total profit over the planning period.

2. If the change is more than a minimum percent increase (TOLSPT), go to 1. If not, go to 3 unless have moved some amount from local at the last iteration. In this case stop.

3. Move a percent of national expenditure (PSPLIT) to local, use "allocate" to find best local allocation, and determine the change in total profit over the planning period.

4. If change in profit is greater than minimum percent increase (TOLSPT), go to 3; otherwise stop.

Split is controlled by the percent moved (PSPLIT) and the tolerance (TOLSPT). Split increments from the best pattern of allocation. This pattern recall minimizes the computation in subsequent utilizations of the allocate heuristic. "Search" evaluates a number (NSTEP) of total budget alternatives.

The steps are:

1. Sequentially examine alternatives in increments (DELBG) above reference. Use "split" for each alternative.

2. Return to reference plan and evaluate specified alternatives sequentially in increments (DELBG) below reference. Use "split" for each alternative.

3. Record best alternative.

Search begins at reference and examines the specified alternatives sequentially from the reference. After each alternative it records the best split and
allocation patterns so that the evaluation of the next alternative will begin from the previous best pattern and therefore will require less computation to find the new best plan.

The multi-stage heuristic is implemented in an on-line interactive computer program. In this way a manager or researcher can interact with the power of the heuristic to find a good solution. The man is important since he has experience and pattern recognition capabilities that can be useful in efficiently finding solutions. He can specify or change the starting points, examine desired total budgets and iteratively converge on the best solution by changing the search increments (DELBG, PSPLIT, DELALL) and tolerances (TOLSPT, TOLALL). The multi-stage feature is useful if a budget is prespecified and only the split between national and local media is to be considered at a given time. Another example is the use of the system by a regional manager who only wants to allocate a local budget for his region. The on-line feature of the system also allows parameters to be changed at any stage so sensitivity testing can be conducted on parameters that are in doubt.

The output of the heuristic when applied to the model is a best total budget, split between national and local expenditure, and budget allocation for each geographic area. By changing distribution levels, the interaction between advertising strategy and distribution can be assessed. Similarly, the on-line change mechanism fosters a sensitivity analysis. The interactive program and model structure are designed to promote application by involving

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6See [6].
managers and data in a reasonable description of the market structure so as to improve the determination of local strategies. The model can also function as a control mechanism when applied on a continuing basis. With continuing experimentation the model could be imbedded in an adaptive planning system.  

Limitations and Extensions of the Model:

This model encompasses the major factors and phenomena in the budgeting problem, but the consideration of some of them is quite simple and could be extended in future work. For example, the competitive effects are included indirectly in each area's retention rate (see Eq. 4) along with brand loyalty. This assumes competitors will behave as they have in the past despite our advertising changes. The future evolutionary development of the model could separate the competitive effect. This could be accomplished by making advertising response (ADVRSP, Eq. 2) a function of our expenditure relative to competition or, more eloquently, by creating a ratio of our advertising to the sum of all firms' advertising, each raised to a sensitivity exponent.  

With an explicit competitive formulation, payoffs could be found for various competitive strategies, and Bayesian decision or game theory procedures could be utilized. Other extensions to the model could be made to include other variables such as price and promotion through additional response functions and to include overlaps between areas.

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7See [1].
8See [5] for an application of such a form.
The heuristic could be extended to explicitly include the sequence of advertising levels. In its current form, the heuristic takes the sequence as given and increments it. Although the sequence could be changed on-line and the heuristic re-run, it would be useful to consider both the level and pattern of expenditure. Although simple, it is hoped that this model is a good base for future developments that would produce a multi-variate local strategy model with a dynamic heuristic and statistical support system.

APPLICATION

Input and Parameter Estimation:

This budgeting and allocation model has been applied to a frequently purchased consumer product marketed by a medium sized firm. The data base available was: 1. sales data for 60 months, 2. advertising contract buys for 60 months, 3. percent of outlets carrying product in each year, 4. audiences of media by area by year, and 5. contribution margin by area by year.

Monthly time periods were used for the input analysis. The areas used were the 20 largest market areas and the residuals of non-top 20 areas by region. The data on advertising was combined by calculating the weighted advertising (see Eq. 1) and used as a measure of advertising pressure in each area in the statistical analysis to determine advertising response.

In all the statistical analysis, the data for the last 12 months was not examined. It was "saved" so that the model could be compared to the actual results.

Exploratory plots and regressions of the monthly sales, advertising and distribution data were conducted in an effort to diagnose problems and
overall relationships in the data. The plots indicated (1) a strong seasonal pattern with highest sales in the summer, (2) advertising did not follow the seasonal pattern — largest advertising generally occurred in the fall, (3) a growth trend in most areas, (4) considerable fluctuation in sales from trend and seasonal patterns, and (5) no specific one or two month lag in sales and advertising (that is, advertising increases in period \( t \) did not seem to always produce a sales increase in \( t+1 \)).

Exploratory regressions were run in each area on a step-wise basis. Distribution did not enter the equations due to the yearly nature of the distribution and its small variation. Log-linear response forms did not improve the exploratory power or significance of the regressions over the range of data, so linear equations were adopted as the standard form. These linear equations provide an estimate of the marginal response to advertising in the data range. The exploratory analysis indicated that a distributed lagged term \( \text{sales}_{t-1} \) was significant and large (70% of the areas had a value greater than .8). Although a specific lag was not present, an exponential decay of sales was present. The values of advertising response in these exploratory equations were generally negative. This was counter intuitive, but was due to the contra-cyclical nature of advertising. Low advertising with high sales (summer) and high advertising with lower sales (fall). This indicated future regressions would have to carefully separate seasonal sales and advertising effects.

The formal time series regression analysis used the following equation:
\[ \text{Sales}_t = b_0 + b_1 D_1 + b_2 D_2 + b_3 \text{advertising}_t + b_4 \text{sales}_{t-1} \]

\[ D_1 = \begin{cases} 1 & \text{in summer} \\ 0 & \text{otherwise} \end{cases} \]

\[ D_2 = \begin{cases} 1 & \text{in winter} \\ 0 & \text{otherwise} \end{cases} \]

\( D_1 \) and \( D_2 \) are dummy variables that are specified in an attempt to
deseasonalize the data. The results of these regressions are shown in
Table 1. All the F statistics were significant at 10\% level. The retention
rates are generally significant and positive, but the advertising coefficients
are low when positive and sometimes significantly negative. The retention
rates were acceptable, but it was suspected that advertising and sales effects were
confounded. In an attempt to separate the seasonal effects, a prior seasonal
model was developed for each area. This model de-seasonalized the data on
the basis of an index derived from areas where advertising did not change
over the seasonal pattern. Unfortunately, this effort was not successful.
Finally, a twelve month change model was run for each area. In this model
the change in sales in month "t" minus sales twelve months earlier was
regressed against the change in advertising between those periods. This
again was unsuccessful in separating the effects of the seasonal sales and
advertising response.

Although the advertising response values were counter intuitive
due to the confounding of the seasonal and advertising response, the
regression model was good on a predictive test against the saved 12 months
of data. The error of month by month estimates was 11 percent. This is low
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$S$ = significant at 10% level on basis $t$ statistic for coefficients, all F statistics significant at 10% level

$i$ = insignificant at 10% level

**TABLE 1: Time Series Regression Results**
for a monthly test and lends confidence to the estimates of the retention rates. The retention rates were also robust to changes in structure in the regression equations such as the removal of current period advertising or seasonal effects. The retention rates from the time series regressions in Table 1 were adopted.

Since the advertising response was confounded with the seasonal pattern, cross sectional regressions on yearly data were conducted in an attempt to find the response. Since the presumption in building a local allocation model is that area responses are different, one certainly could not regress across all areas. However, some areas may be basically similar and represent a basis for cross sectional regression. In order to find such groups, the average response of total sales to total advertising over four years was considered. On the basis of the average responses and the retention rate estimates, three groups were formed. One group had high retention rates and high average response and was called the "high" response group. The second group had low average response but a high retention rate and was called the "medium" response group. The "low" response group had a low average response and low retention rates. The composition of the groups was examined by checking the year to year marginals in each area. The final group compositions were specified after two adjustments on the basis of the examination of marginals.

Regressions of sales and advertising were run for each of the last three years in each group. The results of these regressions are in Table 2. The high group is significant with a marginal of about .7 units of sales per dollar expenditure. The medium group is less significant and lower in
<table>
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<tr>
<td></td>
<td>$b_1$</td>
<td>$R^2$</td>
<td>F</td>
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<td>.62</td>
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<td>.75</td>
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<td>Third year</td>
<td>.67$^S$</td>
<td>.44</td>
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</table>

$S$ = significant at 10% level  
$i$ = insignificant at 10% level

**TABLE 2: Cross sectional regression response coefficient for advertising**

response. The low response group has a high marginal response, which is unexpected. But plots of the data indicated two groups in the low response area and that each had a marginal response of .1. In the first year the low response sub-grouping did not appear, so apparently the low response areas were opened with high response, but over time become less responsive. The cross sectional values in the regressions were supported by the values of the year-to-year marginals in each area.

At this point satisfactory estimates were available for the retention rates from the time series regressions and for the marginal responses from the cross sectional analysis. These were combined with the subjective judgments provided by five managers. The managers gave sales estimates for a minimum, fifty percent above reference, and saturation levels of advertising. The data and judgments were discussed and a consensus of the best estimate for each area was reached.
The distribution response function was estimated by a cross sectional plot of percent of outlets carrying the product to the percent of market coverage. One non-linear curve fit most points, but a lower curve was defined for one area and a higher curve for three others. The remainder of the input defined in Figure 2 was obtained from company and agency records.

**Model runs:**

All runs were conducted as if the time was period 48. The first run was based upon planned advertising and forecasted sales for the four quarters in months 49 to 60. This evaluation was merely a detailed calculation of sales and profits for the reference plan.

Next the "allocation" capability of the model was used to try to find a better pattern of allocation given the reference total budget and national expenditure. The model found an allocation strategy that increased the two year contribution profit by 17 percent. The allocation changes are shown in Table 3. The allocation heuristic with on-line guidance took about 20 CPU seconds of 360-67 time.

The "split" capability was used to examine the advisability of shifting more expenditure to national media from local media or vice versa. It was found that within a tolerance of .1 percent, the planned split of about 37 percent to national media was best.

The model's "search" capability was used to examine alternate total budgets. The profit of each budget reflected the best split and allocation. The best budget was found to be 95 percent above the planned level. The profit at the best budget, split, and allocation was 50.5 percent greater than the profit of the original plan. The allocation pattern for the larger
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**TABLE 3: Allocation Changes**

The budget was similar to the initial allocation of the planned budget. The split pattern was changed with 41 percent going to national media. The search heuristic used about 40 seconds of 360-67 time per total budget level examined and 11 levels were evaluated in a series of three iterations.

The indications of the runs based on the best input estimates were to increase advertising 95 percent, put more money in national media, and re-allocate the local expenditure. The model indicated this would increase profits 50 percent (over 5 million dollars). But the input was not perfect, so it is logical to ask how the recommendation would be affected by changes in the input. Answering this question involves sensitivity testing of the model. The most critical parameter and least certain is the advertising response curve. To test the impact of errors in this curve, the marginal response to increases in advertising was reduced 25 percent. The model was again run. The output was a total budget increase of 70 percent and a 32 percent
increase in the profit level. The sensitivity of the recommendation to a substantial error in the estimate does not seem to be great.

A model should be a guide to decision making so the results of the runs should indicate directions for changes in budget and allocation, but not a binding solution. In this spirit, a recommendation of a 50 percent increase in total advertising and limited re-allocation in the directions indicated was tested. The recommended re-allocation reduced no area by more than 50 percent nor increased any area by more than 75 percent of its previous expenditure. The profit from this recommendation was 40 percent greater than the reference profit. The model seems to be robust in its recommendation for a higher budget and its pattern of re-allocation.

Limited Testing of Model:

It must be appreciated that this is a strategy model and not a forecasting model. It begins with a reference sales forecast and budget. From this starting point it finds a better advertising strategy and produces a revised sales forecast. The test of the model should depend upon whether or not recommendations, in fact, lead to higher profits. The strategic features of the model can therefore be tested only by experiments or actual implementation of the recommendations. Since the recommendations for periods 49-60 were not reflected in the saved data, the recommendations could not be tested. A limited test of the model would be to examine the forecasted sales and actual sales. The difference between these two will reflect the error in the starting point forecast of sales and the error in the model response curve. If the actual advertising equals the budgeted advertising, the error will be due entirely to the starting point. In the period of months 49 to 60 the planned advertising and actual advertising
were not equal, so the limited test could be conducted if the two sources of error could be separated.

The separation of the starting point forecast and the response curve error was implemented by finding quarters and areas where budgeted and actual advertising were equal within $300. Seventeen quarterly observations were found. The actual and predicted sales were compared and the error was used to estimate the starting point forecast error. Based on this procedure the reference forecast understated actual sales by 11.5 percent. In addition, another source of error was found. This was that the total actual national media expenditure was 1/3 less than budgeted value. This would further increase the starting point error. It was estimated that this reduction in actual expenditure would increase the starting point error 3.1 percent in the 17 observations. The total starting point error was estimated as an understatement of 14.6 percent (11.5 + 3.1).

The starting point error was used to correct the model reference input by increasing the reference values by 14.6 percent. Then the actual advertising was inputted to the model and evaluated to give predicted sales. A comparison of the actual total yearly sales to the reference sales indicated a 2.3 percent error in the prediction. This is encouraging, but the comparison of actual and predicted by area and for each quarter produced an error of 15 percent. This is acceptable, but does indicate that although the overall performance of the response curves is good, there is a considerable need to improve the exact shape of each curve. This could be done by market experimentation.
CONCLUSION

This paper has developed a relatively simple initial model to aid in setting a total advertising budget, national media split, and local geographic allocation. The model uses weighted advertising as a measure of pressure in each area and then links this to sales by a response function in each area. Distribution and carryover effects are added through a distribution response function and a retention rate that reflects brand loyalty and competition. The best budget and allocation are found by an on-line multi-stage heuristic.

An application of this model and heuristic to a real product indicated that profit could be increased 17 percent by a better allocation and 50 percent by a 95 percent higher total budget, if it were split and allocated in the best manner. The recommended increase was robust with respect to errors in the marginal response. A limited test of the model was encouraging, but indicated the need for market experimentation to improve the accuracy of the area response curves. This experimental design phase is now being implemented and the model will eventually be placed in an adaptive control system. The cost of the study was less than $50,000 and seems to have been a worthwhile expenditure since the forecasted profit increases are in millions of dollars and it appears that the model will gain acceptance and continuing usage.
REFERENCES


